

MACQUARIE UNIVERSITY

# Term Structure Modeling, Forecasting and Implications for Monetary Policy

by

Chamadanai Marknual

A thesis submitted in partial fulfillment for the  
degree of Doctor of Philosophy

in the

Faculty of Business and Economics  
Department of Economics

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# Declaration of Authorship

I, Chamadanai Marknual, declare that this thesis titled, ‘Term Structure Modeling, Forecasting and Implications for Monetary Policy’ and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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MACQUARIE UNIVERSITY

# *Abstract*

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This thesis examines the macro-finance-fiscal term structure model to incorporate fiscal instability variables and the term spread to understand the impact of the sovereign debt crisis on the evolution of the yield curve. My findings reveal financial instability increases the term spread associated with the expectation of higher sovereign default risk and consequently signals economic agents to reduce their spending, and thus worsens economic activity. Secondly, I also investigate whether the dynamic factor model with nonparametric factor loadings is more accurate relative to other term structure models by employing the dynamic semiparametric factor model (DSFM). The empirical results indicate that a better in-sample fit is provided by the dynamic semiparametric factor model. However, the overall forecasting results are not encouraging. The dynamic semiparametric factor model provides accurate results in forecasting a persistent trend while the dynamic Nelson-Siegel model is more suitable to fit more volatile series. Thirdly, I use a Sheen-Trueck-Wang business conditions index for term structure modeling and forecasting. I find the cross-sectional yield provides guidance to anchor the yield in the next period. The prediction performance of the model is enhanced by using the index since it includes information on frequently released or more recent available data. The index is significantly related to the slope factor, which suggests the forward-looking information from the index influences the adjustment in the yield slope. Lastly, I examine the effectiveness of the US quantitative easing (QE) policy with a Bayesian structural vector autoregressive (B-SVAR) model with sign restrictions. I find the transmission mechanism of the Federal Reserve asset purchase effectively expands output and averts deflation through a compression in the yield spread.

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# Chapter 1

## Introduction

This thesis is positioned in the research area represented at the intersection of term structure estimation and the modeling of macroeconomic and monetary policy. The main contribution to the existing literature is the investigation on different aspects of the term structure and developing new methods for estimating and forecasting. Furthermore, the forecasting performance of several models of the term structure is assessed and also examined for its forecasting stability. Another main contribution of this thesis is to analyze the informational content of yield curve and examine its influence on the economy.

My thesis comprises a collection of four papers, which make empirical and methodological contributions to the empirical financial and monetary economics and dynamic term structure modeling of the yield curve term structure. In Chapter 2, I propose a macro-finance-fiscal term structure model to incorporate fiscal variables and yield spread in examining the impact of fiscal instability on the entire yield curves and macroeconomic variables. In Chapter 3, I compare the forecasting accuracy of the dynamic semiparametric factor model and the dynamic Nelson-Siegel term structure model and further investigate the forecasting instability of both models. In Chapter 4, I present the term structure model with the Sheen-Trueck-Wang (2014) business conditions index to analyze the linkage between the forward looking information contained in the index and the evolution of term structure. Finally, I use a Bayesian structural vector-autoregressive model with sign restrictions in Chapter 5 to analyze the effectiveness of unconventional monetary policy in changing yield slope and macroeconomic variables at the zero

lower bound interest rate.

In the first paper entitled “Spanish Sovereign Term Structure: Implications of the Sovereign Debt Crisis” as presented in Chapter 2, I introduce a macro-finance-fiscal term structure model to study the dynamic relation between fiscal instability and the shape of the entire sovereign yield curves for Spain. The term structure of sovereign bond yields is estimated by the [Nelson and Siegel \(1987\)](#) parametric model, using a state space specification and Kalman filtering for maximum likelihood estimation as suggested by [Diebold and Li \(2006a\)](#) and [Diebold et al. \(2006\)](#). The model is developed in a vector-autoregressive (VAR) model with unobserved yield factors, fiscal indiscipline proxies, macroeconomic variables and yield spread, which is specifically used as an signal of instability in fiscal stance. I apply this model to Spanish monthly data over the period between January 1988 to August 2011.

My main results show that fiscal indiscipline significantly determines the term spread. An increase in public debt or a worsened net government budget position widens yield spread associated with sovereign default intensities. However, the government budget position shock is found to generate a significant response of the yield spread, whereas a shock of the public debt to GDP entails no significant reaction of yield spread since it takes more than a year to react. The results implies fiscal instability drives expected default probabilities and thereby term spread. Corresponding, I suggest that fiscal discipline can be considered as a necessary condition for fiscal policy to effectively stimulate economy.

The second paper entitled “Term Structure Forecasting - A Comparison between the Dynamic Semiparametric Factor Model and the Dynamic Nelson-Siegel Model” is presented in Chapter 3. In this paper, I compare the in-sample fit and out-of-sample forecasting performance of the dynamic semiparametric factor model, the dynamic Nelson-Siegel model with other competitors, including the random walk. I consider the dynamic semiparametric factor model and show how the smoothness from non-parametric estimation can be combined with the dynamic factor model. The dynamic factor model without a pre-specified functional form is able to fit a wide range of yield curves.

The assessments are conducted using monthly data of Australian zero-coupon bond yields over the period from April 1999 to March 2013. My results indicate that the dynamic semiparametric factor model provide a better in-sample fit relative to dynamic Nelson-Siegel model. However, the overall forecasting results are not encouraging to find a model that dominates all competitors and overcome the random walk. Comparing between the dynamic semiparametric factor model and the dynamic Nelson-Siegel model, I find the relative performance of these models vary over time. The dynamic Nelson-Siegel model perform better in relatively volatile periods, especially the global financial crisis during 2008 to 2009. However, the dynamic semiparametric factor model is more suitable to fit more persistent period. The difference in forecasting performance is partly due to structural breaks. I then conduct the [Giacomini and Rossi \(2010\)](#) fluctuation test which is statistically confirmed the uncertain environment resulting from the global financial crisis. forecasting instabilities of the individual models during the period of study. The dramatic lowering of yields during the global financial crisis lessened the predictability performance of both the dynamic semiparametric factor model and the dynamic Nelson-Siegel model against the random walk.

The third paper entitled “Term Structure Forecasting with a Business Condition Index” is then presented in Chapter 4. I explore the role of forward looking information regarding the business conditions in improving the understanding of the term structure. I propose to use the Sheen-Trueck-Wang business conditions index as an additional information to estimate and forecast the term structure. [Sheen et al. \(2014\)](#) extended the [Aruoba et al. \(2009\)](#) business conditions index for the closed economy to a small open economy and used the Kalman filter to measure economic activity from different frequencies. This index represents real time economic activity and also contains predictive content of the future economy. The term structure modeling and forecasting are estimated by using Australian monthly data from March 1999 to April 2013.

Based on term structure estimation, I find the incorporating information about the current state of the economy as well as forward-looking information contained in the Sheen-Trueck-Wang business conditions index offers an anchor for cross-sectional and in-sample term structure model. Furthermore, the forecasting performance of the term structure model with the Sheen-Trueck-Wang business

conditions index is then assessed relative to the two most common survey-based Australian indicators: the the Melbourne/Westpac leading index and the Melbourne/Westpac consumer sentiment index. I find the informational advantage of the forward-looking information carried by my proposed model provides better out-of-sample predictive accuracy. My findings are consistent with the expectation hypothesis of the term structure that assumes the evolution of the yield curve is driven by expectations about the future state of the economy.

The last paper of this thesis entitled “The Economic Impact of Quantitative Easing on the US Economy: A Structural VAR with Sign Restriction Analysis”, I present it in Chapter 5. This paper analyzes the effectiveness of unconventional monetary policy in changing the long-end of term structure, in other words, the yield slope. In this perspective, I employ a Bayesian structural vector autoregressive (SVAR) model with sign restrictions to assess the economic impact of quantitative easing (QE) measure. The structural innovation of unconventional monetary policy is identified by a zero restriction on the policy interest rate and positive sign imposed output growth on inflation. The effect on yield slope is unrestricted to examine the transmission mechanism through long term rate. The Bayesian SVAR model is estimated using monthly data over the period from January 2003 to August 2013, covering the period of the Federal Reserve’s quantitative easing implementation programs: QE1, QE2 and Operation Twist, during November 2008 to September 2012.

I find that unconventional monetary policy shock leads to a significant increase in output growth and inflation by compressing yield slope. The increase in the size of Federal Reserve’s balance sheet provides additional loan for financial institutions. In turn, it restores confidence and causes term premia charged on long term rate to decline. Nonetheless, the relative effects of unconventional monetary policy appear to be quite smaller in comparison to conventional monetary policy. That implies that the attempt of the central bank to stimulate the economy requires much larger assets. However, the objective of unconventional monetary measures to avert deflationary pressure at the zero bound interest rate appears to be more effective to raise inflation. The effectiveness of unconventional monetary policy even robust for a sample period which is limited to the pre-crisis period. My findings suggest that unconventional monetary policy is effective to boost output and



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avert deflation while the conventional monetary policy is relatively silent when the economy is at a zero-lower-bound interest rate.



## Chapter 2

# Spanish Sovereign Term Structure: Implications of the Sovereign Debt Crisis

### 2.1 Introduction

We analyze the impacts of fiscal instability on yield spreads and the economy over the period that covers the global financial crisis, focusing on Spain, one of the peripheral European countries suffering high government debt following the crisis. It is the only one among other vulnerable countries that had not obviously suffered from a chronic structural budget deficit prior to the crisis. The widening of the spread was solely caused by an increase in the probability of a sovereign default associated with a sharp drop in public revenue and a surge in public debt from the bailout of debt held by commercial banks. This Spanish study provides a good example for other countries to avoid an unsustainable fiscal policy implementation that could subsequently lead to a sovereign debt crisis following the onset of the financial crisis.

In this paper, we introduce a macro-finance-fiscal term structure model to study the dynamic relation between fiscal instability and the shape of the entire yield curves for Spain. The term structure of sovereign bond yields is estimated by

the Nelson and Siegel (1987) parametric model, following a state space specification and Kalman filtering for maximum likelihood estimation as suggested by Diebold and Li (2006b) and Diebold et al. (2006). We apply our model to Spanish monthly data over the period between January 1988 to August 2011. The yield latent factors, term spread, macroeconomic and fiscal variables are related in a country-specific vector autoregressive (VAR) model, which then allows us to examine the dynamic relation by means of impulse response function and forecast error variance decomposition.

The sovereign yield spread between the bonds of two countries is an indicator of perceived relative credit risk. When the perceptions of default risk increase, markets will raise premia and widen spreads until sufficiently high enough to cover their obligations. The persistence of budget deficits and rising public debt could raise sovereign risk premia as the default probability increases. Thus, the remarkable rise in the yield spread indicates that markets penalize fiscal profligacy resulting from financial indiscipline. In turn, policy makers could consider a higher yield spread as a signal of unsustainable fiscal positions. This may ensure more prudent fiscal policy implementation to prevent a sovereign debt crisis.

The question whether fiscal indiscipline has a statistically significant widening effect on yield spreads has been investigated from several existing literatures. The most common method is regression of yield spreads on a set of variables, including fiscal position variables. Following this approach, fiscal deficits and government debt are found to be significant to explain spreads at the long-end of the yield as mentioned by Von Hagen et al. (2011) and Bernoth et al. (2012). There are a few studies (see Borgy et al. (2011), Afonso and Martins (2012) and Dewachter et al. (2015)) that have thoroughly assessed the dynamic relations between fiscal developments and the whole shape of the sovereign yield curve. In this approach, there may be a possible empirical linkage between fiscal position variables and latent factors which are extracted from the panel of yields across time. The unobserved yield factors and the fiscal variables are related together with other macroeconomic variables in a vector-autoregressive (VAR) model. This allows the yield curve to be driven by unobserved yield factors as well as fiscal variables that contain information about risk premia. Therefore, we can then examine the economic impact

of fiscal indiscipline on the whole yield curve through yield latent factors.

[Borgy et al. \(2011\)](#) and [Dewachter et al. \(2015\)](#) combine an affine term structure model with public debt-to-GDP and other macroeconomic variables to investigate how the government bonds react to the change in fiscal variables across the entire yield curve. They find a significant increase in sovereign yields, albeit there is no further exploration of the potential impact on output and inflation. A study which is even more focused on the interrelation between fiscal variables and yield curve factors was conducted by [Afonso and Martins \(2012\)](#). They set-up a macro-finance term structure model based on the more parsimonious [Nelson and Siegel \(1987\)](#) framework. However, their study fails to observe any statistically significant effects of fiscal instability on yield latent factors and the monetary policy interest rate.

The aforementioned literatures discussed so far do not provide a sufficient explanation of how fiscal indiscipline affects yields for the entire range of maturities through widened yield spreads that would eventually have an influence on macroeconomic variables. We complement the literature by the inclusion of fiscal variables in macro-finance term structure model, and further replace the policy interest rate with the term spread. Our macro-finance-fiscal term structure model gives a more specific link between the development of yield spreads and the evolution of fiscal indiscipline. It is based on the expectation theory concerning fiscal instability that drives the potential term premium to compensate for expected sovereign default risk and expected loss of income from recession. This model leads to the use of a term spread in a macro-finance term structure model, which is more closely related to expected default risk than the use of the policy interest rate as used in [Afonso and Martins \(2012\)](#). The yield spread can therefore be considered as an instrument to signal the stance of fiscal policy for the transmission mechanism. It enhances the macro-finance model by avoiding the constraints from the zero lower bound of the interest rate and non-autonomous monetary regime, which makes the earlier mentioned literature entirely silent on the effects of fiscal indiscipline.

Our main results for Spain from 1988 to 2011 show that fiscal indiscipline significantly determines the term spread. Both fiscal variables; the net change in the government budget position to GDP and the change in public debt to GDP,

are statistically significant in explaining the sovereign term spread. When public debt increases or the net government budget position worsens, there would more likely be a rise in the yield spread. However, we find the evidence of asymmetric responses obtained from these two alternative variables that gauge fiscal indiscipline. A shock of the government budget position is found to generate a significant response of the yield spread, while a shock of the public debt to GDP does not immediately bring about a significant reaction. In fact, the impulse response to a shock on public debt takes more than a year to react. The results indicate that market participants penalize a worsened government budget position, whereas higher public debt does not seem to urgently matter for the yield spread because of the substantial lag response. This finding is consistent with the expectation hypothesis of the term structure according to which a deteriorating government budget means that a higher term premium may be required to compensate for higher perceived sovereign default risk. Our empirical study confirms the results of [Bernoth et al. \(2012\)](#), who found evidence that European countries have generally focused on deficits to measure fiscal stability and often ignored public debt. We also redesign the macro-finance-fiscal term structure model using a term spread instrument and in contrast to [Afonso and Martins \(2012\)](#)'s study and find that fiscal indiscipline actually has a significant impact on yield spreads and affects macroeconomic variables later. It is revealed that the unpromising results from earlier studies are due to a zero lower bound constraint that kept interest rates at a prevailing low level over a period of study.

The remainder of the paper is organized as follows. Section [2.2](#) presents an overview of the literature. Section [2.3](#) summarizes the stylized fact about Spanish fiscal imbalance and sovereign debt crisis. Section [2.4](#) explains the methodology to estimate the [Nelson and Siegel \(1987\)](#) parametric model and the macro-finance-fiscal term structure model used to determine the influence of fiscal indiscipline on term spread and macroeconomy. Section [2.5](#) presents the data collected and Section [2.6](#) discusses the term structure estimation results. Section [2.7](#) analyses the VAR estimation, impulse response function and forecast error variance decomposition of the macro-finance-fiscal model and Section [2.8](#) concludes the paper.

## 2.2 Review of Literature

There are number of empirical papers on the relationship between yield spreads and fiscal variables. Most of these studies, including [Schuknecht et al. \(2009\)](#), [Von Hagen et al. \(2011\)](#), [Bernoth et al. \(2012\)](#), [Eichler and Maltritz \(2013\)](#), [Georgoutsos and Migiakis \(2013\)](#), [Aristei and Gallo \(2014\)](#) and [Afonso and Nunes \(2015\)](#); mainly focus on investigating determinants of the yield difference between a risky and a risk-free sovereign bond, particularly yield differentials on 10-year government bonds issued by European countries against the German bond. They regress yield spreads at certain maturities on several candidate explanatory variables that may represent default risk premia and liquidity premia in the absence of exchange rate risk since the common currency was introduced in 1999. A common finding in this literature is that the European sovereign yield spreads are significantly influenced by country-specific factors related to sovereign default risk from fiscal distress. However, there is no consensus on the relative importance of the determining factors on yield spread. Nonetheless, fiscal condition factors do become more important to explain widened term spreads after the global financial crisis in 2008 because of adverse market sentiment. The factors that are normally used to capture the government probability's of default are the ratio of public debt-to-GDP and net budget position (surplus or deficit) to GDP. Debt-to-GDP represents a country's leverage ratio. If the outstanding public debt exceeds a critical level, the interest rate begins to rise and the term spread increase. The increased cost of borrowing would then penalize fiscal indiscipline and encourage the government to re-balance its fiscal position. The rise in yield spreads is associated further with an increasing national level of leverage. Another proxy for fiscal imbalance is a negative government budget to national income, in other words the fiscal deficit-to-GDP. A larger deficit may put a greater upward pressure on interest rates due to the crowding out effect on private spending. By running a deficit, the revenue side of the government budget would be expected to decline dramatically and could become unsustainable. The yield spread may amplify with the increased risks of exposure to default in anticipation of continuing inadequate revenue. In this respect, an increase in credit risk may entail a significantly upsurge in term spreads in order to correct for government irresponsibility.

Although the relevant literature provides that net government budget position and

the debt-to-GDP are most used as a gauge of default risk premia, it is not unanimous about how these fiscal imbalances affect interest rates. Earlier studies by [Gale and Orszag \(2004\)](#) and [Laubach \(2009\)](#) find government bond yield spreads depend positively on the government budget deficit. An empirical investigation for European countries show that the indiscipline of government spending became more influential after the start of the European Monetary Union (EMU) for 1999. [Bernoth et al. \(2012\)](#) find the market reaction to fiscal deficits much stronger after the provisions of the EMU Stability and Growth Pact (SGP), were enacted in 17 June 1997, advocating a balanced budget in the longer term and specifying a ceiling for deficit spending of 3 percent of GDP for each member country. [Von Hagen et al. \(2011\)](#) also report that financial markets pay less attention to public debt; however, they reacted more strongly to the deficit-to-GDP than they previously did after the default of Lehman Brothers in September 2008. These findings confirm that the needs for maintaining sustainability of the government budget has considerably increased. However, [Afonso and Nunes \(2015\)](#) observe that the deficit-to-GDP loses its statistical significance in explaining a widened yield spread after the onset of the Greek sovereign debt crisis from October 2009. At that time, the Greek government unveiled drastic revisions to its deficit figures which in turn destroyed its government's credibility. Investors overlooked the budget statistics and public debt regained interest for them. In fact, term spreads respond more significantly to changes in national indebtedness than the level of debt. [Eichler and Maltritz \(2013\)](#) state that investors actually proxy the burden of debt by the growth rate of sovereign debt-to-GDP instead of the current level of debt. They claim that a larger debt burden implies the expectation of a worsening solvency problem that particularly boosts long-term yield spread. This finding suggests the growth rate of debt mainly affects the long-end of yield curve. The evidence of a significantly higher a long-term yield associated with an increasing debt-to-GDP is reported by [Gruber and Kamin \(2012\)](#) and [Marattin et al. \(2011\)](#).

Fiscal conditions have been confirmed by many empirical studies as a major contributor in determining the yield spread. Yet, the relation between yield and fiscal imbalance can be sharpened by using a term structure model to exploit the information contained in the yield latent factors. [Oliveira et al. \(2012\)](#) use the Heath-Jarrow-Morton (1992) multi-factor interest rate model to extract yield latent factors and use them together with other fiscal variables to explain the changes in term spread. Their study reveals that yield factors as well as the change in public



debt-to-GDP appear to influence yield spreads. Interestingly, yield factors become less important after the global financial crisis while the change in debt-to-GDP is found to be more significant. [Borgy et al. \(2011\)](#) follow [Dai and Philippon \(2005\)](#) to combine a [Duffee \(2002\)](#) and [Ang and Piazzesi \(2003\)](#) arbitrage-free affine term structure model with a set of macroeconomic variables; including fiscal variables, in a vector-autoregressive (VAR) model to trace the effect of a fiscal variable shock on the yield latent factors and other macroeconomic variables. Since their model is built on a dataset of European countries which all have a common policy interest rate, the effect of fiscal variables is therefore expected to drive the term spread at the long-end of the yield. They find debt-to-GDP is the main determinant for the sharp increases in the European yield spreads after the global financial crisis in 2008. Another affine term structure model augmented with macroeconomic variables was used by [Dewachter et al. \(2015\)](#). Unlike [Borgy et al. \(2011\)](#)'s study, they not only explore the effect of macroeconomic variable on the term structure, but also utilize [Joslin et al. \(2014\)](#)'s spanned factors framework to derive unobserved principal components from a group of macroeconomic variables, including debt-to-GDP, and examine their effect on yields at different maturities. They find a positive innovation to the first (level) and second (slope) components of macroeconomic variables significantly increase bond spread. For the investigation on the effect of fiscal distress on yield spreads, they observe a significant increase in long-term yields after a positive shock in debt-to-GDP. Apart from the affine term structure model, [Afonso and Martins \(2012\)](#) apply a Nelson-Siegel (1997) macro-finance term structure model, as proposed by [Diebold et al. \(2006\)](#). They extend this model with fiscal variables to analyze the impact of public debt shock on the yield latent factors and macroeconomic variables. Unfortunately, their study only reports a short-lived decrease in the curvature factor, however there is no significant responses on other yield latent factors and the monetary policy interest rate.

## 2.3 Spain fiscal imbalance and sovereign debt crisis

Spain is the fourth largest economy in the Eurozone which uses the euro as its currency. It is one of five Eurozone nations known as PIIGS; Portugal, Ireland, Italy, Greece, and Spain itself, which were fiscally vulnerable after the eruption of

the sovereign debt crisis in 2009. Comparing with other countries in this group, Spain is relatively different and markets did not expect a dramatic surge in its term spread. During the 1990s, its public debt significantly declined and reached the Treaty of Maastricht ceiling level at 60 percent of GDP in the second half of the 1990s. Italy and Greece still, however, were above 90 percent debt-to-GDP. In terms of budget deficit-to-GDP, Spain also maintained its level below the 3 percent Maastricht criteria and even ran a budget surplus from earning extra tax revenue on housing in the years from 2005 to 2007. In contrast, Greece, Portugal and Ireland encountered a structural budget deficit during 2001 to 2007 due to their chronic macroeconomic imbalance. Evidently, Spain had never experienced fiscal instability before the global financial crisis emerged.

Indeed, joining the Eurozone helped Spanish banks to raise funds from other member countries without any exchange rate risk. The excessive optimism of a sound economy drives a massive capital inflow to fuel domestic consumption and property-related borrowing. In turn, the accumulated external debt actually made Spain into a vulnerable economy. In the aftermath of the global financial crisis, Spain lost competitiveness from real exchange rate appreciation. As a result, its current account deficit reached more than 10 percent of GDP in 2008 to 2009. A collapse in its housing bubble and together with the huge burden of external debt in Spanish banking system caused the Spanish central bank to bailout and subsequently transfer private debt into public debt. Therefore, the sovereign debt crisis in Spain was originally generated by the private sector, while Portugal, Greece and Italy were mainly led by public debt.

To illustrate the evolution of the Spanish sovereign debt crisis we will explore development of public debt and the fiscal deficit and their impact on yield spread in more detail. During the 1990s, Spain experienced a period of sustained economic growth and boosted tax revenue, so that outstanding public debt declined. After the global financial crisis started in 2007, the Spanish public debt-to-GDP began to climb once again due to a sizable government bailout of the banking system.

As shown in Figure 2.1, Spanish public debt-to-GDP continuously declined from around 65 percent of GDP in 1998 and reached its lowest level at 38 percent of GDP in 2007. Afterward, it started to sharply rise again and returned to the high

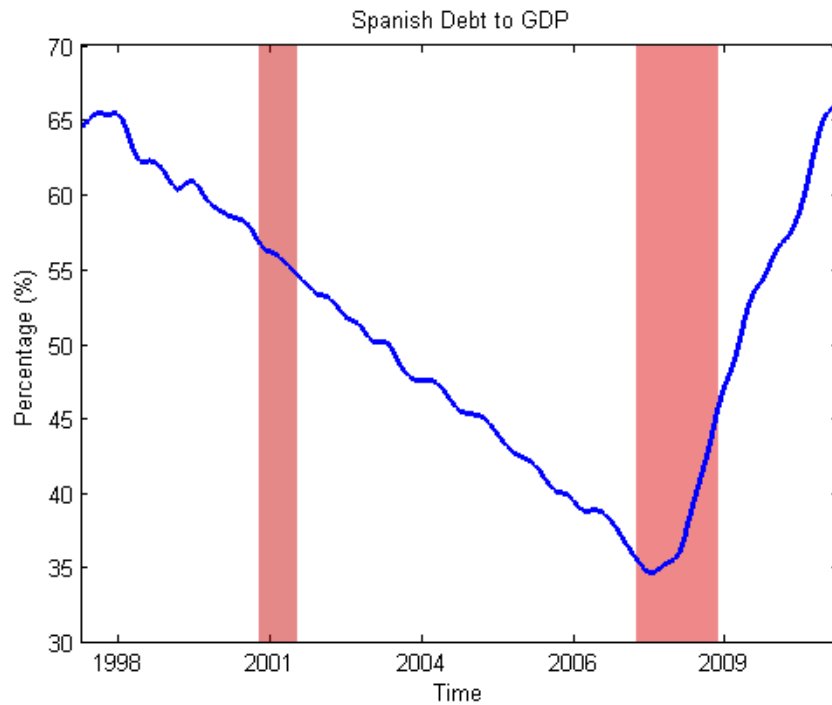


FIGURE 2.1: Spanish Debt to GDP

levels of the 1990s in 2011. Following the Greek's austerity package announcement after its above 3 percent deficit limit was disclosed in October 2009, fiscal distress intensified throughout peripheral Europe and subsequently Spanish public debt-to-GDP surged from 36 percent in 2008 to reach 65 percent in 2011.

As mentioned before, a drastic increase in public debt was the result of fiscal transfer from private debt. For over a decade until 2007, Spain experienced a credit boom that later generated a bubble in housing and the real estate sector. The housing boom was actually financed by foreign lenders with a low interest rate and risk premium. After the onset of global financial crisis in 2007, the housing bubble finally burst which caused a plunge in asset prices. The markets started to ponder whether the crisis could increase further defaults and losses on mortgage loans. Such concerns raised default risk premia. In turn, it became more difficult for borrowers to refinance their existing debt obligation. The Spanish share of non-performing loans to total gross loans jumped from 0.8 percent in 2004 to 4.1 and 6.0 percent in 2009 and 2011 respectively. In this crucial time, commercial banks were required to raise more capital to resolve their large stock of non-performing loans. Since commercial banks lacked suitable collateral, the Spanish central bank therefore needed to temporarily remove bad debts from the banks and recapitalize

them. The central bank bailout by considerable purchasing private debt hence increased the burden of Spanish public debt.

During the credit and housing booms from 2003 to 2007, Spain earned ballooning tax revenues through the non-indexation of many tax categories. The crash in housing prices combined with the global financial crisis brought budget position to below the threshold of 3 percent of GDP beyond 2008.

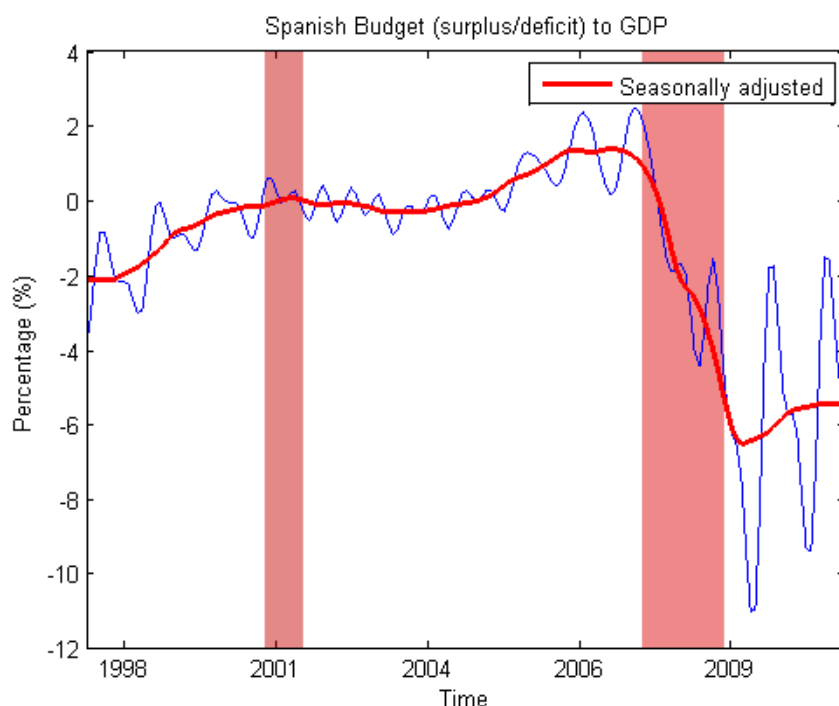


FIGURE 2.2: Spanish Budget (Surplus/Deficit) to GDP

As Figure 2.2 shows the seasonally adjusted (solid thick line) and the non-seasonally adjusted (thin line) Spanish net budget position to GDP. Over the period from 1998 to 2007, the net budget position to GDP varied in a range between -3.0 percent deficit and 2 percent surplus-to-GDP. With a growing budget revenue from housing bubble, the budget position to GDP ratio even turned into a surplus between 2005 to 2007. Nevertheless, the housing bubble burst and the global financial crisis began, leaving a deep fiscal deficit that exceed the Maastricht ceiling of 3 percent of GDP after 2007.

Evidence of the housing boom and bust cycle in the 2000s indicates a failure of tight fiscal policy to counter a speculative bubble forming. If the Spanish government had adopted an indexation tax, it would help to reduce housing demand when the economy was booming. A counter-cyclical fiscal policy could have prevented a deep recession and avoided adverse deficit dynamics at the cost of moderate growth. As discussed before, a collapse in house prices exacerbated unexpected loan losses. The financial institution had to cut back their loans which led to a credit crunch. In severe financial distress, the Spanish economy deteriorated further into a deep recession. Tax revenue from residential construction and dwelling sectors declined. With lower government revenue, the deficit-to-GDP increased larger-than-expected and breached the prescribed ceiling of 3 percent in the last four-year period of study from 2008 to 2011. A concern about the violation of the EMU fiscal rule had a negative impact on sovereign bond yields since investors penalized this fiscal indiscipline with higher risk premium.

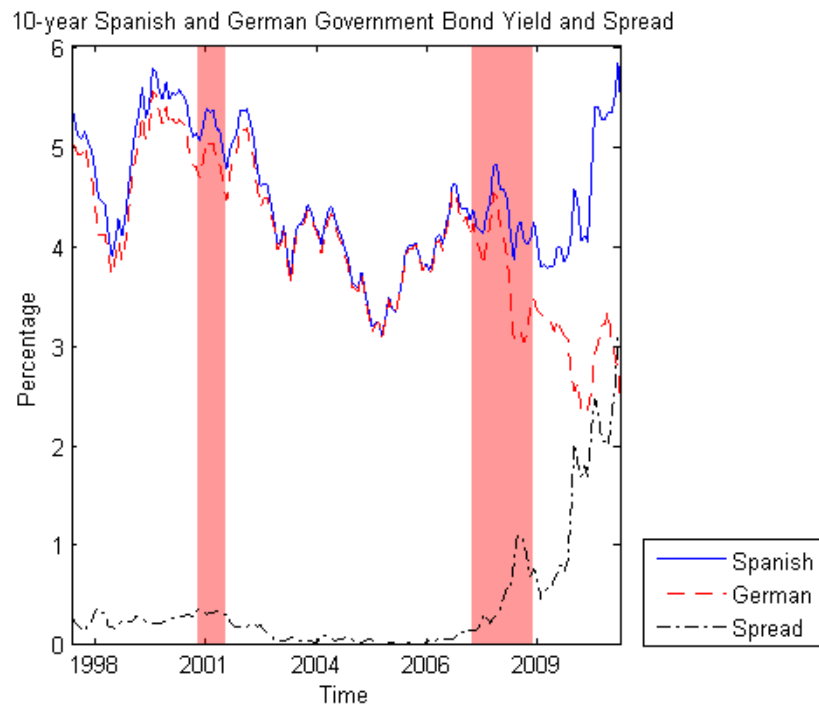


FIGURE 2.3: 10-year Spanish and German Government Bond Yield and Spread

In Figure 2.3, we plot the 10-year Spanish spread relative to Germany government bond from 1998 to 2011. Between 1998 and 2001, the yield spread hovered around 30 basis points. After 2001, the yield spread was typically below 20 basis points and stayed in the only 1-5 basis points range during 2005 to 2006. With

the burst of the Spanish housing bubble and global financial crisis in 2007, yield spreads widened to over 100 basis points. From the late 2009 onwards, fears of a the European sovereign debt crisis developed and the yield spread spiked sharply to reached above 300 basis points in the late 2011.

Basically, the evolution of yield spreads reflects heightened risk and/or investors' risk aversion. For Spain and other Eurozone members, yield spreads are all denominated in a common currency. With the elimination of exchange rate risk, the yield spreads therefore mainly represent market perception of the unobserved sovereign default risks. During 1999 to 2000, the average yield spread had somewhat widened because of market illiquidity. However, a more integrated and liquid secondary market gradually developed which caused lower spreads observed between 2001 to 2006. These would represent market perception of a fully credible EMU commitment to the bail-out of unpaid debt of its member states. With the collapse of the housing bubble and subsequent effect from global financial turmoil, yield spreads were substantially pushed upward. In 2009, the European sovereign debt crisis intensified associated with the deterioration of deficit budget and public debt, yield spreads increased further and reached values exceeding 300 basis points in 2011. The rising spread implies that investors rebalanced their portfolios away from riskier bonds.

## 2.4 Methodology

In this section, we give an overview of the [Nelson and Siegel \(1987\)](#) exponential-polynomial parametric yield curve model and further discuss about the dynamic Nelson-Siegel term structure model, which are introduced by [Diebold and Li \(2006a\)](#) and [Diebold et al. \(2006\)](#). Then, we propose a macro-finance-fiscal term structure model to incorporate additional fiscal variables as proxies to measure fiscal indiscipline and also introduce term spread as a signal of transmission mechanism, instead of using policy interest rate. Afterwards, we explain the Kalman filter algorithm for maximum likelihood estimation that is used to obtain all model parameters in one step. This technique was claimed by [Diebold et al. \(2006\)](#) among

others as an effective technique to avoid spurious regression and enhance statistically efficient of term structure estimation.

### 2.4.1 The Dynamic Nelson-Siegel Term Structure Model

We firstly employ the exponential-polynomial parametric [Nelson and Siegel \(1987\)](#) function to model sovereign term structure with a limited set of unobserved factors spanning the entire curve. It allows us to present Spanish government bond yields in a parsimonious and flexible model which is able to capture a variety of shapes.

#### 2.4.1.1 The Nelson-Siegel Yield Curve Estimation

[Nelson and Siegel \(1987\)](#) proposed a model to fit the yield curve at a given date with a mathematical as a constant plus a Laguerre function, which is a polynomial times an exponential decay term. Even though the Nelson-Siegel model was designed to be a static model, [Diebold and Li \(2006b\)](#) reformulate the original Nelson-Siegel expression as a dynamic latent factor for the yield curve as

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad (2.1)$$

where  $y_t(\tau)$  denotes the set of (zero-coupon) yields at each period  $t$  and  $\tau$  is the corresponding maturity. The three parameters are  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$  and their respective loadings are given by 1,  $\frac{1-e^{-\lambda\tau}}{\lambda\tau}$  and  $\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda}$ . The parameter  $\lambda$  governs the exponential decay rate.

[Diebold and Li \(2006b\)](#) interpret  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$  as time-varying unobserved factors that govern the yield shape together with the exponential decay rate  $\lambda$  at each period  $t$ , while the three factors loading are illustrated as a function of time-to-maturity. The evolution of the yield curve is driven by 3 components; short-term, medium-term and long-term components. The long-term component is the factor

loading on  $\beta_{1,t}$  which is equal to one and constant for every maturity. The short-term component is the factor loading on  $\beta_{2,t}$  which is designated as  $\frac{1-e^{-\lambda\tau}}{\lambda\tau}$ . The value of short-term loading starts at one, and then decays monotonically to zero at an exponential rate. The medium-term component is the factor loading on  $\beta_{3,t}$  which is defined as  $\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda}$ . This loading starts at zero. Then, it increases for to capture medium-term maturities, and finally decays to zero.

The three factor coefficients are labeled by [Diebold and Li \(2006b\)](#) as level, slope and curvature factors. The reasons are related to what extend of the entire yields are affected by each factor. The long-term factors  $\beta_{1,t}$  determine the yield curve level. An increase in the value of  $\beta_{1,t}$  will lead to a consistent increase in the level of the yield curve for all time-to-maturities. The short-term factor  $\beta_{2,t}$  is related to the yield curve slope, which is defined as the ten year-to-maturity yield minus the three months-to-maturity yield. Alternatively, yield curve slope can be defined as  $y_t(\infty) - y_t(0)$  which is equal to  $-\beta_{2,t}$ . Finally, the medium-term factor  $\beta_{3,t}$  affects the yield curve curvature since it has the greatest impact on medium-term yields.

Using the Nelson-Siegel framework, we can estimate the cross-sectional yield curve with the method of least square to derive regression coefficients of latent factors  $\hat{\beta}_i$  where  $i = 1, 2$  and  $3$  from a series of yield observations at any specific period  $t$ .

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + u_t \quad (2.2)$$

The disturbances are assumed to be independent with mean zero and constant variance for a given time  $t$ . With the cross-sectional estimated factors  $\hat{\beta}_i$ , we can fit them into the Nelson-Siegel model to obtain the value of estimated yield  $\widehat{y_t(\tau)}$  for each period  $t$ .

#### 2.4.1.2 The State Space Specification

Suppose these three latent factors; level  $\beta_{1,t}$ , slope  $\beta_{2,t}$  and curvature  $\beta_{3,t}$  factors, which follow a first order vector autoregressive VAR(1) process, [Diebold et al.](#)



(2006) proposed to cast a dynamic factor model into a state-space representation, which allows us to obtain maximum-likelihood estimates from the Kalman filter. The state-space model is typically a linear dynamical system of the measurement and state equations. Given a set of observed yields, measurement equation relates actual yield observations with the three latent factors, while state equation describes the evolution of latent factors.

Let  $Y_t$  be the vector of yields, which contains  $N$  different maturities and let  $\beta_t$  be the vector of latent factors.

### (1) Measurement equation

The measurement equation specifies relationship between the yield vector  $Y_t$  and latent factor vector  $\beta_t$  at a given period  $t$ . For this purpose, the cross-sectional Nelson-Siegel yield curve estimation is rewritten as.

$$Y_t = X_t \beta_t + u_t \quad u_t \sim N(0, Q_t) \quad (2.3)$$

$X_t$  is the  $(N \times 3)$  factor loading measurement matrix where its  $(i, j)$  element is given by .

$$X_{i,j}(\lambda) = \begin{cases} 1 & \text{if } j = 1 \\ \frac{1-e^{-\lambda\tau}}{\lambda\tau} & \text{if } j = 2 \\ \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda} & \text{if } j = 3 \end{cases} \quad (2.4)$$

The disturbance vector  $u_t$  is Gaussian white noise and assumed to be independent across maturities. The variance of the yield  $\sigma^2(\tau_i)$  is constant for each maturity but different across maturities.

### (2) State equation

The state equations represent the evolvement of latent factors. We follow [Diebold et al. \(2006\)](#) to specify the dynamics of factors with the first-order vector autoregressive VAR(1) process with mean  $\mu$ .

$$\beta_t = \mu + T\beta_{t-1} + \omega_t \quad \omega_t \sim N(0, H_t) \quad (2.5)$$

$T$  is transition matrix.

### (3) The Variance-covariance Matrix

The vector  $u_t$  and  $\omega_t$  are serially uncorrelated, normal distributed error terms with mean zero and positive definite covariance matrices  $Q_t$  and  $H_t$ .

$$\begin{pmatrix} \omega_t \\ u_t \end{pmatrix} \sim WN \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] \quad (2.6)$$

If the decay parameter  $\lambda$  is fixed as a priori known, one can estimate latent factors through a linear least squares regression for each cross-sectional period. Then, we can model the dynamics of latent factors from the previous stage. This two-step estimation procedure is obviously simple and widely used by practitioners. However, [Diebold et al. \(2006\)](#), [Morales \(2010\)](#) and [Laurini \(2014\)](#) argued this two-step method is in fact inefficient since it does not take into account the estimated latent factor in the first step of yield curve estimation and therefore generates inaccurate results.

Another estimation technique is to assume that decay parameter  $\lambda$  is still constant but rather unobserved. Therefore, we can simultaneously estimate it with latent factors in one-step through maximum likelihood estimation using Kalman filter. In our study, we follow [Diebold et al. \(2006\)](#) to apply this method to enhance statistical efficiency.

### 2.4.2 Macro-Finance-Fiscal Term Structure Model

Based on the yield latent factors from the [Nelson and Siegel \(1987\)](#) yield curve estimation and the VAR(1) dynamics as mentioned in the previous subsection, we now assess the role of fiscal indiscipline and macroeconomic variables in the yield curve dynamics. Following [Diebold et al. \(2006\)](#) and [Afonso and Martins \(2012\)](#), we expand the state-space framework of the Nelson-Siegel dynamic term structure model to draw explicit connections between the latent factors that drive the yield curve dynamics and observable fiscal and macroeconomic variables that characterize the state of the economy. In this analysis, we focus on the impact of fiscal instability on yield spread and the role of sovereign default risk in explaining the yield dynamics and macroeconomy.

We propose a macro-finance-fiscal term structure model to study the transmission mechanism of fiscal stance that signal fiscal instability through the yield spread and affect macroeconomic variables. [Afonso and Martins \(2012\)](#) attempted to include fiscal variables into the Nelson-Siegel term-structure model to examine the impact of fiscal shocks. However, they found fiscal variables do not significantly affect the yield curve dynamics. Their study also left unexplored the connections between the term spread and yield latent factors as well as macroeconomic dynamics.

In order to bridge this gap, we construct a dynamic term structure model based on the Nelson-Siegel yield curve, fiscal variables and macroeconomic variables, which allows for an explicit feedback from fiscal indiscipline variables to term spread, yield latent factors and macroeconomy. Meanwhile, the inclusion of yield spread that reflects expected default risk can also help us to model the propagation of the economic environment in the transmission mechanism and other dynamic changes of the entire term structure.

A straightforward extension of the state-space model is to augment the additional fiscal and macroeconomic variable to the set of state equations, which leads to following system of simultaneous equations.

### 2.4.2.1 Measurement equation

The measurement equation in the macro-finance-fiscal model is extended to include macroeconomic and fiscal instability variables to analyze the dynamic interaction between the latent factors determining shape of the yield curve, macroeconomy and effect from fiscal instability. In spite of that the macroeconomic and fiscal variables are represented through a minimum set of variables required to assess the macro dynamic to maintain parsimony of the yield curve model. There are four macroeconomic and fiscal variables included; the growth rate of industrial production index (GIPI) as a measure of economic activity, the inflation index (INF) as a measure of nominal growth and the change in yield spread (SPRD) as an instrument of instability in fiscal stance, the fiscal instability variable (FIS) which can be either the net government budget position to GDP (GBTG) or the growth rate of public debt to GDP (GDTG), and the three yield curve latent factors, level (LEV), slope (SLP), and curvature (CUR). Now, our macro-finance-fiscal model replaces state variables of the previous state space specification with  $Z_t = (GIPI, INF, SPRD, FIS, LEV, SLP, CUR)$ .

$$\begin{bmatrix} Y \\ Z \end{bmatrix} = \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \beta \\ Z \end{bmatrix} + \begin{bmatrix} \epsilon \\ 0 \end{bmatrix} \quad (2.7)$$

where  $Y$ ; is the vector of yields,  $X$  is factor loading matrix and  $\epsilon$  is the measurement errors. Similar to a previous dynamic latent factor models of the yield curve, the yields at all maturities load only on the unobserved yield factors.  $Z$  is the vector of observed macroeconomic and fiscal variables, and  $I$  is a identity matrix.

### 2.4.2.2 State equation

The state equation relates the dynamics of yield latent factors, macroeconomic and fiscal variables. We assume the transition follows the first-order vector autoregressive VAR(1) process that allows for exploiting the dynamic relationships between fiscal instability, economy and term structure. The VAR model can be

written as.

$$\xi_t = \mu + V\xi_{t-1} + \eta_t \quad (2.8)$$

where  $\xi_t$  is the vector notation of yield latent factors, macroeconomic and fiscal variables.  $\mu$  is a vector of intercept terms.  $V$  is the matrix of autoregressive coefficients and  $\eta_t$  is the vector of random disturbances.

The ordering of the variables in the model is based on [Diebold et al. \(2006\)](#) and others as in the Nelson-Siegel macro-finance term structure literatures. We place the yield curve latent factors and then fiscal and macroeconomic variables are followed. Any changes in the shape of the yield curve, represented by latent factors, will affect fiscal and macro variables and vice versa. A shock to fiscal stance of instability may impact the term spread due to a change in perceived probability of default risk. The adjustment of macroeconomic variables will then work through the reaction of economic agents to the term spread signal.

#### 2.4.2.3 The Variance-covariance Matrix

Following the state space presentation in a simple model from the previous section, the innovations of both measurement equation and state equation are assumed to be normally distributed and mutually uncorrelated. Additionally, the measurement and transition disturbances are assumed to be orthogonal to each other. This assumption is used to avoid numerical difficulties by reducing the number of coefficients and obtain computational tractability.

$$\begin{pmatrix} \omega_t \\ u_t \end{pmatrix} \sim WN \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] \quad (2.9)$$

where  $Q$  is non-diagonal matrix of variance-covariance of the innovations for the measurement equation and  $H$  is the diagonal matrix of variance-covariance matrix for the state equation.

In comparison with the standard model, the measurement equation in our macro-finance-fiscal model remains fully described by the three latent factors. For the state equation, however, the yield latent factors are appended to include macro and fiscal variables. Hence, the inclusion of macroeconomic and fiscal instability information affects the dynamic term structure only through unobserved yield factors.

### 2.4.3 The One-step Estimation Procedure Based On Kalman Filter

In order to estimate the model, we implement a simultaneous estimation of the measurement and state equations by using the Kalman filter method. The one-step estimation performed in the Kalman filter gives maximum likelihood estimates of unobserved parameters conditional on the past and current observations. The Kalman filter is a recursive algorithm for updating linear projections for a dynamic system in state-space representation. This method carries out the prediction error decomposition of the likelihood function. We have to use numerical procedure of optimization in order to get the best parameters.

By present the dynamic Nelson-Siegel model as a linear Gaussian state space model, the state vector of unobserved factors  $\beta_t$  can be extracted conditional on the past and concurrent observations of yields by the Kalman filter.<sup>1</sup>

Assuming normality in all distributions of the state space system, the conditional distribution of latent factor  $\beta_t$  derived from yield  $Y_t$  can be characterized by its expected mean  $\hat{\beta}_t$  and covariance  $\widehat{P}_{t+1}$ .

---

<sup>1</sup>Filter is a term used to describe an algorithm that allows recursive estimation of unobserved, time varying parameters rely on information up to time  $t$ . Once a new observation  $y_t$  becomes available, it will update information.

$$\beta_{t+1} = E(\beta_{t+1}|Y_t) \quad (2.10)$$

$$P_{t+1} = Var(\beta_{t+1}|Y_t) \quad (2.11)$$

$$(2.12)$$

The mean of the conditional distribution of  $\beta_{t+1}$  represents an optimal estimator of the state vector at time  $t + 1$  based on the observations up to yield  $Y_t$ . It minimizes the mean squared error (MSE) matrix  $E[(\beta_{t+1} - \mu_{t+1})(\beta_{t+1} - \mu_{t+1})']$  for all  $\beta_{t+1}$ .

To estimate unobserved factors  $\beta_t$  conditional on yield  $Y_t$ , it needs to have a prior guess of the initial state  $\beta_0$  under the assumption that it is a Gaussian random variable with expected mean  $E(\beta_0|Y_{t-1}) = \bar{\beta}_0$  and variance  $Var(\beta_0|Y_{t-1}) = P_0$ .

#### 2.4.3.1 The Prediction Equation

Under the assumption that latent factor  $\beta_t$  given information of yield  $Y_t$  is normally distributed with expected mean  $\hat{\beta}_t$  and covariance  $P_t$  implies that  $\hat{\beta}_{t+1}$  and  $P_{t+1}$  can be calculated recursively from  $\hat{\beta}_t$  and  $P_t$  as.

$$\hat{\beta}_{t+1} = \mu_t + T\beta_{t-1} \quad (2.13)$$

The corresponding optimal predictor of  $y_t$  given information at  $t - 1$  is.

$$Y_{t|t-1} = X_t\beta_{t|t-1} + u_t \quad (2.14)$$

The prediction error is

$$\nu_t = \eta_{t|t-1} = y_t - y_{t|t-1} = y_t - X_t\beta_{t|t-1} \quad (2.15)$$

and its variance of prediction error is

$$f_{t|t-1} = E[(\hat{\beta}_{t|t-1} - \beta_{t-1})(\hat{\beta}_{t|t-1} - \beta_{t-1})'] \quad (2.16)$$

Equivalently

$$f_{t|t-1} = TP_{t|t-1}T' + Q \quad (2.17)$$

### 2.4.3.2 The Updating Equation and Filtered Estimation

When new observation  $y_t$  is available, the optimal predictor  $\hat{\beta}_{t|t-1}$  and its variance are updated. The filtered estimate of  $\beta_t$  is  $\hat{\beta}_t$  and is updated from  $\hat{\beta}_{t|t-1}$

$$\hat{\beta}_t = \hat{\beta}_{t|t-1} + P_{t|t-1}T'f_{t|t-1}^{-1}(y_t - X_t\beta_{t|t-1}) \quad (2.18)$$

where  $\kappa_t = P_{t|t-1}T'f_{t|t-1}^{-1}$  is denoted as the Kalman gain. It is a weight given to new information contained in the prediction error.

### 2.4.3.3 The Smoothed Estimates

The smoothed estimate of  $\beta_t$  updates estimation based on the whole set of  $T$  observation  $Y_t$  when they are available.

$$\hat{\beta}_{t|T} = \hat{\beta}_{t|t-1} + \kappa_t\eta_{t|t-1} \quad (2.19)$$

The smoothed estimates are recursively calculated backwards to the last value of the filtered estimate with covariance.

$$P_{t|T} = P_{t|t-1} + \kappa_tTP_{t|t-1} \quad (2.20)$$

Denote  $\theta$  as the parameters of the state space model. The Kalman Filter produces the prediction errors which can be decomposed into log-likelihood function of the



prediction error.

$$\ln L(\theta|y) = -\frac{NT}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^T \ln |F_t \theta| - \frac{1}{2} \sum_{t=1}^T \nu_t'(\theta) F_t^{-1} F_t^{-1}(\theta) \nu_t(\theta) \quad (2.21)$$

Estimation of parameters is based on the numerical maximization of the log-likelihood function. A Nelder-Mead optimization method is employed for solving the optimal value of parameters.

## 2.5 Data

The dataset used here consists of end-of-month Spanish zero-coupon bond yields of 22 maturities; 3, 6, 9, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114 and 120 month-to-maturity, from January 1998 through September 2011. We also use Spanish macroeconomic and fiscal variables for vector-autoregressive analysis, which are the growth rate of public debt-to-GDP (GDTG), net government budget position-to-GDP (GBTG), change in yield spread between Spanish and German 10-year government bond (DSPRD), the growth in industrial production index (GIPI) and inflation rate (INF).

## 2.6 Empirical Results

In this section, we provide summary statistics for the data set and perform preliminary analysis that gives a foundation for the subsequent estimation. In Subsection 2.6.1, we present descriptive statistics of the yields, macroeconomic and fiscal variables. Then, we conduct an in-sample fit assessment to evaluate the performance of the maximum likelihood estimation with Kalman filter in Subsection 2.6.2 and report the empirical results of the estimated latent factors in Subsection 2.6.3.

## 2.6.1 Descriptive Statistics

Prior to fitting the term structure model, we report summary statistics for Spanish government bond yields at representative maturities and provide descriptive statistics of the macroeconomic and fiscal variables afterwards.

### 2.6.1.1 Yield Statistics

In this part, a three-dimensional plot of yields for various maturities are presented and discussed, following by summary statistics of the yields over the period of 1998 to 2011.

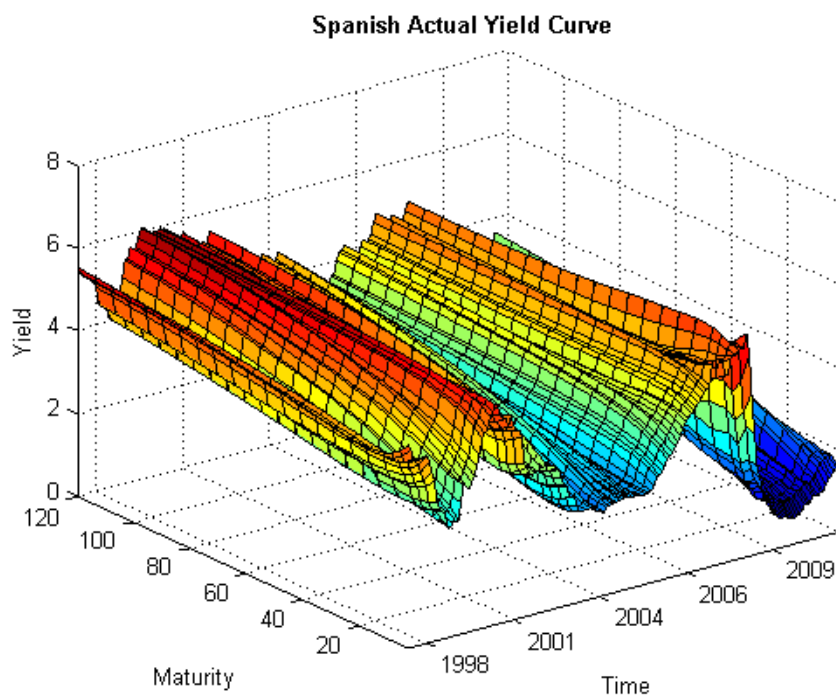


FIGURE 2.4: The Spanish Actual Yield Curve

Figure 2.4 graphically shows that the Spanish yields exhibit substantial variation in their level while their slope and curvature are less varied. For the reference period, yield curves are typically upward sloped, humped and monotonically concave. As can be seen the in time series plots of Figure 2.4, the yields with longer time to maturity tends to be less volatile.

Table 2.1 reports yield statistics for different maturities. For each maturity, we report mean, standard deviation, minimum, maximum and a selection of autocorrelation. The empirical statistics for proxies of the level, slope, and curvature of the yield curve also presented.

TABLE 2.1: Descriptive statistics of the Spanish government bond yield

| <b>Maturity</b> | <b>Mean</b> | <b>Std Dev</b> | <b>Min</b> | <b>Max</b> | $\rho(1)$ | $\rho(12)$ | $\rho(30)$ |
|-----------------|-------------|----------------|------------|------------|-----------|------------|------------|
| 3               | 3.05        | 1.36           | 0.63       | 5.38       | 0.98      | 0.38       | -0.32      |
| 6               | 3.09        | 1.26           | 0.93       | 5.36       | 0.98      | 0.40       | -0.34      |
| 9               | 3.09        | 1.25           | 0.92       | 5.37       | 0.98      | 0.41       | -0.34      |
| 12              | 3.15        | 1.22           | 1.09       | 5.38       | 0.98      | 0.41       | -0.35      |
| 18              | 3.24        | 1.17           | 1.17       | 5.47       | 0.98      | 0.42       | -0.32      |
| 24              | 3.33        | 1.12           | 1.22       | 5.52       | 0.97      | 0.43       | -0.28      |
| 30              | 3.43        | 1.08           | 1.28       | 5.58       | 0.97      | 0.44       | -0.23      |
| 36              | 3.53        | 1.05           | 1.36       | 5.61       | 0.97      | 0.44       | -0.20      |
| 42              | 3.62        | 1.01           | 1.45       | 5.64       | 0.97      | 0.45       | -0.17      |
| 48              | 3.71        | 0.99           | 1.53       | 5.67       | 0.97      | 0.46       | -0.14      |
| 54              | 3.79        | 0.96           | 1.62       | 5.70       | 0.96      | 0.47       | -0.10      |
| 60              | 3.86        | 0.94           | 1.71       | 5.72       | 0.96      | 0.48       | -0.07      |
| 66              | 3.94        | 0.92           | 1.79       | 5.75       | 0.96      | 0.48       | -0.05      |
| 72              | 4.01        | 0.91           | 1.88       | 5.78       | 0.96      | 0.50       | -0.02      |
| 78              | 4.07        | 0.89           | 1.95       | 5.81       | 0.96      | 0.51       | 0.01       |
| 84              | 4.14        | 0.88           | 2.02       | 5.83       | 0.96      | 0.52       | 0.03       |
| 90              | 4.19        | 0.87           | 2.09       | 5.86       | 0.96      | 0.53       | 0.06       |
| 96              | 4.25        | 0.87           | 2.15       | 5.90       | 0.96      | 0.54       | 0.08       |
| 102             | 4.30        | 0.86           | 2.21       | 5.95       | 0.96      | 0.55       | 0.09       |
| 108             | 4.34        | 0.85           | 2.26       | 6.00       | 0.97      | 0.56       | 0.11       |
| 114             | 4.39        | 0.84           | 2.31       | 6.04       | 0.97      | 0.56       | 0.12       |
| 120             | 4.43        | 0.84           | 2.36       | 6.07       | 0.97      | 0.57       | 0.13       |
| Slope           | 0.74        | 0.67           | -0.94      | 1.98       | 0.94      | 0.00       | -0.16      |
| Curve           | -0.66       | 0.46           | -1.45      | 0.40       | 0.93      | 0.26       | -0.28      |

The descriptive statistics reveal that the yield curve tends to be upward sloping and concave. The volatility of the yields decreases with longer maturity. Time series of yields are persistence given by the first order autocorrelation above 0.96 for all maturities. The empirical level, slope, and curvature are also persistent but the curvature is least persistent.

### 2.6.1.2 Macroeconomic and Fiscal Variable Statistics

The descriptive statistics of the macroeconomic and fiscal variable used in the macro-finance-fiscal term structure model are reported in Table 2.2. For each variable, we report the mean, standard deviation, minimum, maximum, autocorrelation coefficient at various displacements and the Augmented Dickey-Fuller test statistics for unit-root.

TABLE 2.2: Descriptive statistics of macroeconomic variables

| <b>Maturity</b> | <b>Mean</b> | <b>Std Dev</b> | <b>Min</b> | <b>Max</b> | $\rho(1)$ | $\rho(12)$ | $\rho(30)$ | <b>ADF</b>   |
|-----------------|-------------|----------------|------------|------------|-----------|------------|------------|--------------|
| GDTG            | 0.36        | 12.08          | -10.80     | 36.01      | 0.99      | 0.54       | -0.02      | <b>0.20*</b> |
| GBTG            | -1.23       | 2.63           | -10.98     | 2.27       | 0.95      | 0.64       | -0.05      | <b>0.76*</b> |
| DSPRD           | 0.11        | 0.45           | -1.14      | 1.80       | 0.92      | 0.10       | 0.16       | <b>1.21*</b> |
| GIPI            | -0.10       | 5.95           | -21.61     | 10.96      | 0.93      | 0.17       | -0.12      | <b>0.27*</b> |
| INF             | 2.77        | 1.22           | -1.40      | 5.30       | 0.96      | -0.11      | 0.02       | <b>0.48*</b> |

*Notes:* Bold numbers imply variables are found to be stationary by augmented Dickey-Fuller: ADF test and \* indicates statistical significance at 5 percent level

Concerning the macroeconomic and fiscal variables, we use monthly data from January 1998 to August 2011 for the growth rate of public debt-to-GDP (GDTG), net government budget-to-GDP (GBTG), change in yield spread between Spanish and German 10-year government bond (DSPRD), the growth rate of in industrial production index (GIPI) and inflation rate (INF). All five variables are measured as percentages for two main reasons: first, for the stationarity consideration; and secondly, for consistency with the yield spreads which for all maturities are measured in annual percent format.

As shown in Table 2.2, the change in yield spreads range between -1.14 and 1.80 percent points and average about 11 basis points. For fiscal variables; the net government budget position to GDP is less volatile, compared with the growth rate of debt-to-GDP, as witnessed by their standard deviations. In fact, the growth rate of public debt to GDP is the highest volatility among macroeconomics and fiscal variables due to its unprecedented rise after the run-up to the global financial crisis, staying high during the European sovereign debt crisis afterwards. The autocorrelation coefficients at a one-month lag show all variables are strongly positive autocorrelated. We also examine whether the time series are unit roots by

applying the augmented DickeyFuller (ADF) test and find all variables are stationary.

## 2.6.2 Model Estimation

In this subsection, we report and discuss the accuracy of the dynamic Nelson-Siegel model estimated by maximum likelihood with Kalman filter and then present the estimation of yield curves at some selected dates.

### 2.6.2.1 Estimation Accuracy

To assess the ability of the dynamic Nelson-Siegel model with the Kalman filter estimation to fit the yield data, we report the statistics that describe the in-sample fit and the residuals from the estimation procedure as shown in Figure 2.5 and Table 2.3.

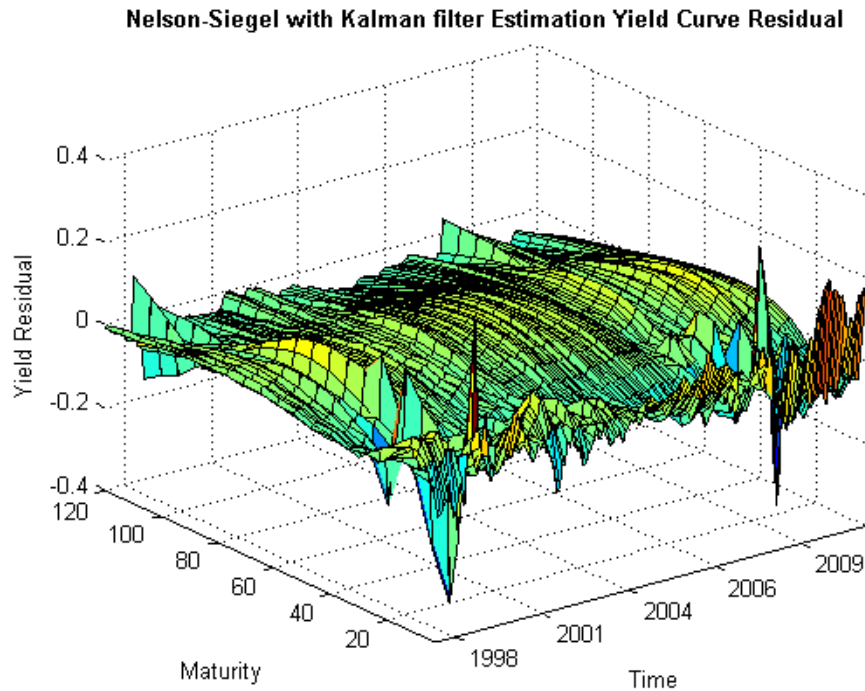


FIGURE 2.5: The Residual of the Spanish Yield Curve Estimation

Figure 2.5 shows the good fit of the Nelson-Siegel term structure model. There is not a large difference between the estimated yields and the historical yields. For short maturity yields, we find minimal residuals which confirms the stylized facts that short-end yields have more variation.

Table 2.3 provides descriptive statistics for the residuals. The goodness of fit also measures by root mean square errors of the estimated yields from various maturities.

TABLE 2.3: Descriptive statistics of the yield curve residuals, estimated by the Nelson-Siegel model with Kalman filter estimation

| Maturity | Mean  | Std Dev | Min   | Max  | RMSE | $\rho(1)$ | $\rho(12)$ | $\rho(30)$ |
|----------|-------|---------|-------|------|------|-----------|------------|------------|
| 3        | 0.04  | 0.08    | -0.29 | 0.36 | 0.09 | 0.79      | 0.21       | -0.11      |
| 6        | -0.01 | 0.04    | -0.20 | 0.14 | 0.04 | 0.71      | 0.36       | 0.00       |
| 9        | 0.01  | 0.05    | -0.20 | 0.31 | 0.06 | 0.78      | 0.10       | -0.29      |
| 12       | -0.02 | 0.06    | -0.36 | 0.10 | 0.06 | 0.75      | 0.18       | -0.02      |
| 18       | -0.04 | 0.04    | -0.25 | 0.11 | 0.06 | 0.57      | 0.05       | -0.01      |
| 24       | -0.03 | 0.03    | -0.10 | 0.18 | 0.04 | 0.39      | 0.04       | -0.14      |
| 30       | -0.02 | 0.03    | -0.17 | 0.10 | 0.03 | 0.59      | 0.30       | 0.13       |
| 36       | -0.01 | 0.02    | -0.07 | 0.17 | 0.03 | 0.55      | 0.11       | -0.06      |
| 42       | 0.01  | 0.03    | -0.10 | 0.11 | 0.03 | 0.76      | 0.13       | -0.29      |
| 48       | 0.02  | 0.02    | -0.05 | 0.12 | 0.03 | 0.78      | 0.06       | -0.26      |
| 54       | 0.03  | 0.02    | -0.04 | 0.13 | 0.04 | 0.75      | -0.02      | -0.21      |
| 60       | 0.03  | 0.02    | -0.05 | 0.12 | 0.04 | 0.71      | -0.06      | -0.24      |
| 66       | 0.04  | 0.02    | -0.05 | 0.11 | 0.04 | 0.68      | -0.06      | -0.19      |
| 72       | 0.03  | 0.02    | -0.03 | 0.09 | 0.04 | 0.60      | -0.11      | -0.18      |
| 78       | 0.03  | 0.01    | -0.02 | 0.06 | 0.03 | 0.58      | -0.15      | -0.08      |
| 84       | 0.02  | 0.01    | -0.03 | 0.04 | 0.02 | 0.51      | -0.02      | 0.03       |
| 90       | 0.01  | 0.01    | -0.04 | 0.02 | 0.01 | 0.56      | 0.12       | -0.04      |
| 96       | 0.00  | 0.01    | -0.05 | 0.02 | 0.01 | 0.71      | 0.17       | -0.08      |
| 102      | -0.01 | 0.01    | -0.07 | 0.03 | 0.02 | 0.76      | -0.01      | -0.24      |
| 108      | -0.03 | 0.02    | -0.11 | 0.03 | 0.03 | 0.66      | -0.05      | -0.19      |
| 114      | -0.03 | 0.02    | -0.13 | 0.07 | 0.04 | 0.61      | -0.06      | -0.14      |
| 120      | -0.05 | 0.03    | -0.16 | 0.10 | 0.05 | 0.58      | -0.06      | -0.08      |

Statistics of the residuals indicates that the estimates from the Nelson-Siegel fit the Spanish yield data well. The residual sample autocorrelations imply the dynamics are persistent. The estimated means and standard deviations of the residuals are negligible. Root mean square errors indicate a good in-sample fit at all maturities.

