MATHEMATICAL PATTERNING IN EARLY CHILDHOOD:
AN INTERVENTION STUDY

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DEDICATION

In memory of my father, Vladimir

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who had a passion for books,

a love of learning and

who would have been very proud of both this thesis

and my achievements.

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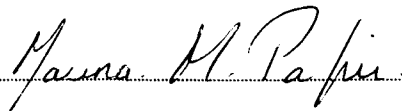
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STATEMENT OF CANDIDATE

I hereby certify that this work has not been submitted for a higher degree to any other university or institution.

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SYNOPSIS

Patterns are widely recognised as the foundation of mathematics. However, it is not yet fully understood how patterning influences the development of representation, symbolisation, abstraction and generalisation in young children's mathematical thinking. A central problem is that patterning has not been considered critical to the development of key mathematical concepts and processes, or early algebraic thinking.

It is believed that children in the elementary grades are not capable of mathematical generalisation until formal algebra instruction in the secondary school (Carraher, Schliemann, Brizuela, & Earnest, 2006). Recent studies provide evidence that students' later difficulties in algebra may not be a result of developmental constraints after all, but rather, from the limited approach to teaching elementary mathematics (Carraher et al., 2006).

The study raises four key questions: What are the characteristics of mathematical patterning young children develop naturally prior-to-school? In what ways does an intervention program promoting mathematical patterning impact on the complexity of children's patterning concepts and skills and the development of other mathematical processes such as multiplicative thinking? Is the influence of such an intervention maintained after one year of formal schooling? If so, in what ways? What is the role of patterning in the development of early algebraic thinking?

This study describes the patterning skills young children develop prior-to-formal schooling and implements an intervention that promotes the development of a broad range of patterns: *Repeating Patterns*, *Spatial Patterns* and *Growing Patterns*. The study is significant because it identifies how children as young as four years-of-age construct and represent simple and complex patterns using a *unit of repeat*, and how they apply this to other forms of pattern. The design allows the monitoring of 53 young children's pre-algebra (patterning) skills from preschool to the end of the first year of formal schooling. Case-studies of two preschools ('Intervention' and 'Non-intervention') are compared in order to examine the influence of a mathematics intervention promoting children's patterning over a 6-month period. One-to-one task-based interviews were conducted at three intervals over an 18-month period. The study was designed as an intervention employing a mixed-method approach: integrating a traditional constructivist-based teaching experiment (Hunting, Davis & Pearn, 1996) with more contemporary aspects of a design study (Dede, 2005).

The Intervention comprised three distinct components: Structured individual and small group work on pattern-eliciting tasks, 'patternising' the regular preschool program, and observing children's patterning in free play. Using a *Framework of Assessment and Learning,* children were placed on an individual 'learning trajectory' and progressed through an increasingly complex series of tasks. Analysis of children's progress focused on levels of structure and abstraction. Further, the Intervention provided on-going professional development of the importance of pattern and structure in early mathematical learning, which assisted teachers in modifying the emergent curriculum to incorporate patterning skills.

Intervention children could successfully identify, construct and abstract the element within *Repeating Patterns* and calculate the number of repetitions. This was the dominant strategy used by Intervention children at Assessment 2 and sustained at Assessment 3 (12 months later). Many children used their knowledge of *unit of repeat* to extend and represent patterns in other forms. They were also able to draw complex repetitions from memory. The development of structural thinking about simple repetition, not just the modelling of simple repetition, advantaged the Intervention children. When dealing with *Spatial Structures* such as arrays of dots, Intervention children could perceive the structure of the patterns. In comparison, Non-intervention children's responses lacked any structural features. Another critical learning process observed during the Intervention was the children's development of transformation skills; they successfully used rotation to construct *Hopscotch* patterns and visualised simple and complex repetitions from different orientations.

The assessment of counting and arithmetic development provided by the *Schedule for Early Number Assessment* (SENA 1), administered at the third assessment, showed that Intervention children's numerical strategies were more advanced than Non-intervention children. Some were quite advanced in their arithmetic strategies, using known facts and other non-count-by-one strategies. Further analysis of SENA interview data indicated that Intervention children recognised the structure of the patterns and partitioned the patterns into parts rather than counting individual items.

Intervention children successfully symbolised, abstracted and transferred complex *Repeating Patterns,* and with no apparent exposure to *Growing Patterns,* many of these children could construct, extend, represent and justify these patterns 12 months after the Intervention. In contrast, Non-intervention children were unable to identify or extend G*rowing Patterns.* They saw these exclusively as 'items' in simple repetitions in the same

way as the simple repetitions that they were familiar with. These findings support those found by Warren (2005a), where 9-year-olds had greater difficulty with *Growing Patterns* than with *Repeating Patterns*. It was inferred that the difficulty with *Growing Patterns* was not necessarily the absence, or predominance of *Repeating Patterns* in early mathematics curricula. Rather, the inadequate or inappropriate development of repeating patterns without a sound understanding of the unit of repeat, limited, and possibly impeded the development of *Growing Patterns*. Children may be able to copy and extend patterns, but they may not necessarily identify a *unit of repeat*.

The findings support Blanton and Kaput's (2004) conclusion that early algebraic learning is not developmentally constrained; young children have natural powers of generalisation and an ability to express generality (Mason, 1996). This study recommends that experiences in the first year of schooling focus on identifying, justifying and transferring various patterns, and using a variety of materials. Further, the study suggests repeating patterns should include not just "recognising, copying, continuing and creating" (Board of Studies, NSW, 2002, p. 73) simple linear patterns but rather, identifying the element within repeating patterns, the number of repetitions, drawing from memory, viewing patterns from different orientations, extending a pattern in multiple directions, and transferring a pattern to a different medium. Professionals must be aware of the natural patterning experiences in children's play and ask appropriate questions that will promote mathematical thinking. This can only be achieved through programs that integrate effective professional development that build teachers' knowledge and expertise and provide them with the necessary conceptual structures to take ownership of their planning and teaching.