

# An Analysis of Bitcoin Spot and Futures Markets

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## Abstract

Bitcoin is a decentralised virtual currency that uses cryptography to remove the need for trust between agents and intermediaries. Though it is deemed a currency, the cryptographic nature of bitcoin gives it characteristics similar to commodities, for example, gold. This thesis examines the relationship between spot and futures prices in the bitcoin market. It also applies the well-known Gibson-Schwartz model to the pricing of bitcoin futures contracts. A sample from March 2013 to February 2015 is considered, including two separate periods of substantial increase in the bitcoin spot market. Findings suggest that the futures market was significantly in contango, with convenience yields often below -100% p.a. and significant arbitrage profits readily available. The convenience yield is found to be mean-reverting and decreasing in magnitude since 2013, with the market inefficiently pricing convenience yield risk. A negative relationship between the convenience yield and spot price volatility is observed, drawing comparisons with the electricity market. Overall, the Gibson-Schwartz model is found to provide a good fit to nearest-term futures contracts, while it fails to appropriately price bitcoin futures with longer maturities.

*Keywords:* Bitcoin, commodity pricing model, futures, convenience yield

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## 1. Introduction

Bitcoin<sup>1</sup> is a decentralised and peer-to-peer version of electronic cash which allows transactions to be non-reversible via cryptographic treatment of transaction information, so that the need for trust between agents and intermediaries, such as financial institutions, is eliminated. Bitcoin is an opensource and completely digital currency that uses cryptography as the method of validating transactions and controlling the creation of new money, so has thus been labelled a cryptocurrency. Since its inception in January 2009, bitcoin has become more widely used and has seen rapid growth, especially since 2012, as indicated by the price of bitcoin and number of transactions using bitcoins (see Figure 1), as well as the size of companies now adopting or experimenting with its use, such as Dell, Expedia, Overstock, Microsoft and Ebay. Its popularity has made it currently the largest digital currency by both market capitalisation and the number of daily transactions, with Bloomberg now reporting the spot price due to public interest (see Bloomberg (30 Apr 2014)).

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<sup>1</sup>See <https://bitcoin.org/>

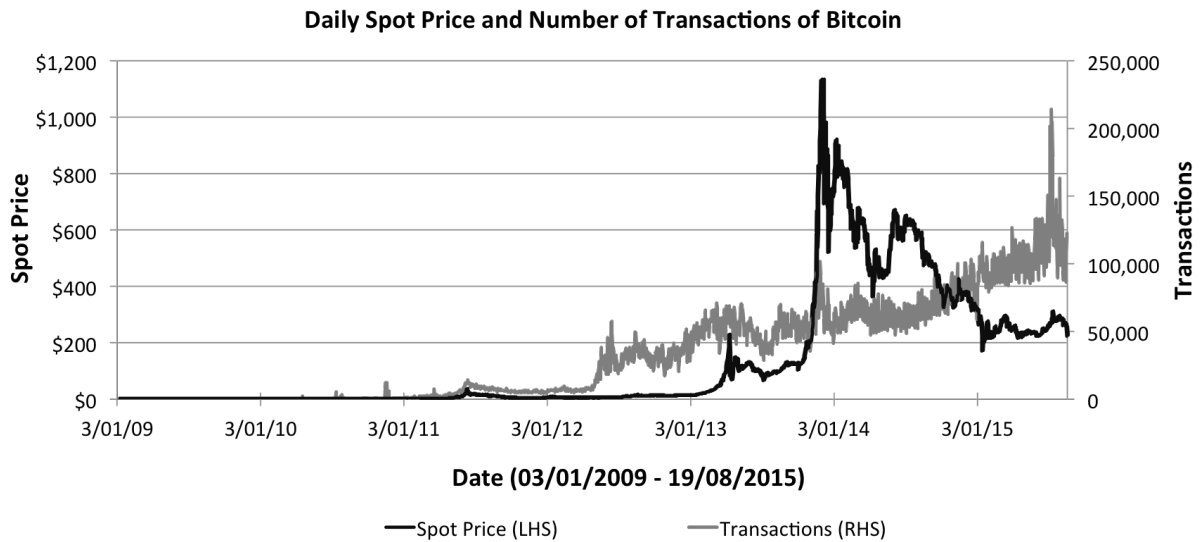


Figure 1: The spot price of bitcoin in USD and number of bitcoin transactions (per day) since inception (Data from <https://blockchain.info/charts>).

This increased popularity of bitcoin, which could be called a new asset class, is attracting investors: Trading strategies are being created (for example, see Shah and Zhang (2014)); hedge funds are considering the use of bitcoin within their strategies (for example, Bitcoin Fund, Bitcoin Investment Trust and the Winklevoss Bitcoin Trust<sup>2</sup>); and investment is occurring to create bitcoin currency exchanges, mining and hardware companies<sup>3</sup> and even *insurers* (see Bloomberg (10 Apr 2013), IB Times (10 Mar 2014) and Business Insider (14 Aug 2014)). But despite alluring many investors, the dynamics of bitcoin are still relatively unknown. Furthermore, as derivatives markets start to become established,<sup>4</sup> the need to better understand these dynamics and their interactions with derivatives markets increases. Orderbook, a bitcoin futures market established in 2012, shows

<sup>2</sup>See <http://bitcoinfund.eu/index> and <http://grayscale.co/bitcoin-investment-trust/>

<sup>3</sup>For example KNC Miner, Butterfly Labs and Avalon

<sup>4</sup>For example, OrderBook, OKCoin and BITVC

a particularly unusual occurrence. Figure 2 shows the bitcoin spot price and futures quotes for contracts ending in March and September 2014. It is clear that futures prices are significantly above spot prices for nearly the entire contract period. This, in theory, leads to an investment opportunity: buy and hold bitcoins, short futures contracts and sell the bitcoins at the maturity of the contracts. This strategy of going long bitcoin and short bitcoin futures regularly shows annualised returns exceeding 100% p.a., well above borrowing and transaction costs incurred. A natural question to ask is: why do these contracts deviate so severely from the cost-of-carry relationship, that is, the arbitrage relation between the futures price, spot price and the cost of carrying the asset?

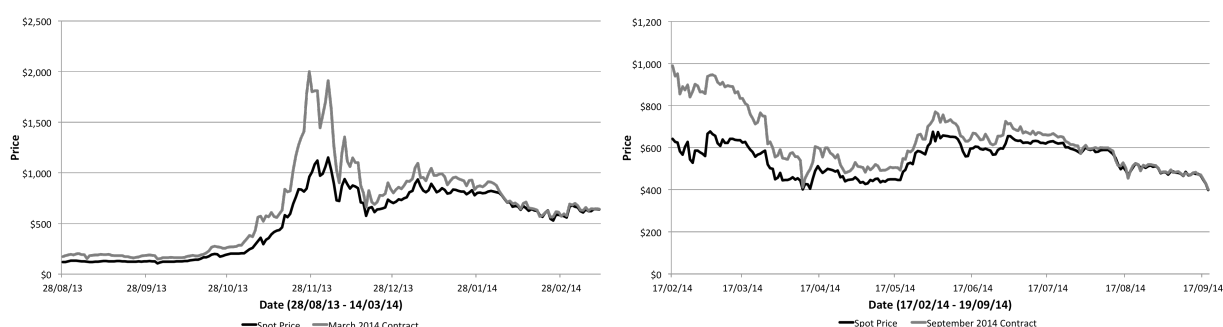


Figure 2: Bitcoin futures quotes and spot prices for futures contracts expiring March 2014 and September 2014 (Data from <https://blockchain.info/charts> and <https://orderbook.net/>).

For other commodities, analysis of spot and futures markets and the convenience yield has been widespread. Commodities including oil, various metals and agricultural products have been shown to have had spot prices above futures prices, a situation called backwardation, see, e.g., Bodie and Rosansky (1980), Chang (1985), Fama and French (1987) and Pindyck (2001). On the other hand various electricity and CO<sub>2</sub> markets have been shown to have had spot prices below

futures prices, a situation called contango, see, e.g. Longstaff and Wang (2004) and Trück et al. (2014). In addition, Pindyck (2001) found that backwardation in oil futures markets increases during periods of high volatility, whereas Bessembinder and Lemmon (2002) found the opposite to be true for electricity futures markets during periods of high demand volatility. Other studies have included investigating the relationship between inventories and the convenience yield, see, e.g., Fama and French (1987), and Gibson and Schwartz (1990) present a two-factor model using the spot price and the convenience yield to price commodity contingent claims, applying the model to oil futures.

Motivated by this discussion, the aim of this paper is to analyse the bitcoin spot and futures markets by applying the well-known Gibson-Schwartz stochastic commodity pricing model, and to use this to value arbitrary maturity bitcoin futures, as well as explain deviations from typical results for other commodities by relating the obtained results to market structures and operational and counterparty-default risks<sup>5</sup> in the bitcoin spot and futures markets. To the best knowledge of the author, this is the first study in the area of bitcoin futures, and is of interest for several reasons. Firstly, it will provide a view of the dynamics occurring in the bitcoin spot and futures markets, which are currently unknown, but required for investment decisions. Secondly, derivatives are of key importance in any market, and without a pricing mechanism for derivatives, the risk undertaken in investment decisions is unknown. Given bitcoin derivatives markets are in their infancy, it is reasonable to start with futures contracts (of which data is available, unlike other derivatives).

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<sup>5</sup>For example, OrderBook does not act as the central counterparty for contracts. This, strictly speaking, places OrderBook contracts somewhere between forwards and futures, making them either a standardised and tradeable forward with margin requirements or a future with non-zero counterparty-default risk (though margin requirements ensure profitable positions still provide a non-zero profit in the event of a counterparty defaulting. See <https://orderbook.net/about/rules> Section 3.3.5). Operational risks apply to exchanges and bitcoin e-wallets, from malware, hacking, fraud and errors

Thirdly, it will assist in determining the levels of risk in the bitcoin spot and futures markets. Finally, it will add to the discussion confirming or opposing the view of bitcoin as its own asset class, part of another asset class or neither.

In approaching this last point, bitcoin is a cryptocurrency, but its cryptographic nature gives it characteristics not unlike commodities. Many users genuinely use it in preference to domestic currencies, but it is significantly used as an object for investment and speculation, and experiences major price fluctuations with no central bank able to appreciate or depreciate it. But the most obvious reason for comparison to commodities is because of the mining process. The number of bitcoins is limited (21 million fungible bitcoins) and distributed to miners in compensation for work completed validating transactions, resembling a typical commodity, and indeed this construction intentionally resembles a commodity like gold, see, e.g., Nakamoto (2008). One point of difference between typical commodities and bitcoin is that bitcoin is inexhaustible, but this is not a unique feature, since electricity is also an inexhaustible commodity. Therefore, it will be assumed that bitcoin can be treated as a commodity. Note that this does not contradict any notion that bitcoin may be a currency, since commodities can be currencies also, such as gold, but provides the benefit that theory developed for commodity markets can now be used to investigate the dynamic behavior of bitcoin spot and futures contracts.

On a more fundamental level, bitcoin provides an interesting market. It gives a view of a low-regulation environment where agents are allowed to determine prices relatively unhindered, whether it be chaotically, harmoniously or otherwise. This is because bitcoin is decentralised (that is, has no central bank) and bitcoin and bitcoin derivative *exchanges* are unregulated. It also has small barriers to entry, the factors of production (a computer, electricity and internet connection)

are easily obtainable, taxes are avoidable, and market information is free and instantly available. These are key assumptions of financial markets. On the other hand, there are various caveats of this immature market that weigh against these benefits, such as difficulties shorting the bitcoin, a lack of comprehensive derivatives markets, heavy speculation and a small market capitalisation, where large investors may gain market power. In addition, though an individual government can not impose regulation on bitcoin directly, it can impose other restrictions, such as banning its use as an accepted payment method.

Despite these caveats, bitcoin provides an opportunity for new perspectives and insights across a broad range of social sciences, as well as finance, but first a greater understanding of bitcoin markets is required. However, previous studies on bitcoin have been focussed on computer science and cryptography, with limited study, and therefore literature, of the markets themselves. Having said this, it has been considered before: Shah and Zhang (2014) attempt to model the price of bitcoin using Bayesian methods, creating a trading strategy; ECB (2012) and ECB (2015) provide investigatory pieces on bitcoin and other virtual currencies and their effect on the global economy; Barber et al. (2012) provides an overview of bitcoin with some mild economic considerations; Ron and Shamir (2013) look at the complete bitcoin transaction history to investigate the behaviour of users; and Cheah and Fry (2015) apply a one-factor stochastic pricing model with jumps to look at the spot price of bitcoin and the affect of speculation on the bitcoin spot price. The work of Cheah and Fry (2015) use a similar approach to model the bitcoin price, but do not consider the convenience yield or bitcoin derivatives, and are focussed on speculation and the fundamental value of bitcoin rather than commodity contingent claims and investment decisions.

The remainder of this paper is set out as follows. In the second section an introduction to

bitcoin, how it works and its risks is provided. The third section discusses commodity pricing models, the convenience yield, and model parameter estimation, specifically looking at the Gibson-Schwartz model. It also provides derivations of partial differential equations of futures prices that have closed form solutions, that is, the futures price expressed in terms of the model parameters. Section four introduces the data used and provides a market history for each market. Section five contains the empirical results: the convenience yield, examples of arbitrage, parameter estimations and the pricing of futures contracts. Finally, section six concludes.

## 2. An Introduction to Bitcoin

This section will discuss bitcoin, how it operates, its risks and market history. Bitcoin was the first cryptocurrency created, based on a paper by Satoshi Nakamoto<sup>6</sup> in 2008, see Nakamoto (2008), but many other cryptocurrencies have since emerged with varying levels of success, such as litecoin and peercoin. Digital currencies have been of academic interest for several decades; furthermore, digital currencies have existed before bitcoin, such as E-gold, Linden Dollar and Ven,<sup>7</sup> but most were only able to last a short time before collapsing or being shut down, see, e.g. Miers et al. (2013), Reid and Harrigan (2013) and ECB (2012). A major point of difference between previous digital currencies and cryptocurrencies like bitcoin is the existence of a trusted third party to validate transactions and stop *double-spending*, where multiple transactions occur using the same electronic cash. Digital currencies before bitcoin needed a trusted third party to do this,

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<sup>6</sup>Satoshi Nakamoto's identity, curiously, can not be confirmed, and there are many who believe the name is a pseudonym

<sup>7</sup>E-gold was a gold denominated virtual currency that ran from 1996-2009 (see <http://blog.e-gold.com/>). The Linden Dollar is the virtual currency of the virtual world *Second Life*, created in 2003 (see <http://secondlife.com/>). Ven was established in 2007 as part of social networking site *hubculture.com* and has a value based on a basket of currencies, commodities and carbon futures (see <http://ven.vc/>)

but bitcoin was designed to be decentralised (that is, have no central bank) and allow a peer-to-peer network to do this validation cryptographically via a proof-of-work system which can be fully trusted, easily checked and is irreversible. In this section a brief overview of how bitcoin operates will be provided, but for a more detailed view of bitcoin and the literature from a computer science perspective the reader is referred to Nakamoto (2008), Karame et al. (2012), Barber et al. (2012), Babaioff et al. (2012), ECB (2012), Reid and Harrigan (2013), Christin (2013), Ron and Shamir (2013), Miers et al. (2013) and ECB (2015).

The concept underlying transactions of bitcoins is that the current owner of bitcoins can transfer them to the next owner by digitally creating a history of the previous transaction of those bitcoins along with the public key of the next owner. Each transaction is logged to a public ledger of all transactions via the *block-chain*, ensuring that the transfer of bitcoins can be followed along a chain of transactions. As transactions occur they get grouped into blocks that must get validated by nodes in the peer-to-peer network that run software designed to solve a cryptographic problem posed as part of the proof-of-work system, by which is meant a system where the answer to the problem is difficult (time-consuming) to find, but easily verified as correct, see Nakamoto (2008), Karame et al. (2012) or Miers et al. (2013). Each time a node solves the problem, the block is added to the block-chain, the single chain of all validated blocks and source of truth: this is bitcoin's solution to the double-spending problem. This process is illustrated in Figure 3. Each transaction in a block can not exist in any other block and each time a new block is created it is timestamped via a hash function.<sup>8</sup> The node is then ready to be validated by nodes in the network. Once solved by a node, the block is published and the next block will contain the hash from

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<sup>8</sup>A hash function is a function sending an input of arbitrary length to an output of set length

the most recently published block, allowing a single chain of blocks to eventuate which is easily traceable. Each validated block then contains a list of transactions that are regarded as finalised. By including the hash function output, which acts as a timestamp for each block, only the first transaction of any double-spending attempt will be accepted.<sup>9</sup> Karame et al. (2012) explain the validation process succinctly as follows:

1. New transactions are broadcasted by peers in the network.
2. When a new transaction is received by a peer, it checks whether the transaction is correctly formed, and whether the bitcoins have been previously spent in a block in the block chain. If the transaction is correct, it is stored locally in the memory pool of peers.
3. Peers work on constructing a block. If they find a [solution to the proof-of-work], they include all the transactions that appear in their memory pool within the newly-formed block. Peers then broadcast the block in the network.
4. When peers receive a new block, they verify that the block hash is valid and that every transaction included within the block has not been previously spent. If the block verification is successful, peers continue working towards constructing a new block using the hash of the last accepted block in the “previous block” field.