### 2.6.2.2 Cross-sectional in-sample fit

To further examine the fit of the Nelson-Siegel model, we examine the estimated yield curves from both models against the actual yields at particular days as shown in Figure 2.6. We plot the yield curves on 30 November 1998, 28 April 2006, 29 August 2006 and 31 August 2009. These four selected dates are examples of the various different term structure shapes that occur in the data.

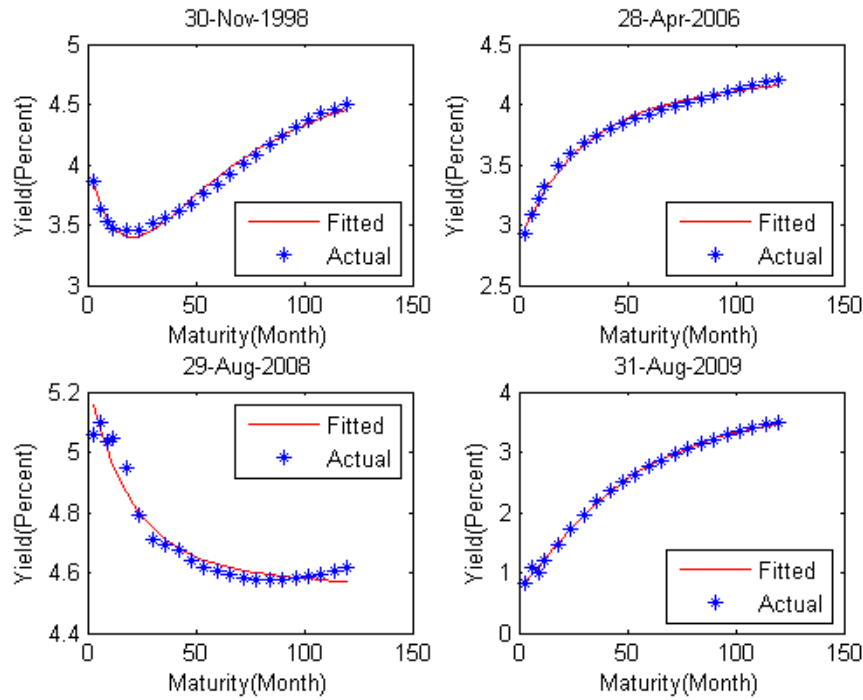


FIGURE 2.6: Fitted yield curve for specific months

In Figure 2.6, the plots demonstrate that the term structure curve shapes can vary over time. On the 30 November 1998 curve is a J-curve shape and the curve for 29 August 2008 is an inverted yield curve. The shapes of the yields are concave upward for 28 April 2006 and 31 August 2009, despite the first one in April being much steeper for short maturities. As expected, the Nelson-Siegel model seems flexible enough to fit complex curves, for example, a J-curve shape.

### 2.6.3 Dynamic Latent Factor Estimates

In this subsection, we report the descriptive statistics of the estimated latent factors and provide correlation coefficients between the latent factors and macroeconomic variables. Then, we plot the time series of estimated latent factors, their corresponding proxies and potentially related macroeconomic variables.

#### 2.6.3.1 Latent factor statistics

The statistical properties of the estimated latent factors from the Nelson-Siegel model are presented in Table 2.4.

TABLE 2.4: Descriptive statistics of the latent factors, estimated by the Nelson-Siegel model (NS)

|     | Mean  | Std Dev | Min   | Max  | $\rho(1)$ | $\rho(12)$ | $\rho(30)$ | ADF   |
|-----|-------|---------|-------|------|-----------|------------|------------|-------|
| LEV | 4.98  | 0.82    | 2.99  | 6.66 | 0.96      | 0.64       | 0.30       | -0.85 |
| SLP | -1.82 | 1.15    | -3.65 | 0.79 | 0.97      | 0.22       | -0.26      | -1.20 |
| CUR | -2.43 | 1.43    | -5.07 | 1.32 | 0.87      | 0.13       | -0.20      | -0.90 |

Our regression provides us with three latent factors and a set of residuals. The descriptive statistics of latent factors are summarized in Table 2.4. The yield curve dynamics are characterized, by the evolution of the estimated latent factors. We found the level factor (LEV) is positive with a mean of 4.98. In contrast, the slope (SLP) and curvature (CUR) take negative mean values, fluctuating around -1.82 and -2.43 respectively. Based on autocorrelation coefficients with lags of 1, 12 and 30 months, all factors demonstrate high positive autocorrelation. The slope factor is the most persistent amongst other estimated factors. This strong autocorrelation across the sample indicates the future values of these factors should be forecasted with their own lagged values. In terms of volatility, we observed the curvature factor has highest standard deviation even it is the least persistent. With negative means and other quite similar statistical properties, the slope and curvature factors are noticeably correlated.



### 2.6.3.2 Latent Factor Correlation

We present the correlation coefficients between the latent factors and macroeconomic variables, including fiscal positions in Table 2.5.

TABLE 2.5: Correlation coefficients of the estimated latent factors and their empirical proxies

|              | LEV   | SLP   | CUV   | GDTG  | GBTG  | DSPRD | GIPI | INF  |
|--------------|-------|-------|-------|-------|-------|-------|------|------|
| <b>LEV</b>   | 1.00  |       |       |       |       |       |      |      |
| <b>SLP</b>   | -0.09 | 1.00  |       |       |       |       |      |      |
| <b>CUV</b>   | -0.17 | 0.42  | 1.00  |       |       |       |      |      |
| <b>GDTG</b>  | -0.41 | -0.47 | -0.21 | 1.00  |       |       |      |      |
| <b>GBTG</b>  | 0.29  | 0.43  | 0.27  | -0.83 | 1.00  |       |      |      |
| <b>DSPRD</b> | -0.49 | -0.11 | -0.03 | 0.46  | -0.48 | 1.00  |      |      |
| <b>GIPI</b>  | 0.23  | 0.12  | 0.20  | -0.51 | 0.38  | -0.37 | 1.00 |      |
| <b>INF</b>   | 0.15  | 0.48  | 0.28  | -0.70 | 0.55  | -0.04 | 0.42 | 1.00 |

In regard to the relation among yield factors, we find the correlation between all pairs of latent factors are not strongly correlated with factor variables. For the relation between the curvature and slope factors, their correlation coefficient is found to be non-negligible at 0.42. There are apparently some strong bilateral relations between latent factors and macroeconomic variables. Indeed, there is a negative correlation between yield level factor and term spread because a lower level of interest rates are expected to offset the excessive increase in borrowing costs caused by the widening term spreads. We also find that the yield slope is positively correlated with inflation expectations. The yields on long-term bonds tend to be higher if investors expect a future increase in inflation, thus yield slope will rise with inflationary pressure.

We observe the growth rate of public debt-to-GDP and the net position of government budget to GDP are significantly and negatively correlated. A worse budget position may lead government to become highly indebted borrowers. More interestingly, some negative correlations between yield latent factors and growth in public debt-to-GDP are observed. Higher debt could cause a worsened recession. A deflationary pressure would probably decrease the expected policy interest rate and therefore lessen long-term yields. Since expected interest rates are lower, the yield curve level falls together with an inverted yield slope. Another interesting

point we should mention is there exists positive correlation between the growth of public debt and term spread. High growth of public debt is assumed to entail higher yield spread associated with a higher perceived default risk. We also observe the evidence for the presence of a negative relation between the growth of public debt-to-GDP and economic activities. As we expect, growth in government debt-to-GDP would increase more concern about an unsustainable government and then cause an even more output and price level deterioration.

### 2.6.3.3 Latent factors, Empirical yields and Macro variables

In this part, we explore the empirical relevance of our estimated latent factors, their corresponding proxies and potentially related macroeconomic variables. A plot of each factor against its empirical counterpart can help us to investigate whether our model can accommodate the stylized facts of yield factors. We also compare the evolution of latent factors with some seemingly related macroeconomic variables to examine the comovements among them. Figure 2.7 plots the time series of the estimated latent factors against those of their empirical proxies and selected macroeconomic variables. The upper panel shows level (LEV), empirical level (E-LEV) and yield spread (SPRD), while slope (SLP), empirical slope (E-SLP) and inflation (IFL) are plotted in the middle panel. In the lower panel, we just only plot curvature (CUR) and empirical curvature (E-CUR) since relationships between curvature and macroeconomic variables are minimal.

We follow [Diebold et al. \(2006\)](#) to define the empirical proxy for the level factor as the long-term (10-year) bond yield. The empirical slope is represented by the difference between the long-term (10-year) and short-term (3-month) yield and the empirical curvature is determined as two times the 2-year yield minus the sum of the 10-year and 3-month yields.

From the upper panel of Figure 2.7, the estimated level factor is closely related to its empirical proxy and yield spread. This evidence is consistent with the expectation hypothesis of the term structure that the long-term rate reflects the anticipation of future short-term rate plus a risk premium. A higher term premium drives up the long-end of the yield and thereby widens the yield spread.

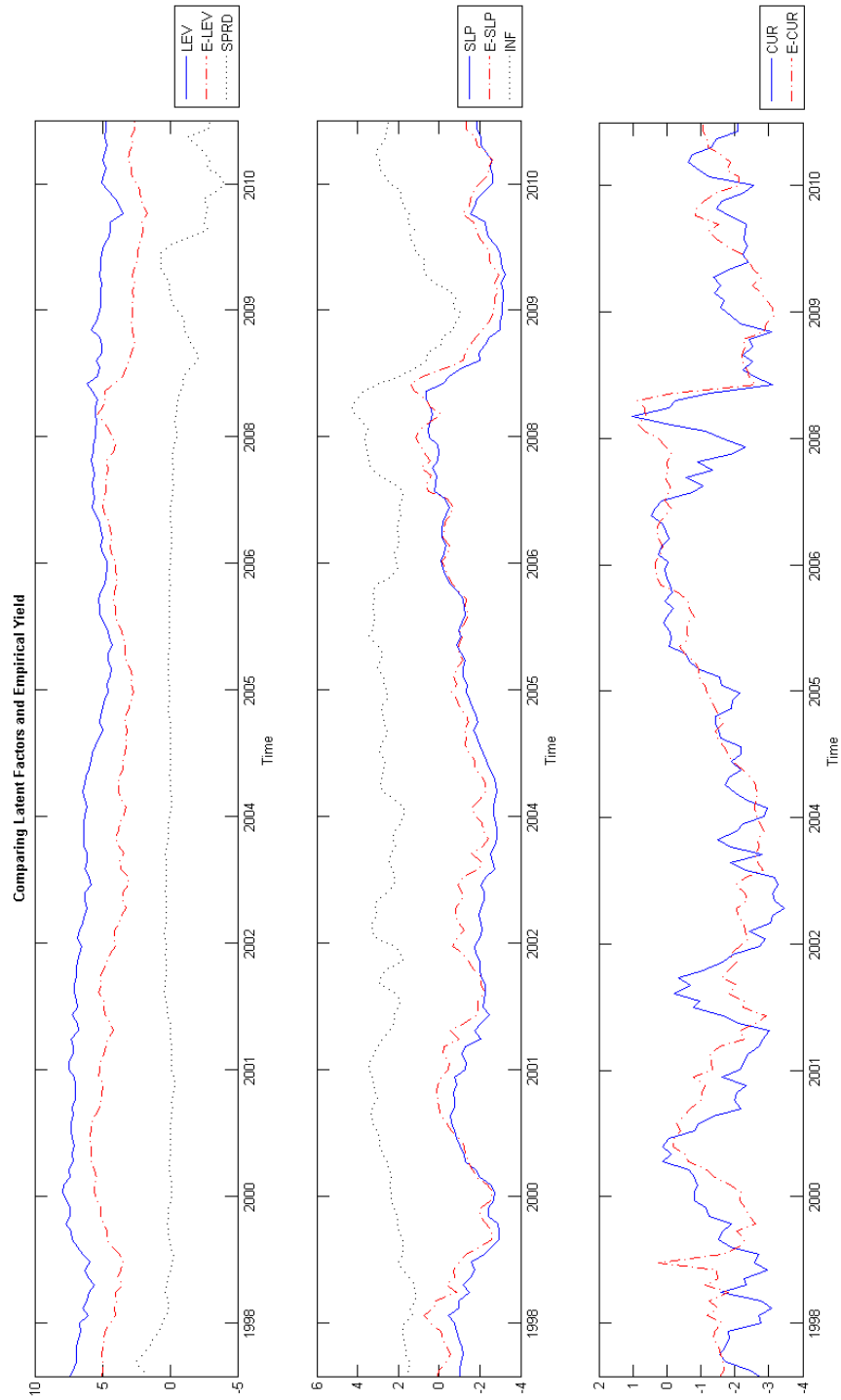


FIGURE 2.7: Estimated latent factors and empirical factors

We find that over our sample period, the movement of the level factor and its corresponding proxy are persistent, particularly after the formation of EMU in 1999. Even if there are minimal effects from a drastic increase in yield spreads after the financial turmoil in 2008, we observe evidence of structural change when the tensions of a sovereign debt crisis intensified. Our finding is consistent with previous studies in the macro-finance literature, for example, [Lange \(2013\)](#) and [Ullah et al. \(2013\)](#) who found that level factor movements are related to monetary variables. Nonetheless, we use the yield spread as a channel of a fiscal transmission mechanism to financial markets instead of the policy interest rate since it is not independently determined by the Spanish central bank and ran into the zero lower bound after the global financial crisis.

In the middle panel of Figure [2.7](#) we observe a close link between the estimated slope factor, empirical slope factor and inflation. The slope of the yield curve can indicate the link between expectations of inflation and the long-term yield, which is also found in [Lange \(2013\)](#). When the markets anticipate that inflationary pressure is rising, there would be a steeper slope of the yield curve that require an increase in long-term rate. For the Spanish case, rising real estate prices in the early 2000s led the yield slope to rise associated with the increase in expected inflation. After reaching the peak level in 2008, yield slope fell steeply since the fear of delation had been triggered after the global financial crisis. From 2009 onwards, the higher-than-expected sovereign debt then returned the yield slope to an upward trend. Our findings suggest the variation in yield slope is closely explained by state of economy, and indicate the long-term interest rate became an important possible monetary instrument in the absence of monetary policy independence.

For the last panel of Figure [2.7](#), we can observe the estimated curvature factor and its yield proxy are correlated even though the estimated factor is the more volatile relative to its proxy. Although there is no evidence of a relation between the curvature factor and other macroeconomic variables, we find the movement of curvature factor follow the same pattern as the slope factor, though it is relatively less persistent.

## 2.7 Macro-Finance-Fiscal Model Analysis

In order to integrate fiscal instability variable in the macro-finance-fiscal term structure model, we have two alternative fiscal proxies represented fiscal indiscipline; the growth rate of public debt to GDP (GDTG) and net government budget position-to-GDP (GBTG). Indeed, public debt and net government budget position could convey relevant information regarding to sovereign credit risk that might explain a widening term spread. Our choice of separating the macro-finance-fiscal term structure model is based on the argument that a different fiscal instability proxies would generate different estimates obtained from macro-finance-fiscal model .

In this section, we estimate the vector-autoregressive (VAR) model in a unique state-space model with different fiscal proxies for fiscal instability; public debt to GDP (GDTG) and net government budget position-to-GDP (GBTG) respectively. We report the estimated coefficient matrices for each model. Then, we present the impulse response functions (IRFs) to positive innovation of yield latent factors, macro and fiscal variables with magnitude of one standard deviation. To provide a more intuitive and quantified description of the dynamic reaction, we conduct forecast error variance decomposition (FEVD). The following results are based on the estimates of the individual model which has been described in the previous section.

### 2.7.1 Macro-Finance-Fiscal Model with Debt-to-GDP as a Proxy for Fiscal Instability

We are now present the estimation results for the model in the case of using the growth rate of public debt to GDP (GDTG) as fiscal instability proxy.

#### 2.7.1.1 VAR(1) Parameter Estimates

Table 2.6 reports the estimates of the parameters which represent the interaction between the yield latent factors and macroeconomic variables, including the

change in debt-to-GDP.

TABLE 2.6: Estimates of the Macro-finance-fiscal Model with a change in debt-to-GDP proxy for fiscal instability

|                | <i>L t-1</i> | <i>S t-1</i> | <i>C t-1</i> | <i>GDTG t-1</i> | <i>DSPRD t-1</i> | <i>GIPI t-1</i> | <i>INF t-1</i> | <i>u</i>    |
|----------------|--------------|--------------|--------------|-----------------|------------------|-----------------|----------------|-------------|
| <i>L t</i>     | <b>0.97</b>  | 0.00         | <b>0.02</b>  | <b>-0.01</b>    | 0.07             | 0.00            | <b>-0.04</b>   | <b>0.30</b> |
|                | <i>0.02</i>  | <i>0.02</i>  | <i>0.01</i>  | <i>0.00</i>     | <i>0.05</i>      | <i>0.00</i>     | <i>0.02</i>    | <i>0.13</i> |
| <i>S t</i>     | 0.01         | <b>0.93</b>  | <b>0.07</b>  | <b>0.01</b>     | -0.08            | <b>0.01</b>     | <b>0.05</b>    | -0.16       |
|                | <i>0.03</i>  | <i>0.02</i>  | <i>0.02</i>  | <i>0.00</i>     | <i>0.06</i>      | <i>0.00</i>     | <i>0.03</i>    | <i>0.16</i> |
| <i>C t</i>     | -0.08        | -0.03        | <b>0.87</b>  | <b>-0.02</b>    | 0.26             | 0.02            | <b>-0.16</b>   | 0.47        |
|                | <i>0.08</i>  | <i>0.06</i>  | <i>0.04</i>  | <i>0.01</i>     | <i>0.16</i>      | <i>0.01</i>     | <i>0.07</i>    | <i>0.42</i> |
| <i>GDTG t</i>  | <b>0.28</b>  | <b>0.13</b>  | 0.04         | <b>0.96</b>     | <b>0.54</b>      | <b>-0.16</b>    | -0.10          | -0.72       |
|                | <i>0.10</i>  | <i>0.07</i>  | <i>0.05</i>  | <i>0.01</i>     | <i>0.19</i>      | <i>0.01</i>     | <i>0.08</i>    | <i>0.52</i> |
| <i>DSPRD t</i> | -0.02        | 0.02         | <b>-0.01</b> | <b>0.00</b>     | <b>0.93</b>      | <b>0.00</b>     | 0.02           | 0.08        |
|                | <i>0.02</i>  | <i>0.01</i>  | <i>0.01</i>  | <i>0.00</i>     | <i>0.03</i>      | <i>0.00</i>     | <i>0.01</i>    | <i>0.08</i> |
| <i>GIPI</i>    | -0.15        | <b>-0.54</b> | <b>0.23</b>  | 0.00            | <b>-0.93</b>     | <b>0.92</b>     | -0.08          | 0.56        |
|                | <i>0.24</i>  | <i>0.18</i>  | <i>0.13</i>  | <i>0.02</i>     | <i>0.49</i>      | <i>0.03</i>     | <i>0.21</i>    | <i>1.30</i> |
| <i>INF t</i>   | -0.01        | -0.02        | <b>0.04</b>  | 0.00            | 0.08             | <b>0.02</b>     | <b>0.90</b>    | <b>0.35</b> |
|                | <i>0.04</i>  | <i>0.03</i>  | <i>0.02</i>  | <i>0.00</i>     | <i>0.08</i>      | <i>0.01</i>     | <i>0.03</i>    | <i>0.21</i> |

*Notes:* The table displays matrix of the macro-finance-fiscal model's coefficients and standard errors. Bold numbers indicate statistical significance at 5 percent level while italic numbers refer to standard errors.

The results from the estimation shows a total of 16 coefficients are significant, 7 diagonal and 9 off-diagonal. The diagonal coefficients reveal the yield latent factors and other macro variables, including change in debt-to-GDP are highly persistent. Considering the lagged influence of the government debt position on current yield curve factors, this previous period fiscal variable explains negatively the level and curvature factors but positively the slope factor. Since recession causes a steep deterioration in government finances and a rapid rise in government debt, the default risk premia on long-term government bonds consequently tend to be high. Meanwhile, the expected short-term rate tends to be lower and causes the yield level as well as the curvature to decrease. As the long end of the yield curve rises whereas the short end rate goes down, the yield slope therefore increases due to high potential exposure to government bond losses.

For the coefficients showing the influence of the lagged change in debt-to-GDP on macroeconomic variables, only the current period growth of government debt and the changes in yield spread are significantly and positively affected, albeit with a

minimal impact on the spread. Another main finding is that the lagged value of the spread between 10-year Spanish bonds over German bonds influences current changes in debt-to-GDP and changes in the industrial production index as well as its own lagged rate. In particular, a positive impact of lagged yield spread on the change in debt-to-GDP indicates that government liabilities would increase as the bonds are being paid back more which is associated with a higher default risk premium. As for the change in debt-to-GDP, not only the increase in the lagged yield spread and its own lag significantly raise debt position, but also the yield level and slope positively affect the change in government debt. The higher expected policy interest rate and higher difference between the long-term and short-term rate, represented by the yield level and slope, increase government debt payments.

It should be stressed that the relationships between public debt, yield latent factors and other macroeconomic variables are mainly represented by the interaction between the change in debt-to-GDP, term spreads and the growth in output. In fact, the change in the yield spread is influenced by the change in the public debt position. The higher debt position signals even lower expected policy interest rate, in other words, level factor. At the same time, long-term yield is raised up in line with higher term premium. As a result, the yield slope increase. A higher term spread also signals a recession, reflecting a lower economic activity. Our findings are consistent with the expectations of future monetary policy being actually related to fiscal variable.

#### **2.7.1.2 Impulse Response Functions**

Figure 2.8 shows the impulse response functions that summarize the response of the macroeconomic variables to shocks in the term-structure factors or vice versa.

A positive innovation to the change in debt-to-GDP with a magnitude of one standard deviation initially leads to a fall in the yield curve factors. These reactions are consistent with a monetary policy response that depresses the term structure in response to expected lower output. Among these latent factors, the dynamic path of the level factor appears to be more persistent. Nonetheless, the impact on yield curve factors are insignificant. Moreover, it can be observed that the

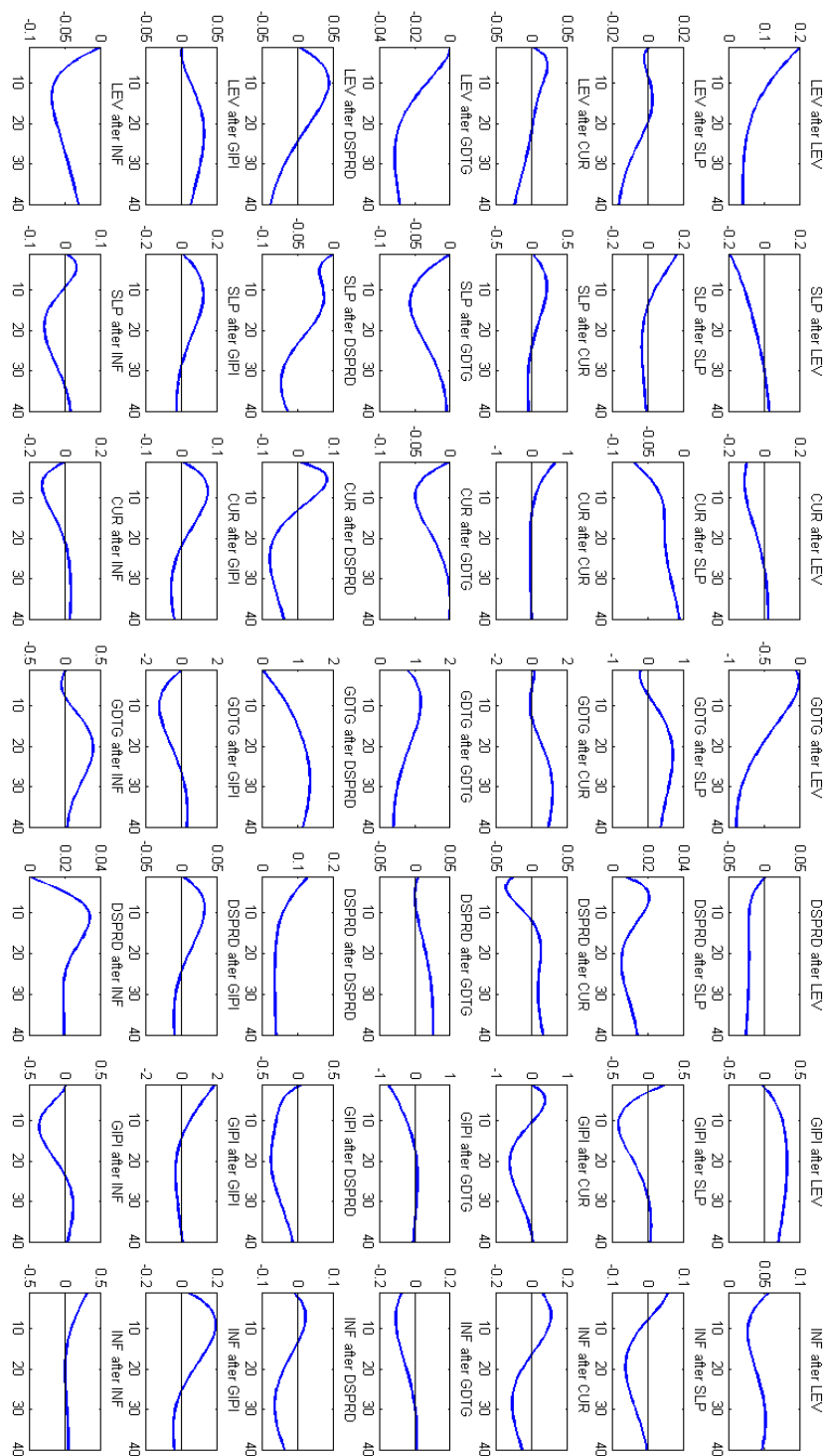


FIGURE 2.8: Impulse response function of the Macro-finance-fiscal Model with a change in debt-to-GDP proxy for fiscal instability



positive innovation of a change in the debt ratio also affects the yield spread even if it does not react immediately and only around 10 months later begins to pick up. This movement implies that the response of the long-term yield is lagged and takes time for investors to require a compensation for the higher risk premium. As for the effect on economic activity, we observe a significantly immediate drop in output growth for almost two years. In addition, the innovation of the change in government debt position also generates a deflation for over two years even if it is an insignificant impact. A key implication of this finding is that any changes in debt position can be used as a fiscal policy instrument to significantly affect the real economy. This interpretation is consistent with the idea of unconventional monetary policy and quantitative easing that targets the long-term yield.

Apart from the dynamic adjustments created by a change in debt ratio, we also find an innovation to change in the yield spread significantly raises the change in debt-to-GDP. The public debt gradually increases and reach a peak within around two and a half year, followed by sustained high levels of public debt. A positive innovation to growth in industrial production index significantly lessen debt-to-GDP for over two years. This reaction confirms the fact that the government debt position is pro-cyclical.

### **2.7.1.3 Forecast error variance decomposition**

Table 2.7 presents the results of forecast error variance decomposition of the change in public debt-to-GDP, yield spread, level and slope factors at 6, 12, 24 and 40 months horizons.

Regarding the variance of the error in forecasting the change in the debt-to-GDP ratio, we find an innovation of the government debt position is the most important variable in explaining the variation of the debt ratio, contributing a 60 percent share of the variations at a 6-month horizon. In addition, the shock in industrial production index also influences around one-third of the fluctuations in government liabilities. One year later, the innovation of the industrial production index becomes more important and almost equally contribute to the variation of forecasting errors of the debt ratio, roughly 40 percent. It should be noted that the

TABLE 2.7: Forecast error variance decomposition of the Macro-finance-fiscal Model with a change in debt-to-GDP proxy for fiscal instability

| Variable            | Horizon | LEV  | SLP  | CUR  | GDTG | DSPRD | GIPI | INF  |
|---------------------|---------|------|------|------|------|-------|------|------|
| <b><i>GDTG</i></b>  | 6       | 0.00 | 0.03 | 0.00 | 0.60 | 0.04  | 0.33 | 0.00 |
|                     | 12      | 0.01 | 0.02 | 0.00 | 0.45 | 0.10  | 0.42 | 0.00 |
|                     | 24      | 0.05 | 0.06 | 0.05 | 0.31 | 0.24  | 0.27 | 0.02 |
|                     | 40      | 0.11 | 0.06 | 0.15 | 0.19 | 0.32  | 0.15 | 0.01 |
| <b><i>DSPRD</i></b> | 6       | 0.01 | 0.02 | 0.09 | 0.00 | 0.83  | 0.03 | 0.02 |
|                     | 12      | 0.03 | 0.03 | 0.07 | 0.00 | 0.73  | 0.07 | 0.06 |
|                     | 24      | 0.07 | 0.03 | 0.06 | 0.01 | 0.66  | 0.07 | 0.10 |
|                     | 40      | 0.10 | 0.03 | 0.06 | 0.06 | 0.60  | 0.06 | 0.11 |
| <b><i>LEV</i></b>   | 6       | 0.94 | 0.00 | 0.01 | 0.00 | 0.02  | 0.00 | 0.03 |
|                     | 12      | 0.84 | 0.00 | 0.01 | 0.00 | 0.04  | 0.00 | 0.10 |
|                     | 24      | 0.71 | 0.00 | 0.01 | 0.02 | 0.05  | 0.02 | 0.19 |
|                     | 40      | 0.63 | 0.01 | 0.01 | 0.05 | 0.06  | 0.03 | 0.21 |
| <b><i>SLP</i></b>   | 6       | 0.47 | 0.24 | 0.21 | 0.01 | 0.00  | 0.05 | 0.01 |
|                     | 12      | 0.33 | 0.13 | 0.38 | 0.03 | 0.00  | 0.12 | 0.01 |
|                     | 24      | 0.28 | 0.10 | 0.37 | 0.04 | 0.02  | 0.16 | 0.04 |
|                     | 40      | 0.25 | 0.10 | 0.36 | 0.04 | 0.07  | 0.14 | 0.04 |

unexpected change in yield spread turns out to be the the main contributor to the fluctuation in the debt-to-GDP ratio by accounting for 32 percent of total variation in 40 consecutive months. This decomposition indicates that the unanticipated change in spread would be the main shock that predominantly contributes to the total variation in the debt-to-GDP in the medium run or after one year.

While we find the yield spread successfully influences the variation of debt-to-GDP in the medium run, there is no significant bilateral effect from the debt position on the yield spread. In fact, spread innovations initially explain around 80 percent of total variation in yield spread in 6 month period. However, the variations which are explained by an unexpected change in spread gradually fall to 60 percent more than three years later, albeit remaining the most important variable in explaining a fluctuation in yield spread. Meanwhile, shocks in latent factors and inflation all together turn to account for around 30 percent of total variation in yield spread. However, shock in debt position still makes little contribution to variance of the forecasting errors in yield factors.

For the variation of yield level forecasting errors, an unanticipated innovation to the level factor itself plays a major role in explaining the forecasting errors,

accounting for 94 percent of total variation at a 6-month horizon. Although it remains the major contributor to explain fluctuations in the yield level 40 consecutive months later, a shock in inflation is now able to account for 21 percent of the variation of level factor. This finding evidently affirms the influence of expected inflation in determining the expected interest rate and the level factor.

Surprisingly, the variance of forecasting errors for the yield slope is not mainly explained by its own innovation. Notwithstanding, the unexpected innovations in the yield level and curvature factors account for 68 percent (47 percent from the level and 21 percent from the curvature factor) of total variation, while the innovation of slope factor itself is able to contribute only 21 percent of the variation in the yield slope. After two years, the shock in the slope factor itself can only explain 10 percent of total variation. However, the shock in the curvature factor becomes the most important driver of yield slope forecasting errors, sharing 37 percent of all variation. In addition, the contribution of the industrial production index innovation reaches the peak of a 16 percentage share of total variation in yield slope and then reduces to 14 percent within 40 months.

### **2.7.2 Macro-Finance-Fiscal Model with Budget-to-GDP as a Proxy for Fiscal Instability**

For the case of using net government budget position-to-GDP, we report the estimation results as follows. It allows us to visualize and assess the relationship between net government budget position proxy and yield spread together with yield latent factors and other macroeconomic variables.

#### **2.7.2.1 VAR(1) Parameter Estimates**

Table 2.8 reports the estimates of the parameters which represent the interaction between net government budget position-to-GDP, yield spread, yield latent factors and other macroeconomic variables.

TABLE 2.8: Estimates of the Macro-finance-fiscal Model with a change in government budget position-to-GDP proxy for fiscal instability

|                | <i>L t-1</i> | <i>S t-1</i> | <i>C t-1</i> | <i>GBTG t-1</i> | <i>DSPRD t-1</i> | <i>GIPI t-1</i> | <i>INF t-1</i> | <i>u</i>    |
|----------------|--------------|--------------|--------------|-----------------|------------------|-----------------|----------------|-------------|
| <i>L t</i>     | <b>0.97</b>  | 0.00         | 0.02         | 0.01            | 0.07             | 0.00            | -0.03          | <b>0.26</b> |
|                | <i>0.02</i>  | <i>0.02</i>  | <i>0.01</i>  | <i>0.01</i>     | <i>0.05</i>      | <i>0.00</i>     | <i>0.02</i>    | <i>0.12</i> |
| <i>S t</i>     | -0.02        | <b>0.92</b>  | <b>0.07</b>  | -0.01           | -0.06            | <b>0.01</b>     | 0.03           | 0.04        |
|                | <i>0.03</i>  | <i>0.02</i>  | <i>0.02</i>  | <i>0.01</i>     | <i>0.06</i>      | <i>0.00</i>     | <i>0.02</i>    | <i>0.16</i> |
| <i>C t</i>     | -0.01        | -0.01        | <b>0.85</b>  | <b>0.09</b>     | <b>0.50</b>      | <b>0.02</b>     | <b>-0.13</b>   | 0.13        |
|                | <i>0.07</i>  | <i>0.05</i>  | <i>0.04</i>  | <i>0.03</i>     | <i>0.14</i>      | <i>0.01</i>     | <i>0.06</i>    | <i>0.40</i> |
| <i>GBTG t</i>  | <b>-0.02</b> | <b>-0.04</b> | -0.00        | <b>1.00</b>     | -0.01            | <b>0.02</b>     | -0.10          | 0.02        |
|                | <i>0.01</i>  | <i>0.01</i>  | <i>0.01</i>  | <i>0.00</i>     | <i>0.02</i>      | <i>0.00</i>     | <i>0.01</i>    | <i>0.06</i> |
| <i>DSPRD t</i> | -0.02        | 0.02         | -0.01        | <b>-0.02</b>    | <b>0.91</b>      | <b>0.00</b>     | 0.02           | 0.09        |
|                | <i>0.02</i>  | <i>0.01</i>  | <i>0.01</i>  | <i>0.01</i>     | <i>0.03</i>      | <i>0.00</i>     | <i>0.01</i>    | <i>0.08</i> |
| <i>GIPI</i>    | -0.21        | <b>-0.58</b> | <b>0.25</b>  | 0.02            | -0.87            | <b>0.92</b>     | -0.10          | 0.95        |
|                | <i>0.25</i>  | <i>0.19</i>  | <i>0.14</i>  | <i>0.11</i>     | <i>0.51</i>      | <i>0.03</i>     | <i>0.19</i>    | <i>1.35</i> |
| <i>INF t</i>   | -0.02        | -0.03        | <b>0.04</b>  | 0.00            | 0.08             | <b>0.02</b>     | <b>0.91</b>    | <b>0.40</b> |
|                | <i>0.04</i>  | <i>0.03</i>  | <i>0.02</i>  | <i>0.02</i>     | <i>0.08</i>      | <i>0.01</i>     | <i>0.03</i>    | <i>0.22</i> |

*Notes:* The table displays matrix of the macro-finance-fiscal model's coefficients and standard errors. Bold numbers indicate statistical significance at 5 percent level while italic numbers refer to standard errors.

For the estimates of the effect of yield curve factors and macroeconomic variable on net government budget position, we find significantly negative relation between yield factors and the government budget position. The negative change in the net government budget position is associated with higher yield latent factors since an increased interest rate generates a higher cost of borrowing and debt repayment. In turn, it significantly deteriorates the net government budget. As a result, the net government budget is negatively related to yield latent factors.

We also examine the relationship between the current term spread and lagged values of the net government budget position. Our estimation reveals a significantly negative relation between the term spread and the net government budget position. This relation implies possible fiscal instability with a worsened net government budget position, significantly widening yield spreads. Unlike the model with the debt-to-GDP ratio, the adverse effect of fiscal instability on the yield spread, represented by a deterioration in the net government budget position, becomes more important as the coefficient increases in magnitude. Yet, the negative impact of a higher yield spread on the growth rate of the industrial production index

or economic activity is insignificant. In fact, we observe a significantly negative effect from the yield slope factor on the growth rate the industrial production index. The increase in term spread is perceived as higher default risk which eventually raises the long-term interest rate and yield slope factor. Consequently, economic agents will reduce their spending and reduce the industrial production index. However, there is no significant evidence of bilateral feedbacks from macroeconomic variables to yield latent factors. This finding reveals the transmission mechanism from the fiscal instability stance that pass through yield spread signal could affect the entire yield curve shape and macroeconomic variables afterwards.

### 2.7.2.2 Impulse Response Function

Figure 2.9 shows the impulse response functions that summarize the dynamic response of the term spread, yield latent factors, macroeconomic and fiscal variables.

With a positive innovation of one standard deviation to net government budget, we found an improvement of the net government budget-to-GDP significantly narrow the difference between Spanish and German 10-year government bond spread since the perceived default risk on long-term public debt is lessened. It drops to around 3 percentage points and reaches the bottom of 5 percentage points after around ten months and gradually reverts back to steady state after 40 consecutive months. As compared to the previous VAR model with debt-to-GDP proxy, the dynamic response of a change in the term spread is now significant whereas a shock to a debt-to-GDP generates a lagged response. In addition, the responses of the change in industrial production index is reflected by hump-shaped curves, which are actually less than the impact produced by the debt-to-GDP innovation.

For the response of economic activity after a positive innovation to spread, we find an inverted U-shaped decline in the industrial production index, similar to the result from the model with debt-to-GDP proxy. To explore the change in net government budget-to-GDP in response to real economy, we examine the dynamic adjustment of response of net government budget position to a positive innovation of industrial production index and find the improvement in net government budget

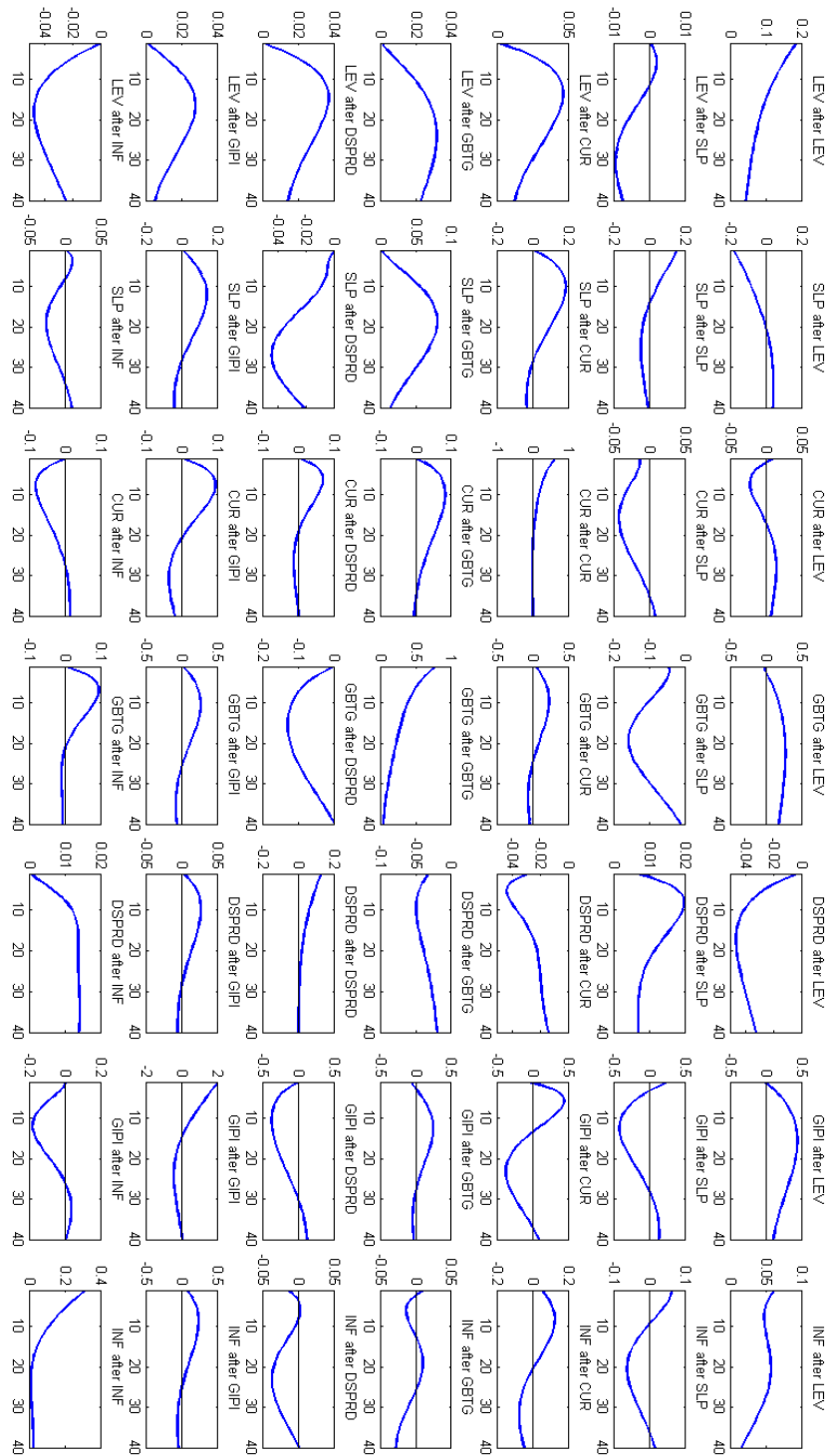


FIGURE 2.9: Impulse response function of the Macro-finance-fiscal Model with a change in government budget position-to-GDP proxy for fiscal instability

is actually transient and just remains only around two years.

### 2.7.2.3 Forecast error variance decomposition

Table 2.9 summarizes the results of forecast error variance decomposition of the net government budget position-to-GDP, yield spread, level and slope factors at 6, 12, 24 and 40 months horizons.

TABLE 2.9: Forecast error variance decomposition of the Macro-finance-fiscal Model with a change in government budget position-to-GDP proxy for fiscal instability

| Variable     | Horizon | LEV  | SLP  | CUR  | GBTG | DSPRD | GIPI | INF  |
|--------------|---------|------|------|------|------|-------|------|------|
| <b>GBTG</b>  | 6       | 0.01 | 0.01 | 0.04 | 0.89 | 0.01  | 0.04 | 0.01 |
|              | 12      | 0.04 | 0.02 | 0.08 | 0.72 | 0.02  | 0.10 | 0.01 |
|              | 24      | 0.13 | 0.05 | 0.08 | 0.58 | 0.04  | 0.11 | 0.01 |
|              | 40      | 0.20 | 0.06 | 0.07 | 0.50 | 0.04  | 0.11 | 0.01 |
| <b>DSPRD</b> | 6       | 0.03 | 0.01 | 0.11 | 0.12 | 0.71  | 0.02 | 0.00 |
|              | 12      | 0.08 | 0.02 | 0.12 | 0.17 | 0.56  | 0.03 | 0.01 |
|              | 24      | 0.17 | 0.02 | 0.12 | 0.22 | 0.42  | 0.04 | 0.01 |
|              | 40      | 0.24 | 0.02 | 0.12 | 0.22 | 0.34  | 0.03 | 0.02 |
| <b>LEV</b>   | 6       | 0.96 | 0.00 | 0.02 | 0.00 | 0.01  | 0.00 | 0.01 |
|              | 12      | 0.87 | 0.00 | 0.05 | 0.01 | 0.03  | 0.01 | 0.04 |
|              | 24      | 0.75 | 0.00 | 0.07 | 0.03 | 0.05  | 0.02 | 0.08 |
|              | 40      | 0.70 | 0.00 | 0.08 | 0.05 | 0.05  | 0.02 | 0.10 |
| <b>SLP</b>   | 6       | 0.47 | 0.27 | 0.19 | 0.01 | 0.00  | 0.06 | 0.00 |
|              | 12      | 0.30 | 0.15 | 0.36 | 0.03 | 0.00  | 0.17 | 0.00 |
|              | 24      | 0.20 | 0.11 | 0.38 | 0.08 | 0.01  | 0.22 | 0.01 |
|              | 40      | 0.19 | 0.12 | 0.35 | 0.10 | 0.03  | 0.21 | 0.01 |

The variance decompositions of the errors in forecasting the change in the net government budget to GDP reveals that an innovation of the net government budget itself is able to explain most of the variation of forecasting errors, accounting for 89 percent at 6 months horizon. Its own innovation still contributes over 50 percent in the following three years. In comparison with the previous model with a debt-to-GDP proxy, the innovation of changes in the net government budget predominantly influence the total variation of forecasting errors whereas the innovation of the change in debt-to-GDP shares account for only 60 percent of total variation in errors to forecast itself.

Concerning the variation of yield spread forecasting errors, the unexpected change in yield spread substantially explain more than 70 percent of total variation in forecasting itself for the initial a 6 months horizon. When we expand our forecasting horizon to 40 months, the forecasting errors of yield spread to predict its own value just account for only one-third of total variance. Interestingly, the net government budget position is able to explain 12 percent of total variation in yield spread at 6 months horizon and eventually 22 percent of total variation at 24 to 40 months horizon. Our forecast error variance decomposition result confirms the significant and immediate impact of the net government budget position on the term spread from the impulse response analysis.

As for the variance of yield factors' forecasting errors, we find the innovation of the level factor contributes over 95 percent of total variation of its forecasting errors in the first 6 months. After 40 months, inflation and the curvature factor are able to explain 15 and 14 percent of the errors in forecasting the latent factor. Similar to the model with the debt-to-GDP proxy, we also find the innovation of the curvature and level factors are the main drivers for the variation of yield slope forecasting errors, accounting for more than 65 percent, while the yield slope itself can explain 27 percent of total variation at the 6 months horizon.

## 2.8 Conclusion

This paper analyzes whether term spreads are affected by fiscal indiscipline and subsequently impact upon aggregate macroeconomic variables. Our study focuses on Spain which was notably vulnerable to fiscal deterioration after the global financial crisis. We apply the [Nelson and Siegel \(1987\)](#) yield curve parameterization and follow [Diebold and Li \(2006a\)](#) and [Diebold et al. \(2006\)](#) to estimate a macro-finance term structure model based on maximum likelihood estimation in a state-space specification and Kalman filtering. With the addition of fiscal stability variables (the net change in government budget position to GDP and the change in public debt to GDP), and the inclusion of the term spread, we propose a macro-finance-fiscal term structure model to assess the impact fiscal indiscipline on the entire yield curve space and examine the macroeconomic linkages among those variables through the yield spread. The use of term spread can help us to



unveil how the transmission mechanism of fiscal signals can be explained.

Our results indicate that fiscal indiscipline significantly influences the term spread. From the estimated coefficients of VAR model, the fiscal instability variables are found to be statistically significant in affecting the yield spread, albeit with minimal impact. We also find that the term spread significantly determines the change in industrial production index. These empirical results can be explained within the context of rational expectation theory. When fiscal positions become weaker than expected, risk averse investors require an additional term premium as compensation for the higher default probability. Term spreads are then widened and signal markets about a possible coming recession. A growing concern about the probability of economic downturn influences economic agents to cut down aggregate demand, exacerbating by a further recession. We also investigate the responses of yield spreads to shocks of fiscal instability variables. Either a net change in the budget position or in public debt are found to substantially alter yield spreads. In fact, markets seem to pay more attention to government budget than debt since the change in government budgets have immediately generated a significant response on spread while the reaction from debt is lagged. Notwithstanding, both fiscal variables have statistically significant effects on output.

The evidences from forecast error variance decompositions (FEVD) consistently complement estimates of VAR coefficients and impulse response function (IRF) analysis. Responses of government debt innovation show that a change in the industrial production index and a change in the yield spread are accounted for variation in debt. However, there seems to be a negligibly reverse relation from government debt to alter term spreads, whereas macroeconomic and yield latent factors tend to be more important in explaining spread variation. In contrast, the variance of net change in government budget position is predominantly attributed by its own innovation. More importantly, we find yield latent factors and yield spread jointly contribute to substantial variation in yield spread for the model using government budget as a proxy for fiscal instability. At the medium horizon of 40 months, yield factors and spread share more than 40 percent of underlying variation in the term spread.

Findings from the VAR model reveal that yield latent factors and macroeconomic variables have a crucial role in determining the variance of the yield spread. Indeed, these influences become more important in the medium run. It means information contained in unobserved yield factors and economic conditions would take a long time to affect the yield spread. However, yield latent factors and government budget deterioration appear to provide non-negligible contributions to the fluctuations in yield spread, even at the early 6-month horizon. Results of the government budget variance decomposition confirm a statistically significant response of the yield spread following a shock to the budget position. A potential increase in sovereign default risk from a worsening budget immediately raises the term spread and the yield level factor in anticipation of higher interest rates. Therefore, we can infer that information of the budget position is more quickly passed through yield spread rather than public debt variable. Our empirical results provide further explanation to support the previous literature, for example [Dai and Philippon \(2005\)](#) and [Bernoth et al. \(2012\)](#), who argue government budget measure is more significant in explaining yield spread. More importantly, yield spread is found to be the key variable which signals further fiscal stance. Markets are likely to penalize fiscal indiscipline in action with higher term premium. An increase in the spread associated with the expectation of higher sovereign default risk and consequently signals economic agents to reduce their spending and thus even further worsen the the fiscal position.

Based on our empirical results, we offer two main perspectives on fiscal policy implementation. On the one hand, fiscal stimulus is likely to be less effective in the presence of fiscal indiscipline. As a matter of fact, the increase in public debt or budget deficit would have negative impacts on output if loose fiscal policy widens the yield spread due to higher default risk. On the other hand, expansionary fiscal policy might successfully induce aggregate demand once it is accompanied by credible commitments to fiscal discipline. From these different perspectives, fiscal discipline can be considered as a necessary condition for fiscal policy to effectively boost the economy and avoid recession; otherwise this stimulus can make the recession somewhat more severe. This interpretation also supports the idea of alternatively conduct quantitative easing or fiscal policy in the absence of response from conventional monetary policy when the policy interest rate reach zero lower bound.

## Chapter 3

# Term Structure Forecasting - A Comparison between the Dynamic Semiparametric Factor Model and the Dynamic Nelson-Siegel Model

### 3.1 Introduction

In recent years, significant progress has been made in term structure forecast by combining nonparametric curve fitting with dynamic latent factors to capture the evolution of yield curves. Yet, there are still few studies, for example, [Härdle and Majer \(2012\)](#), [Hays et al. \(2012\)](#) and [Jungbacker et al. \(2013\)](#), that investigate whether the dynamic factor model with nonparametric factor loadings is more accurate relative to other term structure models. In this paper, we employ the dynamic semiparametric factor model, which was developed by [Fengler et al. \(2007\)](#) and [Park et al. \(2009\)](#) to model the yield term structure and examine its forecasting performance relative to the dynamic Nelson-Siegel and other competitive models, including the random walk. In particular, we evaluate the in-sample fit of model estimation and compare individual out-of-sample prediction accuracy with its competitors. In order to assess this, we use a sample of Australian zero-coupon bond yields consisting of monthly data over the period from April 1999 to March

2013. To gauge the out-of-sample performance, we construct yield forecasts for short and long term horizons and compare the results with forecasts from several competitor models. Our forecasting exercises are based on a rolling window estimation with fixed size, in which parameters are re-estimated at each stage. In addition to model comparisons, we use the pairwise [Diebold and Mariano \(1995\)](#) test against the random walk benchmark and other competitors. We also examine the robustness of the forecasting ability and investigate the structural break effect from the global financial crisis during 2007 to 2008 by separating the data into three sub-samples and re-estimating the yields for each sub-sample. For statistical evaluation, we implement the [Giacomini and Rossi \(2010\)](#) fluctuation test to assess the forecasting instability environment.

Our empirical results indicate that a better in-sample fit is provided by the dynamic semiparametric factor model. A particular empirical finding is that the dynamic semiparametric factor model is able to fit a wide range of yield curves very accurately. The standard errors of the estimated yield are much lower than for the dynamic Nelson-Siegel model. This finding implies that the dynamic semiparametric factor model based on nonparametric B-spline factor loadings is sufficiently flexible to match cross-sectional yield. The overall forecasting results for the individual models over the period from 2006 to 2013 are not very encouraging for yield curve prediction to overcome the naive random walk. The only exception is the dynamic semiparametric factor model with an AR(1) specification for 6-month maturity at a 1-month and 3-month ahead horizon, which outperforms the random walk. This also shows that the dynamic semiparametric factor model with an AR(1) specification provides more accurate forecasts for 6-month maturity at 1-month and 3-month ahead with a statistically significant Diebold-Mariano test against the dynamic Nelson-Siegel counterpart. Additionally, the AR(1) specification seems to be the proper stochastic process for the dynamic latent factor. However, for nearly all maturities at every forecasting horizon, the random walk produces more accurate results than other models. Our findings affirm the superiority of the random walk for out-of-sample term structure forecast.

As for the robustness check and the forecasting instability assessment on a particular sub-sample periods, we observe the dynamic semiparametric factor model, the dynamic Nelson-Siegel and the principal component model all perform poorly

compared to the random walk for most maturities and forecasting horizons. During the sub-sample period from 2003 to 2006, the observed yields increase sharply and become highly volatile, and accompanied by a substantial widening of term spreads. The sub-sample period from 2006 to 2009 has the observed yields declining dramatically during 2008 to 2009. In these two periods, the dynamic Nelson-Siegel model with VAR(1) provides more accurate prediction compared to the dynamic semiparametric factor model. This result indicates that the dynamic Nelson-Siegel model is a more suitable fit with more volatile periods. In contrast, there is a persistently downward trend in yields throughout the sub-sample period from 2009 to 2013. The forecasting ability of the dynamic semiparametric factor model with AR(1) specification tends to outperform the dynamic Nelson-Siegel model with VAR(1). It seems that each model may play a complementary role in forecasting, at least during the subperiods in this study. Model uncertainty is troublesome if one has hopes of obtaining a single model for forecasting. The results from the [Giacomini and Rossi \(2010\)](#) fluctuation test also confirm the forecasting instabilities of the individual models during the period of study. It is clear that the uncertain environment resulting from the global financial crisis lessened the predictability performance of both the dynamic semiparametric factor model and the dynamic Nelson-Siegel model against the random walk.

In the remainder of this paper, the related literatures are reviewed in Section [3.2](#). We further present the methodology of the dynamic semiparametric factor model and the Nelson-Siegel model in Section [3.3](#) and [3.4](#). Section [3.5](#) presents some stylized facts and a principal component analysis of the Australian yields, follows by an estimation of the dynamic semiparametric factor model and the Nelson-Siegel model in Section [3.6](#). In Section [3.7](#) and [3.8](#), we compare the in-sample fit estimation and out-of-sample prediction accuracy together with the detection of the structural break effect for the whole sampling and sub-sample periods. We then give the conclusion of our study in Section [3.9](#).

## 3.2 Review of literature

The use of the term structure of interest rates in finance and macroeconomics has been an active line of research. The term structure carries information about

expected inflation and the business cycles, which are very important for policy makers and investors. The term structure yield curve represents a collection of interest rates that relates to the yield rates of different maturities for any particular period as well as to the evolution of the yield rates for bonds with the same maturities over time. It shows the dynamic of the bond yields that evolve across the periods by linking a yield curve structure in specific cross-section periods. As such, modeling the term structure of interest rates is challenging. In practice, the term structure model can be estimated by a statistical model that fits the yield curve pattern. There are numerous statistical approaches to model the yield curve, including principal components as in [Litterman and Scheinkman \(1991\)](#), nonparametric splines interpolation as in [McCulloch \(1971\)](#) and the parametric exponential-polynomial model as in [Nelson and Siegel \(1987\)](#). Among these several statistical models, the Nelson-Siegel which was further extended by [Diebold and Li \(2006b\)](#) as a dynamic factor model, is widely used by practitioners and in academia. The dynamic Nelson-Siegel can capture both the cross-sectional variation of the yields in different maturities and the dynamic evolution of the yield curve through times with a parsimonious structure.

However, the functional Nelson-Siegel model does not allow for capturing more complicated yield curves, such as when there are multiple changes in the slope and curvature. It may also be misspecified due to a preselected parametric model. To avoid the imposition of a predetermined parametric form, there are several non-parametric methods to estimate yield curve, such as the use of splines as in the pioneer work of [McCulloch \(1971\)](#) and [Vasicek and Fong \(1982\)](#), or kernel estimation as in [Linton et al. \(2001\)](#) and [Jeffrey et al. \(2001\)](#). Nonparametric methods are able to minimize incorrect specification and provide accurate cross-sectional fit for each observed period. Unfortunately, the nonparametric estimation ignores the evolution of the yield curve over time. There should be some possible latent factors that explain the variation of the yield curve in different time periods. Thus, a semiparametric technique has been introduced to handle the complexity of the dynamic properties of interest rates without arbitrary restrictions imposed by a pre-specified parametric functional forms. [Ghysels and Ng \(1998\)](#) proposed the semiparametric term structure model to estimate a cross-sectional nonparametric function for the yield curve. They use the estimated nonparametric basis function as a factor loading to obtain yield curve factors over time by parametric least square estimation that minimizes the errors from the actual yields. Among several

nonparametric techniques, spline estimation is considered as a proper method to accommodate the non-linearity of yield corresponding to maturity without flattening at the boundary as in kernel smoothing. [Bowsher and Meeks \(2008\)](#), [Jarrow et al. \(2004\)](#) and [Krivobokova and Kauermann \(2007\)](#) employed the spline technique to fit the cross-sectional dimensions of the yield and derive the yield factors associated with the coefficient of the nonparametric loading functions to estimate yield curves for all time periods. Thus, the dynamic evolution of term structure is captured through the latent factors without assuming the parametric structure of the cross-sectional dimension of yields.

Basically, the dynamic specification of yield latent factors can be interpreted as a dynamic factor model (DFM) in the spirit of [Geweke \(1976\)](#) and [Sargent and Sims \(1977\)](#). The DFM explains a panel of a high-dimensional data set by a small set of unobserved dynamic factors. [Fengler et al. \(2007\)](#) and [Park et al. \(2009\)](#) applied the DFM and imposed a smoothness condition on the factor loading that allows latent factors to represent the evolution of the multidimensional data series. The proposed method, which is called the dynamic semiparametric factor model (DSFM), combines the DFM framework with a smooth flexible function for factor loading coefficients. The smooth factor loadings and time series of latent factors are simultaneously estimated. The DSFM technique achieves dimension reduction and the corresponding dynamic factors can then be used for forecasting. Within the context of yield curve forecasting, the yield dynamics depend on the latent factors that are modeled by a dynamic stochastic process. [Bowsher and Meeks \(2008\)](#) developed the cubic spline loading function of the dynamic factor model, which they labeled the functional signal plus noise (FSN), to estimate the US term structure from 1984 to 2000. To forecast the term structure, they specified the dynamic factors in a cointegrated vector autoregressive process or error correction model (ECM) and found that it outperformed the dynamic Nelson-Siegel model and the random walk at the one-month ahead horizon. While their approach provided an accurate term structure forecast, this method was disputed by [Koopman et al. \(2010\)](#) that the cointegrated factors may be associated with some loss of economic interpretation.

Since the DFM was proposed by [Geweke \(1976\)](#), it increasingly has played a major role in term structure modeling and forecasting. Apart from a co-integrated DFM

term structure forecasting, the dynamic process can be also presented in an autoregressive model. There are some recent empirical studies that investigate whether the DFM with an autoregressive representation is effective for term structure forecasting. [Härdle and Majer \(2012\)](#) employed the dynamic semiparametric factor model for four European countries' term structure estimation during the period 1999 to 2009 by specifying the dynamic process of the yield factors as a first-order autoregressive process VAR(1). They found that the dynamic semiparametric factor model did a better job than the dynamic Nelson-Siegel model in providing more precise short and long maturities forecasts for a short-term (6-month ahead) horizons. Nonetheless, the prediction performance became worse for longer horizons. However, they did not reveal the results for their forecasting exercise at shorter (1-month ahead) or longer (12-month ahead) horizons. This study claimed that the difficulty in forecasting may be caused by a structural break from financial distress.

[Hays et al. \(2012\)](#) proposed to use a functional data analysis, as in [Ramsey and Silverman \(2002\)](#), to synthesize factor loading and facilitate dynamic factor modeling of the US term structure over the period 1985 to 2000. They assumed the dynamic latent factors follow an AR(1) process and found that their functional dynamic factor model (FDFM) outperforms the dynamic Nelson-Siegel model for both 1-month and 12-months ahead forecasting horizons despite its failure to do better in predicting short-term or 3-month and 1-year maturity yields for a 6-month horizon.

[Jungbacker et al. \(2013\)](#) developed a maximum likelihood procedure for imposing cubic spline smoothing restrictions on the factor loadings of dynamic factor model of the US term structure. They forecasted the US term structure from 1994 to 2009 and showed their smooth dynamic factor model (SDFM) with VAR(1) representation was highly competitive with the dynamic Nelson-Siegel model and the Bowsher-Meeks functional signal plus noise (FSN) model in producing precise term structure forecasts, especially for 3-year, 5-year and 10-year maturity at 6-month, 12-month and 24-month ahead horizons. This study also examined the robustness of the forecasting improvement over the periods 1994 to 1998, 1999 to 2003 and 2004 to 2009. They observed that the smooth dynamic factor model (SDFM) outperforms the dynamic Nelson-Siegel model over the first two sub-samples periods, while in the last sub-sample from 2004 to 2009, the functional signal plus noise



model performs remarkably better.

All of these recent papers choose to compare their dynamic factor model with the dynamic Nelson-Siegel model. Actually, the dynamic Nelson-Siegel model can also be regarded as a dynamic factor model. Its functional factor loadings are pre-specified as fixed parametric curves, whereas other loadings are unknown smoothing functions. The dynamic Nelson-Siegel is also popular amongst practitioners and academics, and so it serves as a benchmark model. However, [Hays et al. \(2012\)](#) and [Jungbacker et al. \(2013\)](#) did not compare their models' forecasting performance with the random walk, which is indeed difficult to beat. Even the dynamic factor model with smooth loadings seems to outperform its competitors, especially the dynamic Nelson-Siegel model. The results are still inconsistent and vary across forecast horizon, maturity and period of study.

For recent Australian term structure studies, there are some empirical studies that tried to model the term structure and provided insight analysis regarding the effects of expectation and structural break on the yields and term premia. [Chiarella et al. \(2009\)](#) estimated the dynamics for interest rate processes with the multi-factor Heath, Jarrow and Morton (HJM) specification (as in [Heath et al. \(1992\)](#)). This study found the three-factor model is the proper interest rate model for Australia. The three factors are labeled as the level, the slope and the twist effect. The contribution of each factor towards overall variability of the interest rates were claimed to be considerable in explaining the Australia term structure. [Finlay and Chambers \(2009\)](#) used the affine term structure model to fit Australian government bonds with the aim of decomposing forward rates into expected future overnight cash rates plus term premia. They found the expected future short rates derived from the model fluctuate around the average observed short rates. However, term premia appeared to have a decline in levels and displayed smaller fluctuations after the implementation of inflation targeting that stabilized inflation expectations. This suggests that the market has become less uncertain about the expected interest rates. Prior to the emerging of the global financial crisis, term premia have been negative, which actually indicated the concern over potential risk from financial distress. This study was claimed to provide a useful decomposition of changes in the expected path of interest rates and term premia.

Further development were done to incorporate structural break with term structure modeling. [Suardi \(2010\)](#) investigate the structural breaks on the cointegrating relationship implied by the linear expectations hypothesis of the term structure of interest rates. They found that structural break generated a shift in the cointegration and altered the information content of the term structure. Their findings provided an implication for policy makers to take into account a regime shift in the cointegrated term structure modeling. [Elliott et al. \(2011\)](#) proposed a Markov regime-switching affine term-structure model to include the impact of structural changes on the term structure dynamics. They introduced a double Esscher transformation to determine a price kernel which was actually defined by the product of two density processes that measure changes in the interest-rate process and the Markov chain. As a result, their model takes into account both the market risk and the long-term economic risk that are useful for term structure modeling.

Although much progress has been made in the Australian term structure modeling, there is still room for in-depth investigations on the accuracy of the modeling and forecasting, especially the systematic comparison between the parametric Nelson-Siegel model and the semiparametric model that are built on the dynamic factor type approach. For a term structure study in general; there remain relatively few studies that consider the effects of the economic environment uncertainty on term structure forecasting accuracy, for example; [Altavilla et al. \(2013\)](#), [Araújo and Cajueiro \(2013\)](#), [Exterkate et al. \(2013\)](#) and [Dijk et al. \(2014\)](#), particularly during the global financial crisis.

### 3.3 The dynamic semiparametric factor model

The dynamic semiparametric factor model (DSFM) provides a general method for modeling and forecasting time series data that captures the dynamic evolution of high-dimensional time series by a non-parametrically estimated lower-dimensional factor. It has the ability to flexibly fit various shapes of the cross-sectional data while providing time-varying factors that describe the dynamics of the time series. This method was proposed by [Fengler et al. \(2007\)](#) on their implied volatility surface study. Due to its virtue of parsimony and parametric interpretability, the dynamic semiparametric factor model is widely used in many areas of research,

including [Giacomini et al. \(2009\)](#) on risk neutral density, [Härdle and Trück \(2010\)](#) on hourly electricity price, [Trück et al. \(2012\)](#) on spot and futures  $CO_2$  emission allowance prices, [Härdle et al. \(2012\)](#) on liquidity supply, [Härdle and Majer \(2012\)](#) on bond yield term structure modeling and [Choros-Tomczyk et al. \(2013\)](#) on collateralized debt obligations surface dynamics. Detailed discussion on the dynamic semiparametric factor model specification is given below.

