Understanding Mathematical Self-Efficacy, Student Approaches to Learning and Conceptions of Mathematics in Learning Mathematics

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"This thesis is presented for the degree of Doctor of Philosophy"

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THIS THESIS IS DEDICATED

To my parents,
who imbued the core value of higher education.

To my husband and daughter,
who have shown me the meaning of perseverance
and supported me in many ways.

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Declaration Page

I certify that the work in this thesis entitled

Understanding Mathematical Self-Efficacy, Student Approaches to Learning and Conceptions of Mathematics in Learning Mathematics

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has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree to any other university or institution other than Macquarie University.

I also certify that the thesis is an original piece of research and it has been written by me. Any help and assistance that I have received in my research work and the preparation of the thesis itself have been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis. The research presented in this thesis was approved by Macquarie University Ethics Review Committee, reference numbers:

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IN EVERYTHING
give *Thanks*

1 Thes. 5:18

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List of Key Abbreviations

AG	Achievement goal
ACT	American College Test
ASI	Approaches to Studying Inventory
ASSIST	Approaches and Study Skills Inventory for Students
CM	Conceptions of mathematics
EFA	Exploratory factor analysis
ESMI	Experiences of Studying Mathematics Inventory
IPENZ	Institution of Professional Engineers New Zealand
ISPSQ	Interest, Self-Perceived Competence, and Study Strategies Questionnaire
LPQ	Learning Process Questionnaire
MQ	Macquarie University
MIT	Manukau Institute of Technology
MSLQ	Motivated Strategies of Learning Questionnaires
MSE	Mathematical self-efficacy
NCEA	National certificate of educational achievement
OECD	Organization for Economic Cooperation and Development
RSS	Refined Self-Efficacy Scale
SAL	Student approaches to learning
SETLQ	Shortened Experiences of Teaching and Learning Questionnaire
SMRB	Students' mathematics-related beliefs
SOLO	Structure of observed learning outcomes
SPQ	Study Process Questionnaire
STEM	Science, technology, engineering and mathematics education
TEC	Tertiary Education Commission

Abstract

My research provides a unique contribution to the field of mathematics education by advancing our understanding of the nature of mathematical self-efficacy, student approaches to learning, and conceptions of mathematics. Influenced by theoretical frameworks of self-efficacy (Bandura, 1977, 1997), student approaches to learning (Biggs, 1987; Marton & Säljö, 1976, 2005) and students' mathematics-related beliefs (Op't Eynde, De Corte, & Verschaffel, 2002), my research aims to investigate the nature of and inter-relations of these constructs with examination performance. This research is important because successful completion of mathematics courses is a priority for higher education providers, whose goals are to improve mathematical skills and knowledge in business, science, technology, engineering and mathematics (STEM) education.

This thesis incorporates three studies in Australia and New Zealand. Surveys are carried out with around 300 engineering and business students who study mathematics courses as service subjects. Three noteworthy findings would be of relevance to lecturers: firstly, that strong mathematical performance is predicted by mathematical self-efficacy (Study 1; N=67), secondly, that successful mathematics performance is strongly associated with deep approaches to learning, organised approaches to learning, and a cohesive conception of mathematics (Study 2; N=291), and thirdly, a low-level secondary mathematics education is associated with high examination scores in first-year mathematics courses (Study 3; N=73). These research findings would have practical implications on the development of mathematical self-efficacy, guided mastery experiences, deep learning strategies, real-life applications of mathematics, and authentic assessments for higher education students.

Chapter 1

Introduction

I now believe that there are two effectively different subjects being taught under the same name, 'mathematics'.

—Skemp, 1978, p.11

I have much sympathy with Skemp's view that mathematics has two faces—'instrumental mathematics' and 'relational mathematics' (Skemp, 1978). On the one hand, an expert mathematics educator may teach 'relational mathematics' by explaining how and why formulae and proofs apply in real life situations. On the other hand, another university lecturer may be familiar with 'instrumental mathematics' so they teach formulae and calculations without explanations and reward students for giving only correct answers. A prominent mathematician, Felix Klein viewed this phenomenon as 'double discontinuity' because the

> *young university mathematics student, found himself, at the outset confronted with problems which did not suggest, in any particular, the things with which he has been concerned in school. Naturally, he forgot these things, quickly and thoroughly. (Klein, 1932, p. 1).*

This phenomenon seems to occur when mathematics students transition from secondary schools to universities. In higher education, the discontinuity between 'relational mathematics' and 'instrumental mathematics' became apparent when the same individuals, who become lecturers in higher education, teach "traditional elementary mathematics in the old pedantic way" (p.1). It seems that the epistemological gap between school and university mathematics should be addressed as the quality of teaching and learning is affected in higher education.

Over the last two decades, reforms in university mathematics education have been driven by research showing that university mathematics students are familiar with procedural learning from school, less inclined to develop conceptual understanding of mathematics at university and increasingly under-prepared for higher education and may lack adequate mathematical knowledge to grasp advanced mathematical concepts. These findings have been demonstrated in Australia (Belward, Mullamphy, Read, & Sneddon, 2007; Varsavsky, 2010), Canada (Kajander & Lovric, 2005), Hong Kong (Luk, 2005), Ireland (Hourigan & O'Donoghue, 2007), New Zealand (Thomas et al., 2010), South Africa (Engelbrecht & Harding, 2008), Sweden (Brandel, Hemmi, & Thunberg, 2008). Mathematics education researchers have reported that successful students apply mathematics to other fields of studies whereas less successful students tended to memorise mathematical formulae without applying these to real-life applications (Crawford, Gordon, Nicholas, & Prosser, 1994, 1998a, 1998b). Furthermore, student learning researchers have reported that successful higher education students have a tendency to exhibit high levels of self-efficacy in learning mathematics (Hailikari, Nevgi, & Komulainen, 2007; Hall & Ponton, 2005; Marcou & Philippou, 2005; Pajares & Kranzler, 1995; Pajares & Miller, 1994; Skaalvik & Skaavik, 2011; Stevens, Olivarez, Lan, & Tallent-Runnels, 2010). When mathematics students receive appropriate academic support to address specific areas of mathematical learning, they are likely to advance into further education (Carroll & Gill, 2012; Dowling & Nolan, 2006; Patel & Little, 2006; Symonds, Lawson, & Robinson, 2007). These empirical findings raise questions of whether student approaches to learning, self-efficacy and worldviews of mathematics are related to mathematical achievement, and how universities could improve mathematical achievement. These questions should be addressed because higher education providers are increasingly focused on improving graduate attributes, engagement in STEM courses and successful outcomes in mathematical learning.

My research aims to conceptualize mathematical learning and recommend new ways of supporting mathematics students in higher education. I will focus on affective factors (self-efficacy, conceptions of mathematics), cognitive factors in learning mathematics (student approaches to learning, mathematics results) and personal factors (prior mathematics, age and gender). For definitions of some key constructs such as mathematical self-efficacy (MSE), student conceptions of mathematics and approaches to learning and successful performances, refer to Sections 1.2 and 1.3. Using a sample of around 300 first-year engineering and business students who study mathematics as service subjects in Australia and New Zealand, my research examined the inter-relations between mathematics results, conceptions of mathematics, mathematical self-efficacy, student approaches to learning, and prior mathematics. The key findings were: firstly, mathematics examination results were best predicted by self-belief in motivation, cognitive and selection processes and self-belief for self-regulated learning; secondly, deep approaches and Level 3 life conceptions of mathematics have positive correlations whereas surface approaches to learning were negatively correlated with cohesive conception of mathematics; thirdly, mathematics students tended to adopt deep approaches to learning rather than surface approaches to learning; fourthly, low prior mathematics background was positively related to high mathematics results.

My research would be of interest to higher education providers and policymakers, whose priorities are to develop human capital growth, particularly of mathematical skills and knowledge that are prerequisites in business, science, technology, engineering, and mathematics education. Moreover, my research would pose theoretical as well as practical significances to mathematical learning and teaching. First, the theory of self-efficacy (Bandura, 1997) highlights the nature of self-efficacy and its performance enhancement role in learning mathematics, which will be discussed in Chapter 2. Next, drawing upon the frameworks of student approaches to learning (Marton & Säljö, 1976) and students' mathematics-related beliefs (Op't Eynde, De Corte, & Verschaffel, 2002), my research examines how and why students study mathematics and the complex structure of mathematics-related beliefs. Further, my research also poses practical significance because lecturers need to consider how teaching programmes influence student beliefs in learning and mathematical knowledge. As stated by Schoenfeld (2013), powerful mathematical teaching is about creating "opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics" (Schoenfeld, 2013, p. 11). When students are provided with opportunities for developing agency and authority in learning mathematics, they develop a mathematical identity as learners. Lastly, my research provides a unique contribution to mathematical teaching and learning because it identifies the applications of mathematical self-efficacy, deep approaches to learning and cohesive conceptions of mathematics in mathematics education. This is an important step to promoting conceptual understanding in learning mathematics or 'relational mathematics'. Another step is to investigate the factors that influence mathematics examination results.

1.1 Research context

Mathematics is ubiquitous in undergraduate education and working life. In higher education, mathematics is embedded in professional fields such as accounting, finance, engineering, business, medicine and social sciences. According to Bolstad et al. (2012), 21st century learning is about building one's sense of identity, becoming self-reliant, critical and creative thinkers with the ability to use knowledge in a global world. This can clearly be seen in engineering and business education. The Institution of Professional Engineers New Zealand (IPENZ) has developed a national plan to ensure that engineering graduates meet local economic needs. Following the guidelines of the Washington Accord (International Engineering Alliance, 2013), IPENZ accredits engineering programs that adhere to a well-defined knowledge profile and a set of graduate attributes. The Washington Accord knowledge profile of a graduate focuses on "conceptually-based mathematics, numerical analysis, statistics and formal aspects of computer and information science to support analysis and modelling applicable to the discipline" (p.8). The graduate attributes in the Washington Accord emphasise the importance of applying mathematics in complex engineering problems and problem-solving skills and of mathematical modelling and computations using software.

Mathematics is also important in business undergraduate programmes. According to El Nemar (2013), mathematical modelling is necessary in commerce to maintain a competitive edge in the industry. Software is used to carry out inventory control, forecasting and data analyses. Mathematical skills are not only required to interpret the data but also help adapt the procedures in computing packages. High level mathematical skills are required to solve complex problems, and to present mathematical models in risk management, biosecurity, economics, population trends and environmental issues. Hence, mathematics permeates different professions and is a fundamental component of many undergraduate courses.

Internationally, higher education providers promote participation in STEM-related degrees in order to contribute to growth of human capital. In response to technological changes, policymakers have sought to address the shortage of skilled labour. In New Zealand, the Ministry of Education has identified a lack of skilled workers in the areas of information technology, engineering, building and health (Earle, 2008). This implies more skilled employees with advanced qualifications are required to meet the demand created by the increasing application of technology in the workforce and greater investment in the building and construction fields of education. The Ministry suggested that this shortage was likely to be exacerbated by difficulties in hiring school teachers who teach STEM subjects. According to the Organization for Economic Cooperation and Development (2014), these episodic shortages of new entrants for STEM were evident in many developed countries, including the United Kingdom (8%), New Zealand (7%) and Australia (9%), with statistics indicating enrolments below the OECD average of 15% for higher education new entrants in the engineering, building and construction field of education. By prioritising STEM education, higher education providers play an important role in human capital growth because STEM graduates will be able to engage in workplace systems that require quantitative skills, analytical skills and scientific knowledge. Thus, higher education providers would indirectly address labour shortage by improving participation in STEM education, a priority which reflects the significant role of mathematics education in economic reform.

Higher education providers are increasingly accountable for student performance through fiscal policies and local auditing system. In New Zealand, student performances in the university, institutes of technology and polytechnic sectors are currently benchmarked against Educational Performance Indicators (EPIs) including course completion and qualification completion rates (New Zealand Tertiary Education Commission, 2014, 2016, 2017). In order to get the best returns for the average $4 billion in university funding, the New Zealand TEC proposed the strategy of increasing higher education sector performances by means of successful student outcomes, better academic support and sector performances (New Zealand Tertiary Education Commission, 2014). They recommended that higher education providers support and encourage student performances. Some researchers claimed that when higher education providers support students throughout their studies, the government would obtain greater economic returns for funding if these graduates gain employment in the early stage of adulthood, particularly with the science and engineering graduates (Park, Mahoney, Smart, & Smyth, 2014).

In Australia, the local regulator, The Tertiary Education Quality and Standards Agency (2011) provides Learning and Teaching Standards for higher education providers. The Learning Standards describe the desired areas of knowledge and skills and the levels of attainment required for graduation and for awarding grades at pass level or above. The Teaching Standards include curriculum design, the quality of teaching, student learning support, and the infrastructure which directly supports the processes of teaching and learning. These are the aspects of institutional provision or educational delivery that are commonly accepted to have an effect on the quality of student learning. Thus, in accordance with performance indicators and strategic priorities, higher education providers are incentivised to improve student performance.

National reports in Australia and New Zealand (Brown, 2009; Marginson, Tytler, Freeman, & Roberts, 2013; Thomas et al., 2010) have reported a downward trend in mathematics preparedness of first-year university students. One possible reason identified was that fewer students have completed advanced mathematics in secondary schools. An Australian mathematics performance report found that 10% of Year 12 students had opted for advanced mathematics in the final year of secondary education (Marginson et al., 2013). A study of New Zealand secondary mathematics students (Nuffield Foundation, 2013) found that even though participation rate in advanced mathematics (Year 13 National Certificate of Educational Achievement level 3) of 17/18 years old was the highest in advanced countries internationally, the completion rates were somewhat lower than the enrolments in 2011. The study suggests that while high participation is a result of the New Zealand system flexibility in which 17/18 years old students could opt for mathematics domain courses (such as mathematics with statistics mathematics with calculus), their level of preparation for mathematics in higher education was somewhat low as indicated by poor completion rates. Other studies have shown that higher education students faced difficulties in performing basic mathematical knowledge without the aid of calculators in Australia (Brown, 2009) and in New Zealand (Thomas et al., 2010). Moreover, they have found that advanced mathematics students also lacked understanding of logical proofs and appreciation of assumptions in mathematical principles.

Internationally, mathematics education researchers have found that first-year higher education mathematics students are likely to have weak mathematical knowledge and numeracy skills in Australia, England, Ireland, Hong Kong, South Africa, Sweden, United Kingdom, United States (Belward et al., 2007; Brandel et al., 2008; Engelbrecht & Harding, 2008; Hourigan & O'Donoghue, 2007; Kajander & Lovric, 2005; Luk, 2005; Varsavsky, 2010; Walker & Plata, 2000). While some researchers have suggested this problem is due to poor alignment of mathematics curriculum and assessments (Belward et al., 2007; Thomas et al., 2010) and lower university entry requirements (Brown, 2009; Marginson et al., 2013), others cited low self-confidence and surface learning in schools (Thomas et al., 2010). To address misalignment of curriculum and assessments, some researchers have recommended curriculum reform and better communication between secondary school teachers and university lecturers (Barton, Clark, & Sheryn, 2010; Belward et al., 2007).

To support mathematics students, some higher education providers provide early teaching interventions such as after-class study groups (Solomon, Croft, & Lawson, 2010), basic mathematics support and diagnostic testing (Heck & Van Gastel, 2006; Hieb, Lyle, Ralston, & Chariker, 2015; Warwick, 2010; Wilson & MacGillivray, 2007). Others have offered mathematics support in preparation for non-mathematics majors or service mathematics courses (Belward et al., 2011; Broadbridge & Henderson, 2007; Taylor & Morgan, 1999) and the workplace (Wood, 2010; Wood, Mather, et al., 2012). Thus, to address the problem of lack of preparedness, lecturers are increasingly requiring their students to master basic mathematics skills in preparation for future mathematics studies. This raises the question of how lecturers could guide students to develop deep strategies in order to master basic mathematical skills, a question that arises in my research implications.

Besides offering student support, a broader question concerns the goals of mathematics education in higher education. One of the goals of mathematics lecturers could be to encourage relational understanding to improve outcomes. Skemp (1987) states that relational understanding involves a cognitive schema of interconnected concepts, enabling students to appreciate why and how mathematical procedures work and connect prior knowledge with new knowledge in conceptual development whereas procedural understanding involves remembering the rules without reasons so one may show step-by-step procedures and use formulae to work out the answers without comprehending the underlying meaning behind the numbers. He further argues that self-confidence could promote relational understanding when students experience the freedom from feeling anxious and incompetent and instead experience to feelings of security and competency. This positive emotion occurs when relation understanding takes place. To promote relational understanding, mathematics lecturers could also build students' confidence in learning mathematics.

Another goal of mathematics education is to provide opportunities for students to develop cognitive and affective aspects of learning. Students not only face cognitive challenges but also their disposition to learning affects their problem-solving skills. Schoenfeld (2013) argues that the success and failure in one's ability to do mathematical problem-solving are determined by both cognitive aspects (including mathematical knowledge and resources, access to heuristic skills to tackle challenging problems) and affective aspects of learning (including disposition to do mathematical problem solving, mathematics-related belief system, monitoring learning and self-regulation). He suggests that mathematics is a sense-making and human activity that is grounded in human practices. In other words, when solving challenging problems, successful students are able to use appropriate heuristic skills, monitor their learning, form their own belief about mathematics as either a pure or applied subject and apply mathematics knowledge onto realistic problem-solving situations. In order to enhance student learning, it is important that lecturers consider these goals in mathematics education.

1.2 Research scope

In the next section, I discuss key student learning research relating to the scope of my research, including theoretical frameworks and research paradigms.

Mathematical self-efficacy (MSE)

Albert Bandura's theory of self-efficacy (1997) defines self-efficacy as people's belief about their capabilities to organise and execute specific tasks in learning. In learning mathematics, students make judgements of the extent of their capabilities in performing mathematical tasks. Hence, the theory suggests that students with high mathematical self-efficacy tend to expect successful performances because they will make an effort to persevere in learning mathematics and meet their goals. Based on my literature review, educational researchers have made considerable progress in establishing the importance of self-efficacy in mathematical achievement in secondary education (Skaalvik & Skaavik, 2011; Stevens et al., 2010; Williams & Williams, 2010), in higher education (Hall & Ponton, 2002, 2005; Pampaka, Kleanthous, Hutcheson, & Wake, 2011), in engineering education (Parsons, Croft, & Harrison, 2011) and through classroom interventions (Clutts, 2010; Falco, Summers, & Bauman, 2010; Fast et al., 2010; Peters, 2013). These research studies suggest that mathematics students with high levels of self-efficacy are more likely to succeed in learning mathematics than those with low levels of self-efficacy.

However, a review of the literature also suggests that university mathematics research could place more emphasis on investigating the predictive value and correlates of self-efficacy rather than the psychological functioning of self-efficacy (Marat, 2005, 2007; Parsons et al., 2011). A small-scale study of secondary mathematics students (Marat, 2005) has found that beliefs in using cognitive processes (solving mathematical problems, using mathematical processes for problem-solving), motivational processes (goal setting), selection processes (time management), beliefs for self-regulated learning is related to mathematical achievement. This evidence suggests that self-efficacy is not only about one's personal judgement but also shapes one's behaviour in self-regulated learning, goal setting and mathematical thinking. Past research has tended to focus on one dimension of mathematical self-efficacy with limited consideration of how it can be applied in higher education. In order to understand the processes of self-efficacy, Bandura (1997) cautioned against using an idiographic approach, rooted in trait theory but proposed a personal determinant approach to future research, founded on the relations between personal, environmental and behavioural factors. This approach will illuminate the nature of self-efficacy and its practical applications. In brief, drawing on the psychology of mathematical learning, I will examine the predictive nature of self-efficacy and its implications on mathematics teaching and learning.

From the perspective of student affect, I will highlight that MSE may be conceptualised as an affective subset of students' mathematics-related beliefs (SMRB) in Chapter 2. Op't Eynde et al. (2002) described the term, student mathematics-related beliefs as

> *the implicitly or explicitly-held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context. These beliefs determine, in close interaction with each other and with students' prior knowledge, their mathematical learning and problem solving in class" (p.27).*

SMRB are relatively stable traits and have cognitive and affective structures. Further definitions of student beliefs and descriptions of the framework of SMRB can be found in Chapter 2.

Conceptions of mathematics (CM)

Another subset of SMRB is conceptions of mathematics that reflect one's beliefs about learning mathematics and mathematics education. A seminal study of student conceptions of mathematics was investigated by Leigh Wood from Macquarie University, Australia and her team of researchers from Australia, Canada, South Africa, Brunei and Ireland. They surveyed a sample group of higher education students across these countries to examine their conceptions of mathematics, asking them to respond to the question, "What is mathematics?" (Houston et al., 2010; Petocz et al., 2007; Wood, Mather, et al., 2012; Wood, Petocz, & Reid, 2012). Based on a phenomenological analysis of participants' responses, they have identified three levels of conceptions of mathematics:

- Level 1 numbers and components which is about a set of procedures or techniques in problem-solving, doing calculations and manipulations of number;
- Level 2 models is defined as a mathematical representation of a specific real life situation and conceptual thoughts;
- Level 3 life is described as connecting mathematics to their own lives and engaging in a mathematical way of thinking.

Pertinent to my research, this hierarchical structure of student conception reflect complex ways in which students view mathematics. Further studies have been carried out that validate these results. For example, Houston et al. (2010) reported that many university mathematics students were likely to hold low levels mathematical conception: seeing mathematics as calculations with numbers (Level 1 Number) (9.2%, N=109) and a toolbox of techniques used to solve problems (Level 1 Components) (43.6%, N=515) whereas fewer students viewed mathematics as a way of thinking about reality and an integral part of life (Level 3 Life) (6%, N=71). These findings provide a starting point for more investigation in the context of Australian and New Zealand higher education.

Producing somewhat consistent results, a phenomenological study of first-year mathematics students (Crawford et al., 1994, p. 336) concluded that students were likely to have fragmented conceptions because a majority of students (77%, N=226) reveal their conceptions of mathematics as being about numbers, rules and formula in problem-solving. The remaining sample (23%, N=67) hold cohesive conceptions of mathematics, which is defined as a complex, logical way of thinking and solving complex problems, and having insights for understanding the world. They have also found that high mathematics achievement cross-tabulated with cohesive conceptions and deep approaches to learning. In another example, Macbean (2004) found that many successful first-year mathematics students were more likely to adopt fragmented conceptions (31%, N=143) than cohesive conceptions (6.8%, N=31), a phenomenon that was not detected for the sample group of failing students. Thus, these results suggest that CM may be associated with other aspects of learning.

Researchers who are studying student beliefs of mathematics have described the nature of student beliefs. While many educational studies have assessed the relationships between student conceptions of mathematics and student achievement (Macbean, 2004), student approaches to learning (Crawford et al., 1994, 1998b; Liston & O'Donoghue, 2009, 2010), teaching and assessments (Cano & Berbén, 2009; Cano & Berbén, 2014), others have examined beliefs about themselves (students) as mathematicians with respect to task-value beliefs (Craig, 2013; Flegg, Mallet, & Lupton, 2012; Khiat, 2010; Matic, 2014), self-efficacy beliefs (Di Martino & Zan, 2011; Op’t Eynde, De Corte, & Verschaffel, 2006) and goal orientation (Eley & Meyer, 2004; Gordon & Nicholas, 2013; Meyer & Eley, 1999). Of these studies, student conceptions of mathematics have been conceptualised across different fields: educational psychology (Cano & Berbén, 2009; Cano & Berbén, 2014), student learning in mathematics education (Crawford et al., 1994; Crawford, Gordon, Nicholas, & Prosser, 1998b; Liston & O'Donoghue, 2009), teaching and learning in higher education (Eley & Meyer, 2004; Gordon & Nicholas, 2013; Meyer & Eley, 1999), student affect in mathematics education (Di Martino & Zan, 2011; Op’t Eynde et al., 2006) and engineering education (Craig, 2013; Flegg et al., 2012; Khiat, 2010; Matic, 2014). To date, some mathematics education researchers have not only framed student conceptions of mathematics in terms of student beliefs in learning and perceptions but also on the procedures and concepts taught in mathematics as a discipline. Hence, my review of past literature serves to frame student conceptions of mathematics and map empirical results onto the framework of students' mathematics-related beliefs (Op't Eynde et al., 2002) (Chapter 2).

Student approaches to learning (SAL)

Previous research has shown how SAL and CM impact on learning outcomes. This raises the question: What can researchers discover about how and why students learn higher education mathematics? Coined by Ference Marton and Roger Säljö (1976), student approaches to learning are identified to be of three kinds: deep, surface and achieving. A surface approach to learning is driven by one's fear of failure and rote-learning and results in low quality learning outcomes. An achieving approach (alternatively, referred as an organised approach in my research) is driven by one's need for achievement and how one makes use of space and time to achieve a task. A deep approach to learning produces the most complex learning outcome and involves the motive of intrinsic interest and strategy to maximise meaning. In line with Marton and Säljö (2005) notions of approaches to learning, a learner makes decisions about his learning based on his intended outcomes of learning.

In Chapter 2, my review will highlight that researchers in student learning have used different terminologies to describe SAL: conceptions of learning (Marton & Svensson, 1979), deep or surface approaches or achieving approaches (Biggs, 1987; Marton & Säljö, 1976), instrumental and relational learning (Skemp, 1987). These terminologies arise from different perspectives on student learning. Furthermore, SAL researchers have long assumed both a phenomenographic approach (Marton, 1981; Marton & Säljö, 1976; Säljö, 1979; Svensson, 1977) and cognitive processing and constructivist perspectives to student learning (Biggs, 1985, 1987). Phenomenographic researchers contextualise their research by extrapolating student conceptions of learning through student interviews. Säljö (1979) has reported that when they carried out academic tasks, Swedish adult learners revealed five main conceptions of learning: A) An increase of knowledge, B) Memorising facts, C) The acquisition of facts or procedures, D) The abstraction of meaning and E) Applying ideas to reality. As stated by Marton and Svensson (1979), phenomenographic researchers have adopted a contextualised approach to frame SAL in a qualitative way. But many quantitative studies have produced generalised descriptions of SAL (Cano & Berbén, 2009; Cano & Berbén, 2014; Clercq, Galand, & Frenay, 2014; Crawford et al., 1998a; Entwistle, Nisbet, & Bromage, 2005; Liston & O'Donoghue, 2009). For example, a study of mathematics students by Crawford et al. (1998a) has demonstrated a negative loading on final marks with a surface approach, fragmented conception, inappropriate workload and assessments; and a positive high loading on cohesive conception, good teaching and deep approach. This study shows the generalised relations between these specific constructs. Hence, while phenomenological studies produce rich data, quantitative studies reveal external relations between SAL and other learning constructs. For discussion about the research paradigms, refer to chapter 2.

Further, Biggs (1985) argues that student approaches to learning operate as products of meta-learning from a cognitive processing perspective. This refers to the cognitive processes of how one goes about the tasks in which both intention and process co-exist based on the ‘psych-logic’ of human behaviour in academic setting. To illustrate ‘psych-logic’ behaviour—if a person intends only to pass the course (surface motive), they may undertake surface learning using minimal effort and rote learning (surface strategies) to meet the assessment requirements. If an individual is interested in the subject (deep motive), they may adopt deep strategies such as understanding and relating the topics with real life problems, regardless of any assessment which might ensue. The task is a means for achieving their aims and is dependent on the situation (e.g., teaching and assessment). If the task requires understanding the materials and relating the concepts, they will engage in deep approaches to learning. Based on this perspective, many quantitative studies (Cano & Berbén, 2009; Cano & Berbén, 2014; Crawford et al., 1998a; Liston & O'Donoghue, 2009, 2010; Macbean, 2004) have used the Study Process Questionnaire (SPQ) (Biggs, 1987) or its modified version, Approaches to Studying Inventory (ASI). Furthermore, Biggs (1987) conceptualised the presage-process-product (3Ps) model of student learning to assess the nature of learning given an appropriate class intervention. In this constructivist model, SAL are assessed during the presage phase (prior learning) and the process phase (during learning). Other latter studies (Prosser & Trigwell, 1999; Streitwieser & Light, 2010; Trigwell & Prosser, 1991a, 1991b, 1991c) have examined the impact of teaching and student perceptions on student approaches to learning.

In terms of teaching interventions, student learning researchers have focused on successful teaching interventions (Albano & Pierri, 2014; Carroll & Gill, 2012; Engelbrecht & Harding, 2015), beliefs about teaching (Biggs & Tang, 2007; Hounsell et al., 2005a; Norton, Richardson, Hartley, Newstead, & Mayes, 2005), professional development of teachers (Evans, 2014; Isvoran, Pitulice, Ostafe, Craciun, & Asproniu, 2011), curriculum development (Bruner, 1977) and formative assessments (Biggs, 1995). A review of these studies will be included in Chapter 2. Based on my review, I observed that to date, many educational studies have investigated SAL in the context of 3Ps model of student learning and its relation with mathematical achievement but few have investigated the interrelations of SAL and CM and explain why students were not achieving in mathematics and how to improve this situation.

1.3 Research aims and questions

Researchers have made good progress in conceptualising student beliefs in learning mathematics and student approaches to learning. However, such conceptualisations have raised a number of questions concerning the role and nature of these constructs and whether they are predictive of mathematical achievement. My research will focus on the nature of learning (SAL), student beliefs (CM, MSE) and personal characteristics (prior learning, gender and age). Understanding student beliefs about learning is a positive approach of eschewing the deficit model of learning and teaching. Valencia (2010) argues that this deficit model of learning and teaching is institutionally created and promotes a culture of blame. If higher education providers only emphasise student performance, then some mathematics lecturers may focus on students' deficiencies in learning, resulting in efforts to improve student grades that may not address the underlying causes of poor performance. The main rationale of my research is to frame these constructs in the context of higher education and recommend ways of supporting mathematics students. In keeping with this rationale, firstly, my research aims to understand the nature of mathematical self-efficacy (MSE) and its predictive nature in terms of mathematical achievement. Based on the theory of self-efficacy by Albert Bandura (1997), the sub-constructs are:

- *Cognitive processes or strategies are described as thinking processes which involves the acquisition, organization and use of information.*
- *Motivational processes or strategies include causal attributions, outcome expectancies, and cognized goals.*
- *Selection processes or strategies involve choices that people make in the social and physical environment and types of activities that they judge themselves to be capable of handling.*
- *Self-regulation learning strategies entail planning and organizing instructional activities, utilising resources, adjusting one's own motivation and using metacognitive skills to evaluate the adequacy of one's strategies and knowledge.*

Secondly, I will investigate the nature of conceptions of mathematics as situated within the framework of students' mathematics-related belief (Op't Eynde et al., 2002). This framework has been applied to another study of secondary mathematics students (Op't Eynde et al., 2006) to elucidate their epistemic dimension. To conceptualise CM, the same terminologies (as shown below) have been used following an international study of higher education mathematics students (Wood, Petocz, et al., 2012) as shown in Section 1.2.

Thirdly, my research aims to investigate the interrelations between SAL, CM and learning outcomes. The key sub-constructs are as follows (Marton & Säljö, 1976):

- *A surface approach to learning is driven by one's fear of failure and rote-learning and results in low quality learning outcomes.*
- *An achieving approach is driven by one's need for achievement and how one makes use of space and time to achieve a task.*
- *A deep approach to learning produces the most complex learning outcome and involves the motive of intrinsic interest and strategy to maximise meaning.*

Lastly, besides investigating these constructs, my research aims to examine whether personal characteristics affect examination performances. Demographic characteristics of students included are prior mathematics, age and gender. Prior mathematics is based on the New Zealand secondary qualification such as National Certificate of Educational Achievement (NCEA) Levels 1, 2 and 3 (equivalent to Grades 10th, 11th and 12th respectively). Age is categorised as 'under 25 years old' or 'over 25 years old'. Gender is labelled as 'Male' and 'Female'. The performance indicator refers to examination mathematics results, ranging from 0 to 100 marks. In my research, 'success' is associated with passing or attaining more than 50 marks in the mathematics examinations. Based on the course outlines (See Chapter 3), students, who passed the mathematics course, would be able to apply mathematical knowledge in problem-solving and develop procedural as well as conceptual understanding in learning mathematics. This notion of success seems to be a conventional indicator of examination performances in higher education. In line with the abovementioned aims, my research questions are outlined below.

Study I

1. *What is the nature of student mathematical self-efficacy?*
2. *To what extent does mathematical self-efficacy predict mathematics results?*

Study II

1. *What is the nature and extent of student approaches to learning?*
2. *What are the characteristics of students' conceptions of mathematics?*
3. *To what extent are student approaches to learning and conceptions of mathematics related?*
4. *How are they related to performance?*

Study III

1. *To what extent do mathematical self-efficacy, student approaches to learning and conceptions of mathematics predict mathematics performance?*
2. *How are prior mathematics, age and gender differences related to mathematics results?*

Based on my literature review (Chapter 2), for Study I and Study III, my hypothesis is that MSE predicts high scores in examinations. For Study II, I posit that if the students view mathematics as a discipline that teaches mathematical modelling and life applications (as a cohesive conception of mathematics), they are likely to engage in deep approaches to learning and relational understanding. Conversely, when they view mathematics as formulae and procedural calculations (as a fragmented CM), they tend to adopt surface approaches to learning and procedural understanding. For Study III, based on previous literature (see Chapter 2), I postulate that successful students are likely to have completed NCEA level 3 mathematics, males and older students are likely to succeed in mathematics.

1.4 Research methods

Sample

My research sample included approximately 300 mathematics students at the Macquarie University (MQ), Australia and Manukau Institute of Technology (MIT), New Zealand. In 2014, I conducted a quantitative study of 67 MIT students to investigate the role of MSE (Study I). In a separate study (Study II), I investigated the nature of CM and SAL using two samples of 183 MQ students and 93 MIT students who completed Likert-style questionnaires at the same time. In another quantitative study (Study III), 73 MIT students participated in the questionnaires which investigated the relations between MSE, CM and SAL, personal characteristics and mathematics results. Both groups of mathematics students (MQ and MIT) enrolled in first-year mathematics courses, which comprised of algebra and calculus but differed in terms of mathematical applications in engineering and business. More details of the courses will be discussed in Chapter 3.

Instruments

In my quantitative studies, I use five-scale Likert-style questionnaires: the original and abridged versions of Refined Self-Efficacy Scale (RSS) (Marat, 2005), the Shortened Experiences of Teaching and Learning Questionnaire (SETLQ) (Hounsell et al., 2005b) and the Short Form of Student Conception of Mathematics survey (Wood, Petocz, et al., 2012), included in Appendices 1a, 1b, 2 and 3 respectively. According to Patton (2002), using quantitative data can be advantageous because the data is gathered from an independent source and analysed according to hypotheses and theoretical underpinnings. The strengths of conducting quantitative surveys include objectivity based on simple random sampling, access to students' self-reports of learning with the aid of statistical comparisons and the use of aggregated scores. Since the categories in the surveys are pre-determined and numerical scales are assigned to each category, the researcher can analyze the categories and make conclusions of these categories. Scaling can be defined as "the assignment of objects to numbers according to a rule"(Trochim, 2006, para. 1) As such, the Likert-style scale enables participants to give their responses to a set of items, using assigned single numbers, which represent a person's overall attitude or belief. For more descriptions about these scales, refer to Chapter 3.

1.5 Research findings

Study I

Based on the survey data, the participants (N=67) show the highest scores in self-efficacy in solving numerical and measurement problems. Multivariate regression data show the model (Beta=0.482, t=2.335, p=0.027) indicates that the appropriate predictors of strong examination performance are self-belief in motivational, cognitive and selection strategies and self-belief for self-regulated learning. These results have been published in a conference paper (P. Murphy & Wood, 2017), included in Appendix 4.

Study II

This study reports that the participants (N=291) had high mean scores in conceptions of mathematics Level 2 (3.94), Level 1 conception of mathematics (3.88), deep approach to learning (3.88), and organised approach to learning (3.61); and lower scores in surface approach to learning (3.22) and a Level 3 conception of mathematics (3.42). The Chi-square data show that deep approaches to learning and Level 3 life CM were related (χ^2=5.657, df=1, p=0.017). Correlation data show a negative correlation between Level 2 models CM and surface approaches to learning and positive correlations between high examination mathematics scores, deep approaches to learning and cohesive CM. These findings have been published in a book chapter (P. Murphy, 2017), included in Appendix 6.

Study III

Using the sample (N=73), the multivariate regression data reveal that an appropriate predictor of successful examination performance is self-belief in selection processes. The model (Beta=0.599, t=2.413, p=0.019) accounts for 34.7% (R square) of the variation of results. Positive correlations are reported of these constructs: deep approaches to learning, organised approaches to learning and MSE; strong performance Level 2 models CM and Level 3 life CM, Level 3 life CM, deep approaches to learning and organised approaches to learning. There is a negative correlation between surface approaches and Level 2 models CM. The univariate analysis of variance data show that age and gender are not significant factors of examination results but prior mathematics is. These results have been recorded in a journal article (Appendix 7) and a poster presentation.

1.6 Research significance

This research has theoretical and practical significances. It provides a unique theoretical contribution by advancing our understanding of the nature of mathematical self-efficacy, approaches to learning and mathematics-related beliefs in learning mathematics. Based on the psychological framework of self-efficacy (Bandura, 1997), the framework of student approaches to learning (Marton & Säljö, 2005) and the framework of students' mathematics-related beliefs (Op't Eynde et al., 2002), I will explore the role of these constructs and their influence on mathematical performances in higher education. Next, my research findings would be of interest to higher education practitioners since these constructs are related to mathematical performances. In line with Skemp's (1978) notion of relational understanding, which is an important goal of mathematics education, my research findings would be useful for identifying new ways of learning mathematics and teaching interventions.

1.7 Summary and going forward

In summary, the primary goals of my research were to investigate the nature of mathematical self-efficacy, student approaches to learning, conceptions of mathematics and their relationships with mathematics examination results. My research would pose practical and theoretical significances in mathematics education as well as fulfil the broader objective of developing human capital in undergraduate business and STEM education.

My thesis incorporates six chapters: introduction (Chapter 1) literature review (Chapter 2), methodology (Chapter 3), research portfolio (Chapter 4), research discussion (Chapter 5) and conclusion (Chapter 6). Keyword definitions, theoretical frameworks, past literature based on research paradigms and findings are presented in Chapter 2. In Chapter 3, my ontological research perspective, research design, methodological consideration and statistical methods of analysis are discussed. Here there is a description and an in-depth methodological analysis of questionnaires. Next, my research portfolio is outlined in Chapter 4. This is followed by a summary of notable research findings and implications on mathematical teaching and learning

(Chapter 5). My final chapter is an outline of my personal reflections and future research opportunities (Chapter 6). Finally, the appendices section includes four questionnaires (Appendices 1a, 1b, 2 and 3), the full texts of a conference paper (Appendix 4), a book chapter (Appendix 6), a couple of journal articles (Appendices 5 and 7) and the participant information and consent form for students and lecturers (Appendix 8).

Chapter 2
Literature Review

Using quantitative methods of study, my research objectives are three-fold:

- To investigate the nature of mathematical self-efficacy, student approaches to learning and conceptions of mathematics
- To ascertain whether these constructs and other personal factors (prior mathematics, age, gender) influence mathematics examination results
- To recommend appropriate teaching interventions for enhancing mathematical self-efficacy and deep approaches to learning

My review will discuss theoretical frameworks of self-efficacy (Bandura, 1997), student approaches to learning (Marton & Säljö, 1976), student learning (Biggs, 1985) and Student Mathematics-Related Beliefs (Op't Eynde et al., 2006). Next, I will synthesise previous literature based on significant findings and quantitative research paradigms. In doing so, my review will identify research gaps and inform my research questions.

2.1 Mathematical self-efficacy

The purpose of my first study is to examine the nature of mathematical self-efficacy and its relationship with mathematical performances. This section will describe a theoretical framework of self-efficacy.

Theoretical framework of self-efficacy

According to Bandura (1997), self-efficacy is about human enablement. Bandura (1977) states that "a strong sense of efficacy to regulate one's motivation and instructional activities undergirds belief in one's academic efficacy and aspirations" (p.231). People with strong self-efficacy are likely to attain successful learning outcomes. People with strong self-efficacy affirm themselves by drawing on self-knowledge based on prior mastery experiences and adapting their knowledge and skills to successfully accomplish future tasks. As such, self-efficacy plays a self-enhancement role in academic outcomes. In other words, a student with high self-efficacy will try to attain their goals and work hard in order to achieve high grades.

In the face of failure, one can either be resilient or feel discouraged, depending on whether one has the means of mastering the skills required in certain performances. According to Bandura (1997), past performances can either improve or undermine learning through self-efficacy because “efficacy beliefs contribute to the acquisition of knowledge and development of sub-skills, as well as the construction of new behaviour patterns (Bandura, 1997, p. 61). On one hand, high performance in a particular task promotes self-efficacy, which in turn, emboldens individuals to work harder and develop further skills necessary for attainment in future tasks. On the other hand, repeated failure lowers self-efficacy, at the early stage of learning when there is a lack of effort. For instance, when students fail their first algebra test, they can either choose to study hard and improve their algebraic skills or not. Successful students tend to work hard and increase skill mastery whereas unsuccessful students tend to expend less effort in building their basic algebraic skills. This self-efficacy trajectory in learning is often observed in skill development.

Framed within social cognitive theory, self-efficacy is the function of several causally-related determinants – personal factors (cognitive, biological and affective events), environmental factor and behavioural factor. In other words, human behaviour is determined by reciprocal interplay of personal and environmental factors. As people are agents of their own actions, they can adapt to the environment or change it to make things happen. Self-efficacy is not just about having appropriate knowledge and skills but also about what individuals can do under a variety of circumstances. Those with positive self-belief that their personal actions determine positive outcomes will develop a sense of self-efficacy and work hard to succeed. Conversely, a lack of belief generates apathy in doing one’s task or performance. However, in a responsive environment that values accomplishment, such individuals will increase their self-belief so that their renewed effort and participation in activities are productive. Depending on the environment, people vary their strength in self-efficacy. In a nurturing teaching environment, lecturers could help student to develop strong self-efficacy whereas in a teaching environment with fewer resources (such as lack of professional development, insufficient technology), this may promote low self-efficacy. This personal determinant approach shows that self-efficacy not only predicts outcomes but is governed by personal, environmental and behavioural factors.