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<sup>9</sup>Karame et al. (2012), however, suggests that *fast transactions*, where goods or services are received quickly after purchase, may still suffer from double-spending attacks

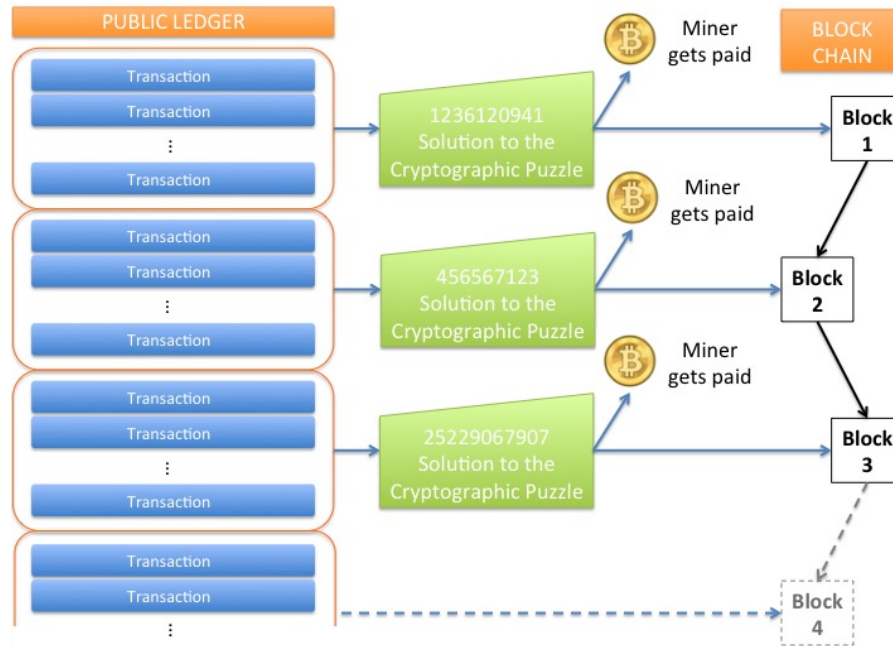


Figure 3: A depiction of the bitcoin mining process.

The process of validating transactions by solving the cryptographic problems posed is called mining, and users of the software are called miners. It is this process that makes bitcoin decentralised, since the network is peer-to-peer and anybody with the hardware and software (which can be as simple as a home computer) can become a node. As an incentive to miners, they are rewarded with bitcoins each time they successfully add a new block to the block chain, which also acts as the process of distributing new currency into the market. But this process is not left to chance, with bitcoin being designed to change the difficulty of the cryptographic problem posed so that the expected time for the network to solve the problem is 10 minutes.<sup>10</sup> Furthermore, the

<sup>10</sup>The difficulty is changed every 2016 blocks (approximately 2 weeks, since  $\frac{2016 \cdot 10}{60 \cdot 24 \cdot 7} = 2$ ) so that, under the new difficulty, the previous 2016 blocks would take  $2016 \cdot 10 = 20160$  minutes to solve if the network of the miners were to be unchanged. See <https://bitcoin.org/en/developer-guide#proof-of-work>

size of the reward for miners halves every 210,000 blocks, so that a limit of 21 million bitcoins<sup>11</sup> will be in circulation approximately by the year 2140.<sup>12</sup> It is expected that miners will charge transaction fees as compensation for computing costs once this limit is reached. This method also has the benefit of providing a predictable money supply, which is usually done via a central bank for domestic currencies. In February 2015, the number of bitcoins in circulation was larger than 13 million with a market capitalisation near 3 billion USD, after an all time high near 14 billion USD market capitalisation one year earlier during the largest of bitcoins numerous price spikes.<sup>13</sup>

An interesting observation in the bitcoin market are the large spot price increases (akin to a bubble) followed by lesser in magnitude, but still large, decreases that have occurred on numerous occasions in the past, including June 2011, August 2012, April 2013 and December 2013 (see Figure 1). In particular, the March 2013 international bailout of Cyprus drove the price of bitcoin up 500% to \$238 from one month earlier, only to decrease substantially in April 2013 after concerns of an attack on bitcoin exchange Mt. Gox by hacker, see Fox (22 Mar 2013) and Bloomberg (11 Apr 2013). Similarly, in October/November 2013 Baidu's (China's largest internet search engine) acceptance of bitcoin coupled with positive news from the US Federal Reserve saw the price rise to \$1,151, 900% higher than at the beginning of October 2013, only to see another substantial fall from December 2014 to February 2015 as China imposed restrictions and Mt. Gox was hacked and consequently filed for bankruptcy, see Bloomberg (7 Dec 2013), New York Times (22 Nov 2013), Bloomberg (17 Dec 2013) and Reuters (28 Feb 2014). Notably, during these astronomical price increases, futures prices increased at the same or greater pace than spot prices, showing

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<sup>11</sup>Based on 50 bitcoins as the starting reward,  $50 \cdot 210000 \cdot (1 + \frac{1}{2} + \frac{1}{4} + \dots) = 21000000$

<sup>12</sup>See Nakamoto (2008) and <https://bitcoin.org/en/developer-reference#serialized-blocks>

<sup>13</sup>See <https://blockchain.info/charts>

over-optimism about long term expectations of bitcoin spot prices. This is illustrated, for example, in Figure 2 for the March 2014 futures contract. This agrees with the findings of Cheah and Fry (2015), who find a significant speculative component in bitcoin spot prices.

To finish this overview, some of the risks inherent in bitcoin will be mentioned. Some of these risks impact the long term viability of bitcoin and some are of immediate concern; indeed, some have already occurred. The main exogenous threat posed to bitcoin, and potentially the largest overall, is due to malware, hackers and fraud. There have been several attacks by hackers on exchanges and e-wallets. For example, Mt. Gox, the worlds largest bitcoin exchange at the time of its closure, was forced to close due to hackers (see Bloomberg (25 Feb 2014)). Further, the lack of regulation locally and globally on bitcoin and companies dealing in bitcoin provides opportunities for fraud to occur, as well as other illegal practises such as black-market purchases. For example, see Christin (2013) for details on *Silk Road* and FINRA (2014) for some recent fraudulent activities in the US.

One fundamental risk inbuilt in bitcoin is the maximum of 21 million bitcoins, which is distributed to miners as an incentive to mine. Modern economic theory leads central banks of major economies to aim for a positive level of inflation, so a limit on the total number of bitcoins means the price of bitcoin will continue to appreciate indefinitely, if it were to become widely accepted. This is referred to as the *deflationary spiral* and increases the incentive to hold bitcoins to realise these gains instead of using them for transactions (Barber et al., 2012). This *hoarding* of bitcoins would also apply downward pressure on the number of transactions and thus downward pressure on fees generated for miners, decreasing the incentive of miners and increasing the potential for so-called history-revision attacks. Furthermore, many commodities serve other purposes (for ex-

ample, gold and oil in manufacturing), which bitcoin does not. This means inventories of bitcoin currently have little use other than for capital gains, especially since its use as a currency is still in its infancy. In addition, the increased price of bitcoin would increase the incentive for fraudulent behaviour, hacking and malware.

Another fundamental issue regards lost and unrecoverable bitcoins through lost public keys, referred to as *zombie coins* (Barber et al., 2012), of which there are informally estimated to be tens of thousands. These can occur, for example, through forgotten bitcoin e-wallets, deceased accounts or errors caused by exchanges or companies managing e-wallets. For example, bitomat, the third largest bitcoin exchange at the time of its closure, was closed due to a manual error which lost client details (see New York Observer (19 Aug 2011)). In addition to these are risks of: history-revision attacks attempting to redefine the block-chain, and thus the transaction history of bitcoin, which would question the trust given to the network of miners and thus the feasibility of bitcoin (Barber et al., 2012); and the 10 minute confirmation time for blocks to be solved, which potentially allows double-spending for quick transactions (see, in particular, Karame et al. (2012)). Further discussion on the risks bitcoin faces can be found in the references listed above and at the start of this section; in particular, see, Babaioff et al. (2012), ECB (2012) and Reid and Harrigan (2013).

### **3. Commodity Pricing Models**

#### *3.1. Commodity Pricing Models and the Gibson-Schwartz Model*

The commodity pricing models discussed in this paper were founded by the ideas associated with *real options*, that is, the application of derivatives pricing (or contingent claims analysis)

by, e.g. Black and Scholes (1973), Merton (1973), Black (1976), Cox et al. (1981) and Fama and French (1987) to real investment problems, see also Lund and Øksendal (1991), Dixit and Pindyck (1994), Schwartz (1997) and Hull (2012). For example, to have the option to build a mine on an undeveloped oil field at any point in the future can be considered as similar to an infinite maturity American option. The owner has the choice to receive (build) an oil mine (the underlying asset) at the cost of its development (settlement price) at any date in the future. Another example is having the option to start-up a bitcoin mining operation or any claim contingent on the price of bitcoin.

The theory of pricing such options and that of capital budgeting is sometimes done using a discounted cash flow approach, estimating cash flows and discount factors (often determined via a capital asset pricing approach at each period) to apply to these cash flows (Dean, 1951; Bierman and Smidt, 1960; Hull, 2012). Although work by Bogue and Roll (1974), Brennan (1973) and Constantinides (1978) tries to improve these methods, the literature suggests that this pricing method was somewhat unsatisfactory considering the uncertainty that exists in future cash flows and appropriate discount rates. This lead to more advanced approaches to be developed. A natural place to start is, as suggested by the previous example, estimating future cash flows that are dependent on commodity prices. This lead to treating the spot price as stochastic and, furthermore, applying an approach similar to that of Black and Scholes (1973) and Merton (1973), where expected future cash flows are equated to a portfolio of securities, such as derivatives, that can be priced, and, therefore, allowing the commodity contingent claim to be priced. Such examples of this are provided by Ross (1978), who examines, in a general fashion, using this approach to value streams generated by risky assets, and Paddock et al. (1988) who considers specifically the valuation of offshore petroleum leases.

Of particular interest here, capital budgeting decisions are also considered when using this method. Dothan and Williams (1980) examine capital budgeting decisions with stochastic interest rates, as well as term-structures. Pindyck (1980) looks at a fairly general case where there is a stochastic level of commodity reserves, including extraction and exploration costs. Brennan and Schwartz (1985) then investigate capital budgeting decisions where the so-called *convenience yield* of a commodity, which acts as a kind of dividend to the holder of a commodity over the holder of a futures contract,<sup>14</sup> is proportional to the spot price. This is somewhat related to the work of Pindyck (1980), because it is seen that the level of reserves and the convenience yield are intimately linked, as indicated by Fama and French (1987). Brennan (1991) then generalised this further by considering multiple types of functional forms for the convenience yield, including a mean-reverting stochastic convenience yield: an assumption postulated to be true for oil by Gibson and Schwartz (1991).

This hypothesis allowed the development of what is now known as the Gibson-Schwartz model (Gibson and Schwartz, 1990; Schwartz, 1997), which provides a framework for explaining the movements between the convenience yield and the spot price of a commodity. The Gibson-Schwartz model is a two-factor model where the spot price follows a standard geometric Brownian motion and the convenience yield follows a mean-reverting Ornstein-Uhlenbeck process. The model can be described by the following joint stochastic process, where  $s$  is the spot price and  $\delta$

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<sup>14</sup>Because of the convenience of holding the commodity over a futures contract. For example, holding the commodity gives the option for use in regular production processes in the cases of unexpected demand or commodity shortages, unlike a futures contract. This has also been called the *rate-of-return shortfall* (McDonald and Siegel, 1986). The convenience yield is explained in more detail in Section 3.2.

is the instantaneous convenience yield,

$$\begin{aligned}
ds &= \mu s dt + \sigma_s dz_s, \\
d\delta &= \kappa(\alpha - \delta)dt + \sigma_\delta dz_\delta, \\
dz_s dz_\delta &= \rho dt.
\end{aligned} \tag{1}$$

The spot price  $s$  has a drift (or long-term spot price return) of  $\mu s$  and volatility of  $\sigma_s s$  (also called the diffusion). Equivalently,  $\mu$  and  $\sigma_s$  can be considered as the trend and volatility of the rate of return of  $s$ . The convenience yield  $\delta$  is mean-reverting around the long-term average convenience yield  $\alpha$ , with mean-reverting coefficient  $\kappa$  and volatility  $\sigma_\delta$ . Each  $z_{[\cdot]}$  here denotes a standard correlated Brownian motion with the correlation described in the final equation of the model via the correlation coefficient  $\rho$ . To express model 1 under the risk-neutral measure a market price of convenience yield risk,  $\lambda$ , must be included, since convenience yield risk cannot be hedged. Furthermore, defining  $X = \ln s$  and using Ito's Lemma on the spot price process results in the joint process under the risk-neutral measure:

$$\begin{aligned}
dX &= (r - \delta - \sigma_s^2/2) dt + \sigma_s dz_s^*, \\
d\delta &= (\kappa(\alpha - \delta) - \lambda) dt + \sigma_\delta dz_\delta^*, \\
dz_s^* dz_\delta^* &= \rho dt.
\end{aligned} \tag{2}$$

Note that each of the variables in model 1 (or equivalently, model 2) could be dependent on any of the other variables within the model and outside the model. However, within this paper, they will be assumed constant, as is often done in practise. This model can then be used to price claims

contingent on the price of the commodity, such as futures contracts. This will be discussed and the equations will be derived in Section 3.3.

In addition to this two-factor model, many extensions and modifications have emerged in the literature. For example, Schwartz (1997) adds the interest rate as a third stochastic factor (following the work done by Vasicek (1977)); Casassus and Collin-Dufresne (2005) further extend this by considering the dependence of the convenience yield on interest rates and the spot price, including time varying risk-premia, and considering jumps in the stochastic processes; Cortazar and Schwartz (2003) add the long term spot price return  $\mu$  as a third stochastic factor; and Schwartz and Smith (2000) create an equivalent two-factor model that may include a mean-reverting (with zero mean) short-term price deviation and a long-term equilibrium price that follows a geometric Brownian motion. These models primarily use oil as the commodity of choice, as it tends to exhibit the postulated behaviour described in the models, but also because data on oil spot and futures contracts is more comprehensive and of wide interest. Considering the accuracy and appropriateness of each model, using oil, copper and gold data, Schwartz (1997) compares model 1 to the same model with interest rates added as a third stochastic factor, suggesting that the two- and three-factor models provide similar results.<sup>15</sup>

For our study of the bitcoin futures market, only a very limited number of futures contracts is available. Furthermore, as a result of the very low US interest rates during the sample period, changes in interest rates are also very small and such have little impact on the bitcoin convenience yield (because the convenience yield is largely negative). Thus, a model with greater simplicity that focuses on key variables may be ideal to investigate the behaviour in this market, and the three-

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<sup>15</sup> Although the three-factor model may provide better results valuing long-term futures contracts.

factor models of Schwartz (1997) and Casassus and Collin-Dufresne (2005) may not provide any benefit over a simpler two-factor model. However, the extension proposed by Casassus and Collin-Dufresne (2005) that makes  $\delta$  dependent on  $s$  may be worth consideration in future work, but will not be attempted here. As explained above, small changes in  $r$  may have little significance on  $\delta$ , but  $s$  may be significant, since larger negative convenience yields are seen during the bitcoin price spikes in November and December 2013 (see Figure 4).

### 3.2. *The Convenience Yield*

Given the use of the convenience yield in the models discussed in Section 3.1, this section will provide a brief discussion of the convenience yield. The convenience yield is usually derived via a no-arbitrage or cost-of-carry argument which is based on a hedged strategy of a short position in a futures contract and holding the underlying asset of the futures contract until maturity, which is funded by a short position in the money market, see, e.g., Geman (2005), Hull (2012) and Pindyck (2001). Let  $s_t$  be the spot price of a commodity at time  $t$  and  $F_{t,T}$  be the futures price of that commodity at time  $t$  with maturity at time  $T$ . The cost-of-carry model gives the following no-arbitrage situation

$$F_{t,T} = s_t e^{(r_{T-t}+u)(T-t)}, \quad (3)$$

where  $r_{T-t}$  denotes the risk free rate at time  $t$  referring to a time period  $T - t$  and  $u$  is the rate of cost of storing one unit of the commodity. In most commodity markets, however, this equality does not hold, due in part to the convenience<sup>16</sup> of holding the underlying commodity and the inability of investors and speculators to short the commodity. Thus, equation 3 is modified by including a

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<sup>16</sup>See footnote 14

gross convenience yield  $c_{T-t}$ ,

$$F_{t,T} = s_t e^{(r_{T-t} + u - c_{T-t})(T-t)}, \quad (4)$$

where  $c_{T-t}$  denotes the gross convenience yield seen at time  $t$  referring to a time period  $T - t$ , that is, for a futures contract with maturity at time  $T$ . The *net* convenience yield is the gross convenience yield after storage costs,  $c_{T-t} - u$ . In this case, bitcoin has no storage costs, as online companies provide free e-wallet services<sup>17</sup>, so  $u = 0$  and the gross and net convenience yields are equal. Solving for  $c_{T-t}$  gets the following equation for the convenience yield:

$$c_{(T-t)} = r_{T-t} - \frac{\ln(F_{t,T}) - \ln(s_t)}{T - t}. \quad (5)$$

It is explained by Pindyck (2001) that the convenience yield obtained by holding a commodity can be viewed in a similar way to the dividend yield obtained from holding a company's stock. Since the price of a stock can be valued as the present value of the expected future flow of returns from dividends, the price of a commodity can be considered as the present value of the expected future flow of returns from convenience yields.

