GRADUATE VOICES

THE NEXUS BETWEEN LEARNING AND WORK

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2006

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2006

This thesis is presented for the degree of Doctor of Philosophy

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This work has not been submitted for a higher degree to any other institution. The work is mine and other sources have been identified.

Ethics approval for the research was granted by the Human Ethics committees of the University of Technology, Sydney and Macquarie University.

Ethics approval numbers: UTS HREC 03/113A, Macquarie University 03/1359

Abstract

GRADUATE VOICES: THE NEXUS BETWEEN LEARNING AND WORK

The aim of this study is to inform curriculum change in the mathematical sciences at university level. This study examines the transition to professional work after gaining a degree in the mathematical sciences. Communication is used as the basis for the analysis of the transition because of the importance of language choices in work situations. These experiences form part of the capabilities that become part of a person's potential to work as a professional. I found a subtle form of power and, of the opposite, lack of power due to communication skills. It is not as obvious as in, say, politics but it is just as critical to graduates and to the mathematical sciences.

There were 18 participants in the study who were graduates within five years of graduation with majors in the mathematical sciences. In-depth interviews were analysed using phenomenography and examples of text from the workplace were analysed using discourse analysis. Descriptions of the process of gaining employment and the use of mathematical discourse have been reported in the thesis using narrative style with extensive quotes from the participants.

The research shows that graduates had three qualitatively different conceptions of mathematical discourse when communicating with a non-mathematical audience: jargon, concepts/thinking and strength. All participants modified their use of technical terms when communicating with non-mathematicians. Those who held the *jargon* conception tried to simplify the language in order to explain the mathematics to their audience. Those who held the *concepts/thinking* conception believed that the way of thinking or the ideas were too difficult to communicate and instead their intention with mathematical discourse was to inspire or sell their ability to work with the mathematics. The *strength* conception considers the ethical responsibility to communicate the consequences of mathematical decisions. Not one of the participants believed that they had been taught communication skills as part of their degree.

Participants gained a 'mathematical identity' from their studies and acquiring a degree gave them confidence and a range of problem-solving skills. Recommendations are made about changes in university curriculum to ensure that graduates are empowered to make a high-quality transition to the workplace and be in a position to use their mathematical skills. Mathematical skills are necessary but not sufficient for a successful transition to the workplace. Without the ability to communicate, graduates are unable to release the strength of their knowledge.

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ACKNOWLEDGMENTS

PUBLICATIONS ARISING FROM THIS WORK

Full texts are attached in Appendix A. (CD)

Wood, L.N. (2004). University Teaching and Learning. *MERGA Review of Research in Australasia 2000-2003*, 219-242. (Chapter 3)

Wood, L.N. & Reid, A. (2005). Graduates' initial experiences of work. *Proceedings of the Australian Association for Educational Research*. [Online]. (Chapter 5)

Wood, L.N. & Reid, A., (2005). Graduate voices: new mathematics graduates in industry. *DELTA05*, 125-131. (Chapter 6)

Wood, L.N. & Kaczynski, D. (2005). University students in USA and Australia: anticipation and reflection of the transition to work. *Proceedings of the 4th International Conference on Researching Learning and Work*. Sydney, December 2005. [Online]. (Chapter 9)

Wood, L.N. & Reid, A. (2006). Conversations with graduates: reflections on learning mathematics. *ICTM3*. [Online]. (Chapter 8)

GLOSSARY

Advanced mathematical discourse: the uses of language in university mathematics learning and teaching and in professional life. (Wood & Perrett, 1997).

Communication: a term used in natural language. Here communication refers to discourse.

Discourse: in the broad sense of a 'communicative event', including conversational interaction, written text, as well as associated gestures, facework, typographical layout, images and any other 'semiotic' or multimedia dimension of signification. (van Dijk, 2001, p. 98). The sort of language used to construct some aspect of reality from a particular perspective, for example the liberal discourse of politics (Chouliaraki & Fairclough, 1999, p. 63); a system of discursive practices that constitute their objects of knowledge (Howarth, 2000, p. 68); and a relational identity whose identity depends on differentiation from other discourses (Howarth, 2000, p. 102).

Discourse analysis: discourse analysis is an umbrella term used to describe a variety of approaches to analysis of language (in the broad sense), most of which have developed since the 1970s.

Genre: genre is a relatively stable set of conventions that is associated with, and partly enacts, a socially ratified type of activity, such as informal chat, buying goods in a shop, a job interview, a poem or a scientific paper. A genre implies not only a particular text type, but also a particular process of producing, distributing and consuming texts. (Fairclough, 1992, p. 126).

Outcome space: the outcome of a phenomenographic study is a hierarchical set of logically related categories, from the narrowest and most limited to the broadest and most inclusive. This is referred to as the outcome space for the research.

Phenomenography: phenomenography examines how different people experience the same phenomenon. (Marton & Booth, 1997).

Hegemony: a mode of domination which is based upon alliances, the incorporation of subordinate groups, and the generation of consent. (Fairclough, 1992, p. 9).

Intertextuality: the condition whereby all communicative events draw on earlier events. One cannot avoid using words and phrases that others have used before. (Jørgensen & Phillips, 2002, p. 73).

Systemic functional linguistics: systemic functional linguistics (SFL) is a theory of language centred around the notion of language function. While SFL accounts for the syntactic structure of language, it places the function of language as central (what language does, and how it does it), in preference to more structural approaches, which place the elements of language and their combinations as central. SFL starts at social context, and looks at how language both acts upon, and is constrained by, this social context. (Halliday, 1994).

Text: any product whether written or spoken, so that the transcript of an interview or a conversation, for example, would be called a 'text'. (Fairclough, 1992, p. 4).

Voice: the language used by a particular category of people and closely linked to the identity, for example the medical voice (Chouliaraki & Fairclough, 1999, p. 63).

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ACRONYMS AND ABBREVIATIONS

AMS: American Mathematical Society

ARC: Australian Research Council

AustMS: Australian Mathematical Society

CEQ: Course Experience Questionnaire www.graduatecareers.com.au/content/view/full/1787

CUDOS: Centre for Ultrahigh Bandwidth Devices for Optical Systems. www.cudos.org.au

DEST: Department of Employment, Science and Technology (Australian Federal Government department that controls universities)

GDS: Graduate Destination Survey

HEFCE: Higher Education Funding Council for England

ICMI: International Committee for Mathematics Instruction, a committee of United Nations Educational Scientific and Cultural Organisation (UNESCO)

IT: Information Technology

NCM: National Committee for Mathematics, sub-committee of the Australian Academy of Sciences

NVivo: Qualitative software package. www.qsrinternational.com/

NZ: New Zealand

OHT: Overhead transparency

SAS: Statistical software package. www.sas.com/

SFL: Systemic functional linguistics

SPSS: Statistical software package. http://www.spss.com/

UK: United Kingdom

USA: United States of America

UTS: University of Technology, Sydney

VB: Visual basic (a computer programming language)

1 INTRODUCTION

The aim of my research is to inform curriculum change, particularly in Australian universities, but applicable to similar tertiary education systems. I will show that mathematical communication skills are central to the ability of graduates to use their mathematical knowledge in their workplaces. Furthermore, lack of understanding of the power of the mathematical sciences in the wider community (especially employers in this case) is making it difficult for graduates to find appropriate employment.

This study examines the transition to professional work after gaining a degree in the mathematical sciences. I examine graduates' early experiences in the workforce, influences that helped their adjustment to the workplace and their reflections on their learning. Communication is used as the basis for an examination of graduates' transition to work because of the importance of language choices in work situations. These experiences form part of the capabilities that become part of a person's potential to work as a professional. The research indicates that mathematical skills are necessary but not sufficient for a successful transition to the workplace. Without the ability to communicate graduates are unable to release the power of their mathematical knowledge at work.

There have been numerous studies on the transition from secondary school to university mathematics study (summarised in Wood, 2001). There are also general studies of graduates and requisite graduate attributes, such as Scott (2003). The transition to university and its challenges are well recognised (DEST, 2006). What is less understood is the transition from university to professional work; how people move from being students to developing a professional identity and professional skills and attitudes. This latter transition is the context of this study.

The results were not as I anticipated. During my exploration of the area through literature, I had read about power, hegemony, and thought that it would not be an important consideration in this study as I was studying mathematics not politics or social science. What I found, however, was a subtle form of power and, of the opposite, lack of power in the workplace. It is not as obvious as in, say, politics but it is just as critical to graduates and to the mathematical sciences.

The study involved 18 graduate participants who were within five years of graduation with majors in the mathematical sciences. I undertook in-depth interviews focusing on the participants' experience of their transition to work. Those experiences were analysed using phenomenography and examples of their work-related texts were analysed using discourse analysis. Descriptions of the process of gaining employment and the use of mathematical discourses have been reported here in narrative style using extensive quotes from the participants.

The results of the research indicated that graduates found it difficult to get work. Many were not prepared for the process of getting a job or coping with the work environment. Several had moved from mathematics into other fields. Those who stayed in fields that relied on mathematics were often the only mathematicians in their area, which provided challenges for them personally and in terms of their work-related communication. The participants' initial work experiences were important influences on the successful transition to the workplace, especially where they had the support of their manager.

Participants gained a 'mathematical identity' from their studies and the achievement of gaining a degree gave them confidence and a range of problem-solving skills. They reflected positively on their studies overall, but had some robust criticism of teaching and teaching methods used in mathematics. In Chapter 9, recommendations are made about changes in teaching and learning practices based on those criticisms. Three examples of mathematical discourse are discussed in depth in the light of the participants' conceptions of discourse. The skills needed by recent graduates in the workplace are complex, and I make recommendations for teaching and learning tasks that will help with the transition to the discourse of the workplace.

I make proposals about changes in university mathematics curriculum to ensure that graduates are prepared and empowered to make a high-quality transition to the workplace and to be in a position to use their mathematical skills. University departments of mathematical sciences should work with employers and organisations so that they are aware of the benefits that mathematicians could bring. As one participant said:

> It's actually really easy to see areas in industry where you think you could make a difference; it's extraordinarily difficult to get past that recruitment filter and to actually find yourself in one of those jobs.

Overview

Chapter 2 reviews literature related to the two methodologies, phenomenography and discourse analysis, that are used and demonstrates their appropriateness to this research. Chapter 3 is a literature review of research related to university mathematics teaching and learning, and to the transition to the workplace. The first section demonstrates how the investigation of mathematical communication has developed from the detailed analysis of the components of mathematical language to the examination of real texts and how professionals interact and learn to work with those texts. The second section critically summarises the last four years of research in university teaching and learning in Australasia and considers the impact on curriculum. The third section briefly examines the recent literature on graduate attributes and capabilities and finally there is a small section on contemporary learning theories in higher education. Chapter 4 describes the design of the study, the interview process and introduces the participants. Chapter 5 offers a description of the data in a narrative style, exploring the experiences of the participants as to how they gained employment. Chapter 6 examines how these professionals use mathematical discourse and their experiences of it. A phenomenographical outcome space (a hierarchical set of logically related categories), to describe graduates' conceptions of mathematical discourse with non-mathematicians, is developed from the data. An outcome space of how graduates learnt discourse skills is also developed. Chapter 7 examines texts supplied by three participants using discourse analysis. Discourse analysis (in this context) investigates the circumstances of the production and use of the texts. Texts for management, teaching and industry are analysed. Chapter 8 describes what the graduates gained from their studies. Suggestions from the participants about changes to university programs are considered. Chapter 9 begins with my reflection on the research and the methodologies used. The implications for curriculum design and the development of professional communication skills in mathematics are described. Recommendations for professional development for mathematics lecturers as well as ideas for making teaching and learning more accessible are considered.

Mathematical sciences

The report, *Mathematical Sciences: Adding to Australia*, commissioned by the Australian Academy of Sciences (National Committee for Mathematics, 1996, p. ix), gives this description of modern mathematics:

> Mathematics is the study of measurements, forms, patterns, variability and change.
> It evolved from our efforts to understand the natural world …

> Over the course of time, the mathematical sciences have developed a rich and intrinsic culture that feeds back into the natural sciences and technology, often in unexpected ways. The mathematical sciences now reach far beyond the physical sciences and engineering; they reach into medicine, commerce, industry, the life sciences, the social sciences and to every other application that needs quantitative analysis.

This shows the wide range of areas that the mathematical sciences cover. They are entwined within the natural sciences and further discipline areas. What I find interesting in the context of this study is the idea that mathematics has developed its own culture as distinct from the disciplines that it works within. Certainly, Burton (2004) found that university academics form a community of practice (Wenger, 1998). Graduates working in disperse situations may find it more difficult.

While the following quote is written in the Australian context, comparable reviews of mathematics in Canada and UK find that mathematics is critical to economic prosperity and that mathematics is underutilised in industry (Hoyles et al., 2002; Fields Institute Annual Report, 2005). The Australian Academy of Sciences report (National Committee of Mathematics, 1996, p. x) states:

> The mathematical sciences are critical to Australia's economic competitiveness and quality of life, and will become more so. The mathematical sciences are generic and enabling technologies. They are essential to the prosperity of many value-adding industries in Australia.

The Australian Academy of Sciences and the Australian Government (Nelson, 2005) agree that the mathematical sciences are important to the economic prosperity of Australia. Rubinstein (2006, online), in his article entitled *The crisis in maths in Australia*, describes the importance of mathematical ideas to industry but he also states that:

> The paradox is that although there is no "mathematical industry" similar to the chemical industry or earth sciences in mining, it is also true that there is no non-mathematical industry. Every area requires or can benefit from mathematicians and statisticians to increase efficiency.

This illustrates a difficulty with the mathematical sciences. They can contribute to all industries however the form of the contribution is not as clear as, say, a mining engineer to a mining project. It is harder to form a community and more difficult for others to see what you do as a mathematician.

This problem has been identified by mathematicians worldwide (such as Krantz, 1999). Despite the importance of the mathematical sciences, few students are enrolling for degrees in mathematics and

those who graduate are having difficulty getting appropriate jobs. The Australian Government graduate destination surveys of 2004 show that of the mathematics graduates who want full-time employment, 64.4% are in full-time employment and 35.6% are seeking full-time employment. This is at a time of general high employment and skill shortages. In all the graduate fields surveyed, mathematics had the second-lowest full-time rate for students graduating in 2004 (the lowest was Visual and Performing Arts with 57%), (Graduate Careers Council of Australia, 2005). Nonetheless, the starting salaries for mathematics graduates in employment are high.

Half the students studying mathematics and statistics at university are at first year level (Figure 1.1, DEST, 2005). These figures are compiled from data of the DEST website for each year and graphed in Figure 1.1. Note that in Figure 1.1, the bachelor degree numbers include the commencing students. For example, at the University of Sydney, Australia, Taylor (2005, p. 299) reported that there were more than 2500 students studying mathematical sciences in first year, 200 completed mathematics or statistics majors and 25 studied a fourth year of honours.

In terms of research, the numbers of postgraduate research students in the mathematical sciences has fluctuated slightly. In Australia, Johnston (2005, p.320) reports that the numbers of honours students was 138 in 2004, slightly down from approximately 160 in the previous three years. The numbers have been similar since 1970. There have been approximately 60 PhD completions each year since 1995, up from around 40 from 1970–1995 (Johnston, 2005, p. 321). The DEST figures are slightly different (Table 1.1) because they are enrolments rather than completions and the Johnston figures are those self reported by mathematics departments. The numbers for those studying behavioural sciences reveal an entirely different pattern, where there is a much stronger conversion rate from undergraduate to postgraduate study (Table 1.1). I find it remarkable that over 18 000 students study mathematics at university level (Figure 1.1) and around 150 (Johnston, 2005) of these complete honours! Clearly mathematics is not hooking its students and retaining them in the discipline.

Figure 1.1 Commencing, degree and postgraduate mathematics at Australian universities 2000–2004 (DEST, 2005)
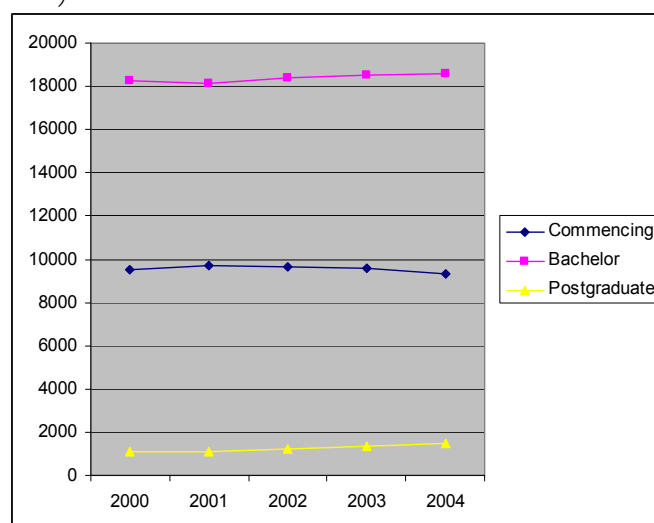
Table 1.1 Undergraduate and postgraduate enrolments 2004 (DEST, 2005)

Narrow Discipline Group	Doctorate by Research	Doctorate by Coursework	Master's by Research	Master's by Coursework	Other Postgraduate	Sub-total Postgraduate	Bachelor
Natural and Physical Sciences							
Mathematical Sciences	416	14	54	727	286	1,497	18,556
Physics and Astronomy	623	0	74	109	61	867	3,966
Chemical Sciences	766	0	52	21	20	859	6,934
Earth Sciences	676	0	83	119	65	943	2,574
Biological Sciences	3,086	1	289	331	266	3,973	27,401
Behavioural Science	1,411	328	105	1,626	1,498	4,968	18,499

Hall (2004, online) states:

> There is little dispute that the mathematical sciences in Australia are in decline. In some fields the fall is precipitous. My own, statistics, is a case in point. Indeed, it is doubtful I could adequately fill as many as half of the Chairs of Statistics that are currently vacant in Australia, or which will become vacant during the next decade.

He goes on to describe how the decline of Australian mathematics can be quantified. For example, the number of mathematicians working in Australian universities is today between 60 and 70% of what it was in the mid 1990s. Johnston (2005) concurs, the number of honours mathematics graduates in the five-year period from 1997 to 2001 was only three-quarters of what it had been in the previous five years. Hall states that the number of departments of statistics remaining in Australia is today only three; fifteen years ago there were at least three times that number, when

there were fewer than half as many universities as now, and when the demand by employers for trained statisticians was far less than at present.

Whom do mathematicians think of as a mathematician? As an example, I consider the Australian Mathematical Society (AustMS) which has three levels of accreditation as a mathematician; a Graduate Member (degree), an Accredited Member (postgraduate degree plus three years appropriate experience) and a Fellow.

> A **Fellow** shall be a person who currently has or previously has had the qualifications of an Accredited Member and who, in addition, is deemed by the Accreditation Committee (see Paragraph 19(a)) to have demonstrated a high level of attainment or responsibility in an area of mathematics and to have made a substantial contribution to mathematics or to the profession of mathematician or to the teaching or application of mathematics. (AustMS, 2006).

Finding out who is a mathematician in professional work is not easy. To quote from the AustMS website (www.austms.org.au/AdminDir/ accessed 23/3/06): *Unfortunately, finding information about members of the Society who reside outside of Australia, or who do not work in an Australian mathematics department is not so easy at present.* One could even argue the opposite is true. Very few graduates join the Society.

All of this illustrates that the job of "mathematician" or "mathematical scientist" is not obvious, visible or well defined. Rather, it encompasses a wide range of careers that are related through using the tools and ideas of mathematics. For most students, the nature of mathematics in workplaces is not at all clear, and hence it is not easy for them to make a connection between what they are learning at university and what they will be doing as a mathematician (Reid et al., 2003).

Furthermore, this study will show that many employers do not appreciate the insights that a mathematics graduate can bring to their business and industry.

Graduates

The experience of graduates in the workforce has rarely been studied. When I conducted a survey of mathematics education papers published in Australasia (or with Australasian authors) for the years 2000–2003 inclusive (Wood, 2004; attached in Appendix A), there were:

- 45 papers investigating the transition to tertiary study and first year mathematics teaching and learning,

- 31 papers on technology,

- 9 papers investigating higher-level mathematical study,

- 14 papers on service mathematics learning and teaching,

- 2 papers studying lecturers, and

- 1 paper (Wood & Petocz, 2003) discussing the transition to the workforce.

The research effort is focussed on the transition *to* tertiary study and teaching and learning issues at that level. This is true internationally (Holton, 2001). Given the high numbers of students in first year mathematical sciences and the concentration of teaching effort for those groups, the concentration of research resources at this level is understandable, but, I contend, not desirable for the long-term health of mathematics. More research needs to focus on the critical needs of novice workers and the manner in which university curriculum can help them make a successful transition into the workforce.

On the other hand, the specific mathematical needs of several disciplines, such as nursing (Hoyles, Noss & Pozzi, 2001) and engineering (Kent & Noss, 2003) have been studied and can lead us towards a closer interpretation of the relationship between the disciplines' needs, employers' perceptions and university curriculum. The University of London, Institute of Education (Hoyles et al., 2002) has surveyed employers about the mathematics needs of industry and:

> A key finding of the study was that although the ubiquitous use of information technology in all sectors has changed the nature of the mathematical skills required, it has not reduced the need for mathematics. The authors of the report refer to these mathematical skills and competencies, framed by the work situation and practice and the use of IT tools, as "mathematical literacy". The term partly reflects the skills needed by individuals in relation to business goals, but also reflects the need to communicate mathematically expressed decisions and judgements to others. On the basis of detailed case studies, the report concludes that there is an increasing need for workers at all levels of organisations to possess an appropriate level of mathematical literacy. (Smith, 2004; §1.8)

The Mathematics Association of America has published a summary of research in the area (Selden & Selden, 2001) and found similar needs to the Hoyles et al. study. A detailed report from the UK (HEFCE, 2003) examines how higher education enhances the employability of five discipline

groups of graduates (biology, computing, history, design studies, business studies). Mathematics graduates were not investigated.

Industry in Australia is asking for different skills from mathematics graduates. Rubinstein (2006, online) was involved in the 2006 national review of the mathematical sciences in Australia and makes the point that:

> As part of the national review, we are hearing that employers want more mathematics and statistics graduates with better skills in communication, team and project work and numerical and computational methods.

The number of industry representatives who took part in the review was small. Nevertheless it points to the need for curriculum change in Australian universities.

Internationally and in Australia, there is a dearth of research literature on graduate employment from the graduates' perspective (Johnston, 2003). The exception is the Journeyman project in Sweden (Albrandt Dahlgren et al., 2005), which follows the transition to the workplace for engineering and psychology students. Johnston argued that there is a need for research focussing on experiences of graduates in early employment, including relationships between higher education and work, working conditions, work expectations and satisfaction.

The effort and expense, noted by others (Yorke & Knight, 2003), of finding information about graduates partly explains the lack of research in this area. Finding information on graduates further out from their degree becomes even more problematic.

Communication

Communication is *the imparting or interchange of thoughts, opinions or information by speech, writing or signs* (Macquarie Dictionary, 1991). In this study I will be more precise and use the term *discourse* (see Chapter 2) in the analysis. However, when interviewing participants I will use the term *communication* or *language* to uncover their interpretations of their experience with discourse.

From the point of view of an organisation, an employee who can communicate well with others makes a more significant contribution to that organisation. Employers see communication skills as the most important graduate attribute (Hoyles et al., 2002). Communication skills are listed in graduate capabilities for every undergraduate degree. From the perspective of the individual, the

power of language choices can help you get a job, adapt to the job and progress in your career (Burton, 2004).

Academic teaching staff (who comprise the majority of mathematicians who are members of the Australian Mathematical Society) go through a strong apprenticeship in research. They complete an honours degree (or equivalent) then a PhD and perhaps several post doctoral fellowships. They generally have good publication records by the time they are appointed as an academic. There is considerable pressure to publish and continue to publish throughout an academic career. Each of these career stages helps develop an understanding of mathematical communication within a very specific, academic, context. As pointed out by Burton (2004), research mathematicians do not receive training in writing and the expectations of writing are not clear. McGrail, Richard & Jones, 2006, p. 24) make a similar point:

> Even though the ability to write for publication is a key skill for an academic staff member to possess, most staff will not at any stage of their career, whether as a student or as a staff member, be directly taught how to write for publication in refereed literature. In most cases, it is expected that they will have already attained a medium level of written communication, and will be able to learn on-the-job the more specific academic writing skills needed. However, this is not always the case and some universities have introduced writing courses, believing that their staff will benefit by attending these.

Several ideas have been suggested to increase the quality and quantity of academic writing. McGrail, Richard & Jones (2006) published a review of interventions to increase academic publication rates. Participants were provided with didactic and written information about the writing and publication process. Most required development of a draft manuscript during the course. A couple of universities provided professional writing coach for staff to assist with one-to-one development of writing. For all of these interventions, increases in publication rates were noted. It is these experiences of mathematical and academic communication that form the basis of the skills that are implicitly and explicitly embedded within mathematics curriculum. Many of these academic mathematicians have never worked outside a university context and, perhaps, are unable to provide pedagogic activities that will prepare graduates for the workforce. By examining how mathematics is used in workplaces, results from this study will give guidance and examples of curriculum and pedagogical change that will enhance the transition to the workplace.

Research aims

1. To describe and analyse mathematics graduates' experience of mathematical communication skills required in their workplace,

2. To investigate the perceptions of the graduates as to the requirements of their profession and how they acquired them,

3. To document graduates' transition and adaptations to work and the development of a professional identity,

4. To describe the implications for curriculum design in the mathematical sciences.

Methodologies

The methodologies used in my study depend on the research question being examined (Bishop, 1992). I take the view that different methodologies are necessary in different contexts. I use the approach of phenomenography, because it looks at how people experience, understand and ascribe meaning to a specific situation or phenomenon (Marton & Booth, 1997). It is a qualitative methodology that is often used to describe the experience of learning and/or teaching. The outcome of a phenomenographic study is a hierarchical set of logically related categories, from the narrowest and most limited to the broadest and most inclusive. This is referred to as the outcome space for the research. Phenomenography defines aspects that are critically different within a group involved in the same situation. Participants are interviewed in depth. The questions posed are designed to encourage the participants to think about why they experience the phenomenon in certain ways and how they constitute meaning of the phenomenon. In my study, the outcome space derived in the phenomenographic study is augmented by the use of discourse analysis to examine textual artefacts of three participants' work situations. The interplay between the participants' experiences and how they describe their experiences in relation to the texts gives valuable insight into the development of meaning in communication.

Originality

My original contribution to the field is the investigation of:

1. Types of discourse used in mathematics at university level and in the profession using:

    a. discourse analysis to examine mathematically based workplace artefacts and

    b. phenomenographical methods to examine the perceptions of the participants in the field

2. The linking of the phenomenographic outcome space with learning and professional experience.

3. The development of an outcome space, which describes graduates' views of mathematical discourse in the workplace.

4. The linking of workplace needs with explicit directions for curriculum change.

2 EXPERIENCE AND EXPRESSION

This study uses two methodologies to investigate the transition to the workplace:

1. Phenomenography

2. Discourse analysis

This chapter describes each of the methodologies and gives an overview of their development by considering the major proponents and examples of the use of their application in mathematics education. Phenomenography is described first, as the study starts with the experiences and perceptions of the participants, and then moves to an in-depth analysis of textual artefacts of work experiences using elements of discourse analysis.

Phenomenography

Phenomenography is both a theory and a qualitative method of research. It centres on people's interpretation of their experience of the phenomena they encounter. An essential focus in phenomenography is describing the variation in qualitatively different views and in how people perceive the surrounding world. Any phenomenon from specific experiences to experiences of more general aspects of the surrounding world can be investigated using the phenomenographic approach.

 "Phenomenography looks at how people experience, understand and ascribe meaning to a specific situation or phenomenon …" (Loughland et al., 2002, p. 190). As Ashworth and Lucas (2000, p. 295) explain: "It seeks to identify the qualitatively different ways in which individuals experience such aspects of their world as teaching, learning or the meaning of disciplinary concepts." Marton and Booth (1997, p. 14) express this as:

In phenomenography, individuals are seen as the bearers of different ways of experiencing a phenomenon, and as the bearers of fragments of differing ways of experiencing that phenomenon. The description I reach is a description of variation, a description on the collective level, and in that sense individual voices are not heard.

Trigwell et al. (2000, p. 159) give a summary of the approach:

This approach maps the essential variation in the understanding of a particular phenomenon in any given population. From a phenomenographic perspective, understandings of a phenomenon are not seen to reside within individuals, but are relationships between individuals and a particular task and context. They are not stable constructs, but dynamic and context dependent.

Richardson (1999) describes the beginnings of phenomenography in Sweden in the mid 1970s as, "… a program of investigations carried out by Marton and his colleagues that was concerned with qualitative differences between individual students in the outcome and process of learning." (p. 54) He notes that in their first study of learning (based on students' recall of the context of academic texts alongside their conceptions of learning):

Marton and two independent judges found it fairly straightforward to classify the students' attempts to recall the article into four categories of learning outcome, reflecting qualitatively different ways of comprehending the text. In addition, the entailment relations among these categories defined a partial ordering, so that each category defined a hierarchy in terms of the depth of the learning outcome. (Richardson, 1999, p. 54)

From this first study on learning, then replicated in others, derive the central tenets of phenomenography.

The methodology has since been particularly popular in investigating both students' and teachers' experience of learning and teaching in tertiary settings (Ashworth & Lucas, 2000, p. 295):

It originally developed out of investigations into students' experiences (in particular, their approaches to learning), and has extended into the investigation of how students understand disciplinary concepts and how lecturers experience their teaching. … As a research method within higher education, it has been very influential.

There are now many publications detailing various studies using phenomenographic methods into learning (such as Marton, Watkins & Tang, 1997; Prosser, Walker & Millar, 1996) and into conceptions of teaching (such as Trigwell et al., 2000).

The most popular technique for data collection is semi-structured in-depth interviews. Semi-structured interviews start with a small number of questions and then the interviewer probes for meaning. The interviews are audio-taped, then transcribed and analysed. In this sense, it falls within the outlines of discourse analysis, since it is participants' *recounting* of their experience (reality) that is analysed, via discourse (analysis of interview transcriptions). Trigwell et al. (2000, p. 159) describe the data analysis phase thus:

> When the conceptions have been identified within a context they are then decontextualised and the key dimensions of the thinking about the phenomenon mapped out. … The main outcome of the analysis is a set of categories that are very precisely constituted in terms of the most distinctive characteristics of the variation in the range of understandings of the key phenomenon and the relationship between these characteristics. What is consequently mapped on to a matrix, usually called an 'outcome space', is the essential variation in ways of understanding the phenomenon. The categories are hierarchical with higher order categories, incorporating lower order ones.

Loughland et al. (2002), in their study of conceptions of the environment, give a description of analysis and outcomes within phenomenographic research:

> The outcome of a phenomenographic study is a set of logically related categories …
> The categories are usually reported in order of their inclusivity and sophistication, and they are defined by their qualitative *difference* form the other categories. … it is the structure of the variation across the *group* that emerges through individuals' descriptions of their experience." (p. 190; original emphasis)

They also point out that the conceptions do not represent a process, but rather, "… a snapshot of the experience of the participants at that particular time." (Loughland et al., 2002, p. 191)

What the researcher seeks to find is not the 'objective' truth, but rather how different people experience a certain phenomenon from their own perspective. The field has expanded to include the description of conceptions across a wide range of areas, including health care (Sandström & Stålsby, 2004).

This next section examines the process of a phenomenographic study in detail.

The idea of phenomenography

Ference Marton was a main progenitor of the concept of phenomenography and he has produced many publications, alone and with others, based on its precepts. The major underlying principle of phenomenography is that:

> a capability for *acting* in a certain way reflects a capability [of] *experiencing* something in a certain way. … You cannot act other than in relation to the world as you experience it. … The unit of phenomenographic research is *a way of experiencing something* … and the object of the research is the *variation* in ways of experiencing phenomena (Marton & Booth, 1997, p. 111; original emphasis).

Research starts with the individual's experience (note that in Marton's previous writings, "conception" had been used instead of "experience"):

> The different ways in which they experience the text, the presentation, the problem, or the phenomenon are observed to be logically related to each other and to form together a complex that I have called the outcome space. … 'A way of experiencing something' is a way of discerning something from, and relating it to, a context. The meaning of something for someone at a particular point in time corresponds to the pattern of parts or aspects that are discerned and are simultaneously objects of focal awareness. … The variation between different ways of experiencing something, then, derives from the fact that different aspects or different parts of the whole may or may not be discerned and be objects of focal awareness simultaneously. (Marton & Booth, 1997, p. 112)

Phenomenographers focus on the variation in experience: "They seek the totality of ways in which people experience, or are capable of experiencing, the object of interest and interpret it in terms of distinctly different categories that capture the essence of the variation, a set of categories of description from the second-order perspective." (Marton & Booth, 1997, p. 121). This set of categories forms the outcome space, that is, the complex of categories of description comprising distinct groupings of aspects of the phenomenon and the relationships between these categories. The categories can generally be ordered hierarchically, in terms of increasing complexity.

Marton and Booth describe criteria for this set of categories:

> The first … is that the individual categories should each stand in clear relation to the phenomenon of the investigation so that each category tells us something distinct about a particular way of experiencing the phenomenon. The second is that the categories have to stand in a logical relationship with one another, a relationship that is frequently hierarchical. Finally, the third criterion is that the system should be parsimonious, which is to say that as few categories should be explicated as is feasible and reasonable, for capturing the critical variation in the data. (Marton & Booth, 1997, p. 125)

Svensson (1997) suggests that although phenomenographic methods are continually being modified for each object under consideration, there are features in common. The most significant characteristics of this research approach are the aiming at categories of description, the open explorative form of data collection and the interpretative character of the analysis of data. Svensson

proposes that description is fundamental to phenomenography, which is linked to assumptions about knowledge and that the importance of description is related to an understanding of knowledge as a matter of meaning and similarities and differences in meaning. Marton and Booth (1997) also describe the search for meaning, for descriptions and categorising of descriptions of phenomena as opposed to the phenomena themselves which could be seem as the object of science research.

Entwistle (1997, p. 127) introduces 'the idea of differing conceptions' and suggests that:

> We do not store definitions [of words] in memory, but that the meaning resides within the interconnections of remembered instances, and has to be reconstituted in providing an explanation. Moving from a relatively straightforward concept to a more problematic one … immediately demonstrates the somewhat idiosyncratic way in which we each understand abstract ideas. Phenomenography seeks to explore these different conceptions, or *structures of awareness* …, which people constitute from the world of their experience.

A way of experiencing something is related to how a person's awareness is structured (Pang, 2002). The categories in phenomenography come from the structural aspect of ways of experiencing the relationship between the different aspects of a phenomenon. Marton and Booth (1997) call these the internal horizon and the external horizon. The internal horizon refers to the parts and their relationship, together with the part-whole structure discerned therein. The external horizon refers to the way the experience is related to the context:

> To experience something in a particular way, a person must discern a whole from the context, and at the same time understand its relationship to the context as well as to other contexts. The external horizon extends from the immediate boundary of the experience through all other contexts in which similar and related happenings have been experienced. (Pang, 2002, p. 7)

Pang discusses various studies into learning and notes that the main theme is an interest in variation of experience of a phenomenon. The variation is then experienced by the learner but described by the researcher. Overall, these two senses of variation represent the different facets of the same object of research in phenomenography. (Pang, 2002, p. 18)

The role of the researcher

Marton and Booth (1997) make a distinction between a first-order perspective where statements are made about the world and phenomena directly and a second-order approach. They claim that a first-order perspective is what occurs in science while phenomenography takes a second-order

approach where the researcher is investigating the ways of experiencing of the phenomenon, not the phenomenon itself. These perspectives carry with them requirements for the phenomenographic researcher:

> It means taking the place of the respondent, trying to see the phenomenon and the situation through her eyes, and living her experience vicariously. At every stage of the phenomenographic project the researcher has to step back consciously from her own experience of the phenomenon and use it only to illuminate the ways in which others are talking of it, handling it, experiencing it, and understanding it. (Marton & Booth, 1997, p. 121)

So the researcher must step back from their own experiences and elucidate the experiences of the informants.

Number of participants

In performing a phenomenographic study, 15 – 20 informants is typically regarded as sufficient, as data saturation is usually achieved within this number. Data saturation means that no new information is added by new interviews. It happens when you have covered the breadth of your data (Richards, 2005, p. 136) and one can examine the depth of the data. Indeed Koslander and Arvidsson (2005) found that data saturation occurred after 8 interviews, that is, no new categories were added to the outcome space. Lindgren, Patriksson and Fridlund (2002) found that saturation occurred after 11 interviews. Others, such as McDowell (2002) found sufficient data from 12 interviews.

Data collection

Data collection generally is in the form of an interview where the interviewee reflects on his or her experiences. The interview can be seen as operating at two levels:

> On one level there is the situation of interpersonal contact in which the interview resembles a social discourse, in structure if not in content. On the second level, a metalevel, the interview is more like a therapeutic discourse inasmuch as the interviewer is trying to free the interviewee of hitherto unsuspected reflections. (Marton & Booth, 1997, p. 130)

In setting up this metalevel, the interviewer can use various techniques, such as alternative questions, bringing the interviewee repeatedly back to the focus for reflection, or it might be approached through offering interpretations of ideas the interviewee has said earlier in the interview.

Entwistle (1977) also describes the interview process in phenomenographic studies in higher education. He says that it is essential that the questions are posed in a way which allows the students to account for their actions within their own frame of reference, rather than one imposed by the researcher. Ashworth & Lucas (2000, p. 300) describe the process of the interview and reiterate the concerns of Entwistle:

1. The selection of participants should avoid presuppositions about the nature of the phenomenon or the nature of conceptions held by particular 'types' of individual while observing common-sense precautions about maintaining 'variety' of experience.

2. The most appropriate means of obtaining an account should be identified, allowing maximum freedom for the research participant to describe their experience.

3. In obtaining experiential accounts the participant should be given the maximum opportunity to reflect, and the questions posed should not be based on researcher presumptions about the phenomenon or the participant, but should emerge out of the interest to make clear *their* experience.

4. The researcher's interviewing skills should be subject to an ongoing review and changes made to interview practice if necessary. …

5. The transcription of the interview should be aimed at accurately reflecting the emotions and emphases of the participant.

There will be specific descriptions of the participants and the interview process for this study in Chapter 4.

Analysis

Marton and Booth (1997) discuss the approaches to data analysis and note that it is an ongoing process beginning early in the study. Analysis is described in terms of viewing 'a pool of meaning':

> The pool contains two sorts of material; that pertaining to the individual and that pertaining to the collective … The analysis starts by searching for extracts from the data that might be pertinent to the perspective, and inspecting them against the two concepts; now in the context of other extracts drawn from all interviews that touch upon the same and related themes; now in the context of the individual interview. (Marton & Booth, 1997, p. 133)

Åkerlind (2002) uses a set of quotes from the main practitioners to illustrate some of the variations in the practice of analysis. She suggests that one source of variation is in managing the data, and a number of different foci from previous research are presented:

Focusing on the referential or structural components of the categories of description;

Focusing on the 'how' or 'what' of the phenomenon;

Focusing on similarities and differences within and between categories and transcripts associated with particular categories;

Attempting to resolve or understand mismatches or inconsistencies between the interpretations of different researchers involved in the project;

Focusing on borderline transcripts and those transcripts in which there are aspects that do not fit the proposed categories of description; and

Looking for the implications for all of the categories of description of a change in any one category. (Åkerlind, 2002, p. 9)

Entwistle (1997) also describes the process of analysing the data and says that care must be taken in establishing the categories in ways which most fairly reflect the responses made. Discussions with others is an important safeguard in forming the categories. Having established the categories of description, phenomenography explores the relationships between them. This stage involves the analysis of the meaning of each category in relation to every other one, a consideration of individual variations in the ways each category is exemplified by individual respondents, and a thorough logical analysis of the meaning of these differences. (Entwistle, 1997, p. 132)

Ashworth and Lucas (2000) clarified methodological points in phenomenographic research and they emphasise that:

The categories of description must depend upon an earlier evocation of students' *very own* descriptions of their relevant experience. It is, therefore, a paramount requirement for phenomenography to be sensitive to the individuality of conceptions of the world – it must be grounded in the lived experience of its research participants. Without this, the descriptions of students' experience will be unsound and the categories of description will be arbitrary. (Ashworth & Lucas, 2000, p. 297; original emphasis)

Ashworth and Lucas (2000) show the need for bracketing and empathy though many researchers do not agree with their ideas. They describe a phenomenological technique of bracketing, that is, acknowledging presuppositions that must be restrained and where the researcher must suspend judgment. Ashworth and Lucas suggest that empathy may be the key to avoiding these presumptions: "Empathy requires a detachment from the researcher's lifeworld and an opening up to the lifeworld of the student." (Ashworth & Lucas, 2000, p. 299). They state:

1. The analysis should continue to be aware of the importation of presuppositions, and be carried out with the maximum exercise of empathic understanding.