According to Bandura (1997), self-efficacy produces learning outcomes through major processes known as cognitive, motivational and selection processes. Firstly, cognitive processes are described as thinking processes which involves the acquisition, organization and use of information. As a function of self-appraisal of capabilities, goal setting resides in forethought which translates into purposive actions. People with high self-efficacy mediate through cognitive processes by visualising success, which in turn provides cognitive support and guides for attainment. The stronger the self-efficacy, the higher the goals individuals set themselves to attain performances. Secondly, self-efficacy plays a key role in self-regulating motivation. Motivational processes include causal attributions, outcome expectancy, and cognized goals, corresponding with the attribution theory, expectancy-value theory and goal theory. In causal attribution, people with high self-efficacy will attribute poor outcomes to lack of effort whereas those with low self-efficacy attribute failure to low ability. Further, expectancy theory states that people expect their behaviour and actions to bring about valued outcomes. People with high self-efficacy are more likely to persevere and attain successful outcomes. Also, goal setting is governed by the cognitive processes of motivation. Those with strong self-efficacy will endeavour to reach their goals through effort and persistence. Thirdly, driven by selection processes, people are partly the product of their environment because they choose the social and physical environment and types of activities that they judge themselves to be capable of handling. In theory, these metacognitive processes determine self-efficacy and indirectly affects the outcomes of learning.

In metacognitive terms pertaining to self-efficacy, self-belief for self-regulated learning promotes both skill mastery and learning strategies. According to Bandura (1997), self-regulation entails skills and strategies for planning and organizing instructional activities, utilising resources, adjusting one's own motivation and using metacognitive skills to evaluate the adequacy of one's strategies and knowledge. Students who have strong belief in using self-regulation strategies tend to have better mastery of mathematics skills and performances. Other self-regulation theorists (Boekaerts & Cascallar, 2006; Zimmerman, 1989) claim that self-regulation is also used in cognitive processes through goal-setting, monitoring progress and use of cognitive strategies. These strategies involved orienting oneself before an assignment, collecting relevant resources, integrating ideas and monitoring progress in learning. As such, these strategies would enable individuals to steer their learning processes, to self-regulate their motivation for learning and amount of effort. Using the social cognitive theory of self-regulation, Zimmerman (1989) also supports that the triadic influences of personal (cognitive and emotional), environmental and behavioural factors affect their learning outcomes. This study of forty grade 10 students reported positive impact of self-regulation strategies such as seeking information, goal setting, seeking social assistance, on test performances. The result suggests that if a mathematics student successfully solves an algebra problem using their self-regulated strategies, they show the correct procedures (behavioural) with the help of their peers (environmental) and plan long-term goals in solving more complex problems (personal). By using metacognitive strategies in self-regulated learning, student self-efficacy increases.

Mathematics performance is seen to be determined by four sources of self-efficacy: mastery experiences, vicarious experiences, verbal persuasion and physiological processes. According to Bandura (1997), the most influential source is mastery experiences as they provide the most authentic evidence of skill mastery that involves the acquisition of cognitive and self-regulatory tools for performing the activities. These mastery experiences provide evidence of competence and are organised hierarchically such that complex skills are broken down into easily mastered sub-skills. Next, vicarious experiences show the effects of modelling on self-efficacy depending on how the information is cognitively processed by individuals. Self-efficacy increases when one models behaviour of people with similar attributes (such as age, gender, educational levels, socioeconomic levels and ethnicity). Thirdly, self-efficacy improves through verbal persuasion. When a mathematics teacher offers positive feedback about learning mathematics, the recipient's self-efficacy in learning mathematics improves as they have confidence in the person who provides feedback. Lastly, physiological processes refer to mood states including positive and negative affect. A positive mood creates thoughts and feelings of past successes, improving one's self-efficacy in solving a mathematical problem. Negative affect conjures past failings, reducing one's appraisal of self-efficacy in learning mathematics. Both kinds of mood set in motion either an upward or downward cycle of accomplishment or poor performance. Based on the self-efficacy theory, the four sources of self-efficacy can be performance-enhancing tools in teaching and learning mathematics.

Previous literature

My first study will investigate the role and nature of mathematics self-efficacy in learning mathematics. In line with these objectives, the following sections will highlight the predictive role of self-efficacy and metacognitive components of self-efficacy (such as cognitive, motivational and selection processes, self-regulated learning).

Role of self-efficacy

In line with my research objective, I will consider the role of self-efficacy in learning mathematics. Empirical studies have revealed a positive relationship between strong self-efficacy in solving mathematics problems and high mathematics performance. Yet, some researchers have suggested a need to examine bi-directional relationships and learning factors. In an international study, Williams and Williams (2010) argued that causal relationships between self-efficacy and mathematics performances have been difficult to prove as researchers tend to assume one position or other when using recursive statistical models to estimate the model. To illustrate this point, their structural equation modelling data showed bi-reciprocal relationships between self-efficacy and achievement of secondary mathematics students in twenty-four out of thirty-three nations.

Many researchers have shown that self-efficacy predicts success in mathematics performance (Hailikari, Nevgi, & Komulainen, 2007; Hall & Ponton, 2005; Marcou & Philippou, 2005; Pajares & Kranzler, 1995; Pajares & Miller, 1994; Skaalvik & Skaavik, 2011; Stevens et al., 2010). Marcou and Philippou (2005) reported that motivational beliefs as a function of self-efficacy correlated with problem-solving performances of fifth and sixth graders. A study of middle and high school mathematics students has found that self-efficacy was a better predictor of mathematics achievement than prior achievement (Skaalvik & Skaavik, 2011). This result was also evident for higher education students of calculus in a study by Hall and Ponton (2005) who found that university calculus students who reported high self-efficacy gained better results than other remedial students who also had low prior experience and/or achievement. In another study, path model data showed a positive relationship between mathematical achievement and self-efficacy in problem-solving of ninth-grade and tenth-grade mathematics Caucasian students (Stevens et al., 2010). Pajares and Kranzler (1995) concluded that students had high self-efficacy because they exhibited more effort and perseverance in challenging problem-solving situations. Although these findings are mixed, these studies serve to conceptualise the self-enhancement role of self-efficacy. However, such investigations of the way self-efficacy affects mathematical performance in higher education have been limited. Hence, more research is warranted to understand the psychological functions of self-efficacy in learning mathematics, particularly in higher education.

Correlates of mathematical self-efficacy

Cognitive, motivational and selection processes

Few researchers have conceptualised the nature of self-efficacy in learning mathematics in accord with the theoretical framework of self-efficacy. In a New Zealand quantitative study of secondary mathematics students, Marat (2005) investigates the determinants of self-efficacy on mathematics performances to model the personal determinant approach. Using the method of discriminant analysis, Marat (2005) reported that student self-beliefs in cognitive processes (solving mathematical problems, using mathematics processes for problem-solving), self-beliefs in motivational processes (goal setting), self-beliefs in selection processes (time management) and self-beliefs for self-regulated learning were positively related to strong mathematical achievement. This study also reported positive correlations between excellent mathematics grades (equivalent to A grade) and high scores in self-efficacy in solving algebra problems, self-belief for self-regulated learning as well as beliefs in selection processes and motivational processes. These findings suggest that self-efficacy is governed by cognitive, motivational and selection processes which positively impact on mathematical achievement. Another qualitative study of engineering mathematics students in United Kingdom by Parsons et al. (2011) including interviews of seven students at the Harper Adams University College found that the provision of student support has somewhat helped students to develop their cognitive processes. Confident students set high goals of mastering all the topics whereas less confident students avoided doing the difficult mathematics. They also developed a low self-belief in motivational processes as they were less motivated to work hard and tried to avoid difficult mathematics questions, which lowered their self-confidence and made them choose alternative questions in the examinations. Further results showed that selection processes were reflected by their deliberate choices to study mathematics. Although it may not be possible to generalise from these local studies, these empirical results shed some light on the metacognitive determinants of self-efficacy in learning mathematics.

Self-regulated learning

Another mechanism of self-efficacy is self-regulated learning which is considered here for its potential as a metacognitive component in my research. A few researchers have conceptualised self-efficacy in learning mathematics using metacognitive perspective of self-regulation. For instance, Mulat and Arcavi (2009) have reported that university mathematics students attributed their success to using self-regulation strategies such as studying without distraction, completing homework, seeking peer and teacher support, paying attention in class, preparing well for examinations, persistence in solving challenging tasks, and making concerted effort on school tasks. Interestingly, they pointed out that independent study was also considered as a self-regulation strategy as students found individual learning less disruptive and were able to engage in deep thinking and understanding. Other educational researchers (Pintrich & De Groot, 1990; Wolters, Yu, & Pintrich, 1996) have integrated research on cognitive and motivational processes in the form of achievement goals, expectancy-value, self-efficacy and task value in academic settings. For example, Wolters et al. (1996) have reported that both mastery and performance goals had positive effects on self-efficacy, task value and the use of cognitive processes and meta-cognitive strategies. They also found that students, with high levels of mastery goals and low levels of performance goals, tended to have the highest scores in self-efficacy, task value and use of cognitive and meta-cognitive strategies. These results resonate with an earlier study by Zimmerman, Bandura, and Martinez-Pons (1992) who found that goal setting, as part of cognitive processes, was predictive of their achievements when both constructs had increased prediction by 31% compared to prior grades in social studies. As a whole, past research has indicated that when students develop self-regulation strategies, they are likely to increase self-efficacy and improve their outcomes of learning.

Sources of self-efficacy

Understanding the sources of self-efficacy may help to address the issue of how self-efficacy is developed in practical teaching situations. In recent decades, mathematics education researchers have demonstrated self-efficacy as developing through mastery experiences, vicarious experiences, verbal persuasion and emotional states of self-efficacy. In a qualitative study of engineering college students, Parsons et al. (2011) reported that mastery experiences were depicted by one's success in school mathematics. Many students perceived difficulties as challenging experiences and would work harder to obtain positive performances. Similarly, interviews of eighth graders by Usher and Pajares (2009) have found that low grades in mathematics and perceived difficulties of mathematics formed their mastery experiences. Furthermore, the study by Parsons et al. (2011) showed how vicarious experiences in the form of attribute similarity were formed when college students who had completed GCE A-level mathematics gained confidence in the company of those without GCE A-level mathematics background. With younger people, the study by Usher and Pajares (2009) showed that through verbal persuasion, some parents provided negative modelling experiences as they had low aptitude in mathematics. They also reported that some students received little positive feedback and had lower self-efficacy while others were emboldened by verbal encouragement. These studies provide findings which could be generalised to mathematics students in various contexts as learning mathematics requires mastery of skills that are performed in high-stakes assessments and modelling of skills in class or at home. Another study of fourth- to tenth- grade mathematics students examined relationships between mathematics self-efficacy, source of self-efficacy and mathematics anxiety as a form of emotional feedback and performances (Stevens, Olivárez, & Hamman, 2006). They found that the sources of self-efficacy predicted MSE, and indirectly, mathematics performances. These researchers concluded that self-efficacy can help to create powerful learning experiences through mastery experiences, vicariously, verbally and emotionally, which were predictive of mathematics achievement. In a latter section of this chapter (Prior mathematics), I argue that mastery experiences serves as a proxy to prior mathematics, which in turn, influences mathematics results. In terms of self-efficacy programmes, lecturers could considering the four sources of self-efficacy, a topic that will be discussed in another section of this chapter (Teaching interventions to build self-efficacy).

Correlates of mathematical self-efficacy and performance

In the subsequent sections, I will review how prior mathematics knowledge, gender and age differences and teaching interventions relate to self-efficacy and achievement. Despite inconsistencies in some studies, past literature suggests that advanced prior mathematics (equivalent to Year 13 mathematics), age and gender stratifications are related to mathematics performances. To inform my research findings (Chapter 5), previous research on teaching interventions address the broad issue of improving mathematical achievement and learning.

Prior mathematics

As highlighted in Chapter 1, prior mathematics is an area of investigation. Some educational researchers have challenged the notion that self-efficacy is not the sole determinant of achievement. According to Bandura (1997), the discordance between self-efficacy and actual performance is explained by perceived value of learning outcomes, prior performance, other forms of beliefs, knowledge and skills and a need to meet socially desirable norms. These constructs should work in harmony with self-efficacy to predict performance. If students have poor mathematics skills due to low levels of prior mathematics, strong self-efficacy alone may not bring about desired performance. Instead, more effort and persistence in learning basic mathematics can help to improve mathematical skills. Bandura (1997) states that self-efficacy beliefs are developed by prior performance known as enactive attainment. Successful enactive attainment is about mastery experiences gained from prior learning. As one of the four sources of self-efficacy, mastery experiences provide the most authentic evidence of skill mastery that involves the acquisition of cognitive and self-regulatory tools for performing activities. Since prior performance develops self-efficacy, Bandura (1997) argues that researchers will need to control for prior performance in order to ascertain whether self-efficacy has a greater effect. Mastery experiences are necessary in skill development. In learning mathematics, mastery experiences are important because the levels of mathematics competence are organised hierarchically such that complex skills are broken down into easily mastered sub-skills. For instance, at the University of Mississippi, Ponton, Edmister, Ukeiley, and Seiner (2001) claimed that creating mastery experience was important in engineering education so that students would develop skills necessary for practising professionals in engineering. Mapping the four sources of self-efficacy onto mathematical learning, a qualitative study by Parsons et al. (2011) reported that mastery experiences of engineering college students depicted their past successes in school mathematics. When they perceived difficulties as challenging experiences, they were willing to work harder. Another study of first-year engineering mathematics students by Hutchison, Follman, Sumpter, and Bodner (2006) reported that some students cited grades as a form of mastery experiences. To sum up, these studies have indicated that prior mathematics is not only about mastery experiences of self-efficacy but is also reflected in school grades.

Many researchers have reported that advanced prior school results determines success in college and university mathematics (Carmichael & Taylor, 2005; Cybinski & Selvanathan, 2005; Engler, 2010a; Faulkner, Hannigan, & Fitzmaurice, 2014; Hailikari, Nevgi, & Komulainen, 2007; Hailikari, Nevgi, & Lindblom-Ylänne, 2007; Hall & Ponton, 2002, 2005; Harwell, Post, Medhanie, Dupuis, & Lebeau, 2013) and engineering mathematics (Gynnild, Tyssedal, & Lorentzen, 2005; Parsons, Croft, & Harrison, 2009). For instance, Hall and Ponton (2005) found that university calculus students reported higher self-efficacy and better results than students who had less prior experience based on American College Test (ACT) mathematics sub-scores. But in a longitudinal two-year study of economically disadvantaged higher education mathematics students, Pampaka et al. (2011) reported that students who scored an 'A grade' in GCSE mathematics had better self-efficacy scores whereas those who had attained a C grade, produced lower self-efficacy scores. These findings suggest that a greater exposure to secondary mathematics could have a positive influence on self-efficacy and future mathematical performances. However, there may be other intervening factors that lead to inconsistencies between prior mathematics and future mathematics achievement. As a case in point, Hailikari, Nevgi, and Komulainen (2007) had used a seven-item academic self-beliefs scale to examine the causal relationships between prior knowledge, self-efficacy and prior success in mathematics in predicting mathematics achievement. Of these factors, structural equation modelling data indicated that advanced prior mathematics knowledge was the strongest predictor of future mathematics achievement and that self-efficacy was a good predictor of achievement via prior knowledge. Because the authors had treated self-efficacy as a global measure of success, this may have had an impact on the effects of self-efficacy on achievement.

Prior school mathematics is not always an entry prerequisite to higher education. In order to attract diverse students who study mathematics as service subjects, higher education providers seem to adopt a minimalist approach to admitting newly-enrolled mathematics students. These students enrol in mathematics courses which may be compulsory subjects in arts, engineering, science and business undergraduate programmes. In Australia, Belward et al. (2011) reported that of the 17 higher education providers, five did not require mathematics in science degree programmes. In New Zealand, higher education providers set their admission requirements by stipulating a specified number of mathematics credits but provide differentiated pathways for students with relatively weak mathematics backgrounds (those who do not study mathematics up to NCEA level 3) and those with strong mathematics backgrounds (NCEA level 3 mathematics or equivalent to twelfth-grade). In these cases, higher education providers tend to downplay the importance of Level 3 NCEA mathematics course requirements. This practice may not be consistent with the finding of a New Zealand government report that success rate of completing first-year higher education mathematics courses was high for nearly all the NCEA level 3 students, who had scored at least above-average school mathematics at NCEA level 2 (equivalent to eleventh-grade) (Engler, 2010a). Likewise, other evidence indicates that the quality of achievement in NCEA level 3 mathematics with calculus correlates with successful completion of first-year engineering, management, commerce and mathematical sciences courses (Engler, 2010b) and of first-year university mathematics (James, Montelle, & Williams, 2008). In Australia, Rylands and Coady (2009) also concluded that performance in senior secondary school mathematics correlates with attaining success in first-year mathematics. In another large scale three-year study of 1000 undergraduates studying service mathematics in the field of science, technology, business, computers and engineering, further evidence of those at risk of failing university mathematics (science, technology, business, computers and engineering) in these fields were predicted by low diagnostic results as well as prior mathematics background (Faulkner et al., 2014). While these studies have consistently shown that prior mathematics matters, another study by Varsavsky (2010) reported that senior secondary students, who had completed elementary mathematics (with no calculus) and intermediate mathematics, (with little calculus) were performing equally well in university mathematics. Moreover, their performances were on par with those who did not complete mathematics in the twelfth grade. This inconsistency is surprising because it challenges the common assumption that higher secondary mathematics qualifications are indicative of prior knowledge and skills in learning mathematics. Institutional factors come into play in considering the relations between prior mathematics and future mathematical achievement.

Given the increasing number of higher education students with weak mathematics backgrounds, some researchers have addressed the need for students to seek early academic interventions. For example, Warwick (2010) has argued that lack of basic mathematics knowledge tends to result in inadequate mathematical understanding and poor performances, which could be improved by offering additional academic intervention. Since the entry requirement for the course was a C grade, the mathematics students had to pass a ‘driving test’, which serves as a diagnostic tool and attend in-house extra mathematics sessions if they failed the test. However, the driving test led to more anxiety for students. Their study concluded that it was more appropriate to conduct a survey at the beginning of the course in order to identify students' expectations and attitudes than assess their mathematical knowledge, which caused greater student anxiety and a barrier to learning and that mathematics support was a useful way of helping students to build self-efficacy. Another study by Wilson and MacGillivray (2007) concluded that algebra-based mathematics was an appropriate pre-requisite subject in order for students to develop basic algebraic knowledge in learning statistics. These studies indicate the importance of academic support and foundation or bridging mathematics education for academically disadvantaged higher education students. Furthermore, results pose significant implications for the nature of academic support in mathematics education.

Age

Another factor related to self-efficacy and achievement is age. Challenging the popular notion of self-enhancement, which was mentioned earlier, Carmichael and Taylor (2005) have argued that age and advanced levels of prior mathematics were confounding factors of self-efficacy. They found that despite having lower self-efficacy, non-traditional students could perform better in their examinations than traditional students (who were between 18 and 25 years old). Even though older students had less experience in higher education, they were more realistic about their own capabilities and had a better awareness of the skill set required for the tasks. They also reported that traditional students were less likely to outperform those with higher prior knowledge. Another reason why non-traditional students were performing well was interest in learning. Forgasz and Leder (2000) have found that they were less critical of the higher education mathematics environment than the younger students although more students enjoyed mathematics in school than at university. Other reasons cited were better academic preparation in bridging higher education courses (Liston & O'Donoghue, 2010) and a sense of confidence and enjoyment in learning (Miller-Reilly, 2006). According to Knowles (1984), adult students were able to overcome their poor confidence and are motivated to succeed because these align with the principles of andragogy in adult learning theory, stating that such learners (non-traditional group) were intrinsically motivated to learn and succeed. With maturity, they develop a strong self-concept and become self-directed learners. In short, in the light of the adult learning theory and past literature, being non-traditional student may not be a disadvantage.

Gender

Self-efficacy is further influenced by gender**.** Current debate about gender differences raises equity issues of under-achievement in mathematics education. With the aid of advanced statistical analyses and data from the Trends in International Mathematics and Science Study and the Programme for International Student Assessment, Else-Quest, Hyde, and Linn (2010) conducted a meta-analysis of gender differences in mathematics achievement, attitudes, and affect across 69 nations throughout the world on over 490000 students, 14–16 years of age. The authors have argued that societal gender stratification caused by lack of opportunities and resources could lead to poor mathematical performances of women. With more resources for females and gender equity, they could compete with males on a level playing field. This study has reported that in Australia and New Zealand, there have been gender differences, favouring the males, in mathematical achievement and self-confidence. In brief, this international study raises the point about inequity because the males seem to have more opportunities than the females.

Associated with lack of opportunities, some female students have experienced barriers to learning in the engineering industry. In New Zealand, a national government report (Ayre, 2011) has claimed that the barriers to entry into engineering professions were male dominance in engineering profession, gender-biased classroom practices, inappropriate career counselling in secondary schools and faulty perceptions of females in engineering. In Australia, a government report about international comparisons of STEM education (Marginson et al., 2013) stated that low participation of women in mathematics and science education was considered as a waste of economic resources. In order to grow the pipeline of women graduates in mathematics education, the Australian report has recommended better inclusion of women in mathematics education by ensuring that school principals, school career counsellors, media publicists, higher education providers and industry professionals work collectively towards a common goal of attracting the females in engineering education. To sum up, these Australian and New Zealand reports have shown that lack of opportunities and barriers to entry into engineering education tend to perpetuate gender inequities and privilege males, challenges that may be addressed through educational, political and corporate partnerships.

Some literature on self-efficacy has shown gender differences in achievement and self-efficacy. For instance, a study by Carmichael and Taylor (2005) has found that females reported lower levels of confidence in solving specific mathematics topics and questions than males. However, they also indicated that there were no significant differences in the prior knowledge of males and females nor were there any in mathematical performances, suggesting that differences in mathematics self-efficacy between males and females did not account for mathematical performances. Due to teaching interventions, some quantitative studies reported that male mathematics students displayed higher MSE scores (Betz & Hackett, 1983; Falco et al., 2010; Peters, 2013). Falco et al. (2010) have reported that male students had higher self-efficacy than females after completing a nine-week counselling programme of time-management, goal-setting, mathematics study skills, and help-seeking skills. This finding suggests that the counselling programme could have a positive impact on self-efficacy. However, Nielsen and Moore (2003) have reported inconsistent findings when MSE was assessed under different test and class conditions. They found no significant differences between the MSE scores for high school male and female mathematics students who completed the self-efficacy scales under both classroom conditions and test conditions. Similarly, a dissertation research by Clutts (2010) has used the MSE Survey (Betz & Hackett, 1989) to investigate whether age and gender differences existed in developmental mathematics courses, intended for at-risk students at a community college. Their study found no age and gender effects due to variations in the levels of self-efficacy levels in the sample. In short, these inconsistencies challenge the notion of gender inequities and suggest that other factors such as teaching interventions might be at play.

In line with practical significance of this research and the association between teaching and learning (See Chapter 1), the following sections describe teaching interventions related to self-efficacy and professional development of lecturers. Bandura (1997) stated that while the learning contexts, abilities, past educational performance, gender, attitudes towards activities are important considerations, academic performances are best predicted by the way MSE is affected in self-efficacy training. The greater the transformation in self-efficacy, the better the academic attainment. As students develop better learning skills, they tend to attain successful outcomes in learning. In accord with the performance-enhancing role of self-efficacy, these sections highlight some examples of teaching interventions, which will be valuable for future discussion (See Chapter 5).

Teaching interventions to build mathematical self-efficacy

Teaching and Assessment

There is a growing body of literature which demonstrates the positive effects of teaching interventions on MSE and learning. For example, Nielsen and Moore (2003) have reported that high school mathematics students showed higher MSE in the classroom context than under test conditions as well as significant differences between self-efficacy and scores in mathematics Those with high self-efficacy had scored better in class than in a test situation. This study indicates that the nature of teaching and assessments might influence the nature of self-efficacy change. In another example, some researchers (Pampaka et al., 2011) have reported that the type of mathematics course has a significant effect on the nature of MSE. They found that those students with a traditional mathematics course at college level had developed self-efficacy in applying formulae. However, those who had completed a non-traditional course, known as Use of Mathematics, gave them more opportunities for problem-solving applications of mathematics concepts, had increased their self-efficacy in higher-order thinking skills in problem-solving. These findings suggest that students' self-efficacy might be responsive to changes in learning contexts, which adds weight to my argument that self-efficacy can be fostered through teaching interventions.

Classroom instructional design

Based on the conceptual framework of self-efficacy, students build self-efficacy in learning mathematics through four sources of self-efficacy. This theory is relevant to my study because it addresses the wider issue of implementing self-efficacy programmes through instructional design. Some researchers have described some practical ways of applying self-efficacy theory in teaching situations. For example, in the University of Mississippi, Ponton et al. (2001) have formulated teaching guidelines for engineering lecturers to improve students' self-efficacy. One strategy was to provide opportunities for mastery experiences. Hence, they proposed that engineering lecturers develop engineering-related skills by asking some pertinent questions "1) What exactly do we want the students to master? and 2) How are we going to let them know?" (Ponton et al., 2001, p. 249). In line with four sources of self-efficacy (Bandura, 1997), they also recommended that firstly, in order to create mastery experiences, the assigned problems were to be based on engineering contexts so that students could develop capabilities in solving engineering problems. Secondly, lecturers have to create opportunities for vicarious experiences so that students could see others perform the skills. Next, through verbal persuasion, lecturers could provide encouragement and constructive feedback in order to raise students' self-belief in accomplishing specific tasks and skills. Lastly, lecturers have to recognize stressful periods and provide coping strategies to manage their stress. Based on these guidelines, lecturers need to be aware of how they provide feedback and classroom instructions. Another study by Hoffman and Spatariu (2011) demonstrated that students, who were given prompting in problem-solving, had increased their problem-solving efficiency through cognitive reflection and strategy knowledge, an aftermath that reflects how classroom instructions had a positive influence on developing self-efficacy and self-regulated learning in problem-solving. Hence, such teaching interventions which not only build upon the sources of self-efficacy but also focus on teaching instructions and student feedback, may provide some avenues for increasing self-efficacy.

Self-efficacy programmes

In the context of university mathematics, there has been scarce research about successful implementation of self-efficacy training. This may be due to a lack of collaboration between university researchers and lecturers. Nevertheless, Hanlon and Schneider (1999) have found that their self-efficacy training programme helped pre-college mathematics students to be mathematically proficient. They also found that by offering a five-week summer programme, consisting of problem-solving skills, small group tutoring and regular meetings with instructional coordinators, the participants got more involved in goal-setting and self-monitoring activities and had better self-efficacy levels than the non-participants. In another example, within a primary education setting, Falco et al. (2010) have reported that the benefits of a nine-week self-efficacy counsellor-led training on sixth-grade mathematics students were taught time-management, goal-setting, mathematics study skills, and help-seeking skills. They demonstrated that since the skills training was developmental in nature, the students could gain some opportunities for mastery experiences by sequencing cognitive strategies and skills from the relatively simple (time management) to the more complex skills (help-seeking skills), which allowed them to apply these skills in learning mathematics. Other positive effects were an improvement in students' attitudes toward mathematics learning, particularly for females, and improved mathematics performances for both males and females. To sum up, then, these researchers have demonstrated some positive effects of self-efficacy training in their local context. This raises an important point that if the mathematics lecturers appreciate the benefits of developing soft skills in their mathematics programmes, they may consider self-efficacy training to be worthwhile for their students.

Teacher-centred or learner-centred Teaching

Researchers have described the positive impact of self-efficacy interventions. Yet they have produced inconsistent findings about learning in different educational settings. For example, Peters (2013) has demonstrated that university algebra students and teachers perceived that teacher-centred classroom has a greater impact on MSE than learner-centred classroom using the hierarchical linear modelling. Another study of fifth- and sixth-grade students by Fast et al. (2010) concluded that student-centred classroom was mediated by MSE and mathematics achievement. However, at the Kentucky Community College in United States, a dissertation study by Clutts (2010) reported that self-efficacy did not promote student performances in their developmental courses, an outcome that contradicted the notion that self-efficacy would enhance mathematical skills. These mixed findings suggest more investigations are needed to replicate these studies in other similar contexts. More importantly, quantitative researchers may find it challenging to attribute the underlying reasons for changes in self-efficacy. Such unfounded causes of self-efficacy may be a trivial omission to researchers but not for higher education providers who not only need to justify self-efficacy programmes but also address the wider issue of under-achievement.

Professional development for building self-efficacy

In order to apply theory of self-efficacy to teaching practices, teaching staff could collaborate with university researchers as part of professional development. For instance, an unpublished mathematics project was led by a researcher, Tara Stevens with four higher education providers, an independent school district and three Texas Educational Service Centre Regions in the United States, and whose main purpose was to help teachers understand some practical ways of building self-efficacy in the classroom (Stevens, 2009). Drawing insights from previous research studies in mathematics self-efficacy of middle school students in West Texas (Stevens et al., 2006; Stevens, Wang, Olivárez, & Hamman, 2007), they have concluded that mathematical self-efficacy, sources of self-efficacy, and emotional feedback (e.g., anxiety) were better predictors of mathematics performance than general mental ability. Based on these research outcomes, Stevens (2009) has designed a professional development programme to promote teacher self-efficacy by matching four sources of self-efficacy with teaching strategies. In short, Stevens (2009) has customised this training guide for secondary teachers and their training protocol may offer useful applications in higher education.

Mathematical self-efficacy: Summary

This review discusses the theoretical framework of self-efficacy, highlighting key terms: self-efficacy, self-belief in motivational, cognitive and selection processes, self-regulated learning and sources of self-efficacy (Bandura, 1997). In line with the conceptual framework of self-efficacy, previous literature have reported the predictive role and mechanism of self-efficacy and metacognitive components (such as self-regulated learning, cognitive, motivational and selection processes) (See Previous literature). These findings suggests that successful students have high self-efficacy levels. Some factors of self-efficacy and mathematical performance, including prior mathematics, age and gender, and teaching interventions (See Correlates of mathematical self-efficacy and performance) were also reviewed. Some notable findings have indicated that students with advanced prior secondary mathematics, non-traditional and male students tended to be high achievers in mathematics and confident learners. In terms of teaching, research has shown positive development of self-efficacy depending on the teaching styles, course design and assessments, self-efficacy programmes and quality of professional development (See Teaching interventions to build self-efficacy).

2.2 Conceptions of mathematics

Another purpose of my research is to investigate the nature of student conceptions of mathematics. This section defines conceptions of mathematics within the broad domain of affect in mathematics education. Some key constructs are discussed: beliefs, meta-affect, 'the quasi-logical structure and the framework of Student Mathematics-Related Beliefs (Op't Eynde et al., 2006).

Definitions

McLeod (1989) defines an affective system as made up of subdomains of affective representation: emotions (states of feeling), attitudes (moderately stable predispositions towards ways of feelings, involving a balance of affect and cognition), beliefs (See Table 1) and values, ethics and morals (deeply-held preferences, highly cognitive and affective in nature, and characterised by personal truths). In my review, I will focus on the belief aspect of the affective system.

Table 1 Meaning of beliefs

Authors	**Definitions**
(Rokeach, 1968)	Beliefs are observable behavioural consequences.
(Schoenfeld, 1985)	Belief systems are one's mathematical world view that influence mathematical learning and problem-solving.
(McLeod, 1994)	Students form beliefs about mathematics and oneself as a learner. Their low level (such as mathematics is about rules and formulae) beliefs can come into conflict with the central goals of problem-solving and cause negative reactions to developing problem-solving abilities.
(Op't Eynde et al., 2002, p. 27)	"Student mathematics-related beliefs are "the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context. These beliefs determine, in close interaction with each other and with students' prior knowledge, their mathematical learning and problem solving in class."
DeBellis and Goldin (2006, p. 135)	"The attribution of some sort of external truth or validity to systems of propositions or other cognitive configurations. Beliefs are often highly stable, highly cognitive, and highly structured – with affect interwoven in them, contributing to their stabilization."
Goldin, Rösken, and Törner (2009, p. 9)	"The process of sense making and the genesis of beliefs go hand in hand – the learner searching for sense and meaning develops beliefs about "small objects" (the mathematical objects being studied), as well as beliefs about "larger objects" (e.g., the role of meaning in mathematics)."

Within the affective system, student conception of mathematics is a form of belief. As shown in Table 1, beliefs are described as stable traits, mathematical world views and made up of cognitive and affect systems (DeBellis & Goldin, 2006; Goldin et al., 2009; Op't Eynde et al., 2002; Schoenfeld, 1985). The notion of meta-affect is interwoven in student belief (DeBellis & Goldin, 2006). They describe an individual belief system as an elaborate belief structure that stems from socially shared belief system and is characterised by varying degrees of validity or viability. They argue that powerful affective representation involves both affect and meta-affect in the structure, which fosters mathematical success. For instance, in learning mathematics, because student belief is highly cognitive, a successful mathematician may consider that learning mathematics in engineering is socially viable but this belief may not be valid in another context of education. Furthermore, beliefs can stabilise the meta-affect. To illustrate this point, a strong belief in mathematics can encourage a successful mathematician to develop speed and accuracy in problem-solving. This 'cognitive' belief of one's ability to do problem-solving establishes a meta-affective context when the student receives more validation in winning mathematical awards in problem-solving competitions. A succession of positive outcomes causes one to encode the information that problem-solving is satisfying. In future, the student develops more interest in solving complex real-life problems. This repeated success creates a stable belief that one could successfully perform problem-solving in any situations. So this idea of meta-affect within the belief system conceptualises the underlying beliefs about successful outcomes in learning. This concept of meta-affect addresses the notion that lecturers could create powerful and emotional learning experiences by providing opportunities for the students to develop positive affect in learning mathematics.

In line with the notion of sense-making and development of beliefs (Goldin et al., 2009), some researchers (Op't Eynde et al., 2002) conceptualised the framework of students' mathematics-related beliefs (SMRB), which represents students' conceptions about the nature and the structure of knowledge and knowing in mathematics. In other words, their mathematical thinking and knowledge are closely related to their thinking about learning and teaching. In essence, they argue that since mathematics education and epistemological studies on student beliefs have been studied in isolated ways, this framework would help researchers to identify epistemological differences and investigate student beliefs in context. The framework of SMRB is sub-divided into three subsets of beliefs (about mathematics education, about themselves as mathematicians, and about the mathematics class context). Firstly, beliefs about mathematics education include mathematics, mathematical learning and teaching. Secondly, beliefs about themselves as mathematicians consist of intrinsic and extrinsic goals, task value, effort management and self-efficacy. Thirdly, beliefs about mathematics class contexts is about the socio-mathematical norms and role of students and teachers. Some explanations were extracted from the text in descriptions (Table 2).

Table 2 The framework of students' mathematics-related beliefs (Op't Eynde et al., 2006, p. 63)

Subset 1. Beliefs about mathematics education	**Descriptions (Op't Eynde et al., 2002, p. 28)**
1.1 Beliefs about mathematics 1.2 Beliefs about mathematical learning and problem solving 1.3 Beliefs about mathematics teaching	1.1 Formal mathematics has got nothing to do with real thinking or problem solving. 1.2 Mathematics learning is memorisation. 1.3 A good teacher explains the theory and gives an example of an exercise before he asks to solve mathematical problems
Subset 2. Beliefs about themselves as mathematicians	**Descriptions (Op't Eynde et al., 2002, p. 30)**
2.1 Intrinsic goal orientation beliefs 2.2 Extrinsic goal orientation beliefs 2.3 Task-value beliefs 2.4 Control beliefs 2.5 Self-efficacy beliefs	2.1 The most satisfying thing for me in this mathematics course is to try to understand the content as thoroughly as possible. 2.2 Not shown in the document 2.3 It is important for me to learn the course material in this course. 2.4 If I study in appropriate ways, then I will be able to learn the materials in the course. 2.5 I am confident I can understand the most difficult material presented in the readings of this mathematical course.
Subset 3. Beliefs about the mathematics class contexts	**Descriptions (Op't Eynde et al., 2002)**
3.1 Beliefs about the role and the functioning of their teacher 3.2 Beliefs about the role and the functioning of the students in their class 3.3 Beliefs about the socio-mathematical norms in their own class	3.1-3.3 Perceptions of the roles of teachers and students and about classroom culture that are specific to mathematical activities

The framework of SMRB may be conceptually similar to the notion of beliefs (DeBellis & Goldin, 2006). Both groups of authors argue that student beliefs are relatively stable traits and have cognitive and affective structures. In the framework of SMRB, beliefs are revealed in specific mathematical situations and classroom interactions. Through their actual experiences, they develop a sense of what it means to do mathematics and how they view teaching and learning. In other words, students form their beliefs depending on their perceptions of knowledge and teaching and learning situations. Here, according to Green (1971) coins the term 'the quasi-logical structure of beliefs'. It represents tacit knowledge of individuals. Every person has their own tacit knowledge which is not publicly known and may be connected with other belief systems. It can be argued that knowledge systems are based on logical reasoning and conclusions. Because each individual has made their own logical connections in their belief system, personal beliefs within a belief system do not have generally accepted ways of reasoning. Hence each individual has a unique belief structure, known as ‘quasi-logical’ which depends on their cognitive and affective dimensions. ‘The quasi-logical structure of beliefs’ is important to my research because it will help to increase conceptual clarity and address my research question about the nature of conceptions of mathematics.

Previous literature

To fulfil my research objective of understanding the nature of student conceptions of mathematics, this section will conceptualise conceptions of mathematics by distinguishing the following sub-constructs: fragmented and cohesive conceptions, Level 1 numbers and components conceptions of mathematics, Level 2 models conceptions of mathematics and Level 3 life conceptions of mathematics (Table 3). Next, I will utilise the framework of SMRB (Table 2) for identifying patterns in past research findings. A summary of research articles is shown in Table 4. In mathematics education research, the following research patterns have been detected:

- Research about student beliefs have focused on two main areas: beliefs about mathematics education and beliefs about themselves as mathematicians.
- Research about engineering students has demonstrated task-value beliefs in learning mathematics.

Nature of conceptions of mathematics

Conceptions of mathematics is defined by a complex belief system. Using the framework of SRMB (Table 2), Op’t Eynde et al. (2006) reported multi-faceted structure of students’ beliefs about competence of Flemish junior high mathematics students (N=365) based on the following:

1. “the role and the functioning of their own teacher” (Table 2 subset 3.1 beliefs about the role and the functioning of their teacher)
2. “beliefs about the significance of and their own competence in mathematics” (Table 2 subset 2.5 self-efficacy beliefs)
3. “mathematics as a social activity” (Table 2 subset 3.3 beliefs about the socio-mathematical norms in their own class)
4. “mathematics as a domain of excellence” (Table 2 subset 1.1 beliefs about mathematics) (p.65)

In general, this study shows that findings (1), (3) and (4) are linked to beliefs about mathematics education and beliefs about the mathematics class contexts whereas finding (2) revealed self-efficacy beliefs (subset 2 beliefs about themselves as mathematicians). Furthermore, a study of mathematics students (grades 1 to 13) (Di Martino & Zan, 2011) found that student perceived competence was associated with both instrumental and relational view of learning and perceived mathematical usefulness. In this instance, student beliefs about competence are linked to beliefs about mathematical learning and problem solving (subset 1.2) and task-value beliefs (subset 2.3). Hence, studies like these reveal complex structure of students' beliefs.

Having illustrated the complex structure of student beliefs, I will highlight research findings related to students' beliefs about mathematics. This will serve to define the sub-constructs of conceptions of mathematics. Several researchers (Houston et al., 2010; Petocz et al., 2007; Wood, Mather, et al., 2012; Wood, Petocz, et al., 2012) used the same research question, "What is mathematics?" to examine student beliefs about mathematics education. These studies reveal that majority of mathematics students tended to eschew a high level of conception of mathematics. For instance, Houston et al. (2010, p. 73) reported that students had high scores in level 1 conceptions of mathematics described as mathematics is "calculations with numbers (*number*) (9.2%, N=109) and a toolbox of techniques used to solve problems (*components*)" (43.6%, N=515) and in level 2 conceptions of mathematics defined as mathematics is "models (*modelling*) (19.9%, N=235), and abstract structures and a logical system (*abstract*)" (14%, N=165). They also found that the level 3 conception of mathematics as "a way of thinking about reality and an integral part of life (life)" (6%, N=71) was less evident. To sum up, these researchers have detected a hierarchical structure (Levels 1 to 3) of conceptions of mathematics and that mathematics students were less likely to view life applications of mathematics (Level 3 life conception of mathematics).

This hierarchical format of conceptions of mathematics can be distinguished by fragmented and cohesive conceptions. It was argued that students develop strong identities of being mathematicians if they adopt holistic conception of mathematics to apply mathematics in their future studies and career (Wood, Petocz, et al., 2012). In a large-scale study, using a sample of over 1000 non-mathematics majors students (Engineering, Arts and Business) across five countries and six universities, a group of researchers led by Leigh Wood shed new epistemological perspectives of students' conceptions of mathematics (Wood, Petocz, et al., 2012). Their study defined conceptions of mathematics as an individual's interpretation of the discipline of mathematics. In this sense, people construe specific meanings that are attached to phenomena (such as mathematics) and these meanings elicit responses. Their phenomenographical research analyses found that conceptions of mathematics are sub-divided into three levels (Wood, Petocz, et al., 2012). Conceptually, this structure differed from an earlier study that found fragmented and cohesive conceptions (Crawford et al., 1994). Table 3 shows the results of both research studies. Wood et al. (2012) found that undergraduate mathematics students perceived that mathematics is about numbers and components (Level 1 numbers and components); mathematics is about modelling and abstraction (Level 2 models); and mathematics is relevant to life (Level 3 life). The Level 1 conception of mathematics as a study of numbers, components, or techniques overlaps with a fragmented conception of mathematics. Level 2 is akin to a cohesive conception, whereby mathematics is a complex logical system which can be used to solve complex problems. Level 3 is a higher level in which mathematics is understood as being insights for understanding the world. As the earlier study only investigated first-year mathematics students, the Level 3 conception was not detected among their participants. In short, these studies highlight the dual nature of conceptions of mathematics.

