

MACQUARIE UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

*Star Formation and
the Hall Effect*

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Doctor of Philosophy (Physics)

June 2011

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Abstract

Magnetic fields play an important role in star formation by regulating the removal of angular momentum from collapsing molecular cloud cores. Hall diffusion is known to be important to the magnetic field behaviour at many of the intermediate densities and field strengths encountered during the gravitational collapse of molecular cloud cores into protostars, and yet its role in the star formation process is not well-studied. This thesis describes a semianalytic self-similar model of the collapse of rotating isothermal molecular cloud cores with both Hall and ambipolar diffusion, presenting similarity solutions that demonstrate that the Hall effect has a profound influence on the dynamics of collapse.

Two asymptotic power law similarity solutions to the collapse equations on the inner boundary are derived. The first of these represents a Keplerian disc in which accretion is regulated by the magnetic diffusion; with an appropriate value of the Hall diffusion parameter a stable rotationally-supported disc forms, but when the Hall parameter has the opposite sign disc formation is suppressed by the strong diffusion. The second solution describes the infall when the magnetic braking is so efficient at removing angular momentum from the core that no disc forms and the matter free falls onto the protostar.

The full similarity solutions show that the size and sign of the Hall parameter can change the size of the protostellar disc by up to an order of magnitude and the accretion rate onto the protostar by $1.5 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ when the ratio of the Hall to ambipolar diffusion parameters moves between the extremes of $-0.5 \leq \tilde{\eta}_H/\tilde{\eta}_A \leq 0.2$. These variations (and their dependence upon the orientation of the magnetic field with respect to the axis of rotation) create a preferred handedness to the solutions that could be observed in protostellar cores using next-generation instruments such as ALMA.

Hall diffusion also determines the strength of the magnetic diffusion and centrifugal shocks that bound the pseudo and rotationally-supported discs, and can introduce subshocks that further slow accretion onto the protostar. In cores that are not initially rotating Hall diffusion can even induce rotation, which could give rise to disc formation and resolve the magnetic braking catastrophe. The Hall effect clearly influences the dynamics of gravitational collapse and its role in controlling the magnetic braking and radial diffusion of the field would be worth exploring in future numerical simulations of star formation.

Statement of Candidate

I certify that the work in this thesis, entitled “Star Formation and the Hall Effect”, has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree to any other university or institution other than Macquarie University.

I also certify that this thesis is an original piece of research and that it has been written by myself. Any help and assistance that I have received in my research work and the preparation of the thesis itself have been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis.

Catherine Braiding (30615399)

15 / 7 / 2011

Acknowledgements

Writing this thesis has been the hardest thing I have ever done; over the course of the last few years my self-esteem and sense of self-worth have plummeted, and I could not have finished without the support of a great number of people. To all of them I give my heartfelt thanks, but in particular I would like to acknowledge:

- my supervisor, Mark Wardle, who is brilliant and funny and has always supported me, even when he was overburdened with too many PhD students to supervise and theses to read. You inspire me, and if I continue on this path it will be thanks to you.
- Gemma and James, for all the hugs, tears and unconditional love, as well as the gifts of backup drives. You make me a better person; when times were rocky you were there for me and I love you both.
- Korinne, Sarah and Anna, the best PhD buddies and officemates anyone could ask for. Thanks for the songs, the cupcakes, the hugs, and making it through to the end with me — I'm proud of all of us.
- Korinne again, because she proofread this beast twice. Thanks, crazy lady.
- my family, for everything. I promise I'll get a real job soon.
- the department of physics and astronomy and others, particularly Alan, Judith and Carol, for the moral and occasional monetary support.
- Raquel, Pandey, and the participants of the *DDP* and *CC2YSO* workshops in 2009 and 2010, for the engaging and stimulating discussions.
- finally, my closest friends: Brendan, Heidi, Tony, Nilanka, Roberto and Drew; the West Ryde knitters; and all the other friends, online and off, who have listened to my rants and offered sympathy as needed. May all of you continue to be awesome.

