

**THE EFFECT OF A MULTIFACETED VARIABLE APPROACH  
ON THE LEARNING AND TEACHING OF ALGEBRA  
IN THE JUNIOR SECONDARY SCHOOL**

by

Salma Tahir, B.Sc., B.Ed., M.Sc., M.Phil.

Department of Education,  
Faculty of Human Sciences  
Macquarie University

This thesis is presented for the degree of Doctor of Philosophy

October, 2011



# TABLE OF CONTENTS

TABLE OF CONTENTS	i
LIST OF APPENDICES	vii
LIST OF TABLES	viii
LIST OF FIGURES	x
DECLARATION	xi
ACKNOWLEDGEMENT	xii
ABSTRACT	xiv
CHAPTER 1 Introduction	1
1.1 Teaching algebra	2
1.2 A proposed new approach	3
1.3 Aim of the study	4
1.4 Significance of the study	4
1.5 Outline of the thesis	5
CHAPTER 2 Student Difficulties in Algebra	6
2.1 Introduction	6
2.2 Difficulties in transition from arithmetic to algebra	6
2.3 Misconceptions regarding the concept of a variable	8
2.4 Problems related to the representation and solution of a linear equation	11
2.4.1 Errors in representation	12
2.4.2 Errors in simplification of algebraic expressions	13

2.4.3	Problems with the interpretation of equality and Equivalence	14
2.4.4	Moving from arithmetic to algebraic solution strategies for linear equations	15
2.5	Issues of engagement	18
2.6	Summary	18
CHAPTER 3 Teaching Approaches		20
3.1	Teaching approaches	20
3.1.1	Problem-solving and modelling approaches	20
3.1.2	Generalisation approaches	24
3.1.3	Functional approaches	26
3.2	Essential elements of a successful algebra program	28
3.2.1	Developing the concept of a variable	29
3.2.2	Promoting algebraic reasoning	30
3.2.3	Breadth and integration	31
3.2.4	Making algebra meaningful and interesting	31
3.3	The Multifaceted Variable Approach	32
3.3.1	The MVA in the present study	35
3.3.1.1	Patterns	36
3.3.1.2	Algebraic techniques	36
3.3.1.3	Linear relationships and coordinate geometry	36
3.4	Summary	37

CHAPTER 4 Methodology	39
4.1 Introduction	39
4.2 Sample	39
4.3 Phase I	41
4.3.1 Professional Development Workshop I	41
4.3.2 The algebra teaching program	44
4.3.3 Lesson observations	45
4.3.4 First and Second Algebra Tests and interviews	46
4.4 Phase II	47
4.4.1 Professional Development Workshop II	49
4.4.2 Algebra teaching	49
4.4.3 Lesson observations	50
4.4.4 Third Algebra Test and interview	51
4.5 Justification of methodology	52
4.6 Summary	57
CHAPTER 5 Results: The Teachers	58
5.1 Introduction	58
5.2 Professional Development Workshop I	58
5.2.1 Teachers' beliefs about mathematics teaching and learning	58
5.2.1.1 Rosa	59
5.2.1.2 Mona	59

5.2.1.3	Amy	60
5.2.1.4	Ben	60
5.2.2	Teachers beliefs about algebra and teaching algebra	61
5.2.2.1	Rosa	61
5.2.2.2	Mona	62
5.2.3	Common student errors and usual sequence of algebra lessons	63
5.2.4	Workshop evaluation	64
5.3	Lesson observations	64
5.3.1	Rosa	65
5.3.2	Mona	68
5.3.3	Amy	69
5.3.4	Ben	71
5.4	Professional Development Workshop II	73
5.5	Summary	74
CHAPTER 6	Results: The Students (Phase I)	77
6.1	Introduction	77
6.2	Learning (Phase I)	77
6.2.1	First Algebra Test	78
6.2.2	Student Interview I	85
6.2.3	Yearly Examination	89
6.2.4	Second Algebra Test	90
6.3	Summary	95