Table 3 Comparing terminologies

Crawford et al. (1994)	**Wood, Petocz and Reid (2012)**
Fragmented	Level 1: Mathematics is about topics, numbers, techniques (Numbers and Components)
Cohesive	Level 2: Mathematics is about modelling and/or abstract structures (Models)
	Level 3: Mathematics is about life and career (Life)

To illustrate these studies against the framework of SMRB (Table 2), this dualistic notion of student conceptions of mathematics is associated with subset 1.1 beliefs about mathematics (Houston et al., 2010; Petocz et al., 2007; Wood, Mather, et al., 2012). Specifically, Houston et al. (2010) observed that 40% of the students (N=472) tended to perceive Level 2 and level 3 CM whereas most students had Level 1 CM (52.7%, N=623. Similarly, another study (N=1182) by Petocz et al. (2007) reported that the majority of university students (53%), who were studying mathematics in business, education, science, engineering and computing science degrees, developed a narrow conception of mathematics. Fewer students adopted cohesive conceptions such as modelling and abstract conception (34%) and life conception (13%). Studies like these outlines the multi-faceted nature of student conceptions of mathematics. Table 4 outlines key research publications that are related to the framework of SMRB. For each subset, the articles have predominantly reported specific beliefs.

Table 4 Mapping past research findings onto the framework of students' mathematics-related beliefs

Subset 1.Beliefs about mathematics education	**Articles**
1.1 **Beliefs about mathematics**	(Houston et al., 2010; Mura, 1995; Petocz et al., 2007; Wood, Mather, et al., 2012; Wood, Petocz, et al., 2012)
1.2 Beliefs about mathematical learning and problem solving	(Cano & Berbén, 2009; Crawford et al., 1994; Liston & O'Donoghue, 2009; Macbean, 2004)
1.3 Beliefs about mathematics teaching	(Bingolbali & Ozmantar, 2009; Mura, 1993)
Subset 2.Beliefs about themselves as mathematicians	
2.1 Intrinsic goal orientation beliefs	(Meyer & Eley, 1999)
2.2 Extrinsic goal orientation beliefs	(Gordon & Nicholas, 2013)
2.3 Task-value beliefs	(Craig, 2013; Flegg et al., 2012; Khiat, 2010; Matic, 2014)
2.4 Control beliefs	
2.5 Self-efficacy beliefs	(Di Martino & Zan, 2011; Op't Eynde et al., 2006)
Subset 3.Beliefs about the mathematics class contexts	
3.1 Beliefs about the role and the functioning of their teacher	(Op't Eynde et al., 2006)
3.2 Beliefs about the role and the functioning of the students in their class	
3.3 Beliefs about the socio-mathematical norms in their own class	

Based on the complex belief structure (Table 4), students' beliefs about mathematics may be inextricably linked to beliefs about mathematical learning. For example, some research of non-mathematics majors students have reported positive correlations of fragmented and cohesive conceptions of mathematics with surface and deep approaches to learning (Cano & Berbén, 2009; Crawford et al., 1994; Liston & O'Donoghue, 2009; Macbean, 2004). These studies revealed that the majority of first-year non-mathematics major students perceived the importance of fragmented conceptions of mathematics and surface approaches to learning. To illustrate this point, Crawford et al. (1994) found that 77% of mathematics students (N=226) held fragmented conceptions as mathematics is numbers, rules and formula. Furthermore, they found that 91% of the students who had a fragmented view of high school mathematics had adopted a surface approach to learning. The remaining group (23%, N=67) adopted cohesive conceptions of mathematics, viewing mathematics as "a complex, logical way of thinking", "about solving complex problems" and "insights for understanding the world" (p.336). Interestingly, those with a cohesive conception of mathematics and a deep approach to learning tended to achieve at a higher level after a year of university study. Another large-scale study of first-year science, technology and engineering mathematics students (N=607) (Liston & O'Donoghue, 2009) showed positive correlations between cohesive conceptions and deep approaches (R=0.32, p=0.01) and between fragmented conceptions and surface approaches

($R=0.14$, $p=0.01$). Furthermore, in a larger study (N=680), Cano and Berbén (2009) reported positive associations between good teaching, clear goals, appropriate workload and assessments, deep approaches to learning and cohesive conceptions of mathematics. In short, these studies suggest that successful mathematics students tend to adopt both cohesive conceptions and deep approaches to learning.

Lecturers also perceive the importance of having cohesive conceptions of mathematics in teaching service mathematics. An earlier study by Mura (1993) revealed that mathematics university lecturers held holistic conceptions of mathematics because they believed that mathematics was about "design and analysis of models abstracted from reality, …applications, and a means of understanding phenomena and making predictions" (29.1%, N=30), "logic, rigour, accuracy, reasoning, especially deductive reasoning; the application of laws and rules" (25.2%, N=26) and "the creation and study of formal axiomatic systems, of abstract structures and objects, of their properties and relationships" (24.3%, N=25) (pp. 389-390). Similarly, another study of mathematics university lecturers (N=51) (Bingolbali & Ozmantar, 2009) perceived the importance of teaching theoretical fundamentals of the concepts and applying these concepts in engineering studies. Hence, both studies showed that for lecturers, cohesive conceptions of mathematics are associated with beliefs about mathematics and mathematical teaching.

Another significant finding of mathematics education research is that mathematics students tend to perceive the usefulness of mathematics (subset 2 beliefs about themselves as mathematicians). As shown in Table 2, students develop beliefs about themselves as mathematicians when they develop goal orientation beliefs and task-value beliefs. To illustrate some research findings based on the framework of SMRB (Table 4), mathematics education researchers have found evidence of students exhibiting "intrinsic goal orientation beliefs" (subset 2.1 Table 2) (Meyer & Eley, 1999), "extrinsic goal orientation beliefs" (subset 2.2 Table 2) (Gordon & Nicholas, 2013) as well as "task-value beliefs" (subset 2.3 Table 2) (Craig, 2013; Flegg et al., 2012; Khiat, 2010; Matic, 2014; Wood, Petocz, et al., 2012). Specifically, a study of mathematics and non-mathematics majors students (Meyer & Eley, 1999) revealed that mathematics students were intrinsically motivated through enjoyment and interest whereas non-mathematics major students valued beauty ("Mathematics is a universal language of reality"), truth ("I prefer solving problems in which there is well established procedure to follow") and procedures ("I prefer solving problems in which there is a well-establish procedure to follow") (p.203). By contrast, a study of pre-degree mathematics students (Gordon & Nicholas, 2013) reported that students created extrinsic goals because they were more concerned about getting a degree than developing mathematical thinking and self-development.

However, some researchers adopt a cognitivist perspective to show how engineering students develop mathematical understanding and skills (Craig, 2013; Flegg et al., 2012; Khiat, 2010; Matic, 2014). For instance, Khiat (2010) reported that their data revealed how, why and when first-year engineering mathematics students use formulae, highlighting the significant role of mathematics in engineering education. The study found that students' levels of understanding ranged from the lowest conceptual understanding (how the formula is derived, proofs and assumptions of the formula), functional understanding (the use of the formula), procedural understanding (how to use the formula), disciplinary understanding (relate maths to discipline) to the highest associative understanding (ability to apply and solve engineering problems). Studies like these shed new light on cognitive understanding which is associated with students' beliefs about the usefulness of mathematics (Table 2 subset 2.3 task-value beliefs) and beliefs about mathematical learning (Table 2 subset 1.2 beliefs about mathematical learning and problem solving).

Some engineering mathematics students develop task-value beliefs, a form of belief that is pertinent to this research. For example, a large study of engineering and science students (N=174) by Matic (2014) found that 93% of engineering students and 76% of science students perceived that mathematics was important in technical and natural sciences, 90% of students from both groups reported that knowledge in basic mathematical disciplines was necessary for students in these sciences, and engineering students have more positive beliefs about the role of mathematics in their study program. A small study of engineering students (N=15)(Craig, 2013) detected that students perceived the importance of learning mathematics because they develop conceptual skills (such as problem solving), professional skills (such as mathematics is seen as playing a substantial role in the workplace) and designated identities as engineers. These studies reveal that engineering students tend to have a pragmatic view about learning mathematics.

A potential difficulty with analysing students' conceptions of mathematics is that student beliefs develop and change depending on how, when and where teaching and learning occur. The authors of the framework of SMRK have tried to addresses this difficulty. Theoretically, mathematics-related beliefs are susceptible to changes given 'the quasi-logical structure of beliefs' as discussed earlier. This particular characteristic may explain contradictory findings about mathematics-related beliefs. For example, Flegg et al. (2012) found that even though engineering mathematics students perceived the importance of mathematics, they did not understand the relevance of mathematics in their likely careers and future studies. This inconsistency sheds light on most engineering students who may be realistic about studying mathematics but do not regard it as a useful subject. Their conclusion warrants further investigation of engineering mathematics students in similar contexts. Therefore, these mixed results may be caused by 'the quasi-logical structure of beliefs', a challenge that researchers should consider in their quest for epistemic clarification.

Conceptions of mathematics: Summary

In summary, my research purposes are to investigate the nature of student conceptions of mathematics and their relationships with mathematics results. Theoretically, in mathematics education research, conceptions of mathematics is conceptually akin to student beliefs which is characterised as relatively stable traits, world views of mathematics education and 'the quasi-logical structure of beliefs' (See Definitions). I found that while some notable studies reported a hierarchical structure, others revealed fragmented/cohesive conceptions of mathematics (Table 3). Specifically, fragmented CM refers to Level 1 CM whereas cohesive conceptions is akin to Level 2 and Level 3 CM. Its multi-dimensional framework of SMRB (Table 2) serves as an appropriate tool for identifying patterns in mathematics education research (Table 4). Based on my review, I concluded that past research findings were mainly about beliefs about mathematics, beliefs about mathematical learning and task-value beliefs. More importantly, these findings suggest that successful mathematics students are likely to adopt both cohesive conceptions and deep approaches to learning whereas less successful mathematics students perceive the importance of fragmented conceptions of mathematics. These findings will help to address the issue of low mathematical achievement and wider implications on teaching and learning (Chapter 5).

2.3 Student approaches to learning

The aims of my second study are to examine the nature of and relationships between surface, deep and achieving student approaches to learning and mathematics performances. To address these objectives, this section will provide a historical overview of conceptions of research in student approaches to learning. This overview will shed light on using a phenomenographic approach to research, cognitivist processing and psychological perspectives of learning. Concurrently, I will explain the following key constructs: student approaches to learning, meta-learning, 3Ps model of student learning, constructive alignment, instrumental and relational understanding.

Theoretical framework

The term 'student approaches to learning' was first coined by Marton and Säljö (1976) to conceptualise how students are studying. In a seminal study of young students in a reading class by Marton and Säljö (1976)**,** they established a structure in the variety of individual learning conceptions known as the *outcome space.* The outcome space constitutes both intentions of learning and processes (later adopted as approaches to learning). The main focus of their study was to investigate how the students had arrived at those qualitatively different ways of understanding the article. They found that some students did not understand the text because they concentrated on discrete bits of information in an atomistic manner, while others were more concerned in a holistic fashion to make sense of the text. They concluded that motives co-existed with approaches of learning which consequently, impact on the quality of learning outcome. Deep approaches to learning were found to be associated with an intention to understand the materials whereas surface approaches to learning were related to an intention to reproduce information. Another achieving approach is described as the intention to achieve the highest possible grades by means of appropriate level of effort, effective study skills and time management.

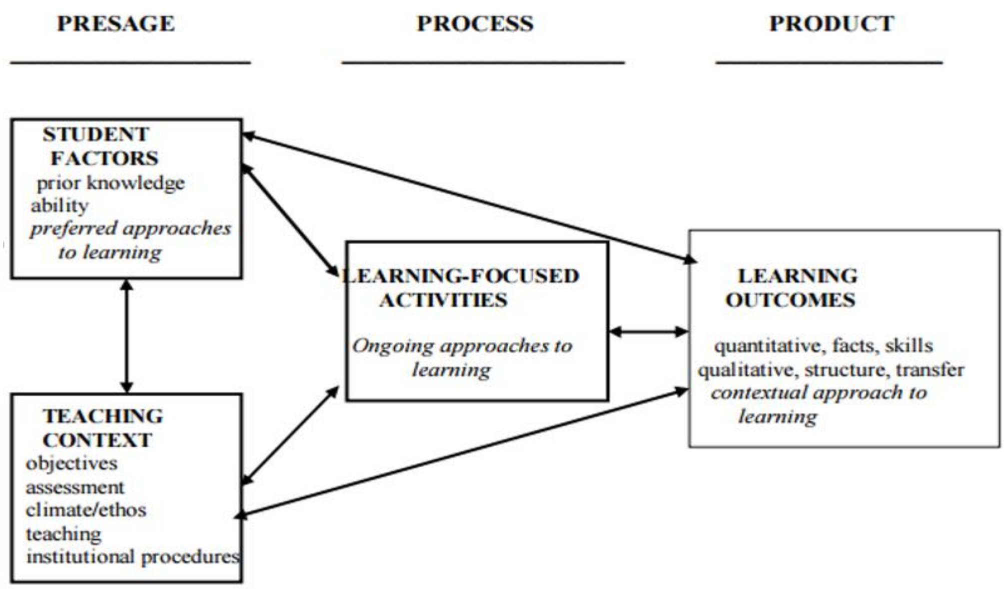
Since the late 1970s, most qualitative research in student learning have utilised phenomenographic approaches (Marton, 1981; Marton & Säljö, 1976; Säljö, 1979; Svensson, 1977) and cognitive system approaches (Biggs, 1987, 1993, 1995). Using phenomenographic research, Säljö (1979) reported that Swedish adult learners revealed five main conceptions of learning (A to E) when they carried out academic tasks. They defined conceptions of learning as established beliefs about what people have prior to learning the subject. They were interviewed about their learning experiences and study techniques and responded to a question about what they had meant by learning. The conceptions of learning were reported as:

A. An increase of knowledge (learning)
B. Memorising facts (surface-level processes)
C. The acquisition of facts or procedures (surface-level processes)
D. The abstraction of meaning (deep-level processes)
E. Applying ideas to reality (deep-level processes)
F. Developing as a person

In a later study, a sixth conception (F), "developing as a person" was also reported (Marton, Dall' Albat, & Beaty, 1993). While conceptions A means learning and conception F is about personal growth in understanding the world and atypical in higher education, they conclude that the other conceptions are distinguished by surface-level and deep-level processes. As surface-level processes, both conceptions B and C show the learner as a passive recipient of new knowledge. As deep-level processes, Conceptions D and E depict the learner as processing new knowledge in a specific task, interpreting the meaning of the ideas and applying them to a real life situation The studies concluded that relationships between the conceptions of learning are symbolic. In short, these studies imply that surface learning uses a 'consumption' approach in learning as students take in information and reproduce what they have learned in a repetitive way whereas deep learning uses a 'production' approach as students produces new knowledge and change their view of the world and learning as a phenomenon. This distinction in learning is relevant to my research, which aims to establish the relationships between surface and deep approaches to learning and mathematics results.

Another approach to research in student learning is cognitive processing. From a cognitive processing perspective, Biggs (1985) argues that SAL framework operates as a product of meta-learning, which refers to the cognitive processes of how one goes about the tasks in which both intention and process co-exist based on the 'psych-logic', as illustrated earlier as logical reasoning of human behaviour in academic setting. Biggs (1987) conceptualised the presage-process-product (3Ps) model of student learning to assess the nature of learning (Figure 1). In relation to my research, the 3Ps model of student learning depicts the relationship between prior mathematics (presage), approaches to learning (process) and mathematics performances (product). Assuming that there are continuous interactions between presage, process and product factors and that students construct their knowledge through processes of assimilation (integrate new knowledge with old knowledge) and accommodation (change their knowledge in order to adapt to the situation), the 3Ps model of student learning serves as a tool to understand variations in student learning.

Figure 1 The 3Ps model of student learning (Biggs, 1987, p. 96)
Used with permission by John Biggs



Each part is independently constituted but does not show a causal process. To illustrate this point, the model shows that student characteristics and teaching (presage) are related to student learning processes (process), and the outcomes of learning (product). Based on the concept of systems theory by Von Bertalanffy (1968), it indicates an interactive system which reaches equilibrium when all the components are in balance. At the *presage* level, students' learning preferences, abilities such as verbal and information processing abilities and prior approaches to learning are considered within the teaching environment (e.g. objectives, assessment, climate, teaching and instructional procedures). Students focus on their learning as a form of metacognitive activity (Biggs, 1985) at the *process* level. Specific tasks are handled by using current deep, achieving and surface approaches. At the *product* level, the learning approaches result in academic performance which is understood to be a learning output described as either correctly reproducing details or comprehending the structure wherein the detail is applied. This model depicts bi-reciprocal relationships between the components of presage, process and product. Using this model, I argue that researchers in student learning will consider the characteristics of the student and the unique experiences provided by the institutions. Some researchers tend to emphasise one aspect of learning or one group of learners (Biggs, 1993). In some cases, the curriculum is taught with a minimal regard for student abilities and differences in learning. The teacher is blamed for poor results and the students are labelled as poor learners. To address this phenomenon of deficit learning, the 3Ps model of student learning will be considered in my research discussion to establish a framework of teaching and learning mathematics (Chapter 5).

In the previous decade, other university researchers have adapted the SAL framework by applying the theory of constructive alignment. Led by Dai Hounsell and his colleagues (2005a), was conducted a research project in the United Kingdom. Based on the constructivist approach to teaching, they used the theoretical framework of constructive alignment (Biggs, 1996), to develop conceptual knowledge in the courses when lecturers match the stipulated learning outcomes for their courses with teaching approaches and assessment goals. In line with this notion of constructive alignment, it is important to understand how this concept applies in mathematics education so one of my research objectives is to propose new ways of learning. As such, my book chapter (P. Murphy, 2017) recommends the application of constructive alignment in university courses in order to bring about a paradigmatic shift in teaching and learning.

The SAL framework of surface and deep learning is conceptually similar to the dual nature of instrumental and relational understanding. From a psychological perspective of learning, Skemp (1987) claims that instrumental understanding is caused by backwash effects of assessments, a desire to pass assessments and surface learning. This form of understanding is advantageous to students who prefer to remember rules and obtain the answers in problem-solving, and requires less effort and fewer teaching instructions. However, he argues that problem-solving tends to be more challenging for students who are only familiar with instrumental learning since instrumental understanding creates an inability to apply the same rules to new problems. By contrast, relational understanding is about making mental connections by integrating simple mathematical knowledge with complex information in order to solve problems efficiently. In other words, students are able to solve mathematical problems easily when they comprehend and apply all the rules and knowledge. In this sense, relational understanding is formed by deep learning. However, according to Skemp (1987), relational understanding can be difficult to teach as more instructions are required and students will need more time and effort to seek new materials and apply the concepts in different problems. This dual perspective of understanding has not only shed new light on the psychological aspects of learning but also shown how learning and teaching are both mutually enhancing of each other. Influenced by both cognitive processing framework of SAL and psychology of learning mathematics, my research will examine SAL (such as deep and surface approaches to learning) and their relationships with mathematics performances and implications for teaching and learning implications in mathematics education at tertiary level.

Previous literature

To fulfil my research objectives, this section reviews research about the nature of student approaches to learning and their relationships with mathematical performances. Here Table 5 provides an overview of research publications. Based on these publications which vary by mathematics courses, quantitative instruments and sub-constructs, I will comment on the patterns of sub-constructs and research paradigms. By providing some illustrations, I support the view that quantitative researchers tend to adopt generalised conceptualisations and a variable-centred method of study. I will further comment on the benefits of each method of study in mathematics education research.

Table 5 Research findings by mathematics courses, method of study and sub-constructs of student approaches to learning

Articles	Mathematics courses	Method	Deep	Achieving	Surface
(Bälter, Cleveland-Innes, Pettersson, Scheja, & Svedin, 2013)	U	ASSIST	+	+	-
(Bernardo, 2003)	Other	LPQ	+	+	-
(Biggs, Kember, & Leung, 2001)	Other	Revised SPQ	- (Surface)	X	- (Deep)
(Cano & Berbén, 2009; Cano & Berbén, 2014)	SU	Revised SPQ	+		-
(Crawford et al., 1994, 1998a)	U	ASI	+	X	+
(Entwistle et al., 2005)	UE	LSQ, ETLQ	+ (Achieving)	+ (Deep)	0
(Gynnild et al., 2005)	U	A local student survey	+	+	
(Liston & O'Donoghue, 2009, 2010)	ESU	Revised SPQ	– (Surface)	X	- (Deep)
(Mji, 2003)	U	ASI	0		0
(Phan, 2011)	A, Ed	MSLQ SPQ	+	X	-
(Senko & Miles, 2008)	Psy	ISPSQ	0	X	0

Approaches and Study Skills Inventory for Students (ASSIST); Learning Process Questionnaire (LPQ), Approaches to Studying Inventory (ASI); Experiences of Studying Mathematics Inventory (ESMI); Study Process Questionnaire (SPQ), Motivated Strategies of Learning Questionnaires (MSLQ), Interest, Self-Perceived Competence, and Study Strategies Questionnaire (ISPSQ)

Note: Studies are listed in chronological order of publications.
A=Arts, B=Business, E=Engineering, Ed=Education, S=Science, Psy=Psychology

+ statistically positive significant relationship with performance
– statistically negative significant relationship with performance
0 statistically insignificant relationship with performance

X not examined
P Primary mathematics students
S Secondary mathematics students
U University mathematics students
Other A variety of educational courses

Nature of student approaches to learning

According to Entwistle , McCune , and Walker (2014), it was more important to contextualise SAL conceptual definitions than to match the meaning of SAL to fit the courses. As such, they recommended that the processes within the specific approach have to be defined within each discipline. These processes in learning, which reflect individual differences in learning experiences, could also be used to predict the likely outcomes of learning. Performance is an outcome of learning. Based on this research perspective, I will comment on key publications that have investigated SAL and mathematical performance.

A review of previous SAL literature (Table 5) reveals that both deep and achieving approaches to learning have positive relationships with mathematical performances, which are negatively associated with surface approaches to learning. Some quantitative researchers have reported positive relationships between deep approaches, achieving approaches and mathematical performances (Bälter et al., 2013; Crawford et al., 1994; Gynnild et al., 2005), between avoidance of surface approach and mathematics performance (Cano & Berbén, 2009; Cano & Berbén, 2014) and between deep approaches and achieving approaches (Entwistle et al., 2005). But some have reported a negative relationship between surface approaches and mathematical performances (Bälter et al., 2013; Crawford et al., 1994) and between deep approaches and surface approaches (Crawford et al., 1998a; Liston & O'Donoghue, 2009). Here, using the Learning Process Inventory (LPQ), Bernardo (2003) indicated that weak positive correlations between deep motive, achieving motives and academic performances and negative correlations of surface motives with positive learning outcomes. Yet, for the weaker students (who scored below 2.0 Grade Point Average for all the subjects in the first year), the data could not be rotated, suggesting that the LPQ inventory was not suitable for non-achieving students but was theoretically acceptable for investigating the achievers whose GPA scores were more than 2.0. The factor analysis data showed two factor solutions of deep and surface approaches to learning with 57% cumulative variance.

By contrast, based on research in educational psychology, Senko and Miles (2008) analysed their structural equation modelling data to report that student course grades were jeopardised due to mastery goals and positive grades predicted performance approach goals. Also, they also found that neither deep nor surface learning strategy predicted grades. According to Dweck and Leggett (1988), mastery goals are about developing one's ability and knowledge in a course; one's performance is thereby assessed through self-referential standards, whereas performance goals involve evaluating one's performance with normative standards as people try to outperform their peers. These findings raise the question about how the data was conceptualised and analysed.

Research paradigms

In this section, I argue that it is important to consider research paradigms in SAL research. Some researchers support the view that when mathematics education researchers recognise the conceptions of research in student learning, they could make informed choices about methodological, theoretical and conceptual frameworks (Schoenfeld, 2002; Simon, 2009). Furthermore, according to Marton and Svensson (1979), to develop SAL conceptual framework, researchers will not only need to be aware of specific learning situations but also understand both generalised and contextualised conceptualisation of research approaches. These approaches impact on how data is analysed and its replicability in other education settings. They argued that the way the data are conceptualised and analysed are intrinsically related and described aspects of learning as either internally related or externally related. In generalised conceptualisation, the results are externally related because the researcher starts with the defined meaning of the terms, examines the learning activity and outcome separately and then tries to relate these components, as part of a system. They have claimed that these results may not be replicable in different educational settings. In other words, when a researcher conducts a survey which has a set of assigned sub-scales, they assume that each sub-scale has a specific meaning which applies to different educational contexts. In reality, each individual may not interpret the scale in the same way. In contextualised conceptualisation, the results are the meaning of the categories and their interrelations. This suggests that the researcher attempts to find the meaning of terms and the internal relationships between them. The data on learning activity and outcome are delimited to each other and then understood and interpreted in relation to each other until they arrive at a new categorisation which represents the internal relation between activity and outcome.

Many contemporary researchers tend to employ generalised conceptualisations rather than contextualised conceptualisations. Marton and Svensson (1979) predicted that future research would shift from generalised to contextualised conceptualisations, an apparent trend in current mathematics education research. To support this claim, here, I will describe some noteworthy studies (Table 6).

Table 6 Research paradigms by publications

	Generalised	**Contextualised**
		Crawford et al. (1994)
Variable-centred analysis	Crawford et al. (1994) Crawford et al. (1998b) Cano and Berbén (2009) Entwistle et al. (2005)* Liem, Shun, and Youyan (2008)** Phan (2011)***	
Person-centred analysis	Crawford et al. (1998a) Cano and Berbén (2014) Clercq et al. (2014)	

*Engineering students
** English students
*** Arts and Education

To show how researchers employ contextualised conceptualisation, a mixed-method study by Crawford et al. (1994) examined the learning beliefs of university mathematics students by asking the question, “What is mathematics?”. Using the phenomenographic method of analysis, the outcome space showed the concepts of both fragmented and cohesive conceptions of mathematics: fragmented conceptions were described as numbers, rules and formula, solve problems using rules whereas cohesive conceptions were about a way of thinking about mathematics, solving problems using modelling and relating to the world. Their study not only found rich descriptions of the nature of learning in higher education but also formulated the subscales of the Conceptions in Mathematics Scale. Conceptually, these researchers argue that focusing one’s attention on one aspect without the other would defeat the purpose of learning mathematics because conceptions of mathematics and student approaches to learning were internally-related phenomena. Their analysis reveals an internal relation between CM and SAL.

However, many quantitative researchers who employ generalised conceptualisation tend to show external relations between constructs in their analyses. The aforementioned research (Crawford et al., 1994) also examined SAL and outcomes of learning using the Approaches to Learning Questionnaire. Their cross-tabulation data revealed that 75.8% of mathematics students ($N=179$, $p<0.001$) perceived that they tended to develop fragmented conception and surface learning (reproduction of discrete information) and poor learning outcomes whereas 15.3% of the students ($N=36$, $p<0.001$) perceived that they tended to adopt cohesive conception and deep learning (understanding, doing difficult problems, applying the theory) and better learning outcomes. In another quantitative study of higher education mathematics students, Crawford et al. (1998b) explained the external relations between CM and SAL by analysing significant correlational results between fragmented conception and surface approach and between cohesive conception and deep approach on both pre-tests and post-tests. Using cluster analysis, another study of higher education mathematics students by Crawford et al. (1998a) reported different outcomes of two main groups of students. One group was characterised by fragmented conceptions, were unsatisfied with their learning environment, had low achievement in mathematics and used surface approaches. By contrast, the other group showed cohesive conceptions, were satisfied with their learning environment, adopted deep approaches to learning and had high mathematics achievement. These studies not only provided new insights about the external relations between student approaches to learning and conceptions of mathematics in higher education but their study have been replicated in other research contexts in Ireland (Liston & O'Donoghue, 2009, 2010) and United Kingdom (Macbean, 2004).

One advantage of analysing data using generalised conceptualisation is that other researchers would attempt to replicate their studies in other educational settings and utilise new methods of analysing SAL to predict learning outcomes (Marton & Svensson, 1979). As such, I argue that many quantitative researchers would adopt a variable-centred approach e.g., factor analysis, regression analysis and Pearson’s correlation instead of a person-centred approach e.g., cluster analysis and discriminant analysis. Comparing both perspectives, Vanthournout, Donche, Gijbels, and Van Petegem (2014) have defined a person-centred research perspective as a means of identifying variations in approaches to learning and underlying developmental processes of a student or groups of students as units of analysis. The key difference is that an individual score on a single dimension derives its meaning from scores that the same individual has on other dimensions. By contrast, they stated that researchers, who have variable-centred perspectives, can identify relations between variables and predict outcomes based on the variables. The unit(s) of analysis is a variable or subgroups of variables and the meaning of each dimension is derived by the scores other individuals have on the same dimension. The following section will distinguish research perspectives of key quantitative studies as highlighted in Table 5.

SAL and CM: Person-centred analysis

A person-centred approach to research can be used to identify patterns in learning outcomes. Here, using the method of hierarchical cluster analysis, the sample of university mathematics students was used to assess SAL, CM and Achievement Goal (AG). Cano and Berbén (2014) reported six groups of students whose learning patterns were labelled as follows:

1) disintegrated learning pattern (deep approach, surface approach, fragmented conception, performance avoidance)

2) mastery-oriented approach (high positive scores on mastery approach, mastery avoidance) and a cohesive conception

3) disengaged learning with low scores on deep approach, cohesive conception, mastery approach and mastery avoidance

4) reproduction and performance-oriented learning with high positive scores on surface approach, fragmented conception, performance avoidance

5) mastery and meaning-oriented learning with high positive scores on deep approaches, cohesive conception and mastery approach and goals

6) meaning-oriented learning with high scores on deep approach and class mastery, low scores on surface approach and performance avoidance

Using univariate analysis of variance, cluster 5 and cluster 6 showed the best academic performance whereas cluster 3 and cluster 4 were the worst performers. These data revealed that on one hand, deep approaches to learning, cohesive conceptions, mastery goals and strong performance were related. On the other hand, surface approaches to learning, fragmented conceptions and low performances were associated to one another.

SAL and CM: Variable-centred analysis

Here in variable-centred analysis, similar patterns of learning were reported using canonical correlations. A study by Cano and Berbén (2009) also reported significant canonical variates with 71.7% of the variation accounted by the Achievement Goal (AG) and SAL frameworks. The first set of variates included good teaching, appropriate workload and assessments, mastery approach, negative performance avoidance, positive deep approach, negate surface approach cohesive conceptions. The second set of constructs was inappropriate teaching, deep approach, appropriate workload, cohesive conception of mathematics, mastery approach/avoidance, and performance approach. The third set correlates were good teaching, fragmented conception, surface approach and performance avoidance. One limitation of this study was it did not relate these constructs with mathematical performance.

Using variable-centred analysis, another study of 365 electronic engineering students showed different findings. Entwistle et al. (2005) have reported four key factors (contributing to 49.7% of the variance) in their factor analysis. The first factor showed positive perceptions of the teaching and learning environment. The second factor had high loadings on organised studying together with effort and concentration. The third factor loaded positively on deep approaches, monitoring studying and negatively on a change in deep approach score (before and after their course). The fourth factor revealed surface approaches were positively related with lack of purpose and negatively with a change in surface approach score (before and after their course). These variable-centred findings suggest that students tend to adjust their approaches to learning depending on the teaching contexts. This study is significant because it showed how perceptions of teaching and learning contexts influence learning, a finding which was not detected in previous studies.

To date, few researchers have addressed the relations between SAL and MSE in mathematics education. Below is a summary of three research studies that have investigated the relations between these constructs. The findings were somewhat inconsistent because researchers adopted different theories to explain student learning. On one hand, it seems that Clercq et al. (2014) had generalised the external relations between motivational and cognitive factors. On the other hand, Liem et al. (2008) found that their results were incongruent with the theory of achievement motivation. Of these three studies, Phan's (2011) study appears to generate substantial evidence of learning.

SAL and MSE: *Person-centred analysis*

A study of engineering students at the Universite Catholique de Louvain in Belgium (Clercq et al., 2014) investigated the relations between motivational factors (self-efficacy beliefs and achievement goals) and cognitive factors (learning strategies and self-regulation) with mathematics tasks and overall course mark. The mathematics tasks involve scientific concepts, solving a mathematics problem and solving a contextualised engineering problem. The hierarchical regression data showed that cognitive processes had an impact on specific measure of mathematics tasks whereas motivational processes have an impact on global measure (overall course percentage). Here, the study did not establish any relations between SAL and MSE which I posit is an area of limitation in this study and confirms my earlier claim.

SAL and MSE: Variable-centred analysis

In a study of university Arts and Education students, Phan (2011) reported that self-efficacy and learning approaches were inter-related. Using the multivariate growth curve method of analysis, they found that high levels of deep processing are related to greater change in self-efficacy and change in surface processing over time. Conversely, the initial levels of self-efficacy and surface processing impacted positively on the change in deep processing. Also, prior learning experience made a positive impact on the initial level of surface processing. These results suggest that there were positive relationships between high self-efficacy, low surface processing and high deep processing.

Another study showed contrasting results. Liem et al. (2008) investigated the effects of self-efficacy, peer relationships, task value, prior achievement, mastery and performance goals, deep learning approaches and surface approaches on English results of English secondary students in Singapore. Using the structural equation modelling (SEM), the results were advanced prior achievement, high levels of self-efficacy, high level of deep learning and low level of surface learning predicted high scores in English, positive effect of mastery goals on surface learning than on deep learning and positive effect of performance approach goal on deep learning. These findings contradict the theory of achievement motivation which states that deep learning is driven by the motive of intrinsic interest which corresponds with the need to improve one's competence (mastery goal) whereas performance approach goals are based on a referential standard of competing with peers, which suggests taking short cuts in learning (surface learning).

In summary, one advantage of employing generalised conceptualisation is that researchers provide a generalised meaning to each dimension. They can infer a specific meaning to each dimension and apply this meaning to all students who give the same response, irrespective of what they say in other items and categories in the questionnaires. Few mathematics education research studies produce rich descriptions of learning because qualitative descriptions of learning are not easily obtained through generalised conceptualisations. Both variable-centred and person-centred analyses are useful for clarifying relationships between learning constructs. The person-centred analysis emphasises characteristics of individuals whereas the variable-centred analysis shows the generalised dimensions of learning constructs. The latter approach has been increasingly popular in mathematics education quantitative research. Both approaches help researchers to understand student learning by focusing on the extent and the nature of learning approaches of individuals and how the generalisations are arrived.

Other correlates of student approaches to learning

In this section, I will review key educational research by considering student perceptions of teaching and teaching interventions. Here I will focus on five main foci in mathematics education research: successful interventions, beliefs about teaching, professional development, curriculum and assessments. These findings will address my research question about teaching and learning implications (Chapter 5).

Student perceptions of teaching environment

Student perceptions of teaching environment can determine student approaches to learning. De Corte (2003) showed how some students who are familiar with deep approaches to learning in school will consider that teaching promotes deep learning in higher education. This perception leads to further use of deep approaches to learning in higher education. Conversely, if teachers promote surface approaches to learning and good grades, some students change their approaches from deep to surface. Students generally develop tendencies for surface learning before they are taught to use deep approaches in higher education, but may decide to change to deep approaches in a conducive environment. Some empirical studies (Trigwell & Prosser, 1991b, 1991c) found that when university students perceived inappropriate assessments, involving low-order thinking, they would adjust from deep to surface approaches. This backwash effect of assessments has resulted in surface learning as students got the cues from lecturers who offered some rewards for rote learning to pass the course and have taught specific topics procedurally. On the other hand, they found that high-quality assessments that promote relational understanding led to deep learning. These studies are congruent to the 'psycho-logic' characteristic of SAL and constructivist perspective of learning.

Teaching interventions for building deep approaches to learning

To align student learning theory with practice, Mason (2010) argues that the outcome of student learning research is to not only validate theories, test hypotheses but also provide ways of addressing issues related to poor learning processes. He suggests that in planning interventions to improve student learning, the more choices students are allowed to make in their learning process, the more valuable it will be. This perspective echoes the notion of emancipatory functions of teaching (Marton & Svensson, 1979). In an emancipatory teaching scenario, teachers tend to align study skills with course outcomes and design assessments for relational understanding of the course. By contrast, technically-driven teachers provide piecemeal attempts in offering study support such as general note-taking and time management classes but pay little attention to its practical contribution to the course and to student learning.

Successful interventions

In recent years, research on teaching and learning in higher education has placed a great emphasis on teaching methods to develop conceptual understanding in learning mathematics. One such study is a study of first-year mathematics students at the University of Pretoria. Engelbrecht and Harding (2015) reported nine interventions to support engineering mathematics students across commerce, science, social science faculties: a refresher course preceding the first semester, ongoing adaptation of the curriculum, at-risk students have meetings with faculty learning advisors, online homework system before the lectures, online examinations after the lectures, supplemental instruction tutor system, overview of lectures, introduction of summer and winter schools. The most successful interventions were summer and winter schools, which comprise of eight days of teaching followed by a quiz. Consequently, in this study these emancipatory forms of teaching initiatives demonstrated having equipped at-risk students adequately with conceptual mathematical knowledge and skills.

As discussed earlier in Chapter 1, stakeholders, including both the engineering lecturers and professionals, tend to regard conceptual understanding to be an important goal in engineering education. Some studies reported how innovative assessment practices could result in deep learning. For instance, engineering mathematics students had to create mathematical learning materials for role play activities in line with the competency frameworks by the European Society for Engineering Education (Albano & Pierri, 2014). Consequently, these students shifted from surface to deep approaches to learning. In another successful study (Loch & Lamborn, 2016), advanced mathematics students had to record videos of professional engineers to show how mathematics was applied in real life situations. However, at the University of Louisville, one study (Hieb et al., 2015) found that conceptual understanding in engineering education may not always entail numerical learning and reported that the summer mathematics course improved their algebra examination core but did not predict better engineering analysis scores. Instead, they reported that time and study environment management, internal goal orientation, and test anxiety had significantly predicted exam scores. In another study of professional engineers, Bergsten, Engelbrecht, and Kågesten (2015) found that professional engineers felt that conceptual mathematical approach was highly relevant to the engineering work and a general engineering understanding was important but mathematical calculations were not necessarily performed in these aspects of engineering. To sum up, when students got involved in real life and skill-based engineering activities, they could successfully develop conceptual learning. Also, some engineering workplace practices require students to have strong conceptual understanding so that they can apply mathematical concepts in engineering.

Hence, to promote conceptual understanding, lecturers should place more emphasis on engineering applications than numerical calculations in the mathematics refresher courses.

In higher education, a formal system of student support could also be provided to improve the quality of student learning. For example, at the University of Limerick, Carroll and Gill (2012) have reported that the Mathematics Learning Centre has the potential to provide both cognitive and affective support for students who are struggling with mathematics. It was found that students had improved in their confidence in learning mathematics and had made reference to deep learning. Some effective learning support strategies were availability of the extra help such as one to one help, extra tutorials and one-week revision programme. Other reports showed similar findings (Mac an Bhaird, Morgan, & O'Shea, 2009; Pell & Croft, 2008). Furthermore, other studies (Carroll & Gill, 2012; Dowling & Nolan, 2006; Patel & Little, 2006; Symonds, Lawson, & Robinson, 2007) showed that students were interested in continuing their mathematics education at advanced levels. Hence, extra mathematics support is perceived to be an effective form of intervention, enabling students to develop interest and self-confidence in learning mathematics and conceptual understanding at their own pace.

Belief about teaching

Besides offering student support, higher education providers will need to adjust pedagogical practices of mathematics lecturers in order to develop conceptual understanding. Some studies have shown that this change warranted a pedagogical shift from transmissive to constructivist way of teaching. For example, Trigwell and Prosser (1996) examined the relations between approaches to teaching and conceptions of teaching of 24 first-year science teachers in Australia found that a majority (more than half) reported teacher-centred approach. Teachers, who held a student-centred approach to teaching, were found likely to adopt a learning-oriented conception and those, who adopted a teacher-centred approach to teaching, tended to use a content-oriented approach. Interestingly, further evidence from another study (Norton et al., 2005) had reported that teachers could change from learner-centred teaching to teacher-centred teaching but did not change their teachers' conceptions of teaching despite several years of teaching experiences. Therefore, these suggests that transforming one's conceptions of teaching may be more difficult than one's approach to teaching.

Biggs and Tang (2007) recommended that in order to produce high-quality learning, teachers can provide student-centred teaching by changing their learning activities and assessments and helping students to monitor deep learning based on the stipulated outcomes of learning.

According to Biggs (1996), the concept of constructive alignment is a framework for evaluating pedagogical ‘goodness-of-fit’ between intended learning outcomes, approaches to teaching and assessment depending on the constraints and opportunities within a given course setting. To implement student-centred teaching, Biggs (1996) argued that teachers needed to consider the constructive alignment framework for teachers to guide their decision-making at all stages of instructional design. He proposed that teachers can review course objectives with an emphasis on promoting deep learning as an indicator of performances. By asking themselves what levels of deep understanding in typical activities and assessments are expected for a course, they will be able to adjust their classes to fit with the constructivist outcomes of learning. One successful example of constructive alignment was the Experiences of Teaching and Learning research project led by Dai Hounsell and his colleagues (2005a), who collaborated with teaching staff to promote deep learning in electronic engineering, biological sciences, economics, history. Their study had contributed a theoretical framework of constructive alignment for student learning in a cross-section of university science and social science courses (Biggs & Tang, 2007; Hounsell et al., 2005a).