2. Analysis should avoid premature closure for the sake of producing logically and hierarchically-related categories of description.

3. The process of analysis should be sufficiently clearly described to allow the reader to evaluate the attempt to achieve bracketing and empathy and trace the process by which findings have emerged. (Ashworth & Lucas, 2000, p. 300)

Research outcomes

The outcome of phenomenographic research is an outcome space of logically connected categories of qualitatively different experiences. The structural relationships between the categories are important, therefore,

> The researcher aims to constitute not just a set of different meanings, but a logical structure relating the different meanings. On this basis, it is possible in principle for the researcher to make a professional judgment about the optimal structure of the outcome space that may go beyond what is present in the data (as long as it is not inconsistent with the data). (Åkerlind, 2002, p. 10)

A structure may not be possible in all cases. Inconsistencies that may arise can be dealt with in various ways. Åkerlind (2002) suggests that one option is to describe them as representing *non-critical* variation within one or more ways of experiencing; another is to describe them as representing sub-categories of a primary category of description. Where neither of these seems appropriate the research outcomes may be represented as an incompletely structured outcome space, accompanied by speculation as to what a coherently structured outcome space might look like with more complete data. Conversely, the researcher could choose to present the most plausibly structured outcome space, while explicitly indicating where there is and is not active empirical support for the outcomes.

The aim is to get the best description possible and to reduce the data to a summary of the content or meaning as close to the data as possible. Thus some data will be abstracted from the rest and condensed as to their meaning and then grouped under categories. These categories are not stand-alone concepts but are relative (Svensson, 1997). The categories:

are based on the formulations in the interview … The content is … not primarily considered in terms of meanings of linguistic units, but from the point of view of expressing a relation to parts of the world. Furthermore fundamental characteristics of this relation are focussed on. This makes the specific forms of language used, although the basis for the analysis, subordinate to their expressed content. What counts as the 'same' conception may be expressed in many linguistically different ways and what counts as different conceptions may be expressed in a very similar language. (Svensson, 1997, p. 170)

The categories of description need to be presented with sufficient quotes from informants to illustrate the meaning of the category fully, and also to show, where appropriate, the contextual relationships which exist. The meaning resides in the essence of the comments from which the category has been constituted (Entwistle, 1997).

One technique suggested by Ashworth and Lucas, (2000, p. 305) is to develop a profile of each participant to identify 'central points of focus.' They note that in one study, as well as producing profiles and a set of categories in analysis, they produced a set of themes: "… it was found that the production of (a) individual profiles, (b) themes and (c) categories of description were mutually supportive in providing both an overview and the detail relating to the lifeworlds [of the participants]…" . Following this suggestion, I have presented employment profiles of the study participants in Chapter 4.

Limitations and criticisms

Each study only deals with a sample of individuals: "… the system of categories presented can never be claimed to form an exhaustive system. But the goal is that they should be complete in the sense that nothing in the collective experience as manifested in the population under investigation is left unspoken." (Marton & Booth, 1997, p. 125)

In terms of improving the capacity to generalise from the results, "… the sample should be chosen for *heterogeneity*, rather than for *representativeness* in terms of distribution along demographic and other lines." (Åkerlind, 2002, p. 12; original emphasis). Furthermore, within the limits of a qualitative form of research, "… to the extent that the *variation* within the sample reflects the *variation* within the desired population, it is expected that the *range* of meanings within the sample will be representative of the *range* of meanings within the population …" (Åkerlind, 2002, p. 12) Thus the results could be generalised to a similar population.

It is noted that there is some dispute about whether the descriptions or the experiences themselves are the object of research, and Entwistle comments that:

> The validity of a response made to an interview question certainly depends on the degree of overlap between the discourse practices of the participants, and it is entirely appropriate to make discourse the focal point of an enquiry. Yet, that set of responses can also be used as a starting point for analysing actions and experiences, as long as the limitations of individual accounts are also taken into account." (Entwistle, 1997, p. 133)

Cope (2002) is also concerned with determining the validity and reliability of phenomenographic studies. He suggests that validity is established by the researcher's justification for presenting the outcome space and claims based on those results. Justification of validity lies in a full and open account of a study's method and results. He sets out considerations for establishing validity when reporting results:

1. the researcher's background should be acknowledged … Describing the researcher's scholarly knowledge of a phenomenon is a means of providing a reader with the context within which the analysis took place;

2. the characteristics of the participants should be clearly stated, providing a background for any attempt at applying the results in other contexts;

3. the design of interview questions should be justified;

4. the steps taken to collect unbiased data should be included;

5. attempts to approach data analysis with an open mind rather than imposing an existing structure should be acknowledged;

6. the data analysis method should be described;

7. the researcher should account for the process used to control and check interpretations made throughout the analysis process …

8. the results should be presented in a way which permits informed scrutiny. Categories of description should be fully described and adequately illustrated with quotes … (Cope, 2002, p. 2)

It is noted that reliability in qualitative studies differs from that in quantitative studies and refers to whether results can be replicated: "In phenomenographic studies, this interpretation would refer to replicability of the outcome space(s)." (Cope, 2002, p. 2) However, in phenomenography this is difficult to achieve: "… although broad methodological principles are adhered to, the open, explorative nature of data collection and the interpretative nature of data analysis mean that the intricacies of the method applied by different researchers will not be the same. … Consequently, replication of outcome spaces by different researchers is unlikely." (p. 2) Nonetheless, "… the

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outcome space should be described and illustrated so that the variation in the data is communicated to other researchers." (p. 2) Involving a number of researchers in classification against the categories of description means, "Reliability of categories of description can be claimed on the basis of the percentage agreement between all the researchers' classifications before and after consultation. Säljö … believes an agreement of 80 to 90% after consultation is appropriate." (Cope, 2002, p. 2)

Richardson (1999) offers some benefits and disadvantages of phenomenography:

> The approach to qualitative research that was developed by Marton and his colleagues during the 1970s revolutionized the way in which both researchers and teachers think about the process and the outcome of learning in higher education. Unfortunately, this research was felt to lack a clear conceptual basis. 'Phenomenography' represented the attempt to provide an ad hoc and post hoc underpinning for the methodology that Marton and his colleagues had employed with such apparent success. (p. 72).

Although the intent is to illuminate the way people experience the world, Richardson states that phenomenography depends on other people's discursive accounts of their experience. Phenomenographers are describing the world as others describe it to them. He also criticises the analysis of the interview data in Marton's original investigation:

> Marton's original research seems to have been based upon verbatim transcripts …
> These transcripts were then subjected to an iterative and interactive process to
> identify fundamental categories of description in the data. (Richardson, 1999, p. 70)

The categories should emerge from comparisons conducted within the data and part of the reason for not developing categories in advance is based on the phenomenological concept of 'bracketing', or holding in check any preconceived notions that might contaminate one's immediate experience. Richardson asserts that these concepts are not unique to phenomenography: "However, precisely the same approach to the analysis of qualitative data is to be found in 'grounded theory'." (p. 70). This theory was first developed by Glaser and Strauss in the late 1960s, and its basis can be seen as similar to phenomenography. The essential difference is the emphasis in phenomenography on variation and the development of an outcome space.

Conclusion

The phenomenographic approach has been applied to illuminate conceptions of particular subject areas, for instance Crawford et al. (1994) investigated university students' conceptions of mathematics. They analysed the written responses to a questionnaire (containing open-ended

questions) using phenomenographic techniques; this then led to development of a closed-question survey.

Phenomenography has been used to investigate single phenomena and applying the analysis to a complex situation may result in a complex outcome space. Nevertheless, it is a useful methodology for situations where learners are reflecting on their learning and therefore appropriate for use in this study. Phenomenography works well for analysing lived experience and the outcome space (should one emerge) is useful for identifying key conceptions. It is suitable given that a reliable statistical analysis would not be possible, due to the difficulty in finding graduates and quantitative methods would not be desirable due to the need for in-depth analysis. Care will be taken to avoid the pitfalls of phenomenography listed above by being precise with the transcripts and by critically examining prior biases. In order to analyse the communication used in the workplace, there is a need for a different methodology and so discourse analysis provided a suitable methodology.

Discourse analysis

In this study, discourse analysis is used to interpret texts and how these texts illustrate the position of the graduate in their work situation. It provides a different viewpoint from which to analyse the professional situation. Three examples of texts will be discussed in Chapter 7.

Discourse analysis is an umbrella term used to describe a variety of approaches to analysis of language (in the broad sense), most of which have developed since the 1970s. Its themes present a useful background to inform the concerns of this study. Amongst the variety of methods (or viewpoints of discourse analysis), I have selected critical discourse analysis as providing the fullest range of both a theoretical framework and techniques to undertake the investigation of the fields of endeavour under review. Within critical discourse analysis – itself divided on the basis of individual theorist's philosophies – the approach of Fairclough (1992) (which represents an attempt at melding linguistics and social theory) has been chosen as a model for a research method.

A core feature of Fairclough's methodology is text analysis, for which the techniques of systemic functional linguistics offer the most systematic method. Systemic-Functional Linguistics (SFL) is a theory of language centred around the notion of language function. While SFL accounts for the syntactic structure of language, it places the *function* of language as central (what language does, and how it does it), in preference to more structural approaches, which place the elements of language

and their combinations as central. SFL starts at social context, and looks at how language both acts upon, and is constrained by, this social context (Halliday, 1994).

The field of discourse analysis grew out of largely European discussions on deconstructionist/poststructuralist views of social organisation and language, which could be seen as a reaction to the previously accepted structuralist theories, the main exemplars of which are Marx and Freud (and Saussure to some extent).

A corollary to poststructural ideas is that perceptions of social interactions and society are created and modified through language (or discourse); there is no social reality that is not constructed and then altered through the filter of language, in a dialectical exchange. As Fairclough (1992) put it: "The dialectical perspective sees practice and the event as contradictory and in struggle, with a complex and variable relationship to structures which themselves manifest only a temporary, partial and contradictory fixity." (p. 66) For some theorists, there has been a convergence of views of society and of linguistics; as MacLure (2003) noted, a distinction can be made between, "… two broad discourse traditions, distinguished by their intellectual lineage. One stems from European philosophical and cultural thought and is associated with poststructuralism. The other has its origins in Anglo-American linguistics." (p. 174)

Discourse

As for the term "discourse", it is used differently by the many writers on the subject. A few descriptions are presented here. Van Dijk (2001) submits perhaps the clearest definition:

> … in the broad sense of a "communicative event", including conversational interaction, written text, as well as associated gestures, facework, typographical layout, images and any other "semiotic" or multimedia dimension of signification. (van Dijk, 2001, p. 98)

Jørgensen and Phillips (2002) describe discourse as "… a particular way of talking about and understanding the world (or an aspect of the world)." (p. 1)

Fairclough's (1992) definition of discourse is:

> More commonly, "discourse" is used in linguistics to refer to extended samples of either spoken or written language. … This sense of "discourse" emphasizes interaction between speaker and addressee or between writer and reader, and therefore processes of producing and interpreting speech and writing, as well as the situational context of language use. (Fairclough, 1992, p. 3)

He expands on this to describe discourse as being widely used in social theory and analysis, for example in the work of Michel Foucault, to refer to different ways of structuring areas of knowledge and social practice. Discourses in this sense have particular ways of using language and other symbolic forms such as visual images (Fairclough, 1992) or, indeed, mathematical symbols.

MacLure (2003) describes the purpose of discourse practices thus: "They can be thought of … as practices for producing meaning, forming subjects and regulating conduct within particular societies and institutions, at particular historical times." (p. 175). The mathematical sciences have done this over the centuries and have a distinct form of discourse.

Discourse analysis

Discourse analysis is thus a weaving together of ideas gleaned from various writers and streams of thought in order to make sense of human thinking and social interaction, as evidenced in communicative events. Van Dijk (2001) sums up the underlying first principles of discourse as being: "… what we say and how we say it depends on who is speaking to whom, when and where, and with what purposes." (p. 108) He views as fundamental to any research based on discourse analysis,

> … the intricate text-context relationships. Just an analysis of text or talk added to some cognitive and/or social study will not do. … Adequate discourse analysis at the same time requires detailed cognitive and social analysis, and vice versa, and that it is only the integration of these accounts that may reach descriptive, explanatory and especially critical adequacy in the study of social problems. (van Dijk, 2001, p. 98)

Van Dijk describes society as including, "… both the local, microstructures of situated face-to-face interactions, as well as the more global, societal and political structures …" (van Dijk, 2001, p. 98)

Jørgensen and Phillips (2002) propose that the underlying principles of discourse analysis are founded on the notion that, "… our ways of talking do not neutrally reflect our world, identities and social relations but, rather, play an active role in creating and changing them." (p. 1) Further, "… our access to reality is always through language. With language we create representations of reality that are never mere reflections of a pre-existing reality but contribute to constructing reality." (pp. 8-9). That is, a foundation concept is, "… the idea that signs derive their meanings not through their relations to reality but through internal relations within the network of signs." (p. 10) On the concept of semiotic change, they state that, "… signs still acquire their meaning by being different to other signs, but those signs from which they differ can change according to the context in which

they are used …" (p. 11) Thus for analysis of discourse, "The starting point is that reality can never be reached outside discourses and so it is discourse itself that has become the object of analysis." (p. 21) In summing up the theory, they note, "… therefore that 'reality' is socially created, that 'truths' are discursively produced effects …" (p. 21).

Mathematics notation also changes over time as discussed in Chapter 3, with Cajori (1928, 1929) as the definitive writer on the historical development on mathematical notations before computers. Certainly the changes in context and the advent of computational tools have caused difficulty for programmers and students alike. For example, in the computer algebra system *Mathematica* there are three equals signs; = meaning assignment, == meaning solve and := to define a function.

Jørgensen and Phillips (2002) also note that the different theories of discourse analysis, "… have in common the aim of carrying out *critical* research, that is, to investigate and analyse power relations in society and to formulate normative perspectives for which a critique of such relations can be made with an eye for possibilities for social change." (p. 2) Most approaches rest on the notion that, "… the functioning of discourse – discursive practice – is a social practice that shapes the social world." (p.18) They point out that within both critical discourse analysis and discursive psychology, it is understood that, "Through producing new discourses in this way, people function as agents of discursive and cultural change." (p. 17)

Critical discourse analysis

 'Critical discourse analysis' is used by Fairclough as a working description of the approach he has taken. I have discussed in general the place of linguistics and social enquiry within discourse analysis, but this section aims to focus on the directions taken by those who would describe themselves as critical discourse analysts, with particular reference to the works of Fairclough.

Jørgensen and Phillips (2002) give an excellent overview of discourse analysis in general and devote a chapter to a discussion of Critical Discourse Analysis (CDA) in particular, with a focus on Fairclough as a major exponent. They offer suggestions for conducting CDA research projects with the following fundamental principle: "Discursive practice – through which texts are *produced* (created) and *consumed* (received and interpreted) – are viewed as an important form of social practice which contributes to the *constitution* of the social world including social identities and social relations." (Jørgensen & Phillips, 2002, p. 61; original emphasis) They acknowledge CDA's interest in power relations (or hegemonic relations) and propose what could be regarded as a neat summary

of CDA: "The research focus of critical discourse analysis is accordingly *both* the discursive practices which construct representations of the world, social subjects and social relations, including power relations, *and* the role that these discursive practices play in furthering the interests of particular social groups." (Jørgensen & Phillips, 2002, p. 63; original emphasis) Their view of CDA's methodology is that analysis of a discursive event should include:

- Analysis of the discourses and genres which are articulated in the production and the consumption of the text….

- Analysis of the linguistic structure …

- Considerations about whether the discursive practice reproduces or, instead, restructures the existing order of discourse and about what consequences this has for the broader social practice. (Jørgensen & Phillips, 2002, p. 69)

They describe Fairclough's prescriptions for research design and methods in some detail, but point out that, "It is not necessary to use *all* the methods or to use them in exactly the same way in specific research projects. The selection and application of the tools depend on the research questions and the scope of the project." (Jørgensen & Phillips, 2002, p. 76; original emphasis) They emphasise, however, that in any research project, "The governing principle is that discursive practices are in a dialectical relationship with other social practices: discourse is socially embedded." (Jørgensen & Phillips, 2002, p. 78) On this basis, when analysing results, the research should, "… give insight into the ways in which texts treat events and social relations and thereby construct particular versions of reality, social identities and social relations." (Jørgensen & Phillips, 2002, p. 83) The aim of research should be based on the following precept:

> Critical language awareness should give people insight into the discursive practice in which they participate when they use language and consume texts and also into the social structures and power relations that discursive practice is shaped by and takes part in shaping and changing. (Jørgensen & Phillips, 2002, p. 88)

Methodology

A major part of Fairclough's interest is to develop a methodology for analysing discourse and discursive practices, underpinned by the principle that: "Analysis of a particular discourse as a piece of discursive practice focuses upon processes of text production, distribution and consumption. All of these processes are social and require reference to the particular economic, political and institutional settings within which discourse is generated." (Fairclough, 1992, p. 71) In this analysis, "The central concern is to trace explanatory connections between ways (normative, innovative, etc.)

in which texts are put together and interpreted, how texts are produced, distributed and consumed in a wider sense, and the nature of the social practice in terms of its relation to social structures and struggles." (Fairclough, 1992, p. 72) These concepts can be rendered by a three-dimensional conception of discourse.

Figure 2.1 Three levels of discourse (Fairclough, 1992, p. 73)
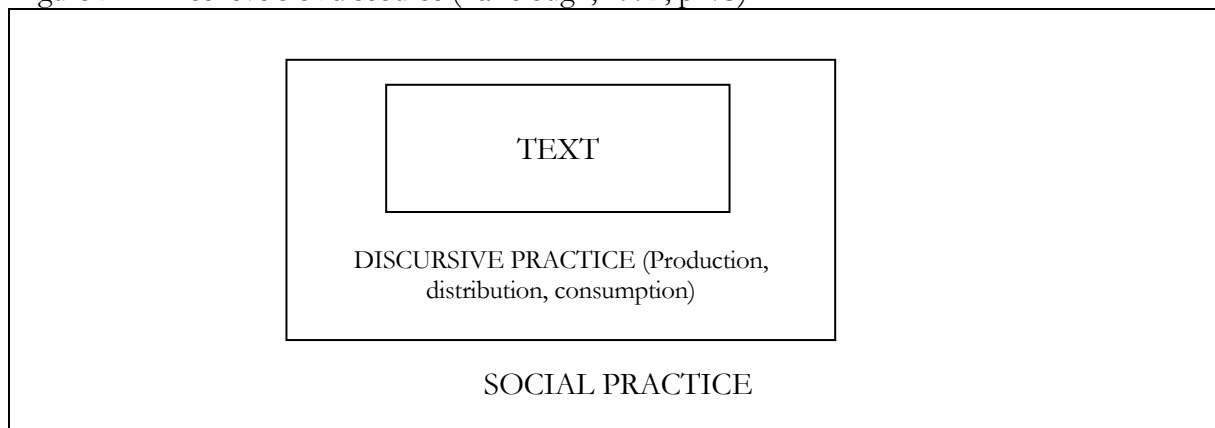
Fairclough (1992, p. 72) describes this representation as combining,

> three analytical traditions, each of which is indispensable for discourse analysis. These are the tradition of close textual and linguistic analysis within linguistics, the macrosociological tradition of analysing social practice in relation to social structures, and the interpretivist or microsociological tradition of seeing social practice as something which people actively produce and made sense of on the basis of shared commonsense procedures.

In discussing the analysis of texts, Fairclough notes that, "Texts are made up of forms which past discursive practice, condensed into conventions, has endowed with meaning potential [which is] generally heterogeneous, a complex of diverse, overlapping and sometimes contradictory meanings … so that texts are usually highly ambivalent and open to multiple interpretations." (1992, p. 75) Therefore, in practice analysts, "… usually reduce this potential ambivalence by opting for a particular meaning, or a small set of alternative meanings." (Fairclough, 1992, p. 75)

Titscher et al. (2000) sum up Fairclough's schema:

> Fairclough's method is based on the three components *description*, *interpretation* and *explanation*. Linguistic processes are described, the relationship between the productive and interpretative processes of discursive practice and the text is interpreted, and the relationship between discursive and social practice is explained … (Titscher et al., 2000, p. 153; original emphasis).

Of particular relevance to both the use of language in mathematics and in universities is Fairclough's point that texts are created as a result of previous history:

> Texts vary in the relative weight of these pressures depending upon their social conditions, so that some texts will be relatively normative whereas others are relatively creative. Centripetal pressures follow from the need in producing a text to draw upon given conventions, of two main classes; a language, and an order of discourse – that is, a historically particular structuring of discursive (text-producing) practices …(Fairclough, 1995, p. 7)

Fairclough reiterates one of his particular concerns – and one that is the foundation for this study – that text cannot be analysed in isolation:

> The analysis of discourse practice involves attention to processes of text production, distribution and consumption. This feature of the framework encapsulates what I think is an important principle for critical discourse analysis; that analysis of texts should not be artificially isolated from analysis of institutional and discoursal practices within which texts are embedded. (Fairclough, 1995, p. 9)

Jørgensen and Phillips (2002) note that a development presented in Chouliaraki and Fairclough (1999) is to, "distinguish between non-discursive and discursive moments of a social practice and propose that these moments adhere to different kinds of logic." (p. 70); that is, that there is a range of mechanisms at work in society and not all are discursive (economics, for example). Any discursive mechanism is thus, "… working in combination with other mechanisms … to constitute a social practice." (Jørgensen and Phillips, 2002, p. 71)

Chouliaraki and Fairclough (1999) propose that, "It is an important characteristic of the economic, social and cultural changes of late modernity that they exist as *discourses* as well as processes that are taking place outside discourse, and that the processes that are taking place outside discourse are substantively shaped by these discourses." (p. 4; original emphasis) This accords with Fairclough's earlier notions of discourse constituting social practice, but here Chouliaraki and Fairclough suggest that, in reference to the social changes now under way:

> Language is relevant not only in the discursive construction of the changing practices of late modernity – what is changing in these practices is in part also language. For example, 'flexibility' in the practices of the workplace is partly a matter of the increasing prominence of 'team work', and 'team work' is partly constituted by new forms of dialogue which for instance transcend old divisions between shopfloor and management. (Chouliaraki & Fairclough, 1999, p. 5)

Wood and Petocz (2002) put forward a similar argument: "… there has been a large-scale restructuring of employment with major implications for the linguistic demands of work. The modern workplace is requiring more interpersonal communication skills, as the emphasis shifts from isolated workers to teams." (p. 70) They point out that a similar transformation is occurring in the professions generally, where professionals are being required to adapt to the language needs of their clients.

Systemic functional linguistics (SFL)

Fairclough (and others) have used the tools of systemic functional linguistics (also known as functional grammar) as an intrinsic part of analysing discourse. As with discourse analysis in general, the primary emphasis of SFL is on the interpretation of meaning embodied by texts, rather than the structures of language per se. A writer embeds meanings in a text by selecting from the options available, from both the vocabulary and the potential functions of the lexicogrammar. The importance of context – for writer and reader – is intrinsic to the approach. SFL is used in computer algorithms that mimic natural language.

Central to SFL is the use of system networks to represent the choices present in producing a text. For example, a simplified lexicogrammatical network could be (in decreasing complexity):

> Clause: finite, non-finite
>
> Group: nominal group, adjectival/adverbial group
>
> Word

The choices on each level are constrained by those on others. Thus the decision to use a nominal-group (which is a noun phrase), rather than a clause, to express a semantic process will be determined by both the structure of the text as a whole, and also by the social context (for example, nominalisation is more functional in a science text than in casual conversation). This idea of language choice is critical to the ideas of my study.

In discussing spoken language, Halliday (1994, p. xxiii) compares it with written language:

> The potential of the system is more richly developed, and more fully revealed, in speech. … spoken language responds continually to the small but subtle changes in its environment, both verbal and non-verbal, and in so doing exhibits a rich pattern of semantic, and hence also of grammatical, variation that does not get explored in writing.

He suggests that, "what much of the written language achieves lexically is achieved by the spoken language through the grammar." (p. xxiv) Thus written language is often lexically dense but grammatically simple, whereas spoken language may be the opposite: "… much more of the meaning is expressed by grammar than by vocabulary. As a consequence, the sentence structure is highly complex …" (Halliday, 1994, p. xxiv). As spoken language is used to a large extent in this study, these features of spoken language become apparent.

SFL and mathematics

Morgan's (1998) book, *Writing Mathematically: the Discourse of Investigation*, arose from many of the same concerns as are addressed in this study, a primary motivation being an interest in improved communication of mathematics and curriculum design. Morgan states:

> Written communication is not a simple matter of transmitting what is in the writer's mind onto paper and thence into the reader's mind. Both writer and reader have their own perceptions of and relations to the subject matter and to each other, all of which influence the construction and interpretation of the text. (Morgan, 1998, p. 76)

Since possible interpretations by readers can have effects on writers (for instance, for an academic submitting an article to a journal, or for a student submitting a report to a teacher), "It is, therefore, of importance to writers to know what forms of language are likely to perform the functions they want and what forms are likely to be highly valued within the particular discourse." (p. 76)

The approaches Morgan elucidates in the book are highly relevant and she offers some useful expansions of the basic tools presented here – namely, critical discourse analysis and systemic functional linguistics. The primary difference in topic area is that Morgan focuses on secondary school mathematics while this study focuses on tertiary students and professional mathematicians.

In terms of identifying a text as mathematical, Morgan suggests that:

> It seems likely … that any text that is commonly identifiable as mathematical will share at least some linguistic characteristics with other texts that are also considered to be mathematical. This is not necessarily to say that there are core characteristics common to all mathematical texts but that there are likely to be areas of 'overlap' that contribute to our ability to identify a particular text as mathematical. (p. 8)

Features that academic mathematical texts may have in common include: "A formal impersonal style, including an absence of reference to human activity, is one aspect that mathematical writing appears to share with many other academic areas …" (p. 11) The type of modality is another

feature: "… high modality (i.e. a high degree of certainty and an absence of such human frailties as doubt or expressions of attitude) …" (p. 14) Morgan also refers to the use of nominalisation in science writing: "One source of the 'distant authorial voice' is the use of nominal rather than verbal expressions." (p. 14) She elaborates on this to say, "The ability to represent processes as objects and hence to operate on the process-objects themselves is part of the power of mathematics; at the same time, it increases the impersonal effect, strengthening the impression that it is these process-objects that are the active participants in mathematics rather than the human mathematicians." (Morgan, 1998, p. 15)

This impersonality is common in academic writing in general: "It is clear that an impersonal style is an accepted convention in much academic writing, particularly in the sciences, and analysis of the linguistic features contributing to this style (e.g. passive voice, use of personal pronouns and choice of tenses) is a major focus of research in this area." (p. 15) It is possible, however, that there is variation in these features between genres (such as textbooks or academic papers).

Morgan suggests that mathematical writing differs from other forms of science writing in its use of the imperative: "A human presence does, however, intrude into mathematical text in a way that is not so characteristic of academic writing in other sciences through the use of imperatives, conjuring up a human actor. Thus, the reader of higher mathematics texts is likely to be frequently enjoined to *suppose*, *let*, or *define*." (p. 16) The type of argument employed may also differ from that of other disciplines: "In academic mathematics high value is placed on deductive reasoning as a means of both 'discovering' knowledge and providing its warrant." (p. 17) An example of this is the formal mathematical proof: "A characteristic of the proofs … is the occurrence of strings of statements thematizing both the fact that an act of reasoning is occurring (i.e. starting with words such as *hence* or *but*) and the previously established facts which act as the bases for the deductions." (Morgan, 1998, p. 17)

Morgan noted that,

> The analysis of non-verbal parts of texts is less fully developed in the literature; analyses of the syntax of mathematical symbols … have paid little attention to the functions that might be fulfilled by choices between alternative symbolizations or between verbal and symbolic forms, while discussion of graphical elements in mathematics texts … has tended to consider only the difficulties that these may cause for student-readers … (Morgan, 1998, p. 77)

The device of nominalisation can obscure human action: "… the obscuring of agency; the transformation of process into object removes the grammatical need to specify the actor in the process … [which] fits in with an absolutist image of mathematics as a system that exists independently of human action." (Morgan, 1998, p. 82) Objects may also be presented as actors: "A similar function is performed by the use of representational objects as actors in verbal processes, i.e. *the table shows that …* rather than *I have shown in the table that …*, which obscures the writer's presence as author as well as mathematician." (p. 83; original emphasis) Passive voice can have the same effect: "The use of passive voice rather than active forms of verbs is a further way of obscuring agency that is much used in academic writing." (p. 83) The way in which causal relationships are depicted builds on this: "Here again the presence or absence of humans as causal agents is significant in the extent to which mathematics is seen as an autonomous system." (Morgan, 1998, p. 83)

In discussing the interpersonal function, Morgan notes the tendency of mathematics writers to use 'we' as the Actor extensively, whether the authorship is a number of writers or a single author, "… thus suggesting that the author is not speaking alone but with the authority of a community of mathematicians …" (p. 85) It can also suggest that the reader is a participant 'constructing the argument'. The use of the second person pronoun can enhance this effect of involvement: "Addressing the reader as *you* may indicate a claim to a relatively close relationship between author and reader or between reader and subject matter." (p. 85) Another common phrase is 'you will notice', which gives the impression that, "… the author is addressing an individual reader personally and directing her attention with a degree of authority; it also suggests that the reader ought to be interested in the details of the mathematics presented in the text." (Morgan, 1998, p. 85)

Regarding textual function, "Given the high status of deductive reasoning in the mathematics community, we might expect to find expressions of logical reasoning thematised. For example, the presence in a report of mathematical activity of a large number of themes expressing reasoning (e.g. *Hence, Therefore …*) would serve to construct the text as a deductive argument, while a predominance of temporal themes (e.g. *First, Next …*) would construct a story or report recounting what happened or, if used with imperatives, would construct an algorithm." (p. 87) Reasoning can be expressed in a number of ways: "… through the use of conjunctions …, nouns (*the reason is …*),

verbs (*X causes Y*) or prepositions … It may also be expressed less explicitly through the juxtaposition of causally related statements." (Morgan, 1998, p. 88; original emphasis)

Non-verbal elements are raised as a significant characteristic: "Non-verbal features play an important part in most mathematical texts. In particular, the system of mathematical symbolism plays a crucial role in the activity of doing mathematics …" (p. 88) Algebra represents a unique use of symbolism: "Algebraic symbolism is rather different from other non-verbal features, not only because it can be translated into words and read in a linear way similar to verbal text but because of the significance of its role within mathematics." (pp. 88-89) The use of symbols has a number of effects, for instance, "By representing an object, quantity, action or relationship by a symbol in a mathematical text, it is declared to be 'mathematical' and thus of significance. At the same time symbolizing is an act of abstraction allowing the writer and the reader to focus only on the formal properties of the symbol itself …" (p. 89) It also serves as a form of nominalisation by turning the symbol into an object: "Mathematics itself thus appears as a domain in which the main activity is manipulation of symbols rather than of concepts or 'real world' objects." (p. 89) The use of symbols can heighten the impression of the writer as an authority.

Some of the features shown by Morgan are reflected in the investigation of university mathematics teaching and learning by Wood and Smith (2004), which is described in detail in the next chapter, and by the case studies in Chapter 7.

Conclusion

A general description, for our purposes, of contemporary discourse analysis incorporates the following principles:

- Reality is interpreted through the filter of language, and social interaction and structures are constituted (constructed) by discourse

- Truth is not absolute, but dependent on each individual's context

- Knowledge equally is not absolute, but depends on cultural and historical context

- Discourse consists of all types of symbols (signs), including gestures, and everything that is written or spoken may be of relevance to a study of meaning

- Textual meaning cannot be absolute since it is determined by context and because all discourse is two way – even for written text, there is always an (at least implied) audience who will alter the meaning of the text

- There is continuous social change, which is shaped by discourse.

I will discuss the texts provided by the participants in relation the above principles and to Fairclough's three components in Chapter 7.

Links between phenomenography and discourse analysis

Webb (1997), in his critique of phenomenography, says that, "Phenomenographic explanation is prone to reproduction of the discourses it studies." This may explain possible connections between discourse analysis and phenomenographic analysis. Phenomenography uses discourse to explore and categorise conceptions. Participants use language in interviews to describe their experiences and this is then transcribed to make the text that is used for the phenomenographic analysis.

Richardson (1999, p. 64) also makes the point that analysis of discourse forms the basis for data collection. He notes that, "… conceptions of reality are not psychological entities somehow residing in the minds of individuals. Rather, they represent discursive practices that are used as resources in particular communicative encounters." (p. 72). In other words:

> There is in fact an alternative way of interpreting the accounts provided by participants in research interviews, which is to regard them as examples of people's discursive practices, without making any assumptions as to their evidential status. It is this kind of approach that is adopted in discourse analysis … Indeed, it is possible to go further and argue that the entities that figure in such accounts are merely artefacts that are constituted in social interactions and have no independent existence … (Richardson, 1999, p. 67)

Hasselgren and Beach (1997) described five forms of phenomenography. They suggest that one form of phenomenography, a discursive form, began with Dahlgren in 1979, since his study of price formation was not related to learning per se, that is, he used: "… a form of phenomenography which was not directly related (experimentally or otherwise) to an evaluation of the outcomes of specifically pre-directed learning. Dahlgren's interest in price formation has an educational origin, but was treated in the investigation in a way that did not go beyond the knowledge interest of phenomenographic investigation itself." (p. 197) This form, "… uses discourse without regard for the rules of discourse production and analysis to 'produce expressions of conceptions' which can be analysed phenomenographically." (Hasselgren & Beach, 1997, p. 197)

Hasselgren and Beach (1997) named another form 'naturalistic', which, "… alludes to the possibilities for collecting empirical material for phenomenographic analysis from 'authentic' situations … It is about recording what is actually said or happens in a given situation without direct manipulation or involvement from the researcher and then analysing that data phenomenographically." (p. 197). This data collection method is very similar to that often used in discourse analysis but the methods of analysis are different.

Conclusion

Phenomenography and discourse analysis provide ways to investigate the transition to the professional workplace by looking at workplace practice (discourse analysis) and participants' reflections on the experience of that practice (phenomenography). Phenomenography is particularly useful in examining professional situations as the participants have the skills and insight to contribute deep reflections on their practices and experiences. It is also useful in situations where an idea of the depth of experiences is important. It is a qualitative methodology which is ideal for examining the transition to professional work as it is difficult to find a reasonable number of participants for a quantitative study. In terms of curriculum design a small in-depth study will give more insight. In particular, I will look at the features of mathematical discourse, particularly the combination of the personal (we, you) and the formal (eg nominalisation, extended noun phrases, high lexical density), which is very confusing for students to read and even harder to duplicate themselves.

Summary of chapter 2

In this chapter, I described how the transition to professional work will be investigated using two methodologies, phenomenography and discourse analysis, supplemented by data from published sources.

3 BECOMING A PROFESSIONAL

Much of the mathematics teaching and learning research at university level considers the components of teaching and learning rather than how the teaching and learning will assist students to connect with their discipline area. The literature about graduate skills and generic capabilities is more critical to the transition to the workplace as it is looking at how a student will be able to fit into a discipline after university study; however, there are few studies in the discipline area of mathematics. Only peer-reviewed publications are included here and those available using library searches. This limits the publications to mainly English language and western sources. Nevertheless, given that the focus of this study is the development of mathematics professionals in Australasia, a focus on western literature is appropriate.

I start by considering the mathematics education literature using the three-level framework of discourse from Fairclough (1992), (Figure 3.1). I will show that most of the university mathematics education literature falls into the components level of Figure 3.1 in that it describes components of learning, such as computing, and artefacts of learning, such as performance on assessment.

Much of the early literature concerning language and mathematics refers to school level mathematics and accordingly just a few significant papers and books are covered here. The review then moves on to summarise the research on the teaching and learning of university mathematics in Australasia over the past four years. This will situate the current study in the context of Australasian research. The literature on graduate capabilities in mathematics is discussed. The final section examines learning theories, which range from theories that look at components (behaviourism) to theories that consider how a person develops an identity (Wenger, 1998).

Framework

Fairclough's model (Fairclough, 1992, p. 73) gives three hierarchical levels of discourse, with each level subsumed in the other. These three levels suggest strong connections with the three conceptions of statistics that were the phenomenographical outcome space of the work of Petocz and Reid (2001) as shown in Coutis and Wood (2002). I make further connections with the levels

of conceptions of mathematics found by Reid et al. (2003), (Table 3.1). Because of the hierarchical nature of conceptions in phenomenography, the higher levels also subsume the lower levels.

It is to be expected that the levels have some relationship with Fairclough and I will discuss links further in Chapter 9. Fairclough developed his hierarchy after extensive experience with discourse analysis and observations of discourse in many workplaces and amongst students. The studies by Petocz and Reid (2001) and Reid et al. (2003) consider undergraduate students who are preparing for professions in the mathematical sciences. In essence, Level 1 deals with the components of the text (syntax, vocabulary and so on) and components for the statistics and mathematics (*t*-tests, algebra, calculus and so on). At Level 2, one is dealing with the *use* of the components; being able to work with text, produce correct text and in statistics and mathematics; being able to use the components to model situations. Level 3 puts the knowledge into a professional context. The main difference between level 2 and 3 is the personal and professional connection that is made with a situation for both text and mathematics and statistics.

Table 3.1 Levels of discourse and conceptions of statistics and mathematics

	Fairclough (1992)	Petocz and Reid (2001)	Reid et al. (2003)
Level 1	How texts are constructed (Components of text)	Extrinsic technical concept of statistics	Components
Level 2	Construct and interact with text	Extrinsic meaning	Models
Level 3	How this all fits into a discipline	Intrinsic meaning	Life

Language and curriculum

The work in systemic functional analysis theory and practice (Halliday, 1985, 1991) had a strong influence on the development of language and discourse initiatives in curriculum development. In Australia, this coincided with the influx of refugees from South-East Asia into the school and tertiary education sectors. These students had fractured mathematics schooling due to conflict in their countries and many teachers worked on innovative approaches to integrating mathematics and

language learning for these students. Halliday and Hassan (1985) used science and mathematics as examples of the language difficulties in these discipline areas.

There are two main aims in the integrating of language and disciplinary studies that have been used to develop curriculum. The first is exemplified by such articles as *Building Bridges: Using Science as a Tool to Teach Reading and Writing* (Nixon & Akerson, 2002), where science and mathematics are used as a vehicle to teach language. The second is where language is used to teach the mathematics/science, for example *Reading and Writing to Learn Science* (Glynn & Muth, 1994). These articles are at primary and secondary school levels and the approach was developed to solve a perceived problem with cohorts of students whose first language is not the language of instruction. Much of this work is anecdotal and many believe that there is considerable room for more empirical investigation (Stoddart, Pinal, Latzke & Canaday, 2002).

Cummins (1994) has been particularly influential by demonstrating that integrating a second language across the curriculum does not disadvantage native speakers, though he has been criticised because his work was in Canada where students had reached high levels of academic literacy in English or French and are learning or studying in the other language. His work does not take into account the notion of linguistic distance, which is based on the difficulty of learning other languages in comparison to one's own language. For example, Chiswick and Miller (2004) studied English in North America and used their measure of linguistic distance in an analysis of the determinants of English language proficiency among adult immigrants in the United States and Canada. They showed that, when other determinants of English language proficiency are the same, the greater the measure of linguistic distance, the poorer is the respondent's English language proficiency.