### 3.3.1 Semiparametric estimation

The dynamic semiparametric factor model estimates yield term structure with a semiparametric procedure. As discussed earlier, the nonparametric methods are used in modeling the yield curve to minimize the problems caused by an incorrect specification of the parametric methods, and additionally allow a more accurate fit to the observed curves. However, they are estimated nonparametrically for each cross-sectional period and thus ignore all existing dynamics in the yield curve. To capture the dynamic term structure, the dynamic latent factor model is introduced to nonparametric estimation. The dynamic semiparametric factor model combine advantages of these two approaches, allowing a nonparametric fit for curves to time and capturing the evolution of curves in time based on latent factor estimation. This method models dynamic yield curves without assuming a parametric structure for the cross-section dimension of the process.

#### 3.3.1.1 The dynamic semiparametric regression

Semiparametric regression imposes some structure but the regression function is still not directly predetermined. However, the structure of the model leaves less flexibility than in the nonparametric case. One motivation for creating this limitation comes from the curse of dimensionality, since in high dimensions nonparametric methods may become infeasible.

Among many possible semiparametric models, we focus on the imposition of the additive property as in [Härdle \(2004\)](#), [Fengler et al. \(2007\)](#) and [Härdle and Majer](#)

(2012). The key assumption is that the regression function has an additive structure of the explanatory variable coordinates. The actual yields are supposed to be linear combinations of high dimensional latent factors. Proposing a suitable statistical model results in the problem of finding an appropriate way of reducing the high dimension without losing too much information on the spatial and dynamic structure of the process. A common way to reduce the dimensionality of multivariate processes is to apply factor decomposition. For instance, a  $J$ -dimensional vector of yield observations  $Y_t = (Y_{t,1}, \dots, Y_{t,J})$  can be represented as an  $L$ -factor model.

$$Y_{t,j} = \sum_{l=1}^L Z_{t,l} m_{l,j}(X_{t,j}) + \epsilon_{t,j} \quad (3.1)$$

where  $Y_{t,j}$  is the yield obtained by holding a bond at time  $t$  to time-to-maturity  $j$ ;  $Z_{t,l}$  are latent factors of the factor  $l$  at date  $t$ ,  $m_{l,j}(X_{t,j})$  is an undetermined smooth function, or so called basis function, that characterizes the loading of factor  $l$  given time-to-maturity  $j$ ;  $X_{t,j}$  are maturity-related variables representing bond yield characteristics at date  $t$ ; and  $\epsilon_{t,j}$  are errors that explain the residual parts. The index  $t$  represents time evolution as  $1, \dots, T$  and index  $j$  is the number of bonds with different maturities  $1, \dots, J$  observed at that time. The corresponding yield curve can be shown in a  $J$ -dimensional vector of yields  $Y_{t,j} = (y_{t,1} \dots y_{t,J})'$ . This high dimension of the cross sectional  $J$  bonds can be reduced to a smaller number of factors  $L \ll J$ . The dynamics of yield through time are then explained by the time propagation of the  $L$  factors and can be estimated through the evolution of the latent factors  $Z_{t,l}$ . The latent factors reflect bond yield characteristics associated with factor-loadings.

### 3.3.1.2 Basis function estimation

The main assumption of the dynamic semiparametric factor model is that the loading coefficients  $m_{l,j}(X_{t,j})$  are unknown nonparametric functions. These functions can be approximated by many classes of smoothing techniques. We follow [Laurini and Hotta \(2010\)](#) and [Härdle and Majer \(2012\)](#) to choose the B-spline function as a nonparametric basis function.

A generic spline is a piecewise polynomial curve, constructed from individual polynomial segments joined at knot points and its first derivatives are continuous at all points. It approximates the cross-sectional unknown basic function using the selected  $K$  knots within the domain of variation of  $X_{t,j}$ . In particular, if there are  $K$  knots positioned at the maturities that are deterministic and fixed over time, then the piecewise polynomial  $m_{l,j}(X_{t,j})$  corresponding to subinterval in the domain of  $X_{t,j}$  can be written as the linear regression function. Therefore, the coefficient associated with the spline basis function can be estimated by the method of least squares.

To calibrate factor loadings,  $m_{l,j}$  are approximated by a series of estimators, given the B-spline basis function. More formally, the loadings  $m_{l,j}(\cdot)$  are linearized with the piecewise-defined smooth polynomial function as

$$m_{l,j}(X_{t,j}) = \sum_{k=1}^K a_{l,k} \psi_k(X_{t,j}) \quad (3.2)$$

where  $X_{t,j}$  are the maturity-related variables representing  $j$  maturity bond yield characteristic at date  $t$ .  $\psi_k(\cdot) = \psi_1, \dots, \psi_K$  denotes the vector of the B-spline basis functions.  $K$  is the number of knots used for the spline functions and is interpreted as a bandwidth parameter, and  $a_{l,k}$  are coefficients that approximate  $m_{l,j}(\cdot)$  with  $A\psi$ .

Suppose that the yield curves are twice continuously differentiable, then the least square error is achievable by the spline approximation to the observed yields with an arbitrary number and positioning of knots. A set of spline polynomial pieces can be viewed as a local approximation to the cross-sectional yield, with the polynomial pieces joined together to form a smooth function overall. Spline functions are used frequently for numerical yield curve approximation because they provide a balance between accurate approximation and smoothness.

### 3.3.2 The dynamic factor model

In this subsection, we present the theoretical background of the dynamic factor model, which is combined with the nonparametric basic function estimation for loadings to form the dynamic semiparametric factor model.

#### 3.3.2.1 Dynamic regression

In time-varying regression, we simultaneously analyze in space and time dimensions. Note that neglecting the time structure would lead to a regression based on pooled data. However, such a simplification could cause a loss of important information. Therefore, we perform the space or cross-sectional regression over time with certain modeling assumptions that build dynamic linkages with the latent factors time dependent.

$$E(X_{t,j}|Y_{t,j}) = \sum_{l=0}^L Z_{t,l} m_{l,j}(X_{t,j}) \quad (3.3)$$

This representation is relatively parsimonious and allows a convenient separation between space and time effects. Time changes are caught by latent factor  $Z_{t,l}$ , which can be afterwards analyzed through times series methods. The space dependence is achieved by functions  $m_{l,j}$ , which operate on exploratory variables  $X_{t,j}$  and do not depend on time. The models assume a linear link between the time and space components. The yield latent factor  $Z_{t,l}$  are common for all observations in moment  $t$  and the basic functions  $m_{l,j}(X_{t,j})$  determine their influence on yields  $Y_{t,j}$ .

#### 3.3.2.2 The dynamic semiparametric factor model

In the case, where the maturity dimension  $J$  of the yield time series  $Y_{t,j}$  is relatively large, it may be better to apply dimension reduction techniques. One possible approach utilizes factor analysis.  $Y_{t,j}$  can be then rewritten to

$$Y_{t,j} = \sum_{l=0}^L Z_{t,l} m_{l,j}(X_{t,j}) + \epsilon_{t,j} \quad (3.4)$$

which is exactly the dynamic semiparametric regression discussed earlier. This representation assumes the existence of co-movements among all components of  $Y_{t,j}$ , which are driven by unobservable factors  $Z_{t,l}$ .

The yield latent factors  $Z_{t,l}$  are time series processes. The usual way is to assume that these processes are first-order autoregressive processes, represented by

$$Z_t = \Phi Z_{t-1} + \omega_t \quad (3.5)$$

where  $Z_{t,l}$  is the yield latent factor;  $m_{l,j}(X_{t,j})$  is a factor loading with deterministic maturity-related variables  $X_{t,j}$ ;  $\Phi$  are parameter matrices and  $\epsilon_{t,j}$ ; and  $\omega_t$  are random components independent of each other. Since the dynamics of the factors are incorporated, the above representation is called the dynamic factor model.

### 3.3.3 Estimation algorithm

This subsection is devoted to the estimation algorithms of the dynamic semiparametric factor model. We present two approaches representation on the discrete grid, and series estimation. Before we discuss in detail the estimating procedures, we recall once more the structure of the model.

#### 3.3.3.1 Series estimators

The dynamic semiparametric factor model of order  $L$  that incorporates all factors and factor loadings can be shown as

$$Y_{t,j} = \sum_{l=0}^L Z_{t,l} \sum_{k=1}^K a_{l,k} \psi_k(X_{t,j}) = Z_t^T A \psi(X_{t,j}) \quad (3.6)$$

where  $Z_t^T = (Z_{t,0}, \dots, Z_{t,L})^T$  and coefficient  $A = a_{l,k}$ . The coefficient matrix  $A$  and time series of latent factor  $Z_t$  can be estimated using least squares. The estimation procedure searches through all estimated matrices  $A$  and time series  $Z_{t,l}$  minimizing the sum of squared residuals.

$$\begin{aligned}
 (\hat{Z}_t, \hat{A}) &= \arg \min_{Z_t, A} \sum_{t=1}^T \sum_{j=1}^J (Y_{t,j} - Z_t^T A \psi(X_{t,j}))^2 \\
 &= \arg \min_{Z_t, A} \sum_{t=1}^T \sum_{j=1}^J (Y_{t,j} - \sum_{l=0}^L Z_t^T a_{l,k} \psi(X_{t,j}))^2
 \end{aligned} \tag{3.7}$$

To find a solution for the minimization problem above, a Newton-Raphson algorithm is used. Following [Borak et al. \(2005\)](#) and [Park et al. \(2009\)](#), the procedure starts by setting the initial estimate  $\hat{Z}_{t,l}^0$  to be equal to a white noise sequence of appropriate length. Next, taking this initial estimate as given, the estimate  $a_{l,k}^0$  is obtained. Then we proceed iteratively switching from  $Z_{t,l}$  to  $a_{l,k}$  and vice versa until convergence is reached.

### 3.3.3.2 Latent factor specification

An important parameter in the model is the number of factors  $L$  and corresponding factor loadings. The choice of  $L$  is based on the explained variance by factors  $EV(L)$  as in [Borak et al. \(2006\)](#) and [Härdle and Majer \(2012\)](#). For different values of  $L$ , the proportion of the variation explained by the model compared to the simple invariate estimate given by the overall mean can be calculated by

$$EV(L) = 1 - \frac{\sum_{t=1}^T \sum_{j=1}^J (Y_{t,j} - \sum_{l=1}^L Z_{t,l} m_{l,j}(X_{t,j}))^2}{\sum_{t=1}^T \sum_{j=1}^J (Y_{t,j} - \bar{Y})^2} \tag{3.8}$$

Since the model is not nested, the whole estimation procedure has to be repeated for different  $L$ 's until the explanatory power of the model is considered to be sufficient.

## 3.4 The Nelson-Siegel Model

In this section, we describe the exponential-polynomial Nelson-Siegel model and the modified Nelson-Siegel approach proposed by [Diebold and Li \(2006b\)](#). [Nelson](#)



and Siegel (1987) proposed to fit the forward rate curve, and thus yields or spot rates, from observed coupon-bond prices at a given date with a flexible, smooth parametric function. They demonstrated that their proposed model is capable of capturing many of the typically observed shapes that the yield curve assumes over time. The Nelson-Siegel model is widely used among academia and policy maker practitioners, and it is ranked as one of the most popular term structure estimation methods.

### 3.4.1 The Nelson-Siegel parametric model

Nelson and Siegel (1987) suggest to fit the forward rate curve at a given date with a class of prespecified parametric functions. The functional form they advocate uses Laguerre functions which consist of the product between a polynomial and an exponential decay term. The resulting Nelson-Siegel approximating forward curve can be assumed to be the following three-factor term structure model.

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t}e^{-\lambda_t\tau} + \beta_{3,t}\lambda_t e^{-\lambda_t\tau} \quad (3.9)$$

To obtain the yield (or spot rate)  $y_t$  on a zero-coupon bond with  $\tau$  periods to maturity, it is necessary to take the equally weighted average of the forward rates.

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \quad (3.10)$$

where  $y_t(\tau)$  is the spot-rate curve with  $\tau$  time to maturity, and  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$  are latent factor parameters, which in dynamic form are referred to as level, slope and curvature and  $\lambda_t$  is referred to the exponential decay parameter.

The three latent factor parameters correspond to the factor loading components on these parameters. The factor loading on the  $\beta_{1,t}$  parameter is 1. As this is a constant, it does not decay to zero and will be the same for all maturities. So, this long term factor  $\beta_{1,t}$  is independent of time to maturity and for that reason it is often interpreted as the long-run yield level. The factor loading that is weighted on  $\beta_{2,t}$  represents the short-term factor with a loading of  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}$ . This function starts at one and decays exponentially to zero if time to maturity  $\tau$  grows.

Therefore, the corresponding latent factor is often denoted as the slope factor.  $\beta_{3,t}$  is also weighted by a function depending on time to maturity  $\tau$ . This function  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t}$ , starts at zero and when the time to maturity  $\tau$  grows it initially increases and then decreases back to zero. Hence this component creates a hump and so it is often denoted as the medium-term component. The  $\lambda_t$  parameter is an exponential decay parameter that determines the rate at which the regressor variables decay to zero. Small values for  $\lambda_t$  result in a slow decay and a better fit for longer maturities, large values of  $\lambda_t$  will result in fast decay and a better fit for short maturities. In addition, the  $\lambda_t$  parameter also governs where the factor loading reaches its maximum.

[Diebold and Li \(2006b\)](#) provide insights to how these three factors representing the long, short and medium components can also be interpreted as the level, slope and curvature of the curve. The factor loading on the long term component  $\beta_{1,t}$  is 1 and the same for all maturities, any increase in  $\beta_{1,t}$  will cause the whole curve to shift upwards and thus it can be seen that this factor represents the level of the curve. The short term factor  $\beta_{2,t}$  can be viewed as the slope of the curve, an increase in  $\beta_{2,t}$  will cause the short rates to increase more than long rates as the short rates load more heavily on  $\beta_{2,t}$ , thus changing the slope of the curve. Finally, the medium-term factor is closely related to the curvature of the curve, as both long and short-term maturities do not load heavily on it, but an increase in  $\beta_{3,t}$  will increase the curve for medium maturities and so increasing the curvature of the curve.

### 3.4.2 Optimal decay parameter and latent factor calibration

In order to obtain these parameter estimates, we follow [Annaert et al. \(2013\)](#) and [Rezende and Ferreira \(2013\)](#) by estimating a linearized Nelson-Siegel model. The parameters have typically been estimated by minimizing the root mean squared errors (RMSE).

Parameter estimation is computed by running a standard optimization technique. We use a grid of different values of exponential decay  $\lambda_t$  parameter, and then run an ordinary least square algorithm to obtain latent factors  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$  parameter estimates. More generally, the parameters of the models can be estimated by minimizing the difference between the model yield rates and the actual yield rates. The optimization problem can be stated as.

$$\hat{\lambda} = \arg \min_{\lambda \in \Omega} \left\{ \frac{1}{T} \sum_{t=1}^T \sqrt{\frac{1}{J} \sum_{j=1}^J (y_{t,j}(\tau) - \hat{y}_{t,j}(\tau, \lambda, \beta))^2} \right\} \quad (3.11)$$

With given  $\lambda$ , the value of latent factor parameters can be computed with the ordinary least squares. This procedure was repeated for a whole grid of  $\lambda$  values ranging in  $\Omega \in (0.0000, 1.0000)$ . The estimates with the lowest root mean squared errors (RMSE) were then chosen as the optimal parameter set.

### 3.4.3 The dynamic Nelson-Siegel model

[Diebold et al. \(2006\)](#) proposed a dynamic term structure of the Nelson-Siegel model by specifying first-order autoregressive processes for the latent factors. [De Pooter \(2007\)](#) generalized the dynamic Nelson-Siegel model as the dynamic latent factor model, given by the Nelson-Siegel model and the dynamic process of the latent factors.

The Nelson-Siegel model is

$$Y_t(\tau) = X_t(\tau)\beta_t \quad (3.12)$$

and the stochastic process of the latent factor

$$\beta_t = \mu + \Phi\beta_{t-1} + \varepsilon_t \quad (3.13)$$

The first dynamic factor equation above specifies the vector of yields, which contains  $T$  different maturities. The Nelson-Siegel yield curves are those discussed in the previous subsections with  $\beta_t$  being the vector of factors and  $X_t$  as the matrix

of factor loadings, given by the estimated decay parameter  $\lambda$ .

To estimate the dynamic Nelson-Siegel model, we first solve for the optimal decay parameter  $\lambda$ . By doing so, the Nelson-Siegel model is treated as a linear cross-sectional model and we can calculate the latent factors for every particular period from the ordinary least squares estimation. Then, we estimate the factor dynamics by specifying the stochastic process of latent factors and hence the dynamic yield curves. The dynamic latent factors are assumed to be a first-order univariate autoregressive process AR(1) for each factor as well as a first-order multivariate autoregressive process VAR(1).

In fact, the Nelson-Siegel decay parameter  $\lambda$  and latent factors  $\beta_t$  parameters are able to be estimated in one step by using the Kalman filter as in [Diebold et al. \(2006\)](#). However, this simultaneous estimation is achieved with the expense of more computational efforts. Instead, we follow [Diebold and Li \(2006b\)](#) to estimate the dynamic Nelson-Siegel model by the two-step approach. [Diebold and Li \(2006b\)](#) and [Koopman et al. \(2010\)](#) argued the two-step forecasting approach does better than directly estimate all parameters and dynamic term structure, especially for the longer maturities.

## 3.5 Australian yield statistics and stylized fact

In this section, we present the empirical and stylized fact of the Australian yield data. The details of the data sets are reported in Section [3.5.1](#). The stylized fact and descriptive statistics are then presented in Section [3.5.2](#) and Section [3.5.3](#). We also conduct principal component analysis of the Australian yields in Section [3.5.4](#).

### 3.5.1 Data

For the empirical study, we use the Australian zero-coupon bond yields provided by Thomson Reuter Datastream. The data set consists of monthly yield series of

11 maturities government bond over the period April 1999 to March 2013. The maturities are 6, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 month-to-maturities.

### 3.5.2 Yield stylized fact

During the period of analysis, the Australian government bond yields exhibit a sizable inter temporal variation. Figure 3.1 illustrates a three-dimensional plot of the data set and Figure 3.2 depicts time-series plots for a subset of the maturities and shows how yield vary substantially throughout the sample.

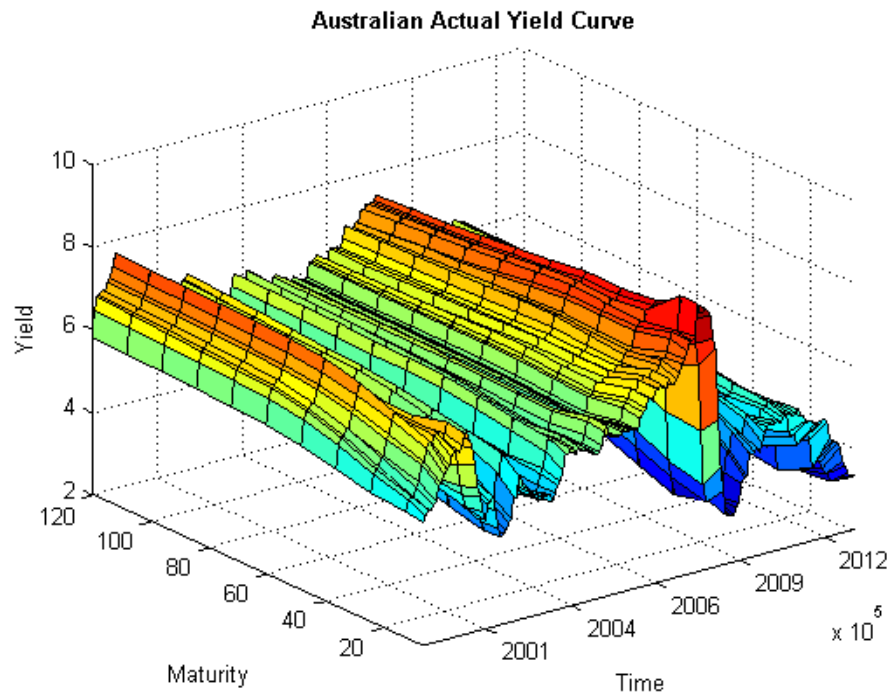


FIGURE 3.1: A panel of monthly Australian government bond yields  
*Notes:* The figure shows a 3-dimension plot of the Australian government bond yields. Sample period is April 1999 to March 2013 (168 months).

In Figure 3.1, the three-dimensional plot shows that the yield series vary heavily over time for each of the maturities. However, there is a strong common pattern in the 11-maturity bond series over time. For most months, the yield curve is the upward sloping and concave. Casual observation of this figure shows the upward trend of yield for different maturities over the period 1990–2000, 2003–2007 and 2008–2009, revealing a concentration on stabilized inflation. Though on average

it is upward sloping, there are periods when it is downward sloping. During 2000–2001, 2007–2008 and from 2010 onwards, the yield curves were on a downward trend, responding to concern about the dot-com crisis, the global financial crisis and the European sovereign debt crisis respectively. It is also observed that yield dynamics are persistent and the short end of the yield curve is more volatile than the long end. In other words, volatility tends to be lower for the yields of bonds with a longer time to maturity. These findings are supported by the time series plots of the 6-month, 2-year and 10-year bonds in Figure 3.2.

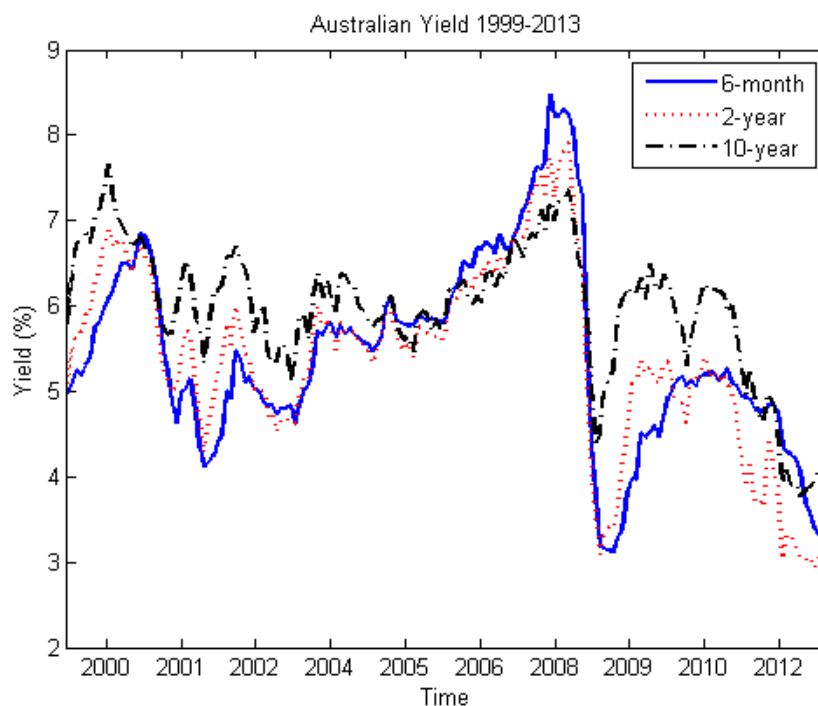


FIGURE 3.2: The evolution of the Australian Yields from April 1999 to March 2013

*Notes:* This figure shows time series plots of 6-month, 2-year and 10-year Australian government bonds yields over the period of study, covering the 2000-2001 dot-com crisis, the 2008-2010 global financial crisis and the 2010-2012 European sovereign debt crisis.

As can be seen in Figure 3.2, the 6-month, 2-year and 10-year bond yields were exposed to a number of changes during this period from 1999 to 2013. From 1999 to 2000 and 2001 to 2005, long-term yields were higher than short-term yield, which indicates raising expected short-term interest rates and longer-term interest rates in response to inflationary pressure. In the aftermath of the global financial crisis from 2008 to 2010, long-term yield rose once again and widened spread over short-term yields. Investors anticipated greater volatility and uncertainty in the future, which increases risk premium and influences the yield spread. Prior to the

global financial crisis and the European sovereign debt crisis, short-term yields were higher than long-term yields and caused the yield curve to be inverted. This means the market expected falling interest rates which are historically followed by periods of recession.

### 3.5.3 Yield statistics

To summarize the yield information over the period from 1990 to 2013, the descriptive statistics of government bonds at different maturities and for the yield curve empirical level, slope and curvature factors are reported. Table 3.1 provides mean, standard deviation, minimum, maximum and some autocorrelation coefficients for bonds at different maturities and proxies of empirical level, slope and curvature in accordance with Diebold and Li (2006b).

TABLE 3.1: Descriptive statistics of the Australian government bond yield

| Maturity    | Mean    | Std Dev | Min     | Max    | $\rho(1)$ | $\rho(12)$ | $\rho(30)$ |
|-------------|---------|---------|---------|--------|-----------|------------|------------|
| 6           | 5.4305  | 1.1364  | 3.1230  | 8.4747 | 0.9635    | 0.1035     | -0.0595    |
| 12          | 5.4169  | 1.1963  | 2.9999  | 8.4761 | 0.9596    | 0.1413     | -0.0031    |
| 24          | 5.3795  | 1.1206  | 2.9261  | 7.9124 | 0.9551    | 0.1834     | 0.0295     |
| 36          | 5.5132  | 1.0676  | 3.0509  | 7.8198 | 0.9520    | 0.2210     | 0.0273     |
| 48          | 5.6867  | 1.0120  | 3.2013  | 7.8413 | 0.9481    | 0.1843     | 0.0114     |
| 60          | 5.7641  | 0.9664  | 3.3207  | 7.7188 | 0.9462    | 0.1752     | 0.0081     |
| 72          | 5.8167  | 0.9179  | 3.4359  | 7.6321 | 0.9426    | 0.1570     | 0.0038     |
| 84          | 5.8661  | 0.8766  | 3.5386  | 7.5947 | 0.9406    | 0.1468     | 0.0012     |
| 96          | 5.9011  | 0.8451  | 3.6285  | 7.6153 | 0.9373    | 0.1391     | -0.0024    |
| 108         | 5.9295  | 0.8188  | 3.7083  | 7.6355 | 0.9343    | 0.1345     | -0.0024    |
| 120 (Level) | 5.9540  | 0.7974  | 3.7713  | 7.6596 | 0.9314    | 0.1303     | 0.0000     |
| Slope       | 0.5236  | 0.8186  | -1.2409 | 2.7027 | 0.9497    | 0.0849     | -0.1761    |
| Curve       | -0.6255 | 0.5742  | -2.2786 | 0.3583 | 0.8919    | 0.2713     | 0.1259     |

*Notes:* The table shows summary statistics for the Australian government bonds yields. The results shown are for annualized yields (expressed in percentages). The sample period is April 1999 to March 2013 (164 observations). Reported are the mean, standard deviation, minimum, maximum, and the 1st, 12th and 30th sample autocorrelation.

From Table 3.1, we see that the average yield curve is upward sloping. Volatility decreases by maturity, with the exception of the 12-month maturity being more volatile than the 6-month maturity. Important for econometric analysis, yields for

all maturities are very persistent. The persistence is most notable for short term bonds. As we can see, the first-order autocorrelation of the 6-month bill is 0.9635, representing highly persistent yields. The slope, level and curvature proxies are persistent but to a lesser extent. The curvature proxies is least persistent compared to other factor proxies.

### 3.5.4 Principal component analysis of the yield

Before we proceed with the empirical estimation of the dynamic semiparametric factor model and the Nelson-Siegel model, we explore how latent factors based on principal component analysis contribute to understanding variations in the yields.

#### 3.5.4.1 Principal component analysis

Principal component analysis can retrieve a  $L$  number of common factors spanning the term structure of the yields  $Y_{t,j}(\tau)$ , denoted as  $PC$ , for  $t = 1, \dots, T$ . This can be done by the spectral decomposition of the variance-covariance matrix of  $Y_{t,j}(\tau)$ , for maturities  $\tau = 1, \dots, J$ , denoted as  $\Sigma_Y$

$$\Sigma_Y = \Omega \Theta \Omega' \quad (3.14)$$

where  $J > T$ ,  $\Theta$  is a diagonal matrix with its elements as the eigenvalues of matrix  $\Sigma_Y$ .  $\Omega$  is an orthogonal matrix whose columns are the eigenvectors corresponding to the eigenvalues of matrix  $\Sigma_Y$ .

Given estimates of  $\Omega$  and  $\Sigma$ , the principal component factors  $PC$  can be retrieved from the yields as follows.

$$PC_t = \Omega'(Y - \bar{Y}) \quad (3.15)$$

where  $\bar{Y}$  is the sample mean of the yield. Note that  $PC$  may not correspond one-to-one to unobserved factors, but they will be very highly correlated.



### 3.5.4.2 Factor specification based on principal component analysis

In this subsection, we use a common non-parametric statistical technique principal-components analysis (PCA) to explore the dynamic behavior of bond yields in Australia.

The objective is to estimate yields and extract factors that govern yield curve dynamics. Before estimating the term structure, the principal component analysis is conducted to characterize the number of latent factors and the general pattern of the factor loadings.

TABLE 3.2: Percentage of variation in yields explained by the first  $L$  principal components

|                          | $L = 1$ | $L = 2$ | $L = 3$ | $L = 4$ | $L = 5$ |
|--------------------------|---------|---------|---------|---------|---------|
| Explained Variation (EV) | 93.13   | 99.56   | 99.86   | 99.95   | 99.98   |

*Notes:* This table reports explained variation (EV) in yields. The numbers in the table are the percentage variation in yields explained by the first  $L$  principal components.

The first three principal components obtained through this analysis, explain almost all the variation of the term structure of yield; over 99.86 percent. The first factor explains the largest part of this variation, which is 93.13 percent. The next part, which is 6.73 percent, is explained by the second and the third factors. However, the fourth and fifth components contribute negligible increments to explained variation.

As can be seen from Figure 3.3, the loadings of the three principal components behave as a horizontal, an invert and a hump shape curve.

The loading of the first principal component is nearly horizontal. This pattern means that changes in the first principal component correspond to parallel shifts in the yield curve. The loading of the second principal component is downward sloping. Changes in the second principal component thus rotate the yield curve. The loading of the third principal component is hump shaped. The hump occurs

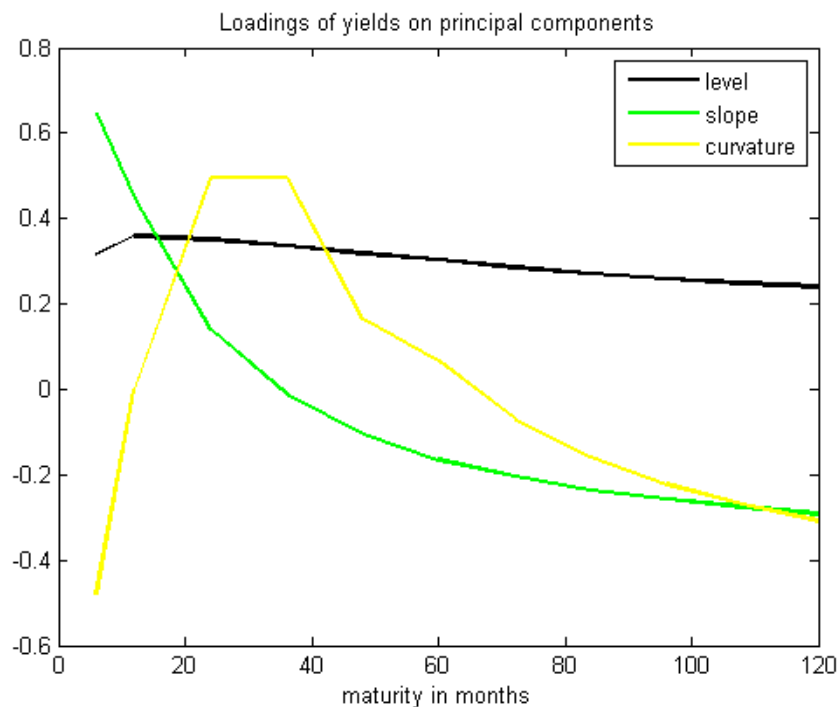


FIGURE 3.3: Estimated loadings of yields on principal components

*Notes:* This figure provides a plot of yield loadings implied by response of yields to changes in the first three principal components; level, slope and curvature. The loadings of three components are plotted as a function of yield-to-maturity.

at intermediate maturities. The interpretation of these principal components in terms of level, slope and curvature is consistent with the principal component analysis study by [Litterman and Scheinkman \(1991\)](#) and the Nelson-Siegel model study by [Diebold and Li \(2006b\)](#)

## 3.6 Model estimation

In this section, we present the empirical results of our term structure estimation. The dynamic semiparametric factor model (DSFM) estimation is provided in Subsection 3.6.1. Then, we present the estimation results for the Nelson-Siegel model in Subsection 3.6.2.

### 3.6.1 The dynamic semiparametric factor model estimation

The DSFM yield curve was calibrated to the data set comprising the entire period for the term structures. Following [Härdle and Majer \(2012\)](#), this study specifies the knots as the time to maturity grid and the order of tensor B-splines is set to 1. The factors  $L$  are selected according to their contribution to the total variation. The higher the number of factors, the better the general fit, however this at the cost of parsimony and robustness of the model.

#### 3.6.1.1 Factor identification

An important parameter in the model is the number of factors  $L$  and corresponding factor loadings. The choice of  $L$  is based on the explained variance by factors  $EV(L)$  as in [Borak et al. \(2005\)](#) and [Härdle and Majer \(2012\)](#).

TABLE 3.3: Share of variance explained by the dynamic semiparametric factor model with different number of factors

|                          | $L = 1$ | $L = 2$ | $L = 3$ | $L = 4$ | $L = 5$ |
|--------------------------|---------|---------|---------|---------|---------|
| Explained Variation (EV) | 0.9109  | 0.9861  | 0.9984  | 0.9994  | 0.9998  |

*Notes:* This table summarize the explained variation (EV) of the dynamic semiparametric factor model with different number of factors. The numbers in the table are the percentage variation in yield explained by the first  $L$  factors. Notice that the first three factors together explain 99.84 of the total variation of the yields

As can be seen from Table 3.3, the percentage of cumulative is explained variance for each factor; 91.1 percent in the case of the one factor and 98.61 percent in the case of the two factors, while the three factors can explain 99.84 percent of variance. The inclusion of the third factor improves the explanatory power of the fit and, therefore, the three factors model is appropriate. However, the fourth and fifth factors indicate the superfluous inclusion provided by their increment explained variation.

### 3.6.1.2 Estimated factor loading

The series estimators of  $m_{l,j}$  were extracted by approximating with tensor B-splines. Since the model is identifiable up to the choice of factor, for this study, we choose function  $m_{l,j}(\cdot)$  based on  $L = 3$  factor with respect to the variance of  $Z_t$ . In the dynamic semiparametric factor model, the number of factors are determined up to rotation transformations that contain the most underlying information. We follow Myšičková et al. (2011) and Härdle and Majer (2012) to model the  $J$ -dimensional random vector with  $L$ -factor that are allowed for nonorthogonal rotation. The nonorthogonal rotation greatly simplifies the clusters between factors and maturities which are not necessary to orthogonal to each other. The estimate  $m_1$ ,  $m_2$  and  $m_3$  are plotted in Figure 3.4.

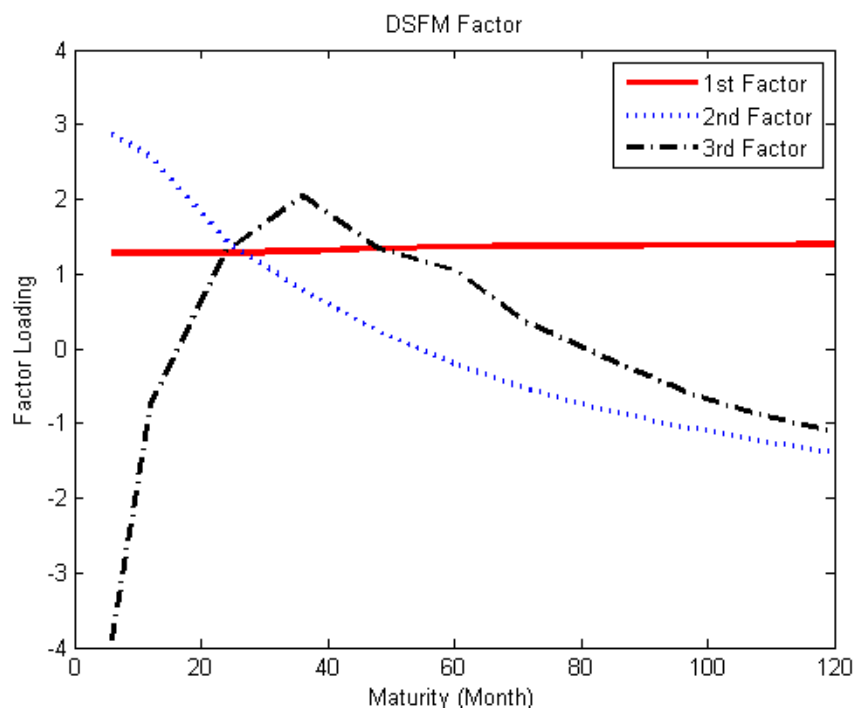


FIGURE 3.4: Estimated loadings of the dynamic semiparametric factor model  
*Notes:* This figure presents estimated loadings of the first, second and third factors extracted by the dynamic semiparametric factor model. The structure of loadings are able to interpreted as level, slope and curvature factor, respectively.

The loadings in Figure 3.4 represent the level, slope and curvature, as in the spirit of the ? term structure model. The loading  $\hat{m}_1$  is constant, horizontal and may be viewed as a long-term factor. This pattern means that changes in the first loading correspond to parallel shifts in the yield curve. This loading is therefore

called the level factor loading. The loading  $\hat{m}_2$  is downward sloping and decays monotonically and quickly so that it may be viewed as a short-term factor. This shape implies changes in the second factor loading by the rotation, yield curve, and therefore it may be called a slope factor.

The loading  $\hat{m}_3$  is hump shaped. It starts at a negative value and increases, then decays to be negative again. The hump normally occurs at intermediate maturities, so it may be viewed as a medium-term factor. The interpretation of these three factors in terms of level, slope, and curvature is consistent with previous yield curve studies as in [Litterman and Scheinkman \(1991\)](#), [Diebold and Li \(2006b\)](#) and [Piazzesi \(2010\)](#).

### 3.6.2 The Nelson-Siegel model estimation

To fit the yield curve to the Nelson-Siegel model, we estimated the optimal decay parameter and other corresponding latent factors. Most researchers have fixed the shape parameter with the pre-specified value of the decay parameter and have estimated the latent factors. However, we chose the optimal decay parameter that minimizes the sum of squared errors by using the ordinary least square over a grid of decay parameters.

We searched over a grid of possible value of decay parameters to find the best-fitting value by using linear least squares. We found the optimal decay parameter over the whole sample is 0.0822.

Next, we plotted the factor loadings based on the optimal decay parameter 0.0822 as a function of maturity in order to understand intuitively what the Nelson-Siegel factor loadings look like and provide the reason why the factors they load are interpreted as level, slope and curvature.

We observed that the decay parameter governs the exponential rate of growth of the slope factor as well as the rate of growth and decay of the curvature factor. Thus small lambda produces slow decay and can better the yield curve at long

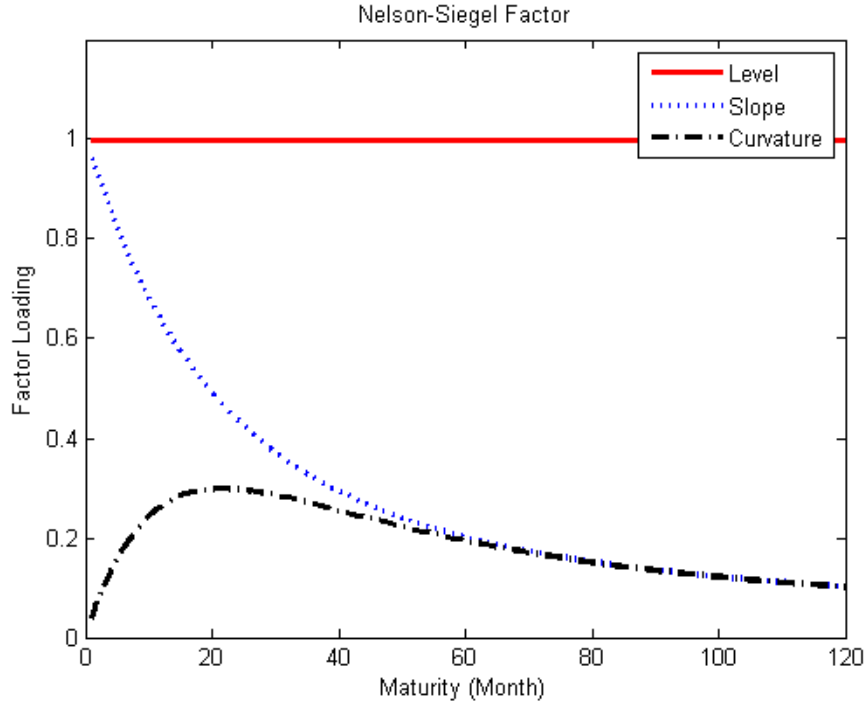


FIGURE 3.5: Estimated Loading Factors of the Nelson-Siegel Model

Notes: Factor loadings plotted with an optimal decay factor lambda  $\lambda$  value of 0.0822 where the factor loading on level factor is 1, the factor loading on slope factor is  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}$  and the factor loading on curvature factor is  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t}$ .

maturities, whereas large  $\lambda$  produces fast decay and therefore yields with maturities less than a year better.

According to Nelson and Siegel (1987),  $\beta_{1,t}$  should be regarded as a long-term interest rate and can be interpreted as a level parameter.  $\beta_{2,t}$  should be regarded as the difference between long-term and short-term interest rates and be interpreted as a slope parameter.  $\beta_{3,t}$  represents the medium-term interest rate and could be interpreted as a curvature parameter. Finally,  $\lambda$  is a decay parameter, measuring the rate at which the parameters decay to zero. For low values, the parameters will decrease slowly and for high values fast. Therefore, low and high values of  $\lambda$  could be used to fit the yields at long and short-term maturity respectively.

From the yield equation formula we notice that the loading of parameter  $\beta_{1,t}$  is constant and equal to 1. This means that  $\beta_{1,t}$  does not decay to zero as time to maturity approaches infinity and hence it will affect the yield curve at all possible maturities. Therefore, it is indeed appropriate to regard it as a long-term factor.

Further, since the loading of  $\beta_{1,t}$  is constant at all maturities, the changes of the factor will affect all the yields in the same way, changing the level of the yield curve.

The loading of parameter  $\beta_{2,t}$  is equal to  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}$ . For fixed values of  $\lambda$ , it is a decreasing function of  $\tau$  starting from 1 when  $\tau$  converges to zero and decreasing to zero when  $\tau$  ends to infinity. Since the loading of this parameter decays to zero faster than the one of  $\beta_{3,t}$ , it is appropriate to regard it as a short-term factor. Also, since the loading of  $\beta_{2,t}$  is higher for short-term maturities, changes of this factor will affect more the short-term yields, changing the slope of the yield curve.

Finally, the loading of parameter  $\beta_{3,t}$  is equal to  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t}$  and it starts at zero when  $\tau$  converges to zero, increases and then decreases again, converging to zero as  $\tau$  ends to infinity. The fact that the loading starts and finishes with zero indicates that this parameter does not affect the short term and the long-term yields, but only the medium-term yields. Hence, it is appropriate to regard it as a medium-term factor and changes of the factor will change the curvature of the yield curve. Figure 3.5 presents the loadings of each one of the three factors for the fixed value of 0.0822.

## 3.7 Model estimation comparison

In this Section, we compare the estimation accuracy and latent factor of the dynamic semiparametric factor model and the Nelson-Siegel model. To evaluate the performance of both models, we first conducted an in-sample fit assessment and compare the performance between the two models for the whole period of study from April 1999 to March 2013 in Subsection 3.7.1. In Subsection 3.7.2, we present the empirical result of the estimated latent factors as well as their empirical proxies.

### 3.7.1 Term structure estimation accuracy

First, we assess in-sample fit performance of the dynamic semiparametric factor model (DSFM) and the Nelson-Siegel model. We compare the root mean square

error (RMSE) and the explained variation between both models in Table 3.4. Then, we present the descriptive statistics of the yield curve residuals of both models for all the maturities in Table 3.5. We also graphically compare the actual yields with estimated term structure at some specific dates to facilitate in-sample fit comparisons of the estimated model in Figure 3.6.

### 3.7.1.1 Root mean square error and explained variation

The performance of the estimates can be assessed by fitting the latent factor with the time-invariant factor loading at a given point of time and detect the root mean square error (RMSE) as well as the explained variation of the latent factors. The dynamic semiparametric factor model (DSFM) is expected to outperform the Nelson-Siegel (NS) since it is more parsimonious and flexible.

TABLE 3.4: The root mean square error (RMSE) and explained variation (EV)

|             | all           | 6-m    | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          | EV            |
|-------------|---------------|--------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| <b>NS</b>   | 0.0477        | 0.0439 | 0.0830        | 0.0641        | 0.0494        | 0.0499        | 0.0129        | 0.0417        | 0.9978        |
| <b>DSFM</b> | <b>0.0398</b> | 0.0455 | <b>0.0680</b> | <b>0.0513</b> | <b>0.0421</b> | <b>0.0413</b> | <b>0.0125</b> | <b>0.0350</b> | <b>0.9984</b> |

*Notes:* This table reports root mean square error (RMSE) for the in-sample fit and explained variation (EV) of the evaluated models where NS and DSFM denote the Nelson-Siegel and the dynamic semiparametric factor model respectively. Bold numbers indicate DSFM outperforms NS

As shown in Tables 3.4, up to 99.84 percent of the explained variation in term structure curves can be explained the three-factor dynamic semiparametric factor model, whereas the three-factor Nelson-Siegel model can explain 99.78 percent of the term structure variance. For the overall in-sample fit assessment, the dynamic semiparametric factor model outperforms the Nelson-Siegel model in providing a relatively lower root mean square error (RMSE). Thus, the dynamic semiparametric factor model latent factors do a better job than the Nelson-Siegel model in capturing the cross-sectional variation of the yield and providing in-sample-fit with lower root mean square error (RMSE). This result agrees with Laurini (2014) who finds the functional dynamic factor model, which is based on the use of non-parametric penalised splines, outperforms the Nelson-Siegel model to produce a better in-sample fit. Härdle and Majer (2012), who also compares the in-sample fit



performance between the dynamic semiparametric factor model and the Nelson-Siegel model, observes that the dynamic semiparametric factor model outperforms the Nelson-Siegel model in estimating Greek term structure. They find it fails to provide lower root mean square error (RMSE) relative to the Nelson-Siegel model for term structure estimation in Italy, Portugal and Spain. Indeed, several studies, including [De Pooter \(2007\)](#), [Koopman et al. \(2010\)](#) and [Laurini and Hotta \(2010\)](#) find the more flexible and less-restricted models can beat the Nelson-Siegel model in achieving better in-sample fit.

### 3.7.1.2 Relative residual statistics

In this part, we compare the in-sample fit performance between the dynamic semiparametric factor model and the Nelson-Siegel model by presenting the descriptive statistics of the residual for all the maturities. We report the residual mean, standard deviation, maximum value, minimum value, root mean square error (RMSE) and autocorrelation at various displacements of the residuals in [Table 3.5](#).

The results show that the dynamic semiparametric factor model (DSFM) greatly outperforms the Nelson-Siegel model in producing negligible mean errors at all maturities. The RMSE and standard deviation produced by the dynamic semiparametric factor model is also less than those from the Nelson-Siegel model, except for the 6-month maturity. Both models seem to provide better fit for the medium-term maturities, whereas yields for short and long maturities have a slight worse fit, particularly for the shortest or 6-month maturity. According to the residual autocorrelation, both models produce high error term autocorrelation, especially at lag one with ranges from 0.5 to 0.8, and it almost disappears at lag 12. These results clearly show that the dynamic semiparametric factor model gains more accurate estimation than the Nelson-Siegel model for almost all maturities. This is also evidence that the more flexible model, in particular the spline-based dynamic factor model as in [Laurini \(2014\)](#), is able to outperform the Nelson-Siegel model.

TABLE 3.5: Descriptive statistics of the yield curve residuals, estimated by the dynamic semiparametric factor model (DSFM) and the Nelson-Siegel model (NS)

| <b>Maturity</b> | <b>Mean</b> | <b>Std Dev</b> | <b>Min</b> | <b>Max</b> | <b>RMSE</b> | $\rho(1)$ | $\rho(12)$ | $\rho(30)$ |
|-----------------|-------------|----------------|------------|------------|-------------|-----------|------------|------------|
| <b>DSFM</b>     |             |                |            |            |             |           |            |            |
| 6               | 0.0003      | 0.0454         | -0.2008    | 0.1057     | 0.0453      | 0.6747    | 0.0144     | -0.0247    |
| 12              | -0.0006     | 0.0682         | -0.2150    | 0.2846     | 0.0680      | 0.6376    | 0.0273     | -0.0423    |
| 24              | 0.0002      | 0.0513         | -0.0979    | 0.1813     | 0.0511      | 0.8469    | 0.0563     | 0.2830     |
| 36              | 0.0005      | 0.0422         | -0.1299    | 0.1076     | 0.0421      | 0.8709    | -0.0286    | -0.1773    |
| 48              | -0.0002     | 0.0433         | -0.1783    | 0.0683     | 0.0431      | 0.8088    | 0.2205     | 0.2735     |
| 60              | 0.0000      | 0.0414         | -0.1347    | 0.0921     | 0.0413      | 0.7571    | 0.0625     | 0.1901     |
| 72              | -0.0002     | 0.0280         | -0.0794    | 0.1192     | 0.0279      | 0.5999    | 0.1606     | 0.1157     |
| 84              | 0.0000      | 0.0125         | -0.0338    | 0.0339     | 0.0125      | 0.6731    | 0.1816     | -0.0763    |
| 96              | -0.0001     | 0.0119         | -0.0324    | 0.0418     | 0.0119      | 0.6151    | -0.0931    | -0.0489    |
| 108             | 0.0000      | 0.0214         | -0.0417    | 0.0797     | 0.0213      | 0.7463    | 0.1157     | 0.1802     |
| 120             | 0.0001      | 0.0352         | -0.0806    | 0.1255     | 0.0351      | 0.8087    | 0.2848     | 0.1812     |
| <b>NS</b>       |             |                |            |            |             |           |            |            |
| 6               | 0.0148      | 0.0415         | -0.1644    | 0.1163     | 0.0439      | 0.6565    | 0.0024     | -0.0039    |
| 12              | -0.0413     | 0.0723         | -0.2315    | 0.2591     | 0.0830      | 0.6622    | -0.0233    | 0.0127     |
| 24              | 0.0425      | 0.0482         | -0.0587    | 0.2054     | 0.0641      | 0.8743    | 0.3213     | 0.0082     |
| 36              | 0.0255      | 0.0425         | -0.1102    | 0.1341     | 0.0494      | 0.8740    | 0.0364     | -0.1457    |
| 48              | -0.0364     | 0.0453         | -0.2082    | 0.0446     | 0.0580      | 0.8025    | 0.2475     | 0.2019     |
| 60              | -0.0234     | 0.0441         | -0.1400    | 0.0700     | 0.0499      | 0.7427    | 0.1089     | 0.0091     |
| 72              | -0.0061     | 0.0306         | -0.0801    | 0.0943     | 0.0311      | 0.6460    | 0.1517     | 0.0256     |
| 84              | -0.0019     | 0.0129         | -0.0359    | 0.0378     | 0.0129      | 0.6866    | 0.0828     | -0.0932    |
| 96              | 0.0050      | 0.0125         | -0.0298    | 0.0483     | 0.0134      | 0.5789    | -0.0571    | 0.0315     |
| 108             | 0.0096      | 0.0245         | -0.0423    | 0.0895     | 0.0262      | 0.7468    | 0.1652     | 0.0496     |
| 120             | 0.0117      | 0.0402         | -0.0840    | 0.1326     | 0.0417      | 0.8119    | 0.3038     | 0.0561     |

*Notes:* This table presents summary statistics of the residuals for different maturities of both evaluated models; the dynamic semiparametric factor model (DSFM) and the Nelson-Siegel (NS) using monthly data, from April 1999 to March 2013. Std Dev and RMSE are standard deviation and root mean squared error respectively.  $\rho$  denotes the sample autocorrelations at displacements of 1, 12 and 30 months.

### 3.7.1.3 Cross-sectional in-sample fit

To further examine the fit of the dynamic semiparametric factor model to the data compared with the Nelson-Siegel model, we examine the estimated yield curves from both models against the actual yields at particular days as shown in Figure 3.6. We chose to plot the yield curves on 29 February 2000, 31 March 2004, 29 September 2006 and 30 November 2009. These four selected dates are examples of the various different term structure shapes that occur in the data.

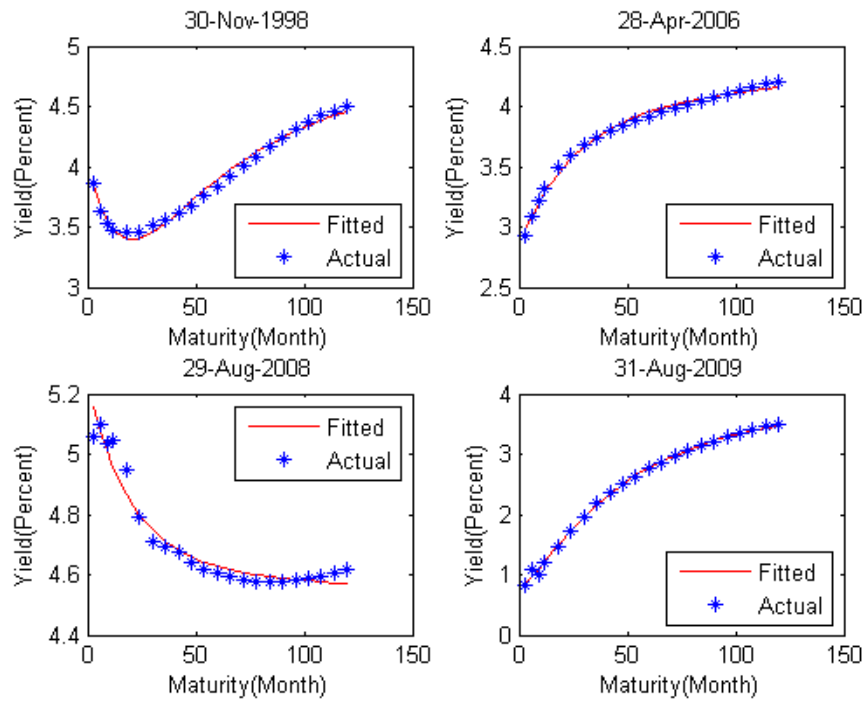


FIGURE 3.6: Fitted yield curve for specific months

*Notes:* The graph depicts the actual yield curve (blue asterisk) and the fitted yield curve for the DSFM (red dashed line) and the NS (black solid line). Shown are four months from April 1999 to March 2013: [a] February 29, 2000, [b] March 31, 2004, [c] September 29, 2006 and [d] November 30, 2009.

In Figure 3.6, the shapes of the yields are concave upward for 29 February 2000 and 30 November 2009, despite the 29 February 2000 curve much steeper for short maturities and then turn to be flattening for long maturity. Whereas the 31 March 2004 curve is a J-curve shape and the curve for 29 September 2006 is an inverted yield curve with two hump phases. The plots demonstrate that the term structure curve shapes can vary over time. As expected, due to more flexibility, the dynamic semiparametric factor model work better to fit wide ranges of yield curve shapes, while the Nelson-Siegel model does not seem flexible enough to fit more

complex curves. The most striking observation in the plots are the J-curve type and the downward sloping with two-hump shape which the Nelson-Siegel model has difficulties to fit. The better fit is obtained with the dynamic semiparametric factor model at the cost of a less smooth fit. In contrast, the curve fitted by the Nelson-Siegel model has a smoother fit than the actual yields.

Given the characteristics of the yield curve fitting, the dynamic semiparametric factor model is more accurate and more flexible to fit the actual yields. The results obtained by dynamic semiparametric factor model indicate that this method has good properties for the term structure estimation with the robustness to misspecification problems from the parametric nature of the Nelson-Siegel model.

### 3.7.2 Latent factors

Next, we analyze and compare the factors of the dynamic semiparametric factor model (DSFM) and the Nelson-Siegel model. The three-factor identification based on the explained variance criteria shows that the dynamic semiparametric factor model is quite similar to the Nelson-Siegel model with three factors; level, slope and curvature. The time series of the factor estimates as well as the series of their empirical proxies, which have been constructed from the yields directly, are plotted through the time period from 1999 to 2013 in Figure 3.7. The estimated value for the factors are standardized for simplicity in order to compare the level change.

Furthermore, to examine whether latent factors extracted from model estimation are legitimately called a level, slope and curvature factors, we follow [Diebold and Li \(2006b\)](#) to construct an empirical level, slope and curvature from the yields data and compare them with the estimated latent factors. The empirical level of the yield is defined as the 120 month yield; the slope is close to spread of 6 month over 120 month yields and the curvature is worked out as two times the two-year yield minus the sum of the twenty-five-year and three-month zero-coupon yields. Then, we compare the descriptive features of the estimated factors across models and their empirical factor proxies in Table 3.6. To provide some insight on the latent factors, we also investigate the the correlations between the latent factors

and the principal components as well as their correlation with the empirical proxies in Table 3.7.

### 3.7.2.1 Latent factor and empirical yield

The objective of using the dynamic semiparametric factor model and the Nelson-Siegel is to explain the term structure through latent factors. The latent factors obtained from the model can be compared with their empirical proxies. Each of the factors should agree with their data-based proxies. To get a first impression whether the estimated factors from the term structure models are able to capture their corresponding proxies, we present the time series from the three latent factors associated with their proxies in Figure 3.7.

Comparing the graphs for latent factor estimates from different models all give rather similar estimates for the level, slope and curvature factors. Moreover, the latent factors follow almost the same pattern as the empirical factors. As can be observed, there is a sharp decline in level as well as slope factor the slope factor during 2008 to 2009, which is followed by a gradual recovery process. The evolution of the level and slope factors are closely related to the Australian monetary policy regimes corresponding to the global financial crisis. In late 2008, the Australian economy was more likely heading into recession and financial instability. In order to avoid a severe recession, the aggressive monetary expansion was accommodated, which produced a greater negative slope factor and lower yield level. This evidences suggest the latent factor movement is related to the interest rate regime and the business cycle.

The evolution of the interest rate can be described by the propagation of the latent factors during the period of study from April 1999 to March 2013. This time span covers the great moderation since the mid-1980s to 2002, the oil price hike and asset price bubble during 2003-2007, the global financial crisis during 2008-2009, the mining boom during 2009-2010 and the European sovereign debt crisis which began in late 2009. Throughout 1999 to 2005, bond yields show a downward trend reflecting success in stabilizing the Australian economy towards a low and stable inflation. The dot-com crisis which emerged in the early 2000s caused

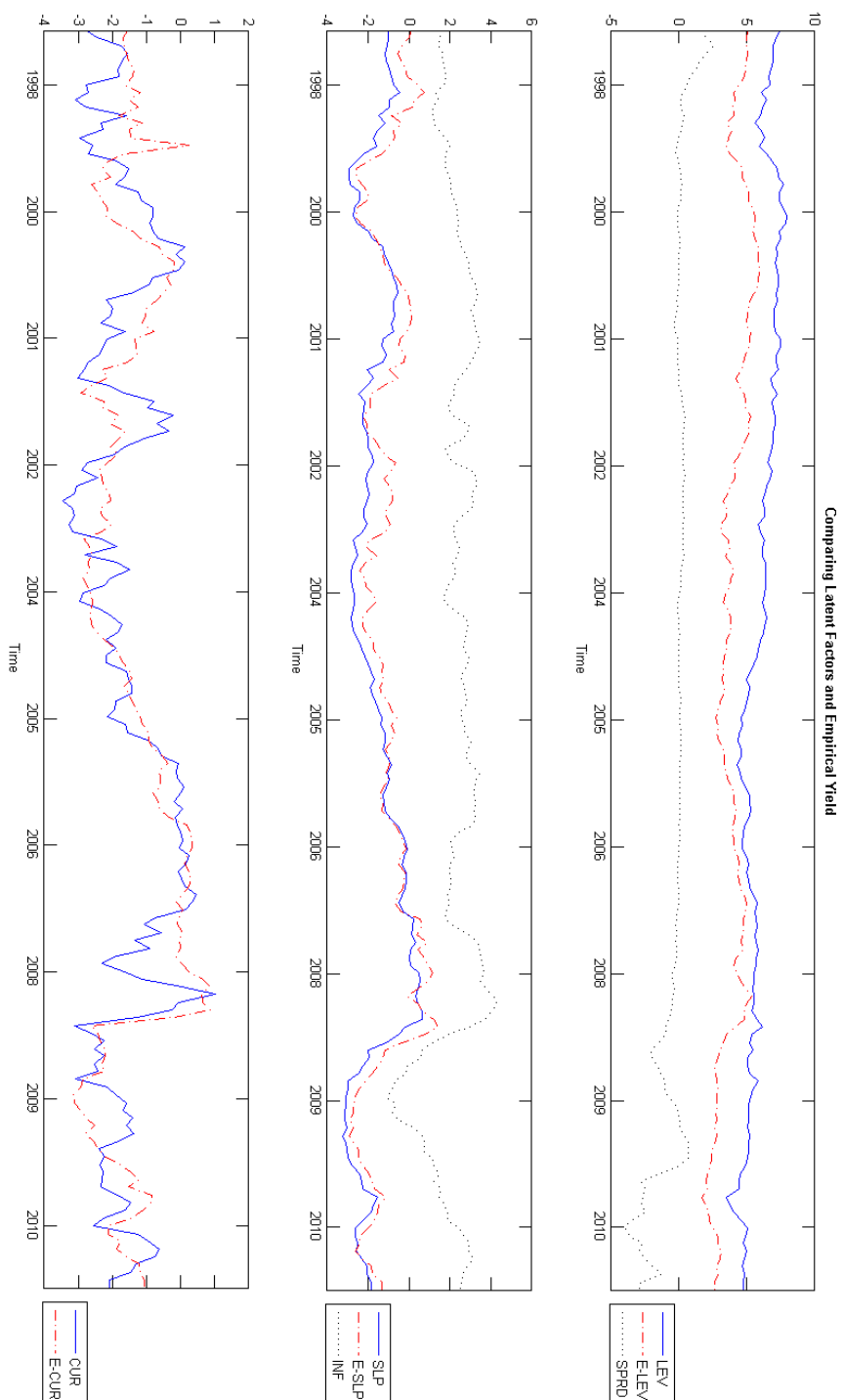


FIGURE 3.7: Estimated latent factors and empirical factors

*Notes:* The empirical proxy for level is the longest maturity yield (120 months), for slope it is the longest (120 months) minus the shortest (6 months) maturities, and for curvature it is two times the 24-month yield minus the 6-month and 120-month yields. In relative valuation, the value of estimated latent factors and empirical factors are normalized for comparable results.

the global economy to slow down. In response to the crisis, the Reserve Bank of Australia (RBA) applied the countercyclical monetary policy by lowering target rate to avoid a recession. After the introduction of a stimulus-induced interest rate regime, the yield curve shifted downward. This situation can be explained by the first and the second latent factor components; a downward parallel shift produced by the decline in the level factor and the greater negative value of slope factor. Expansionary monetary policy generates a bubble in asset prices which can presage a run up in price levels for the next period. From 2003 to 2007, Australia experienced inflationary pressures driven by an oil price hike and housing market boost. The RBA raised the target rate which can be explained by the increase in the level factor and positive value of the slope factor. In late 2008, during the onset of the global financial crisis, the RBA decided to cut the target rate sharply. The level factor declined significantly in line with a marked decrease in the slope factor that caused the yield curve to be quickly decayed. In the aftermath of the global financial crisis, the RBA raised the target rate once again to cope with mining boom which would push the economy into inflationary pressure. The upswing of the interest rate is clarified by the sharp increase in level factor and slope factor. By late 2009, the Australian economy was hit by the European sovereign debt crisis and the RBA accommodated expansionary monetary policy by again reducing target rate. The level factor decreased dramatically while the slope factor slightly increased. However, the negative value of the curvature factor was added, which created a hump shape of the yield curve. The opposite direction between level and slope factor implies higher long-term rate which means the market was anticipating a rise in term premium due to more risky sovereign debt. Regarding the above empirical results, it can be concluded that the movement of Australian latent factors is consistent with the RBA policy regime and macroeconomic condition.

Overall, the latent factor evolution appears to be associated with the business cycle. Whereas the time series of latent factors mimic the empirical factors quite closely across the time. The cross relation between latent factors and their empirical proxies are then assessed by comparing correlation coefficient statistics.

### 3.7.2.2 Latent factor statistics

The statistic properties of the estimated latent factors from the dynamic semiparametric factor model and the Nelson-Siegel model as well as the empirical factor proxies are presented. Table 3.6 shows the descriptive statistics of the estimated latent factors along with the empirical counterparts, which have been constructed from the yields directly.

TABLE 3.6: Descriptive statistics of the latent factors, estimated by the dynamic semiparametric factor model (DSFM) and the Nelson-Siegel model (NS), compared with the empirical proxies

|             | Mean    | Std Dev | Min     | Max    | $\rho(1)$ | $\rho(12)$ | $\rho(30)$ |
|-------------|---------|---------|---------|--------|-----------|------------|------------|
| <b>DSFM</b> |         |         |         |        |           |            |            |
| Z(1)        | 4.2427  | 0.6896  | 2.5542  | 5.7063 | 0.9479    | 0.1613     | 0.0122     |
| Z(2)        | -0.0156 | 0.1921  | -0.5617 | 0.4378 | 0.9561    | 0.1537     | -0.1126    |
| Z(3)        | -0.0081 | 0.0692  | -0.2175 | 0.1385 | 0.8970    | 0.1873     | 0.0207     |
| <b>NS</b>   |         |         |         |        |           |            |            |
| $\beta(1)$  | 6.2909  | 0.6446  | 4.4561  | 7.8099 | 0.8927    | 0.0771     | -0.0674    |
| $\beta(2)$  | -0.7804 | 1.1716  | -4.3102 | 1.7277 | 0.9500    | 0.0718     | -0.1606    |
| $\beta(3)$  | -1.4866 | 1.7592  | -6.4267 | 1.6954 | 0.8895    | 0.2996     | 0.0717     |
| <b>EMP</b>  |         |         |         |        |           |            |            |
| E(1)        | 5.9540  | 0.7974  | 3.7713  | 7.6596 | 0.9314    | 0.1303     | 0.0000     |
| E(2)        | -0.5236 | 0.8186  | -1.2409 | 2.7027 | 0.9497    | 0.0849     | -0.1761    |
| E(3)        | -0.6255 | 0.5742  | -2.2786 | 0.3583 | 0.8919    | 0.2713     | 0.1259     |

*Notes:* This table presents summary statistics of the latent factors estimated by the dynamic semiparametric factor model or Z(i), the Nelson-Siegel or B(i) and the empirical proxy (EMP) latent factors; 120-month yields for level, 120-months minus 6 months yields for slope and two times the 24-month yield minus the 6-month and 120-month yields for curvature or E(i). Denote  $i = (1,2,3)$  as latent factors. For all three factors; 1 = level, 2 = slope and 3 = curvature. The last three columns contain the sample autocorrelations at displacements of 1, 12 and 30 months. Std Dev stands for standard deviation.

