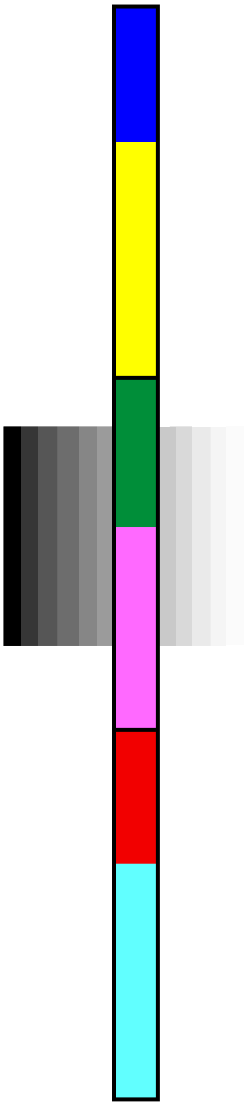

Roles of Complementary Colours in Colour Perception

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DECLARATION BY CANDIDATE

I state that this thesis has not been submitted for a higher degree to any other university or institution.

Ralph W. Pridmore 1 May 2008

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SUMMARY

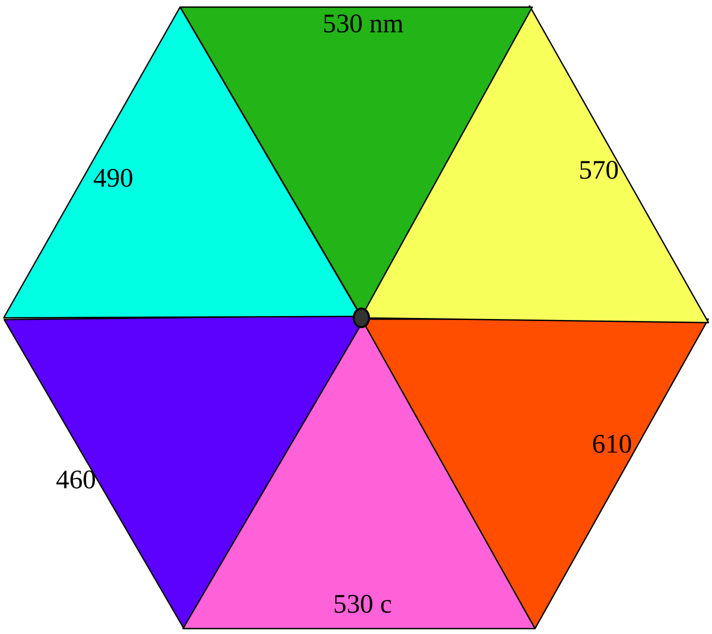
Complementary colours have been studied in science for over 300 years but little is known of their role in colour vision. Complementary colours are defined as a pair of colour stimuli which, with appropriate complementary wavelengths and ratios of radiant powers, admix a selected white (e.g. that of the illuminant). The thesis aim is to find and describe roles of complementary colours in colour perception. Only one role is generally accepted in the scientific literature: a colorimetric role in colour mixture and matching. A second role, in chromatic induction, was widely accepted until (erroneously) usurped 50 years ago by the opponent colours theory of chromatic induction. This thesis stemmed from signs that the visual system structures various visual functions from complementary colours.

Complementarism is shown to structure ten functions. For six of the functions, math models are formulated from complementary colours. The first three, closely related, papers describe three roles in colour constancy (see Appendix: Definitions), hue cycle structure, and hue cycle relative wavelength metric. Paper 1 describes complementary wavelengths' central role in a spectral mechanism of colour constancy, where constant hue and its complementary constant hue form parallel straight lines. Paper 2 establishes the roles of complementary colours in forming hue cycle structure and its wavelength-based metric. Dominant wavelength is extended into the nonspectrals/purples to form a *relative wavelength* scale over the whole cycle. Paper 3 utilises this scale to extend Paper 1's spectral mechanism of colour constancy into the nonspectrals to complete the global mechanism. Paper 4 argues that chromatic induction is governed by complementary colours rather than opponent colours. Paper 5 formulates math models for six roles, in chromatic induction, wavelength discrimination, uniform hue, spectral sensitivity, saturation, and chromatic adaptation, from the ratio of *either* complementary wavelength intervals *or* of complementary powers; a seventh role, in three-dimensional color discrimination, is described. The total eleven varied roles (including the known colorimetric role) have a wide influence over the visual process from cones to cortex. These discrete roles indicates an overall role of complementary colours is structural, in shaping functions to a trimodal framework (RGB peaks and complementary CMY troughs), whose implicit purpose is to adapt functions to the illuminant. Thus, it is concluded the general role is colour constancy.

INTRODUCTION

Definitions of Complementary Colours

Complementary colours were well known to artists as expressions of simultaneous contrast and successive contrast (e.g. after images; see Appendix: Definitions, this chapter), and were first described in detail by the artist and scientist Chevreul in 1839,[1] followed by Rood[2] and later by Wilson and Brocklebank.[3] A pair of complementary colours was initially defined as the focal colour and its induced after image. For example, if one gazes at the red triangular area in Figure 1 for 30 seconds, and then transfers the gaze to a white or grey area, one will perceive an after image of the same triangular shape but a cyan (aqua or turquoise) colour, i.e. the complementary hue to red. The



after

Figure 1. Trimodal structure (Red-Cyan, Green-Magenta, Blue-Yellow) of complementary colours, showing approximate wavelengths, nm. This figure illustrates the complementary basis of after images. Gaze at the central black dot for 30 seconds and then at a sheet of white or grey paper to see

the (fainter) complementary after images.

image is fainter than the focal, or inducing, colour.

The first scientific definitions, dating from about 1936,[4] state complementary colours are a pair of colour stimuli, of appropriate wavelengths and power ratio, that mix (by additive, subtractive, or any reliable method) a selected achromatic colour (e.g. the white or gray of the light source). Wilson and Brocklebank[3] demonstrated that complementary colours produced from the admixture of lights are slightly different from those perceived as after images; however, that difference seems mainly due to hue shift effects (Abney effect)[5] from the desaturated after image.

The term “complementary” derives from the Latin “complementum” meaning something “which fills up or completes.” The Oxford Dictionary defines “complementary” as “Something, which when added, makes up a whole; each of two parts which mutually complement each other ...”

Early Research of Complementary Colours

Little is known of the roles of complementary colours in vision, despite some 300 years of research. Isaac Newton was the first to study light and colour on a scientific, experimental, basis.[6,7,8] In a letter to the Royal Society (1673),[9] following his first paper,[6] he discusses the admixture of pairs of lights to form white, that is, complementary colour pairs (to use the modern term): “But yet there is an experiment or two mentioned in my (previous) letter ... by which I have produced White out of two colours alone, and variously, as out of Orange and full Blew, and out of Red and pale Blew, and out of Yellow and Violet, as also out of other pairs of Intermediate colours.”

Yet, in his major treatise on light and colour (Opticks, 1704),[8] Newton appears to have forgotten these experiments because he says, “For I could never yet by mixing only two primary (*spectral*) Colours produce a perfect white.” Given these two conflicting statements, the matter of Newton’s experimenting with complementary colours is controversial.[10] However, his well known colour circle diagram,[8] shown in Figure 2, where saturated colours lie on the circumference and white lies at the centre, shows that diametrically opposite colours mix white. Hence, certainly in theory, Newton accepted that complementary pairs of colours mix white. This part of Newtonian theory on colour mixture was accepted by subsequent major European colour scientists, for example Helmholtz,

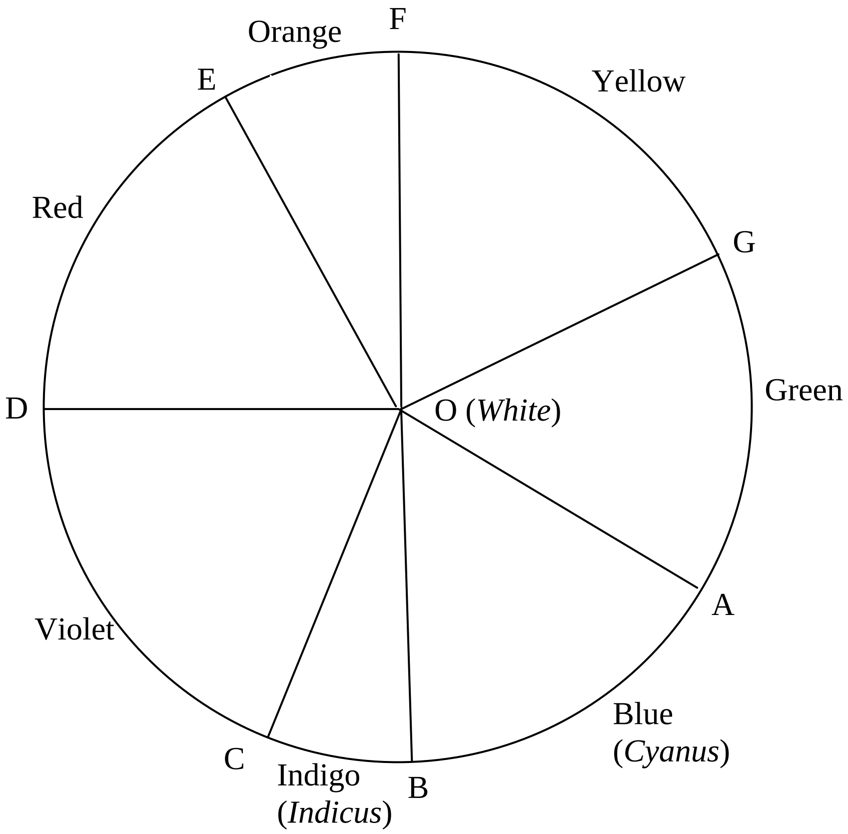


Figure 2. Diagram illustrating relations between hues and centre of gravity principle of colour mixture, after Newton.[8] O is centre of circle, representing White. Text in brackets added by the present author, from Newton's text elsewhere. Nonspectral purples (mixed from Red and Violet) seem to be omitted.

Rumford, Grassman, and Maxwell,[10] all experimenting in the mid 19th century on establishing theories of additive colour mixture (including complementary pairs) and subtractive colour mixture.

It is probable that Newton's difficulty in mixing a "perfect white" may have been due to his lack of adequate equipment to adjust the pair of wavelengths until exactly complementary, and/or adjust the ratio of powers until exactly complementary. Even today, though the latter ratio (of powers) can be calculated from formulae (see MacAdam and Pridmore, below), there is no such formulae to directly calculate correct pairs of complementary wavelengths, other than a complex algorithm employing CIE (Commission d'Eclairage Internationale) data and the CIE colour mixture diagram.

There is other evidence that Newton investigated and understood the complementary nature of colour. Newton studied and reported the phenomenon that bears his name as "Newton's rings". These are seen when two pieces of glass are pressed together. Newton describes this in Part I of "The Second Book of Opticks".[8] The relevant passage is "Observ. 9", quoted below:

"Observ. 9. By looking through the two contiguous Object-glasses, I found that the interjacent Air exhibited Rings of Colours, as well by transmitting Light as by reflecting it. ... Comparing the colour'd Rings made by Reflexion, with these made by transmission of the Light; I found that white was opposite to black, red to blue (*cyan*), yellow to violet, and green to a Compound of red and violet. That is, those parts of the Glass were black when looked through, which when looked upon appeared white, and on the contrary. And so those which in one case exhibited blue (*cyan*) , did in the other case exhibit red. And the like of the other Colours." (*Italicised parentheses added by the present author*.)

Newton’s description of the reflected colour and the transmitted colour clearly amounts to complementary colour pairs, including the “white ... opposite to black”. Given the Latin and English meanings, above, of “complement”, it follows that the complement to white is black and vice versa, where the admixture produces a grey. This, like all complementary colours, may be seen in after images: Observing from indoors a bright sky (effectively white) through a window with a dark crosspiece or window divider, for a few moments and then closing the eyes, produces an after image of a black square (the sky) and a white crosspiece.

Rumford[11] was probably the first (in 1802) to use the word *complementary* in the colour science sense of one colour being the complement of another in admixing white or grey.

Helmholtz,[12] despite his vastly superior equipment relative to Newton’s prisms, found only one pair of complements, as yellow and blue (but later acknowledged other complementary pairs).[13] However other workers, such as Grassman,[14,15] soon corrected the situation to show that every colour has a complementary colour, as predicted by Newtonian theory (see Fig 2 above), and that the mid-spectrum wavelengths (in the green area) have no complementary wavelength but are complemented by a pair of short and long wavelengths (appearing purple). Not only was Helmholtz mistaken about the number of complementary pairs (which is infinite) but he rejected the physiological theory of Thomas Young,[16] of three fundamental colour sensations in colour vision, nowadays universally accepted as the trichromatic theory of three cone types in the retina. Helmholtz, the leading name in

European colour science in the 19^{th} century, at times displayed a poor understanding of colour and vision and erroneously contradicted such scientists as Young, Grassman, and Hering.[17] To the first two, he later acknowledged his mistake but he never understood the (now widely accepted) opponent colours theory of Hering.

Grassman was a little known Sanscrit scholar who came into prominence with his mathematically analytical approach to colour mixture.[14,15] He entered the debate initially, and controversially, by criticising the 1852 colour theory of the famous physicist Helmholtz, particularly Helmholtz's eccentric assertion of only two spectrum colours that admix white. Grassman showed purely by logic that such an assertion was nonsensical and was contrary to Newtonian theory that every colour has its complementary, and even contrary to some of Helmholtz's own work. Grassman's main body of work is now known as Grassman's laws of additive colour mixture, and it was he who finally supplied a mathematical basis to Newton's colour mixture concept. His vector algebra was the first in colour science and led later to Cohen's brilliant matrix-R theory of colour mixture (below).[10]

James Clerk Maxwell studied colour mixture, particularly complementary colours, and the associated fundamentals of colour vision.[18,19,20,21] He at first used the rotary method (spinning a disc with coloured papers affixed) to admix colours. Later he constructed his major colorimetric equipment, a so-called colour box for colour matching, where prisms produced near-monochromatic colour stimuli. Subsequently he produced two experimental sets (triplets) of colour matching functions, one from his wife as the subject and one from himself. This represented the first attempt to systematically construct RGB (Red, Green, Blue) colour matching functions from experimental colour matching data.[19] His data were later found by Judd (1964)[22] to closely approximate CIE 1931 RGB colour matching functions.[23] Maxwell further developed Newton's system of colour mixture, assisted by Grassman's mathematical foundation.

Almost all Maxwell's research involved complementary colours to some degree. His study of colour blindness touched on the complementary relationship between the pair of colours [e.g. red and bluish-green (cyan in modern terms) in protanopia] confused with grey by colour-blind subjects. In his colour matching functions and similar three-part equations for mixing grey or white, the sum of any two (of the three) primaries is complementary to the third primary.

Maxwell studied and recorded more about complementary colours than any other scientist up to the present, but he never specified the overall or particular roles of complementary colours other than their role in colour mixture.

Modern Research of Complementary Colours

Helmholtz[13] first made the observation that the relative luminance and relative saturation of spectral colours were reciprocal. In the early 20th century, Sinden[24] developed this idea further by demonstrating experimentally that complementary spectral stimuli were reciprocal in luminance: a yellow, say 570 nm, has high luminance whereas the complementary colour, a blue of 460 nm, has reciprocally low luminance. (See Figure 1 for colours of those approximate wavelengths.) About the same time, Judd, amongst his many interests, recorded the spectral energy distributions of various complementary colour pairs.[25]

Soon after, MacAdam in 1938 formulated the basic photometric relationships between complementary colours,[26] including the ratio of complementary powers (i.e. the power of a colour required to neutralise its complementary colour of 1 Watt) for spectral complementary pairs. MacAdam's data were later extended to nonspectral colours by Pridmore,[27] to complete the hue cycle and to fill in the mid-spectrum gap for the green hues whose complementaries are nonspectral.

Pridmore[28,29] further utilised the complementary powers data to define optimal colour stimuli for aperture colours (see Appendix: Definitions) over the complete hue cycle. Pridmore's work usually concerns the total hue cycle rather than only the spectrum colours (which, according to colour appearance systems such as Munsell and the Swedish *Natural Colour System*, amount to some 70% of the hue cycle, leaving the remaining 30% to nonspectral hues). Optimal spectral stimuli lie in the range 442-613 nm ±1, for all CIE illuminants, from 2,500 degrees Kelvin (*K*) to 25,000 *K* colour temperature.[23,27,28] Beyond those limits, the photometric efficiency of monochromatic colours (i.e. single wavelengths) in colour mixture reduces rapidly to nil at the spectrum extremes which by definition are invisible. Optimal nonspectral stimuli (again for aperture colours) are compound colours comprising 442+613 nm ±1 (see Figure 3). Similar pairs produce optimal nonspectral stimuli for object colours. Pridmore, developing the ideas of Helmholtz and Sinden, later published a mathematical model of saturation and brightness, based on complementary colours.[30] The brightness model, defined in terms of luminance, was simple for monochromatic stimuli. But for less saturated colours (i.e. all object colours), the model was too complex to be of much practical use, though it accurately predicted the experimental data.

Jozef Cohen, an academic psychologist with a mathemetical bent, was working on colour

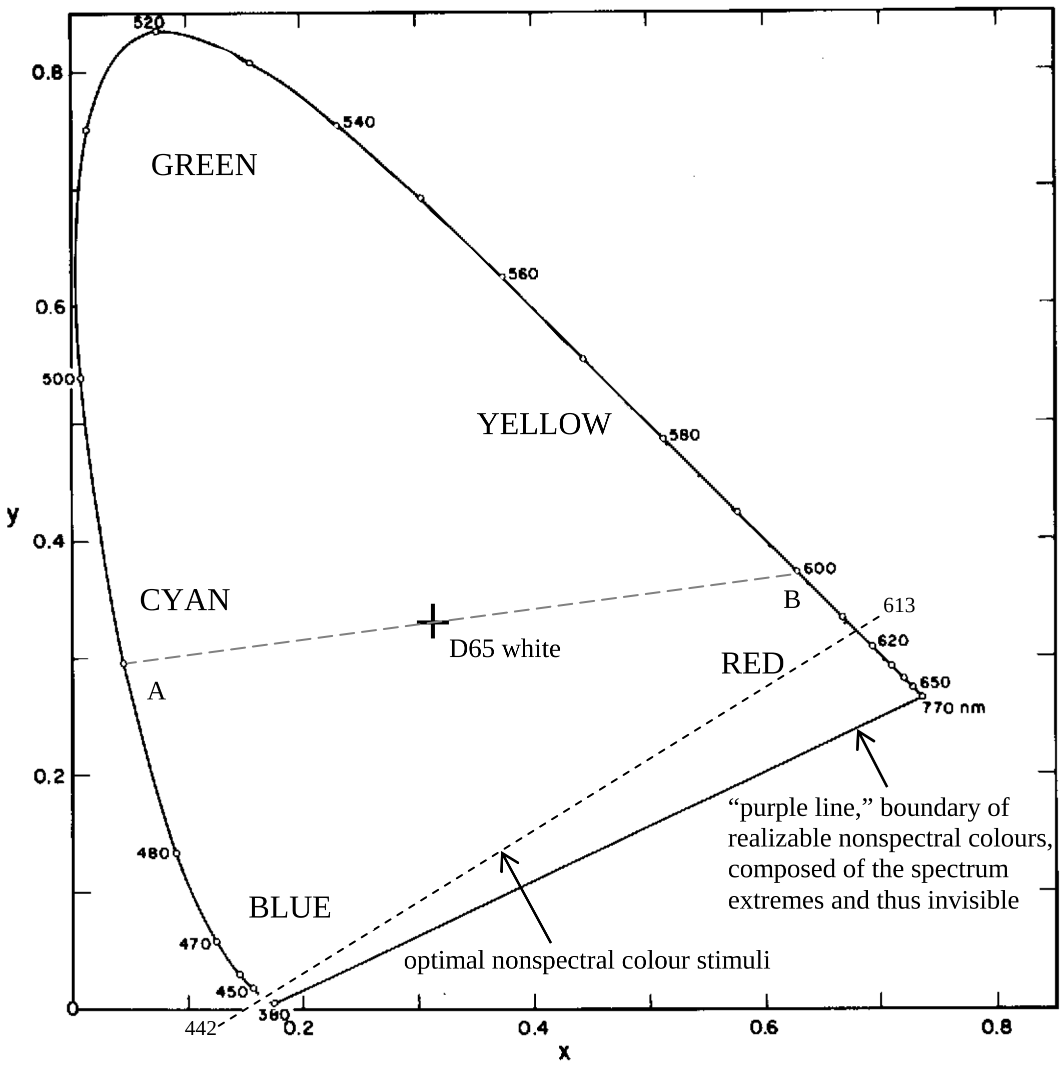


Figure 3. CIE 1931 chromaticity diagram, showing (dashed grey line) an example pair of complementary wavelengths A and B (490 and 600.5 nm) as opposites through the white point ("+")

for CIE standard Illuminant D65, and the locus (dotted line from 442-613 nm) of optimal aperture-colour nonspectral stimuli. Colour names indicate approx colour areas.

mixture and metamerism (Appendix: Definitions), which includes the special case of complementary colours, before and concurrently with Pridmore. The two communicated about complementary colours and hue cycle structure by correspondence. Cohen discovered new analytical and predictive methods using vector and scalar analysis, which he published in psychological journals.[31,32,33] But due probably to his radical methodology, Cohen was unable to publish his work in the colour science literature until 1988.[34] Cohen died in 1995 and his groundbreaking book, which was largely about metamerism including several chapters on complementary colours and their mathematical fundamentals, was completed by his colleagues and published in 2001.[10]

In the CIE system of colorimetry,[23] complementary wavelengths and chromaticities play an essential role in the design of chromaticity (or colour mixture) diagrams (see Figure 3), where complementary pairs are located as opposites on a line through the white point. In the case of nonspectral purples, which have no single wavelength, their chromaticities are denoted by their spectral complementary wavelengths, e.g. as 530 c (c for complementary) rather than 530 nm.

To summarise: Despite scientific research of complementary colours for 300 years, the role of complementary colours in vision is little understood other than: (1) a role in the basic colorimetry of colour matching and mixture; and (2) a role in chromatic induction (e.g. after images), though this role is currently disputed by the theory of *opponent colours* (Appendix: Definitions). The purpose of the present thesis is to redress this lack of knowledge on the roles of complementary colours in vision.

In contrast, the role of opponent colours in colour vision has been amply described and quantified in the Hering-Hurvich-Jameson theory.[35,36,37,38] Opponent colours occur in the later part of the visual process (concerning colour sensation), according to zone models of colour vision.[35]

Current Worldview of the Role of Complementary Colour in Vision

Muller and Judd types of zone models, published in early and mid 20th century,[39,40,41] ascribe complementary colours to the first zone (the retina's receptoral layer), corresponding to the traditional

Young-Helmholtz trichromatic theory[13,16,35] including basic colorimetry[23] (e.g. colour matching functions and data). The second or third zone corresponds to Hering's opponent colours theory[15,36,37] and concerns colour appearance, that is, the conscious perception or sensation of colours.