In terms of curriculum design at university, there are students and staff with a range of language and cultural backgrounds. The idea of linguistic distance may make it more difficult for native Chinese speakers (say) than native French speakers (say) to learn mathematics at an English medium university and to develop professional discourse skills. The work of Cummins (1994) showed that those who reached high level analytical skills in one language were able to transfer them to another language and I believe that this would be true for professional discourse skills.

Barton et al. (2005) have shown that English as an additional language (EAL) mathematics learners at university have more difficulty in learning mathematics at university than previously thought and

that this corresponded to a significant reduction in the students' grades. They therefore recommended increasing the language proficiency required to enter mathematics courses at university. In terms of curriculum design, it means that language has an important dimension in learning. I show in this study is that communication is also important for professional formation.

As detailed in Chapter 2, a major study of mathematical communication at secondary level was by Morgan (1998). Morgan points out that differences in mathematical texts often reflect the differences between the experience of students and professional mathematicians:

> On the whole, students have worked only on relatively short, routine problems for which little elaboration or explanation is required. Their writing has been addressed only to their teacher … In contrast, 'real' mathematicians tend to work on relatively substantial and often original problems. Their anticipated audiences are expected to be genuinely interested in knowing the results and to need to be persuaded of the correctness of the results. (Morgan, 1998, p. 2)

There has been insufficient attention paid to this divergence within curriculum design. This is important to my study and is similar to the previous findings in that the secondary curriculum aims (if it teaches language at all) are to teach the mathematics through the language or teach language through the mathematics, but not teach the discourse of mathematics itself. In ignoring these possibilities, I believe that curriculum designers are missing an opportunity to provide graduates and students with the skills necessary to function well in a mathematical workplace. They are also missing out on the highest level skills of integrating discourse with their discipline.

Raymond Duval (1999, 2001) argues that, "systems of semiotic representation for mathematical thinking is essential because, unlike other fields of knowledge (botany, geology, astronomy, physics), there are no other ways of gaining knowledge to the **mathematical objects** but to produce **some semiotic representations**." (1999, p. 4, original emphasis). This important, and controversial, idea is that we can only gain access to mathematical objects through their representations, and it has led to a research school that investigates the ways that students develop their mathematical understandings with different representations. He contends that mathematics contains a series of registers (such as both the equation of the linear function $y = 2x$ and the graph of that function) and that learning mathematics is a translation between registers. He states, "The characteristic feature of mathematical activity is the simultaneous mobilization of at least two registers of representation." (2001, p. 3). He presents data to show that students are more able to

move in one direction between different registers (for example, if given a function then sketch the graph) than in the opposite direction (2001, p. 6).

At university level, researchers influenced by Duval such as Durand-Guerrier (2004), have been investigating how details in mathematical discourse, such as the way letters change status over the course of a proof, can lead to deep misunderstanding in students. She argues that the detailed use of language has not been seriously addressed in mathematics education. I agree that these difficulties have not been addressed but believe, as implied by Morgan (1998), that an enhanced course of action is to investigate how practicing mathematicians use mathematical discourse and develop teaching and learning strategies on this basis to induct students into the discourse of the discipline. The work of Durand-Guerrier and others may change practices within the mathematical community, as their discourse practices are made explicit.

Other work at tertiary level has investigated the language needs of students studying mathematics and statistics at university (Wood & Smith, 2004; Coutis & Wood, 2002; Wood & Petocz, 2002; Wood, 2004) and then the design of materials to assist with language development (Wood & Perrett, 1997; Wood & Petocz, 2003a). These studies use discourse analysis to examine the language used in lectures, tutorials, textbooks and computer packages. One study (Wood & Smith, 2004) will be discussed below in detail in order to demonstrate the types of discourse used in mathematics teaching and learning at university. In Chapter 7, examples of workplace texts will be analysed.

Discourse analysis of university mathematics learning

This section is adapted from Wood and Smith (2004) to illustrate the types of mathematical discourse that students are exposed to at university. It is an interesting contrast to the types of discourse required in the workplace. The analysis uses systemic functional linguistics to examine the function of the words and phrases in the texts.

For the discourse analysis of mathematics learning, a 'typical' lecture was video and audio taped. The accompanying textbook and computer help files from the computer algebra system *Mathematica* were also analysed. Because discourse analysis can be detailed, the discussion here centres on one topic: De Moivre's Theorem. To summarise these findings, I will discuss some linguistic aspects of the language used in order to lecture effectively in the area of mathematics and provide some examples of each in turn. The different modes would be considered by Duval (1999) as semiotic representations.

Spoken and written modes

Students and lecturer alike are working on both spoken and written mode. The textbook and computer help files are written, the lecture itself is a mix of oral and written, in that the lecturer is constructing overhead transparency (OHT) versions of the presentation as he talks. It is difficult to make sense of the transcribed oral data without reference to the OHTs and the non-verbal context.

The lecturer is constantly referring to context using spoken language:

> Just before the break I did this result there.
>
> If you take z squared with z equal to the following (writing)...
>
> If you multiply the thing together you find that z squared is cos theta plus theta plus i sin theta plus theta using exactly the same thing as I've done over here with theta one equal to theta two and r one equal to r two equal to one which of course simplifies down to something like (writing) that …

The lecturer is working in a number of modes: oral language, written language, mathematical notations, visual diagrams, mathematical notation and is organizing the students' attention through to each through both verbal and non-verbal means.

Differences between spoken and written language

The data samples involve both written and spoken modes. An extract from the textbook is shown in Figure 3.1, the OHTs in Figure 3.2, the corresponding spoken text constructed in the course of the lecture in Figure 3.3 and the computer help files (Figure 3.4).

What grammatical features stand out in Text 1 (Figure 3.1) and how do they contribute to its overall tone? The overall tone of the text is impersonal, with uses of the passive supporting this:

> It can be shown that
>
> The result is known as De Moivre's theorem
>
> ... complex numbers can be used to derive results...

where the doer or subject is suppressed in order to prioritise different aspects of the clause.

Another way that this effect is achieved is through non-human subjects:

> There is an important special case of this result.
>
> The result ... has many uses.

The first (use) is an example of the way in which complex numbers can be used …
and complex nominal groups:

an important special case of this result,

results which are only about real numbers

Again the text is multi-mode (semiotic representations), switching between natural language and algebraic notation: $z^2 = r^2 cis\, 2\theta$ as well as a geometrical diagram.

Figure 3.1 Written text 1: the textbook

Suppose now that z_1 and z_2 are complex numbers with polar forms

$$z_1 = r_1 cis\theta_1, \quad z_2 = r_2 cis\theta_2$$

Then

$$z_1 z_2 = r_1(\cos\theta_1 + i\sin\theta_1) . r_2(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1 r_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2))$$

$$= r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$$

$$= r_1 r_2 cis(\theta_1 + \theta_2)$$

Thus the geometric interpretation of multiplication is:

multiply moduli and add arguments.

There is an important special case of this result. Let $z = r\, cis\,\theta$. Then

$$z^2 = r^2 cis\, 2\theta.$$

Further

$$z^3 = z^2 z$$

$$= r^3 cis\, 3\theta,$$

and in general, for any positive integer n,

$$z^n = r^n cis\, n\theta.$$

It can be shown that this result is also true if n is a negative integer or zero. The result is known as *de Moivre's Theorem* and has many uses, of which we shall consider two. The first is an example of the way in which complex numbers can be used to derive results which are only about real numbers.

In addition to the impersonal grammatical constructions, the instance of one pronoun "we" ("we shall consider two [uses]") is evident. This is an impersonal use of the pronoun, (used to construct a

kind of writer/reader complicity or perhaps with overtones of the royal "we"?), certainly to diffuse the "I" of the writer.

Functional grammar analyses the verb of traditional grammar into a number of process types which are: material process (doings and happenings), relational process (being and having), verbal process (saying) and mental process (thinking/knowing/feeling). In the written text, the grammar of the algebraic notation seems to privilege relational meaning over the operations needed, for example, to move from: $z = rcis\,\theta$ to $z^2 = r^2cis\,2\theta$, which would be expressed linguistically through the grammar of material process. To sum up, evidence from the grammar of the written text seems to support an impersonal tone in which the subject is edited out in the way that Walkerdine and others characterise mathematics as a discourse (Walkerdine, 1988). Mathematics writers follow the effect of science writers (Halliday, 1991) in turning processes into objects.

Figure 3.2 Written text 2: the overhead transparency

What is interesting about Written text 2 (Figure 3.2) is that the data consist of not just the finished product, but also the way that it was constructed orally, since part of the data is a video recording of the lecture in which the lecturer constructs the overhead transparency as he talks. Taken purely as written text, there are some of the impersonal features of Text 1 such as the use of the impersonal subject:

> It turns out that this result is also true for negative integers. (Text 2)

Yet here there is a subtle shift in the tone, achieved by the colloquial/informal use of "turns out". The difference in tone is clear when comparing the OHT text with the nearest equivalent in the lecture notes:

> It can be shown that this result is also true if *n* is a negative integer or zero. (Text 1)

Text 2 is constructed dialogically, for example in the selection of imperative verbs "consider", "show", which position the reader as required to do something. In additional to the impersonal forms, there is the authorial use of the pronoun "we", noted in Text 1:

> In general we find...

as well as the multi-mode embedding of algebraic notation in natural language, typically as a projection.

When considering the grammatical features of Text 2, note that the grammar functions to support and summarise the talk. The fact that the grammar is often "note-form" with the use of visual organizers, rather than the linear sequencing of text, is a product of this.

Spoken Text

The spoken text (Figure 3.3) is the lecturer's oral presentation of the material corresponding to Texts 1 and 2, the introduction of De Moivre's theorem. The transfer to written text to spoken text often makes the spoken language look disorganized, full of hesitations, reformulations and repetitions. This is an effect of the transfer to written form and the increase of redundancy is generally a feature of oral language. In the following example, the lecturer introduces the notion that de Moivre's theorem holds for negative as well as positive integers.

Figure 3.3 Spoken text.

> ... the implicit assumption here is that n is actually a positive integer and in fact I should make that explicit (WRITES) "n a positive integer" but it turns out in fact that it holds for all n and it's fairly easy to prove for negative integers and it's quite obviously true for n zero n a fraction is a little more difficult and in fact there's a sort of way round that which you'll deal with shortly it turns out and I don't I won't prove (   ) I'll just state the result so it turns out that this result also holds for negative (WRITING) "negative integers"

I will not do a complete analysis of what makes this a typically spoken text, but will comment on two features: clause structure and logical connectors. What the sentence is for written language, the tone group is for spoken. Technically we can refer to both as clauses and talk about the clause

structure. The clause structure of the above is typical of spoken language, with a piling on of clauses, linked by everyday logical connectors (*and, but, so*).

When considering the role of interpersonal adjuncts, Text 1 was quite impersonal. In the spoken text however, the lecturer makes use of a range of words like *actually, fairly, obviously* to personalise and introduce values and judgments into the presentation. These are highlighted below.

> … the implicit assumption here is that n is **actually** a positive integer and in fact I should make that explicit (WRITES) "n a positive integer" but it turns out in fact that it holds for all n and it's **fairly** easy to prove for negative integers and it's quite **obviously** true for n zero n a fraction is a little more difficult and in fact there's a sort of way round that which you'll deal with shortly it turns out and I don't I won't prove … I'll just state the result so it turns out that this result also holds for negative (WRITING) negative integers.

Computer help files

Figure 3.4 Help file from the computer algebra system *Mathematica* (4.0)

3.3.8 Expressions Involving Complex Variables

Mathematica usually pays no attention to whether variables like x stand for real or complex numbers. Sometimes, however, you may want to make transformations which are appropriate only if particular variables are assumed to be either real or complex.

The function ComplexExpand expands out algebraic and trigonometric expressions, making definite assumptions about the variables that appear.

This expands the expression, assuming that x and y are both real.

ComplexExpand[Tan[x + I y]]

There are several ways to write a complex variable *z* in terms of real parameters. As above, for example, *z* can be written in the "Cartesian form" Re[*z*] + I Im[*z*]. But it can equally well be written in the "polar form" Abs[*z*] Exp[I Arg[*z*]].

The option TargetFunctions in ComplexExpand allows you to specify how complex variables should be written. TargetFunctions can be set to a list of functions from the set Re, Im, Abs, Arg, Conjugate, Sign . ComplexExpand will try to give results in terms of whichever of these functions you request. The default is typically to give results in terms of Re and Im.

This gives an expansion in Cartesian form.

ComplexExpand[Re[z^2], {z}]

Here is an expansion in polar form.

ComplexExpand[Re[z^2], {z},TargetFunctions -> {Abs, Arg}]

Figure 3.4 is the computer help file that you get if you ask for de Moivre's theorem. Notice it does not state the theorem specifically and students would have to realize that to apply de Moivre's theorem you need the complex number expressed in polar form. It is not immediately clear that `Abs[z]Exp[I Arg[z]]` is equivalent to $re^{i\theta}$ with $r =$ `Abs[z]` and $\theta =$ `Arg[z]`. To be able to move between the different modes students would need to know that:

$$z = x + iy = re^{i\theta} = r(\cos\theta + i\sin\theta) = rcis\theta = \texttt{Re[z]+IIm[z]=Abs[z]Exp[I Arg[z]]}$$

The textbook and the OHT use of i is similar but different to either use in the computer program.

The written language used in the Help file is direct and reads as a recipe with direct commands in active voice. There is use of quantifiers and qualifiers, *sometimes, if, usually*. While the program authors have attempted to make the language direct and use short sentences, the mathematical complexity of the ideas presented and the notation used causes difficulty for the beginning student.

This example of De Moivre's theorem illustrates the way students learn to change between different semiotic representations of the same mathematical object. The reading, listening and writing skills required are complex. The induction into the discourse of the discipline is generally done implicitly and it is assumed that by learning the mathematics, students will also learn the discourse. I test this assumption in this study.

Professional mathematics

At the professional level, Burton and Morgan (2000) examined the ways that academic mathematicians write. They argue that writing plays a critical role in many mathematical practices:

> The stakes involved in producing mathematical texts that are seen as acceptable are often high, both for students who are likely to be assessed on the basis of their written work, and for professional mathematicians whose status within the community and even job security may depend on the quality (and publishability) of their writing. (p. 430)

Burton and Morgan believe that, "knowledge of the forms of language that are highly valued within mathematical discourses and the effects that may be achieved by various linguistic choices … would empower them" to make choices, to break conventions and express their own personality (p. 431). While they quote some published guidelines for the development of mathematical writing they conclude that, "the training of mathematicians does not appear to include any systematic attention to the development of writing skills" (p. 448).

The aim of the present study is to broaden the work of Burton and Morgan (2000) to investigate the discourse used by new mathematics graduates in industry and their perceptions of how they acquired these skills. I share the beliefs of Burton and Morgan about the power of language choices and have used some similar analytical methods, such as aspects of discourse analysis and the use of in-depth interviews. I have also investigated graduates' perceptions of how they use this discourse and how they learnt the discourse. I need therefore to consider what is meant by mathematical discourse.

Advanced mathematical discourse

This section considers aspects of communication in advanced mathematics. I use the phrase *advanced mathematical discourse* to refer to the uses of language in university mathematics learning and teaching and in professional life (Wood & Perrett, 1997).

Communicating in or about the field of mathematics involves taking part in mathematical discourse, whether by reading, writing, listening or speaking. Discourse is a broader concept than language because it also involves all the activities and practices that make up mathematics as a profession. The discourse of mathematics includes all the ways that mathematics is done: through language, textbooks, computer packages, seminar presentations, mathematicians talking to each other and to a wider public, and through popularisation and application of mathematical knowledge.

Symbolic language and natural language

Mathematical discourse is distinct from other types of discourse simply because mathematics is quite distinct from other subjects or disciplines, and the things that mathematicians do are distinct from what other academics and professionals do. They require specific methods and forms of communication. The discourse is often highly symbolic and has been developed because of advantages over natural language such as:

- it is more economical,

- it is more precise,

- it encapsulates concepts in a way that clarifies connections,

- it is easier to manipulate the mathematical objects,

- it is less open to ambiguity and multiple interpretations.

Mathematical symbolic language is not universal and unchanging: it has been invented and developed by mathematicians to do a particular job. The *History of Mathematical Notations* (Cajori, 1928, 1929) catalogues these developments. Two examples show the power of notation: the adoption of Hindu-Arabic numerals (Cajori, 1928, pp. 74–99) and the systematic development of notation by Leibniz (Cajori, 1929, pp. 180–196), which allowed mathematics to develop into such a powerful tool. Mathematical notation has progressed since 1929, particularly with the use of computers, but the foundations were laid earlier as catalogued by Cajori.

Symbolic language is easier to follow if it is expressed in ways which follow the conventions of the natural language. Mathematicians use natural language to help other people understand mathematics. If it were not the case, everything to do with mathematics could be written as strings of symbols. What we find is that mathematicians mix symbolic language with English (or another language) in different proportions and in different ways depending on what they think their audience will be comfortable with. In his popular book *A Brief History of Time*, Stephen Hawking (1988) was advised that every equation in a book halved the audience. He only included $E = mc^2$ but I don't actually believe it helped the readability! It is still an extremely difficult read. Mathematicians could well have found equations easier.

Using natural language does have reasons; it can make a point more memorable, to show how something is important, for clarification or to fill in useful background knowledge. There is a danger of becoming so over-trained in the art of concise note making that mathematicians regard the natural language around the central symbolic language as "padding". It is usually more than padding, it is used with clear aims, and advanced users of mathematical discourse can recognise and interpret these aims in the light of what they know about how mathematics is practised. They can also use appropriate natural language in their own papers and seminars, adapting it according to their own aims and to what they believe their audience knows about how mathematics is practiced.

Will the participants in this study be able to use symbolic and natural language to meet their aims? I will investigate case studies in Chapter 7.

University mathematics learning and teaching

In 2004, I published a summary of Australasian research into university teaching and learning for the years 2000 – 2003 (Wood, 2004; full text in Appendix A). It is important to consider the current state of research in the light of curriculum design at university level mathematics learning and teaching. The following section contains a summary of pertinent research and areas that are in need of further investigation, such as graduates' attributes.

Mathematics learning in the adult education and vocational educational sectors has a different focus to my study; however research in this area informs the debate about mathematics use in the workplace and its impact on curriculum. An important contribution to this area is the work of FitzSimons (2002), who examined historical, sociological, and practical elements of mathematics within vocational education considering the impact of technology. Benefits to national economies and the personal benefits of successful contributions to the workplace are emphasised. Differences between the educational institution and the workplace are raised as sources of tension.

Other studies have also investigated the impact of information and communication technology on the teaching and learning of mathematics at many universities. Students' attitudes towards technology and learning in mathematics are the subject of several Australasian studies (e.g., Cretchley & Galbraith, 2003; Galbraith & Haines, 2000). Fogarty, Cretchley, Harman, Ellerton, and Konki (2001) report on the validation of Galbraith and Haines' (2000) questionnaire designed to measure general mathematics confidence, general confidence with using technology, and attitudes towards the use of technology for mathematics learning. They found that the scales—on the basis of factor analysis—demonstrated high internal consistency, reliability and divergent validity. Other studies, such as Cretchley and Galbraith (2002) and Cretchley and Harman (2001), also investigated students' confidence and motivation using the scales developed by Galbraith and Haines (2000). These studies considered the affective domain including confidence, attitudes, motivation and engagement. They found that mathematics attitudes (both confidence and motivation) correlated strongly with achievement in mathematics and that attitudes about using technology in mathematics learning correlated far more strongly with computer attitudes than with mathematics attitudes. This important work contrasts with those who introduce technology into mathematics teaching and learning at university without consideration of the consequences for groups of students. There should be more informed debate about technology and its use.

Despite the increasing depth of research on these topics, there are still areas that are significantly under-researched in the tertiary domain. The following are the conclusions I identified from my survey of research (Wood, 2004) and the references are included in the article reproduced in Appendix A.

Researching the teachers. There were few articles that explored the attitudes and conceptions of university lecturers of mathematics. Considering the important role that lecturers have in designing the teaching and learning process, this lack of research should be addressed. Internationally this is also true, with the exception of teacher education where teachers are regularly studied.

Higher mathematics. There was very little research on the teaching and learning of higher levels of mathematics, such as the learning of abstract algebra, and graduate studies, such as Graduate Diplomas in Operations Research.

Cross-disciplinary mathematics teaching and learning. This is an important area for the survival of university mathematics departments. Internationally, much of the learning and teaching research is done in cross-disciplinary teams which include academics from the serviced departments, but this was not evident in the Australasian studies. It is time to work in multidisciplinary teams in Australasia.

Studies of graduates of mathematics and those who use mathematics. There is scope to investigate the effects of our teaching on graduates. The present study addresses part of the gap in graduate research.

Emotion and motivation in mathematics learning. Why do students stay with mathematics? What is the motivation for students to engage more deeply with mathematics at all levels? A small study in the UK by Rodd (2002) examines emotion in undergraduate mathematics learning, but there are no Australian studies. The study by Forgasz and Leder (2000) about perceptions of learning mathematics comes the closest.

Alternative teaching and assessment methods. Collaborative learning and peer teaching are under-utilised in mathematics teaching and learning at university level. These should be examined for optimal implementation. Assessment practices need to be explored to ensure they encourage deep learning.

As there are few studies of mathematics graduates, the next section will examine more general literature on graduates and the transition to professional work.

Graduate capabilities

This section reviews the literature investigating what happens when students leave university as graduates and the skills they have acquired (or should have acquired). In studying this area researchers are hampered by lack of data. To study the experience of the large numbers of students entering first year we have good data on students' entry levels (school results and so on), on students' demographic backgrounds and on their progress through university. These data are available at our fingertips on most university computer systems. Information on alumni, who are spread far and wide, is difficult to track. Graduates are no longer a captive audience and any data that are collected are voluntary. In Australia, the course experience questionnaire (CEQ), (Graduate Careers Australia, 2006), and the graduate destination survey (GDS), (Graduate Careers Australia, 2006) are answered by students a few months after they finish their degrees.

There is some confusion about terminology here. Are graduates developing skills, attributes, capabilities, dispositions or all of these? Are these generic or discipline specific? At this point I will use the terms interchangeably even though there are subtle differences in the way they are used. I will visit the idea of generic or discipline specific in Chapter 9.

Bowden et al. (2000) considers generic graduate attributes to be:

> The qualities, skills and understandings a university community agrees its students should develop during their time with the institution. These attributes include but go beyond the discipline expertise or technical knowledge that has traditionally formed the core of most university courses. They are qualities that also prepare graduates as agents of social good in an unknown future.

I agree with Barrie (2004) who suggested that much of the literature on graduate attributes is not research-based, which makes it difficult to monitor whether graduates have actually achieved the attributes that are listed by universities and indeed whether the listed attributes are actually those needed by graduates. As I have stated previously, studying graduates is difficult and expensive.

Graduate skills and employability

What is the main aim in teaching students at university? Are universities inducting them into a discipline, or preparing them for a specific career or the general workforce? Do institutions care about employability? Yorke and Knight (2003) describe views of the role of higher education in regards to employability. Firstly, higher education can be a preparation for a profession, so employability can be defined as how well students are prepared for that profession. Secondly, there

is a view that university prepares students for any job by developing generic achievements, so that employability is enhanced by the development of excellent generic achievements. A major report by the higher education funding body in the UK (HEFC, 2003) considered how much higher education enhanced the employability of graduates. The report examined five discipline areas, 34 universities and interviewed graduates and their line managers, in a similar way to the work of Scott (2003) in Australia. They also surveyed academic staff in the departments for their opinions about the development of graduate capabilities. The results were inconclusive.

To identify the generic achievements and get agreement on them by stakeholders is not easy. A large Australian study reported by Hambur, Rowe and Luc (2002) tested graduates over a range of graduate skills. They selected 5 cognitive dimensions to assess. Critical thinking, problem solving and interpersonal understandings were each tested using 30 multiple-choice items; argument writing and report writing were assessed using a writing task. The items were changed for context in different disciplines. These cognitive dimensions were selected after consultation with universities and other stakeholders such as employer groups and professional bodies. Employers preferred skills that helped their organisations with their goals, especially personal and interpersonal skills, which they listed as self-management, effective oral communication, problem solving, logical and orderly thinking, creativity and flair in business, entrepreneurship, teamwork and leadership (Hambur, Rowe & Luc, 2002, p. 24). The universities focused more on academic skills and qualities related to citizenship. The cognitive skills investigated in the study were chosen because they were measurable and appeared to be components of other skills. Major findings included those that may be expected, for example, Arts/Humanities students performed better on 'critical thinking' and 'interpersonal understandings', whereas Engineering and Architecture students did relatively better on 'problem solving'.

The report is an important contribution to the discussion on graduate skills. Mathematics and Science students (grouped for the report) perform around average for all domains, slightly higher for problem solving and slightly lower for argument writing, and with less variability than students in other domains. Student-specific variables such as motivation and ability appeared to account for much of the variance in the scores. Performance did not seem to be related to gender, age or language background. Another useful finding was that scores on the domains tested were significantly higher in later years of study. The authors express caution concerning this result, as the reasons for it are not clear. The numbers tested in the later years were smaller, and those who did

not have the required skills may have dropped out, or there could be a variety of other explanations, such as maturity. However, if the finding is correct, and students are improving on these graduate skills, then this is a positive result for teaching and learning. Further research with larger and matched samples will assist with understanding these findings.

Other recent research focuses on successful graduates (Scott & Yates, 2002; Scott, 2003). For a particular field of study, several employers were selected and asked to nominate a group of their most successful recent graduates, about 20 in all. The graduates and supervisors were then interviewed in depth to ascertain the attributes that had contributed to the graduates' success. From his research, Scott has developed a 'framework of professional capability' (Scott, 2003, p. 5). Scott's research points out that it is when things go wrong, when an unexpected or troubling problem emerges, that professional capability is most tested, not when things are running smoothly or routinely. It is at times like these that the individual must use the combination of a well-developed emotional stance and an astute way of thinking to 'read' the situation and, from this, to figure out ('match') a suitable strategy for addressing it, a strategy which brings together and delivers the generic and job-specific skills and knowledge most appropriate to the situation. An example used by Scott is that if a professional is unable to remain calm and work with staff when things go wrong, then how much they know or how intelligent they are may be irrelevant.

Another avenue of research is to consider those graduates who fail to find professional employment after graduation. In a small study of unemployed graduates, Knight (2003) found that lack of work experience, unrealistic aspirations, competition for jobs, poor degree results and poor career planning were given as reasons for their unemployment. They referred to the 'degree-work mismatch', feeling they had learned to execute a limited number of academic procedures well but remained deficient in areas such as self-presentation, self-motivation and communication. Many felt that they would need further qualifications before getting a job, and suggested that lecturers could include discussion about employability and careers as part of the curriculum from first year.

Several professional societies have grappled with the development of graduates, in particular in the area of ethics and professional responsibility. Engineers Australia and the Statistical Society of Australia both have codes of conduct for their members. Engineers Australia has a graduate program where they consider the professional education of their graduates. Graduates have a mentor and development program that emphasises the competence and responsibility of an

engineer. This approach could be beneficially used in other professional areas including science disciplines.

Developing professional capability and finding professional employment is important for graduates, but so too is the influence of these ideas on learning at university. The following section considers research on how ideas of future profession influence learning.

Conceptions of future profession and approaches to learning

As discussed below, evidence is mounting that perceptions of a students' subject area and their ideas of the type of work that they will be doing influence learning at university.

In an in-depth study of 22 undergraduate mathematics students' conceptions of mathematics and the influence on learning, Reid et al. (2005) found that the links between students' conceptions of mathematics, their approach to learning and the anticipated outcome of their studies were compelling (Table 3.2). The model built on and expanded upon earlier descriptions of students' learning approaches, such as the surface and deep approach of Marton and Säljö (1976) and the 3P model of Biggs (1999). This demonstrates that ideas of mathematics and the perceived outcomes have an important influence on students' approaches to learning.

Table 3.2 Students' conceptions of learning mathematics (Reid et al., 2005)

Aspect Orientation	Intentions	Approaches	Outcomes
Techniques	1a. To pass the subject or course	1. Focus on course requirements and expectations	1a. A pass, degree, qualification
	1b. To get a [better] job, status, money		1b. A [better] job, status, money
	1c. To acquire mathematical tools and skills		1c. Acquire mathematical tools and skills
Subject	2a. To understand mathematics, practice, theory, applications	2. Focus on mathematical components	2a. Understanding mathematics, practice, theory, applications
	2b. To help others with mathematics		2b. Help people using mathematics
Life	3a. To acquire a mathematical way of thinking or philosophy	3. Focus on understanding beyond mathematics	3a. A mathematical way of thinking or philosophy
	3b. To open one's mind, to satisfy intellectual curiosity		3b. Satisfaction of intellectual curiosity, personal growth

In another study of undergraduates' conceptions of mathematics, about 1200 students at five universities on five continents responded to three open-ended questions, including *What part do you think mathematics will play in your future career?* (Wood et al., 2006). The responses were coded using phenomenographic methods and grouped into a hierarchy of conceptions in a similar manner to

the previous study (Reid et al., 2005). Open-ended questions and the students' responses were used in preference to a closed form survey in order to elicit the students' ideas rather than being restricted to those of the researchers.

Students were unclear about the part mathematics would play in their future career and many answered *I really don't have any ideas. Sorry!* Twenty-two percent of students stated that they would need mathematics for their future career but were unable to articulate why or how the mathematics would be used in their career (for example: *It will play a huge role in my future career because I am focusing on an actuarial career.*). Several students (4.3%) expressed the view that *I have to know some basic mathematics but the others, the computers and calculators will do it for me.* The uncertainty of many students is demonstrated in their word use, such as "should" (*It should play an important role in my future since it's used in many fields*), "maybe" (*Maybe I'll only use basic calculation*) and "certain amount" (*I cannot answer that question as of right now since I don't know what I will be doing in the future just yet. But in almost every career mathematics has a certain amount of importance.*).

All the research leads to the conclusion that, "while technical expertise is a necessary capability … it is certainly not sufficient", to produce a successful graduate (Scott & Yates, 2002). Students are vitally interested in their careers, but often feel that the technical skills are the ones that will give them employment, not the collateral skills acquired through their degree. This is demonstrated in the career conceptions study where the majority of students focussed on the use of mathematics as a tool or did not have any specific idea about how they were going to use the mathematics. In his study of unemployed graduates, Knight (2003) found that, "resistance to wise messages helps to explain the unemployed situation in which these graduates find themselves". This study also highlighted an interesting dilemma: two of the ex-students were unemployed due to ethical issues. Their ideals were restricting their choice of employers. For areas such as mathematics, where many of the jobs are in defence or finance, ethical considerations might also narrow students' employment choices.

Transition to work

There is a growing body of general literature investigating the transition to professional work and the development of identity and a community of practice (Wenger, 1998). Burton (2004) describes and analyses the practice of established academic mathematicians. She interviewed 35 male and 35 female mathematicians and showed that they could be described as a 'community of practice' (Wenger, 1998) with the features of participation and reification. (Reification refers to abstractions

such as tools, symbols, discourse and concepts developed by the community.) Burton also used facets of discourse analysis to analyse the academic writing of her participants. Though Burton's study did not examine the transition to the profession, it did establish that, for academic mathematicians, a community of practice could be identified.

Adbrandt Dahlgren et al. (2005), as part of a larger study on the transition from higher education to work, interviewed 12 students in each of two programs (Psychology and Mechanical Engineering) before they started work and then 18 months later as novices in the workplace. They established that in the psychology program there was a high degree of continuity between being a student and a professional novice and that the transition to work was relatively easy. This was attributed to the use of problem-based learning in the university degree program. The Mechanical Engineering students, on the other hand, were only participating on the periphery of their profession after 18 months. Their studies had opened the door to the profession and demonstrated their technical ability but did not, of itself, confirm their participation in the community of practice. It may that it is the *field* of practice that is important.

The comparative study by Wood and Kascynski (2005) showed differences related to the field of practice. The students in juvenile justice programs participated in internship programs that contextualised their studies and, as a result, students felt more confident in their ability to make the transition and were more committed to their field of study. The participants in this study were students anticipating their transition rather than graduates. The experiences of the other participants reported in the Wood and Kaczynski comparison will be discussed in chapter 5.

There is a dearth of research literature on graduate employment from the graduates' perspective (Johnston, 2003). Johnston argues that there is a need for research focussing on experiences of graduates in early employment, including relationships between higher education and work, working conditions, expectations and satisfaction. This study aims to fill that gap for mathematics graduates.

Learning

This study is investigating the transition to the professional workplace and how curriculum can be designed to achieve a better transition. In the context of a graduate becoming an functioning member in the workplace, this section briefly examines various current theories of learning.

Constructivism

Constructivism is the most prevalent theory within mathematics education research and is originally based on Piaget's ideas (1954). It suggests that a learner develops their mental structures based on prior knowledge and attitudes when challenged with new tasks. The pedagogy implied is task oriented and can be done individually or in social settings. As an example, John Mason from the Open University in UK designs mathematics materials that embody the constructivist learning pedagogy (for example, Mason, 2004).

The constructivist theory associated with Vygotsky is centred on the social aspects of learning where a learner is guided (scaffolded) by an adult or more capable peers. The notion of zone of proximal development (ZPD) is the gap between a learner's current development level determined by independent problem-solving and the learner's potential level of development.

> the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers (Vygotsky, 1978, p. 86).

This theory of learning goes some way to supporting the development of active participants in the workplace. I believe it falls into the level 2 of Table 3.1 but falls short of fitting the discipline into the wider social context. The focus is on the subject itself and its development in the learner (Biggs, 1999).

Behaviourism

Behaviourism (Skinner, 1974) is often maligned as being connected to rats and mazes and Pavlov's dog salivating. One important aspect of this theory of learning is that the learner is viewed as adapting to the environment and learning is seen largely as a passive process in that there is no explicit mental processes, that is an observable response such as factorising a quadratic. The learner responds to the 'demands' of the environment, such as a test. Knowledge is viewed as given and absolute.

It is, I contend, the basis for much mathematics teaching and learning in use today. Drill and practice, using formulas in 'plug and chug' situations, are the common teaching methods for mathematics in school and higher education systems. Lecturers believe that students should arrive from the secondary school system being able to do routine operations of algebra and calculus

without thinking. This falls into the level 1 categorisation illustrated in Table 3.1. The focus is on components of the discipline.

Many teachers and lecturers would insist they do not use behaviourism but would argue they are teaching the 'basics'. The 'Math wars' in the USA are examples of conflicts between behaviourist and constructivist learning theories as Sowder (1998) explains:

> There is today in California a serious debate taking place concerning the mathematics that should be taught in schools. One group is saying that the traditional mathematics curriculum, the curriculum most of us experienced, is not working for most children, and never has. Another group is saying that abandoning the traditional curriculum, a curriculum that can best be described as a focus on developing computational and symbolic manipulation skills, will be the downfall of our educational system.
>
> This debate crystallized in the recent action of the California State Board of Education to adopt Mathematics Standards that support a computationally driven curriculum, an adoption supported by many mathematicians and parents in California. The immediate and strong negative reaction from the State Superintendent of Education, the California Mathematics Council, the President of the National Council of Teachers of Mathematics, the Assistant Director of the National Science Foundation, and countless others is but the latest manifestation of what has become known by many as "The math wars in California."

Aspects of behaviourism appear inappropriate to the development of active participants in the workplace; however it is clear that some dexterity with mathematical tools is required for a mathematician.

Other theories

Activity theory (Engeström, 1987) looks at the whole societal, educational and personal situation and considers many influences upon a learner. This theory has contributed much to educational literature. As an example, FitzSimons (2005) has used activity theory as a theoretical framework for informing developers of technology-mediated education for undergraduates. Organisational learning theories (Senge et al., 1999; Clegg, Hardy & Nord, 1996) describe how organisations learn and develop. Organisational theory has not been used as yet in mathematics education literature. These theories are not appropriate for this study as I am concentrating on particular learning within the individual and only tangentially on the organisation. Learning through work is another area of study with a range of theoretical underpinnings (Boud & Carrick, 1999). These studies consider the interplay between work, learning and the individual. One particularly interesting idea is that skills learnt at work can spill over to improve workers' personal lives.

Communities of Practice

The theory that is most relevant to this study is an aspect of Wenger's *Communities of Practice* (Wenger, 1998), in particular the idea of *identity*. Burton (2004) used Wenger's criteria to show that academic mathematicians form a community of practice. In this study I will consider how doing a mathematics degree has developed a graduate's sense of identity.

> *Identity*: a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities. (Wenger, 1998, p. 5)

The pedagogical implication of this is the need to create learning and teaching practices which will enhance students' development of a mathematical identity within their work. This connects with the higher level conceptions of mathematics as 'life' and the high level of discourse in Fairclough's model. The aim of a mathematics degree is to develop graduates who will be mathematicians and be able to successfully function as a mathematician in whatever workplace or society they wish to be part of.

Conclusion

The starting point for much of the mathematics communication and mathematics education literature is a focus on details. Consideration is given to language features of mathematics such as words, syntax or features of a student such as learning style or gender. There are many articles on specific features of mathematics or of learners. From the viewpoint of classification, I would put these articles at an *external extrinsic* level of conception (Petocz & Reid, 2001) or the first level of conception from Fairclough (1992, p. 73). Recent studies on the transition process are moving to consider the graduate, or professional in the work situation, Level 3 in the model. The development of an identity as a mathematical person is a way of examining the transition process to becoming a professional.

In the area of science, Florence and Yore (2004, p. 637) investigated a program of assisting novice scientists to learn to write for publication: "The novice scientists came to appreciate that the writing, editing, and revising process influenced the quality of the science as well as the writing." This is to the core of my research: learning to write clearly improves the discipline as well as the writing. A successful professional needs the Level 3 discourse skills as well as level 3 discipline skills, and needs to be able to integrate and switch between these depending on the situation.

Previous findings (Morgan, 1998) in the secondary curriculum show that the aims are to teach the mathematics through the language or teach language through the mathematics, but not teach the discourse of mathematics itself. In a professional situation, there is a third way: integration between the discipline and the language, where neither is privileged. This intertwining between discourse and discipline is not done explicitly at secondary level and rarely at university. In ignoring these possibilities, I believe that curriculum designers are missing an opportunity to provide graduates and students with the skills necessary to function well in a mathematical workplace. They are also missing out on the highest level skills of integrating discourse with their discipline.

The idea of developing a mathematical identity is interesting in following the narrative of new graduates.

Summary of Chapter 3

The transition to the workforce is an important area for research. Perceptions of the workforce and expectations about working as a professional undoubtedly influence students' learning whilst at university. The graduate skills and attitudes developed have the possibility of being transferable across discipline boundaries. The evidence that this perception of future work influences a student's learning is mounting. The evidence that lack of knowledge of future work possibilities can adversely influence subject and degree choice is also clear.

Current research on tertiary education is looking at the development of graduate skills and attributes, features of successful graduates and of unemployed graduates, how teaching and learning influence the development of graduate attributes, and conversely how the concept of work influences learning. However, the collection of data from graduates is difficult (and expensive).

In the next chapter, I describe the study design and introduce the participants.

4 STUDY DESIGN

The study uses two forms of data, supplemented by evidence from published sources.

1. Interviews with graduates, analysed using phenomenographic methods

2. Samples of mathematical texts, investigated using discourse analysis

The participants

The aim was to interview mathematics graduates who were working in diverse jobs, preferably in business and industry. Participation in the study was by invitation, issued through two university databases of graduates (University of Technology, Sydney and Macquarie University), the email listing of the Society of Young Statisticians and through word of mouth. Recent graduates who had completed their mathematics study in the five years 1999-2003 were the focus of the study. It would have been possible to interview many graduates who were working in banking, as volunteers had a good network of colleagues; however, I only interviewed those from different banks with different job descriptions.

Ethics clearance was obtained from the ethics committees of the University of Technology, Sydney and Macquarie University (Appendix B). There were issues raised about confidentiality of contact details until a graduate had decided to participate in the study; therefore, all initial contact with graduates was through an administrative assistant and the alumni associations of the two universities.