Comparing the mean, standard deviation and other descriptive features of the estimated factors across models shows that both models give rather similar estimates for the magnitude of level, slope and curvature factors. The magnitudes and signs of the estimated parameters are consistent with theoretical predictions. For example, the value of level factors are typically positive while the value of the slope and curvature factors are negative on average. From the autocorrelations of



the three factors, the slope factor is more persistent than the other two factors for both models. The results suggest the high persistency and low volatility of the slope factor. Whereas the other two factors also show high autocorrelation at the first lag. This finding is quite different from other term structure studies prior to the global financial crisis, which found that the level factor is the most persistent. The high auto-correlated in slope factor represents the persistent yield spread of the long-term bond over the short-term bond that played a major role in the term structure evolution during the early 2000s and the aftermath of the crisis. Furthermore, the time series statistics of the estimated latent factors as well as the series of their empirical proxies are potentially correlated. This evidence implies that the estimated factors can represent the empirical factors quite closely.

In general, the highly positive autocorrelation amongst estimated factor time series at the first lag across the sample demonstrate that these factors in turn should produce accurate forecasting results. The future values will depend greatly on their recent historical values. We now examine the forecasting accuracy of the dynamic semiparametric factor model, the Nelson-Siegel model together with other competitive models based on the stochastic process of their latent factors.

### 3.7.2.3 Latent factor correlation

As mentioned earlier, the latent factors from the dynamic semiparametric factor model and the Nelson-Siegel model alongside the empirical proxies are potentially correlated. It is interesting to investigate the relationship of the individual factor associated its corresponding factor calculated from other model and empirical proxy. For this purpose, we present the correlation coefficients between the latent factors and their empirical factors in Table 3.7.

In general, the individual factor is most highly correlated with its counterpart factor. For example, the dynamic semiparametric factor model level factor, quoted by  $DL$ , is highly related to the Nelson-Siegel model level factor, represented by  $NL$ , with 0.88 correlation coefficient. The level factor seems to have a slightly

TABLE 3.7: Correlation coefficients of the estimated latent factors and their empirical proxies

|    | NL           | NS           | NC           | DL           | DS           | DC           | PL          | PS          | PC    | EL          | ES    | EC   |
|----|--------------|--------------|--------------|--------------|--------------|--------------|-------------|-------------|-------|-------------|-------|------|
| NL | 1.00         |              |              |              |              |              |             |             |       |             |       |      |
| NS | -0.34        | 1.00         |              |              |              |              |             |             |       |             |       |      |
| NC | <b>0.63</b>  | -0.01        | 1.00         |              |              |              |             |             |       |             |       |      |
| DL | <b>0.88</b>  | 0.07         | <b>0.84</b>  | 1.00         |              |              |             |             |       |             |       |      |
| DS | 0.05         | <b>0.87</b>  | 0.48         | 0.50         | 1.00         |              |             |             |       |             |       |      |
| DC | <b>-0.74</b> | 0.50         | <b>-0.87</b> | <b>-0.72</b> | 0.01         | 1.00         |             |             |       |             |       |      |
| PL | <b>0.85</b>  | 0.13         | <b>0.84</b>  | <b>1.00</b>  | 0.56         | <b>-0.69</b> | 1.00        |             |       |             |       |      |
| PS | 0.49         | <b>-0.98</b> | 0.05         | 0.07         | <b>-0.82</b> | -0.53        | 0.00        | 1.00        |       |             |       |      |
| PC | 0.21         | 0.14         | -0.53        | 0.01         | -0.09        | 0.49         | 0.00        | 0.00        | 1.00  |             |       |      |
| EL | <b>0.97</b>  | -0.16        | <b>0.78</b>  | <b>0.97</b>  | 0.28         | <b>-0.78</b> | <b>0.95</b> | 0.30        | 0.07  | 1.00        |       |      |
| ES | 0.21         | <b>-0.98</b> | -0.18        | -0.23        | <b>-0.94</b> | -0.32        | -0.29       | <b>0.95</b> | -0.04 | 0.01        | 1.00  |      |
| EC | <b>0.62</b>  | -0.07        | <b>0.99</b>  | <b>0.82</b>  | 0.42         | <b>-0.89</b> | <b>0.81</b> | 0.11        | -0.56 | <b>0.77</b> | -0.11 | 1.00 |

*Notes:* This table summarizes the correlation matrix between the latent factors estimated by the Nelson-Siegel model (N-); the dynamic semiparametric factor model (D-), the principal components (P-) and the empirical proxy (E-) latent factors. For all models and empirical proxy; the suffix -L, -S and -C refer to level, slope and curvature. Bold numbers indicate high correlation between factor within the model and across different models coefficient

higher positive cross-correlation with the curvature factors. This may cause multicollinearity between the level and curvature factors. However, there is low cross-correlation between level and slope factors.

The correlation of the estimated latent factors with their empirical counterpart factors is extremely high. These correlations assure the economic interpretation of estimated latent factors from the dynamic semiparametric factor model and the Nelson-Siegel model as the yield level, slope and curvature respectively. Unfortunately, the correlation between the third-component of the principal component analysis (PCA) model with other curvature counterparts is not so high.

Considering the dynamic semiparametric factor model, the latent factors are mostly correlated with the empirical counterpart factors, while less correlated with the Nelson-Siegel model counterpart factors. Nonetheless, its curvature factor is less related to the empirical counterpart factor. Even the Nelson-Siegel model has 99 percent correlation. Interestingly, its level factor is highly related with the first

component of the principal component analysis model.

In essence, the estimated factors remain to keep their interpretation as the level, slope and curvature despite the fact that they are derived from a different method. Next, we will investigate the forecasting performances of the dynamic semiparametric factor model, the Nelson-Siegel model and other competitive models.

## 3.8 Model forecasting comparison

For the out-of-sample forecasting performance, we examine the predictive ability of the dynamic semiparametric factor model and the dynamic Nelson-Siegel with other competitor models in a rolling-window out-of-sample forecasting experiment using the Australian government bond yields. While the dynamic semiparametric factor model outperforms the dynamic Nelson-Siegel in providing better in-sample fit, there is no clear-cut gain in forecasting. This is because there may be a trade-off between in-sample and out-of-sample performance. The models that provide a better in-sample fit do not necessarily have to perform well out-of-sample because of the risk of over-fitting.

Before documenting the results of the forecasts, we briefly describe how the models are specified for forecasting in Subsection 3.8.1. Then, we provide details of the out-of-sample forecasting exercise implementation in Subsection 3.8.2. In Subsection 3.8.3 and 3.8.4, we explain the statistical accuracy assessment and forecasting accuracy test across the models and discuss the overall forecasting results in Subsection 3.8.5 and sub-sample forecasting results in Subsection 3.8.6.

### 3.8.1 Model Specification

In this Subsection, we present the methodology for forecasting the dynamic semiparametric factor model, the dynamic Nelson-Siegel and the other competitor models.

### 3.8.1.1 The dynamic semiparametric factor model

The dynamic semiparametric factor model (DSFM) provides a general method for modeling and forecasting yield curve from the panel of yield data. Once the cross-sectional yield curve fitting has been estimated, the dynamics of the time series of the yields can be further conducted. In this study, the B-spline is used to model the underlying yield at a specific period and the dynamic evolution of the yield is assumed to be driven by a stochastic process of a small dimension of latent factors. The resulting DSFM combines the virtues of parsimony and dimensional reduction and allows us to forecast the evolution of the yields.

As mentioned earlier, the cross sectional yield is estimated by a B-spline that corresponds to the knots (time-to-maturities) of the spline at time point  $t$ . The spline basis function is essentially a piecewise smoothing function with pieces that join together to form a twice continuously differentiable function overall. The yield curve is estimated as the knots positioned at the maturities. The spline interpolates to yields with a stochastic process of the time-varying latent factors. Suppose the dynamic process of the factor is identified, the  $h$ -step ahead forecast of the latent factors and dynamic yields will be based on the dynamic factor specification. In particular, the  $h$ -month ahead prediction of a  $j$ -maturity bond yield in period  $t$  is a relationship between latent factor  $l$  at the  $h$ -month ahead and factor-loading non-parametric function of the knots at different maturities.

$$Y_{t+h,j} = \sum_{l=0}^L Z_{t+h,l} m_{l,j}(X_{t,j}) + \epsilon_{t,j} \quad (3.16)$$

Where  $Z_{t+h,l}$  is the latent factor  $l$  at the  $h$ -month ahead, given the information of latent factor up to period  $t$  and  $m_{l,j}(X_{t,j})$  is factor loading.

Basically, the cross-sectional estimation of the yield will extract latent factors at a particular date and provide the whole range of latent factors over the period of study. Given the estimated time-series of latent factors from the first step, the dynamic latent factors are modeled by the specifying stochastic process and the  $h$ -step ahead forecast  $Z_{t+h,l}$  is then used with the estimated factor loading  $m_{l,j}(X_{t,j})$  to derive the out-of-sample yield forecasts. In this study, the dynamic evolution

of latent factors is assumed to be driven by the first order autoregressive process, which can be either an univariate autoregressive AR(1) process or a multivariate autoregressive VAR(1) process. The first order autoregressive process is motivated by the numerous studies that successfully model the dynamic yield curve as in Diebold and Li (2006) and De Pooter (2007) among others. The stochastic process can be written as.

$$Z_{t+h,j} = \Xi_{t,l} + \Phi_{t,l}Z_{t,l} + \nu_{t,l} \quad (3.17)$$

where  $Z$  contains the contemporaneous and lagged observations of the latent factors explaining the variation of the yields corresponding to the cross-sectional basis function at each period up to  $t$ . The coefficients  $\Phi_{t,l}$ ,  $\Xi_{t,l}$  and  $Z_{t,l}$  are computed by the AR(1) or VAR(1) process that minimizes the sum of squared fitted errors of the model by using the set of  $Z$  up to date  $t$ .

### 3.8.1.2 The dynamic Nelson-Siegel model with optimal decay parameter

For the dynamic Nelson-Siegel model, we use rolling window estimation and re-estimate the model at every step by choosing the optimal decay parameter rather than using a fixed decay parameter to a pre-specified value as [Diebold and Li \(2006b\)](#) suggested.

Suppose the sub-sample  $T_s$  is given for the  $h$ -step ahead forecasting exercise at period  $t$ . The algorithm selects the optimal values for the decaying parameters  $\lambda$  and simultaneously defines a time series of estimated latent factors based on the procedure described previously.

$$\widehat{\lambda_{t+h}} = \arg \min_{\lambda \in \Omega} \left\{ \frac{1}{T_s} \sum_{t=1}^T \sqrt{\frac{1}{J} \sum_{j=1}^J (y_{t+h}(\tau) - \hat{y}_{t+h}(\tau, \lambda, \beta))^2} \right\} \quad (3.18)$$

By minimizing the difference between the model yield rates and the actual yield rates, we run an ordinary least square algorithm for a whole grid of  $\lambda$  values to

obtain the optimal exponential decay  $\lambda_t$  parameter and latent factors  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$ .

Based on the underlying stochastic process for the dynamic latent factor, the yield forecast is given by

$$\hat{y}_{t+h}(\tau) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t} \left( \frac{1 - e^{-\lambda_{t+h}\tau}}{\lambda_{t+h}\tau} \right) + \hat{\beta}_{3,t+h} \left( \frac{1 - e^{-\lambda_{t+h}\tau}}{\lambda_{t+h}\tau} - e^{-\lambda_{t+h}\tau} \right) \quad (3.19)$$

and the stochastic process can be written as.

$$\hat{\beta}_{i,t+h} = \hat{\Upsilon} + \Gamma_i \hat{\beta}_{i,t}, i = 1, 2, 3 \quad (3.20)$$

where  $y_{t+h}(\tau)$  is the  $h$ -step ahead forecasting yields with  $\tau$  time to maturity, and  $\beta_{1,t+h}$ ,  $\beta_{2,t+h}$  and  $\beta_{3,t+h}$  are  $h$ -step ahead forecasting latent factor parameters, which in dynamic form are referred to as level, slope and curvature and  $\lambda_{t+h}$  is referred to the exponential decay parameter.

### 3.8.1.3 Other competitor models

We compare the forecasting performance of the dynamic semiparametric factor model and the dynamic Nelson-Siegel model to those of several competitor models. In particular, these are a principal component model, the random walk of the yield levels, the simple AR(1) on yield levels and the unrestricted VAR(1) on yield levels. The naive random walk of the yield levels model is expected to be the most challenging competitor as reported by several studies. In the following, we briefly sketch the individual competitor forecasting models.

#### (1) The principal component model

We investigate whether the forecast based on the data-driven factor extraction technique, the principal component analysis, can compete with the dynamic semiparametric factor model and the dynamic Nelson-Siegel model. The idea behind

principal component analysis is to determine the linear combination of variables that has the highest variance. This linear combination of variables forms component or factor variables and the loadings or coefficients of the linear combination.

Recall the principal component of factor that extracted from the yields.

$$PC_t = \Omega'(Y - \bar{Y}) \quad (3.21)$$

Where  $\bar{Y}$  is the sample mean of the yields and  $\Omega$  is the eigenvector matrix of variance-covariance matrix of the yields.

Inverting the factor equation, we get a regression equation of the factors onto the yields. This regression explains how principal components can be used to reduce the dimensionality of the yields.

$$Y_t = \Omega PC_t + \bar{Y} \quad (3.22)$$

Due to the above equation, this regression explains all of the variance in yields. This equation lies in the fact that the principal components are independent or orthogonal.

Having a time series of principal components or factors  $PC_t$ , the h-step ahead out-of-sample forecast of the yields can be achieved by assuming the stochastic process of the factors.

$$\hat{PC}_{i,t+h} = \hat{\psi} + \Psi_i \hat{PC}_{i,t} \quad i = 1, 2, 3 \quad (3.23)$$

Based on the underlying stochastic process for the dynamic latent factor, we forecast the yield at h-step ahead by

$$\hat{Y}_{t+h} = \Omega \hat{PC}_{t+h} + \bar{Y} \quad (3.24)$$

where  $\bar{Y}$  is the mean of the in-sample yields.

## **(2) The random-walk of yield-level model**

Many previous studies have suggested that the evolution of interest rates might be well described by random walk processes. The random walk therefore remains a common benchmark for interest rate prediction models and is also used as a competitor here. We assume a random walk model for interest rates implies a simple no-change forecast of individual yields. Hence, in this model the h-months ahead prediction of an J-maturity bond yield in period t is simply given by its time t observation

$$Y_{t+h}(\tau) = Y_{t+h-1}(\tau) \quad (3.25)$$

In this model, any h-step ahead forecast is equal to the most recent observed value. This assumption implies that interest rates can roam around freely and do not revert back to a long-term mean, which contradicts the central bank's monetary policy targets.

## **(3) The first-order univariate autoregressive model on yield-level**

Assuming that the yield of maturity J follows a first-order autoregression, its h-step ahead forecast is given by

$$Y_{t+h}(\tau) = \mu(\tau) + \phi Y_{t+h-1}(\tau) \quad (3.26)$$

Simple autoregressive processes constitute another natural benchmark for modeling the time variation of bond yields.

## **(4) The first-order multivariate autoregressive model: Y-VAR(1)**

In this model, forecasts of yields are obtained according to.



$$Y_{t+h}(\tau) = \Delta(\tau) + \Lambda Y_{t+h-1}(\tau) \quad (3.27)$$

where  $\Delta$  and  $\Lambda$  are estimated by regressing the vector  $Y_t$  onto a constant and its  $h$ -months lag. A well-known drawback of using an unrestricted VAR model for yields is that forecasts can only be constructed for those maturities used when estimating the model. As we want to construct forecasts for 11 maturities, this results in a considerable number of parameters that need to be estimated. As an attempt to mitigate estimation error, and subsequently, to reduce the forecast error variance, we summarize the information contained in the explanatory vector by replacing it with a small number of common latent factors that drive yield curve dynamics.

### 3.8.2 Forecast procedure

We choose to evaluate the prediction accuracy of the term structure models on the basis of their out-of-sample forecasting performance for different yields. In this way, we will have a uniform ground to systematically compare models. We base our forecasting comparison exercise on a rolling window estimation with fixed size, in which parameters are re-estimated at each stage. This study divides the full data into the training period; April 1999 - March 2006 (84 observations) and the forecasting period; April 2006 - March 2013 (84 observations). Next to gauging the models predictive accuracy over the full sample, we also examine the robustness of the forecasting improvement and assess the instability of the structural change from the global financial crisis during 2008-2009 and European sovereign debt crisis during 2010-2012. We split the sample into three parts; the pre-crisis period, starting from April 1999 to March 2006 (84 observations), and the crisis period, starting from October 2002 - September 2009 (84 observations) and the crisis period, starting from April 2006 - March 2013 (84 observations). For the pre-crisis part, we divide them into the training sub-sample; April 1999 - August 2002 (42 observations) and forecasting sub-sample; September 2002 - March 2006 (42 observations). Likewise, the crisis part is separated into the training sub-sample; October 2002 - February 2006 (42 observations) and forecasting sub-sample; March 2006 - September 2009 (42 observations). Lastly, the post-crisis part is separated

into the training sub-sample; April 2006 - August 2009 (42 observations) and forecasting sub-sample; September 2009 - March 2013 (42 observations).

By doing this, it allows us to compare how the dynamic semiparametric factor model, the dynamic Nelson-Siegel counterpart and other competitors perform in the normal and crisis period. All the models are estimated with a rolling window by moving the sample forward with a fixed sample size and re-estimating the model iteratively until the  $h$ -step ahead out-of-sample forecast is obtained. We consider four forecast horizons,  $h = 1$  month as well as 3, 6 and 12 months ahead.

### 3.8.3 Forecasting accuracy performance

To assess the prediction accuracy of the out-of-sample forecast of the the dynamic semiparametric factor models, the dynamic Nelson-Siegel model and other competitors, we use a standard forecast error evaluation criteria. The predictive performance of the models are statistically evaluated by the root mean squared prediction error (RMSPE), which is widely used to assess forecasting accuracy of the models at particular maturities. We also compute the trace root mean squared prediction error (TRMSPE) of the models for all maturities as in [Hördahl et al. \(2006\)](#) and [De Pooter et al. \(2010\)](#). It combines the forecast errors of all maturities and summarizes the performance of each model, thereby allowing for a direct comparison between the models.

#### The root mean squared prediction error (RMSPE)

Given a sample of  $T$  out-of-sample forecasts with  $h$ -months ahead forecast horizon, we compute the RMSPE for a  $\tau$  time-to-maturity as follows:

$$RMSFE(\tau) = \sqrt{\sum_{t=1}^T \frac{[Y_{t+h}(\tau) - \hat{Y}_{t+h}(\tau)]^2}{T}} \quad (3.28)$$

where  $\hat{Y}_{t+h}$  is the forecasted yield in period  $t$  for  $t + h$  period and  $[Y_{t+h} - \hat{Y}_{t+h}]^2$  is the forecast errors at  $t + h$  for the yields.

### The trace root mean squared prediction error (TRMSPE)

For each forecast horizon, the trace root mean squared prediction error (TRMSPE) measures the aggregate forecast errors of all yields in  $J$  maturities. Given a sample of  $T$  out-of-sample forecasts of  $J$  distinct maturities with  $h$ -months ahead forecast horizon, we compute the TRMSPE as follows.

$$TRMSFE = \sqrt{\sum_{j=1}^J \sum_{t=1}^T \frac{[Y_{t+h} - \hat{Y}_{t+h}]^2}{JT}} \quad (3.29)$$

The RMSPE and the TRMSPE for the dynamic semiparametric factor model and the Nelson-Siegel model are reported for both the specifications of latent factors stochastic process; the AR(1) and VAR(1) for all forecasts horizons.

### 3.8.4 Forecasting accuracy test

In order to assess the relative accuracy of forecasts derived from the dynamic semiparametric factor model (DSFM), the dynamic Nelson-Siegel model and the principal component analysis model, we employ the [Diebold and Mariano \(1995\)](#) test whether the random walk benchmark is significantly superior to those models' forecasts. We also conduct a pairwise comparison between the DSFM with the dynamic Nelson-Siegel model and the principal component analysis model. The Diebold-Mariano test makes a direct comparison between these term structure models with the random walk and among themselves for each maturity and each forecast horizon. The testing is based on the null hypothesis of equal predictive ability to generate mean squared prediction error for each pairwise comparison.

We also examine the stability over time of the out-of-sample forecast over the period of study which includes the global financial crisis. The methodology we used is based on the [Giacomini and Rossi \(2010\)](#) test of whether the two models can produce consistent forecasts, which means they do not suffer from structural breaks during the crisis.

### 3.8.4.1 Diebold-Mariano test

The main feature of the [Diebold and Mariano \(1995\)](#) test of forecast accuracy lies in its direct applicability to quadratic loss function of the multi-period forecast. We define the squared forecast errors  $e_t$  as

$$e_t = [Y_{t+h}(\tau) - \hat{Y}_{t+h}(\tau)]^2 \quad (3.30)$$

recall  $Y_{t+h}$  is the actual yield and  $\hat{Y}_{t+h}$  is the forecasted yield at  $t+h$  for the yields.

To determine if one model predicts better than another, one can observe the loss difference  $d_t$  of the squared forecast errors as

$$d_t = e_{1,t} - e_{2,t} \quad (3.31)$$

where  $e_{1,t+h}$  and  $e_{2,t+h}$  are the quadratic loss functions of the two competing models. In this case, the forecasting accuracy of the DSFM, the dynamic Nelson-Siegel model and the principal component analysis model are tested against the random walk. We also compare the forecasting ability of the DSFM with the dynamic Nelson-Siegel model and the principal component analysis model. The null hypothesis of equal predictive accuracy is then  $H_0 : E(d_t) = 0$  against the alternative hypothesis  $H_1 : E(d_t) \neq 0$ .

Assuming covariance stationarity, the Diebold-Mariano test statistics is computed as

$$DM = \frac{\bar{d}}{\sqrt{\frac{\hat{\sigma}_{\bar{d}}}{T}}} \quad (3.32)$$

where  $\bar{d}$  is the mean loss difference and  $\hat{\sigma}_{\bar{d}}$  is a consistent estimate of the asymptotic (long-run) variance of  $\sqrt{T}\bar{d}$ . This test corrects for the autocorrelation of the multi-period forecast errors by a [Newey and West \(1987\)](#) heteroskedasticity and

autocorrelation consistent standard errors for sample variance of the loss differential to account for this concern.

We apply the Diebold-Mariano test to forecast errors of the DSFM, the dynamic Nelson-Siegel model and the principal component analysis model with the random walk as well as among these term structure models and then report the results in Table 12 and 13. A negative and statistical significance of the Diebold-Mariano test statistic means that the DSFM, the dynamic Nelson-Siegel model and the principal component analysis model provides smaller in magnitude errors than the random walk, and thus it rejects the null hypothesis that they have the same forecasting ability as the random walk. For the test of DSFM against the dynamic Nelson-Siegel model and the principal component analysis model, a negative and statistical significance indicates the DSFM outperform the dynamic Nelson-Siegel model or the principal component analysis model in provides more accurate forecast.

#### **3.8.4.2 Giacomini-Rossi fluctuation test**

Since the period of study covers the global finance crisis, it is plausible that the forecasting performance of the DSFM, the dynamic Nelson-Siegel model, the principal component analysis model and the random walk may change over time. To analyze the stability of the forecasting performance in the presence of instabilities, we implement the fluctuation test proposed by [Giacomini and Rossi \(2010\)](#).

The Giacomini-Rossi statistic can be used to test whether the out-of-sample forecast performance statistics are able to break down, due to unforeseen structural breaks. The test is based on the idea that due to a potentially unstable environment, possibly as a consequence of the crisis, the forecast performance of the DSFM, the dynamic Nelson-Siegel model and the principal component analysis model relative to the random walk may change. Likewise, the relative prediction accuracy of the DSFM in comparison with the dynamic Nelson-Siegel model or the principal component analysis may also be uncertain. Therefore, the assessment of a local loss difference over time may supply additional information about the structural break rather than concentrating on the global loss difference as in the

Diebold-Mariano test.

To implement the fluctuation test, the Giacomini-Rossi test statistics is computed on the principle that, if the forecast performance of a model does not break down, then there should be no difference in the predictive accuracy for each moving window. The loss difference are calculated as the Diebold-Mariano type and the Giacomini-Rossi fluctuation test is defined as.

$$GR = \frac{\bar{d}^{local}}{\sqrt{\frac{\hat{\sigma}_{\bar{d}^{local}}}{Q}}} \quad (3.33)$$

where  $\bar{d}^{local}$  is the mean local loss difference of the sub-sample for each rolling window,  $\hat{\sigma}_{\bar{d}^{local}}$  is the heteroskedasticity and autocorrelation consistent standard errors for sample variance of the loss differential in the sub-sample window and  $Q$  is the length of the sub-sample window size.

We thus provide the Giacomini-Rossi test of the null hypothesis that the DSFM, the dynamic Nelson-Siegel model or the principle component analysis model perform equally to the random walk in predicting yields at each point in time against the alternative that there is a one-time break in the relative performance. We also test for stable superior accuracy of the DSFM against the dynamic Nelson-Siegel model or the principle component analysis model. To investigate such potential instabilities, we measure the local relative forecasting performance of the models, and test whether it equals zero at each point in time subject to rolling windows. Following [Giacomini and Rossi \(2010\)](#), we plot the standardized sample path of the relative measure of local performance, together with critical values, which measured by the Diebold-Mariano statistics. If the Giacomini-Rossi test statistic is negative and statistical significant; the DSFM, the dynamic Nelson-Siegel model or the principle component analysis model are outperformed the random walk at some points in time. For pairwise test between the DSFM and the dynamic Nelson-Siegel model or the principle component analysis model, the DSFM outperforms the other two models whenever the test statistics are negative and significant. Graphically, if any plots of local performance cross above the positive critical value or below the negative critical value, one of the models significantly outperforms its competitor at that point of time. In other words, the predictive

performance between two specified models are unequal. It implies the unstable environment may affect the relative forecast performance.

### 3.8.5 Overall forecasting results

Next, we report the RMSPE of the multiple steps ahead term structure forecast of the Australian government bond yields. The forecasts produced by the dynamic semiparametric factor model (DSFM) are compared with the dynamic Nelson-Siegel model, the principal component analysis model, the yield first-order autoregressive AR(1), the yield first-order vector autoregressive VAR(1) and the random walk. The results for the whole range of forecasting period from April 2006 to March 2013 are presented in Tables 3.8 - 3.11. To visualize these results, Figures 3.8 to 3.15 show the actual yields and those predicted by the DSFM and the dynamic Nelson-Siegel specification by the AR(1) or VAR(1) process, compared with the random walk benchmark for some selected maturities.

#### 3.8.5.1 One-month ahead

In Table 3.8, the one-month-ahead forecasting performance of the DSFM is compared with the dynamic Nelson-Siegel model, the principal component analysis model and other competitors.

The results for the one-month-ahead horizon are not very encouraging. For nearly all maturities, the random walk and the AR(1) for yield-level show better statistics than any of the models. There only six-month bond yield forecasts produced by the DSFM with AR(1) specification, the Nelson-Siegel model with VAR(1) specification and the principal component analysis model with AR(1) and VAR(1) specification are clearly accurate and do better than the random walk. Over all, the principal component analysis model with AR(1) and VAR(1) specification for the dynamic latent factors obviously works better than those of the DSFM and the Nelson-Siegel counterpart in terms of lower RMSPE.

TABLE 3.8: Out-of-sample one-month-ahead forecasts for the period 2006 to 2013

|             | TRMSPE        | RMSPE         |               |               |               |               |               |               |
|-------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|             | all           | 6-m           | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          |
| <b>RW</b>   | <b>0.2932</b> | 0.3054        | <b>0.3403</b> | <b>0.3324</b> | <b>0.3118</b> | <b>0.2866</b> | <b>0.2667</b> | <b>0.2654</b> |
| Y-AR(1)     | 0.3034        | 0.3146        | 0.3497        | 0.3428        | 0.3230        | 0.2974        | 0.2771        | 0.2749        |
| Y-VAR(1)    | 0.3342        | 0.3382        | 0.3917        | 0.3879        | 0.3634        | 0.3354        | 0.2974        | 0.2885        |
| <b>DNS</b>  |               |               |               |               |               |               |               |               |
| AR(1)       | 0.3392        | 0.3385        | 0.4085        | 0.3902        | 0.3727        | 0.3376        | 0.3061        | 0.2924        |
| VAR(1)      | 0.3142        | <b>0.2891</b> | 0.3773        | 0.3561        | 0.3408        | 0.3201        | 0.2867        | 0.2741        |
| <b>DSFM</b> |               |               |               |               |               |               |               |               |
| AR(1)       | 0.3337        | <b>0.2935</b> | 0.3926        | 0.4183        | 0.3785        | 0.3223        | 0.2956        | 0.3022        |
| VAR(1)      | 0.3304        | 0.3108        | 0.3768        | 0.3996        | 0.3744        | 0.3289        | 0.2945        | 0.2932        |
| <b>PCA</b>  |               |               |               |               |               |               |               |               |
| AR(1)       | 0.3057        | <b>0.3000</b> | 0.3784        | 0.3535        | 0.3334        | 0.2970        | 0.2743        | 0.2684        |
| VAR(1)      | 0.3090        | <b>0.2961</b> | 0.3695        | 0.3477        | 0.3362        | 0.3091        | 0.2822        | 0.2733        |

*Notes:*. This table summarizes the overall trace root mean squared prediction errors (TRMSPE) and the root mean squared prediction errors (RMSPE) for each particular maturity obtained from out-of-sample yield forecasts made for the period April 2006 to March 2013. RW refers to the random walk; Y-AR(1) refers to the first-order univariate autoregressive model of yield level; Y-VAR(1) refers to the first-order multivariate autoregressive model of yield level; DNS refers to the dynamic Nelson-Siegel model; DSFM refers to the dynamic semi-parametric factor model and PCA refers to the principal component analysis model. Bold numbers indicate the best performing model

Between the VAR(1) and AR(1) specification for latent factor dynamics of the DSFM and the Nelson-Siegel model, the VAR(1) specification outperforms the AR(1) specification for one-month ahead prediction. However, it is difficult to outperform the random walk. This evidence is consistent with other term structure forecast studies.

To further illustrate how the forecasting performance of different models varies over time, we plot the 1-month ahead forecast horizon for the DSFM, the principal component analysis model and the Nelson-Siegel model with AR(1) specification and VAR(1) specification.

Figure 3.8 plots the yield forecasts with AR(1) specification for the 1-month ahead forecast horizon. According to these plots, the DSFM with AR(1) specification



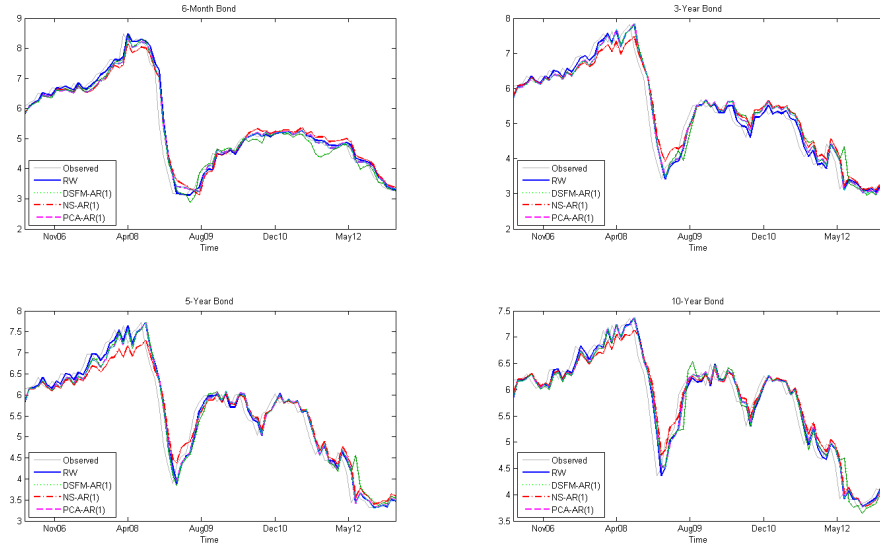


FIGURE 3.8: Observed and 1-month ahead predicted yields with AR(1) specification

*Notes:* This figure provides plots of the observed and 1-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with AR(1), NS with AR(1) and PCA with AR(1) model, respectively.

forecast the persistent movements of yields quite well, especially the 6-month yield forecast over the period of 2006-2008. However, it fails to produce less RMSPE during 2011 to 2012. Whereas the Nelson-Siegel model with AR(1) specification forecast demonstrates the effectiveness in tracking the actual time series better than the the DSFM, in particular, since the aftermath of the global financial crisis. Even the Nelson-Siegel with AR(1) specification underestimate the forecasted yield over the period of 2006-2008, it turns to outperform the DSFM by producing a minimal errors. Nonetheless, the principal component analysis model with AR(1) specification provide the better track for the actual yield across the entire forecasting period from 2006 to 2013.

Figure 3.9 present the actual yield and the 1-month ahead predicted yield by VAR(1) specification. The dynamic Nelson-Siegel model and the principal component analysis model achieve accurate prediction while the DSFM predicts more variation than actual yields. Comparing between the AR(1) and VAR(1) specification, the VAR(1) tend to overshoot at the turning point. However, the VAR(1)

appears to better perform the AR(1) in overall.

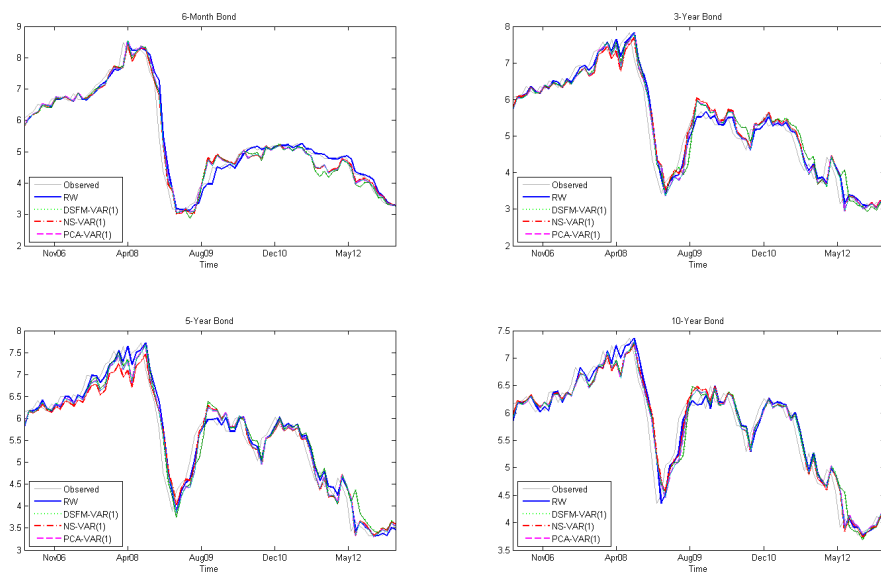


FIGURE 3.9: Observed and 1-month ahead predicted yields with VAR(1) specification

*Notes:* This figure provides plots of the observed and 1-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

As can be seen, the VAR(1) forecasts overstate the actual yields on 2009 and the six-month yield forecasts predict a severe drop during 2011. Though, the VAR(1) specification provide accurate out-of-sample forecast at the beginning and at the end of the forecasting period. The dynamic Nelson-Siegel model and the DSFM with VAR(1), in particular, forecast more accurate yield rather than the AR(1) specification.

### 3.8.5.2 Three-month ahead

Table 3.9 shows the three-month horizon for all models. Now, the DSFM with AR(1) specification becomes the preferable model and does better than the Nelson-Siegel model with VAR(1) specification.

TABLE 3.9: Out-of-sample three-month-ahead forecasts for the period April 2006 to March 2013

|             | TRMSPE        | RMSPE         |               |               |               |               |               |               |
|-------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|             | all           | 6-m           | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          |
| RW          | <b>0.6364</b> | 0.7534        | <b>0.7814</b> | <b>0.7389</b> | <b>0.6802</b> | <b>0.6177</b> | <b>0.5711</b> | <b>0.5401</b> |
| Y-AR(1)     | 0.6929        | 0.8132        | 0.8412        | 0.7966        | 0.7403        | 0.6781        | 0.6284        | 0.5871        |
| Y-VAR(1)    | 0.7818        | 1.0098        | 1.0961        | 0.9472        | 0.8331        | 0.7282        | 0.6417        | 0.5826        |
| <i>DNS</i>  |               |               |               |               |               |               |               |               |
| AR(1)       | 0.7332        | 0.8591        | 0.9116        | 0.8626        | 0.8041        | 0.7153        | 0.6534        | 0.6031        |
| VAR(1)      | 0.7119        | 0.8380        | 0.8876        | 0.8311        | 0.7808        | 0.7006        | 0.6313        | 0.5773        |
| <i>DSFM</i> |               |               |               |               |               |               |               |               |
| AR(1)       | 0.7071        | <b>0.7489</b> | 0.8743        | 0.8412        | 0.7871        | 0.7101        | 0.6361        | 0.5780        |
| VAR(1)      | 0.7276        | 0.8036        | 0.8965        | 0.8437        | 0.8086        | 0.7431        | 0.6537        | 0.5747        |
| <i>PCA</i>  |               |               |               |               |               |               |               |               |
| <i>PCA</i>  |               |               |               |               |               |               |               |               |
| AR(1)       | 0.6973        | 0.7936        | 0.8787        | 0.8256        | 0.7724        | 0.6846        | 0.6183        | 0.5667        |
| VAR(1)      | 0.7138        | 0.8364        | 0.8933        | 0.8251        | 0.7816        | 0.7052        | 0.6350        | 0.5791        |

*Notes:.* This table summarizes the root mean squared errors obtained from out-of-sample yield forecasts. RW refers to the random walk; Y-AR(1) refers to the first-order univariate autoregressive model of yield level; Y-VAR(1) refers to the first-order multivariate autoregressive model of yield level; DNS refers to the dynamic Nelson-Siegel model; DSFM refers to the dynamic semiparametric factor model and PCA refers to the principal component analysis model. Bold numbers indicate the best performing model

As presented in Table 3.9, it is clear that the DSFM and the principal component analysis model with AR(1) specification produce more accurate results for the entire time-to-maturities as compared to the Nelson-Siegel. It is interesting that the DSFM with AR(1) specification not only fits the term structure very well, but also still produces accurate 6-month maturity yield forecasts. The DSFM with VAR(1) specification relatively outperform the dynamic Nelson-Siegel counterpart in producing lower RMSPE for short and long maturities.

Surprisingly, the dynamic Nelson-Siegel with AR(1) specification is now fail to outperform the DSFM. This results can be clearly seen from the Figure 3.10.

The main point to take form the graphs is that the DSFM with AR(1) specification forecast is more accurate than the Nelson-Siegel. Specifically, the DSFM

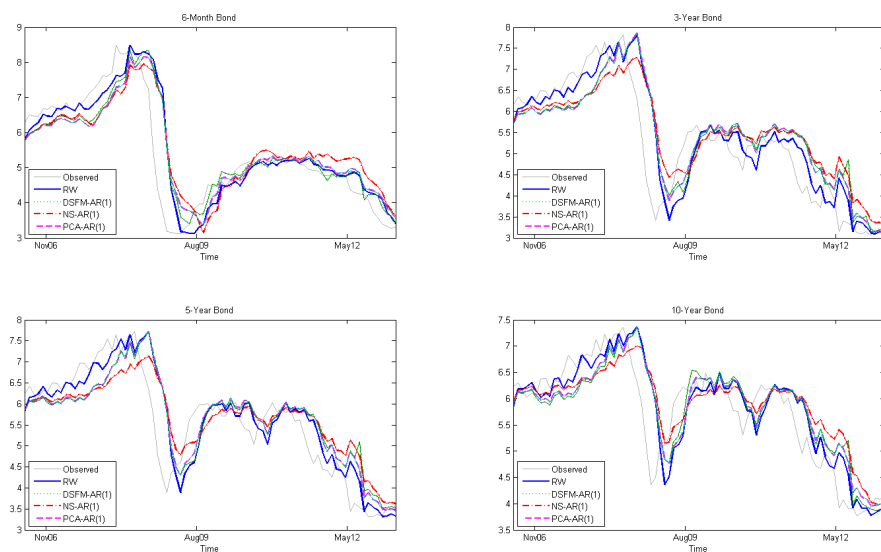


FIGURE 3.10: Observed and 3-month ahead predicted yields with AR(1) specification

*Notes:* This figure provides plots of the observed and 3-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

successfully tracks the 3-month ahead forecast for 6-month bonds during almost four years from 2009 to 2012. Unfortunately, it is difficult for the DSFM to give a precise prediction once the economy experiences a structural break or regime shift. Figure 3.10 reveals the fact that the DSFM forecasts for 3-month ahead yields somewhat lag behind the actual yields, more specifically, after the yield hit the bottom in 2009. In addition, the principal component analysis model with AR(1) specification produces somewhat optimal forecast track and outperforms the DSFM and the dynamic Nelson-Siegel model.

Having look at Figure 3.11 gives a clear cut to explanation of why the DSFM with VAR(1) specification turns to beat the dynamic Nelson-Siegel counterpart and other models with VAR(1) specification.

As Figure 3.11 shows that the dynamic Nelson-Siegel and the DSFM with VAR(1) specification overstate the increase in yields by several months over 2009 to 2010. This pattern indicates the forecasting errors due to the sharp structural shock, causing the sharp decline in interest rates and the widening spread during this

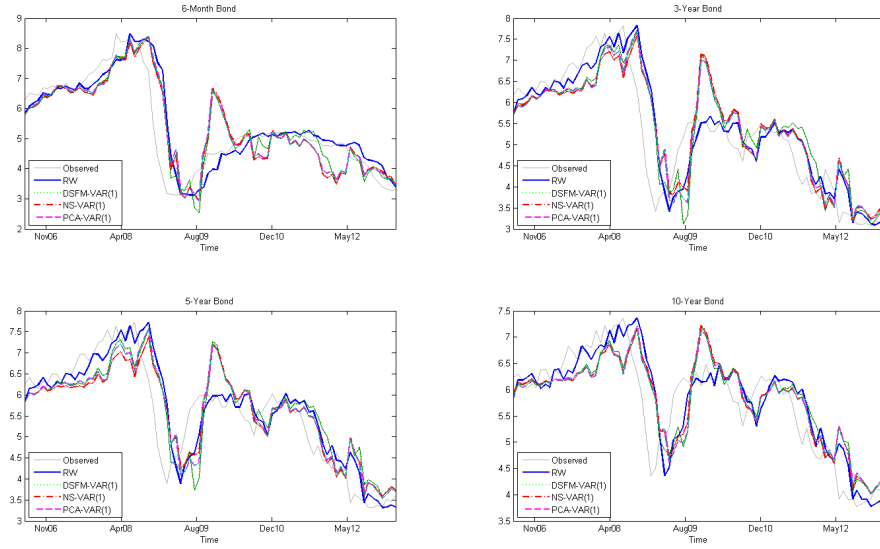


FIGURE 3.11: Observed and 3-month ahead predicted yields with VAR(1) specification

*Notes:* This figure provides plots of the observed and 3-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

period.

### 3.8.5.3 Six-month ahead

Table 3.10 present the 6-month ahead yield curve forecast produced by the DSFM, the dynamic Nelson-Siegel model and the principal component analysis model, together with their competitors.

For a 6-month horizon, the Nelson-Siegel with AR(1) specification turns out to do better job as compared to the DSFM for all maturities. Noticeably, the models with VAR(1) specification are struggling to predict yield for medium term maturity. Even though, the principal component analysis model, particularly the model with VAR(1) specification, produces accurate forecasts. In fact, the principal component analysis and the Nelson-Siegel model with VAR(1) specification exhibit poor prediction for short maturities but they provide more accurate forecasts for

TABLE 3.10: Out-of-sample six-month-ahead forecasts for the period April 2006 to March 2013

|             | TRMSPE        | RMSPE         |               |               |               |               |               |               |
|-------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|             | all           | 6-m           | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          |
| RW          | <b>1.0255</b> | <b>1.3198</b> | <b>1.3160</b> | <b>1.2257</b> | <b>1.1101</b> | <b>0.9990</b> | <b>0.9159</b> | <b>0.8438</b> |
| Y-AR(1)     | 1.1782        | 1.5028        | 1.4972        | 1.3663        | 1.2594        | 1.1645        | 1.0732        | 0.9768        |
| Y-VAR(1)    | 1.5415        | 2.2096        | 2.2520        | 1.9084        | 1.6677        | 1.4197        | 1.2359        | 1.0816        |
| <i>DNS</i>  |               |               |               |               |               |               |               |               |
| AR(1)       | 1.1471        | 1.4348        | 1.4350        | 1.3402        | 1.2493        | 1.1328        | 1.0509        | 0.9719        |
| VAR(1)      | 1.1586        | 1.5148        | 1.5052        | 1.3972        | 1.2841        | 1.1223        | 1.0178        | 0.9258        |
| <i>DSFM</i> |               |               |               |               |               |               |               |               |
| AR(1)       | 1.1867        | 1.4246        | 1.5031        | 1.4017        | 1.3055        | 1.1937        | 1.0837        | 0.9743        |
| VAR(1)      | 1.2123        | 1.5700        | 1.6083        | 1.4505        | 1.3410        | 1.1927        | 1.0593        | 0.9322        |
| <i>PCA</i>  |               |               |               |               |               |               |               |               |
| AR(1)       | 1.1922        | 1.4373        | 1.5350        | 1.4371        | 1.3383        | 1.1891        | 1.0691        | 0.9621        |
| VAR(1)      | 1.1488        | 1.5223        | 1.5420        | 1.3844        | 1.2708        | 1.1109        | 0.9907        | 0.8861        |

*Notes:.* This table summarizes the root mean squared errors obtained from out-of-sample yield forecasts. RW refers to the random walk; Y-AR(1) refers to the first-order univariate autoregressive model of yield level; Y-VAR(1) refers to the first-order multivariate autoregressive model of yield level; DNS refers to the dynamic Nelson-Siegel model; DSFM refers to the dynamic semiparametric factor model and PCA refers to the principal component analysis model

long maturities in terms of RMSPE. Overall, the Nelson-Siegel with AR(1) specification gains the lowest root mean square prediction errors on average, compared to other models, except for the random walk.

Figure 3.12 presents the corresponding actual and predicted yields for the 6-month ahead. As figure shows, the Nelson-Siegel with AR(1) specification is actually less volatile while the DSFM suffers from higher variation in yield curve forecast. From 2010 onwards, the Nelson-Siegel with AR(1) specification demonstrates large errors in tracking the actual yield whereas the DSFM with AR(1) specification predicts more impressive results in reducing the RMSPE. Nonetheless, the VAR(1) specification clearly fluctuates more and overstates the volatility of the yield, as presented in Figure 3.13.

From Figure 3.13, the spike yield prediction from VAR(1) estimation, especially

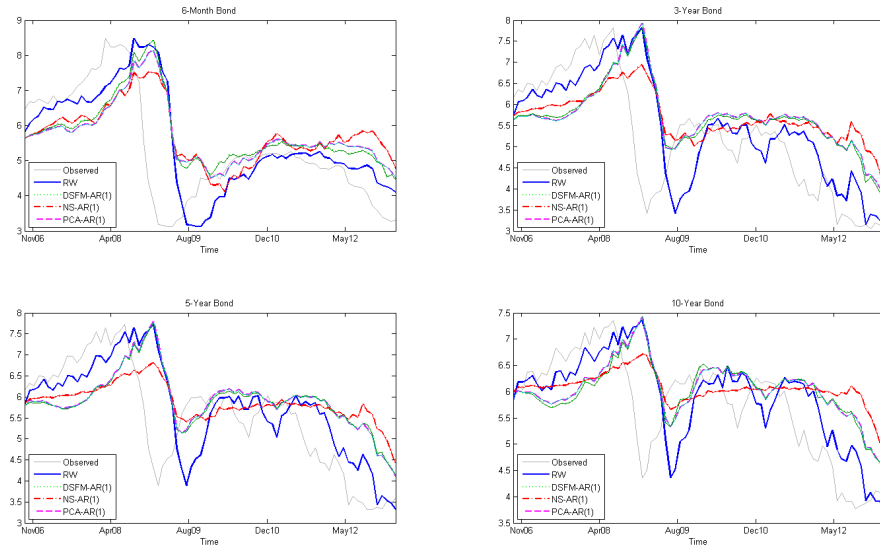


FIGURE 3.12: Observed and 6-month ahead predicted yields with AR(1) specification

*Notes:* This figure provides plots of the observed and 6-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

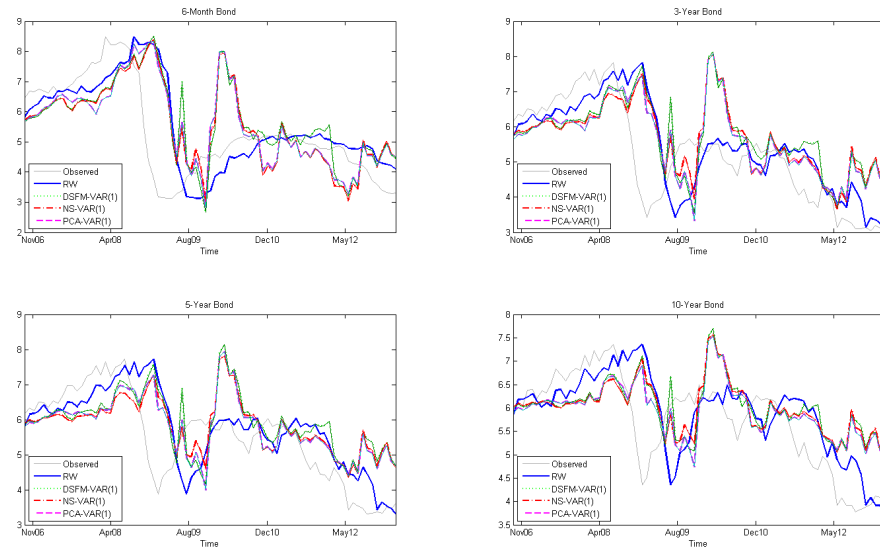


FIGURE 3.13: Observed and 6-month ahead predicted yields with VAR(1) specification

*Notes:* This figure provides plots of the observed and 6-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

during 2009, become more evident for longer step ahead forecasts. Yet, the principal component analysis model with VAR(1) specification outperforms the AR(1) counterpart and other competitors, except for the Nelson-Siegel with AR(1) specification in tracking the actual yield and forecast over the entire period.

### 3.8.5.4 Twelve-month ahead

In Table 3.11, the twelve-month ahead forecast for the DSFM, the Nelson-Siegel model, the principal component analysis and their competitors are reported.

TABLE 3.11: Out-of-sample twelve-month-ahead forecasts for the period April 2006 to March 2013

|             | TRMSPE        | RMSPE         |               |               |               |               |               |               |
|-------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|             | all           | 6-m           | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          |
| RW          | <b>1.3653</b> | <b>1.9582</b> | <b>1.9346</b> | <b>1.7009</b> | <b>1.5049</b> | <b>1.3551</b> | <b>1.2205</b> | <b>1.0904</b> |
| Y-AR(1)     | 1.6268        | 2.4643        | 2.4244        | 2.0809        | 1.8337        | 1.5653        | 1.3615        | 1.1732        |
| Y-VAR(1)    | 2.1330        | 3.2889        | 3.1153        | 2.6560        | 2.3535        | 2.0867        | 1.8111        | 1.5661        |
| <i>DNS</i>  |               |               |               |               |               |               |               |               |
| AR(1)       | 1.4854        | 2.1934        | 2.0999        | 1.8933        | 1.7073        | 1.4367        | 1.2886        | 1.1628        |
| VAR(1)      | 1.5709        | 2.2382        | 2.1820        | 1.9891        | 1.7812        | 1.5396        | 1.4024        | 1.2825        |
| <i>DSFM</i> |               |               |               |               |               |               |               |               |
| AR(1)       | 1.6179        | 2.1818        | 2.2426        | 2.0333        | 1.8880        | 1.6625        | 1.4536        | 1.2637        |
| VAR(1)      | 1.6539        | 2.2890        | 2.3183        | 2.0554        | 1.8782        | 1.6728        | 1.4880        | 1.3135        |
| <i>PCA</i>  |               |               |               |               |               |               |               |               |
| AR(1)       | 1.6143        | 2.1963        | 2.2658        | 2.0468        | 1.8796        | 1.6270        | 1.4396        | 1.2756        |
| VAR(1)      | 1.5059        | 2.1269        | 2.1411        | 1.9041        | 1.7207        | 1.4910        | 1.3291        | 1.1901        |

*Notes:.* This table summarizes the root mean squared errors obtained from out-of-sample yield forecasts. RW refers to the random walk; Y-AR(1) refers to the first-order univariate autoregressive model of yield level; Y-VAR(1) refers to the first-order multivariate autoregressive model of yield level; DNS refers to the dynamic Nelson-Siegel model; DSFM refers to the dynamic semiparametric factor model and PCA refers to the principal component analysis model

As can be seen from Table 3.11, it is still very difficult for any model to provide forecasts that are more accurate than the random walk. The Nelson-Siegel model with AR(1) specification produces low RMSPE and does better than other models,



except the random walk, to anticipate yield in the next twelve month for all maturities. Apparently, the Nelson-Siegel model, the principal component analysis model and the DSFM produce poor forecast for long period ahead horizons.

To visualize the result, Figure 3.14 shows the actual and those predicted yields by AR(1) and VAR(1) specification for the Nelson-Siegel, the principal component analysis model and the DSFM respectively.

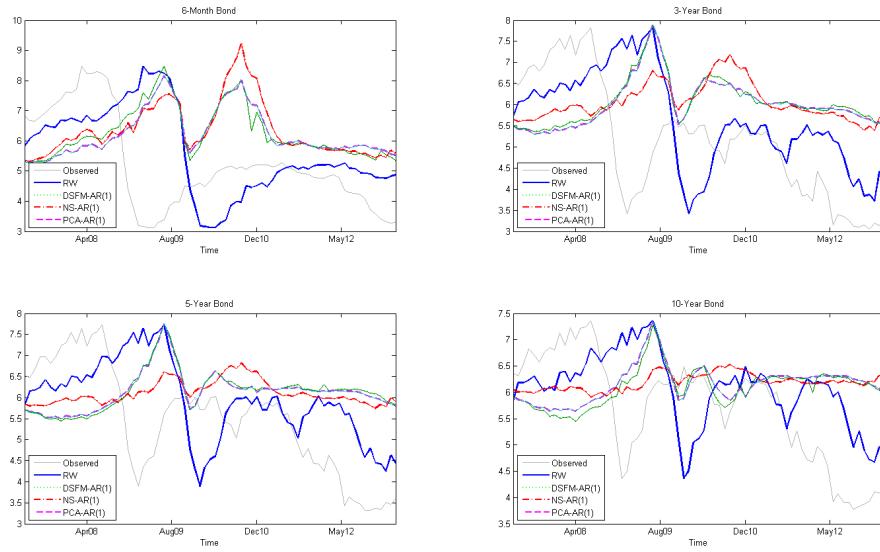


FIGURE 3.14: Observed and 12-month ahead predicted yields with AR(1) specification

*Notes:* This figure provides plots of the observed and 12-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively. Noticeably, the DSFM and the principal component analysis model with AR(1) produce a spike yield curve for short (6-month) maturity during the global financial crisis.

From Figure 3.14, the DSFM, the principal component analysis model and the Nelson-Siegel model are struggling to provide accurate forecast for long horizons. In particular, the DSFM and the principal component analysis model with AR(1) produce a spike yield curve that widens the gap from actual yield and reduces its predictability. Whereas the Nelson-Siegel with AR(1) specification gives a smoother yield curve in producing yield for long maturity. The forecasting errors become larger when the VAR(1) are used to model the dynamic process of latent

factors.

Figure 3.15 reveals that the VAR(1) specification for the DSFM, the principal component analysis model and the Nelson-Siegel produce even more overshoots, as compared to the AR(1) specification.

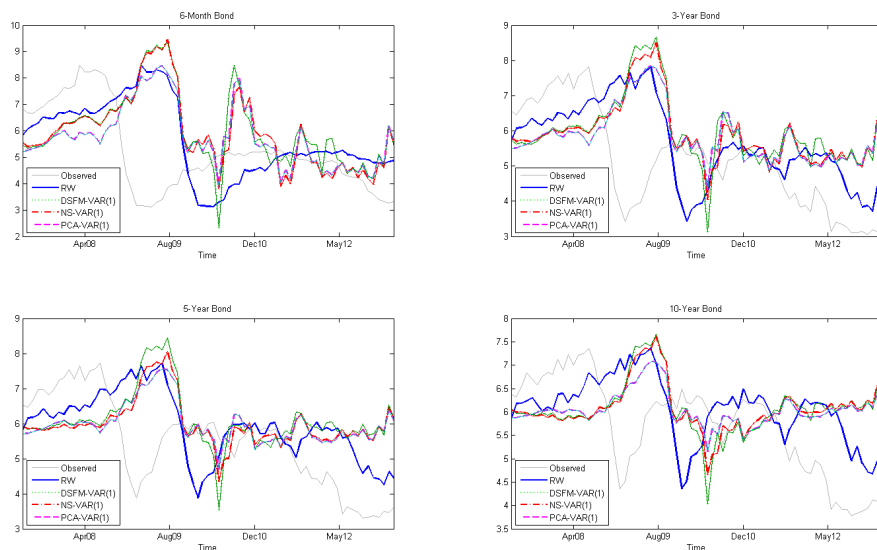


FIGURE 3.15: Observed and 12-month ahead predicted yields with VAR(1) specification

*Notes:* This figure provides plots of the observed and 12-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

As presented in Figure 3.15, the DSFM suffers from structural break and yield volatility, resulting in larger errors than the dynamic Nelson-Siegel and the principal component analysis model. The DSFM with VAR(1) specification overstates the prediction, especially at the turning point during the first-half of 2009 and the second-half of 2010.

For the overall multi-step ahead prediction over the period from 2006 to 2013, the DSFM and the dynamic Nelson-Siegel are not encouraging with consistent accurate forecasts. The performance of each model is contingent to the forecast horizon, maturity and period of the sample. This may be attributed to a the out-of-sample period we use. De Pooter et al. (2010) noticed that including the

period from 2000 onwards may fail to achieve the promising out-of-sample results. Over the 2003-2006, yield spreads were widened and then sharply declined after the onset of the global financial crisis during 2007 to 2009. In contrast, the yield term structure during the second half of the 1990s to the early 2000s was rather stable and produced promising results for several studies undertaken on this period, as in [Duffee \(2002\)](#), [Ang and Piazzesi \(2003\)](#), [Diebold and Li \(2006\)](#) and [Hördahl et al. \(2006\)](#). Based on findings over the period of 2006 to 2013, the Nelson-Siegel typically produces more accurate forecast for 1-month, 6-month and 12-month ahead, compared with the DSFM which outperforms Nelson-Siegel in providing less prediction errors for 3-month ahead forecast. These findings imply that the DSFM may probably be suffering from the volatility and regime shifts while the Nelson-Siegel can do better.

#### **3.8.5.5 Forecasting accuracy test**

In this section, we examine the forecasting ability of the DSFM, the principal component analysis model and the dynamic Nelson-Siegel model relative to yields implied by the random walk. As noted in the literatures, for example [De Pooter \(2007\)](#) and [Koopman et al. \(2010\)](#), the random walk is the hard model to beat in term structure forecasting. In order to statistically confirm the superior forecasting ability of the random walk model, we employ the [Diebold and Mariano \(1995\)](#) statistics to assess the prediction performance of the DSFM, the principal component analysis model and the dynamic Nelson-Siegel model, compared with the random walk. A negative and statistically different from zero of the quadratic loss means that the DSFM, the principal component analysis model or the dynamic Nelson-Siegel model can provide smaller in magnitude errors than the random walk, and thus it rejects the null hypothesis that the prediction models have the same forecasting ability as the random walk. Another main interest is in examining whether the DSFM can produce more accurate results, compared with the dynamic Nelson-Siegel model and the principal component analysis model. For these pairwise tests, the negative value with statistical significance of the Diebold-Mariano statistics show that the DSFM outperforms its competitor models.

Besides the Diebold-Mariano test to evaluate the forecasting superiority of the random walk to other models, we also apply the [Giacomini and Rossi \(2010\)](#) fluctuation test to statistically check whether the superior prediction ability of the random walk is consistent over the forecasting period and does not suffer from structural break. To carry out the Giacomini-Rossi fluctuation test exercise, we recursively estimated the DSFM, the principal component analysis model or the Nelson-Siegel model against the random walk, by adding one observation and then re-estimating three models until the end of the rolling window.

### (1) Diebold-Mariano test

Since the random walk has superior predictive ability compared to the DSFM, the principal component analysis model and the dynamic Nelson-Siegel model, we perform the Diebold-Mariano test to forecast errors of two pairs of the models; specifically, the DSFM, the principal component analysis model or the dynamic Nelson-Siegel model with the random walk and present the results in [Table 3.12](#) - [3.13](#). In the first step, we compare the forecast accuracy of the AR(1) factor dynamic specification of the DSFM, the principal component analysis model or the dynamic Nelson-Siegel model with the random walk, and subsequently make a comparison of the VAR(1) latent factor specification of both models in comparison with the random walk for each maturity and each forecast horizon.

The results in [Table 3.12](#) and [Table 3.13](#) indicate that the random walk provides better term structure forecasts. The positive values of Diebold-Mariano statistics indicate that the DSFM, the principal component analysis model and the Nelson-Siegel model produce higher in magnitude forecast errors than the random walk model, especially at medium-term and long-term maturities for all forecasting horizon. The reported values of Diebold-Mariano test statistic are consistent with the RMSPE results. These confirm the random walk has superior prediction ability as compared with other three models.