The theory of convenience yields is usually derived for forward contracts, because a typical forward contract has no obligations until the delivery date. On the other hand, futures contracts are marked-to-market and require margins to be posted and regularly updated, which can influence costs incurred or profits gained, and thus the price of the contracts (Black, 1976; Fama and French, 1987). There are many studies on the differences between futures and forward contracts, such as Cox et al. (1981) and French (1983), which theoretically and empirically show that they

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<sup>17</sup>For example, see <https://bitcoin.org/en/choose-your-wallet>

are not identical, and Cornell and Reinganum (1981), Hodrick and Srivastava (1987), Baum and Barkoulas (1996) and Pindyck (2001) which show that this difference is near or equal to zero. In particular, with a constant interest rate forward prices are equal to futures prices. In addition to this, it is usually seen that futures and forward prices are approximately equal and many studies, including de Roon et al. (1998), Trück and Weron (2015) and many others assume that the convenience yield theory for forwards also holds for futures because of these reasons. This assumption will also be made here. The benefit of this is that futures contract data can be used, which is available for bitcoin, whereas data on bitcoin forward contracts is either non-existent or rare and difficult to obtain.

If a futures curve is a decreasing function of time-to-maturity then the futures market is said to be in (*normal*) *backwardation* (Geman, 2005; Hull, 2012). From equation 4 it can be seen that this occurs when  $r_{T-t} + u - c_{T-t} = r_{T-t} - c_{T-t} < 0$ , or equivalently when

$$r_{T-t} < c_{T-t} = r_{T-t} - \frac{\ln(F_{t,T}) - \ln(s_t)}{T - t} \implies F_{t,T} < s_t,$$

that is, when the futures price is below the spot price or the convenience yield is larger than the risk-free rate. If a futures curve is an increasing function of time-to-maturity then the futures market is said to be in *contango*. This occurs when  $r_{T-t} - c_{T-t} > 0$ , or equivalently when  $F_{t,T} > s_t$ , that is, when the futures price is above the spot price or when the convenience yield is below the risk-free rate.

The difference between spot and futures prices of a commodity can be considered via the example of hedging: when the market is in backwardation, commodity *producers* are hedging against

falling prices; and when the market is in contango, commodity *consumers* are hedging against rising prices. Backwardation was first suggested in Keynes (1930) with the idea that backwardation assumes that hedgers (for example, producers who fear a price drop) tend to hold short positions as insurance against their cash position and consequently must pay speculators a premium to hold long positions in order to offset their risk, that is, observed futures prices  $F_{t,T}$  with delivery at time  $T$  are below the expected spot price  $E_t(s_T)$ . Thus, backwardation is equivalent to a positive *risk premium*<sup>18</sup>, since the risk is transferred to the long position in the futures contract. Contango was first mentioned in Hicks (1946) and states the reverse, that contango occurs when hedgers (for example, consumers who fear a price rise) are long and speculators are paid a premium to be short, so that observed futures prices  $F_{t,T}$  are above the expected spot price  $E_t(s_T)$ . Contango is therefore equivalent to a negative risk premium. Modern treatment of these definitions, as is done here, often uses the spot price instead of the expected future spot price.

The above theory postulates that commodity futures markets usually exhibit backwardation, or a convenience yield larger than the risk-free rate, but empirical studies on backwardation and contango in commodity markets show cases of both occurring. Studies such as Bodie and Rosansky (1980), Chang (1985), Fama and French (1987), Pindyck (2001), Considine and Larson (2001a,b) and Milonas and Henker (2001) find evidence supporting backwardation for several commodity futures markets and portfolios of commodity futures, including wheat, corn, soybeans, metals, heating oil, crude oil, natural gas, and other major mining and agricultural commodities. No-

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<sup>18</sup>The *risk premium* is the reward for holding a risky investment rather than a risk-free one. In this case, the risk premium is the difference between the expected spot price (the best estimate of the future price of the commodity) and the futures price (the price a trader is prepared to pay today for delivery of the commodity in the future) (see Geman (2005) and Pindyck (2001))

tably, Pindyck (2001) also found that the level of backwardation increases during high volatility for crude and heating oil. On the other hand, recent studies such as Longstaff and Wang (2004), Weron (2008), Botterud et al. (2010), Bierbrauer et al. (2007) and Trück et al. (2014) find cases of contango, i.e. convenience yields that are negative or lower than the risk-free rate, in various electricity and CO<sub>2</sub> markets. They reason that the non-storability of electricity makes electricity price fluctuations a risk to consumers, especially compared to other commodities where inventories can be established. This increases consumers' incentive to hedge by going long in futures contracts, pushing the market into contango (aligning with the reason detailed in Hicks (1946)). This is an unusual example of a commodity that is difficult to store, thus deviating from the usual negative relationship seen between the level of inventories and the convenience yield (for example, see Fama and French (1987) and Pindyck (2001)).

### *3.3. Derivation of Contingent Claims*

As mentioned in Section 3.1, the Gibson-Schwartz model can be used to price claims contingent on the price of a commodity. This section will briefly derive two partial differential equations for these prices, one for the present value of a commodity delivered at time  $T$  and another for the futures price. These derivations follow similarly those of the bond pricing model developed by Brennan and Schwartz (1979), where a joint stochastic process is used to model the term structure of interest rates. These derivations were originally presented by Gibson and Schwartz (1990) and Brennan (1991), and the reader is referred to these sources for further details.

Firstly, the price of a contingent claim  $B(s, \delta, t)$  dependant on the price of bitcoin, convenience yield and time is considered. Using Ito's lemma, the following stochastic differential equation can

be attained, assuming  $B$  is twice differentiable in the spot price and convenience yield and once differentiable in time,

$$dB = \left( \frac{\partial B}{\partial s} \mu s + \frac{\partial B}{\partial \delta} \kappa(\alpha - \delta) + \frac{\partial B}{\partial t} + \frac{1}{2} \frac{\partial^2 B}{\partial s^2} \sigma_s^2 s^2 + \frac{\partial^2 B}{\partial s \partial \delta} \rho \sigma_s \sigma_\delta s + \frac{1}{2} \frac{\partial^2 B}{\partial \delta^2} \sigma_\delta^2 \right) dt + \frac{\partial B}{\partial s} \sigma_s s dz_s + \frac{\partial B}{\partial \delta} \sigma_\delta dz_\delta. \quad (6)$$

For notation, let  $\partial B / \partial [\cdot] = B_{[\cdot]}$  denote partial derivatives and  $\tau = T - t$  be the time until maturity, so that  $B_\tau = -B_t$  and

$$dB = \left( B_s \mu s + B_\delta \kappa(\alpha - \delta) - B_\tau + \frac{1}{2} B_{ss} \sigma_s^2 s^2 + B_{s\delta} \rho \sigma_s \sigma_\delta s + \frac{1}{2} B_{\delta\delta} \sigma_\delta^2 \right) dt + B_s \sigma_s s dz_s + B_\delta \sigma_\delta dz_\delta. \quad (7)$$

Assuming no arbitrage opportunities the expected return from this contingent claim, which can be called  $\mu_B$ , must be equal to the risk-free rate plus compensation for any risks taken at the market price of risk. Here, the risks are associated with the spot price and convenience yield, as indicated by the  $dz_s$  and  $dz_\delta$  components. Assuming the market price of risk for these are  $\lambda_s$  and  $\lambda_\delta$  respectively,  $\mu_B$  can then be expressed as

$$\mu_B = r + \lambda_s \frac{B_s \sigma_s s}{B} + \lambda_\delta \frac{B_\delta \sigma_\delta}{B}. \quad (8)$$

Now, an example of a contingent claim on the spot price of bitcoin is going long in one unit of bitcoin. In this case  $B = s$ , so  $\mu_B = \mu_s$ , and note that the expected return from holding bitcoin is

comprised of a capital gain,  $\mu$ , and a convenience yield,  $\delta$ . Equation 8 then becomes

$$\mu + \delta = r + \lambda_s \frac{\sigma_s s}{s} \implies \lambda_s = \frac{\mu + \delta - r}{\sigma_s}. \quad (9)$$

The market price of spot price risk,  $\lambda_s$ , has thus been solved for analytically. The market price of convenience yield risk,  $\lambda_\delta$ , however, can not be solved for analytically. Substituting equation 9 into equation 8, letting  $\lambda_\delta = \lambda$  and equating this to the drift term in equation 7 then leads to the following partial differential equation for the price of the contingent claim  $B$ ,

$$\frac{1}{2} B_{ss} \sigma_s^2 s^2 + B_{s\delta} \rho \sigma_s \sigma_\delta s + \frac{1}{2} B_{\delta\delta} \sigma_\delta^2 + B_s s(r - \delta) + B_\delta (\kappa(\alpha - \delta) - \lambda \sigma_\delta) - B_\tau - rB = 0. \quad (10)$$

This equation can be used to find the price of any contingent claim of this type, where equation 8 holds, and given boundary conditions. For example, for calculating the present value of a bitcoin at time  $T$  in the future, where the boundary condition would be  $B(s, \delta, T) = S$ . In general, solutions to equation 10 depend on the boundary conditions and can not always be found analytically.

A type of contingent claim that is not of this type is a futures contract,  $F$ , since there is no initial investment and thus a hedged portfolio,  $P$ , must be considered. In this case, consider going long a bitcoin and short  $1/F_s$  futures contracts, and assume  $F$  depends on the spot price, convenience yield and time. The rate of return from this portfolio comes from the capital gain of the commodity  $ds/s$ , the convenience yield  $\delta/s$  and the change in the futures price  $dF/s$ . Using Ito's lemma to

calculate  $dF$ , as in equation 7, the return from this portfolio can be written as

$$\frac{ds + \delta - \frac{1}{F_s}dF}{s} = \frac{1}{s} \left[ \delta - \frac{1}{F_s} \left( \frac{1}{2} F_{ss} \sigma_s^2 s^2 + F_{s\delta} \rho \sigma_s \sigma_\delta s + \frac{1}{2} F_{\delta\delta} \sigma_\delta^2 - F_\tau + F_\delta \kappa (\alpha - \delta) \right) \right] dt - \frac{F_\delta \sigma_\delta}{s F_s} dz_\delta. \quad (11)$$

As above, assuming no arbitrage opportunities the expected return from this portfolio must be equal to the risk-free rate plus compensation for any risks taken at the market price of risk. This risk is associated with the convenience yield, as indicated by the  $dz_\delta$  component, because unlike spot price risk convenience yield risk cannot be hedged. Therefore, the following relationship holds:

$$\mu_P = r + \lambda \frac{F_\delta \sigma_\delta}{(-s F_s)}. \quad (12)$$

Equating this to the drift term in equation 11 then leads to the following partial differential equation for the price of a futures contract,

$$\frac{1}{2} F_{ss} \sigma_s^2 s^2 + F_{s\delta} \rho \sigma_s \sigma_\delta s + \frac{1}{2} F_{\delta\delta} \sigma_\delta^2 + F_s s (r - \delta) + F_\delta (\kappa (\alpha - \delta) - \lambda \sigma_\delta) - F_\tau = 0. \quad (13)$$

Unlike equation 10, which does not have a general solution, Jamshidian and Fein (1990) and Bjerksund (1991) found a solution for the futures price in equation 13 which takes the following

form:

$$F = s \cdot \exp \left[ -\delta \left( \frac{1 - e^{-\kappa T}}{\kappa} \right) + A(T) \right], \quad (14)$$

where

$$A(T) = \left( r - \alpha + \frac{\lambda}{\kappa} + \frac{1}{2} \frac{\sigma_\delta^2}{\kappa^2} - \frac{\sigma_s \sigma_\delta \rho}{\kappa} \right) T + \frac{1}{4} \sigma_\delta^2 \frac{1 - e^{-2\kappa T}}{\kappa^3} + \left( \alpha \kappa - \lambda + \sigma_s \sigma_\delta \rho - \frac{\sigma_\delta^2}{\kappa} \right) \frac{1 - e^{-\kappa T}}{\kappa^2}. \quad (15)$$

### 3.4. Model Parameter Estimation

The method used to estimate the model parameters follows the method used by Gibson and Schwartz (1990). This method first creates a linear discretised version of Model 1 and regresses these equations to estimate the parameters  $\kappa, \alpha, \sigma_\delta$  and  $\sigma_s$ . The residuals of the two regressions then determines  $\rho$ , and  $\lambda$  is calculated by equating the modelled futures prices given by equation 14 to the observed futures prices. The spot price return,  $\mu$ , is not required in the futures pricing formula, but can be calculated via the standard method of taking the mean value of  $(s_t - s_{t-1})/s_{t-1}$  over time.

More specifically, the following procedure is implemented: for each day  $t$  model 1 can be discretised by looking at

$$\begin{aligned} \ln \left( \frac{s_t}{s_{t-1}} \right) &= a + b \ln \left( \frac{s_{t-1}}{s_{t-2}} \right) + \epsilon_s, \\ \delta_t - \delta_{t-1} &= \kappa \alpha + \kappa \delta_{t-1} + \epsilon_\delta, \end{aligned} \quad (16)$$

for some  $a$  and  $b$ .<sup>19</sup> Regressing the second equation will recover  $\kappa$  and  $\alpha$ , and  $\sigma_\delta$  and  $\sigma_s$  can be calculated by taking the standard deviations of  $\delta_t - \delta_{t-1}$  and  $\ln\left(\frac{s_t}{s_{t-1}}\right)$ , respectively. The correlation of the two residuals,  $\epsilon_s$  and  $\epsilon_\delta$ , then determines  $\rho$ . Finally, solving equation 14 for  $\lambda$  allows a daily value of  $\lambda$  to be calculated. The mean of these daily values is then taken as the final value of  $\lambda$  over the period to minimise the difference between the estimated and actual futures prices. For more details on this procedure refer to Gibson and Schwartz (1990).

Finally, it must be noted that there are more advanced methods for calculating these parameters. In particular, Schwartz (1997) uses the Kalman filter to maximise a log likelihood function and Cortazar and Schwartz (2003) use excel to implement a mean squared error minimising procedure. Both of these procedures were tested, but failed to provide reasonable results. The Kalman filter procedure lead to a flat log likelihood function where a global maximum was unable to be attained. Varying the initial parameters and bounds of the procedure often caused the estimated parameters to vary greatly, with most results providing implausible estimates for the parameters. Furthermore, despite the varying parameter estimates, the final log likelihood values were very close, often within several decimal places of each other. The mean squared error procedure suggested by Cortazar and Schwartz (2003) also showed a similar problem. Varying the initial parameters and bounds of the procedure again lead to estimated parameters varying greatly, even though the mean squared errors were very small. It is unclear why these procedures did not work in this case, but given the inappropriate results obtained from them, the estimation approach of Gibson and Schwartz (1990) is used in the following sections.

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<sup>19</sup>Using bitcoin data,  $b$  is not significantly different from zero, as was also found for oil by Gibson and Schwartz (1990).

## 4. Data and Market History

The data used consists of daily bitcoin spot and futures price data and daily US treasury bond yield quotes over a period of nearly two years, from 3 March 2013 to 18 February 2015. The spot data used is from *bitstamp*, one of the world's largest and most liquid bitcoin exchanges,<sup>20</sup> and the futures data used is from *Orderbook*, the first bitcoin futures market provider. All data is converted to daily prices and the calculation of the convenience yield on each day uses quotes from the same day. The remainder of this section will be used to briefly introduce and provide a recent overview of the markets used, that is, the bitcoin spot and futures market, as well as the US treasury bond market.

### 4.1. Bitcoin Spot Price

The bitcoin spot data<sup>21</sup> used is from *bitstamp*, one of the world's largest and most liquid bitcoin exchanges and the same exchange used by *Orderbook* in the determination of futures settlement prices from April 2014 onwards. Spot price tick data was converted to a daily price by taking the last trade price at the end of each day (based on Coordinated Universal Time) and every day of the year is a trading day. Between 3 March 2013 and 18 February 2015 there were three days, from 6 January 2015 until 8 January 2015, that showed no traded volume.<sup>22</sup> For this non-traded period linear interpolation was used from 5 January 2015 until 8 January 2015.