Macquarie University has several degrees that result in mathematics majors (BSc, BComm, BA) and the alumni database did not identify a student's major so it was not possible to select students who had graduated majoring in mathematics. I therefore sent invitations to those who studied mathematics or mathematics-based units in their final year. The 1000 names and contact details supplied by Macquarie University were mixed as to graduation year and degree program so letters of invitation were sent to the first 200, excluding those with international or interstate addresses.

Several letters were returned with incorrect addresses and some of the graduates may have excluded themselves because they did not have a full mathematics major.

The University of Technology, Sydney (UTS) has an email listing of graduates and were able to identify those with particular majors, mainly due to their degree structure. The graduate office sent 224 emails inviting graduates to participate in the interviews and, from their tracking, 84 were opened.

An email invitation to participate was also sent on the Young Statisticians network of about 100 people.

In all, 18 agreed to participate. One person replied from the Young Statisticians email list, five from the Macquarie University mail-out, ten from the UTS email list (however four were interstate or international so not useful for interviews) and six participants were recruited by word of mouth (friend of another volunteer or known by a lecturer). As noted in Chapter 2, the total number of participants is suitable for a qualitative study. In any case, given the difficulty of contacting a sufficient number of graduates, a quantitative study would not have been appropriate.

Attributes of the participants

The variety of the respondents was high. There was a range of employment, work situations, age, ethnic background and academic background. Some graduates had laboured through their degrees with many failures while others had high grades. Some had done well (by their own reckoning) in the workplace; others had struggled. Participants had graduated from five universities and had a range of degrees with mathematics majors. Most of the participants were in their early to mid twenties. There were three people who had studied mathematics as their second degree and one had come to university as a mature-aged student. There were 10 males and 8 females, and 8 spoke languages other than English as their home language (Table 4.1).

The names of the participants have been changed for confidentiality. For the same reason, the names of places of employment have been omitted.

Table 4.1 Descriptions of participants

ID	Sex	Age	Majors	Job description	Work area
Angie	F	Early 30s	Mathematics, Finance	Loan advisor	Banking
Boris	M	Mid 20s	Pure, applied mathematics	Cryptographer	Security research
Christine	F	Late 20s	Pure, applied mathematics	Police constable	Police
David	M	Early 30s	Mathematics, Finance	Dealer, bank's treasury	Banking
Evan	M	Mid 20s	Mathematics, Finance	IT	Banking
Fredrik	M	Mid 30s	Mathematics, Physics	Technical officer	Hospital research
Gavin	M	Mid 20s	Applied mathematics	Climate modelling	University
Heloise	F	Early 20s	Statistics, Operations Research	Logistics analyst	Industrial
James	M	Mid 20s	Mathematics, Finance	Corporate treasury	Insurance
Kay	F	Mid 20s	Statistics	Statistician, tutor	University
Leah	F	Early 50s	Statistics	Clerk	Government
Melanie	F	Late 20s	Applied mathematics	Jazz violinist	Self-employed, Entertainment
Nathan	M	Late 20s	Applied mathematics, IT	IT development	Self-employed, Industry
Paul	M	Mid 20s	Mathematics, Finance	Trading risk management	Banking
Roger	M	Mid 20s	Pure, applied mathematics	Modeller, programmer	Geological survey
Sally	F	Early 20s	Statistics	Statistician	Insurance
Thi	F	Mid 20s	Telecommunications Engineering, mathematics	Loan support	Self-employed, Bank services
William	M	Late 20s	Actuarial	Instructional designer	University

The interviews

Graduates were interviewed at their workplaces, at coffee shops close to their workplaces or on a university campus, that is, a place of their choosing. Graduates were asked basic information about themselves, such as which degree program they had completed and when, and then were asked the questions for the study. The interview questions are in Table 4.2. The interviews were audio taped and transcribed. Each tape took approximately 7 hours to transcribe, due to the technical vocabulary used and the desire for accuracy. The interviews ranged from half an hour to one hour and the transcripts contained between 6 000 and 20 000 words.

The questions

The interview can be described as being in three parts. Firstly were descriptive questions, then an analysis of their work situation and finally participants were asked to reflect on their learning. After general information on their background, each participant was asked to describe the work they do and the way mathematics is used in that work. I did all the interviews and was able to probe the participant to find the level and type of mathematics used. I then moved to analytical questions where graduates first described how they used mathematics to communicate and then analysed the differences between mathematics and other forms of communication. Participants reflected on their learning of communication and the transition process from university.

Table 4.2 Interview questions

Thank you for helping us with our research.
I am with a group of teachers who are interested in finding out graduates' views of how mathematics is used in the workplace, with emphasis on the ways that mathematics is communicated. We would like to see how these views might fit in with your ideas about how you learnt these skills. Our purpose in doing this research is to improve the way we teach mathematics at our universities.
This interview should take about 1 hour. I have a few short questions first about your background (such as what work you are doing), and then I ask you for your opinions.
I will be taping this interview and then transcribing the tape. For the transcription, we will use a pseudonym so the information will be confidential.
First a few questions about your background.
What university did you attend?
What was your degree program?   What was your major?
What year did you finish?   ☐ Full-time ☐ Part-time?

What language do you speak at home?

How would you describe the work that you do? In what ways do you use mathematics in this?

In what ways do you use mathematics to communicate ideas? Examples for prompting if necessary: reading graphs, using formulas in Excel. From others? Produced by you?

Is mathematical communication different to other forms of communication?

How did you learn these communication skills?

In what way has studying mathematics at university level prepared you for work?

What else helped you to make the transition to professional work?

What do you think should be in a university course to help you to make the transition to professional work?

Anything else?

The use of interview

Phenomenography mainly uses interviews to uncover the outcome space of participants' experiences. Here, the participants in the study are professionals who use mathematics in different ways. They each have a perspective to add to the research study; therefore a semi-structured interview process met the objective of gaining maximum information from each participant. The advantage of the semi-structured interview is that the interviewer is in control of the process of obtaining information from the interviewee, but is free to follow new leads as they arise (Bernard, 1988, cited in Partington, 2001). Hitchcock and Hughes (1989, p. 83, cited in Partington, 2001) describe the "semi-structured interview" as one:

> which allows depth to be achieved by providing the opportunity on the part of the interviewer to probe and expand the interviewee's responses. ... Some kind of balance between the interviewer and the interviewee can develop which can provide room for negotiation, discussion, and expansion of the interviewee's responses.

The semi-structured interview process requires the interviewer to create empathy, to listen, to seek clarification (perhaps by restatement) and to persist by probing to elucidate shared meaning. Here is an example of a discussion with David to find shared meaning for the word "transparent".

> David: ... And plus it's very transparent. Something like that is very transparent and it's very readily and easily explainable. Anything that cannot be readily explained and anything that's not transparent is not well accepted by management.
>
> I: Transparent: define what you mean.

David: Transparent. It's just like doing a weighted average, that's very transparent because you can very easily, very, very easily see what it means. You put numbers on paper and very easily show somebody in a very easy example as to how the whole process works. Quite honestly coming up with some of the stuff, I mean some of the stuff is readily available on Excel, some of the statistical functions that are readily available on Excel that I've used at uni, just descriptive statistics, they're not well understood by people. People, they just can't, they can't visualise it, they need to be able to visualise it to understand it.

I: So transparent is kind of like if you are able to explain something that your manager understands, that's transparent.

David: That's right. And he's an intelligent man, but he's not, he doesn't have, his background's not quantitative, but he's quite comfortable with quantitative ideas. But I don't think I'd be able to take him to the […] calculus, that's not his cup of tea.

The circumstances of the interviewees were diverse. There were graduates who had moved from their mathematics degrees into other fields of work. They could answer questions only in general whereas those using extensive mathematics in their profession needed to be probed differently. Here are three examples of questioning for different circumstances. Firstly, asking Paul about the use of graphs for mathematical communication:

I: What about graphs?

Paul: No. In the most recent role at the […] bank, there was a plan that [if] you ever handed across numbers it's only ever graphs basically because you can't sell, people, if they are not maths literate, want to see a picture. Probably at […] doing structured finance, never did any graphs. Where I am, very occasionally if I want to pretty up a report or if it's going to say, Treasury's level or something then, yeah, but apart from that, no.

I. But you don't use graphs at all now because people understand the maths?

Paul. Well people are more interested in the numbers than anything else.

Here is an example talking to Thi about the use of graphs. Because she sells financial services (mainly loans), we were discussing how she communicates with her clients:

I: OK And what about graphs, do you show them graphs?

Thi: We have the ability, we have the ability to show them graphs because there's a lot of online material that you can use that is already available on bank sites and any other finance sites. It shows their payments on how it is reduced, and, if you pay this much per month this is how you can reduce your payments and this is the amount of money that you can save by the end of thirty years or twenty-five years.

I: And how do they find that?

Thi: How do they find that? They like the graphical representation more than the…
just the numerical.

Whereas it would be inappropriate to ask Leah, who answers the following question in the way she has, how she uses graphs in her work:

I: Alright. So in what ways do you use your maths or stats or…?

Leah: Bugger all!

Face-to-face interviews are the best way to collect high-quality data (Mathers, Fox & Hunn, 2002, p. 3). They allow for establishing rapport, more probing and prompting and ways to follow up ideas that are difficult for other methods, such as questionnaires. Because the area of mathematical communication in the workplace is not well recognised, I believed that participants would not have thought deeply about the ideas and would need to be probed. This was borne out in the interviews, for example with Heloise:

I: So, how did you learn to talk differently to different people?

Heloise: I don't know. I just kind of picked it up. Mainly I'd respond by the way they spoke to me. If they speak analytically to me… with my manager, sometimes he's even too analytical for me, I just go, 'what? Run that past me again'. So, I know that I can speak like that to him, type thing, and then again with the sales lady and with my director, he's a director, he just wants the bottom line, as all directors want.

I: But how did you figure that out?

Heloise: Just kind of picked it up.

During the interviews, three participants brought examples of texts that demonstrated, in their opinion, good and bad use of mathematical communication. They also brought (or supplied afterwards) examples of texts that they had produced themselves. Part of their interviews includes the participant describing the attributes of good and bad mathematical communication using the mathematical texts as prompts. Other participants were invited to contribute texts but had trouble with workplace confidentiality. I would have liked an example of a tender document (Nathan) but his office would not allow it. Paul was invited to contribute an email text but it was against company policy. I endeavoured to get a wide range of text content and modalities though all were written. Excerpts from the interviews are used as illustrations of spoken texts. There are examples of overhead projector transparencies from a talk, textbooks and articles used in industry. The texts are on the attached compact disc, Appendix C.

Approach to analysis

In this study, using the phenomenographic paradigm, I read the transcripts of the 18 participants as a whole and identified evidence that was emerging from the data according to the variation and differences demonstrated. I was mindful of exploring what made one way of experiencing mathematical communication in the workplace qualitatively different from another. I used the qualitative data analysis program NVivo 2.0. (QSR, 2002) as a tool to assist the analysis. The program allows textual data to be coded and then grouped; for example, one of the ideas that emerged was the influence of the boss and office politics. These groupings can be rearranged and statements can be coded into more than one area. As I worked through the transcripts, I was able to add topics that described aspects of variation as they emerged. I then went back through the earlier transcripts to see if the new aspect was present and then coded them accordingly. Several of the ideas overlapped and may not have formed part of the eventual outcome space, but this was a useful technique to deal with the mass of data. Several of the groupings corresponded with the questions asked, as would be expected.

The interview data supplemented the texts that were supplied by three participants. These were analysed using discourse analysis in relations to the interviews.

Qualitatively different responses

Coding brings together components that can inform the progress towards a description of qualitatively different experiences. Considering all the responses, I then looked for qualitative differences with a view to developing an outcome space that described the nature of the differences of graduates' experiences of communicating mathematics in their workplaces and their experiences of learning mathematical communication.

Summary of each transcript

For a holistic view, I then went back to each transcript and considered its contribution to the outcome space and wrote a brief summary for each. The summary enabled me to consider the important components of an individual's experiences whilst facilitating a global understanding of the variation found within the group.

Quotations from participants

In phenomenography, quotations from participants are used to describe and illustrate the outcome space and develop the explanatory arguments. The quotes selected are verbatim with some deletion

of "ums" and repetitions. Spoken English is full of grammatical errors and these have generally not been corrected. Quotes have been shortened by omitting sections that are not relevant to the particular context– these are shown by the use of an ellipsis. At all times care has been taken to minimise changes to the sense of what a participant is saying. A few quotes are relevant in different contexts so have been repeated where necessary, or reference is made to the earlier quote.

In the next section I will introduce the participants of the study. As this is a personal study describing how people experience the transition to work, I consider how they themselves describe their work situations. Traditionally phenomenographers consider the participants as a group and this implies a certain homogeneity, at least in the sense that they are all involved in the same thing. In the earliest phenomenographic studies, for instance, the group comprised students in the same class. The phenomenon under exploration in this project is a more complex situation. Whilst the phenomenon is their experience of communicating mathematically in a workplace, the workplaces and the sorts of mathematics they do, can be quite different. For this reason, the individual graduates will be presented so that the reader can 'see' the contextual variation as well as the variation within the group of the phenomenon.

Meet the participants

Ashworth and Lucas (2000) suggest developing a profile of each participant to identify 'central points of focus'. They note that in one study, as well as producing profiles and a set of categories in analysis, they produced a set of themes: "… it was found that the production of (a) individual profiles, (b) themes and (c) categories of description were mutually supportive in providing both an overview and the detail relating to the lifeworlds …" (Ashworth & Lucas, 2000, p. 305) Cope (2002) also considers that the characteristics of the participants should be clearly stated to assist with the validity and generalisability to other contexts.

In this segment, individual profiles of the participants are developed as they describe their work situations in their own words. For confidentiality, references to the names of banks and other places of employment have been removed from the quotes. This begins the voice of the graduates as they unfold their experiences of their transition to professional work.

Angie

Angie works for a major bank preparing documents for housing loans. She has an honours degree in Mathematics and Finance and has a previous qualification and experience in nursing. This is how she describes her work:

> My work is not challenging. It's basically administration. I prepare documents for security. I have learnt how business loans work but it doesn't challenge me in a way that I am prepared, educationally, to be challenged. I liaise with the customer managers and the solicitors. I prepare settlements. I draw down loans. I organize registration [of the loans].

Boris

Boris studied engineering for his first year at university, switched to mathematics and completed an honours degree in applied mathematics. His role is to work with engineers and theoreticians to produce security tools for the market. His is a theoretical job where he needs to be up to date with the latest mathematics. Boris has a speech impediment and English is his second language so, as interviewer, I often repeated what he had said to be sure that I had the correct meaning and so that it would be easier to transcribe the tape afterwards.

> I'm actually doing … cipher design, to be able to design that, you need the mathematical theory to prove security bounds.

Christine

Christine completed a degree with a major in mathematics. She became a police officer and furthered her study with training at the Police Academy. She is extremely proud of having completed her mathematics degree and believes that it gives her a different way to look at a situation, for example, *one time someone gave three letters and a number, and to me that sounded like a number plate, a car registration.* Here is her description of her job:

> I'm a police officer, currently attached to … Transit Police, meaning that I travel the trains proactively, seeing, talking to people, um, asking particulars when there's people, arresting people who have committed offences, and, in the broader sense, um, oh it's only a secondment to transit but also just general duties policing, responding to jobs that come over. It could be anything from a noise complaint to an assault that's happening, just responding, going and talking to whoever's made the complaint and trying to do something about it.

David

David studied mathematics and finance as a part-time student. He works in the quantitative section of a large bank. During his studies he worked in customer service in a bank (basically as a teller) but

still found the transition to professional work a revelation: *Going through university definitely gave me a very different perspective on what I did in the workforce.* Here is David's description of his job:

> I work as a dealer in group treasury. OK, well, what we're responsible for is, we're responsible for basically finding funding for the bank, and our purposes are finding funding in the short-term wholesale markets for the bank. That involves basically going and borrowing money for the bank in the markets with maturities of less than one year. In a nutshell.

Evan

Evan studied mathematics and finance and is studying a Master's degree in Finance. He works with the IT section of a bank to implement technical solutions for the bank. He is highly competitive and has progressed up the ladder in a large bank since finishing his bachelor's degree. He works long hours and stresses the importance of, *getting something right, being interrupted every 5 minutes, and having a deadline.* Here is his work description:

> I write trading software for the equity markets and equity derivatives business at … bank. We primarily take care of their risk management systems, their derivative pricing systems and, to a lesser extent, their general IT infrastructure. We're dedicated to the equity markets business; we're split up into account teams round the bank such that we have no other affiliation with any other part of the bank except for the business with which we work with. Our strength is not necessarily in IT, but in the business line that we have, as in we're trained mostly to work for the IT side of it. And we're a team of about 15 people, with relatively interchangeable skill sets across the board. That's pretty much all of it.

Fredrik

Fredrik studied a Science degree with majors in mathematics and physics. He was a mature-aged student who previously had a career as a musician. This was his second job since finishing his degree. The first was in research and development, however, the company went into receivership and he was retrenched. He is working in research support at a large teaching hospital and has considerable freedom to develop his own designs. He is not as happy in his present employment, which he sees as at a lower level to the previous job:

> I design circuits and do modification for instruments, and design instruments, and what else, do software or put together software to drive those instruments, um and, there's just a lot of stuff that I do. Take care of the computing, all the computers for my research group as well, I do ordering, purchasing, for whatever it is that the group requires in terms of equipment that the researchers need. I also help the postgrads doing their research, and all that kind of stuff. They come to me for technical assistance and advice

Gavin

Gavin is a PhD candidate after completing a mathematics degree with honours. He is tutoring mathematics part-time through his studies. Here is his description of his work:

> I'm doing a PhD, and I'm now, I'm just starting my third year, and up until I guess a year and a half through the PhD I was teaching maths as well, doing tutes, and working in the numeracy centre, but these days I've stopped doing that just for time reasons. My PhD is sort of vaguely maths related, it's working with a climate modelling group on, what they call data assimilation, which is ways of incorporating observations, real observations to better kind of models, I guess, so, using neural networks, using multi-dimensional calculus stuff, using, so it's sort of applied maths really. But a lot of other stuff that's not really maths, that's sort of, plant physiology has to get in there as well and, that kind of stuff.

Heloise

Heloise is a new young graduate in her first professional job. She has a Bachelor of Science degree with a major in operations research and a submajor in finance. She worked hard to obtain this job in logistics. Here is how she describes her job:

> My position is a logistics analyst which involves a lot of monthly reporting and repetitive stuff which involves… 'cause I work for a gas company and I'm in the supply division so we supply the whole of Australia, all of our branches, with gas. We import it and distribute it to all of our branches and our customers. So I keep an eye on stock levels within all of the branches and, 'cause we actually have an underground facility which holds sixty thousand tonnes of gas, so we … keep an eye on what's happening there and all of our branches. The other part of the monthly stuff that I do is pricing, set all the pricing forecasts for, not only our customers but within our company, pricing for the company. I've only been there for seven months and I've just gotten my head around how all of my monthly stuff works. The rest of what I do is sort of a work in progress. They've got models set up and developed and they want me to learn more about the models, to either improve them, change them and things like that. I haven't really gotten into all of that yet 'cause we're taking it, sort of, one step at a time.

James

James studied mathematics and finance as a part-time student. He previously completed a course with the Securities Institute and realised he needed a degree to advance his career. He works for a large insurance company in their bank section. Here is his work description:

I work in a bank treasury. So I work at … and we have a credit business that writes mortgages and we do the funding and the interest rate, risk management for that. So, yeah, in a bank treasury, so it's very financial, obviously very quantitative. … So, any particular day, the retail business, the people out there selling the mortgages etcetera, we get the cash flow from them. So, on a particular day they might have sold five or so many million dollars of mortgages or 300 000. The risk of that depends on whether it is a fixed rate mortgage or a variable rate mortgage, obviously the amounts of it. … One of the longer term, as time goes on as we build up these portfolios of mortgages. They get up to quite a sizeable amount we actually do mortgage secularization now where we pull those mortgages into a pool of assets that investors like to buy.

Kay

Kay completed a Science degree with honours in statistics and is studying for a Master's degree in Science. Before her present job, she worked as a casual tutor and started her own consulting firm. She is enjoying herself in the present position. Here is how she describes her work:

I'm an associate lecturer in biology. My job is split into, I guess, teaching and research and then also contributions to the faculty and the university, which is in the form of committees and things like that and doing talks for students and whatever else needs doing in terms of recruitment and promotion for the faculty. So teaching during semester takes up the majority of my time. So that involves a unit of study coordination, lecturing, preparing material, preparing solutions, briefing casual teachers, securing casual teachers, all that sort of stuff, and research is, I guess setting down roads of interest and just getting things that you're interested in and presenting at conferences and posters and doing journal articles.

Leah

Leah returned to university after several years' work in the IT industry and has recently graduated with a major in statistics. Her previous qualification was in IT and she wanted to change to a field where the content was more stable. She is very disappointed with the position she has been able to attain as she feels she is not using her skills:

At the moment I am working in an area called dispatch and collection control which is mainly about making sure that surveys get dispatched on time and in a hundred percent correct manner so that there's no chance of anybody getting a form that doesn't belong to them. There are a couple of surveys where you get into the survey in one quarter and some information that you provide in that quarter is actually relayed back to you in the next quarter. So, you know, it's crucial that we don't swap things around and it's … my role tends to involve, sort of, print procurement and at the moment it's involving a little bit of investigation of doing cost-benefit analyses of whether or not we are going to switch to using a different size envelope on the basis that that will reduce our postage costs. It's a very cost-sensitive area so a lot of my, a lot of the focus is on efficiency at the moment.

Melanie

Melanie studied mathematics (BSc Hons) and completed a PhD. She plays jazz and works part-time as a mathematics tutor.

> I'm a small-business owner, or partner in a small business, playing jazz music for weddings! So it has nothing to do with maths, now […] cos after my PhD I did a 2 year um, diploma in jazz performance, now I'm a jazz musician.

Nathan

Nathan studied mathematics and IT and was employed as a web programmer straight from university. With a downturn in the industry he was retrenched and is now working freelance. He works for his old company as a contractor as well as having his own clients.

> My role's a bit diverse. I don't do as much programming anymore, I do a lot more account management and training of our clients, as well, so I've steered away from me doing as much programming although I still do a fair bit and often the new guys that come on board I'll sort of look after them and steer projects that they're involved with.

Paul

Paul has an honours degree in Mathematics and Finance and works for a bank hedging risk in a similar fashion to James.

> First thing in the morning three days a week I need to give, my part of where I track the risks is for the treasury, so the people managing [the bank's] balance sheet. First thing, say 7.30, I'll get in 3 days per week and provide them with a hedged risk position so [the bank] has assets and liabilities and equity. Some of that you can hedge using swaps and things and you can change your exposure to interest rates on that. So these guys basically try making money out of changing [the bank's] balance sheets exposure and I track the risk of their activity. … Apart from that, some of it is the sort of mundane, 'can you look into this?' and you go and play around in a database and try and reconcile some numbers or something like that. Some of it is research, say, at the moment I'm writing a paper on modelling prepayments which is for our NZ operations, some of it is regression modelling basically because if you are going to track someone's risk you've got to have a model of how something behaves first so a lot of it is we've just gone through a process of updating our regression models.

Roger

Roger gained a Bachelors degree in pure mathematics with honours. He works for a mining company in the research and development section. He had never done any computing before but was confident he could do it (and he did).

Well, we worked in the geophysics department, and so, what they did is, they wished to prospect for minerals but without digging, so they now they just wanted to put machines and instruments on an aeroplane and fly over the area taking measurements and from this data deduce what if anything was there. So they would have instruments to measure magnetic field, and obviously iron, or metallic, comes forward, give little blips in the data so if you could get, extract that, then you would have a really good indication that there was some metallic object under the surface. And so then they would go and prospect further. So basically it was a statistical thing where you would take this data, and then remove all effects, such as the plane flying through the atmosphere obviously is buffeted so, the data will be a bit skewed by that, so you have to subtract all these factors, and be left only with the pure data. And also subtract the surrounding rock and everything. So it was a statistical thing and a programming task as well, because they had this data, and then you had to write your own software for it because they didn't have any, obviously, like that.

Sally

Sally studied a Bachelor of Science degree with a major in statistics. She is excited to have the job she has now, her first professional job since finishing university. She had been there for 2 months at the time of the interview. She is working in a small team and they look at accident and claims data for an insurance company. Because she has been there a short time, she is doing the tedious work for her boss. This involves statistical work on data such as comparing car accidents with anti-lock and non-anti-lock braking systems in wet conditions. Sally uses Excel mainly and is doing a SAS (statistical software) course soon. Sally has had to test distributions to check for normality and so on. She spends much of the day in front of the computer, which is something she did not want to do when planning her career.

Thi

Thi studied telecommunications engineering and mathematics. She did very well in her studies but was unable to obtain a suitable job due to the downturn in the telecommunications industry. She is now using her mathematics and has set up her own company selling mortgages. She is very ambitious and sets her own goals.

I'm actually in finance now. I have my own business. It's property and finance. The business name is …. I've had it for a year and a few months now. It's been running fine. The components that are technical would be the analysis of loans. … I know how to manipulate Excel to work out everything for me. So I can work backwards and get an equation and put it in Excel and get it to work everything for me from then on so I wouldn't have to use the equation again and again and again. Whereas, in finance, they'd just do that, by hand, on a calculator, over and over again. I've been able to create these databases that work everything out and to the point that when I meet competitors (other people that do the same as me) they've asked me to give them quotes to create the same databases and the same Excel spreadsheets for them.

William

William studied actuarial science and had a cadetship. He found it difficult to fit into the office environment and he then trained as a teacher of mathematics. He is now designing educational software (courseware). Here is his work description:

I am assisting in the creation of an online graduate diploma of information technology for release at [a Sydney] University in 2005. I'm doing curriculum development and platform development in terms of what software and tools are going to be used to teach this subject, probably subjects, and also a lot of pedagogical development, thinking about how … the best ways to teach humans online.

Overview of participants' work situations

Two graduates in this study, Christine and Melanie, had moved by choice from their mathematics background into different areas (though Melanie has since obtained a full-time job as an associate lecturer in mathematics and is now a part-time musician). William moved from his actuarial science background because he did not fit with the work environment. Leah is working in a job that does not directly use her statistical skills and is having difficult finding appropriate employment.

David, James and Paul all work for different banks (or banking sections of insurance) and have similar job descriptions. They do not know each other. The level of mathematics, finance and computing required is medium to high. Good communication skills are required as they all work with teams of diverse backgrounds. Evan also works in a bank providing IT services and requires a similar level of skills in these areas.

Angie and Thi are both working in the banking sector in mortgage applications and approvals; Angie for a large bank doing documentation and Thi for herself. Angie requires low-level mathematical skills, good organisational skills and very little communication as she does not deal with customers and works mainly by herself. Thi requires low-level mathematics skills, high-level

skills in computing tools and very high-level business skills to successfully run her own business. Her communication skills need to be high to liaise with employees and clients.

Melanie, Nathan and Thi all set up their own businesses, either by choice (Melanie), because of retrenchment (Nathan) or inability to find a suitable position (Thi). Kay had also set up her own company before her present job. Given the industrial relations climate in Australia (and internationally), the number of self-employed is likely to increase. Other participants were in positions where they had to report to "the boss" and this influenced their discourse significantly. They saw themselves in relation to those they had to report to. However, Kay and Fredrik talked about their "colleagues'" and gave the impression that they were equal team members even though they both had junior positions.

Boris and Roger worked in industrial research and development. Of these, the highest level of mathematics use was Boris, who regularly searched academic literature for new developments. He was a consumer, not producer, of academic texts. Roger believed that he was only using second-year university level mathematics and he demonstrated high-level communication skills. Sally could also be considered to be working in research and development in insurance, using statistics to test hypotheses about risk in insurance. She was very new in the position and was adjusting to her role by attending training courses and depending on her university textbooks. Gavin used high-level mathematical modelling but saw himself as a student rather than a professional. Fredrik worked in research support and was able to develop his own ideas and techniques. The skills required appeared high in terms of communication and technical skills.

Heloise was another new graduate in her first position using her Operations Research major. She demonstrated high-level computing and analytic skills but was yet to use her mathematical skills, though she could see that she would be able to use them in the future. Her communication skills were naïve.

Conclusion

These participants cannot be considered as a 'community of practice' (Wenger, 1998) in the way that academic mathematicians can be so considered (Burton, 2004). In fact it is difficult to see if there is anything that ties this group together other than mathematics study at university. Was it perhaps the academic workplaces that defined the community of practice in Burton (2004), rather than the mathematics? I will return to this idea in later chapters.

Phenomenography and discourse analysis provide a way to investigate the transition to the professional workplace by looking at workplace practice (discourse analysis) and participant's reflections on the experience of that practice (phenomenography).

Summary of chapter 4

In this chapter, I described how the transition to professional work will be investigated using two methodologies, phenomenography and discourse analysis, supplemented by data from published sources. Eighteen participants in diverse situations were interviewed in depth using a semi-structured interview protocol. I described the interview process, and introduced the participants and their work situations, using their own words.

In the next chapter, I will use a narrative style to illustrate the graduates' reflections on the start to their careers.

5 GRADUATES' EXPERIENCES: A NARRATIVE

I am investigating how mathematics graduates move from their mathematical studies into the workforce using mathematical communication as a way of examining that transition. This chapter follows the process of acquiring a job, adjusting to the work situation and considers some of the discourse concerns that graduates have identified. It unfolds in a narrative form following the graduates' stories.

They have taken their learning experiences at university and it has changed their way of looking at the world – their studies have mostly found them a 'foot in the door', a start to their careers. Their jobs have given them a different perspective again. Their comments here are set within their experiences of employment.

Getting a job

The experience of the graduates shows that the transition to work is difficult. Each graduate has a story and it is valuable to consider a few accounts here to appreciate the experience of the process of moving to work:

> Nathan: I pretty much got a job straight out of uni, full-time and I started as a programmer for a web-development company and after about fourteen months, or so, a bit over a year, the market sort of did a bit of a downturn and the company retrenched nearly half its staff and I was one of those, so I was out of a job for about three months or so and then they took me back as a contractor so I started just working for them maybe ten to twenty hours a week …

> Heloise: When you do go into interviews you gotta sell the degree because … especially talking just with an agency who has no knowledge of maths and how far you can go with it, you've gotta really prove to them what you can do with it. Because it's not like an accounting degree where, 'OK, you're an accountant'. It's not as straightforward as that. You've gotta say, 'well, with what I've done, I can do this, I can do that, I can do that'. And then they'll go … you'll see how people just go, 'Really? You can do that?' People don't realize how much you can do with a maths degree.

Thi: … there's not that much work out there and when I came out, in 2002, the telecommunications industry was hitting rock-bottom, was hitting absolute rock-bottom, that is why I'm where I am now because of the lack of work.. … One of my goals in life was to start my own business, anyway. It just accelerated the choice and it's a huge risk.

Sally: I knew what I wanted, and I wasn't gonna settle for anything less so I was always searching and stuff. But I think they will maybe, cos you do feel, you do get down on yourself after like, a few months without getting any work and you're at home, and you know, and you're like, ooh.

Angie: I've got the skills. Given the opportunity I'm more than capable of doing the job. However, having the piece of paper itself hasn't helped me get a job. So, that's a negative problem. Once I get the job I've got plenty of theory and knowledge and skills to go ahead and do it but it has been a very real hurdle trying to get the position.

Leah: I don't think that my studies prepared me for a subsequent career in statistics at all well. As a matter of fact I think it was very poor and I think it had … My getting this job here had a lot more to do with me just using my initiative than having anything, than applying anything I've learnt at university. … It's actually really easy to see areas in industry where you think you could make a difference, it's extraordinarily difficult to get past that recruitment filter and to actually find yourself in one of those jobs.

So some graduates fell into work but had to reappraise their situations due to changes in the workforce, others were struggling at the entry stage to get a job that they felt was commensurate with their knowledge. They believed that they had skills and knowledge that would make a difference in industry – and could see how to apply their knowledge – but could not get into those positions where they could make a difference. There is a clear need for training about applying for jobs, as Leah suggests: *I think it would be humane for somebody … to have some kind of in-your-face training … I don't know if training's the word, just exposure to what happens when you go for a job.*

Initial work experiences

Getting the job is one objective, adapting to the situation is another. Graduates had a range of initial work experiences which were often mediated by the people that they worked with, particularly their managers:

Heloise: I think the biggest help was the people I work with. When you're comfortable, especially with your manager, the person you report to, you're comfortable with that person, it makes a big difference, I think. I just … I've taken a step back and had a look sometimes and, if I'd had a different manager I don't think I could have gotten as far as I have and I've only been there seven months.

William: I ended up leaving after a year … when I sort of started my cadetship because that was when I first entered an office and worked in an office and I'd expected there to be much more … I wasn't prepared for the environment.

Paul: When you first come out of uni and you try to explain, you're really keen, oh I've got this great idea and here, blah blah blah, and your boss just looks at you blankly and says that's too technical, I don't understand.

Evan: The underlying thing of coming out of uni is not … you're not going to be CEO in 3 years' time! It's something we had to beat out of me in the first 3 months! I guess one of the main things is the responsibility is just given very much in piecemeal, and built up over time … you might have got HDs in everything, but you've still gotta prove yourself in a particular context. And that's not uni any more.

Initial work experiences, particularly a graduates' relationships with their manager and their workmates, have a strong influence on their transition to the workforce.

Use of mathematics

How do these graduates use mathematics in their early careers? The responses range from 'bugger all' through 'the most simple mathematics equations' to sophisticated mathematical modelling. All graduates felt that they knew more mathematics than was required for their positions, though most used their textbooks when they had forgotten details of the mathematics or needed to develop new work.

Christine: I can tell you how many wheels there are in a train, for example. It's mostly just the way that I look at it … because I'm a logical person and the logic applies in the work that I do. As far as mathematics itself is concerned, I'm not using my degree in any way, shape or form.

Leah: It [statistics] just gives you a lot of points of view to start looking at problems, whereas statistics is very much about, 'Who says?' 'Where's the evidence?' "What are you basing that on?', 'Is there a real trend?', 'Is it just a one-off?', 'Back it up'.

Roger: I think the hardest thing that I ever had to actually use, in my job was second year maths, these Riemann sums, that was about the hardest. But it really helped to know a lot more, because you can more freely work with the things.

James: …When I have a particular project or something I like to go and refresh my memory on it things around, for example, I had to use the t-distribution the other week and I wanted to go back and read and make sure I was using the correct distribution. I should have been using chi-squares, in my head I went this is what I should use, I just wanted to check.

Paul: A couple of weeks ago I was given a new swaption, this weird derivative to price, and how do you do it? And I kind of sat down and had a think about it and worked through, pulled out an old derivatives securities textbook and worked through it like a problem, that was it.

Gavin: [I'm] working with a climate modelling group on what they call data assimilation, which is ways of incorporating observations, real observations to better kind of models, I guess, so, using neural networks, using multi-dimensional calculus stuff, using, so it's sort of applied maths really. But a lot of other stuff that's not really maths, that's sort of, plant physiology has to get in there as well and, that kind of stuff.

Boris: I'm actually doing … cipher design, to be able to design that, you need the mathematical theory to prove security bounds.

These graduates are using mathematics in different ways. The ones who are not explicitly using the mathematical procedures that they learnt at university nevertheless believe that they have taken on the characteristics of a 'mathematical person', such as logical thinking and being more aware of numerical and logical situations around them. They have a 'mathematical identity'.

Working as a mathematician

Many mathematics graduates are isolated in their workplaces in that they are the only ones with a mathematical background. There is a degree of loneliness with being the sole member of your work team who is the mathematician. Often you are unable to discuss details with any of your colleagues or supervisors. Other issues about working as a mathematician emerged in that their bosses may be demanding results that are simply not possible from the mathematics or the data available. Roger, in particular, expressed serious frustration with the way mathematics was used in his workplace and had to change his ideas of working as a mathematician. He was incredulous about the demands made on him (and mathematics) and the way he was expected to work. Both Evan and Roger talk about having to change your ideas about mathematics and how it is used in the real world, to relax your assumptions. Evan says: *this is theory and this is what really happens.* He is making connections between the perfect mathematics and financial theory he has learnt in the classroom and the reality of dealing with real financial situations.

James: I was really fortunate up until about 12 months ago I had a guy who did honours degree in stats and so that was great I used to bounce ideas off each other quite a lot and became good friends so probably no one else in that I work with in my work area has a maths degree …

Gavin: None of my supervisors are particularly well informed in that respect, which makes it quite difficult sometimes, because a lot of what I'm doing, probably most of it, isn't really supervised I guess. I'm working by myself …

Roger: So, basically, there were many absurdities of this nature where they expected you from minimal data to extract more information than the data could provide. So that was quite irritating. And there was no stopping them demanding it. If they think that they're paying you enough, and that's it.

David: [Talking about his boss] That's right. And he's an intelligent man, but he's not, he doesn't have, his background's not quantitative, but he's quite comfortable with quantitative ideas. But I don't think I'd be able to take him to the […] calculus, that's not his cup of tea.

Roger: One problem for people with pure maths, well that I certainly had, is it is quite a shock to them in the real world to see how maths is used in this strange way where assumptions are made left and right, whereas in pure maths one dare not. But it is very important to accept that, if you are going to go and work in the real world so to speak, one has to be able to just allow oneself to make assumptions and so on, and even though it is completely against what one is trained to do, in pure maths, which is everything is to be verified, perfectly, so you must then get rid of that idea.

Evan: Maths has a whole heap of simplifying assumptions that say, you know, this is what you've, these are all your assumptions, and they're all fixed or they're all ah, perfect-world constraints, but then you have a real American option, that has dividends, that you know, has a share split, on a real stock. And so it's taking those compromises into the real world, and I guess seeing all these pure maths, when I say pure maths I just mean not tainted by finance! All these pure maths and saying, OK, now these are the sort of things that we need to relax in a real-world context, and these are the sort of ah, this is theory and this is what really happens, and that's one of the things that you learn as a, yeah, absolutely correct, but, that's not the way it's going to happen. And so that's very important to get that understanding.

Paul: I work in a very mathematical area, basically everyone has either a maths or a computer science or an econometrics background. The regression modelling for example, that's all maths, the prepayment modelling again is all maths. I guess what I'm using is the statistics side of what I did at uni and also some of the financial maths that I did so the derivatives pricing and things like that. Coming up there's going to be a project where I'm going to get into the stochastical modelling, basically because rather than using deterministic risk models we're going to use stochastic ones, because all the banks, the way that they are managing risk is changing because of new regulation changes and there's an opportunity there to try and lower the amount of capital we've got to keep aside by managing risk more effectively.

Using mathematics to communicate ideas

This is where interesting differences between graduates emerge. In other parts of the interview, graduates are in familiar territory. They are describing how they got their jobs, their initial

experiences and how they use mathematics. The majority of the graduates had not considered the use of mathematics to communicate ideas – or that they used mathematics to communicate ideas – or that they were communicating at all. Their language becomes more hesitant with additional pauses and fillers.

> Boris: I can always stand up, go to somebody['s] office and start talking about the mathematical ideas when we write on the whiteboard basically.

> Christine: Yeah, um. I dunno if it's, I don't know if it's the degree and the skills I have influencing the way I talk or if it's just the person I am influenced the skills that I got, but um, then, it impacts I guess how I look at a situation.

> Gavin: I'm working by myself, so when I communicate with my supervisors, it's broad brushstrokes.

> Melanie: Yeah, yeah. You've kind of somehow got to convey to somebody who thinks that maths is just like, a couple of, like, numbers and equations maybe, and a tick or a cross, like something's right or wrong, there's nothing mysterious about it, there's nothing unknown, like everything's right or wrong, like it's kind of trying to get over that hurdle and say, OK, well the maths that you know about isn't actually what maths is.

> Heloise: The sales manager, she's from a sales background, so not as analytical as myself and my manager and a lot of times when I talk to her, I just try not to be as mathematical, or as analytical, about something. The good thing is she's very open, she'll just say, 'English please!' 'Cause she knows that we don't mean it, even when my manager talks to, we don't mean … that's just how we talk. She's like, 'hang on, just run that past me again!'