Comparison of the DSFM with AR(1) and VAR(1) specification against the Nelson-Siegel model and the principal component analysis model at six-month maturity for one-month and three-month ahead show the DSFM as statistically better in

TABLE 3.12: Diebold-Mariano (1995) test statistics of the models with AR(1) specification and the random walk over the period 1999 to 2013

|                       | Diebold-Mariano (1995) test statistics |         |         |         |         |         |         |
|-----------------------|--|---------|---------|---------|---------|---------|---------|
|                       | 6-m                                    | 1-y     | 2-y     | 3-y     | 5-y     | 7-y     | 10-y    |
| <i>1-month ahead</i>  |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | -0.5510                                | 2.1603* | 3.5162* | 3.1893* | 2.0284* | 2.0029* | 2.1048* |
| <i>NS/RW</i>          | 1.5374                                 | 2.5950* | 2.3376* | 2.9840* | 2.5325* | 2.2027* | 1.4358  |
| <i>DS/NS</i>          | -2.6382*                               | -0.7794 | 1.1023  | 0.2486  | -0.5991 | -0.4693 | 0.4571  |
| <i>PCA/RW</i>         | -0.3203                                | 1.7960  | 1.7405  | 2.7218* | 1.8144  | 2.3999* | 0.6798  |
| <i>DS/PCA</i>         | -0.5131                                | 1.7765  | 2.8133* | 2.5193* | 1.5149  | 1.4945  | 2.0555* |
| <i>3-month ahead</i>  |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | -0.1039                                | 2.2631* | 2.2142* | 2.4821* | 2.2984* | 1.7603  | 1.1788  |
| <i>NS/RW</i>          | 1.7262                                 | 1.8265  | 1.8485  | 1.9688  | 1.4695  | 1.3259  | 1.1105  |
| <i>DS/NS</i>          | -2.7396*                               | -0.6851 | -0.6570 | -0.5114 | -0.1378 | -0.5166 | -0.7495 |
| <i>PCA/RW</i>         | 0.7867                                 | 2.2371* | 1.8059  | 2.2639* | 1.9936* | 1.7080  | 1.1403  |
| <i>DS/PCA</i>         | -2.4196*                               | -0.3839 | 0.8872  | 0.8767  | 1.4117  | 1.4279  | 0.9519  |
| <i>6-month ahead</i>  |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | 1.6813                                 | 2.0405* | 1.4080  | 1.4581  | 1.3285  | 1.1936  | 1.0154  |
| <i>NS/RW</i>          | 0.6634                                 | 0.5840  | 0.5962  | 0.6426  | 0.5584  | 0.5675  | 0.5539  |
| <i>DS/NS</i>          | -0.0808                                | 0.4568  | 0.5574  | 0.4717  | 0.5271  | 0.2855  | 0.0202  |
| <i>PCA/RW</i>         | 1.1767                                 | 2.0683* | 1.6222  | 1.6581  | 1.3450  | 1.1411  | 0.9429  |
| <i>DS/PCA</i>         | 0.8366                                 | 0.7765  | 0.7704  | 0.7524  | 1.0916  | 1.2056  | 1.2114  |
| <i>12-month ahead</i> |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | 1.2791                                 | 1.6213  | 1.3738  | 1.3716  | 1.0293  | 0.7978  | 0.6475  |
| <i>NS/RW</i>          | 0.6434                                 | 0.4390  | 0.5260  | 0.5366  | 0.2367  | 0.2116  | 0.2427  |
| <i>DS/NS</i>          | -0.0596                                | 0.7934  | 0.9565  | 1.4709  | 2.4668* | 2.0510* | 1.2259  |
| <i>PCA/RW</i>         | 1.0157                                 | 1.4257  | 1.3761  | 1.3464  | 0.9427  | 0.7835  | 0.7064  |
| <i>DS/PCA</i>         | -0.2465                                | -0.4457 | -0.4948 | 0.4463  | 2.6063* | 0.8447  | -0.4301 |

*Notes.*: The table presents Diebold-Mariano (DM) forecast accuracy comparison test results of the dynamic semiparametric factor model (DS), the dynamic Nelson-Siegel model (NS) and the principal component analysis (PCA) model with AR(1) specification against the benchmark random walk (RW), DSFM against benchmark NS and DSFM against benchmark PCA model. The null hypothesis is that the two forecasts have the same root mean squared error. Value with a asterisk indicate the statistical significance at the 95 percent. For each pair, the negative value of DM statistic indicate that first model provides smaller in magnitude forecast errors than the benchmark model. Four forecast horizons are evaluated at 1, 3, 6 and 12 months ahead, for yields observed at maturities of 1, 2, 3, 5, 7, and 10 years. To correct contemporaneously correlated and serially correlated in forecast errors, we modified the Diebold-Mariano test with heteroscedasticity and autocorrelation consistent (HAC) estimation

TABLE 3.13: Diebold-Mariano (1995) test statistics of the models with VAR(1) specification and the random walk over the period 1999 to 2013

|                       | Diebold-Mariano (1995) test statistics |         |         |         |         |         |         |
|-----------------------|--|---------|---------|---------|---------|---------|---------|
|                       | 6-m                                    | 1-y     | 2-y     | 3-y     | 5-y     | 7-y     | 10-y    |
| <i>1-month ahead</i>  |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | 0.1544                                 | 1.9624  | 2.6038* | 2.7063* | 2.2046* | 1.7913  | 1.5173  |
| <i>NS/RW</i>          | -0.4357                                | 1.3728  | 1.2267  | 1.7326  | 1.7265  | 1.3724  | 0.8115  |
| <i>DS/NS</i>          | 2.1587*                                | -0.0242 | 1.5774  | 1.4868  | 0.4615  | 0.5795  | 1.3399  |
| <i>PCA/RW</i>         | -0.3168                                | 1.5579  | 0.9707  | 1.4601  | 1.3804  | 1.0655  | 0.6574  |
| <i>DS/PCA</i>         | 1.4928                                 | 0.7102  | 1.9756* | 1.7885  | 1.2636  | 1.3386  | 1.6418  |
| <i>3-month ahead</i>  |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | 0.4273                                 | 1.5082  | 1.8178  | 1.7483  | 1.6487  | 1.3938  | 0.7028  |
| <i>NS/RW</i>          | 0.6609                                 | 1.0344  | 1.3076  | 1.4673  | 1.2588  | 1.1192  | 0.8284  |
| <i>DS/NS</i>          | -1.2044                                | 0.1822  | 0.2673  | 0.5750  | 0.8632  | 0.7215  | -0.1829 |
| <i>PCA/RW</i>         | 0.7851                                 | 1.4939  | 1.5790  | 1.7081  | 1.4757  | 1.2027  | 0.7827  |
| <i>DS/PCA</i>         | -1.1699                                | 0.1022  | 0.4861  | 0.7463  | 1.1426  | 0.9217  | -0.3000 |
| <i>6-month ahead</i>  |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | 2.3381*                                | 2.3256* | 1.8772  | 1.5670  | 1.1794  | 0.8918  | 0.5842  |
| <i>NS/RW</i>          | 1.4296                                 | 1.2294  | 1.0642  | 0.9664  | 0.6798  | 0.6082  | 0.5399  |
| <i>DS/NS</i>          | 0.6212                                 | 0.9591  | 0.8609  | 1.1074  | 1.4067  | 1.2913  | 0.3774  |
| <i>PCA/RW</i>         | 1.4393                                 | 1.5239  | 0.9642  | 0.8538  | 0.5954  | 0.4262  | 0.2577  |
| <i>DS/PCA</i>         | 0.3431                                 | 0.8290  | 1.3757  | 1.5750  | 2.1015* | 2.0286* | 1.7564  |
| <i>12-month ahead</i> |  |         |         |         |         |         |         |
| <i>DS/RW</i>          | 5.0376*                                | 5.6264* | 9.6226* | 9.1699* | 2.3022* | 1.5769  | 1.2369  |
| <i>NS/RW</i>          | 3.4420*                                | 3.4198* | 4.7663* | 2.6930* | 2.0251* | 2.2918* | 1.8785  |
| <i>DS/NS</i>          | 5.6676*                                | 4.8655* | 5.2226* | 3.1332* | 1.0813  | 0.9405  | 0.9605  |
| <i>PCA/RW</i>         | 2.2962*                                | 3.3914* | 3.3433* | 1.5985  | 0.6961  | 0.5119  | 0.4606  |
| <i>DS/PCA</i>         | 0.8758                                 | 1.1333  | 1.4022  | 1.5785  | 2.0030* | 2.6540* | 3.7778* |

*Notes:*. The table presents Diebold-Mariano (DM) forecast accuracy comparison test results of the dynamic semiparametric factor model (DS), the dynamic Nelson-Siegel model (NS) and the principal component analysis (PCA) model with AR(1) specification against the benchmark random walk (RW), DSFM against benchmark NS and DSFM against benchmark PCA model. The null hypothesis is that the two forecasts have the same root mean squared error. Value with a asterisk indicate the statistical significance at the 95 percent. For each pair, the negative value of DM statistic indicate that first model provides smaller in magnitude forecast errors than the benchmark model. Four forecast horizons are evaluated at 1, 3, 6 and 12 months ahead, for yields observed at maturities of 1, 2, 3, 5, 7, and 10 years. To correct contemporaneously correlated and serially correlated in forecast errors, we modified the Diebold-Mariano test with heteroscedasticity and autocorrelation consistent (HAC) estimation

forecasting yields at short maturities for short-horizons. However, as judged for all tested forecasting horizons across all maturities, the DSFM, the Nelson-Siegel model and the principal component analysis model are worse than the random walk.

## (2) Giacomini-Rossi fluctuation test

To evaluate whether potential instabilities may affect the forecast performance of the random walk relative to the DSFM, the Nelson-Siegel model and the principal component analysis model, we check the structural stability by the Giacomini-Rossi fluctuation test which assesses whether the predictive ability changes over time.

Figure 3.16 shows the Giacomini-Rossi fluctuation test for the 6-month, 3-year, 5-year and 10-year bond yields over the rolling windows during March 2008 to March 2013. The figure plots the relative forecasting performance for the DSFM, the Nelson-Siegel model and the principal component analysis model against the random walk, together with the relative forecasting performance for the DSFM against the Nelson-Siegel model and the principal component analysis model at the 5 percent critical values. Since the values of the statistic are below the (negative) critical value, we reject the null hypothesis of equal predictive ability at each point in time and conclude that the test model forecasts better than the benchmark model in particular periods.

The graph suggests that the global financial crisis (represented by shaded area) deteriorated the term structure predictability of the DSFM, the Nelson-Siegel and the principal component analysis model. However, from the end of the year 2008 to the first half of 2009, the forecasting ability of the DSFM, the Nelson-Siegel model and the principal component analysis model start to recover and even beat the random walk by producing a negative value of the Giacomini-Rossi fluctuation test statistics for 6-month yield forecast. It is clear that the DSFM significantly produces better forecasts for 6-month bonds relative to the random walk by attaining some negative Giacomini-Rossi fluctuation test statistics below the critical value. Since the European sovereign debt crisis emerged from the end of 2010,

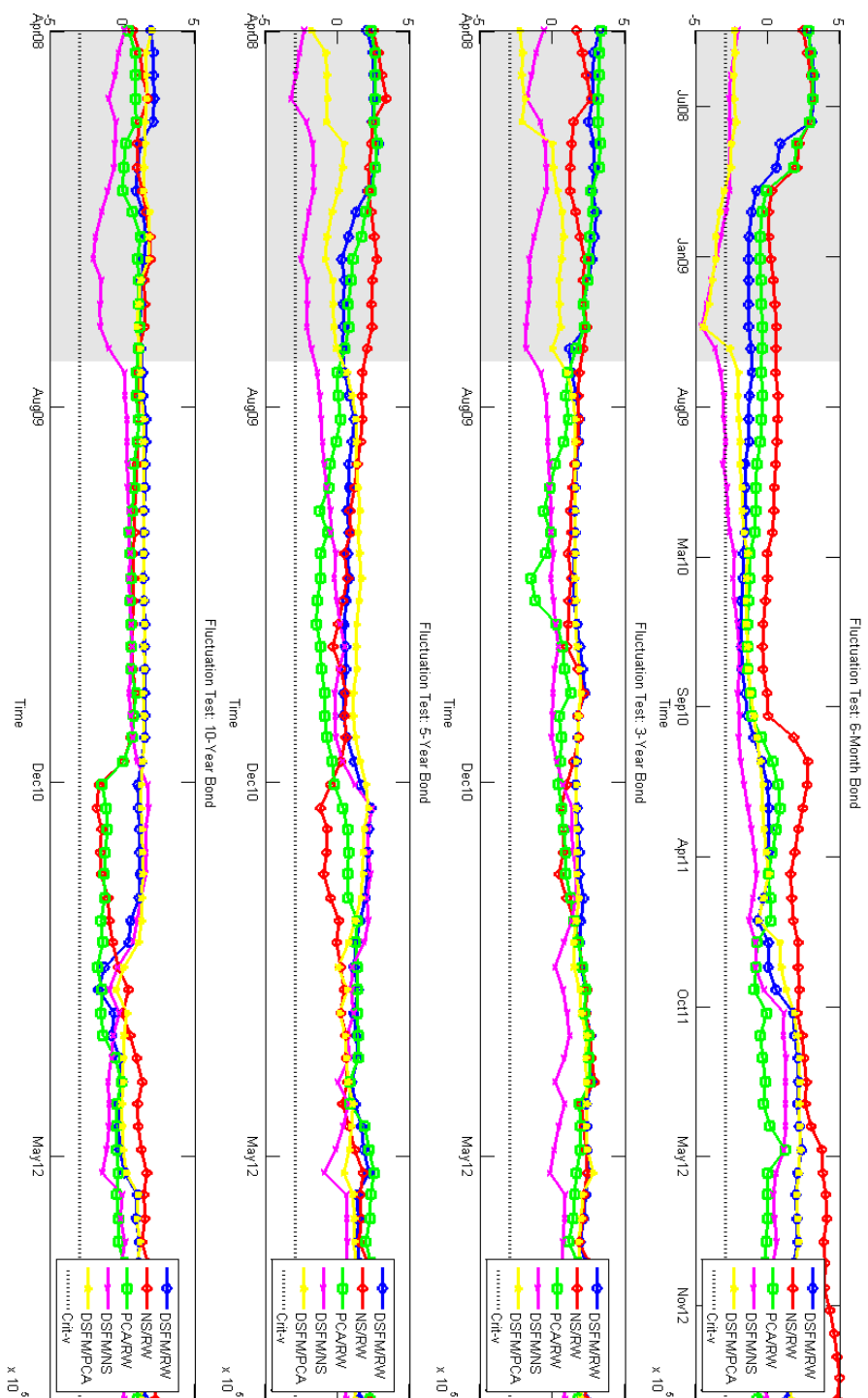


FIGURE 3.16: Giacomini-Rossi (2010) fluctuation test for one-month ahead forecasting with AR(1) specification over the period 2008 to 2013

*Notes:* The figure reports Giacomini-Rossi (GR) fluctuation test statistic for comparing forecasts of the dynamic semiparametric factor model (DSFM), the Nelson-Siegel model (NS) and the principal component analysis (PCA) model with AR(1) specification against the benchmark random walk (RW), DSFM against benchmark NS and DSFM against benchmark PCA model. The black dotted line indicates the critical value of the GR test statistic  $\pm 2.248$  at 95 percent level of confidence. If the estimated test statistic is above or below this line, the forecasting accuracy of the specified model is significantly different than its benchmark. The GR statistics of the DSFM, NS and PCA against RW are plotted by blue line with circles, red line with diamonds and green line with squares respectively, while DSFM against benchmark NS and DSFM against benchmark PCA model represented by pink line with cross and yellow line with star. Negative values indicate that specified model forecasts better.



the DSFM, the Nelson-Siegel and the principal component analysis model start to suffer from a structural break and interest rate volatility corresponding to higher term premium and yield spread that turns the fluctuation test statistics to be positive.

Further support for structural instability in forecasting term structure can be also observed by Figure 3.17 which presents Giacomini-Rossi fluctuation test statistics based on the VAR(1) specification.

From Figure 3.17, the plot of Giacomini-Rossi fluctuation test statistics clearly shows the effect of the global financial crisis in 2008 and the European sovereign debt crisis from the end of 2010 which weakens the prediction ability of the DSFM, the Nelson-Siegel and the principal component analysis model to compete with the random walk. The values of fluctuation test statistics are raised and remained positive after the structural break which indicates the predictability of the three models is worsened by the unstable environment.

### 3.8.6 Sub-sample forecasting

As the structural break may be a possible explanation for the inconsistent results in predicting the term structure, we examine the robustness of the predictive performance of the DSFM and the dynamic Nelson-Siegel as well as try to detect the affect for the periods that covers the crisis or regime shifts. The overall out-of-sample period was initially estimated from 2006 to 2013 and thus directly falls into the 2007 to 2008 global financial crisis period. Therefore, we carry out a forecasting experiment by comparing the forecasting results over three sub-samples; the first sub-sample from 2003 to 2006, the second sub-sample from 2006 to 2009 and the third sub-sample from 2009 to 2013. The second sub-sample will cover the global financial crisis.

For each sub-sample, we report RMSPE for the DSFM and the dynamic Nelson-Siegel with the AR(1) and VAR(1) specification and illustrate 1-month ahead yield forecasts for 6-month, 3-year, 5-year and 10-year bonds and compare them with

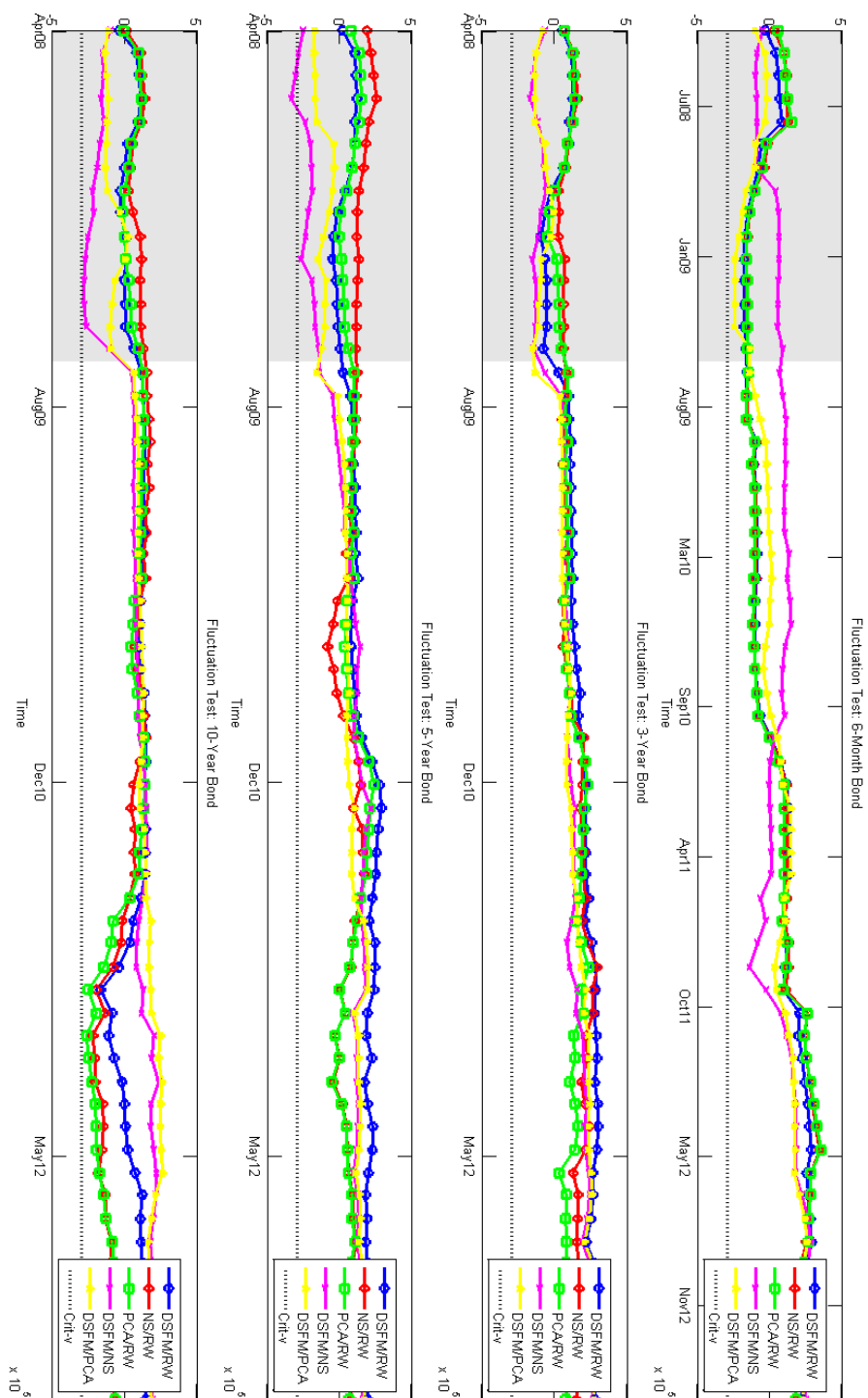


FIGURE 3.17: Giacomini-Rossi (2010) fluctuation test for one-month ahead forecasting with VAR(1) specification over the period 2008 to 2013

*Notes:* The figure reports Giacomini-Rossi (GR) fluctuation test statistic for comparing forecasts of the dynamic semiparametric factor model (DSFM), the Nelson-Siegel model (NS) and the principal component analysis (PCA) model with AR(1) specification against the benchmark random walk (RW), DSFM against benchmark NS and DSFM against benchmark PCA model. The black dotted line indicates the critical value of the GR test statistic  $\pm 2.248$  at 95 percent level of confidence. If the estimated test statistic is above or below this line, the forecasting accuracy of the specified model is significantly different than its benchmark. The GR statistics of the DSFM, NS and PCA against RW are plotted by blue line with circles, red line with diamonds and green line with squares respectively, while DSFM against benchmark NS and DSFM against benchmark PCA model represented by pink line with cross and yellow line with star. Negative values indicate that specified model forecasts better.

the actual yields.

### 3.8.6.1 Sub-sample 2003-2006

We start the discussion for each sub-sample period by considering the individual models performance in terms of the RMSPE. Table 3.14 provides the root mean squared forecast errors of the different models for the out-of-sample prediction during the period from 2003 to 2006.

In general, the RMSPE in every individual model and every maturity during the sub-sample period are lower than those for the overall out-of-sample forecast from 2006 to 2013. In addition, the AR(1) specification clearly outperforms the VAR(1) dynamics for all multi-period ahead forecasts while the results over 2006 to 2013 are mixed. These results show that forecasting during the normal period is more accurate and more consistent than the period that includes a structural break. Analyzing over the specific period ahead forecast, the DSFM with AR(1) specification presents the superior forecasting performance at the 3-month and 6-month ahead, especially for the longer maturities. On the other hand, the Nelson-Siegel specification demonstrates the accurate forecast for shorter (1-month ahead) and longer (12-month ahead) horizons.

To demonstrate the forecasting performance corresponding to time period, Figure 3.18 shows the 1-month ahead prediction of the models with an AR(1) specification for selected bonds in different maturities while Figure 3.19 depicts the prediction of the models with a VAR(1) specification. During this sub-period, yields initially decline until the first quarter of 2003. Afterward, these yields rise and continue on the upward trend, accompanied with a substantial widening of spreads.

As can be seen from the Figure 3.18, the model with an AR(1) specification forecast the persistent movement of the yield, especially after 2003 until the end of sub-sample period. In particular, the Nelson-Siegel with an AR(1) specification provide better forecast performance than the DSFM with an AR(1) specification,

TABLE 3.14: Out-of-sample forecasts for the subperiod September 2003 to March 2006

|                       | TRMSPE        | RMSPE         |               |               |               |               |               |               |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|                       | all           | 6-m           | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          |
| <i>1-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | 0.2085        | <b>0.1293</b> | <b>0.1583</b> | <b>0.1814</b> | 0.2019        | 0.2139        | 0.2267        | 0.2389        |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | <b>0.2057</b> | 0.1481        | 0.1846        | 0.1866        | <b>0.2013</b> | <b>0.2130</b> | 0.2164        | <b>0.2273</b> |
| VAR(1)                | 0.2100        | 0.1589        | 0.2138        | 0.1932        | 0.2039        | 0.2120        | <b>0.2159</b> | 0.2286        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.2103        | 0.1444        | 0.1784        | 0.1936        | 0.2071        | 0.2214        | 0.2210        | 0.2350        |
| VAR(1)                | 0.2187        | 0.1768        | 0.2007        | 0.1996        | 0.2114        | 0.2274        | 0.2262        | 0.2393        |
| <i>3-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | <b>0.3159</b> | <b>0.2531</b> | <b>0.3068</b> | <b>0.3273</b> | <b>0.3432</b> | <b>0.3375</b> | 0.3272        | 0.3235        |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.3561        | 0.3242        | 0.3826        | 0.3858        | 0.3938        | 0.3774        | 0.3539        | 0.3390        |
| VAR(1)                | 0.3453        | 0.3793        | 0.4124        | 0.3599        | 0.3647        | 0.3523        | 0.3314        | 0.3225        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.3276        | 0.3181        | 0.3688        | 0.3634        | 0.3640        | 0.3428        | <b>0.3168</b> | <b>0.3035</b> |
| VAR(1)                | 0.3549        | 0.4069        | 0.4218        | 0.3795        | 0.3781        | 0.3624        | 0.3355        | 0.3225        |
| <i>6-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | 0.4075        | <b>0.3834</b> | <b>0.4446</b> | 0.4563        | 0.4654        | 0.4486        | 0.4271        | 0.4083        |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.4114        | 0.4618        | 0.5212        | 0.4735        | 0.4647        | 0.4327        | 0.4059        | 0.3853        |
| VAR(1)                | 0.5017        | 0.6830        | 0.6749        | 0.5679        | 0.5424        | 0.5012        | 0.4722        | 0.4491        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | <b>0.3738</b> | 0.4921        | 0.5162        | <b>0.4509</b> | <b>0.4250</b> | <b>0.3788</b> | <b>0.3393</b> | <b>0.3102</b> |
| VAR(1)                | 0.4748        | 0.6553        | 0.6586        | 0.5418        | 0.5079        | 0.4659        | 0.4413        | 0.4204        |
| <i>12-month ahead</i> |               |               |               |               |               |               |               |               |
| RW                    | 0.4937        | <b>0.5523</b> | <b>0.6194</b> | 0.6167        | 0.6203        | 0.5924        | 0.5494        | 0.5180        |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | <b>0.3324</b> | 0.7262        | 0.6810        | <b>0.4191</b> | <b>0.3241</b> | <b>0.2320</b> | <b>0.2139</b> | <b>0.2326</b> |
| VAR(1)                | 0.3807        | 0.8776        | 0.7467        | 0.4250        | 0.3237        | 0.2662        | 0.2724        | 0.2960        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.3696        | 0.6772        | 0.6489        | 0.5026        | 0.4396        | 0.3638        | 0.3021        | 0.2549        |
| VAR(1)                | 0.4116        | 0.7799        | 0.7124        | 0.5126        | 0.4440        | 0.3851        | 0.3592        | 0.3461        |

*Notes:.* This table summarizes the overall trace root mean squared prediction errors (TRMSPE) and the root mean squared prediction errors (RMSPE) for the random walk (RW); the first-order univariate autoregressive model of yield (Y-AR(1)); the first-order multivariate autoregressive model of yield (Y-VAR(1)); the dynamic Nelson-Siegel model (DNS); the dynamic semiparametric factor model (DSFM) and the principal component analysis model (PCA). The results are made for subperiod 2003 to 2006. For each model, the RMSFEs are reported for 6-month and 1-, 2-, 3-, 5-, 7- and 10-year maturities, and for 1-, 3-, 6- and 12-month-ahead horizon. Bold numbers indicate the best performing model.

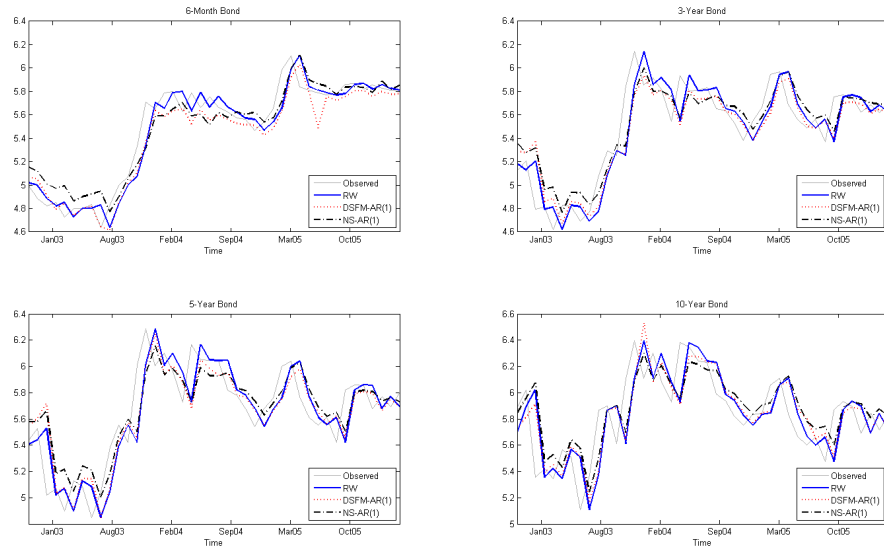


FIGURE 3.18: Observed and 1-month ahead predicted yields with AR(1) specification for the subperiod 2003 to 2006

*Notes:* This figure provides plots of the observed and 1-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

except for the 6-month bond yield.

The forecasting results for the VAR(1) models are presented in Figure 3.19. According to this plot, the VAR(1) specification of the DSFM and the Nelson-Siegel tend to overshoot the fluctuation of the yields whereas the AR(1) provide more persistent results.

During 2003 to 2004, the VAR(1) specification of both models understate the predicted yields around the turning point of the term structure dynamics. However, the DSFM with VAR(1) specification appears to miss the particular dynamics by few months or are lagged relative to the actual yields, while the Nelson-Siegel does a better job.

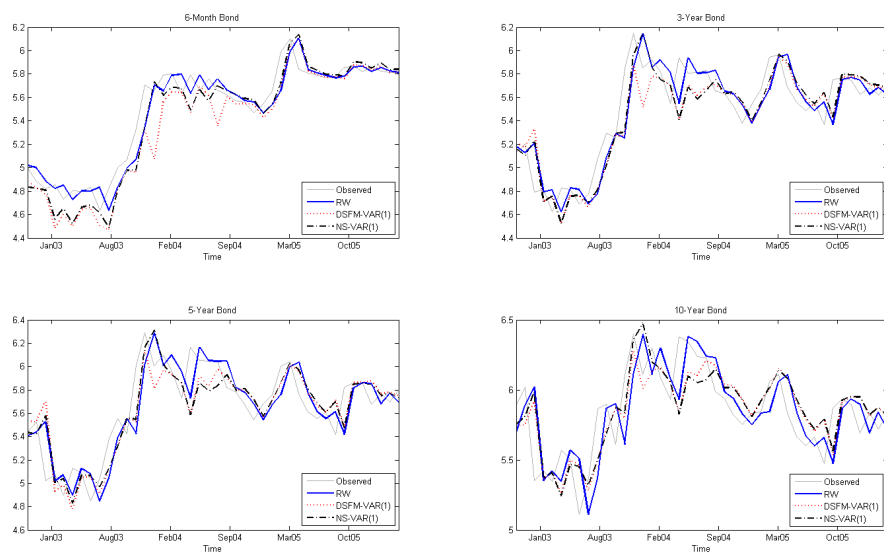


FIGURE 3.19: Observed and 1-month ahead predicted yields with VAR(1) specification for the subperiod 2003 to 2006

*Notes:* This figure provides plots of the observed and 1-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

### 3.8.6.2 Sub-sample 2006-2009

This sub-period runs from 2006 to 2009 and thus includes the global financial crisis. We expect the reason why the prediction accuracy performance for the overall forecasting period is unpromising and inconsistent may be due to the structural break. This examination may shed more light on this claim. Forecasting results for individual models over the multi-step ahead are reported in Table 3.15.

The results for this sub-period shows that the DSFM and the Nelson-Siegel model with an AR(1) and VAR(1) specification produce higher RMSPE than the overall forecasting period from 2006 to 2013, and even higher than the previous sub-sample over 2003 to 2006 which does not include the crisis. Despite the fact that both models suffered from the crisis and produced less accurate forecast over the crisis, the Nelson-Siegel model performs a better job than the DSFM, in particular the 1-month, 6-month and 12 month ahead. Interestingly, the dynamic semiparametric factor model with an AR(1) specification outperforms the Nelson-Siegel model for the three-month horizon forecast. It can also beat the random walk for

TABLE 3.15: Out-of-sample forecasts for the period March 2006 to September 2009

|                       | TRMSPE        | RMSPE         |               |               |               |               |               |               |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|                       | all           | 6-m           | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          |
| <i>1-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | <b>0.3387</b> | 0.4043        | <b>0.4261</b> | <b>0.3817</b> | <b>0.3521</b> | <b>0.3220</b> | <b>0.2920</b> | <b>0.2886</b> |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.4025        | 0.4912        | 0.5541        | 0.4816        | 0.4155        | 0.3713        | 0.3296        | 0.3179        |
| VAR(1)                | 0.3673        | <b>0.3869</b> | 0.4774        | 0.4497        | 0.4128        | 0.3736        | 0.3383        | 0.3399        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.3819        | 0.3965        | 0.4905        | 0.4688        | 0.4138        | 0.3719        | 0.3241        | 0.3175        |
| VAR(1)                | 0.3850        | 0.3880        | 0.4516        | 0.4261        | 0.3919        | 0.3768        | 0.3215        | 0.2973        |
| <i>3-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | <b>0.7942</b> | 1.0273        | <b>1.0472</b> | <b>0.9457</b> | <b>0.8572</b> | <b>0.7624</b> | <b>0.6892</b> | <b>0.6389</b> |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.9545        | 1.3066        | 1.3330        | 1.2056        | 1.0500        | 0.8852        | 0.7739        | 0.6888        |
| VAR(1)                | 0.9382        | <b>0.9976</b> | 1.1459        | 1.1547        | 1.0733        | 0.9393        | 0.8545        | 0.7974        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.8905        | 1.0971        | 1.2062        | 1.1340        | 1.0050        | 0.8627        | 0.7467        | 0.6662        |
| VAR(1)                | 1.7470        | 1.8962        | 1.9762        | 1.8679        | 1.8024        | 1.8007        | 1.7226        | 1.6875        |
| <i>6-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | <b>1.2846</b> | <b>1.8205</b> | <b>1.7910</b> | <b>1.6096</b> | <b>1.4421</b> | <b>1.2639</b> | <b>1.1304</b> | <b>1.0172</b> |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 1.4288        | 2.2237        | 2.1112        | 1.8327        | 1.5763        | 1.3223        | 1.1755        | 1.0601        |
| VAR(1)                | 1.4518        | 2.1676        | 2.1216        | 1.8724        | 1.6052        | 1.3432        | 1.2247        | 1.1443        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 1.7366        | 2.6599        | 2.7189        | 2.2960        | 1.9352        | 1.5867        | 1.3687        | 1.2031        |
| VAR(1)                | 1.4434        | 2.2349        | 2.2215        | 1.8617        | 1.5697        | 1.3134        | 1.1719        | 1.0575        |
| <i>12-month ahead</i> |               |               |               |               |               |               |               |               |
| RW                    | <b>1.6021</b> | <b>2.7816</b> | <b>2.6965</b> | <b>2.2652</b> | <b>1.9541</b> | <b>1.6219</b> | <b>1.3709</b> | <b>1.1469</b> |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 1.7627        | 2.9441        | 2.8743        | 2.5470        | 2.1959        | 1.7936        | 1.5467        | 1.3433        |
| VAR(1)                | 2.0715        | 3.1036        | 3.1287        | 2.9305        | 2.6128        | 2.2222        | 1.9877        | 1.7892        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 2.1102        | 3.1460        | 3.1605        | 2.8826        | 2.6375        | 2.3286        | 2.0619        | 1.8145        |
| VAR(1)                | 2.1425        | 3.3672        | 3.3561        | 2.9380        | 2.6441        | 2.3070        | 2.0203        | 1.7582        |

*Notes:.* This table summarizes the overall trace root mean squared prediction errors (TRMSPE) and the root mean squared prediction errors (RMSPE) for the random walk (RW); the first-order univariate autoregressive model of yield (Y-AR(1)); the first-order multivariate autoregressive model of yield (Y-VAR(1)); the dynamic Nelson-Siegel model (DNS); the dynamic semiparametric factor model (DSFM) and the principal component analysis model (PCA). The results are made for subperiod 2003 to 2006. For each model, the RMSFEs are reported for 6-month and 1-, 2-, 3-, 5-, 7- and 10-year maturities, and for 1-, 3-, 6- and 12-month-ahead horizon. Bold numbers indicate the best performing model.

six-month bond yield at one-month ahead horizon.

Comparing the AR(1) and VAR(1) specifications, the AR(1) still demonstrates better forecasting performance for the sub-sample period, while the VAR(1) specification with the Nelson-Siegel model only provides better results for the 1-month ahead forecast. This evidence also confirms that the AR(1) specification outperforms the VAR(1) for sub-sample estimation as documented before. The results also suggest that both models have performed better at the longer maturity, except for the 1-month horizon, which provides more accurate forecast at both short (6-month) and long (10-year) maturities.

The results discussed above are shown in Figure 3.20 and 3.21 for the forecasting results by the AR(1) and VAR(1) specification over the period 2006-2009 at the 1-month ahead horizon. In this subsequent period, the yield curve experienced a structural break. From 2006 onwards, yields persistently increased and reached the peak before the eruption of the global financial crisis at the end of the year 2008. Then, they sharply declined during the distress period and up-swung back after the economy started to recover at the second-half of the year 2009. Figure 3.20 plots the outcome for the DSFM and the Nelson-Siegel model with an AR(1) specification.

As shown in Figure 3.20, the DSFM with an AR(1) specification outperforms the Nelson-Siegel in forecasting the persistent upward trend from the beginning of the sub-sample period until the outbreak of the global financial crisis. Unfortunately, it fails to track the yield curve by providing a somewhat lag in time series of the yields since the 2008 crisis. This forecasting pattern causes the estimation to have higher RMSPE than those produced during the period prior to the crisis. The unpromising results appear to become worse when the VAR(1) specification is undertaken as presented in Figure 3.21.

From Figure 3.21, the Nelson-Siegel model with VAR(1) specification forecasts the term structure is more precise than the DSFM with VAR(1) specification. The DSFM is more volatile and overstates the forecasted yield around the turning



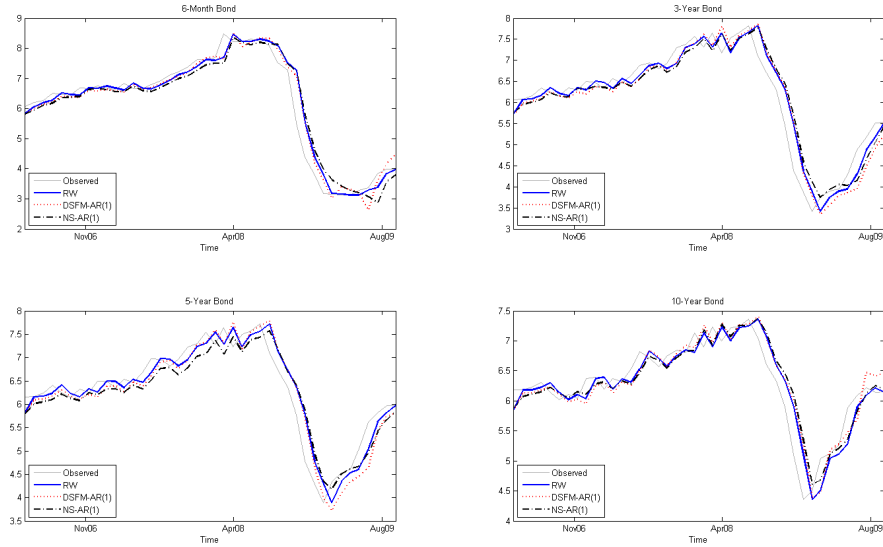


FIGURE 3.20: Observed and 1-month ahead predicted yields with AR(1) specification for the subperiod 2006 to 2009

*Notes:* This figure provides plots of the observed and 12-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

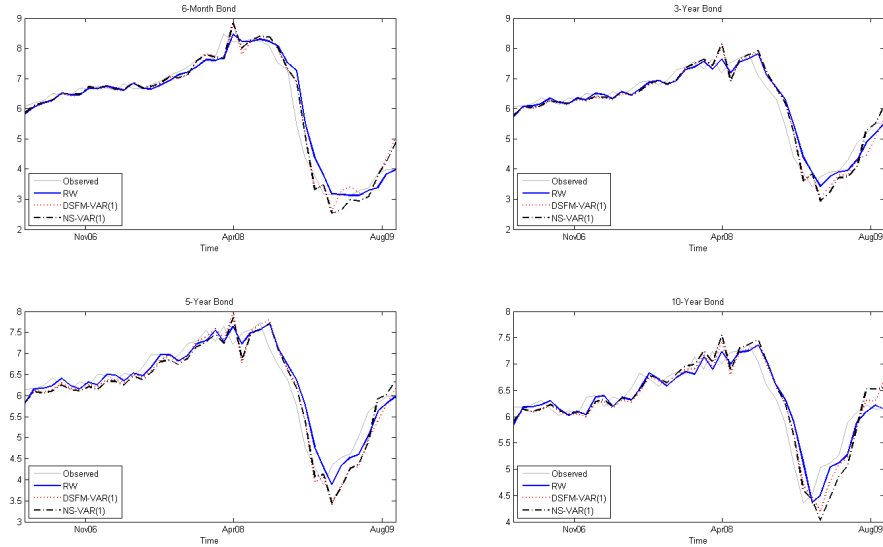


FIGURE 3.21: Observed and 1-month ahead predicted yields with VAR(1) specification for the subperiod 2006 to 2009

*Notes:* This figure provides plots of the observed and 12-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

point. Both models also suffer from the structural break after the crisis by producing lag values that miss the yield dynamic.

### 3.8.6.3 Sub-sample 2009-2013

This sub-sample period is the ex-post crisis era and therefore, there is no structural break. The prediction performance from individual models can be used to examine the robustness by comparing with the first sub-sample over the period from 2003 to 2006, which does not cover the 2008 global financial crisis. From the results, we are also able to evaluate the structural breaks effect on the term structure forecast when the crisis is included.

The RMSPE for the DSFM and the Nelson-Siegel model, together with their competitors are reported in Table 3.16. The absolute size of the prediction errors found the post-crisis are much less than those reported during the crisis. This result appears to be the reason that the term structure forecast tends to outperform once it encounters the structural break. In spite of that, the prediction errors in this sub-sample period are as quite small in magnitude as the pre-crisis figures.

As reported the prediction accuracy for this sub-sequent period in Table 3.16, the AR(1) process is still better to specify the dynamic latent factors for each sub-sample period, rather than the VAR(1) specification. The AR(1) process gives better forecasting performance in producing lower RMSPE, particularly for the 3-month, 6-month and 12-month ahead; but not for 1-month ahead. Another key result from this sub-sequent period is that the DSFM with AR(1) specification still outperforms the Nelson-Siegel model to predict 3-month and 6-month ahead forecast as it does for the pre-crisis period. Its 6-month maturity yield prediction also does a better job than the random walk at 3-month and 6-month ahead forecast. Interestingly, the DSFM with AR(1) specification presents better performance beyond the Nelson-Siegel counterpart for 12-month ahead predictions, even it still fails to overcome the Nelson-Siegel in anticipating the one-month ahead yield. The important point is that the DSFM with AR(1) is suitable for predicting yield in the period without a structural break and in particular, for the longer step ahead.

TABLE 3.16: Out-of-sample forecasts for the period September 2009 to March 2013

|                       | TRMSPE        | RMSPE         |               |               |               |               |               |               |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|                       | all           | 6-m           | 1-y           | 2-y           | 3-y           | 5-y           | 7-y           | 10-y          |
| <i>1-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | <b>0.2392</b> | <b>0.1521</b> | <b>0.2237</b> | <b>0.2743</b> | <b>0.2654</b> | 0.2461        | 0.2387        | 0.2399        |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.2731        | 0.1961        | 0.2525        | 0.3119        | 0.3338        | 0.2753        | 0.2660        | 0.2708        |
| VAR(1)                | 0.2582        | 0.2199        | 0.2903        | 0.2930        | 0.3002        | 0.2625        | <b>0.2384</b> | <b>0.2325</b> |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.2623        | 0.1923        | 0.2498        | 0.3097        | 0.2691        | 0.2784        | 0.2557        | 0.2633        |
| VAR(1)                | 0.2682        | 0.2942        | 0.2981        | 0.3022        | 0.2682        | 0.2802        | 0.2453        | 0.2347        |
| <i>3-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | <b>0.4158</b> | 0.3017        | <b>0.3614</b> | <b>0.4554</b> | <b>0.4555</b> | <b>0.4469</b> | <b>0.4389</b> | <b>0.4353</b> |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.6239        | 0.4786        | 0.5624        | 0.6804        | 0.7172        | 0.6621        | 0.6509        | 0.6447        |
| VAR(1)                | 0.5691        | 0.6947        | 0.6723        | 0.6459        | 0.6380        | 0.5589        | 0.5213        | 0.5069        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.5267        | <b>0.2731</b> | 0.4173        | 0.5154        | 0.5577        | 0.6000        | 0.5808        | 0.5675        |
| VAR(1)                | 0.5765        | 0.7234        | 0.6867        | 0.6049        | 0.6214        | 0.5969        | 0.5347        | 0.4928        |
| <i>6-month ahead</i>  |               |               |               |               |               |               |               |               |
| RW                    | <b>0.6041</b> | <b>0.5090</b> | <b>0.5423</b> | <b>0.6327</b> | <b>0.6478</b> | <b>0.6764</b> | <b>0.6785</b> | <b>0.6712</b> |
| <b>NS</b>             |               |               |               |               |               |               |               |               |
| AR(1)                 | 1.1203        | 0.9402        | 1.0529        | 1.2097        | 1.2726        | 1.2555        | 1.2338        | 1.1996        |
| VAR(1)                | 1.1169        | 1.1104        | 1.1736        | 1.2992        | 1.3213        | 1.2127        | 1.1606        | 1.1134        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 0.9130        | 0.4713        | 0.7556        | 0.9283        | 0.9842        | 1.0789        | 1.0608        | 1.0236        |
| VAR(1)                | 1.1131        | 1.0966        | 1.2135        | 1.2615        | 1.3005        | 1.2465        | 1.1542        | 1.0641        |
| <i>12-month ahead</i> |               |               |               |               |               |               |               |               |
| RW                    | <b>0.8779</b> | <b>0.8014</b> | <b>0.9351</b> | <b>0.9862</b> | <b>0.9895</b> | <b>1.0997</b> | <b>1.0775</b> | <b>1.0253</b> |
| <b>DNS</b>            |               |               |               |               |               |               |               |               |
| AR(1)                 | 1.4249        | 2.2016        | 1.9212        | 1.7832        | 1.7136        | 1.5510        | 1.4880        | 1.4148        |
| VAR(1)                | 1.4943        | 1.8502        | 1.8409        | 1.8918        | 1.8685        | 1.7306        | 1.6579        | 1.5762        |
| <b>DSFM</b>           |               |               |               |               |               |               |               |               |
| AR(1)                 | 1.3535        | 2.2788        | 2.0958        | 1.7495        | 1.5548        | 1.4211        | 1.2965        | 1.1890        |
| VAR(1)                | 1.7991        | 2.6403        | 2.6258        | 2.3777        | 2.2217        | 2.0101        | 1.8088        | 1.6179        |

*Notes:.* This table summarizes the overall trace root mean squared prediction errors (TRMSPE) and the root mean squared prediction errors (RMSPE) for the random walk (RW); the first-order univariate autoregressive model of yield (Y-AR(1)); the first-order multivariate autoregressive model of yield (Y-VAR(1)); the dynamic Nelson-Siegel model (DNS); the dynamic semiparametric factor model (DSFM) and the principal component analysis model (PCA). The results are made for subperiod 2003 to 2006. For each model, the RMSFEs are reported for 6-month and 1-, 2-, 3-, 5-, 7- and 10-year maturities, and for 1-, 3-, 6- and 12-month-ahead horizon. Bold numbers indicate the best performing model.

Even so the DSFM and the Nelson-Siegel provide lower prediction errors than those figures produced during the crisis period. The RMSPE in this sub-sample period are not quite similar to the figures from the pre-crisis period. This result can be clearly seen from Figure 3.22 and 3.23 that illustrate the forecasted with the actual yields across the sub-sample period.

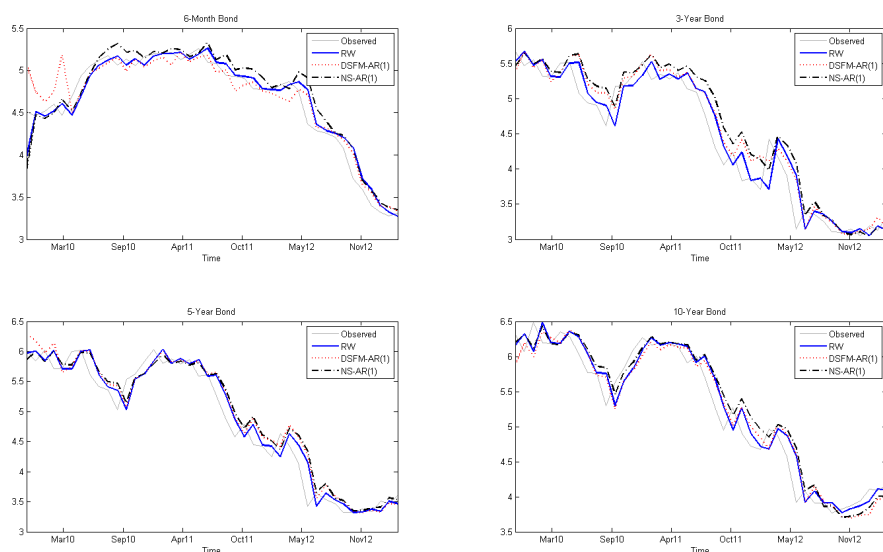


FIGURE 3.22: Observed and 1-month ahead predicted yields with AR(1) specification for the subperiod 2009 to 2013

*Notes:* This figure provides plots of the observed and 12-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

At the beginning of the sub-sequent period, long-term yields start at high levels that produce a wide spread over the the short-term yield due to higher risk premium after the aftermath of the global financial crisis. However, the European sovereign debt that emerged from 2009 raised risk premia and induced higher yields for a few months during 2010 to 2011. The uncertain economic recovery for the European Union and other advanced economies caused the market to expect falling interest rates and produced a downward trend of the term structure from 2011 onwards. Figure 3.22 plots the yield forecast with AR(1) specification for the one-month ahead forecast over the post-crisis period. From the plots, the DSFM with AR(1) specification reflects the persistent evolution of the yield well and produces lower prediction errors as compared to the Nelson-Siegel. Nevertheless, it is still difficult for the AR(1) specification to track the actual yield when

there is a sharp decline in the term structure curve. As depicted in the figure, the DSFM with AR(1) misses the actual yield and predicts the yield that lags behind, it though does better than the Nelson-Siegel. The prediction performance of the individual model with VAR(1) specification compared with the actual yield is presented in Figure 3.23.

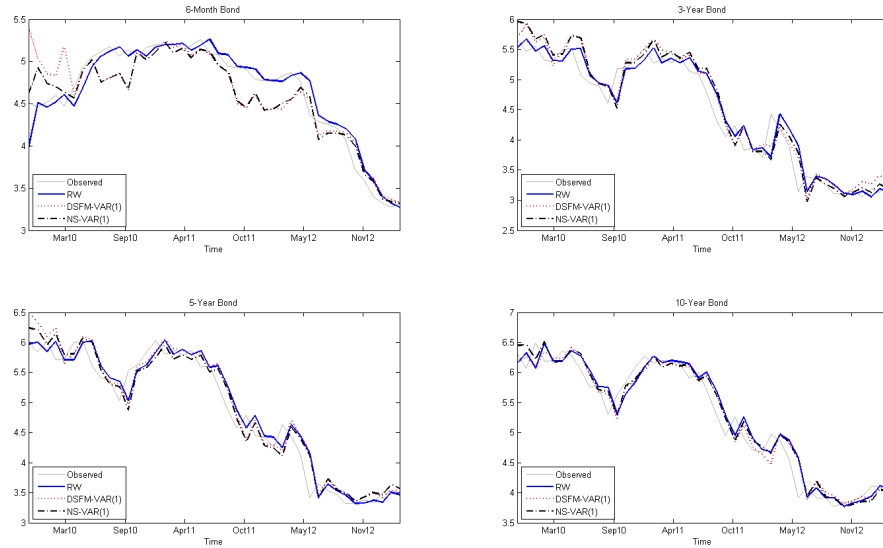


FIGURE 3.23: Observed and 1-month ahead predicted yields with VAR(1) specification for the subperiod 2009 to 20013

*Notes:* This figure provides plots of the observed and 12-month ahead predicted time series for the 6-month, the 3-, the 5- and 10-year maturities. The observed yields are plotted by gray solid lines, whereas blue solid, green dotted, red dash-dotted, and pink dashed lines correspond to predictions of the random walk (RW), DSFM with VAR(1), NS with VAR(1) and PCA with VAR(1) model, respectively.

In Figure 3.23, there is more evidence that confirms the drawbacks of the VAR(1) specification in producing the overstated or understated results at the turning point. For the one-month ahead prediction, the DSFM as well as the Nelson-Siegel model overstates the yield forecast during 2010 and over the time from 2011 to 2012 for only the 6-month bond, in particular. Comparing both models, the Nelson-Siegel model with VAR(1) specification provides a more accurate forecast than the DSFM counterpart.

#### 3.8.6.4 Sub-sample forecasting performance

In summary, the results of the sub-sample analysis show the strong forecast performance of the random walk over the DSFM, the dynamic Nelson-Siegel and the principal component model. The forecasting ability of the DSFM, the dynamic Nelson-Siegel and other competitive models varies during subperiods of study. In particular, for the first sub-sample period from 2003 to 2006, the dynamic Nelson-Siegel model with VAR(1) provide a more accurate prediction as compared to the DSFM. Once the economy had experienced the global financial crisis in the second sub-sample period from 2006 to 2009, the dynamic Nelson-Siegel model with VAR(1) still outperforms the DSFM. However, the DSFM with an AR(1) specification provide better forecast compared to the dynamic Nelson-Siegel model with VAR(1) in the third sub-sample period from 2009 to 2013. Comparing these two models, the dynamic Nelson-Siegel model seems to predict more accurate results during the period of uncertainty with high volatility or sudden structural change, while the DSFM with AR(1) specification better suits term structure forecasting once the yield behaves persistently.

### 3.9 Conclusion

This paper examines the in-sample fit estimation accuracy and out-of-sample forecasting performance of the dynamic semiparametric factor model (DSFM) and the Nelson-Siegel model. The DSFM allows the linking of two classes of methods widely used in financial econometrics; the dynamic factor model and nonparametric estimation. The combination of these features recovers the dynamic structures in curves observed over time without assuming a priori functional forms. We use the B-spline function to estimate smooth factor loadings and then construct a parsimonious semiparametric factor model. To provide a better explanation on yield variation, we use a 3-factor specification instead of only 2 factors as in [Härdle and Majer \(2012\)](#). By doing this, we can interpret the latent factors as level, slope and curvature as [Diebold and Li \(2006b\)](#) did. Our methodology provides means to build a parsimonious dynamic factor model for even high-dimensional time series panels and with factors that can be given a clear economic interpretation. The term structure estimation of the DSFM shows it is superior to the Nelson-Siegel

in producing accurate in-sample fit.

We then compare the prediction accuracy of the DSFM and the dynamic Nelson-Siegel model with the competitors; the principal component model, the autoregressive of yield-level and the random walk model along the maturity spectrum for horizons of 1-month, 6-month, 3-month and 12-month ahead. Our main results can be summarized as follows. The random walk forecasts more accurately than the DSFM, the dynamic Nelson-Siegel model and other competitors, irrespective of any forecasting horizon. Considering between the DSFM and the dynamic Nelson-Siegel model for the entire period as well as several sub-sample periods, we find that the predictive ability of individual models varies over time considerably. The dynamic Nelson-Siegel model seems more accurate in sub-periods during the uncertainty about the future path of the yields; the widened term spread during 2002 to 2004 and the global financial crisis during 2007 to 2008. However, the DSFM with AR(1) specification does particularly well in the sub-period after the global financial crisis which represent a persistent downward trend in yields. The fact that different models forecast well in different sub-periods implies the performance of each model is contingent on the forecast horizon, maturity and period of the sample. The results from the [Giacomini and Rossi \(2010\)](#) fluctuation test also confirm that the uncertain environment from the widen term spread and the global financial crisis caused unstable predictability of the term structure forecasting.





## Chapter 4

# Term Structure Forecasting with a Business Conditions Index

### 4.1 Introduction

We propose to use the Sheen-Trueck-Wang (2014) small open economy business conditions index (as in [Sheen et al. \(2014\)](#)) for term structure modelling and forecasting. This index is the real-time index with a mixed frequency data that captures change in market expectation and provide relevant macroeconomic information for term structure dynamics. Our models are the extension of dynamic factor class of term structure models; the dynamic Nelson-Siegel and the dynamic semiparametric factor model (DSFM), to include business condition index. The estimation is based on a monthly time series of Australian government zero coupon bonds for different maturities together with other macro variables and business condition index from March 1999 to April 2013. We find the in-sample fit estimation that incorporates business condition index is a few less accurate, but its cross-sectional yield at particular period provides guidance to anchor the yield in the next period. We also find that business condition index improve out-off-sample forecasting accuracy of the dynamic semiparametric factor model for medium and long-term maturity at one-month step ahead. Nonetheless, there is no statistically significant between the random walk, the model with factor only, the macro-finance model relative to the model with business condition index for all maturities and over multistep ahead based on the [Diebold and Mariano \(1995\)](#) test. We also find the prediction performance of the model with business condition index can

be enhanced by using the index with more frequently release (daily, weekly) or more recent available (one-week and two-week lag) index. The predictability of the business conditions index also significantly outperform the two most common Australian survey based indicators; the the Melbourne/Westpac leading index and the Melbourne/Westpac consumer sentiment index.

Regarding to forward looking behavior of the yield curve, we find the slope factor significantly relate to the index. This suggests a change in overall economy or business cycle will affect the yield slope. The monetary authority is anticipated to implement a contractionary monetary policy once the economy is expected to be overheating. The yield slope is adjusted corresponding to forward-looking information about the state of economy. Accordingly, a decrease in the level and slope of the yield curve can be considered as a signal of the monetary stimulus to accommodate economy from recession. Thus, the business conditions index significantly improve associated with the expectation of economic recovery.

The remainder of the paper is organized as follows. Section 4.2 discusses the related literature on term structure modeling and forecasting with the inclusion of forward looking information. Section 4.3 provides details about the Sheen-Trueck-Wang business conditions index and theoretical background of the linkage between term structure and expectation. Section 4.4 explains the methodology to incorporate business conditions index with and the term structure model and forecasting technique. Section 4.5 reports the data used in this study and their corresponding descriptive statistics. Section 4.7 shows the estimation and forecasting forecasting results and Section 4.8 gives a conclusion.

## 4.2 Review of Literature

The term structure of interest rates at any moment contains information regarding interest rates that markets expect to prevail later on. This information is of tremendous interest to financial practitioners and policymakers alike. Policymakers carefully monitor this information to infer market-based expectations of future monetary policy and to gauge the effectiveness of their communications strategy.

For practitioners, the availability of accurate interest rate forecasts can be the key to a successful trading strategy. In recent years, a new generation of dynamic term structure models has focused on flexible term structure models, for example, the Nelson-Siegel model or the non-parametric or semi-parametric factor model. These models have shown considerable promise for capturing the entire term structure. However, it is still difficult for these model to provide accurate term structure forecasting. One main reason is that they disregard the relationships between macroeconomic variables and interest rates. Hence, the linkage between term structure evolution and expected path of monetary policy and macroeconomy is not captured. Another reason is that typical data samples used in a dynamic term structure estimation may be too short to represent a mean-reversion and it is hard to provide precise prediction. Given insufficient information, it would help to provide additional relevant information. [Ang et al. \(2007\)](#) and [Kim and Orphanides \(2012\)](#) proposed to incorporate the survey forecasts of short-term interest rates into the term structure models and concluded surveys are informative and facilitate the estimation of underlying term structure models. Studies such as [Mönch \(2008\)](#), [Füss and Nikitina \(2011\)](#), [Dijk et al. \(2013\)](#) and [Koopman and Van der Wel \(2013\)](#) have also shown that the term structure model that adds macroeconomic information, in particular, the macroeconomic factor-augmented vector-autoregression (FAVAR), is beneficial to improve forecasting performance.

Even though the survey are useful proxies for the market expectations implicit in the term structure that can supplement the available interest rate data for the estimation of a dynamic term structure model. However, it may be concerned about the reliability of the survey forecast information. [Chun \(2011\)](#) and [Francis and Hua \(2012\)](#) evaluated the survey-based bond yield forecasts by experts and found the predictive performance is quite poor, relative to statistical model and the naive random-walk. In this paper, we propose to tackle this problem by using the Sheen-Trueck-Wang small open economy business condition index. [Sheen et al. \(2014\)](#) followed [Aruoba et al. \(2009\)](#) to develop the business conditions index which is a mixed-frequency data indicator that combine relevant explanatory macroeconomic variables as the additional information for term structure forecasting. The business conditions index is the real-time indexes that capture changes in market expectations as well as movements in the macroeconomic variables. The business conditions index can be serves as a summary statistics of the information

market participants have received thus far about real activity.

## 4.3 Business Conditions Index and Term Structure

We propose to incorporate the business condition index information in term structure forecast. The basic idea is that this additional information are useful proxies for the market expectations implicit in the term structure at any point in time, they should be a rich source of information that can supplement the available interest rate data for the estimation of a dynamic term structure model. This should help improve the overall precision of the estimated parameters.

### 4.3.1 Business Conditions Index Estimation

Earlier studies in term structure forecast tried to improve forecasting accuracy by using survey information. Unfortunately, they did not yield encouraging conclusions regarding reliability of surveys. Moreover, the interest rate environment has changed from the widened term premium and financial distress during 2000s. Term structure forecast also suffer from the look-ahead bias that arise in the computation of forecast in n-step ahead horizon. One might expect the sophistication of the real-time indicator could help alleviate inaccurate forecast.

[Sheen et al. \(2014\)](#) extended the [Aruoba et al. \(2009\)](#) business conditions index for the closed economy to a small open economy. This index uses the Kalman filter in a dynamic factor model to measure economic activity from different frequencies, in particular high frequency. It represents economic activity in real time, so called now-casts, which provides accurate and timely forecasts of the economy. Since the business confidence plays a major role in driven the indicator, it therefore contains forward-looking information and predictive content that could remedy the look-ahead bias. In this part, we discuss the Sheen-Trueck-Wang small open economy business condition index. The indicator is derived from a dynamic factor

model and is extracted by using Kalman filter and maximum likelihood estimation.

#### 4.3.1.1 State space representation of the mixed frequency factor model

Assume that the macroeconomic variables are measured at high frequencies. Let  $F_t$  denote the unobserved state of the economy or dynamic factors extracted from a mixed frequency data set, and assume these factors evolves with autoregressive process with exogenous variables  $C_t$ . The state equation can be written as.

$$F_t = AC_t + RF_{t-1} + We_t \quad e_t \sim N(0, P) \quad (4.1)$$

where  $e_t$  is white noise with zero mean and  $P$  variance.

Let  $M_t$  denote the economic or financial variable, which depends linearly on  $F_t$  and also on various exogenous variables  $D_t$ . So, the measurement equation is represented by.

$$M_t = GD_t + BF_{t-1} + \eta_t \quad \eta_t \sim N(0, H) \quad (4.2)$$

where the  $\eta_t$  are contemporaneously and serially uncorrelated innovations with zero mean and  $H$  variance. Given the state-space system, we can estimate mixed frequency factors via maximum likelihood using Kalman filtering and prediction error decomposition .

#### 4.3.1.2 Signal extraction and Kalman filter

The Kalman filter and smoother are used to extract the common state using the above state-space representation. The parameters are estimated by maximizing the log-likelihood derived from the Kalman filter equations. The parameter estimates are used to compute the filtered estimate of the state of the economy, which

is further passed through the Kalman smoother to obtain the optimal latent state of the economy. In order to illustrate the use of the Kalman filter, let us write the above two equations as follows.

Let denote  $F_{t|t-1}$  as the predicted states and  $\Sigma_{t|t-1}$  as their variance at time  $t$  conditional on information up to time  $t - 1$ ,  $F_{t|t}$  and  $\Sigma_{t|t}$  as the updated values conditional on information at time  $t$ . Then, the Kalman filter algorithm is conducted by the following equations.

$$F_{t|t-1} = AC_{t|t-1} + RF_{t-1|t-1} \quad (4.3)$$

$$\Sigma_{t|t-1} = R\Sigma_{t-1|t-1}R' + WPW' \quad (4.4)$$

$$F_{t|t} = F_{t|t-1} + K_tv_t \quad (4.5)$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - K_tB\Sigma'_{t|t-1} \quad (4.6)$$

$$K_t = \Sigma_{t|t-1}B'(\sigma^2 + B\Sigma_{t|t-1}B')^{-1} \quad (4.7)$$

$$v_t = (M_t - GD_t - BF_{t|t-1}) \quad (4.8)$$

where  $K_t$  is the Kalman gain and  $v_t$  is the prediction error when using the previous period value of the state. The predicted states  $F_{t|t-1}$  and their corresponding variance  $\Sigma_{t|t-1}$  are made according to the state and measurement equation and the update the predictions given by the newly arrived data at time  $t$ .

The Kalman filter remains valid if the missing data exist in  $M_t$ . We skip updating and the recursion becomes.

$$F_{t|t} = F_t + K_tv_t \quad (4.9)$$

$$\Sigma_{t|t} = \Sigma_t - K_tB\Sigma'_{t|t-1} \quad (4.10)$$

And the measurement equation can be replaced by.

$$M_t^* = GD_t^* + BF_{t-1}^* + \eta_t^* \quad \eta_t^* \sim N(0, H_t^*) \quad (4.11)$$

where  $M_t^*$  is of dimension  $N^* < N$ , containing the elements of the  $M_t$  vector that are observed. The  $M_t^*$  and  $M_t$  are linked by the transformation  $M_t^* = W_t M_t$  where  $W_t$  is a matrix whose  $N^*$  rows are the rows of  $I_N$  corresponding to the observed elements of  $M_t$ . Similarly,  $F_t^* = W_t F_t$ ,  $D_t^* = W_t D_t$ ,  $\eta_t^* = W_t \eta_t$  and  $H_t^* = W_t H_t W_t'$ . In this case, the Kalman filter works the same as described above.

#### 4.3.1.3 Maximum likelihood estimation

The above state space model can be estimated using the Kalman filter. Moreover, maximum likelihood estimation can be carried out using the so called prediction error decomposition method. The log-likelihood ( $L_t$ ) of the model can be evaluated from the prediction error  $v_t$ . Denote the variance of the prediction error as  $\Psi = H + B\Sigma_{t|t-1}B'$ . The log-likelihood is given by.

$$\log L_t = -\frac{1}{2}((N \log 2\pi + \log |\Psi| + v_t \Psi^{-1} v_t')) \quad (4.12)$$

where  $N$  is the number of observations at time  $t$ . We estimate the vector of factors and the hyper-parameters by maximizing the likelihood, equivalently to minimize the prediction errors,  $v_t$ . Given initial conditions, the likelihood is built iteratively. Hyper-parameters are chosen to maximize the likelihood, and then simply plugs the estimates into the system and construct the estimated latent factor.

As can be seen from the estimation method above, it is a challenging task to combine several macroeconomic variables into one index to capture the overview of economic situation. Macroeconomic data are typically low and mixed frequencies released and may not be measured consistently. The business conditions index is used to extract an unobserved common factor from the underlying observed macroeconomic time series with mixed frequencies. The indices are constructed by applying Kalman filter technique that optimizes the updated information on estimation without concerning any missing data. Hence, this index can summarize large amounts of information in an efficient way, such as the ability to incorporate missing and mixed-frequency data and also the ability to update real time

information with the most recently available data. Due to these useful properties, the Sheen-Trueck-Wang index can be used in nowcasting and forecasting other variables of interest, including yield term structure.

### 4.3.2 Empirical Business Conditions Index Results and Statistics

In this subsection, we provide more details which data sources that [Sheen et al. \(2014\)](#) used to construct the Sheen-Trueck-Wang business conditions index. Then, we present the empirical business conditions index.

#### 4.3.2.1 Data and estimation

[Sheen et al. \(2014\)](#) follow [Aruoba et al. \(2009\)](#) to use Kalman filter to construct real-time indicator with mixed frequency factors for Australian economy. Their study use five Australian data series from the Reserve Bank of Australia (RBA); the yield spread between a 10 year Australian Treasury bond and a 3 month Treasury bill, hours worked, the NAB survey-based business confidence index, real GDP and job vacancies. There are also other four external economy data series; the real trade-weighted index from the Reserve Bank of Australia (RBA), term of trade from the Australian Bureau of Statistics (ABS), export-weighted world real GDP index from Thomson-Reuter Datastream and the TED spread between the 3 month US Treasury bill rate and its LIBOR rate from Federal Reserve Economic Data (FRED). These variables are key variables representing the Australian economy and are chosen to construct the index similar to the Aruoba-Diebold-Scotti index. Despite the Sheen-Trueck-Wang index follow quite closely to [Aruoba et al. \(2009\)](#), the [Sheen et al. \(2014\)](#) study extend the index to a small open economy. They find that terms of trade plays a vital roles are played in explaining overall Australian economic activity.



#### 4.3.2.2 Empirical results

In this part, we present the Sheen-Trueck-Wang business conditions index and discuss its implication for term structure. The business conditions index is illustrated in Figure 4.1, compared with time series of the 6-month and 10-year bond yields.