This exclusion of complementary colours from a role in colour appearance is not supported by physiological or psychophysical evidence and seems motivated more by the mid-20th century wave of enthusiasm for opponent colours theory. There were, however, examples where complementary colours were thought important to colour appearance: Chevreul based his colour harmony system on complementary colours, while Ostwald and Munsell based their uniform hue difference scales[42] largely on complementary wavelength pairs. Moreover, the three RGB peaks of complementary colours' relative radiances (at 445, 532, 606 nm, illustrated later below) do not reflect the cone sensitivity peaks (about 445, 530, 565 nm) of the receptor level but instead indicate spectral sharpening (where the R peak of sensitivity shifts from the cone sensitivity peak at 565 nm to about 600 nm),[43] which only occurs at a post-receptoral stage of vision.

Indications of Other Roles for Complementary Colours

There are indications that complementary colours have other roles to play than in basic colorimetry or in the early retinal layers of the visual process. The most obvious signs lie in the fact that colour matching functions (including experimental data on complementary colours) and complementary colours' relative radiant powers both demonstrate spectral sharpening of their RGB peaks,[25] particularly in the Red peak near 600 nm rather than the 565 nm peak of the "Red" cone.[35] This fact alone indicates a post-receptoral role for complementary colours. Further, there are several visual functions, such as wavelength discrimination and spectral sensitivity or responsivity,[44] that share very similar RGB peaks, thus inferring some part played by complementary colours.

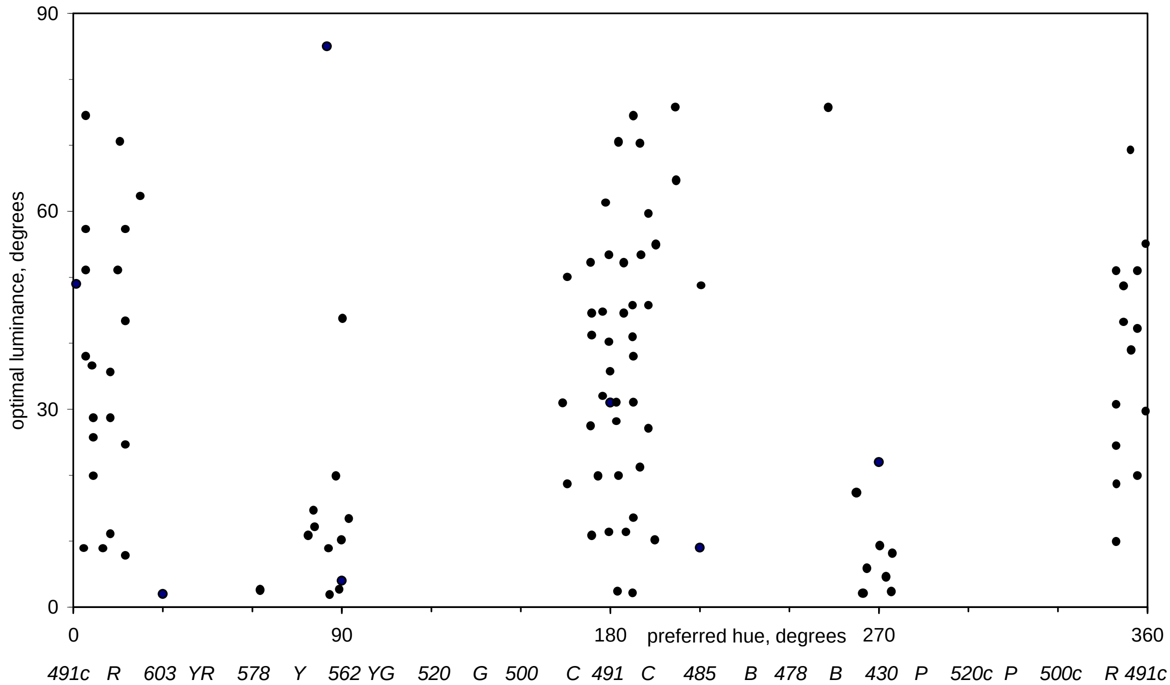
Since about 2000, the author had noticed in his research that data for corresponding (or constant) hues in various illuminants, when plotted in a plane of wavelength and reciprocal illuminant colour temperature (see Appendix: Definitions), tended to lie on straight lines and have complementary wavelengths that also plotted straight lines, practically parallel to the first lines. This level of symmetry, to the author's knowledge, is previously unknown in colour science, and suggests an important role for complementary colours in colour constancy, to be explored in the present thesis.

For three decades, the author has noticed that the neurophysiological data for the responses of so-called opponent colour cells[45,46,47] (more correctly termed spectrally-opposed cells) of the centre-surround type or double-opponent type, in retina and cortex of vertebrate animals, more accurately reflected complementary colours than opponent colours (see Appendix: Definitions). This would indicate the presence of complementary colours in late retina and in cortical processes of vision, a controversial concept at this time.

These physiological data on so-called opponent colour cells when first discovered in the late 1950s and early 1960s attracted wide attention throughout the colour science, neuroscience, and cognitive science, communities and were generally thought to provide experimental evidence[48] for the Hering-Hurvich-Jameson opponent-colour theory of colour vision.[15,36-38] There is now, however, a renewed interest in complementary colours (as noted below) and increasing evidence in the literature that *opponent colour* is too simplistic a description of spectrally-opposed cells,[49] and that the latter in many cases actually reflect complementary colours (see Figure 4). In recent years, complementary colours, in the form of terms (e.g. R-C, G-M) or relations (e.g. opposite chromaticities on a line through the illuminant point), have been used increasingly in the physiological literature.[50,51,52,53]

Figure 4 shows data from some 230 single cells with Y-B and so-called R-G spectrally-opposed responses.[54,55,56] Clearly, the supposed R-G responses in both Figures 4A and 4B are better described as complementary Red-Cyan responses (classing Cyan as about 485-495 nm), and are closely grouped around 0 and 180 degrees azimuth. The angles 0-180 degrees describe a straight line drawn between two complementary wavelengths through the illuminant point in the CIE 1931 diagram, and termed the R-G axis by most authors. However, its dominant wavelengths (491 c and 491 nm) show it is actually the Red-Cyan axis (the term used by a very few authors).[51,53]

Data on triphasic cells,[57] found in vertebrates, are sometimes thought to represent a R-G response. Triphasic cells have response peaks in three wavelength regions: a peak in medium-wavelength (producing green sensation) opposed by peaks in short- and long-wavelengths (admixing a purple sensation). But this clearly is a green-purple or G-M response, which indicates complementary colours (see Figure 1 and the Green-Magenta pair) rather than the Green-Red of opponent colours.



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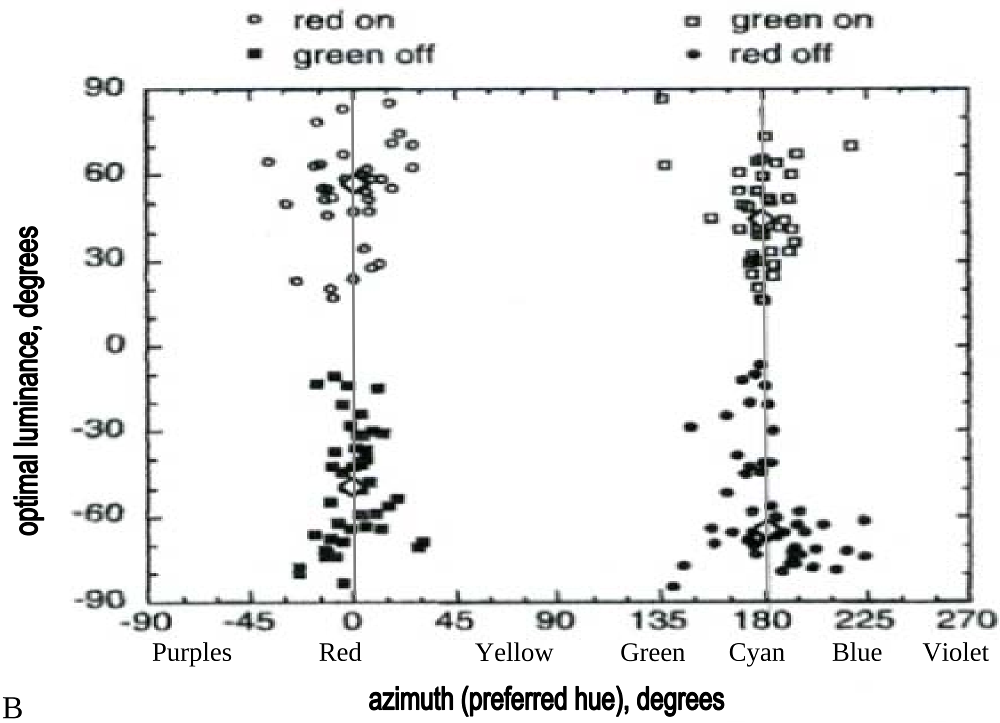

B

Figure 4. Colour tuning of some 230 cells in the macaque LGN. **A.** Adapted from Fig. 4a of Ref.[54], itself adapted from Ref.[55], showing so-called Red-Green (really Red-Cyan) opposed responses clustered at 0/360 degrees and 180 deg (i.e. 491 c and 491 nm), and opposed Yellow-Blue responses clustered at 90 and 270 deg (562 nm and 400-430 nm; the latter wavelengths are bunched in the CIE 1931 diagram). Note both R-G and Y-B pairs are complementary. Data points for some 50 cells are plotted to azimuth deg (representing preferred hues) and optimal luminance, deg. (Hue initials and wavelengths below the x-axis added by the present author.) **B**. As Fig **A** but from Ref.[56], showing so-

called R-G (really Red-Cyan) opposed responses clustered at 0 and 180 deg, for 177 cells.

The variety of neurons, and the variety of their locations in the brain, involved in the above references indicates a variety of roles for complementary colours.

Another indication of the role of complementary colours in cortical processes is chromatic induction (Appendix: Definitions), a common and important effect that has been recorded by artists and scientists for at least two centuries.[1-3,58,59,60,61,62] It is perhaps the most often reported effect involving complementary colours. The effect also occurs in conjunction with other effects, such as neon spreading.[63,64,65,66] Chromatic induction, whether simultaneous or successive contrast (the effects are the same),[67] is affected by attention. For example, an after image is rarely noticed since it rarely has an operational purpose, but is more a residual effect. Most people never notice an after image, unless an artist or scientist draws their attention to it. But one can perceive the after image if one concentrates on the perception. Since attention is a higher cognitive function of the cortex, and cannot enter the retina (in which all signal traffic is one-way, up to the cortex), it can be argued that chromatic induction (and thus the innate complementary colour process) is a process that exists in the cortex. Further, chromatic induction represents perception of colour, a cortical rather than retinal activity.

In colour blindness, the confusion hues, or hues confused with grey or with each other, are complementary in fact,[35,68,69] if not noted as such in the literature. For example, in protonopia, red and blue-green (a complementary pair) are confused with grey. In deuteranopia, green and purplish red (a complementary pair) are confused with grey. And in tritanopia, greenish yellow and blue-violet (a complementary pair) are confused with grey. These pairs have been established largely through subjects with one good eye.[70,71] The above common forms of colour blindness are generally considered to be visual systems that are fully operational in persons of normal vision. Hence the presence of complementarism in these visual systems indicates an operational role in those systems.

The effect of variable purity on hue (Abney effect),[5,72,73,74,75,76,77] and the effect of variable brightness or lightness on hue (Bezold-Brucke effect),[78,79,80.81,82,83,84] plot similar curves when graphed. Each curve has two peaks (about 493 nm and 493 c) and two troughs (about 530 nm and 440-465 nm).[83,85] Note these peaks are complementary as distinct from opponent colours, though the null wavelengths (so-called invariant hues) of the Bezold-Brucke effect (but not the Abney effect) closely represent the four unique hues R, G, Y, B. That the curve peaks are a pair of complementary wavelengths implies a role for complementary colours in both of these well-known and complex effects, which together involve the interaction of all three attributes of colour: i.e. brightness, hue, and purity (or saturation/chroma). These effects appear too complex, and too closely tied to colour appearance, to be restricted to the receptoral layer or even the retina. By structuring (in part) the Bezold-Brucke and Abney effects, complementary colours again indicate a role in colour appearance.

The effect of a monochromatic, or substantially chromatic, light source upon shadows is to make the latter appear the complementary hue to that of the light source. It is known as the Helson-Judd effect;[15,86,87] experimental data were last reported in 1991.[88] For instance, in a bush fire the brown smoke imparts a strong yellow hue to the incoming sunlight, throwing distinctly bluish shadows. The Helson-Judd effect, like negative after images,[1-3] is not commonly noted by naive observers unless their attention is called to the effect. It is an interesting example of how complementarism, often without our noticing it, occurs in such natural conditions as light and (its thrown) shadow, and infers a role for complementarism in chromatic adaptation (as shown in Paper 5). Sensory (i.e. automatic) chromatic adaptation to the light source is thought to initiate early in the retina, so that all chromatic perceptions are adapted to the light source,[56] and its neural basis may possibly be complementarism. The neural process of complementarism (e.g. in successive contrast) is thought to have a simple basis: During observation of a stimulus, the cones become relatively desensitized until the stimulus ceases, when complementary after images result from the remaining cone sensitivities.[68,89,90]

The reciprocal nature of brightness (or lightness) and saturation (or chroma), first noted by Helmholtz,[13,24,30] is familiar to many artists as well as scientists. It’s well known they are not linearly reciprocal, but a recent model, for aperture and object colours, demonstrates that relative brightness/ lightness over the hue cycle equates to 1/log chroma.[91] This reciprocity brings a balance to colour in any stimulus or scene. Those colours with high saturation, e.g. blue, have low lightness whereas those colours with high luminance, e.g. yellow, have high lightness. No single colour can have both high lightness and high saturation. This balance derives from the exact reciprocity of complementary colours in (1) the ratio between complementary wavelength intervals, and (2) the ratio between complementary radiant powers, to be studied in Paper 5 below. A colour with a high ratio of the first type (correlating to high saturation) has a low ratio of the second type (correlating to low lightness and low power required to neutralize its complementary colour), whilst its complementary colour has the exactly reciprocal ratios. Besides the general reciprocity of saturation and brightness (two of the three colour attributes), recall the hues of complementary colours are reciprocal in nature, since their admixture neutralizes each other. That this reciprocity in complementary colours extends to all three attributes of colour suggests an extensive role of complementary colours in colour vision.

This cancellation of all hue between opposed hues, and general reciprocity between relative brightness and saturation, is not the rule in opponent colours. Yet opponent colours are currently far better known to the colour science community than complementary colours. For example, there is no mention of complementary colours in the indices or glossaries of four recent, well known, books on colour science (in their 1st or 2nd editions),[47,56,92,93] compared to older books whose 1st editions were written in the 1970s or earlier, which use or describe complementary colours in detail.[35,42,88,94,95,96]

Several possible roles of complementary colours, including a few of those mentioned above, will be explored in detail in the research papers below.

Aims of the Thesis

The aim of the thesis is to find and describe roles of complementary colours in colour perception. The aim includes the possibility of finding not only minor or discrete roles but the overall role of complementary colours in vision. Specific aims for each paper are defined below.

Introduction to the Research Papers

Two earlier papers helped to provide a basis for the present thesis. Pridmore[27] established that the hue cycle (for aperture colours) is separated into two parts, spectral and nonspectral, at the common boundaries of 442 and 613 nm ±1, for all CIE illuminants. Given that the spectral part (442-613 nm) is an interval of 171 nm, this allows the possibility of estimating or calculating, for example by comparitive analysis, the nonspectral part. Second, Pridmore[97] (supported by a later paper[98] discussing implications in the conventional, 3x3 matrix, chromatic adaptation transform) showed that complementary colours play a role in corresponding colours (or colour constancy) and in hue cycle structure. The latter[97] is described in the Appendix as centred on a linear symmetry of complementary wavelength pairs. Hence, these two papers[27,97] provide a basis for further exploration (in *Papers 1-3*) of the role of complementary colours in *colour constancy* and *hue cycle structure* (see Appendix: Definitions).

Paper 1 aims to demonstrate a wavelength-based mechanism of colour constancy, and the role of complementary colours (specifically complementary wavelengths) in that mechanism.[99] It builds on previously published data on corresponding colours.[97,98] Paper 1 concerns only spectral colours and describes an empirically discovered, as distinct from a postulated, spectral mechanism of colour constancy. The mechanism is notable in being highly symmetrical and consisting of straight near-parallel lines (for constant hues and their complementary constant hues in respective illuminants) in the plane of wavelength (nanometres) and reciprocal illuminant colour temperature (degrees Kelvin).

Paper 2 aims to establish a hue cycle structure and wavelength-based hue cycle scale, and to demonstrate the role of complementary wavelengths in that structure and scale.[100] Besides the importance of finding the hue cycle's innate structure, continuous and coherent for both spectral and nonspectral hues, this aim is prerequisite to describing any functions over the complete hue cycle (including nonspectrals), to a scale equivalent to dominant wavelength. The scale becomes a basis for Paper 3, concerning nonspectral hues and enabling them to be graphed to the same *relative wavelength scale* as spectral hues.

Paper 3 aims to use the relative wavelength scale of Paper 2 to theoretically extend the spectral colour constancy mechanism (described in Paper 1) into the nonspectrals.[101] This would enable a description of the global mechanism of colour constancy (global in the sense that it covers the total hue cycle and the total practicable range of illuminant colour temperature). In doing so, Paper 3 also aims to demonstrate the role of complementary wavelengths in the hue cycle relative wavelength scale and the colour constancy mechanism. To adapt the hue cycle interval found[101] for illuminant D65 to other illuminants, Paper 3 systematizes the hue cycle interval as a constant proportion of a harmonic period,[102,103] which shifts wavelength with illuminant.

Paper 4 aims to establish,[104] by analyses of previous published data, that chromatic induction is governed by complementary colours rather than by opponent colours. The latter has been the convention in colour science (but not art) since about 1960.[61] Chromatic induction is a widely observed effect in art and science, comprehensively reported since 1839,[1,2] and sometimes occurring in combination with other effects.[64-67]

Paper 5 aims to demonstrate that complementary colours provide the basic structure of several important visual functions,[105] by formulating six mathematical models of wavelength discrimination, uniform hue, spectral sensitivity, saturation, chromatic induction, and chromatic adaptation. The latter until quite recently was thought to be based on independently operating RGB responsivities,[106,107] but these are now thought to be interconnected to some degree,[58,108] consistent with the findings of Paper 5. The six models derive from either of two ratios: the ratio of complementary wavelength intervals or the ratio of complementary powers.

The final chapter, “Conclusion,” discusses to what extent these aims have been achieved and draws conclusions. Papers 1 and 4 were submitted to a refereed journal prior to the author’s enrolment in the PhD degree and are now published. They are presented here exactly as published (single space, double column) with references per the Council of Biology Editors (Cambridge University Press, UK); the entire thesis follows this style of references. Papers 2, 3, and 5, were researched and submitted to a refereed journal after the author’s enrolment in the degree. Paper 5 (probably the most significant of the papers) was accepted by Color Research & Application on 4 January 2008, and Papers 2 and 3 were accepted later by the same journal subject to minor revision.

Appendix: Definitions

Here are given briefly some common colour definitions. More detailed definitions/descriptions may be found in References 35, 42, 58, or 94.

Abney effect: the effect of variable purity (or saturation or chroma) on hue.[5,73-78] For example, if a monochromatic red of constant dominant wavelength is desaturated, its hue shifts to purplish-red.

Achromatic: non-chromatic; a white or grey that (strictly speaking) matches that of a given illuminant, e.g. the ambient illuminant. Note: illuminants, if compared, vary from slightly bluish to yellowish

Aperture colours: also known as lights, luminous colours and unrelated colours (the formal CIE term).

Bezold-Brucke effect: the effect of variable brightness/lightness (or intensity of light) on hue.[79-85]

Brightness (applicable to unrelated or aperture colours): Attribute of visual sensation in which an area emits more or less light.[109,110,111] It correlates to lightness of related colours.

Chroma (applicable to related colours: Colourfulness of an area judged as a proportion of the brightness of a similarly illuminated area that appears white or highly transmitting.

Chromatic adaptation: the adaptation of the visual process to the ambient illuminant; its main purpose is colour constancy (which see below).

Chromatic induction:[1-3, 58] a modern term incorporating simultaneous contrast and successive contrast (e.g. after images); see below.

Chromaticness: a general term (covering chroma, colourfulness, saturation, and Natural Colour System chromaticness) for the attribute wherein a perceived colour appears more or less chromatic.[58,94] This is the third attribute of colour other than Brightness and Hue.

CIE: Commission d'Eclairage Internationale, an international standards body publishing chromaticity diagrams, formulas, definitions, and data on lighting and colour science.

Colour: a three-dimensional sensation whose fundamental dimensions, or colour attributes, are hue, brightness (or lightness), and chromaticness (or saturation, chroma, etc).

Colour constancy: the tendency for object colours to remain the same colour in different illuminants. Colour constancy is served by the process of chromatic adaptation.

Colourfulness (related or unrelated colours): Attribute of visual sensation in which the perceived colour of an area appears to be more or less chromatic or colourful.

Colorimetric purity: A colorimetric term approximating chromaticness, it represents the proportion of chromatic luminance in a colour mixture (a mixture of a monochromatic component and an achromatic component), calculated as *C/D*, where *C* is luminance of the monochromatic component (known as chromatic luminance) and *D* is total luminance of the colour mixture.[29,35]

Complementary colour: Defined as a pair of colour stimuli, of appropriate wavelengths and power ratio, that mix (by additive, subtractive, or any reliable method) a given achromatic colour (e.g. the white or gray of the light source). Ref.[96] gives coloured examples. *Complementary wavelengths* lie on opposite sides of the neutral in a chromaticity diagram (Figure 3); the term also refers to CIE nomenclature [e.g. 520 c denotes the (purple) complementary to 520 nm] for nonspectral colours which have no dominant wavelength so are named from their spectral *complementary* dominant wavelength.

Constant wavelength means that the respective wavelength remains invariant or constant regardless of varying parameters such as illuminant.

Corresponding colours denote colours, of a given luminance or lightness, that are perceived to colour-match (e.g. as unique blue) in different illuminants; they are often named as B, G, or Y, etc, corresponding colours after unique hues.

Dominant wavelength of a given colour or chromaticity may be found by drawing a straight line from the CIE diagram's illuminant point (e.g. D65) through the given chromaticity point to intersect the spectrum locus, where wavelengths are labelled or may be looked up, from their x,y coordinates, in CIE tables.[23] Only spectrum-locus colours are monochromatic, i.e. composed entirely of one wavelength (e.g. 590 nm) or nearly so (e.g. 588-592 nm). Other colours have *dominant* wavelengths, that represent the calculated colour mixture of two or more wavelength components (e.g. 560 and 600 nm). Object colours rarely if ever correspond to monochromatic light so their so-called wavelengths usually are *dominant* wavelengths. As a dominant wavelength decreases in purity (or chroma) from spectrum locus to neutral point, its hue usually shifts (the Abney effect).[5]

Excitation purity: A colorimetric term approximating chromaticness, calculated (for a given illuminant and chromaticity diagram) as *A/B*, where *A* is the distance from the illuminant point to a given chromaticity of given wavelength, and *B* is the distance from the illuminant point to the spectrum locus at the same dominant wavelength. (See purity formulas in Wyszecki & Stiles.[35])

Hue Cycle: the complete cyclic series of hues, from a given hue through the series to its repetition.