> Boris: No, no. Actually generally I don't talk about mathematics at all with my friends. With exception for one or two friends that is […] all know nothing mathematics. That's actually because they see mathematics as this hard and difficult, and when you talk you almost, it's like a door that is closing, so that's the main reason why I [am] not talking about mathematics.

Mathematical discourse and general communication

The majority of graduates explain the differences between mathematical and other forms of communication in relation to their work situation. For example, Thi explains that you would need to start again if you were too technical with clients. William explains why mathematics itself is different to natural language and Paul describes how he uses layman's terms when explaining mathematical situations. Angie considers how to communicate technical areas, David considers those who understand numbers and those who don't, while Kay compares mathematical language with describing feelings. She also describes how she has attended many seminars and not understood anything – mathematics does require special communication skills.

Thi: Yes, because if you go into too technical an explanation you lose them and that's it, you have to start all over again.

William: Absolutely: because it's symbolic representation; because of its concise and precise meanings; because of its generally quantitative nature; because of the level of complexity involved and the depth of conceptual understanding that is related to, sometimes, one particular symbol or word could be much more than in, sort of, other domains.

Paul. Yeah … I think because, what I find is that the majority of people are not really maths literate to the point of being maths grads, then, as soon as I start talking, as soon as you drop a technical term, whether what you are saying is technical or not, if you drop anything that sounds technical then straight away they close up, 'I don't understand, too hard for me'. I guess that's what I was getting at when explaining thing in layman's terms, removing anything technical so that a kid could understand what you are saying.

Angie: Well yeah because mathematics is quite technical and you need to have the ability to understand it and then the ability to come down to a normal level and write about it normally without being all technical so that other people know what you're talking about. For instance, if we're doing our statistical report, you don't talk about 'the 'p' value was .05' you've got to explain to them in the report what that means, rather than just giving them the output. So there … it takes a bit more of a sophisticated communication approach.

David: It is different. I've worked in an area of the bank before that was more focused on communication, and it does involve a very different set of skills. It was just recently somebody actually said to me, he said, the difference between people who are bankers and the people who work in financial markets is that the people who work in banking don't understand the value of the basis point, for instance. The people who work in financial markets do understand the value of the basis point, the people who work in banking who go out to … large corporations, they'll happily give up five or ten basis points, maybe .05 or .10 of a percent to get a deal but the people who work in the … who work in financial markets, won't, 'cause they know that every basis point might cost them two and a half thousand dollars, they just simply won't give it up.

Kay: Yes, I think it is. I think that communicating any technical or non-average lifestyle thing is different to communicating my feelings, my emotions, that sort of stuff is not knowledge or content based. I think that something that has a difficult concept to grasp, I think, does take a special type of communication and I think that it really takes you understanding that other person and where they are at because if you are communicating a concept or a mathematical idea, you need to know where that other person's knowledge, or understanding, is at before you start communicating that and I think that just the number of mathematical and statistical seminars that I've been to and not understood is testament to the fact that, yes it does require extra communication skills, just to recognise what type of information you are trying to transmit.

Lack of communication skills

While not directly asked, several participants commented on the consequences of poor communication skills in the workplace. Evan, Nathan and Leah described effects on a person, such as getting a job, advancement and keeping the job, of difficulties with communication. Nathan also discussed the problems for the whole organisation when the technical people and management are not able to talk the same language.

> Evan: At work, you'd be out the door quicker than anything, doesn't matter how good you are, if you are not tactful, or don't know how to talk to someone, you can't have a, provide a client-style consultative relationship with, with people you work with, doesn't matter how good you are.
>
> Nathan: Often people in IT, I've found, that are very technically savvy and they're brilliant at what they do, they often don't advance, career-wise, or don't get the opportunities they deserve because they lack certain vital communication skills.
>
> Leah: I've often thought well at least I wasn't one of those poor kids from the western suburbs with atrocious, you know, verbal communication skills and I often wondered what happened to them and I don't know whether … I mean I really do. I wonder what happened to them.

Working as a mathematician and communicating mathematically

Working as a mathematician requires engagement with mathematical discourse. The links between the experience of working as a mathematician and the experience of communicating mathematics is considered in the following tables. I have categorised the participants through evidence provided in their individual interviews as to their use of mathematics (Table 5.1) and computing (Table 5.2). I have based this categorisation on the position descriptions as given by the participants and on their descriptions of their work situations. In addition I have knowledge of their individual work situations as in many cases the interviews were conducted at their workplace, so I was able to observe the tools and texts that they were using.

No mathematics in their work situation

These participants use mathematical communication in a general way. They believe their mathematical education affected their way of dealing with people and situations. In terms of Fairclough, these participants do not construct, interact or explain any texts in terms of their discipline. This is because they are *not* part of the community of mathematicians. Consider here, quotes from Christine and Leah:

> Christine: As far as mathematics itself is concerned, I'm not using my degree in any way, shape or form …

Christine: But um, and yeah, within, as far as talking to people, the questions I ask might be a bit different to something that another person might ask, just follow a different train of thought.

I asked Leah, *So in what ways do you use your maths or stats or…?*

Leah: Bugger all!

When I asked her about mathematical communication she described how the idea of looking for a pattern in statistics had influenced her way of dealing with colleagues:

Leah: This other woman who's really, really nice but she's really attracted to people problems … whereas my whole way of dealing with it is to wait until I see a pattern. If I don't see a pattern … I never get stressed about the occasional dummy-spit or … something can seem really off in terms of the way people behave but if it is a one-off then I just don't engage at all. Unless a pattern emerges I don't, I just don't go there.

Low mathematics level in their work situation

A low level of mathematics in the job corresponds roughly to the use of first year university level mathematics. These participants use mathematical communication to deal with clients and colleagues. They are applying the mathematics but shielding their colleagues/clients from it. They are not constructing mathematical texts or communicating mathematics directly; they are communicating the results of their use of mathematics. In Fredrik's case, he does this by the circuits he has designed and in Thi's case she gets straight to the bottom line.

Fredrik: The maths degree is not um, directly pertinently to what I do. I mean, it helps a lot … just elementary algebra involved in that and circuit topology … the people in my team come to me and say, look, we have an idea for an experiment, this is what we need done, can you put together some kind of an instrument that would do this, and I'd go off and do some research, and come back and say, yeah, this is possible. I put together a prototype, tweak it, you know, get back to them.

Thi: The equations that I'm now grappling are very simple equations.

Thi: [with clients] Yes, so what we'd do is, we'd say, 'basically, it's going to be 5.9% per annum. There's going to be a 30-year term. Your interest payments will be … your interest plus principal will be $50 a week on $100000.'

I: So that's all … that's as far as you'd go with the client?

Thi: Yes.

Medium mathematics level in their work situation

A medium level of mathematics in employment corresponds roughly to the use of second year university mathematics. Participants who use this level of mathematics in their jobs consume

mathematical texts daily. In many cases they also construct mathematical texts within their discipline and communicate mathematically with management, colleagues and clients. In Chapter 7, I will discuss David and a paper written by him for management.

Below are two quotes; one from Evan and one from James. James has worked in the back room in mainly quantitative areas and has now moved to an area where he requires a more diverse range of discourse skills. Evan emphasises oral communication. The main texts he is consuming and constructing with his team are computer programs. Both Evan and James are very concerned with being correct – they feel responsible for the accuracy of their work. In Evan's case this is further emphasised by company policy, as described in the following quote:

> Evan: If you make a mistake, and excuse the French, but it's the fuck-up jar. You make a mistake, and I'm talking, you've released something you shouldn't have released, you got a configuration so people, you put a configuration out and people can't do something, you make a mistake in something which ends up with something else, you're supposed to have done something you didn't, and it's not like every little thing but it's things that really matter: 2 bucks in the jar.

> James: I think now is that if someone comes out with a maths finance degree, they are a specialist in their area or they have got these special skills and at some point they are going to have to talk about what they are doing to someone who is not a specialist … I think it's something for me just doing the quant stuff behind the scenes up until about 12 months ago. That has changed recently, where I'm still doing that, but now having to stand up and present or having to write a paper and send it off to someone else and I always get concerned when I actually do it to make sure that number one, that it's right, my assumptions of what I'm doing and then how do I present it.

High-level mathematics in their work situation

High level of mathematics in employment corresponds roughly to the use of a third year university level mathematics or higher. Participants who used this level of mathematics in their jobs consume mathematical texts daily. In many cases they also construct mathematical texts within their discipline and communicate mathematically with management, colleagues and clients. I will focus on two of these graduates and discuss their texts in Chapter 7.

This study did not consider the effects of gender due to the small numbers of participants; however, a gender divide in the use of computing is hinted at in Table 5.2 and reveals an area for further investigation. High-level computing is an area for expansion in the mathematical sciences (Australian Academy of Sciences, 1996, p. iv).

Table 5.1 Level of mathematics used in the workplace

0 None	1 Low	2 Medium	3 High
University mathematics study not required	First year university level mathematics	Second year	Third year or higher
Christine	Angie	David	Boris
Leah	Fredrik	Evan	Gavin
Melanie	Nathan	Heloise	Kay
William	Thi	James	Paul
		Sally	
		Roger	

Table 5.2 Level of computing used in the workplace

0 General	1 Standard tools (Spreadsheet, database)	2 Specialist (VB, Mathematica, SAS)	3 Programming (various high-level languages)
Christine	Angie	Fredrik	Boris
Leah	David	Gavin	Evan
Melanie	Heloise	Kay	Nathan
	James	Sally	Roger
	Paul	Thi	William

Conclusion

Graduates move into a wide variety of work situations. The participants found it difficult to gain employment, to adapt to the work situations and to find their role as a mathematician. The majority of the participants were the only people in their workplace who have qualifications in mathematics and who could speak the mathematical language. Participants often had to modify their discourse to deal with managers, clients and workmates. The graduates' initial experiences, particularly with their manager, came out as extremely important in how well graduates adjusted to their work situation.

It is clear that, within their workplaces, these graduates have a mathematical identity but do not form a community of practice. They are modifying their practices to fit in with the dominant workplace situation. Communication skills are seen as an important part of preserving the job and progressing in the workplace.

Summary of Chapter 5

This chapter has described graduates' experiences of finding a job, initial experiences, uses of mathematics and working as a mathematician. I have shown how difficult it is to find appropriate work and how the graduates are using mathematics. Lack of communication skills affects the individual in their career and has a detrimental effect on the organisation. The next chapter describes and analyses participants' reflections on learning mathematics and mathematical communication. I examine links between experiences of the work situations and mathematical communication.

6 REFLECTIONS ON COMMUNICATION

Phenomenographical analyses examine how different people experience the same phenomenon – in this case the experience of transition from university to the workplace. In order to illustrate this transition process, I use communication skills because of the power that good communication skills bring to the person, to the organisation and to society.

The participants' experiences of discourse practices do not reflect the same level of internalisation as their descriptions of how they use mathematics given in Chapter 5. These graduates exhibit a range of experiences with discourse practices and with how they learnt that capability. This chapter will discuss these practices then identify an outcome space for graduates' conceptions of mathematical discourse. I also present an outcome space to describe how graduates perceived that they had learnt communication.

Discourse practices in the workplace

Discourse practices are the ways that people use mathematical communication. Here I observe it in the workplace from the viewpoint of the participants. I consider the four macro skills of discourse; speaking, reading, writing and listening. I then show the different aspects of each of these; for example, speaking can be: presenting a formal seminar, teaching your peers about a particular technique, explaining to your client what you (and your mathematical skills) are able to offer them and so on.

Types of discourse identified by the graduates
Speaking

Presenting seminars and discussing new ideas with colleagues. In this case, graduates are in a work situation where there is an exchange of mathematical ideas. The graduates are combining the skills of listening and speaking in a semi-formal situation. In the following quote from Boris the group has been given an article to discuss, so reading skills are implied. Also of interest in the following quote is the reliance on the whiteboard in mathematical discussions:

Boris: We basically have um, each week we have a seminar basically, and actually not a seminar, it's more a discussion group, then somebody just said, I want to talk about something, and you have an article or something that you hand it out beforehand, and it is actually the one that is supposed to be the most prepared. And you just go over talk to the … and we talk about what we don't understand, and how it affects you. But otherwise um, I can, I can always stand up, go to somebody['s] office and start talking about the mathematical ideas when we write on the whiteboard basically.

Instructing team members. Graduates talked about instructing their peers, junior staff or students:

Heloise: Yeah, and then I'll go, 'OK sorry', and then I'll just re-word and then it might be a matter of just getting on to the whiteboard, just jotting a few things down to say, 'OK, say this, this, that, that,' and she'll go, 'oh, OK,' which really helps.

Evan: Yeah, I guess … you've really gotta determine how much you need to let someone know. There's, there's, I guess there's a difference between giving someone all the information they need, and giving someone enough information for them to get what they need to do done. And that's probably the hardest thing to differentiate. You could go through the whole theory of it, but sometimes it's easier just to get them to the end goal.

Talking with colleagues and management. These interviews were done just after a scandal around foreign exchange losses in a major Australian bank and whilst I did not interview anyone from that specific bank, those at other banks were expressing their concern. In particular the risk operations of the banks were under scrutiny. James has implied, in the following quote, that the mathematics person is on the line and must be able to explain what is going on to management:

James: I've just got to be careful when presenting to different people, that people don't, I think, people just turn off if you start talking maths to them. But I think it's good because you can do a bulletproof argument without being subjective you can just quote the facts. … For example if you work in a bank if you look at the issues right at the moment that are around its losses, if you have someone who is an expert in that field, they are going to be a maths person no doubt, I'm sure everyone in management is asking, 'what's going on here?' You need to be able to communicate in general and in jargon.

Sally worked with a range of people and adapted her language for each person:

Sally: You can't expect a mathematician to speak and the other person to understand, 'cause you really need to go back to an average person's level and explain what your thing is.

Paul found that the degree program that combined two discipline areas allowed him to discuss ideas with a wider range of colleagues:

Paul: It was handy that I did the maths finance course …that blend[s] the maths and the finance and that has put me in an advantage to people who either have one or the other in a work environment because I can approach things from both. I mean I can talk both languages so it's handy that way.

Paul also developed the following method to modify his discourse when working with non-technical colleagues. This looks like an excellent way to create an assignment for university students!

Paul: I have a hard time explaining things without being technical but it's probably partly my nature as much as my background. I think that the majority of ideas, if you sit there and think, 'how would I explain this (and this is how I think about it) to my brother,' then usually if you can work out how to explain it then it will work with my work colleagues.

Roger gives a clear description of how he modifies his discourse for mathematicians as against non-mathematicians:

Roger: Well, fortunately there [at my workplace] both of the bosses were PhDs in maths themselves so they understood. So you could just leave everything in the mathematical form, you didn't have to first interpret it back into some hand-waving thing, you could just leave the equations, and then they could understand that. … If we had to give reports to the non-mathematical people, then we would use um, we would do examples, we would have our, all our own equations in the background but then we would certain examples, like: If the mountain looked like that, for example, then this is what it would give. So in this way we would teach by example, almost, with lots of pictures to show the comparison, that was the easiest. It would have to be quite simple. And mostly examples with pictures, no, as few equations as possible for the general people. But for the, for our mathematician bosses then we could just say exactly what it was without trying to interpret it further.

Negotiating and selling ideas. Part of Roger's position is to negotiate with small companies for work. Roger discusses the difficulty of tendering for work where the clients have little idea of what mathematics is capable of doing. He has also some difficulty in estimating the costs of the work. It is clear that he has no idea about accountancy or other work practices and is feeling his way in dealing with clients:

Interviewer: When you work with the small companies, how do you negotiate for the work?

Roger: Oh, well, you must explain to them in general terms, again you have, you cannot explain it mathematically to them, that is not the point. You have to prove to them, or not prove but, somehow convince them that you are capable of doing this. Of course if you have a high-ish maths degree you are OK. An honours and higher I would say, or master's and higher, you can easily convince them that you are capable of it. They will immediately believe you. But you still have to convince them, you still have to inspire them in some way, to believe that you're going to provide something good. The difficult thing is to judge how much money you should get, because they do not realise how difficult it is, because to them mathematics consists of arithmetic, maybe there's an equation with an x in it that must be solved, but that's about as far as, because they obviously only know their own, everything they have done which is school maths, which is nothing in comparison with even third year maths.

So it is difficult, it is not possible to really convince them that things are so much more difficult, you just have to say it. And it really is, perhaps it is good to do, dazzle them with a matrix equation or two, just to show them! That it's not so easy as they thought. So that, but that again is quite easy to flummox them in that way. But I mean, it is important to stress to them that this is not as easy as they think it is, because that's what they often do, they often think you can write down a simple little equation, and then that will solve the problem, whereas in reality it's not an equation but it's a whole method that has to be applied. So it's important to communicate to them that you can solve it, then that you must, that it is actually difficult, it is not school maths, and yes. So you need basically to inspire them that this is really going to be something worthwhile, because it'll take longer than they think usually.

Paul is talking about selling ideas to management and how he has to do it differently to how he would have proved it to a mathematical audience:

Paul: In both my previous roles since uni, one was in structured finance and that one I was the only maths person, everyone else was finance, accounting something along those lines, and weren't maths literate so if I came up with ideas or something like that I'd have to dumb down the way I was explaining it because … if you try and sell an idea which is, like, here's a proof, and then prove something to them they are just going to look at it and go, well, 'so what? It doesn't mean anything to me'. You've got to actually sell the idea as an idea.

Both Paul and Roger, with the use of the words *dumb down* and *hand waving*, are expressing some attributes of being part of a mathematical community, which has its own symbols and discourse and is only open to those in the know. It is almost arrogance. They both feel more comfortable with the mathematics and dealing with those who are mathematicians.

Legislative requirements. Thi needs to work with clients who are not mathematically literate. These clients are borrowing money and Thi wants to be sure that they understand what they are getting into. This is partly due to legislative requirements associated with loans.

Thi: Imparting the knowledge that you already know so that clients feel that what they're getting into is not above their head.

Teaching/explaining. Kay works as an associate lecturer at a university. She teaches statistics to students who are not statistics majors, so they often do not have specialist mathematical knowledge and may be mainly interested in statistics as applied to their subject area. She also works as a consultant statistician with colleagues. She likens the process of consultation to teaching as she tries to involve the clients in the process of making meaning with statistics. She is educating her colleagues about the statistical process:

> Kay: [teaching undergraduates] I'm trying to communicate a new, or a hard topic area, to people that don't want to know about it and are doing it because they have to and I try and be, I guess, related, as much as I can, to their common experience and just build from what they know and also communicate in such a way that they can see the big picture, where it all lies within their discipline that they're pursuing. … Mathematical ideas, I try and break them down into component parts. If I can find something that they all understand I do it from there. So I just keep going back to the simplest. … I'll just bring it back to a point where I feel like I'm getting a response, that they understand that basic concept and then I'll move to the next thing and then the next thing.

> Kay: So colleagues … I actually … I'll do it the same way, pretty much, I just don't make it quite as simple so they don't feel like I'm treating them like a student but basically that's what I do. …I'll advise them and work through with them what's a good analysis approach, why's it a good analysis approach, what assumptions do we need to make and what sort of data do you have and what do you actually want to say from the … Like, that's probably the biggest is actually at the beginning, figure out what they did, why did they do it and what they want to know and half the time they don't even know what they want to know. So, that is the best … I love that 'cause that's just like problem solving right there.

Writing

Informal writing

While the graduates talked to a large extent about oral communication, there were several instances where it was possible to describe their position on written communication as well: for example, [Heloise] *I'll just re-word and then it might be a matter of just getting on to the whiteboard.* This is an instance of informal writing but the whiteboard was an essential component of communication. Boris also talks about going to colleagues' offices and writing on the whiteboard. Informal writing is a powerful element of mathematical communication. As shown earlier, graphs, often demonstrated on computer, are a part of communication with less mathematical colleagues and clients.

Many participants worked with computer programs. They modified, constructed and applied programs. The development was often in a team environment and required planning and an obligation to meet the needs of the organisation and clients. These participants had to convert mathematical and commercial ideas to computing solutions that were implemented in their workplaces.

Formal writing

More formal writing is necessary for reports, writing tenders, responding to tenders and producing academic articles or theses. No graduates talked about grant writing but this is not surprising given their work situations. It was a common requirement in Burton (2004) for academic mathematicians.

Reports (for boss, for client, for management). The following quote from James shows that he struggles with the need for accuracy and writing for the audience. He has thought about how to do this and has worked out a way to present information without too much detail:

> James: Actually I try, it's a hard balance to be honest, for example I wrote a paper on the stability of the portfolio of deposits that we have and it's pure stats as far as standard deviation 99 and 95% confidence intervals for a report paper for example. So I'm using the maths in that but I wouldn't give them the spreadsheet with all the formulas on it. I'd just say 'using data back for the last 18 months I can be 99% confident that the portfolio won't be more than 3 or 5 million dollars in a day'. So I'm using the maths to get my answer that I'm trying to put it in a form where I'm just state that I'm 95% confident of this. If you want the details I'm happy to provide it. So that's, it's a fine balance because you have got to get the information there but you don't kill them with the detail. So I do use the stats to get the results but then how much of that do I use to explain the result? It depends on the audience.

Thi finds that having the technical background is important for her writing, as she is able to work with the technical material:

> Thi: I'm finding that I have an edge over the competition in that I know how to manipulate databases; I know how to create technical documents.

Nathan finds he is writing in a range of genres:

> Nathan: Pretty much once a week I'd write some sort of a … it might be a quote, it might be a tender response, it might be a progress report.

Paul writes reports on the bank's risk position three times a week and has longer-term writing projects to research areas of interest to the bank. This would be more in the form of a report than

an academic journal article. James is moving into a more legal financial area and expressed the need to develop his formal writing skills.

Listening

Participants attended meetings, negotiated with clients and colleagues and received instruction from others, particularly their managers. In both speaking and writing, graduates showed that they were sensitive to the need to consider their audience: *It's usually that glaze-over effect! When they start glazing over it's time to stop!* [Evan]. This implies that they are listening, at least to the body language of their colleagues. Some of the previous quotes show the difficulty of influencing others to hear their message, to make meaning of their mathematical knowledge. Graduates through their mathematical training have learnt to listen, but not to promote themselves or mathematics.

Reading

Graduates read emails, reports, tender documents and academic papers. Depending on their work situation, they move between a variety of formal and informal genres. Several graduates (Paul, David, Sally, James) mentioned reading and working through sections of their university textbooks. Graduates often read reports consisting mainly of numbers or numerical situations (for example David, below). Kay assesses the readability of textbooks from her students' point of view.

> David: There's a difference between being able to memorise a page of numbers, and being able to understand a page of numbers. I mean, for instance we might very well get reports coming past each day from our settlements department and it's really quite important actually just to quickly look at them, look at the pages of the reports and just pick out the amounts that are going to be relevant whilst keeping mental score of them in your head, add them together and work out what your position's going to be for the day.

Conceptions of professional communication

Participants have described how they use mathematical communication but what is the essential variation that makes one description (experience) different from another? The graduates responded to two questions which shed light on this and form the basis of the phenomenographical analysis of variation in their experience of discourse within a workplace. The first is their use of mathematical discourse in the workplace, mainly discussed above, and the second is their ideas about the differences between mathematical discourse and natural language. Using the phenomenographical paradigm, I discuss the responses from each person's entire transcript and use quotes from participants to illustrate the outcome space. The outcome space

relates to the participants' conceptions of discourse when dealing with non-mathematicians (Tables 6.1 and 6.2). When dealing with mathematicians the categories were the same, but the intentions and actions were different.

The analysis of the data resulted in three ordered categories of graduates' conceptions of mathematical communication.

Level 1: Jargon and notation. Mathematical discourse is about using technical vocabulary. When participants explain mathematics to non-mathematicians they avoid technical vocabulary. People who describe this conception see the components of the texts as being the barrier to communication; that to communicate is to change your vocabulary and perhaps your semiotic representation by using a picture or a graph. This is different to when you talk with mathematicians when you do not have to change your vocabulary and can talk using equations.

> Thi: Talking to colleagues we can go into a more technical discussion about our field and they'll understand the lingo. Predominantly, it's the lingo … whereas; if I was to start an in-depth conversation with a client I'd have to explain to them what every word meant first.
>
> Paul: As soon as you drop a technical term, whether what you are saying is technical or not, if you drop anything that sounds technical then straight away they close up, 'I don't understand, too hard for me'.
>
> Angie: Mathematics is quite technical and you need to have the ability to understand it and then the ability to come down to a normal level and write about it normally without being all technical so that other people know what you're talking about.

Level 2: Concepts and thinking. Mathematical discourse is about difficult concepts and about thinking differently, seeing the world in a different way. Mathematical discourse is different to communicating about other areas of one's life. When some graduates explain mathematics to non-mathematicians they 'dumb down' the ideas, whilst others endeavour to work with their colleagues or clients to explain the concepts by careful exposition of the ideas. Some believe that the different thinking acquired by mathematics training makes it impossible to communicate mathematical ideas to those who have not been so trained. Nevertheless, participants who hold this conception may be required to sell or inspire their colleagues or clients to use mathematics. They are not trying to explain the mathematics but the value of the mathematics to their and the clients' organisations. This is where some participants express frustration with their inability to negotiate meaning (see also the section on lack of power later in this chapter).

Kay: I think that communicating any technical or non-average lifestyle thing is different to communicating my feelings, my emotions, my… that sort of stuff is not knowledge or content based. I think that something that has a difficult concept to grasp …

Melanie: You've somehow got to convey to somebody who thinks that maths is just like, a couple of like, numbers and equations maybe, and a tick or a cross, like something's right or wrong, there's nothing mysterious about it, there's nothing unknown, like everything's right or wrong, like it's kind of trying to get over that hurdle and say, OK well the maths that you know about isn't actually what maths is …[you can] use it to build this huge, amazing thing, and you can build anything you want.

Boris: Also the way they look at it and also the way they think about it. That's completely, actually it's more the way they think about it.

Roger: Oh, yes, it is very different. Because most people do not want to know this. I mean it really does take a long time how to think in maths, so it's not surprising that, if you haven't had the training, then you won't be able to.

So it is difficult, it is not possible to really convince them [clients] that things are so much more difficult, you just have to say it. … So it's important to communicate to them that you can solve it, then that you must, that it is actually difficult, it is not school maths, and yes. So you need basically to inspire them that this is really going to be something worthwhile, because it'll take longer than they think usually.

Level 3: Strength. Mathematical discourse is about the power of mathematical knowledge and the ability to communicate it. Those who hold this conception strive to present correct mathematics and put forward objective arguments. They realize that they have powerful mathematical knowledge and thinking, however they must be able to communicate it otherwise their strength is useless. They consider ethics and responsibility. The mathematics person in an organisation has to be able to make arguments that are justified and will help management (or others) make informed decisions.

James: You can do a bullet-proof argument without being subjective you can just quote the facts. … For example if you work in a bank if you look at the issues right at the moment that are around its losses, if you have someone who is an expert in that field, they are going to be a maths person no doubt, I'm sure everyone in management is asking, 'what's going on here?' You need to be able to communicate in general and in jargon.

Actions and intentions

If graduates hold these different conceptions of mathematical discourse, how do they respond in the workplace? The graduates suggest that it depends on whether their audience has a mathematical

background. Table 6.1 summarises the responses from the participants for communication with non-mathematicians. Notice that because the levels are hierarchical, participants who espouse any specific conception talk about lower actions and intentions as well. When communicating with those who have a mathematical background, the conceptions hold but the actions and intentions are different.

For the two lower level conceptions, some participants were in a work situation where it was important for their colleagues to understand the mathematical details even though they modified their use of words and, perhaps, simplified the ideas. For example, Evan needed his team to be working together to produce software tools. He explained more than was essential so that there would be less opportunity for error. For the highest level conception, no participants required their audience to understand the mathematical details. They have confidence in their mathematical knowledge and their ability to communicate it meaningfully in their workplace. They did not expect their audience to understand the mathematics but the audience needs the use of the mathematical knowledge. As David says about his manager:

> That's right. And he's an intelligent man, but he's not, he doesn't have, his background's not quantitative, but he's quite comfortable with quantitative ideas. But I don't think I'd be able to take him to the … calculus, that's not his cup of tea.

I will analyse a detailed example of David's writing in chapter 7 to demonstrate facets of this level of discourse.

There is a qualitative difference between levels 1 and level 2 in both intention and action and again for level 3 (Table 6.2). For level 1, the conception is that mathematical communication with a non-mathematical audience requires removal of jargon and mathematical notation. This is similar to the idea expressed by Hawking (1988), who was advised to leave out all equations because of the effect on the perceived audience. The intention of the mathematician is to bring the audience into the mathematical discourse by simplifying their language and notations. They may attempt to reword and express themselves in different ways (such as writing on a whiteboard). Their belief is that it is the words and images of the discourse that are hindering the communication and that removal, or simplification, will fix this.

Table 6.1 Conceptions of discourse with a non-mathematical audience

Conception	Intention	Action
1. Jargon/ notation	A. To be more efficient and friendly	A. Omit technical terms and equations
	B. To avoid losing the audience	B. Omit technical terms and equations
	C. To simplify the language to help the audience to understand	C. Avoid technical terms and repeat yourself in different ways – observing the response of your audience
2. Concepts/ thinking	A. Don't explain – the audience will never understand	A. Do nothing – it is too difficult to explain mathematical thinking
	B. To give an impression of a mathematical concept – the audience will never understand the real idea	B. Dumb down ideas, use "hand waving" and pictures
	C. For the audience to understand	C. Careful exposition of ideas, explaining in different ways, teaching
	D. To give only an impression of a mathematical concept – the audience does not want and/or need to understand	D. Careful discourse to give an overview without detail
	E. To win a contract or an argument – audience cannot or does not need to understand details	E. Inspire and sell ideas instead of explaining
3. Strength	Justify the mathematics in an appropriate context so that the audience will understand the consequences of the mathematics. Present ideas ethically and correctly.	Use mathematics and mathematical discourse in a flexible way as the situation requires. Check for accuracy and correctness.

Table 6.2 Outcome space for mathematical communication

Intention / Action	To avoid the 'glaze over' effect	Simplify the ideas, teach / Win an argument/ contract	Justify
Omit technical terms	Level 1 / Jargon/ notation		
'Dumb down' ideas / Explain carefully / Inspire, sell		Level 2 / Concepts/ thinking	
Flexible use of discourse / Ethical responsibility			Level 3 / Strength

For Level 2, participants held the conception that communicating with a non-mathematical audience was different because the mathematical concepts and thinking were hard for those not trained in mathematics. It is not only the words and notation that make the communication different, but also the complexity and abstractness of the ideas and the analytical ways of thinking. The intentions of the participants ranged from not even trying to explain (perhaps hoping someone else would), to giving an impression of a concept (using analogies, pictures or graphs), to carefully explaining the ideas and teaching the concepts. Being able to explain mathematical ideas is critical to success in the workplace for many as they will need to teach their colleagues or students. Some participants intend to win contracts or arguments by selling the outcomes of mathematical ideas – they have to go beyond the mathematics to how mathematics and their skills can add value to an organisation. Their actions ranged from: avoidance through teaching to inspiration. The case study of Kay in chapter 7 examines the level 2 conception in more detail.

Participants who demonstrate level 3 intentions and actions are able to do both; inspire and justify themselves mathematically. There is an ethical compulsion to explain the mathematical consequences correctly so that management can understand and make informed decisions. The case study of David in Chapter 7 will develop this level in more detail.

In all these aspects, it is not the level of mathematics in the workplace that dictates the participants' experiences of mathematical communication, though only those who are using mathematics in their employment have experienced the strength that comes will being able to communicate effectively in the workplace. It is the graduate's capability with the discourse, the confidence and ethical responsibility that dictates it.

I will now examine how these participants learnt their mathematical discourse and other communication skills.

Learning discourse skills

Participants expressed three levels of understanding of how they learnt mathematical discourse, with the key qualitative difference being is the level of control that the graduate perceives. Here I particularly explore mathematical communication, that is, when the graduate is trying to communicate their technical ideas. The first level is one where trial and error is used to work out the right communication technique for the situation. For the second level graduates learnt communication skills from outside themselves, from their bosses, or an outside agency such as a church group or sporting organisation. On the third level, graduates stood back, systematically observed good and bad communication and actively modelled good practice. None of the graduates learned communication skills as part of their studies; in fact more than one graduate made a comment like: *Those sort of people skills I do not think, one certainly cannot learn them at the maths department*! [Roger]. Here are some statements from graduates to exemplify the different levels of conceptions.

Level 1: Trial and error. Heloise is working in a medium-sized company where she and her boss are the 'quants'. She is mirroring the language used by those around her and feels comfortable enough to question if she does not understand. The interview showed that she was not in control of her language skills; she was making it up as she proceeds in the job.

Interviewer: So, how did you learn to talk differently to different people?

Heloise: I don't know. I just kind of picked it up. Mainly I'd respond by the way they spoke to me. If they speak analytically to me … with my manager. … So, I know that I can speak like that to him, type thing, and then again with the sales lady and with my director, he's a director, he just wants the bottom line, as all directors want.

Interviewer: But how did you figure that out?

Heloise: Just kind of picked it up.

Evan works in a team to deliver IT solutions for the financial products of a large bank:

Evan: Trial and error. It's usually that glaze-over effect! When they start glazing over it's time to stop! Trial and error, very much so.

Level 2: Mediated by others and outside situations. People with these experiences have worked with an outside influence or a supervisor who has taught them how to communicate in an appropriate style.

Kay: Yeah, I think … I guess, actually just learning about generic communication skills in the environment that were never related to my academic life, like, I'm quite involved in leadership and things in areas outside uni like in a helping situation, also in church and things like that and I've done a few courses and I've actually read a lot of leadership books and things like that.

Paul: Experience. After, when you first come out of uni and you try to explain, you're really keen, oh I've got this great idea and here, blah blah blah, and your boss just looks at you blankly and says that's too technical, I don't understand or if you get asked to write a report for someone and you run it past your boss, who says it is way too technical, take all the technical stuff out of it and just give them what they can understand, write for the audience. If you hear that and get told that often enough, then sooner or later, it changes the way that you deal with people at work. So it just is trial and error, and going through the process often enough.

Level 3: *Active, detached observation.* People who express these experiences appear to have approached communication in the workforce in a scientific way and have closely observed the good and bad ways that interaction occurs and then modified their behaviour accordingly. They have not used trial and error but have observed rigorously and are making controlled, conscious decisions about how to communicate. The qualitative difference is the self awareness that is demonstrated.

Roger: Oh, well, the, from the people that, like there were two kinds that stood out. Firstly those that stood out in a positive way, and those that stood out in a negative way! So then the ones that stood out in a negative way, they were exactly the sorts of people you would expect to come out of the maths department almost, very dry and techy, … But, so then, that you realise you must avoid, and then on the other hand the interesting people had a different style, you know, they spoke about interesting things, not just in the context, not just in more general human things but even in the context of their work, they'd always try to approach it from the more general perspective. …Yeah, it was quite a learning curve I must say. So, because when I went there I didn't know, obviously it was the first time I'd worked in a, in the so-called real world, which is not as real as people think.

These levels are hierarchical; those who demonstrate level 3 also used trial and error and outside influences and those whose experiences were mediated by others also used trial and error. For example, Paul mentions *trial and error* though he also has the influence of his boss. The difference between these levels is one of control. At level 3, the graduates are exercising control of their environment by using observational skills, making their own judgments and then modifying their behaviour. At level 2 the graduate is being moderated by their boss or outside influences and at level 1 the graduates are not in any position of control and are reacting to the situations in which they are placed using simple coping mechanisms. None of these participants showed that they had formally learnt critical language awareness (Jørgensen & Phillips, 2002), which could have given them insight into the discursive practice in which they participate. The participants who demonstrate level 3 conception of learning communication were using critical language awareness by observing the discourse practices of the workplace and modifying their behaviour in a much more detached and scientific way to those who used trial and error.

Considering the links between conceptions of mathematical discourse and learning mathematical discourse, we can see that those who considered that mathematical discourse was jargon tended to have learnt their discourse skills by trial and error or mirroring those around them. It is a reasonable response because as Paul says, *if you drop anything that sounds technical then straight away they close up, 'I don't understand, too hard for me'*, so the mirror effect is to quickly eliminate the use of jargon when dealing with non-mathematicians. All participants modified their use of jargon when dealing with non-mathematicians.

Links with previous studies

The discourse practices of these graduates differ considerably from those of academic mathematicians analysed in Burton (2004) and Burton and Morgan (2000). There was more emphasis on oral practice and on the need to teach in a team situation or explain to colleagues.

There was considerable importance placed on motivating the audience and the need to sell mathematical ideas as well as what a mathematician can do for the company.

Hierarchy

Burton (2004, p. 161) describes the world of mathematics that she investigated as having role-legitimated hierarchies, such as doctoral student, lecturer, professor and so on, however she quotes one of her participants: *The biggest hierarchy is the one that mathematicians are always putting themselves in.* In my study the participants were placed very much in the role-legitimated hierarchy. A hint of us-and-them hierarchy came when several participants described how they *dumb down* mathematics for a lay audience but this was rare (and restricted to male). The majority of the participants were aware of their role as the mathematics expert and the need to modify their discourse for the audience. This corresponds with Fairclough (1992), who discussed the need for professionals (in his case medical) to learn new forms of discourse to be able to deal with patients and peers appropriately.

Lack of power

The majority of these graduates were not in positions of authority and believed that often the technical people were at a disadvantage when it came to negotiating. This is a reflection of their relatively junior job status and the lack of understanding (by management) of what mathematics can add to an organisation. This situation reflects on the power of mathematics as a discipline to develop its place in society rather than on the individual graduates concerned. Changing this perception of mathematics is the role of the mathematics community. There is also the need for managers to be trained in mathematics and for mathematicians to become managers.

You can sense the feelings of frustration and helplessness in the following two quotes:

> Nathan: The people that are in a position to make a decision on, you know, who to spend with so much money because they have a budget and, they're high up, government, educational, corporate institute, none of them are technical … the people that know, that have the skills, they don't have the decision-making abilities. So, all of a sudden you'd be in a meeting where you have two people who are in a position of power, they are talking about a concept. They don't really understand it yet they're the ones making the decisions on what to promise, how much money to spend and that always causes problems …

> Leah: It's actually really easy to see areas in industry where you think you could make a difference; it's extraordinarily difficult to get past that recruitment filter and to actually find yourself in one of those jobs.

Conclusion

Participants used a wide range of genres and text types. Their audiences ranged from mathematical colleagues through to managers and non-mathematical colleagues. Some were required to sell their mathematical expertise by writing tenders or meeting with non-mathematicians, which they found difficult and some found it frustrating.

Participants saw mathematics communication as:

1. jargon/notation,

2. concepts/thinking,

3. strength.

Of these, the strength conception was the most inclusive, and jargon (or notation) the least inclusive. The first level when explaining mathematics to non-mathematicians was to take out the technical jargon or notation. They next tried to make the concepts more accessible to lay people. One participant described how he imagined his brother as the audience when discussing mathematical ideas in his workplace. (Is this a compliment to his brother? I think not.) Other participants described the different way of thinking of those trained in mathematics and the difficulty of explaining this to others. Those who could explain clearly, gained greater control and strength in the workplace. There was a qualitative difference between each level with the most marked difference being between levels 2 and 3.

None had learnt mathematical communication as part of their undergraduate study. There were 3 levels of experience of how they learnt communication skills:

1. go with the flow, trial and error,

2. mediated by others and outside situations,

3. active, detached observation.

Those who used medium and high-level mathematics in their work situation consumed and produced mathematical texts regularly. All participants who used mathematics in their employment were in positions where they had to communicate mathematics to non-mathematicians. Those who

did not use mathematics in their jobs had taken the logic and thinking skills into their employment but could not be said to be communicating mathematically.

In terms of Fairclough, those who use medium and high-level mathematics are working at the highest level of discourse: fitting their discourse into their discipline. They do not, however, display 'critical language awareness' (Jørgensen & Phillips, 2002).

Summary of Chapter 6

Chapter 6 described the use of mathematical communication in the workplace. Participants saw mathematics discourse as: jargon, concepts, thinking or strength. Of these, the strength conception was the most inclusive and jargon (or notation) the least inclusive. I then moved on to show the three hierarchical levels of graduates' experiences of learning mathematical communication skills and considered how the level of mathematics used in the workplace influences the graduates' use of communicative elements. In Chapter 7, three case studies of mathematical discourse will be presented.