Thank you all,
Catherine

Contents

1	Star Formation	1
1.1	Molecular Clouds	3
1.2	Magnetic Diffusion	9
1.3	Gravitational Collapse	15
1.3.1	Core formation and support	16
1.3.2	Dynamic collapse	20
1.3.3	Late stages of collapse	23
1.4	Rotation and the Angular Momentum Problem	25
1.5	Magnetic Fields and the Magnetic Flux Problem	29
1.6	The Hall Effect in Star Formation	32
1.7	Project Outline	34
2	Self-Similar Gravitational Collapse	37
2.1	Basic Equations	38
2.2	Assumptions	39
2.3	Vertical Averaging	42
2.3.1	Conservation of mass	44
2.3.2	Conservation of radial momentum	44
2.3.3	Conservation of angular momentum	46
2.3.4	Vertical hydrostatic balance	47
2.3.5	z -component of the induction equation	47
2.3.6	Radial field component, $B_{r,s}$	51
2.4	Vertical Angular Momentum Transport	51
2.5	Further Simplifications	56
2.6	Self-Similar Form of the Equations	57
2.7	The Outer Solution	60
3	The Inner Asymptotic Solutions	65
3.1	Derivation	66
3.2	Keplerian Disc Solution	70
3.3	Free Fall Solution	77
3.4	Summary	81

4	Collapse without the Hall Effect	85
4.1	Nonmagnetic Solutions	86
4.1.1	Inner solution	88
4.1.2	Shock position and jump conditions	90
4.1.3	Similarity solutions	94
4.2	Ideal Magnetohydrodynamics Solutions	98
4.2.1	Inner solution	100
4.2.2	Shock position and jump conditions	106
4.2.3	Similarity solutions	108
4.3	Ambipolar Diffusion Solutions	114
4.3.1	Inner solution	116
4.3.2	Shock positions and jump conditions	118
4.3.3	Model construction	123
4.3.4	Similarity solutions	129
4.4	Summary	140
5	Collapse with the Hall Effect	145
5.1	Inner Asymptotic Solution	146
5.2	Numerical Method	148
5.2.1	Simplified model	152
5.2.2	Iterative routine for locating the shock position	153
5.2.3	Subshocks	154
5.2.4	Simplified inner integration	159
5.3	Shocks	160
5.3.1	Magnetic diffusion shock position	160
5.3.2	Centrifugal shock position	165
5.3.3	Jump conditions	166
5.4	Hall Solutions	168
5.5	Summary	176
6	Discussion and Conclusions	179
6.1	Star Formation and the Hall Effect	180
6.2	The Magnetic Braking Catastrophe	188
6.2.1	Case study: ambipolar diffusion collapse	190
6.2.2	Proposed Solutions	194
6.2.3	Hall-driven spin-up of collapsing cores	197
6.3	Future Work	202
6.3.1	Limitations and assumptions	203
6.3.2	Vertical angular momentum transport	204
6.3.3	Scaling the magnetic diffusivities	205
6.3.4	Exploring parameter space	206
6.4	Conclusions	208
	References	211

A	Deriving the Inner Solutions	229
A.1	Region AC	230
A.2	Region AD	237
A.2.1	$p < 1$	238
A.2.2	$p > 1$	240
A.2.3	Coefficients of the solution	241
A.3	Region BC	244
A.4	Region BD	245
B	Parameters and Shock Positions	247
C	Additional Similarity Solutions	251

List of Figures

1.1	Vector diagram of ambipolar and Ohmic drift (Wardle, 2009)	10
1.2	Vector diagram of Hall drift (Wardle, 2009).	11
1.3	Magnetic diffusion regimes (Wardle, 2007)	13
1.4	Star formation by gravitational collapse	17
1.5	Cartoon of a thin disc with pinched magnetic field	33
2.1	Vertical angular momentum transport	53
3.1	The pq -plane	69
3.2	$\tilde{\eta}_A$ vs $\tilde{\eta}_H$ for the Keplerian disc solution where $\dot{M} = 10^{-5} M_\odot \text{ yr}^{-1}$. . .	73
3.3	Σ vs B_z and \dot{M} with varying $\tilde{\eta}_H$ for the Keplerian disc solution	75
4.1	Nonmagnetic slow rotational collapse	95
4.2	Nonmagnetic fast rotational collapse	97
4.3	IMHD slow rotational collapse	111
4.4	IMHD fast rotational collapse	113
4.5	IMHD fast rotational collapse with reduced α	115
4.6	Simple ambipolar diffusion model	125
4.7	Differences between the full and simple AD models	126
4.8	The inner singularity	127
4.9	Ambipolar diffusion collapse	131
4.10	Field behaviour in the magnetic diffusion shock	133
4.11	Slow ambipolar diffusion collapse	135
4.12	Slow ambipolar diffusion collapse with reduced $\tilde{\eta}_A$	137
4.13	Fast ambipolar diffusion collapse	139
5.1	Two-point boundary value problem	149
5.2	Locating the centrifugal shock	155
5.3	Locating the centrifugal shock in the presence of subshocks	155
5.4	Magnetic diffusion subshock when $\tilde{\eta}_H = 0.2$	157
5.5	Centrifugal subshock when $\tilde{\eta}_H = 0.2$	157
5.6	Linear fit to x_d against $\tilde{\eta}_H$	163
5.7	Nonlinear fit to x_d against $\tilde{\eta}_H$	163
5.8	Estimations of x_c against $\tilde{\eta}_H$	167
5.9	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.2$	169
5.10	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.5$	173