CHAPTER 7 Results: The Students (Phase II)	97
7.1 Introduction	97
7.2 Third Algebra Test	98
7.3 Half-yearly examination	105
7.3.1 Errors in simplification	107
7.3.2 Solution of linear equations	109
7.4 Fourth Algebra Test	113
7.5 Student Interview II	116
7.5.1 The experimental group	116
7.5.2 The comparison group	118
7.5.3 Comparing the two groups	121
7.6 Summary	122
CHAPTER 8 Discussion	126
8.1 Students' conceptions of variables	126
8.1.1 Specific unknown quantity	126
8.1.2 Variable quantity (generalised number)	127
8.1.3 Letter as a label (to represent an object)	127
8.1.4 Letters and numbers detached	128
8.2 Comparison between the experimental and the comparison group	129
8.3 Algebraic competence of students	130
8.3.1 Translating word problem to algebraic form	130
8.3.2 Translating tabulated values to line graphs and vice versa	132

8.3.3	Translating tabulated numerical values to linear equations	132
8.3.4	Simplification of algebraic expressions	134
8.3.5	Solving linear equations	136
8.3.6	Reasoning and justifying	138
8.4	The role of the teachers and teaching resources	138
8.4.1	The experimental teachers	138
8.4.2	The comparison teachers	140
8.5	Summary	142
CHAPTER 9	Conclusions and Implications	144
9.1	Summary	144
9.2	Limitations of the study	144
9.3	The research questions	145
9.4	Implications for the algebra curriculum	150
9.5	Implications for teaching	151
9.6	Implications for research	152
	Concluding remarks	153
REFERENCES		155

## LIST OF APPENDICES

Appendix A	Principals' Information and Consent Form	166
Appendix B	Teachers' Information and Consent Form	168
Appendix C	Parents' Information and Consent Form <sup>1</sup>	170
Appendix D	Mathematics Teaching Questionnaire	172
Appendix E	Algebra Questionnaire (I)	174
Appendix F	Algebra Questionnaire (II)	176
Appendix G	Workshop Evaluation Form	180
Appendix H	Examples of Activities from the Resource Book	182
Appendix I	First Algebra Test	186
Appendix J	Yearly Examination	192
Appendix K	Second Algebra Test	194
Appendix L	Lesson Plan Feedback Sheet	196
Appendix M	Student Interview Schedule I	198
Appendix N	Third Algebra Test	200
Appendix O	Half Yearly Examination (Algebra Problems)	206
Appendix P	Fourth Algebra Test	210
Appendix Q	Student Interview II	212
Appendix R	Ethics Approval Letter	214

## LIST OF TABLES

### CHAPTER 2

Table 2.1	Algebraic levels, Piaget's stage of cognitive development, sample items, student understanding of variables	11
-----------	---	----

### CHAPTER 4

Table 4.1	The participating teachers	41
Table 4.2	Student and teacher activities and data collected in Phase I	42
Table 4.3	Student-teacher activities and data collection in Phase II	48

### CHAPTER 6

Table 6.1	Mean marks (as percentage) in number, angles and algebra	78
Table 6.2	Mean marks (as percentage) in algebraic skills for first algebra test	79
Table 6.3	Percentage of students who made errors in Question 2 and Question 4 by category	80
Table 6.4	Categories of student responses	86
Table 6.5	Mean interview scores	86
Table 6.6	Mean marks (as percentage) in algebraic skills for algebra problems in the yearly examination	89
Table 6.7	Mean marks (as percentage) in algebraic skills for Second Algebra Test	91
Table 6.8	Percentage of conjoining errors in Question 2 (part b and part c)	92

## CHAPTER 7

Table 7.1	Mean marks (as percentage) in algebraic skills for Third Algebra Test	98
Table 7.2	Percentage of students making various errors in the Third Algebra Test	101
Table 7.3	Percentage of correct responses in algebraic skills on the Half Yearly Examination	106
Table 7.4	Percentage of students who made simplification errors in the Half-Yearly Examination	109
Table 7.5	Percentage of correct responses of students of each class in solving linear equations	110
Table 7.6	The number of students who used the solution strategy and the number of correct responses	112
Table 7.7	The mean (as percentage) of students who used the solution strategies	114
Table 7.8	Percentage of correct responses by solution strategy	115

## LIST OF FIGURES

### CHAPTER 2

Figure 2.1 Working Backwards 16

### CHAPTER 5

Figure 5.1 Graphs drawn on the white board (a) represents climbing one stair at a time, (b) represents climbing one stair then two stairs, then repeating this pattern 67

## STATEMENT OF CANDIDATE

I hereby certify that the research presented in the thesis entitled “The effect of a Multifaceted Variable Approach on the learning and teaching of algebra in the junior secondary school” is my original work and it has not been submitted for a higher degree to any other university or institution other than Macquarie University.