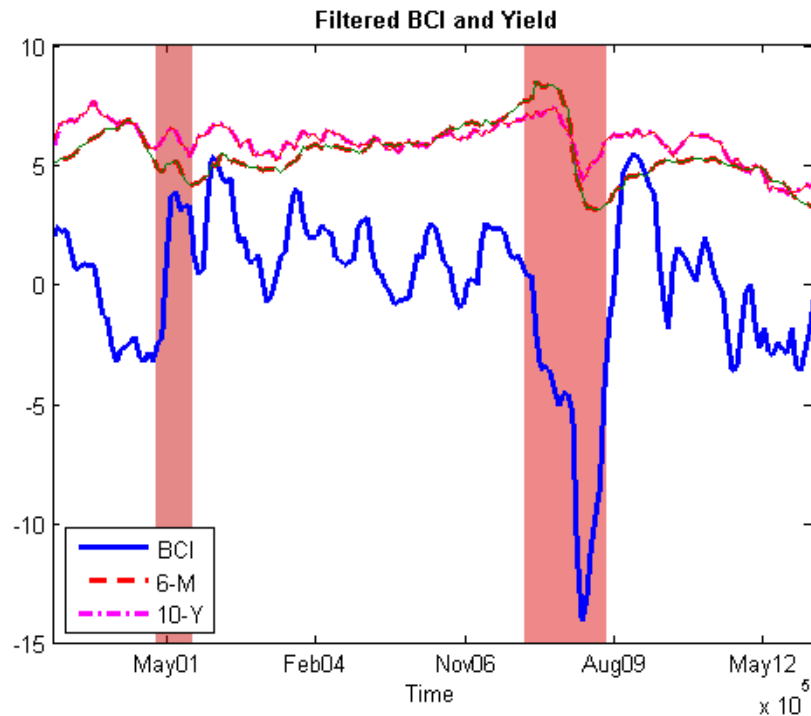


FIGURE 4.1: The business conditions index, short-term (6-month) yields and long-term (10-year) yields

To interpret the value of the business conditions index, we need to compare the index with zero. The positive values imply a better-than-average state of economy, while the negative values imply economic downturn. This index can be used as a leading indicator to predict whether the economy is going into a recession. As in Figure 4.1, the Sheen-Trueck-Wang index shows the two Australian recessions; the dotcom crash during the late 2000 and the global financial crisis after 2009. Therefore, the business conditions index is consistent with the observed Australian business cycle. Comparing with the short-term and long-term yields, there are some relationship between the yields and the business conditions index. The short-term and long-term yields explicitly follow the movement in index. Economic agents have to anticipate state of economy prior to make their decision. Their perception about the economy would evolve overtime as they realize some

additional information and take them into account. The term structure of the yield would mirror their expectation based on economic information with some lead times. As a result, the nowcasting and forward looking information from the business conditions index can be use as the information about expectations that drive the yield evolvement.

### 4.3.3 Term structure and expectation

In term structure literature, yield curve reflects the demand for higher yields to compensate with the risk associated to holding bond and other long-term financial asset as well as the opportunity cost of losing liquidity. The expectation of economic activity could therefore influence the term premium. On the early empirical studies by [Estrella and Hardouvelis \(1991\)](#) and [Estrella and Mishkin \(1998\)](#), they found the feedback from the macroeconomy to monetary policy will impact on the yield curve. [Ang and Piazzesi \(2003\)](#) and [Diebold and Li \(2006a\)](#) proposed the macro-finance term structure model to incorporate macroeconomic variables with the vector autoregressive model of yield latent factors and also found macroeconomic variable explain the variation of the yield. Thus, macroeconomic activity could attribute to yield term structure and a forward looking information should be beneficial for term structure estimation and forecasting.

The macro-finance term structure model typically link the term structure and the real economy by referring to the forward-looking monetary policy rule. The monetary authority selects an optimal short-term policy rate based on the predicted path of future economy. Indeed, the policy interest rate is consistent with investor expectation observed at the time that decision is made. As the macroeconomic variables evolve over time, term structure modeling and forecasting based on the most recent realized expectation are more accurate. [Orphanides and Wei \(2012\)](#) found that the term structure model that takes into account rational expectation should provide better explanation about inflation dynamics, monetary policy decision and generate term structure and macroeconomic forecast more consistent with survey data. In turn, additional information about the up-to-date economic situation can improve the accuracy of term structure model. In term structure literature, the expected return on holding bond must be equal to the risk-free

yield plus a risk premium. The attitude of the market participants towards risk is a forward-looking and thereby reflecting the link between term premium and perception about future economy. This relation can be written as the linearized expectation model that represent the relation between expected return on bond and term premium proposed by [Shiller \(1979\)](#).

$$E_t[Y_{t,j}] = r_t + \phi_{t,j} \quad (4.13)$$

where  $E_t[Y_{t,j}]$  is expected yield on bond with  $j$  time-to-maturity at time  $t$ ,  $r_t$  is the short term interest rate and  $\phi_{t,j}$  is a term premium defined for holding the bond with residual maturity.

The expected yield is not just only about the anticipation about short-term rate in the next period. It also implies about a risk premia or term premia that induce investors to hold long-term nominal bonds. Suppose the expected short rate for  $T$  year ahead years ahead and the  $T$ -year instantaneous forward rate given by  $f_{t,T}$  and the forward term premium  $\phi_{t,T}$  is

$$\phi_{t,T} = f_{t,T} - E_t[Y_{t+T}] \quad (4.14)$$

Expectations are unobserved and the estimation on them is not straight forward. Model misspecification may lead to substantial forecast errors. [Kim and Orphanides \(2012\)](#) and [Wright \(2011\)](#) use survey information to measure expected yield. The survey, for example the US Blue Chip survey, is conducted by asking the respondents to predict short-term interest rates. However, if there is no survey on interest rate expectations, they propose to impute the approximate expected interest rate from forecast on inflation and real GDP growth instead. The link between risk premiums and perceptions about future economic outcomes reflects a forward-looking manner.

## 4.4 Term structure modeling and forecasting with business condition index

In this part, we describe the exponential-polynomial Nelson-Siegel model and the modified Nelson-Siegel approach proposed by [Diebold and Li \(2006b\)](#). [Nelson and Siegel \(1987\)](#) proposed to fit the forward rate curve, and thus yields or spot rates, from observed coupon-bond prices at a given date with a flexible, smooth parametric function. They demonstrated that their proposed model is capable of capturing many of the typically observed shapes that the yield curve assumes over time. As the Nelson-Siegel model is widely used among academia, policy maker practitioners and it is ranked as one of the most popular term structure estimation methods.

### 4.4.1 The Nelson-Siegel Parametric Model

[Nelson and Siegel \(1987\)](#) suggest to fit the forward rate curve at a given date with a class of prespecified parametric functions. The functional form they advocate uses Laguerre functions which consist of the product between a polynomial and an exponential decay term. The resulting Nelson-Siegel approximating forward curve can be assumed to be the following three-factor term structure model.

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t}e^{-\lambda_t\tau} + \beta_{3,t}\lambda_t e^{-\lambda_t\tau} \quad (4.15)$$

To obtain the yield (or spot rate)  $y_t$  on a zero-coupon bond with  $\tau$  periods to maturity, it is necessary to take the equally weighted average of the forward rates.

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \quad (4.16)$$

where  $y_t(\tau)$  is the spot-rate curve with  $\tau$  time to maturity, and  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$  are latent factor parameters, which in dynamic form are referred to as level, slope and curvature and  $\lambda_t$  is referred to the exponential decay parameter.

The three latent factor parameters are corresponding to the factor loading components on these parameters. The factor loading on the  $\beta_{1,t}$  parameter is 1, as

this is a constant, it does not decay to zero and will be the same for all maturities. So, this long term factor  $\beta_{1,t}$  is independent of time to maturity and for that reason it is often interpreted as the long-run yield level. The factor loading that is weighted  $\beta_{2,t}$  on represents the short-term factor with a factor loading of  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}$ . This function starts at one and decays exponentially to zero if time to maturity  $\tau$  grows. Therefore, the corresponding latent factor is often denoted as slope factor.  $\beta_{3,t}$  is also weighted by a function depending on time to maturity  $\tau$ . This function  $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}$ , starts at zero and when the time to maturity  $\tau$  grows it initially increases and then decreases back to zero. Hence this component creates a hump and so it is often denoted as the medium-term component. The  $\lambda_t$  parameter is an exponential decay parameter that determines the rate at which the regressor variables decay to zero. Small values for  $\lambda_t$  result in a slow decay and better fit longer maturities, large values of  $\lambda_t$  will result in fast decay and better fit the curve with short maturities. In addition, the  $\lambda_t$  parameter also governs where the factor loading reaches its maximum.

[Diebold and Li \(2006b\)](#) provide insight to how these three factors representing the long, short and medium components can also be interpreted as the level, slope and curvature of the curve. The factor loading on the long term component  $\beta_{1,t}$  is 1 and the same for all maturities, any increase in  $\beta_{1,t}$  will cause the whole curve to shift upwards and thus it can be seen that this factor represents the level of the curve. The short term factor  $\beta_{2,t}$  can be viewed as the slope of the curve, an increase in  $\beta_{2,t}$  will cause the short rates to increase more than long rates as the short rates load more heavily on  $\beta_{2,t}$ , thus changing the slope of the curve. Finally the medium factor is closely related to the curvature of the curve, as both long and short term maturities do not load heavily on it, but an increase in  $\beta_{3,t}$  will increase the curve for medium maturities and so increasing the curvature of the curve.

Applying ordinary least squares to the yield data for each particular period gives a time series of the estimates of latent factors  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$ . [Diebold and Li \(2006b\)](#) and [Diebold et al. \(2006\)](#) proposed a dynamic term structure of the Nelson-Siegel model by specifying first-order autoregressive processes for the latent factors. [De Pooter \(2007\)](#) generalized the dynamic Nelson-Siegel model as the dynamic latent factor model, given by the Nelson-Siegel model and the dynamic

process of the latent factors.

The Nelson-Siegel model

$$Y_t(\tau) = X_t(\tau)\beta_t \quad (4.17)$$

The stochastic process of the latent factor

$$\beta_t = \mu + \Phi\beta_{t-1} \quad (4.18)$$

The first dynamic factor equation above specify the vector of yields, which contains  $T$  different maturities. The Nelson-Siegel yield curves are those discussed in the previous subsections with  $\beta_t$  being the vector of factors and  $X_t$  as the matrix of factor loadings, given by the estimated decay parameter  $\lambda$ .

#### 4.4.2 The dynamic semiparametric factor model

The dynamic semiparametric factor model (DSFM) provides a general method for modeling and forecasting time series data that captures dynamic evolution of the high-dimensional time series by a non-parametrically estimated lower-dimensional factor. It has the ability to flexibly fit variety shapes of the cross-sectional data while providing time-varying factors that describe dynamics of the time series. This method was proposed by [Fengler et al. \(2007\)](#) on the implied volatility surface study. The detailed discussion on the dynamic semiparametric factor model specification are given below.

Semiparametric regression imposes some structure but the regression function is still not directly predetermined. However, the structure of the model leaves less flexibility than in the nonparametric case. One of the motivation for creating this limitation, comes from the curse of dimensionality, since in high dimensions the nonparametric methods may become infeasible.

Among many possible semiparametric models, we focus on the imposition of the additive property as in [Härdle \(2004\)](#), [Fengler et al. \(2007\)](#) and [Härdle and Majer](#)

(2012). The key assumption is that the regression function has an additive structure of the explanatory variable coordinates. The actual yields are supposed to be linear combination of high dimensional latent factors. By proposing a suitable statistical model results in the problem of finding an appropriate way of reducing the high dimension without losing too much information on the spatial and dynamic structure of the process. A common way to reduce the dimensionality of multivariate processes is to apply factor decomposition. For instance, a  $J$ -dimensional vector of yield observations  $Y_t = (Y_{t,1}, Y_{t,j})$  can be represented as an  $L$ -factor model.

$$Y_{t,j} = \sum_{l=0}^L Z_{t,l} m_{l,j}(X_{t,j}) + \epsilon_{t,j} \quad (4.19)$$

where  $Y_{t,j}$  are the yield obtained by holding a bond at time  $t$  to time-to-maturity  $j$ ,  $Z_{t,l}$  are latent factors of the factor  $l$  at date  $t$ ,  $m_{l,j}(X_{t,j})$  are undetermined smooth function, or so called the basic function, that characterizes loading of factor  $l$  given time-to-maturity  $j$ ,  $X_{t,j}$  are maturity-related variables representing bond yield characteristic at date  $t$  and  $\epsilon_{t,j}$  are errors which explain the residual part. The index  $t$  represents time evolution as  $t, \dots, T$  and index  $j$  is a number of bonds with different maturities  $j, \dots, J$  observed at that time. The corresponding yield curve can be shown in a  $J$ -dimensional vector of yields  $Y_{t,j} = (y_{t,1} \dots y_{t,J})'$ . This high dimension of the cross sectional  $J$  bonds can be reduced to a smaller number of factors  $L \ll J$ . The dynamics of yield through time is then explained by the time propagation of the  $L$  factors and can be estimated through the evolution of the latent factors  $Z_{t,l}$ . The latent factors reflect bond yield characteristics associated with factor-loadings.

This representation assumes existence of comovements among all component of  $Y_{t,j}$ , which are driven by unobservable factors  $Z_{t,l}$ . The yield latent factors  $Z_{t,l}$  can be treated as time series. A usual way is to assume that these processes are first-order autoregressive processes, represented by

$$Z_t = \Phi Z_{t-1} + \omega_t \quad (4.20)$$

where  $Z_{t,l}$  is the yield latent factor,  $m_{l,j}(X_{t,j})$  is a factor loading with deterministic maturity-related variables  $X_{t,j}$ ,  $\Phi$  are parameter matrices and  $\epsilon_{t,j}$  and  $\omega_t$  are

random components independent of each other. As dynamics of the factors is incorporated, these above representation are called the dynamic factor model.

#### 4.4.3 Incorporating business conditions index

The yield curve is closely related to other macroeconomic variables. In terms of term structure forecast, it reflects economic agent and policy maker beliefs about economy. As we discussed before, the business conditions index contains real-time state of economy as well as forward-looking information. It should adapt quickly to the volatility in business cycle and structural breaks, making it a useful instrument for measuring trends. From this perspective, we may consider to link them to predict the yield term structure.

A straight forward approach is to treat the business conditions index as an exogenous variable. In the dynamic semiparametric factor model, we extract latent factors and are these factors are assumed to follow an first-order autoregressive process. Then, we extend the multivariate autoregressive process (VAR) with business conditions index. The specification that incorporate the business conditions index can be written as.

$$Z_{t+h,j} = \Xi_{t,l} + \Phi_{t,0}M_{t,0} + \Phi_{t,l}Z_{t,l} + \nu_{t,l} \quad (4.21)$$

where  $M_{t,0}$  is the business conditions index. We can then use this autoregressive process with business conditions index as exogenous variable to forecast the latent factors and then forecast the term structure of yields. The advantage of this model is the possibility to use information about the economy to forecast the term structure.



## 4.5 Macroeconomic variables and business conditions index statistics

Before getting into the model, this section presents information about the data used in this study. We first provide details of the data sets and then report the descriptive statistics of the data.

### 4.5.1 Data Description

In this part, we describe the data on the yields, the business conditions index and the macroeconomic data that we used in the paper. The data we use are monthly spot rates for zero-coupon bonds of the Australian government bonds, provided by Thomson Reuters Datastream. This data set consists of 11 maturities (in months); 6, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120, over the period from April 1999 to March 2013. Concerning the macroeconomic variables, we use monthly data for capacity utilization from National Australian Bank's Australian business surveys (seasonally adjusted); the overnight interbank rate from the Reserve Bank of Australia and implied inflation rate, calculated from the difference between the 5-year Australian commonwealth government bond yield and the Australian commonwealth government indexed bond yield. All of macro-variables are gathered from Thomson Reuters Datastream. These three macroeconomic variables represent real economic activity relative to potential output, the policy interest rate as monetary policy instrument, and the change in price level. All of these variables are widely used to augment the term structure model as the macro-finance model to capture macroeconomic dynamics.

TABLE 4.1: Overview of Macroeconomic Series

| Variables                 | Frequency                     | Acronym | Transformation       |
|---------------------------|-------------------------------|---------|----------------------|
| Capacity Utilisation Rate | Monthly (Seasonally Adjusted) | CAPU    | First-difference     |
| Overnight Interbank Rate  | Monthly                       | INT     | Log-first-difference |
| Implied Inflation Rate    | Monthly                       | INF     | First-difference     |
| Business Conditions Index | Daily                         | BCI     | No                   |

We also extend the term structure model with business conditions index to investigate the way in which macroeconomic information containing in the index can be used to improve yield curve estimation and forecasting. We apply the Sheen-Trueck-Wang business conditions index and compare the estimation accuracy as well as forecasting performance of the term structure model that includes business conditions index with the macro-finance model. The summary statistics for the macroeconomic variables are reported in Table 4.1.

## 4.5.2 Descriptive statistics

For the empirical study, we presents summary statistics for our macroeconomic variables and the business condition index in Table 4.2 and plot the time series of them in Figure 2.4. For each of the macroeconomic series, we report the mean, standard deviation, minimum, maximum, autocorrelation coefficient at various displacements and as well as the Augmented Dickey-Fuller test statistics for stationarity.

TABLE 4.2: Descriptive statistics of macroeconomic variables and BCI

| Variables | Mean   | Std Dev | Min      | Max    | p(1)   | p(12)   | p(30)   | ADF      |
|-----------|--------|---------|----------|--------|--------|---------|---------|----------|
| CAPU      | 0.8132 | 0.0127  | 0.7824   | 0.8463 | 0.8597 | 0.3091  | 0.0625  | -0.1989* |
| INT       | 5.1951 | 1.1172  | 2.9500   | 7.9600 | 0.9691 | 0.1266  | -0.0145 | -0.8171* |
| INF       | 2.4464 | 0.7036  | 1.1174   | 4.2262 | 0.9551 | 0.2894  | -0.0211 | -0.5832* |
| BCI       | 0.0200 | 3.2594  | -14.0216 | 5.5571 | 0.8933 | -0.1701 | 0.0854  | -2.7895  |

The summary statistics reveal that the business conditions index and the overnight interbank rate are noticeably vary over time. The standard deviations for the business conditions index is around 3.26. It is also clear that all macroeconomic variables are highly persistent at the first lag, especially the overnight interbank rate, but varies relative to its mean. The augmented DickeyFuller tests suggest that the capital utilization, the overnight interbank rate and the implied inflation rate are non-stationary. In order to obtain stationary series, we then transform the monthly recorded of the capital utilization, the interest rate and the implied inflation rate to ensure stationarity by using first difference on the capital utilization and the implied inflation rate as well as log differences on the interest rate.

The empirical statistics are also consistent with Figure 4.2. We plot the capital utilization, the overnight interbank interest rate, the implied inflation rate and the business conditions index from April 1999 to March 2009. All macroeconomic variables move reasonably together and the business conditions index tracks these macro-variables closely with mean-reverting. There was an upward trend for these macroeconomic series after 2001, however, all macroeconomic variables turned to be more volatile after the global financial crisis since 2008 onwards.

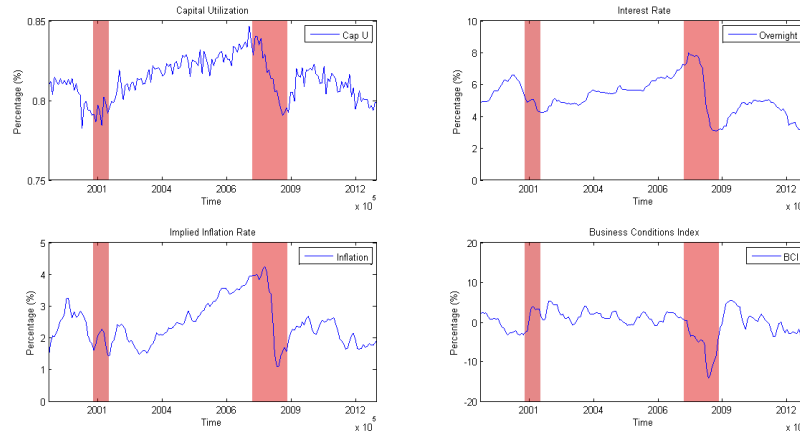


FIGURE 4.2: Time series of macroeconomic variables; capital utilization, interest rate, inflation and business conditions index

Comparing between the business conditions index and other macroeconomic variables, change in index indicates subsequent economic cycle. From Figure 4.2, the business conditions index leads the consecutive movement in macroeconomic series. It should be beneficial to use the index to extract forward looking information from large data sets of many mixed frequencies variables, and thus deliver forecasting gains. The business conditions index also preserve the real-time nature that is crucial in estimating and nowcasting the recent or near future macroeconomic variables and term structure of the yields as well.

To investigate the correlation among macro variables and relationship between the business conditions index and macro variables, we estimate correlation coefficients among them and reported in Table 4.3.

From Table 4.3, inflation is strongly correlated with interest rate, following by relation with capital utilization. Correlation between interest rate and capital

TABLE 4.3: Correlation coefficient

|             | <b>CAPU</b> | <b>INT</b> | <b>INF</b> | <b>BCI</b> |
|-------------|-------------|------------|------------|------------|
| <b>CAPU</b> | 1.0000      | 0.6453     | 0.7047     | 0.2903     |
| <b>INT</b>  | 0.6453      | 1.0000     | 0.8244     | 0.0861     |
| <b>INF</b>  | 0.7047      | 0.8244     | 1.0000     | 0.1533     |
| <b>BCI</b>  | 0.2903      | 0.0861     | 0.1533     | 1.0000     |

utilization is somewhat smaller even highly correlated. On the contrary, the relationship between business conditions index the and macro variables are weak. As mentioned in [Sheen et al. \(2014\)](#), the consumer confidence index and term of trade play a major role to explain the index rather than real economic activity or inflation.

## 4.6 Model estimation with BCI

In this section, we present the results of term structure estimation with the inclusion of business conditions index. First, we present two term structure models that incorporates forward looking information from the business conditions index. Then, we evaluate the estimation accuracy and compare the estimated latent factors with empirical factors and macroeconomic variables, particularly the business conditions index.

### 4.6.1 Estimation and model specification

To examine the accuracy of the dynamic semiparametric factor model and the Nelson-Siegel model augmented with the business conditions index relative to the baseline models without compared to other competitors, we first estimate a simple autoregressive model of the dynamic factor as a baseline model. Next, we add the business conditions index to the baseline model to see to what extent its predictive power is improved by this index. By looking at the baseline model compared with the extension model based on the index could explicitly show how the extension model react with the additional forward looking information is taken into account.

We illustrate the results of the in-sample-fit for the augmented business conditions index with the dynamic semiparametric factor model relative to baseline model against the actual yields at some particular days in Figure 4.3 and the results for the Nelson-Siegel model in Figure 4.4. We chose to plot the yield curves for selected dates; which are 29 February 2000, 31 March 2004, 29 September 2006 and 30 November 2009, to capture different term structure shapes.

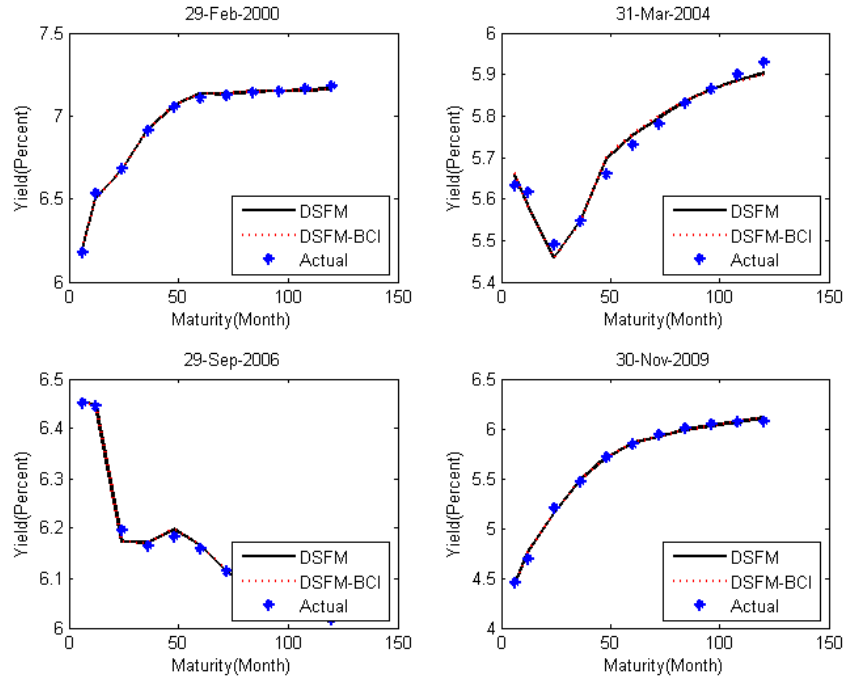


FIGURE 4.3: Cross-sectional term structure estimation by the dynamic semiparametric factor model

As shown in Figure 4.3, the dynamic semiparametric factor model works well to fit the actual yields information, especially the J-curve type at 31 March 2004 and the downward sloping with two-hump shape at 29 February 2000. The inclusion of the business conditions index, represented by the dash line, also provide accurate fit, similar to the simple model without the index. However, the model with the supplement information from the index tends to shift the curve of the simple model towards the trend. For example, the yield curve for 3-year (36 months) to 10-year (120 months) maturity at date 31 March 2004 is tilted up along the upward trend after including the index. In contrast, the downward trend for shorter maturity or 6-month to 2-year (24 month) shift part of the curve downward corresponding to lower realized yields.

For the upward sloping yield curve at 29 February 2000 and 30 November 2009, it is clearly seen that the model with the business condition index supplementation shift the entire curve upward associated with higher long-term yields. This implies the economic agents incorporate the forward looking information about the overheating economy and inflationary pressure. The anticipation about an increasing policy interest rate cause the yield estimated from the index to be hike relative to the baseline model. The response of the model with index cause it to be less accurate for cross-sectional term structure estimation.

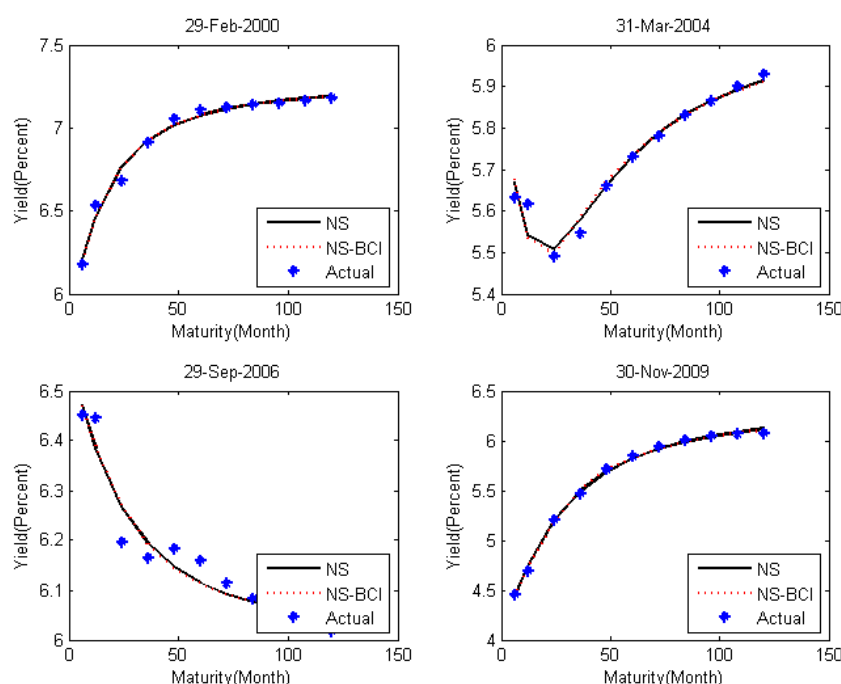


FIGURE 4.4: Cross-sectional term structure estimation by the Nelson-Siegel model

In Figure 4.4, we observe the Nelson-Siegel model is not flexible enough to fit more complex curves, especially the J-curve and the curve with two-hump shape. The estimation accuracy become worsened when the business conditions index is included. As compared to the dynamic semiparametric factor model, the errors are much more pronounced for the the Nelson-Siegel model with an extension of the index. For example, the yield with index at 30 November 2009 overestimate the baseline Nelson-Siegel model, particularly for the medium term maturity (3-year to 5-year bond). The tilted curve from the model with index reflects an anticipation of contractionary monetary policy to stabilize economy from higher expected

inflation.

From the in-sample-fit term structure estimation with the business conditions index, we find the additional information from the index is useful to guide the near term yield since it normally move towards the expectation. Therefore, the business conditions index itself should not be use to improve estimation accuracy of the term structure model. [Kim and Orphanides \(2012\)](#) who used survey data to model the term structure suggested that term structure model based on forward looking information provide the expectation on the yields. Rather than using forward looking information to estimate the term structure, [Altavilla et al. \(2013\)](#) recommended to use this expectation information as an anchor to forecast the term structure. In our study, we use the business conditions index instead of survey data as additional forward looking information to forecast the term structure. If the business conditions index is informative to improve latent factors dynamics, it should produce more accurate forecasting performance as compared to the baseline model without the index.

#### 4.6.2 In-sample-fit

In this subsection, we assess whether the business conditions index provides information beyond that already captured by the latent factors in the baseline model by comparing the trace root mean square error (TRMSE), the root mean square error (RMSE) and the explained variation (EV) of the simple model without the business conditions index and the model with the inclusion of index.

TABLE 4.4: Out-of-sample forecasts of the DSFM with business confidence index compared with other competitors

|                 | TRMSE  | 6-m    | 1-y    | 2-y    | 3-y    | 5-y    | 7-y    | 10-y   | EV     |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| <b>NS</b>       | 0.0477 | 0.0439 | 0.0830 | 0.0641 | 0.0494 | 0.0499 | 0.0129 | 0.0417 | 0.9978 |
| <b>NS-BCI</b>   | 0.0498 | 0.0456 | 0.0841 | 0.0696 | 0.0518 | 0.0503 | 0.0132 | 0.0431 | 0.9976 |
| <b>DSFM</b>     | 0.0398 | 0.0454 | 0.0680 | 0.0512 | 0.0421 | 0.0413 | 0.0125 | 0.0350 | 0.9984 |
| <b>DSFM-BCI</b> | 0.0403 | 0.0458 | 0.0685 | 0.0513 | 0.0428 | 0.0413 | 0.0126 | 0.0361 | 0.9984 |

As shown in Table 4.4, the dynamic semiparametric factor model with the business conditions index extension achieves 99.84 percentage of the explained variation in

term structure curves while the Nelson-Siegel model with the index can explain 99.76 percentage of the term structure variance. Considering with the estimation accuracy measured by the trace root mean square error (TRMSE), the dynamic semiparametric factor model augmented by the business conditions index also outperform the Nelson-Siegel model with the index. The dynamic semiparametric factor model specification that is more flexible and less-restricted makes it outperform the Nelson-Siegel specification in providing more precise in-sample estimation. This finding is consistent with other studies, including [De Pooter \(2007\)](#), [Koopman et al. \(2010\)](#) and [Laurini and Hotta \(2010\)](#) who also find the nonparametric or semiparametric is beneficial for term structure modeling. It should be noted that the inclusion of the business conditions index actually deteriorate the estimation accuracy provided by the baseline models. Forecasting errors produced by models with the index are higher than those of the baseline model without the index. These results confirm the findings we shown in the previous subsection.

### 4.6.3 Yield latent factor, macro variables and corresponding empirical

Next, we compare the latent factors from the dynamic semiparametric factor model and the Nelson-Siegel model. These factors are extracted from the yield estimation and then plotted through the time period from 1999 to 2013 in [Figure 4.5](#) to [4.6](#). The estimated value for the factors are standardized to make it more easy for comparison. The latent factors obtained from the model are presented together with related macroeconomic variables and their empirical proxies. By following [Diebold and Li \(2006a\)](#), the level factor is close to the 120 month yield, the slope is close to spread of 3 month over 120 month yields and the curvature is close to the 24 month yield minus the 3 and 120 month yield. Moreover, the level factor is linked with inflation expectation as suggested by the Fisher equation while the slope factor appears to related with business cycle, represented by capacity utilization. These relations also mentioned in term structure literatures, including [Hördahl et al. \(2006\)](#) and [Dewachter and Lyrío \(2006\)](#). For the curvature factor, it should be related with expectation about macroeconomy as noted in [Orphanides \(2003\)](#) and [Mönch \(2012\)](#). We proposed to use the business conditions index as a macroeconomic variable to represent the anticipation about future economy. The



time series of the latent factors, macroeconomic variables and their empirical proxies, estimated by the dynamic semiparametric factor model and the Nelson-Siegel model are depicted in Figure 4.5 and Figure 4.6 respectively.

Comparing the graphs for latent factors extracted from two different models, we observe these latent factors follow similar pattern as the empirical factors. The level factor is more persistent relative to other two factors, while the slope and curvature are more volatile. The evolution of the factors are also related with macroeconomic situation. In macroeconomic context, we find the level factor is closely related to inflation and consistent with the inflationary expectations hypothesis. The anticipation about higher inflation shifts up the long end of the yield curve up and also drive up short rates as well as expected interest rates. It suggests that the shift of the long end and the shape of the yield curve. Therefore, the variation in inflation expectation is captured by the level factor.

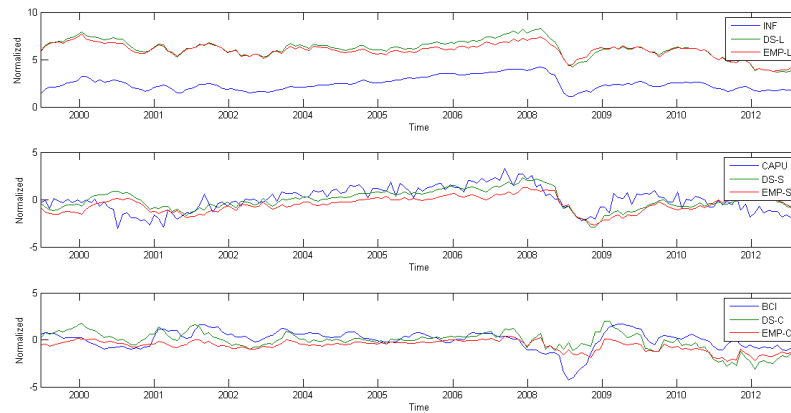


FIGURE 4.5: The dynamic semiparametric factor model latent factors, macro variables and corresponding empiricals

Moreover, there is also a positive relation between the slope factor and macroeconomic activity represented by the capacity utilization. The plots of slope factors are closely related to the Australian business cycle throughout the periods of study. Obviously, there is a declining slope factor that causes the Reserve Bank of Australia to accommodate lower interest rate in response to economic downturn during the global financial crisis in 2008 to 2009. During the expansionary monetary policy, we observe a downward trend in the long rates and the slope of the yield curve. As a result, the greater fall in long rates was larger than the decline

in short rates and alter the yield curve shape.

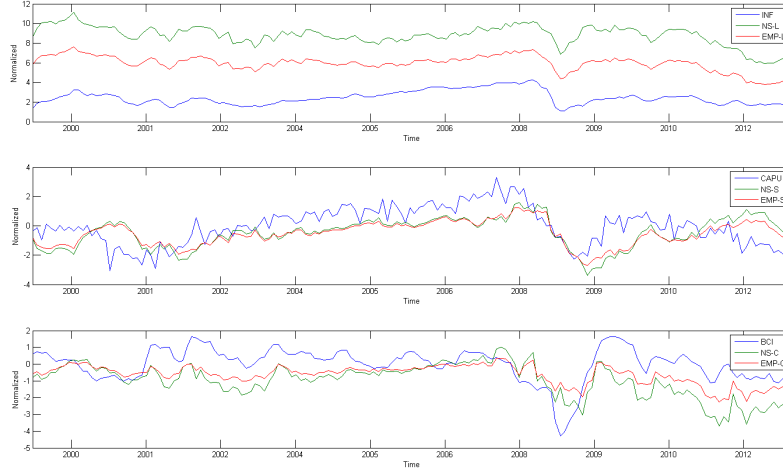


FIGURE 4.6: The Nelson-Siegel model latent factors, macro variables and corresponding empiricals

Although the level and slope factors are found to be related with inflation and output, the curvature is still ignored by term structure studies, such the recent work by [Ullah et al. \(2013\)](#), to search for the linkage with empirical macroeconomic variables. Nonetheless, [Mönch \(2012\)](#) found that unexpected changes of the curvature factor are more informative about the future evolution of the yields and macroeconomy. We propose to link the curvature factor with the business conditions index due to the fact that it represents forward looking information and expectation about state of economy. From [Figure 4.5](#) and [Figure 4.6](#), there is a strong relationship between the curvature factors and the business conditions index. Prior to the global financial crisis, the curvature factor appear to decline and signal the upcoming recession before the onset of the crisis. It also anticipates the recovery earlier than other latent factors. This finding is consistent with the pattern of the business conditions index that represents forward looking information and leads the movement of other macroeconomic variables.

## 4.7 Term structure forecast with business condition index

We examine the predictive ability of the dynamic semiparametric factor model (DSFM) with business confidence index in a rolling window out-of-sample forecasting experiment using the zero coupon bond yield and the Sheen-Trueck-Wang business conditions index for Australia. We conduct rolling window examination for the period from April 2006 to March 2013. The forecasts are made for yields of all maturities, at 1-month, 3-month, 6-month and 12-month horizons ahead. To assess the forecasting accuracy, we compute the root mean square forecast errors (RMSFE) for 6-month, 1-year, 2-year, 3-year, 5-year, 7-year and 10-year maturity as well as the overall trace root mean square forecast errors (TRMSFE) for individual model.

### 4.7.1 Forecast Procedure

We choose to evaluate the prediction accuracy of the term structure models on the basis of their out-of-sample forecasting performance for different yields. In this way, we will have a uniform ground to systematically compare models. We base our forecasting comparison exercise on a rolling window estimation with fixed size, in which parameters are re-estimated at each stage. This study divides the full data into the training period; April 1999 - March 2006 (84 observations) and the forecasting period; April 2006 - March 2013 (84 observations).

By doing this, it will allow us to compare how the dynamic semiparametric factor model, the Nelson-Siegel counterpart and other competitors perform in the normal and crisis period. All the models are estimated with a rolling window by moving the sample forward with a fixed sample size and re-estimating the model iteratively until the  $h$ -step ahead out-of-sample forecast is obtained. We consider four forecast horizons,  $h = 1$  month as well as 3, 6 and 12 months ahead.

### 4.7.2 Forecasting Accuracy Performance

To assess the prediction accuracy of the out-of-sample forecast of the dynamic semiparametric factor models and the Nelson-siegel model, we use a standard forecast error evaluation criteria. The predictive performance of the models are statistically evaluated by the root mean squared prediction error (RMSPE), which is widely used to assess forecasting accuracy of the models at particular maturities. We also compute the trace root mean squared prediction error (TRMSPE) of the models for all maturities as in [Hördahl et al. \(2006\)](#) and [De Pooter et al. \(2010\)](#). It combines the forecast errors of all maturities and summarizes the performance of each model, thereby allowing for a direct comparison between the models.

#### The Root Mean Squared Prediction Error (RMSPE)

Given a sample of  $T$  out-of-sample forecasts with  $h$ -months ahead forecast horizon, I compute the RMSPE for a  $\tau$  time-to-maturity as follows:

$$RMSFE(\tau) = \sqrt{\sum_{t=1}^T \frac{[Y_{t+h}(\tau) - \hat{Y}_{t+h}(\tau)]^2}{T}} \quad (4.22)$$

Where  $\hat{Y}_{t+h}$  is the forecasted yield in period  $t$  for  $t+h$  period and  $[Y_{t+h} - \hat{Y}_{t+h}]^2$  is the forecast errors at  $t+h$  for the yields.

#### The Trace Root Mean Squared Prediction Error (TRMSPE)

For each forecast horizon, the trace root mean squared prediction error (TRMSPE) measure the aggregate forecast errors of all yields in  $J$  maturities. Given a sample of  $T$  out-of-sample forecasts of  $J$  distinct maturities with  $h$ -months ahead forecast horizon, we compute the TRMSPE as follows.

$$TRMSFE = \sqrt{\sum_{j=1}^J \sum_{t=1}^T \frac{[Y_{t+h} - \hat{Y}_{t+h}]^2}{JT}} \quad (4.23)$$

The root mean squared prediction error (RMSPE) and the trace root mean squared prediction error (TRMSPE) for the dynamic semiparametric factor model and

the Nelson-Siegel model are reported for both the specifications of latent factors stochastic process; the AR(1) and VAR(1) for all forecasts horizons.

### 4.7.3 Forecasting Result

In this subsection, we present the empirical results of our forecasting exercises. In the first, we report the estimates of yield latent factor autoregressive process, the augmented dynamic factor model with macroeconomic variables and with business conditions index. Next, we investigate the out-of-sample forecasting performance of the term structure model driven by these different factor dynamics to see what extent their predictive power are improved. The multiple steps ahead forecasting experiments are conducted from rolling windows of the yields and macroeconomic time series. Then, we determine whether the differences in forecasting accuracy are statistically significant to examine the overall quality of forecasting. Finally, we assess the robustness of our forecasting results with different sub-samples and frequencies.

#### 4.7.3.1 VAR estimation and specification

Our goal in this study is to analyze how we use the business conditions indicator as an additional information for term structure modeling and forecasting. To estimate the dynamic term structure model, we specify the multivariate autoregressive process of the unobserved factors and then subsequently fit the dynamic latent factors into the term structure model to capture the yield curve evolution over the times. Following traditional macro-finance term structure model, we use yield latent factor and macroeconomic variables to estimate the vector autoregression model as also did in earlier studies by [Ang and Piazzesi \(2003\)](#), [Diebold and Li \(2006a\)](#), [Hördahl et al. \(2006\)](#) and [Rudebusch and Wu \(2008\)](#). Furthermore, we propose to use the business conditions indicator that summarizes macroeconomic information and compare its predictive performance. The results obtained from the vector autoregression model of the yield latent factors extracted from the Nelson-siegel model and the dynamic semiparametric factor model (DSFM) are reported in Table 4.5, whereas the vector autoregression estimates of the extension

model with macrocosmic variables (macro-finance model) are in Table 4.6 and the vector autoregression model of latent factors with business conditions indicator are in Table 4.7.

TABLE 4.5: Estimated vector autoregression model of the Nelson-Siegel and the DSFM dynamic factors

| NS        |               |               |                |               | DSFM      |                |               |               |               |
|-----------|---------------|---------------|----------------|---------------|-----------|----------------|---------------|---------------|---------------|
|           | CON           | Lt-1          | St-1           | Ct-1          |           | CON            | Lt-1          | St-1          | Ct-1          |
| <b>Lt</b> | <b>1.2337</b> | <b>0.8126</b> | <b>-0.0528</b> | <b>0.0585</b> | <b>Lt</b> | <b>0.6189</b>  | <b>0.8555</b> | 0.1317        | <b>1.4016</b> |
|           | 0.2615        | 0.0398        | 0.0184         | 0.0138        |           | 0.1885         | 0.0434        | 0.1065        | 0.3699        |
| <b>St</b> | -0.7114       | <b>0.1170</b> | <b>0.9678</b>  | 0.0174        | <b>St</b> | <b>-0.1356</b> | <b>0.0313</b> | <b>0.9035</b> | -0.0686       |
|           | 0.3917        | 0.0597        | 0.0275         | 0.0207        |           | 0.0559         | 0.0129        | 0.0316        | 0.1098        |
| <b>Ct</b> | -1.1846       | 0.1492        | -0.0135        | <b>0.8703</b> | <b>Ct</b> | -0.0133        | 0.0027        | -0.0229       | <b>0.8821</b> |
|           | 0.8420        | 0.1282        | 0.0591         | 0.0445        |           | 0.0316         | 0.0073        | 0.0178        | 0.0619        |

In Table 4.5, the estimates of factor coefficient from their own lags along the diagonal of transition matrix from both models are all close to unity, while the estimates of the off-diagonal elements are minimal. The yield slope factor is the most persistent, following by curvature and level factor. For the estimates from the dynamic semiparametric factor model specification, the level factor is significantly determined by the first lag of curvature and its own lag, while the first lag in slope factor of the Nelson-Siegel model also explains level factor. In addition, the slope factor significantly depend on lagged level factor and its lagged value for both models.

The results in Table 4.6 for the yield-macro models show that the level factor is significantly explained by its lagged value, other yield factors and inflation rate. This suggests that the yield curve is raised in response to inflationary pressure. In turn, change in level factors from previous period significantly causes lagged negative effects on inflation. The positive relation between the slope factor and interest rate in the Nelson-Siegel model is also statistically significant, while this relation in the dynamic semiparametric factor model (DSFM) is insignificant. The expectation on lower policy interest rate shifts the yield curve downward with greater negative value of slope factor. The curvature that generates a humped-shape term structure significantly related the a change in monetary policy interest rate in response to inflation. Finally, the economic activity, represented by capacity utilization, is significantly affected by regime change in yield term structure or slope factor.

TABLE 4.6: Estimated vector autoregression model of the DSFM dynamic factors, macro-variables and business conditions index

| NS           |                |                |                |               |                |                |               |
|--------------|----------------|----------------|----------------|---------------|----------------|----------------|---------------|
|              | CON            | Lt-1           | St-1           | Ct-1          | CAPUt-1        | INTt-1         | INFt-1        |
| <b>Lt</b>    | <b>1.1322</b>  | <b>0.8273</b>  | <b>-0.0378</b> | <b>0.0496</b> | 0.5229         | -0.3377        | <b>0.3637</b> |
|              | <i>0.2592</i>  | <i>0.0396</i>  | <i>0.0185</i>  | <i>0.0140</i> | <i>2.3339</i>  | <i>0.5549</i>  | <i>0.1241</i> |
| <b>St</b>    | -0.5599        | 0.0906         | <b>0.9680</b>  | 0.0065        | 1.5497         | <b>2.1362</b>  | -0.1898       |
|              | <i>0.3912</i>  | <i>0.0598</i>  | <i>0.0280</i>  | <i>0.0212</i> | <i>3.5232</i>  | <i>0.8377</i>  | <i>0.1873</i> |
| <b>Ct</b>    | -1.6465        | 0.2234         | 0.0290         | <b>0.8642</b> | 12.5848        | <b>-3.6948</b> | <b>1.0149</b> |
|              | <i>0.8353</i>  | <i>0.1276</i>  | <i>0.0597</i>  | <i>0.0452</i> | <i>7.5220</i>  | <i>1.7885</i>  | <i>0.3998</i> |
| <b>CAPUt</b> | 0.0089         | -0.0015        | <b>-0.0014</b> | 0.0004        | <b>-0.4898</b> | 0.0038         | 0.0012        |
|              | <i>0.0075</i>  | <i>0.0012</i>  | <i>0.0005</i>  | <i>0.0004</i> | <i>0.0678</i>  | <i>0.0161</i>  | <i>0.0036</i> |
| <b>INTt</b>  | 0.0083         | -0.0012        | -0.0065        | 0.0034        | 0.4965         | <b>0.2541</b>  | <b>0.0548</b> |
|              | <i>0.0397</i>  | <i>0.0061</i>  | <i>0.0028</i>  | <i>0.0021</i> | <i>0.3579</i>  | <i>0.0851</i>  | <i>0.0190</i> |
| <b>INFt</b>  | <b>0.4917</b>  | <b>-0.0771</b> | <b>-0.0371</b> | 0.0169        | 0.8077         | -0.0148        | <b>0.4368</b> |
|              | <i>0.1709</i>  | <i>0.0261</i>  | <i>0.0122</i>  | <i>0.0092</i> | <i>1.5389</i>  | <i>0.3659</i>  | <i>0.0818</i> |
| DSFM         |                |                |                |               |                |                |               |
|              | CON            | Lt-1           | St-1           | Ct-1          | CAPUt-1        | INTt-1         | INFt-1        |
| <b>Lt</b>    | <b>0.5148</b>  | <b>0.8788</b>  | 0.1367         | 0.9851        | 2.3777         | -0.3187        | <b>0.3700</b> |
|              | <i>0.1798</i>  | <i>0.0414</i>  | <i>0.0995</i>  | <i>0.3565</i> | <i>1.6051</i>  | <i>0.3813</i>  | <i>0.0853</i> |
| <b>St</b>    | <b>-0.1345</b> | <b>0.0310</b>  | <b>0.9062</b>  | -0.1356       | 0.7949         | 0.1431         | 0.0255        |
|              | <i>0.0558</i>  | <i>0.0128</i>  | <i>0.0309</i>  | <i>0.1106</i> | <i>0.4978</i>  | <i>0.1183</i>  | <i>0.0265</i> |
| <b>Ct</b>    | -0.0330        | 0.0072         | -0.0230        | <b>0.8635</b> | 0.3289         | <b>-0.1746</b> | <b>0.0379</b> |
|              | <i>0.0314</i>  | <i>0.0072</i>  | <i>0.0174</i>  | <i>0.0622</i> | <i>0.2799</i>  | <i>0.0665</i>  | <i>0.0149</i> |
| <b>CAPUt</b> | 0.0093         | -0.0022        | -0.0021        | 0.0264        | <b>-0.4903</b> | 0.0042         | 0.0012        |
|              | <i>0.0076</i>  | <i>0.0017</i>  | <i>0.0042</i>  | <i>0.0151</i> | <i>0.0678</i>  | <i>0.0161</i>  | <i>0.0036</i> |
| <b>INTt</b>  | 0.0041         | -0.0013        | -0.0162        | 0.1303        | 0.4964         | <b>0.2569</b>  | <b>0.0553</b> |
|              | <i>0.0402</i>  | <i>0.0092</i>  | <i>0.0222</i>  | <i>0.0796</i> | <i>0.3584</i>  | <i>0.0852</i>  | <i>0.0190</i> |
| <b>INFt</b>  | <b>0.4978</b>  | <b>-0.1145</b> | 0.0762         | <b>0.9497</b> | 0.7924         | 0.0030         | <b>0.4367</b> |
|              | <i>0.1724</i>  | <i>0.0397</i>  | <i>0.0954</i>  | <i>0.3418</i> | <i>1.5390</i>  | <i>0.3656</i>  | <i>0.0818</i> |

The estimates of yield latent factor corresponding to their lagged values are higher in magnitude as compared to the effect of macroeconomic variables on yield factors. However, some coefficients, particularly the effect of inflation on yield factors, are statistically significant and consistent with economic literature. As all diagonal elements and some off-diagonal elements in contemporaneous matrix are statistically significant, the inclusion of macroeconomic factors in the NelsonSiegel model as well as the dynamic semiparametric factor model (DSFM) specification should improve the estimation accuracy and prediction performance. This finding is consistent with previous macro-finance literature, including the recent studies by [Koopman and Van der Wel \(2013\)](#) and [Ullah et al. \(2013\)](#)

TABLE 4.7: Estimated vector autoregression model of the the dynamic semi-parametric factor model and the Nelson-Siegel model dynamic factors

| NS          |                |                |                |               |               |
|-------------|----------------|----------------|----------------|---------------|---------------|
|             | CON            | Lt-1           | St-1           | Ct-1          | BCIt-1        |
| <b>Lt</b>   | <b>1.2381</b>  | <b>0.8123</b>  | <b>-0.0556</b> | <b>0.0610</b> | -0.0036       |
|             | <i>0.2614</i>  | <i>0.0398</i>  | <i>0.0190</i>  | <i>0.0146</i> | <i>0.0067</i> |
| <b>St</b>   | -0.7431        | <b>0.1186</b>  | <b>0.9873</b>  | -0.0008       | <b>0.0258</b> |
|             | <i>0.3842</i>  | <i>0.0585</i>  | <i>0.0280</i>  | <i>0.0215</i> | <i>0.0099</i> |
| <b>Ct</b>   | -1.2166        | 0.1508         | 0.0062         | <b>0.8520</b> | 0.0261        |
|             | <i>0.8388</i>  | <i>0.1277</i>  | <i>0.0611</i>  | <i>0.0469</i> | <i>0.0216</i> |
| <b>BCIt</b> | <b>2.2911</b>  | <b>-0.3817</b> | <b>-0.3581</b> | 0.0724        | <b>0.9130</b> |
|             | <i>1.1162</i>  | <i>0.1699</i>  | <i>0.0813</i>  | <i>0.0624</i> | <i>0.0288</i> |
| DSFM        |                |                |                |               |               |
|             | CON            | Lt-1           | St-1           | Ct-1          | BCIt-1        |
| <b>Lt</b>   | <b>0.6161</b>  | <b>0.8560</b>  | 0.1354         | <b>1.2603</b> | 0.0063        |
|             | <i>0.1875</i>  | <i>0.0431</i>  | <i>0.1060</i>  | <i>0.3833</i> | <i>0.0048</i> |
| <b>St</b>   | <b>-0.1378</b> | <b>0.0317</b>  | <b>0.9064</b>  | -0.1780       | <b>0.0049</b> |
|             | <i>0.0540</i>  | <i>0.0124</i>  | <i>0.0305</i>  | <i>0.1103</i> | <i>0.0014</i> |
| <b>Ct</b>   | -0.0133        | 0.0027         | -0.0229        | <b>0.8813</b> | 0.0000        |
|             | <i>0.0316</i>  | <i>0.0073</i>  | <i>0.0178</i>  | <i>0.0645</i> | <i>0.0008</i> |
| <b>BCIt</b> | <b>2.2557</b>  | <b>-0.5270</b> | -0.6688        | <b>5.5384</b> | <b>0.9150</b> |
|             | <i>1.1247</i>  | <i>0.2588</i>  | <i>0.6359</i>  | <i>2.2994</i> | <i>0.0286</i> |

In Table 4.7, we propose to use the business conditions index instead of a group of macroeconomic variables. The results also reveal that the yield latent factors and the business conditions index are highly persistent and all diagonal elements of the transition matrix are statistically significant. The inclusion of the index is significantly explained the slope factor. This suggests a change in overall economy or business cycle will affect the yield slope. The monetary authority is anticipated to implement a contractionary monetary policy once the economy is expected to be overheating. The yield slope is adjusted corresponding to forward-looking information about the state of economy. Accordingly, an decrease in the level and slope of the yield curve can be considered as a signal of the monetary stimulus to accommodate economy from recession. Thus, the business conditions index significantly reverses with a change in policy interest rate.

Regarding the statistically significant of the relationship between the business conditions index and slope factor together with the significant impact of yield factors on the business conditions index, we expect the extension of yield factor vector autoregressive model to include the business conditions index should be



an alternative way to incorporate macroeconomic information into term structure model. The forward-looking and mixed frequencies information that contains in the business conditions index should improve the prediction performance of the term structure model relative to the traditional yield-macro model.

#### 4.7.3.2 Out-of-sample forecasting results

Next, we examine the out-of-sample prediction performance of the term structure model with business conditions index in comparison to the model with only yield latent factor and the model augmented with macroeconomic variables. The forecasts are computed for the period from April 2006 to March 2013. We conduct forecasting exercise in multiple steps ahead, covering  $h = 1, 3, 6$  and 12 month ahead, and compute the root mean square forecast errors (RMSFE) for each model and each maturity as well as the overall trace root mean square forecast errors (TRMSFE) for individual model. The forecasting results are reported in Table 4.8 for all forecasting horizons. The leftmost column reports the trace root mean square forecast errors (TRMSFE) and the other columns on the right are the root mean square forecast errors (RMSFE) for 6-month, 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The first row records forecast errors produced by the benchmark random walk.

As reported in Table 4.8, the forecasts produced by the dynamic semiparametric factor model with business conditions index supplement at the one-month ahead provide lower root mean square error (RMSPE) than those with yield latent factor only and with macro variables. However, for nearly all maturities in all  $n$  step ahead, the random walk provides the lowest error amongst those obtained by other models.

For one-month ahead forecast, incorporating business conditions index as an additional source of information improves forecasts for the dynamic semiparametric factor model with the first order vector regressive model, especially for maturities of more than 2 years. This implies a support from additional information contained in the index to enhance predictability of the dynamic semiparametric factor model relative to the model without index and the model with the use of

TABLE 4.8: Out-of-sample forecasts of the dynamic semiparametric factor model and the Nelson-Siegel model with business confidence index compared with other competitorsx

|                       | TRMSE  | 6-m    | 1-y    | 2-y    | 3-y    | 5-y    | 7-y    | 10-y   |
|-----------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| <b>1-month ahead</b>  |        |        |        |        |        |        |        |        |
| RW                    | 0.2932 | 0.3054 | 0.3403 | 0.3324 | 0.3118 | 0.2866 | 0.2667 | 0.2654 |
| <b>DSFM</b>           |        |        |        |        |        |        |        |        |
| VAR(1)                | 0.3304 | 0.3108 | 0.3768 | 0.3996 | 0.3744 | 0.3289 | 0.2945 | 0.2932 |
| VARMAC(1)             | 0.3334 | 0.3195 | 0.3723 | 0.3923 | 0.3633 | 0.3272 | 0.3039 | 0.3143 |
| VARBCI(1)             | 0.3268 | 0.3247 | 0.3772 | 0.3878 | 0.3608 | 0.3223 | 0.2915 | 0.2928 |
| <b>NS</b>             |        |        |        |        |        |        |        |        |
| VAR(1)                | 0.3142 | 0.2891 | 0.3773 | 0.3561 | 0.3408 | 0.3201 | 0.2867 | 0.2741 |
| VARMAC(1)             | 0.3072 | 0.2853 | 0.3586 | 0.3475 | 0.3335 | 0.3130 | 0.2832 | 0.2704 |
| VARBCI(1)             | 0.3195 | 0.2870 | 0.3726 | 0.3425 | 0.3483 | 0.3297 | 0.2975 | 0.2842 |
| <b>3-month ahead</b>  |        |        |        |        |        |        |        |        |
| RW                    | 0.6364 | 0.7534 | 0.7814 | 0.7389 | 0.6802 | 0.6177 | 0.5711 | 0.5401 |
| <b>DSFM</b>           |        |        |        |        |        |        |        |        |
| VAR(1)                | 0.7276 | 0.8036 | 0.8965 | 0.8437 | 0.8086 | 0.7431 | 0.6537 | 0.5747 |
| VARMAC(1)             | 0.7021 | 0.8100 | 0.8856 | 0.8155 | 0.7701 | 0.7044 | 0.6220 | 0.5532 |
| VARBCI(1)             | 0.7181 | 0.8228 | 0.8918 | 0.8240 | 0.7832 | 0.7246 | 0.6436 | 0.5760 |
| <b>NS</b>             |        |        |        |        |        |        |        |        |
| VAR(1)                | 0.7119 | 0.8380 | 0.8876 | 0.8311 | 0.7808 | 0.7006 | 0.6313 | 0.5773 |
| VARMAC(1)             | 0.7015 | 0.7992 | 0.8625 | 0.8057 | 0.7681 | 0.6946 | 0.6339 | 0.5868 |
| VARBCI(1)             | 0.9026 | 0.9289 | 1.0599 | 1.0350 | 1.0127 | 0.9164 | 0.8345 | 0.7720 |
| <b>6-month ahead</b>  |        |        |        |        |        |        |        |        |
| RW                    | 1.0255 | 1.3198 | 1.3160 | 1.2257 | 1.1101 | 0.9990 | 0.9159 | 0.8438 |
| <b>DSFM</b>           |        |        |        |        |        |        |        |        |
| VAR(1)                | 1.2123 | 1.5700 | 1.6083 | 1.4505 | 1.3410 | 1.1927 | 1.0593 | 0.9322 |
| VARMAC(1)             | 1.2414 | 1.6649 | 1.6928 | 1.5000 | 1.3749 | 1.2063 | 1.0562 | 0.9122 |
| VARBCI(1)             | 1.2052 | 1.4645 | 1.5185 | 1.4153 | 1.3334 | 1.2190 | 1.0996 | 0.9764 |
| <b>NS</b>             |        |        |        |        |        |        |        |        |
| VAR(1)                | 1.1586 | 1.5148 | 1.5052 | 1.3972 | 1.2841 | 1.1223 | 1.0178 | 0.9258 |
| VARMAC(1)             | 1.2022 | 1.5611 | 1.5687 | 1.4647 | 1.3431 | 1.1648 | 1.0504 | 0.9499 |
| VARBCI(1)             | 1.5464 | 2.0269 | 2.0093 | 1.8803 | 1.7282 | 1.4957 | 1.3481 | 1.2159 |
| <b>12-month ahead</b> |        |        |        |        |        |        |        |        |
| RW                    | 1.3653 | 1.9582 | 1.9346 | 1.7009 | 1.5049 | 1.3551 | 1.2205 | 1.0904 |
| <b>DSFM</b>           |        |        |        |        |        |        |        |        |
| VAR(1)                | 1.6539 | 2.2890 | 2.3183 | 2.0554 | 1.8782 | 1.6728 | 1.4880 | 1.3135 |
| VARMAC(1)             | 1.5533 | 2.1130 | 2.1467 | 1.9147 | 1.7548 | 1.5831 | 1.4190 | 1.2638 |
| VARBCI(1)             | 1.5850 | 2.1781 | 2.2066 | 1.9654 | 1.7964 | 1.6014 | 1.4355 | 1.2785 |
| <b>NS</b>             |        |        |        |        |        |        |        |        |
| VAR(1)                | 1.5709 | 2.2382 | 2.1820 | 1.9891 | 1.7812 | 1.5396 | 1.4024 | 1.2825 |
| VARMAC(1)             | 1.6273 | 2.3650 | 2.3040 | 2.0833 | 1.8512 | 1.5737 | 1.4228 | 1.2971 |
| VARBCI(1)             | 1.8675 | 2.6546 | 2.6159 | 2.3819 | 2.1234 | 1.8251 | 1.6566 | 1.5084 |

macro variables. The index can also improve the prediction performance of the Nelson-Siegel model for shorter maturity, in particular 6-month to 2-year maturity. Actually, all specification of the Nelson-Siegel dominates either the dynamic semiparametric factor model and even the random walk in providing more accurate forecast. The improvement made by the index is also pronounced when we include it into the Nelson-Siegel model even though the macro variable produce more accurate results.

The results for the three-month ahead forecast horizon are quite similar to those for the 1-month horizon. The inclusion of business conditions index into the dynamic semiparametric factor model is able to improve the accuracy of the yields with 1-year to 7-year maturities as compared to the simple model with only yield latent factors. However, the overall trace root mean square prediction error (TRMSPE) generated by the model with the index fails to overcome the counterpart model with macro variables. For the Nelson-Siegel model, incorporating the business conditions index is not informative and cause more errors relative to the model without the index. It appears to be beneficial for the dynamic semiparametric factor model to include the business conditions index to improve forecasting results while it does not work for the Nelson-Siegel model.

Considering the results of six-month ahead forecast, the dynamic semiparametric factor model with the business conditions index outperforms those forecast with only yield latent factors for the short and medium term yields with less than 5-year maturity. Impressively, it turn to produce more precise results relative to the model with macro variables. Again, the business conditions index worsen the predictability of the Nelson-Siegel model after including it. In this context, the index seems to be informative to incorporate with the dynamic semiparametric factor model rather than the Nelson-Siegel model.

At the twelve-month ahead horizon, the dynamic semiparametric factor model with the business conditions index produce forecasts that consistently outperform the model with only yield latent factors. Relative to the model with macro variable, the business conditions index become less informative for twelve-month horizon forecast. For the Nelson-Siegel model, the accuracy is deteriorating when the business conditions index is included, representing by considerably higher root

mean square prediction error (RMSPE) compared to the model without the index.

To visualize these results, Figures 4.7 and 4.8 provide a visual impression of the one-month ahead out-of-sample forecasting performance of the index-augmented models compared with the actual yields and those predicted by the dynamic semi-parametric factor model and the Nelson-Siegel model with the supplement of yield factors and macro variables. Figure 5 plots the outcomes for the dynamic semi-parametric factor model and Figure 6 for the Nelson-Siegel model.

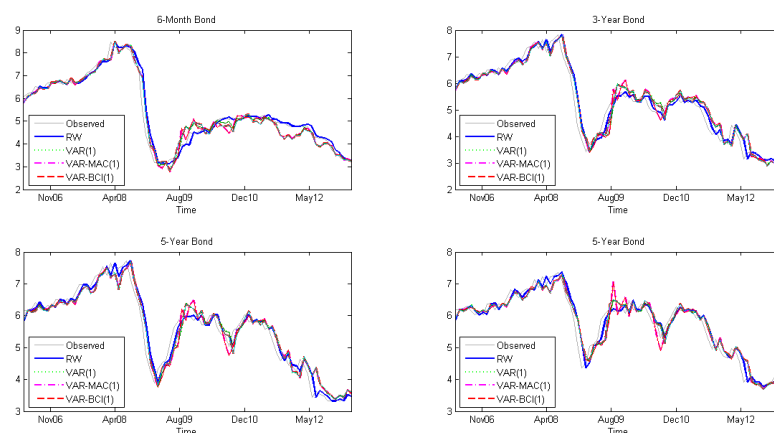


FIGURE 4.7: Term structure forecasting by the dynamic semiparametric factor model

As Figure 4.7 shows, the dynamic semiparametric factor model with business conditions index; VAR-BCI(1) produces less variation than the simple model with only yield latent factors; VAR(1), and the model that includes macro variables; VAR-MAC(1). As can be seen, the model with latent factors and model with macro variables overstate the actual yields during the global financial crisis on 2009. The forecasts of them perform particularly badly in the month after the crisis. In contrast, the model augmented with business conditions index perform impressively by producing more accurate results than others at the turning points and the periods after the crisis. This indicates the forward-looking information from the index enhances the forecasting accuracy of the dynamic semiparametric factor model.

Figure 4.8 reveals that the Nelson-Siegel model forecast the persistent movements of the yield at short-term maturity, in particular 6-month yield, remarkably well.

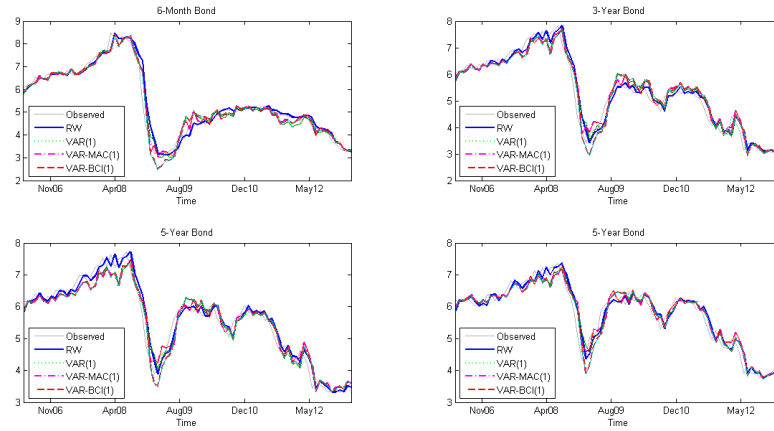


FIGURE 4.8: Term structure forecasting by the Nelson-Siegel model

As compared to the dynamic semiparametric factor model, it more precisely track the actual yields than those provided by the dynamic semiparametric factor model. Unfortunately, the forecasts of the Nelson-Siegel model perform particularly badly in at the turning points relative to the dynamic semiparametric factor model. Interestingly, an inclusion of the business conditions index produces more errors by understating the trough as well as overstating the yields during recovery period.

#### 4.7.3.3 Robustness check

In order to analyze whether the predictive power of the business conditions index varies under different conditions, we conduct three types of robustness checks. We first compare the forecasting accuracy produced by the model with the business conditions index relative to survey-based indicators. Next, we investigate whether the results are affected by adopting different data frequencies. Lastly, we check to what extent the forecasting results are improved by a different lag value of the business conditions index.