Professional development for building deep approaches to learning

Lecturers can develop their emotional and cognitive dispositions to facilitate useful SAL. Evans (2014) has reported that improving the emotional and social domains of trainee teachers should help to promote their dispositions for deep approaches to learning. They found that teachers had changed their dispositions to teach deep learning when they were able to ‘notice’ and be alert to learning opportunities; maintain consistent effort, and solicit peer support through sharing practices and advising one another; manage personal feedback responses; be open to receiving criticisms as well as take risks and apply what they have learnt to new situations. Further post-service training was recommended to build competencies and skills of teachers to enable them to translate deep learning into the curriculum. Isvoran et al. (2011) identified key inter-disciplinary components of mathematics and science curriculum in the context of teacher education with the objectives of encouraging trainee teachers to develop better competencies in promoting deep approaches to learning. The programme included inter-disciplinary nature of the self-content, self-paced learning with the aid of computing technologies, peer teaching involving people with different technical and knowledge expertise to teach the programme and a course for teaching innovative practices in science and mathematics. Therefore, to promote value-driven education, post-service professional development can help to foster appropriate dispositions in teaching and thereby encourage deep approaches to learning among students.

Curriculum development

Lecturers need to establish an important goal of curriculum development in teaching mathematics—relational understanding. Mathematics can be taught in a structured way using the curriculum that promotes understanding. Bruner (1977) stated that a well-structured curriculum is necessary to ensure that fundamental concepts are taught well so that students can comprehend the basics and grasp complex concepts. At- risk learners, who struggle with the basics, tend to be easily thrown off track by poor teaching. By presenting more exposure to basic concepts, they will gain mastery of elementary skills. At the same time, students should be encouraged to develop intuitive thinking. This means that a learner can arrive with the solution without showing the proof or they can use different strategies to solve mathematical problems. Bruner (1977) argued that “usually intuitive thinking rests on familiarity with the domain of knowledge involved and with its structure, which makes it possible for the thinker to leap about, skipping steps and employing short cuts in a manner that requires a later rechecking of conclusions by more analytic means, whether deductive or inductive” (p.58). The point of this quote is when students develop intuitive thinking, they need to be confident in their ability. In this way, they can intuitively correct their mistakes and make concerted effort to solve the problems independently. By creating mathematics curriculum that promotes intuitive thinking, lecturers could increase conceptual understanding and deep learning.

To promote deep learning, lecturers can also seek to create an authentic curriculum. According to Bruner (1977), de-contextualising specific topics is uneconomical in learning as learning discrete concepts makes it difficult for students to generalise the concepts for future applications and it falls short of intellectual excitement in learning if the students are not able to grasp conceptually. By ignoring the applications of knowledge, the learner will be less interested in using the knowledge beyond the classroom. It also results in low cognitive understanding and less retention of knowledge since the information is deemed to be meaningless. Bruner (1977) suggests that mathematics is no longer taught to mathematicians but the subject matter is trivialised as decontextualized topics. Due to poor structure in teaching mathematics, this poses challenges for young learners to develop deep learning in their advanced years of education. Hence, “if the child [was] earlier given the concepts and strategies in the form of intuitive geometry at a level that he could easily follow, he might be far better able to grasp deeply the meaning of the theorems and axioms to which he is exposed later” (Bruner, 1977, p. 39). This quotation underscores the phenomenon of ‘double discontinuity’(Klein, 2014) that could be avoided if lecturers design meaningful curriculum.

Ecologically-designed tasks in assessments

One way of contextualising the curriculum is to design 'ecological' tasks. According to Biggs (1995) ecological tasks have real-life and meaningful applications and are qualitative types of questions such as open-ended questions. When lecturers do not design meaningful ecological tasks, this could lead to backwash effects of assessments (Biggs, 1995). In other words, they tend to encourage surface learning of mathematical formulae and rules in order to pass the assessments. To create ecological tasks, the assessor is expected to carry out careful design and fulfil detailed performance requirements. For the assessor, some leading questions are recommended: "1) What qualities of learning are we looking for? What performances need to be confirmed in the assessment? 2) Should the assessment be decontextualized or situated? 3) Who should set the criteria for learning, provide the evidence and assess how well the evidence addresses the objectives?" (Biggs, 1996, p. 358). Furthermore, the assessor needs to consider how curriculum/course objectives match with teaching activities, assessment tasks and their learning criteria. In particular, for various types of assessment tasks, Biggs (1995) suggests three key dimensions: function of testing, nature of what it is tested, and context in which the item is placed. On one end of the spectrum, Biggs (1995) states that quantitative-standard-de-contextualised assessments (such as multiple choice questions) are overrepresented and may not produce deep learning. As an example, using the Structure of Observed Learning Outcomes (SOLO) taxonomy, short answer questions require low cognitive thinking (pre-structural and unistructural levels), indicating that the students do not understand the concepts well enough to apply higher-order thinking (relational and extended abstract levels). On the other end of the spectrum, qualitative-standard-de-contextualised assessments (such as open-ended questions) promote higher level thinking. In short, the most appropriate ecological-qualitative-situated assessments promote contextual understanding of real-world applications.

Student approaches to learning: Summary

To recap, my research purpose is two-fold to examine the nature of student approaches to learning (surface, deep and achieving student approaches to learning) as well as their relationships with mathematics performances. The practical aspect is to recommend new approaches to learning mathematics in university mathematics education. For theoretical frameworks, I described how SAL researchers used a phenomenographic approach, cognitivist processing as well as psychological perspectives of learning (See Theoretical Frameworks). The key constructs: student approaches to learning, meta-learning, 3Ps model of student learning, constructive alignment, instrumental and relational understanding were discussed. Previous university research has shown consistent patterns of a positive association between a low level of surface approach to learning, a high level of deep approach to learning and high scores in mathematics (See Previous Literature). Considering research paradigms, few mathematics education research have produced rich descriptions of learning because qualitative descriptions of learning are not easily obtained through generalised conceptualisations (See Research Paradigms). By using such a research paradigm, quantitative researchers tend to use one of these methods of analysis —variable-centred and person-centred in order to understand the nature of multiple learning constructs. In teaching mathematics, past studies reveal how teaching interventions can successfully promote conceptual understanding. More importantly, in order to conduct these interventions, research has shown that professional development of lecturers were required for incorporating deep learning and relational understanding in teaching programmes (See Teaching Interventions for Deep Learning and Professional Development).

2.4 Research gaps

There has been a growing body of literature in mathematics education to support the view that self-efficacy enhances learning and future mathematical performance. Based on my review, the aforementioned studies highlight the predictive role of self-efficacy and its metacognitive components (such as beliefs for self-regulated learning, self-beliefs in cognitive, motivational and selection processes), which are in line with the conceptual framework of self-efficacy. It is not obvious that mathematics education researchers have investigated the processes of self-efficacy and their impact on teaching and learning. To further understand the role of self-efficacy in teaching and learning mathematics, I will examine the psychological functions of self-efficacy and self-efficacy development in higher education. Therefore, my research questions are as follows:

- *What is the nature of student mathematical self-efficacy?(Study I)*
- *To what extent does mathematical self-efficacy predict mathematics results? (Study I)*

Next, student conceptions of mathematics are akin to beliefs about mathematics education according to the framework of SMRB. Previous literature has also reported that successful mathematics students are likely to form cohesive conceptions as well as deep approaches to learning whereas some mathematics students are less likely to perceive the importance of cohesive conceptions of mathematics. These findings suggest that student beliefs about learning and beliefs about mathematics are related. By aligning past mathematics education studies with the framework of SMRB, university research has yet to unravel the complex structure of conceptions of mathematics and wider implication of understanding conceptions of mathematics on mathematics education. To conceptualise the nature of students' conceptions of mathematics, my research questions are as follows:

- *What are the characteristics of students' conceptions of mathematics?* (Study II)

Furthermore, my review found that SAL researchers have observed that the relationships between deep, achieving and surface approaches to learning and mathematical performances. Furthermore, past studies have shown that student approaches to learning vary depending on teaching and learning contexts and successful mathematics students are likely to adopt both cohesive conceptions and deep approaches to learning. However, few mathematics education researchers have investigated the nature of and the relations between student approaches to learning and conceptions of mathematics and wider implications on mathematics education in Australia and New Zealand. To address this gap in mathematics education research, I will consider these research questions:

- *What is the nature and extent of student approaches to learning? (Study II)*
- *To what extent are student approaches to learning and conceptions of mathematics related?(study II)*
- *How are they related to performance?(Study II)*

Previous literature has also indicated that prior mathematics, age and gender differences influence mathematical achievement. Within the same educational contexts, few mathematics education researchers have examined the inter-relations between these factors, MSE, SAL, CM and mathematical performances. Therefore, my research questions are as follows:

- *To what extent do mathematical self-efficacy, student approaches to learning and conceptions of mathematics predict mathematics performance? (Study III)*
- *How are prior mathematics, age and gender differences related to mathematics results? (Study III)*

Going forward

In accord with theoretical frameworks, my research goals are to investigate the nature of mathematics self-efficacy, student approaches to learning and conceptions of mathematics and their inter-relations with mathematical performances. As outlined in Chapter 3, my sample includes a group of 300 students who study mathematics in engineering and business programmes. A suite of questionnaires were distributed to these students across two higher education providers in three separate studies (also known as, Study 1, Study II and Study III). Here, I will also discuss my philosophical stance, advantages and disadvantages of quantitative research, descriptions, rationale of my quantitative research design and statistical methods.

Chapter 3
Methodology

In this chapter, I will discuss my philosophical stance, advantages and disadvantages of quantitative research, research design and statistical methods. My overarching research goal is to understand the nature of mathematics self-efficacy, student approaches to learning and conceptions of mathematics in the context of higher education. Surveys have been used to analyse these constructs. In surveys, the participants are expected to recall learning experiences and relate their attitudes and behaviour to meaningful actions. This is important because researchers can obtain first-hand learning experiences when students rate their learning and how they think they learn, which are not easily observed in teaching situations. Based on Marton and Svensson's (1979) notion of research paradigms, from an observational perspective, I 'looked at' the survey data to make inferences about what the data means to me. However for my research, to a lesser degree, from an experiential perspective, I 'looked at' the data 'with' the aid of theoretical frameworks, affective, psychological and cognitive factors in student learning. My observational research perspective is a matter of personal choice and understanding my own biased points of view.

3.1 Philosophical view of research

My observational research perspective is underpinned by the positivist philosophy. As stated by Comte (1896), the positivist philosophy is based on knowledge formation in three stages. First, knowledge can be fictitious or theological. Next, knowledge is metaphysical in nature. Lastly, knowledge is scientific. For instance, talent in mathematics can be nurtured from childhood (fictitious). This talent is shown in the Olympiad mathematics event (metaphysical). In scientific research, mathematical talent is assessed in terms of intelligence levels and ability to solve complex mathematics problems. This scientific paradigm is based on the assumptions that knowledge about the world or reality are objective and value-free and that the learner's observations of his/her own learning form their realities of learning. The world, learner and knowledge are separate entities. The learners gain knowledge as individuals within various classroom contexts of learning (world) and form their worldview of the subject at hand. Based on these assumptions, my epistemological view is that observations about one's learning are obtained in data collection. In data collection, the learner communicates his notions of learning and knowledge of the subject while the researcher investigates these notions as 'truths'. Based on this philosophy, research can reveal the 'truths' by generalising their observations about the phenomenon. Comte (1974) has stated that the mind could associate one event with general facts. These explanations tend to be limited to generalisations of the phenomenon without making inferences about underlying beliefs. For instance, their research has shown that high intelligence quotient level in a test is associated with high mathematical talent but the same research could not prove that high IQ is a product of a nurturing childhood. Therefore, adopting a positivist approach to research allows quantitative researchers to understand the world based on what can be observed and explained in a limited way.

In retrospect, my investigation about student learning may reveal one's subjective opinions about learning. Their opinions tested in terms of research conjectures such as deep learning is negatively correlated with surface learning, low levels of mathematics self-efficacy predicts high failure rate in mathematics examinations and tend to be influenced by my ontological and epistemological perspectives of student learning. Ontologically, my observation of learning was based on student experiences about knowledge and their world-views about learning. This was associated with my epistemological perspective of success in learning higher education mathematics courses, which has somewhat shaped the way learning is assessed in my research. In my quest for quantitative data, I have gathered statistical information about mathematics examinations results as indicators of student performances and several independent variables such as mathematics self-efficacy, deep approach to learning, surface approach to learning, conceptions of mathematics as likely predictors of their results.

At the heart of quantitative research, scientific thinking underpins empirical observations of data, systematic and deliberate methods, and objective procedures of data collection and interpretation (Nardi, 2006). The survey data are collated, recorded and coded statistically. The hypotheses are scientifically proven and findings are value-free as the researcher engages in a deductive style of reasoning. These findings are tested and statistical inferences are made against the theoretical framework of student learning. In the context of mathematics education, my scientific research is framed by theories of self-efficacy (Bandura, 1997), student approaches to learning (Marton & Säljö, 2005) and the framework of student mathematics-related belief system (Op't Eynde et al., 2002). According to Nardi (2006), numerous empirical studies have contributed to these theories as a result of systematic and objective analyses of people's experiences about learning. This is done by selecting appropriate samples and questionnaires, random sampling, measurement reliability and validity. Since the assumption is that researchers are unbiased when they collect and analyse the factual evidence, research procedures are set up so carefully that other researchers can replicate the same study without bias. In this way, when researchers make the same conclusions on the basis of carrying out their research in different settings, their conclusions are treated as 'truths' about human behaviour and such knowledge is built over time. Therefore, in quantitative research, scientific thinking in research is important as people develop scientific ways of measuring patterns of social behaviour and attitudes.

3.2 Advantages of quantitative research

The quantitative method helps to create objective data from an independent source (Nardi, 2006). For instance, my surveys have been used to gather personal information such as gender type, ages, educational background, mathematics results but the real names of participants were not reported. Furthermore, the data were objectively collected by a research assistant who does not have previous working relationships with them. As such, they could code and analyse the data numerically and systematically. A key strength of this research is the way in which researchers obtain and analyse the data without making pre-conceived judgment of individuals.

This objectivity is easily achieved because survey scales are parsimonious, precise and easy to analyse. The scales are pre-determined by researchers based on research conjectures and analysed using deductive reasoning. According to Berends (2006), deductive reasoning is used to gather data numerically and statistical inferences and conclusions are based on the statistics. For example, in my research, the format of coding using the Statistical Package for the Social Science (SPSS) software varies from gender as nominal data, age as ordinal data to mathematics self-efficacy to approaches to learning as scale data. These scale categories or sub-scales had three types of student approaches to learning, five domains of self-efficacy and 3 levels of conceptions of mathematics. For each scale, respondent's ratings using five-point Likert-style questionnaires (Likert, 1931) were aggregated and analysed efficiently with the aid of computer statistical software, which created efficient deductive analyses. Therefore, with the aid of the SPSS software, questionnaires serve as a powerful way of producing useful statistical information for deductive analysis.

Surveys are easy to administer and allow for multiple topics to be assessed efficiently to a large sample. The surveys can be computer-based or web-based so that people can easily access the surveys. At the same time, several items can be included in a survey. People can respond quickly to standard questions about learning without wasting too much time trying to interpret the meaning of the items. They can also decide to complete the surveys at their own pace. When conducting the survey, a researcher can reach a large sample without incurring a huge administrative cost (Nardi, 2006).

3.3 Limitations of surveys

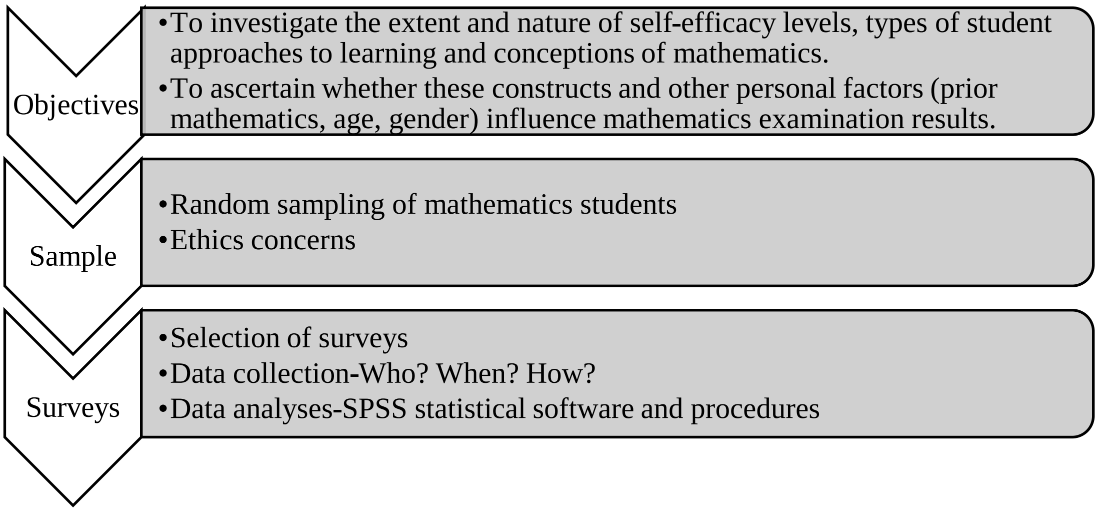
In research design, the validity of the research study could be compromised due to the sampling bias and subjective responses of the respondents. According to Nardi (2006), sampling bias stems from the way the measures are constructed and selected. For example, in designing the self-efficacy scales, Bandura (2006) states that the adequacy of the self-efficacy measures is dependent on how specific the tasks are, whether researchers measure what they intend to measure, and whether research procedures minimise the effects of social desirability. When assessing self-efficacy in problem solving, it is important that the items are task-specific. Otherwise, the respondents may interpret self-efficacy in solving problems in a general way rather than efficacy of a sub-skill required in problem-solving. Also, as respondents provide personal self-appraisals, they tend to be self-aiding or self-limiting in their own judgements. They can provide inaccurate responses about the levels of self-efficacy and their learning experiences due to poor recall, lack of understanding of the tasks, and self-deception. Therefore, faulty assessments in terms of subjective responses and measures could influence the validity of the surveys.

This problem of subjectivity could be associated with the design of questionnaires. According to Nardi (2006), researchers should be aware that with limited scales, not all the factors in questionnaires would fully explain the phenomenon. Bredo (2006) further argues that more data is required to verify the study findings as the findings could be "falsifiable" or disproven (Popper, 1959). As such, researchers have to be critical by carrying out hypotheses testings and find more evidence to disconfirm the findings. Due to the dogmatic design of surveys, Bredo (2006) suggests that this form of quantitative method provides limited descriptions of the phenomenon based on the pre-determined scales but does not explain other underlying factors and causes of the phenomenon. Therefore, quantitative researchers would use their survey data to explain the phenomena at the expense of understanding the underlying issues.

3.4 Research design

Research design involves setting research objectives, selecting appropriate samples and creating a timeline for data collection and data analysis (Figure 2). As my research goal is to examine the nature of constructs considerable preparation in designing questionnaires and data collection took place.

Figure 2 Research design



Sample size and ethics concerns

My sampling design involves a simple random sampling of 300 mathematics students in two Australian and New Zealand higher education providers based on deliberate human ethics decisions. According to Chromy (2006), simple random sampling refers to the process of selecting *n* unique units from the population frame *N*. Each unit has an equal probability of selection. During the sampling process, ethics considerations were evident since there was a formal ethics agreement by parties involved. Furthermore, my data collection was carried out with minimal interference to academic teaching and professionalism among teaching staff, respondents and researchers was maintained.

At the beginning of the research, a written consent was obtained by the students and lecturers The research information and consent forms are shown in Appendix 8. Other ethical concerns related to the way the questionnaires were conducted in my research. According to Nardi (2006), the ethics of questionnaire design are associated with the construction and sequencing of items to meet the desired outcome. In my research, an initial study was carried out to test the validity and reliability of the scales. This form of pre-testing helps to prevent inadvertent response bias and poor wordings in the questionnaire. Some wordings of the items have been customised to suit the intended sample (to be discussed in 'Use of questionnaires' section). The timing of administering the questionnaires was planned purposively in order to minimise disruption of normal teaching times and immerse the participants in appropriate contexts of learning. To illustrate this point, the Refined Self-efficacy Scale (RSS) was conducted in the fourth week of the course so that the participants have had some authentic experiences of learning mathematics in the universities before they appraised their mathematical self-efficacy levels. In the eighth week, the Experiences of Teaching and Learning Questionnaire was administered, assuming that this extra period of learning would allow them to be aware of their motives and strategies of learning. Given practical, ethical and methodological considerations, my research was intended to gather accurate responses from the participants.

Research context

My research project occurred in Australia and New Zealand. One of the higher education provider is Manukau Institute of Technology (MIT) based in Auckland, New Zealand. As a leading vocational institute, it offers a broad range of certificate, diploma and degree programmes to about 20000 full-time and part-time students annually. Another higher education provider is Macquarie University (MQ). Located in the metropolis city, Sydney, it is research-focused and offers undergraduate and postgraduate courses to a large and culturally diverse population of 38000 students in 2017. In general, Mathematics is offered as an academic discipline in non-mathematics major departments in order to fulfil the course requirements for the degree and diploma programmes. At MQ, traditional modes of lecture and tutorials were common in service mathematics courses (such as Math 123 and 130) since they are designed for undergraduates who intend to pursue business, economics and finance degrees. In order to gain entry into university mathematics courses, Australian New South Wales Higher School Certificate (HSC) Mathematics Unit 3 is a recommended course in preparation for higher education mathematics as stipulated by the New South Wales Board of Studies (2008). In the same way, MIT offers service mathematics courses in the following programmes:

- Pre-degree foundation (Mathematics for Engineering Studies),
- Diploma in Engineering and Bachelor in Engineering Technology (Engineering Mathematics)
- Bachelor of Business and Information Technology (Business Statistical Analysis and Programming Precepts)

According to the New Zealand Qualifications Authority (2010), the university entrance requires having completed successfully three approved subjects (14 credits each) at New Zealand National Certificate of Educational Achievement (NCEA) Level 3. To gain entry into these higher education programmes, students need to study mathematics (algebra, calculus or trigonometry) at NCEA level 2 (for diploma students) and NCEA level 3 (for degree students). Table 7 outlines six secondary strands and the curricular alignment between secondary school mathematics and higher education mathematics courses. This secondary-higher education alignment not only suggests that mathematical processes are characteristic of secondary/higher education mathematics but also emphasises the role that mathematics plays in professional degrees. According to the New Zealand Ministry of Education (2010), secondary students develop mathematical processes by

- using problem-solving strategies
- using mathematical and statistical models to solve problems
- making sensible estimates
- using and interpreting data
- evaluating mathematical and statistical information
- communicating ideas. (¶3)

In short, mathematical processes refer to reasoning skills, heuristic strategies and real life mathematical applications. In higher education, these components of mathematical processes are necessary for students to carry out mathematical applications in business, economics, engineering, computer science and social science situations.

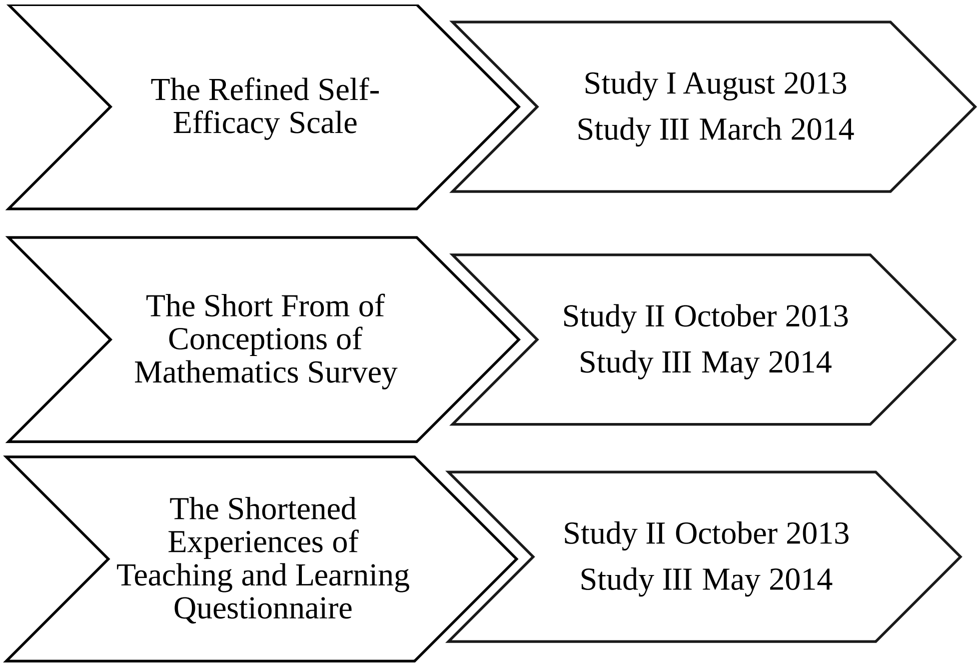
Table 7 Mathematics syllabi and higher education mathematics courses

Strands/ Unit Syllabi	MQ, Australia	MIT, New Zealand
Numbers and Measurements	• Math 130	• Mathematics for Engineering Studies
Algebra	• Math 130 • Math 123	• Engineering Mathematics • Mathematics for Engineering Studies
Calculus	• Math 130 • Math 123	• Engineering Mathematics • Mathematics for Engineering Studies
Geometry	• Math 130 • Math 123	• Engineering Mathematics • Mathematics for Engineering Studies
Probability & Statistics	• Math 123	• Engineering Mathematics • Mathematics for Engineering Studies • Programming Precepts • Business Statistical Analysis
Mathematical processes	• Math 130 • Math 123	• Engineering Mathematics • Mathematics for Engineering Studies

Use of questionnaires

Figure 3 shows the timeline of administering the questionnaires. In the next section, I will also discuss the respective methodology and theoretical background of each questionnaire

Figure 3 Timeline of questionnaires



The Refined Self-Efficacy Scale (RSS)

The RSS (Appendix 1a) was created by Marat (2005) based on the Motivated Strategies and Learning Questionnaire (Pintrich, Smith, Garcia, & McKeachie, 1991) and the social cognitive theory of self-efficacy (Bandura, 1997). As stated in other studies (Marat, 2005, 2007), the RSS was to assess New Zealand secondary and higher education students by incorporating all the strands of New Zealand mathematics curriculum as shown in Table 7. Since then, the Ministry has changed its focus to conceptual understanding and metacognitive skills. According to the New Zealand Ministry of Education (2012), the New Zealand mathematics curriculum has retained the key strands but changed its emphasis to developing procedural learning ("skill in carrying out procedures flexibly, accurately, efficiently, and appropriately"), conceptual understanding ("comprehension of mathematical concepts, operations, and relations") and metacognitive skills ("the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy") (¶1). This highlights the point that these metacognitive skills are important in mathematics education. As such, I have retained the metacognitive component of self-efficacy. As stated by Bandura (2006), the self-efficacy scale should assess specific tasks in learning using a localised rather than a global measure of learning as a general activity. In order to avoid a broad generalization of learning domain, he recommends that there was a need to differentiate the level of competence within each activity domain. The RSS scale consists of multidimensional components of self-efficacy with respect to understanding (motivational), organisation of thought (cognitive), resilience (resource management), independence (self-regulated learning). Therefore, the broad scope of RSS scale aligns with theory and practice.

Moreover, in order to maintain content validity, for my studies, I have kept some original words of the items (shown within the parentheses) without changing the meaning of the items. Some examples are "How well do you believe you can concentrate (on school subjects) on other courses?" (Marat, 2005, p. 65) and "How well do you believe you can motivate yourself (to do school work in mathematics) your studies in mathematics?" (Marat, 2005, p. 66). In addition to these changes, due to my older sample, other scales: "Self-belief in leisure time skills and extra-curricular activities", and "Self-belief to meet other people's expectations", which were designed for secondary students who tended to rely heavily on peer and parental support and be involved in extra-curricular activities, have been omitted. As a result, the RSS has been refined to increase its content and construct reliability (See Table 8). The abridged version is included in Appendix 1b.

Table 8 RSS sub-scales

Sub-scales	
I Self-efficacy in solving numerical problems and measurements	α=.82
II Self-efficacy in solving problems in geometry	α=.84
III Self-efficacy in solving problems in algebra	α=.84
IV Self-efficacy in solving problems in statistics	α=.76
V: Self-efficacy in mathematical processes	α=.76
VI:Self-belief in cognitive, motivational, selection strategies/processes and self-belief for self-regulated learning	α=.86

The Short From of Conceptions of Mathematics Survey (Appendix 2)

Created by Leigh Wood and her international colleagues (2012), it was tested with a large sample of over 1100 mathematics (engineering, business and computer science and mathematics major) students in Australia, Brunei, Canada, Ireland and South Africa. This survey assesses the nature of conceptions of mathematics arranged in hierarchical order. At the lowest level of conception ('Level 1 numbers and component CM'), it defines mathematics as 'numbers and component'. 'Numbers' conception is connected with basic operations, manipulation of numbers, arithmetic and sums whereas 'components' conception has a more coherent structure such as a toolbox of formulae, laws and equations for solving mathematics problems, isolated techniques that were unrelated to real world problems. This level consists of 'numbers' which was described theoretically by the researchers to be a lower conception than ‘components’ though they found no empirical evidence to distinguish both components and numbers. In an earlier study with a group of statistics students, Reid and Petocz (2002) found evidence that mathematics is about numbers and components and both were classed at the same level, so it was decided to combine these aspects as level one. At the next level of conception (‘Level 2 models CM’), mathematics is about ‘models and the abstract’. They found that modelling is associated with applied mathematics which integrates mathematics with the real world. On the same level, abstract mathematics is linked to mathematical ideas that have a logical system or structure. Lastly, at the broadest and highest level (‘Level 3 life CM’), it describes how mathematics is related to life and a way of thinking.

Following their phenomenological study, the new conceptions of mathematics questionnaire was developed and validated using factor analyses and a series of internal consistency reliability. The original survey consisted of 67 items using a five- point Likert scale (‘1’ as strongly disagree to ‘5’ as strongly agree) and its purpose was to find out students’ views about mathematics (part 1) and future use of mathematics in their studies (part 2) and career (part 3). My study examines student conceptions of mathematics so part 1 was used (Table 9).

Table 9 The Short Form of Conceptions of Mathematics survey sub-scales Part 1 (Wood, Petocz, et al., 2012, p. 175)

Level 1 numbers and components CM *[alpha=.92]*
Mathematics is
3. A set of rules and equations
8. Figuring out problems using numbers
9. Using formulas to get results
10. Calculations
11. Numbers being processed
16. The study of numerical concepts
Level 2 models CM *[alpha = .71]*
Mathematics is
2. A way of analysing ideas and problems
4. Basic knowledge for all scientific fields
5. No use to me at all
7. A tool that can be applied in various fields
Level 3 life CM *[alpha = .75]*
Mathematics is
1. A set of models used to explain the world
6. A way to solve problems in my life
10. A way to give humans a more advanced life
11. The language of nature
14. A theoretical framework that describes reality
16. A way to generate new ideas

The Shortened Experiences of Teaching and Learning Questionnaire (SETLQ) (Appendix 3)

As part of the Approaches to Learning and Studying Inventory (ALSI), consisting of the Learning and Studying Questionnaire (LSQ) and the Experiences of Teaching and Learning Questionnaire (ETLQ), the SETLQ was designed by a team of university scholars, Dai Hounsell and his colleagues (Hounsell et al., 2005a) in the Enhancing Teaching-Learning (ETL) project. These questionnaires were designed to assess the nature of teaching and learning in order to enhance teaching and learning environments and achievements in in higher education (Hounsell et al., 2005a). As shown in the ETL project website (Hounsell et al., 2007a, 2007b), the LSQ consists of 19 items to cover learning orientations of programme and course and 36-item to assess approaches to learning. The ETLQ is made up of 18 items for assessing student approaches to learning and studying and 48 items for investigating their experience of teaching and learning. The SETLQ comprises six scales: 'expectations of the course', 'reasons for taking the course', 'approaches to learning and studying, 'experiences of teaching and learning', 'perceived easiness of demands made' and 'knowledge and learning required' (Hounsell et al., 2005b).

Of these six scales, only the 'approaches to learning and studying' (Table 10) is relevant to my research. By definition, a deep approach is used to understand and relate ideas and information, which aligns with constructivist way of thinking; and a surface approach as learning without understanding by memorizing and forming fragmented knowledge. Further, an organized effort/approach is described in terms of effective time management and the amount of effort that is expended on learning the course (Hounsell et al., 2005b). As discussed in Chapter 1, my research uses another term, 'achieving approach', which has similar meaning to 'organised approach'. Since the surveys were administered to mathematics learners, the phrase in item 11 (Table 10) was changed from "…follow the argument" to "… follow the steps/procedures" (Hounsell et al., 2005b, p. 2) The SETLQ was administered to a group of mathematics students in my initial study. After some factor analysis and item reliability testing, the approaches to learning sub-scale has been further reduced from 17 to 10 items. These statistical methods of reducing the items have produced greater item reliability consistency and higher alpha scores.

Table 10 SETLQ: Approaches to learning and studying sub-scales (Hounsell et al., 2005b)

Subscale: Deep approaches *[$alpha_1$ = .75; $alpha_{2=}$.77]*
*2. I've been over the work I've done to check my reasoning and see that it makes sense.
5. In making sense of new ideas, I have often related them to practical or real life contexts.
7. Ideas I've come across in my academic reading often set me off on long chains of thought.
8. I've looked at evidence carefully to reach my own conclusion about what I'm studying.
9. When I've been communicating ideas, I've thought over how well I've got my points across.
*11. It has been important for me to follow the steps/procedures, or to see the reasons behind things.
*13. I've tried to find better ways of tracking down relevant information in this subject.
15. In reading for this course, I've tried to find out for myself exactly what the author means.
*17. If I've not understood things well enough when studying, I've tried a different approach.
Subscale: Surface approaches *[$alpha_1$ = .67; $alpha_2$=.68]*
1. I've often had trouble in making sense of the things I have to remember.
*4. Much of what I've learned seems no more than lots of unrelated bits and pieces in my mind.
*12. I've tended to take what we've been taught at face value without questioning it much.
*16. I've just been going through the motions of studying without seeing where I'm going.
Subscale: Organised effort/ achieving approaches *[$alpha_1$ = .75; $alpha_{2=}$0.82]*
*3. I have generally put a lot of effort into my studying.
*6. On the whole, I've been quite systematic and organised in my studying.
*10. I've organised my study time carefully to make the best use of it.
14. Concentration has not usually been a problem for me, unless I've been really tired.

* Selected items after factor analyses and reliability testings
Before factor analysis -$alpha_1$
After factor analysis - $alpha_2$

Methodological considerations

A key consideration of research design was whether the survey was appropriate for my research. According to Entwistle and McCune (2004), research anomaly may arise from using different terminologies to assess student learning because different meanings are assigned to the same terms and different terms are used to cover the same aspect of learning. Hence, by outlining the conceptual (dis)similarities of SPQ and SETLQ (Table 11), I will discuss why the SETLQ tool was selected for my research.

Table 11 (Dis) similarities of Shortened Experiences of Teaching and Learning Questionnaire and Study Processes Questionnaire

Sub-scales/Features	SETLQ	SPQ
Deep	Intention to understand Relating ideas Use of evidence	Deep motive Deep strategy
Achieving	**Monitoring studying** Monitoring understanding, generic skills, study effectiveness **Organised studying/effort** Time management, study organization, **Effort management** Concentration, effort	Achieving motive Achieving strategy
Surface	Intention to cope with course requirement Memorise without understanding Fragmented knowledge	Surface motive Surface strategy
Purpose	To investigate how specific changes in the teaching-learning environment affect students' approaches to studying.	To assess what students usually do while learning
Interpretation	Approaches to studying are substantially affected by students' perceptions of their teaching-learning environments.	'Psycho-logic' of the student, depends on the context, the task, and the individual's encoding of both
Role of theory in the development of the questionnaire	Derived from SAL framework based on empirical qualitative reports of students	

Both inventories are suitable for university research because they are designed to assess individual differences in learning. The SPQ is commonly used in many quantitative SAL studies. Biggs (1987) has created a 42-item Study Processes Questionnaire (SPQ) to examine the nature of SAL of students in various educational contexts based on Marton and Säljö's (1976) categories of deep, surface, and achieving approaches to learning. Conceptually, the

SPQ subscales are aligned with the ALSI subscales (Table 11). Past researchers have argued that the 3-factor model have practical implications for teaching and learning. But Richardson (2000) criticised that there was no value in using the metacognition dimension since deep and achieving approaches were conceptually similar and that the motive and strategy were incongruent. However, Entwistle, Meyer, and Tait (1991) refuted that the use of factor analysis alone to validate the scales was not justifiable and their rationale was that researchers could gather useful information about student learning in order to improve learning and teaching rather than label students by their approaches to learning. Using the 3-factor model, both tools are useful for investigating how specific changes in the teaching-learning environment affect students' approaches to studying (SETLQ) and to assess what students usually do while learning (SPQ).

Both SPQ and SETLQ have incorporated a metacognition dimension (such as 'monitoring studying', 'organised studying' and 'effort management' in the SETLQ and 'achieving' factor in the SPQ). According to Vermunt (1998), metacognition encompasses beliefs and knowledge about learning, monitoring, regulating and reflecting on learning. Self-regulation also overlaps with this dimension as students are monitoring and regulating their learning. This approach typically describes the way learners organise their tasks and is associated with achievement motivation (Vermunt, 1996, 1998; Zimmerman, 1989). However, a difference between the SETLQ and SPQ is that SETLQ has additional self-regulation components, which have been encapsulated in one sub-scale, 'organised studying/effort' in the abridged version of the ETLQ, whereas the SPQ's 'achieving approach' subscale sub-divides its approach into achieving motive and achieving strategy.

Further, SPQ and SETLQ differ by the way researchers interpret their data. Based on their SETLQ data, Entwistle, McCune, and Hounsell (2003), argued that study effectiveness were drawn from student's perceptions of the teaching and learning environment based on the constitutionalist perspective of prior learning experiences, perceptions, learning and outcomes that occur simultaneously. In a typical scenario, if students had prior experiences of limited conception and surface learning and perceived the situation as not affording deep learning (e.g., learning goals are clear, students have a choice of topics), they would be inclined to use surface learning. This perspective might not apply for SPQ users. Biggs (1987) states that its goal is to assess what students usually do while learning and studying and not what they actually do when engaging a given task in a particular context. Hence, when interpreting the data, researchers should consider the ‘psych-logic’ of the student, depending on the task, context of learning and students’ perception of learning within the SAL framework.

Data analyses

My statistical methods were the univariate analysis of variance, cross-tabulation, chi-square tests, correlation, multivariable regression and exploratory factor analysis. Given the multifaceted variables, these statistical methods have been selected to examine the nature and strength of relationships among these variables and predict the dependent variable. To get started with data analysis, a unique coding system using the SPSS statistical software was generated (Table 12).

Table 12 SPSS software codes

Name	Label	Values
Student	Student ID	e.g. S1(Student 1)
Institution		1=MIT 2=MQ
Mathscourse	Currently enrolled	1 =Foundation 2= Statistics 3= Engineering mathematics degree level 4= Engineering mathematics diploma level 5=Programming Precepts 6=Math 123 7=Math 130 8= Math 131
Year	Year of enrolment	1=Year 1 2=Year 2 3=Year 3 4=Others
Gender	Male Female	1=Female 2=Male
Age	Age group	1=16-20 2=21-25 3=26-30 4=31-35 5=36-40 6=0ver 40
CurrentEnrol	Currently enrolled in a programme	1=Foundation 2=Diploma 3=Degree 4=Postgraduate
HighProgExp	Highest level of programme expected	1=Foundation 2=Diploma 3=Degree 4=Masters 5=Others

HighSchMa	Prior mathematics (only for New Zealand sample)	1= Year 10 2=NCEA level 1 (Year 11) 3=NCEA level 1 (Year 12) 4=NCEA level 1 (Year 13)
Independent Variables		
Efficacy	Self-efficacy (Overall mean Sections I to V)	1= Not well at all 2=Not too well 3=Satisfactory 4=Pretty well 5=Very well
SEI	Self-efficacy in solving mathematical problems	
SEII	Belief in using motivational strategies	
SEIII	Belief in using cognitive strategies	
SEIV	Belief in using selection strategies	
SEV	Belief for self-regulated learning	
DATL	Deep Approaches to Learning	
SATL	Surface Approaches to Learning	
OE	Organised Approaches to Learning	
CON1	Level 1 Components Conceptions of Mathematics	
CON2	Level 2 Models Conceptions of Mathematics	
CON3	Level 3 Life Conceptions of Mathematics	
Dependent Variables		
ExpGrade2	Expected Grade (SETLQ)	1 and 2=Rather badly 3 and 4=Not so well 5=About average 6 and 7=Quite well 8=Well 9=Very well
Marks	Mathematics examination marks	0 to 100

Consistent coding helped to model the data for statistical analysis. To conduct the univariate analysis of variance, the original age categories have been changed to '1' for people who are under 25 years old and '2' for people who are older than 25 years old. Furthermore, to conduct the cross tabulation chi-square analysis, the independent deep learning variable was transformed from an interval scale to a nominal scale. Given the five-scale Likert style scale responses, I assumed that the responses with ‘low level’ of deep learning (1 and 2) indicated disagreement with a deep approach to learning whereas a ‘high level’ of deep learning (3 to 5) signals their agreement. In practice, the dual age categories were chosen as higher education providers tend to consider young school leavers (below 25 year old) as ‘traditional’ students and older students (26-64 years old) as ‘non-traditional’ students.

In the following sections, I will describe the statistical methods for data analysis.

