

**THE EFFECT OF A MULTIFACETED VARIABLE APPROACH
ON THE LEARNING AND TEACHING OF ALGEBRA
IN THE JUNIOR SECONDARY SCHOOL**

by

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STATEMENT OF CANDIDATE

I hereby certify that the research presented in the thesis entitled “The effect of a Multifaceted Variable Approach on the learning and teaching of algebra in the junior secondary school” is my original work and it has not been submitted for a higher degree to any other university or institution other than Macquarie University.

I also certify that the thesis is an original piece of research and it has been written by me. Any help and assistance that I have received in my research work and the preparation of the thesis itself have been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis.

The research presented in this thesis was approved by the Macquarie University Ethics Review Committee, protocol number HE22FEB2008-D05638 (February 2008).

A handwritten signature in blue ink, appearing to read 'Salma', is placed on a light green rectangular background. Below the signature is a horizontal dotted line.

Salma Tahir

Student ID: 41102479

30th September, 2011

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Abstract

Research has shown that students find algebra a difficult subject. One reason may be that they have a limited concept of a variable. To encourage the development of a deeper understanding of variables, a new teaching approach called the Multifaceted Variable Approach (MVA) was designed. The emphasis in the MVA is on understanding relationships between variables using real contexts and their tabular, algebraic and graphical representations. Students experience several facets of the variable concept in parallel: as unknown numbers, generalised numbers and functions. The MVA approach was used to teach algebra in Stage 4 of the syllabus of the NSW Board of Studies. The students were in Years 7 and 8.

The effect of the MVA on students' understanding of variables and their algebraic competence was investigated in a two-year case study in a girls' high school in Sydney. The sample consisted of four streamed classes, called (in decreasing order of ability) Sets 1 to 4, and their teachers. The experimental group (49 students in Sets 2 and 4) was taught using the MVA and the comparison group (54 students in Sets 1 & 3) was taught using a traditional teaching approach. Data were collected from student assessments on six separate occasions and from two rounds of student interviews.

Qualitative and quantitative analysis of the data indicated that, on average, students who were taught via the MVA acquired a deeper concept of "variable" and greater algebraic competency than the comparison group. On average, they demonstrated fewer misconceptions and were more successful in interpreting algebraic expressions, representing word problems in an algebraic form, solving linear equations, and in general algebraic reasoning.

The study also confirmed the strong influence of teachers' concepts of algebra on the quality of student learning. More sustained teacher professional development may be necessary to ensure the change in teaching practice that is necessary to implement the MVA effectively.