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<sup>20</sup>Herein, the word *exchange* will be used in a more general sense. Typically, an exchange is a regulated marketplace, but since bitcoin is not regulated most bitcoin *exchanges* are also not regulated (although some claim to be, such as Tera Exchange). Thus, an exchange here could more accurately be described as an *unregulated exchange*.

<sup>21</sup>Since bitcoin is being considered a commodity the phrases *USD/BTC exchange rate* and *spot price of bitcoin in terms of USD* will be used interchangeably.

<sup>22</sup>This was due to a trading halt caused by hacking, an occurrence that has happened on numerous occasions for other bitcoin exchanges, as will be seen in Section 4.1. The number of bitcoins stolen was reported to be approximately \$5 million USD in total and was reimbursed to traders. See <https://www.bitstamp.net/article/bitstamp-is-open-for-business-better-than-ever/>

Since bitcoin displays high volatility and is not as actively traded as other commodities or currencies, different exchanges usually have different values for the bitcoin spot price (USD/BTC), varying even by up to several dollars. Furthermore, the total market capitalisation of bitcoin in February 2015 was near 3 billion USD,<sup>23</sup> after a high near 14 billion USD one year earlier, which is small enough to be influenced by large trades or for market manipulation to occur. Indeed, it is alleged that attempts to manipulate the market have occurred, for example, in general market trading to create arbitrage opportunities (see AFR (8 Oct 2014)) and during the price spike in November 2013.<sup>24</sup> Thus, when considering the bitcoin spot price, multiple exchanges must be considered. One option for the spot price would be a weighted average of several exchanges, as is provided by Bloomberg, Winkdex and CoinDesk, but since the Orderbook futures contracts from April 2014 onwards use bitstamp to calculate the settlement price, it is natural to use bitstamp as the spot price also. Although the differences between bitstamp and these other measures for the spot price are non-zero, a non-zero difference is expected and when bitstamp is compared to these other measures the difference is within reason.

Figure 4 shows the spot price of bitcoin in USD from 3 March 2013 until 18 February 2015, and in the following a brief history of this period will be provided. The period starts in turmoil due to the March and early April 2013 international bailout of Cyprus. A levy on large bank deposits caused an outflow of cash from Cyprus and the EU, with bitcoin receiving a large inflow, driving the price of bitcoin up 500% from one month earlier. After this, in early April 2013, newly passed US regulations led to the bank account of US bitcoin exchange *bitfloor* being closed, creating

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<sup>23</sup>See <https://blockchain.info/charts>

<sup>24</sup>See <https://willyreport.wordpress.com/>

concerns that user funds would be lost. This caused bitcoin exchange Mt. Gox's trading engine to freeze due to an unexpected increase in trades, causing concerns that Mt. Gox was under attack by hackers (see Fox (22 Mar 2013), Bloomberg (11 Apr 2013) and Reuters (28 Feb 2014)). These two events lead to a large drop in the spot price and to a stabilisation near 100USD until a similar pattern was seen from October 2013 to January 2014. During this period influential news emerged, including: the treatment of bitcoin as a "unit of account" in Germany, allowing it to be used for tax purposes (see The Economist (30 Nov 2013)); Ben Bernanke of the US Federal Reserve stating technologies like bitcoin "may hold long-term promise" (see The Economist (30 Nov 2013)); Yi Gang of the People's Bank of China commenting positively about the long-term potential of bitcoin (see New York Times (22 Nov 2013) and CNBC (29 Nov 2013)); bitcoin being presented with national television coverage in China (see New York Times (22 Nov 2013)); and Baidu's, China's largest internet search engine, acceptance of bitcoin payments (see Bloomberg (7 Dec 2013)). This news was well received, especially in China, driving increased traffic in bitcoin exchanges and causing an astronomical spike in the bitcoin spot price in November and early December 2013 to above 1,100 USD.<sup>25</sup> Over a period of a few days, the Chinese Government placed bans on financial institutions and intermediaries from accepting bitcoin as a currency, clear it as payment or provide clearing services to bitcoin exchanges (see Bloomberg (5 Dec 2013) and Bloomberg (17 Dec 2013)). This helped fuel a halving of the spot price until a rally stabilised the price. These regulations did not stop investors from having the ability to purchase bitcoin, but did stop bitcoin from being accepted as a currency in China.

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<sup>25</sup>It is also alleged that price manipulation occurred during this period, exacerbating the situation. See Footnote 24.

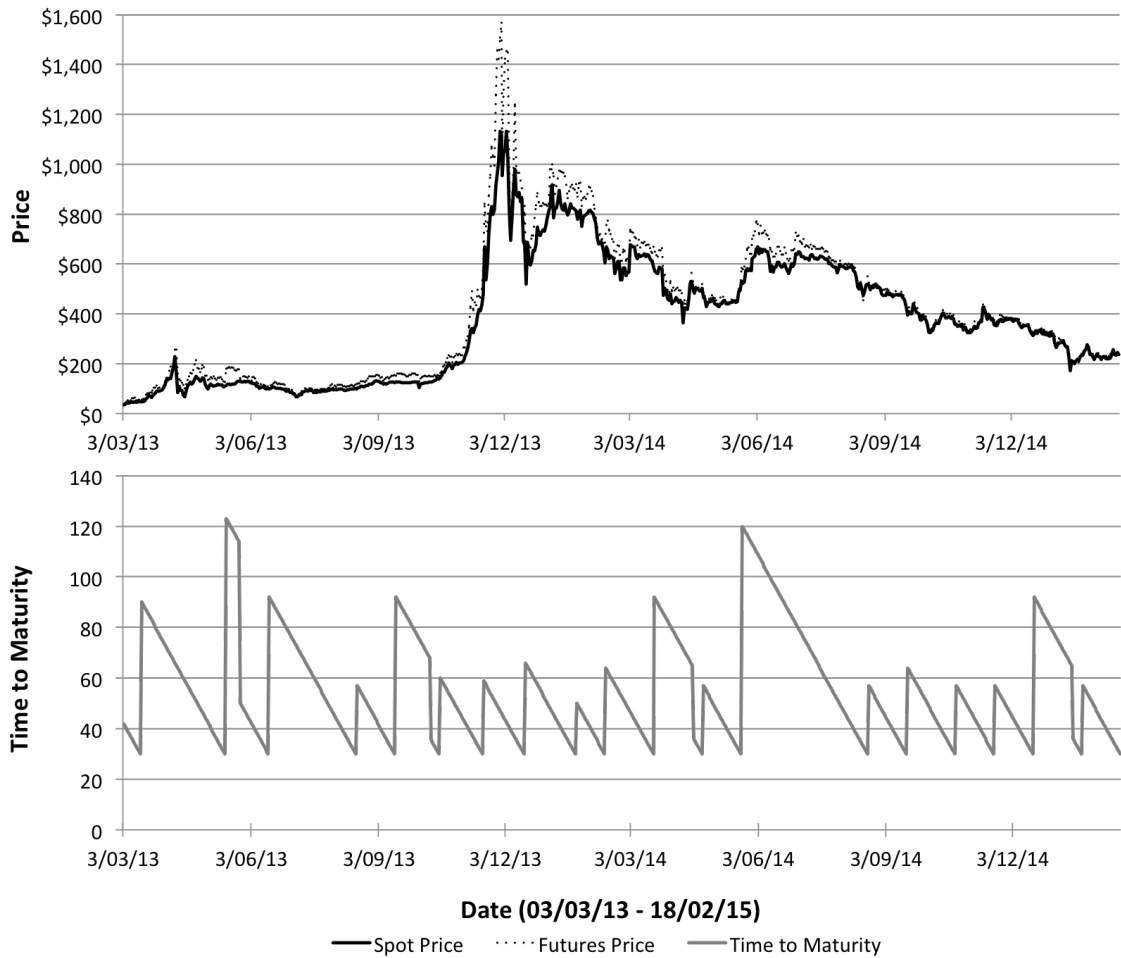


Figure 4: The bitcoin spot price, futures price and time-to-maturity of the futures contract used.

In February 2014 hackers attacked various bitcoin exchanges, causing a drop in the spot price and temporary suspension of several exchanges, including bitstamp (see Reuters (28 Feb 2014), Bloomberg (25 Feb 2014) and Forbes (12 Feb 2014)). While all other exchanges recovered from the attack, Mt. Gox was unable to recover, citing nearly half-billion US dollar losses and eventually filing for bankruptcy. Adding further pressure to the spot price, in March 2014 the US Internal Revenue Service declared bitcoin would be taxed as property and not currency, leading to further pressure on the spot price due to added costs and complexity (see Bloomberg (26 Mar 2014)). The

market recovered in May 2014 and saw a gradual decline until the end of the period in February 2015. The reason for this decline is unclear, but a strengthening USD, over-valuation of bitcoin and new businesses accepting bitcoin payments for goods and services exchanging these payments for USD immediately after receipt may be influencing this decline.

An interesting observation to note is that, during the steady decline in the spot price since June 2014, there has been much news concerning bitcoin, but the spot price has been relatively unaffected by news compared to 2013. This may indicate a stabilisation of the market and a reduction in speculation and over-optimism that has been seen previously. For example, a sell order of 31,000 bitcoins at \$300 USD, \$20 USD below the spot price and totalling \$9.3 Million USD or around 0.2% of bitcoin's market capitalisation, on 6 October 2014 did not provoke any great disturbance in the spot price, whereas this may have triggered a large sell-off previously (see AFR (8 Oct 2014)). Also, news of companies like Dell, Expedia, Overstock, Microsoft<sup>26</sup> and Braintree, a subsidiary of Ebay's Paypal, accepting or experimenting with the use of bitcoin as a payment method showed modest or no impact to the spot market. Similarly, news of regulated bitcoin exchanges and derivatives markets (*Coinbase* and *Tera Exchange*) in the US registered no or a temporary affect in the spot price. This can also be said after reports of a ponzi scheme in Hong Kong which led to investors losing \$387 Million USD and the Hong Kong Central Bank warning investors about investing in bitcoin (see Reuters (9 Feb 2015)), and even after the arrest and trial of Ross Ulbricht, the founder of black market website *Silk Road*, in February 2015. Having said this, there is still an expectation that the bitcoin spot market is dominated by speculators and over-

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<sup>26</sup>Microsoft allows online users to exchange bitcoins to add USD to online user accounts for use in Windows Store or for XBOX purchases

optimistic users, since its use as a currency is still not wide-spread. Therefore, a price spike of a similar magnitude to the ones seen in March and November 2013 would not be unexpected, but perhaps less expected compared to previous times in bitcoin's history.

#### 4.2. *Bitcoin Futures Quotes*

Bitcoin futures data was attained from *Orderbook* (<https://orderbook.net/>), the world's first bitcoin futures market provider. Originally set up as ICBIT in 2012, OrderBook have run markets for various (bitcoin) cash settled BTC futures contracts with underlying assets including USD, S&P500, crude oil, gold, bitcoin difficulty and litecoin. OrderBook developed the first futures contracts for bitcoin, but other futures markets have emerged since 2014, including OKCoin (<https://www.okcoin.com/>), BITVC (<https://www.bitvc.com/>), BitMEX (<https://www.bitmex.com/>), 796 (<https://796.com/>) and Tera Exchange (<http://www.teraexchange.com/>). These markets, however, either do not have USD/BTC contracts or have a much smaller time period to investigate, making Orderbook data both unique and interesting.

The tick data provided by OrderBook for each contract was converted to a daily quote by taking the last trade price at the end of each day (based on Coordinated Universal Time). For days with no trade data linear interpolation was used between the last and next available days with trades, and, notably, every day of the year is a trading day for OrderBook.<sup>27</sup> The futures contract with the smallest time to maturity was then used for the estimate of the convenience yield, but the last thirty days of data for each contract is omitted due to high volatility towards the maturity date;<sup>28</sup> indeed,

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<sup>27</sup>Days without trading volume by contract is summarised in Table 1.

<sup>28</sup>In addition to this, settlement prices before April 2014 use a 30-day weighted average, which would make using futures prices with times to maturity less than 30 days inappropriate, as information about the settlement price is already known. This is discussed ahead

| Contract | Trading Days | No Volume | %  | Max Open Interest |
|----------|--------------|-----------|----|-------------------|
| Apr-13   | 43           | 0         | 0  | NA                |
| May-13   | 31           | 0         | 0  | NA                |
| Jun-13   | 104          | 0         | 0  | NA                |
| Jul-13   | 51           | 0         | 0  | NA                |
| Aug-13   | 32           | 4         | 13 | NA                |
| Sep-13   | 197          | 3         | 2  | NA                |
| Oct-13   | 65           | 12        | 18 | 3,253             |
| Nov-13   | 37           | 1         | 3  | 33,501            |
| Dec-13   | 189          | 0         | 0  | 359,565           |
| Jan-14   | 62           | 0         | 0  | 76,973            |
| Feb-14   | 69           | 7         | 10 | 17,701            |
| Mar-14   | 199          | 0         | 0  | 394,850           |
| Apr-14   | 93           | 0         | 0  | 82,370            |
| May-14   | 37           | 0         | 0  | 11,354            |
| Jun-14   | 99           | 3         | 3  | 38,577            |
| Jul-14   | 27           | 2         | 7  | 12,365            |
| Aug-14   | 31           | 3         | 10 | 5,613             |
| Sep-14   | 215          | 4         | 2  | 137,767           |
| Oct-14   | 94           | 29        | 31 | 7,680             |
| Nov-14   | 129          | 38        | 29 | 4,179             |
| Dec-14   | 208          | 4         | 2  | 62,591            |
| Jan-15   | 104          | 16        | 15 | 7,104             |
| Feb-15   | 36           | 1         | 3  | 1,128             |
| Mar-15   | 101          | 18        | 18 | 8,257             |

Table 1: A summary of days with no trading volume and open interest levels for each of the futures contracts used. The second column, *No Volume*, is the number of days with no trading volume, and this is expressed as a percentage of total *Trading Days* (column one) in the third column.

some contracts reach convenience yields below -1,000% during this period. This lead to only the April 2013 to March 2015 contracts being used from 3 March 2013 until 18 February 2015. Both before 3 March 2013 and after 18 February 2015 there were significant time periods where the time to maturity of the contracts were either less than 30 days or greater than 90 days, data which is not suitable to calculate convenience yields. Thus, these periods were omitted. Figure 4 graphs futures prices of the nearest maturity contracts (excluding quotes when the time-to-maturity of the contract is less than 30 days) against the bitcoin spot price, as well as the corresponding time-to-maturity of the contract. The average time-to-maturity of the contracts used for the futures price is 55.51 days with a standard deviation of 21.21 days. Table 1 shows the number of non-trading days for each of the futures contracts used in the analysis. From the October 2014 contract onwards trading volume declined, as indicated the the decrease in the open interest levels seen for those contracts (also see Figure 8). This may impact results, since linear interpolation is used frequently in the data, but this situation is considered more appropriate than keeping the futures price constant across days with no trading volume (equal to the last day with trading volume), as this latter method may increase volatility.

OrderBook operates a relatively small and immature market with USD/BTC futures contracts having some days without any trading volume, small open interest levels<sup>29</sup> and, prior to 2015, bid-ask spreads often greater than 10USD. Questions were raised in 2012 over the central clearing of OrderBook and their margin calls (see Bitcoin Wiki - ICBIT (2014)). OrderBook does not act as the central counterparty for contracts. This, strictly speaking, places OrderBook contracts

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<sup>29</sup>The total open interest across all futures contracts in the market on a day is often less than 50,000 contracts at 10USD per contract, but has been as high as 800,000 contracts on 5 December 2013.

somewhere between forwards and futures; a standardised and tradeable forward with margin requirements, or a future with non-zero counterparty credit risk (though margin requirements ensure profitable positions still provide a non-zero profit in the event of a counterparty defaulting). OrderBook provides an explanation of margin calls that would leave a negative net balance for one party after closure of a position.<sup>30</sup> In the event of a margin call that would leave one party (the *losing* party) with a negative net position, that is, when the margin held by the losing party does not cover the losses incurred because there are no bids/asks to close the position in the positive or zero, then *force closing* may be performed. In this case, an equal number of contracts are closed from both the losing and *winning* parties such that all available funds from the losing party are transferred to the winning party and the losing party is left with a zero net position. This essentially uses the profits gained from the winning party to close the losing party position back to zero, thus may affect the profitable position negatively and the losing position positively. This would be unlikely to impact the futures price significantly, as it essentially lowers the risk of large losses and equally lowers the *risk* of large profits. Furthermore, since the market was typically significantly in contango, as will be shown, concerns over this clearing method have lessened since early 2013, as OrderBook has become more established and the market become more mature. One other point of difference between a standard futures contract and the ones offered by Orderbook is the calculation of the settlement price. Prior to the April 2014 contract, the settlement price was a volume-weighted average of the previous 30 days from the bitcoin exchange with the largest 30-day volume. This means the settlement price would only approximately equal the spot price at maturity of the contract. This may affect results and is discussed further in Section 5.1. From the

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<sup>30</sup>See <https://orderbook.net/about/rules> Section 3.3.5

April 2014 contract onwards, the settlement price is of less concern as its calculation was changed to a 1-hour volume-weighted average from the bitstamp exchange.