Illuminant colour temperature: CIE daylight and artificial illuminants lie close to the Planckian locus of blackbody radiators in CIE colour space. Hence it is CIE practice to specify illuminants by their blackbody temperatures, in degrees Kelvin.

Lightness (in related colours): The brightness of an area judged relative to the brightness of a similarly illuminated area that appears to be white or highly transmitting.

Metamerism: where different spectral energy distributions evoke identical colour sensations.

Opponent colours: Hering's opponent colours theory[15,35-38] postulates three pairs of opponent colours: White-Black (in the brightness dimension), and Yellow-Blue and Red-Green in the chromatic dimension. These primary chromatic opponent colours are defined as two opposing pairs of unique hues,[112,113] Y-B and R-G, each judged by its appearance: e.g. unique B is judged by an observer as pure blue, neither greenish nor reddish, and unique Y is judged as pure yellow, neither greenish nor reddish. Between the four chromatic primaries are the four binary opponent colours, e.g. BG is judged as appearing equally blue and green, and its opponent colour is YR, judged as equally yellow and red. Mixture of a pair of chromatic opponent colours is not claimed to produce white, but a colour that is neither of the mixed components. For example, a mixture of Red and Green is in equilibrium when it produces a pure yellow, neither reddish nor greenish.[114] However, a mixture of Blue and Yellow is in equilibrium when it produces white or grey;[115] hence B and Y unique hues are not only opponent colours but also complementary colours (though this fact is rarely mentioned in expositions of opponent colours theory).

Psychophysical: the most literal definition is: psychological perceptions arising from physical stimuli. Psychophysical also denotes mental processes leading, directly or indirectly, to psychological perceptions.[58] *Psychophysics* is the study of mental processes by quantitative methods, specifically reports of human subjects of perceptions arising from carefully measured light stimuli.[94]

Related (or Object) Colour: Colour perceived to belong to an object seen in relation to other colours.

Relative Wavelength Scale: A hue cycle wavelength-based scale, where dominant wavelength over the optimally efficient spectrum (442-613 nm)[18,19] is extended over the nonspectrals, i.e. the remaining hue cycle, as “equivalent wavelength”; the total scale is termed “relative wavelength.”

Saturation: The classical general term for chromaticness, but nowadays saturation specifically means the attribute of visual perception which permits a judgement to be made of the degree to which a chromatic stimulus differs in appearance from an achromatic stimulus, regardless of their brightness.

Simultaneous Contrast or Induction: the juxtaposition of different colours causes chromatic shifts in each colour, from each colour area inducing an effect upon its adjacent area. Simultaneous contrast involves images of different colours falling on adjacent parts of the retinal field. Both successive induction and simultaneous induction have the same effect on the induced colour.[68]

Successive Contrast or Induction: observing a colour area successively after observing another (the inducing) colour may cause a chromatic shift in the second colour area. Successive contrast is thought to involve relative desensitization of cone sensitivities and, when the stimulus ceases, complementary after images result from the remaining cone sensitivities.

Unrelated (or Aperture or Luminous) Colour: Colour that appears to be a glowing light or a colour seen in isolation from other colours.[23] (Usually seen on a dark background.)

Value: Munsell Value correlates linearly to lightness (and nonlinearly to relative luminance), and is a 10-step gray scale from black (=0) to white (=10) in ten uniform steps.

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Color Constancy from Invariant Wavelength Ratios: I. The Empirical Spectral Mechanism

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Abstract: The wavelengths of several constant hues over four illuminants (D95, D65, D50, A) are derived from several sets of published data. In the plane of wavelength and reciprocal illuminant color temperature (MK^{-1}), the wavelengths of constant hues plot straight approximately parallel lines whose mean slope is about 87°. Parallel lines give invariant wavelength ratios, hence constant hues in this plane are near-invariant wavelength ratios across illuminants. As recently demonstrated, the complementary wavelengths to a constant hue (across illuminants) represent the complementary constant hue; these complementary wavelengths also plot a near-parallel line to the first constant hue. To confirm and further define the constant slope of these lines, it is shown that complementary wavelength pairs, per CIE data, can only plot parallel straight lines at the angle of 87° ± 1. In summary, near-parallel sloping lines represent constant hues at near-invariant wavelength ratios. This mechanism of color constancy is shown to relate to the well-known theory of relational color constancy from invariant cone-excitation ratios. In the visual process, the latter ratios are presumably the source of the former (invariant wavelength ratios). © 2008 Wiley Periodicals, Inc. Col Res Appl, 33, 238–249, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/col.20405

Key words: color appearance; color constancy; corresponding colors; chromatic adaptation

INTRODUCTION

Color constancy is the constancy of the perceived color of an object despite changes in illumination. In practice, color constancy is only approximate and the degree of constancy is difficult to measure.[1–4] However, precise color constancy would not be desirable since the visual system at times needs to discriminate rather than discount illuminants,[2–4] for example when stormy weather threatens. Theoretically precise color constancy would represent surfaces in one illuminant as matching colors (also known as corresponding colors) in another illuminant. Color constancy is a product of the broad process of chromatic adaptation, which is the visual system's ability to adjust to the color of the illumination so that perceived object colors vary less than would be expected from basic colorimetry or radiometry. Several theoretical approaches to color constancy have been employed, including adaptational mechanisms simulating possible physiological processes,[4–7] and direct computational methods.[8–10] Most ways of addressing practical problems of color constancy rely on chromatic adaptation transforms based on von Kries's theory of coefficient adaptation.[4]

This article takes an empirical rather than computational approach. It analyzes published data on corresponding colors and complementary colors in search of a psychophysical mechanism of color constancy. The adaptational mechanism thus found is remarkably simple.

For the interested but nonspecialist reader some basic colorimetric terms (e.g. dominant wavelength, corresponding colors, complementary colors) are defined in Appendix A.

CORRESPONDING COLORS: DATA AND ANALYSIS

Most of the experimental data on corresponding colors concern illuminants D65 (daylight) and A (tungsten light), and mostly involve surface (reflective) colors. In convention, the data are graphed and analyzed in color appearance terms and spaces (e.g. CIE LAB) rather than in physics-based terms. Here I use physics-based terms, par-

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ticularly dominant wavelength and illuminant correlated color temperature (CCT), to illustrate a different and promising perspective of color constancy.

The dominant wavelength of a surface color depends on the spectral power distribution (SPD) (i.e. the spectral concentration of radiant power as a function of wavelength) of the surface's reflected light and of the light source, as measured by a colorimetric instrument (e.g. spectroradiometer) to output the surface color's tristimulus values and chromaticity coordinates. Dominant wavelength is readily determined from the latter. Most colorimetric data on corresponding colors[11] are given in the form of tristimulus values and/or chromaticity coordinates derived from spectroradiometry rather than spectrophotometry.

Tristimulus values and thence dominant wavelength may also be determined from surface colors' reflectance spectra (the ratio of reflected energy to the incident energy as a function of wavelength, measured by spectrophotometry) and the light source SPD. The importance of reflectance spectra to color constancy is indicated by computational methods[8–10] of estimating color constancy utilizing only reflectance spectra of objects without information of the light source. However, in the present study, reflectances are not directly relevant to calculating color constancy. An advantage of predicting color constancy from dominant wavelength rather than SPD or reflectance is that metamerism (a problem for computational color constancy) is avoided. Different SPDs or reflectance spectra may produce a metameric match in one illuminant but not necessarily another, whereas the reverse applies to different dominant wavelengths: they cannot produce a match in any one illuminant but can do so in different illuminants (if they represent corresponding colors). In fact, one purpose of this study is to determine sets of wavelengths which each specify a set of corresponding hues in various light sources.

This study uses experimental data collected and made available by CIE committee TC 1-52 (on Chromatic Adaptation Transforms) via the Leeds University Colour & Imaging Group website[11] or through the archived data still held at Derby University.[11] The stimuli are mainly surface (i.e. reflective) colors but some data sets include transparency colors and CRT monitor colors. The aim of TC 1-52 is to review certain chromatic adaptation transforms. These are normally evaluated using corresponding color experimental data sets in which each color is defined by two sets of tristimulus values under two illuminants. TC 1-52 has accumulated a comprehensive collection of data sets for evaluating color appearance models and chromatic adaptation transforms, and has made these data sets (see Table I, after Ref. 11) publicly available.

Figure 1 shows four data sets on corresponding colors for illuminants D65 and A, downloaded from Ref. 11 and plotted to CIE chromaticity diagrams. Figures 1(A)–1(C) are for 2° visual fields (plotted to the CIE 1931 diagram) and Fig. 1(D) is for 10° fields (plotted to the CIE 1964 diagram). Figures 1(A)–1(C), show the Colour Science Association of Japan (CSAJ) data (104 observers),[12] the

TABLE I. Summary of the 14 corresponding-color data sets assembled by CIE committee TC 1–52.[11]

			Experimental conditions						
			Illuminant						
Data Set	No. of Phases	No. of Samples	Test	Ref.	Illuminance (lux)	Background (Y%)	Sample Size	Medium	Experimental Method
CSAJ-C*	1	87	D65	A	1000	20	S	Refl.	Haploscopic
CSAJ-Hunt Effect	4	20	D65	D65	10–3000	20	S	Refl.	Haploscopic
CSAJ-Stevens Effect	4	19	D65	D65	10–3000	20	S	Refl.	Haploscopic
Helson*	1	59	D65	A	1000	20	S	Refl.	Memory
Lam & Rigg	1	58	D65	A	1000	20	L	Refl.	Memory
Lutchi (A)*	1	43	D65	A	1000	20	S	Refl.	Magnitude
Lutchi(D50)*	1	44	D65	D50	1000	20	S	Refl.	Magnitude
Lutchi (WF)	1	41	D65	WF	1000	20	S	Refl.	Magnitude
Kuo&Luo(A)*	1	40	D65	A	1000	20	L	Refl.	Magnitude
Kuo & Luo (TL84)	1	41	D65	TL84	1000	20	S	Refl.	Magnitude
Breneman-C*	9	107	D65, D55	A, P, G	50–3870	30	S	Trans.	Magnitude
Breneman-L	3	36	D55	D55	50–3870	30	S	Trans.	Haploscopic
Braun & Fairchild*	4	66	D65	D30,D65 D93	129	20	S	Monitor & Refl.	Matching
McCann	5	85	D65	R,Y,G	14–40	30	S	Refl.	Haploscopic

*The seven sets used in this study.
S, small (2–4°) visual field; L, large (10–15°) field; refl., reflective; trans., transparency.

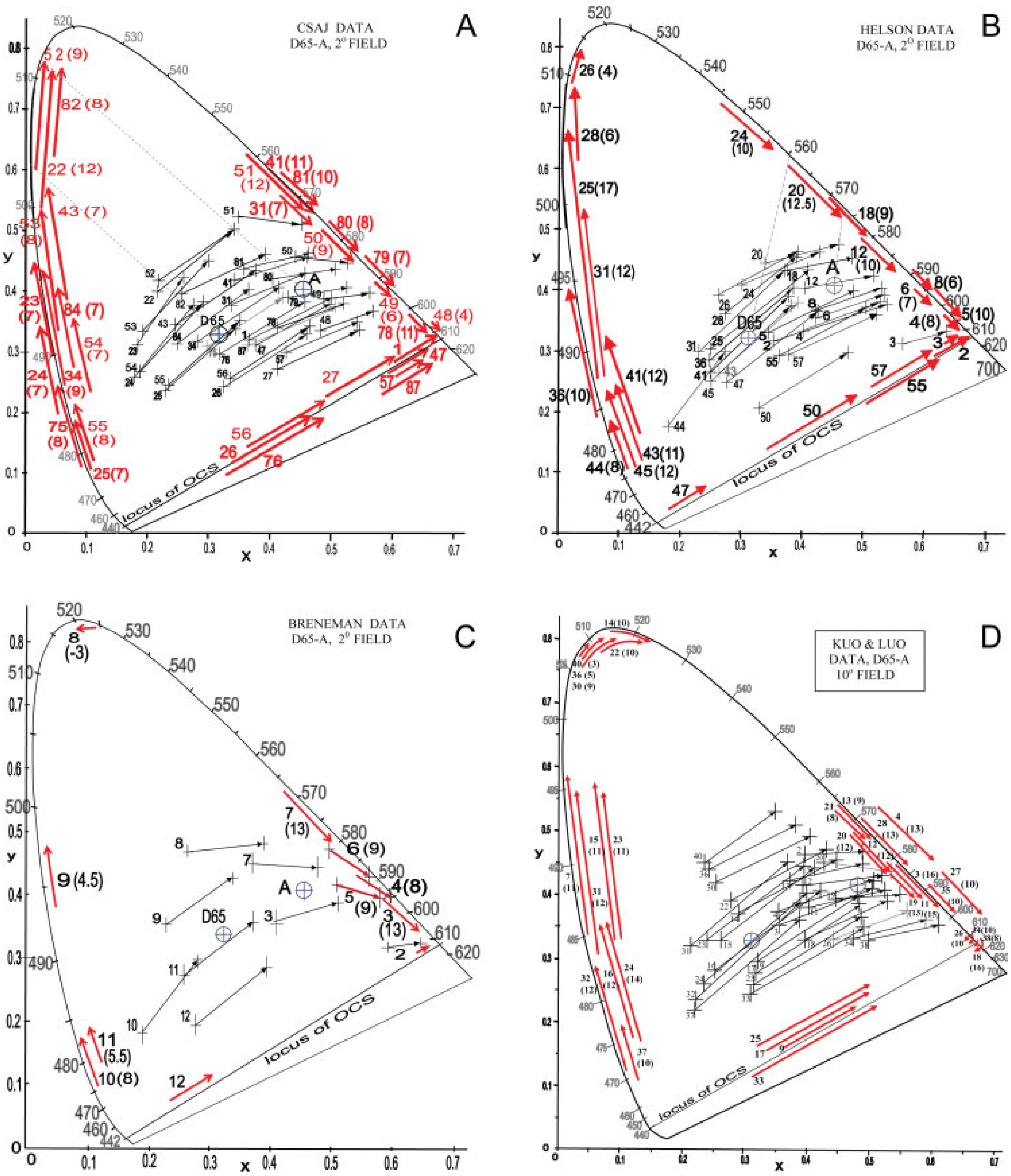

FIG. 1. Experimental data on corresponding colors for test illuminant D65 and reference illuminant A. Data points "+" and thin black arrows (from illuminant D65-A) show selected samples (numbered) of original data, generally of higher chroma. Thick red arrows around the color boundary: chromaticity shift from illuminant D65 to A, with wavelength shift nm in parentheses. "Locus of OCS" indicates nonspectral optimal color stimuli. (A) CSAJ data,[12] 2° visual field, in CIE 1931 chromaticity diagram. (B) As Fig A but Helson data[13] (his Experiment 1, high luminance, 1500 cd/m2). (C) As Fig A but Breneman data[14] (through lack of space, thick red arrows for samples 4, 5, 6, also represent thin black arrows for original data). (D) Kuo & Luo data,[19] 10° field, in CIE 1964 diagram.

Helson data (6 observers),[13] and the Breneman data[14] (7 observers for Experiment 1, 1500 cd/m^2,), all for small-field (2°) stimuli, plotted in the CIE 1931 chromaticity diagram. A line drawn from the relevant light source neutral point (blue cross within a circle) through the sample data point (black cross) to intersect the boundary locus indicates the dominant wavelength, as illustrated by dashed gray lines in Fig. 1(A) for sample pair #52 or in

TABLE II. Wavelength shifts of corresponding colors from illuminant D65 to illuminants A, D50, and D95, from 7 data sets.[11]

Dataset	CSAJ[12]	Lutchi[15,16]	Helson[13]	Bren'n[14]	Mean	Kuo[19]	Lutchi[15,16]	Braun[20]
Illums	D65-A	D65-A	D65-A	D65-A	D65-A	D65-A	D65-D50	D65-D95
Field	2°	2°	2°	2°Exp.1	2°	10°	2°	2°, RIT1
Samples	31–87	13–43	23–59	11–12	–	32–40	13–44	11–17
% used	36%	30%	39%	92%	49%	80%	30%	65%
Subjects	104	6–7	6	7	31	5	6–7	5
470–490	7.5	13	11	7	9.6	12	5	−13
490–505	7.5	12.5	13	4.5	9.4	11.25	6	–
505–525	9.8	16	5	−3	10.3[a]	7.5	17(?)	–
545–560	–	–	10.5	–	10.5	–	–	−3
560–575	10	11	12.5	13	11.6	10	5	−1
575–590	8.5	12	10	9	9.9	11.9	4	−1
590–605	6.5	9	8	8.5	8	12	4	−3
605–620	7.5	10	8	13	9.6	11	4	−3
Means	8.2	11.9	9.7	7.4	9.9	10.8	4.6[b]	−4
Predicted					11		2.6	−2.6

'Samples' row shows number of sample pairs (e.g. for D65-A) graphed in Figs 1 or 2, over the total number of sample pairs for the respective experiment and data set; next row shows the ratio as a percentage; low chroma samples and eccentric data (e.g. hue shifts in wrong direction) are generally omitted; both Lutchi data sets (D65-A and D65-D50) are graphed in Ref 17. Column 1 shows the approximate wavelength area; where Refs 12–16 show 2 or more samples (at different chromas) for the same wavelength area, the mean is shown here. The bottom row shows predicted hue shifts per Eq. (1) for small-field data.

[a] This number, in the Means column, denotes 'mean without Breneman.'

[b] This number, in the D65-D50 column, denotes 'mean without the eccentric 17 nm shift in the green area.'

Fig. 1(B) for sample pair #20. The chromaticity shifts are indicated by heavy red arrows around the boundary locus (some arrows are curved to follow the locus), with the wavelength shifts shown in parentheses.

To avoid clutter and improve reliability of data, Fig. 1 omits low chroma samples in the respective data set since these tend to be less reliably judged for hue. The CSAJ data [Fig 1(A)] represent an extensive experiment for illuminants D65 and A, with 104 observers and 87 sample pairs. Figure 1(A) shows only 30 sample pairs, arbitrarily omitting most of the lower chroma samples and low luminance (40 Y) samples, i.e. #58–74.

Figure 1(B) shows 23 sample pairs of Helson's total 59, mostly higher chroma samples plus two low chroma samples #2 and #5. Low chroma samples tend to include some eccentric shifts in amount and direction. Figure 1(C) shows 11 of Breneman's total 12 samples for Experiment 1, since all but sample #1 (gray) are moderately saturated. These three data sets [Figs. 1(A)-1(C)] are summarized in Table II, together with the LUTCHI data (6–7 observers),[15,16] recently graphed in Pridmore.[17] Its wavelength shifts are generally larger than the CSAJ data. The wavelength shifts for the four sets are generally in mutual agreement other than the negative wavelength shift (−3 nm) for green in the Breneman data. For these four data sets, Table II indicates the mean wavelength shift from illuminant D65 to A is almost 10 nm. Some detailed deductions on the wavelength shifts are in Ref. 17 (see Ref. 18 for math implications).

Figure 1(D) shows large-field (10°) samples from the Kuo & Luo data set,[19] plotted in CIE 1964 diagram. The samples are fairly saturated so 32 sample pairs out of a total 40 are plotted. Eight sample pairs were omitted due to eccentric shifts (e.g. samples #2 and #6 shift in reverse direction to the usual) or to avoid clutter. Some sample points (e.g. #21, 27, 28, 35) are omitted for lack of space but their wavelength shifts (heavy red arrows) are graphed around the boundary locus. The wavelength shifts in Kuo & Luo data are rather more consistent than the LUTCHI and Helson data. These large-field data[19] are included in Fig. 1 and Table II to allow comparison with small-field data. Like the latter, they indicate that shifts are approximately uniform across the spectrum. The large-field data shifts may differ systemically from small-field data, so are excluded from the Means column in Table II.

Also shown in Table II are a further two data sets on small-field stimuli: LUTCHI data for illuminant D65-D50,[15,16] recently graphed in Ref. 17 and Braun & Fairchild data[20] for illuminant D65-D95, Experiment RIT1, graphed in Fig. 2. The data in Fig. 2 represent the difficult task of matching monitor colors with reflective colors, and are clearly eccentric including opposed directions of hue shift for the same hue (see samples #5, 6, 12, 17, in Fig. 2). Mean shift over the spectrum for the 11 sample pairs graphed (including "0") is −2.6 nm or −4 nm if calculated for the six wavelength areas of Table II; i.e. the shift is about −3 nm, toward shorter wavelength as expected since D95 is a higher CCT than D65.

Figure 2 omits sample points for Experiment RIT2 but shows the wavelength shifts, as thick gray arrows around the boundary locus; these shifts are often in the wrong direction, e.g. sample #4 shifts 10 nm longer instead of shorter. For both RIT1 and RIT2, several samples are wildly eccentric in amount or direction so are omitted from Fig. 2; e.g. RIT2's sample #10 (not shown) shifts from purple 505 c through the reds to orange 590 nm. Even excluding eccentric samples such as #10, RIT2's mean wavelength shift over the spectrum is slightly posi-

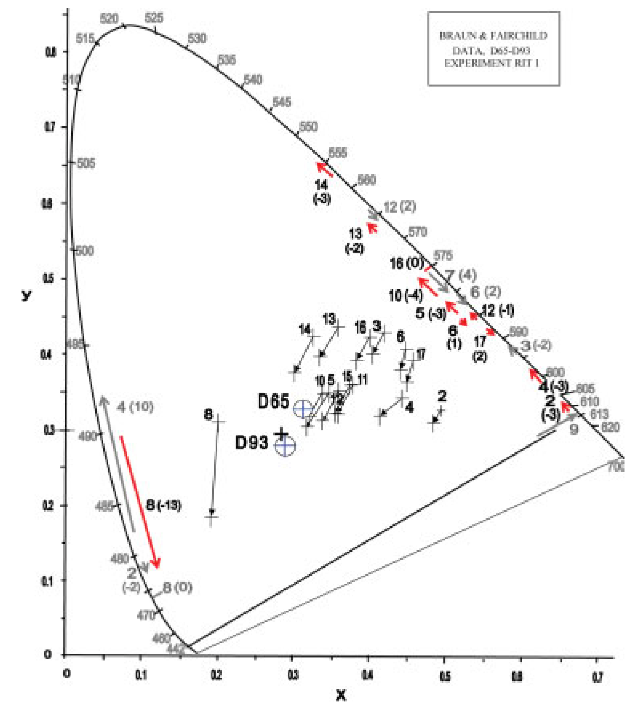

FIG. 2. As Fig 1(A) but Braun and Fairchild data[20] for illuminant D65-D93, showing experiment RIT 1. Also shown, as grey arrows (and associated grey numbers), are shifts for experiment RIT 2. Heavy black cross above, and significantly different from, the D93 simulator (cross inside circle) is CIE illuminant D93. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

tive (0.5 nm, i.e. in the wrong direction), which is not a credible result; however the authors noted RIT2 was less reliable than RIT1.