7 EXAMPLES OF TEXTS

In this chapter, examples of texts supplied by three participants are analysed. I will discuss the features that impinge on the graduate's ability to function effectively in their workplace. The examples consist of texts for management, teaching and industry.

In Chapter 2, I developed a general description of contemporary discourse analysis as incorporating the following principles:

- Reality is interpreted through the filter of language, and social interaction and structures are constituted (constructed) by discourse

- Truth is not absolute, but dependent on each individual's context

- Knowledge equally is not absolute, but depends on cultural and historical context

- Discourse consists of all types of symbols (signs), including gestures, and everything that is written or spoken may be of relevance to a study of meaning

- Textual meaning cannot be absolute since it is determined by context and because all discourse is two way – even for written text, there is always an (at least implied) audience who will alter the meaning of the text

- There is continuous social change, which is shaped by discourse.

These principles will be used implicitly as I examine the delicate interplay between the participant, their professional situation and their written communications with their audience.

I will also consider Fairclough's (1992) three components of discourse: *description* (how texts are constructed), *interpretation* (construction and interaction with text) and *explanation* (how this fits into a discipline). Discourse analysis gives insights into the texts that graduates are actually using in their workplaces. In Chapter 6, the participants described the types of communication that they were using and now these three diverse examples will give more depth to understanding the needs of the graduates. The interplay between the concrete artefact of their workplace (the text) and the interview brings out interesting differences, particularly in the participants' awareness of communication.

Texts for management – David

David described a situation in a bank treasury where statistics were being used in a way that could have misrepresented the true situation and how a simple, clear example convinced management to change their mathematical practices. The way data had been presented to management was based on historical practice and it took one clear example for his argument to be convincing. The knowledge has been developed in an historical context – this is the way the data have previously been presented. In the past, computing tools have not been sophisticated enough so a 'near enough' approach had been appropriate. Now with improvements to tools and closer attention to risk management it is possible to present better information. It does, however, require a mathematically trained professional to notice the need and act upon it. Here is how David describes the situation from where he sits:

> One of the problems we see with management information is the lack of real information that it often contains. Should someone see a statistic merely because there is a readily available Excel function that produces it? The answer is no. The reality is that most managers outside the specialist quantitative fields do not understand anything beyond basic descriptive statistics. Even the concept of standard deviation is alien to many.

David is the mathematical expert faced with a position where he sees a risk to the bank and must act but his managers are not mathematicians. However, this is an attribute of mathematics that the graduates found powerful. If they could pitch the argument at the right level and with a clear example for management then they had a *bulletproof argument* [James] that was *transparent* [David]. These graduates hold the *strength* conceptions of mathematical discourse. It is not the level of mathematics that is important. It is their ability to communicate it.

In Figure 7.1, David has given an example that emphasises the effect of how one large promissory note skews the situation as reported to management. This was not done to deliberately obscure, but was the way it had been calculated historically – it is not incorrect. Because the actual numbers are confidential, he has presented an abbreviated outsiders' version of what was presented to management and I have also deleted the name of the bank involved. David has made use of mathematical elements: an Excel printout, tables of values and percentages used in parentheses through the text. His use of conversational tone (such as, *winning the day)* and liberal use of financial jargon is appropriate for his audience.

The positioning of the text is interesting. David is confident that his audience will understand the banking jargon (though he has defined terms for our benefit) but does not believe that management will understand the mathematics so he has devised a simple numerical example. The motivation for changing the way of calculating the bank's position is a desire to present a more accurate picture to management, particularly the Board of Directors. This is different to how information had previously been reported, so a case had to be argued. To a mathematician, the argument is obvious but I will now examine how David has presented the argument.

He sets up the problem and uses plenty of financial jargon so that he is not talking down to his audience. He is situating the document in the realm of banking not mathematics. He uses an Excel printout that is not at all necessary for the mathematical argument but is familiar to the audience – again making them feel comfortable. Then he reaches the nub of the argument. He presents the table showing the old information and the underlying deals. He then poses the question, *Do you see a problem?* By setting the argument up in this fashion, David is using the knowledge that his audience is sensitive to numbers (that is, money) and will see the drawback with the way it had previously been reported.

The key mathematical argument is presented at the end and David is confident that his audience will accept it as he says: *It is clear in this table that we have removed the bias …* In the end it is the mathematics that is the argument, but at the start he has set up the conditions for his audience to believe the mathematical argument.

The text contains features of a report, such as impersonal tone and limited use of personal pronouns. The bank is the agent. It is when you come to the argument that it is personified, *Do you see a problem?* At the end it is not David who has won the argument it is *common sense statistics winning the day.* The text has been constructed to position it in the discipline of banking.

Where is this graduate situated? What are the power relationships? David is reporting to the Board of Directors who has ultimate responsibility for the management of the bank. However, he understands mathematics so he has the power to present information in an accurate way. But he must battle with the historical way that information has been presented and he needs to present a mathematical argument in a way that is *transparent.* Here is how he described the process in the interview:

It's very simple, but in order to get there what you have to do is you have to question, well, what am I doing, am I just simply filling in the numbers on the spreadsheet that goes off to the Board, what am I doing, what is it that I'm actually trying to do, what is the Board about? They're about managing funding risk for the bank. What is it that I'm actually giving them, well I'm giving them this, but when you look at this, and when you look at that, the two, there is a bit of a disconnect there which the statistics are driving. So you actually have to come up with a solution. What is the solution for the problem? Weight it.

The key to David's strength is his ability to be aware of the problem (mathematical skills required), to question the current practice (professional responsibility), and to communicate the problem and solution to management (communication skills). He has balanced the context and the technical attributes in his writing. This contrasts with parts of his interview where he demonstrates a lack of awareness of communication skills; that in fact his communication with management is building relationships and trust in the same way as a client relationship.

If you work in an area of the bank that's more involved in communications like doing deals with corporate clients, you obviously need a very different set of skills than if you're sitting down here trading foreign exchange or trading … you need a very different set of skills to be able to do that, if you're going to be with clients you need to be able to communicate effectively with the clients and be able to understand what they need, understand what we might be able to do for them, build a relationship, build trust.

I: And you don't need to do that?

D: No.

I: What about with your team?

D: Certainly when we work together we do, we need to be able to understand each other and understand what each of us are doing. We need to be able to communicate what's going on with each other, but that's very different to a professional one-to-one.

When David was asked about his learning at university he talked about 'bad math' and 'good math' and that university had taught him the skills and awareness to make a judgment about this in a professional situation:

David: I think it's [university] probably given me an appreciation of the good math and bad math, in some ways. … And certainly having a background in statistics at uni has definitely given me a better appreciation of ooh, this little statistic here I produced actually tells a lie, it doesn't really tell the truth about the situation. And certainly going to uni has definitely given me appreciation of that.

What he had not been taught was the fact that he also needed the ability to communicate his judgments with others to be effective in the workplace. He had developed sufficient of these in his

professional life to be able to write well and win arguments. He did not demonstrate awareness of the communication process. If he had learnt this, either at university or in his professional life, he may have become even more effective.

Figure 7.1 Example of reporting to management

Promissory Note Program

Promissory notes are short-term money market securities issued by companies to raise funds. Dealers at banks make a market in the securities buying the securities from issuers such as [bank] and selling them to the investors.

Promissory notes are traded in the market at a margin to the benchmark short-term interest rate in the Australian dollar market. The benchmark rate is called BBSW (Bank Bill Swap Rate). This benchmark interest rate is "set" every business day by polling the largest market participants. This rate reflects the levels that banks would pay to borrow funds for terms of 1 month, 2 months, 3 months. This is a picture of the recent BBSW history.

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Quote Browser: Item Address: QBBSX
Q  BBSX
08:12 13FEB04     AUSTRALIAN BANK BILL SWAP REF RATES    AU17832              BBSX
                                   HISTORICAL
DATE   1MTH     2MTH     3MTH     4MTH     5MTH     6MTH     9MTH     1YEAR
13/02  5.4650   5.5033   5.5517   5.5550   5.5800   5.6233   5.6500   5.7200
12/02  5.4450   5.5000   5.5333   5.5400   5.5600   5.5817   5.6333   5.6950
11/02  5.4383   5.5183   5.5483   5.5600   5.5750   5.6000   5.6633   5.7317
10/02  5.4467   5.5150   5.5433   5.5583   5.5767   5.5967   5.6500   5.7150
09/02  5.4367   5.5150   5.5433   5.5500   5.5650   5.5750   5.6300   5.6933
06/02  5.4317   5.5150   5.5800   5.5800   5.5983   5.6117   5.6717   5.7400
05/02  5.4317   5.5067   5.5700   5.5683   5.5833   5.6017   5.6583   5.7267
04/02  5.4417   5.5100   5.5800   5.5900   5.6083   5.6383   5.6800   5.7433
03/02  5.5350   5.5750   5.6267   5.6300   5.6400   5.6600   5.7133   5.7900
02/02  5.5250   5.5717   5.6133   5.6233   5.6317   5.6467   5.7150   5.7933


* Denotes insufficient contributions to calculate fixing
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Companies that borrow in the promissory note market generally pay a margin over the BBSW rate reflecting the fact that they have a greater chance of going bankrupt than banks. The longer the term of the promissory note, the greater the margin would be to the BBSW. The reason that companies pay a greater margin for issuing longer term promissory notes is because the investor takes a greater risk of the company going bankrupt and not repaying the note over a six month term rather than a one month term.

Illustrations of promissory note pricing on 13/02/04.

If the [bank] were issuing 1 month paper the rate would be 1 month BBSW (rounded to 5.47%) + 1 basis point (0.01%) to give a rate of 5.48% (5.47% + 0.01% = 5.48%).

If the [bank] were issuing 3 month paper the rate would be 3 month BBSW (rounded to 5.55%) plus 3 basis (0.03%) points to give a rate of 5.59% (5.56% + 0.03%)

If the [bank] were issuing 6 month paper the rate would be 6 month BBSW + 6 basis points (5.62% + 0.06% = 5.68%).

Management of the [bank] is concerned with the measuring the performance of the program. The margin for issuing longer term paper is greater, however the risk of margins blowing out over the period is avoided as you will have locked in funding for a longer period. There can be periods of distress in the market when the margin could increase in the 3 month area from 3 basis points over BBSW to say 10 basis points over BBSW. There is a trade-off that

companies make when they rely on this funding. When the board meets each month they want to get some indication of how the promissory note program is going. Remember the program represents our short-term funding!

Here is an example of some information that management used to get.

Total Issued	Average term	Average BBSW	Average margin
$125,000,000	60 days	5.51%	0.02%

Here are the underlying deals for the month that give the above statistics.

Face value	Term	BBSW	Margin
$100,000,000	1 month (30 days)	5.47%	0.01%
$20,000,000	2 months (60 days)	5.50%	0.02%
$5,000,000	3 months (90 days)	5.55%	0.03%

Do you see a problem? The deal for $100,000,000 is only 30 days. That means that of all the deals done in the month, most of the funding raised by the promissory notes will have to be replaced in the next month when the $100,000,000 matures. The basic statistics here are really letting the board of directors down. The information that we have given them does not really capture the fact that there is a bias in the deals to the shorter maturity date.

We overcame this problem by producing weighted average data. This overcame the biases inherent in this type of data and gave the board of directors a clearer picture of our performance.

Face value	Term * Face Value	BBSW * Face Value	Margin * Face Value
$100,000,000	30*$100,000,000 = $3,000,000,000	5.47*$100,000,000 = 547,000,000	0.01*$100,000,000 = 1,000,000
$20,000,000	60*$20,000,000 = $1,200,000,000	5.50*$20,000,000 = 110,000,000	0.02*$20,000,000 = 400,000
$5,000,000	90*$5,000,000 = $450,000,000	5.55*$5,000,000 = 27,750,000	0.03*$5,000,000 = 150,000
Totals	4,650,000,000	684,750,000	1,550,000

If we divide by the total face value issued $125,000,000, then we will get weighted averages.

Total Issued	Weighted Average term	Weighted Average BBSW	Weighted Average margin
$125,000,000	37.2 days	5.478%	0.0124%

It is clear in this table that we have removed the bias created by the large 30-day issue for $100,000,000. This is a real life example of common sense statistics winning the day.

Texts for teaching – Kay

Kay supplied four examples of texts. The first two were the handouts and accompanying overhead transparencies for first year agricultural science students (excerpts in Figures 7.2 and 7.3, full texts in Appendix C). The second group consists of extracts from – in Kay's opinion – a good textbook and from an unsatisfactory textbook for these first year university students (Figures 7.4 and 7.5). The covers of the two textbooks are shown in Figure 7.6. Kay is considering her students as the audience so her idea of an unsatisfactory text depends on her view of the students she is dealing with, but it also reflects the way she likes to learn and the purpose for which she uses her mathematics. She believes her students are more comfortable with applications than with the mathematical details. She shields her students from notation (*I actually try and avoid math notation as much as I can because I find it scares them*) so she filters textbooks and creates notes that reflect this. She

has the *concepts* conception of mathematical discourse and her intention is for her students to be able to understand the statistical ideas and to be able to use the techniques.

Kay developed communications skills through her interest in leadership and teaching. She is an active member of a church group and has done leadership training in that context. This has led her to read and reflect about leadership, which she believes has helped with her communication skills. She describes her motivation as:

> The fact that I wanted people to understand and that I will just keep trying different things until someone understood, and then I started figuring out what was a key and some patterns to helping people understand and actually picking up when they didn't understand. Just being attuned to that and then going "OK, well there's no point going any further if they don't understand that", so then just going back and trying to think of another five different ways of explaining that same concept so that … in a way that that person can relate to, but I guess I was just driven to keep trying different approaches because I felt that teaching was important and that people understanding what they were doing was important.

The texts in Figures 7.2 and 7.3 are related. The overhead transparencies describe the example to accompany the written handout. Kay would have been speaking and interacting with her audience at the same time. Here there is only the written part of the communication as well as Kay's reflections and motivations.

The grammatical features shown in the text of Figure 7.2 are almost the same as those described in the earlier text example of a mathematics lecture shown in Chapter 3. The overall tone of the text is impersonal, with uses of the passive voice supporting this where the doer or subject is suppressed in order to emphasise different aspects of the clause. Another way that this effect is achieved is through non-human subjects:

> The experimental unit is the unit to which a treatment is applied
>
> Blocking involves restricting the randomization in some way

and complex nominal groups:

> A more advanced treatment structure
>
> Randomised complete block design

In addition to the impersonal grammatical constructions, notice the use of the pronoun 'we' (*We randomly allocate* …). The use of the impersonal 'we' in mathematical writing has been noted by others (Pimm, 1987; Wood & Smith, 2004), but in this instance the author has made a decision to

use the pronoun to include the audience and to avoid the use of passive voice. For example, *We randomly allocate treatments to experimental units …* could have been written, *Experimental units were randomly allocated treatments …* This gives agency to the speaker and, by implication, the audience.

The evidence from the grammar of the written text (Figure 7.2) seems to support an impersonal tone in which the subject is omitted. Kay is following in the footsteps of science writers and previous university science and mathematics teaching in a similar way to the example in Chapter 3. The written notes are based on the historical way that science and mathematics textbooks have been presented – and probably the way that Kay was taught.

The readability index in the written text (Figure 7.2) is also high. The Gunning Fog Index is one of the best-known readability indexes. Typical newspaper articles have an index of around 10–12 and the recommendation is that technical reports aim for between 10 and 15, and should never be above 18 (Miles, 1989, p. 280). There are caveats on the use of readability indices but they give a general idea of difficulty. The Fog index roughly equates to the number of years of schooling required to read the document. Miles (1989) gives examples of a research article (Fog Index 18.45), an article from *Scientific American* (Fog Index 15.6) and part of a high school text (Fog Index 9.97). The formula is:

$$(\frac{\text{Number of words}}{\text{Number of sentences}} + \frac{\text{Words of three syllables or more} \times 100}{\text{Number of words}}) \times 0.4$$

For the first section of Figure 7.2, the readability index is 14.7, which is similar to *Scientific American*. The complex nominal groups and the high Fog Index indicate that the materials would be difficult for Kay's target audience of first year students who, "don't want to know about it and are doing it because they have to".

Contrast this with the initial impression given by the title 'Shrubbery, beasties & dirt … our observations of them'. The title is colloquial, non-technical and appealing to students. The overhead transparencies (Figure 7.3) put in plain words the complex ideas and language of the written text. There are fewer, shorter words and concrete examples, *wheat varieties* and *protein content*, are used to put the abstract ideas into context for the audience. In fact the notes do not have examples; they have been left for the live presentation. This allows the author to change the example for the audience but may increase the formal nature of the notes. Kay is assisting her

audience to learn formal reading skills by motivating them to engage with the material (catchy title) and then explaining the use of the formal text in the presentation with a concrete example and a diagram.

Kay *constructed* these texts in order to teach students about experimental design. She has not used notation or graphics in the written notes but has introduced some simple notation and diagrams in the overhead transparencies. I suggest that the reading level required for the notes would make it very difficult for first year students to understand without the demonstration on the overheads.

Figure 7.2 Extract from written teaching materials

"Shrubbery, beasties & dirt... our observations of them"

The experimental unit is the unit e.g. plot, pot etc to which a treatment is applied.

Replication is repeating the application of each treatment on >1 experimental units.
Replication helps you to separate:
- the variation due to differences in the experimental units; and
- the variation due to treatments.

We randomly allocate treatments to experimental units so the probability of a particular treatment being allocated to a particular unit is the same for all treatments and units. Randomization makes it more likely that the observed effects are due to the treatment as opposed to some other cause.

Blocking involves restricting the randomization in some way. If we have reason to believe that the experimental units are not homogeneous, then we would group the units together that are similar. We then allocate at random each treatment to one unit in each of the groups. Examples where blocking is required: soil fertility trends; non-constant conditions within a glasshouse; differing weights among animals.

The intention is to remove a source of variation from the results to make the comparison between treatments more clear.

B. Treatment structure is concerned with the choice of different treatments to be included in the experiment.

Simple treatment structures:
- Unstructured
- Factorial

A more advanced treatment structure is the fractional factorial (also known as partial factorial).
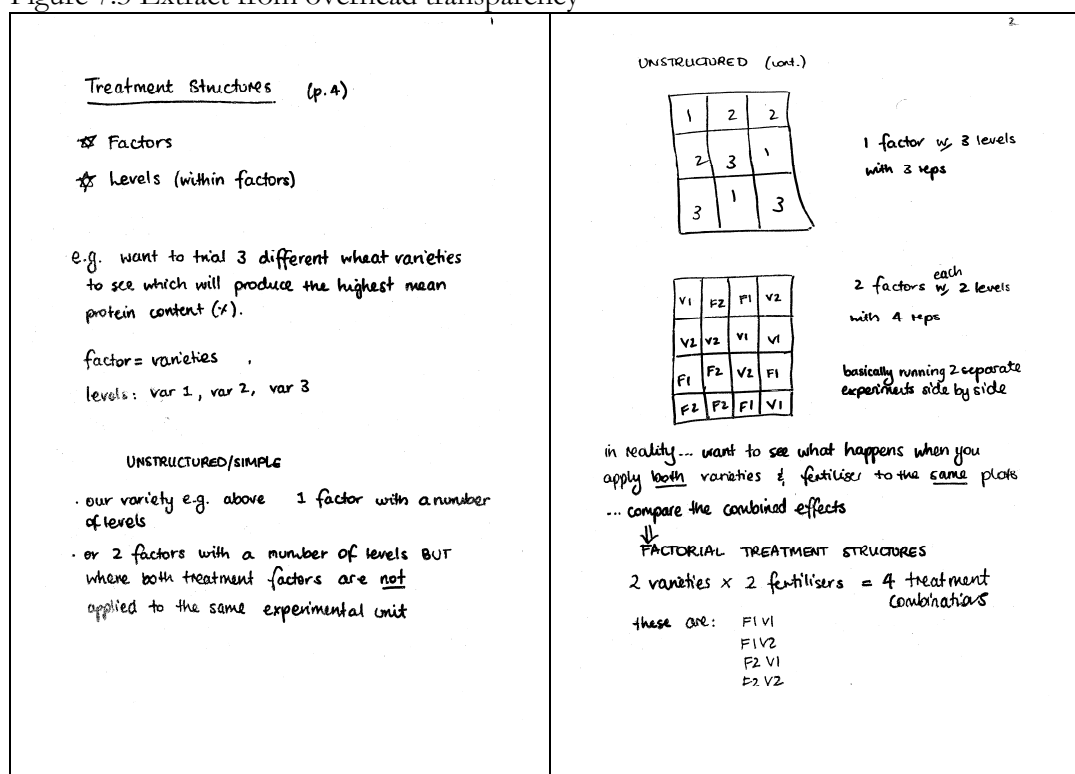
4. Experimental Design – Common Designs

- Completely randomized design (CRD)
- Randomised complete block design (RCBD)
- Split plot design
- Latin square design

More advanced designs: incomplete block, lattice, "nearest neighbour"

Sketch each of these common designs below.

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Figure 7.3 Extract from overhead transparency

The other texts that Kay supplied were two textbooks she had reviewed for her teaching (Figures 7.4 and 7.5). Kay teaches both statistics and differential equations to biology and agriculture students. In Figure 7.6, both covers show schools of fish and it may give the notion that each book is aimed at biology students. Despite this impression the books are very different on the inside and are aimed at different audiences. The mathematical level required for Glover and Mitchell (2001) is much less than that required to work with the King, Billington and Otto (2003) textbook. The first textbook is applications-based with the mathematics supporting the applications, whereas in the second the mathematics takes central stage. As a consumer of textbooks, she sees the first as more appropriate for her students because of the applications focus. This is subjective based on her experience.

There are also language differences. The notation in Figure 7.5 is more complex (something Kay wishes to avoid but is the nature of the different mathematics levels) and the grammar use more formal. Both texts use the 'if … then …' construction common in mathematics texts. Note the start of the last sentence in each:

Remember that when p is very small, … (Figure 7.4)

Recall from complex number theory (see Appendix 6) that, … (Figure 7.5)

Even the use of the word 'recall' is more formal than 'remember'. The first text is more conversational in tone. Both texts use passive voice and typical mathematical constructs: *Let X be a binomial random variable* … and *We define the finite complex Legendre polynomials* … These complex nominal groups make it difficult for novices. They need to understand the meaning of binomial, random and variable or finite, complex, and polynomials, and be able to put them together. Each of these terms is used as jargon. Consider the work *complex*. I have used it in *complex nominal groups* to mean convoluted but in Figure 7.5 it is used to mean complex variables (in a strictly mathematical sense). Both texts use passive voice, impersonal constructions and conciseness; all of which make them difficult to read. The first, however, is friendlier in tone and that is what has influenced Kay to prefer this type of text for her students.

Figure 7.4 Extract from appropriate textbook (Glover & Mitchell, 2001, p. 82)

Normal Approximation to the Binomial Distribution

An additional property of the normal distributions is that they provide good approximations for certain binomial distributions. Let X be a binomial random variable with parameters n and p. For large values of n, X is approximately normal with mean $\mu = np$ and variance $\sigma^2 = np(1 - p)$.

A simple rule of thumb is that this approximation is acceptable for values n and p such that $np(1 - p) > 3$. For example, if $p = 0.5$, then n should be at least 12 before the approximation may be used, since $12(0.5)(1 - 0.5) = 3$. For values of p other than 0.5, the sample size must be larger before the normal approximation is acceptable. For example, if $p = 0.1$, then n should be at least 34 since $34(0.1)(1 - 0.1) = 3.06$. Remember that when p is very small, the Poisson distribution may also be used to approximate the binomial.

Figure 7.5 Extract from inappropriate textbook (King, Billington & Otto, 2003, p. 40)

Rodrigues' formula can also be used to develop an integral representation of the Legendre polynomials. In order to show how this is done, it is convenient to switch from the real variable x to the complex variable $z = x + iy$. We define the finite complex Legendre polynomials in the same way as for a real variable. In particular Rodrigues' formula,

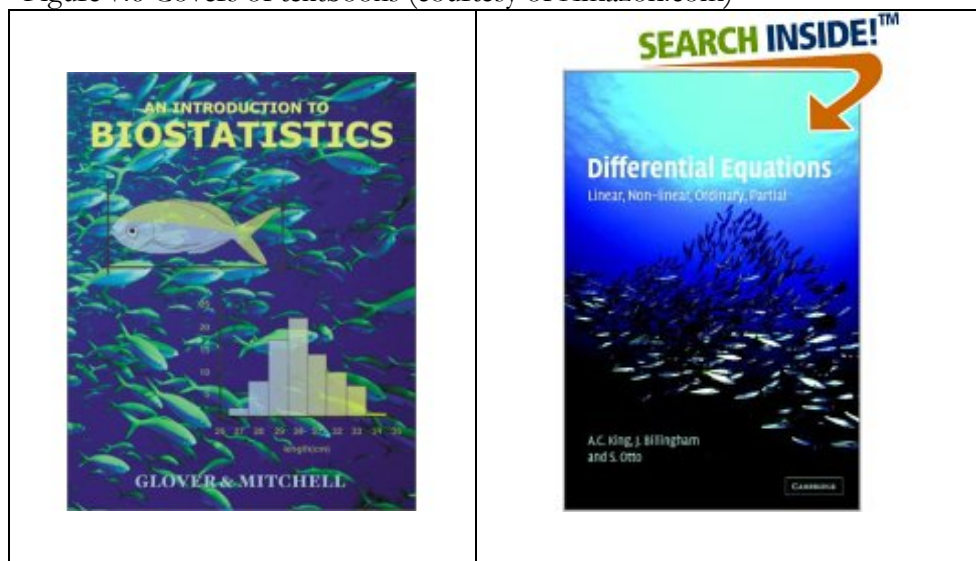$$P_n(z) = \frac{1}{2^n n!} \frac{d^n}{dz^n} \left(z^2 - 1 \right)^n,$$

will be useful to us here. Recall from complex variable theory (see Appendix 6) that, if $f(z)$ is analytic and single-valued inside and on a simple closed curve $\mathcal{C}$,

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_{\mathcal{C}} \frac{f(\xi)}{(\xi - z)^{n+1}} d\xi,$$

for $n \geqslant 0$, when z is an interior point of $\mathcal{C}$.

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Figure 7.6 Covers of textbooks (courtesy of Amazon.com)

Texts for industry – Boris

Boris's conception of mathematical discourse is that of *thinking*. He works with a mixed team of mathematicians, computer scientists and engineers. Their aim is to design ciphers for the security industry. His role is to work through the mathematical texts to find possible algorithms, and then collaborate with the engineers and computer scientists to make products. Boris is a consumer of texts and interprets them for members of his team so that they can apply the results in a different way. In a similar way to Kay, Boris is filtering his texts for his audience and extracting and passing on the salient parts in a form that the audience understands:

> Boris: [I] can start talking about the mathematical ideas when we write on the whiteboard basically.
>
> I: Can you talk mathematics *technically* with everybody?
>
> Boris: No, not, no. The mathematicians OK, but with the engineers and computer scientists depending on who the person is … almost just from our group really to which you can talk to.

His way of working out the mathematical facility of his team's members is:

> By seeing who is attending the weekly seminar of ours, and seeing what types of remarks they make, and the questions they ask.

And what is his aim in his work?

> We are only eager actually to find the new stuff.

Boris supplied examples of texts (extracts in Figures 7.7 and 7.8, full texts in Appendix C) that he has used in his development of security codes. These texts are both academic and commercial. The developments that are made in cryptography are profitable and important in the protection of information and communication networks. Boris believes that some authors deliberately make their articles difficult and leave out parts of the argument in order to keep commercial secrecy.

> I would say in this case that actually if you look at it from the purely mathematical point of view that it wouldn't matter to that. But if you want to apply the article to design a cipher, that would prevent […] you.

Both the texts supplied by Boris have the features of mathematical academic texts (Burton & Morgan, 2000; Wood & Perrett, 1997). There is the use of specialised vocabulary and symbolism, extreme economy of expressions and density of meaning. Note that Boris is reading in English, his second language, but it is not the words that he finds difficult. I have not corrected his quotes for grammatical errors.

> Actually I don't think I ever complained to the language what they used in the article, usually it's just very hurtful, the structure they used for it, they assume you know some knowledge that I don't have.

Boris reads the article concentrating on the storyline, then the definitions and theorems. He works through the proofs. Reading formal mathematics is an iterative process, he says:

> And with each iteration I try to understand the proofs. But if I don't understand I go as far through the proof as I can then I skip that theorem and go to the next one. And then it's just an iterative process and if someone try to, sometimes I zoom into one theorem, and read article from the beginning and go just up to that point to where I understand.

Boris supplied what he considered to be a poor article (Figure 7.7) and a good article (Figure 7.8). The choices were subjective and provided a way to assist in the discussion of features of mathematical communication. Here we discuss the article in Figure 7.7.

> I didn't [have] any knowledge about the theory that he used. And then I find what I call a decorrelation matrix. And because it's, I have to, reason why we look at it was not good theory mathematical. It was also for application of mathematics, I couldn't actually get the feeling of what this definition actually means practical. And 'cause it was looking afterwards it was just extremely difficult.
>
> I: All right. So, the initial definition …

Boris: … useless, almost useless, like! …to get exactly what he means you must look how he uses the definition in different theorems … actually I also look at the theorems, without the theorems I can't actually see what the storyline is.

Boris: That's actually the most difficult part I have. If I at least have the storyline of the article, the rest it's almost just um, putting the theorems, relations as they're supposed to be, giving the mathematical proof of sometimes that's also difficult, and then just explain your storyline. And you're putting your words in between the theorems.

Boris: Actually, I don't think I know what the aim of the author was!

This is the crux of the difficulty. The author did not communicate a clear aim in writing the article and Boris was unable to follow the mathematical storyline. The words and notation themselves are not the issue. As noted above, this is a typical mathematical text, but the definitions are not well defined mathematically and this makes it difficult to follow the rest of the argument. Mathematical writing should have the features of natural language with an aim, introduction, clear storyline and conclusion. Using the features of mathematical discourse, such as correct symbols and concise form, does not make it a good piece of mathematical writing.

Figure 7.7 Extract from inappropriate article (Vaudenay, 2003)

2 Decorrelation

2.1 Block Ciphers, Random Functions, Distribution Matrices

In what follows, we consider ciphers as random permutations C on a message-block space $\mathcal{M}$. Since we are considering block ciphers, and for simplicity reasons, messages are considered as elements of $\mathcal{M}$ which is assumed to be a finite set. In most of practical cases, we have $\mathcal{M} = \{0,1\}^m$. We emphasize on C being a *random* permutation. Here the randomness comes from the random choice of the secret key. In particular, for any (fixed) permutation c over $\mathcal{M}$, there is a probability $\Pr[C = c]$ that the C instance is equal to c.

Definition 1. *Given a random function F from a given set $\mathcal{M}_1$ to a given set $\mathcal{M}_2$ and an integer d, we define the d-wise distribution matrix $[F]^d$ of F as a $\mathcal{M}_1^d \times \mathcal{M}_2^d$-matrix where the (x,y)-entry of $[F]^d$ corresponding to the multipoints $x = (x_1, \ldots, x_d) \in \mathcal{M}_1^d$ and $y = (y_1, \ldots, y_d) \in \mathcal{M}_2^d$ is defined as the probability that we simultaneously have $F(x_i) = y_i$ for $i = 1, \ldots, d$. We denote it $[F]_{x,y}^d = \Pr[x \xrightarrow{F} y]$.*

Basically, each row of the d-wise distribution matrix corresponds to the distribution of the d-tuple $(F(x_1), \ldots, F(x_d))$ where $(x_1, \ldots, x_d)$ corresponds to the index of the row. Intuitively, every experiment (or attack) on C with d samples will provide some information on some simultaneous equations $C(x_i) = y_i$. The experiment probability will thus correspond to a cell in the $[C]^d$ matrix.

2.2 Perfect Decorrelation

The d-wise distribution matrix of a random function intuitively defines its *d-wise decorrelation*. There is no precise definition of decorrelation, only ways to *compare* some, and models for *perfect decorrelation*. Two random functions have the same d-wise decorrelation if, and only if their d-wise distribution matrices are equal.

A random function (or a random permutation) will be compared with an ideal version of it which will have to be specified. Then, we will be able to compare the decorrelations of the function (or permutation) with its ideal version. For example, a block cipher C over $\mathcal{M}$ is compared with the ideal block cipher C^* over $\mathcal{M}$ which is defined to be a random permutation over $\mathcal{M}$ with uniform distribution. Note that for $\mathcal{M} = \{0,1\}^m$, we need $\log_2(2^m!) \approx m2^m$ bits in order to specify fully an instance of C^*, which is enormous. If C and C^* have the same decorrelation to the order d, we say that the d-wise decorrelation of the cipher C is *perfect*.

Similarly a random function F from $\mathcal{M}_1$ to $\mathcal{M}_2$ is compared with a uniformly distributed random function F^* from $\mathcal{M}_1$ to $\mathcal{M}_2$. We say that the d-wise decorrelation of the random function F is *perfect* if F and F^* have the same d-wise decorrelation.

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Consider the article in Figure 7.8 and why Boris believes that it is a good article for his purposes. To help with reading the quotes below, I should explain that Diffie-Hellman is a key agreement protocol in cryptography. It was the first practical solution to the key distribution problem, that is, to enable two parties who had never met to establish a shared secret key by exchanging information over an open channel (Lenstra & Verheul, 2000).

> I: All right, now why is it a good article?
>
> Boris: It has a introduction, it tells you about the … protocol, and the security about that, then it teach you more about around the variants of the different elements … that has been published, and the security, and also in such a way that you can really write that with Diffie-Hellman. … And afterwards, it goes through, it's all what he does, and he also say it's a … variant and also the security parameters and in such a way that you can also relate that to Diffie-Hellman. So when you start you have a nice big … on what you want to do, and then afterwards it just has some, some, a systematical purpose you actually have to follow.
>
> I: OK, so it gives you an argument for you to follow, to do his algorithm.
>
> Boris: Yes. And also um, really does not actually go into theory that he use, he gives you a reference. And that reference is for example an article that's not actually what you can call a standard article, instead he gives a small sentence or two just to give you a taste of actually of the result that's in that article. So if your series of that resembles that sentence, and you don't understand, you know where to go to read about it.

For a non-mathematician (or even a mathematician who does not work in cryptography), the two texts are obvious examples of mathematical writing even if the details seem to have no obvious meaning. It is not clear unless using the texts for a particular purpose that the discourse of one is better than the other – the content may be mathematically more important or more elegant but the writing is done for a specific purpose and for an implied audience. Burton (2004) examines ideas about publishability and what reviewers look for in mathematical articles and the results were far from consistent. Some reviewers focused on correctness, some on rigour, and some on elegance.

Boris found that not all published mathematical writing is well written and this made his job much more difficult. Again I believe this is an example of the mathematical writers not having the power to communicate adequately with their audience. Mathematical writing can have clear aims and be well structured. As part of mathematics curriculum at university these skills of reading and writing mathematics need to be taught. It is not enough to be able to write in a general context for a general audience. Mathematics has discipline-specific ways of writing that should be included in mathematics learning and teaching.

Figure 7.8 Extract from a good article (Lenstra & Verheul, 2000)

In Section 2 we describe XTR, and in Section 3 we explain how the XTR parameters can be found quickly. Applications and comparisons to RSA and ECC are given in Section 4. In Section 5 we prove that using XTR does not have a negative impact on the security.

2 Subgroup representation and arithmetic

2.1 Preliminaries

Let $p \equiv 2 \bmod 3$ be a prime such that the sixth cyclotomic polynomial evaluated in p, i.e., $\phi_6(p) = p^2 - p + 1$, has a prime factor $q > 6$. In subsection 3.1 we give a fast method to select p and q. By g we denote an element of $\mathrm{GF}(p^6)^*$ of order q. Because of the choice of q, this g is not contained in any proper subfield of $\mathrm{GF}(p^6)$ (cf. [11]). Many cryptographic applications (cf. Section 4) make use of the subgroup $\langle g \rangle$ generated by g. In this section we show that actual representation of the elements of $\langle g \rangle$ and of any other element of $\mathrm{GF}(p^6)$ can be avoided. Thus, there is no need to represent elements of $\mathrm{GF}(p^6)$, for instance by constructing a sixth or third degree irreducible polynomial over $\mathrm{GF}(p)$ or $\mathrm{GF}(p^2)$, respectively. A representation of $\mathrm{GF}(p^2)$ is needed, however. This is done as follows.

From $p \equiv 2 \bmod 3$ it follows that $p \bmod 3$ generates $\mathrm{GF}(3)^*$, so that the zeros α and α^p of the polynomial $(X^3 - 1)/(X - 1) = X^2 + X + 1$ form an optimal normal basis for $\mathrm{GF}(p^2)$ over $\mathrm{GF}(p)$. Because $\alpha^i = \alpha^{i \bmod 3}$, an element $x \in \mathrm{GF}(p^2)$ can be represented as $x_1\alpha + x_2\alpha^p = x_1\alpha + x_2\alpha^2$ for $x_1, x_2 \in \mathrm{GF}(p)$. In this representation of $\mathrm{GF}(p^2)$ an element t of $\mathrm{GF}(p)$ is represented as $-t\alpha - t\alpha^2$, e.g. 3 is represented as $-3\alpha - 3\alpha^2$. Arithmetic operations in $\mathrm{GF}(p^2)$ are carried out as follows.

For any $x = x_1\alpha + x_2\alpha^2 \in \mathrm{GF}(p^2)$ we have that $x^p = x_1^p\alpha^p + x_2^p\alpha^{2p} = x_2\alpha + x_1\alpha^2$. It follows that p^{th} powering in $\mathrm{GF}(p^2)$ does not require arithmetic operations and can thus be considered to be for free. Squaring of $x_1\alpha + x_2\alpha^2 \in \mathrm{GF}(p^2)$ can be carried out at the cost of two squarings and a single multiplication in $\mathrm{GF}(p)$, where as customary we do not count the cost of additions in $\mathrm{GF}(p)$. Multiplication in $\mathrm{GF}(p^2)$ can be done using four multiplications in $\mathrm{GF}(p)$. These straightforward results can simply be improved to three squarings and three multiplications, respectively, by using a Karatsuba-like approach (cf. [10]): to compute $(x_1\alpha + x_2\alpha^2) * (y_1\alpha + y_2\alpha^2)$ one computes $x_1 * y_1$, $x_2 * y_2$, and $(x_1 + x_2) * (y_1 + y_2)$, after which $x_1 * y_2 + x_2 * y_1$ follows using two subtractions. Furthermore, from $(x_1\alpha + x_2\alpha^2)^2 = x_2(x_2 - 2x_1)\alpha + x_1(x_1 - 2x_2)\alpha^2$ it follows that squaring in $\mathrm{GF}(p^2)$ can be done at the cost of two multiplications in $\mathrm{GF}(p)$. Under the reasonable assumption that a squaring in $\mathrm{GF}(p)$ takes 80% of the time of a multiplication in $\mathrm{GF}(p)$ (cf. [4]), two multiplications is faster than three squarings. Finally, to compute $x * z - y * z^p \in \mathrm{GF}(p^2)$ for $x, y, z \in \mathrm{GF}(p^2)$ four multiplications in $\mathrm{GF}(p)$ suffice, because, with $x = x_1\alpha + x_2\alpha^2$, $y = y_1\alpha + y_2\alpha^2$, and $z = z_1\alpha + z_2\alpha^2$, it is easily verified that $x * z - y * z^p = (z_1(y_1 - x_2 - y_2) + z_2(x_2 - x_1 + y_2))\alpha + (z_1(x_1 - x_2 + y_1) + z_2(y_2 - x_1 - y_1))\alpha^2$. Thus we have the following.