#### *4.3. US Treasury Bond Yields*

The risk-free rates used in this paper are daily US treasury bond yield quotes. This is because most bitcoin exchanges conduct USD/BTC markets and USD/BTC trading was the most predominant type until CNY/BTC markets became more heavily traded in late 2013. Furthermore, the futures contracts used are USD/BTC contracts, so it also seems appropriate to use US interest rates. Daily yield quotes for bonds with maturities of 1, 3 and 6 months are used and linear interpolation is used to calculate the yield curve from 1 to 6 months. This was considered satisfactory, since the futures market is significantly in contango and minor changes in the risk-free rate will be immaterial.<sup>31</sup> Further to this, daily data is used, and not an average over the period. For non-trading days (in this case, weekends and public holidays) linear interpolation is again used between the last and next available trading days for each maturity from 1 to 6 months. The time-to-maturity of the bond used for the risk-free rate is the same as the time-to-maturity of the futures contract (with monthly accuracy). So with 1 month remaining on the futures contract the yield from a 1-month bond is used, with 2 months remaining on the futures contract the yield from a 2-month bond is used and so on. Also, daily bond yield quotes are used, so that all data used to calculate daily convenience yields are quoted from the same day; that is, the spot price, futures quote and relevant bond yield are all quoted from the same calendar date. Even though interest rates only show minor changes during the period 2013-2015 inclusive, this method is considered more theoretically robust.

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<sup>31</sup>The highest yield during the period was 0.32% on 15 October 2013, whereas convenience yields are sufficiently negative to make these yields insignificant, as will be shown in Section 5.1

Figure 5 shows the yields for 1, 3 and 6-month US treasury bonds, which are mostly stable and close to zero, in line with long-term expectations during the quantitative easing the US Federal Reserve undertook after the financial crisis in 2007 and 2008. The spikes seen in the yields during October 2013 and February 2014 were caused by concerns that the US debt ceiling crises would affect payments to short term bondholders (see Austin and Levit (2014), CNBC (9 Oct 2013) and Bloomberg (5 Feb 2014)). During these periods the 1-month yield was close to or above the 6-month yield, but only for short periods of time. The increase in the 6-month yield from October to December 2014 was caused by the end of quantitative easing undertaken by the Federal Reserve and expectations of a rate rise in mid-2015 (see Bloomberg (16 Dec 2014) and Wall Street Journal (18 Dec 2014)). The 6-month rate subsequently fell as this rate rise was expected to be delayed to late 2015 or 2016.

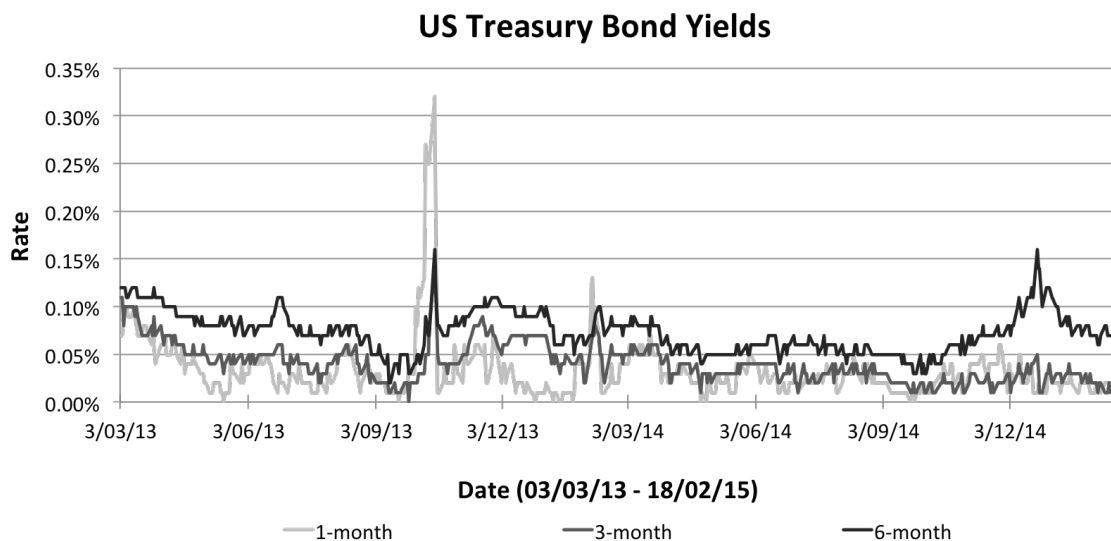


Figure 5: US treasury bond yields for bond maturities of 1, 3 and 6 months.

## 5. Empirical Results

### 5.1. Convenience Yields and the Spot Market

Figure 6 shows the observed daily convenience yield estimates and Table 2 provides some summary statistics. It is seen that there is a significant level of contango, with a mean convenience yield of -72.7% and median of -48.3%. Furthermore, the convenience yield is less than zero nearly 95% of the time, less than -100% more than one quarter of the time and when the market exhibited backwardation it was only for small time periods. Of particular interest is that the two troughs in the convenience yield to below -200% occurred during the times of the spot price spikes seen in April 2013 and November 2013 and the succeeding high levels of spot price volatility. Figure 7 shows log spot price returns and log spot price return volatility<sup>32</sup> compared to the convenience yield, and displays that when spot market volatility is high the convenience yield decreases, that is, there is a negative relationship between spot price volatility and the convenience yield. A negative relationship is also seen in electricity<sup>33</sup> and CO<sub>2</sub> emissions markets (Longstaff and Wang, 2004; Trück and Weron, 2015), but is unlike what happens to other major commodities, such as oil, where increases in spot price volatility typically cause the convenience yield to increase due to increases in inventories (Pindyck, 2001). Since bitcoin is not used in production processes and companies that accept bitcoin payments for goods and services usually alter the bitcoin amount charged regularly to equal the equivalent amount in the domestic currency, holding inventories of bitcoin is of

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<sup>32</sup>The volatility of spot price returns is calculated via an exponentially weighted moving average (EWMA) as follows, where  $s_t$  is the price on day  $t$  and  $r_t = \ln(s_t/s_{t-1})$  is the daily log spot price return of day  $t$ ,

$$VOL_t = \sqrt{\epsilon VOL_{t-1}^2 + (1 - \epsilon)r_t^2} \text{ for } t > 1 \text{ and } VAR_1 = r_1^2.$$

The value of  $\epsilon$  is taken as 0.94, as is standard in RiskMetrics.

<sup>33</sup>But note that the typical cost-of-carry model can not apply to electricity markets since electricity is non-storable.

little benefit other than to speculators, so it is not unexpected that a positive relationship between spot price volatility and the convenience yield does not occur. Having said this, in typical commodity markets sudden increases in volatility are usually caused by spot price decreases. In the case of bitcoin, they have been caused by spot price increases. This unusual occurrence may be a result of a high number of speculators and over-optimism in the market (which is also discussed by Cheah and Fry (2015)), and the negative relationship between spot price volatility and the convenience yield is also likely a result of these over-optimistic expectations of the future spot price and thus futures prices.

|                       | Mean   | Median | St. Dev | Min     | 5 <sup>th</sup> Per | 95 <sup>th</sup> Per | Max      |
|-----------------------|--------|--------|---------|---------|---------------------|----------------------|----------|
| Spot Price (\$)       | 377.58 | 365.60 | 245.92  | 34.33   | 85.24               | 814.77               | 1,132.01 |
| Log Returns (%)       | 0.27   | 0.18   | 6.51    | -66.39  | -8.03               | 9.56                 | 33.75    |
| Futures Price (\$)    | 415.03 | 379.88 | 280.79  | 35.00   | 93.96               | 912.92               | 1,568.00 |
| Convenience Yield (%) | -72.71 | -48.29 | 69.08   | -376.48 | -217.78             | 0.44                 | 52.66    |

Table 2: Summary statistics of the observed daily spot price, log spot price return, futures price convenience yield from 03/03/2013 until 18/02/2015. 5<sup>th</sup> Per and 95<sup>th</sup> Per refer to the 5<sup>th</sup> percentile and 95<sup>th</sup> percentile, respectively.

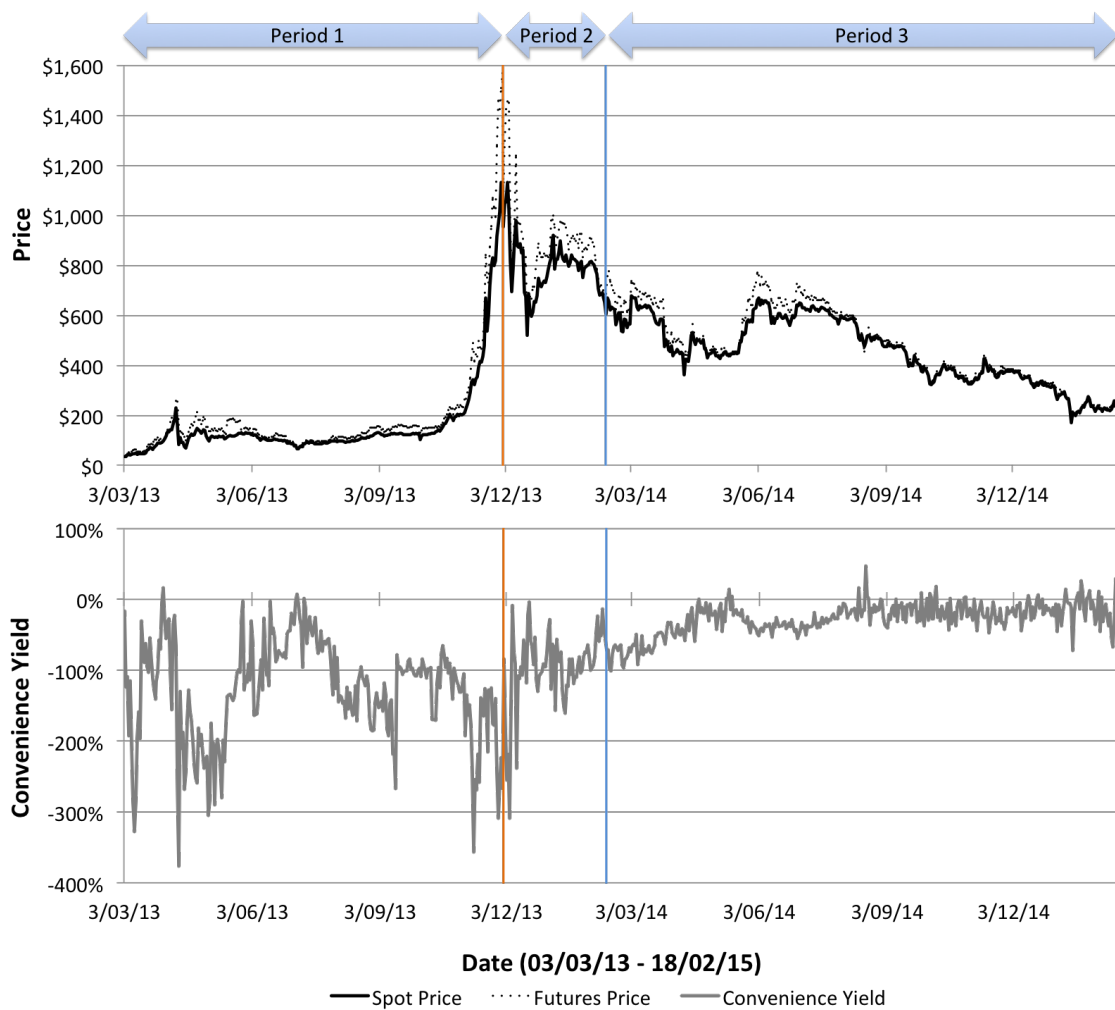


Figure 6: Daily convenience yields, futures quotes and bitcoin spot prices from 3 March 2013 until 18 February 2015.

The orange line indicates the end of the major price spike (1 December 2013). The blue line indicates the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

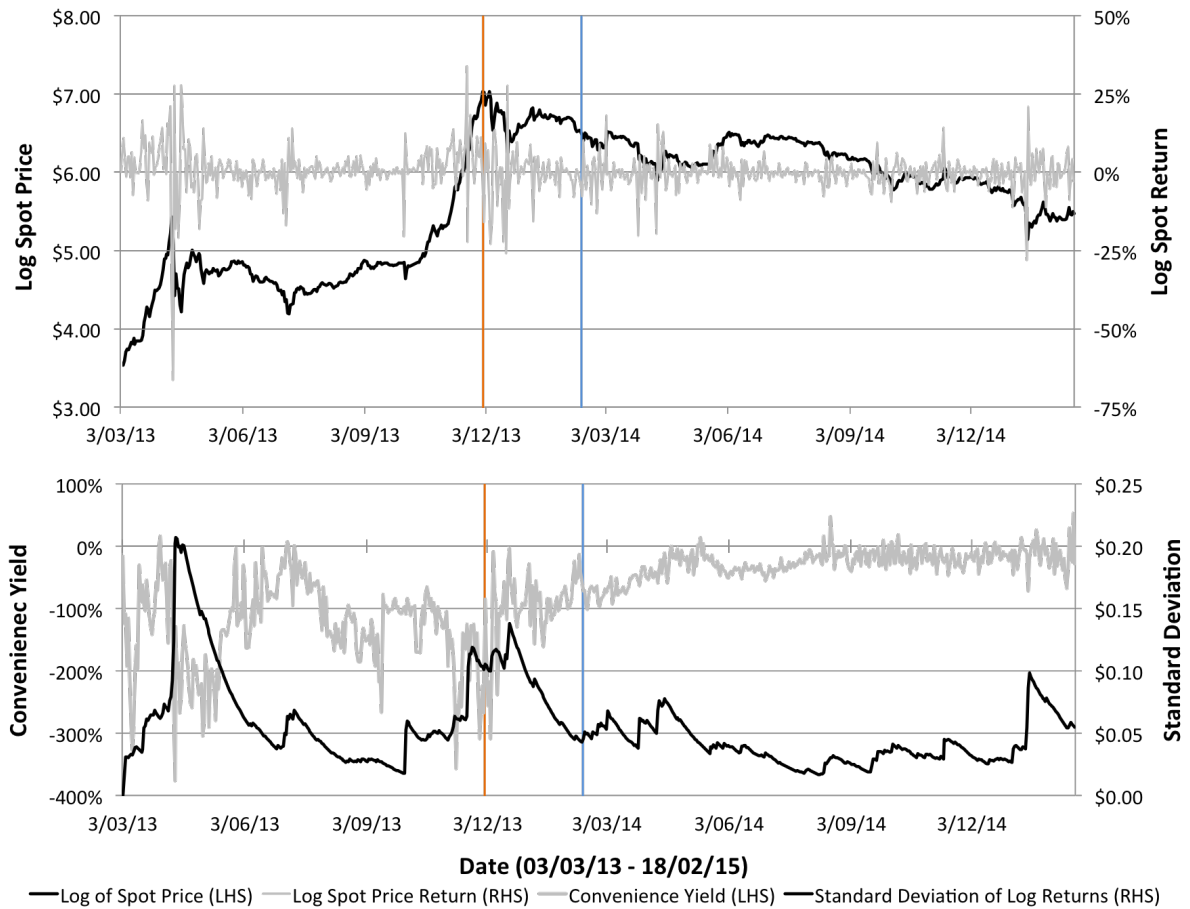


Figure 7: Log spot prices and log spot returns (top) and the standard deviation of log spot returns (bottom). The orange line indicates the end of the major price spike (1 December 2013). The blue line indicates the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

It is seen that from March 2014 the convenience yield, though still negative, trends towards zero. There are two likely reasons for this. One reason is the stabilisation of the spot price. As mentioned above, there seems to be a negative relationship between the convenience yield and spot price volatility, and indeed during periods of lower volatility, for example from June 2013 to September 2013 and after March 2014, the convenience yield is near zero (see Figure 7). Bessembinder and Lemmon (2002) show that, in the electricity market, high expected demand or high

demand volatility causes the futures price, or more specifically the futures risk premium, to increase<sup>34</sup> due to positive spot price skewness.<sup>35</sup> For bitcoin, demand as well as the volatility of demand can be looked at via the spot price and trade volume, similar to the approach taken by Longstaff and Wang (2004), and indeed times of increased trade volume (high demand), such as April 2013 and November 2013, are when the convenience yield is at its lowest and times of lower trade volume, such as after May 2014, are when the convenience yield is near zero (see Figure 8). Furthermore, both bitcoin and electricity are inexhaustible commodities with markets that have high volatility of spot prices, frequent spot price spikes and positive spot price skewness, and although storage of electricity is generally not possible, the high risks in storing bitcoin discussed in Sections 2 and 4.2 suggest a similarity between these commodities, since in practise bitcoins stored in e-wallets may be lost due to hacking or other risks. However, as Bessembinder and Lemmon (2002) mention, this theory is developed based on the assumption that it is industry participants rather than speculators, and this is not clear in the bitcoin futures market. Nonetheless, the futures risk premium and its positive relationship to spot price volatility (or the negative relation between the convenience yield and spot price volatility) may be one reason explaining the recent trend of the convenience yield closer to zero.