To illustrate the global situation for small-field data for illuminant correlated color temperature (CCT) from 2850–9500 K (i.e. A to D95), Table II's mean wavelength shifts from illuminant D65 to A, D65 to D50, and D65 to D95, are plotted to the plane of wavelength and reciprocal illuminant CCT (MK^{-1}) in Fig. 3. This plane is significant for reasons discussed later. For illuminants D65-D95 (for Experiment RIT1), Fig. 3 shows, for all hue areas, the mean hue shift over the spectrum because the RIT1 data contain minor eccentricities (Fig. 2) which make better sense when averaged. The main lesson (say, Empirical Observation 1) from Fig. 3 is that constant hue plots approximately straight and parallel lines in this plane. The slope of the lines averages about 87° in the full scale of the plane (where unit y and unit x are equal spatial scale), and is defined by (unit y)/(unit x); i.e. (1 reciprocal MegaKelvin)/(1 nm). The 87° angle has no great significance since it would change with the selected scale of the graph or the size of the x- and y-axis units. Figure 3 is vertically compressed to save space (but see Fig. 4). Hence, constant hue appears to adapt linearly to MK^{-1}, and is simply modeled by Eq. (1) (slightly modified from Ref. 17) where s is wavelength shift nm of a constant hue from illuminant 1 to illuminant 2, and t_1 and t_2 are the two illuminants' reciprocal CCTs in MK^{-1}.

$$s = (t_2 - t_1)/17.8 \tag{1}$$

A worked example of Eq. (1) follows: Say the two illuminants 1 and 2 are D65 and A, of ($1/6504\ K \times 10^6 =$) 153.75 and ($1/2856\ K \times 10^6 =$) 350.14 MK^{-1}. Then per Eq. (1), $s = (350.14 - 153.75)/17.8 = 196.4/17.8 = 11$, wavelength shift nm of any constant hue from illuminant D65 to A.

The bottom row of Table II gives predicted hue shifts per Eq. (1). They agree with the experimental data to within 1 or 2 nm. The *Mean* shift for D65-A (for four data sets) is 9.9 nm, whilst the *Predicted* shift is 11 nm, a difference of only 1.1 nm. For the three illuminant pairs D65-A, D65-D50, and D65-D95, the average of the three *Mean* shifts is 3.5 nm and that of the three *Predicted* shifts is 3.7 nm. The LUTCHI data shifts for D65-A and D65-D50 seem a little large in the green hue.

A corollary of Grassman's linearity laws[21] was recently derived[17,18] which states: If a number of colors have a single corresponding color appearance **A** in various illuminants, then their complementary colors have a single corresponding color appearance **B** (different from, and complementary to, **A**) in those illuminants. Another expression of the corollary* is: If stimulus *A* matches stimulus *B*, and stimulus *C* matches stimulus *D*, and if any two of the four stimuli are complementary colors, the remaining two are complementary colors. The intuitive logic to this corollary (the *complementary linearity law*) is: (1) a perceived color can have only one complementary color, according to colorimetry; (2) a pair of corresponding colors have the same complementary color; and thus (3) the complementary colors to the corresponding colors will themselves be a pair of corresponding colors. The same may be applied to any number of illuminants, by Grassman's transitivity law.[21] Hence the complementary wavelengths to constant hues may be plotted if required in Fig. 3 to represent further constant hues (subject to the original constant hue data accuracy). These would graph as near-parallel lines to the existing constant hues in Fig. 3, as exemplified by the blue and yellow unique hues (labeled *B and *Y, which are complementary as well as opponent hues) already plotted in Fig. 3. Empirical Observation 2 is stated as: Constant hue and complementary constant hue plot near-parallel lines in the plane of wavelength and MK^{-1} (Fig. 3).

Given that constant hues in Fig. 3 plot near-parallel lines, or are modeled in Eq. (1) as parallel lines, it fol-

*Corollary: Strictly speaking, this Grassman corollary depends on two commonly made assumptions (detailed in Ref. 18): (1) The tristimulus transformation on reflected light incurred by illuminant change is a 3 × 3 linear matrix (guaranteed if, as assumed in many color-constancy models, all reflectances are linear combinations of the same three basis functions including a white); and (2) Chromatic adaptation by the visual system also incurs a 3 × 3 linear matrix tristimulus transformation (e.g. von Kries).

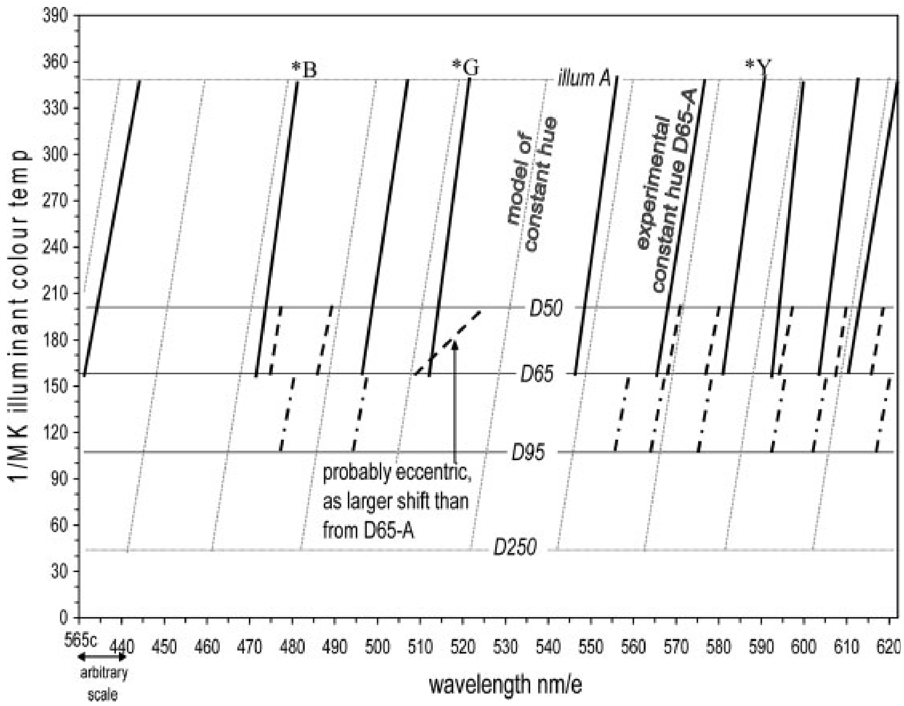

FIG. 3. Adaptation of constant hue to illuminant color temperature, plotted as wavelength versus reciprocal MegaKelvin (1/MK). Solid sloping black lines connect the wavelengths of constant hue for illuminants D65 and A, as the means of four data sets;[12–16] unique hues Blue and Yellow are labeled *B and *Y. Dashed black lines for D65-D50 represent LUTCHI data. Dash-dot black lines for D65-D95 represent the mean shift for Braun & Fairchild data. Dotted parallel grey lines show Eq. (1) model of constant hue for any illuminant from D250 to A.

lows (say, Empirical Observation 3) that the ratio of wavelengths for two constant hues is practically invariant with illuminant MK^{-1}. For example, consider the dotted gray lines which model constant hue in Fig. 3. The extreme right line and the adjacent line (to its left) are 618 and 599 nm (ratio 1.032) for illuminant A, 611 and 591 nm (ratio 1.034) for D50, 609 and 588 nm (ratio 1.036) for D65, and 606 and 585 nm (ratio 1.036) for D95. These ratios vary by only 0.4%. To obtain precisely invariant ratios, the dotted lines would need to vary slightly from strictly parallel.

It is required to confirm the above empirical observations. They depend on, and can be tested by, Empirical Observation 2 (i.e. constant hue and complementary constant hue plot near-parallel lines in the plane of wavelength and MK^{-1}). The test shall be CIE colorimetry on complementary wavelengths.

COMPLEMENTARY WAVELENGTH PAIRS

It was demonstrated in Ref. 17 that in any illuminant there is a unique pair of complementary wavelengths that give the minimum complementary interval (MCI). Consider a dominant wavelength *A* and its complementary wavelength *Ac*. The wavelength interval between *A* and *Ac* varies with hue. The interval is largest between the red end of the spectrum and its complementary cyans (e.g. 622 and 492 nm, interval 130 nm) or between the violet end of the spectrum and its complementary yellow-greens (e.g. 437 and 567 nm, interval 130 nm), and is smallest between blues and their complementary yellows; e.g. 480 and 578 nm, an interval of 98 nm, is the MCI for illuminant D65.

From CIE colorimetry,[22] the MCI pair of complementary wavelengths vary by illuminant, as graphed in Fig. A2 of Ref. 17. The peak of each curve in that figure is formed necessarily by the pair of wavelengths of least wavelength difference, that is, of minimum complementary interval, since axes *x* and *y* represent complementary wavelengths. The wavelengths of the MCI pairs for seven illuminants are plotted as two sloping lines labeled MCI in the plane of wavelength and MK^{-1} in Fig. 4. Remarkably, they are precisely straight and near-parallel, and infer this plane and the MCI lines are of special importance to the physiology. The mean of the MCI pair of wavelengths is labeled the hue cycle midpoint because it forms a center of symmetry in various functions of complementary wavelength. Hue cycle midpoints across CIE Daylight illuminants 4,000–25,000 K plot straight line G. Now, the angle of line G is 86.75° from horizontal, in the correct scale plane shown in Fig. 4, very similar to the mean angle (87°) of constant hue lines plotted in Fig. 3 from experimental data.

It can be shown that pairs of complementary wavelengths can plot straight parallel or near-parallel lines in this plane at only one angle, 86.75° ± 1. Several pairs of such lines are shown in Fig. 4, plotted as follows: a first line was drawn parallel to midpoint line G; the complementary wavelengths were plotted for respective illuminants and found to plot a near-parallel line to the first line; if the second line were slightly curved, the angle of the first line needed to be adjusted slightly, and the process repeated until two near-parallel lines resulted. It is impossible to plot two complementary, straight and near-parallel lines except at 87° ± 1 tolerance. This is illustrated by grey dashed/dotted lines in Fig. 4. One such line is labeled "1" and is significantly over 87°, i.e. 90°; its

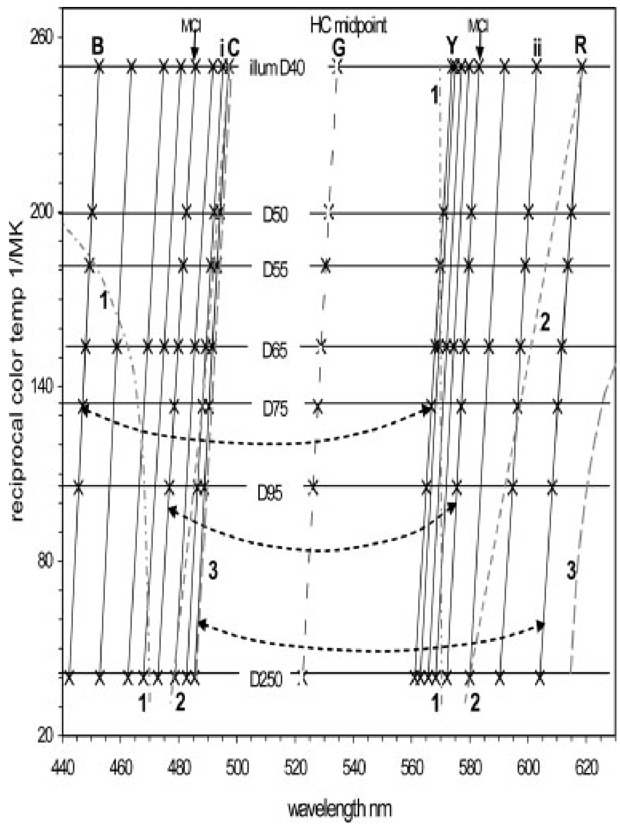

FIG. 4. The plane of wavelength and reciprocal illuminant color temperature (MK^{-1}), to correct scale where unit y equals unit x in spatial scale; e.g. 60 reciprocal MK (y-axis) occupies same scale interval as 60 nm (x-axis). Seven CIE Daylight illuminants are labeled from D40-D250. Two sloping lines labeled "MCI" indicate MCI complementary wavelength pairs (data points "x") plotted for respective illuminants, e.g. 479.8 and 578.2 nm for illuminant D65. Notably they form straight lines. Their mean wavelengths are termed hue cycle midpoints (dashed black line G). Any straight lines drawn parallel to G, e.g. solid black line B, has complementary wavelengths (respective to illuminant) that plot data points "x" on another straight and practically parallel line, e.g. solid black line Y. Grey lines 1, 2, 3: see text. Grey straight lines 1 and 2 (on right half of graph) exemplify how nonparallel lines have complementaries that produce curved lines (in left half). Grey dashed lines 3 exemplify that if complementaries are outside the range 440–615 nm they do not form parallel or straight lines but curves.

complementary wavelengths (per CIE data) plot the curved grey dash-dot line "1", on the left side. Straight grey dashed line "2", on the right, is significantly less than 87° and gives complementary wavelengths that plot curved grey line "2" on the left. Finally, straight grey dashed line "3", on the left, is the correct 87° but gives complementary wavelengths that plot a curve (grey line "3" on the right). This is presumably because line "3" on the right moves significantly outside the 442–613 nm limits to optimal monochromatic stimuli for aperture/unrelated colors.[23] Wavelengths outside these limits cannot form optimally efficient monochromatic stimuli, but are best represented by compound colors comprising 442 + 613 nm. Similar limits and wavelengths apply to object/related colors.

In summary, Fig. 4 demonstrates a principle (say, the invariant ratio rule): complementary wavelength pairs can plot two straight near-parallel lines across illuminants, with near-invariant wavelength ratios, only if the lines are 87 ± 1°. Hence in Fig. 3, if the paired lines of constant hue and complementary constant hue are approximately parallel (as they are in general), they can only be 87 ± 1°. This applies to all complementary pairs, whether constant hues or otherwise (though it can be shown that the particular class of complementary wavelength pairs that obey the invariant ratio rule comprise no more and no less than the constant hues, which themselves are complementary hue pairs).

The two sets (Figs. 3 and 4) of near-parallel lines are shown to be equivalent by yet another method. Both sets have three conditions in common: (1) each set comprises complementary wavelength pairs; (2) in each set, complementary pairs of lines are near-parallel; (3) in each set, complementary pairs of lines approximate 87°. Therefore both sets are equivalent. The set in Fig. 4 is the colorimetric basis (quantifiable for respective illuminants by CIE colorimetry) of the set in Fig. 3.

The wavelength ratio between a line and its complementary line in Fig. 4 varies from almost to precisely invariant across illuminants, as shown in Table III which lists the complementary wavelength pairs for four of the pairs of lines in Fig. 4. The slope of near-parallel lines in Fig. 4 can be seen to decrease slightly from left to right (except the MCI lines) from about 87.5 to 86° apparently in order to preserve invariance of wavelength ratios. Because of constant hues comprising complementary pairs of constant hues (per the complementary linearity law in Ref. 17), these invariant or near-invariant ratios will necessarily apply to the constant hue lines in Fig. 3. One may therefore impose Fig. 4's structure of near-parallel lines, derived from CIE colorimetry, directly upon Fig. 3. Thus it is true to say that the wavelengths of two constant hues across illuminants form near-invariant ratios. Table III indicates the ratios are invariant to within ±1%.

DISCUSSION

Having corroborated Empirical Observation 2 (that constant hue and complementary constant hue plot near-parallel lines in the Fig. 3 plane), Empirical Observations 1 and 3 above are also corroborated: that is, constant hue plots near-parallel straight lines of near-constant slope, and constant hues across illuminants are related by near-invariant wavelength ratios.

Invariant wavelength ratios for complementary pairs or for constant hues are not previously reported in the literature to my knowledge. Such ratios are remarkable but particularly so in a plane (Fig. 4) where they display as exactly straight, near-parallel lines in a highly symmetrical structure or grid. Figure 4 illustrates a mechanism that

TABLE III. Complementary wavelength pairs and ratios for four indicated pairs of lines in Fig. 4 (MCI wavelength pairs, B & Y pairs, C & R pairs, and pairs labeled i and ii) for seven CIE Daylight illuminants.

Illum	MCI λ	MCI λ	& ratio	B	Y	& ratio	C	R	& ratio	i	ii	& ratio
D250	472.9	572.2	0.826	442.4	561	0.7886	485.5	604.1	0.8037	482.7	590.3	0.818
D95	476.8	575.6	0.828	445.6	565.05	0.7886	488.8	608.26	0.8036	486.4	594.7	0.818
D75	478.5	577.1	0.829	447	566.8	0.7886	490.4	610.2	0.8037	488.1	596.4	0.818
D65	479.8	578.2	0.830	448	568.1	0.7886	491.5	611.6	0.8037	489.3	597.4	0.819
D55	481.5	579.7	0.831	449.35	569.8	0.7886	493.2	613.65	0.8037	491	598.6	0.820
D50	482.6	580.6	0.831	450.25	570.95	0.7886	494.2	614.9	0.8037	492.1	600.2	0.820
D40	485.8	583.2	0.833	452.65	574	0.7886	497.3	618.6	0.8038	495.2	603.0	0.821

These pairs plot straight, near-parallel, lines in the plane of wavelength and illuminant MK^{-1} (Fig. 4). Wavelength ratios between lines vary from practically invariant (for the MCI pair of lines) to exactly invariant for the B & Y pair of lines.

assures invariant or near-invariant wavelength ratios between complementary constant hues or indeed any two constant hues. This psychophysical mechanism of invariant wavelength ratios regulates the wavelengths of any given constant hue over the global range of illuminant MK^{-1}; in other words, it adapts constant hue to illuminant, using physics-based quantities wavelength and MK^{-1}. Presumably, from the notable symmetry of Fig. 4, the corresponding physiology does the same rather than create (at this early stage) psychological appearance terms. The adaptation is executed in a very simple symmetrical method (Fig. 4). The method is based on a plane of wavelength and MK^{-1} and their linear relationship. It is hard to imagine a simpler or more elegant mechanism. The plane of wavelength and MK^{-1} (Fig. 4) seems especially important to the physiology and color science due to its linear display of the above-described relations in complementary colors and color constancy.

What drives this mechanism of adaptation? The driving force may be the automatic (sensory) adaptation of complementary colors to illuminant, as discussed in Appendix B.

The described empirical mechanism of constant hue as invariant wavelength ratios can be tentatively related to Foster and Nascimento's well-known theory of relational color constancy from invariant cone-excitation ratios.[6] Several workers have demonstrated that wavelength may be calculated by the postreceptoral physiology comparing cone excitation curves.[24–26] Nobel Prize-winner Hubel[27] postulates the process as follows: "If we subtract the sensitivity curves of two cones (they are logarithmic curves, so we are really taking quotients), we get a curve that is independent of intensity. So the two cones together now constitute a device that measures wavelength." Note that Hubel is comparing any two of the three human cones, and that the term "wavelength," used below and by Hubel above, may imply a wavelength correlate rather than literal wavelength.

Now if, in the case of constant colors, the excitation ratios between any two of the cones were invariant across different illuminants,[6] subtraction of the excitation curves would give invariant wavelength ratio across illuminants. Hence, invariant cone-excitation ratios imply invariant wavelength ratios across illuminants. After suggesting this concept to David Foster, (Foster DH. Personal communication dated 25 Oct 2005) he proposed the relationship (invariant wavelength ratios) may be formalized as follows.

For two surfaces 1 and 2 under illuminant E, suppose they produce, respectively, cone excitations a_1 and a_2 in receptor class "a" and excitations b_1 and b_2 in receptor class "b". The Hubel mechanism gives a correlate of the (dominant) wavelengths w_1 and w_2, say, of light from surfaces 1 and 2 by taking the respective quotients a_1/b_1 and a_2/b_2. Suppose that now the illuminant is changed from E to E'. This produces new dominant wavelengths w_1' and w_2', with respective correlates a_1'/b_1' and a_2'/b_2'. How, then, does the ratio of the wavelengths w_1/w_2 under E compare with the ratio of the wavelengths w_1'/w_2' under E'? This question can be expressed in terms of the correlates, that is, how does the ratio $(a_1/b_1)/(a_2/b_2)$ under E (corresponding to w_1/w_2) compare with the ratio $(a_1'/b_1')/(a_2'/b_2')$ under E′ (corresponding to w_1'/w_2')? Trivially, these expressions can be rearranged:

$$(a_1/b_1)/(a_2/b_2) = (a_1/a_2)/(b_1/b_2) \tag{2}$$

$$(a_1'/b_1')/(a_2'/b_2') = (a_1'/a_2')/(b_1'/b_2') \tag{3}$$

But from the observation that cone-excitation ratios from pairs of surfaces are almost exactly constant under changes in illuminant,[6] the ratio $a_1/a_2 = a_1'/a_2'$ for receptor class a, and $b_1/b_2 = b_1'/b_2'$ for receptor class b. This means that the right-hand sides of Eqs. (2) and (3) are identical, which in turn means that the correlates of wavelength ratios w_1/w_2 [left-hand side of Eq. (2)] and w_1'/w_2' [left-hand side of Eq. (3)] are also identical. I am much indebted to Foster's formalisation above, which gives theoretical support to the present empirical evidence of invariant wavelength ratios, and tentatively relates this newly found mechanism to a well documented one.[6]

Of course, the above logic is subject to the extent to which (CIE) dominant wavelength identifies with Hubel's wavelength correlate, namely, a ratio of excitations in different receptor classes. The above logic indicates in principle that the theory of color constancy from invariant cone-excitation ratios[6] and the present theory of color constancy from invariant wavelength ratios mutually support one another. The mechanism of invariant wavelength ratios presumably develops physiologically from the (necessarily earlier) mechanism of invariant cone-excitation ratios.

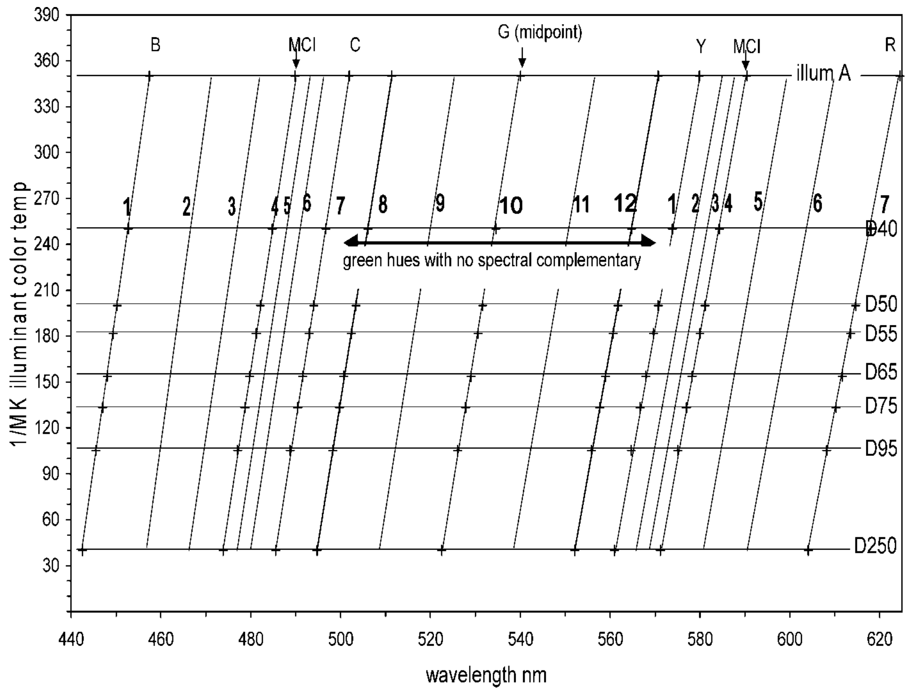

FIG. 5. The spectral mechanism of color constancy. Sloping near-parallel lines (numbered) represent some of the infinite number of spectral constant hues; any pair of lines represent near-invariant wavelength ratios. Each numbered line has a complementary line indicated by the same number, except greenish hues (lines 8–12, arrowed) whose complementaries are nonspectral constant hues (not shown). This figure is not to scale.