5.11	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = +0.2$	175
5.12	Variance of shock positions with $\tilde{\eta}_H$	177
6.1	Comparison of Σ & B_z with r at $t = 10^4$ years	181
6.2	Centrifugal shock radius against $\tilde{\eta}_H/\tilde{\eta}_A$	185
6.3	Central mass accretion rate against $\tilde{\eta}_H/\tilde{\eta}_A$	187
6.4	Strong braking solution from Krasnopolsky and Königl (2002)	189
6.5	Velocity profiles from Mellon and Li (2009)	191
6.6	Velocity profiles from Krasnopolsky and Königl (2002)	191
6.7	Non-converged initially nonrotating similarity solution	199
A.1	The pq -plane	231
C.1	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.5$	252
C.2	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.4$	253
C.3	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.3$	254
C.4	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.2$	255
C.5	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.1$	256
C.6	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.01$	257
C.7	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = -0.001$	258
C.8	Ambipolar diffusion collapse with $\tilde{\eta}_H = 0$	259
C.9	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = 0.001$	260
C.10	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = 0.01$	261
C.11	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = 0.1$	262
C.12	Hall and ambipolar diffusion collapse with $\tilde{\eta}_H = 0.2$	263

List of Tables

1.1	Physical properties of clouds, clumps and cores (Klessen et al., 2011) . . .	4
1.2	Characteristic values of specific angular momentum (Bodenheimer, 1995)	25
2.1	Outer boundary conditions	64
4.1	Estimated vs actual values of x_c in the nonmagnetic model	91
4.2	Estimated vs actual values of x_c in the IMHD model	107
4.3	Estimated vs actual values of x_d in the AD model	120
4.4	Estimated vs actual values of j_{pl2} and x_c in the AD model	123
4.5	Parameters of the ambipolar diffusion similarity solutions	129
5.1	Linear fit parameters to the magnetic diffusion shock position x_d	162
5.2	Nonlinear fit parameters to the magnetic diffusion shock position x_d . .	164
5.3	Boundary conditions and parameters for the Hall similarity solutions . .	168
6.1	Σ and B_z at $r = 1$ AU, and M_c , M_{disc} and R_c when $t = 10^4$ years for similarity solutions with $\tilde{\eta}_H = 0, \pm 0.2$	180
6.2	Comparison of initial conditions and parameters between AD models . .	193
6.3	Estimated vs actual values of j_{pl} in the Hall spin-up calculations	201
B.1	Shock position in the nonmagnetic and IMHD similarity solutions	247
B.2	Shock positions in the Hall diffusion similarity solutions	248
B.3	Variables at x_m for the Hall diffusion similarity solutions	249

“It seems to me, that if the matter of our sun and planets, and all the matter of the universe, were evenly scattered throughout all the heavens, and every particle had an innate gravity towards all the rest, and the whole space throughout which this matter was scattered, was finite, the matter on the outside of this space would by its gravity tend towards the matter on the inside, and by consequence fall down into the middle of the whole space, and there compose one great spherical mass. But if the matter were evenly disposed throughout an infinite space, it could never convene into one mass; but some of it would convene into one mass and some into another, so as to make an infinite number of great masses, scattered great distances from one to another throughout all that infinite space. And thus might the sun and fixed stars be formed, supposing the matter were of a lucid nature.”

- Newton to Bentley (December 10, 1692), quoted by Jeans (1928)