Cross-tabulation and chi-square tests

In Study II, Chi-square (χ^2) tests and cross-tabulation of variables were used. In order to ascertain the relationship between nominal variables (deep learning, organized effort and surface learning, mathematics conception levels 1, 2 and 3), I used contingency tables or cross-tabulations and $\boldsymbol{\chi^2}$ tests. When analysing the SPSS data, following Nardi (2006), two assumptions for chi-square tests were considered: that the categories for the observations should not overlap; and that each category must have an expected frequency of at least 5. If the probability (p) value of obtaining a chi-square value is less than 0.05, then the null hypothesis is rejected, suggesting that both variables are related. These methods are useful for showing bivariate data. Contingency tables are visual representations which depict the discrete occurrences of both variables at the same time. Hence, the observed value of the first variable is shown in the columns and the value of the second variable is observed in each row simultaneously. I used 2 by 2 contingency tables to cross-tabulate each type of conception (e.g. high versus low levels of mathematics conception level 1) to each type of learning approaches (e.g. high versus low levels of deep learning), so the df is 1.

To illustrate this method, refer to Tables 13 and 14. My SPSS cross-tab results show the frequencies of deep learning (shown by the columns) and mathematics conception level 3 (depicted in the rows) (Table 13), revealing that the observed frequency of deep learners who have Level 3 life conception is 259 (90%).

Table 13 SPSS Cross-tabulation deep approaches and Level 3 life conceptions of mathematics

Crosstab

			Level 3 Life CM		
			Low level Mathematics is Life	High level Mathematics is Life	Total
Deep approach	Low level Deep approaches to learning	Count	3	5	8
		Expected Count	.7	7.3	8.0
	High level Deep approaches to learning	Count	21	259	280
		Expected Count	23.3	256.7	280.0
Total		Count	24	264	288
		Expected Count	24.0	264.0	288.0

The relationship of both variables are further tested by using the χ^2 tests. Karl Pearson's χ^2 test is based on the logic that observed frequency of variables is close to the predicted frequency of the same variables. This logic determines the null hypothesis that two variables e.g. deep approaches to learning and conceptions of mathematics level 3 are independent. According to Nardi (2006), the two assumptions for chi-square tests were considered (i.e. categories for the observations should not overlap and each category must have an expected frequency of at least 5). If the probability (p) value of obtaining a chi-square value is less than 0.05, then the null hypothesis is rejected, suggesting that both variables are related. In SPSS, the p-value is denoted by the "Asymp. Sig." (2 tailed). Table 14 shows that p=0.017 for a chi-square value of 5.657 with degrees of freedom =1. This result suggests that the null hypothesis is rejected and shows that both low/ high levels of both deep learning and cohesive mathematics conception are associated. The coefficient of the symmetric measure known as, Phi (φ) =0.178 has a significant p value (p=0.02) further suggests that there is a relationship between deep learning and cohesive conception. Given the significance level of the test is 0.05, this suggests that there is a 1 in 20 chance of being wrong (i.e., false positive) and the significant relationships did not necessarily mean that it was true. Nevertheless, according to Nardi (2006), the statistics were reliable and valid as the overall total was more than 40 and only 1 cell had a low frequency (less than 5) count. Although the chi-square test showed that deep learning and cohesive conception were both related, I could not conclude that deep learning was stronger or weaker than cohesive conception.

Table 14 Chi-square tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	9.164[a]	1	.002		
Continuity Correction[b]	5.657	1	.017		
Likelihood Ratio	5.457	1	.019		
Fisher's Exact Test				.022	.022
Linear-by-Linear Association	9.132	1	.003		
N of Valid Cases	288				

a. 1 cells (25.0%) have expected count less than 5. The minimum expected count is .67.
b. Computed only for a 2x2 table

Correlation coefficient

In my studies, I have created correlation matrices to indicate the correlation coefficients of student approaches to learning, conceptions of mathematics and self-efficacy with student examination results. While correlation statistics measure the strength of the relationships between bivariate data and has a predictive value, the results do not show the extent of the differences of different groups within the sample. Nardi (2006) claimed that the r coefficient can be low but significant due to large sample sizes. Hence, it is usually better to focus on the correlation coefficients rather than the significance levels. For my studies, the rule of thumb for interpreting the strength of relationships are based on Dancey and Reidy's (2004) categorisation of the strength of correlation. Strong correlations range from R =0.7 to 0.9, moderate range from 0.4 to 0.6, and weak correlations range from 0.1 to 0.3.

Univariate analysis of variance (ANOVA)

For Study III, this method satisfies my research goal of examining the possible effects of independent variables or factors (e.g., age, gender, prior mathematics) on a dependent variable (mathematics examination results). According to Mardia (1980), the univariate general linear model (GLM) is used to model the situation where there is one dependent variable and one or more independent variables. The GLM test shows the differences of effects of each factor on the output. If age has a significant influence on results, the GLM model could show whether age differences have an effect on results (main effect) by using the technique of ANOVA significance level of testing. Using the same statistical parameters, if both the effects of gender and current courses (interaction effect) are considered simultaneously, controlling the effect of gender differences, one can assess the main effect of the type of current course on the results and infer whether one factor or both have an interaction effect on the results.

In Study III, my data have confirmed the homogeneity of variance assumption using the Levene's test of inequality of variance procedure, the normality assumption as indicated by the

Kolmogorov-Smirnov D test and the independence test based on the guidelines (Mardia, 1980). Using my null hypotheses that there are no mean differences in the results between male and females and there are no mean differences between the levels of high school mathematics, I examined the effects of gender and mathematics background on results, and observed that the F value is 1.045 but the significance value is 0.311. So I could not reject the null hypothesis and deduced that male and female students exhibit no score differences (Table 15). Conversely, the 'high school mathematics' (HighSchMa) or prior mathematics category shows a high F value (3.452) and low significance value (0.014). Therefore, I rejected the null hypothesis and inferred that there are mean score differences between students who have different mathematics background. However, I concluded that the effect of the high school mathematics background on results was significant but minimal given that the eta squared value is 0.195.

Table 15 GLM 2-way ANOVA table

Tests of Between-Subjects Effects Dependent Variable: Exam Marks

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	8508.920[a]	7	1215.560	2.433	.030	.230
Intercept	87836.772	1	87836.772	175.839	.000	.755
Gender	522.085	1	522.085	1.045	.311	.018
HighSchMa	6896.725	4	1724.181	3.452	.014	.195
Gender * HighSchMa	141.874	2	70.937	.142	.868	.005
Error	28473.183	57	499.530			
Total	203992.980	65				
Corrected Total	36982.102	64				

a. R Squared = .230 (Adjusted R Squared = .136)

Multivariate regression

I have used multivariate regression statistical method to examine whether MSE, CM, SAL predict performances in both Study I and Study III. In multivariate data analysis, the extent of the relationships between independent and dependent variables can be determined by multiple regression. The multiple regression statistics are represented by the overall multiple correlations or determinants of coefficient (R^2) and the coefficient weight for each independent variable. According to Hair, Black, Babin, Anderson, and Tatham (2006), the predictive power is shown statistically in the form of linear and non-linear associations between the variables given the requirement that the individual variables meet the assumptions of the multiple regression. According to Hair et al. (2006), one advantage of multiple regression is its predictive role in understanding the relationships of the multiple independent variables on dependent variables. To maximise the overall predictive power of the independent variables, the statistical regression table produces an equation or variate: $y = a + bx_1 + bx_2$, which shows the estimates (b) of the regression coefficients for each independent variable (x_1, x_2). The predictive accuracy ensures the validity of independent variables and the statistical tests are used to measure the predictive power at specific significance level.

Another advantage of multiple regression is its explanatory power (Hair et al., 2006; Nardi, 2006). In multiple regression, the extent of the impact of independent variables on dependent variables is assessed. The relative contribution of each variable to the dependent variable are determined by regression coefficients which serve as indicators of the relative influence and importance of independent variable. These indicators are means of measuring the magnitude and direction of each independent variable's relationship to the dependent variable. For instance, the beta weight shows extent (how strong or weak) of the relationships between the independent variables and the dependent variables. High positive beta scores suggest strong relative importance of the independent variables. Conversely, low beta coefficients suggest lower importance. As the independent variables are simultaneously assessed, some variables may have diminishing effects on the outcome variable and appear to be redundant. In such cases, researchers may query whether the variables need to be eliminated or not and make statistical inferences for the strong predictors of the dependent variable.

Exploratory factor analysis (EFA)

In Study I, the EFA method of analysis was used as a tool for theoretical and methodological purposes. Using this method, one of my research contribution is testing and validation of RSS for Study I, which is included in my journal article (Appendix 5). My quantitative analysis has shown that EFA, as a statistical method of analysis, serves to conceptualise mathematical self-efficacy in higher education but also increases scale reliability and construct validity.

Summary and going forward

Influenced by my belief that knowledge is objective and separate from the learner, I argue that quantitative surveys are inexpensive, practical and easy to administer. Conversely, they are subjected to sampling bias and subjective responses of the respondents which could result in faulty assessments. In terms of research design, the aforementioned surveys have been purposively selected so that they could produce reliable data (See Methodological considerations). With the aid of SPSS, these data were analysed using cross-tabulation, chi-square tests, univariate analysis of variance, multivariate regression and exploratory factor analysis (See Data analyses). I have provided a brief description of my research contribution—testing and validation of RSS using the powerful method of exploratory factor analysis. I will further discuss my research contribution in the next chapter (Research portfolio).

Chapter 4
Research Portfolio

In this chapter, I discuss how each study contributes to the overall thesis and significance to mathematics education research. To recap, my research aims to examine the nature of mathematical self-efficacy, conceptions of mathematics and student approaches to learning and whether these constructs and personal factors are related to mathematics examination performances. I also examined the implications of my findings on mathematical teaching and learning. A preliminary quantitative study (Study I) was carried out to investigate the nature of mathematical self-efficacy (MSE). Subsequently, a series of surveys were administered to investigate the nature of and inter-relations between student approaches to learning (SAL) and conceptions of mathematics (CM) (Study II), MSE, CM, SAL, personal factors and mathematics results (Study III). Following these studies, I have outlined some key research contributions below

a) *Conceptualised theoretically-based dimensions of mathematical self-efficacy, student approaches to learning and conceptions of mathematics*

b) *Tested and validated the Refined Self-Efficacy Scale*

c) *Proposed a teaching framework based on the Structure of Observed Learning Outcome taxonomy in higher education*

d) *Recommended the presage-position-process-product model of student learning*

My research publications were a double-blind peer-reviewed conference paper and a book chapter. Furthermore, a poster has been presented to over 3000 international conference participants in order to obtain constructive feedback for my journal article. In future, a couple of journal articles will be subjected to double-blind peer review by editors in high standard mathematics education journals.

4.1 Study I

Firstly, my research contribution was published as a double-blind peer-reviewed research paper (See the abstract below and the full paper in Appendix 4). I was given the opportunity to provide an individual 40-minute research presentation at the 41st Conference of the International Group for the Psychology of Mathematics Education. This is a high-quality mathematics education conference which provides opportunities for early career researchers, like myself, to present empirical or theoretical research findings on topics that relates to the main goals of PME. The conference organisers stated that "research reports are intended:

- to promote international contacts and exchange of scientific information in the field of mathematics education;
- to promote and stimulate interdisciplinary research in the aforesaid area; and
- to further a deeper understanding of the psychological and other aspects of teaching and learning mathematics and the implications thereof." (Singapore National Institute of Education, 2017, p. 14).

Murphy, P., & Wood, L. (2017). Understanding the nature of self-efficacy. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 289-296). Singapore: PME.

Abstract: This paper investigates the importance of self-efficacy in learning tertiary mathematics using quantitative measures. In line with Bandura's (1997) theoretical framework of self-efficacy, multiple regression data show that metacognitive self-efficacy (Self-belief in using cognitive, motivational, selection processes and Self-belief for self-regulated learning) are key predictors of success in learning mathematics. Further results reveal a positive correlation between self-efficacy in problem-solving and mathematics results. Therefore, an important point for practitioners to consider is to introduce these ways of developing self-efficacy in mathematics curriculum and student support in accord with the theory of self-efficacy.

After receiving feedback from the conference, I wrote a journal article, which was edited by Professor Leigh Wood from the Macquarie University, Australia. The full text is included in Appendix 5. It is anticipated that the article will be subjected to double-blind peer-review by referees of the *International Journal of Mathematical Education in Science and Technology*. Hence, the referencing style has been adjusted to meet the journal requirements. Future publication would add theoretical understanding of mathematical self-efficacy and create a practical tool for assessing mathematical self-efficacy in higher education.

Murphy, P. (to be submitted). Identifying factors for self-efficacy in learning mathematics. *International Journal of Mathematical Education in Science and Technology*, x (x),1-20.

Abstract: This study was intended to validate the Refined Self-efficacy Scale (RSS) (Marat, 2005) survey as well as clarify the construct of self-efficacy in mathematics education. The RSS survey was utilised to examine the self-efficacy levels of 67 higher education mathematics students. Using exploratory factor analysis method of data analysis, five determinants of self-efficacy in learning mathematics have been extracted, confirming the original RSS survey structure. These findings also matched Albert Bandura's (1997) theory of self-efficacy and the RSS dimensions were reduced by 33% (reduced 81 items to 54 items). These new results could pose both methodological and conceptual significance for future self-efficacy investigations.

4.2 Study II

I have published the findings of Study II in a book chapter. The book chapter (Appendix 6 for full text) was edited and double-blind peer-reviewed. It would be valuable for mathematics lecturers or practitioners as it added empirical knowledge about student approaches to learning and conceptions of mathematics and their applications in mathematical teaching and learning.

Murphy, P.E.L. (2017). Student approaches to learning, conceptions of mathematics, and successful outcomes in learning mathematics. In L. N. Wood, & Y.A. Breyer. (Eds), *Success in Higher Education: transition to, within and from university (pp.75-94)*. Singapore: Springer Nature Pte Ltd.

In summary, this study is a micro-analysis of student conceptions of mathematics, student approaches to learning and student performances in examinations. The book chapter highlights practical implications for mathematics in higher education, draws from the comprehensive report of the Enhancing Teaching-Learning Environments project by Hounsell et al. (2005), and extends the international research on student conceptions of mathematics by Wood, Petocz, and Reid (2012). In my study, surveys were used to investigate the learning experience of a random sample of first year mathematics students in Australia and New Zealand (N=291). The study highlights two key findings of relevance to teachers and curriculum developers: firstly, that successful mathematics performance was strongly associated with deep approaches to learning, organised approaches to learning, and cohesive conceptions of mathematics; and secondly, that surface approaches to learning were negatively related to Level 2 models conceptions of mathematics. Based on these results, I have recommended the 'constructive alignment' (Biggs, 1996) and the Structure of Observed Learning Outcome taxonomy (Biggs & Collis, 1982) frameworks for developing holistic conceptions and deep learning in curriculum development.

4.3 Study III

I was invited to present a peer-reviewed poster to over 3000 conference participants at the 13th International Congress in Mathematical Education Conference. This conference was founded under the auspices of the *International Commission on Mathematical Instruction* to improve the quality of teaching and learning worldwide. I incorporated feedback by the conference participants in my journal article (See the abstract below and full text in Appendix 7). This journal article was edited by Professor Leigh Wood and will be submitted to a high standard journal. In short, my poster presentation and journal article are important contributions to mathematics education research because they would create a new conceptual model of mathematical learning in higher education.

Murphy, P. (2016). A model of student learning. *Poster presented at the 13th International Congress in Mathematical Education Conference*, Hamburg, Germany.

Murphy, P. (To be submitted). Relating mathematics self-efficacy, student approaches to learning and conceptions of mathematics to mathematics results. *Mathematics Education Research Journal*, x(x), 1-8.

Abstract: Mathematics learning is influenced by personal factors, ongoing approaches to learning, mathematical self-efficacy, and conceptions of the subject matter. This quantitative study investigates these constructs in relation to examination results of higher education mathematics students in New Zealand (N=73). The study used the Refined Self-Efficacy Scale (Marat, 2005), the Short Form of Student Conceptions of Mathematics Survey (Wood, Petocz, et al., 2012) and the Shortened Experiences of Teaching and Learning Questionnaire (Hounsell et al., 2005b). The key findings were: self-belief in selection processes was the best predictor of examination results; deep approaches, cohesive conceptions and mathematical self-efficacy correlated positively with examination grades; individuals who had attained Year 11 secondary mathematics qualification (or equivalent to Grade 10) and pre-degree mathematics qualification scored better than students with Year 12 and Year 13 qualifications. This evidence is a basis for modelling mathematical learning in higher education.

Chapter 5

Discussion

This chapter is a summary of my research findings. It is interesting that there were common patterns in research findings across three separate studies, included in Figure 4 below. Some notable findings were related to these constructs: MSE and mathematics performance, Level 3 life CM, deep approaches and performance, Level 2 models CM and surface approaches, organised approaches, deep approaches and mathematics performance, prior mathematics and mathematics performance. Based on my analyses, I will propose a conceptual model of student learning to address the wider implications of mathematical teaching and learning for higher education students.

Figure 4 Overview of notable research findings

Study I ***N=67***	**Study II** ***N*=291**	**Study III** ***N*=73**
• **Predictors of positive course outcomes** -Self-belief in cognitive, motivational, selection strategies and for self-regulated learning • ****Positive correlations** Self-efficacy in solving numerical and geometry problems and grades (Note:**Significance level p<0.05 or p<0.01. More details in the correlation data section.)	• **Chi-square statistics** +DA and Level 3 life CM are positively related +OA and Level 3 life CM are positively related • ****Positive correlations** +Examination results, DA and OA +OA and DA +Examination results and Level 3 life CM +Level 3 life CM, DA and OA • ****Negative correlation** SA and Level 2 models CM	• **A predictor of positive course outcomes** -Self-belief in selection strategies • **Positive correlations** **+***MSE and examination results +Examination results and DA +OA and DA +DA and *MSE + Examination results and Level 2 models CM • ****Negative correlation** SA and Level 2 models CM • **Univariate Analysis of Variance** Low prior mathematics is a determinant of strong mathematics results but age and gender differences are not.

Notes: * All five types of MSE (self-efficacy in solving mathematics problems, self-belief in cognitive, motivation, selection strategies and self-belief for self-regulated learning); DA = a deep approach; SA= a surface approach; OA= an organised approach

5.1 Research findings

Average scores: Mathematical self-efficacy

In Study I, the overall average of self-efficacy in problem solving of numerical, geometry, algebra, statistics problems and mathematical processes was 3.47 (out of 5) and self-efficacy in motivation, cognitive, selection strategies, self-belief for self-regulated learning was 3.55. In Study III, the participants reported average scores of 3.38 for self-efficacy in problem solving of numerical, geometry, algebra, statistics problems and mathematical processes and 3.50 for self-belief in motivational, cognitive, selection strategies, and self-belief for self-regulated learning. These results indicated that the students tended to perceive the importance of motivation, cognitive, selection processes and self-regulated learning.

Average scores: Conceptions of mathematics and student approaches to learning

In Study II, the students had high mean scores in Level 2 models CM (3.94 out of 5), Level 1 numbers and components CM (3.88), a deep approach (3.88), and an organised approach (3.61) but lower scores in a surface approach (3.22) and Level 3 life CM (3.42). In Study III, the high mean scores were self-belief in motivational strategies (3.66), a deep approach (3.96), an organised approach (3.77) and Level 1 numbers and components CM (3.98) and Level 2 models CM (3.96) and low mean scores were surface approaches (3.16), self-efficacy in solving mathematical problems (3.38), self-belief in cognitive strategies (3.38) and Level 3 life CM (3.44). Interestingly, the general pattern is that students appear to have lower scores in surface approaches and Level 3 life CM but higher scores in Level 1 numbers and components CM, deep approaches and organised approaches. While the differences in scores were minimal, these mixed results were not surprising as first-year mathematics students might not easily develop Level 3 life CM and deep learning strategies.

Regression model

Next, the regression models in Study I (N=67, Beta=0.482, t=2.335, p=0.027) and Study III (N=73, Beta=0.599, t=2.413, p=0.019) have accounted for 32.7% and 34.7% (R square) of the variation of results. The regression data revealed that the appropriate predictors of mathematics performances were self-belief in motivational, cognitive, selection strategies and self-belief for self-regulated learning in Study I whereas self-belief in selection strategies is the only predictor of success in Study III. This distinction is somewhat unexpected given that both studies had similar sample sizes and courses.

Correlation data

Furthermore, the correlation data yielded some new relationships. These correlations indicated the strength of relationships but not causal relationships. I observed that the negative significant correlation between a surface approach and Level 2 models CM was consistently weak in both Study II ($R=-.25$, $p<0.01$) and Study III ($R=-.272$, $p<0.05$). In Study II, the other sub-constructs showed mixed relationships. Firstly, there were moderate significant correlations between

- Deep approaches and organised approaches ($R=.57$, $p<0.01$)
- Level 1 (components) and Level 2 conceptions (models) ($R=.64$, $p<0.01$)
- Level 1 (components) and Level 3 conceptions (life) ($R=.51$, $p<0.01$)
- Level 2 (models) and Level 3 conceptions (life) ($R=.43$, $p<0.01$)

Secondly, there were also weak significant correlations between

- Final grade and deep approaches ($R=.25$, $p<0.01$)
- Final grade and organised approaches ($R=.30$, $p<0.01$)
- Final grade and Level 3 life conception ($R=.15$, $p<0.05$)

In Study III, there were moderate significant correlations between

- Examination results and Self-efficacy in solving mathematics problems, self-belief in cognitive, motivational and selection strategies and self-belief for self-regulated learning ($.39<R<.52$, $p<0.01$)
- Deep approaches and organised approaches ($R=.63$, $p<0.01$)
- Deep approaches and MSE* ($.37<R<.47$, $p<0.01$)

But there were weak significant correlations between

- Examination results and deep approaches ($R=.27$, $p<0.05$)
- Examination results and Level 2 models CM ($R=.23$, $p<0.05$)

Overall, the salient findings were a negative significant correlation between a surface approach and Level 2 models CM; positive significant correlations between an organised approach, a deep approach and examination performances; a positive significant correlation between a deep approach and MSE; a positive significant correlation between Level 2 models CM and examination results.

Chi-square statistics data

In Study II, the Chi-square statistics showed a low statistical *p* value (lower than .05) suggested that deep approaches and Level 3 life CM are related ($N=291$, $\chi^2=5.657$, $df=1$, $p=0.017$). Furthermore, organised approaches and Level 3 life CM were statistically related ($\chi^2=5.091$, $df=1$, $p=0.024$). These results indicated that both deep and organised approaches were positively related to a Level 3 life CM, suggesting that students were likely to adopt a cohesive CM as well as deep approaches to learning.

Univariate ANOVA

In Study III, contrary to my expectations, the univariate ANOVA data showed that the estimated marginal means were significant (N=73, F=4.002, p=0.007) and pairwise comparisons revealed that those who had completed mathematics at NCEA level 1 (65 marks) were more likely to score higher examination marks than the others with NCEA Level 2 (47 marks) and Level 3 (50 marks). Surprisingly, the data showed that age variations (F=2.632, p=.111) and gender types (F=.265, p=.609) were not related to examination results.

A key finding was the predictive role of MSE in cognitive, motivation, selection processes and self-regulated learning on mathematical performances Theoretically, having such self-belief generate strong effort, proximal goals, willingness to accept challenges, time management and high expectations (Bandura, 1997; Zimmerman, 1989), which are considered to be indicators of success in learning. Moreover, the finding supports the triadic influences of personal, environmental and behavioural factors of learning outcomes, which has been reported in previous studies (Marat, 2005; Parsons et al., 2009), Therefore, this salient finding suggests that to develop the students' sense of resilient self-efficacy and future mathematical accomplishments, a nurturing teaching environment may promote self-regulated learning and processes of self-efficacy.

Students' adaptability in learning mathematics is driven by inherent dynamic characteristics of SAL and CM. As evident in my study, many first-year mathematics students were likely to employ deep approaches to learning and fragmented CM rather than surface approaches to learning and cohesive CM. To a certain extent, these findings differ with previous studies (Crawford et al., 1998a; Houston et al., 2010; Petocz et al., 2007; Wood, Mather, et al., 2012; Wood, Petocz, et al., 2012). These contrasting findings may be due to the fact that students could be perceptive enough to adjust their approaches to learning, from deep to surface or their beliefs about mathematics, from fragmented to cohesive. Conceptually, these apparent shifts match the 'psych-logic' of SAL (Marton & Säljö, 2005) and quasi-logical structure of student beliefs (Op't Eynde et al., 2002). Previous studies have found that students were likely to adapt well to varying learning outcomes and teaching context (De Corte, 2003; Prosser & Trigwell, 1999; Trigwell & Prosser, 1991a, 1991b, 1991c). Therefore, my findings imply two things: firstly, lecturers may need to produce high-quality mathematics assessments to assess both procedural and conceptual understanding; secondly, to increase their chances of success, mathematics students may adapt well to teaching and learning conditions by navigating seamlessly between cohesive and fragmented conceptions of mathematics and between deep and surface approaches to learning.

Some new findings have emerged. First, Level 3 life CM was positively related to deep approaches to learning; second, Level 3 life CM was positively related to organised approaches to learning; third, Level 2 models CM was negatively correlated with surface approaches to learning; fourth, MSE was positively correlated with deep approaches. These results are similar to previous studies that have reported positive relations between fragmented /cohesive CM and surface/deep SAL (Cano & Berbén, 2009; Crawford et al., 1994, 1998a, 1998b; Liston & O'Donoghue, 2009; Macbean, 2004; Mji, 2003).

Theoretically, my findings focused on beliefs about mathematics education and beliefs about mathematical learning, which are situated within the framework of SMRB (Op't Eynde et al., 2002). This indicates that student approaches to learning may stem from students' belief about learning, which has an affective nature. Associated with this is the positive relationship between MSE and deep approaches. Although a study of engineering students (Clercq et al., 2014) has found that learning strategies influence local mathematical tasks and that self-efficacy predicts one's global mathematical performances, few mathematics education studies have examined the relations between these constructs. These new results present some evidence of the inter-relations between the constructs in higher education.

Another unexpected finding was that low prior mathematics, as a presage factor, was a significant factor of high examination scores. In contrast to a study (Faulkner et al., 2014), my data revealed that those students who had completed NCEA level 1 (equivalent to grade 10) scored higher examination scores than those with NCEA level 2 and 3 (equivalent to grades 11 and 12). While this finding matches the result of a large-scale Australian study (Varsavsky, 2010), it differs from previous New Zealand studies (Engler, 2010a, 2010b; James et al., 2008), another Australian study (Rylands & Coady, 2009) and previous self-efficacy studies (Hailikari, Nevgi, & Komulainen, 2007; Hall & Ponton, 2005; Hutchison et al., 2006; Pampaka et al., 2011; Parsons et al., 2011). Other factors could be at play. Some reasons were that the participants with lower prior mathematics, might obtain additional student support or complete a pre-degree mathematics courses. Since they left school early to seek employment, they also might be familiar with mathematical applications in the workplaces. Therefore, this finding could indicate that participants with the lowest level of secondary mathematics qualifications are likely to succeed in mathematics since their studies are clearly related to their personal and career goals. This is clearly an area for further study in similar educational settings in order to improve generalisability of research.

My univariate ANOVA data showed that age differences (below 25 category versus 25-64 year old category) were not indicative of performance variations. Previous literature contradicts this result because some studies have observed that non-traditional students tended to have a better academic preparation in foundation studies (Liston & O'Donoghue, 2010), develop a sense of confidence and enjoyment in learning (Carmichael & Taylor, 2005; Miller-Reilly, 2006) and accept challenges in learning (Forgasz & Leder, 2000). Hence, contrary to my expectations, age was not a determinant of success in learning mathematics.

My study did not reveal that gender differences have an impact on performances variations. Considering my study sample, males (80%) were overrepresented so this could affect the data. This result differs from past self-efficacy studies, favouring male students (Betz & Hackett, 1983; Falco et al., 2010; Hernandez-Martinez et al., 2008; Peters, 2013). Also, a national report of engineering students showed that more New Zealand males were likely to study engineering than females (Ayre, 2011). Another international study (Else-Quest et al., 2010) found that males tend to receive additional resources in learning mathematics. However, this finding in my study suggests that gender differences do not have an impact on learning outcomes.

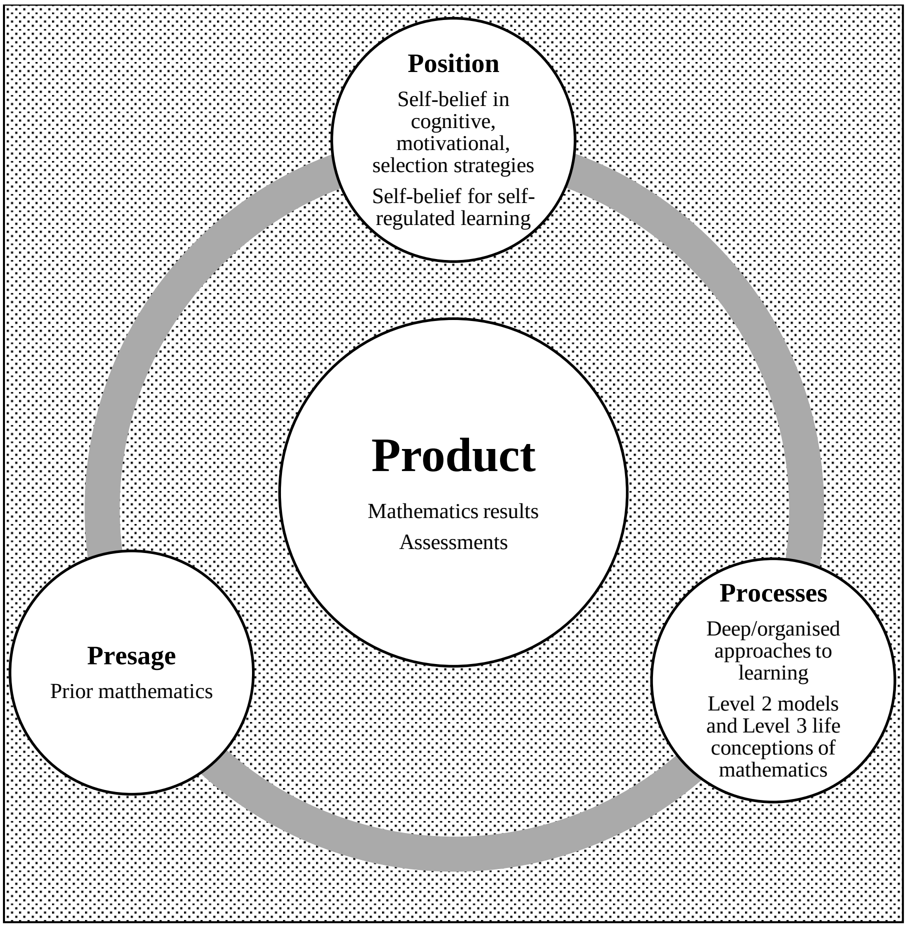
My data suggest that mathematics students are likely to develop relational understanding and apply mathematics in their lives. In line with previous studies (Wood, Petocz, et al., 2012) and the SAL framework (Marton & Säljö, 1976), these findings are not new: strong mathematics results correlated positively with Level 2 models conception, strong mathematics results correlated positively with deep approaches to learning and organised approaches to learning. As reported by Wood, Petocz, et al. (2012), mathematicians were likely to perceive the importance of Level 2 models and Level 3 life CM. This idea of mathematical application is relevant in higher education. As my participants were studying mathematics in order to gain qualifications in social science, engineering and business, their mathematics courses were intended to promote conceptual understanding. As such, they were expected to model mathematical concepts in real-life situations. This would require deep and achieving learning strategies. Therefore, these data suggest that mathematics students who employ cohesive conceptions of mathematics are likely to use deep and organised approaches to learning.

Congruent with past literature (Bälter et al., 2013; Crawford et al., 1994, 1998a; Gynnild et al., 2005) and the SAL framework (Marton & Säljö, 1976, 2005) other findings were that successful mathematics students tended to use both deep approaches and organised approaches to learning. In the 3Ps model of student learning, my participants' SAL has been assessed during their mathematics courses (process) and their examinations focused on applications of mathematical concepts (product). To successfully manage their studies, students would require conceptual understanding, time management and strong effort. Hence, these results suggest that successful students develop conceptual understanding and are able to manage their resources.

5.2 Implications for future practice: A new model of student learning

Based these research findings, I will use an analogy of driving to conceptualise an affective approach for teaching and learning mathematics. Driving a car is not only about knowing how to turn on the ignition and how to put one's foot on the accelerator. A driver is assessed on whether they are able to navigate the car under different driving conditions with confidence and awareness of the outcomes of their decisions. In learning mathematics, successful students can 'steer' their learning experiences and cultivate a sense of accomplishment by selecting appropriate learning strategies, adapting to different learning situations and developing strong self-belief in applying relational mathematics. Such capabilities in learning may be developed when higher education practitioners place more emphasis on affective development of mathematics students. Figure 5 illustrates a new model of student learning. Adapting Biggs' (1987) model of student learning, I have included a new component, 'position' for mathematical self-efficacy, including Self-belief in cognitive, motivation, selection strategies and Self-belief for self-regulated learning. Similar to Biggs' model, the first component, 'presage' includes prior mathematics; the second one, 'processes' consists of sub-constructs of SAL and CM. However, the third, 'product' includes both mathematics results and assessments. These components will be discussed in the next sections.

Figure 5 Proposed 4Ps model of student learning



Presage

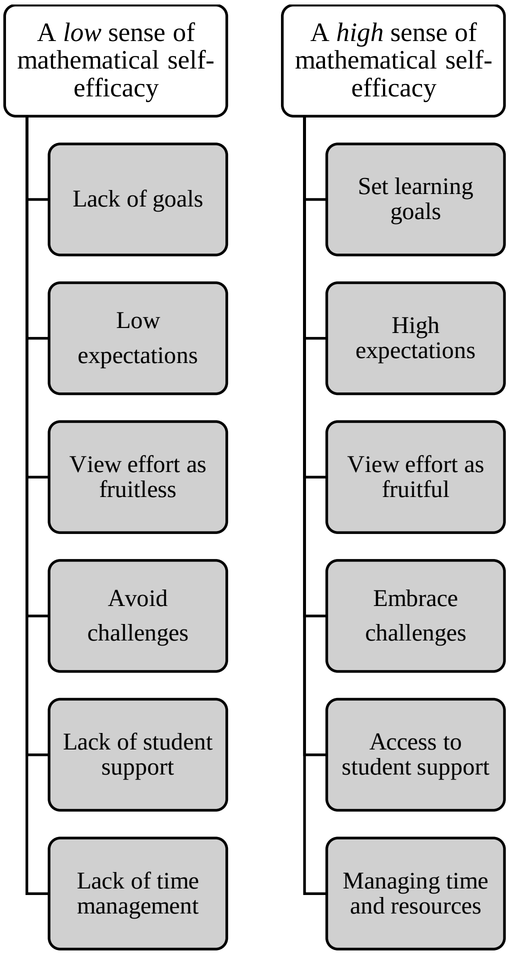
My study finding about prior mathematics indicate two things: firstly, first-year students may need support in learning mathematics; secondly, advanced prior mathematics do not determine future success in learning mathematics. Previous studies have shown that those with inadequate prior mathematics were supported by building their basic mathematics skills (Wilson & MacGillivray, 2007) and confidence in learning (Warwick, 2010). But, in accord with self-efficacy theory, prior mathematics is a proxy to mastery experiences (Bandura, 1997). As learning mathematics is about developing competencies, efficacy beliefs affect the extent to which people act on their outcome expectations. One way of managing outcome expectations is to be able to manage cognized goals, which are personal standards that are based on cognitive comparison of perceived performance to one's personal standard and persistence to reach these goals will develop self-satisfaction or discontent. Guided mastery experiences which equip people with the skills, knowledge and beliefs of self-efficacy, can motivate learners and help them to make choices and manage their negative emotions by using self-relaxation strategies. This is important for students who may have experienced poor mastery experiences but have completed advanced mathematics in schools. Therefore, to enhance mathematical performances, lecturers could allocate resources for guided mastery experiences in teaching mathematics.

Position

My research revealed the predictive role of mathematical self-efficacy on mathematics performances. This implies that lecturers may need to consider the importance of self-efficacy development in enhancing mathematical performances. Past literature have shown how self-efficacy training of secondary students and teachers have improved classroom instructions, student-centred learning and assessments (Falco et al., 2010; Fast et al., 2010; Hoffman & Spatariu, 2011; Pampaka et al., 2011; Peters, 2013; Stevens, 2009) and recommended professional development that are grounded in self-efficacy theory (Ponton et al., 2001; Stevens, 2009). According to Bandura (1997), while the learning contexts, abilities, past educational performance, gender, attitudes towards activities are important considerations, academic performance is best predicted by the extent to which self-efficacy alters in self-efficacy training. The greater the transformation, the better the academic attainments.

An important point to consider in such self-efficacy programmes is the application of self-regulation theories (Bandura, 1997; Boekaerts & Cascallar, 2006; Zimmerman, 1989). By implication, some practitioners may design mathematics programmes based on learning strategies that reflect the covert processes of cognitive, motivational and selection and self-regulation. These strategies include seeking information, goal setting, seeking social assistance, help with test preparation, managing time, making effort and taking risks in learning. Figure 6 shows an outline of these strategies based on low and high levels of MSE. For example, on the one hand, successful mathematics students with high levels of self-efficacy tend to uphold high expectations of themselves as learners, show great effort, receive academic support, overcome challenges and manage their resources. On the other hand, unsuccessful students with low self-efficacy are less likely to study hard, receive academic support from their lecturers, have high expectations and choose difficult tasks. Therefore, in order to improve student performances, some practitioners may incorporate self-regulation learning strategies and processes of self-efficacy in teaching mathematics.

Figure 6 Applications of self-regulation theory



From a psychological perspective of learning mathematics, the lecturer may need to promote student self-efficacy in teaching mathematics. Skemp (1987) states that confident learners can better manage their risks when they assimilate old knowledge with new knowledge in the cognitive schema. As they learn new and difficult concepts, they are willing to make mistakes and take the opportunity to deepen their understanding. Therefore, when lecturers create a clear concept map of the mathematical knowledge required to teach a topic, it is likely to enable learners to master basic skills before grasping advanced concepts. In this way, the students can form their own learning goals which are driven by a series of actions to consolidate their learning. By attaining positive outcomes in their actions, they may increase competence in mathematics and develop greater confidence in learning mathematics.

Processes

My research study revealed that a successful mathematician was likely to develop cohesive conceptions of mathematics and conceptual understanding. According to Op't Eynde et al. (2002), student mathematics-related beliefs have a quasi-logic structure so students do not consciously or explicitly recognise their own conceptions of mathematics. It is more difficult to identify student conceptions than learning strategies so it is difficult to observe the effects of conceptions on performances. In the context of higher education, cohesive conceptions are not explicitly taught but they may help students to develop relational understanding. This is in line with Bruner's (1977) notion of conceptual development. He states that surface learning of discrete topics sow the seeds of fragmented conceptions. This may be a starting point for developing conceptual understanding.

One way of promoting conceptual understanding is the practical applications of mathematical concepts. According to Bruner (1977), de-contextualising specific topics is uneconomical in learning as learning discrete concepts makes it difficult for students to generalise the concepts for future applications and it falls short of intellectual excitement in learning if students are not able to grasp conceptually. By ignoring the applications of knowledge, the learner is less interested in using the knowledge beyond the classroom. It also results in low cognitive understanding and less retention of knowledge since the information is deemed to be meaningless. In this respect, this raises the question of what forms of understanding are necessary for developing cohesive conceptions of mathematics. Hounsell (2005) argues that lecturers should anchor knowledge in meaningful contexts so that they could help students to relate theory with real-life situations in order to improve conceptual understanding and interest in the subject matter. In my book chapter (Appendix 6), I have argued that when teachers design objectives, they can focus on developing higher levels of understanding at all stages of the learning, teaching, and assessment cycle. This notion of constructive alignment (Biggs, 1996) may be found appropriate by using the SOLO taxonomy (Biggs & Collis, 1982). It may be that higher levels of understanding (relational and extended abstract) are achieved by adopting deep approaches as well as Level 2 (models) and Level 3 (life) conceptions of mathematics. Conversely, it may be that lower levels of understanding (pre-structural, unistructural, multistructural) are associated with surface approaches and Level 1 (components) conceptions of mathematics. Therefore, to promote cohesive conceptions of mathematics, practitioners may need to consider how relational and abstract thinking can be developed in the mathematics curriculum.

To promote conceptual understanding, practitioners will need professional development. According to Skemp (1987), this change may be difficult because lecturers tend to spend more time on instructing students who tend to be more familiar with procedural learning. Some researchers (Evans, 2014; McCune & Entwistle 2011) recommend that post-service training for teachers is necessary to help them develop a better understanding of the subject matter and their emotional and social dispositions for deep learning. To promote deep learning, other university researchers have reported that practitioners have successfully introduced role play mathematical activities (Albano & Pierri, 2014), inter-disciplinary curriculum (Isvoran et al., 2011), recording practices of professional engineers (Loch & Lamborn, 2016), student conceptions in classes (Hounsell, 2005), mathematics refresher courses (Engelbrecht & Harding, 2015) and learning support (Carroll & Gill, 2012). However, some researchers have reported that teaching conceptions were more difficult to change than teaching strategies (Norton et al., 2005; Trigwell & Prosser, 1996) and that students with inadequate algebra knowledge were unable to apply mathematical concepts and develop deep learning (Hieb et al., 2015). For practitioners, the aforementioned teaching initiatives may create unexpected personal difficulties in teaching but professional development may help to develop their collective goals in teaching and learning.

Product

The results of an assessment is considered as a 'product' of learning because learning outcomes are measurable and specific in an assessment. To create high-quality assessments, some practitioner may require targeted professional development to promote deep learning and applications of mathematical concepts. As mentioned earlier, Biggs (1995) points out that when lecturers teach mathematics in a procedural way, it contributes to the problem of backwash effect of assessments, characterising poor conceptual understanding and an inability to apply mathematics to real-life situations. To prevent the backwash effect of assessments, lecturers need to design high-quality assessments. To plan such assessments, Biggs (1995) recommends three areas of consideration: function of testing, nature of what it is tested, and context in which the item is placed. Some examples are de-contextualised types (context) of short answer questions (nature) requiring low cognitive thinking (function), de-contextualised-open-ended questions (context, nature) promoting higher level thinking (function) and ecological-qualitative-situated open questions (context and nature) encouraging contextual understanding of real-world applications (function). Therefore, to promote conceptual understanding and real-life applications of mathematics, lecturers may need to change their mode of assessments, from de-contextualised-quantitative to ecological-qualitative in their curriculum.