Figure 5 derives from Fig. 4 and Empirical Observations 1–3. As in Fig. 4, each sloping line represents a constant hue. Any pair of sloping lines represents a near-invariant wavelength ratio at all the indicated temperatures MK^{-1}. The *x*-axis is limited to the approximate range 440–620 nm, limits to monochromatic optimal color stimuli (see Fig. 4)[23]; beyond those limits, optimal color stimuli are compound colors (effectively nonspectral). Every constant hue has a complementary constant hue, indicated by equal-numbered pairs of lines. However, complementaries to the greenish constant hues between (approximately) lines C and Y, arrowed, are nonspectral constant hues (not shown since nonspectrals have no wavelength). The structure evident in Fig. 5 is symmetrical, simple, and elegant. As the DNA researcher Crick has said,[28] "structure is the natural path to understanding function." The structure here is evidently a mechanism adapting constant hue wavelength to illuminant, to preserve color constancy. In other words, the structure represents a spectral mechanism of color constancy.

Given Fig. 5 axes of wavelength versus MK^{-1}, one may expect that nonspectral constant hues occupy straight sloping lines like spectral constant hues. This raises interesting questions: (1) Given the simplicity of the spectral mechanism of color constancy, does the mechanism's symmetry extend into the nonspectrals to complete the hue cycle? And (2), is there a color space similar to Fig. 5 somewhere in the visual process, incorporating spectral and nonspectral constant hues to the same (or similar) *x*-axis scale? If so, the units of the *x*-axis could not be physical or dominant wavelength but some psychophysically equivalent scale. Such a scale, if derivable from Fig. 5, would be useful. The matter of nonspectral constant hues is beyond the present article's scope but is intended for a later article.

The present analysis of corresponding colors data indicated that low chroma samples tend to give less consistent results, in terms of dominant wavelength shift between light sources, than higher chroma samples. This is presumably due to observers finding the hue of low chroma samples more difficult to judge.[29] This may seem to be contradicted by the fact that color difference ellipses[30] (of equal luminance) in a color space (say CIELUV) are no bigger in areas of low purity than high purity. But consider two equal distances (two straight lines *A* and *B* of equal length) in *u, v* space, with line A along the boundary locus (monochromatic color) and line *B* parallel but near the neutral point. Now, line *B* represents a much greater hue angle and thus greater hue difference than does line A. So the two similarly long lines *A* and *B*, if taken as the major axes in color difference ellipses, would give similar sized ellipses *A* and *B* although *B* represents a greater perceived hue difference.

It has been suggested that highly chromatic color stimuli are not color constant, and so have a high color inconstancy index.[31] This does not necessarily conflict with the present analysis. Consider a highly saturated color sample *S* in illuminant D65, say 510 nm dominant wavelength and, to an individual observer, appearing unique green. To do so in illuminant A, the sample would need to shift 10 nm (according to Table II above) to 520 nm. To do that would require a color sample's instrumentally measured chromaticity coordinates and dominant wavelength to shift between illuminants exactly as does the constant color, i.e. +10 nm from D65-A (per Table II). Unfortunately, that is not the case according to Part 1 of the LUTCHI color appearance data[15,16] (although, interestingly, it is roughly the case for blue and yellowish colors). In the case of green, where the constant color shifts +10 nm, the sample card *S* shifts little or not at all. An observer would probably perceive the difference, i.e. see the sample card *S* as slightly too bluish to be unique green. On the other hand, consider a very desaturated sample *D* of the same approximate hue as the saturated sample *S*. The observer would probably judge sample *D* as a similar green in both D65 and A, because the hue of the desaturated green is harder to distinguish, technically because the hue angle difference is the same for both samples *S* and *D* but the hue angle in low purity, near the neutral point, occupies a much smaller *u, v* distance in the color appearance space. The terms *probably* and *similar green* reflect stochastic reasons: the present analysis of corresponding colors data indicates similar green would result as the mean of several sessions some of which may have given eccentric hue shifts in amount or direction.

It is worth noting Thornton's hypothesis[32] that the dominant wavelengths of constant hues are invariant in varying illuminant. That is, constant hues in Figs. 3 and 4 would plot 90° (vertical) lines. However, the experimental data sets (see Fig. 3) all indicate a significant wavelength shift with illuminant, closely supporting the present theory rather than Thornton's hypothesis. The present article's indicated wavelength shifts with illuminant seem minor but have major effects, primarily in that 87° slopes allow a wavelength shift that follows, to a small but important degree, the shift of illuminant SPD towards longer wavelength with lower CCT (i.e. higher MK^{-1}).

The role played by complementary wavelengths in the process of color constancy seems crucial; their physiological basis is considered in Appendix B. Previously, the role of complementary wavelengths in color vision was thought to be mainly in color mixture and matching (exemplified by their position as opposites through the neutral point in CIE color mixture diagrams), and in chromatic induction (e.g. after images).[33]

CONCLUSIONS

In the plane of wavelength versus 1/CCT (MK^{-1}), it was shown that (a) wavelengths of constant hues plot approximately straight parallel lines, whose mean slope (in Fig. 4 axes scale) is about 87°; see Figs. 3–4 and Eq. (1); (b) such near-parallel lines give near-invariant wavelength ratios; (c) the pair of complementary wavelengths of minimum complementary interval (MCI) for respective illuminants plot two straight almost parallel lines of $87 \pm 1°$ slope, and in fact the complementary wavelengths to any line of $87 \pm 1°$ slope plot a straight almost parallel line (see Fig. 4); and (d) wavelengths of any two parallel or near-parallel lines give almost invariant ratios (see Table III).

Given (a)–(d) above and that the complementary wavelengths to a constant hue represent the complementary constant hue,[17] it was deduced that the set of approximately parallel lines in Fig. 3 (for constant hues) and the set of near-parallel lines in Fig. 4 (for lines of $87 \pm 1°$ slope and their complementary wavelengths) are equivalent sets.

Hence it was concluded that constant hue and complementary constant hue plot near-parallel lines (in the plane of wavelength and MK^{-1}) of near-constant 87° slope, and that any two constant hues, complementary or not, are related by near-invariant wavelength ratios. This empirical mechanism of spectral color constancy (within the overall chromatic adaptation process) has been derived from basic colorimetry and experimental data on constant hues, formulated by Eq. (1) and illustrated in Fig. 5. The mechanism is notably simple and symmetrical, and was shown to relate to Foster and Nascimento's well-known theory[6] of relational color constancy from invariant cone-excitation ratios.

APPENDIX A: DEFINITIONS

Here are given some brief definitions for the nonspecialist reader (Refs. 2, 17, 21 give fuller definitions/descriptions). *Dominant wavelength* of a given color or chromaticity (usually defined by CIE chromaticity diagram *x, y*, chromaticity coordinates) may be found by drawing a straight line from the CIE diagram's illuminant point (e.g. D65) through the given chromaticity point to intersect the spectrum locus (the locus of monochromatic colors), where wavelengths are labeled or their *x,y* coordinates may be looked up in CIE tables.[21,22] Only spectrum-locus colors are monochromatic, i.e. composed entirely of one wavelength (e.g. 590 nm) or nearly so, e.g. a small wavelength range such as 588–593 nm. Other colors have dominant wavelengths, that represent the calculated color mixture of two or more wavelength components; e.g. appropriate luminances for 560 nm and 610 nm may produce a chromaticity of say, 590 nm dominant wavelength. Object colors rarely correspond to monochromatic light so their so-called wavelengths usually are dominant wavelengths. As a dominant wavelength varies in purity or chroma from spectrum locus to neutral point, its hue usually varies (the Abney effect).[2,21,34] *Constant wavelength* means the (e.g. dominant) wavelength remains invariant or constant regardless of varying parameters such as illuminant.

Corresponding colors denote colors, of a given luminance or lightness, that are perceived to color-match (e.g. as unique blue) in different illuminants; they are often named as B, G, or Y, etc, corresponding colors after unique hues.

Complementary colors are defined as a pair of color stimuli, of appropriate wavelengths and power ratio that mix (by additive, subtractive, or any reliable method) a given achromatic color (e.g. the white or gray of the light source). Complementary wavelengths (the basis of complementary colors) lie on opposite sides of the illuminant point in a chromaticity diagram; the term also refers to CIE nomenclature (e.g. 520 c rather than 520 nm) for nonspectral colors which have no dominant wavelength so are named from their complementary dominant wavelength (e.g. as 520 c).

APPENDIX B. PHYSIOLOGY OF COMPLEMENTARY COLORS

The grid of lines (representing some of the infinite number of lines) in Figs. 4 or 5 represents a regulatory mechanism assuring invariant wavelength ratios between any two constant hues over the global range of illuminant MK^{-1}, i.e. regulating adaptation of constant hue to illuminant. What force drives this chromatic adaptation? It may be said generally that chromatic adaptation is initiated or driven by sensory responses in the physiology to change of chromatic surround or light source; but required here is the particular response that drives constant hue's chromatic adaptation. Arguably, the driving force is the automatic (sensory) adaptation of complementary colors to illuminant, since the key to the straight parallel lines in Fig. 4 are pairs of complementary constant hues, e.g. lines B and Y.

What is the physiological basis of complementary colors' chromatic adaptation? The physiological data indicate several types of neural cells with (1) opposed responses resembling complementary colors, and (2) adaptation to light source or chromatic surround. These are the so-called opponent color cells[27,35] or spectrally opposed cells,[7] whose single cell responses (e.g. to a color sample and its surround) typically peak at the wavelengths of complementary colors, for example, Blue-Yellow (about 475 and 575 nm), Red-Cyan (about 610 or longer, and 490 nm), and Green-Magenta (about 500–530 nm and two peaks about 440 and 610 nm forming nonspectral purple; these responses typify the triphasic cells,[35] common in vertebrates). The initial perception among physiologists (led by Svaetichin who enthusiastically but simplistically coined the term "opponent color cells")[27,36] was to categorize the opposed responses as opponent color pairs (e.g. Blue-Yellow and Green-Red). This was despite the "Green" peak in the data (including Svaetichin's famous data[36] on horizontal cells in fish) usually being about 490 nm, cyan, short of the 500–530 nm range for unique green; and in other cases, the so-called "Red" being one response peak of three (about 440, 500–530, 610 nm) in triphasic cell responses representing Green-Magenta. The latter pair (G-M) are not an opponent color pair but a complementary color pair (the difference is detailed in Ref. 33). Most complementary colors may be categorized as three opposed pairs B-Y, G-M, R-C, because yellowish, magenta, and cyan colors complement the remaining hue cycle. There is no opponent color pair able to represent the triphasic cell responses,[35] but only the complementary color pair G-M.

Some physiologists today think the "opponent color" tag is an over-simplification,[37] and treat the opposed responses as opposites (i.e. complementaries) through the neutral point in a chromaticity diagram, or use complementary color terms Cyan-Red (or "blue-green" and red), Green-Magenta, and Blue-Yellow.[38–41] (Note unique blue and unique yellow are opponent colors and also complementary colors).[33] A full exposition of evidence that response spectra of spectrally opposed cells represent complementary colors more closely than opponent colors is beyond this article. At this point in time, from both physiological and psychophysical evidence, one may reasonably suspect that the basis of chromatic adaptation for constant hue is closely concerned with complementary colors.

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FORTHCOMING MEETINGS

Inter-Society Color Council 2008 Annual Meeting

The Inter-Society Color Council (ISCC) will hold its 77th Annual Meeting in Baltimore, Maryland on Sunday, September 14 and Monday, September 15, 2008. Baltimore is a historic city known for its Inner Harbor, National Aquarium, and the resting place of Edgar Allen Poe. General Chairs, Cameron Miller with the National Institute of Standards and Technology, and Carl Andersen with the Federal Highway Administration will be organizing the meeting.

On Tuesday September 16, ISCC will sponsor a "Safety Color Expert Symposium." This one day event will cover all facets of safety colors including the perception, measurement and standardization of regular, fluorescent and photoluminescent materials, and safety light signaling.

ISCC/IS&T Special Topics Meeting on Black and White

Two of the most important colors in any imaging application are white and black. White is normally supplied by the media in printing applications and black or "key" is supplied by a traditional pigmented ink. These two visual concepts hold critical roles in the processes that constitute the graphic arts workflow.

Recently, there has been a renewed interest in these concepts as exhibited by activities within the IDEAlliance Print Properties subcommittee on paper characterization, the SIS Workshop on Paper on optical properties of paper, CIE Publication 163 on the Effect of Fluorescence in the Characterization of Imaging Media, and in papers at the Color Imaging Conference.

This has resulted in a number of research committees and standardization committees being formed to try to understand better the scope of this problem – especially as it relates to international standards on the measurement and communication of color in image reproduction, such as ISO 2469, 3664, 13655, 12647, GRACoL 7 and the G7 press/proofer calibration methodology.

We invite you to join the ISCC and the IS&T in a special 1-day meeting after the 2008 Color Imaging Conference highlighting the recent achievements in the measurement of white and black. Invited and keynote papers will come from the research and standards areas described above and contributed articles will certainly fill in the practical understanding between the requirements of the documentary standards. Topics will include measuring and predicting the media white point when it contain fluorescent brightening agents, predicting the visual impact of the media white on gray balance of an image, correlating instrumental readings to visual judgments under various D50 simulations, the effect of geometry on the measurement of white, an objective assessment and quantification of the percept known as whiteness, the relationship between whiteness and brightness in perceived color gamut, and the relationship between the standard optical properties of paper (ISO 2469) and the end use properties of the media (ISO 12647).

This is going to be a timely and focused look at the issues plaguing the passage of graphic reproduction from a pure art form to an engineering discipline. Please consider staying a day or two after the Color Imaging Conference for this meeting – especially if you are a developer of ICC profiles or software tools for an ICC-based workflow. You will find these topics very relevant and you will want to share your experiences in this area.

For more information on either of these meetings please go to the ISCC website: http://www.iscc.org

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Relative wavelength metric for the complete hue cycle: Derivation from complementary wavelengths

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Abstract. In recent research, it has been increasingly necessary to employ (with a brief description) an extended wavelength metric to cover the complete hue cycle so as to research or represent data as a function of wavelength rather than a psychological scale such as CIELAB or Munsell hue angle. Some problems cannot be solved by treating merely the spectrum but only by treating the whole situation, i.e. the hue cycle. It is timely to fully describe this relative wavelength metric and its derivation from complementary wavelength functions, particularly because much research (mine and possibly others) depends on such a relative wavelength metric to treat nonspectrals in the same coherent scale as spectral hues. The metric provides a useful psychophysical, wavelength-based, ratio scale for the hue cycle. Several indicators, e.g. color order hue cycles, infer the (optimally efficient) spectral hues comprise about 71%, and the nonspectrals about 29%, of the hue cycle interval. This gives a hue cycle whose relative wavelength interval is about 240 nm for illuminant D65. To relate to standard CIE colorimetry, the (spectral) complementary wavelengths to the nonspectrals' relative wavelengths are identified for seven illuminants including D65, D50, C, and A.

Key words: colorimetry, complementary colors, hue, metric, scale, wavelength

INTRODUCTION

In a light stimulus striking the eye, wavelength is one of only two physical variables (radiant power and wavelength) necessary and sufficient to cause the psychological sensation of the three color attributes (hue, brightness/lightness, colorfulness/chroma). The cones do not directly discriminate wavelength but the post-receptoral physiology can calculate wavelength from comparing two cone responsivities.[1,2] The importance of wavelength to the visual physiology is supported by the the plane of wavelength and reciprocal illuminant color temperature where, remarkably, constant hues have recently been shown to plot straight parallel lines.[2] Further, the fact that any constant hue is related to another constant hue by a wavelength ratio that is constant between CIE illuminants (Planckian or Daylight) again indicates the importance of wavelength to the physiology.[2]

In cognitive science or in human information processing, it is recognised that the application of knowledge (such as research and problem solving) first requires the categorisation of knowledge. Color may be categorised by three broad stages in the visual process: (1) as physical stimuli (radiant power and wavelength of light striking the eye); (2) as psychophysical functions relating the physical to the psychological (e.g. luminous efficiency); and (3) as psychological sensations. The latter may be further categorised as three principal attributes:[1] hue, brightness/lightness, and colorfulness/chroma. Each of these comprises the already-mentioned physical, psychophysical, and psychological categories or stages, which are extensively reported (as experimental data) and inter-related (as models or theories) in the literature. Hue is arguably the principal dimension of color (to the nonspecialist, hue *is* color). However, the categorisation of hue poses difficulties. Though the direct physical stimuli (e.g. wavelength) to a phenomenon are normally the easiest to measure (in one coherent scale), unfortunately such is not the case with hue. This has been a problem in measuring and understanding color from the very beginning.[3]

Hue is a cyclic continuum, known as the hue cycle and often represented as a geometric circle. Wavelength is the physical stimulus for hues over the visible spectrum (say 400-700 nm) but the remaining hues of the cycle are nonspectral (the purple hues) and are not stimulated by any single wavelengths but by pairs of short and long wavelengths. Hence science lacks a continuous wavelength scale, or any physical scale, covering the complete hue cycle.

To overcome the problem to some degree, CIE colorimetry employs a psychophysical metric known as dominant wavelength (which represents the resultant wavelength of a mixture of wavelengths) over the spectrum, and which represents the nonspectral hues by their complementary wavelengths (denoted as “c” rather than “nm”, e.g 530 c). This allows specification of color according to wavelength. However, the metric is not one continuous numerical series: It comprises one series for dominant wavelength (from say 400-700 nm) and another series for complementary wavelengths (from about 493 c to 567 c). The two series are incoherent, disallowing spectral and nonspectral hues from being measured, graphed, analyzed, or otherwise treated, as a single cycle or numerical system. And yet, it would not be difficult in principle to develop a psychophysical wavelength metric by finding and formulating some psychophysical structure in the spectrum and extending the structure and numerical wavelength series into the nonspectral hues. Something similar possibly occurs in the visual process, since it needs some sort of wavelength scale over the complete hue cycle to provide a common coherent scale linking spectral and nonspectral hues. Evidence for such a wavelength scale includes invariant wavelength ratios reported between spectral constant hues[2] and used (in the following paper)[4] to predict nonspectral constant hues.

Hence, science presently lacks important categories of hue cycle: a physical wavelength scale, and lacking that, a psychophysical or relative wavelength scale. Nothing can be done about the former, but something can be done about the latter. In some graphical or analytical applications, a physical or psychophysical scale (e.g. wavelength) is necessary or preferable, where a psychological or color appearance scale such as uniform hue difference (e.g. CIELAB or Munsell) would be irrelevant or inappropriate. In the many sciences that employ color (e.g. optics, spectroscopy, astronomy, neuroscience, psychology), graphs conventionally employ wavelength as an axis. Consequently, the nonspectral hues are usually omitted from analyses and graphs of wavelength-based data, even in color science. A wavelength-based metric for the hue cycle would overcome these problems, and for the first time allow data analysis with wavelength as a ratio scale (from one end of the hue cycle to the other).

Presumably the physiology treats the spectral and nonspectral parts of the hue cycle in a coherent scale. Hence, if the physiology has a wavelength scale for the spectral hues, it must have a wavelength-based scale for the total hue cycle, derived from the spectral hues and extended into the nonspectrals. It is required to find a method (hopefully the physiology's method) of relating the nonspectral part of the hue cycle to the spectral part, and the resultant relative wavelength scale. The only apparent method is to employ complementary wavelength relationships, since even the nonspectrals have complementary wavelengths. On this basis, the present paper formulates the structure of complementary wavelength pairs in the spectrum and extends the structure and numerical (wavelength) series into the nonspectral hues, thus deriving a wavelength-based scale for the entire hue cycle, for use with CIE standard illuminant D65, and 2-4 degree visual field. The method was largely developed in previous articles over some years,[5,6,7] integrated and amplified here as a stand alone paper. Appendix B gives data (from the following paper)[4] to extend the use of the wavelength-based metric to several other CIE illuminants and to the 10 degree visual field.

Complementary colors are defined[8,9] as a pair of color stimuli whose admixture color-matches a selected achromatic color stimulus (usually the light source). Complementary colors depend on appropriate complementary wavelength pairs and radiant power ratios, both of which systematically vary with illuminant. Dominant wavelengths and power ratios of complementary colors may be looked up in tables or calculated from CIE colorimetry,[9,10] or complementary wavelength pairs may be found graphically as opposites through the white point in CIE *x, y,* chromaticity diagrams.

The proposed wavelength-based scale will consist of dominant wavelength nanometers (nm) for the spectrals, and where it is extended into the nonspectrals it will be termed *equivalent wavelength* nm (indicated by "e" whenever necessary to descriminate from dominant wavelength "nm"). The whole scale (of dominant wavelength and equivalent wavelength) is a psychophysical scale which may be termed a *relative wavelength* scale since the equivalent wavelength portion is not factually wavelength (or dominant wavelength) but is *relative* to wavelength in the manner to be defined below.

To determine a relative wavelength scale for the hue cycle, two steps are necessary.

Step 1: Determine the interval of the hue cycle, and its limits (or ends), in terms of relative wavelength. That step will provide *equivalent wavelength* numbers to the nonspectral hues.

Step 2: In order to give those equivalent wavelengths a colorimetric identity, their (spectral) complementary wavelengths must be identified; these will vary by illuminant.

Each step will employ at least two different methods of estimation or calculation, giving results in approximate agreement so as to ensure a credible and useful degree of accuracy. The aim is to determine a reasonably accurate scale for use as a standard psychophysical metric across spectral and nonspectral hues, rather than to attempt complete accuracy (which is probably neither achievable nor measurable); however, significant errors will be evident if different estimation methods give significantly different results. To simplify the problem, stimuli will be optimal aperture colors: their monochromatic stimuli occupy the range 442-613 nm;[11,12] shorter or longer wavelengths than that range do not give optimally efficient monochromatic stimuli nor do they admix optimally efficient compound color stimuli (nonspectrals). Optimal color stimuli for the remainder of the hue cycle (nonspectral hues from 613 nm through the purples to 442 nm) are compound stimuli comprising additive mixtures of 442+613 nm. Similar limits and compound stimuli apply to object colors and the MacAdam limits.[9] The line 442-613 nm in color space (see Fig 1) defines the locus of optimal compound stimuli (nonspectrals). This hue cycle omits the spectrum extreme wavelengths as do color appearance systems, because such wavelengths are practically invisible and play no part in normal color vision.

HUE CYCLE INTERVAL

Required is a known function or functions which graph a distinctive and easily analyzed curve over the spectrum, from which the nonspectral portion of curve between the spectrum ends can be predicted by calculation or interpolation. Ideal candidates are functions concerning complementary wavelength pairs, since the nonspectral portion of the function is related in colorimetry to the complementary spectral portion. A characteristic of complementary colors is that three hues (Cyan, Magenta, Yellow, or CMY) complement the remaining hue cycle (Red, Green, Blue hues, or RGB). Several functions related to complementary colors (e.g. wavelength discrimination, and color-matching functions) are similarly trimodal, i.e. with three RGB peaks and three complementary troughs in CMY.

