

# CONTROL AND ENGINEERING OF OPTICAL NONLINEARITIES IN MULTI-LEVEL QUANTUM SYSTEMS

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Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

The contribution by the author to this research lies in the models used for Chapter 2, Chapter 5 and Chapter 6, the results presented in these Chapters, and in the background research required to fit models to physical systems.

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## List of Publications

- J-H. Schönfeldt, J. Twamley and S. Rebić, *Optimized control of Stark-shift-chirped rapid adiabatic passage in a  $\Lambda$ -type three-level system*, Phys. Rev. A **80**(4), 043401 (2009).





# Abstract

We investigate methods related to achieving interaction between photonic qubits. The first general approach to achieve this is to transfer photonic qubits to solid state systems, where the qubit interactions can then take place. The other approach is to enhance the strength of the photon-photon interaction.

We first robustly study methods related to the first approach: in particular we focus on how to convert the photonic quantised excitation into an atomic excitation. The Stark-shift-chirped rapid adiabatic passage (SCRAP) technique in a three level  $\Lambda$ -type system is a coherent population transfer (CPT) technique similar to Stimulated Raman Adiabatic Passage (STIRAP), which in itself is closely related to electro-magnetically induced transparency (EIT). SCRAP has been shown to perform CPT with a high fidelity for a range of different detunings making SCRAP far more robust when there is large inhomogeneous broadenings present in the ensemble. We make use of optimum control techniques in order to optimise the standard SCRAP pulses so as to minimise the decrease in fidelity brought on by inhomogeneous broadenings of the transitions. Our result is that we can improve the average fidelity of population transfer over a wide range of detunings for both the ground to excited state detuning and the ground to target state detuning (two-photon detuning).

Finally we consider the enhancement of photon-photon interactions and the implementation of an optical quantum controlled not (CNOT) gate via a cavity-QED setup. We explore the Nitrogen-vacancy centre in diamond as a suitable four-level tripod system in which to generate the cross-Kerr nonlinearity required to facilitate the strong interaction between the two fields (control and target) held in a cavity. We show that with an ultra-high quality factor cavity, and only a single NV centre strongly coupled to the trapped light, a sufficiently large interaction can be generated between the two fields to obtain a conditional phase shift (CPS) in excess of the required  $\pi$  for the successful implementation of a CNOT gate. We also show that it is possible to use this system to create an entangled state of two macroscopically distinguishable states, that is a Schrödinger cat state, by using two weak coherent fields as input and making a measurement on the second field.



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# List of Acronyms

The following list is neither exhaustive nor exclusive, but may be helpful.

<b>CNOT</b>	Controlled Not
<b>CPHASE</b>	Controlled Phase
<b>C-sign</b>	Controlled Sign Flip
<b>CPS</b>	Conditional Phase Shift
<b>CPT</b>	Coherent Population Transfer
<b>CVD</b>	Chemical Vapour Deposition
<b>EIT</b>	Electromagnetically Induced Transparency
<b>KLM</b>	Knill, Laflamme and Milburn <a href="#">[1]</a>
<b>NV</b>	Nitrogen Vacancy
<b>Q-factor</b>	Quality-factor
<b>QED</b>	Quantum Electrodynamics
<b>QPG</b>	Quantum Phase Gate
<b>QND</b>	Quantum Nondemolition
<b>SCRAP</b>	Stark-shift Chirped Rapid Adiabatic Passage
<b>STIRAP</b>	Stimulated Raman Adiabatic Passage
<b>WGM</b>	Whispering Gallery Mode
<b>XPM</b>	Cross-Phase Modulation
<b>ZPL</b>	Zero-Phonon Line

