

MACQUARIE UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

# *Star Formation and the Hall Effect*

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## Abstract

Magnetic fields play an important role in star formation by regulating the removal of angular momentum from collapsing molecular cloud cores. Hall diffusion is known to be important to the magnetic field behaviour at many of the intermediate densities and field strengths encountered during the gravitational collapse of molecular cloud cores into protostars, and yet its role in the star formation process is not well-studied. This thesis describes a semianalytic self-similar model of the collapse of rotating isothermal molecular cloud cores with both Hall and ambipolar diffusion, presenting similarity solutions that demonstrate that the Hall effect has a profound influence on the dynamics of collapse.

Two asymptotic power law similarity solutions to the collapse equations on the inner boundary are derived. The first of these represents a Keplerian disc in which accretion is regulated by the magnetic diffusion; with an appropriate value of the Hall diffusion parameter a stable rotationally-supported disc forms, but when the Hall parameter has the opposite sign disc formation is suppressed by the strong diffusion. The second solution describes the infall when the magnetic braking is so efficient at removing angular momentum from the core that no disc forms and the matter free falls onto the protostar.

The full similarity solutions show that the size and sign of the Hall parameter can change the size of the protostellar disc by up to an order of magnitude and the accretion rate onto the protostar by  $1.5 \times 10^{-6} \text{ M}_{\odot} \text{ yr}^{-1}$  when the ratio of the Hall to ambipolar diffusion parameters moves between the extremes of  $-0.5 \leq \tilde{\eta}_H/\tilde{\eta}_A \leq 0.2$ . These variations (and their dependence upon the orientation of the magnetic field with respect to the axis of rotation) create a preferred handedness to the solutions that could be observed in protostellar cores using next-generation instruments such as ALMA.

Hall diffusion also determines the strength of the magnetic diffusion and centrifugal shocks that bound the pseudo and rotationally-supported discs, and can introduce subshocks that further slow accretion onto the protostar. In cores that are not initially rotating Hall diffusion can even induce rotation, which could give rise to disc formation and resolve the magnetic braking catastrophe. The Hall effect clearly influences the dynamics of gravitational collapse and its role in controlling the magnetic braking and radial diffusion of the field would be worth exploring in future numerical simulations of star formation.

## Statement of Candidate

I certify that the work in this thesis, entitled “Star Formation and the Hall Effect”, has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree to any other university or institution other than Macquarie University.

I also certify that this thesis is an original piece of research and that it has been written by myself. Any help and assistance that I have received in my research work and the preparation of the thesis itself have been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis.

Catherine Braiding (30615399)

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“It seems to me, that if the matter of our sun and planets, and all the matter of the universe, were evenly scattered throughout all the heavens, and every particle had an innate gravity towards all the rest, and the whole space throughout which this matter was scattered, was finite, the matter on the outside of this space would by its gravity tend towards the matter on the inside, and by consequence fall down into the middle of the whole space, and there compose one great spherical mass. But if the matter were evenly disposed throughout an infinite space, it could never convene into one mass; but some of it would convene into one mass and some into another, so as to make an infinite number of great masses, scattered great distances from one to another throughout all that infinite space. And thus might the sun and fixed stars be formed, supposing the matter were of a lucid nature.”

- Newton to Bentley (December 10, 1692), quoted by Jeans (1928)