Most readers will be familiar with the trimodality of wavelength distribution in CIE chromaticity diagrams. The distribution for CIE LUV is shown in Fig 1; complementary pairs of wavelengths are opposites through the neutral point and this oppositeness causes the trimodality; note that most wavelengths are compressed into three areas generalized as RGB, and their opposites/ complementaries are the CMY hues. The central circle shows varying angle per constant wavelength interval (5 nm), characterised by large angles in Y and C areas relative to small angles in RGB areas. The indicated 5 nm intervals are limited to the wavelength range 440-615 nm, approximate limits to monochromatic optimal color stimuli.[6] Optimal color stimuli for the remaining hue cycle (nonspectral hues) are compound colors comprising admixtures of 442 and 613 nm, shown on the line 442-613 nm. In Fig 1, the nonspectral or Magenta area opposite/ complementary to the green hues about 510-550 nm (with their highly compressed angles for 5 nm intervals) might be expected to have large angles similar to the Yellow and Cyan areas but, of course, nonspectral hues have no physical wavelength. If they had, their 5 nm intervals may be expected to look rather like the dashed lines in Fig 1. (These and the labelled

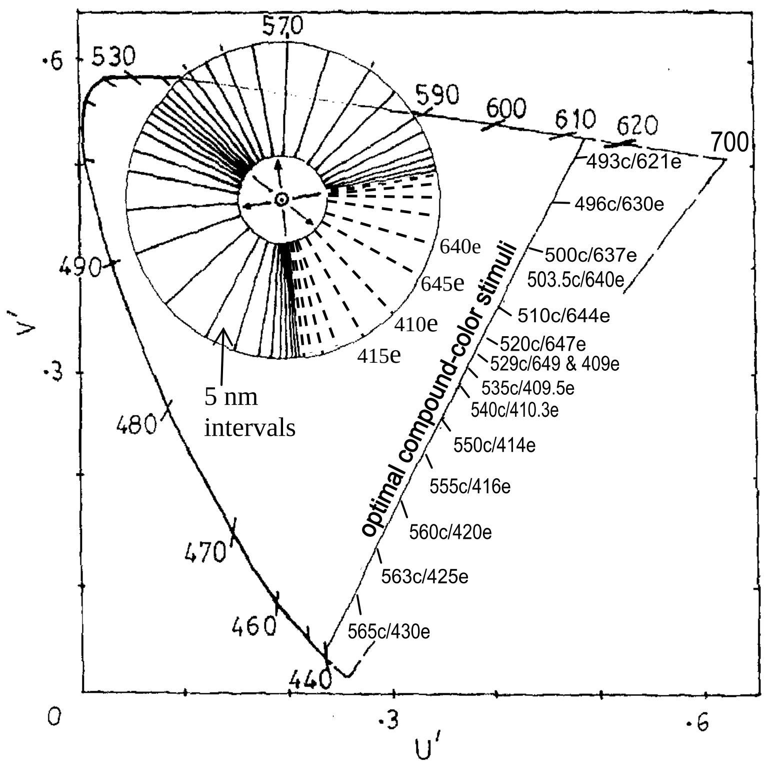


Figure 1. CIE LUV chromaticity diagram. The circle, centered on the illuminant point for D65, shows the angular distribution of wavelengths for constant 5 nm intervals, over the wavelength range 440-615 nm (approx limits to optimal monochromatic stimuli). The frequency distribution is trimodal, with maxima in RGB areas and minima in the opposite (and complementary) CMY areas. Complementary wavelength pairs (e.g. 490 and 600.5 nm) are defined by CIE as opposites through the illuminant point; hence such pairs vary with illuminant. Optimal nonspectral (or compound color) stimuli for all illuminants lie on the line 442-613 nm. Nonspectral Magenta lacks physical wavelengths but if it had, their distribution may resemble the dotted lines (at 5 nm intervals), with large angles similar to C and Y areas. The labeled “equivalent wavelengths” (e.g. 430 e) derive from Fig 3.

equivalent wavelength numbers, e.g. 640 e, will be described later below.)

Complementary Intervals Ratio

It is required to extend the (photometrically efficient) spectrum's wavelength scale over the nonspectrals. It is clear from Fig 1 that distribution of complementary wavelength pairs over the hue cycle will be generally trimodal, with frequency distribution peaks in the RGB hues and troughs in the complementary CMY hues, but a formal expression of the trimodal structure is required. This distribution of angle per 5 nm interval (Fig 1) or wavelength nm per 5 deg hue angle (Fig 2) varies with CIE (or other) chromaticity diagram, but the fundamental relationship in the distribution of complementary wavelength pairs is the ratio of a wavelength interval (nm) to its complementary interval (nm). This *complementary intervals ratio* (CI ratio) is independent of chromaticity diagram as previously noted,[13] and allows the trimodality of complementary wavelengths to be formalised as the ratio of a wavelength interval to its complementary interval of 1 nm. The function is shown in Fig 2 (left *y*-axis). In Fig 2, only the filled-diamond data points are known, as only these have spectral complementaries. The green area's CI ratios are unknown because their complementaries are nonspectral purples, but the relative CI ratio is inferred by the angular distribution of wavelength described above in Fig 1 and shown by the dashed curve (gray line) in Fig 2. This latter function gives valuable assistance by indicating relative CI ratios, e.g. peak CI ratios will occur at or near the RGB peaks.

The CI ratio for the green hues can be deduced in outline by a method using Fig 2 as follows. The curves at the 442 and 613 nm peaks must continue left and right into the nonspectral area until they join, because the curves at 442 and 613 nm cannot rise infinitely higher without implying that the complementary hues' CI ratios (at yellow and cyan troughs) drop to infinitely lower, without the curves ever joining. Hence the curves at the peaks (442 and 613 nm) must become lower while progressing into the nonspectrals, while the curves near the 492 and 568 nm troughs must rise. In this way, the CI ratio curves will (a) rise to join at a peak in the mid-green area about 530 nm, and (b) decrease to join at a trough in the mid-nonspectral area, as suggested by the open-diamond data points.

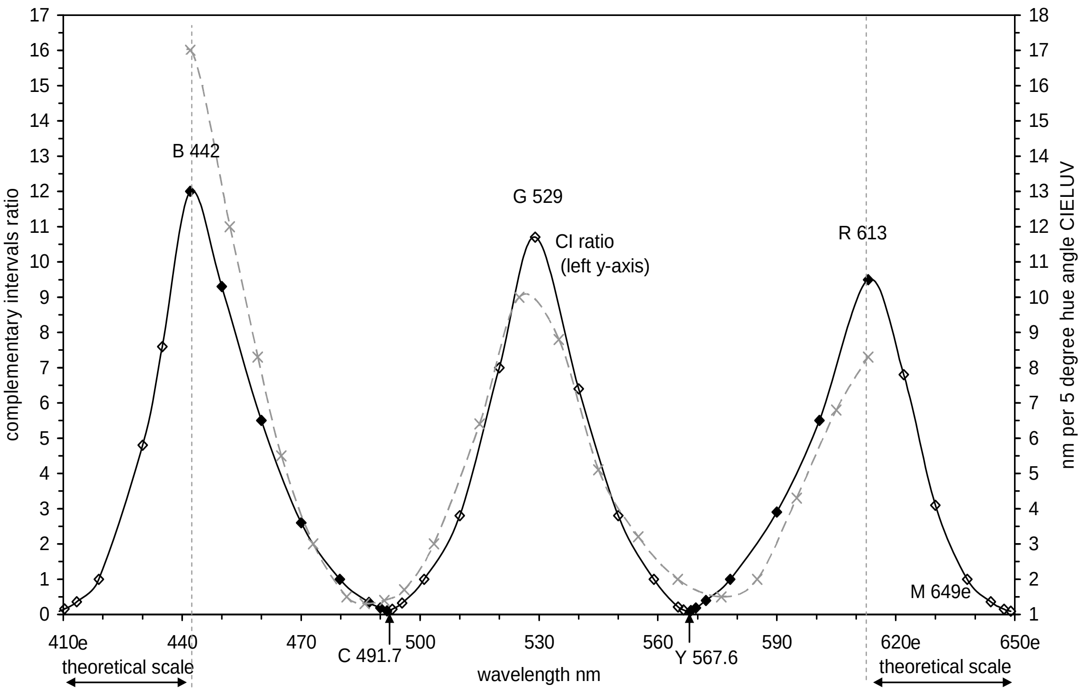


Figure 2. Solid curve (left y-axis): complementary intervals (CI) ratio. Filled-diamond data points are known from CIE data; open-diamond data points are estimates. RGB hues are high CI ratio, and CMY hues are low CI ratio. Wavelength pairs of 1:1 ratio (eg, 480 & 578 nm) are curve peaks in Fig 3. Dashed gray curve (right y-axis): nm per 5 degree hue angle in the CIE LUV diagram.

This general shape of curve is indicated also by the dashed grey line in Fig 2, factually representing (at the right y-axis) the distribution of wavelength nm per 5 degree hue angle in the CIE LUV diagram (see Fig 1), for the spectral range 442-613 nm. This distribution varies between CIE diagrams, but always incorporates the same CI ratios.[13] From Fig 1, the G peak of the dashed gray curve is about 530 nm and intermediate to the B and R peaks in amplitude.

Now, to estimate the CI ratios for the green and nonspectral areas with any accuracy, we need first to estimate the nonspectral interval or alternatively the total hue cycle interval. Four simple methods of estimating these intervals in terms of wavelength are described briefly below. The *first method* is to draw by hand, in Fig 2, smooth curves representing CI ratios between the C and Y troughs through a G peak, assumed (from the gray dashed curve) to be about 530 nm and 10.7 CI ratio (between the B and R peaks at 11.5 and 9.5). The interval from C trough to Y trough is 76 nm, so half that interval, i.e. 38 nm, represents the mean of the intervals from G peak to C trough, and from G peak to Y trough (whatever wavelength the G peak is). What then would be the complementary intervals, from peak (at B or R) to the common trough in magenta (M, at 529 c)? Analysis of the known complementary pair of intervals from B peak to C trough and from R peak to Y trough indicates that a greater range of CI ratios (relative to the complementary interval's CI ratios) gives a slightly bigger wavelength interval than the complementary interval. Hence, the interval from G peak to Y trough (38.6 nm, with a slightly greater range of CI ratios than the interval from G peak to C trough) is expected to be a greater interval (say plus 1 nm) than the complementary interval from B peak to M trough, hence estimated as about 37.5 nm. Similarly, the interval from G peak to C trough (37.3 nm) should be slightly smaller (say minus 1 nm) than the complementary interval R peak to M trough, thus estimated as about 38.5 nm. This gives a total hue cycle interval of 247 nm (171+38.5+37.5) in terms of relative wavelength. However, note that if the differences (e.g. *plus* 1 nm and *minus* 1 nm) are the same absolute amounts, however large, they mutually cancel and make no difference to the overall hue cycle interval. So this method amounts to estimating the nonspectral interval is the same as the green interval between C and Y troughs, i.e. 76 nm. This gives a hue cycle interval of (171+76=) 247 nm.

The *second method* is to estimate the interval of the nonspectrals relative to the known spectral range (442-613 nm) of monochromatic optimal color stimuli. For daylight illuminants D65 or C, the 442-613 nm range occupies 242-286 degrees (or 0.67-0.79 proportion) of the 360 degrees hue cycle in nine uniform color spaces (UCS) listed in Table I, comprising Munsell, OSA-UCS,[4] DIN,[4] CIE LUV,[10] CIE LAB,[10] Farnsworth,[14] MacAdam,[15] Judd,[16] and Judd,[17] UCSs (Kuehni[18] illustrates many of these spaces). That proportion (*p*) represents the ratio of spectral interval to total hue cycle interval *h*. Given the spectral interval 442-613 nm is 171 nm interval, then 171 times (1/*p*) gives *h*:

$171(1/p)=h$ Eqn (1)

Since the proportion *p* varies from 0.67-0.79, the cycle interval *h* varies from 254 nm (for MacAdam) down to 216 nm (CIE LAB, where the nonspectrals are severely compressed). The mean *p* of the nine hue cycles is 0.725, which from Eqn (1) gives a hue cycle interval of 236 nm.

The third method utilises CIE data on complementary wavelengths to estimate the maximum possible relative wavelength interval for the hue cycle. The limits to the photometrically effective spectrum are 442 nm at the short wavelength end, and 613 nm at the long wavelength end. For illuminant

Table I. Interval of hue cycle predicted by Eq (1), for 9 color appearance or uniform chromaticity spaces: Munsell, OSA-UCS, DIN, CIE LUV, CIE LAB, Farnsworth's 1958 transform of CIE 1931 space,[14] MacAdam's 1965 geodesic UCS[15] Judd's 1932 transform of CIE 1931 diagram,[16] and Judd's 1935 UCS.[17] The CIE 1931 color mixture diagram (not a UCS) is merely listed for comparison. The CIE illuminant is C except D65 for OSA-UCS, CIELUV, CIELAB, and CIE 1931.

	442-613 nm, angle	*ratio of hue cycle*	*hue cycle nm per Eq 1*
Munsell	267°	0.74	231 nm
OSA-UCS	265°	0.736	232 nm
DIN	264°	0.73	234 nm
CIE LUV	262°	0.728	235 nm
CIE LAB	285°	0.79	216 nm
Farnsworth 1958 Transform	270°	0.75	228 nm
MacAdam 1965 geodesic UCS	242°	0.67	254 nm
Judd 1935 UCS	255°	0.708	242 nm
Judd 1932 Transform	243°	0.675	253 nm
Mean (of above 9 systems)	*261°*	*0.725*	*236 nm*
CIE 1931 (not a UCS)	242°	0.67	254 nm

D65, the complement to 442 nm is 567.5 nm (a complementary interval of 125.5 nm) and the complement to 613 is 491.7 nm (a complementary interval of 121.3 nm). Given 442 and 613 nm are peaks of CI ratio (see Fig 2), and their complements (567.5 and 491.7 nm) represent minima of CI ratio, the complementary intervals (125 and 121 nm approximately) presumably represent the maximum possible complementary intervals, that is, a half cycle (in both directions around the hue cycle; a smaller-than-half-cyle interval in one direction will give a bigger-than-half-cycle interval in the opposite direction). Both half cycles will be the same, so the smaller of the two (i.e. 121 nm) is the appropriate half cycle. Twice 121 gives 242 nm cycle interval. In this argument, 242 nm is thus the maximum hue cycle interval (for illuminant D65). This number (242 nm) is in fair agreement with the first two estimates (247 and 236 nm).

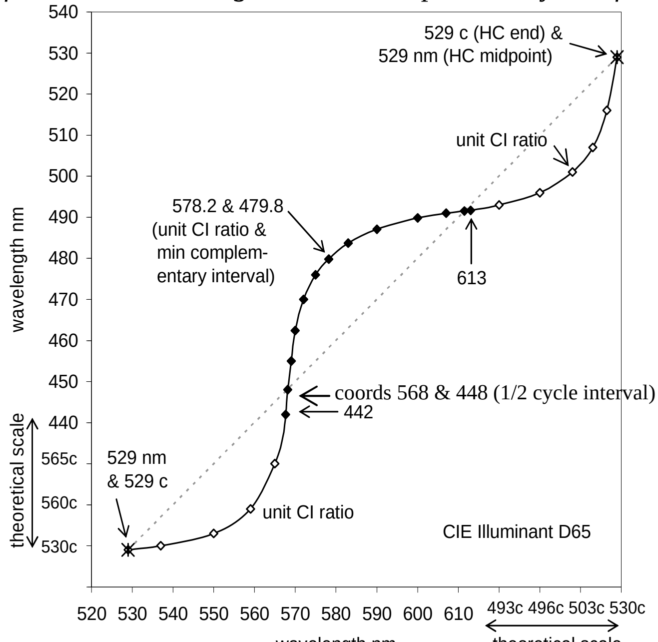
The *fourth method* is to estimate the hue cycle interval from the wavelengths of the RGB peaks of Power Ratios (Watts) of monochromatic complementary stimuli;[7,8,9,11,12] i.e. the power required to neutralise the complementary color of 1W. These peak wavelengths are almost constant with illuminant (the R peak shifts the most, as 607 nm ± 1 nm), indicating a particularly stable function. The (recently re-calculated) peaks for illuminant D65 are 445.5 (B), 531.5 (G), and 606 nm (R),[7] or 160.5 nm overall interval, representing 80.25 nm mean intervals from B to G and from G to R. Assuming the same interval, in terms of relative wavelength, for the third interval from R to B, gives a total 240.75 nm hue cycle interval. The mean of the four methods is 241.4 nm, say 240 nm relative wavelength as a round figure. All four methods are in close agreement, from 236-247 nm, indicating 240 nm is a reasonable estimate of hue cycle interval.

A previous study,[19] including estimation methods different from the above, also deduced a hue cycle interval of 240 nm for illuminant D65. This wavelength scale over the nonspectrals is termed *equivalent wavelength*; it is a psychophysical scale and does not imply physical wavelength. [The 240 nm interval compares with 242 for Judd's 1935 UCS (Table I),[17] the first UCS in color science other than projective transformations of CIE color mixture diagrams.]

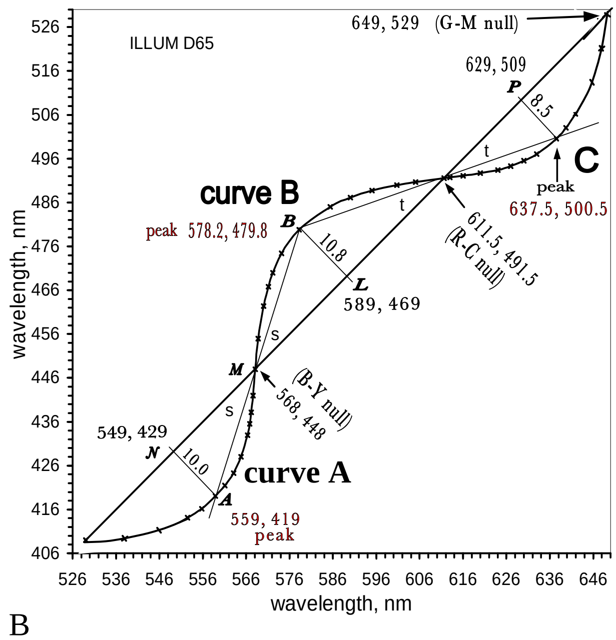
Incidentally, note that the 240 nm hue cycle interval gives a maximum complementary interval (between a complementary pair) of 120 nm, i.e. half cycle. Now the only factual complementary wavelength pairs of 120 nm complementary interval, from the 1931 CIE data,[10] are 448 and 568 nm, and 611.5 and 491.5 nm. This suggests 448 nm (rather than 442) and 611.5 nm (rather than 613) are the true CI ratio peaks, which is possible since the CIE 1931 chromaticity coordinate data at 1 nm intervals were smoothed from larger experimental intervals. *This possibility is of theoretical interest only, and in this study the CI peaks will remain at 442 and 613 nm. However, the theoretical peaks at 448 and 611.5 nm provide a *fifth method* as an interesting version of the *first method*, above, of estimating the nonspectral interval in Fig 2. Given that a CI ratio for a wavelength is reciprocal for the complementary wavelength, then a wavelength interval from CI peak to CI trough (e.g. G peak 529 nm to C trough 491.7 nm, interval 37.3 nm; or G peak 529 nm to Y trough 567.6 nm, interval 38.6 nm) may be assumed to be effectively the same interval for the complementary interval, e.g. M trough to R peak, or M trough to B peak. Hence, if 611.5 nm is the R peak, then the M trough should be at (611.5+37.3=) 648.8 nm. If 448 nm is the B peak, then the Magenta trough should be at (448-38.6=) 409.4 nm. This gives hue cycle ends at 409.4 and 648.8 nm, interval 239.4 nm, very close to the 240 nm interval concluded above from several methods.

Given a hue cycle interval of 240 nm/e, the next problem is to carefully determine the cycle

*Footnote: * So it is possible that max CI ratio is at 448 rather than 442 nm, say about 11.5:1, and then reduces to say about 10.5:1 at 442 nm (limit to optimal monochromatic stimuli), beyond which the CI* *ratios for shorter wavelengths increase exponentially to infinitely large at the spectrum extreme.*



A



B

Figure 3. A. Wavelength plotted to complementary wavelength as *x* and *y* coordinates (solid-diamond data points) per CIE data for illuminant D65. Open-diamond points are wavelength and nonspectral complementary, e.g. 530 nm and 530 c; the latter are correct numerically but their locations on the axes are estimates; i.e. the indicated complementary wavelengths were chosen to fit the labeled 10 nm intervals (in the linear scale) to give a smooth curve (and to match CI ratios in Ref 1). Two aste-risks represent the complementary pair 529 nm and 529 c (hue cycle midpoint and hue cycle ends, at 409 and 649 nm *equivalent wavelength*), representing 240 nm hue cycle interval. **B.** As Fig 3A but showing features of the geometric method of calculating coordinates of peaks of curves A and C. Curve B is fully spectral, comprising spectral-spectral complementary wavelength pairs. Curves A and

C comprise (mostly) spectral-nonspectral pairs. Each axis represents relative wavelength, nm.
ends, or alternatively the cycle midpoint if the latter is defined (see below) as the mean of the cycle ends. Hence we need only to determine one (the midpoint or the ends) to find the other.

A useful perspective of the structure of complementary wavelength pairs is shown in Fig 3A. (The same graph is basic to Cohen's mathematical study[20] of complementary colors.) The filled data points represent complementary wavelength pairs (from the CIE 1931 data) as *x* and *y* coordinates, within the limits 442 and 613 nm, and thus the curve represents the achromatic locus in color space. This "white curve" corresponds to the illuminant white point in a CIE diagram but allows more analysis and information than a point. Note the peak of the curve represents the minimum interval between a pair of complementary wavelengths (termed *minimum complementary interval*), and marks a 1:1 complementary intervals (CI) ratio, for the respective illuminant. This unique wavelength pair varies between illuminants,[5] but for illuminant D65 it is 578.2 and 479.8 nm, an interval of 98.4 nm. The mean of the pair, 529 nm for D65, represents the center of symmetry in the complementary wavelengths structure for the respective illuminant.

This center of symmetry is the ideal candidate for a *hue cycle midpoint*, defined here as follows: it is (1) primarily, the mean of the complementary wavelength pair of *minimum complementary interval*; and (2) the mean wavelength of the cycle ends, which are adjusted to make their mean coincide with the hue cycle midpoint. Note that 529 nm agrees closely with the 530 nm peak of the dashed gray line's G curve in Fig 2, is located in mid-spectrum, and its complementary (529 c) is located in Fig 1 at about the middle of the nonspectral range, where one might expect to find the hue cycle ends. Given the cycle midpoint is defined as the mean wavelength of the cycle ends, then the latter are each a half-cycle from the midpoint, ie, 120 nm, given the estimated cycle interval of 240 nm/e. Hence the cycle ends are 409 and 649 e, as shown in the *x*-axis of Fig 2.