Summary

To sum up, my research studies have reported that successful mathematics learners were likely to perceive the importance of mathematical self-efficacy, deep approaches to learning, organised approaches to learning and cohesive conceptions of mathematics. By implication, for lecturers or practitioners, as key 'drivers' of change, this may warrant a shift towards developing mathematical self-efficacy, deep learning and cohesive conceptions of mathematics. Table 16 is a summary of my agenda for paradigmatic changes.

Table 16 Proposed presage-position-process-product model of student learning

PRESAGE	
Prior mathematics	• Allocate resources for guided mastery experiences in learning mathematics
POSITION	
Mathematical self-efficacy	• Incorporate theory-based self-regulation and processes of self-efficacy in mathematical teaching and learning • Design a well-structured mathematics curriculum that enhances students' self-efficacy in learning mathematics
PROCESSES	
Student approaches to learning	• Encourage innovative teaching interventions to promote deep learning strategies
Conceptions of mathematics	• Encourage conceptual understanding to promote cohesive conceptions in teaching practices • Promote high-order relational and abstract thinking by applying the SOLO taxonomy and constructive alignment in mathematics curriculum • Offer professional development opportunities for practitioners to develop innovative teaching practices
PRODUCT	
Assessment	• Review the context, function and nature of assessments and design ecological assessments

Chapter 6
Conclusion

There is nothing like looking, if you want to find something. You certainly usually find something, if you look, but it is not always quite the something you were after.

—JRR Tolkien

It is a tale
Told by an idiot, full of sound and fury,
Signifying nothing.

—Shakespeare

An overarching outcome of mathematics education research is changing one's perspective of learning and teaching. This happens because the outcomes of research are unknown at the start. As stated figuratively using Tolkien's quotation, the outcomes depend on how the researcher conceptualises their research data. Mathematics education research is not only about obtaining research knowledge but similarly is about its applications in teaching mathematics. Following Shakespeare's quotation, what is stated of 'life' may hold true for much research; it may end up as "a tale", signifying nothing when researchers do not make an effort to reflect on their research. Schoenfeld (2008) argues that researchers can view the same phenomenon but 'see' very different things. Despite varied interpretations about learning mathematics, he suggests that it is important for researchers to undergird their conceptual framework with sound theoretical frameworks in order to maintain specificity, rigour, be led by the learners' voices and be agents of change.

To avoid educational myopia, mathematics education researchers need to translate the rhetoric of new research knowledge to educational praxis through self-reflection. As mentioned before, student learning researchers tend to exhibit educational myopia when they concentrate on weaknesses in knowledge and skills of students in content areas rather than examining the significance of the affective domain in learning. By concentrating on gaps in student knowledge and skills rather than the development of student affect, the stakes are high—failure and lack of cognitive preparedness in learning mathematics. To address this problem of under-achievement in learning mathematics, fruitful research needs to happen. To improve mathematical learning and teaching, my research aimed to understand the nature of mathematics self-efficacy, student approaches to learning and conceptions of mathematics, drawing upon the theories of self-efficacy (Bandura, 1997), SAL (Biggs, 1987; Marton & Säljö, 2005) and SMRB (Op't Eynde et al., 2002). To recap the research questions,

Study I

1. What is the nature of student mathematical self-efficacy?
2. To what extent does mathematical self-efficacy predict mathematics results?

Study II

1. What is the nature and extent of student approaches to learning?
2. What are the characteristics of students' conceptions of mathematics?
3. To what extent are student approaches to learning and conceptions of mathematics related?
4. How are they related to performance?

Study III

1. To what extent do mathematical self-efficacy, student approaches to learning and conceptions of mathematics predict mathematics performance?
2. How are prior mathematics, age and gender differences related to mathematics results?

Based on my research findings as shown in Figure 4 (Chapter 5), I have concluded that learning mathematics involves developmental processes in affect and cognition, shifting between self-efficacy *in* learning mathematics and self-efficacy *about* learning mathematics; progressing from surface learning to deep learning; from fragmented to cohesive conceptions of mathematics. In addition, both students and lecturers should play critical roles in bringing about this paradigmatic shift in learning and teaching. As stated by Biggs (1993), such changes may prove to be difficult but not impossible as long as lecturers interact with students to work towards their common goals.

In the next two sections, I will reflect on current research quality in accord with Schoenfeld's (2008) three-dimensional framework (predictability, trustworthiness and importance) of research methods in education. Some leading questions are: Is my study important to research in mathematics education? Are the results of my study trustworthy? Is my study explain the underlying issue? In short, my research strengths are high trustworthiness and high importance but my current research is somewhat constraint by low explanatory power or predictability. Lastly, I will discuss new research ideas to add rigor and robustness in future research.

6.1 Research quality

High trustworthiness

According to Schoenfeld (2008), trustworthiness is determined by the methods used, extent of sampling error, consistency and richness of data and replicability of the study. My research has a high degree of trustworthiness. Firstly, my research samples were randomly selected from mathematics classes in NZ and Australian higher education providers. Secondly, the questionnaires have high validity and reliability. Thirdly, Study III has used appropriate methods of statistical correlation, cross tabulation and linear regression methods of analysis. It appears that these procedures produced the same results when past researchers used the same methods on similar samples. Using the same line of inquiry, my surveys could be easily replicated by other university mathematics researchers. Lastly, in line with past studies and theoretical frameworks, my findings confirmed the predictive role of self-efficacy on mathematical performances and positive correlations between deep approaches to learning and holistic beliefs about mathematics. Therefore, the rigour of statistical analyses and consistency of data support my claim of a high degree of trustworthiness.

High importance

Associated with research worthiness is the level of importance of research in theory-building and practice. My mathematics education research is important within the fields of student affect and university learning because it contributes to theory building in mathematics education research and helps to create a better understanding of mathematical teaching and student learning. Firstly, to frame CM and SAL, I have attempted to clarify some terminologies. Following the framework of MSRB established by Op't Eynde et al. (2002), student conceptions of mathematics is not only about what students think about mathematics but it also involves a complex cognitive and affective structure. I have also discovered that both mathematical self-efficacy and student approaches to learning have an affective structure as these constructs can be explained in terms of specific beliefs of students about themselves as mathematicians and beliefs about mathematics learning. For example, in study II, I found that some successful mathematics students are likely to shift from fragmented to cohesive conceptions of mathematics. This mixed result might be explained by the cognitive and affective nature of student beliefs in accord with the framework of SMRB. This has enhanced my understanding of student beliefs about mathematics and their relations with mathematical self-efficacy and student approaches to learning.

Secondly, mathematics education researchers could advance their knowledge in theory building by considering different research traditions. While all these constructs have an affective undertone, I have also viewed my research from a constructivist perspective of student learning and psychological perspective in learning mathematics, contributing to the field of student learning. Based on my studies, I have found that mathematical self-efficacy was the best predictor of examination results. This result is important as it creates new conceptual knowledge about the psychological functions of self-efficacy in learning mathematics and their alignment with the reciprocal determinants of self-efficacy theory (Bandura, 1997). From a constructivist perspective, it is significant because it conceptualises learning as a ‘position’ about oneself as mathematicians. Extending the 3Ps model of student learning (Biggs, 1987), I have concluded that mathematical self-efficacy, as affective component of learning, plays a predictive role on learning outcomes.

My research is important because the empirical results imply new ways of teaching and learning from presage to product stages. As discussed in my previous chapter, my main argument is that lecturers are key ‘drivers’ because they can design the curriculum and assessments to promote relational understanding, cohesive conceptions of mathematics and students’ mathematical self-efficacy. To sum up, the student may need guided mastery experiences (presage). But the social cognitive theory of self-efficacy implies that students, on their own, do not form self-belief for self-regulated learning and increase their mathematical self-efficacy. Hence, lecturers could design self-regulation and self-efficacy programmes that are theoretically grounded (position). Next, they need professional development to promote deep learning and cohesive conceptions of mathematics (processes). Some cited examples are the application of constructive alignment, the SOLO taxonomy in curriculum development, and high-quality assessments (product).

Low predictability

In contrast to high importance, my research has a low predictive value due to its lack of explanatory power. According to Schoenfeld (2002), the problem of ‘true’ predictability is lack of explanatory power (degree of explanation of how and why things work). Because my quantitative data showed the numerical responses about student learning, they did not indicate underlying causes of their beliefs about mathematics education and mathematical learning. Other conditions of teaching and social influences (e.g., classroom instructions, assessments, teaching styles, variations in specific courses, formative assessments and circumstances in which the assessments were set, peer learning) were assumed to be the same but not examined. While these teaching aspects might have been virtually impossible to control and were intrinsically intertwined with learning, it was difficult to use the data to explain the contexts of student learning and the psychological functioning of self-efficacy. As such, my statistical data could not explain how student learning affected their outcomes and how student beliefs were formed.

6.2 Future research opportunities

Mathematics education research is about linking theory with practice. This research supports the view that improving relational understanding in mathematical learning would help to develop problem-solving skills, which is considered as a graduate attribute, as mentioned in Chapter 1. My research participants, who study mathematics in business and engineering courses, perceived the importance of applying mathematics to real-world problems and engaging in deep learning of mathematical concepts. This suggests that students who have cohesive conceptions of mathematics and use deep learning strategies are likely to develop relational understanding in mathematical learning. As discussed in Chapter 5, these results might help to improve the quality of mathematical teaching and learning. In line with this goal of mathematics education, future research may explore the development of values, which is another dimension of student affect. Mathematics researchers, as well as practitioners, may consider how students develop creativity, engagement and intuition since innovative and creative skills are workplace requirements. They may seek to implement and evaluate a pedagogical intervention aimed at enhancing student engagement in mathematics courses and increasing their creativity. One possible intervention is the use of non-routine problems, which do not have ready-made solutions and require some creativity and originality for problem-solving based on real-world puzzle-based learning concept. This new research may contribute towards the development of graduate attributes as well as the quality of mathematical teaching and learning since practitioners are given research opportunities under the guidance of researchers who inform teaching practices with theoretical knowledge. In this case, this research may have a high predictive value since it explains student learning based on the development of creativity, engagement and intuition.

Mathematics education research is also about exploring new ways of doing research in specific educational settings. As mentioned in Chapter 5, Study III showed that participants with low mathematical background were likely to perform better in higher education mathematics courses, indicating that other factors might influence student performances. Quantitative researchers may explore other advanced statistical methods such as structural equation modelling method (Phan, 2010, 2011) and latent growth analysis (Phan, 2012) in order to examine the effects of intervening variables on prior mathematics. Furthermore, they can create a mixed-method research design, which employs both quantitative and qualitative methods of study in order to provide more compelling evidence about student learning. In line with a qualitative inquiry, case studies capture rich information about individual differences and diverse experiences of the participants. This creates room for holistic analysis in which researchers treat case studies as a particular event in context and extrapolate the findings to other situations under similar conditions. In some countries, such inquiries may benefit specific communities, who experience low economic status and poor educational background. In such scenarios, other non-educational factors of prior mathematics, might only be observed through personal interviews. Therefore, the introduction of different research tools and qualitative case studies may allow explanatory and causal relationships to be identified.

Concluding remarks

My research unravels new conceptual knowledge about the psychological functions of self-efficacy in mathematical learning. First, quantitative measures are used to investigate the nature of mathematical self-efficacy and their relationships with examination results. In Study I, I found that cognitive and metacognitive self-efficacy are positively correlated with performances. Multiple regression data show that metacognitive self-efficacy (self-belief in using cognitive, motivational, selection processes and self-belief for self-regulated learning) are key predictors of success in learning mathematics. Along with cognitive self-efficacy (self-efficacy in solving mathematical problems), these metacognitive determinants of self-efficacy are extracted from the Refined Self-Efficacy Scale using the method of exploratory factor analysis. Further validation of this scale was successfully carried out in a subsequent study. Hence, multiple regression data from Study III show that self-belief in selection processes is the best predictor of examination results. Therefore, using the Refined Self-Efficacy Scale, the findings reveal the multi-dimensional nature of mathematical self-efficacy, which is consistent with the theory of self-efficacy (Bandura, 1997).

Second, further investigation of the nature of student approaches to learning and conceptions of mathematics and their inter-relations with examination results helps to create new theoretical knowledge as well as validate previous empirical knowledge. Adding new theoretical knowledge, the framework of students' mathematics-related beliefs (Op't Eynde et al., 2002) reveal that students' beliefs about mathematics, which is characterised by relatively stable traits and worldviews of mathematics, is conceptually akin to conceptions of mathematics. However, previous literature distinguished the construct of conceptions of mathematics by types (fragmented or cohesive) or by levels (Levels 1, 2, and 3). Consistent with other studies (Crawford et al., 1994, 1998b), chi-square statistical data show that successful mathematics performance is strongly associated with a deep approach to learning, an organized approaches to learning, and a cohesive conception of mathematics (Level 3 life conception of mathematics). Moreover, correlational data indicates that successful mathematics students tend to develop cohesive conceptions of mathematics (Level 2 models and Level 3 life conception of mathematics). Moreover, in line with previous studies (Wood, Petocz, et al., 2012), Study II reports that the students tend to develop fragmented conceptions of mathematics (Level 1 numbers and components conceptions of mathematics). While these findings are not new, they could pose practical significance to mathematical teaching and learning.

My research adds new empirical knowledge which could be used to enhance mathematical teaching and learning in higher education. In Study III, I found that a deep approach to learning and a cohesive conception of mathematics are positively related to strong examination results. But more importantly, some reports show that mathematical self-efficacy is the most appropriate predictor of strong examination performances compared to the other constructs (Studies I and III). These results imply that students could attain success in mathematical learning when they develop strong mathematical self-efficacy, deep learning strategies, and perceive the importance of mathematical modelling and real-life mathematical applications. In order to conceptualize mathematical learning, I propose a presage-position-process-product model of student learning. First, students could form mastery experiences in mathematical learning so that they experience a measure of success and develop a high sense of self-efficacy (presage). Second, lecturers could design theoretically grounded self-regulation and self-efficacy programs (position). Next, they could promote deep learning strategies by applying the concept of constructive alignment, incorporating high-order relational and abstract thinking in the mathematics curriculum (processes) and creating high-quality assessments (product).

In conclusion, both the mathematics lecturer and the mathematics student play major roles in determining the quality of mathematical teaching and learning. The lecturer demonstrates excellent teaching practices whereas the student focuses on personal growth and career development. The lecturer, whose goal is to promote relational understanding, could introduce new powerful tools in mathematical teaching, a ‘relational mathematics’ course that incorporates processes of self-efficacy, deep learning strategies as well as real-life mathematical applications. In this way, the lecturer has an added responsibility of encouraging the student to develop relational understanding. But the student, of their own volition, also bears a measure of responsibility in engendering strong mathematical self-efficacy and a deep approach to learning. Eventually, the mathematics student is

> *the author and artist of her own self-belief, but is advanced in that patterning of self-creation by the encouragement of her [lecturers] and any other significant others in her educational endeavours (Barnett, 2007, p. 59).*

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Appendix 1a The Refined Self-Efficacy Scale

Below are a number of questions about your **self-efficacy levels in learning mathematics this semester**.

The scale consists of six sections and 81 items.

Please answer each item. Do not spend a long time on each item; your first reaction is probably the best one. Do not worry about projecting a good image. Your answers are **CONFIDENTIAL** and will not be divulged to anyone teaching this course.

Thank you for your co-operation.

SECTION I OF VI
The focus here is about the belief in your capability to solve NUMERICAL problems and problems in MEASUREMENT.

For each item there is a row of numbers (1 - 5) corresponding to a five point scale. A response for an item is shown by circling ONE of the five numbers. The numbers stand for the following responses:
1 . . . this item is **not well of at all** true of me when I study mathematics
2 . . . this item is **not too well** true of me when I study mathematics
3 . . . this item is **satisfactorily** when I study mathematics
4 . . . this item is **pretty well** true of me when I study mathematics
5 …..this item is **very well** true of me when I study mathematics

1.How well do you believe you can calculate accurately numerical problems mentally?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

2.How well do you believe you can calculate accurately numerical problems on paper?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

3.How well do you believe you can estimate and make approximations?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

4.How well do you believe you can interpret the accuracy of results and measurements?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

5.How well do you believe you can calculate the effects of change in variables using mathematical models?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

6.How well do you believe you can predict the rate of change of variables using mathematical models?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION II OF VI

The focus is in your belief in your capability to attempt successfully problems in GEOMETRY .

7.How well do you believe you can recognise the geometrical properties of objects in daily life?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

8.How well do you believe you can use geometrical models to solve practical problems in daily life?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION III OF VI

The focus is on your belief in your capability to attempt successfully problems in ALGEBRA

9.How well do you believe you can recognise patterns and relationships in mathematics and generalise from these?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

10.How well do you believe you can think abstractly and use symbols to communicate mathematical concepts, relationships and generalisations?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

11.How well do you believe you can think abstractly and use graphs and diagrams to communicate mathematical concepts, relationships and generalisations?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

12.How well do you believe you can use algebraic expressions to solve practical problems?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION IV OF VI

The focus here is about your belief in solving problems in STATISTICS.

13.How well do you believe you can analyse statistical data as reports and summaries?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

14.How well do you believe you can interpret data presented in charts, tables and graphs?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

15.How well do you believe you can estimate probabilities?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION V OF VI

The focus here is about your belief in using mathematical processes

16.How well do you believe you can use logical and systematic thinking in mathematical contexts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

17.In a mathematical problem solving situation, how well do you believe you can critically reflect on the method you have chosen?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

18.How well do you believe you can use information technology in mathematical contexts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

19.How well do you believe you can be part of a problem solving team, expressing your ideas, listening and responding to others?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

20.How well do you believe you can use the knowledge and skills in mathematics to interpret presentations of mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

(The following question is to be attempted only by those students who are bilingual i.e. use their ethnic language for communication on a daily basis at home. Other participants can kindly proceed to the next question)

21.How well do you believe you have developed skills in using your own ethnic language to express mathematical ideas?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION VI OF VI

(Please note that questions are mathematics- specific and also applicable in general. Hence some might seem repetitive.)

SELF BELIEF IN MOTIVATION STRATEGIES

22.How well do you believe you can study in appropriate ways that you will be able to learn mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

23.How well do you believe that if you try hard enough you will be able to understand the different concepts in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

24.How well do you believe that you understand the most complex concepts in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

25.How well do you believe that you can master the skills taught in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

26.How well do you believe that you can do an excellent job on the assignments and tests in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SELF BELIEF IN COGNITIVE STRATEGIES

27.When studying mathematics how well do you believe you can set goals for yourself to direct your activities?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

28.When you study mathematics how well do you believe you can outline the material to help organise your thoughts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

29.When you study mathematics how well do you believe you can formulate questions to focus your thoughts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

30.When studying mathematics how well do you believe you can go through your notes and readings to find out the most important concepts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

31.When studying a new mathematical concept how well do you believe that you can skim it to see how it is organised?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

32.When studying mathematics how well do you believe you can think through the topic to decide what it is you are supposed to learn rather than just reading it over?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

33.When studying mathematics how well do you believe that you can use information from different sources such as class notes, text books and discussions?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

34.When studying mathematics how well do you believe that you can ask yourself questions to make sure that you have understood the material?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

35.When studying mathematics how well do you believe that you can change the way of study to fit the requirements of the topic?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

36.When studying mathematics how well do you believe you can memorise key words to help recall important concepts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

37.When studying mathematics how well do you believe you can summarise concepts of the topic of study?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

38.When studying mathematics how well do you believe you can determine the concepts you have not understood well?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

39.When studying mathematics how well do you believe you can relate ideas from mathematics to other subject/s?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

40.When studying mathematics how well do you believe you can try to relate material to what you already know?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

41.When studying mathematics how well do you believe you can sort out confusion which arises over missing note taking in class?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SELF BELIEF IN RESOURCE MANAGEMENT STRATEGIES

42.How well can do you believe you can explain a topic in mathematics to your classmate or friend?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

43.How well do you believe you can work on your own, even if you have trouble learning the material in mathematics class?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

44.How well do you believe you can use your study time for mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

45.How well do you believe you can work with your classmates to complete the course assignments?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

46.How well do you believe you can work in class even if you don't like what is being done?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

47.How well do you believe you can stick to your study schedule?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

48.How well do you believe you can seek clarifications from your mathematics teacher when you do not understand a concept?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

49.How well do you believe you can persist on a topic in mathematics when you find the material difficult?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

50.How well do you believe you can ask a peer or another student in class for help in mathematics when you cannot understand the material being taught?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

51.How well do you believe you can keep up with topics and assignments in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

52.How well do you believe you can manage to keep working in mathematics even when you find the material uninteresting?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

53.How well do you believe you can review your mathematics notes / readings before an exam?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SELF BELIEF FOR SELF-REGULATED LEARNING

54.How well do you believe you can finish your *mathematics assignments* by deadlines?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

55.How well do you believe you can learn mathematics when there are other interesting things to do?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

56.How well do you believe you can concentrate on *other courses*?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

57.How well do you believe you can concentrate in mathematics in the classroom?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

58.How well can do you believe you can take notes of class instruction?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

59.How well do you believe you can take notes of mathematics during class instruction?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

60.How well do you believe you can use the library to get information for class assignments?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

61.How well do you believe you can plan your *studies*?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

62.How well do you believe you can organise your *studies*?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

63.How well do you believe you can remember information presented in class and textbooks?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

64.How well do you believe you can remember information presented in class and textbooks in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

65.How well do you believe you can arrange a place to study without distractions?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

66.How well do you believe you can motivate yourself to do *your studies*?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

67.How well do you believe you can motivate yourself to do *your studies* in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

68.How well do you believe you can participate in class discussions?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

69.How well do you believe you can clarify doubts in mathematics in class?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SELF BELIEF IN LEISURE TIME SKILLS AND EXTRACURRICULAR ACTIVITIES *(OPTIONAL)*

70.How well do you believe you can learn sport skills?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

71.How well do you believe you can learn dance skills?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

72.How well do you believe you can learn music skills?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

73.How well do you believe you can do the kinds of things needed to be a member of the school newspaper?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

74.How well do you believe you can do the things needed to be a member of the students' council?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

75.How well do you believe you learn the skills for team sports (for example basket ball, volleyball, swimming, cricket, rugby)?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

BELIEF IN SELF ASSERTIVENESS

76.How well do you believe you can express your opinions when other classmates disagree with you?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

77.How well do you believe you can live up to what you expect of yourself in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

78.How well do you believe you can you stand up for yourself when you feel you are being treated unfairly?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

79.How well do you believe you can stand firm to someone who is asking you to do something unreasonable or inconvenient?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

80.How well do you believe you can live up to what you expect of yourself?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

81.How well do you believe you can deal with situations when others are annoying you or hurting your feelings?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

Source: Marat, D. (2005). Assessing mathematics self-efficacy of diverse students from secondary schools in Auckland: Implications for academic achievement. *Issues In Educational Research*, 15(1), 37-68. http://www.iier.org.au/iier15/marat.html

Appendix 1b. The Refined Self-Efficacy Scale (abridged version)

Below are a number of questions about your **self-efficacy levels in learning mathematics this semester**.

The scale consists of six sections and 54 items.

Please answer each item. Do not spend a long time on each item; your first reaction is probably the best one. Do not worry about projecting a good image. Your answers are **CONFIDENTIAL** and will not be divulged to anyone teaching this course.

Thank you for your co-operation.

SECTION I OF VI
The focus here is about the belief in your capability to solve NUMERICAL problems and problems in MEASUREMENT.

For each item there is a row of numbers (1 - 5) corresponding to a five point scale. A response for an item is shown by circling ONE of the five numbers. The numbers stand for the following responses:
1 . . . this item is **not well of at all** true of me when I study mathematics
2 . . . this item is **not too well** true of me when I study mathematics
3 . . . this item is **satisfactorily** when I study mathematics
4 . . . this item is **pretty well** true of me when I study mathematics
5 …..this item is **very well** true of me when I study mathematics

1.How well do you believe you can calculate accurately numerical problems mentally?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

2.How well do you believe you can estimate and make approximations?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

3.How well do you believe you can interpret the accuracy of results and measurements?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

4.How well do you believe you can calculate the effects of change in variables using mathematical models?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

5.How well do you believe you can predict the rate of change of variables using mathematical models?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION II OF VI

The focus is in your belief in your capability to attempt successfully problems in GEOMETRY .

6.How well do you believe you can use geometrical models to solve practical problems in daily life?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION III OF VI

The focus is on your belief in your capability to attempt successfully problems in ALGEBRA

7.How well do you believe you can recognise patterns and relationships in mathematics and generalise from these?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

8.How well do you believe you can think abstractly and use symbols to communicate mathematical concepts, relationships and generalisations?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

9.How well do you believe you can think abstractly and use graphs and diagrams to communicate mathematical concepts, relationships and generalisations?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION IV OF VI

The focus here is about your belief in solving problems in STATISTICS.

10.How well do you believe you can analyse statistical data as reports and summaries?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

11.How well do you believe you can interpret data presented in charts, tables and graphs?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION V OF VI

The focus here is about your belief in using mathematical processes

12.How well do you believe you can use logical and systematic thinking in mathematical contexts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

13.In a mathematical problem solving situation, how well do you believe you can critically reflect on the method you have chosen?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

14.How well do you believe you can use information technology in mathematical contexts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

15.How well do you believe you can be part of a problem solving team, expressing your ideas, listening and responding to others?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

16.How well do you believe you can use the knowledge and skills in mathematics to interpret presentations of mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

(The following question is to be attempted only by those students who are bilingual i.e. use their ethnic language for communication on a daily basis at home. Other participants can kindly proceed to the next question)

17.How well do you believe you have developed skills in using your own ethnic language to express mathematical ideas?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SECTION VI OF VI

(Please note that questions are mathematics- specific and also applicable in general. Hence some might seem repetitive.)

SELF BELIEF IN MOTIVATION STRATEGIES

18.How well do you believe you can study in appropriate ways that you will be able to learn mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

19.How well do you believe that if you try hard enough you will be able to understand the different concepts in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

20.How well do you believe that you understand the most complex concepts in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

21.How well do you believe that you can master the skills taught in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

22.How well do you believe that you can do an excellent job on the assignments and tests in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SELF BELIEF IN COGNITIVE STRATEGIES

23When you study mathematics how well do you believe you can outline the material to help organise your thoughts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

24.When studying mathematics how well do you believe that you can use information from different sources such as class notes, text books and discussions?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

25.When studying mathematics how well do you believe that you can ask yourself questions to make sure that you have understood the material?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

26.When studying mathematics how well do you believe that you can change the way of study to fit the requirements of the topic?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

27.When studying mathematics how well do you believe you can memorise key words to help recall important concepts?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

28.When studying mathematics how well do you believe you can summarise concepts of the topic of study?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

29.When studying mathematics how well do you believe you can determine the concepts you have not understood well?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

30.When studying mathematics how well do you believe you can relate ideas from mathematics to other subject/s?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

31.When studying mathematics how well do you believe you can try to relate material to what you already know?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

32.When studying mathematics how well do you believe you can sort out confusion which arises over missing note taking in class?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SELF BELIEF IN RESOURCE MANAGEMENT STRATEGIES

33.How well can do you believe you can explain a topic in mathematics to your classmate or friend?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

34.How well do you believe you can work on your own, even if you have trouble learning the material in mathematics class?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

35.How well do you believe you can use your study time for mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

36.How well do you believe you can work in class even if you don't like what is being done?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

37.How well do you believe you can stick to your study schedule?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

38.How well do you believe you can seek clarifications from your mathematics teacher when you do not understand a concept?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

39.How well do you believe you can persist on a topic in mathematics when you find the material difficult?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

40.How well do you believe you can ask a peer or another student in class for help in mathematics when you cannot understand the material being taught?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

41.How well do you believe you can keep up with topics and assignments in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

42.How well do you believe you can review your mathematics notes / readings before an exam?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

SELF BELIEF FOR SELF-REGULATED LEARNING

43.How well do you believe you can learn mathematics when there are other interesting things to do?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

44.How well do you believe you can concentrate on *other courses*?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

45.How well do you believe you can concentrate in mathematics in the classroom?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

46.How well can do you believe you can take notes of class instruction?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

47.How well do you believe you can take notes of mathematics during class instruction?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

48.How well do you believe you can remember information presented in class and textbooks?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

49.How well do you believe you can remember information presented in class and textbooks in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

50.How well do you believe you can arrange a place to study without distractions?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

51.How well do you believe you can motivate yourself to do *your studies*?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

52.How well do you believe you can motivate yourself to do *your studies* in mathematics?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

53.How well do you believe you can participate in class discussions?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

54.How well do you believe you can clarify doubts in mathematics in class?

1	2	3	4	5
Not well at all	Not too well	Satisfactorily	Pretty well	Very well

Finally, how well do you think you're doing in this course unit as a whole?

Please try to rate yourself objectively, based on any marks, grades or comments you have been given. Circle ONE number

very well well quite well about average not so well rather badly

9 8 7 6 5 4 3 2 1

Source: Marat, D. (2005). Assessing mathematics self-efficacy of diverse students from secondary schools in Auckland: Implications for academic achievement. *Issues In Educational Research*, 15(1), 37-68.

Appendix 2. The Short Form of Conceptions of Mathematics Survey

Please TICK the boxes below corresponding to which of the following indicates your level of agreement with each statement.

5 – I STRONGLY AGREE (SA), 4 – I AGREE (A),

3 – NEUTRAL (N), 2 – I DISAGREE (D)

1 – I STRONGLY DISAGREE (SD)

Mathematics is …

	1 (SD)	2(D)	3(N)	4(A)	5(SA)
1. A set of models used to explain the world	☐	☐	☐	☐	☐
2. A way of analyzing ideas and problems	☐	☐	☐	☐	☐
3. A set of rules and equations	☐	☐	☐	☐	☐
4. Basic knowledge for all scientific fields	☐	☐	☐	☐	☐
5. No use to me at all	☐	☐	☐	☐	☐
6. A way to solve problems in my life	☐	☐	☐	☐	☐
7. A tool that can be applied in various fields	☐	☐	☐	☐	☐
8. Figuring out problems using numbers	☐	☐	☐	☐	☐
9. Using formulas to get results	☐	☐	☐	☐	☐
10. A way to give humans a more advanced life	☐	☐	☐	☐	☐
11. Calculations	☐	☐	☐	☐	☐
12. Numbers being processed	☐	☐	☐	☐	☐
13. A theoretical framework that describes reality	☐	☐	☐	☐	☐
14. The study of numerical concepts	☐	☐	☐	☐	☐
15. A way to generate new ideas	☐	☐	☐	☐	☐

Appendix 3 The Shortened Experiences of Teaching and Learning Questionnaire

This questionnaire has been designed to allow you to describe, in a systematic way, your reactions to the course you have been studying and how you have gone about learning it. Please respond truthfully, so that your answers will describe your **actual** ways of studying, and work your way through the questionnaire quite **quickly**. It is important that you respond to **every** item, even if that means using the UNSURE category. Your answers will be **confidential**. Put a **TICK** in the box to indicate how strongly you agree with **each** of the following statements.

	Disagree	Disagree somewhat	Unsure	Agree somewhat	Agree
1. I've been over the work I've done to check my reasoning and see that it makes sense.					
2. I have generally put a lot of effort into my studying.					
3. Much of what I've learned seems no more than lots of unrelated bits and pieces in my mind.					
4. On the whole, I've been quite systematic and organised in my studying.					
5. I've organised my study time carefully to make the best use of it.					
6. It has been important for me to follow the procedure/steps, or to see the reasons behind things.					
7. I've tended to take what we've been taught at face value without questioning it much.					
8. I've tried to find better ways of tracking down relevant information in this subject.					
9. I've just been going through the motions of studying without seeing where I'm going.					
10. If I've not understood things well enough when studying, I've tried a different approach.					

Appendix 4

UNDERSTANDING THE NATURE OF SELF-EFFICACY IN LEARNING MATHEMATICS

Priscilla E.L. Murphy — Manukau Institute of Technology, New Zealand
Leigh N. Wood — Macquarie University, Australia

This paper investigates the importance of self-efficacy in learning tertiary mathematics using quantitative measures. In line with Bandura's (1997) theoretical framework of self-efficacy, multiple regression data show that metacognitive self-efficacy (Self-belief in using cognitive, motivational, selection processes and Self-belief for self-regulated learning) are key predictors of success in learning mathematics. Further results reveal a positive correlation between self-efficacy in problem-solving and mathematics results. Therefore, an important point for tertiary practitioners to consider is to introduce these ways of developing self-efficacy in mathematics curriculum and student support in accord with the theory of self-efficacy.

OVERVIEW

This paper is influenced by an extensive study in New Zealand led by Mike Thomas, which reported that several practitioners perceived that first-year tertiary students lacked confidence in learning (Thomas et al., 2010). A pertinent result was that the high-achieving tertiary students felt that their level of confidence in mathematics was lower at tertiary level than in their secondary education, which indirectly lowered their level of preparedness in learning mathematics. Following their research, this paper will investigate the self-efficacy levels of first-year mathematics students in a New Zealand (NZ) tertiary institution. Our research questions are 1) What is the nature of self-efficacy? 2) In what way does self-efficacy predict success in learning mathematics?

THEORETICAL BACKGROUND

Self-efficacy is concerned with human enablement rather than personal judgement of one's ability (Bandura, 1997). People with high self-efficacy tend to make an effort and overcome difficulties because they are driven by personal affirmation which draws on one's self-knowledge (based on prior mastery experiences) and adapt their knowledge and skills to successfully accomplish future tasks. This sense of efficacy increases one's determination to succeed as well as promotes the use of self-regulation strategies for planning and organizing instructional activities, utilising resources, adjusting one's own motivation. It has been observed that having a strong belief in using self-regulation strategies determines academic success. Students are agents of their own learning so when they develop self-belief in using these strategies, they become more self-regulated learners. Mulat and Arcavi (2009) have reported that university mathematics students attributed their success to using self-regulation strategies such as, studying without distraction, completing homework, seeking peer

2017. In Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, p. 289-296. Singapore: PME.

and teacher support, paying attention in class, preparing well for examinations, persistence in solving challenging tasks, and making concerted effort on school tasks. Their results suggest that using self-regulated strategies reflect the students' metacognitive belief in learning which in turn, translates their will to achieve into learning processes and effort to produce positive outcomes.

In mathematics education research, Cretchley (2008) stated that to advance affect research, it is important to clarify the terms based on Bandura's theoretical framework since past researchers tend to generalise its concepts rather than assess it within specific contexts of learning, which tends to result in misconceptions in self-efficacy research. Therefore, this study conceptualises the nature of self-efficacy in mathematics education. In his theory, Bandura (1997) states that self-efficacy beliefs produce learning outcomes through cognitive, motivational and selection processes. First, cognitive processes are described as thinking processes which involve the acquisition, organization and use of information. These processes underpin purposive learning behaviour, which is a function of self-appraisal of capabilities, resides in forethought and in the self-regulation mechanisms by which forethought is translated into incentives and guides for purposive actions. The stronger the self-efficacy, the higher the goals individuals set themselves to attain performances. People with high self-efficacy mediate through cognitive processes by visualising success, which in turn provide cognitive support and guides for attainment. Secondly, self-efficacy plays a key role in the self-regulation of motivation via motivational processes. These include causal attributions, outcome expectancies, and cognized goals. In causal attributions, Bandura (1997) states that people with high self-efficacy attribute poor outcomes to lack of effort whereas those with low self-efficacy attribute failure to low ability. Next, in outcome expectancies, people expect their behaviour and actions to bring about valued outcomes so people with high self-efficacy are more likely to persevere and attain successful outcomes because their goal setting is governed by the cognitive processes of motivation. Thirdly, in selection processes, individuals are partly the product of their environment because they choose the social and physical environment and types of activities that they judge themselves to be capable of handling. In a nurturing learning environment, people are predisposed to achieving their goals and make deliberate choices to manage challenging activities in these situations. Therefore, based on the abovementioned processes in self-efficacy, this study aims to conceptualise these metacognitive forms of self-efficacy and examine their relationships with outcomes of learning mathematics.

Empirical studies have revealed a positive relationship between strong self-efficacy in solving mathematics problems and high mathematics performance but some researchers suggested that there was a need to examine their bi-directional relationships and factors of learning. In an international study, Williams and Williams (2010) argued that causal relationships between self-efficacy and mathematics performances have been difficult to prove as researchers were forced to assume one position or other when they used recursive statistical models to estimate the model. To illustrate this point, they have modelled the concept of reciprocal determinism, which refers to the psychological functioning involving behavioural, cognitive and environmental elements (Bandura, 1986) using structural equation modelling to report bi-reciprocal relationships between cognitive form of self-efficacy and achievement of secondary mathematics students in twenty-four out of thirty-three nations.

Other researchers have shown that self-efficacy predicts success in mathematics performance (Hailikari, Nevgi, & Komulainen, 2007; Hall & Ponton, 2005; Marcou & Philippou, 2005; Pajares & Kranzler, 1995; Pajares & Miller, 1994; Skaalvik & Skaavik, 2011; Stevens, Olivarez, Lan, & Tallent-Runnels, 2010). Marcou and Philippou (2005) reported that motivational beliefs as a function of self-efficacy correlated with problem-solving performances of fifth and sixth graders. In line with the social cognitive theory of self-efficacy (Bandura, 1997), Marat (2005) investigated determinants of self-efficacy with secondary mathematics students. Their discriminant analysis showed a positive correlation between high achievers in mathematics and high scores in self-efficacy in solving algebra problems, belief for self-regulated learning, selection and motivation strategies. A study of middle and high school mathematics students have found that self-efficacy was a better predictor of mathematics achievement than prior achievement (Skaalvik & Skaavik, 2011). This result was also evident for tertiary students of calculus in study by Hall and Ponton (2005) wherein it was found that university calculus students who reported high self-efficacy gained better results than other remedial students who also had low prior experience. To take another case in point, the path model data showed that there was a positive relationship between mathematical achievement and self-efficacy in problem-solving of ninth-grade and tenth-grade mathematics Caucasian students (Stevens et al., 2010). By comparison, Hispanic students scored poorly in mathematics and their confidence level, which suggests that some students succeeded in mathematics due to their high abilities and confidence. Pajares and Kranzler (1995) have concluded that students had high self-efficacy because they exhibited more effort and perseverance in challenging problem-solving situations. The abovementioned studies suggest that investigations of the way self-efficacy affects mathematical performance (at tertiary level) have been limited. Hence, more research is warranted to understand the psychological functions of self-efficacy in learning mathematics, particularly in tertiary education.

Literature suggests that positive self-efficacy breeds success whereas negative self-efficacy spawns failure in learning mathematics. Conversely, past successes increase self-efficacy levels and past failure diminishes it. In reality, this phenomenon might reflect a misconception of tertiary mathematics students. On one hand, lecturers might perceive first-year mathematics students to be confident. On the other hand, for many under-prepared students, the reverse is true. While lecturers focus on teaching mathematical concepts in class, such students become disenfranchised with the lack of opportunities to increase self-efficacy and possibly experience failure in learning mathematics. Nevertheless, some university studies have investigated the development of self-efficacy. Parsons, Croft, and Harrison (2011) interviewed seven engineering mathematics students at the Harper Adams University College, who reported that the provision of student support has somewhat helped students to develop their cognitive processes. Hence, the confident students set high goals of mastering all the topics whereas the less confident students avoided doing the difficult mathematics. They also developed a low self-belief in using motivational processes as they were less motivated to work hard and tried to avoid difficult mathematics questions, which lowered their self-confidence and made them choose alternative questions in the examinations. Further results showed that selection processes were reflected by their deliberate choices to study mathematics. More positive results were reported by Falco, Summers, and Bauman (2010). Their study skills programme was effective because their students developed greater self-efficacy, self-regulated learning, interest and engagement in learning mathematics and achieved better achievement scores. Therefore, although these studies were carried out in specific educational settings, these findings are likely to have important consequences for the broader domain of affect in mathematics education because understanding the role of self-efficacy sheds new light on its applications in learning mathematics.

METHOD

For this study, the participants were 166 tertiary students enrolled in the Business and Engineering programmes in a NZ tertiary institution. With ethics approval and participants' consent, their final assessment results were collected and linked to their survey responses. Originally designed by Marat (2005), the Refined Self-efficacy Scale was appropriate because it accords with the Motivated Strategies and Learning Questionnaire (Pintrich, Smith, Garcia, & McKeachie, 1991) and the social cognitive theory (Bandura, 1997). This survey consists of five-point Likert type scales which has Cronbach's alpha ranging between 0.76 and 0.91. The sub-scales included cognitive self-efficacy: Self-efficacy in solving numerical and measurement problems (SEI), geometry (SEII), algebra (SEIII), statistics (SEIV), Self-efficacy in using mathematical processes (SEV) and metacognitive self-efficacy: Self-belief in motivational, cognitive, selection strategies, Self-belief for self-regulated learning (SEVI). At the end of the scale, students had to assess how well they were doing of the course using a 9 -point numeric scale (1 as 'Very Badly'; 5 as 'about average'; 9 as 'Very well').

FINDINGS

Of the 166 students, 67 students (40%) completed the Refined Self-efficacy Scale (Marat, 2005). The majority of the participants were young (17-25 years old) and male (55.3%). Of those who had passed mathematics examination (79%), the same proportion of participants scored either A or C grades (31%). Considering each subscale SEI-VI, we found that the participants had the highest scores for SEI (3.87), Following the aforementioned sub-scales, the overall average of cognitive self-efficacy level (SE I –V) was 3.47 and metacognitive self-efficacy level (SE VI) was 3.55.