I also certify that the thesis is an original piece of research and it has been written by me. Any help and assistance that I have received in my research work and the preparation of the thesis itself have been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis.

The research presented in this thesis was approved by the Macquarie University Ethics Review Committee, protocol number HE22FEB2008-D05638 (February 2008).

A handwritten signature in blue ink that reads "Salma". The signature is written in a cursive style and is positioned above a dotted line.

**Salma Tahir**

**Student ID: 41102479**

**30<sup>th</sup> September, 2011**

## **Acknowledgement**

In completion of this study I have benefited from the encouragement and support of two institutions, my supervisors, professional friends and my family.

I am grateful to the Department of Education, Macquarie University for providing me with additional financial support and a learning environment conducive for research and professional development.

I am indebted to AusAID who assisted me by providing me with a scholarship to complete the Doctoral program.

I am grateful to both of my supervisors Dr Michael Cavanagh and Dr Mike Mitchelmore whose visionary approach and professional expertise guided me at every stage of this study.

Michael's encouragement and support facilitated me right from planning to the final stage of this study. He always guided me with enthusiasm, dedication and professionalism.

Mike's advice and feedback was invaluable. His expertise in finding patterns and relationships was enlightening to say the least, when the data were analysed.

My special thanks to Dr John Hedberg, Head of Department of Education, Macquarie University who provided moral encouragement and support when it was needed during the course of this project.

My special thanks to the staff and students of the school who participated in the study and welcomed me into their community.

I am extremely grateful for the invaluable discussions, comments and feedback provided by my colleague Debbie Bautista who was always there for me in time of need and also for providing me with technical support to restore my endnote file and in making copies of my lesson recordings.

I am thankful to Dr Heather McMaster who edited this thesis.

I also thank Dr Joanne Mulligan who encouraged and supported me and all the staff of Department of Education, in particular to Alison and Marissa for their facilitation in providing me with logistic support.

On a personal level I would like to thank my husband Tahir Iqbal whose continuous support and encouragement made it possible for me to complete this study.

## **Abstract**

Research has shown that students find algebra a difficult subject. One reason may be that they have a limited concept of a variable. To encourage the development of a deeper understanding of variables, a new teaching approach called the Multifaceted Variable Approach (MVA) was designed. The emphasis in the MVA is on understanding relationships between variables using real contexts and their tabular, algebraic and graphical representations. Students experience several facets of the variable concept in parallel: as unknown numbers, generalised numbers and functions. The MVA approach was used to teach algebra in Stage 4 of the syllabus of the NSW Board of Studies. The students were in Years 7 and 8.

The effect of the MVA on students' understanding of variables and their algebraic competence was investigated in a two-year case study in a girls' high school in Sydney. The sample consisted of four streamed classes, called (in decreasing order of ability) Sets 1 to 4, and their teachers. The experimental group (49 students in Sets 2 and 4) was taught using the MVA and the comparison group (54 students in Sets 1 & 3) was taught using a traditional teaching approach. Data were collected from student assessments on six separate occasions and from two rounds of student interviews.

Qualitative and quantitative analysis of the data indicated that, on average, students who were taught via the MVA acquired a deeper concept of "variable" and greater algebraic competency than the comparison group. On average, they demonstrated fewer misconceptions and were more successful in interpreting algebraic expressions, representing word problems in an algebraic form, solving linear equations, and in general algebraic reasoning.

The study also confirmed the strong influence of teachers' concepts of algebra on the quality of student learning. More sustained teacher professional development may be necessary to ensure the change in teaching practice that is necessary to implement the MVA effectively.