3

Conclusion

"Critical language awareness should give people insight into the discursive practice in which they participate when they use language and consume texts and also into the social structures and power relations that discursive practice is shaped by and takes part in shaping and changing." (Jørgensen & Phillips, 2002, p. 88)

The examples in this chapter demonstrate that the participants do not have critical language awareness. David shows he can develop text appropriate for his audience, however, from his interview it is clear that he is doing this intuitively, from experience, rather than from a critical awareness. Kay is attempting to produce texts that meet her aims but, I argue, is constrained by the historical way texts have been produced in mathematics. By filtering her textbooks for her students she is working towards gaining awareness of the consumption of text by her students. This is done from *her* perspective, not from systematic observations of students working with different texts. She has motivation and a student-centred approach and would be assisted by directed readings in the area of student learning. Boris is using the consumption of texts to develop his understanding of good writing for his purpose. Boris found that not all published mathematical writing is well written and this made his job much more difficult.

The graduates discussed in this chapter have developed their mathematical writing through experience and modelling. They can distinguish between good writing for their own purposes and inappropriate writing. These are successful graduates sharing their mathematical knowledge in the workplace. How many of their classmates have not been able to do this?

As part of mathematics curriculum at university these skills of reading and writing mathematics need to be taught. It is not enough to be able to write in a general context for a general audience nor is it enough to be able to write clear mathematical texts. Mathematics has discipline-specific ways of reading and writing that should be included in all mathematics learning and teaching. Burton (2004) made similar recommendations for the development of academic mathematicians. Chapter 9 gives ideas of how to include these strengths into mathematics curricula.

Summary of Chapter 7

Participants supplied texts for analysis and I have described these as texts for management, teaching and industry. The purposes of the texts were to convince management, to fascinate students and to develop new commercial commodities. Graduates dealt with many different genres without

adequate support to develop appropriate reading and writing skills. There are also some hints that the texts used by our participants are not serving as good models for the development of discourse skills.

In Chapter 8 we will consider graduates' experience of learning at university and how this has affected their working lives.

8 REFLECTIONS ON LEARNING AND TEACHING

In Chapters 6 and 7 I presented the participants' conceptions of mathematical discourse and the different ways these graduates have developed their communication skills for the workplace. In this chapter, the graduates reflect on their learning of mathematics, how this has affected their working lives and how they have developed a 'mathematical identity'. They also consider outside influences in their background that helped them in the transition to the workplace. They reflect on what should have been in their degree program to aid their transition to the workplace. The ideas will be presented as a selection of themes supported by quotes.

This study complements analysis of the experiences of undergraduates (Chapter 2) where they anticipated their transition to the workplace.

Identity

I have shown that these graduates do not form a community of practice as they are too dispersed in type of employment and backgrounds. However, members of this group have developed mathematical identities. Notice how they are drawing boundaries around themselves and their colleagues – they have 'got it' and others don't:

> Christine: I'm very proud about it [my degree] and I'll sometimes, you know, flaunt it and people look at me strangely, and I love that look! And so I think it makes me who I am.

> David: It is, it's very subtle, it's often in the job we do, you sometimes see examples of, it's very difficult to explain, but people who've got it and people who don't.

> Evan: I think that with it [maths], I mean my boss probably has different opinions, but I'm a lot more valuable!

> Heloise: So me and the supply manager we work together … the sales manager, she's … not as analytical as myself and my manager.

> James: Some of the people I work with will shake their head sometimes at how I'll do something or why I'll do something, stick with something logically and get the answer out.

Identity is considered by Wenger (1998, pp. 189-190). He states that identity formation is a tension between investment in various forms of belonging (membership) and the ability to negotiate meaning in those contexts. Our participants clearly demonstrate one facet of identity; they have drawn boundaries between themselves as having the mathematical knowledge and those who do not. Not all of the participants were able to become full members of their workplace, for example William, because of their difficulty in being part of the work environment, the work group:

> William: I ended up leaving after a year or so … they gave me a cold shoulder and I didn't quite understand, at the time, what was going on but it was more … I think, I wasn't prepared for the environment.

The other half of identify formation is negotiation and ownership of meaning (Table 8.1) and here is where our graduates find it most difficult in their workplaces. It is where their communication skills are critical. It is where the power of their ability to negotiate meaning is critical. In Chapter 6 there were descriptions of situations where participants felt they were ignored and how that led to frustration. There were also counter-examples of those who were able to successfully negotiate meaning and have their ideas adopted. Clearly those with mathematics degrees have developed a mathematical identity, but not all have been able to successfully negotiate with that identity in the workplace. This is a significant area that should be addressed in curriculum design.

Table 8.1 Negotiability in the ownership of meaning (Wenger, 1998, p. 190)

Identities of participation	Identities of non-participation
Having one's ideas adopted	Marginality through having one's ideas ignored

Technical skills

On the base level, graduates stated that they had learnt specific skills to get them a job and they particularly mentioned technical skills. Many of the jobs required a degree and some required a specific set of skills that were guaranteed in a mathematics degree (or mathematics and finance, computing and so on):

> James: I wouldn't have the role I'm in today without it, that goes without saying.

Nathan: But what I did learn at uni which I couldn't have really got elsewhere was that the skills set where I could sort of lay a foundation and, you know, put my foot out in the door and, you know, sort of go into the workplace and move from there.

Problem solving

The participants described how they developed ways of solving problems and ways of thinking that they used in their working and, in some cases, personal lives. Their university education taught them to think analytically and abstractly, to define the edges of a problem, to look for more optimal solutions and to model problems in a mathematical way. For some , these ways of thinking and logic were seen as more general and widely applicable to life in general. These are general skills – not connected to particular content but gleaned over the whole degree.

Mathematical problem solving

The term 'problem solving' was used by the participants in a particularly mathematical way, that is, it was mostly applied to technical problems in their workplace.

Sally: Just maths, like, you gotta do this, that goes before you do that, so that then.

Gavin: Another useful idea that I found from maths that I use a lot is the idea of, given a particular, it's just the idea of independence and dependence of variables in problems. I often think of, you know, things like in this act of having a problem that you put into a context, and cut down, a really important thing is to say, you know, can you establish dependence relationships between this and this, or is that independent of that, I guess that's one of the ways you cut it down.

Heloise: Well it got me the job for starters! I think what really helped me was, not so much remembering every theory and every formula that I'd learnt, it was just the way of thinking that kind of helped me … 'look at everything … here's a problem, translate it to mathematical problem, break it down, try and get a solution and then translate it back'. Just that way of thinking has really helped.

Paul: A couple of weeks ago I was given a new swaption, this weird derivative to price, and how do you do it? And I kind of sat down and had a think about it and worked through, pulled out an old derivatives securities textbook and worked through it like a problem, that was it. So that's another way uni had helped.

Way of thinking

Graduates were able to consider abstract ways of thinking that go beyond mathematical problem-solving skills:

William: Just abstract thinking skills and just, sort of, a tangential understanding of scientific process and just the domain of logical thought.

Fredrik: Without that grounding [university], you don't approach problems, the best way, to being able to solve them. It's not the most or the optimal way to solve a problem. … it's [university] taught me that, there's an easy way to do something, and there's a hard way to do something. And sometimes the two aren't easy to, well there can be many options, sometimes it's not easy figuring out which one, which road to take. But you know that you're there, and you start looking for a simple way or an easy way to do it.

Melanie: Problem solving and just thinking about a problem logically is, you know, transferable to anything.

James: I think it helps me out for the future roles that I want to take, not because necessarily you go to a particular class and you've learnt how to do a particular thing but just the different way of thinking about problems.

Personal

Mathematical skills can flow over into the graduates' personal life and they have given them a way of working with other problems and people. Leah looks for patterns in people's behaviours and does not react until a pattern emerges (quoted earlier). Gavin has taken his way of working with mathematical problems into his daily life:

Gavin: I guess it, if you were gonna have any kind of success in mathematics as an individual, you need to develop the skills to recognise where the boundaries of a particular problem are, what's involved in a problem, how do you split the problem up, so that I suppose, at least that attitude carries over into problems in everyday life … That I definitely use, over and over again, and I think I've learnt that through mathematics.

Motivation and value

What motivates industry? What value is placed on the knowledge of the graduate? The motivation is seen as different between the goals espoused at university and the needs of industry. The ideas of elegance and beauty (Burton, 2004, p. 69) are not a motivation in industry where graduates learnt that it is the applications and the bottom line that are imperative.

Evan: That the maths will hold value but it needs to be applied … 'cause there's just, people a) won't understand that you know it, and b) you've gotta be able to talk and understand what's going on in context.

Roger: It is a different way of working. And the goal is different. And in fact usually the motivation for people to do pure maths is how beautiful it is, it all fits together so perfectly and so on, and then that's not at all how the real world works, or indeed industry does not care how beautiful the maths is. They only care if they can get answers that help.

Personal attributes and advancement

Finishing a degree in mathematics demonstrated that the graduates had determination and had applied themselves to complete a difficult program. The following quote by Christine shows how influential failure was for her. It defined her experience of university and changed her attitude to learning. The quote has evoked interesting discussion at conferences, with a mathematics professor approaching me after a presentation saying that he was pleased that the quote was included because he had failed all his subjects in first year. Failure can be a crucial event and potentially excellent motivation.

> Christine: I probably wouldn't say that going to university helped me as far as work is concerned, it was more failing for the first year that really hit home that I had to work for what I wanted, so that, that helped me. But the fact that mathematics is not the sort of subject you can just, you know, figure it out in the last week of semester, read a book and you'll be right, because of the type of subject it is, you need to be working throughout the whole thing, so by the time I eventually finished my degree, I suppose that sort of taught me … up until a point I had just managed to coast through but then it just got too hard so then I had to work for what I was getting. So, that … university in general could have taught me that, but because I was doing mathematics it was more, more of an effort, so …

James studied the degree part-time and found that his managers looked very favorably on his achievement:

> James: So my answer is the other way around, what made me come to uni and how does it help me, for example, in the next level up in the job. I think doing it part-time helps because people look at it and say you went there part-time off your own bat you must have a bit of desire to go forward so it helped.

Broadening horizons

University changed the participants and gave them a view of themselves as professionals. For those who have found employment using their skills this has been liberating, but for those who have not found their niche the journey has been frustrating.

> David: It made me appreciate the bigger picture. It sort of opened up avenues of my career that I never would have ever imagined could have existed. I mean, certainly when I started university, for the bank I was working in customer service, more sort of in a branch of the bank rather than this, I mean this whole world of what goes on outside of where I'd worked, I had no idea about. I mean, I just always thought working in a bank was about counting out money and saying, would you like that in twenties or would you like that in fifties, so it was …

Paul: Uni kind of changed my mind-set a bit into, before uni, because I was in clerical roles, you've got a clerical mind-set, where you're not decision making and you're not overly keen on taking responsibility for making decisions whereas after uni I found I was given the shits if I wasn't given the responsibility to make decisions because you're thinking 'I've been to uni, I've done this time and trained for this'.

Roger: You have to know mathematics on a much deeper level than you will ever really use. So these third, these analysis courses with epsilons and deltas, whereas in reality you will never use an epsilon and a delta because naturally such things don't exist in real-world measurements. Nevertheless it is worthwhile to know it, because then you understand why many algorithms involve approximations and so on, so you can understand. And so instead of just remembering one or two algorithms and … uses, you can perhaps even invent your *own* one. And you will be able to understand why.

Confidence and pride

University education develops more than technical skills. Several participants described the way that their education changed them, not only broadening their career horizons but changing the way they think about themselves:

Christine: I'm very proud of it [my mathematics degree]. Absolutely. Because, because it is not easy, it wasn't easy to finally get it, I'm very proud about it.

Paul: Well, it gives you a background in something and a confidence in your background so you know that you've got confidence in yourself and it gets you into a problem-solving mind-set where you want to take on a problem 'oh this is interesting' and it's stretching my mind a little bit so you want to take on that kind of work.

Roger: It is a kind of confidence and experience that you don't feel bound by rules.

Angie: Well, I think it gives … finishing a course like Mathematics and Finance gives you a great boost in confidence and a big edge.

Personal characteristics

Several participants described the arduous task of finding a job and convincing the employers that they were suitable for the job (see Chapter 5). As part of the application process most applicants were required to do a psychology test to indicate their fit with the organisation and team. Those who were happy with their jobs indicated a fit with their personality and their employment.

Heloise: I like making things work. That's why I think I enjoy working for a gas company, rather than a bank, or an insurance company, or anything like that because it's more physical. It's like, 'we have this much gas, we need to get this much gas to there, there, there and there, we've got this many trucks, it costs this much to go there, we're going to put that on the same route as that'…

Graduates pointed to particular personal characteristics that helped in their transition to the professional workplace. They talked about motivation, initiative and tenacity:

Thi: One of the most important traits that people have, something that they have within themselves is, to be successful, in my opinion, is the willingness to work and work and work and just continue doing it and not giving up. I think it goes into everything that you do in life: uni; friends, even; networking; business. Just striving and working and continuing to work until you're satisfied that this is all that you can do and you cannot do any more and just doing that. Just actually going for it.

William: Personal attributes and initiative and motivation are pretty crucial in securing some employment.

Non-technical study at university

Several of the graduates studied psychology, business, languages or anthropology as well as other technical subjects such as physics, IT and engineering. Many describe how the different teaching methods, particularly in non-technical subjects, helped them in the transition to the workforce. They were obliged to talk to classmates, read and write essays and think in different ways. They had a broader range of learning experiences while at university.

Other influences

University was not the only influence in the graduates' journey to professional work. There were three main other influences: life experience, initial work experiences and the graduates' personal characteristics.

Life experience

The participants' previous work experience, hobbies and home background had a beneficial effect on their transition to professional work. Participants mentioned parents, school, sport, church and previous employment, usually part-time while they were studying:

Christine: I worked for a phone company so those jobs were, you know, pleasing people and that sort of thing, so that comes across in the way that I do my current job, I need to sort of judge how people are feeling.

William: I think my school studies helped me a lot. [*William had taken Business Studies at school*].

Sally found that employers were looking for over-achievers, those who had a range of hobbies and other activities that demonstrated leadership and teamwork skills so she joined activities to augment her résumé:

> Sally: Extra-curricula activities. Oh, working in different casual jobs with the communications skills and stuff like that, being in, just trying to fit in like, the people … they [the employers] want some of that … So you just try and find, I try and like took up lots of different hobbies, joined different activities.

Nathan worked as a tennis coach and found that the teaching aspect of the coaching taught him to read people better. Teaching as a preparation for the workplace, for communication skill development and as a way of learning about oneself was described by several participants. Along with the need to teach as part of professional work (see Chapter 6), this points to good opportunities for assessment in undergraduate programs.

> Nathan: I was a bit lucky in that I did do tennis coaching which is quite different [to working in McDonalds]. … I was able to read people a lot better after coaching because you are dealing with people from kids to parents, you're learning about their habits and also how you communicate with them because I was in a situation where I was teaching something. To be a good teacher you have to try and find good ways of communicating, like making your point across and that often changes from student to student. So it sort of kept me on my toes and kept me thinking and, yeah, it was good experience although I never really wanted to do it as a career. But it was good.

Initial work experience

Participants' initial experiences were described in Chapter 5 so here I will only reiterate how important these experiences seem to be to the adjustment to the workplace. These experiences demonstrate the importance of good mentoring for new graduates:

> Nathan: I started working for a relatively small company where I had close interaction with the two directors and one of them, he's sort of the sales director, has a sales background and I've learnt an awful lot from him in terms of how the whole sales side of things works in an IT company, or just generally in business how you negotiate with other suppliers or customers, in all those sort of things that, you know, form part of the business cycle.

Reflections on university

After describing how university and other experiences had helped with their transition to professional work, participants were asked to reflect on what they believe could have been in their university program to help with the transition to work. There were suggestions about content,

computer skills and personal skills. Some ideas were the obvious domain of academics (that is, academic content) and other suggestions were concerned with actually getting a job, such as interview skills and knowledge of the employment market. There was appreciation of the diversity of jobs that universities are preparing students for and that the role of universities is not necessarily vocational; that the broad aim is learning. Nevertheless, there was robust criticism of teaching and the way content is delivered at university, in particular the lack of overall coherence and a failure to link areas of knowledge. Especially strong was the perceived failure to link areas of knowledge to real situations. Good teaching, as always, makes a difference. When talking about why he studied mathematics Gavin says: *It was a good lecturer. And I wasn't doing as well in maths as I was doing in physics, but, a good teacher makes it much more exciting.*

Technical content

Very few of the graduates used all the mathematical content that they had learnt at university. Many had to learn more or different mathematics, for example, Roger had learnt pure mathematics and needed applied mathematics for the work he was doing. In Figure 8.1, I have used the information from Table 6.1 and compared it with the level of mathematics learned, which emphasises the gap between learnt and used. Table 6.1 showed the level of mathematics that participants used in their workplace. Level 0 showed no use of mathematics, Level 1 was use of first year university mathematics, level 2 showed the use of second year mathematics and level 3 referred to third or higher year mathematics.

Figure 8.2 illustrates the information on the level of computing used in the workplace. Level 1 is general computing (such as word processing and email) and is assumed to be learnt at school or elsewhere; level 2 consists of the use of standard tools, such as spreadsheets; level 3 is specialist software such as SAS and *Mathematica*; and level 4 is the use of programming in high-level languages. Figure 8.2 shows that for this group of graduates the level of computing taught is less than that required for the workplace.

In general, graduates felt that they knew far more mathematics than was needed for their jobs (Figure 8.1). This was not always seen as a negative as Roger says: *It is always much better to know a great deal more than what you actually use. It is a kind of confidence and experience that you don't feel bound by rules.* Others were frustrated about having so much knowledge that they were not using. Some participants needed to revise content they had covered in their courses when needed on the job and others had to learn new content. Curriculum designers should note the need for students to

practice learning new material without formal instruction, and that the technical content is important but not critical to the success of the graduates.

Figure 8.1 Mathematics needed in the workplace and learnt at university

Mathematics learnt and needed

Number of participants / Years of university mathematics

Legend: Learnt, Needed
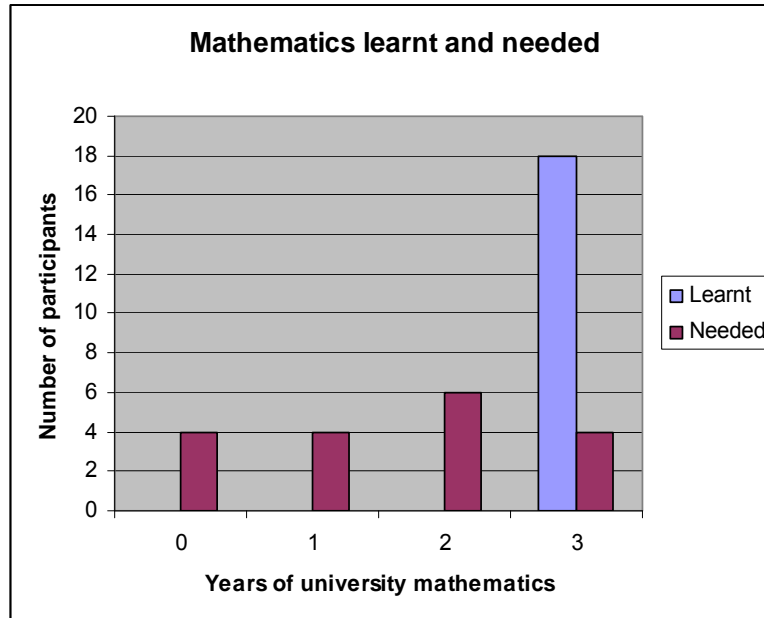
Figure 8.2 Computing needed in the workplace and learnt at university

Computing learnt and needed

Number of participants / Computing level required (see text)

Legend: Learnt, Needed
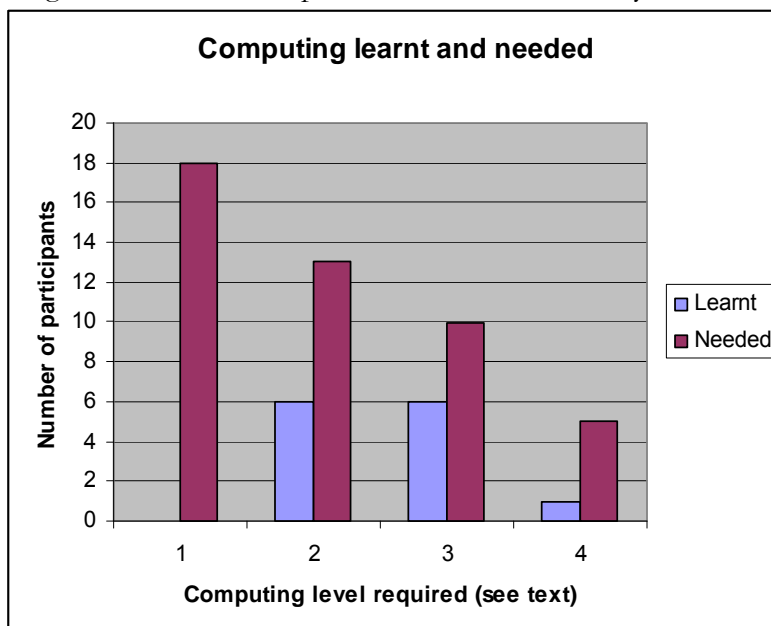
The graduates made several suggestions about changes to technical content. The majority of these were to do with specific computer products, such as Excel, Visual Basic or SAS (a statistical package). The following quote is typical:

> James: As far as transition for work, everywhere uses standard products like Excel, and if you come out of a maths degree, I wasn't really taught to use Excel all that much here [at university] and I think it's really a tool of the trade.

A university degree will never cover all the content required for the range of workplaces that graduates may move into. Work requirements change, indeed mathematics changes and the tools used to support mathematics changes. The point I am making here is that less mathematical content could be taught and more high-level mathematical computing should be taught (these are not mutually exclusive).

Teaching mathematical discourse

Chapters 6 and 7 examined the types of mathematical discourse used by the participants in the workplace and then examined the difficulties faced when communicating with non-mathematicians. The graduates did not think that communicating with fellow mathematicians was a problem – it must have come as somewhat of a relief not to have to be careful about what you say. The graduates believed that they had not been formally taught mathematical discourse at university, though many mentioned statistics laboratories as helping them learn mathematics. These laboratories are also teaching mathematical discourse, though that may not have been made explicit to the students and the graduates did not mention this aspect of their learning.

> David: And lab is vital, I think, having the labs is absolutely vital in statistics. I certainly think that one of the more valuable learning experiences I did have, was labs in statistics, and computing.

Teaching at university

The criticism of mathematics learning and teaching at university level was disappointing. However, the research on mathematics teaching and learning at university level has increased in the past 10 years (see for example, the *ICMI study on University Teaching and Learning*, Holton, 2001) and it takes time for developments to filter through to the chalk face. Burton (2004) too, regrets the lack of action on learning and teaching mathematics at all levels. Indeed her book finishes with a plea for changes in the teaching and learning of mathematics, and I will make proposals in that direction here and in Chapter 9.

Group work

Group tasks were strongly recommended by graduates as an important way to prepare for the workforce. Graduates who had studied in other disciplines where a greater range of teaching methods was used felt that this was an advantage in the workplace:

Nathan: …the maths studies that I did were a lot more, what's the word, individualised. So you get a project or an assignment and you work on it on your own, pretty much, and that's it and then you do your exams and you pass or you fail. So, it's very much an individual's sport, if you like. Whereas, doing something like psychology or business studies is a bit more of a team sport.

Christine: I hate group work, and I hate to encourage group work, but probably group work'd actually, because being mathematics you can do an assignment the night before it's due if you have to, and you can not go to bed if you have to, but if you've got group work then you've got people that are counting on you which is probably more realistic.

Kay: I think that the more group work that we're doing we're forcing people to learn to communicate.

Nathan: Maybe do more presentations or group assignments, something that's going to give them more confidence and be able to… step up to explaining in a group. … I mean most of them would be so scared to do that but I think it would do them a world of good if you had that as part of the study.

Peer teaching

Thi (and others) believe that you really learn a topic if you teach others and participants recommended that opportunities for teaching be incorporated into learning tasks. This resonates with the work done on peer teaching by Ken Houston at the University of Ulster (Houston & Lazenbatt, 1999) and the many studies on collaborative learning, summarised in D'Souza and Wood (2003a, 2003b).

Thi: I think we should be able to know how to communicate our knowledge to others so that they can comprehend it in a way that's easy and I don't think we know how to do that yet.

Structural changes

Many of the changes suggested by graduates were structural. Some believed that the lecture situation is not conducive to learning, others suggested formal work experience as part of their degree and James suggested a third year subject on transition. Laboratory learning for statistics and computing received positive commendations.

Heloise: Maybe an option, to do a six-month formal work experience but if you didn't want to do a full six months of it, maybe a few weeks … the difficulties of setting something like that up … but just a bit of hands-on … I just found it [university study] very theoretical.

James: Maybe like a transition to employment, like a third year subject, whether it's a subject or something else.

Paul: Probably more putting it in a professional context, ... so making something more work relevant would be useful so maybe a report writing course or something like that but for business rather than for academics.

Links between subjects

There was a constant theme about subjects not being related to each other and students having to make connections themselves. In joint programs, such as Mathematics and Finance (which is a set degree with half mathematics and half finance) students expected integration between the subject areas but this did not happen. Even for those who did straight mathematics degrees, the graduates perceived that the subjects were not linked or put in the wider context of mathematics as a discipline. What is clear from this study is that, for this group of graduates, links between subject areas were not made explicit.

> Gavin: That's the problem with most university courses is that you're not introduced to the philosophy of the course, you know, you're not introduced to the motivation of the course, you just go straight onto the content.
>
> Evan: Ah, I guess at uni the thing that was hard for me was um, and that may have changed since I did it, but there is the maths stream and there is the finance stream, and there's not a lot of crossover. ... you have to marry them up yourself.
>
> Boris: Actually, I'm take it for courses a bit more integrated with each other, I realise. In my honours year, there were honours subjects and in the beginning you get the idea that they are starting to deal with each other, and at the end of him it's actually have idea why they have something to do with each other, I think that in the middle of the next semester you actually realise why they have actually something to do with each other.

Friendly approach

Students are often isolated in mathematics classes. Graduates made a plea for more interaction in class and less content-driven curriculum:

> Christine: the classes aren't big, you've got maybe 20 students in a class, but I couldn't have told you a single name of someone in some of those classes. I've just, there was no interaction with other students, for a lot of the time, ... so perhaps if you start up with a bit of the touchy-feely stuff, and just getting to know everyone, and I don't know if other people do that sort of thing, but that perhaps would have been a bit nicer. Just to make, encourage more discussion in the class, cos I think there's so much to get through in the course that it's just, Day 1, right, this is my name, this is my contact details and all that sort of thing, right let's begin. And then it's just scrambling through it for the rest of the semester.

Melanie believed that many of her mathematics lecturers did not have good communication skills and made the following suggestion. It would really be an indictment of mathematics teaching if we had to implement her idea:

> Melanie: This is a left field idea as well, but like if the very first lecture of any course was given by somebody who wasn't actually going to be the lecturer of that course, but just somebody who had good communication skills and could actually put the whole course in context.

Real world experience

Graduates wanted more exposure to real-world situations as part of their learning:

> William: … a graduated approach, where you might start by learning some theory, then be working a bit with, say, a lecturer in a mock team situation on a realistic project, then having industry experts come and work with you to maybe do real project, … So, I suppose, if university was able to forge stronger co-operative project-base work with industry, that would be a really helpful thing to make that transition.
>
> Nathan: I think that you could probably, if you had a lot more exposure to the 'real world' as part of your learning process, I think you can't go wrong.

Managing career expectations

Graduates are often given career advice by their lecturers and this can lead to grief! As Paul says, excitement is good but it needs to be tempered by accurate information about the job market. This is an area where mathematics lecturers may not be able to give an accurate assessment and should leave this to university career services (or equivalent). The various mathematics professional associations, such as the Australian Mathematical Society and the American Mathematical Society, have websites with mathematics jobs listed.

> Sally: Yeah, you should give an accurate idea of what potential jobs are out there, instead of finding out at the end.
>
> Paul: To be honest in first year we were told that this course is designed to send everyone into dealing rooms and that stuff. What I have found is that maybe 2 or 3 people in each year will get a job in a dealing room so managing the expectations of everyone in the course could be done more effectively. Probably by not overselling things early. Trying to be more realistic early. Like it's good to get everyone excited about what they are studying but not so excited that they expect that when they graduate they are going to be this, because a lot of people will end up disappointed or frustrated that they don't get to do that.

Links to studies of undergraduate mathematics students

Outcomes identified by undergraduate students anticipating their transition to professional work are shown in Table 8.2 (Reid et al., 2005). All of the participants in my study had passed their degrees and had acquired mathematical tools and skills. Not all had achieved their desired job – though it was a significant reason for studying the degree. Those who had mathematical employment had outcomes that fitted with level 2 orientations; they understood mathematics and most were in work situations where they were helping others use mathematics. This assistance requires significant use of mathematical discourse to be able to help people. The level 3 orientations observed in undergraduate students were not so evident amongst this group of professionals. Their objectives were much more oriented towards those of their organisations. It may be that undergraduates have difficulty considering life as a mathematics professional and are unable to articulate or anticipate this role.

Table 8.2 Outcomes section of Table 3.2 (Reid et al., 2005)

Orientation	Outcomes
Techniques	1a. A pass, degree, qualification
	1b. A [better] job, status, money
	1c. Acquire mathematical tools and skills
Subject	2a. Understanding mathematics, practice, theory, applications
	2b. Help people using mathematics
Life	3a. A mathematical way of thinking or philosophy
	3b. Satisfaction of intellectual curiosity, personal growth

Conclusion

University changed these graduates. They gained technical skills, problem-solving and thinking tools, confidence and pride in themselves. They developed a mathematical identity. When they reflected on their learning they were modifying their reflections based on the employment and professional experiences they had now gained.

Reading the transcripts as a whole, one of the astonishing facts to come through is the lack of knowledge that was displayed by these graduates about the breadth of mathematics. There seemed to be little awareness of applied mathematics and statistics by those who had studied pure mathematics and vice versa. This was true not only of the techniques but also for the ways of thinking used in different areas. The graduates did not know a good deal about the subject they have majored in.

Another astonishing omission (and one found in other studies) is that most of these graduates were using mathematics on a daily basis but only Roger made the observations that, *you could perhaps even*

invent your own *one* [algorithm]. It is not even clear that he is referring to himself. None of the others believed that they could push the boundaries of mathematics itself. All the graduates were users and explainers of known mathematics but did not express the opinion that they could develop new mathematics. This is a strong criticism of university mathematics teaching.

Their reflections on teaching were not positive and there was robust criticism of teaching practices in mathematics. Graduates made suggestions as to changes that would have assisted their transition to the workplace, such as more group work, more orientation to the workplace and better interpersonal relations.

Summary of Chapter 8

Graduates reflected on their transition to the workplace and what they had gained from their university study. They made suggestions for changes to university programs to assist with their progress to the workplace.

The next chapter will contain my reflections on the research methodologies and results and discuss implications for curriculum design.

9 REFLECTIONS AND IMPLICATIONS

In this chapter, I reflect on the research methodologies and the process of researching the transition to the professional workplace. I also present changes to university programs in the mathematical sciences that will enhance the experience of mathematics learning. I make suggestions as to how the mathematics community can improve connections with graduates and enhance the connections of mathematics with the broader community, particularly industry and government.

Most of these recommendations are developed from the experiences of this group of mathematics graduates. Others come from research on students' experiences of learning mathematics at university. As shown in Chapter 3, there are many studies about facets of mathematics learning at university but many of these lack sufficient evaluation or the student perspective on the outcomes. Ways to incorporate mathematical discourse into curriculum are discussed in detail. The outcome space is used as a framework for the recommendations.

Most students who study mathematics at university do not proceed to gain a major in the mathematical sciences. Those who gain a mathematics major generally do not go on to become academic mathematicians. For these graduates, the basis of this investigation, the amount of technical mathematics that they use in their workplaces is less than what they have studied in the three years at university. Nonetheless, they have taken on a mathematical identity and have gained confidence and a range of problem solving skills that are transferable to their work and personal lives.

This study has shown that mathematics learning and teaching at university are not preparing students for professional life after university. Many graduates are unable to release the strength of their mathematics because they have not been taught how to communicate mathematically. Many are not given the opportunity because of the low level of mathematics required for their work. Some are unable to adapt to the work environment. Their technical mathematics skills are high but many did not use these skills or had to learn others. Graduates required more computing skills than were taught as part of their degrees.

Participants have suggested changes to content, learning methods and structure of university programs to assist with their transition to the workplace. They have also suggested that employers be made more aware of the importance of mathematics so that more mathematical jobs will be opened. Implementation of this latter suggestion is the role of the mathematical sciences community, industry and government. These results complement studies of employer mathematical needs by Kent et al. (2004) which found an increase in the requirement for mathematical literacy amongst employers and for skills of mathematical communication in the workforce.

In terms of curriculum design, we need to consider the whole learning experience of undergraduates, not just what is directly taught and assessed. An integrated approach to course design across an undergraduate program seems sensible, but one of the main barriers to the introduction of explicit teaching of graduate attributes is historical. The curriculum has developed and evolved over time and focuses on content-based subjects within one or two disciplines. The content of subjects is revised frequently, but perhaps not teaching methods, styles of assessment tasks or broader generic skills. At many institutions, there is choice in a science degree so students have some freedom to choose their majors and electives; hence, ensuring a structured series of subjects could be problematic. These issues are not insurmountable and, if addressed, could reap rewards with greater numbers of students completing majors in mathematics.

In order to assist with the development of teaching and learning of future graduates, departments and universities should improve connections with their alumni to track the destinations of all graduates.

Methodology

Before I move onto suggestions for implementation of the findings of this study, let me take some time out to consider the two methodologies which I used, that is, phenomenography and discourse analysis. What are the links between them and how have they assisted with the research aims? Table 9.1 shows that the first level found using each methodology is an emphasis on components; text itself, techniques of mathematics or the jargon of mathematical language. The next level considers the subject itself, whether it is mathematics or mathematical discourse or the production and distribution of text. The top level, Level 3, considers the subject within its social setting. The people involved have identities and are able to use texts as required to meet their personal and society's aims. The subject and the discourse are part of their life, their identity, and they are able to use and link them in appropriate ways.

The methodologies are different. Discourse analysis is founded on the study of artefacts of texts in social settings, while phenomenography focuses on the experiences of participants. Critical discourse analysis examines hegemony and the positioning of the text in society whereas phenomenography concentrates on the variation of experience and creates a hierarchy of outcomes based on the reflections of those experiences. That the two should give results that can be connected is interesting and compelling. I was particularly surprised at the strength conception in the outcome space for mathematical discourse. Whilst all the readings in discourse analysis discuss the importance of hegemony I had not expected an idea like that to emerge from these data. It is an unusual finding for a phenomenographic study and emphasizes the value in studying a complex area such as the transition to work. It is also interesting because these graduates had moved from the learning setting where they took a student role into a workplace setting where they had another, more diverse role. They had moved into society and were experiencing the altered power relationships and extra responsibility of the workplace.

In Chapter 3 the links between discourse analysis and phenomenography were noted by several researchers, particularly the notion that phenomenographic data collection and analysis resides in the discourse of the interviewer and participants. The discourse link may explain the ease with which connections between critical discourse analysis and the phenomenographic analysis can be made. Phenomenography uses discourse to explore and categorise conceptions. Participants use language to describe their experiences and this is then transcribed to make the text that is used for the phenomenographic analysis.

These methodologies have been a valuable way to explore the transition to the workplace. To be successful in the workplace, to become a professional, graduates need to be able to enter the discourse of their discipline and work within it. They also need to be able to translate the discourse for different audiences and different situations. I have been able to explore this with the use of these two methodologies and found that the interplay between the artefacts and the interview with the participant particularly illuminating. It was successful partly because we had the discourse of mathematics in common so participants could discuss their mathematical situations with me and I could make judgements about the type and level of mathematics and computing used because of my background. We did not share the discourse of communication as the majority of participants had not reflected on its role in their becoming a professional. There were many interpretations of communication by the participants, which made for an interesting study!

I believe that phenomenography is a practical methodology for reflection on learning. The categories for describing critical variations in experience are very useful. We all see the world differently based on our experiences. Even if we have what appears to be the same experience, we view it differently. Phenomenography describes these differences clearly so that lecturers and students are aware of the ways of seeing of others. Some approaches are more inclusive than others and these are the methods that curriculum developers should encourage by their design of learning experiences.

Table 9.1 Links between discourse and phenomenographic outcomes

Methodology	Author	Level 1	Level 2	Level 3
Discourse analysis	Fairclough (1992)	Text	Discursive practice (production, distribution, consumption)	Social practice
	Jørgenson & Phillips, (2002)	Text	Texts are produced and consumed	Constitution of the social world including social identities and relations
Phenomenography	Reid et al., (2005)	Techniques	Subject	Life
	Discourse outcome space (Table 6.1)	Jargon/Notation	Concepts/ Thinking	Strength

The following sections cover: changes to university mathematics programs, creating a supportive learning environment, modelling professional work and learning mathematical discourse. Each section contains ideas for implementation.

Curriculum design

How can curriculum developers design learning experiences so that graduates will make a successful transition to the workplace? Table 9.1 gives ample guidance. Graduates will be prepared for the workplace if they reach level 3 conceptions. As each level subsumes the previous, this means that they will be able to incorporate the lower level skills as well as the Level 3 skills an understanding.

While it is important that graduates are able to perform mathematical and computing techniques and know the relevant jargon and notations, it is essential that they are able to communicate their knowledge in a variety of circumstances and work in multidisciplinary teams in the workplace. This is achieved through modelling an extensive range of professional activities using authentic materials. The graduates who demonstrated the strength conception had been able to align their knowledge with the goals of their organisation. In academia the goal of mathematics research is often beauty and simplicity whereas the goals of industry and business are to make money – the goal of these participants was to get a good job.

The fact that these graduates did not demonstrate a broad knowledge of mathematics (in fact, they demonstrated the opposite) or the idea that they could develop new mathematics suggests that these are serious areas of omission in the curriculum. Even if students are going to specialise in an area of mathematics, they should be exposed to the breadth of their subject at an early stage. This should help alleviate the frequent complaint about lack of connections between the subjects studied. These participants believed that all the mathematics is known and that they are just applying it, that there are no opportunities for innovation.

Implications for curriculum design

The modifications recommended here come directly from this study, backed by literature in cases where appropriate articles are available. Some changes appear to be common sense; however this study and others have shown that changes are not occurring or that changes made have not been sufficient to make a difference to the perceptions of graduates. The fact that many suggestions seem like common sense confirms the need to continue to monitor the needs of graduates to enure that they are equipped for industry.

Computing

Participants recommended an increase in the amount of computer programming, such as Visual Basic and the teaching of industry-standard computing tools, such as Excel, SAS, SPSS.

Mathematical sciences

In response to the observation about graduates not knowing much about their subject area, curriculum designers should educate students about the breadth of mathematics and how the different parts fit together. There can be opportunities to discuss open questions in mathematics and where mathematics is heading. All students need information about mathematics careers and the role of mathematics in industry – even non-mathematics majors who should be aware of what mathematicians can offer. Links between topic areas should be made more explicit.

Teaching methods

Group work and collaborative learning are good ways to develop teamwork and interpersonal skills for the workplace and for creating a more friendly environment in the classroom (D'Souza & Wood, 2003a; 2003b; 2004). These participants recommended creating a personal environment in the classroom such as doing introductions to fellow class members. Academics should consider other teaching styles besides lecture/tutorial/laboratory, in particular peer teaching.

Assessment

The type of assessment given to students implies to them what is valued by the academic staff. Therefore it is important to incorporate a range of skills in assessment (for example, Smith et al., 1996). It is also good to vary the types of assessment tasks, for example, examinations, projects, posters, and laboratories. Variety ensures that graduates will have been exposed to different ways of working with mathematics and mathematical discourse. The assessment tasks should also test whether a student has achieved professional levels in academic discourse and graduate attributes. Assessment tasks should model professional work or use professional work itself

Creating a supportive learning environment

Focusing on the technical content of a mathematics degree and even the learning techniques used within subjects is only part of the equation. The attrition rate between first and third year university mathematics study is enormous. This is more prevalent for mathematics than other areas (DEST, 2005). Arguments, such as, that some students are only doing mathematics as part of another

degree like engineering do not hold water. The fact that these students have to do mathematics is a golden opportunity to capture their minds and convert them to the delights of mathematics.