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<sup>34</sup>Bessembinder and Lemmon (2002) discuss this in the context of a *forward risk premium* (forward price minus the spot price) rather than the convenience yield, but this idea is equivalently applicable to futures or the convenience yield.

<sup>35</sup>In a market where expected demand or demand volatility is high, positive skewness adds risk to short positions in futures contracts, since the spot price is more likely to spike, so futures prices increase to compensate for this risk.

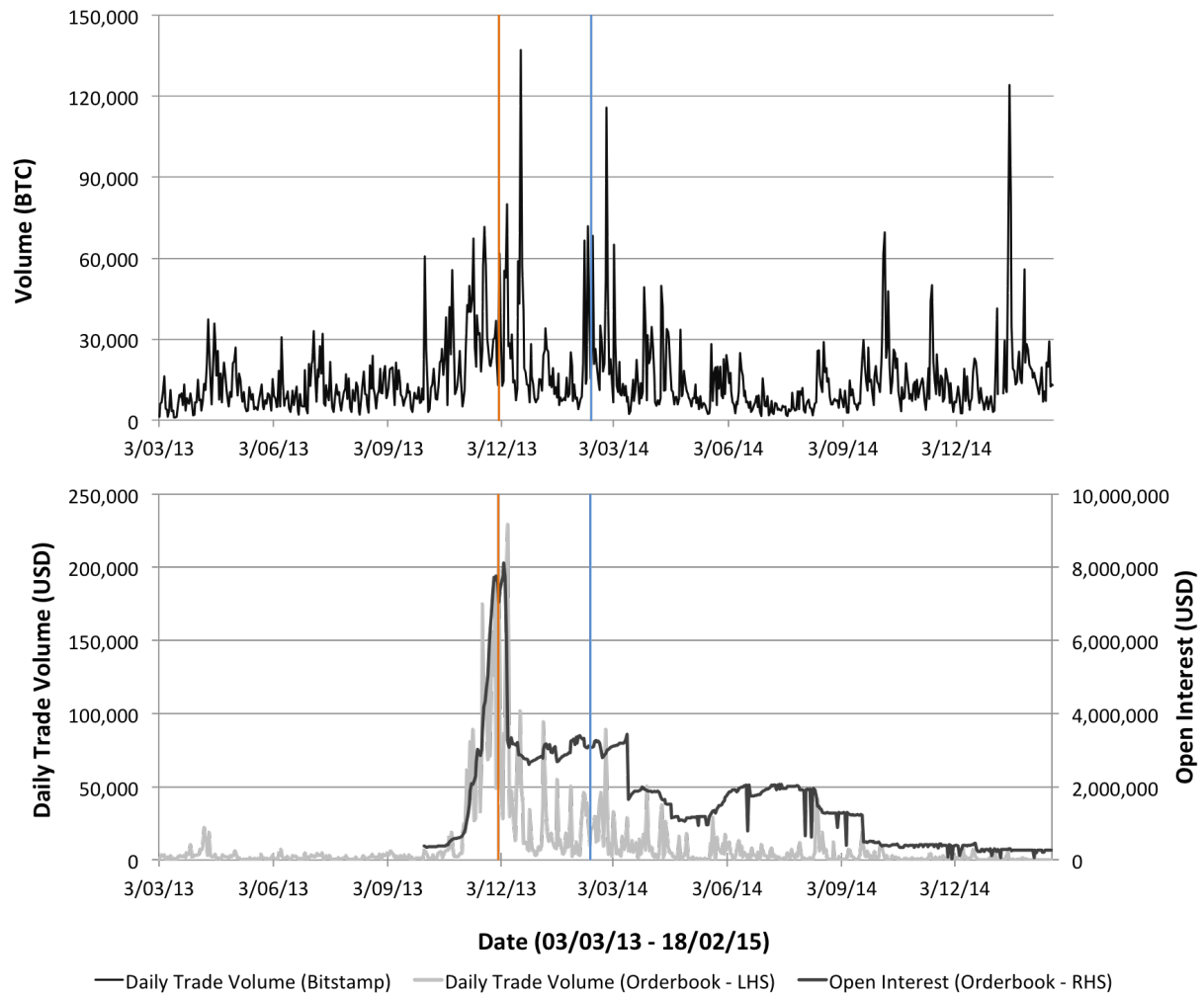


Figure 8: Aggregated daily trade volume (BTC) of bitstamp exchange (top) and daily trade volume (USD) and end of day open interest (USD) of all Orderbook futures contracts (bottom). Note that open interest data started to be recorded on 2 October 2013. The orange line indicates the end of the major price spike (1 December 2013). The blue line indicates the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

Another reason the convenience yield approached zero may be that, as the futures market matured, arbitrage opportunities were exploited, bringing the convenience yield closer to zero. Although the number of trades per day and open interest in each contract do not show any increases

| Period       | Mean   | Median | St. Dev | Min    | 5 <sup>th</sup> Per | 95 <sup>th</sup> Per | Max  | Obs |
|--------------|--------|--------|---------|--------|---------------------|----------------------|------|-----|
| All          | -72.7  | -48.3  | 69.1    | -376.5 | -217.8              | 0.4                  | 52.7 | 718 |
| Sub-Period 1 | -128.3 | -118.2 | 71.2    | -376.5 | -265.6              | -26.0                | 16.2 | 273 |
| Sub-Period 2 | -95.3  | -90.7  | 52.6    | -309.1 | -210.2              | -22.2                | -3.8 | 74  |
| Sub-Period 3 | -27.3  | -25.7  | 22.9    | -100.8 | -72.3               | 3.7                  | 52.7 | 371 |

Table 3: Summary statistics of the observed daily convenience yields from 03/03/2013 until 18/02/2015 (all units are in percent p.a.). 5<sup>th</sup> Per and 95<sup>th</sup> Per refer to the 5<sup>th</sup> percentile and 95<sup>th</sup> percentile respectively. The sub-periods are displayed in Figure 6 and are 3 March 2013 to 30 November 2013 (Sub-Period 1), 1 December 2013 to 12 February 2014 (Sub-Period 2) and 13 February to 18 February 2015 (Sub-Period 3).

during this period (see Figure 8), such large negative convenience yields were unlikely to stay unchanged with considerable arbitrage profits available. This is particularly likely due to the method of calculating the settlement price changing from a 30-day to a 1-hour volume-weighted moving average for the April 2014 contract onwards. Despite a 30-day average reducing the volatility of the expected settlement price, when close to maturity it would reduce the impact of the futures price due to changes in the spot price, increasing volatility in the convenience yield. Furthermore, a 30-day average settlement price means a typical buy and hold bitcoin and short bitcoin futures arbitrage strategy is not riskless due to basis risk, whereas a 1-hour average settlement price would, in practise, be accurate enough to implement this arbitrage strategy, driving the convenience yield closer to zero.

The change in the settlement price calculation is a significant change, so it is of interest to see how this affected the convenience yield. In addition, it is clear from Figure 6 that the contango in the market was more severe during the first half of the period than the second half, in particular, before the price spike seen in November 2013. Hence three sub-periods are considered: the period up to and including 30 November 2013, during the large spot price spikes (sub-period 1); the period

from 1 December 2013 until 12-February 2014 inclusive,<sup>36</sup> after which the method of calculating the settlement price changed from a 30-day to a 1-hour average (sub-period 2); and the period from 13 February 2014 onwards (sub-period 3). These three sub-periods are shown in Figure 6.

Summary statistics for the three sub-periods are shown in Table 3. Although each sub-period still exhibits large levels of contango, they are seen to be very different, with median convenience yields of -118.2%, -90.7% and -25.27% for sub-periods 1, 2 and 3 respectively. Volatilities are also seen to reduce in magnitude in each of the sub-periods, going from 71.2% in sub-period one to 52.6% in sub-period two and 22.9% in sub-period three. This reduction is likely due to the same reasons that have brought the convenience yield closer to zero and are discussed above, that is, reduced arbitrage opportunities and a decrease in volatility of the spot price.

## 5.2. Arbitrage Profits

Naturally, one thinks of exploiting negative convenience yields to acquire a profit using the standard approach of buying and holding bitcoins, shorting the appropriate number of futures contracts, using the purchased bitcoins as margin and selling the bitcoins at the maturity of the (bitcoin settled) contracts. Looking at an average scenario (similar to the mean convenience yield and mean time to maturity of the contracts used), on 21 February 2014 the spot price was \$581.35 and there were 56 days until maturity of the April 2014 futures contract, which had a price of \$650.00, corresponding to a convenience yield of -72.72%. Following the standard arbitrage strategy would have generated a profit of \$9.97 for each \$100 invested (85.77% p.a.). This is inclusive of transaction costs: a bitstamp fee of 0.25% on USD/BTC spot transactions; and orderbook fees that vary from

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<sup>36</sup>The April 2014 contract data started to be used in calculations from 13 February 2014 onwards.

|                                            | 19 Nov 2013 | 21 Feb 2014 | 28 Sep 2014 |
|--------------------------------------------|-------------|-------------|-------------|
| Spot Price at Time Zero (\$)               | 536.01      | 581.35      | 375.35      |
| Futures Price at Time Zero (\$)            | 750.00      | 650.00      | 390.00      |
| Days to Maturity                           | 57          | 56          | 54          |
| Convenience Yield (% p.a.)                 | -215.04     | -72.72      | -25.87      |
| Spot Price at Maturity (\$)                | 841.14      | 483.11      | 351.20      |
| Settlement Price (\$)                      | 864.90      | 484.00      | 356.00      |
| Orderbook Transaction Fee                  | 0.00001     | 0.0001      | 0.0003      |
| <i>Profits excluding transaction costs</i> |             |             |             |
| Profit (\$)                                | 35.49       | 11.66       | 2.62        |
| Profit (% p.a.)                            | 599.31      | 105.15      | 19.14       |
| <i>Profits including transaction costs</i> |             |             |             |
| Profit (\$)                                | 34.66       | 9.97        | -0.09       |
| Profit (% p.a.)                            | 572.23      | 85.77       | -0.62       |
| Transaction Costs (\$)                     | 0.73        | 1.59        | 2.68        |
| <i>Profits from borrowing on margin</i>    |             |             |             |
| Profit (\$)                                | 32.35       | 7.65        | -2.34       |
| Profit (% p.a.)                            | 501.67      | 61.66       | -14.77      |
| Transaction Costs and Margin Fees (\$)     | 3.13        | 3.94        | 4.93        |

Table 4: Summary statistics of the results from a \$100 investment in a standard arbitrage strategy on different dates. Borrowing on margin is assumed to be at the volume-weighted average rate of 20.14% p.a. and charged daily, and the Orderbook transaction fee is charged per \$10 USD contract per trade, whereas the bitstamp transaction fee is charged at a flat rate of 0.25% of the transaction.

0.00001 to 0.003 BTC per contract per trade dependent on the contract throughout 2013 to 2015.

In a more extreme case, on 19 November 2013 the spot price was \$536.01 and there were 57 days until maturity of the January 2014 futures contract, which was priced at \$750.00, a convenience yield of -215.04% (similar to the 5<sup>th</sup> percentile of the convenience yield). Following the standard arbitrage strategy with a \$100 investment would have generated a profit of \$34.66 (572.23% p.a.). Although the return from this is extreme, the difference between the settlement price and spot price at maturity indicates the substantial basis risk that was present before the change in the settlement price calculation. However, a settlement price of \$1051.70 (\$209.56 higher than the spot price) would have been required to reduce the profit to zero.

On the other hand, using a more recent case during period 3 (after the settlement price calculation change which removed most basis risk), where convenience yields moved closer to zero, shows a different result. On 28 September 2014 the spot price was \$375.35 and there were 54 days until maturity of the November 2014 futures contract which was priced at \$390.00, a convenience yield of -25.87% (similar to the mean and median of the convenience yield during period 3). Following the standard arbitrage strategy would have generated a profit of \$2.62 (19.14% p.a.) for each \$100 invested excluding transaction costs. However, transaction costs in this scenario are enough to nullify profits, with the return on a \$100 investment being -\$0.09 (-0.62% p.a.) including these costs, indicating further that during period 3 arbitrage opportunities were exploited in the market. An overview of the results from these three scenarios is shown in Table 4. In addition, bitcoin margin loans,<sup>37</sup> that are often charged at a rate above 20% p.a., are considered and are seen to still give large positive return for the first two scenarios (on 21 Feb 2014 and 19 Nov 2013): thus, arbitrage opportunities were still available during periods 1 and 2 for traders without capital.

A major problem with the above hedged buy and hold strategy is clearly the settlement price, which was previously a 30-day volume-weighted moving average. This means that the hedged buy and hold strategy is not riskless, as it has basis risk, and the settlement price is unlikely to equal to the spot price at the maturity date. This, however, is less of an issue post the change to a 1-hour average and the market seems to have reacted to this by, on average, removing arbitrage profits. More generally, there are two caveats with attempts to generate arbitrage profits from this market. First, as was mentioned in Section 4, the size of both the bitcoin spot market (by market

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<sup>37</sup>Bitcoin margin loans are offered on a peer-to-peer basis by both Bitfinex and OKCoin. Using Bitfinex historical data, the volume-weighted average rate of margin lending from 1 April 2013 to 18 February 2015 was 20.14%.

capitalisation) and bitcoin futures market (by open interest) is relatively small compared to other developed markets; thus, arbitrage strategies will have limited scaling ability. Second are the risks described in Sections 2 and 4.2. These are associated with the typical risks of bitcoin, including hacking, and the counterparty risks associated with OrderBook futures contracts.

### 5.3. Parameter Estimations

The estimations of the parameters of the Gibson-Schwartz model using the procedure of Section 3.4 are given in Table 5. The results of the parameters relating to spot price movements,  $\mu$  and  $\sigma_s$ , are consistent with expectations. A high mean daily spot return,  $\mu$ ,<sup>38</sup> of 0.48% is seen over the entire period (466.60% p.a.). This is clearly due to the two price spikes in sub-period 1, which led this sub-period to have a  $\mu$  of 1.60%, as opposed to the negative values of -0.21% and -0.15% seen in sub-period's 2 and 3, as the market was adjusting after the November 2013 price spike (see Section 4.1). The volatility of daily spot returns,  $\sigma_s$ , is also large at 6.51% over the entire period. As expected from Figures 6 and 7, sub-period 3 shows a drop in the volatility, which reduces to 4.47%.

The long run mean convenience yield,  $\alpha$ , and volatility of the daily change in convenience yield,  $\sigma_\delta$ , show results consistent with Table 2: a highly negative overall mean convenience yield which approaches zero over time; and a high  $\sigma_\delta$  across the entire period with sub-period 3 showing a large decrease. However, unlike the results in Table 2, sub-period 2 reports the highest  $\sigma_\delta$ .<sup>39</sup> This is again due to the market corrections seen after the price spike in November 2013. Interestingly, the speed of mean reversion,  $\kappa$ , is lowest over the entire period (0.14) compared to each

<sup>38</sup> $\mu$  is not needed to calculate futures prices, but is calculated for completeness of the model parameters.

<sup>39</sup>Note that the volatility in Table 2 refers to the convenience yield itself, whereas the volatility in Table 5 refers to the change in the convenience yield, so these two volatilities will not be the same.

|                     | All    | Sub-Period 1 | Sub-Period 2 | Sub-Period 3 |
|---------------------|--------|--------------|--------------|--------------|
| $\mu$ (%)           | 0.48   | 1.60         | -0.21        | -0.15        |
| $\sigma_s$ (%)      | 6.51   | 8.08         | 7.88         | 4.47         |
| $\kappa$            | 0.14   | 0.23         | 0.47         | 0.33         |
| $\alpha$ (%)        | -72.89 | -131.82      | -94.97       | -26.93       |
| $\sigma_\delta$ (%) | 36.89  | 47.98        | 50.82        | 18.55        |
| $\lambda$ (%)       | -0.32  | -0.79        | -1.49        | 0.16         |
| $\rho$              | 0.16   | 0.31         | -0.16        | 0.26         |
| Obs                 | 718    | 273          | 74           | 371          |

Table 5: Estimations of the variables in the Gibson-Schwartz model. The estimations are calculated via the method described in Section 3.4.

sub-period (0.23, 0.47 and 0.33): This is to be expected considering the large change in  $\alpha$  over time. The large negative  $\alpha$  of -72.89% seen over the entire period is well below the convenience yields seen during sub-period 3, so during this sub-period reversion to a *mean* of -72.89% is not occurring. Similarly, -72.89% is above the  $\alpha$  for sub-period 1, so reversion to -72.89% is also not occurring. In comparison, the convenience yield during each sub-period will lie relatively close to their respective  $\alpha$ , increasing  $\kappa$ , as the convenience yield drifts around  $\alpha$ . Indeed, the results show this with a higher  $\kappa$  in each sub-period.<sup>40</sup>

The market price of convenience yield risk,  $\lambda$ , is slightly negative at -0.32%, but is close to zero across all periods. A negative  $\lambda$  implies that there is a positive market price placed on convenience yield risk, or a positive risk premium, and this premium is paid to the holder of the commodity via an increase in the convenience yield.<sup>41</sup> A negative value of  $\lambda$  is expected, since the convenience yield is risky, but  $\lambda$  is not significantly different from zero, implying that no risk premium is present

<sup>40</sup>Furthermore, when investigating more regularly updated parameters,  $\kappa$  trended upwards over time, as  $\alpha$  increased towards zero.