Self-efficacy in mathematics and grades

The statistical software IBM SPSS Statistics 21 was used to analyse the quantitative results. Correlational analyses showed a direct correlation between self-efficacy in mathematics and grades at 0.01 and 0.05 significance levels. Based on Dancey and Reidy (2004)'s categorisation of the strength of correlation, strong correlations range from R = 0.7 to 0.9, moderate to be 0.4 to 0.6, weak as ranging from 0.1 to 0.3. In this study, mathematics examination results are a proxy of mathematical performance since the summative work constitutes 50% course weighting and is a uniform yardstick for assessing students' performance. Table 1 shows that the main findings were SEVI correlated more strongly with the expected grades (R=0.64, p=0.000) than the actual grades (R=0.30, p=0.018). In terms of mathematics self-efficacy, there were moderate but significant correlations between SEI (R=0.44, p=0.001) and SEII (R=0.35, p=0.035) with the expected grades whereas there were weaker correlations between SEIII (R=0.28, p=0.028) and SEIV (R=0.29, p=0.018) with the actual performances. Their expected results were correlated strongly with the actual grades (R=0.55, p=0.000).

	Exam marks	Expected grades	SEI	SEII	SEIII	SEIV	SEV	SEVI
Exam marks	1							
Expected grades	.55**	1						
SEI	.10	.44**	1					
SEII	.35**	.35*	.19	1				
SEIII	.28*	.22	.14	.58**	1			
SEIV	.29*	.23	.000	.52**	.52**	1		
SEV	.079	.030	-.079	.28*	.45**	.57**	1	
SEVI	.30*	.64**	.30	.53**	.46**	.41**	.38**	1

p<**0.05, p<**0.01

Table 1 Pearson Correlations (*n*=55)

Predictors of student performance

There were six predictors of success, we used to understand the concept of self-efficacy and how this affects the results of students. According to Nardi (2006), "regression analysis does not tell [us] about one particular respondent, since the statistics are based on aggregated data. ….Mostly what [we] do with regression is to construct a profile of characteristics related to the dependent variable from past data and use that to explain what already exists or to predict subsequent outcomes" (p.208).

In order to establish a profile of successful students, we set up the independent variables self-efficacy (SE I to SEVI) and examination scores as a dependent variable and chose the linear regression model which assumes that the error term has a normal distribution with a mean of 0, the variance of the error term is constant across cases and independent of the variables in the model. When conducting the regression analyses, we tested if the linearity, normality and data independence assumptions of the dependent variables were satisfied. This method of analyses produced a model summary, which shows 32.7% of the variation in results is a result of the factors. We found that the low p value (p=0.040) in the analysis of variance table (F=2.715), suggests that the model is a better fit than using the mean of the sub-scales and that self-efficacy in using cognitive, motivation and selection strategies, self-regulated learning are significant predictors of the model (Beta=0.482, t=2.335, p=0.027).

DISCUSSION AND IMPLICATION

In response to the initial research question, some noteworthy results were positive correlations between the cognitive (Self-efficacy in solving mathematical problems), metacognitive self-efficacy (Self-belief in using cognitive, motivation and resource management strategies, Self-efficacy for self-regulated learning) and performance as measured by grades. Associated with this predictor was the finding that there was a positive correlation between expected marks and examination marks. This aligns with the theory that greater self-efficacy raises one's expectation to achieve high marks, which in turn, projects the actual performances. The correlation data somewhat matched past literature (Hailikari et al., 2007; Hall & Ponton, 2005; Marcou & Philippou, 2005; Pajares & Kranzler, 1995; Pajares & Miller, 1994; Parsons et al., 2011; Skaalvik & Skaavik, 2011; Stevens et al., 2010), suggesting that self-efficacy is not only about having a strong belief in problem-solving but a disposition to develop cognitive and metacognitive processes in learning. According to Bandura (1997), high performance in a particular task promotes self-efficacy, which in turn, emboldens individuals to work harder and develop further skills necessary for attainment in future tasks. Bandura further explains that in skill development "efficacy beliefs contribute to the acquisition of knowledge and development of sub-skills, as well as drawing upon them in the construction of new behaviour patterns" (Bandura, 1997, p. 61). The point is that having an expectation of positive outcomes and cognitive self-efficacy, alone are not sufficient, other functions of self-efficacy need to work in concert with it so that successful students gain mastery of mathematics skills.

To investigate the next research question, a significant linear regression finding was that the most appropriate predictor of successful performance in mathematics was self-efficacy in using cognitive, motivation, selection strategies and belief for self-regulated learning. Consistent with past research (Marcou & Philippou, 2005; Mulat & Arcavi, 2009), this result suggests that success in learning mathematics is determined by metacognitive dimensions of self-efficacy. In line with Bandura's notion of self-efficacy, by forming selection, cognitive and motivational processes, students could manage their learning by taking ownership of their own learning through self-regulated learning behaviour, goal setting, expenditure of effort and intrinsic motivation. Evidence of under-prepared tertiary mathematics students in Thomas et al's (2010) study further confirms the role of self-efficacy in the teaching and learning environment. Therefore, enhancing student learning is about overcoming low self-belief in learning as well as forming motivational, cognitive, selection and self-regulatory strategies in learning.

These findings raise another question: How could educators increase their chance of achieving success in learning? Given greater political impetus to improve student achievement, practitioners should seriously consider the influence of self-efficacy on their learning. Ultimately, the value of such programmes could outweigh the high cost of failure borne by students and staff. Teaching faculties could incorporate both metacognitive and cognitive forms of self-efficacy in their curriculum, which were shown to be somewhat effective in previous studies (Falco et al., 2010; Parsons et al., 2011). In order to produce desired outcomes in affect development, tertiary institutions could offer more incentives for developing self-efficacy in mathematics programmes in order to raise mathematical achievement.

CONCLUSION

Our study findings show that self-efficacy enhances mathematics results. This tends to shift the onus of learning onto tertiary students who may receive appropriate support for learning mathematics. With improved self-efficacy, these students tend to succeed in learning mathematics, which can serve as a gatekeeper in engineering and business programmes. Although the correlation data did not show causal relationships, we found that cognitive and metacognitive self-efficacy were positively correlated with performance and the most appropriate predictor of success was metacognitive self-efficacy. In this respect, our results suggest that as a result of lower self-efficacy, first-year students may be at-risk of failing mathematics. However, if students develop their metacognitive and cognitive forms of self-efficacy, their chances of achievement will increase. Therefore, an important point for practitioners to consider is to introduce new ways of developing self-efficacy in mathematics curriculum and student support in accord with the theory of self-efficacy.

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Identifying factors for self-efficacy in learning mathematics

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This study was intended to validate the Refined Self-efficacy Scale (RSS) [1] survey as well as clarify the construct of self-efficacy in mathematics education. The RSS survey was utilised to examine the self-efficacy levels of 65 tertiary mathematics students. Using Exploratory Factor Analysis (EFA) method of data analysis, five determinants of self-efficacy in learning mathematics have been extracted, confirming the original RSS survey structure. These findings also matched Albert Bandura's [2] theory of self-efficacy and the RSS dimensions were reduced by 33%. (reduced 81 items to 54 items) These new results could pose both methodological and conceptual significance for future self-efficacy investigations.

Appendix 5

Identifying factors for self-efficacy in learning mathematics

This study was intended to validate the Refined Self-efficacy Scale (RSS) [1] survey as well as clarify the construct of self-efficacy in mathematics education. The RSS survey was utilised to examine the self-efficacy levels of 65 tertiary mathematics students. Using Exploratory Factor Analysis (EFA) method of data analysis, five determinants of self-efficacy in learning mathematics have been extracted, confirming the original RSS survey structure. These findings also matched Albert Bandura's [2] theory of self-efficacy and the RSS dimensions were reduced by 33%. (reduced 81 items to 54 items) These new results could pose both methodological and conceptual significance for future self-efficacy investigations.

Keywords: self-efficacy; mathematics; tertiary education

Subject classification codes: include these here if the journal requires them

1. Introduction

Self-efficacy in learning mathematics is an integral part of mathematics education. Internationally, several researchers [3-10] have reported that in the secondary-tertiary transition, mathematics students are increasingly under-prepared for tertiary education. This raises a new question of how tertiary educators bridge the gap in learning tertiary mathematics. As stated by Schoenfeld [11], the success and failure in one's ability to do mathematical problem-solving are determined by both cognitive aspects (e.g. mathematical knowledge and resources, access to heuristic skills to tackle challenging problems) and affective aspects of learning (e.g. disposition to do mathematical problem solving, monitoring learning and self-regulation). To provide student support, practitioners need to understand both cognitive difficulties as well as affective aspects of learning mathematics.

To enhance student learning, my study aims to conceptualise the nature of mathematics self-efficacy (MSE). In order to understand the processes of self-efficacy, Bandura [2] cautions against using idiographic approach, rooted in trait theory but proposed a personal determinant approach to future research, founded on the relations between personal, environmental and behavioural factors. This approach will illuminate the nature of self-efficacy and its practical applications. Therefore, this study will attempt to identify domains of MSE in learning mathematics.

2. Theory of Self-Efficacy

According to Bandura [2], self-efficacy is defined as people's belief about their capabilities to organise and execute specific tasks in attaining particular outcomes. In learning mathematics, students with a positive sense of MSE will work hard to succeed whereas their lack of MSE conjures apathy in doing one's task. Framed within the social cognitive theory, self-efficacy acts upon personal factor (cognitive, biological and affective events), environmental factor and behavioural factor, determinants that are causally-related to each other. As people are agents of their own actions, they can adapt to the environment or change it to make things happen. In a responsive environment that values accomplishment, such individuals will increase their self-belief so that their renewed effort and participation in activities are fruitful. Depending on the types of environment, one's behaviour can promote varying strength in self-efficacy. If one's behaviour is empowering, this can give rise to strong self-efficacy but demoralising behaviour do not. This personal determinant approach shows that self-efficacy does not only predict outcomes but is governed by personal, environmental and behavioural factors.

Bandura [2] states that, self-efficacy can determine learning outcomes by means of self-regulation of cognitive processes, motivational processes and selection processes. By definition, self-regulation involves goal setting and action plans to attain the goals and the level of self-satisfaction and personal reflection as to whether one has accomplished the goals. First, cognitive processes involve personal goal setting, visualising successful scenarios, appraising and self-motivation to accomplish the activities. Those with MSE will set high goals and challenge themselves. Second, individuals with motivation processes will self-monitor their own learning, appraise their self-efficacy, set personal goals, establish outcome expectations and manage self-reactions. Those with high efficacy may be able to adjust and attain goals based on one's progress (goal theory), expect to produce certain outcomes and place a high value on those outcomes (expectancy-value theory) and attribute their failure to low effort instead of low ability (attribution theory). Third, selection processes are based on choices and decisions people make which determine the course of actions that they take in completing the activities. Those with low MSE will choose to avoid activities as they are overcome by the complex nature of the activities which exceeds their coping capabilities. Hence, they set low aspirations and are less committed, attribute failure to low ability, are likely to dwell on personal deficiencies, adverse outcomes and give up easily. On the other hand, those with strong efficacy are more likely to view difficult tasks as challenges, stay focussed and make an effort, recover from their setbacks and attribute failure to low effort. These three types of processes influence self-efficacy.

Based on this theory, Bandura [11] states that scaling self-efficacy includes both dimensions (generality, strength, levels) and domains (cognitive, motivation, behavioural, social). In terms of generality, people may judge themselves efficacious across a wide range of topics (e.g. algebra, arithmetic) and mode of actions (e.g. behavioural, cognitive, affective). Efficacy beliefs also vary in strength. Weak belief is possibly linked to past experiences of failure whereas strong belief is associated with successful accomplishments mixed with a desire to overcome adversities. The level of belief is measured in terms of the simple or complex nature of activities as perceived by the individual. If the individual considers an activity to be complex, the person will develop low level of self-efficacy and fail to accomplish the task.

3. Previous literature

Past mathematics education studies have mostly revealed the strengths and correlates of self-efficacy than its domains. Many researchers have confirmed the predictive role of mathematics performances [13-17]. In an international study of secondary mathematics students in twenty-four out of thirty-three nations, using structural equation modelling data, Williams and Williams [17] found that this relationship was bi-reciprocal. Other studies have examined correlates of self-efficacy: teaching interventions [19-21], prior mathematics [13, 14, 22-24], age differences [23] and gender [19,20]. These studies highlight how self-efficacy can improve mathematical performances, through self-efficacy programmes and learner-centred teaching, and of those with advanced prior mathematics, non-traditional and male students.

Few studies have reported about the domains of self-efficacy. In a study of secondary mathematics students in New Zealand, Marat [1] investigated the determinants of self-efficacy using the personal determinant approach to research. Using the method of discriminant analysis, they reported that self-efficacy in using cognitive (solving mathematical problems, using mathematics processes for problem-solving), self-efficacy in using motivation processes (goal setting), self-efficacy in using selection processes (time management), self-belief for self-regulated learning were positively related to strong mathematical achievement. They also reported a positive correlation between excellence mathematics grades and high scores in self-efficacy in solving algebra problems, self-belief for self-regulated learning, self-efficacy in using selection processes and motivation processes. These findings suggests that the processes of self-efficacy have a positive impact on mathematical achievement, Another qualitative study[25] of engineering mathematics students at the Harper Adams University College in United Kingdom reported that the provision of student support has somewhat helped students to develop their cognitive processes. Hence, the confident students set high goals of mastering all the topics whereas the less confident students avoided doing the difficult mathematics. They also developed a low self-belief in using motivational processes as they were less motivated to work hard and tried to avoid difficult mathematics questions, which lowered their self-confidence and made them choose alternative questions in the examinations. Further results showed that selection processes were reflected by their deliberate choices to study mathematics. These studies, though not generalizable, sheds some light on the determinants of self-efficacy in learning mathematics.

4. Description of the study

As part of a larger study, this study will evaluate the RSS survey based on the data obtained from a sample of 67 mathematics students from a New Zealand tertiary institution, Manukau Institute of Technology. Created by Marat [1], the RSS survey was built upon the social cognitive theory [10] and adapted from the Motivated Strategies for Learning Questionnaire (MSLQ) [26]. It consists of 81 five-point Likert style questions to assess the following sections: "Self-efficacy in solving numerical and measurement problems" (Questions 1-6), "Self-efficacy in solving geometry problems (Questions 7-8), "Self-efficacy in solving algebra problems" (Questions 9-12), "Self-efficacy in solving statistics problems" (Questions 13-15), "Self-efficacy in using mathematical processes" (Questions 16-21), "Belief in using motivation, cognitive, selection processes" and Belief for self-regulated learning (Questions 22-81). With the help of SPSS software, each item with positive meaning was positively coded whereas those which had negative meaning were coded reversely. Using the EFA method, the variables have been analysed by sections: 1-21, 22-41, 54-69 and 70-81 based on their allocated categories. To find the RSS survey, refer to the webpage [1].

5. Definition of EFA

According to Fabrigar and Wegener [28], EFA is a tool for researchers to understand the data and allow them to make analytical decisions and relate the data with theory. Since EFA was first introduced more than a century ago by Spearman [29], this factor analytic method has been widely used in educational research. In practice, EFA can be used to develop and validate measures such as, self-efficacy scale [30]. On a conceptual level, when conducting factor analysis to process the data and to identify common factors (construct), the variables (questionnaire items) are assumed to be influenced by the factors. Based on these new constructs, Fabrigar and Wegener [28] suggest that researchers can make conclusions regarding the nature of the constructs and theoretical assumptions in construct definition and scale development.

In general, factor analysis is a statistical method for measuring interdependence between the items in a battery. A battery is defined as a set of measured variables (any variables that can be measured) for factor analysis. If researchers have large sets of measures that produce several bivariate correlations (for instance, 80 measures has 3160 correlations), factor analysis can help to overcome such challenges. To address these challenges, the statistical procedures of factor analysis are designed to determine the number of distinct constructs required to account for the pattern of correlations among a set of measured variables assuming that the researcher did not make prior assumptions of the constructs to be used. The correlations between the variables and construct or factor refer to factor loadings. High factor loadings indicate that the variable is strongly influenced by the factor. Some unobserved (latent) constructs presumed, which account for the structure of correlations among measures, are known as factors.

6. Data Analyses

Based on the guidelines by Fabrigar and Wegener [28], Hair et al. [31] and Child [32], I will analyse the data based my key questions for EFA procedures: 1) What is the required sample size? 2) How many factors should be retained? 3) How should the factors be interpreted? 4) How does EFA support theoretical development? To produce a reliable factor analytic solution, an appropriate sample size is considered. Larger sample size decreases sampling error and produce more stable solutions [31]. Despite the small sample size, my data can still produce good estimates because my data meets these requirements as stated by some researchers [28,32]: high factor loadings (greater than 0.60), the communalities of the measured variables are an average of .70 or higher and each factor has at least 3 to 5 measured variables with substantial loadings on each factor. Therefore, the sufficiency of sample size could be determined by the stability of the data produced in the analysis.

Next, in data extraction, all the variables in my data fulfilled the requirements for factor extraction because they have met the assumptions of EFA [31]. First, the initial extraction showed that the Kaiser-Meyer- Olkin measure of sampling adequacy (KMO MSA) is more than 0.50 for survey variables 1 to 21. Next, the Bartlett's test of sphericity (significance >.05) was used to test the factorability of the variables (Table 1). The other variables have also satisfied this assumption.

Table 1KMO and Bartlett's Test

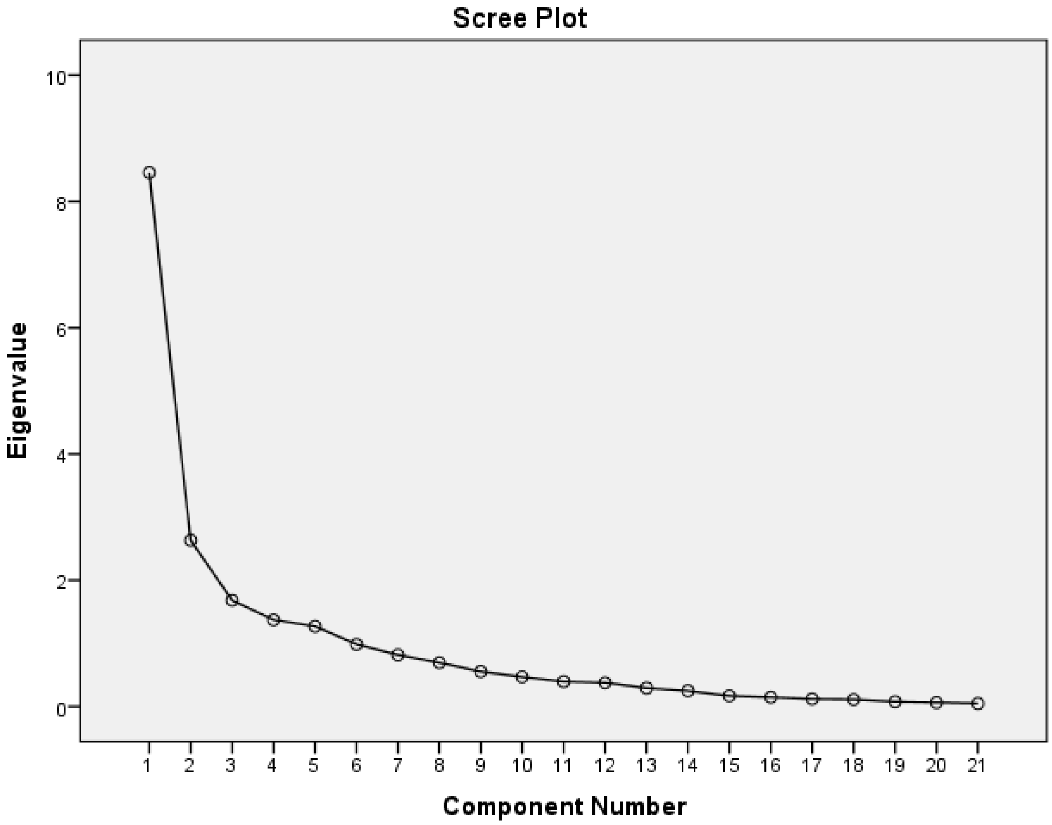
KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.750
Bartlett's Test of Sphericity	Approx. Chi-Square	654.826
	df	210
	Sig.	.000

Next in data reduction, because the factorability of the data has been confirmed, the next step was to determine how many factors should be retained. The process of data reduction is determined by the Principal Components Analysis (PCA), the scree test and the eigenvalue. First, the method of PCA is appropriate for data reduction, which focuses on the minimum number of factors that account for the maximum portion of the variance found in the original set of variables. According to the researchers [28, 31, 32], this method is used to account for the variances of measured variables rather than to explain the correlations among the variables The first factor normally accounts for the greatest variance and this decreases with each subsequent factors. The most significant factors are chosen whereas the other factors are eliminated if they are not significant and theoretically relevant. Once the PCA method is used, the graphical representation of the factors and the corresponding eigenvalues is shown by the scree plot (Table 2). Cattell's [33] guideline calls for retaining factors above the elbow and rejecting those below it. Another criterion (known as, Kaiser's Criterion) for determining the initial number of factors is its eigenvalue should be greater than 1. For survey variables 1 to 21 (Table 2), it appears as though 5 factors fell above the elbow. In summary, we could extract 1) five factors (Variables 1-21), 2) four factors (Variables 22-41), 3) three factors (Variables 42-53), 4) four factors (Variables 54-69) and 5) four factors (Variables 70-81). These factors were labelled as 1)'Self-efficacy in solving mathematics problems', 2)'Self-efficacy in using mathematical processes', 3)'Belief in using motivation, cognitive, selection processes', 4)'Belief for self-regulated learning' and 5)'Belief in using leisure time and belief for self-assertiveness'.

Table 2 Scree plot

Appendix 5



Child [33] argues that it is necessary to select a method to rotate the initial factor analytic solution in order to get a solution that can be easily interpreted. To interpret the factors, I have used the varimax rotation method which is an orthogonal rotation of factors in order to simplify the factor structure. When interpreting the rotated factor matrices, the following Hair et al.'s [31] guidelines for identifying and eliminating variables have been employed: 1) Factor loadings is .60 or higher for statistical significance (α=.05), 2) Communality value is .50 or higher, 3) Cronbach's alpha (0.7) and 4) Factors can be labelled. Hair et al. [31] also states that those variables with higher factor loadings (greater than .60) are considered more important and have greater influence on the name or label selected to represent a factor. For sections 'Self-efficacy in solving mathematical problems' and 'Self-efficacy in using mathematical processes', Table 3 presents the matrix of selected variables that satisfy the first factor loading guideline. But we decided to remove variables 12 and 15 because their Cronbach's alpha values were lower than 0.7. The rest of the sections have been analysed following these guidelines.

Appendix 5

Table 3 Rotated Component Matrix[a]

I to V. Self-efficacy in solving mathematical problems (1-15) and mathematical processes (16-21)	Component				
	1	2	3	4	5
1	**.613**	.376	-.042	.464	-.106
2	.547	.436	-.020	.432	.118
3	.075	.232	.082	**.838**	.034
4	.295	.199	.249	**.757**	.122
5	.041	.163	**.649**	.458	.232
6	.237	.089	**.758**	.409	-.059
7	.471	.196	.322	.442	-.090
8	**.722**	-.088	.307	.242	-.060
9	**.771**	.125	.270	.078	.312
10	**.774**	.065	.406	.011	.048
11	**.696**	.177	-.069	.117	.374
12	.470	-.217	.122	-.110	**.608**
13	.314	.251	**.658**	-.087	.427
14	.243	.249	**.664**	.015	.325
15	.068	.180	.238	.189	**.802**
16	.036	**.744**	.027	.437	.157
17	.228	**.739**	.372	.269	-.016
18	.180	**. 717**	.483	-.140	-.267
19	.381	**.605**	.184	.259	.109
20	.157	**.770**	.284	.295	-.129
21	-.147	**. 746**	-.057	.032	.290

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.[a]
a. Rotation converged in 14 iterations.

Besides adhering to Hair et al.'s guidelines, more variables have been removed in order to maintain construct validity for future research. We found that because the fifth category assessed student beliefs in leisure time and belief for self-assertiveness, this might not be appropriate for the sample of tertiary mathematics students. In a previous study [1] using the same scale, these variables have social and emotional nuances which were appropriate for the secondary school sample. By taking into account the study context, these variables were removed from the scale. Based on the aforementioned data analyses, a new RSS scale (Table 4) was generated, reducing its dimensionality by 33%, from 81 to 54 variables, and emphasising beliefs in using motivation, cognitive, selection processes and for self-regulated learning.

Table 4 Before and after factor analysis

Original sub-scales	**Variables *before* factor analysis**	**Deleted variables *after* factor analysis**	**New variables *after* factor analysis**
I. Self-efficacy in solving numerical and measurement problems	1 to 6	2	1, 3-6
II. Self-efficacy in solving geometry problems	7 to 8	7	8
III. Self-efficacy in solving algebra problems	9 to 12	12	9,10,11
IV. Self-efficacy in solving statistics problems	13-15	15	13,14
V. Self-efficacy in using mathematical processes	16- 21		16- 21
VI. Belief in using motivation, cognitive, selection processes Belief for self-regulated learning	22-81	27, 29-32, 45, 52,54, 60-62,70-81	22-26, 28, 33-44, 46-51, 53, 55-59, 63-69.
Total variables	81	27	54

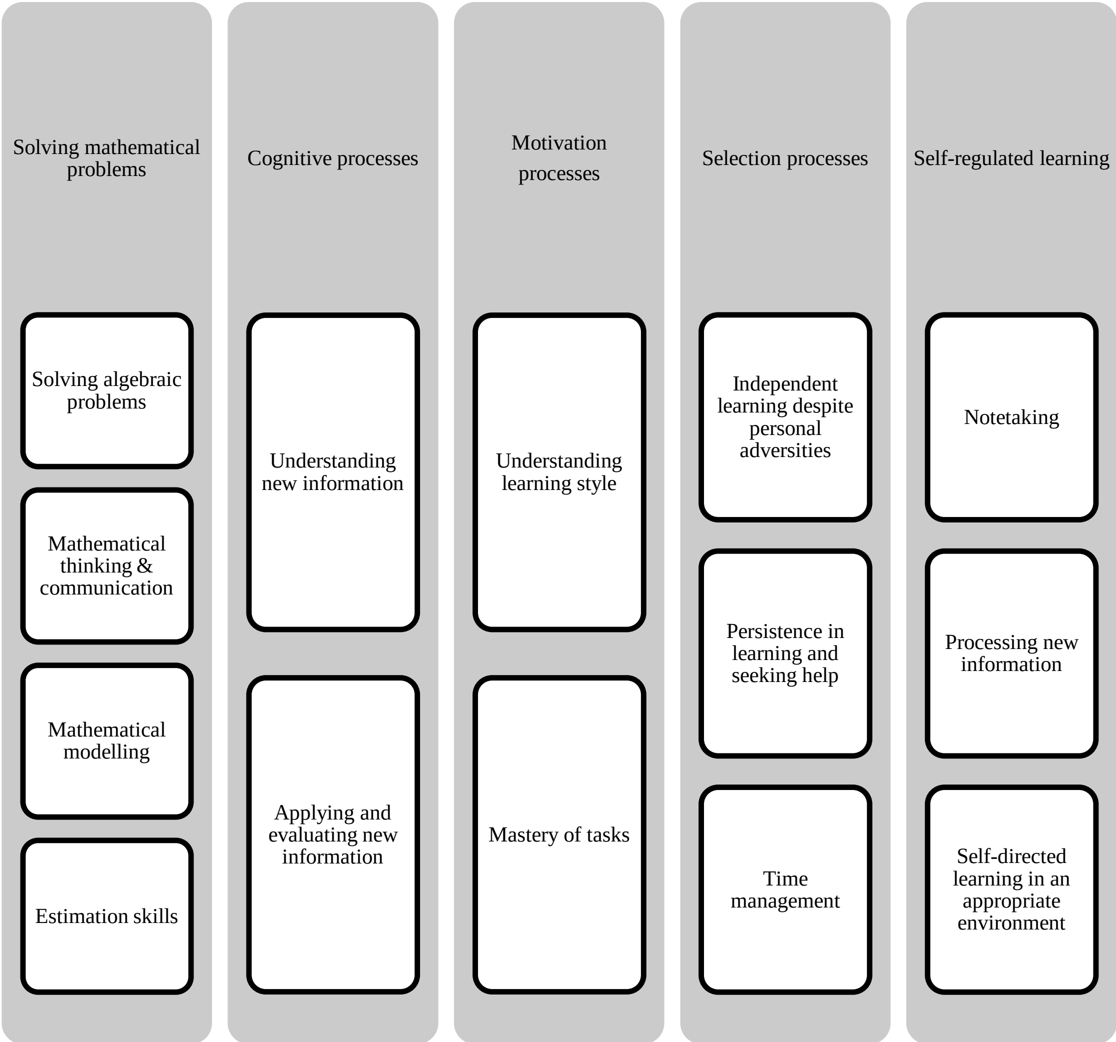
7. Discussion

One of the outcomes of EFA is scale reduction. A key advantage of using the factor analytic tool is scale dimensionality. Factor analysis provides “a clear method for testing the dimensionality of a set of items and determining which items appropriately belong together as part of the same scale or subscale” (p.23) [31]. Moreover, it provides information of the psychometric properties of specific items. For instance, an item which is strongly influenced by a factor has a high factor loading. Another item that have high factor loadings could also influenced by the same factor. This suggests both the items with high loadings are effectively capturing the intended construct. In doing so, data reduction can be achieved by identifying representative variables from a larger set of variables for subsequent multivariate analyses. Factor analysis can provide such information to help researchers to describe the variables that are influenced by the factors for the new scale and later, generate new hypotheses for future research.

The new RSS survey appears not to compromise theoretical underpinnings [2] but simplifies the structure. Based on the EFA data analyses and the newly generated variables (Table 4), Figure 1 outlines a set of five factors and their descriptions. The extracted factors are “Self-efficacy in solving mathematical problems”, “Belief in using cognitive processes”, “Belief in using motivational processes”, “Belief in selection processes” and “Belief for self-regulated learning”.

Figure 1Descriptions of factors

Appendix 5



As discussed in the previous section, the decisions in labelling the factors are methodological as well as conceptual. This is important because these purposeful decisions are critical to theoretical development in factor analysis [28]. The items with the highest factor loadings for each factor influence the labels and are representative of the latent construct [31]. One of the five proposed factors is "Self-efficacy in solving mathematical problems". This factor is about problem-solving strategies such as, item 3 estimation skills, item 4 solving algebraic problems, item 10 mathematical thinking and item 20 communication skills.

Table 5 Self-efficacy in solving mathematical problems

Item	
3	How well do you believe you can estimate and make approximations?
4	How well do you believe you can predict the rate of change of variables using mathematical models?

10	How well do you believe you can think abstractly and use symbols to communicate mathematical concepts, relationships and generalisations?
20	How well do you believe you can use the knowledge and skills in mathematics to interpret presentations of mathematics?

The next two factors (Table 5) are “Belief in using cognitive processes” and “Belief in using motivational processes”. On one hand, items 34 and 41 are related to one’s ability to understanding and evaluating new information, which aligns with self-efficacy in using cognitive processes. On the other hand, item 22 is about using appropriate learning style whereas item 25 is about task mastery. Both describe one’s belief in using motivation processes.

Table 6 Belief in using cognitive processes and Belief in using motivational processes

Items	
22	How well do you believe you can study in appropriate ways that you will be able to learn mathematics?
25	How well do you believe that you can master the skills taught in mathematics?
34	When studying mathematics how well do you believe that you can ask yourself questions to make sure that you have understood the material?
41	When studying mathematics how well do you believe you can sort out confusion which arises over missing note taking in class?

Next, the proposed factor is “Belief in using selection processes” (Table 6). Based on the theory of selection processes, item 47 shows how students can manage their time using a study schedule. Items 43 and 47 indicate self-directed learning and persistence in learning. All these items are driven by one’s personal choices to learn mathematics independently.

Table 7 Belief in using selection processes

Items	
43	How well do you believe you can work on your own, even if you have trouble learning the material in mathematics class?
47	How well do you believe you can stick to your study schedule?
48	How well do you believe you can seek clarifications from your mathematics teacher when you do not understand a concept?

Lastly, the fifth proposed factor was “Belief for self-regulated learning”. Through self-regulated learning, students set learning goals and plan their course of actions to achieve their goals. Students with a strong belief for self-regulated learning have the propensities for active learning [2]. Table 7 shows items 58 and 64 indicate active learning through the use of study skills such as, note-taking skills and processing new information. Such learning will happen in an appropriate learning environment as shown in item 65.

Table 8 Belief for self-regulated learning

Items	
58	How well can do you believe you can take notes of class instruction?
64	How well do you believe you can remember information presented in class and textbooks in mathematics?
65	How well do you believe you can arrange a place to study without distractions?

8. Study Limitation

This study has a small sample size which compromises the subject to variable ratio. As the survey was time-consuming, the response rate was lower than expected. To encourage student participation, an online tool can be used. Furthermore, to triangulate the quantitative data, qualitative researchers can conduct student interviews to elicit rich information about their beliefs of themselves as mathematicians.

9. Conclusion

This study has identified and described the determinants of self-efficacy on the basis of statistical and theoretical justifications. EFA is a powerful statistical method that helps us to conceptualise the theory of self-efficacy as well as reduce the scale dimensions of RSS survey to five factors: cognitive, motivational and selection processes, self-regulated learning and mathematical problem-solving. These factors are consistent with the domains and generality of self-efficacy [2]. Therefore, the EFA is a useful tool that provides information to help quantitative researchers make decisions on the construction of the scale, which could further test the theoretical assumptions of self-efficacy and conceptualise self-efficacy in learning mathematics.

Appendix 5

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Appendix 5

Appendix 6

Student Approaches to Learning, Conceptions of Mathematics, and Successful Outcomes in Learning Mathematics

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Abstract

In this chapter, I concentrate on success as the completion of a degree and investigate mathematics as a key component in that success. I examine the connections between approaches to learning, conceptions of mathematics, and student performance as measured by their grades. This study highlights practical implications for mathematics in higher education, draws from the comprehensive report of the Enhancing Teaching-Learning Environments (ETL) project by Hounsell and Entwistle (2005), and extends the international research on student conceptions of mathematics by Wood, Petocz, and Reid (2012). Surveys were used to investigate the learning experience of a random sample of first year mathematics students in Australia and New Zealand. This chapter highlights two key findings of relevance to teachers and curriculum developers: firstly, that successful mathematics performance was strongly associated with deep approaches to learning, organised approaches to learning, and a cohesive conception of mathematics; and secondly, that surface approaches to learning were negatively related to modelling and the abstract conceptions of mathematics.

Context

New Zealand is a developed country in the South Pacific Ocean. It offers a vibrant mix of cultures resulting from strong Māori and Pacific Island traditions and European migration. It has eight universities, three Wānanga (a public institution that provides education in a Māori cultural context) and eighteen Institutes of Technology and Polytechnics (ITPs). In this chapter, my sample is taken from Manukau Institute of Technology (MIT). Located in Auckland, the University has 16,000 students (Manukau Institute of Technology, 2014), of whom 17% are Maori and 35% Pacific Island. In order to fulfil the NZ Tertiary Education Commission's (TEC) strategy of improving the learning outcomes of priority learners (Māori, Pacific Island, and under 25-year-olds), tertiary institutions have to attain four measures of student success: a) increased participation, b) improved success and retention, c) improved employability and progression, and d) enhanced experience and satisfaction (New Zealand Tertiary Education Commission, 2014).

A neighbouring Pacific nation is Australia. My sample also includes students from Macquarie University (MQ), based in the business and technological hub of Macquarie Park in Sydney. Similar to NZ, the Australian higher education sector is made up of universities, vocational education and training (TAFE), and private providers and is driven by teaching and learning frameworks mandated by the Tertiary Education Quality and Standards Agency (TEQSA). TEQSA's learning standards describe the scope of knowledge and skills and the level of attainment required for graduation purposes (Department of Education Employment and Workplace Relations, 2011).

Introduction

In order to increase the pipeline of mathematics students in professional programs in higher education, my study investigates mathematics performance in relation to student learning processes and provides recommendations for educators.

My research questions are:

- What is the nature and extent of students' approaches to learning mathematics?
- What are the characteristics of student conceptions of mathematics?
- To what extent are learning approaches and conceptions of mathematics related?
- How are these related to student results?
- What are the implications for teaching mathematics in higher education?

These research questions are underpinned by several key constructs based on previous research in student learning: approaches to learning, conceptions of mathematics, and student performance.

Approaches to Learning

This idea was originally coined by Marton and Säljö (1976) to refer to a co-existence of intention and process of learning. A *deep approach to learning* produces complex learning outcomes, involving the motive of intrinsic interest and using learning strategies that maximise meaning, whereas a *surface approach to learning* is driven by fear of failure and rote learning. An *achieving* or *organised approach to learning*, which overlaps with a deep approach to learning, is driven by the need for achievement and the use of space and time to achieve a task. The intentions of those who adopt achieving strategies are to strive to gain high grades. They seek to complete the tasks by making greater effort and managing their time. Approaches to learning may be influenced by learning tasks, teaching, and maturity.

Conceptions of Mathematics

This refers to one's interpretation of the discipline of mathematics. People construe specific meanings that are attached to phenomena (such as mathematics), and these meanings elicit responses. As reported in Wood, Petocz, and Reid (2012), conceptions can be described in three levels and in Crawford et al. (1994) as two types: fragmented and cohesive conceptions. Wood et al. (2012) found that undergraduate mathematics students perceived that mathematics is about numbers and components (Level 1 components); mathematics is about modelling and abstraction (Level 2 models); and mathematics is relevant to life (Level 3 life). The Level 1 conception of mathematics as a study of numbers, components, or techniques overlaps with a "fragmented" conception of mathematics. Level 2 is akin to a cohesive conception, whereby mathematics is a complex logical system which can be used to solve complex problems. Level 3 is a higher level in which mathematics is understood as being insights for understanding the world. As Crawford et al. (1994) only investigated first year students, the Level 3 conception was not evident among their participants. Table 1 summarises this research.

Crawford et al. (1994)	**Wood, Petocz, and Reid (2012)**
Fragmented	Level 1: Mathematics is about topics, numbers, techniques (Components)
Cohesive	Level 2: Mathematics is about modelling and/or abstract structures (Models)
	Level 3: Mathematics is about life and career (Life)

Table 1: Conceptions of mathematics

Student Performance

Student performance can be seen as represented by quantifiable learning outcomes such as assessment marks as a standard indictor of success in learning, assuming that examinations are designed to test higher level learning. For example, for participants in my study a mathematics course in a business faculty requires them to develop mathematical knowledge in algebra and calculus as well as apply the principles, concepts, and techniques learned to solve practical and abstract problems.

Even though institutions define success with broad-brush measurements such as program completion and retention rate, and assessment is often aligned with funding and auditing purposes (New Zealand Tertiary Education Commission, 2014; Tertiary Education Quality and Standards Agency, 2011), my study posits that students view "success" or "failure" in learning mathematics as a nominal outcome of either a Pass or Fail grade in mathematics. For example, if a faculty reports that a mathematics course had a low completion rate, this suggests that several mathematics students had failed due to either early withdrawal from the course, or a Fail grade in their examinations. My study suggests that students who fail mathematics are likely to use surface learning and fragmented conceptions of mathematics, whereas successful students tend to adopt deep learning and a cohesive conception of mathematics, assuming that teaching and assessments promoted either surface or deep learning.

This study adds to existing research in mathematics education. My results extend international research on student conceptions (Wood et al., 2012) and contribute to enhancing teaching and learning in higher education, and particularly to the goal of producing mathematics graduates with skill sets that are well suited to their future careers. In mathematics education research, conceptions of mathematics in student learning have been investigated since the 1980s, but more research is warranted on broadening the notion of learning mathematics in the context of business and engineering.

How students develop deep learning and cohesive conceptions in mathematics and the impact of these on the development of graduate capabilities are emerging as significant questions in the current higher education context. In general, deep learning, work readiness, work relevance, and analytical skills are valued by stakeholders in the international labour market (Organization for Economic Cooperation Development, 2013). Accrediting bodies for engineering and business programs have regarded mathematical skills as essential to the development of graduate attributes. The Institution of Professional Engineers New Zealand (IPENZ) has developed a national plan to ensure that engineering graduates meet New Zealand's economic needs by applying knowledge of mathematics to the solution of complex engineering problems, following the guidelines of the Washington Accord (International Engineering Alliance, 2009). In mathematics education, both the OECD report and IPENZ guidelines imply that mathematics educators should improve the quality of learning through deep learning and the application of mathematics to daily life, in order to adequately prepare mathematics undergraduates for the demands of complex problems faced in workplaces. This further highlights the key role that tertiary educators play in ensuring high-quality learning processes and understanding variations in student learning experiences, which are at the centre of my study.

Background

My study addresses the transition from secondary to tertiary mathematics education by investigating learning processes and recommending teaching strategies for faculty staff to promote successful outcomes. Research has shown that tertiary mathematics students face difficulties with performing basic mathematical calculations without the aid of calculators, and advanced mathematics students also lack understanding of logical proofs and appreciation of assumptions in mathematical principles in Australia (Brown, 2010) and in New Zealand (Thomas et al., 2010). Australia is facing a downward trend in preparedness of first year undergraduate mathematics students, partly because secondary students have tended to choose easier options in senior mathematics and also due to a shortage of mathematics teachers. Consequently, universities tend to offer remedial courses for first year mathematics students (Brown, 2010). Likewise, New Zealand tertiary mathematics students tend to adopt surface approaches to learning in secondary school due to an overemphasis on high-stake assessments (Thomas et al., 2010). It is clear that inadequate preparation in mathematics as well as poor learning approaches, particularly with first year mathematics students, create barriers to achieving success in higher education.