From the symmetry of Figs 1 and 2, it is reasonably assumed that the hue cycle ends are not only equidistant but complementary to the cycle midpoint (or very nearly so). Knowledge of the hue cycle ends facilitates a more accurate estimate of CI ratios, described below together with the design of the uniform wavelength scale and the complementary wavelength scale (Fig 4).

COMPLEMENTARY INTERVAL RATIOS AND WAVELENGTH SCALES

The location of the hue cycle ends facilitates a more accurate estimate of the CI ratios of the green and nonspectral curves in Fig 2 (solid black line). Assuming 10.7 CI ratio for the G peak (an arbitrary number, slightly more than the mean of B & R peaks, 11.5 & 9.5), one can proceed to determine simple curves for the green and the nonspectral areas (supported by the symmetries in Fig 3B, discussed below). These curves represent balanced compromises to such factors as CI ratios in Fig 2 and the geometry of, and curve-fitting in, Fig 3.

In Fig 3A the cycle ends representing 529 c are half-cycle intervals from midpoint 529 nm on both the x- and y-axes; an asterisk plots each 529 nm & 529 c complementary pair. Each asterisk represents a complementary interval of 120 nm (e.g. 409 & 529 nm), as therefore does the dotted gray line (or *curve mean*) drawn between the two asterisks. This is the maximum possible interval (*in both directions around a cycle*) between two complementary wavelengths (termed the *max complementary interval*) and may be expected to represent (or at least approximate) the theoretical max CI ratio, relative to one axis (*x* or *y*), and the reciprocal minimum CI ratio for the other axis. Similarly, the *minimum complementary interval* (for 578.2 and 479.8 nm, or the coordinates for any other curve peak) represents unit CI ratio, as already mentioned.

Given the *max complementary interval* is 120 nm, the appropriate complementary wavelength pairs can be determined from CIE data. For illuminant D65, only two such pairs exist: 448 & 568 nm, and 491.5 & 611.5 nm. Both pairs fall within the wavelength range 442-613 nm, and both lie on the intersection of the white curve with the *curve mean* (dotted gray line), shown in Fig 3A. Both pairs are close to the max CI ratios, found from CIE data to be at 442 and 613 nm (Fig 2), limits to monochromatic optimal color stimuli. Note how, in Fig 3A, the curve from 448 to 442 nm becomes increasingly closer to a straight vertical line: this indicates a slight asymmetry in the CI ratio math function. Between the max CI ratio at 442 nm (approx 11.5:1 ratio) and the minimum CI ratio at 409 nm (i.e. 529 c, 1:10.7 ratio), both on the *y*-axis of Fig 3A, there is necessarily a curve peak with unit CI ratio.

The coordinates of this curve peak may be found geometrically, with the aid of Fig 3B. A perpendicular from the peak of curve B (which consists wholly of known, spectral, complementary

Table II. Complementary intervals (CI) ratio for indicated wavelengths (λ) nm or e, for illuminant D65 (see Fig 2). Each row shows a pair of complementary wavelengths (λ and λc), with reciprocal CI ratios. Ratio represents 1 nm intervals as *x*:1. Unit CI ratios occur at six λs. Max and min CI ratios (shown in bold) per CIE data occur at three λs and the complementary three λs, respectively. 409 e & 649 e are hue cycle ends (same magenta hue), complementary to 529 nm (hue cycle midpoint). Wavelengths outside the range 442-613 nm are "equivalent λs".

λ nm/e	*CI ratio*	*λc nm/e*	*CI ratio*
409	**0.094**	**529**	10.7
410.5	0.156	540	6.4
413.4	0.36	550	2.8
419	1	559	1
430	4.8	565.1	0.21
435	7.5	566.5	0.13
442	**11.5**	**567.6**	**0.087**
450	9	568.3	0.11
460	5.5	569.6	0.18
470	2.5	572.1	0.4
479.8	1	578.2	1
487.1	0.35	590	2.9
490	0.18	600.7	5.5
491.7	**0.1**	**613**	**9.5**
493	0.143	622	6.8
495.5	0.32	630	3.1
501	1	638	1
510	2.8	644	0.36
520	6.8	647.3	0.143
529	**10.7**	**649**	**0.094**

wavelength pairs) in Fig 3B to the curve mean axis divides the curve interval (between nulls) into unequal "halves" at *L* (coordinates labeled). This point is exactly 3/12 of HC interval from HC end or midpoint, predicting *N* and *P* at symmetrical 1/12 and 5/12 HC intervals (coordinates labeled). Adjacent "halves" in curves *A* and *B*, either side of the B-Y null at *M*, contain corresponding (ie, proportional) triangles *BML*, *AMN*, from two equal angles "*s*" and right-angles at *N* and *L*, allowing calculation of curve *A*'s height from proportionality, as 10 nm. Added to the known coordinates of N, gives the peak's coordinates as 559 and 419 nm. Similarly the white curve *C*, from 611.5 nm (max CI interval) to 649 e (ie, 529 c), both on the x-axis, necessarily contains a unit CI ratio and curve peak, whose coordinates may be calculated similarly. Given the peaks of curves *A* and *C* (whose heights and intervals are now known), the remaining "half" curves (from peak to null) may be interpolated by curve-fitting, from assuming curves *A* and *C* are proportional to known spectral curve *B*.

When these theoretical curves are finalized (see open-diamond data points in Figs 2-3A), they may be considered quite accurate since they accomodate several conditions: they must fit not only in Fig 3 but also in Fig 2, where open-diamond data points indicate the final CI ratios listed in Table II.

Final Scales

Fig 4A illustrates the *uniform wavelength scale* over the hue circle, derived above with the help of Figs 2-3. Fig 4B illustrates the *complementary wavelength scale*, also derived from the structures in Figs 2-3. In this circular scale, every diametrically opposite pair is complementary. Certain characteristics of this scale were deduced and made standard as follows, so the complementary wavelength circle can be constructed for other illuminants to the same standards as Fig 4B.

The guiding principle is to modify Fig 4A only where necessary to accord with complementary wavelength structures (Figs 2-3). Hence, the Fig 4B hue circle is centered (at top center) on the *hue cycle midpoint* 529 nm. The main deductions are as follows. *First*, because this midpoint is defined as the mean of the complementary pair (479.8 and 578.2 nm) of *minimum complementary interval* (MCI), the midpoint wavelength lies exactly intermediate to them; its complementary (529 c) lies opposite to 529 nm, of course. Hence, these four wavelengths lie at 90 degree intervals over the circle.

Second, the two other complementary pairs (501 & 638 e, and 559 & 419 e) of *minimum complementary interval* (and of unit CI ratio) which form the curve peaks in Fig 3 (from the geometric method illustrated in Fig 3B) are placed at equal 60 deg intervals around the circle, starting from the pair 479.8 and 578.2 nm. This 60 deg placement assumes the wavelengths of the three pairs of *minimum complementary interval* lie at 1/12, 3/12, 5/12, 7/12, 9/12, and 11/12 of the hue circle, measured clockwise from the hue cycle ends 409/649 e. This follows from the positions of 479.8 and 578.2 nm as exactly 3/12 and 9/12 of the hue circle (Fig 4B). This assumption becomes a standard for the complementary wavelengths circle for other illuminants.

Third, the three half-cycle complementary pairs 448 & 568 nm, 491.5 & 611.5 nm, and 529 nm & 409/649 e occupy the same positions as in Fig 4A, since there is no reason to modify them.

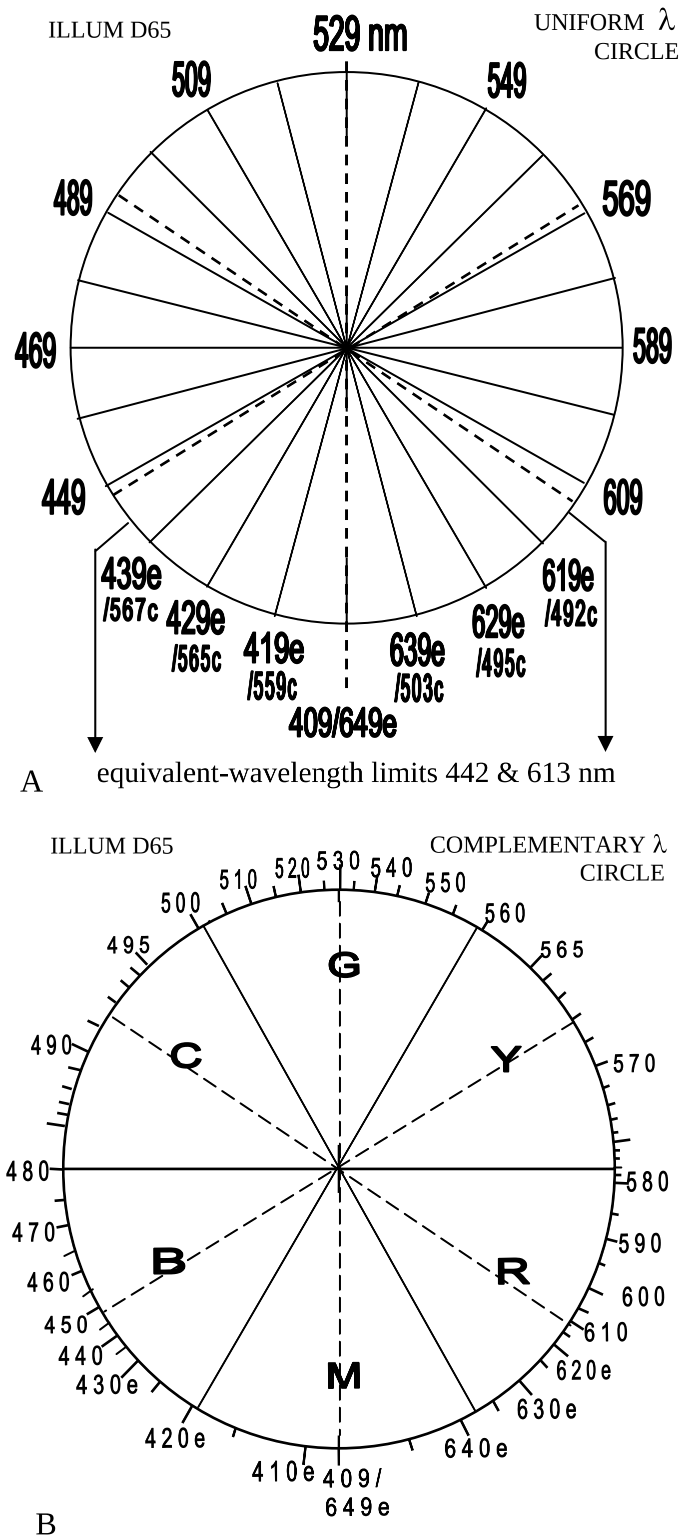


Figure 4. A. Hue circle in uniform wavelength scale, for a 240 nm/e cycle interval from 409-649 e relative wavelength, for illuminant D65. Dotted lines: only 3 opposite wavelength pairs are complementary: 448&568 nm, 491.5&611.5 nm, 529 nm & 409/649 e. **B**. Hue circle in complementary wave-

length scale, from Fig 3. All opposed pairs are complementary. 1/2-cycle complementaries (dashed lines) 448&568 nm, 491.5&611.5 nm, 529 nm & 409/649 e, are same angles as in Fig A.

These wavelengths are almost uniformly spaced, at 60±2 degrees. The three pairs are shown by dashed line (as in Fig 4A), and the three pairs of *minimum complementary interval* are shown by solid line. Hence, the scale is regulated at 12 intervals, for 6 standard types of complementary wavelength pairs. In the intermediate intervals (about 30 degree angles), wavelengths are distributed as opposites to their complementaries but also at the correct CI ratios (Table II) and correct angles (Appendix A).

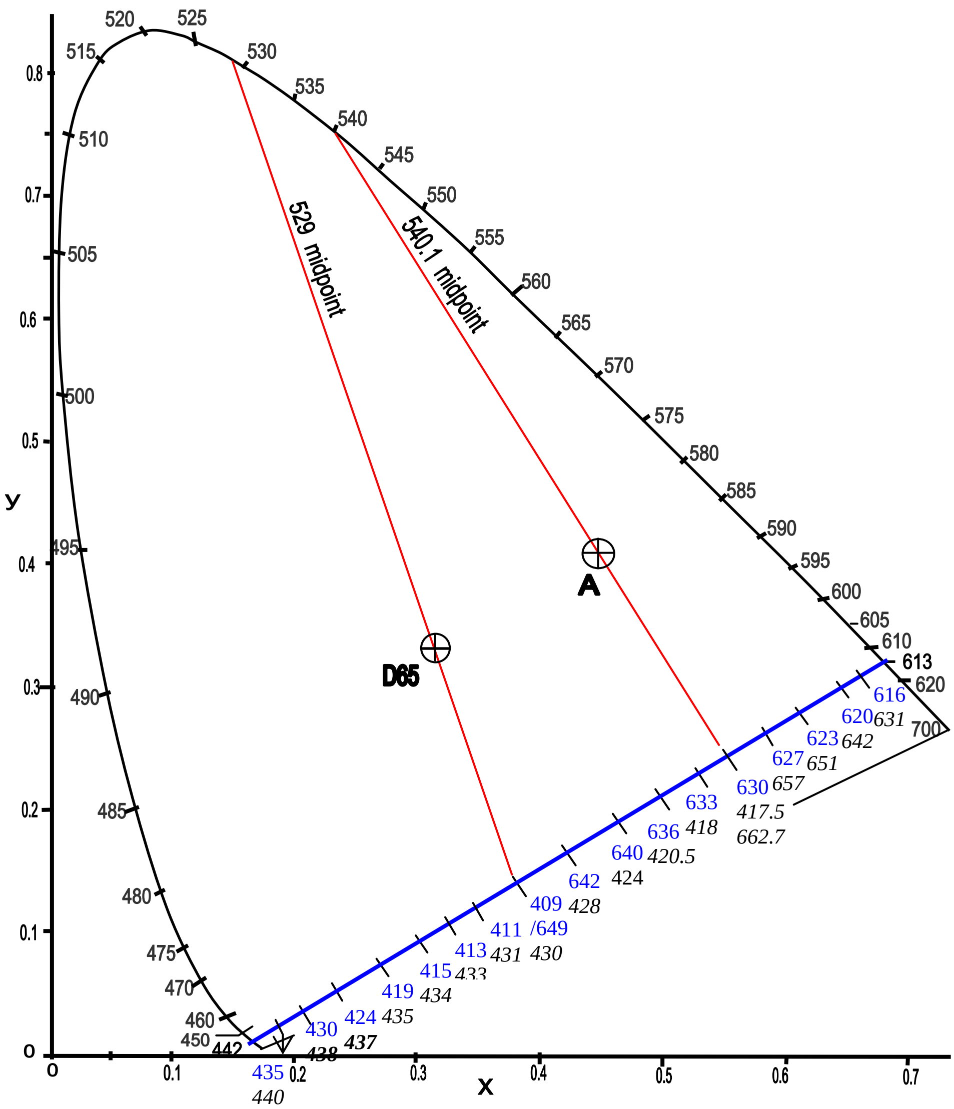
The above transposition of complementary wavelength pairs in Fig 3B to the correct complementary wavelength distribution in Fig 4B (correct in angle as well as opposite pairs) is best achieved by means of Eqn (2) in Appendix A, which explains the equation's derivation and use.

In Appendix B, Tables III and IV (from Ref.4 where hue cycle intervals for other illuminants are calculated) give relative wavelength data for other illuminants and for 2^o and 10^o visual fields. (These data are shown here, ahead of Ref. 4, because this paper and Ref. 4 are interdependent, and were submitted and reviewed together, and were intended to be printed together. The data also give this paper a stand-alone utility.)

CORROBORATION OF METRIC

The calculation of hue cycle intervals and complementary pairs for illuminants D65 and A,[4] listed in Table III below, allows the determination of (equivalent) wavelength shifts for nonspectral corresponding colors for illuminants D65 and A. This in turn allows a test, and potential corroboration, of the relative wavelength metrics for those illuminants. Up to now, these nonspectral shifts could not be quantified,[2] since they needed relative wavelength metrics for both illuminants D65 and A.

Fig 5 shows the CIE 1931 diagram with nonspectrals' equivalent wavelengths for illuminants D65 and A (Table III), labeled on the locus of nonspectral optimal color stimuli (the line 442-613 nm). Fig 6 shows the same but is superimposed on Fig. 1A from Ref. 2, which illustrates CSAJ data (from 104 subjects) on corresponding colors and their wavelength shifts from illuminant D65 to A.[2] Note that the chromaticity shift of a constant hue (e.g. the yellowish hue labeled 51) from its position (a black cross) in illuminant D65 to illuminant A (another cross) is shown by a fine black arrow between



the two crosses, and its direction of shift around the boundary of optimal colors is shown by heavy red

Figure 5. 1931 CIE diagram showing equivalent wavelengths for illuminants D65 and A., labeled on the locus of optimal compound color stimuli (line 442-613 nm).

arrows. Strangely, the direction of shift from the hue's dominant wavelength in illuminant D65 to its dominant wavelength in illuminant A, reverses direction over the nonspectral hues, relative to the clockwise direction (of heavy red arrows) around the spectrum. Yet this reversal does not affect constant hues' positive and fairly uniform wavelength shift from illuminant D65 to A. Over the spectrum, this is about 10 nm positive shift and remains about the same for nonspectrals despite the reversed direction of chromaticity shift from clockwise to anticlockwise. This phenomenon, on first inspection, is surprising but is explained by the appropriate numerical difference of *equivalent wavelengths* for the two illuminants.

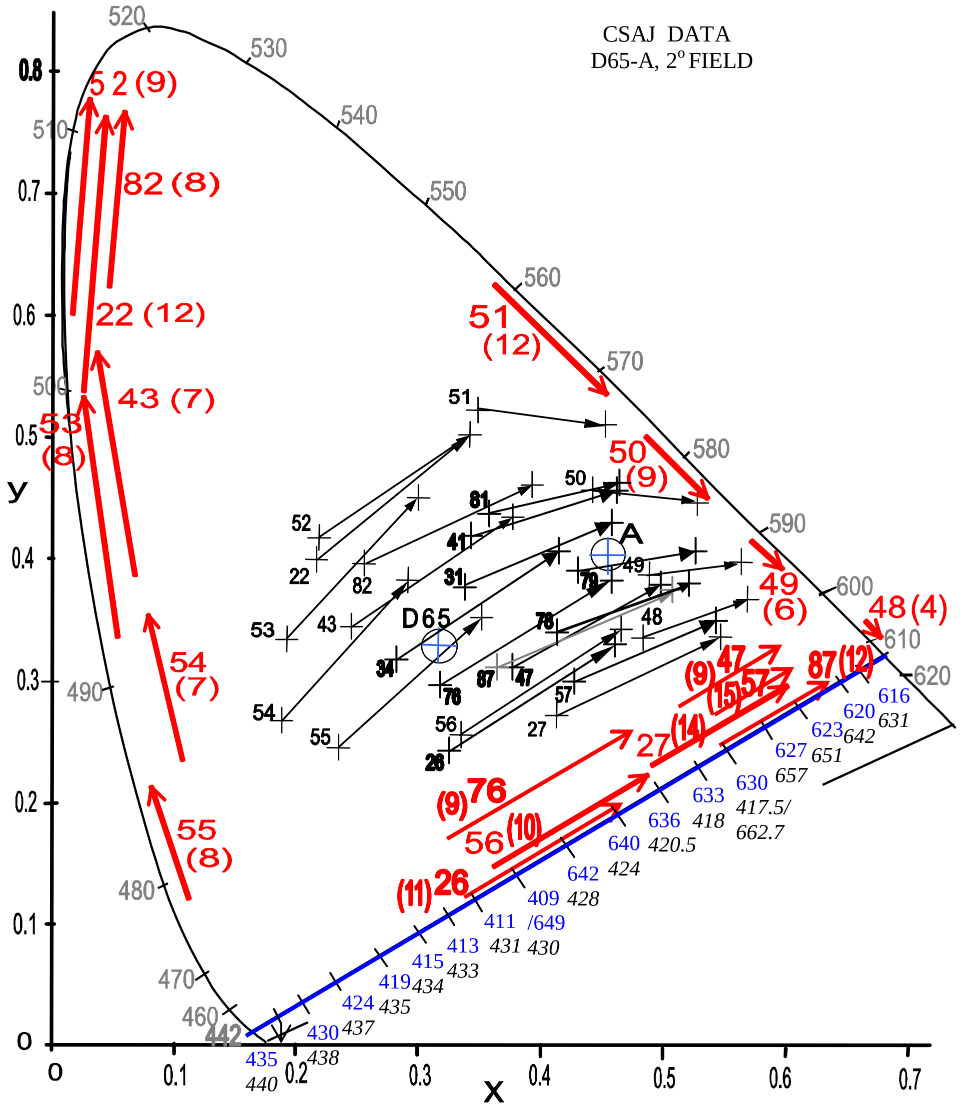


Figure 6. As Figure 5 but showing wavelength shift (numbers in parentheses) of corresponding colors from illuminant D65 to A, including nonspectrals; from CSAJ data in Ref. 2. A line drawn from illuminant point through a (nonspectral) chromaticity point (black cross) to intersect the locus of optimal compound color stimuli (blue line) indicates the nonspectral's equivalent wavelength.

The nonspectral equivalent wavelength shifts for the CSAJ data vary from 9 to 15 e (shown in parentheses for hues #26, 27, 47, 56, 57, 76, 87, in Fig 6), with a mean of 11.4 e. The nonspectral shifts from D65-A for the other three data sets graphed in Refs. 2 and 5 are as follows: (1) LUTCHI D65-A: mean 9.6 e, varying from 8-12 e (hues #9, 40, 41)[5]; (2) Helson et alia: mean 9.9 e, varying from 4 to 14 e (hues #3, 50, 55, 57);[2] Breneman: mean 8 e, varying from 6-10 e (hues #2, 12).[2] The grand mean for the nonspectral shifts of the four data sets D65-A is 9.8 e. This compares with the 9.9 nm mean spectral shift of the four data sets for D65-A (Table II of Ref. 2), indicating that mean non-spectral and spectral wavelength shifts of constant hues are practically the same, as one would expect in a relative wavelength scale that is coherent over the complete hue cycle (Fig. 4).

This result provides a satisfactory corroboration of the general accuracy of the relative wavelength metrics for illuminants D65 and A, including: (a) the respective hue cycle intervals (240 nm/e for D65 and 245 nm/e for A), and (b) the spectral-nonspectral complementary pairs for the respective illuminants (Table III, adapted from Ref. 4).

Just as some spectral hues were omitted from illustration in the figures[2,5] due to eccentric or obviously erroneous data, three nonspectral hues were similarly omitted: Hue #1 in CSAJ data, and hues #2 and 47 in Helson data, are located near the 442 and 613 nm limits where the direction of chromaticity shift reverses, and where the spatial size of the arrowed shifts is very sensitive; a slightly eccentric shift here can mean a large error (e.g. ± 10 or 15 e) in wavelength shift.