<sup>41</sup>To see this, consider equation 2 and see that a negative  $\lambda$  implies a higher expected value of  $d\delta$  via a higher drift term. This can also be seen directly in equations 14 and 15, where a negative  $\lambda$  decreases the value of  $A(T)$  and thus the futures price  $F$ , increasing the convenience yield. Equivalently, a positive  $\lambda$  implies that a risk premium must be paid to hold convenience yield risk, that is, to hold convenience yield risk the holder of the commodity must pay a premium via a decrease in the convenience yield.

for holding convenience yield risk. Daily values of  $\lambda$  are shown in Figure 9, showing that  $\lambda$  drifts with high volatility around zero and indeed that convenience yield risk is not efficiently priced in the bitcoin futures market.

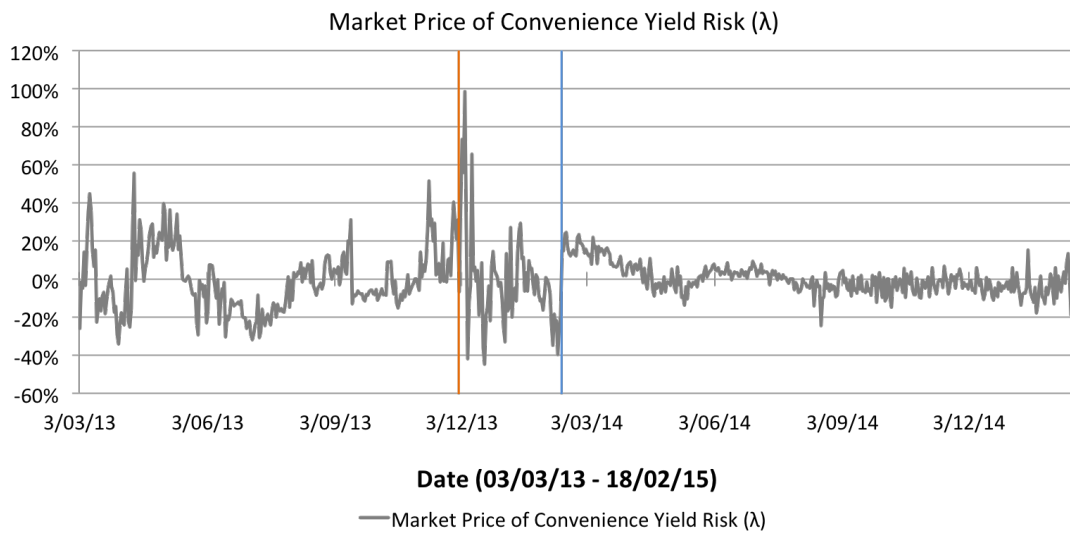


Figure 9: The market price of convenience yield risk,  $\lambda$ , per day and by sub-period. The orange line indicates the end of the major price spike (1 December 2013). The blue line indicates the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

Finally,  $\rho$  shows a positive overall correlation of 0.16 between the two error terms. Period's 1 and 3 show a correlation greater than 0.26, but period 2 shows a negative correlation of -0.16. This is likely because of the joint occurrence of unexpected spot price decreases and the reduction of over-optimistic futures prices, increasing the convenience yield, after the price spike in November 2013, causing a temporarily negative relationship. This is also expressed in Figure 10, which displays the residuals from the regressions of the estimation procedure, and a 30-day moving correlation between these residuals. In December 2013 the correlation between the residuals turns negative before going positive again in January 2014. It is also interesting to see other times with

negative correlations, but these are of lesser magnitude and duration and not unexpected in a highly volatile market.

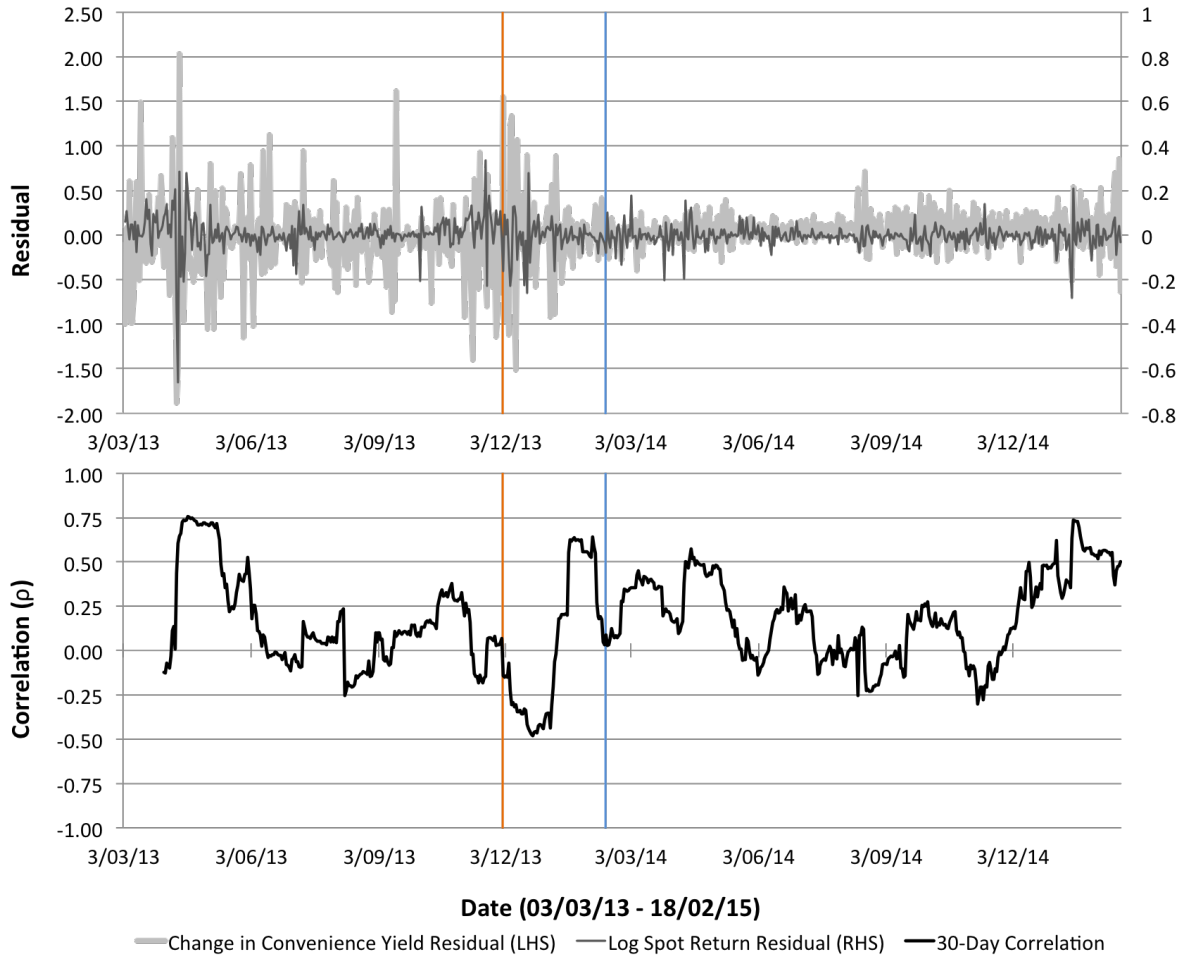


Figure 10: Residuals from the regressions of the estimation method described in Section 3.4. *Change in Convenience Yield Residual* refers to  $\epsilon_\delta$  in Model 16 and *Log Spot Return Residual* refers to  $\epsilon_s$  in Model 16. The first month of correlations is removed due to a low number of observations. The orange line indicates the end of the major price spike (1 December 2013). The blue line indicates the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

#### 5.4. Model Accuracy and the Pricing of Futures Contracts

To test the accuracy of the model, both in sample (residuals of the model) and out of sample (longer term maturity contracts) testing was considered. Both models were considered: the *overall model* which uses estimates over the entire time period (the first column in Table 5); and the *period model* which uses estimates based on the sub-periods defined earlier (the second, third and fourth columns in Table 5). It is noted that *out of sample* testing is still within the same time-period as the data that is gathered. This is because after 18 February 2015 there are numerous months where only one contract is being traded at a time, meaning the futures price used to estimate the convenience yield is the same futures price that is being estimated for testing. The out of sample testing done here uses futures contracts, although in the same time period, of longer maturities than the futures used to calculate the convenience yield. In particular, six contracts are considered and the futures price of these contracts is estimated from the start date to finish date of these contracts:<sup>42</sup> September 2013, December 2013, March 2014, September 2014, December 2014 and September 2015 (up until 18 February 2015).

Results of the in sample testing, that is, residuals of the model (the model estimates compared to the observed futures prices used to create the model), are shown in Table 6 and Figure 11. The statistics used to compare model estimates to observed futures data are the Root Mean Squared Error (RMSE), normalised root mean squared error, Mean Pricing Error (MPE) and normalised

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<sup>42</sup>There will be approximately one to two months where these contracts will have be used to estimate the convenience yield also, and this will become obvious in the results.

|                      | All   | Period 1 | Period 2 | Period 3 |
|----------------------|-------|----------|----------|----------|
| <i>Overall Model</i> |       |          |          |          |
| RMSE (\$)            | 0.59  | 0.57     | 0.66     | 0.60     |
| RMSE (%)             | 0.14  | 0.29     | 0.07     | 0.12     |
| MPE (\$)             | 0.11  | -0.21    | -0.26    | 0.42     |
| MPE (%)              | 0.03  | -0.11    | -0.03    | 0.09     |
| <i>Period Model</i>  |       |          |          |          |
| RMSE (\$)            | 0.84  | 0.62     | 1.92     | 0.60     |
| RMSE (%)             | 0.20  | 0.32     | 0.21     | 0.12     |
| MPE (\$)             | -0.12 | -0.02    | -0.15    | -0.18    |
| MPE (%)              | -0.03 | -0.01    | -0.02    | -0.04    |

Table 6: *Root Mean Square Errors* (RMSE (\$)) and *Mean Pricing Errors* (MPE (\$)) for estimated futures prices of in sample futures, that is, residuals of the model, from 03/03/2013 until 18/02/2015 using the overall and period models. *Normalised RMSE* and *MPE* are also displayed ((RMSE (%)) and (MPE (%)), which are the RMSE and MPE divided by the average futures price over the relevant period.

mean pricing error.<sup>43</sup> These statistics are used to look at both the overall accuracy (RMSE) and bias (MPE), that is, under- or over-pricing, of the model. As expected, the estimated futures prices are very close to the actual futures prices over the entire time period. When plotted in Figure 11, the two modelled futures prices and the observed futures prices are indistinguishable. Each normalised RMSE is within 0.32% of the observed futures price and each normalised MPE is within 0.04%, indicating that the model fits the data well.

<sup>43</sup>RMSE is defined as, for futures price  $F_t$  on day  $t$  and estimated futures price  $\hat{F}_t$ ,

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{n=1}^N (\hat{F}_t - F_t)^2}.$$

Normalised RMSE, also called *Coefficient of Variation of the RMSE*, is defined as  $\frac{\text{RMSE}}{\bar{F}_t}$ , where  $\bar{F}_t$  is the average of  $F_t$  from  $t = 1$  to  $N$ . Similarly, MPE is defined as

$$\text{MPE} = \frac{1}{N} \sum_{n=1}^N (\hat{F}_t - F_t)$$

and normalised MPE is defined as  $\frac{\text{MPE}}{\bar{F}_t}$ .

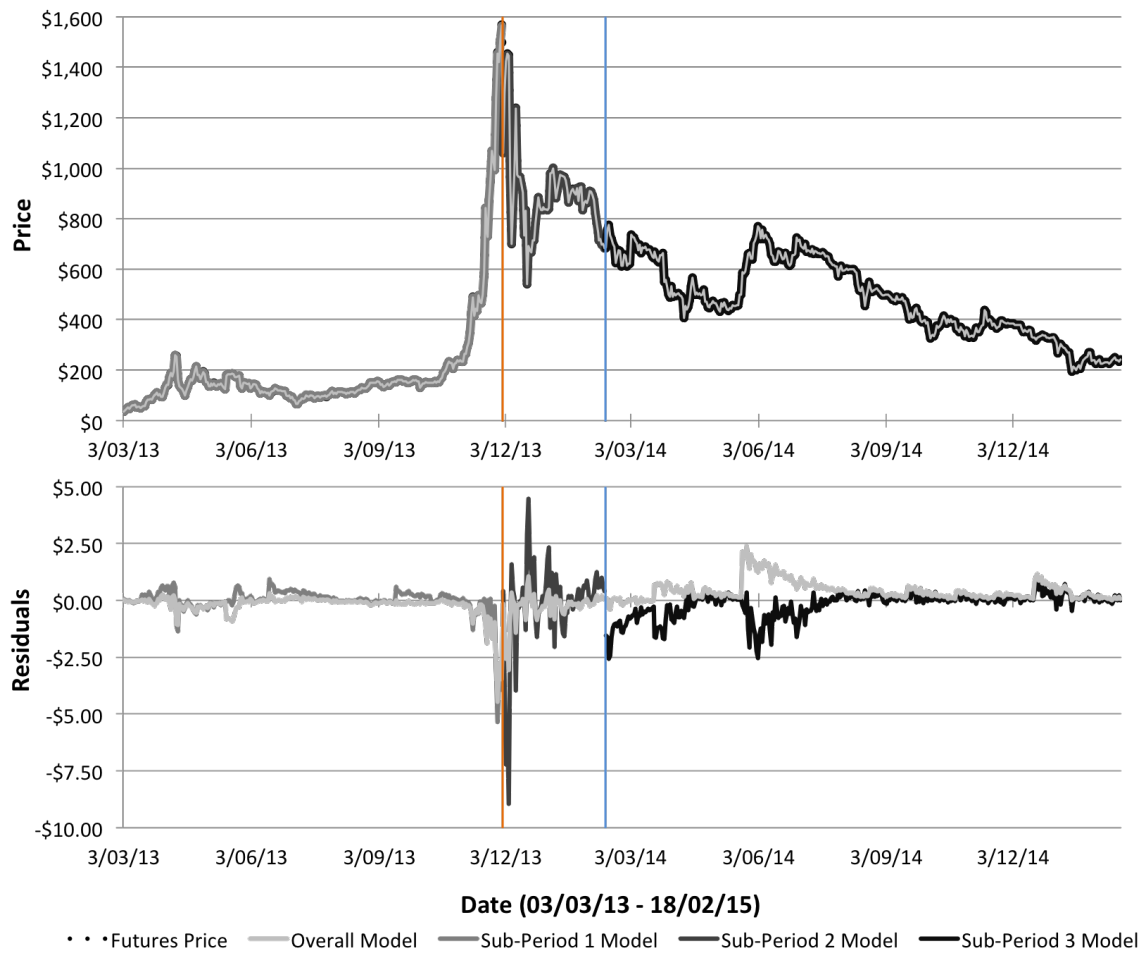


Figure 11: Futures prices and residuals from in sample testing of the overall and Sub-Period models. The orange line indicates the end of the major price spike (1 December 2013). The blue line indicates the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

It is interesting that the overall model performs slightly better over the entire period than the period model. Table 6 shows that the RMSE is lower for the overall model than the period model across all periods, but the MPE is lower for the period model than the overall model across all periods. One would expect that more regular updating of the parameters would provide a more accurate model, but Figure 11 shows that the residuals in sub-period 2 are particularly volatile

|                      | Sep 13 | Dec 13 | Mar 14 | Sep 14 | Dec 14 | Sep 15 |
|----------------------|--------|--------|--------|--------|--------|--------|
| <i>Overall Model</i> |        |        |        |        |        |        |
| RMSE (\$)            | 47.90  | 37.30  | 123.57 | 25.86  | 17.99  | 50.30  |
| RMSE (%)             | 36.15  | 12.66  | 19.16  | 4.10   | 3.37   | 13.88  |
| MPE (\$)             | 19.80  | 1.22   | 50.65  | 0.03   | -2.67  | 19.11  |
| MPE (%)              | 14.94  | 0.41   | 7.86   | 0.00   | -0.50  | 5.27   |
| <i>Period Model</i>  |        |        |        |        |        |        |
| RMSE (\$)            | 47.76  | 37.57  | 120.57 | 23.92  | 18.66  | 43.05  |
| RMSE (%)             | 36.04  | 12.75  | 18.70  | 3.80   | 3.50   | 11.88  |
| MPE (\$)             | 20.25  | 1.89   | 50.96  | -4.76  | -5.60  | 10.50  |
| MPE (%)              | 15.28  | 0.64   | 7.90   | -0.75  | -1.05  | 2.90   |

Table 7: *Root Mean Square Errors* (RMSE (\$)) and *Mean Pricing Errors* (MPE (\$)) for estimated futures prices of various out of sample futures contracts, that is, longer term maturity contracts, from 03/03/2013 until 18/02/2015 using the overall and period models. *Normalised RMSE* and *MPE* are also displayed ((RMSE (%)) and (MPE (%)), which are the RMSE and MPE divided by the average futures price over the relevant period.

for the period model, which may be due to a limited number of observations used to estimate the parameters compared to the other periods.