Many institutions have addressed the need to prepare first year commencing students diagnosed with low levels of basic mathematics (algebra and arithmetic) through bridging and foundation courses. These initiatives include developmental mathematics programs for engineering students at the University of Southern Queensland, Australia (Taylor & Morgan, 1999); mathematics support at the Loughborough University in the UK (Croft, Harrison, & Robinson, 2009); and after-class study groups for mathematics undergraduates (Solomon, Croft, & Lawson, 2010). In several institutions, while mathematics bridging programs are introduced to support first year commencing undergraduates, more pressure is placed on mathematics teaching staff to enhance teaching and offer mathematics support. With this in mind, my study suggests that mathematics educators do consider revisions to curriculum design in courses that support students transitioning from secondary to tertiary education in order to raise their level of mathematical achievement.

My investigation aims to validate research findings on learning processes within the context of mathematics students in business and engineering programs. Studies of student learning from the 1990s report that the majority of mathematics students tend to use surface approaches to learning and demonstrate fragmented conceptions. These students display poor-quality learning outcomes, whereas those who adopt deep learning and cohesive conceptions show high-quality learning outcomes (Crawford et al., 1994; Crawford, Gordon, Nicholas, & Prosser, 1998a; Liston & O'Donoghue, 2009; Macbean, 2004). Using phenomenological approaches, their findings are important to understanding how learning approaches and conceptions are related. However, it is noted that learning processes are not always consistent in the context of teaching and learning. Liston and O'Donoghue (2009) report that students' conceptions of mathematics may not be consistent with the approach to learning that they adopt. As an example, a student who focuses on surface approaches may score high marks and recognise the importance of deep learning. In international mathematics research, Wood et al. (2012) found that more than half the undergraduate mathematics students surveyed (56%) adopted fragmented conceptions with fewer students developing cohesive conceptions (44%) and, to a lesser extent, life conceptions (6%). In the context of engineering education, Khiat (2010) reports that engineering students in their study were less likely to form associative understandings which allowed them to relate mathematics to engineering problems. He argues that surface learners who tend to develop procedural understanding in their use of formulae and doing mathematical calculations are unable to apply mathematical concepts in solving engineering problems. Prior empirical findings in mathematics education therefore suggest that tertiary mathematics students tend to focus on surface learning and display fragmented conceptions. However, given variations in assessment and small sample sizes, it is difficult to generalise learning outcomes in relation to learning processes without considering the context of the studies. As such, my study investigates the extent of relationships between learning processes and mathematics results in the context of learning mathematics in business and engineering.

Learning processes such as approaches to learning and conceptions of learning are relatively stable traits but influenced by the learning and teaching context. If students perceive that teaching promotes deep learning strategies, they may also follow the same agenda in their learning intentions and learning processes even if they still use surface learning (Prosser & Trigwell, 1999; Ramsden, 2005; Richardson, 2005). A study by Prosser and Trigwell (1999) reports that high-performing students are quick to adapt to their learning environment given that they generally adopt deep approaches to learning and, thus, new situations will further evoke similar deep approaches. Once they perceive that the current task or assessment requires deep learning, they tend to adopt strategies which enable them to understand the concepts. If they are aware that the task or assessment requires surface learning, they tend to adopt surface approaches such as memorising information and studying to the test. In order to attain high marks, students are willing to change their learning strategies from deep to surface at the expense of achieving high-quality learning outcomes. It has been found that the backwash effect of inappropriate quantitative assessments, focusing on lower cognitive levels of thinking, could be counter-productive for students, who prefer deep approaches to learning (Lai & Biggs, 1994).

Another determinant of learning approaches is goal orientation. Senko and Miles (2008) found that 260 American university students' results were jeopardised because they tended to engage in deep learning, to adopt mastery goals and to focus too much on understanding their preferred topics. Interestingly, those who had performance-oriented goals and used surface learning strategies were more likely to achieve better results, but displayed less interest in learning the course. Hence, when students place too much emphasis on improving one's performance in examinations, they tend to perceive deep learning as a barrier to getting good grades.

With respect to age differences and workload issues, Biggs (1987) reports that young college students (less than 18 years old) tend to adopt surface approaches to learning, whereas older university students (over 22 years old) who had a heavier workload, tend to switch from deep approaches to surface approaches in their final year of undergraduate studies. In teaching mathematics, several studies (Cano & Berbén, 2009; Crawford, Gordon, Nicholas, & Prosser, 1998b; Entwistle, 2005, September; Entwistle, Nisbet, & Bromage, 2005; Fenollar, Román, & Cuestas, 2007) have found that good teaching environments are related to deep approaches and clear teaching goals, mastery goals, and appropriate assessments; whereas fragmented conceptions are associated with surface learning due to fear of failure, heavy workload, lack of purpose and inappropriate assessments. Although teaching factors will not be investigated in my study, prior findings about mathematics teaching suggest that approaches to teaching and learning coincide with each other. Building on this body of research on student learning, the contribution of my study is to link approaches to learning, conceptions of mathematics, and performance for students commencing tertiary study.

My study is influenced by Biggs' model of constructive alignment (Biggs, 1996) that underpins the Enhancing Teaching-Learning Environments (ETL) project (Hounsell et al., 2005). Hounsell and his colleagues applied principles of constructive alignment in developing undergraduate courses in Edinburgh, Durham, and Coventry universities in the UK in order to enhance the teaching and learning environment. In developing new curricula for each discipline, focusing on how learners can develop better understanding and engage in deep approaches to learning, lecturers designed learning, teaching, and assessment activities that promoted higher levels of learning behaviour. Although my study is concerned with student approaches to learning and their conceptions of mathematics, this research has helped me to recognise that learning experiences are complex and influenced by teaching and learning factors.

Methodology

My sample consisted of 291 business and engineering mathematics students from Manukau Institute of Technology and Macquarie University. At the time of the data collection, students were enrolled in first year mathematics courses, covering basic algebra and calculus concepts and problem-solving applications in engineering and business. Questionnaires were used to assess student approaches to learning and their conceptions of mathematics. This way of obtaining information was practical for a large number of students. My intention was to raise awareness of student learning and implement new teaching initiatives that would foster better learning outcomes. I used five-scale Likert-style questionnaires to investigate students' conceptions of mathematics and approaches to learning mathematics. After gaining ethics approval, questionnaires were sent to the students at both universities. The Short

Form of Conceptions of Mathematics (SCM) consists of 16 items as shown in Table 2.

Level 1 Components [*alpha* = *.92*]
Mathematics is
1. A set of models used to explain the world
6. A way to solve problems in my life
10. A way to give humans a more advanced life
11. The language of nature
14. A theoretical framework that describes reality
16. A way to generate new ideas
Level 2 Models [*alpha* = *.71*]
Mathematics is
2. A way of analysing ideas and problems
4. Basic knowledge for all scientific fields
5. No use to me at all
7. A tool that can be applied in various fields
Level 3 Life [*alpha* = *.75*]
Mathematics is
1. A set of models used to explain the world
6. A way to solve problems in my life
10. A way to give humans a more advanced life
11. The language of nature
14. A theoretical framework that describes reality
16. A way to generate new ideas

Table 2: Short Form of Conceptions of Mathematics Scale (Wood et al., 2012)

The second questionnaire, Shortened Experiences of Teaching and Learning Questionnaire (SETLQ) was developed by Hounsell et al. (2005). The original inventory consists of the Learning and Teaching Questionnaire and Experiences of Teaching and Learning Questionnaire. As part of my pilot study, I performed a factor analysis of the ETLQ scale, which showed that the shortened version (SETLQ) was well validated and that item reduction had improved the scale reliability. In this study, I utilised a revised version (10 items), focusing on the learning approaches sub-scale (Table 3).

Deep approach [*alpha* = *.77*]
1. I've been over the work I've done to check my reasoning and see that it makes sense.
6. It has been important for me to follow the procedure/steps, or to see the reasons behind (Original statement-It has been important for me to follow the argument, or to see the reasons behind things. things.

8. I’ve tried to find better ways of tracking down relevant information in this subject.
10. If I’ve not understood things well enough when studying, I’ve tried a different approach.
Surface approach [*alpha* = *.68*]
3. Much of what I’ve learned seems no more than lots of unrelated bits and pieces in my mind.
7. I’ve tended to take what we’ve been taught at face value without questioning it much.
9. I’ve just been going through the motions of studying without seeing where I’m going.
Organised approach [*alpha* = *.82*]
2. I have generally put a lot of effort into my studying.
4. On the whole, I’ve been quite systematic and organised in my studying.
5. I’ve organised my study time carefully to make the best use of it.

Table 3: Shortened Experiences of Teaching and Learning Questionnaire (Hounsell & Entwistle, 2005)

At the beginning of the data collection, students recorded their demographic details (university ID, gender and age) and self-rated their expected mathematics examination performances (ranging from 1 as “Rather badly” to 9 as “Very well”). Final examination mathematics results were used as a measure of mathematical performance. In line with the ethics protocol, all data were confidential. Using IBM SPSS software (Statistics 22), the mean score for each sub-scale was tabulated. The responses from the positive statements were coded from 1 – strongly disagree to 5 – strongly agree. One negative statement (that is, “Mathematics is of no use to me”) was coded in reverse to match with the coding of the positive statements. Descriptive, correlation and cross-tabulation tables were extracted to investigate relations between conceptions of mathematics, study approaches, and performance.

Findings

Student Results

Omitting missing data (N=15) from my analyses, Table 4 shows the distribution of examination performance for each demographic category. The majority of students who passed their examinations were typically male school-leavers (16–20 years old) and mature students (over 30 years old).

FAIL	GENDER	AGE	*N*	PASS	GENDER	AGE	*N*
MIT	Female 2	16-20	8	MIT	Female 19	16-20	32
Fail *N*=15	Male 13	21-25	5	Pass *N*=78	Male 59	21-25	22

		26-30	2			26-30	8
		Over 30	0			Over 30	14
MQ	Female 9	16-20	17	MQ	Female 70	16-20	91
Fail *N*=23	Male 14	21-25	6	Pass *N*=160	Male 90	21-25	49
		26-30	0			26-30	7
		Over 30	0			Over 30	10

Table 4: Age and gender by types of performance and institution (N=276)

Student Approaches to Learning, Conceptions of Mathematics, and Results

In order to investigate the relationship between sub-scale s and results, correlation coefficients of each category were calculated (Table 5). Weak and moderate correlations were generally found between the constructs.

Sub-scale	1	2	3	4	5	6	7	8
Final course grade	1							
Expected grade	.48**	1						
Deep approach	.25**	.31**	1					
Surface approach	-.077	-.10	.11	1				
Organised approach	.30**	.42**	.57**	.21**	1			
Level 1 conceptions of mathematics (components)	.013	.12*	.14*	-.07	.12*	1		
Level 2 conceptions of mathematics (models)	.12	.17**	.15*	-.25**	-.003	.64**	1	
Level 3 conceptions of mathematics (life)	.15*	.27**	.28**	.017	.26**	.51**	.43**	1
Mean	5.14	6.23	3.88	3.22	3.61	3.88	3.94	3.42
Standard Deviation	2.42	1.68	.674	.90	.97	.69	.74	.76

Table 5: Correlation coefficients of sub-scale s (N=291); * $p<0.05$ and ** $p<0.01$.

The data show that the students had high mean scores in conceptions of mathematics Level 2 (3.94), Level 1 conception of mathematics (3.88), deep approach to learning (3.88), and organised approach to learning (3.61); and lower scores in surface approach to learning (3.22) and a Level 3 conception of mathematics (3.42). Based on Dancey and Reidy's (2004) categorisation of the strength of correlation, strong correlations range from R =0.7 to 0.9, moderate range from 0.4 to 0.6, and weak correlations range from 0.1 to 0.3. In analysing correlation matrixes, low correlation coefficients can be significant with large sample sizes. Hence, it was more meaningful to look at the strength of correlations than to focus on their significance levels. In summary, the foregoing data display significant positive correlations between these sub-scale s:

Weak correlations:

- Final grade and deep approaches (R=.25, $p<0.01$)
- Final grade and organised approaches (R=.30, $p<0.01$)
- Final grade and Level 3 conception (life) (R=.15, $p<0.05$)
- Expected grade and Level 1 conception (components) (R=.12, $p<0.05$)
- Expected grade and Level 2 conception (models) (R=.17, $p<0.01$)
- Expected grade and Level 3 conception (life) (R=.27, $p<0.01$)
- Expected grade and deep approaches (R=.31, $p<0.01$)
- Deep approaches and Level 3 conception (life) (R=.28, $p<0.01$)
- Surface approaches and organised approaches (R=.21, $p<0.01$)
- Organised approaches and Level 3 conception (life) (R=.26, $p<0.01$).

Moderate correlations:

- Final and expected grades (R=.48, $p<0.01$)
- Expected grade and organised approaches (R=.42, $p<0.01$)
- Deep approaches and organised approaches (R=.57, $p<0.01$)
- Level 1 (components) and Level 2 conceptions (models) (R=.64, $p<0.01$)
- Level 1 (components) and Level 3 conceptions (life) (R=.51, $p<0.01$)
- Level 2 (models) and Level 3 conceptions (life) (R=.43, $p<0.01$).

It can be seen from the above that students' expectations of their grades correlated positively with the three categories of conceptions of mathematics and only moderately with final grades. The higher the expected grade, the more cohesive the conception of mathematics. Moreover, there were significant correlations between expected grades and deep and organised approaches to learning. These patterns of significant correlation with organised approaches, deep approaches, and a cohesive conception were also observed in relation to the final grades.

To further establish relations between conceptions of mathematics and approaches to learning, I cross-tabulated the corresponding sub-scale s of these constructs by using 2 X 2 contingency tables (Table 6). When analysing the SPSS data, following Nardi (2006), two assumptions for chi-square tests were considered: that the categories for the observations should not overlap; and that each category must have an expected frequency of at least 5. If the probability (*p*) value of obtaining a chi-square value is less than 0.05, then the null hypothesis is rejected, suggesting that both variables are related. My initial cross-tabulation showed low expected frequencies (less than 5) in some cells, which violated the second chi-square testing assumption. Hence, I adjusted to two categories (low and high levels) by recoding the SPSS codes. For example, 1 (strongly disagree) and 2 (disagree) were classified as low levels, whereas 3 (neutral), 4 (agree), and 5 (strongly agree) were categorised as high levels for the purpose of creating the contingency tables. Each table shows approaches to learning (by row) and conceptions of mathematics (by column).

Level 1 Conception (components)		
Surface	Low	High
Low	1	57
High	10	220
Deep	Low	High
Low	0	8
High	11	269
Organised	Low	High
Low	1	35
High	10	242
Level 2 Conception (models)		
Surface	Low	High
Low	2	56
High	11	219
Deep	Low	High
Low	0	8
High	13	267
Organised	Low	High
Low	1	35
High	12	240
Level 3 Conception (life)		
Surface	Low	High
Low	7	51
High	17	213
Deep*	Low	High

Low	3	5
High	21	259
Organised*	Low	High
Low	7	29
High	17	235

Table 6: Conceptions of mathematics versus approaches to learning (N=288); *Chi-square statistics (significant at $p<0.05$, df=1)

In Table 6, a low statistical *p* value (lower than .05) suggested that deep learning and Level 3 (life) conceptions were related ($\chi^2=5.657$, df=1, p=0.017). The actual count, which was similar to the expected count, confirmed that 90% of respondents had adopted deep learning and a cohesive mathematics conception. Furthermore, organised approaches to learning and “mathematics is about life” conceptions were statistically related ($\chi^2=5.091$, df=1, p=0.024).

By contrast, insignificant chi-square results were noted in the relations between surface learning, deep learning, and Level 1 and 2 conceptions of mathematics.

Discussion

Mathematics Results

On one hand, my prediction about relations between deep approaches to learning, cohesive conceptions (Level 3 life), and high grades was true. On the other hand, my hypothesis about the relations between surface approaches to learning, fragmented conceptions, and low grades was untrue and warrants further investigation. These results are important to curriculum developers who intend to teach mathematics in undergraduate courses and to motivate students to succeed in learning mathematics. Contrary to Senko and Miles (2008), my findings suggest that in order to succeed in mathematics, students should view mathematics as a discipline that has essential application to their lives, adopt deep approaches to learning, be capable of managing their time well, and demonstrate effort in their studies. These findings are consistent with previous research (Crawford et al., 1994; Liston & O’Donoghue, 2009; Macbean, 2004). To a certain extent, they differ from the findings of Crawford et al. (1994) because my study found a significant relationship between deep approaches and cohesive conceptions whereas there was no significant association between surface and fragmented conceptions. Due to the quantitative nature of my results, further investigation is warranted to understand why surface learners tend not to perform well in examinations given that their sole purpose in studying mathematics is to pass, without making real connections between mathematics and their future studies.

Conceptions of Mathematics and Approaches to Learning

If educators place too much emphasis on remembering rules and formulae in assessments, this may encourage a low-level conception of mathematics and a surface approach to learning. My study shows that one group of students held lower level conceptions, seeing mathematics as about numbers and components. We found that these conceptions were significantly correlated to the conceptions that mathematics is about models and life (Levels 2 and 3). Although participants held a lower level conception, they could eventually develop a higher conception of mathematics. As suggested by Wood et al. (2012), there is scope for students to develop higher level conceptions over time because these conceptions are developmental in nature, with higher conceptions building upon lower ones. By implication, in order to achieve higher quality learning outcomes, a student who adopts a fragmented conception prior to tertiary studies should be encouraged to develop a more holistic and cohesive conception of mathematics in the first year of tertiary education. In order to improve students' higher level conceptions of mathematics, applications of constructive alignment in mathematics curricula (Hounsell et al., 2005) suggest that lecturers should teach mathematics as a connected set of topics and concepts which relate meaningfully to people's lives.

Relevance of the Findings to Educators

Mathematics Curriculum

The curriculum can be used to promote cohesive conceptions and deep learning. Constructive alignment is a useful framework for lecturers to assist their students to develop deeper learning of mathematics through applications of concepts in real-life situations, as well as by focusing their awareness on the conceptual aims and learning demands of the subject (Hounsell et al., 2005). According to Biggs (1996), constructive alignment guides the alignment of curriculum goals with teaching and learning activities (TLAs) and assessment goals. One measure of the effectiveness of the constructive alignment model is the SOLO (Structure of Observed Learning Outcome) taxonomy (Biggs & Collis, 1982), which specifies five levels of understanding. As an example, Biggs (1996) evaluated the use of constructive alignment in a psychology course for pre-service teachers at the University of Hong Kong and found that 37% of pre-service teachers reached an “extended abstract” level of understanding and 40% reached a “relational” level of understanding.

As shown in Table 7, the five levels of understanding in the SOLO taxonomy can be shown as parallel to different approaches to learning and conceptions of mathematics: it may be that higher levels of understanding (relational and extended abstract) are achieved by adopting deep approaches as well as Level 2 (models) and Level 3 (life) conceptions of mathematics. Conversely, it may be that lower levels of understanding (pre-structural, unistructural, multistructural) are associated with surface approaches and Level 1 (components) conceptions of mathematics.

SOLO taxonomy	Approaches to learning	Conceptions of mathematics
1. Pre-structural The student has not understood the tasks. 2. Unistructural The student has applied and used one or few aspects of the tasks. Understanding refers to knowing bits of information.	Surface	Components
3. Multistructural Aspects of the tasks are understood and treated separately. Understanding is knowing about each component.	Surface	Components
4. Relational The components are integrated into a whole. Understanding is forming relationships between components.	Deep	Models
5. Extended Abstract Abstraction of ideas and generalisation to a new topic. Understanding involves transfer and metacognition.	Deep	Models, Life

Table 7. SOLO taxonomy and learning processes

Biggs (1996) suggests that when teachers design objectives, they can focus on developing higher levels of understanding at all stages of the learning, teaching, and assessment cycle. For instance, lecturers in his psychology course first intended that pre-service teachers develop “extended abstract” levels of understanding by evaluating their own teaching practices with reference to theories of teaching. Next, to meet class objectives, lecturers should ask themselves: “What activities are standard teaching methods most likely to elicit?” (p. 353) Teaching and learning activities could include teacher-controlled activities (such as formal tutorials involving cooperative learning); peer-controlled activities (for example, students applying teaching theories in group work); or self-controlled activities (such as taking notes from a text before a lecture to understand psychological concepts). Then, to assess whether students have achieved specified levels of understanding, lecturers should ask: "What forms of understanding (based on the SOLO taxonomy) are called for in assessments?" Assessments which promote higher levels of understanding include diary entries, portfolio work which shows changes in practices, and concept maps of readings. To date, although few mathematics education studies have investigated the use of constructive alignment in the mathematics curriculum, it is clear that the principles of constructive alignment have the potential to promote higher levels of mathematical understanding.

As well as developing a connected curriculum which promotes understanding through constructive alignment, educators should aim to overcome barriers to deep learning. One such barrier is an over-emphasis on high-stake assessments. My study found a significant negative correlation between surface approaches to learning and modelling conceptions of mathematics. Participants were studying algebra and calculus topics in order to apply mathematical concepts and modelling to business and engineering problems. Drawing on findings from research by Crawford et al. (1998a), if lecturers focus too heavily on surface learning in high-stake assessments, students will aim to pass the course and view mathematics as a set of isolated topics. One possible reason is that students perceive assessments and TLAs as promoting surface or deep learning, so they would pursue the same agenda in their approaches to learning (Prosser & Trigwell, 1999; Ramsden, 2005; Richardson, 2005). By implication, if students perceive that assessments are designed to test their procedural skills in mathematics despite exposure to deep learning in the classroom, they will study to the test and reproduce their notes. Similarly, if they receive inadequate teaching that promotes deep learning and are given high marks for reproducing notes in inappropriate assessments, they will use surface approaches and attain high scores. In such cases, the danger of attaining high marks is that achievement, in conventional terms, tends to mask real understanding of mathematical concepts. Therefore, in order to promote deep learning in mathematics education, lecturers should be aware of students' perceptions of TLAs and assessment goals and provide opportunities for deep learning. Hence, in the long run, lecturers might find it more productive to be aware of students' perceptions of teaching and learning contexts in order to encourage deep learning and modelling conceptions of mathematics.

Students at Risk

In order to increase student success in mathematics, my data suggest that students who have high expectations of achieving success are more likely to attain better results. My data show low achievement by younger participants: compared to mature students (over 25 years old), a higher proportion (13%) of younger students (below 25 years old) failed mathematics. This is not consistent with a study by Biggs

(1987), which found that younger students performed better than older students because older students used deep approaches to learning whereas younger students were more interested in improving their performance.

Role of Tertiary Educators

Tertiary educators play an important role in ensuring high-quality learning outcomes in mathematics education. My research found significant correlations between Level 2 and 3 conceptions, deep approaches, and an organised approach to learning. These empirical findings from undergraduate mathematics students supported a positive association between deep and organised approaches found by Marton and Säljö (1976) and variations in conceptions of mathematics in Wood et al. (2012). Deep learning is about applying one's mathematical knowledge to various fields and requires understanding of mathematical concepts. Such learning can only happen if individuals adopt organised learning approaches by managing their time well and expending productive effort. As Bruner (1966) observes, learning mathematics is about knowing mathematics as a process of gaining knowledge, not as an end-product of knowledge. This process of "knowledge-getting" suggests that knowledge is not simply accumulated but understood, applied, and constructed by the learner. We teach a subject not to produce little living libraries on that subject, but rather to get students to think mathematically for themselves (p. 72). Therefore, lecturers play an important role in ensuring that students exercise autonomy in developing mathematical understanding through deep learning.

Recommendation for Future Research

Analysis of the correlational data showed relationships between student approaches to learning, conceptions, and results. Student-learning research in education is complex because there are several intervening variables at play. One important variable, which is beyond the scope of our study, is the impact of the teaching environment on deep learning, which could be explored in future studies. As learning and teaching could be considered as two sides of the same coin, research in student learning should also take into account teaching interventions, teaching approaches, assessment and the alignment of the curriculum, and principles of constructive alignment.

Conclusion

Tertiary educators are aware of the need to increase the pipeline of successful science, technology, engineering, and mathematics graduates. One way of increasing the graduate pipeline is to ensure that tertiary institutions improve students' learning outcomes in mathematics, as this subject is a gatekeeper for engineering and business programs. My findings showed that strong mathematical performance was positively correlated with deep approaches to learning, cohesive conceptions, and organised approaches to learning. Compared to younger learners, non-traditional mature students tended to be more successful. Moreover, students who studied a mathematics foundation subject at university were more successful than students who had studied mathematics at high school. This may explain the success of mature students, and demonstrates that students can compensate for knowledge not developed in secondary education. In order to ensure that first year tertiary mathematics students succeed in learning mathematics, educators should be aware of variations in students' learning approaches and conceptions of mathematics. More importantly, they should consider ways of teaching mathematics in order to engage students in deeper approaches to learning and to provide them with more opportunities to integrate knowledge. Students can succeed when universities offer targeted foundation knowledge taught in ways that develop deep learning and structured conceptions of mathematics.

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Appendix 7

RELATING MATHEMATICS SELF-EFFICACY, STUDENT APPROACHES TO LEARNING AND STUDENT CONCEPTIONS OF MATHEMATICS TO MATHEMATICS RESULTS

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Mathematics learning is influenced by personal factors, ongoing approaches to learning, mathematical self-efficacy, and conceptions of the subject matter. This quantitative study investigates these constructs in relation to examination results of higher education mathematics students in New Zealand (N=73). The study used the Refined Self-efficacy Scale (RSS) (Marat, 2005), Conceptions of Mathematics Form (SCM)(Wood, Petocz, & Reid, 2012) and Shortened Experiences of Teaching and Learning Questionnaire (SETLQ)(Hounsell & Entwistle, 2005). The key findings were: self-belief in selection processes was the best predictor of examination results; deep approaches, cohesive conceptions and mathematical self-efficacy correlated positively with examination grades; individuals who had attained Year 11 (or equivalent to Grade 10) *secondary mathematics qualification, pre-degree mathematics qualifications scored better than students with Year 12 and Year 13 qualifications. This evidence is a basis for modelling mathematical learning in higher education.*

INTRODUCTION

This study examines students' perceived capability in learning mathematics, personal intentions and processes in learning mathematics, Students' beliefs about mathematics and their impact on mathematics results. Building upon the presage-process-product model of student learning (Biggs, 1987), the key constructs of this study were prior mathematics (Presage), approaches to learning (Process) and mathematical performances (Product). The other key constructs were mathematical self-efficacy and conceptions of mathematics. The model stems from a constitutionalist perspective in which an individual's learning experience is a result of an internal relationship between the learner and the world through their awareness of the world, not known by the researcher (Marton & Booth, 1997). This study is important as it serves as a basis of understanding how and why mathematics students succeed in tertiary education. Our research in student learning can serve as a conduit for researchers and educators to improve research methodology and enhance teaching and learning programmes.

PRESAGE: PRIOR MATHEMATICS AND AGE

Prior knowledge is defined as the highest mathematics qualification gained at school. A New Zealand (NZ) report by Engler (2010) argued that gaining mastery of the skills taught in secondary mathematics could improve advancement in tertiary education. In order to achieve higher levels of university performance, students should achieve a level of understanding that leads to proficiency in the use of those skills and knowledge. As expected, attaining the highest mathematics secondary education (Year 13) is advantageous for future success in tertiary education (Henderson & Broadbridge, 2009).

In particular, young people (15-24 years old) were targeted by the New Zealand Tertiary Education Commission (2013) as a priority group for increasing Science, Technology, Engineering and Mathematics (STEM)-related qualifications. As such, raising mathematics performances of young people could contribute to STEM-related careers in NZ. An empirical study of tertiary students by Carmichael and Taylor (2005) found that while there was no significant difference in mathematical performances of traditional males and females, non-traditional students (over 25 years old) could perform better than the younger counterparts due to greater self-efficacy levels. Furthermore, a study by Miller-Reilly (2006) showed that academic support helped non-traditional learners to develop greater confidence in learning mathematics and improved their grades. We posit that older students (over 25 years old) could perform better in their examinations since they were more mature and committed to learning.

PROCESS: APPROACHES TO LEARNING

Student approaches to learning, originally coined by Marton and Säljö (1976), refer to co-existence of intention and process of learning. A deep approach involves the motive of intrinsic interest and strategy to maximise meaning, whereas a surface approach to learning is driven by one’s fear of failure and a process of rote-learning. An achieving or organised approach, which overlaps with a deep approach, is driven by one’s need for obtaining good grades and how one makes use of space and time to achieve a task. These learning approaches affect the quality of learning outcomes.

CONCEPTIONS OF MATHEMATICS

Another factor that influences success in learning mathematics pertains to conceptions of mathematics. Conceptions are based on interpretations of personal learning experiences (Schmeck, 1988) . A study by Wood et al. (2012) reported that students at level 1 perceived mathematics to be about numbers and components (53%); at level 2, mathematics is considered to be about modelling and abstraction (34%); and at level 3, mathematics is perceived to be relevant to life. The level 1 conception of Mathematics as a study of numbers, components or techniques that can be used to solve problems overlaps with ‘fragmented conception’ of Maths as a set of numbers, rules and formulae which can be applied to solve problems (Crawford, Gordon, Nicholas, & Prosser, 1994). In contrast, cohesive conception, whereby Maths is a complex logical system which can be used to solve complex problems and provides insights used for understanding the world, was identical in meaning to how mathematical modelling is used to solve real life problems (level 2) and mathematics is applicable in people's lives (level 3). These holistic/cohesive conceptions are formed by mathematicians. Moreover, Crawford et al. (1994) reported that fragmented conception was related to a surface approach and unsuccessful outcomes whereas cohesive conception of mathematics corresponded with a deep approach and positive outcomes.

SELF-EFFICACY

According to Bandura (1997), self-efficacy is a personal judgement of one's ability to do mathematics. Self-efficacy is mediated by a large set of self-regulatory mechanisms such as, cognitive, motivation and selection processes. First, cognitive processes are a function of thought in which inferential judgements are made about how actions affect outcomes. Second, motivation processes are enunciated by the attribution theory stating that people, who are faced with difficult tasks, attribute their successes to personal capabilities and failure to insufficient effort. Third, selection processes are based on choices that people make in terms of time management and their use of resources in learning mathematics. Self-efficacy is also a function of self-regulated learning. Self-regulated learners are said to be agents as they exercise their freedom of self-influence to make things happen.

Self-efficacy plays an important role in predicting mathematics achievement. A study by Jaafar and Ayub (2010) found that self-efficacy was positively related to calculus performances. Another study by Carmichael and Taylor (2005) reported that younger students with low prior knowledge performed poorly at tertiary level whereas mature students at the same level of prior knowledge were willing to work hard and succeed due to higher self-efficacy. A study by Phan (2011) found that deep learning approaches were positively related to changes in self-efficacy and surface approaches. Conversely, the positive effect of self-efficacy beliefs determined deep processing strategies and motives (Fenollar, Román, & Cuestas, 2007; Liem, Shun, & Youyan, 2008).

PRODUCT: EXAMINATION RESULTS

We advocate that mathematics examination results are appropriate products of learning given that summative assessments fulfil a broad range of learning, ranging from mathematical calculations and comprehension to applications of knowledge in the course learning outcomes (Manukau Institute of Technology, 2013). To date, there has been insufficient research in higher education in NZ, which relates conceptions of mathematics, approaches to learning mathematics, mathematical self-efficacy, and personal qualities to mathematics performances. We posit that when students initially develop a strong sense of self-efficacy and have a strong mathematical background, they tend to persist in deep learning, avow holistic conception of mathematics and produce better results. As such, our research questions are as follows:

Q1.What is the nature of relations between mathematics (examination) results and mathematics self-efficacies in five areas (problem-solving, cognitive, motivational, selection processes, and self-regulated learning), deep/organised/surface approaches to learning, as well as Levels 1 to 3 conceptions of mathematics?

Q2.Which factor(s) is/are the most salient predictor(s) of mathematics performances?

Q3.To what extent do age differences, course type and highest level of secondary mathematics determine high mathematics achievement?

SAMPLE

Seventy-three (37% of cohort) mathematics students in a New Zealand tertiary institution participated. The sample consisted of males (80%, N=58) and females (20%, N=15). Their ages were 18-24 years old (74%, N=54) and over 25 years old (26%, N=19). The majority had achieved National Certificate of Educational Achievement (NCEA) Level 3 (30%, N=22) or an overseas qualifications (29%, N=21). Some had completed NCEA Mathematics Level 1 (8%, N=6), NCEA Mathematics Level 2 (15%, N=11) and Mathematics at Cambridge and International Baccalaureate (IB) levels (7%, N=5). The tertiary courses were Engineering Mathematics 1 (1st year), Engineering Mathematics 2 (2nd year), Programming Precepts and Business Statistical Analysis (1st year) and Foundation Mathematics (1st year). In this institution, once they pass mathematics examinations, first-year Mathematics Foundation students could enrol into further mathematical studies taught within the Engineering and Business faculties.

METHOD

During one semester, the RSS(Marat, 2005), SETLQ (Hounsell & Entwistle, 2005) and SCM (Wood et al., 2012) were distributed in March (RSS) and May (SETLQ and SCM) respectively, using five-point Likert style questionnaires (Likert, 1931). On both occasions, the participants completed their personal details and expected grades using a nine-point scale in which the lowest rating score of '1' indicates that 'they think they're doing rather badly in this course unit as a whole'. In July, the summative examination marks were collected. Using the IBM SPSS 22 statistical software, correlational studies, linear regression and general linear model were used to analyse the data.

FINDINGS

Q1. Relationships between variables

Using the categorisations of the strength of correlations (i.e., strong correlations range from R =.7 to .9, moderate to be .4 to .6, weak as ranging from .1 to .3) (Dancey & Reidy, 2004), moderate correlations were found between examination results and self-efficacy in problem-solving and using motivational, cognitive, selection strategies ($0.41<R<0.52$, $p<0.01$); deep and organised approaches ($R=0.63$, $p<0.01$); deep approaches and self-efficacy in five domains ($0.37<R<0.47$, $p<0.01$). Weak and positive correlations (two-tail significance) were reported between results and self-belief for self-regulated learning ($R=0.39$, $p<0.01$); results and expected grades (0.35, $p<0.05$); results and deep approaches ($R=0.27$, $p<0.05$); results and models conceptions of mathematics (R=0.23, p<0.05). The highest mean scores were 'Self-belief in using motivation strategies' (3.66), 'Deep Approaches to Learning' (3.96) and 'Conceptions of Mathematics Level 1' (3.98) and 'Conceptions of Mathematics Level 2' (3.96).

Q2. Factors predicting performance

Considering all the predictors (self-efficacy in the five domains, modelling/abstract conceptions, deep approaches), the most significant predictor was self-belief in selection processes given that the regression assumptions were not violated (Hair, Black, Babin, Anderson, & Tatham, 2006). The F ratio of the model mean square to error mean square was 4.702 (df=7, Sign=0.000).The model (Beta=0.599, t=2.413, p=0.019) accounts for 34.7% (R square) of the variation of results.

Q3. Personal factors predicting performance

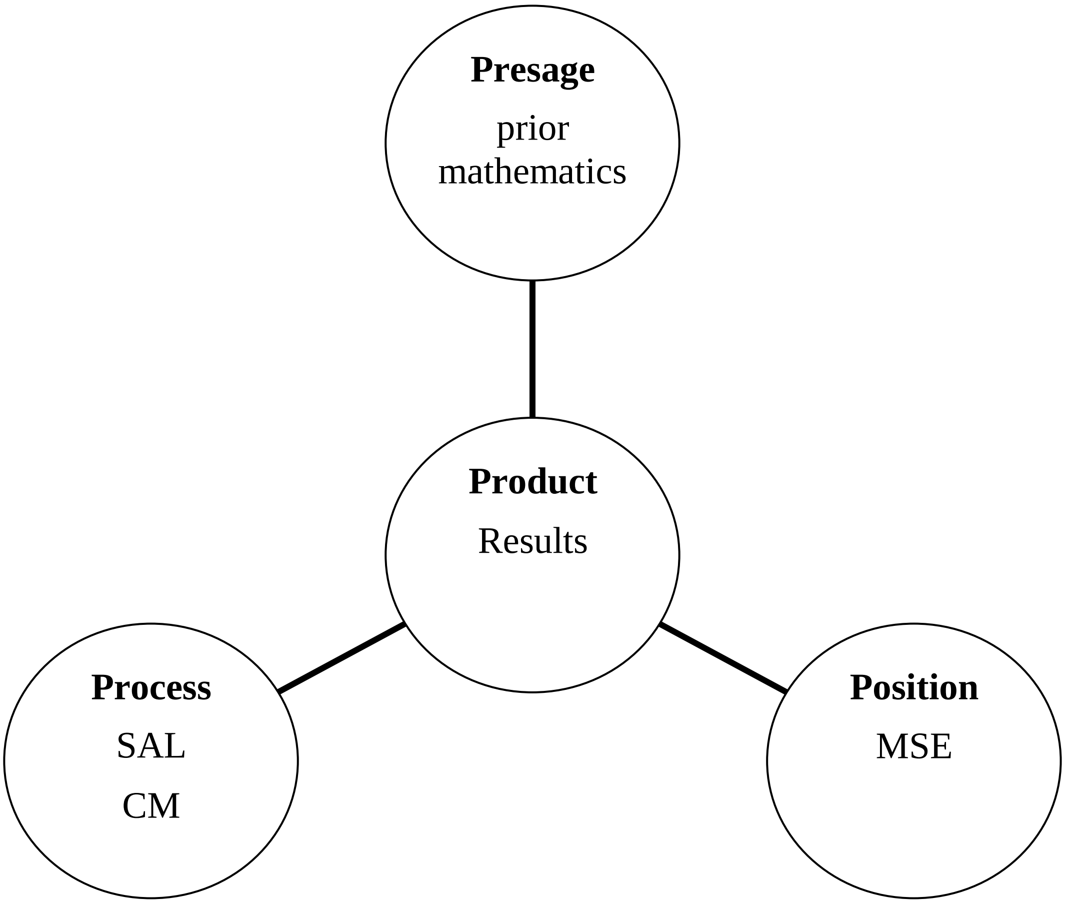
Our univariate variance of analyses showed significant effects (Sign < 0.05) of current mathematics course and mathematics background (Mardia, 1980). The univariate general linear model 2-way Anova table shows the F value (3.452) and low significance value (0.014). The estimated marginal means and significant (F=4.002, p=0.007) and pairwise comparisons showed that those, who were studying Engineering Mathematics 2 (84 marks) and Foundation Mathematics (74 marks), had completed mathematics at NCEA level 1 (65 marks), Cambridge and IB (65 marks) and overseas students (68 marks), were more likely to score higher examination marks than those with NCEA Level 2 (47 marks) and Level 3 (50 marks). Contrary to our expectations, age variations (18-25 years old and over 25 years old) were not significant factors of examination results (age variations (F=2.632, p=.111) and gender types (F=.265, p=.609)).

DISCUSSION

Our data reflected perceptions of students' ability in learning mathematics as their examination results were positively associated with mathematics self-efficacy, modelling conception of mathematics and deep approaches. Despite relatively weak/moderate correlations, these data were validated by our previous pilot study and high average scores in this study. Moreover, our data matched previous research in regards to deep learning and positive outcomes (Marton & Säljö, 1976); deep learning and cohesive conception of mathematics (Crawford et al., 1994; Wood et al., 2012) and self-efficacy (Carmichael & Taylor, 2005; Jaafar & Ayub, 2010); deep approaches to learning and self-efficacy (Fenollar et al., 2007; Liem et al., 2008); and self-efficacy and organised approaches to learning (Phan, 2011). Therefore, these relationships confirmed our hypotheses that deep/organised approaches learning are linked to mathematical self-efficacy and that positive learning outcomes are associated with deep learning, mathematics self-efficacy and cohesive conceptions.

Our study presented a new result that student beliefs in using selection processes was the best predictor of examination results. According to Bandura (1997), individuals develop self-efficacy through the optimal use of resources to accomplish certain tasks. By having strong beliefs about using selection strategies (e.g., time management, effort), individuals can adapt to the teaching and learning environment and are equipped with the necessary means for task completion. Therefore, given aforementioned evidence, we propose an additional 'Position' component in the model of student learning. Students' positions about their abilities differ as they make personal judgements of their abilities to do mathematical problem-solving and to use motivation, cognitive and selection processes in learning mathematics (Bandura, 1997).. Their positions of mathematical self-efficacy could influence results. This proposed model of learning implies that students are more likely to succeed if tertiary institutions provide mathematical tools and activities for raising mathematical self-efficacy, high expectations of their learning outcomes, deep learning and a holistic view of mathematics as a useful subject.

Figure 1: A new model of student learning



Other determinants of success are prior mathematics background and mathematics courses. Inconsistent with other studies (Engler, 2010; Henderson & Broadbridge, 2009) and contrary to our expectations, students with low mathematics background (equivalent to Year 11 Mathematics) scored better than learners with NCEA level 2 and 3 mathematics qualifications (equivalent to Years 12 and 13). As part of the university course requirements, mathematical abilities were dependent on completion of Year 12/13 mathematics courses. The data suggested that foundation students and second-year mathematics students could perform well in the examinations. This was not surprising since the goal of bridging and first-year education was to equip them with mathematical skills and knowledge. Interestingly, our findings suggested that despite higher (Years 12 and 13) NCEA secondary qualifications attained by first-year tertiary students, they appeared to be less prepared for tertiary mathematics.

FUTURE RESEARCH

In order to develop a robust conceptual framework of student learning, learning is best assessed in situations when tasks are provided and teaching factors are considered. Semi-structured interviews could be conducted to further probe the individuals about the origins of learning as mathematicians and self-efficacy. This will enable researchers to study developmental aspects of self-efficacy, learning approaches, conceptions of learning and teaching components.

CONCLUSION

Research in student learning can help researchers to model learning and teaching. Our study reported that student beliefs in managing resources was the most appropriate predictor of successful examination performances. Other factors were secondary mathematics qualification (equivalent to Year 11 mathematics), mathematical skills and knowledge in first-year tertiary mathematics, the use of deep learning, and view of mathematics as an abstract and modelling subject. By implication, a new presage-process-position-product model of student learning incorporates self-efficacy as ‘student positions’. Future research should be used to validate the proposed model of learning.

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