Several researchers have studied the atmosphere of university mathematics learning in depth. Forgasz and Leder (2000) studied perceptions of the tertiary mathematics environment using a large-scale questionnaire survey of 1883 students and 71 interviews with undergraduate mathematics students. They found considerable variations in the quality of teaching and student support available at different mathematics departments. Their students suggested the need for:

> Better teachers and tutors, better course advice, more social contact within departments, a 'space of their own' and raising awareness of potential career paths. (Forgasz & Leder, 2000, p. 42)

Students made unfavourable comparisons between school and university mathematics learning and that the access to staff was much less and teaching was worse that they had had at school. Several students believed there was gender and race discrimination displayed by members of the mathematics department. Rodd (2002) followed a small group of students through a mathematics degree in the UK with in-depth interviews in each year and found the same unfavourable comparisons with school mathematics. Here is a quote from one of her participants:

> Real teachers explain things so you can understand and real teachers help you. Like the teachers I had at school, they helped you but here you've just got to help yourself 'cos they're too busy doing their lecture and writing on the board, they don't even look at you. That's something I've learned and I never thought it would be like that. Some are good but some, doing the lecture is all that seems to matter to them. They come in, some of them just come in and start, like they can't wait to get going and if you've not got your pen out and that, you've missed the start. They don't speak to you or anything, they just start like they've never been away, like it's just a continuation of where they left off. That's hard to get used to. I don't like it, I don't know why they do it. (Rodd, 2002, p. 6)

Rodd found that students were struggling with their identity as mathematics learners and that is was a very emotional time for them as they adjusted to university life. Many were the first in their families to attend university. It is possible in my study that several of the participants were the first in their families to more into a professional career but I did not ask. This may have made it more difficult for them to adjust – and all the more reason to make the teaching of professional skills more explicit. Our work on undergraduates confirms these ideas (Wood et al., 2006). Students are very unsure about how they are going to use their mathematics in their study and future work.

Implications for curriculum design

To create a supportive learning environment, curriculum designers need to pay attention to the transition to first year and develop support services for all students but particularly for those with different backgrounds. Possibilities such as creating a space for mathematics students, perhaps with peer tutors and learning resources have worked in many areas. Academics need to watch for discrimination, implicit and explicit.

To help students create their own mathematical identity, mathematics departments can invite students to seminars and conferences which give overviews of mathematics and show links between areas. Some mathematicians have found it helpful to include history of mathematics in their courses (Furinghetti, 2000).

Modelling professional work

The traditional approaches to introducing professional work into undergraduate studies include fieldwork, excursions, laboratory work, DVDs and computer simulations. Using modelling such as in-class conferences to develop communication skills is described in Wood and Perrett (1997) and reproduced in Appendix D.1. The development of communication skills and quantitative analysis skills for reading research that uses statistics is published in Wood and Petocz (2003a, 2003b) and described in Wood and Petocz (2002). These materials use real sources with carefully designed questions that encourage students into broader conceptions of their subject and link their study to professional work. Questions cover areas of critical reading, ethics and critically examining the use of quantitative analysis. Coutis and Wood (2002) evaluated this model in the context of teaching statistics to optometry students. The lecturers were able to point to the requirements of the professional society that expected quantitative literacy skills in their graduates. Students could clearly see that graduates in the field needed the work they were being assessed on at university.

Modelling of professional work situations, such as fieldwork, working in teams, writing reports and so on, is useful for the development of students' perceptions of work. To quote two students who had watched one of our statistics videos (Petocz & Wood, 2001), *For the first time in my life I saw and understood what the jobs that I might do when I graduate actually look like*, and *That's the first time it's convinced me that we actually use that [probability] in real life.* An essential task for the lecturer is to make such learning situations explicit so that students can make the connections between the learning situation and their future professional careers.

In areas where it is too time consuming, expensive or dangerous to take students into the field, one option is to bring the outside world into the classroom. This is particularly effective for motivation and for demonstrating professional roles, such as that of an engineer (Wood, Petocz & Smith, 2000) or a statistician (Petocz, Griffiths & Wright, 1996). Such videos model the role of the professional using visual means to motivate the learning of professional skills, and the variety introduced by the use of a different medium has proved popular with students (Petocz & Wood, 2001). Other ways to bring the professional world into the classroom are case studies or guest lecturers/experts.

Assessment is a major area where professional work can be modelled. Students can work in teams, with real data (often from the internet), preparing explanations for different audiences and discussion of results (Petocz & Reid, 2003). Explicit teaching of teamwork skills, communication skills, negotiation skills and project management will contribute to graduate attributes listed by employers. Additionally, opportunities can be created for broader discussions in class about aspects such as learning approaches, conceptions of the subject and the profession. Other professional skills and concerns, such as sustainability and ethics, can be discussed and form part of assessed work.

There is a clear need here for universities to be constantly updating their computing tools to fit in with the needs of industry. The amount of mathematics studied in a degree seems to be more than what is needed for most employment. There is space in the mathematics curriculum to develop other skills that graduates need and to bring in real world experience.

Implications for curriculum design
The aim is to model professional activities using real sources where possible.

- Create opportunities for students to interact with industry, such as industry placements, excursions, DVD presentations

- Keep contact with alumni (not just those with honours) and invite them to talk with undergraduates

- Consider a graduated approach, with the 'reality' level of projects increasing as students' skills improve

- Embed the mathematics in a context (Appendix D.3)

- Maintain close contact with the careers service and encourage students to use it

Learning mathematical discourse

I have shown the importance of mathematical discourse and how the lack of power with using appropriate discourse puts the mathematics graduate in a weak position in the workplace. Discourse skills can be taught by including them within the curriculum. As done with mathematics, a graduated approach can be used with the tasks based on the variety of discourse needs in the workplace. This would not only develop discourse skills but bring students into the community of mathematics and help change their perceptions from student to professional.

At Level 1 of Fairclough's hierarchy focussing on the components of text, the combination in mathematical discourse of the personal (we, you) and the formal (such as nominalisation, extended noun phrases and high lexical density) is very confusing for students to read and even harder to then duplicate themselves. In this study, I have shown that few of the graduates need to reproduce formal mathematics but they do need to be able to read and understand it, and then translate for their non-mathematical colleagues. Considering the situation of Boris in Chapter 7, he had to read the articles, find suitable algorithms and present it to his computer science and engineering team to make into products. Kay was grappling with ways to present the formal mathematics, particularly the extended noun phrases, in a meaningful way to her audience. Durand-Guerrier (2004) looked at finer points of mathematical discourse and, as an example, showed how undergraduate students have difficulty with the way mathematicians change the status of letters over the course of a proof. Some of the difficulties are to do with the way mathematicians use discourse (which can be improved) and some are caused by unfamiliarity with the discourse.

How can the results of this study be used to design curriculum to develop mathematical discourse skills? Firstly we can develop the variety of discourse situations used by these graduates in the workplace as described in Chapter 6. This does not require a reduction of mathematics in the curriculum but rather a placing of mathematics into the context of a real situation. As well as looking at the *types* of discourse that a graduate requires, curriculum designers can examine ways for graduates to reach the depth needed to successfully participate in the workforce using their technical knowledge. Here is where the phenomenographic analysis can be a guide.

The teaching suggestions here are not exhaustive and one method is not better than another. Integrated learning and transfer of skills will need a variety of approaches. For example, in a mathematics major made up of several subjects you may want to cover all the communication skills. For that reason, one subject may require the students to present seminars, another may have students working in teams on a project, another may involve collecting data from a field trip and writing up a report. Above all, there is a need for recognition of the importance of the concept of professional work and a willingness to engage with such issues.

Curriculum designers can use the three levels to design learning tasks that will guide a student to the strength conception. In Figure 9.1, notice how students are guided through the conception levels of both discourse and the mathematical content using an article by Garavan, (1997) reproduced in Appendix D.2. It is mainly a reading task and students are being guided to read critically. At the lower levels of conception, the discourse and the mathematical activities can be separated but at the higher levels they are intertwined.

Figure 9.1 Example of using the conceptions to develop curriculum

Preliminary reading (using Garavan, 1997)

Do the following pre-reading activities before you settle down to read carefully. This will save time and help you get the most out of your reading.

Identify the background of the article:

      (a) Who is the author?
      (b) What is the title?
      (c) What is the name of the publication it appears in?
      (d) Write out a reference for this article in one of the approved styles.

Skim through the article:

      (a) Set a timer for 5 minutes and skim the whole article. Get as good an idea as you can of what it is about in the time limit.
      (b) Summarise the article in 3-5 dot points.

Now read the whole article carefully and answer the following questions. They will help you identify the reasons why the author has used the research design and statistical techniques.

Identify the research question:

      (a) Write down the research question.
      (b) How is it presented?
      (c) Why has the author presented it this way?

Identify the research methodology used:

      (a) Write down the research methodology used.
      (b) Is it an observational study or an experimental study, or neither?
      (c) What sampling techniques are used (if any)?

How does the author deal with data?

      (a) What data are given in the article?

      (b) How are the data presented or described?

What statistical techniques are used and why?

      (a) List the statistical techniques used or referred to.

      (b) Give a reason for using each of them.

In-depth reading

The preliminary questions considered the techniques used in the original research and then in writing the article. The following set of questions and activities are to help you with thinking about the text and statistical design in more depth.

Aim and audience

      (a) What is the author's major aim in writing the article?

      (b) What clues do the statistical techniques give you about the aim?

      (c) What audience is the author writing for? Give reasons.

Content

      (a) Define hard, soft, tangible and intangible as used in this paper. How do they correlate with their meanings in natural language?

      (b) The first two-thirds of the article is quite different from the last third: in what way? How does the author join the two sections of his article? Do you think that the combination is successful?

Statistical analysis

      (a) The results of the study are summarised numerically in table 1. Explain the meaning of the rows of the table, and also of the columns of the table. Presumably, the numbers without brackets represent means: but means of what? Are the numbers in brackets standard deviations or standard errors of the means? Some information on sample sizes is given earlier in the text: make a reasonable guess about the sample size that applies to each column of figures.

      (b) Let's carry out some more detailed investigations on the data in the "Intervention" columns. Consider two extreme cases. In one of them, 37 people give a rating of 4 and one person gives a rating of 3. In the other, 19 people give a rating of 1, and another 19 people give a rating of 5. Find the mean, standard deviation and standard error of the mean in each case (enter the ratings into a computer package and ask for summary statistics). What does this tell you about the numbers in brackets in the table? Carry out the t-tests comparing the experimental and control hotels during the "intervention" phase. Do your results agree with those of Garavan?

      (c) Read through the results section and check some of the statements made against the information given in the table. Can you find any places where the written statements and the numerical results don't seem to match?

      (d) The previous investigations suggest that you can't believe everything you read, even in a refereed journal! In general, if the written description of results and the numerical presentation of results in an article do not match, which would you believe, and why?

Activity

The section describing the study procedures makes it clear that the receptionists in this study did not give their "informed consent" for the study. Would you be happy to participate in a study of this type at your (present or future) workplace under these conditions? What aspects of the study would have to be changed to incorporate "informed consent" of the participants?

Implications for curriculum design

In this section, the four macro skills of language are discussed. Clearly they are linked and in many cases should be taught in a linked manner. Nevertheless it is worth considering each separately and make suggestions for links between activities. The activities of the graduates will be used as examples of types of discourse to be modelled.

Speaking

Presentation of ideas

- Miniconference (materials provided, Appendix D.1). This task requires students to take given materials (such as a popular science book) and change the form of the material to make it suitable for a given audience. They then present the material as a miniconference to their peers and staff. This is generally a group work task with individual components.

- Miniconference (own materials). This is similar to the above except that students find their own materials for the conference. For example, they may be studying regression analysis so they will need to find data, analyse them and present their findings in a miniconference.

- Poster sessions. Similar to the above but presenting their findings in poster form.

Teaching

- Pedagogy subject. The idea here is to introduce mathematics students to aspects of teaching and learning theory. It is often used for third year students who are preparing to do honours in order to have well trained tutors for first year subjects. However, a pedagogy subject does more than prepare tutors; it helps the participants to learn better themselves (Oates et al., 2005).

- Peer teaching. Participants in this study were often required to teach their colleagues so peer teaching at university would prepare students to be able to explain mathematics to others using appropriate materials and language.

- Conducting tutorials. Again, students can be required to learn materials themselves and take it in turns to conduct tutorials.

- Preparing teaching materials. This is a mixed writing and teaching activity: for example Appendix D.4 shows an example that has mathematical content and asks them to place the content in an area that interests them by finding an article. It then requires the skills of researching the topic and presenting it to a specified audience.

Negotiating and selling ideas

One of the discourse needs that was often mentioned by participants was the need to inspire and to sell ideas to a non-mathematical audience. Indeed one of the problems with even getting employment in the first place was the need to sell the degree to employers. Ways to deal with this range from debates (for example, 'Why is calculus essential in engineering?') to interview practice (How would your mathematics training add value to our organisation?; How would you use your mathematics in the Department of Defence?)

Listening

Formal listening skills are developed in mathematics learning at university by attending lectures and tutorials. These are a particular type of skill. Graduates in the workplace also needed to be able to perform a broader range of listening behaviours, such as listening and then asking appropriate questions. Formal listening tasks could include:

- Writing a summary of a talk and drafting suitable questions that could be asked

- Writing down the three main points after listening to a lecture

Informal listening skills are also required and these are bound up with listening cues such as body language. It may be useful for lecturers to get in a guest lecturer for a session to discuss body language and other cues with final year students. It may increase their success in gaining employment and succeeding when they are there.

Writing

Writing is an important part of professional life. For graduates in industry it is not as critical as Burton (2004) found as for academic mathematicians nevertheless, many graduates will be judged in part on their written work. Reports are common as are writing manuals, help files and what could be described as teaching materials such as directions for others to follow (see Appendix D.4 for examples that develop this skill).

An important area for participants was the need to write quotes and responses to tenders. This is an area that has rarely been incorporated into mathematics teaching and learning and has outstanding potential for curriculum design. It brings together the ideas of the worth of mathematical knowledge, project and time management and has a real connection with the workplace. In Appendix D.5, I have included a GANTT chart and an Excel spreadsheet of a simplistic response to a tender including a quote and time line. This was developed for the design of a questionnaire but could be adapted for a variety of situations. Ideally students could find an appropriate tender in the press and write a proposal for fulfilling the tender.

In a similar fashion, grant writing and ethics applications are good ways to show students that their mathematical knowledge has a value in the workplace.

Reading

Participants were required to have high-level reading skills. Firstly they needed to be able to comprehend the materials and translate it into other forms, or summarise. Figure 9.1 gives an example of critical reading, where we start with the components of the text (word usage) and moving on to summarising the article and the aims of the author (construction and interaction with the text) to the ethical considerations of the generation of the text (how the text fits into the discipline). This is guiding the students to the strength conception as they work with the jargon and notations, the mathematical concepts and on to being able to place the mathematics in their discipline area.

Other critical reading requires graduates to be able to find appropriate reference materials for their purposes. Notice that both Kay and Boris were doing this. In addition both of these participants were distinguishing between good and bad writing for their purposes. Appropriate learning tasks include comparing textbooks. This is a good task for first year students as it gets them to read different accounts of a topic and consider how they themselves learn. Appendix D.6 gives an example which includes ideas of the use of definition in mathematics as well as developing mathematical skills and comparing the ways that mathematics can be presented.

Professional development for academic staff

Many university academic staff have been through a lecture/tutorial learning apprenticeship and may need training in different learning and teaching methods. They also may need support in implementing new computer packages into their teaching and learning (Houston et al., 2006).

Policy support from their institution, their heads of department and colleagues is important. Mathematicians tend to see themselves and their subject area as different to other areas and requiring different teaching methods. This insular way of viewing their subject area is not conducive to the adoption of new teaching methods.

Houston et al. (2006) surveyed Heads of Departments of mathematics in several countries and found that, in the UK, there is a call for early career teaching development that is subject specific in mathematics. In other countries the requests for discipline-specific professional development is not so clear and it may depend on the quality of the generic professional development programs and the links made within those programs to the subject areas.

Houston's survey found that all universities valued research, however, there was a divide between the cultures of universities which valued teaching. New staff received some training in teaching and induction to the university, but this was not common for sessional teaching staff. Professional development for experienced staff in teaching was neither encouraged nor discouraged but left up to the individual, with encouragement coming from teaching awards, conferences and grants in some institutions. Most saw developing and spending time on teaching and learning as taking time away from research. This attitude will make curriculum and teaching change difficult in universities.

As has been demonstrated in this study, content knowledge is important nevertheless the need for changes in learning and teaching is also critical. Professional development for new and continuing staff to enhance their teaching and learning practices should be mandatory. Both subject specific and general teaching and learning principles should be included in professional development.

Competitions

Competitions can play a role in focusing students' attention to the advantages of good writing. Competitions have a limited, but valuable role in improving mathematical discourse by focussing the students' attention on the possibility of rewards for good writing. Ultimately, all undergraduate and postgraduate students should be in a position to write well in their discipline area for a variety of audiences. This should be part of the skill set of all graduates.

The advertisement for the students writing competition (Figure 9.2) is aimed at postgraduate students of the Australian Research Council (ARC) research group Centre for Ultrahigh Bandwidth Devices for Optical Systems (CUDOS). The competition is similar to the international competition

run each year by the *New Scientist* magazine, which is aimed at postgraduate students of science and technology. Notice how the advertisement gives details of how to write for this audience, including sentence length and the need for a 'hook'. Evidently the organizers do not believe that the writers would know this already.

Figure 9.2 Student writing competition

2006 CUDOS student competition

All CUDOS students are strongly encouraged to participate in the 2006 CUDOS student competition. Your challenge is to write a popular article on your thesis work, aimed at the intelligent lay person, which would be suitable for publication in New Scientist, the science section of the Economist, COSMOS magazine, your university's newspaper, or the CUDOS website.

Entries will be judged by David Ellyard from the National Council for the Australian Science Communicators, Sara Phillips from COSMOS magazine, and Ross McPhedran from CUDOS. The winner and runner up, who will receive cash prizes of $1000 and $250, respectively, will be announced during the CUDOS workshop to be held in Hervey Bay on 9 and 10 August. COSMOS has kindly offered a cap and poster to the winner, and free magazines to the top entries. In addition, every entry will be considered for publication on their web site, and outstanding ones will be considered for publication in the magazine. All submissions commended by the judges will be awarded a certificate.

Your article should engage the reader immediately: start with a "hook," such as showing how their lives will be affected by the research or maybe giving a gee whiz overview of the science, followed by a description of the research. Try to take a "big picture" view of your work, avoiding narrow technicalities and conveying some of the excitement of the field. Do so by putting yourself into the story, rather than by describing what happened.

An illustrative figure or photograph is encouraged, but the article should be largely self-explanatory. It is important that you write clearly, preferably in an engaging conversational manner, as though you are talking to somebody, avoid jargon, acronyms (well OK, perhaps one if you really must), and overly long sentences, to make sure that the reader does not forget the beginning of the sentence when finally making it to the end. Explain all science concepts clearly. Try to find a catchy title to lead the reader in.

Your entry should be 500 1000 words in length.

COSMOS' website can be found at http://www.cosmosmagazine.com/

Professional societies

Only one participant in this study was a member of a mathematical professional society. In Australia, the Australian Mathematical Society and the Statistical Society of Australia have relatively narrow membership bases. Measures are being taken to improve this situation with the establishment of the Australian Mathematical Sciences Institute (AMSI, www.amsi.org.au), modelled on the Canadian Fields Institute (www.fields.utoronto.ca/). By contrast, high school mathematics teacher societies have an active membership and wide coverage of secondary mathematics teachers. In Australia this is mainly done at a state level due to state education systems.

Consideration should be given to free membership of the Australian Mathematical Society for all third year mathematics majors as well as free online access to the Society's journals. With the web tools available now, societies could have members update their own information and so keep track of members whatever their new employment circumstances. Professional societies encourage good writing in their journals and publishing practical guidelines for authors.

Professional societies, Academies of Sciences and Institutes of Mathematical Sciences play an important role in alerting industry and government to the opportunities that mathematics brings to society. The societies, along with the universities, need to make sure that mathematics graduates have the range of professional skills, alongside the mathematical skills, to potentially make a significant contribution. I believe that the promise has been there but in many cases the graduates have not been able to deliver because of their limited experience in communication and interpersonal skills.

Farewell to the participants

These graduates volunteered to participate in this study and their contributions have shed light on the transition between work and learning. They seemed to enjoy the interview experience and freely gave of their ideas and reflections. They were very thoughtful and wanted to help those who will come after to make a smoother transition. Several have kept in contact to let me know how they are progressing in their careers. Doing a research study is an excellent way to meet some of our talented and reflective alumni. Perhaps we should do it more often.

Conclusion

In an in-depth study like this one, I am examining the perceptions of a small group of graduates. Their experiences are important and can inform curriculum development. For example, even if lecturers believe that the curriculum is integrated and subjects are connected, that is not the experience of this group. The connections were not sufficiently explicit.

The work situations that the graduates entered taxed their resilience and some did not make it in the environment. Many have changed careers and others are frustrated with their circumstances, as Leah said: *if I haven't found a niche in the next couple of years then I'll start to get really shitty.* Most believed that university changed them and their expectations, such as David: *It made me appreciate the bigger picture. It sort of opened up avenues of my career that I never would have ever imagined could have existed.*

The study revealed serious areas of under-preparedness for the workforce. Graduates were not ready for the office environment or to deal with colleagues and managers. Their first experience, particularly with their manager, set the scene for their adjustment. Many of the mathematicians were alone with their area of expertise and had to adjust to the language of those around them with no training for this from university. Most were ill-equipped for the job-seeking process itself and had to educate employers on their skill set in order to show they could do the job. Graduates were unaware of language choices and how to communicate at different levels.

Many of these problems could be alleviated by changes in curriculum. Firstly, course designers and lecturers should make connections between subject areas explicit – explicit to the students. Other teaching and learning suggestions should be seriously considered. A well-designed third year *Transition to the Workforce* subject could lead to the study of a project, consulting or peer teaching and would develop many of the generic team and communication skills needed for the workforce as well as consolidating mathematical skills. This could be complemented by smaller 'real' projects in previous subjects. Universities can coordinate opportunities for work experience in the university vacations. There is a need for all agencies of the universities to work together to assist in the transition to the workforce.

The outcome space developed in this study is an excellent guide for curriculum designers as they can be clear as to the aims of their curriculum and the discourse analysis tells them how to do this. It is also easy to make it clear to the students as to the aims of assessment tasks and learning experiences.

Challenges for the mathematical and statistical societies include increasing the membership base and maintaining contact with those who are not in academic positions. There needs to be a strong community of practice in the mathematical sciences. Employers and governments must be encouraged to see how mathematics can contribute to the efficiency and quality of their organisations.

Given that these graduates are not using all the mathematics that they learnt at university, there is opportunity to reduce the amount of content and increase the development of generic capabilities and general learning skills. After all, mathematics will continue to change and graduates will need the skills to be able to learn new areas that are not in current degree programs.

The mathematical sciences are important for the future of Australia and internationally. Our graduates need a broader range of skills to succeed in the marketplace and employers need to be educated as to the skills and ideas that mathematics graduates can offer their organisations. There is a crucial need for curriculum reform to assist with transition to the workforce in technical areas so that we do not squander people who have learnt technical skills, but not job seeking and generic skills that will help them secure employment and thrive in their careers.

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Appendix A. Publications from this study (CD)

1. Wood, L.N. (2004). University Teaching and Learning. *MERGA Review of Research in Australasia 2000-2003*, 219-242.

2. Wood, L.N. & Reid, A. (2005). Graduates' initial experiences of work. *Proceedings of the Australian Association for Educational Research.* [Online].

3. Wood, L.N. & Reid, A.. (2005). Graduate voices: new mathematics graduates in industry. *DELTA05*, 125-131.

4. Wood, L.N. & Kaczynski, D. (2005). University students in USA and Australia: anticipation and reflection of the transition to work. *Proceedings of the 4th International Conference on Researching Learning and Work.* [Online].

5. Wood, L.N. & Reid, A. (2006). Conversations with graduates: reflections on learning mathematics. *ICTM3* [CD].

Appendix B. Ethics approval, invitation letter and consent form

Ethics approval

20 December 2003

Ms Leigh Wood
Department of Mathematics
Level 15, Tower Building
Broadway Campus

Dear Leigh,

UTS HREC 03/113 - WOOD, Ms Leigh, REID, Dr Anna – "Communicating mathematics"

Thank you for your response to my letter dated 18 December 2003. Your response satisfactorily addresses the concerns and questions raised by the Committee, and ethics approval is now granted.
Your approval number is UTS HREC 03/113A.

Please note that the ethical conduct of research is an on-going process. The *National Statement on Ethical Conduct in Research Involving Humans* requires us to obtain a report about the progress of the research, and in particular about any changes to the research which may have ethical implications. The attached report form must be completed at least annually, and at the end of the project (if it takes more than a year), or in the event of any changes to the research as referred to above, in which case the HREC Secretariat should be contacted beforehand.

I also refer you to the AVCC guidelines relating to the storage of data. The University requires that, wherever possible, original research data be stored in the academic unit in which they were generated. Should you submit any manuscript for publication, you will need to complete the attached *Statement of Authorship, Location of Data, Conflict of Interest* form, which should be retained in the School, Faculty or Centre, in a place determined by the Dean or Director.

Please complete the attached (green) report form at the appropriate time and return to the HREC Secretariat in the Research and Development Office, Broadway. In the meantime, if you have any queries please do not hesitate to contact either myself, or the Research Ethics Officer, Ms Louise Abrams on 02 9514 9615.

Yours sincerely,

Associate Professor Jane Stein-Parbury
Chairperson, UTS Human Research Ethics Committee

Invitation letter and invitation email

14th January 2004

Communicating mathematics

Dear Graduate,

You are invited to participate in a study of mathematics and how it is communicated. The purpose is to improve the design of university teaching and learning programs.

The study is being conducted by Leigh Wood, Department of Mathematics, University of Technology, Sydney, phone: 9514 2268 and Dr Anna Reid, Centre for Professional Development, Macquarie University, phone 9850 9780. The study is part of the requirements for the degree of Doctor of Philosophy.

We would like to interview you individually for about one hour.
We will come to a place convenient to you.

If you decide to contribute, you will be interviewed about your professional communication skills and needs. This interview will be audiotaped. Any information or personal details gathered in the course of the study are confidential. No individual will be identified in any publication of the results.

If you would like to participate, please contact the research assistant, Ms Glyn Mather (Glyn.mather@uts.edu.au) to arrange an appointment. Interview times are available from 10th February 2004 until 27th February 2004. Day or evening times are available.

We look forward to your involvement in this important project to better connect university learning with skills needed in the workplace.

Best wishes

Leigh Wood
Project Coordinator

Dear Graduate,

Have you graduated in the last 5 years?
Did you major in maths/stats or a related area?

We need you!

We are a group of lecturers who are working to improve mathematics learning at university.

You are invited to participate in a study of mathematics and how it is communicated. The purpose of the study is to improve the design of university teaching and learning programs. The study is being conducted to meet part of the requirements for the degree of Doctor of Philosophy under the supervision of Dr Anna Reid, Centre for Professional Development, Macquarie University, phone 9850 9780.

The study is being conducted by Leigh Wood, Department of Mathematics, University of Technology, Sydney, phone: 9514 2268 and Dr Anna Reid, Centre for Professional Development, Macquarie University, phone 9850 9780.

We would like to interview you individually for about one hour.
We will come to a place convenient to you.
For more information, please phone or email
Leigh Wood 9514 2268
leigh.wood@uts.edu.au

Consent form

Name of Project: **Communicating mathematics**

You are invited to participate in a study of mathematics and how it is communicated. The purpose of the study is to improve the design of university teaching and learning programs.

The study is being conducted by *Leigh Wood*, Centre for Professional Development, Macquarie University, phone: 9514 2268. The study is being conducted to meet part of the requirements for the degree of Doctor of Philosophy under the supervision of Dr Anna Reid, Centre for Professional Development, Macquarie University, phone 9850 9780.

If you decide to participate, you will be asked to participate in an interview asking you about your communication skills and needs. This interview will be audiotaped. The interview should take about 1 hour and can be done at any place that is convenient for you.

Any information or personal details gathered in the course of the study are confidential. No individual will be identified in any publication of the results. The transcripts will be coded with a pseudonym and the original tapes stored in a locked filing cabinet. Only Dr Anna Reid, Leigh Wood and a research assistant (typing the transcripts) will have access to the original tapes.

If you decide to participate, you are free to withdraw from further participation in the research at any time without having to give a reason and without consequence.

I, _____have read and understand the information above and any questions I have asked have been answered to my satisfaction. I agree to participate in this research, knowing that I can withdraw from further participation in the research at any time without consequence. I have been given a copy of this form to keep.

Participant's Name:
(block letters)

Participant's Signature: _____Date:

Investigator's Name: _____LEIGH WOOD_____
(block letters)

Investigator's Signature: _____ Date:

The ethical aspects of this study have been approved by the Macquarie University Ethics Review Committee (Human Research). If you have any complaints or reservations about any ethical aspect of your participation in this research, you may contact the Ethics Review Committee through its Secretary (telephone 9850 7854; email kdesilva@vc.mq.edu.au). Any complaint you make will be treated in confidence and investigated, and you will be informed of the outcome.

Appendix C. Texts for discourse analysis (CD)

Texts for teaching

C.1 Shrubbery, beasties & dirt … our observations of them

C.2 Overhead transparencies

Texts for industry

C.3 Vaudenay, S. (2003). Decorrelation: A Theory for Block Cipher Security, *J. Cryptology*, *16*, 4, 249-286.

C.4 Lenstra, A.K. & Verheul, E.R. (2000). *Advances in Cryptography – Crypto 2000*. Springer-Verlag.

Appendix D. Examples of discourse activities

D.1 Miniconference. Task and marking scheme

Aim: To develop skills of oral and written presentation and working in groups.

This task is a group assessment. You will be assessed on your individual presentation and on your group work. This is a long-term assignment and needs at least 6 weeks for adequate preparation.

Topic: The conference will be on *How to Read and Do Proofs* by Daniel Solov. You will work in groups of three and take one chapter of the book for each group.

Section 1: Organising the whole class

(a)	Elect or assign a group leader, deputy and editor. Decide on their roles.
(b)	Decide on editorial guidelines for the layout and referencing of summaries. These will be handed out to the audience.
(c)	Allocate referees for summaries. Each group will referee the summaries of one other group.
(d)	Decide on a timetable for completion of summaries and keep to the deadlines. (summaries due for refereeing on 11th May)
(e)	Give summaries to the editor. (25th May)
(f)	Organise an agenda for the conference and appoint chairpeople and timekeepers. (25th May)
(g)	Organise equipment. Do you need overhead projectors, slide projectors, videos, computers and so on? (tell the tutor what you will need)
(h)	Last-minute checks on equipment and rooms.

Section 2: Group tasks (three people per group).

(a)	Present a 15 minute conference talk, 5 minutes each person.
(b)	Write a three page summary/handout to accompany the presentation, approximately one page per person. This will be printed and distributed with the conference agenda.
(c)	Referee draft summary of another group. Comments can be written on their papers. As the refereeing will be assessed, copies of the draft with your comments are handed to your marker.

Marking: Out of 60 then divide by 2 for assessment mark. Students who do not participate in the planning sessions will receive 0 for the class mark section of the assessment.

Conference organisation (Class mark)

0	1	2							The organisation of the topic and sub-topics work well
0	1	2							The timing and links between speakers work well
0	1	2	3						The conference style for summaries is good
0	1	2	3						The class works well together

Conference presentation (Group mark)

0	1	2							Your presentation is within the set time limits
0	1	2							Your presentation shows careful preparation
0	1	2	3						Your presentation is appropriate for the audience
0	1	2							The group works well together
0	1	2	3						Questions were answered correctly

Conference presentation (Individual mark)

0	1	2							Good use of eye contact and body language
0	1								Voice is audible
0	1	2							Aids are readable
0	1	2							Use of equipment is appropriate
0	1	2	3						Your presentation is effective
0	1	2	3						Your mathematics is correct
0	1	2							Your presentation is entertaining

Conference summary/handout (Group mark)

0	1	2							The style of the summary/handout follows the agreed format
0	1								The referencing is correct and consistent
0	1	2	3	4	5	6	7	8	The content is well researched, accurate and informative
0	1	2							The summary/handout communicates effectively
0	1	2							Your use of layout, language, spelling and punctuation is correct

Refereeing (Group mark)
Please hand in the papers that you refereed with your comments

0	1	2	3	4	5	6	7	8	Your comments were appropriate. You found all errors. You made constructive criticisms about layout and content

35100 Mathematical Practice

Peer Assessment Form

Be objective in your assessment. This assessment is for the whole semester and includes the weekly sessions and the conference.

Name: _____ Student number: _____ Group code: _____

Please award each group member, including yourself, a mark out of 5 for each of the following factors:

MARKS:
5 = much higher than the rest of the group
4 = higher than the rest of the group
3 = same as rest of group
2 = less than rest of group
1 = much less than rest of group
0 = no contribution in this way.

Write surnames of group members here → Factors ↓	S E L F			
Level of attendance at group meetings:				
Level of contribution at group meetings:				
Level of contribution outside group meetings:				
Idea suggesting (eg getting started, getting around difficulties, forms of validation):				
Extracting something useful from the ideas (eg. identifying workable approaches, tying ideas together, getting useable equations, examples):				
Keeping the momentum going (eg smoothing group operations, good planning):				
Performing tasks (eg writing up, calculating, computing, research):				
OTHER: (Specify here any other useful roles performed by members.) --				

D.2 Example using the three levels of discourse and conceptions of discourse

See Figure 9.1 for the questions. The article referred to in the questions is on the attached CD.

Garavan, T.N. (1997). Interpersonal skills training for quality service interactions. *Industrial and Commercial Training, 29*, 3, 70-77. (CD)

D.3 Example placing mathematics in context

Question 1 (10 marks)

(a) Define a tridiagonal matrix (reference your sources)
(b) Define a symmetric matrix
(c) Define an upper triangular matrix.

(d) Consider the matrix $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$.

    a. Use row reduction to find the upper triangular matrix
    b. Find the value of the determinant.

(e) Consider the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$.

    a. Use row reduction to find the upper triangular matrix
    b. Find the value of the determinant.

(f) Consider the matrix $\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$.

    a. Use row reduction to find the upper triangular matrix
    b. Find the value of the determinant.

(g) Consider the $n \times n$ matrix $\begin{pmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & 0 & 0 & \vdots \\ 0 & -1 & \ddots & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & -1 & 0 \\ \vdots & 0 & 0 & -1 & 2 & -1 \\ 0 & \cdots & \cdots & 0 & -1 & 2 \end{pmatrix}$.

    a. Find the value of the determinant.
    b. Prove your answer.

Question 2 (10 marks)

Search the Web and the library to find an application of a tridiagonal matrix . You may need to look at a few sources before you find one that you can understand.

(a) In about 2 A4 pages, summarise the application. You can use diagrams, mathematics and words. Consider that your classmates are the audience.
(b) Include references to the material you have used.

D.4 Example of developing teaching skills

35212 Linear Algebra: Assignment 1

Due Wednesday 12th October

Please hand in your assignment in groups of 1 or 2 students.
Only hand in one assignment per group.

This assignment deals with WAVELETS. Wavelets are used in image processing and other areas. You will need to define terms used in your work. For example, if you find an example that uses a *sparse* matrix you would need to define what a sparse matrix is. You need to reference any material you have found on the Internet or in books – including definitions.

Question 1 (12 marks)

    (a) What is a wavelet?
    (b) Find an article that uses wavelets in an area that is interesting to you. Write a 500-word summary of the article. Hand in the summary and the article.
    (c) Answer questions 1–7 in the attached handout (Not attached here. These were mathematical exercises using matrices, including the process of finding wavelets)

Question 2 (8 marks) Bonus marks may be awarded for exceptional work.

Imagine you are tutor teaching our class about wavelets. In about 2–4 pages, design a handout to teach this topic to the class. Consider your fellow students to be the audience for this handout.

D.5 Examples of GANTT Chart and budget for a survey project

From lecture notes (Darcy, UTS)

Month	August				September				Break	October					November		Hours
Week	1	2	3	4	5	6	7	8	Break	9	10	11	12	13	14	15	Hours
Week commencing	30-Jul	6-Aug	13-Aug	20-Aug	27-Aug	3-Sep	10-Sep	17-Sep	24-Sep	1-Oct	8-Oct	15-Oct	22-Oct	29-Oct	5-Nov	12-Nov	
Research proposal	█	█	█	█													20
Review literature & secondary data	█	█				█	█	█									35
Liaison	█				█		█				█		█		█		25
Observation & subsequent analysis				█													8
Questionnaire design & coding			█	█	█	█											24
Street survey							█	█									200
Data entry & processing											█	█					25
In-depth interviews										█	█						4
Analyse & transcribe interviews												█					4
Write report								█	█	█	█	█	█	█			40
Presentation														█	█		5
Submit final report to client																█	
Total Hours																	390

193

BUDGET BREAKDOWN

TASK	HOURS	HR RATE	COST
STAGE: 1 SURVEY DESIGN			
Literature search	24	$50.00	$1,200.00
Study Design	8	$100.00	$800.00
Questionnaire design	12	$100.00	$1,200.00
Liaison	8	$100.00	$800.00
STAGE 11: DATA COLLECTION			
Literature review/secondary	50	$50.00	$2,500.00
Phone questionnaire survey based on 4 per hour	100	$25.00	$2,500.00
In depth interviews (5)	10	$50.00	$500.00
Training and supervision	30	$50.00	$1,500.00
Liaison	6	$100.00	$600.00
STAGE 111: DATA ANALYSIS			
Data entry	50	$25.00	$1,250.00
Coding of questionnaires	16	$25.00	$400.00
Data processing (SPSS)	5	$25.00	$125.00
Analysis	16	$50.00	$800.00
Transcripts	20	$25.00	$500.00
Liaison	5	$150.00	$750.00
STAGE IV: REPORT			
Recreational facilities review	16	$50.00	$800.00
Needs analysis	16	$50.00	$800.00
Draft report	16	$50.00	$800.00
Final report	12	$50.00	$600.00
Liaison	4	$100.00	$400.00
Supervision	4	$100.00	$400.00
Total hours	428		
MISCELLANEOUS COSTS			
Printing (pages)	1200	$0.05	$60.00
Phone (number of calls)	1700	$0.20	$340.00
Car/km	250	$0.50	$125.00
Subtotal			**$19,750.00**
GST 10% flat rate			$1,975.00
Total cost			**$21,725.00**

D.6 Critical reading skills

Aim: To develop skills in reading textbooks.

Section 1: For this task you will need several mathematics textbooks covering approximately the same content. You can find examples in the library. Choose recent editions.

(a)   Choose three textbooks at a similar level and within your mathematical expertise. Spend about 15 minutes looking through each book. From this brief inspection, state which book is the most student-friendly, with reasons.

(b)   Textbooks are written for a particular audience. Who is the audience for each of the textbooks you have examined?

Section 2: Using the same textbooks, this task requires you to look more closely at the presentation of topics and exercises.

(a)   Go to the index and look up the word **function**. Find where the word is first defined and write down the definition of function given in each textbook. Also note whether any illustrations or examples are used to explain the concept.

(b)   Following from section 2(a), compare the way the definitions are explained in the different textbooks and give your opinion, with reasons, as to which is the most successful.

(c)   Choose one set of exercises on the same topic in each textbook, such as **composite functions** in the calculus textbooks or any appropriate topic in other books. Look at the balance between procedural questions, conceptual questions and applications. Work through all the exercises in the section. Which set of exercises best helped you understand the topic? Why?

Marking:

0	1	2				1(a)	The reasons given support the position taken
0	1	2	3			1(b)	Each audience is correctly identified, with appropriate reasons
0	1	2	3	4	5	2(a)	The definitions, illustrations and examples are correctly identified
0	1	2	3	4	5	2(b)	The reasons given support the position taken
0	1	2	3	4	5	2(c)	The reasons given support the position taken