##### (1) Comparison with survey-based indicators

We assess the predictive power of the business conditions index relative to the two most common Australian survey-based indicators; the Westpac-Melbourne Institute leading index of economic activity and the Westpac-Melbourne Institute

consumer sentiment expectations index. The Westpac-Melbourne Institute leading index reports the movements in the leading indicators and is a gauge of Australian business cycle. This business conditions index is estimated from the ASX200 stock index, Westpac-Melbourne Institute consumer sentiment expectations index, US industrial production, the Reserve Bank of Australia's commodity price index, dwelling approvals, yield spread and aggregate monthly hours worked. Unlike the leading index that signals the near term economic performance based on various macroeconomic data, the consumer sentiment index reports the expectations of participants about the future economy. It reflects household financial situation, consumption and macroeconomic projection.

TABLE 4.9: Out-of-sample forecasts of the dynamic semiparametric factor model with business confidence index compared with other competitors

|                    | TRMSE  | 6-m    | 1-y    | 2-y    | 3-y    | 5-y    | 7-y    | 10-y   |
|--------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| RW                 | 0.2932 | 0.3054 | 0.3403 | 0.3324 | 0.3118 | 0.2866 | 0.2667 | 0.2654 |
| <b><i>DSFM</i></b> |        |        |        |        |        |        |        |        |
| VAR-BCI(1)         | 0.3268 | 0.3247 | 0.3772 | 0.3878 | 0.3608 | 0.3223 | 0.2915 | 0.2928 |
| VAR-LDI(1)         | 0.3325 | 0.3075 | 0.3803 | 0.4026 | 0.3775 | 0.3326 | 0.2966 | 0.2938 |
| VAR-CCF(1)         | 0.3324 | 0.3171 | 0.3784 | 0.3992 | 0.3744 | 0.3308 | 0.2970 | 0.2955 |
| <b><i>NS</i></b>   |        |        |        |        |        |        |        |        |
| VAR-BCI(1)         | 0.3195 | 0.2870 | 0.3726 | 0.3425 | 0.3483 | 0.3297 | 0.2975 | 0.2842 |
| VAR-LDI(1)         | 0.3206 | 0.2956 | 0.3831 | 0.3591 | 0.3477 | 0.3269 | 0.2939 | 0.2817 |
| VAR-CCF(1)         | 0.3150 | 0.2930 | 0.3793 | 0.3561 | 0.3403 | 0.3201 | 0.2872 | 0.2751 |

Table 4.9 shows that the business conditions index augmentation enhances the predictability of the dynamic semiparametric factor model to provide better out-of-sample forecasts, relative to the leading index and the consumer sentiment index. However, the supplementary information from the business conditions index worsens the prediction power of the Nelson-Siegel model so that it fails to compete with other two competitive indices. For the Nelson-Siegel model, the inclusion of the consumer sentiment index contributes to the achievement with a lowest trace root mean square prediction error (TRMSPE) compared to other indicators. The consumer sentiment index is also the second best indicator to be augmented with the dynamic semiparametric factor model. It is not surprising that the consumer sentiment index is relatively successful to forecast the yields, especially with the Nelson-Siegel extension. It contains forward looking information and particularly helps the Nelson-Siegel level factor for term structure forecasting with inflation expectation as mentioned in the literature, including [Dijk et al. \(2013\)](#). The leading

index is relatively unsuccessful at forecasting the term structure for both models. Comparing the leading index with the business conditions index, the leading index only concentrates on domestic economy and may not represent the overall economic performance and therefore it is not informative enough for term structure forecasting.

## (2) Data frequency

As noted earlier, term structure forecasting is theoretically based on expectation and is forwarding looking. Participants continuously update their expectation on the new information. To take advantage of the high frequency of the business conditions index that is available for daily and weekly, we conduct a forecasting exercise daily and weekly to investigate whether the predictive performance is improved by higher frequency information.

TABLE 4.10: Out-of-sample forecasts of the dynamic semiparametric factor model with business conditions index compared with other competitors

|         | <b>TRMSE</b> | <b>6-m</b> | <b>1-y</b> | <b>2-y</b> | <b>3-y</b> | <b>5-y</b> | <b>7-y</b> | <b>10-y</b> |
|---------|--------------|------------|------------|------------|------------|------------|------------|-------------|
| Daily   | 0.1206       | 0.0868     | 0.1191     | 0.1256     | 0.1274     | 0.1313     | 0.1179     | 0.1265      |
| Weekly  | 0.1421       | 0.0984     | 0.1083     | 0.1274     | 0.1422     | 0.1521     | 0.1500     | 0.1600      |
| Monthly | 0.3268       | 0.3247     | 0.3772     | 0.3878     | 0.3608     | 0.3223     | 0.2915     | 0.2928      |

Table 4.10 presents the term structure forecasting of the dynamic semiparametric factor model and the Nelson-Siegel model with the business conditions index at different frequencies. High frequency information, in particular of the daily business conditions index, significantly improves the forecasting performance of both models. We also observe the weekly index also significantly enhances the predictability of the term structure forecasting. We suggest using the business conditions index at high frequency (daily or weekly) to link the yields information with the macroeconomic situation rather than traditional macro variables that are normally used in macro-finance term structure model.

## (3) Choice of lags BCI

One important benefit from the business conditions index is its availability. The supplementary information from the index is basically based on the index at the end date of the previous month. Since the index contains forward looking information, forecasting made by the index implies the trend or the anchor for the yields in the future. We consider different lags of the business conditions index. The most recent available index may probably improve the forecasting accuracy of the term structure model. We examine the term structure forecast with different lag periods from one-day lag, two-day lag until one-month lag. The forecasting results measured by the root mean square prediction error (RMSPE) is not sensitive to any lags with less than one-week. We report the results produced by the inclusion of the index at one-week lag, two-week lag in comparison to one-month lag.

TABLE 4.11: Out-of-sample forecasts of the dynamic semiparametric factor model with business confidence index compared with other competitors

|             | <b>TRMSE</b> | <b>6-m</b> | <b>1-y</b> | <b>2-y</b> | <b>3-y</b> | <b>5-y</b> | <b>7-y</b> | <b>10-y</b> |
|-------------|--------------|------------|------------|------------|------------|------------|------------|-------------|
| 1-week lag  | 0.2713       | 0.2562     | 0.2907     | 0.2912     | 0.2883     | 0.2792     | 0.2595     | 0.2520      |
| 2-week lag  | 0.2710       | 0.2565     | 0.2908     | 0.2911     | 0.2880     | 0.2787     | 0.2590     | 0.2514      |
| 1-month lag | 0.2710       | 0.2564     | 0.2910     | 0.2913     | 0.2882     | 0.2790     | 0.2590     | 0.2512      |
| 1-day lag   | 0.2714       | 0.2563     | 0.2907     | 0.2912     | 0.2884     | 0.2794     | 0.2597     | 0.2521      |

Table 4.11 shows that the incorporating of the business conditions index at a two-week lag significantly improves forecasting accuracy and provides the lowest root mean square prediction error (RMSPE) relative to the choice of one-week lag and the baseline model with one-month lag. As mentioned earlier, the index with less than one-week lag is not informative. This implies the forward looking information contained in the index is actually updated with the more recent available information. Hence, the only two-week lag provides better results compared with one-month (four-week lag). It also suggests the adjustment of the yield curve shape in regard to the recent economic situation takes some time to revise expectation and affect the yields.

## 4.8 Conclusion

In this paper, we propose to use the Sheen-Trueck-Wang business conditions index to incorporate forward looking information into the dynamic Nelson-Siegel and



dynamic semiparametric factor model for yield curve estimation and forecasting. Our method offers an anchor for the cross-sectional and in-sample term structure model that is very useful for term structure forecasting by providing the expectation about the future term structure.

The high frequency nature of the business conditions index allows us to produce more frequent term structure forecasts and also utilize the latest information. We find that the inclusion of the business conditions index helps to reduce the forecasting errors relative to other models. Comparing with other survey-based indicators, the business conditions index is found to be a good candidate that provides a very promising new source of data to forecast term structure. It is more relevant to economic activities and therefore predictive power. Unfortunately, it is hard to beat the random walk, especially when forecasting long-term interest rates.



## Chapter 5

# The Economic Impact of Quantitative Easing on the U.S. Economy: A Bayesian Structural VAR (B-SVAR) with Sign Restrictions Analysis

### 5.1 Introduction

We examine the effectiveness of the US quantitative easing (QE) policy that was adopted when the short-term fed funds interest rate became constrained by the zero lower bound by exploring the dynamic effects of a shock to commercial bank credit on the bond yield slope, inflation and output growth. The effects are estimated by a Bayesian structural vector autoregressive (B-SVAR) model with sign restrictions using monthly data over the period of January 2003 to August 2013. Within this B-SVAR, we identify an innovation of the unconventional monetary policy shocks by a shift in the Federal Reserve's balance sheet size that provides a massive credit supply shock and has impacts on the long-term yield, economic activity and inflation. We find that an exogenous increase in the Federal Reserve's assets leads to a drop in long-term interest rates that compresses the long-term bond yield spread over the short-term yield, or yield slope, and raises inflation as

well as output growth.

There are three main contributions to the existing literature. First, this study provides evidence for the effectiveness of the unconventional monetary policy for the US economy. This result suggests that unconventional monetary policy measures adopted by central banks during a liquidity trap can provide temporary support to their economies. Second, using the B-SVAR with sign restrictions to impose a theoretical structure and establish some relevant stylized facts allows us to be agnostic about how macroeconomic activity responds to quantitative easing. To credibly identify our quantitative easing shock, the yield slope, which is the vehicle for central bank asset expansion to affect other macro-variables, is unrestricted to let it freely respond to the data. Most of the earlier literature implemented event studies to analyze the effects of unconventional monetary policy on particular financial market variables such as the long term yield or yield spread. However, this does not imply that unconventional monetary policy will in general have positive macroeconomic effects. Third, we propose the transmission mechanism of unconventional monetary policy by characterizing the macroeconomic consequences of the central banks asset purchases via a rise in the central bank balance sheet, which causes a decline in the long-term rate. The compression in the long-term yield spread leads to the expansion of output and higher inflation. Based on these results, we conclude that large-scale purchases of long-term government bonds constitute a viable policy option to provide additional monetary policy accommodation in a zero-lower-bound environment that enables central banks to achieve their mandate of stimulating the economy in the absence of conventional monetary policy.

The remainder of the paper is structured as follows: Section 5.2 discusses the related literature on unconventional monetary policy, Section 5.3 describes the Bayesian structural vector autoregressive (B-SVAR) model with sign restrictions and proposes identification strategies for conventional monetary policy, unconventional monetary policy and other aggregate shocks. Then, we describe the conceptual framework about unconventional monetary policy and how the Federal Reserve has implemented unconventional monetary policy after the global financial crisis in 2008 in Section 5.4. The data description and descriptive statistics are reported in Section 5.5. Afterwards, the results are presented and discussed

in Section 5.6, as well as a comparison exercise to assess the effectiveness of the unconventional monetary policy relative to the conventional monetary policy. We also conduct robustness checks in Section 5.7 and finally give conclusion in Section 5.8.

## 5.2 Review of literature

After the global financial crisis in the late 2000s, the implementation of non-standard monetary policies and the analysis of their effects became more interesting for policy analysts and academia. Earlier studies prior to the crisis were conducted to examine the effectiveness of any alternative monetary policy other than changing the standard monetary policy interest rate. Most earlier studies focused on the effects on financial variables of a large scale purchase in long term assets by the central bank. The assessment of the effects of unconventional monetary policy on financial variables, especially the long-term interest rate and yield spread, was mainly conducted by event study methods. [Bernanke et al. \(2004\)](#) examined financial market reactions to a non-standard monetary policy to change the relative supply of the government securities and found the extension of central bank assets provides excess reserves while maintaining the zero lower bound of the short-term interest rate, and in fact generated lower long term yields. The determination to pursue a very low short term policy rate was realized by economic agents and transmitted into the long-end of the term structure. The decline in long-term yields implies higher expected inflation and income and therefore induces aggregate demand.

Since then, there have been a series of papers that investigated the effect of unconventional monetary policy on the yield term structure through the expectation channel. For example, [Okina and Shiratsuka \(2004\)](#), [Baba et al. \(2005\)](#), [Ugai \(2007\)](#) and [Oda and Ueda \(2007\)](#) explored the extension of central bank assets in Japan and found a negative impact on yield spreads. However, these studies are not informative for analyzing the effect of the extension of central bank's assets during the 2000s financial crisis, particularly for the case of the US. [Gagnon et al. \(2010\)](#), [Krishnamurthy and Vissing-Jorgensen \(2011\)](#), [Hamilton and Wu \(2012\)](#)

and [DAmico et al. \(2012\)](#) assessed the Large Scale Asset Purchases (LSAPs) implementation programs by the Federal Reserve in the US after the global financial crisis. They found the massive long-term assets purchasing through LSAP1 (during 2008-2009), LSAP2 (during 2010) and Operation Twist (during 2011-2012), effectively lessened long-term interest rates mainly through the reduction in liquidity premia and duration risk. However, these studies found minimal negative effects on yield spreads and even less for the LSAP2 once economic agents had already anticipated the impact before the program was enacted. As noted, these studies employed the event study method to evaluate the effectiveness of unconventional monetary policy on the yield spread and compared the counterfactual impact when there is no such implementation. Nonetheless, there are small but acceptable samples of historical data available for doing a qualified study. Moreover, these empirical studies only focus on the link between the operation of unconventional monetary policy and the yield term structure but ignore the consequences of changes in the yield spread on macroeconomic variables, particularly output and inflation.

To analyze the wider effects on macroeconomic variables, [Wright \(2012\)](#), [Joyce et al. \(2012\)](#) and [Chung et al. \(2012\)](#) estimated the impact of unconventional policy measures on asset prices or the term structure of yields, and then plugged-in the reaction of financial variables into a macroeconomic model, a vector autoregressive model. However, this two-step method as argued by [Hamilton and Wu \(2012\)](#), probably leads to biased estimation. Furthermore, unconventional monetary policy may actually affect the economy through a lower long-term yield and transmit by the channel of a yield slope compression. Therefore, the effect of unconventional monetary policy should be estimated as an agnostic change in the yield spread simultaneously with other monetary policy instrument and other macro economic variables.

For these reasons, there is a growing literature that applies the structural vector autoregressive model (SVAR) to uncover the macroeconomic effects of unconventional monetary policy and analyze the transmission mechanism based on impulse response analysis. This approach investigates a non-standard monetary policy shock, associated with the central bank's assets purchasing, on long-term yield or yield spread, given a restricted policy interest rate unchanged at the zero-lower

bound. There are a few studies that apply the SVAR techniques to estimate the macroeconomic impacts of the standard monetary policy in the aftermath of the global financial crisis.

SVAR techniques analyze the effect of unconventional monetary policy innovation by identifying certain restrictions on the contemporaneous interactions between monetary policy instruments, the yield spread channel, economic activity, inflation and other financial or macroeconomic variables. Among several identification approaches, sign restrictions has a strong theoretical rationale and therefore has been used in recent SVAR studies on non-standard monetary policy measures, including [Peersman \(2011\)](#), [Kapetanios et al. \(2012\)](#), [Schenkelberg and Watzka \(2013\)](#), [Baumeister and Benati \(2013\)](#) and [Gambacorta et al. \(2014\)](#).

One of the early studies on the macroeconomic consequence of non-standard monetary policy initiatives within the context of the the financial crisis after 2008 was done by [Peersman \(2011\)](#). This study estimated the SVAR model for the Euro area over the period of 1999 to 2009 to examine the effectiveness of the massive increase in credit supply shocks, orthogonalized to a zero lower bound policy rate. It was found that a positive innovation on unconventional monetary policy significantly generate a humped shape increase in output and persistently increases inflation. These effects are passed through a persistent decline in the yield spread. Comparing with a negative shock for conventional monetary policy, the impulse responses from the unconventional monetary policy are more sluggish.

[Baumeister and Benati \(2013\)](#) also investigated the macroeconomic effects of a compression in the yield spread during 1954 to 2011 in the U.K. and U.S. Unlike [Peersman \(2011\)](#), this study directly analyzed the impact of a change in the yield spread on economic activity and inflation regardless of the transmission from any monetary policy instruments such as credit supply or central bank's assets. The compression in the yield spread was found to play an important role to stimulate output growth and inflation when the conventional monetary policy is constrained by a zero lower bound interest rate. The unconventional monetary policy measures also confirmed by a counterfactual simulation the effectiveness to avert risk of a prolonged recession and deflation. This study also found the impacts of the non-standard monetary policy on output growth and inflation become stronger after

the global financial crisis when allowing for time-variant parameters. Even though they conducted counterfactual simulation and applied the time-varying parameter structural vector autoregressive (TVP-SVAR) with sign restrictions estimation, the transmission mechanism was still unclear since monetary policy instruments were disregarded in this study.

In order to provide better understanding about the interaction between the monetary policy instrument and macroeconomic variable, [Gambacorta et al. \(2014\)](#) straightforwardly chose the central bank's balance sheet as an instrument to accommodate unconventional monetary policy in their study. They employed a panel SVAR with sign restrictions for eight industrial economies: Canada, the Eurozone, Japan, Norway, Switzerland, Sweden, the U.K. and the U.S., to investigate the effectiveness of non-standard monetary policy on output, price level, stock market volatility and central bank's balance sheet over the crisis period from 2008 to 2011. An extension in central banks assets achieved efficacious outcomes to avert stagnation and mitigate concerns about economic instability. Unlike [Peersman \(2011\)](#), the impacts on price level were founded to be quantitatively weaker and less persistent in comparison with output. The empirical results across economies were quite similar, which implies unconventional policy is able to be used as a tailor-made measure for any central banks to stimulate the economy at the zero lower bound interest. Compared with previous studies, it directly quantified the impacts of changes in the central bank's balance sheet on output and the price level. However, the long-term yield or yield spread, which is theoretically explained as a vehicle of unconventional monetary policy, is left out of this study. Another critical caveat is that they did not orthogonalise the policy interest rate to purely assess the response of non-standard policy.

A broader study that characterizes the transmission mechanism from a change in the monetary policy instrument via the long-term yield or yield spread channel towards macroeconomic variables was done by [Schenkelberg and Watzka \(2013\)](#). This study applied a Bayesian SVAR with sign restrictions to investigate the impacts of unconventional monetary policy on the industrial production index, consumer price index and exchange rate for Japan during the period from 1995 to 2010. The transmission mechanism instrument of unconventional monetary policy was identified by a change in the central bank's reserves that passes through



the long-term yield and eventually affects macroeconomic variables. This study proposed a set of identified shocks: unconventional monetary policy, aggregate demand and aggregate supply shock, that allowed it to examine the impacts of non-standard stimulus relative to others. The unconventional monetary policy shock was found to produce positive effects on output and the price level, but less in magnitude relative to aggregate demand shocks. This study argued that the non-standard monetary policy actually failed to induce output growth and avert deflation due to weak and transient responses. Yet, it still ignored the possible caveat about unorthogonalised policy and therefore the results were not properly measured.

Another study on examining whether unconventional monetary policy instruments affect the yield spread and macroeconomic variables was published by [Kapetanios et al. \(2012\)](#). The unconventional monetary policy was identified by a change in the monetary base that passes through the yield spread and then affects output as well as the price level. This study used several techniques: Bayesian VAR (BVAR), time-varying parameters SVAR (TVP-SVAR) and Markov-Switching SVAR (MS-SVAR), to measure the impacts of quantitative easing conducted by the Bank of England based on data from 1993 to 2010. With regards to counterfactual simulation, the BVAR estimation revealed that the U.K. economy would probably decline even more if the unconventional monetary policy was not put in place. The results produced by TVP-SVAR and MS-SVAR also confirmed that quantitative easing effectively stimulated the economy and avoided deflation despite the fact that the magnitude of the impacts markedly varied across the models. Whilst this empirical study supported the idea that central bank could enlarge the monetary base to affect the long-term yield and overall economy at the zero lower bound interest rate, it did not discuss how the non-standard policy works through the yield spread channel to affect the macroeconomy. It also ignored robustness checks for different identification schemes and other possible transmission channels.

It remains unclear in the literature how unconventional monetary policy affects on economic activity and inflation in comparison with conventional monetary policy and aggregated demand stimulus. The transmission mechanism from a change in the monetary policy instrument via the long-term yield or yield spread to macroeconomic variable also requires further in-depth investigation to reveal the

interaction and propagation in the overall economy. Specifically, there are a limited number of SVARs with sign restrictions studies that focus on unconventional monetary policy measures implemented by the Federal Reserve since the global financial crisis.

## 5.3 Methodology

In this Section, we describe the method in specifying the structural vector autoregressive (SVAR) model in subsection 5.3.1 and estimating by Bayesian method in Subsection 5.3.2. We then discuss the identification strategy by means of sign restrictions technique in subsection 5.3.3. We follow the approach that was proposed by Uhlig (2005) to decompose structural shocks. For subsection 5.3.4, we explain the reason why we choose Federal Reserve's balance sheet as a monetary policy instrument and a yield slope as a monetary policy channel for unconventional monetary policy transmission mechanism. We then specify restrictions for the conventional monetary policy, unconventional monetary policy and other traditional aggregate shocks in subsection 5.3.5.

### 5.3.1 SVAR specification

To analyze the effectiveness and transmission mechanism of unconventional monetary policy at the zero lower bound interest rate, we estimate the joint behaviour of the Federal Reserve balance sheet or the FED asset position (FAS), the spread between the 10-year Treasury bond yield and the 3-month Treasury bill rate or the yield slope (YSL), the Fed funds rate (FFR), the industrial production index (IPI) and the consumer price index (CPI) in the framework of a structural vector autoregressive (SVAR) model.

The benchmark vector autoregressive (VAR) model is estimated by the following reduced-form VAR model:

$$Y_t = A_1(L)Y_{t-1} + \dots + A_p(L)Y_{t-p} + \varepsilon_t \quad (5.1)$$

where  $Y_t$  is a  $K$  dimensional vector of endogenous variables of interest,  $A_1, \dots, A_p$  are collected in the vector of coefficients, and  $\varepsilon_t$  is a vector of residuals with variancecovariance matrix  $E[\varepsilon_t \varepsilon_t'] = \Sigma_\varepsilon$  and  $t = 1, \dots, T$ .

Equivalently the model can be written more compactly as:

$$A(L)Y = \varepsilon_t \quad (5.2)$$

where  $A(L)$  is the a matrix polynomial in the lag operator and  $L$  is an autoregressive lag order polynomial.

For a particular period, these variables are affected by exogenous disturbances. To learn about the effect of the shocks, in particular the unconventional monetary policy shock, we rewrite the reduced-form VAR model as the structural vector autoregressive (SVAR) model.

$$B_0 Y_t = B_1 Y_{t-1} + \dots + B_p Y_{t-p} + u_t \quad (5.3)$$

where  $u_t$  denotes a mean zero serially uncorrelated error term. It is also assumed to be unconditionally homoskedastic.

Again, it can be also rewritten in terms of the lag operator:

$$B(L)Y_t = u_t \quad (5.4)$$

where  $B(L) = B_0 - B_1 L - B_2 L^2 - \dots - B_p L^p$  is the autoregressive lag order polynomial. The variance-covariance matrix of the structural error term is normalized such that:

$$E[u_t u_t'] = \Sigma_u = I_K \quad (5.5)$$

These structural shocks are mutually uncorrelated, implying  $\Sigma_u$  is diagonal. The variance of structural shocks are normalized to unity.

The SVAR model can be used to identify shocks and trace out how structural innovations affect the dependent variables in the original model by employing impulse response analysis and forecast error variance decompositions (FEVD), which we will discuss later.

Recall for the structural vector autoregressive (SVAR) model. If we multiply both sides of the structural VAR representation by  $B_0^{-1}$ , we get:

$$B_0^{-1}B_0Y_t = B_0^{-1}B_1Y_{t-1} + \dots + B_0^{-1}B_pY_{t-p} + B_0^{-1}u_t \quad (5.6)$$

This representation actually is the reduced-form representation where  $A_i = B_0^{-1}B_i$ ,  $i = 1, \dots, p$  and  $\varepsilon_t = B_0^{-1}u_t$ .

The matrix  $B_0^{-1}$  governs how a structural shocks  $u_t$  affect  $Y_t$ . We can estimate the reduced-form equation from the data. However, we need to recover the element  $B_0^{-1}$  from the reduced-form equation. Once knowing  $B_0^{-1}$  allows us to reconstruct  $u_t$  from  $u_t = B_0\varepsilon_t$ .

Recalling  $\varepsilon_t = B_0^{-1}u_t$ , the variance of  $\varepsilon_t$  is:

$$E(\varepsilon_t\varepsilon_t') = B_0^{-1}E(u_tu_t')B_0^{-1'} \quad (5.7)$$

Equivalently,

$$\Sigma_\varepsilon = B_0^{-1}\Sigma_u B_0^{-1'} = B_0^{-1}B_0^{-1'} \quad (5.8)$$

The equation  $\Sigma_\varepsilon = B_0^{-1} B_0^{-1'}$  can be solved for the unknown parameters  $B_0^{-1}$  given known  $\Sigma_\varepsilon$  from the reduced form estimation. The number of unknown parameters in the SVAR exceeds the number of known parameters from the reduced form equation, and so are under-identified.

To recover the structural parameters, we need to impose additional restrictions on selected elements of  $B_0^{-1}$ . The most common approach is to impose zero restrictions on selected elements of  $B_0^{-1}$ . Practically, we may impose a short-run restriction by assuming that there is no instantaneous effect from any shocks on output and inflation. However, there is much skepticism about excluding restrictions used to achieve identification. Another solution to disentangle the structural innovations  $u_t$  from the reduced-form innovations  $\varepsilon_t$  is to identify the model by making assumption on the causal ordering of the variables that compose the SVAR. By applying a recursive restriction, the error term in each regression is uncorrelated with the error in the preceding equations. Define  $P$  as the lower-triangular matrix with positive main diagonal. The orthogonalized matrix  $P$  is then related to the error covariance matrix by  $\Sigma_\varepsilon = PP'$ . By taking such a Cholesky decomposition of the variance-covariance matrix, we can recover the structural shocks  $u_t$  since  $P$  is lower triangular, and so all parameters are exactly identified. The recursive restriction approach was popularized to model the monetary policy transmission mechanism following the pioneer works of [Sims \(1980\)](#), [Bernanke and Blinder \(1992\)](#) and [Christiano et al. \(1999\)](#). It is important to be aware that the recursive restrictions depends on the particular successive ordering and may not be supported by economic theory. Without a reasonable economic interpretation, this solution become meaningless.

To overcome the controversy about the right short-run restrictions or the plausible sequence of restriction, we may focus instead on long-run restrictions that are better justified by theory. For example, the aggregate demand shock is assumed to have a zero long run effects on real variables, as in [Blanchard and Quah \(1990\)](#). Nonetheless, it is difficult to find an accurate estimate of the impulse responses at the infinite horizon from a short time span of data. [Faust and Leeper \(1997\)](#) found the small sample bias actually caused substantial distortion when the long-run restriction is imposed. Hence, the imposition of zero restrictions on

the contemporaneous impact or on the long-run cumulative effect is still doubtful.

Instead of placing stringent constraints on a structural VAR model, restrictions on the signs of impulse responses to structural shocks emerge to be more consistent with economic theory and empirical data. The idea is that sign restrictions are imposed on the response of some variables to a shock for a particular period or lag length while leaving the response of the main variable of interest unrestricted. The sign restrictions method as an alternative way for SVAR model identification has been proposed by [Faust \(1998\)](#), [Canova and Nicoló \(2002\)](#) and [Uhlig \(2005\)](#). These studies used pre-identified sign restrictions on the impulse response functions to identify shocks. Unlike other zero restriction approaches, it is not necessary to impose zero constraints on the contemporaneous impact matrix. Instead, it only needs to detect a band of impulse responses which satisfy the desired signs. It can also combine zero restrictions with sign restrictions to identify shocks. More importantly, the restrictions are explicitly consistent with dynamic general equilibrium theory.

To implement sign restriction, we follow [Uhlig \(2005\)](#) to use Bayesian methods for estimation and inference. The Bayesian SVAR is parsimonious approach to capture the rich dynamic relation. Since we estimate numerous structural parameters from impulse responses experiments that agree with identified signs, the Bayesian approach is able to solve the curse of dimensionality by adding prior information to a reduced form VAR, and improve the accuracy of forecasts by combining updated posterior information. The posterior information from data is weighted through coefficient estimates associated with prior information from data. The details about Bayesian SVARs and SVARs with sign restrictions are explained in the following subsections.

### 5.3.2 Bayesian estimation

Bayesian techniques are widely used in macro-econometric analysis with VAR models since the works of [Litterman \(1986\)](#) and [Sims and Zha \(1998\)](#). By adding prior information about the likely value of parameters, the Bayesian approach can

improve forecasting accuracy in the context of vector autoregressive models. Forecasting performance is also enhanced by combining posterior information with a priori information even if the sample period is short. To explore dynamic propagation of macroeconomic shock, Waggoner and Zha (2003) applied Bayesian techniques in their structural VAR model and claimed the Bayesian SVAR is efficient in drawing posteriors and well-suited to prior information. Uhlig (2005) use the Bayesian approach to form prior beliefs and utilize information from the posterior to estimate structural parameters and investigate the effect of monetary policy shocks. The Bayesian SVAR with sign restrictions was found to be an appropriate tool for macroeconomic policy analysis without imposing zero restrictions on the contemporaneous matrix. This approach was also employed by recent studies on the unconventional monetary policy analysis such as Peersman (2011), Kapetanios et al. (2012), Baumeister and Benati (2013), Schenkelberg and Watzka (2013) and Gambacorta et al. (2014).

We estimate the model using the Bayesian structural vector autoregressive (B-SVAR) approach. Let us discuss how the Bayesian method is conducted for SVAR estimation. B-SVAR method starts with rewriting the reduced form VAR as a system of simultaneous equations:

$$Y = X\beta + E \quad (5.9)$$

where  $Y = (Y_1, \dots, Y_T)'$ ,  $X = (X_1, \dots, X_T)'$  with  $X_t = (Y'_{t-1}, \dots, Y'_{t-p})'$ .  $E = (e_1, \dots, e_T)'$  and  $B = (A_1, \dots, A_p)'$  are  $T \times N$ ,  $T \times k$ ,  $k \times N$  and  $T \times N$  matrices respectively. Let's denote  $K = N \times p$  as the number of coefficients. All variables are collected together for each time  $t$ .

In fact, the simultaneous equation for variable  $i$  is  $Y_i = XB_i + E_i$ . We can then transform the reduced form VAR in another useful notation by stacking the column of  $Y_i$  and  $E_i$  into  $NT \times 1$  vectors as following:

$$Y = (I_m \otimes X)\beta + e \equiv X\beta + e \quad (5.10)$$

For this second notation,  $\beta$  is  $kN \times 1$  vector or  $\beta = \text{vec}(B)$ . Each variable represents a time series of it for all periods. Assume that  $e_t$  is independent and identically distributed (i.i.d.). The likelihood function  $L(\beta, \Sigma)$  of a VAR can therefore be decomposed into the product of a Normal density, conditional on the ordinary least square (OLS) estimator as:

$$\beta | \Sigma, Y \sim N((\hat{\beta}), \Sigma \otimes (X'X)^{-1}) \quad (5.11)$$

where  $\Sigma \otimes (X'X)^{-1}$  is the ordinary least square (OLS) estimator of variable  $\hat{\beta}$ , and a Wishart density for covariance  $\Sigma^{-1}$

$$\Sigma^{-1} | Y \sim W(S^{-1}, T - K - N - 1) \quad (5.12)$$

where  $W(\cdot)$  is a Wishart distribution and  $T - K - N - 1$  are degrees of freedom.  $S$  is the sum of squared errors  $S = (Y - X\hat{\beta})'(Y - X\hat{\beta})$  computed by the OLS estimates  $\hat{\beta}$ .

As we can see, the VAR model is not parsimonious since it contains many parameters and it is hard to obtain precise estimates. Based on maximum likelihood estimation, the parameters are treated as fixed unknown quantities, and unbiased estimators could be estimated from the inference on a large number of samples given known distributions. The average value of the sample estimator converges to the true value with the law of large numbers.

Another approach to estimate the VAR model is Bayesian estimation. Parameters are now assumed to be random variables with a probability distribution. Probability measures the degree of beliefs in the estimators that can be summarized by the probability density, which is called the prior. The prior is formed before investigating the data. Then, we use the data to learn and update information about the parameters, which is also known as the posterior. For the SVAR context, we use Bayesian analysis to incorporate beliefs and information from data to estimate parameters that satisfy the identification scheme.



Suppose  $\theta$  denote parameters which are unobserved and we use data  $Y$  to uncover them. From the Bayes' rule, we know that:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)} \quad (5.13)$$

where  $p(\theta|Y)$  is the posterior probability density that explains the probability of  $\theta$  occurring conditional on  $Y$  having occurred. The probability density describes what we know about  $\theta$ , given the data.  $p(Y|\theta)$  is the likelihood function and  $p(\theta)$  is a prior belief on the assumed probability distribution.

The posterior probability becomes:

$$p(\theta|Y) \propto p(Y|\theta)p(\theta) \quad (5.14)$$

which means that the posterior probability density  $p(\theta|Y)$  is proportional to the prior times the likelihood function.

Hence, we need to build a prior for the parameters  $\beta$  and  $\Sigma$  and then use the likelihood function of the VAR to compute the posterior density as well as the conditional mean  $E(\beta|y)$  and the conditional variance  $var(\beta|y)$  afterward. The posterior density is consequently the object of interest in Bayesian estimation.

The assumption about prior probability density or distribution determines whether the posterior distribution can be computed. Some priors require repeated and complicated calculation which raises computational burden. Among several priors, conjugate priors have proved to be convenient to work with and suited for empirical data. The main advantage of conjugate prior assumptions is that combining distributions of a conjugate family results in a new distribution of the same family. As noted in Zellner (1996), if the normal-inverted Wishart prior is conjugate, then the conditional posterior distribution is also normal-inverted Wishart.

The conditionally conjugate prior distribution can then be written as:

$$\beta|\Sigma \sim N(\beta_0, \Sigma \otimes \Omega_0) \quad \Sigma \sim IW(v_0, S_0) \quad (5.15)$$

Note that  $\beta|\Sigma$  is a matrix-variate normal distribution where the prior expectation  $E(\beta) = \beta_0$  and prior variance  $\text{var}(\beta) = \Sigma \otimes \Omega_0$ . The prior variance matrix has a Kroneker structure where  $\Omega$  is the variance matrix of the disturbances.

As mentioned earlier, the normal-inverted Wishart prior is conjugate, therefore, the conditional posterior distribution is also normal-inverted Wishart:

$$\beta|\Sigma, Y \sim N(\bar{\beta}, \Sigma \otimes \bar{\Omega}) \quad \Sigma|Y \sim IW(\bar{v}, \bar{S}) \quad (5.16)$$

where  $\bar{v}$  and  $\bar{S}$  denote that as parameters of the posterior distribution.

To perform statistical inference and forecasting from Bayesian estimation, we then simulate the posterior distribution of the parameters conditional on the data. Drawing from the conditionals  $\beta|\Sigma, Y$  and  $\Sigma|Y$  would eventually produce a sequence of draws from the joint posterior and the marginal posteriors distribution.

Based on the estimated posterior distribution for the VAR coefficients, we calculate the impulse responses and keep those which are compatible with the sign restriction and ultimately calculate summary statistics of interest, particularly the median and probability bands.

### 5.3.3 Sign-restriction identification

To investigate the impact of structural shock on endogenous variables, we apply a Bayesian SVAR with sign restrictions as used in Uhlig (2005) to decompose structural shocks whose impacts are theoretically reasonable. The idea is to disentangle the reduced-form errors  $\varepsilon_t$  that summarizes statistical relationships into a set of orthogonal structural disturbances, described by economic innovation  $u_t$ . The vector of structural innovations  $u_t$  is assumed to be independent so that  $E(u_t u_t') = \Sigma_u = I_K$ . We need to find a matrix  $H$  such that  $Hu_t = \varepsilon_t$ . This matrix  $H$  can be estimated using the information given by the covariance matrix of the reduced form:

$$\Sigma_u = E(u_t u_t') = H E(\varepsilon_t \varepsilon_t') H' = H H' \quad (5.17)$$

We need at least  $n \times (n - 1)/2$  restrictions on  $H$  to achieve identification.

Let  $q$  be a random orthonormal matrix, orthogonal decomposition of which satisfies  $qq' = I$ . The multiplicity of  $Hqq'H' = HH'$  is also an admissible decomposition. Denote  $a = Hq$ , then we have:

$$\Sigma_u = E(u_t u_t') = HH' = Hqq'H' = aa' \quad (5.18)$$

where  $a$  is not lower triangular anymore.

This decomposition produces a new set of uncorrelated shocks without imposing zero-type restrictions on the model. In order to estimate the structural model, we follow Uhlig (2005), Mountford and Uhlig (2009) and Fratzscher et al. (2010) to find vector  $a$  where  $a \in \mathbb{R}^n$ , given there is an  $n$  dimensional orthogonal vector  $q$  so that  $a = \tilde{H}q$  where  $\tilde{H}\tilde{H}' = \Sigma_\varepsilon$  and  $\tilde{H}$  is a lower triangular Cholesky factor of  $\Sigma_\varepsilon$ .

Solving for structural shock can be conveniently transformed into the problem of choosing elements in an orthogonal set in responses to one particular shock. Rewrite the VAR in reduced form vector moving average representation.

$$Y = [I - A(L)]^{-1} \varepsilon_t \quad (5.19)$$

$$Y_t = \sum_{s=0}^{\infty} C_s \varepsilon_{t-s} \quad (5.20)$$

where the  $C_s$  matrices represent the dynamic multipliers or impulse responses. The impulse response to the  $i - th$  one-step ahead prediction error is given by the  $i - th$  column of the  $C_s$ .

The impulse response  $r_s$  of all variables at horizon  $s$  to the  $i - th$  structural shock is then given by:

$$r_s = C_s a \quad (5.21)$$

The vector  $a$  is called an impulse vector which contains the contemporaneous responses of the endogenous variables to the primary shock.

Equivalently:

$$r_s = [I - A(L)]^{-1} a_j \quad (5.22)$$

Based on the coefficients  $A(L)$  in the reduced form VAR, the impulse responses for  $n$ -variables up to  $s$  horizons can be simulated for a given impulse vector  $a$  to a  $j$  shock. We impose sign restrictions on a subset of the  $n$  variables over horizons up to  $S$  associated with a particularly identified shock of interest. We then check whether the signs of simulated impulse responses satisfy a set of a priori identification and construct a distribution of the solutions that agrees with the restrictions, while discarding any responses that violate the signs.

The estimation and inference is carried out by using the Bayesian SVAR approach to handle sign restriction as in the pioneering work of Uhlig (2005). As discussed before, we firstly estimate the reduced form VAR to form a prior. Using the Normal-Wishart prior in  $(A(L), \Sigma_u)$  implies that the posterior is also the Normal-Wishart for  $(A(L), \Sigma_u)$  times the indicator function on  $a = \tilde{H}q$ .

Then, we take a joint draw from the posterior of the Normal-Wishart for  $(A(L), \Sigma_u)$  and a draw from the unit sphere to obtain candidate  $q$  vectors. The draws from the posterior are used to calculate the Choleski decomposition  $H$  from  $\Sigma_u = HH'$  afterward. Each random orthonormal  $q$  drawn from the uniform distribution is taken together with Choleski decomposition  $H$  to compute impulse vector  $a$ .

Given a structural impulse vector  $a$ , we calculate impulse responses at period  $s$  to the  $i - th$  shock obtained by Choleski decomposition  $\Sigma_u = HH'$ . The impulse response  $r_a(s) \in \mathbb{R}^n$  at  $s$  horizon corresponding to impulse vector  $a$  is given by:

$$r_a(s) = \sum_{i=1}^n q r_i(s) \quad (5.23)$$

From the impulse response  $r_a(s)$  of all variables to the  $i - th$  structural shock at horizon  $s$ , we can identify the impulse vector  $a$  corresponding to the specific  $i - th$  structural shock, in which the impulse responses  $r_a(s)$  is satisfied with the imposed sign for the time interval.

In practice, we take a joint draw from the posterior of the Normal-Wishart for  $(A(L), \Sigma_u)$  and obtain a candidate random orthonormal vector  $q$ . The impulse responses associated with the joint draw  $(A(L), \Sigma_u), q$  are evaluated when the impulse vectors  $\tilde{a}$  satisfy the restrictions and then kept, otherwise discarded. This procedure is repeated until a 1000 draws that satisfy the restriction are obtained.

### 5.3.4 Monetary policy instrument and transmission channel

The VAR model in our study is estimated by means of Bayesian methods using monthly data over the period January 2003 to September 2013. The period of study cover the implementation of unconventional policies under the Troubled Asset Relief Program (TARP) start from 3 October 2008. In the course of this action, the federal funds rate was cut to 0.25 on December 2008 and the 10-year bond yield fell from 3.89 on September 2008 to 2.42 on December 2008. Unlike [Schenkelberg and Watzka \(2013\)](#) who used the monetary base and [Peersman \(2011\)](#) who used commercial bank credit as the monetary policy instrument, we follow [Gambacorta et al. \(2014\)](#) to treat Federal Reserve assets as the unconventional monetary policy instrument.

As mentioned by [McCallum \(1988\)](#), when the economy reaches the zero lower bound interest rate, conventional monetary policy become ineffective and has to

be replaced by a quantitative reaction function. [Gambacorta et al. \(2014\)](#) suggested to use central bank assets as instrument of a quantitative aggregate instead of reserve, the monetary base or commercial bank credit supply since it evidently more accurately gauges unconventional monetary policies during the crisis than others. A large-scale purchase of long term bonds and private securities supplies liquidity for financial market for lending and brings down risk spreads in money markets. Consequently, a higher price on financial assets cause lower long-term interest rates and compresses the yield spread.

The motivation of the unconventional policy intervention is a narrowing down of the yield spread in order to induce economic activity and inflation by reducing risk and borrowing costs. Several empirical studies such as [Rudebusch et al. \(2007\)](#) and [Gilchrist et al. \(2009\)](#) found that a decline in the term premium of 10-year Treasury yields tends to boost real economic activity. The injection of liquidity provides more credit supply and reduces lending and therefore the yield spread charged by banks. As long-term yields decrease, economic agents may anticipate that the central bank will accommodate lower interest rates and expect higher inflation. The impact of unconventional monetary policy eventually stimulates output and raises consumer prices. To examine the transmission mechanism through the change in long term rate, we propose to use the yield spread as the transmission channel and investigate time lag of the impact on output and price level, relative to the impact from a decline in the conventional policy rate. Although [Baumeister and Benati \(2013\)](#) also investigate the impact of unconditional monetary policy on yield spread, their study did not include central bank assets or any monetary instruments so that the transmission mechanism is still unclear.

### 5.3.5 Identification based on sign restrictions

We next discuss what restrictions are imposed on conventional monetary policy, unconventional monetary policy, demand and supply shocks. The propagation of four individual shocks on endogenous variables - industrial production index (IPI), consumer price index (CPI), fed funds rate (FFR), yield slope (YSL) and the Federal Reserve assets (FAS) - are identified by standard sign restrictions.

For the unconventional monetary policy shock, we assume a large scale purchasing of Federal Reserve assets generates higher output and inflation while restricting to zero contemporaneous impact on the fed funds rate, but an unrestricted effect on the yield slope. To order to assess the effectiveness of the non-standard monetary policy in comparison with the standard monetary policy, we identify the conventional monetary policy innovation as a negative shock to the fed funds rate that raises output growth and inflation through purchasing Treasury bills and short-term bonds. By doing so, the Federal Reserve assets increase. The yield spread is still kept unrestricted. In addition to the monetary policy shocks, we also investigate the impacts of two standard aggregate shocks; a demand and a supply shock. A positive demand shock is supposed to stimulate output and inflation at the expense of a higher fed funds rate, while a positive supply shock will increase higher output with lower inflation. Each of these shocks are assumed to affect economic activity and prices whereas the yield spread is left unrestricted and data-agnostic. The restricted contemporaneous responses are set for the initial period. All of the shocks are orthogonal to each other so that the impacts from individual shock are investigated separately. The identified sign restrictions are summarized as in Table 5.1.

TABLE 5.1: Identifying sign restrictions

|                                      | IPI      | CPI      | FFR      | YSL | FAS      |
|--------------------------------------|----------|----------|----------|-----|----------|
| Conventional Monetary Policy Shock   | $\geq 0$ | $\geq 0$ | $\leq 0$ | ?   | $\geq 0$ |
| Unconventional Monetary Policy Shock | $\geq 0$ | $\geq 0$ | $= 0$    | ?   | $\geq 0$ |
| Demand Shock                         | $\geq 0$ | $\geq 0$ | $\geq 0$ | ?   | ?        |
| Supply Shock                         | $\geq 0$ | $\leq 0$ | ?        | ?   | ?        |

Notes:

- 1) The table displays sign restrictions on the responses of the variables in the model after conventional monetary policy, unconventional monetary policy, demand and supply shock
- 2) **Endogenous variables:** IPI: growth of industrial production index, CPI: consumer price index inflation, FFR: fed funds rate, YSL: yield slope, FAS: federal reserve assets
- 3) **Restricted Sign:**  $\leq 0$  less than or equal to 0,  $\geq 0$  greater than or equal to 0,  $= 0$  equal to 0, ? unrestricted

It is worth explaining in more detail about the identification scheme for each shock.

### 5.3.5.1 Unconventional monetary policy shock

The unconventional monetary policy shock is assumed to be an exogenous innovation to Federal Reserve assets. Typically, this non-standard monetary policy is implemented when the policy interest rate approaches a zero-lower bound. Therefore, we need to isolate the effect of conventional monetary policy by combining zero restrictions along all finite horizons. We orthogonalize the fed funds rate for the purpose of disentangling macroeconomic variables from the conventional monetary policy so that they will only be affected by the unconventional monetary policy. The objective of the unconventional policy measures is to compress the yield spread by lowering the long-term yield, given the zero short-term rate. The large scale purchase of government bonds and private securities will influence business confidence and lower term premia. As a result, borrowing costs fall spurring real economic activity and inflation. Hence, the compression in the yield slope requires a zero restriction on the policy interest rate. The identification of a pure unconventional monetary policy shock allows us to examine the impact of a compression of the yield spread within the environment that the policy rate is bound to zero during the period of study. Yet, some empirical studies on the unconventional monetary policy did not orthogonalize the policy interest rate such as [Schenkelberg and Watzka \(2013\)](#) and [Gambacorta et al. \(2014\)](#).

During normal periods, the yield spread compression can be thought of as a lower expected inflation or term premium that lessens the long-term yield, given the central bank still maintains the policy interest rate unchanged. The impact from unconventional monetary policy innovations to growth in output and inflation are allowed to have an immediately positive effect, which is common a assumption for monetary transmission, particularly in the non-standard monetary policy studies, including [Kapetanios et al. \(2012\)](#) and [Baumeister and Benati \(2013\)](#). The dynamic effects on these macroeconomic and financial variables could be used to assess the effectiveness of the quantitative easing measures as well as the transmission mechanism, compared with the conventional monetary policy.



### 5.3.5.2 Conventional monetary policy shock

While we are mainly interested in the effect of a unconventional monetary policy, we also investigate the impact of a conventional monetary policy interest rate shock and other aggregate shocks in order to compare the main shock of interest with other theoretically plausible restrictions as suggested in [Kilian and Murphy \(2012\)](#). The contemporaneous impact of the interest policy shock increases the Federal Reserve balance sheet from buying treasury bills and short-term assets through open market operations. The response of the output growth and inflation are also supposed to increase after the conventional monetary policy shock. However, the responses of the yield slope from the policy interest rate shock is left unrestricted for agnostic purposes. The lower fed funds rate could imply higher expected inflation and ultimately higher long-term yields once the expectation of economic agents is taken into account. Therefore, the unrestricted yield slope can help to explore the transmission that passes through it along the term structure.

### 5.3.5.3 Demand shock

The sign restrictions on the aggregate demand and aggregate supply shocks are imposed in the typical approach as in macroeconomic theory. We assume that the unexpected increase in exogenous demand will induce growth in output and boost inflation. There is also an increase in the policy interest rate corresponding to the higher inflationary pressure. These effects are consistent with an upward shift in the IS curve due to the increase in aggregate spending. The yield slope and Federal Reserve balance sheet are left unrestricted to examine the transmission mechanism of the positive demand shock relative to monetary policy shocks.

### 5.3.5.4 Supply shock

After a positive supply shock, the growth in output is supposed to increase and consequently reduce inflation rate due to higher productivity and lower costs. Nonetheless, the response on interest rate is left unrestricted since the reaction from the central bank is uncertain. As a consequence, we do not impose any restriction on the policy interest rate and other financial variables: the yield slope

and central bank assets. Hence, the data will determine the sign of these responses after the supply shock. The identification scheme for these aggregate shocks are set up in the same way as [Baumeister and Benati \(2013\)](#), however, their study directly examine the unconventional monetary policy response through the yield spread without including any monetary policy instrument.

## 5.4 Conceptual framework

The main objectives of the unconventional monetary policy response to the financial crisis when the economy approaches the zero lower bound interest rate are enhancing economic stability and facilitating liquidity to financial intermediaries and the private sector. To deal with this situation, the central bank is not able to cut the policy interest rate below the zero bound, instead it needs to introduce the non-standard approach to stimulate economy by changing the size of central bank balance sheet rather than varying the amount of money supply.

### 5.4.1 Definition of the unconventional monetary policy

At the zero lower bound interest rate, the central bank supports the financial market by a mix of purchasing financial assets and a lending program to the malfunctioning credit market. This reaction put more emphasis on the asset side of the central bank balance sheet. Basically, the central bank may directly affect financial conditions at the very low policy interest rate by conducting monetary measures that results in substantial changes in central bank's balance sheet in terms of size and composition. The extension of central bank assets require a large scale purchase in long-term asset and securities instead of buying short-term bills as in conventional open market operation. This tremendous increase in balance sheet of the monetary authority is commonly referred to quantitative easing (QE). In 2001, the Bank of Japan first introduced this measure to avert a deflation spiral. In the aftermath of the global financial crisis of 2007 to 2008, it was adopted by the Federal Reserve, the Bank of England and the European Central Bank to stimulate economy at the zero lower bound.

Bernanke et al. (2004) defined unconventional monetary policy as a change in the size and composition of the central bank's balance sheet to influence asset prices and reduce term premia as well as long-term yields. By increasing the size of central bank assets, the central bank injects broad money through long-term government bonds and securities purchasing and directly influences bond yields with long maturities. An increase in asset side of the central bank balance sheet also leads to the expansion of the liabilities side of its balance sheet in the form of greater reserve holding by the banking system. This excess reserve is served as a buffer for liquidity risk at financial intermediation and enhance confidence in financial markets. The extension to longer term assets further shifts the composition of central bank's asset holding from shorter towards longer maturity assets. In contrast to the conventional way which alters short-term policy rate, the non-standard monetary policy affects long term interest rate and eventually the expectation on short-term rate and inflation. Due to lower returns on long-term government bonds, rational economic agents will rebalance their portfolio to invest more in corporate bonds and private securities. Concerning this, the unconventional monetary policy better suits as a measure to stimulate the economy and avert deflation at the zero lower bound interest rate, comparing with the traditional policy approach.

#### 5.4.2 Unconventional monetary policy transmission mechanism

The transmission mechanism through which the unconventional monetary policy works can be explained by two main processes; first how the central bank's balance sheet extension affect the yield spread as a channel to affect financial intermediation at the zero-lower bound interest rate, and second how the change in yield spread influences aggregate demand and alters economic activity as well as price level afterwards. The large scale purchasing in long-term government bonds causes investors to rebalance their portfolio to corporate bonds and securities. By this process, the central bank is able to inject massive amount of liquidity to financial markets and private companies even while financial intermediation malfunctions. The continuing lower long-term yields also signals the central bank's policy intention perceived by economic agents as the expected policy interest remains lower

together with higher expected inflation. Much of how the unconventional monetary policy might affect the economy through portfolio rebalancing and signalling are discussed as follows.

#### 5.4.2.1 Portfolio rebalancing

The implementation of the unconventional monetary policy basically work through the purchase of long-term government bonds from institutional investors, particularly financial intermediaries such as commercial banks, insurance companies and pension funds. The enormous demand for long-term financial assets raises up prices and consequently move down the long-term yields. This procedure is the first part of the unconventional monetary policy transmission mechanism from a change in central bank's assets that affects yield spreads.

At the zero lower bound interest rate, money and short-term bond are perfectly substitute in the sense that both of them pay no interest rate. Any attempts to increase money supply by conventionally purchasing short-term bond fail to stimulate the economy when it is struggling with a liquidity trap. However, long term bonds are imperfect substitute relative to holding money. As a result, the central bank can purchase long-term bonds to implement quantitative easing. When the central bank purchases long-duration assets, the aggregate amount of duration risk that remains in the market is more likely to reduce and therefore the term premia and associated returns are reduced. Investors will seek to re-invest the money obtained from selling long-term government bonds and search for alternative financial assets - especially corporate bonds or private securities, which now provides better returns. This means that the investors may move away from their preference of a particular segment of the yield curve, which was named by [Modigliani and Sutch \(1966\)](#) as preferred habitat. By doing so, the investors rebalance their portfolio and change the proportion of longer-maturity towards shorter-maturity assets. The portfolio switching has viewed as the crucial mechanism to facilitate the unconventional monetary policy measures. However, if there is no such portfolio rebalance as proposed by [Tobin \(1969\)](#) and [Brunner and Meltzer \(1972\)](#), the unconventional monetary policy might become ineffective. [Eggertsson and Woodford \(2003\)](#) argued if the representative agent cannot distinguish between

long-term government bonds and their own assets, they will not willing to swap any financial assets. [Curdia and Woodford \(2011\)](#) then suggested the central bank should conduct direct lending or credit easing instead.

#### 5.4.2.2 Signalling and wealth effect

A second part of the transmission mechanism is the pass-through process via the yield spread to affect the economy and price level. The information from the long end of the term structure reveals about future policy interest rates and expectations on future inflation. Once economic agents realize this information, it will affect their aggregate spending. The large scale purchasing on financial assets raises up asset prices, which in turn creates more wealth for asset holders. Lower cost of borrowing and higher wealth effects will consequently stimulate economy.

If the central bank convinces economic agents of its commitment to pursue low interest rate at or near the zero lower bound for longer periods, this may help maintain credibility and keep inflation expectations positively anchored. Economic agents take into account this information and anticipate a continuing low interest rate with higher expected inflation and therefore increase their consumption and investment. [Vayanos and Vila \(2009\)](#) and [Greenwood and Vayanos \(2010\)](#) provided New Keynesian models to explain how do investors change their expectation affect yields and macroeconomic activity through signalling channel. [Joyce et al. \(2012\)](#) also offered empirical evidence that quantitative easing measure eventually boost household wealth, especially in the form of pension. As a result, the unconventional monetary policy is not only beneficial for corporate firms from falling cost of funds, but also broadly affects household with greater wealth.

#### 5.4.3 The U.S. unconventional policy measures and implementation

In this subsection, we outline the unconventional monetary policy measures that the Federal Reserve conducted in response to the global financial crisis, especially after the collapse of Lehman Brothers in September 2008. The non-standard

scheme fundamentally focused on purchasing long-term government bonds instead of direct lending to financial intermediations which were malfunctioned. The large acquisition in long-term bonds caused a dramatically extension in Federal Reserve balance sheet and allowed it to provide liquidity to financial market.

The financial markets in the US were evidently suffered from an immense losses from subprime mortgages since 2007. The first sign of financial distress started from Bear Stearns failure to maintain a sufficient level of capital against risks involved in the subprime market. Afterwards, several securities and companies with a huge exposure were downgraded by credit rating agencies. On August 2007, the Federal Reserve announced its effort to provide liquidity to financial markets and then sharply cut fed funds rate from 5.25 to 4.75 percent or by 50 basis points on September 2007. By September 2008, the on-going turmoil reached a critical point when Lehman Brothers filed for bankruptcy and the Federal Reserve took over two mortgage-lending Fannie Mae and Freddie Mac and provided bailout loan to the American International Group (AIG). In response to this, the Troubled Asset Relief Program (TARP) was set up on October 2008 to provided necessary funds to purchase assets and equity from financial institutions under the implementation of unconventional monetary policy.

During November 2008 to March 2009, Federal Reserve mainly purchased residential mortgage-related assets. By November 2008, the first attempt was announced to spend 100 billion dollars in purchasing government-sponsored enterprise (GSE) debt and 500 billion dollars in mortgage-backed securities (MBS). The additional package was subsequently announced on March 2009 to purchase 100 billion dollars in government-sponsored enterprise (GSE) debt, 50 billion dollars in mortgage-backed securities (MBS) and 300 billion dollars in long-term Treasury securities. The initial asset purchase program carried out from 2008 to 2009 is commonly known as a quantitative easing 1 or QE1 program. The QE1 was primarily designed to facilitate liquidity for purchasing houses and real estates and reduce term premia, which in turn improve market confidence and return financial intermediaries to function again.

Although the QE1 was found to abate financial turmoil, there were still concerns

about potentially deflation over the second half of 2010. Besides, economic activity remained sluggish. On November 2010, the Federal Reserve announced to purchase an additional 600 billion dollars in U.S. Treasuries to promote a stronger economic recovery and avert deflation. This consecutive quantitative easing program was commonly called as QE2. It was specifically designed to increase the inflation through lower long-term yield and higher expected inflation. Therefore, Federal Reserve focused on purchasing Treasury bonds with longer maturity. Even though, the QE2 did not much change long-term yield when it was implemented since the market had already anticipated. Before the official announcement, the Federal Reserve signalled to market for its further asset purchases. The expectation of renewed QE package undermined market reaction and thus led to less response.

By June 2011, the Federal Reserve has spent 1.725 trillion dollars for QE1 and 600 billion dollars on QE2. However, the unemployment rate remained elevated. On September 2011, the Federal Reserve announced the implementation of Operation Twist to purchase 400 billion dollars of bonds with maturities of 6 to 30 years and to sell bonds with maturities less than 3 years. This unconventional measure allowed Federal Reserve to use the money from selling short-term assets to purchase longer-term assets without printing more money. Therefore, it avoided inflationary pressure though long-term yields were still lowered to spur economy.

Until the second half of 2012, The Federal Reserve still required sustainable improvement in the labor market. However, it could no longer sell short-term Treasury securities due to insufficient holding available. Hence, Federal Reserve announced on September 2012 to spend an open-ended purchases of 40 billion dollars of mortgage debt per month and additional 45 billion dollars on longer-term Treasury securities per month in a QE3 measure. The Federal Reserve planned to end its QE3 on October 2014.

To conclude, the Federal Reserve's unconventional monetary policy measures that have been conducted after the global financial crisis comprises four distinct programs; quantitative easing 1 (November 2008 to March 2010), quantitative easing 2 (November 2010 to June 2011), Operation Twist (September 2011 to September

2012) and quantitative easing 3 (September 2012 to October 2014). These massive asset purchase aimed to reduce long-term yields and raise economic growth as well as avert deflation when conventional monetary policy was binding at the zero lower bound constraint.

## 5.5 Data and descriptive statistics

Before getting into the model, we present information about the data used in this study. The data description is provided in subsection 5.5.1. In subsection 5.5.2, we then present the descriptive statistics of the data.

### 5.5.1 Data description

Our data set for the B-SVAR model comprises 5 monthly variables namely, the industrial production index (IPI), the consumer price index (CPI), the effective fed funds rate (FFR), the spread between the 10-year Treasury bond yield and the 3-month Treasury bill rate or the yield slope (YSL) and the Federal Reserve's balance sheet measure or Fed assets (FAS), covering the period from January 2003 to August 2013. We chose the seasonally adjusted industrial production index (IPI) as a proxy for monthly output and the seasonally adjusted consumer price index for all items less food and energy to represent core inflation. The industrial production index, consumer price index (CPI), the 10-year Treasury bond yield and the 3-month Treasury bill rate are obtained from Thomson-Reuters Datasstream, while the effective fed funds rates are gathered from the Federal Reserve Economic Data (FRED) and the Federal Reserve's balance sheets are released by the Federal Reserve Board of Governors. The start of the sample is subjected to the fact that the monthly data series of the Federal Reserve's balance sheets begin from January 2003. Our samples cover the pre-crisis period over 2003 to 2008 and the period of Federal Reserve's quantitative easing, starting from the Federal Open Market Committee (FOMC) announcement to implement the first Large Scale Asset Purchases programme (LSAP1) or QE1 on November 2008 up to the third programme (LSAP3) or QE3 executed from September 2012. We also consider the possibility of taking into account the unconventional monetary policy



implementation into our benchmark B-SVAR model. Hence, we include a dummy variable to represent the state implementing the quantitative easing measure from October 2008 onwards.

To analyze the effectiveness and the transmission mechanism of the unconventional monetary policy, we use the Federal Reserve's assets as a monetary policy instrument since they directly represent the amount of asset purchases by the Federal Reserve (instead of using the monetary base, Fed credit or Fed reserves which are subsets of the balance sheet).

### 5.5.2 Descriptive statistics and unit root test

The descriptive statistics of the data series used in the B-SVAR model are reported in Table 5.2 and then plotted in Figure 5.1. For each variable, we report the mean, standard deviation, minimum, maximum, autocorrelation coefficient at various displacements and the Augmented Dickey-Fuller test statistics for stationarity.

TABLE 5.2: Descriptive statistics of macroeconomic variables

| Variables | Mean    | Std Dev | Min    | Max     | p(1) | p(12) | p(30) | ADF   |
|-----------|---------|---------|--------|---------|------|-------|-------|-------|
| IPI       | 94.26   | 4.33    | 83.76  | 100.82  | 0.98 | 0.30  | -0.48 | 0.35  |
| CPI       | 213.16  | 12.62   | 192.40 | 234.30  | 0.98 | 0.72  | 0.32  | 2.55  |
| FFR       | 1.71    | 1.88    | 0.07   | 5.26    | 0.99 | 0.67  | -0.11 | -1.37 |
| YSL       | 2.46    | 1.12    | 0.11   | 4.37    | 0.95 | 0.39  | -0.54 | -0.91 |
| FAS       | 1643.03 | 934.99  | 712.81 | 3644.46 | 0.97 | 0.73  | 0.32  | 2.03  |

As shown in Table 5.2, the Federal Reserve's assets has a much higher volatility than other variables, indicating that it was affected by the large scale increase under the quantitative easing scheme. The autocorrelation coefficients at one-month lag show all variables are strongly positive autocorrelated. We also examine whether the time series are unit root by applying the augmented DickeyFuller (ADF) test. The unit root test results show that all variables have unit roots. The ADF tests fail to reject the null hypothesis that the examined variables are non-stationary. In order to proceed our empirical analysis, we therefore transform the industrial production index (IPI), the consumer price index (CPI) and the

Federal Reserve's balance sheet into log-difference of monthly observations. This transformation allows us to redefine them as output growth rate, inflation rate and rate of change in Federal Reserve's assets. The fed funds rate and yield slope are left untransformed since they have already measured in percentage form. Thus, we can infer the impact from change in policy interest rate or yield slope straightforward.

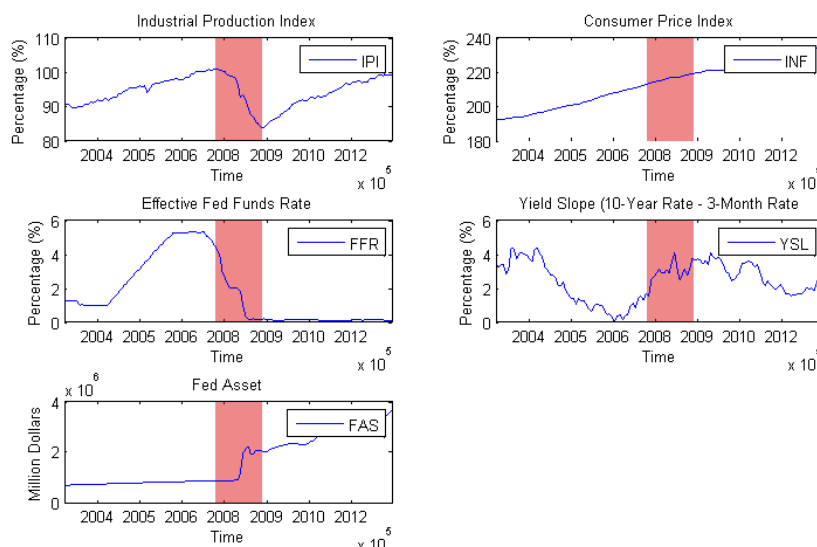


FIGURE 5.1: Time series of macroeconomic variables; industrial production index, consumer price index, effective fed fund rate, yield slope and Federal Reserve assets

Note: Shaded areas indicate the U.S. recession of 2008 to 2009

We also plot time series of all variables in Figure 5.1. As we can see from the graphical presentation, there are clear indications of all variable being non-stationary by having a trend or seasonal pattern. From 2004 to 2008, the industrial production index, consumer price index and fed funds rate continuously evolved upward. However, the eruption of financial crisis during 2008 to 2009 shifted the industrial production index to go down and reach the bottom in the first half of 2009. It also caused the effective fed funds rate curves to the regime of lower rates. This policy interest rate has been bottomed at the zero lower bound since the end of 2008, representing the period in which the Federal Reserve implemented the alternative monetary policy regime. From 2008 onwards, there was a huge surge in the Federal Reserve's assets from the large scale purchase on long-term assets under the quantitative easing (QE) measures. The extension in Federal Reserve's balance

sheet also causes a downward trend in yield slope. As the figures show, there are 3 distinguished periods for the descendent yield spread which are consistent with the precedent quantitative easing time lines; QE1 (November 2008 - March 2010), QE2 (August 2010- June 2011) and operation twist (September 2011 - June 2012). The large scale long-term government bond and securities purchases drove up asset prices and compressed the yield spread once the fed funds rate remained at zero lower bound.

## 5.6 Results

In this section, we provide empirical evidence of unconventional monetary policy, comparing it with conventional monetary policy and other aggregate shocks based on the benchmark specification and identification as explained earlier. First, we estimate impulse responses and discussion on the relative response to conventional and unconventional monetary policy are shown in subsection 5.6.1. Then, the comparison of economic impacts generated by the conventional and unconventional monetary policy is presented in subsection 5.6.2.