Results of the out of sample<sup>44</sup> testing, that is, longer term maturity contracts, are shown in Table 7 and Figure 12.<sup>45</sup> The results are volatile and relatively inaccurate, but large movements in pricing errors are expected due to the small market capitalisation of bitcoin and, in particular, Orderbook. Again, the two models show similar results, but in contrast to the in sample testing the period model now records lower smaller RMSE and larger MPE than the overall model. Both models tend to overprice futures prices with mostly positive MPEs, however, the September 2014 and December 2014 contracts show much more accurate estimates due to these contracts not experiencing periods of high volatility, unlike the preceding contracts (see Figure 7).

<sup>44</sup>It must be noted that each contract, other than the September 2015 contract, shows a period of approximately 1 to 2 months with near zero pricing errors. This is because the futures contracts being used to calculate the convenience yield during these periods are same futures contracts that are being estimated for testing. These periods thus fall under in sample testing. The September 2015 contract, however, is not used at any point to calculate the convenience yield

<sup>45</sup>Figures of the period model have not been provided, as they show similar results to the overall model

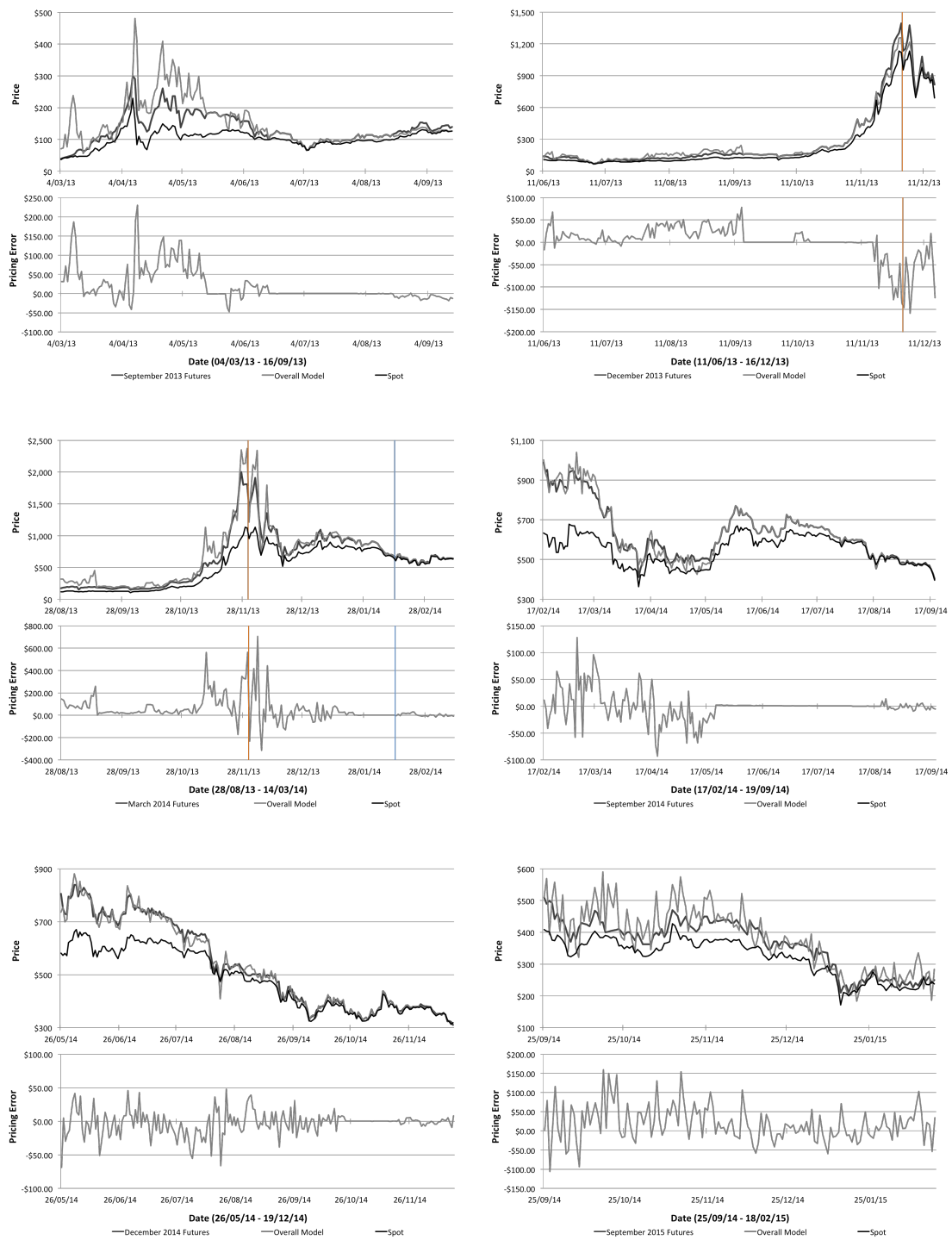


Figure 12: Estimated futures prices and pricing errors of long-term contracts from March 2013 to February 2015 using the overall model. The orange line marks the end of the major price spike (1 December 2013). The blue line marks the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

Older contracts show worse performance than recent contracts, which is again not unexpected due to the high volatility seen during 2013. The normalised RMSE is largest for the September 2013 contract at 36%, representing a substantial mispricing of the futures price for this contract, but the remaining normalised RMSE are all below 20%, and although still substantial, show normalised MPEs of less than 8%, which is surprising given the volatility of these markets. The December 2013, September 2014 and December 2014 contracts all show normalised MPEs of less than 1.05%. The period model noticeably shows superior performance over the overall model for the September 2015 contract, the only contract considered that was not used in the calculation of the convenience yield. Having said this, the results are worse than the preceeding two contracts, with larger and more volatile pricing errors than the September 2014 and December 2014 contracts (see Figure 12).

In summary, although the models fit the data well they do not consistently accurately price longer term contracts, but this is to be expected due to the low market capitalisation of the bitcoin and bitcoin futures markets. However, small MPEs suggest that, given reduced volatility or an increase in market capitalisation or trading volume of these markets, the use of these types of models may provide reasonable results. It is also possible that the parameter estimations for short term contracts fundamentally differ to those for longer term contracts, which would also explain the pricing errors.

### *5.5. Application to Longer-Term Futures Contracts*

The Gibson-Schwartz model was designed to be applied to the valuation of commodity contingent claims, such as the expected return of starting up a bitcoin mining operation or the pricing

of futures contracts, and in this section longer term hypothetical future contracts are considered. Hypothetical futures contracts of 3- and 6-months are priced using the overall model and the results are shown in Figure 13. The estimated futures rise very quickly during the peak periods, with futures of longer time-to-maturities, such as 12-months, reaching several thousand dollars. Although this is almost surely a mispricing of longer time-to-maturity contracts, it is no surprise that modelled futures prices rises rapidly in this hypothetical case when observed futures prices were \$2000 with less than 4 months until maturity (also, \$1568 with 46 days until maturity and \$1395 with 16 days until maturity) when the spot price was half this price.

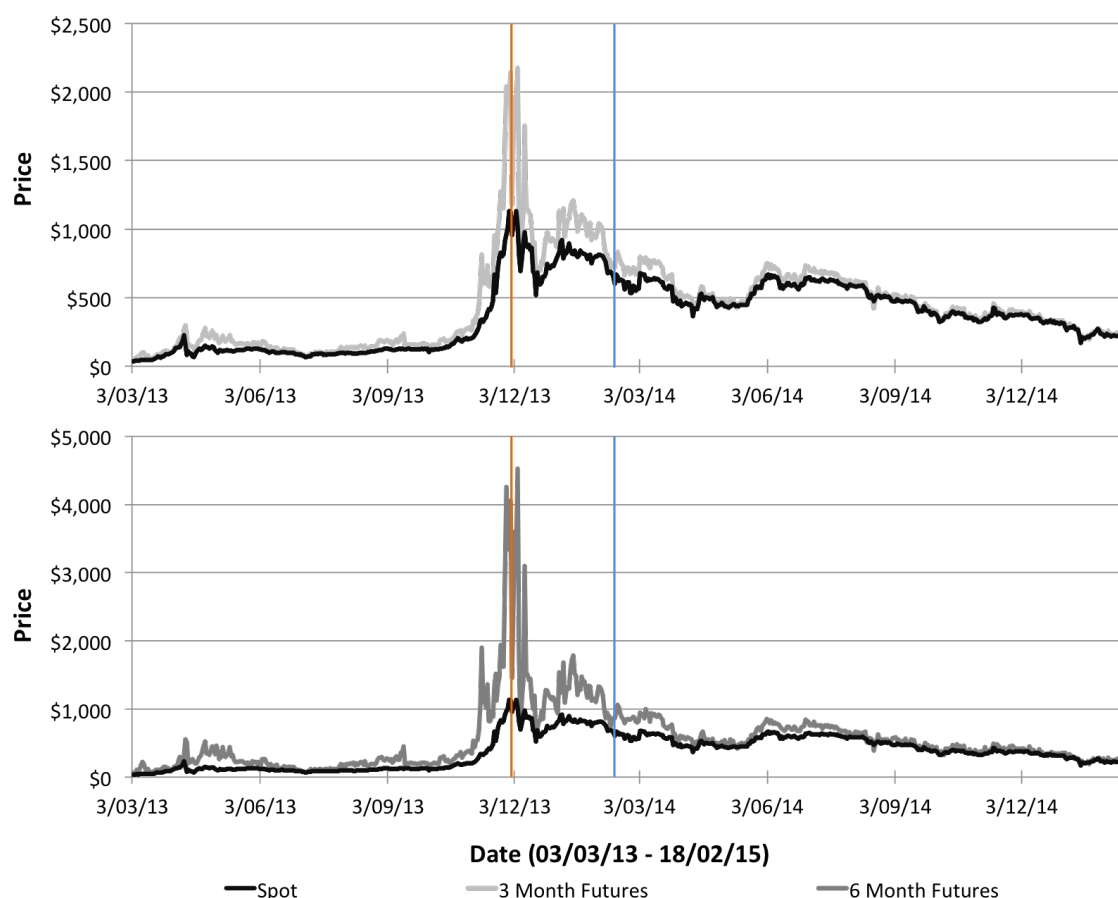


Figure 13: Estimated futures prices of hypothetical 3- and 6- month contracts using the overall model. The orange line marks the end of the major price spike (1 December 2013). The blue line marks the change in settlement price calculation from a 30-day to a 1-hour volume-weighted average (13 February 2014).

Looking specifically at a more recent period, the final 6 months of the time period, when market volatility lessened, shows more reasonable results. Estimated futures prices of hypothetical 3-, 6- and 12-month to maturity futures contracts during the final 6 months are shown in Figure 14. Each of the estimated futures prices shows high volatility, but are more feasible than those estimated during 2013, which, as mentioned, were often priced at several thousand dollars due to a large negative convenience yield, an occurrence no longer seen (marked by sub-period 3 above).

Although, the appropriateness of the model in pricing longer term futures contracts is unclear, this indicates a similar conclusion to Section 5.4, that given more reasonable levels of volatility in the market and higher trading volume and market capitalisation, the use of models of this type may provide reasonable results.

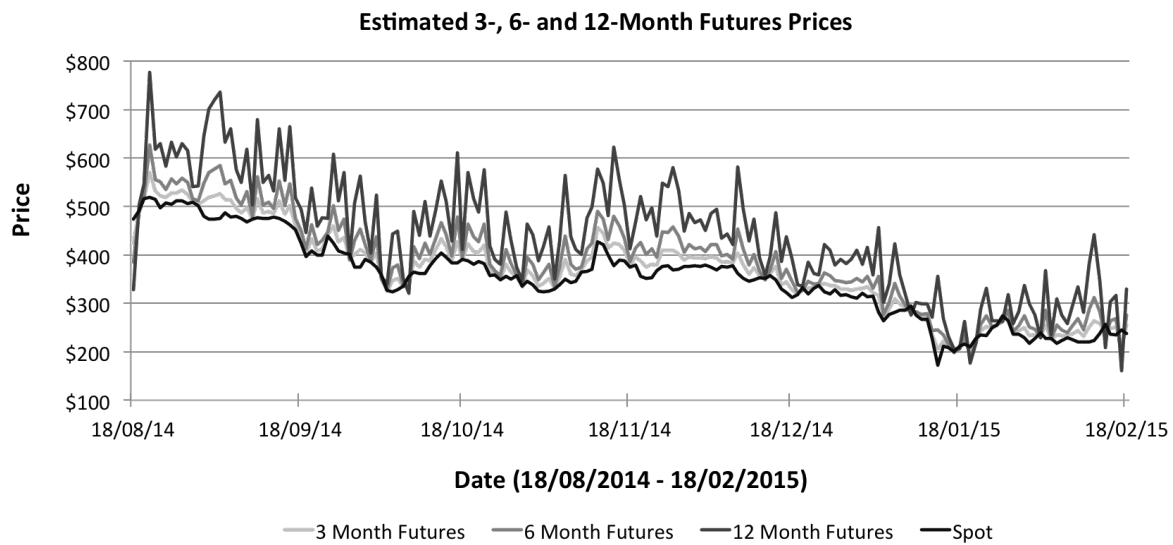


Figure 14: Estimated futures prices of hypothetical 3-, 6- and 12- month contracts for the final 6 months of the time period using the overall model.

## 6. Conclusion

This thesis analysed the bitcoin spot and futures markets and applied the well-known Gibson-Schwartz stochastic commodity pricing model to these markets. To the best knowledge of the author, this is the first study in the area of bitcoin futures since the inception of the world's first bitcoin futures market in June 2012 through OrderBook, and is of interest for several reasons. Firstly, it provides a view of the dynamics occurring in the bitcoin spot and futures markets, which are required for investment decisions. Secondly, it attempts to value futures contracts, which are

required to manage risk undertaken in investment decisions. Thirdly, it assists in determining what risks are in the bitcoin spot and futures markets. Finally, it adds to the discussion confirming or opposing the view of bitcoin as its own asset class, part of another asset class or neither. Furthermore, if the world is heading towards virtual currencies then it is important to know about their markets.

Bitcoin is a virtual currency, but the mining process that removes the need for a central bank gives it characteristics not unlike commodities, such as gold. This provides an argument that the theory developed for commodities can be applied to investigate bitcoin, such as studying deviations from the cost-of-carry relationship between the bitcoin futures and spot markets. Data from the bitcoin spot and futures markets from March 2013 until February 2015 was used for this purpose, an interval that includes two separate periods of substantial increase in the bitcoin spot market. Upon looking at these markets, significant contango was found and large arbitrage profits were readily available, with an average convenience yield of -72.7%. After 13 February 2014 the convenience yield trended towards zero with a much lower average of -27.3% compared to below -100% seen in 2013. This was caused by a stabilisation of the spot price and the exploitation of arbitrage opportunities. When a simple arbitrage strategy of going long bitcoin and short bitcoin futures is conducted during this period in 2014, it is indeed found that the return is near zero and arbitrage opportunities have been exploited (see Table 4). In addition, it is found that there is a negative relationship between spot price volatility and the convenience yield, the market is over-optimistic, and some similarities can be drawn with the electricity market.

In terms of pricing claims contingent on bitcoin, such as futures contracts, the parameters of the Gibson-Schwartz model were estimated. The results indicated that the market shows mean-

reversion of the convenience yield, but inefficiently prices convenience yield risk. The model was found to fit the data well, however it did not accurately price longer term to maturity futures contracts, which is not unexpected given the low market capitalisation and trading volume of the bitcoin and bitcoin futures markets. Given reduced volatility and an increase in market capitalisation or trading volume of these markets, the use of these types of models may still provide reasonable results.

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