The benchmark SVAR is estimated over the period January 2003 to August 2013 with five endogenous variables; the industrial production index, the consumer price index, the difference between the 10-year Treasury bond and 3-month treasury bill yield or the yield slope, the policy interest rate (federal funds effective rate) and Federal Reserve assets. We use a Bayesian approach to estimate posterior distributions of the reduced form VAR. Assuming a Normal-Wishart priors, we take a joint draw from the unrestricted Normal-Wishart posterior for the VAR parameters and a random orthogonal variance-covariance decomposition that allows for constructing associated impulse responses. The impulse responses that satisfy the imposed restrictions are kept. Then we use 1000 successful draws from the posterior to produce the impulse response results. To compare the impacts of unconventional monetary policy to conventional monetary policy, we calibrate one standard deviation of the unconventional monetary policy shock so that the model generates the same change of Federal reserve assets used by the conventional monetary policy shock to increase by 10 basis points (0.1 percentage points) of yield slope and investigate their impulse responses relative to those of the conventional

monetary.

## 5.6.1 Impulse response analysis

Initially, we examine the impact of conventional monetary policy on yield spreads and other macroeconomic variables and compare the results with unconventional monetary policy and standard aggregate shocks. As described in Section 3, we implement Bayesian structural vector autoregression with sign restrictions and examine the impulse responses at horizons up to 40 months. The impulse responses are reported as median responses following a shock equal to one standard deviation. Figures 2-5 show the impulse responses of the industrial production index, the consumer price index, the difference between the 10-year Treasury bond and 3-month treasury bill yield or the yield slope, the federal funds effective rate or policy interest rate and the Federal Reserve assets to one standard deviation of each identified shock based on the specification and sign restrictions explained in the previous part.

In all figures, the middle green lines represent the median impulse responses from a Bayesian estimation with 1000 draws, while the bands indicate the 16 and 84 percentiles of the posterior distribution of the impulse responses. The 68 percent range of the responses represents the confidence bands of successful models that satisfy the identification scheme for each period. Under the assumption of a normal distribution, a one standard deviation of the identified shock would generate a statistically significant response if the confidence intervals associated with the impulse response function (IRFs) do not contain zero for any specific time horizon. Our Bayesian simulation and statistical inference follow the method described in Uhlig (2005), Fratzscher et al. (2010) and Arias et al. (2014).

### 5.6.1.1 Responses to a conventional monetary policy shock

To analyze the effectiveness of quantitative easing during the zero lower bound interest rate, it is useful to examine whether the empirical results from the conventional monetary policy implementation, estimated by the model, is able to

capture structural relationship between the policy interest rate shock and other macro variables.

Figure 5.2 shows the effects of conventional monetary policy. The response of a negative one-standard deviation conventional monetary policy innovation has been restricted to increase growth of output and inflation. The reaction on the federal reserve balance sheet also has to be positive. The yield spread, which is a key variable for transmission mechanism, has been left without restriction.

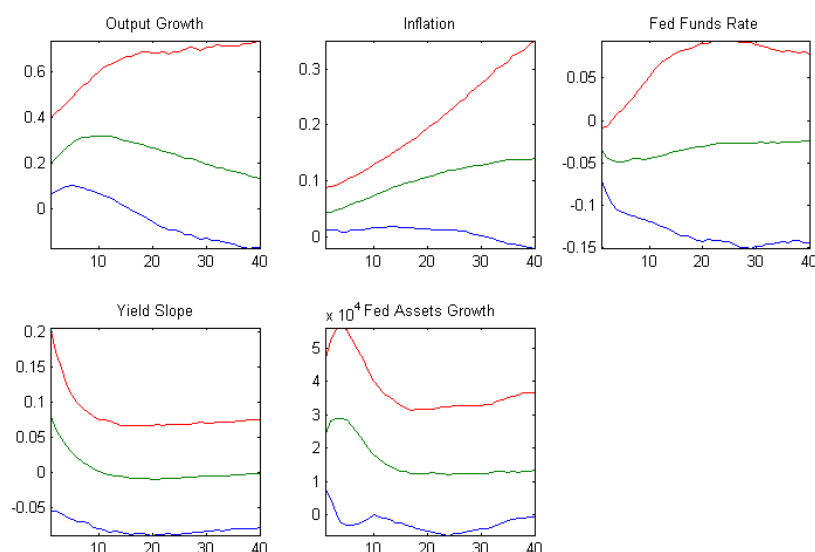


FIGURE 5.2: Median IRFs (green line) of a negative one-standard deviation conventional monetary policy shock together with 16th (blue line) and 84th (red line) percentiles or 68-percent error bands for the estimated median impulse responses

The expansionary monetary policy results in the immediate increase in output growth and inflation as restricted. In particular, the change in the industrial production index and the consumer price index rise up in response to lower interest rate and stay positive throughout 40 months horizon. Especially, the responses on inflation are persistent. For the yield spread that has been left unrestricted, we find the lower interest results in a widened yield spread that suddenly jumps to 5 basis point and then returns to baseline within 2 years. The effect from expansionary monetary policy on the change in Federal Reserve's asset remains positive in line with a negative change in the interest rate throughout 40 consecutive months. To accommodate expansionary monetary policy, the Federal Reserve needs to buy

short-term government securities which leads to higher Federal Reserve's assets over the following 40 months. The impulse response results produced by conventional interest rate innovations are consistent with the early structural vector autoregressive (SVAR) model studies on the monetary transmission mechanism such as [Bernanke and Blinder \(1992\)](#), [Bernanke and Mihov \(1998\)](#) and [Christiano et al. \(1999\)](#) and SVAR with sign restriction on monetary policy, including [Uhlig \(2005\)](#), [Peersman \(2005\)](#) and [Dungey and Fry \(2009\)](#).

### 5.6.1.2 Responses to a unconventional monetary policy shock

The main interest of this study is the effectiveness of unconventional monetary policy intervention. We investigate the consequences of structural shocks to the yield spread and other macroeconomic variables, given the policy interest rate is restricted at zero. The response of the change in Federal Reserve's asset purchase has been restricted to be positive following the shock while growth in output and inflation must have a positive reaction. However, the yield slope which is the main variable of interest that transmits central bank asset shock to macro-variables, has been left unrestricted.

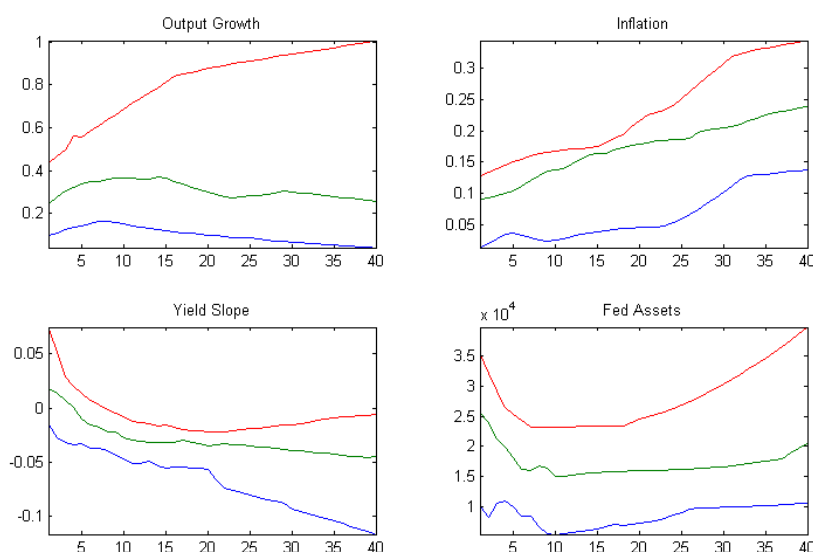


FIGURE 5.3: Median IRFs (green line) of a negative one-standard deviation unconventional monetary policy shock together with 16th (blue line) and 84th (red line) percentiles or 68-percent error bands for the estimated median impulse responses

From Figure 5.3, output growth and inflation increase after the unconventional monetary policy shock. In particular, the growth rate of the manufacturing production index initially increases 0.2 percentage point. Within one year, it attains 0.4 percentage point of growth and maintains this level for three consecutive years. Even though inflation is raised up only by around 0.1 percentage point at the beginning, the impact gradually increases and eventually grows around 0.2 percentage point at the 40 month horizon. Thus, the SVAR with sign restriction suggest that the unconventional monetary policy shock does in fact increase economic activity.

The unconventional monetary policy innovation also causes the growth rate in Federal Reserve balance sheet to jump up significantly and stay positive for over four years. However, the adjustment of the term structure to change the yield curve shape is actually delayed. It takes around a half year for the yield slope to become negative. The figure shows the yield slope gradually decreases from around positive 2 basis points in the first period to reach negative 5 basis points four years later. The yield spread between the 10 year bond and 3 month bill yield continuously compresses so that the long-term yield become less than short-term yield and the yield curve inverts. A downward yield curve indicates the continuing low interest rate policy which then has been realized by economic agents. The expectation that the central bank will keep the interest rate at the zero lower bound and bring down the long-term rate for an extended period potentially stimulates the economy and raises inflation.

Hence, these results indicate that quantitative easing affects the real economy and inflation through lowering long-term rates. Importantly, this effect has been investigated by using an agnostic approach with an unrestricted yield slope. Overall, the results indicate that unconventional monetary policy is effective in stimulating the economy. Both output growth and inflation increase, associated with a compression in yield slope. Compared to the transmission mechanism of conventional monetary policy shocks, the response of macroeconomic variables are qualitatively similar except the long-term rate pass through instead of the short-term interest rate.

### 5.6.1.3 Comparison of impulse responses to conventional and unconventional monetary policy shock

In order to examine the transmission mechanism and the effectiveness of the unconventional monetary policy relative to the conventional monetary policy, we make a comparison on the impulse responses from both policy implementations. For the conventional monetary policy, an unexpected fall in the policy rate is followed by an rise in economic activity and inflation. The similar results on output growth, inflation and central bank balance sheet are also generated by economic non-standard monetary policy. Therefore, unconventional monetary policy is effective in stimulating economy and can be used as an alternative when conventional monetary policy becomes ineffective, especially in a liquidity trap. Nonetheless, the unconventional monetary policy shocks tend to generate a higher impact on inflation compared with standard monetary policy. The transmission mechanism that passes through the yield spread by changing the yield curve shape is in fact delayed. In particular, it takes around half a year for long-term yield to be lower than short-term yield.

Comparing the relative magnitudes of the unconventional monetary policy shock with the traditional monetary policy on the response of output and inflation, we find the initial impact from unconventional monetary policy on the growth rate of the industrial production index is equivalent to the conventional way. The unconventional monetary policy more persistently raises inflation avoiding deflation. We also find the absolute size of the response on inflation is larger than the impact on output growth. This evidence supports the idea to implement unconventional monetary policy to deal with a deflationary spiral, despite it is failure in Japan as mentioned in [Schenkelberg and Watzka \(2013\)](#).

In addition, the compression on yield spread from the lower long-term yield does improve the economy in a liquidity trap and deflation. The large scale purchase in long term government bonds and private securities restore consumer and business confidence. The economic agents realize the on-going stimulus and anticipate lower expected real interest rates due to lower risk and higher expected inflation. As a result, output and inflation will increase and these results will be even more



pronounced and longer lasting because of the expectation. The role of expectations for the transmission mechanism during a zero lower bound interest rate is evidenced by [Eggertsson and Woodford \(2004\)](#). However, we find the long-term yield may be sluggish to fall after the quantitative easing is conducted. Our findings also confirm other previous studies, especially [Lenza et al. \(2010\)](#), [Peersman \(2011\)](#) and [Schenkelberg and Watzka \(2013\)](#), who also noticed a delayed response from non-standard monetary policy. Unlike their studies, we investigate the transmission mechanism through the yield slope and find out the sluggish adjustment is in fact caused by the lagged lowering of the long-term rate.

#### 5.6.1.4 Responses to a demand shock

In addition to examining the responses of conventional and unconventional monetary policy shocks, we also compare the impulse responses of a balance sheet shock with two other standard structural shocks: demand and supply shocks as shown in Figure 5.4 and 5.5 respectively. We follow [Kapetanios et al. \(2012\)](#) and [Schenkelberg and Watzka \(2013\)](#) and investigate the shocks to demand and supply to compare the quantitative importance of unconventional monetary policy with the impacts from aggregate shocks. The two studies mentioned did not identify a monetary policy instrument as by central bank assets in their analyses.

As mentioned before, we impose positive restrictions on output, inflation and the policy interest rate, while the yield slope and the central bank balance sheet are left unrestricted. In Figure 5.4, we find the innovation on aggregate demand increases output, inflation and interest rate as restricted. Noticeably, an increase in demand persistently boosts inflation and economic activity over the 40 months horizon. The interest rate also rises for more than one years, associated with the inflationary pressure. Theoretically, when the central bank raises the policy interest rate by selling treasury bills, the yield spread will be compressed and will cause central bank assets to be smaller. We find the central bank balance sheet and the yield slope evidently decrease, even though the impacts are minimal. Likewise [Kapetanios et al. \(2012\)](#) reported the insignificant effects on yield spread in their study as well. Similarly, [Schenkelberg and Watzka \(2013\)](#) who chose to use the

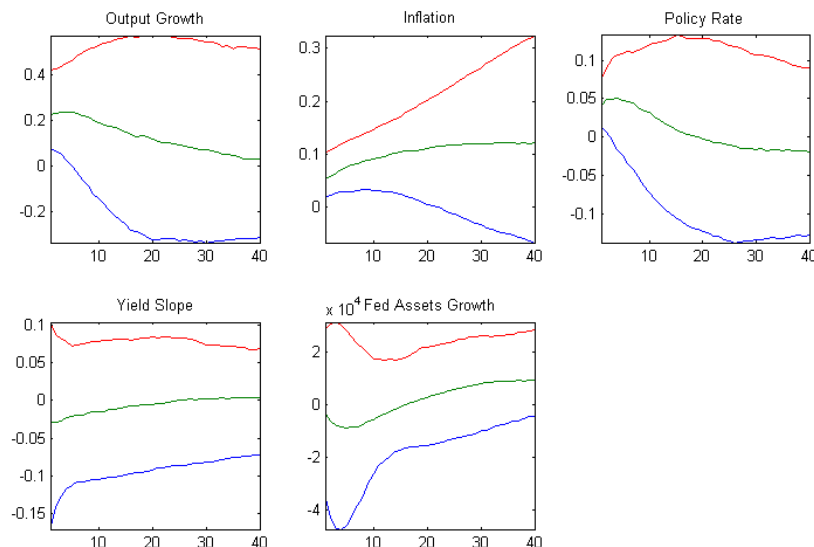


FIGURE 5.4: Median IRFs (green line) to a positive one-standard deviation demand shock together with 16th and 84th percentiles for the U.S. for selected months

long-term yield instead of the yield spread found a minimal decrease in the long-term yield from a demand shock.

#### 5.6.1.5 Responses to a supply shock

For the positive shock in supply, we initially restrict output to increase while inflation decreases. Following [Kapetanios et al. \(2012\)](#) and [Schenkelberg and Watzka \(2013\)](#), the policy interest rate is left unrestricted. In addition, we allow the unrestricted yield slope and central bank balance sheet.

From Figure 5.5, after positive shock in supply, output growth increase whereas inflation decrease, consistent with the imposed restrictions. Over the 40 months horizon, output growth remains positive while inflation stays negative. As the price level decreases, economic agents will anticipate deflation and the policy interest rate is then likely decrease. To lower the policy interest rate, the central bank is required to purchase Treasury bills, so that the central bank balance sheet expands. This policy implementation, in turn, widens the spread between long-term and short-term yields. As we can see, the policy interest rate decreases while the yield slope and the central bank assets increase. It should be noticed that the

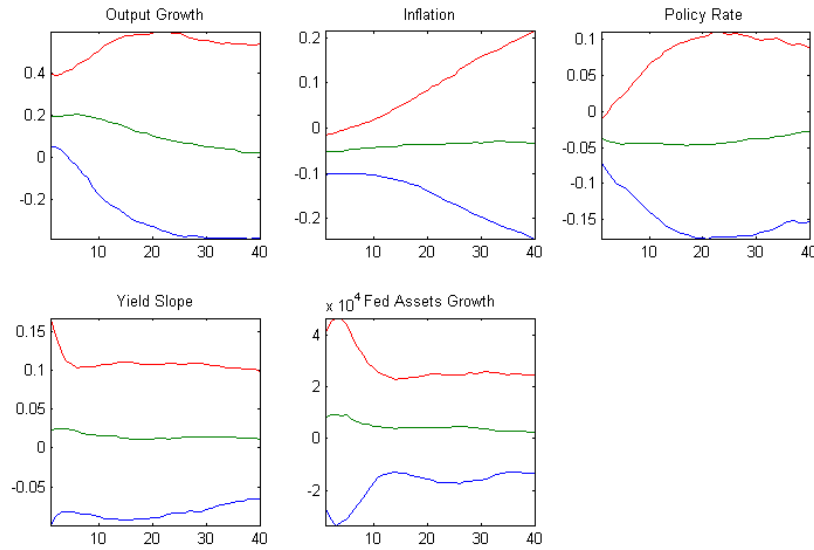


FIGURE 5.5: Median IRFs (green line) to a positive one-standard deviation supply shock together with 16th and 84th percentiles for the U.S. for selected months

effect on output growth and inflation are persistent. [Schenkelberg and Watzka \(2013\)](#) also find a minor effect on the yield slope. However, the effect on output in their study is actually short-lived while we find it is more persistent.

Comparing the relative magnitude of the responses on growth of production and inflation from aggregate shocks with those from the unconventional monetary policy shock, we find the responses produced by the unconventional monetary policy shock by aggregate shocks are on the same level. This findings suggests the unconventional monetary policy is an effective way to stimulate economy in a liquidity trap with the zero lower bound interest rate. It is able to produce higher output growth and inflation in a similar magnitude as an aggregate demand stimulus. Note, [Schenkelberg and Watzka \(2013\)](#) reported the implementation of unconventional monetary policy in Japan could not induce output as much as an aggregate demand shock.

In summary, impulse responses produced by unconventional monetary policy compared with standard monetary policy and other aggregate shocks indicate that unconventional monetary policy measures effectively stimulate economic activity.

The impact on output growth and inflation produced by the unconventional monetary policy is similar to conventional monetary policy. This is in spite of the fact that the transmission mechanism through a compression in yield slope is sluggish. Also, the scale of change in output growth from the increase in central bank assets is also similar to the response produced by the aggregate demand shock. Therefore, the purchasing of long-term bond and securities can be used as an alternative monetary policy measure to stimulate the economy and avoid deflation when standard monetary policy becomes ineffective.

### 5.6.2 Comparison between conventional and unconventional monetary effectiveness

Having examined the effectiveness of unconventional monetary policy compared with conventional monetary policy, we found the non-standard monetary policy produces qualitatively similar responses to the typical monetary policy. In this subsection, we analyze whether the responses created by non-standard monetary policy quantitatively differ from the standard monetary policy. For this purpose, we compare the magnitude of change in endogenous variables through impulse responses following unconventional and conventional monetary policy shocks with the same size of Federal Reserve's assets shock. The only difference in the former operates on the long term rate, and the later on the federal funds rate.

To generate a benchmark for comparison, we extract the value of Federal Reserve's assets following a one standard deviation shock of conventional monetary policy that raises 0.1 percentage point or 10 basis point of the policy rate. Then, we calibrate the unconventional monetary policy shock with the same size of change in Federal Reserve's assets and compare the magnitude of impacts on endogenous variables relative to conventional monetary policy shock. The impulse responses of the two different policies which are created by the same change in balance sheet, are presented in Figure 5.6 and 5.7 as well as the estimated relative responses are reported in Table 5.3.

### 5.6.2.1 Impulse response comparison

We compare the magnitude of dynamic impulse responses to all endogenous variables after a one standard deviation shock to the conventional policy regime and the same increase in Federal Reserve's assets for the unconventional one. The responses to the conventional monetary policy shock are used as benchmarks for comparison with the results produced by the unconventional monetary policy.

#### (1) Responses to a conventional monetary policy shock

As mentioned before, we start our exercise by generating a one standard deviation shock of the conventional monetary policy that raise 10 basis points of yield slope and the implied Federal Reserve's asset jump.

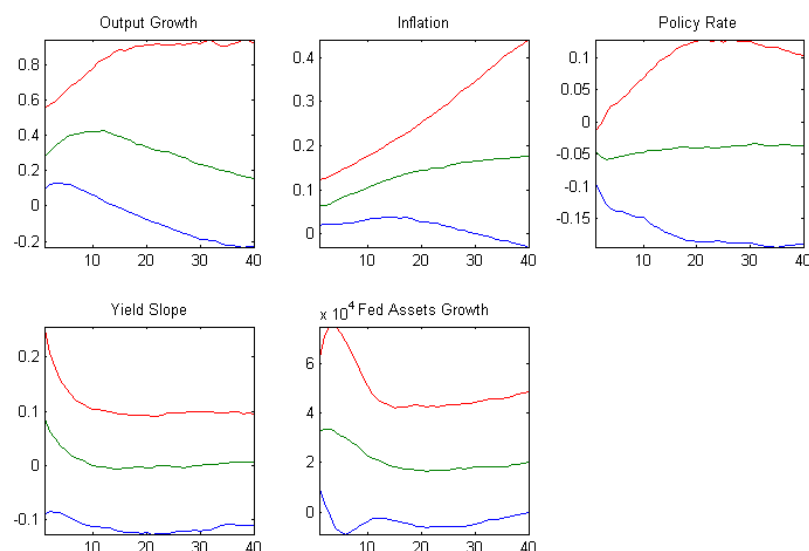


FIGURE 5.6: Median IRFs (green line) of a one-standard deviation shock to a negative conventional monetary policy shock (with the same change in Federal Reserve's assets as conventional monetary policy shock) together with 16th (blue line) and 84th (red line) percentiles or 68-percent error bands for the estimated median impulse responses

From Figure 5.6, a one-standard-deviation shock to the conventional monetary policy that raises 10 basis points in the yield slope requires around 0.03 percentage points of growth in Federal Reserve's assets. The increase in short-term asset purchase through open market operation results in initially lower fed funds rate

by around 0.05 percentage point or 5 basis points and stays negative throughout the 40 month horizons. The lower policy interest rate immediately induces the growth rate of output by around 0.3 percentage points. The growth rate of output gradually reach the peak at 0.4 percentage point in one year later. The initial response of inflation is smaller, about 0.06 percentage point. However, the impact on inflation is more persistent. This exercise also suggests the transmission mechanism is based on the expectation. The initial cut in the fed funds rate by 5 basis points amplifies the yield spread by 10 basis points. The continuing expansionary monetary policy is realized by market, who form an expectation of lower future policy interest rates. Within around one year, long-term yields gradually decrease and compress yield spread towards zero. Thus, the conventional monetary policy works through the policy interest rate channel while the yield slope plays a less important role in transmission mechanism.

## **(2) Responses to an unconventional monetary policy shock**

Now, we assess the magnitude of impulse responses on endogenous variables after a shock of the unconventional monetary policy, given the same change in Federal Reserve's assets as we obtained for the conventional monetary policy. We calibrate the 0.03 percentage points of growth in the Federal Reserve's assets and examine the dynamic adjustment of impulse responses following this unconventional monetary policy shock as shown in Figure 5.7.

The key finding from Figure 5.7, is that the growth of output generated by the unconventional monetary policy shock is in fact a little less than the change in output produced by the conventional monetary policy. A 0.03 percentage points of growth in Federal Reserve's assets induces 0.23 percentage points in output growth whereas we got 0.3 percentage points of output growth from standard policy. If the central bank prefers to achieve higher output growth, it needs to increase Federal Reserve assets more.

Another important result is that the unconventional monetary policy shock effectively averts deflation. By a change of the same size of the central bank balance sheet, the unconventional monetary policy generates around 0.15 percentage points

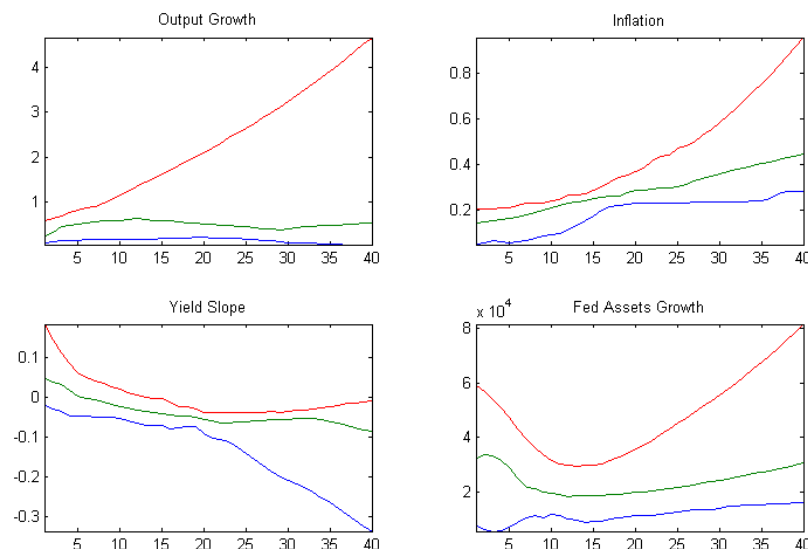


FIGURE 5.7: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock (with the same change in Federal Reserve's assets as unconventional monetary policy shock) together with 16th (blue line) and 84th (red line) percentiles or 68-percent error bands for the estimated median impulse responses

of initial inflation, which persistently increases to reach 0.4 percentage points after 40 months, while the standard policy can only raise initial inflation by 0.05 percentage points. We also find a lag response in the change of shape of the yield curve to an inverted slope. After the unconventional monetary policy shock, the yield spread continuously reduces from its initial level at 5 basis points to become negative in six months and eventually attain -9 basis points at the 40 months horizon. In the context of rational expectation, a reduction in the long-term yield that changes the yield curve to be downward slope shape will be perceived by economic agents as a signal for a continuing zero bound policy rate and higher inflation. The anticipation of low interest rates and expected inflation in the future induces aggregate demand and stimulates the economy.

From the exercise on comparison of the impulse response, we find the effects of the unconventional monetary policy on output growth are lower than conventional monetary policy, given by the same change in central bank's assets. The central bank needs to purchase much more assets than the standard amount in conducting unconventional policy to have the same effect. Notwithstanding, unconventional monetary policy is more effective in averting deflation as compared to conventional

policy.

### 5.6.2.2 Relative responses

To provide quantitative evidence for a comparison between the conventional ( $c$ ) and unconventional ( $u$ ) monetary policy, we calculate relative responses of three endogenous variables after the unconventional monetary policy shock in comparison with conventional monetary policy shock; which are change in growth of Federal Reserve's assets ( $\Delta FAS_c/\Delta FAS_u$ ), change in growth of the industrial production index ( $\Delta IPI_c/\Delta IPI_u$ ) and change in inflation ( $\Delta CPI_c/\Delta CPI_u$ ). The relative responses are separated into three main scenarios: Federal Reserve's assets used for two monetary policy measures are equal ( $FAS_c = FAS_u$ ), industrial production growth index produced by two monetary policy measures are equal ( $IPI_c = IPI_u$ ) and inflation produced by the two monetary policy measures are equal ( $CPI_c = CPI_u$ ). For each scenario, we report the initial relative responses at the first period and the medium run response at 24-month horizons. The results are reported in Table 5.3. The common explanation for all figures in the table is that whenever the number is greater than one this means the unconventional monetary policy requires higher Federal Reserve assets or produces more output or inflation relative to the conventional monetary policy.

The top panel of Table 5.3 presents the relative responses of Federal Reserve assets, growth in output and inflation at the first period and at the 24-month horizon when the conventional and unconventional monetary policy are conducted with the same change in Federal Reserve assets. As we can see, the non-standard monetary policy initially generates relatively less output than the standard monetary policy, represented by relative response ratio equals 0.9. However, the unconventional monetary policy becomes relatively more effective in the medium run or two years later. It also comparably outperforms the conventional monetary policy in the first period with regard to the relative response ratio greater than one for inflation.

From the middle panel of Table 5.3, we compare the relative change in central bank's assets and inflation if both monetary policy regimes are calibrated to generate the same impact on output growth at the first period and the 24 period. If



TABLE 5.3: Relative response produced by conventional and unconventional monetary policy

| Scenario 1: $FAS_c = FAS_u$       |          |           |                                     |          |           |
|-----------------------------------|----------|-----------|-------------------------------------|----------|-----------|
|                                   | Period 1 | Period 24 |                                     |          |           |
| $\Delta FAS_c / \Delta FAS_u$     | 1.00     | 1.31      |                                     |          |           |
| $\Delta IPI_c / \Delta IPI_u$     | 0.90     | 1.45      |                                     |          |           |
| $\Delta CPI_c / \Delta CPI_u$     | 2.43     | 0.89      |                                     |          |           |
| Scenario 2: $IPI_{c1} = IPI_{u1}$ |          |           | Scenario 3: $IPI_{c24} = IPI_{u24}$ |          |           |
|                                   | Period 1 | Period 24 |                                     | Period 1 | Period 24 |
| $\Delta FAS_c / \Delta FAS_u$     | 1.01     | 1.45      | $\Delta FAS_c / \Delta FAS_u$       | 0.32     | 1.13      |
| $\Delta IPI_c / \Delta IPI_u$     | 1.00     | 1.12      | $\Delta IPI_c / \Delta IPI_u$       | 0.32     | 1.00      |
| $\Delta CPI_c / \Delta CPI_u$     | 1.34     | 0.07      | $\Delta CPI_c / \Delta CPI_u$       | 0.43     | 0.02      |
| Scenario 4: $CPI_{c1} = CPI_{u1}$ |          |           | Scenario 5: $CPI_{c24} = CPI_{u24}$ |          |           |
|                                   | Period 1 | Period 24 |                                     | Period 1 | Period 24 |
| $\Delta FAS_c / \Delta FAS_u$     | 0.75     | 2.13      | $\Delta FAS_c / \Delta FAS_u$       | 1.10     | 2.14      |
| $\Delta IPI_c / \Delta IPI_u$     | 0.75     | 1.33      | $\Delta IPI_c / \Delta IPI_u$       | 1.06     | 1.50      |
| $\Delta CPI_c / \Delta CPI_u$     | 1.00     | 0.05      | $\Delta CPI_c / \Delta CPI_u$       | 1.13     | 1.00      |

both policy regimes are supposed to produce the same output in the first period, the unconventional monetary policy costs more Federal Reserve's assets and leads to higher inflation. The non-standard monetary policy even starts with less output when we calibrate both policies to attain the same level of output of the two-year horizon.

In the last panel of Table 5.3, the effects on inflation from both policies are set to be equal at the first and the 24 periods. Given the same initial impact on inflation rate, the unconventional monetary policy provides less output growth in the first month. In the case that two different policies achieve the same effect on inflation two years later, the unorthodox policy outweighs the typical policy to boost economic growth despite the larger size of central bank balance sheet. These findings confirm the economic consequences of the non-standard monetary policy are much more pronounced on inflation than output growth.

This experiment shows that the unconventional monetary policy is more effective to avert deflation when the central bank's balance sheet is changed at the same rate as implementing conventional monetary policy. When considering the impact on output, the non-standard monetary policy requires quite higher central

bank' assets to achieve the same change in output. With larger asset purchase, the unconventional monetary policy is equivalent in terms of impacts and able to substitute the standard monetary policy when the latter policy is ineffective.

## 5.7 Robustness check

In this section, we discuss the robustness of the results whether the identification schemes are varied in subsection 5.7.1. In particular, we impose a zero restriction on output and unrestrict price level. Afterwards, we investigate the results for different sub-samples in subsection 5.7.2, specifically the period before and after the unconventional monetary policy was implemented after the financial crisis. For subsection 5.7.3, we also examine the robustness when we use alternative monetary policy instruments. Moreover, we extend our model to include additional asset market variables, particularly stock index and stock volatility index and explore the transmission mechanism relative to the benchmark model in subsection 5.7.4.

### 5.7.1 Change Identification

For the first robustness check, we investigate whether our empirical results on economic activity and price level regarding the unconventional monetary policy innovation are sensitive to change in restrictions. Previously, we restricted the positive response on the growth of industrial production index and inflation for the shock in the unconventional monetary policy innovation. Now, we impose a zero restriction on industrial production index for the first scenario and then we leave inflation unrestricted for the second scenario.

#### 5.7.1.1 Zero restriction on output

As mentioned before, both industrial production index growth and inflation are assumed to have a positive reaction to the positive Federal Reserve balance sheet

shock. However, previous studies on the unconventional monetary policy implementation, including [Peersman \(2011\)](#) and [Schenkelberg and Watzka \(2013\)](#), found the effect on output actually retards and is transient. Therefore, we impose a zero restriction on output and examine the effect of the unconventional monetary policy. This identification is also more conservative and aligns with the [Sims \(1986\)](#) and [Bernanke \(1986\)](#) identification.

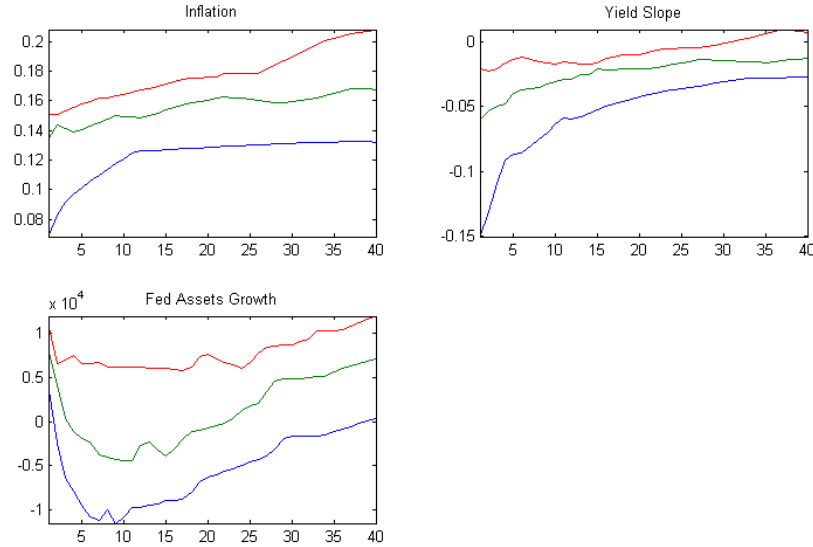


FIGURE 5.8: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock with zero restriction on output together with 16th and 84th percentiles for the U.S. for selected months

Figure 5.8 shows the results of the unconventional monetary policy innovation are insensitive to this change in identification. As we can see, inflation still increases by the increase in balance sheet and there is a compression in the yield slope. However, the quantitative effects on inflation, the yield slope and Federal Reserve's assets are still robust and consistent with the benchmark unconventional monetary policy shock. An increase in the central bank balance sheet actually stimulates output for around 5 months while the increase in price level is more permanent.

### 5.7.1.2 Unrestricted price level

Another alteration for the restriction setup to the unconventional monetary policy identification is an unrestricted effect on inflation. One important objective of quantitative easing is to raise up from deflation, for example as in the deflationary spiral in Japan as mentioned in [Fujiwara \(2006\)](#) and [Ugai \(2007\)](#). It would be instructive to implement an alternative identification by leaving the response of the consumer price index unrestricted so that we can adopt a more agnostic stance regarding the change in inflation. To examine the transmission mechanism through yield slope when we increase central bank asset, we still assume an unrestricted effect on the yield slope after the shock.

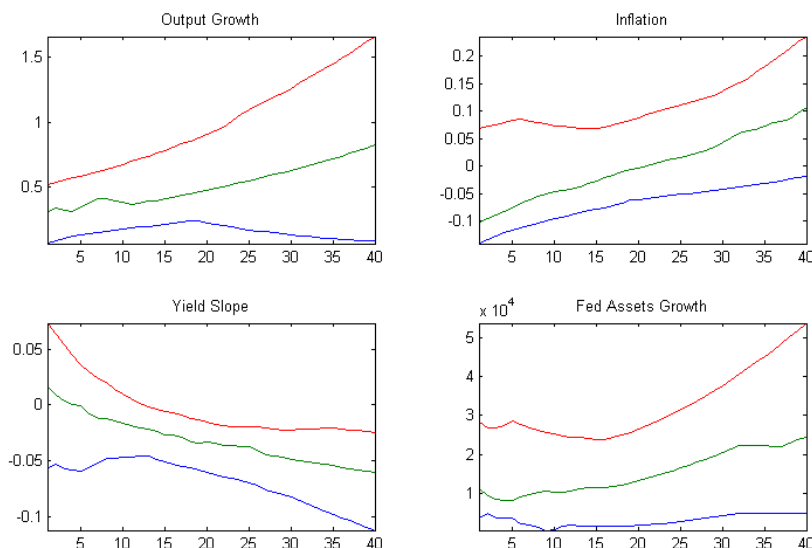


FIGURE 5.9: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock with unrestricted price level together with 16th and 84th percentiles for the U.S. for selected months

As seen in Figure 5.9, the main results remains robust, as they are qualitatively similar to the benchmark impulse responses we did earlier. This implies imposing unrestricted condition on inflation does not lead to a different impact. Again, we still observe persistent effect on output growth and inflation. These findings confirms that inflation is effectively affected by the quantitative easing implementation, as also found in [Schenkelberg and Watzka \(2013\)](#).

## 5.7.2 Sub-sample analysis

Since the quantitative easing began in 2008 in response to the global financial crisis, there is a considerable jump in the Federal Reserve balance sheet. To investigate whether our results are robust accounting for the period prior to the policy was implemented relative to the period after that, we re-estimate the structural VAR specification with respect to two different sub-samples; a period before the unconventional monetary policy implementation (January 2003 to July 2008) and a period after the implementation of the unconventional monetary policy (September 2008 to August 2013).

### 5.7.2.1 Period before the unconventional monetary policy implementation

The first sub-sample period covers the time prior to the unconventional monetary policy implementation. We conduct the impulse response analysis to test for model stability over a sample period that excludes the enhanced central bank asset position. The estimation is based on 68 months (January 2003 to July 2008) prior to the implementation of the unconventional monetary policy.

As depicted the impulse response results in Figure 5.10 confirm that the unconventional monetary policy innovation affects economic activity and inflation through a compression in the yield spread even in the normal time before the liquidity trap. Therefore, the economic consequences of the unconventional monetary policy for the sub-sample period prior to crisis is still robust. Indeed, the unconventional monetary policy does not induce a hump shape of output growth and an upward trend of inflation as we previously got from the whole sample. However, output growth and inflation immediately jump in the first instance and gradually reduce over the period of 40 months. There is no time lag for yield slope to reshape the yield curve that has become inverted as compared to the whole sample exercise.

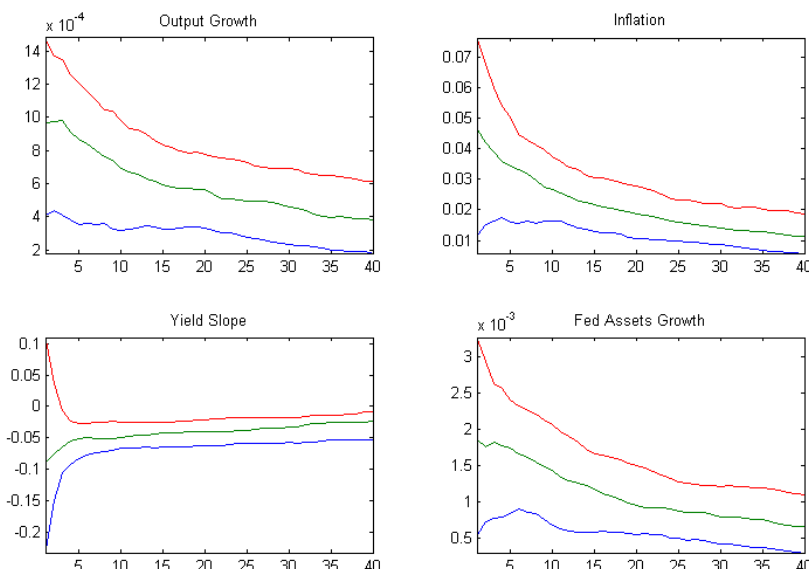


FIGURE 5.10: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock for the period over 2003-2008 together with 16th and 84th percentiles for the U.S. for selected months

### 5.7.2.2 Period after the unconventional monetary policy implementation

After the eruption of the global financial crisis, the Federal Reserve introduced quantitative easing measures to substitute for the ineffective interest rate measure. We therefore investigate the results for the sub-sample period after the non-standard policy is implemented for over 60 months (September 2008 to August 2013).

The impulse response propagations for the period after conducting unconventional monetary policy are similar to those results prior to the implementation. The main results are also robust and consistent with the responses for the whole sample period. We find an immediately sharp decrease in the yield slope after the quantitative easing is introduced. This evidence indicates that the time lag in changing the yield curve shape for the whole sample exercise, in fact relates to the term structure adjustment under the different regimes. Nonetheless, the sub-sample exercise still confirms that unconventional monetary policy works through a compression in yield slope to stimulate the economy and avert deflation.

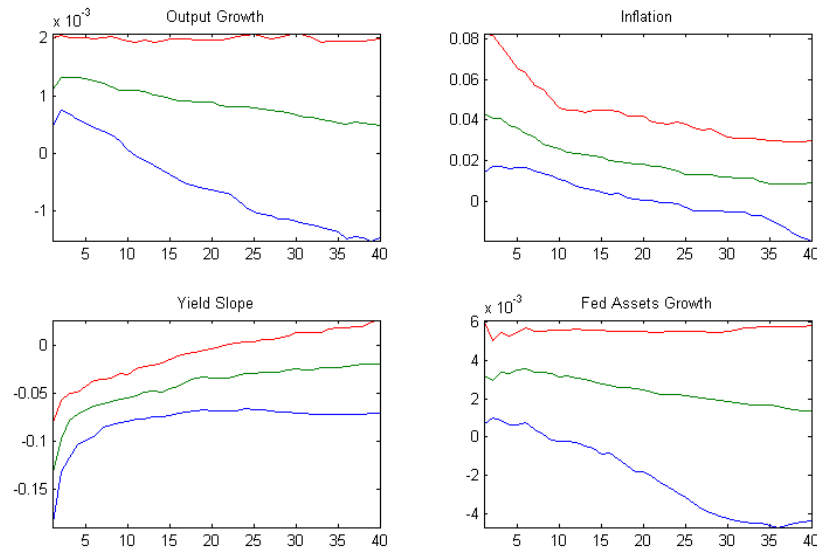


FIGURE 5.11: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock for the period over 2008-2013 together with 16th and 84th percentiles for the U.S. for selected months

### 5.7.3 Alternative measures of the monetary policy instrument

In this subsection, we discuss the robustness of the results for alternative measures of the monetary policy instrument. Rather than using the central bank balance sheet, we now use the monetary base and commercial bank credit as monetary policy measures for the shock in unconventional monetary policy. For previous non-standard monetary policy during the zero-lower bound interest rate, [Schenkelberg and Watzka \(2013\)](#) used the monetary base while [Peersman \(2011\)](#) employed the commercial bank credit as monetary policy instruments.

#### 5.7.3.1 Monetary base

The large increase in the central bank balance sheet due to the unconventional monetary policy generates a large amount of central bank reserves as well as currency. For our analysis, we also restrict the effect from unconventional monetary policy shocks via the monetary base to be orthogonal to the policy interest rate.

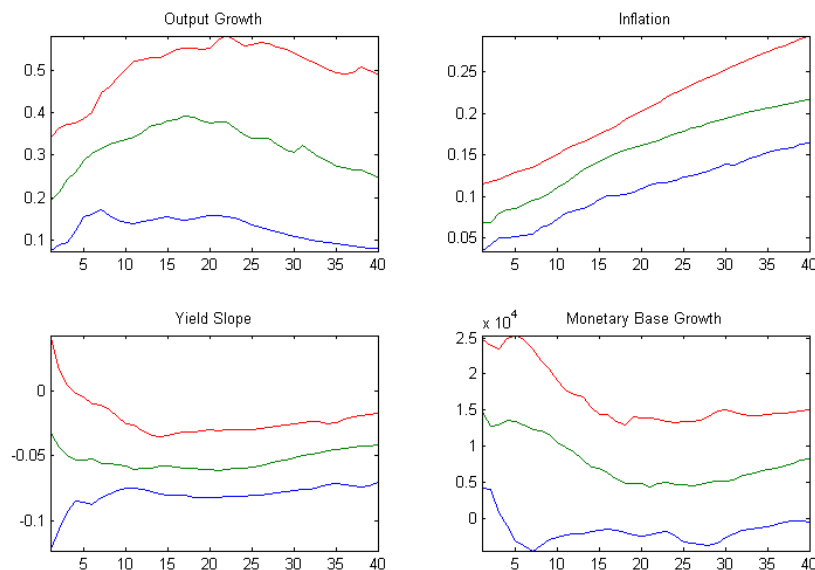


FIGURE 5.12: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock (using monetary base as monetary policy instrument) together with 16th and 84th percentiles for the U.S. for selected months

As can be seen from the impulse responses in Figure ??, an increase in the monetary base provides results that are similar to the responses generated by a central bank balance sheet innovation. The non-standard monetary policy shock through the monetary base effectively increases output growth and inflation. By extending the size of central bank's assets, the monetary authority is able to increase the monetary base, which typically results in a much larger increase in credit supply and eventually aggregate demand. In line with [Schenkelberg and Watzka \(2013\)](#), we find the unconventional monetary policy generates a hump shape impact on output growth and a permanently positive impact on inflation.

### 5.7.3.2 Commercial bank credit

According to the unconventional monetary policy, the large scale increase in the central bank balance sheet through the purchase in long term bond and securities also provides more liquidity to commercial banks thus creating more credit supply. By doing so, the composition of central bank liabilities will increase. We, therefore, investigate an exogenous shock in the volume of commercial bank credit as a monetary policy instrument. Similar to the analysis on monetary base, we



assume the economic impact from the unconventional monetary policy innovation through commercial bank credit to the policy interest rate is still orthogonal.

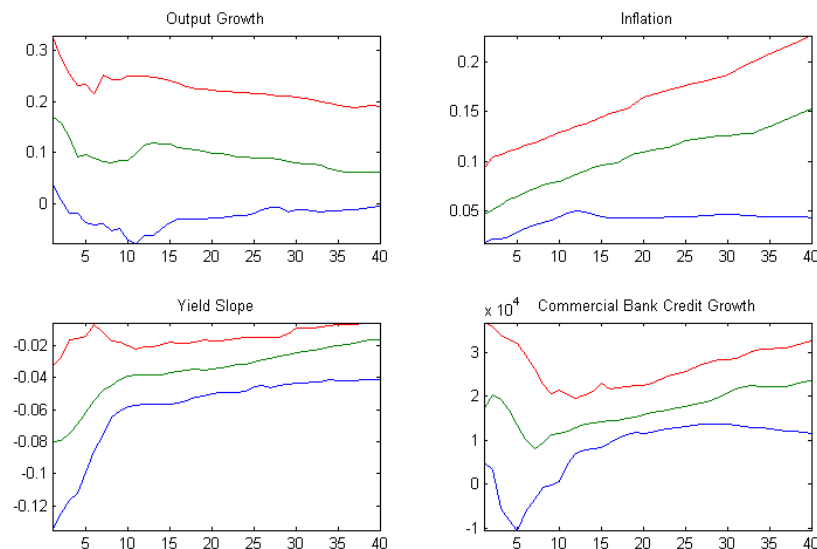


FIGURE 5.13: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock (using commercial bank credit as monetary policy instrument) together with 16th and 84th percentiles for the U.S. for selected months

Figure 5.13 displays the impulse response functions for the unconventional policy monetary innovation via the increase in commercial bank credit. We still observe positive impacts on output growth and inflation which are similar to the benchmark impulse responses with the central bank's balance sheet shock. The non-standard monetary policy, which is here aiming to increase commercial bank credit, effectively stimulates economic activity. Actually, the increase in central balance sheet creates more credit supply and lessens the bank lending rates. As we can see, the yield slope decline, implying the long-term rate and lending rate fall after the shock. This evidence is also consistent with Peersman (2011) who examined the economic effects of unconventional monetary policy using commercial bank credit as the monetary policy instrument.

In summary, the results of our robustness check with alternative monetary policy instruments do not change our main conclusion that quantitative easing measures effectively increase production and raise inflation. In addition, the yield slope is compressed after the unconventional monetary policy innovation, confirming the

effect on the long-term yield is robust for any monetary policy instrument. The increase in commercial bank credit is found to impact yield spread and significantly affects output and inflation. Therefore, a closer inspection of transmission mechanism implies the unconventional expansion of the size of the central balance sheet in fact increases the monetary base, drives up commercial bank credit supply, reduces the bank lending rate and ultimately stimulates economic activity.

### 5.7.4 Additional variables for the asset market reaction

Quantitative easing aims to lower long-term yields and compress the yield slope by directly purchasing long-term financial assets. The liquidity influx to financial markets also restores confidence to make markets function again. In order to have a better understanding about the transmission mechanism, we hence examine the robustness of the unconventional policy monetary by incorporating the stock volatility index and stock market index in our structural vector autoregressive model. We use the implied stock market volatility index (VIX) from Chicago Board Options Exchange to capture market sentiment uncertainly. We also use the NASDAQ 100 stock index to investigate the effect of the quantitative easing on financial asset prices.

#### 5.7.4.1 Volatility index

The implied stock market volatility index (VIX) is a proxy for perception about financial market risk. The inclusion of the volatility index takes into account the reaction of financial markets to the large scale purchase in financial assets under unconventional monetary policy. The expansion of the central bank balance sheet could potentially reduce financial market instability and contribute to economic recovery. The recent unconventional monetary policy study by [Gambacorta et al. \(2014\)](#) examined the consequence of an increase in the central bank asset on stock market volatility by restricting a negative effect on the volatility index. However, their model did not include the policy interest rate. They also ignored the transmission mechanism on the yield slope. Unlike their study, we orthogonalize the impact of the policy interest rate to investigate the pure effect of quantitative

easing and unrestrict the contemporaneous effect on the yield slope, except the volatility index is still kept negatively restricted. This identification scheme allow us to be agnostic about the transmission mechanism of the central bank balance sheet shock on yield spread compression by taking into account financial market sentiment. We expect the unconventional monetary policy shock by purchasing financial assets, especially long-term bond and securities, could induce economic expansion through a lower long-term yield and also enhance financial liquidity to mitigate concerns about economic instability.

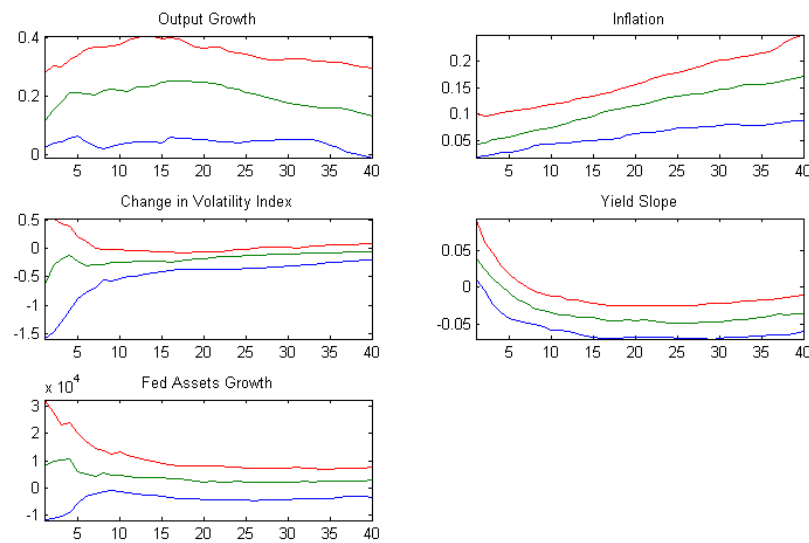


FIGURE 5.14: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock (incorporating the volatility index into baseline model) together with 16th and 84th percentiles for the U.S. for selected months

As in Figure 5.14, our results remain robust with unconventional monetary policy implemented by the massive expansion of central bank asset effectively stimulating the real economy and driving up inflation. The impulse response results also reveal that the unconventional monetary policy innovations significantly reduce financial market risk by enhancing the confidence in economic recovery. Afterward, long term yields decrease and compress the yield spread. The significant impact on financial market sentiment therefore reflects the influence of a purchase of financial asset to reduce the risk premium and lower long term yields. [Gambacorta et al. \(2014\)](#) also found the extension of central bank balance sheet significantly affects the volatility index so that it mitigates economic uncertainty and accommodates effective non-standard monetary policy. Our findings shed light the reason why

unconventional monetary policy produces a lagged effect on the yield slope and economic activity. In fact, the quantitative easing shock initially restores confidence and then reduces the long term yield due to a lower risk premium. Consequently, a compressed yield spread gradually boosts the economy with a persistent inflation. Hence, the effectiveness of unconventional monetary policy is actually explained by the transmission mechanism through lower risk premium and a compression in the yield spread.

#### 5.7.4.2 Stock index

The large scale purchase of long term bonds and securities could lead higher asset prices. Higher stock prices will not just only induce more consumption from the wealth effect, but also will improve the liquidity of the firm to boost investment. [Kapetanios et al. \(2012\)](#) investigated the effect of a change in the monetary base from unconventional monetary policy on economic activity, the price level, policy interest rate and stock index through the yield spread. We follow this study to restrict a zero contemporaneous effect on the policy interest rate and allow an unrestricted condition on the yield spread. Nonetheless, we choose the central bank balance sheet as the monetary policy instrument instead of the monetary base which has limitations to reflect the actual transmission mechanism. We also relax the restriction on the stock index to examine the effect on the stock market from the data.

From Figure 5.15 the impulse response plots confirm similar findings that the expansion of central bank balance sheet effectively increases output growth and inflation. These results are also consistent with [Kapetanios et al. \(2012\)](#) who suggest quantitative easing through a compression in the yield spread is effective to stimulate the economy. While their study also reported that the unconventional monetary policy shock raises the stock index, they did not explain the transmission mechanism through the stock index. In our study, we find a gradual increase in the growth rate of stock index to reach the peak within one year after the shock, given no restriction imposed on stock index. After an unconventional monetary policy shock, the long term yield immediately decreases. This evidence indicates the purchase of long term bonds and securities does not drive financial asset prices

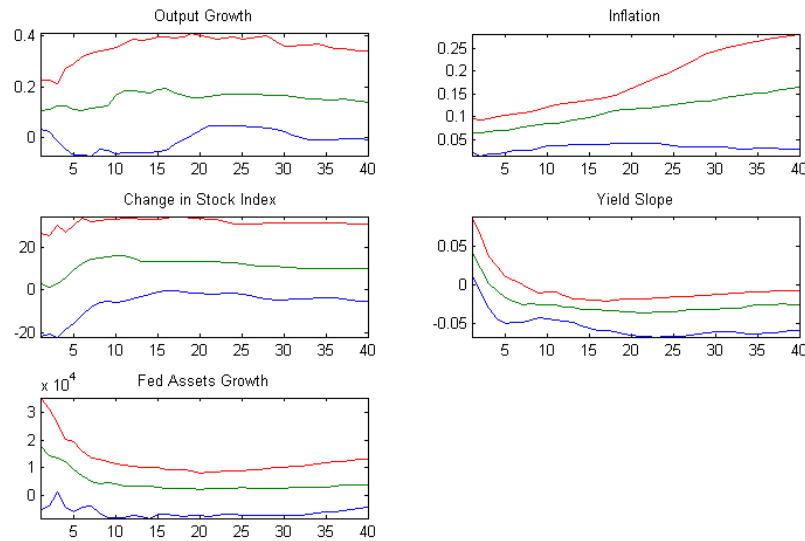


FIGURE 5.15: Median IRFs (green line) of a one-standard deviation shock to a negative unconventional monetary policy shock (incorporating the NASDAQ 100 stock index into baseline model) together with 16th and 84th percentiles for the U.S. for selected months

in the first place. In fact, the immediate decline in long term yield can be explained by the lower risk as mentioned earlier.

The inclusion of the volatility index and the stock index reveals the unconventional monetary policy transmission mechanism basically works through yield spread compression from lower risk premia. The extension of the central bank balance sheet initially enhances confidence in financial markets and consequently reduces the long term yield. Economic agents then anticipate lower real interest rates as well as higher expected inflation. Eventually, aggregate demand starts to increase and return to a normal growth path.

## 5.8 Conclusion

In this paper, we examine the transmission mechanism and the effectiveness of unconventional monetary policy implementation by the US Federal Reserve under its quantitative easing scheme after the global financial crisis in comparison with conventional monetary policy. We use a Bayesian structural VAR (B-SVAR)

with sign restrictions technique to investigate macroeconomic impacts of the four different identifications: the unconventional monetary policy shock through the extension of the Federal Reserve's assets that then lessens the long-term yield and compresses the yield spread; the conventional monetary policy shock by lowering the fed funds rate; as well as typical aggregate demand and aggregate supply shocks during the period of January 2003 to August 2013. To explore macroeconomic impacts of the quantitative easing measures, we identify the unconventional monetary policy shock as an innovation in Federal Reserve's assets in which the fed funds rate is constrained by the zero lower bound, while the yield slope is left unrestricted for an agnostic transmission mechanism.

Overall, we find the exogenous increase in the Federal Reserve's balance sheet at the zero lower bound interest rate effectively stimulates output growth and persistently averts deflation at the expense of a dramatic surge in the Federal Reserve's assets, even though the adjustment of the yield curve may take time to become inverted. This non-standard monetary policy can be used effectively as an alternative measure for expansionary monetary policy when the standard way is ineffective due to the fact that both approaches eventually lead to economic growth associated with higher inflation. Nonetheless, the unconventional monetary policy will quantitatively create less output growth relative to conventional monetary policy, given the same extension in Federal Reserve's assets. In other words, the central bank requires much larger assets to drive the economy when the objective is reducing the long-term yield to conduct a heterodox monetary policy. However, we find unconventional monetary policy is more effective to raise inflation, in contrast to other results that show a failure to avert deflation in Japan, such as in [Schenkelberg and Watzka \(2013\)](#).

In particular, the Federal Reserve is alternatively able to raise the size of its balance sheet as a monetary policy instrument to induce output growth and inflation through a compression in the yield spread channel when standard monetary policy is ineffective. The large scale purchase in long-term bonds lessens the long term yield since it provides liquidity for private equity investment through portfolio rebalancing and therefore restores market confidence accompanied by mitigating risk premia. In addition, the transmission mechanism via lower long-term yields also implies market anticipation on the continuing zero lower bound interest rate

and higher expected inflation. Hence, the quantitative easing measures signal to economic agents to resume their spending, which eventually stimulates the economy and averts deflation. We also check the sensitivity of our results to different measures of monetary policy instruments: monetary base and commercial bank credit. The results do not change the conclusion that unconventional monetary policy effectively raises output growth and inflation. This analysis also reveals quantitative easing indeed increases the monetary base and drives up commercial bank credit supply and reduces the lending rate to stimulate aggregate demand. We further estimate specifications that include a volatility index and a stock index to examine the transmission mechanism through asset markets. Our results are robust to this alteration and confirm the role of the yield spread as a channel to affect economic activity through lower risk premia and gains from higher equity prices.

We also further check the robustness of the results for different sub-samples and find our results are insensitive to the change in sample period. In fact, the Federal Reserve officially began to conduct unconventional monetary policy through a large increase in its asset after the onset of the global financial crisis in 2008. In order to investigate whether the unconventional monetary policy is still effective regardless of any structural breaks in central bank assets, we additionally estimate our unconventional monetary policy specification for the shorter period prior to the implementation of this policy (January 2003 to July 2008) and compare it to the period after the implementation (September 2008 to August 2013). We find the unconventional monetary policy significantly affect output growth and inflation even such a policy is not yet implemented. Indeed, the effectiveness of the non-standard monetary policy is more favorable since the growth rate of industrial production index and inflation significantly increase without time lags, despite [Schenkelberg and Watzka \(2013\)](#) finding this policy ineffective for Japan.

Based on these results, we can support unconventional monetary policy as an accommodating monetary policy option for the monetary authority at the zero lower bound interest rate. It could potentially counter financial risk and restore market confidence to boost aggregate demand. The quantitative easing scheme is also tailored to avert deflation during the liquidity trap. However, this stimulus

measure requires a massive extension of the central bank's balance sheet.

Our analysis also needs further development to cover some caveats. The transmission mechanism involves portfolio rebalancing and needs to better confront imperfect asset substitution. To capture market frictions, a dynamic stochastic general equilibrium (DSGE) framework can be applied to explain the macroeconomic effects of quantitative easing with micro foundations. It is also worth accounting for changes in economic structure due to the financial crisis by using a time-varying parameter specification that allows us to trace the effects of unconventional monetary policy in its effort to stimulate the economy. Another question that could be asked concerns the relative effectiveness of unconventional and conventional monetary policy when the central bank is in a contractionary phase of monetary policy.



# Chapter 6

## Conclusion

This thesis presents four papers to examine different aspects of term structure estimation and forecasting. The main objective of this thesis is to propose an improvement in the area of term structure modeling and forecasting. In Chapter 2, I introduce a macro-finance-fiscal term structure model to explore the impact of fiscal instability on term structure. Besides, I use a Sheen-Trueck-Wang business conditions index as a new strategy to incorporate a forward looking information to estimate yield curve in Chapter 4. This thesis also aims to provide a comprehensive investigation in comparing the relative forecasting accuracy of the dynamic semiparametric factor model, the dynamic Nelson-Siegel model and other competitors. Since the period of study cover the global financial crisis and the zero-lower-bound interest rate, I further test whether unstable economic environment affects the forecasting performance of the underlying models as mentioned in Chapter 3. Another area of my study is to use the information content of the yield curve and analysis their linkage with macroeconomic variables. In Chapter 5, I analyze the impact of a large scale increase in central bank's balance sheet and macroeconomic variables to reveal the transmission mechanism of unconventional monetary policy through yield slope information. This investigation gives a policy recommendation for unconventional monetary policy implementation.

With respect to the methodological contributions, this thesis aims to promote innovative techniques in econometrics. For the dynamic macro-finance-fiscal term structure model in Chapter 2, I use the Kalman filter to extract yield latent factors and simultaneously solve for parameters by using maximum likelihood estimation,

hence improving model efficiency. To provide better understanding on unsolved puzzle whether parametric and nonparametric term structure models become less accurate in comparison to the random walk, I propose to use [Giacomini and Rossi \(2010\)](#) fluctuation test in Chapter 3 to evaluate potential instabilities from dramatic interest rate lowering may affect the forecast performance. In chapters 4, I investigate the effectiveness of unconventional monetary policy based on a modern Bayesian structural vector autoregressive (B-SVAR) model with sign-restrictions. My identification approach is imposed on policy interest rate to be orthogonal with a change in central bank balance sheet. It provide a better interpretation of structural shocks and allows me to be agnostic about how macroeconomic activities respond to quantitative easing.

The first paper entitled “Spanish Sovereign Term Structure: Implications of the Sovereign Debt Crisis” is presented in Chapter 2, I propose a macro-finance-fiscal term structure model as a tool to examine the effect of fiscal instability on yield spread.

My empirical results indicate that yield spread significantly responds to fiscal indiscipline. In particular, a deterioration in net government budget position immediately generated a significant widening of the yield spread. This result may reflect the fact market reaction to fiscal loosening by demanding a higher risk premium due to higher sovereign default risk. Even the reaction from a rise in public debt is more lagging than those generated by a worsening government budget position, I observe the evidence that both fiscal instability variables significantly effect the growth in output. A fiscal indiscipline and high public debt destroy creditworthiness. Consequently, yield spread is raised in the presence of fiscal mismanagement.

A relevant policy implication is that fiscal stimulus package might adverse effect once market participants penalize for the worsening government budget position and public debt. The scepticism about fiscal instability downplays the effectiveness of expansionary fiscal policy in stimulating aggregate demand. Under this perspective, fiscal discipline would be considered as a necessary condition for fiscal policy to be successfully implemented.

In the second paper entitled “Term Structure Forecasting - A Comparison between the Dynamic Semiparametric Factor Model and the Dynamic Nelson-Siegel Model” as presented in Chapter 3, I compare the in-sample fit and out-of-sample forecasting performance of the dynamic semiparametric factor model, the dynamic Nelson-Siegel model with other competitors.

The results show that the dynamic semiparametric factor model outperforms the dynamic Nelson-Siegel model in providing better in-sample fit. With an AR(1) specification, the dynamic semiparametric factor model also statistically produces superior forecasting results than the random walk and the dynamic Nelson-Siegel for 6-month maturity at a 1-month and 3-month ahead horizon. Even though, the random walk turns to overcome other models in predicting the yield curve at longer maturities and horizons over the period from 2006 to 2013.

The comparisons of forecast accuracy are also assessed over three distinct sub-periods that reveal relative performance has changed over time. Indeed, the dynamic Nelson-Siegel model is a more preferable in forecasting for more volatile periods from 2003 to 2006 and 2006 to 2008. By contrast, the dynamic semiparametric factor model has a superior forecasting performance over the period of persistent downward trend following the global financial crisis in 2009. The results from [Giacomini and Rossi \(2010\)](#) fluctuation test also suggest that the evidence of predictability differs across individual predictive models. In particular, forecasts become less accurate when the long-term yields fell while the short-term yields hit the zero-lower bound.

The third paper entitled “Term Structure Forecasting with a Business Condition Index” is presented in Chapter 4. In this paper, I examine the role of forward looking information to estimate and forecast the term structure.

This paper proposes to use the Sheen-Trueck-Wang business conditions index in providing an informational advantage about the current state of economy and forward-looking information. I find the evidence that the inclusion of the business conditions index in a term structure model provides guidance to anchor the yield in the next period. I also find that the business conditions index helps to improve

forecasting accuracy of the dynamic semiparametric factor model for medium and long-term maturity at one-month step ahead. The prediction performance can further be improved with a more recent available or a more frequently released index. The business conditions index is not only useful in improving the nowcasts when the new information becomes available, it is also able to provide more accurate forecast of the term structure.

The last paper entitled “The Economic Impact of Quantitative Easing on the US Economy: A Structural VAR with Sign Restriction Analysis” is presented in Chapter 5. In this paper, I employ a Bayesian structural vector autoregressive (SVAR) model with sign restrictions to analyze the economic impact of unconventional monetary policy.

This paper investigates the effectiveness of unconventional monetary policy adopted by the Federal Reserve. The macroeconomic effects are assessed by estimating the effects of exogenous innovations to balance sheet that compresses slope of the yield curve and eventually stimulate output and inflation. The main finding of this study suggests that unconventional monetary policy measures are actively used to avert a recession in the absence of conventional monetary policy. A large scale purchase of Federal Reserve’s asset significantly reduces yield slope and boosts economic activities.

In fact, it is more effective to use unconventional monetary policy to raise price and avoid deflationary pressures despite the fact that the non-standard monetary policy relatively generates less output growth as compared with conventional monetary policy. Based on these results, unconventional monetary policy provides additional monetary policy measures in response to financial crisis. The policy implication of these findings is that the unconventional monetary policy can be used as a complementary measure when the conventional monetary policy actions become ineffective. The central bank might need a larger assets to reduce long-term yields.

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