The relative wavelength metric, proceeding clockwise around the boundary of optimal colors (the complete hue cycle, Fig 6), is numerically common to all illuminants over the spectrum but not the nonspectrals. The latter's *equivalent wavelengths* vary by illuminant, because complementary wavelengths to given mid-spectrum wavelengths change position on the purple line by illuminant. Just as the wavelength metric proceeds continuously clockwise, so too does the constant hues' wavelength shift from illuminant D65 to A, but the direction of chromaticity shift in color space (Fig 6) reverses without disrupting the positive wavelength shift. This is due to the numerical difference between *equivalent wavelength* numbers (and originally complementary wavelength numbers) for different illuminants at any given position on the purple line, which in turn is due to the shift of illuminant neutral points along the Planckian locus.

Fig. 6 suggests that the relative wavelength metric (in some form) exists innately in the visual system, since the complex relations and finely-differentiated equivalent wavelength numbers (all producing the approximately constant wavelength shift throughout color space in Fig. 6) seem too elegant to represent a man-made artifact. Only a relative wavelength metric, or a very similar extension of dominant wavelength, is able to provide a continuously positive and roughly uniform amount of shift (in any selected psychophysical metric, whether wavelength or another metric, e.g. CIELAB hue angle) of constant hue around the hue cycle, in both spectral and nonspectral hues, while also accomodating a reversed direction of chromaticity shift over the nonspectrals. This relationship can be shown (using Table III data) to apply to wavelength shifts between any two CIE illuminants.

CONCLUSIONS

A relative wavelength metric for the complete hue cycle and CIE daylight illuminant D65 has been derived by extending the dominant wavelength scale (specifically that of the range 442-613 nm) over the nonspectral hues. The method was (1) to estimate the hue cycle interval in terms of relative wavelength, and (2) to formulate the trimodal structure (typified as RGB peaks, CMY troughs) of complementary wavelength pairs in terms of the complementary intervals ratio (Table II), and (3) extend the structure into the nonspectrals. This gave a uniform relative wavelength scale over the hue cycle, including “equivalent wavelengths” for the nonspectrals, which were identified colorimetrically in the CIE system by specifying their spectral complementary wavelengths.

The same process applied to illuminant A, in Ref. 4, provided the necessary data (Table III) to measure equivalent wavelength shifts of nonspectral constant hues from D65-A (Fig 6).

The effectiveness of the metric in quantifying constant hues’ wavelength shifts raises the question: Is the metric effective not only in science but in physiology? Does the physiology use a similar metric to overarch the nonspectrals and bring them into one coherent scale with spectral wavelengths? There seems no other option to such a metric if wavelength (or its psychophysical equivalent) is important to the physiology, as is strongly suggested by the remarkable symmetry and exactly straight lines of complementary wavelength pairs in the plane of wavelength and MK^{-1} (Figs 3-4 in Ref. 2).

It seems that complementary wavelengths, by means of mechanisms not yet well understood, provide a bridge overarching the nonspectrals to complete a coherent psychophysical metric for the hue cycle, which automatically (sensorily) adjusts to any Planckian light source.

APPENDIX A: COMPLEMENTARY WAVELENGTH DISTRIBUTION

The transposition of complementary wavelength pairs in Fig 3B to the exactly correct (in angle) complementary wavelength distribution in Fig 4B may be achieved by means of Eqn (2) below. But first, a preliminary explanation is helpful. Fig 4B transposes the complementary wavelengths function in Fig 3B as follows. Recall that Fig 3B's curve mean axis coordinates (the diagonal line) are all ½ cycle pairs. Every complementary wavelength pair (as *x*, *y*, graph coordinates) on the white curve (labelled as curves A, B, C) is perpendicular to its (say) *base coordinates* on the curve mean axis (the diagonal line). The difference between the latter *x*, *y*, coordinates (as linear ½ cycle pairs) and the complementary pairs is that between linear progression and the exponential progression of the complementary wavelengths function; the math complexity is avoided if treated geometrically as follows. By transposing (converting) the complementary wavelength coordinates to their base coordinates on the curve axis (flat-lining the curve, as it were), the uniform wavelength circle (Fig 4A) is converted to a complementary wavelength circle (Fig 4B). Hence ½ cycle complementary wavelengthss (as nulls) retain same angles in Figs 4A and B but, for example, 479.8 and 578.2 nm (peak of curve B) become 469 and 589 nm, and occupy the latters' angular positions in Fig 4A, as shown in Fig 4B. This transposes all complementary wavelengths to diametrical opposites in Fig 4B. Consequently, MCI wavelengths fall at 60^{o} intervals and the spectral MCI wavelenths (479.8 and 578.2 nm) are horizontal opposites. This angular symmetry in the HC structure is isomorphic in all illuminants.

Fig 3B can be transposed to Fig 4B by Eqn (2), using angles. Hue angle (*h*) is taken to be 180^{o} at HC midpoint, clockwise from *h*=0 at the midpoint's complementary wavelength (the HC ends, bottom center of the circle). Consider a wavelength in the *x*-axis range (Fig 3B), say 556 nm. Hue angle of the wavelength (λ) in the uniform wavelength circle (Fig 4A) is found directly by entering the "λ" inEqn (2). But in the complementary wavelength circle (Fig 4B), "λ" in Eqn (2a) represents not the given *x*-axis wavelength (556 nm) but its base coordinate (on curve mean axis, Fig 3B) in terms of the y-axis; i.e. 546.1 nm. (Use coordinate *y* if the wavelength is in the *y*-axis range.) Where *I* is HC interval nm and *M* is HC midpoint for the illuminant, and *a* is angle between HC midpoint and λ:

$(\lambda\text{-}M)(360/I)=a$ (2a)

$180\pm a=h$ (2b)

In Eqn (2b), *h* gives the angle from 0 degrees at the hue cycle ends (bottom center of hue circle in Fig 4B. For example, consider wavelength 556 nm (x-axis). Its x-axis base coordinate is found to be 546.1 nm. *M* is 529 nm, and *I* is 240. So Eqn (2a) reads: 546.1-529 (360/240)=a= 25.65; so Eqn (2b) reads: 180+26.65=h=205.65 degrees (where the original wavelength, 556 nm, is plotted into Fig 4B).

It has been said, by Rood and by Kuehni,[18] that there is no unique, non-arbitrary, or "correct", system of arranging the relative angles of complementary wavelength pairs in a hue circle or chromaticity diagram. However, Eqn (2) does offer such a system. Given the white curve (and its complementary wavelengths or *x*,*y*, coordinates) in Fig 3B, the correct math method of representing Fig 3B's complementary wavelength distribution in a geometric circle is per Eqn (2). This applies to any illuminant and its respective complementary wavelength pairs; e.g. the same type of graph as Fig 3B is used in Ref. 4 for five illuminants (A, D40, D50, D75, D250).

APPENDIX B: OTHER ILLUMINANTS AND VISUAL FIELDS

The relative wavelength metric will change slightly with illuminant since complementary wavelength pairs change with illuminant color temperature. Ref.4 determines hue cycle intervals and spectral complementary wavelengths to the nonspectral equivalent wavelengths, for various illuminants. These data (for small visual fields, 2-4 deg, and the CIE 1931 observer) are listed here in Table III for several illuminants, to allow workers to apply the relative wavelength metric to other illuminants besides D65. Also listed are complementary wavelength data for the CIE 1964 observer and 10 deg visual fields, in Table IV.

Table III (adapted from Ref 4). Spectral-nonspectral complementary wavelength pairs for seven illuminants, A, C, D40, D50, D65, D75, and D95 (bottom right); some spectral-spectral complementary pairs are also listed, in italics. More data are given for common illuminants (D65, D50, C, A). Half-cycle pairs are shown in **bold** and MCI pairs in **underlined** **bold** (spectral pairs in italics). The first and last wavelength pair listed for each illuminant is the 1st hue cycle (HC) end and HC midpoint, and the 2nd HC end and HC midpoint. Given the HC intervals for the illuminants in this table, complementary pairs are ±0.5 nm uncertainty.

Illum D65 pairs		*Illum D50 pairs*		*Illum C pairs*		*Illum A pairs*		*Illum D40 pairs*		*Illum D75 pairs*	
529	**408.9**	**531.6**	**410.91**	**528.7**	**408.87**	**540.1**	**417.5**	**534.5**	**413.16**	**527.8**	**407.98**
532	409.2	535	411.1	533	408.8	544	417.7	539	413.4	533	408.5
535	409.5	538	411.3	537	409.3	548	418	544	413.8	538	409.3
537	409.8	540	411.6	541	410	552	418.3	550	414.7	543	410
541	410.5	543	412.2	545	410.9	556	418.8	555	416.2	548.7	412.5
546	411.8	547	413.2	548	411.9	560	419.6	560	418.4	553	414.8
549	413	551	414.2	551.5	413.4	564	421	**565.6**	**422.55**	**557.45**	**418.3**
553.5	415	555	415.7	555	415.4	568	423.3	568	425.3	561	423.15
556	416.4	559	418.1	557	417.2	570	425	570.7	429.8	563.6	429
558.9	**419**	**562.1**	**420.65**	**558.8**	**418.5**	**571.7**	**426.8**	572.3	435	565.3	435
561.1	421.5	565	424.6	561	421.3	574.1	429.4	*573.5*	*442*	566	439
562.8	424.3	567.2	428.6	562	422.8	575.8	432	***574***	***452.65***	***566.8***	***447***
564.5	428	568.7	432.5	564.4	427	577.7	435	*574.6*	*460*	*567.8*	*455*
565.8	432	569.6	436	565.9	431.2	578.7	438.4	*575.7*	*469*	*569.25*	*463*
566.6	435.5	570.1	439	567.1	435.8	*579.25*	*442*	***583.2***	***485.8***	***577.1***	***478.5***
567.1	438.2	*570.4*	*442*	567.7	439	*579.39*	*447*	*601.9*	*495*	*584.2*	*483.7*
567.6	*442*	*570.7*	*446*	*568.2*	*442*	*579.51*	*451*	*610.4*	*496.5*	*590*	*486.2*
568.1	***448***	***570.9***	***450.2***	***568.8***	***448.8***	***579.7***	***457.1***	**618.65**	**497.3**	***610.2***	***490.4***
568.8	*455*	*571.6*	*458*	*569.4*	*454*	*580*	*463*	624	497.9	618	491.5
570	*462.4*	*572.5*	*464*	*570.4*	*460*	*580.5*	*469*	630	498.8	622	492.4
572.1	*470*	*573.8*	*470*	*573.1*	*470*	*581.35*	*475*	636	500.6	626	493.5
578.2	***479.8***	***580.6***	***482.6***	***578.7***	***478.7***	***588.75***	***491.45***	641	503	629.5	495
585.35	*485*	*585*	*486.25*	*585*	*483.5*	*597*	*497*	**644.3**	**505.6**	**635.9**	**499.6**
590	*487.1*	*595*	*490.8*	*596*	*487.7*	*606*	*499.9*	647	509	638.8	503
600.7	*490*	*605*	*493*	*605*	*489.7*	616	501.6	649.8	513	641.9	507.9
611.6	***491.5***	**614.9**	**494.2**	***610.4***	***490.4***	**625**	**502.4**	651.8	517	644.1	513
616	492.1	617.5	494.45	*614*	*490.9*	628	502.61	653.3	521.5	645.4	517.8
620	492.7	620.2	494.7	617	491.4	631	502.94	654.5	527	646.4	522.4
624	493.4	625	495.4	622.3	492.6	634	503.3	**655.84**	**534.5**	**647.63**	**527.8**
627	494.3	629	496.4	627	494.1	638	504		*Illum*	*D95*	*pairs*
630	495.5	633	497.7	630.5	495.7	642.1	505	**526.2**	**406.74**	***608.3***	***488.8***
633	497.1	635.5	498.9	633.7	497.5	645.7	506.2	530	407.1	*613*	*489.2*
635.5	498.8	638	500.5	635.43	498.9	648.7	508	534	407.6	618	490.1
637.5	**500.6**	**640.7**	**503**	**637.2**	**500.3**	**651.5**	**510.4**	542	409.2	623	491.4
640	503.6	643.8	507	640	503.6	654.3	513.5	550	412.5	627	493
642	506	646.3	511	642.4	507	656.6	517	**555.5**	**417.3**	630.5	495
643	508	648.1	515	643.9	510	658	520	559	422.1	**633.8**	**498.2**
644	509.7	649.4	518	645	512.6	659.8	525	561.3	427	638.1	504
646	515.2	650.3	521	646.1	516	660.85	529	563.7	437	641.4	511
647	518.6	651	524	647.4	521	661.6	533	*564.65*	*442*	642.7	515
648.7	526	651.5	527	648.3	526	662.25	537	***565.1***	***445.6***	644	519
649.1	**529**	**652.28**	**531.6**	**648.73**	**528.77**	**662.7**	**540.1**	***575.6***	***476.8***	**645.66**	**526.2**

413
433

Table IV. (From Ref 4.) Spectral-nonspectral complementary wavelength pairs for 10 degree visual field (CIE 1964 system) and illuminant D65. Some spectral-spectral complementary pairs are also listed, in italics. Half-cycle pairs are shown in **bold** and MCI pairs in **underlined** **bold** (spectral pairs also in italics). The first and last wavelength pairs listed are the hue cycle (HC) ends and HC midpoint.

Complementary wavelength pairs

	523.05	**404.31**		*567.6*	*465*
	527	404.5		*570.4*	*470*
	530	404.8	MCI	***573.1***	***473***
	533	405.3		*580*	*477.9*
	536	406		*584.6*	*480*
	539	406.7		*591.1*	*482*
	542	407.5		*594.9*	*483*
	545	408.5		***602.85***	***484.11***
	548	410.1		*607*	*484.52*
	551	412.2		*611*	*485*
MCI	**552.96**	**413.98**		615	485.9
	555.35	417		618	486.9
	557.1	420		621	488.1
	558.1	422		624	489.5
	559.3	425		627	491.5
	560.3	428	MCI	**630.74**	**494.52**
	561.2	431		633	497.3
	561.9	434		635	500.3
	562.45	437		637.5	505
	563	440.8		638.6	508
	563.44	***444.7***		639.7	511
	564	*450*		640.5	515
	565	*456.4*		641	518
	565.9	*460*		**641.79**	**523.05**

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Color Constancy from Invariant Wavelength Ratios:

II. The Nonspectral and Global Mechanisms

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Abstract. Given the spectral mechanism of color constancy (Part I of this two-part series), the remaining nonspectral mechanism is formulated in the present article (Part II) by the constraint of correlation with known spectral illuminant-invariant functions, i.e. invariant wavelength ratios between constant hues, which plot straight near-parallel lines in the plane of wavelength and reciprocal illuminant color temperature (MK^{-1}). The same is assumed to apply to nonspectral constant hues in the same plane and dominant wavelength scale, if the latter is extended as "equivalent wavelength" to cover the nonspectrals (see accompanying article "Relative wavelength metric for the complete hue cycle"). To simplify analysis, stimuli are optimal aperture colors; their monochromatic stimuli lie between known limits (442 and 613 nm) which are common boundaries with optimal compound stimuli (nonspectrals). It is shown that the wavelengths and invariant ratios of spectral constant hues can be formulated /predicted exactly ($\pm$0.5 %) from the ratios of an harmonic period, which shifts wavelength systematically with MK^{-1}. The formula implies this color-constant hue cycle is isomorphic across illuminants. Extending the ratios to the nonspectral "equivalent wavelengths" predicts the nonspectral constant hues; however, to identify these colorimetrically, their (spectral) complementary wavelengths are specified for various illuminants. This completes the global color constancy mechanism for the illuminant color temperature range 2,800 to 25,000 K and the complete hue cycle.

Keywords: color constancy; chromatic adaptation; complementary colors; hue cycle.

INTRODUCTION

Part I of this two-part series[1] described a spectral color constancy mechanism derived from experimental data on the dominant wavelengths of constant hues. The mechanism was derived by a novel method utilizing the dominant wavelengths of stimuli whereas most methods,[2] whether chromatic adaptation[3,4,5] or computational models,[6,7,8,9] utilize spectral reflectances from object colors. The mechanism depends on wavelength quantities, but unfortunately wavelength is limited to the spectral hues, i.e. about 73% of the hue cycle according to such color order systems as Munsell and DIN. Hence this paper's task is to extend the mechanism through the nonspectral hues to complete the global mechanism over the complete hue cycle.

In the plane of dominant wavelength and reciprocal illuminant correlated color temperature (as reciprocal megaKelvin, MK^{-1}), constant hues plot straight near-parallel lines as shown in Fig 1, reproduced from Ref[1]. (For the sake of brevity, 'wavelength' will include 'dominant wavelength'.) The wavelength ratio between any pair of constant hues was shown to be invariant or practically invariant across illuminants. These illuminant-invariant ratios and their near-parallel lines offer a means of determining the remaining nonspectral part of the color constancy mechanism, which is schematized by the dashed lines in Fig 1. Because spectral constant hues have invariant wavelength ratios and plot near-parallel lines the same may be expected of nonspectral constant hues in the same plane and wavelength scale (x-axis, Fig 1). This requires the dominant wavelength (linear) scale to be extended into the nonspectrals, where the scale may be termed *equivalent wavelength*. Such a scale may be imagined to represent the visual process's single coherent psychophysical scale for the whole hue cycle, based on dominant wavelength for the spectral hues.

Hue cycle refers to the complete cycle (sometimes represented as a geometric circle) of spectral and nonspectral hues, and may be defined in psychological terms (e.g. a uniform hue difference circle as in Munsell) or psychophysical terms, e.g. dominant wavelength. A psychophysical wavelength-based scale over the hue cycle, comprising dominant wavelength for the spectral part and *equivalent wavelength* for the nonspectral part, has been derived from properties of complementary wavelength pairs, and is described in detail in the preceding article.[10] The scale was initially intended for small

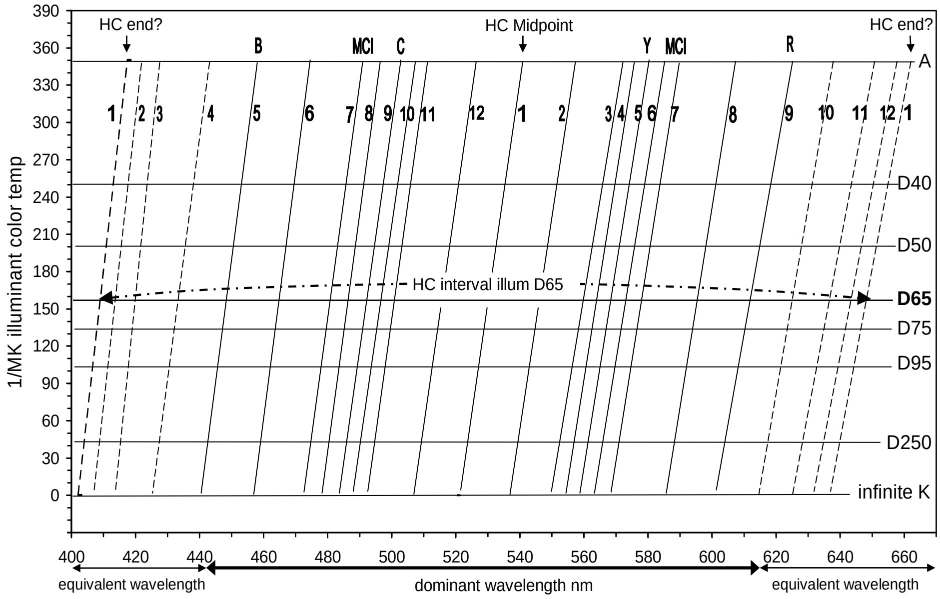

Figure 1. Spectral structure of color constancy (solid straight lines represent constant hues) for small visual fields, in the plane of dominant wavelength and reciprocal illuminant color temperature (MK^{-1}), extended conceptually to nonspectral constant hues (straight dashed lines) to schematize the global structure. This figure derives from Ref [1] and is representative rather than accurate. Number pairs (e.g. 5 and 5) represent a constant hue and its complementary wavelengths, i.e. its complementary constant hue. The two lines (or constant hues) labelled MCI denote the complementary wavelength pair of *minimum complementary interval* (MCI) for respective illuminants, from CIE data. Their mean is the *hue cycle midpoint*, equidistant from, and complementary to, the hue cycle ends.

visual fields (the CIE 1931 observer)[11] but limited data is provided for the CIE 1964 observer. The scale is termed *relative wavelength* and has been used by the author for some years. It was initally developed from optimal aperture color stimuli (or unrelated colors) because their hue cycle is divided into two parts (spectral and nonspectral) at clearly defined common boundaries: optimal monochromatic stimuli occupy the range 442-613 nm; shorter or longer wavelengths beyond that range do not give optimally efficient (in color mixture) monochromatic stimuli nor do they admix optimally efficient compound color stimuli (nonspectrals). The line from 442 to 613 nm in color space (see Fig 1 of Ref 1) defines the locus of optimal compound stimuli,[12] i.e. nonspectrals. The 442-613 nm locus is similar to the MacAdam limits for optimal object colors, depending on levels of luminance. This hue cycle omits the spectrum extreme wavelengths, as do color appearance systems such as Munsell for all

Values higher than about Value 4.

Given that the dominant wavelength scale is linear (Fig 1, x-axis) and easily extended mathematically, in principle it may be extended into the nonspectrals with reasonable psychophysical accuracy if just two conditions are met. First, the dominant wavelength interval of the spectral part (limited to 442-613 nm for reasons above) and the *equivalent wavelength* interval of the nonspectral part of the hue cycle should be in the ratio indicated by color appearance systems and other colorimetric indications. This ratio was determined in Ref[10] to be about 2.45:1 (ie. 71% spectral and 29% nonspectral) and, given that the spectral interval (442-613 nm) is 171 nm, produces a total hue cycle interval of 240 nm [i.e., 171(1/71%)=240];[10] as shown in Fig 1 and Fig 2 for illuminant D65.

Second, and most importantly in practical colorimetry, the (spectral) complementary wavelengths to the nonspectral *equivalent wavelengths* should be determined to identify the latter in CIE colorimetry.[11] The spectral-nonspectral complementary pairs are listed in Ref[10] (and later below).

The *relative wavelength* scale is shown in circular format in Fig 2. The arrowed lines at 442 and 613 nm indicate the boundaries between spectral (monochromatic or nearly monochromatic stimuli) and nonspectral (compound stimuli) parts of the hue cycle. 71% is spectral (from 442-613 nm) and 29% is nonspectral (from 613 nm through the purples to 442 nm). The latter nonspectral range measures a similar percentage of the total hue cycle in most color appearance systems; Fig 2 illustrates the quite similar nonspectral proportions measured from five color appearance systems: CIE LUV, DIN, Judd's UCS,[10] Munsell, and OSA-UCS, from data in Table I of Ref[10]. So the total interval, and spectral/nonspectral proportions, of the *relative wavelength* hue cycle present no surprises, and the *relative wavelength* scale seems a reasonable psychophysical representation of the hue cycle.

Given this hue cycle interval and relative wavelength scale (or metric),[10] the aim of this paper is now feasible: To specify the nonspectral color constancy mechanism and thus complete the global mechanism. To do so, two steps are necessary: (1) Find a formula or algorithm that accurately calculates the wavelengths of spectral constant hues across the useful range of illuminant color temperature (see Fig 1), or in other words formulate the sloping lines of constant hues in Fig 1, such that each constant hue is specified by a unique number that is invariant across illuminants. Next, (2) apply the formula to compute the appropriate illuminant-invariant numbers for nonspectral constant hues, and plot

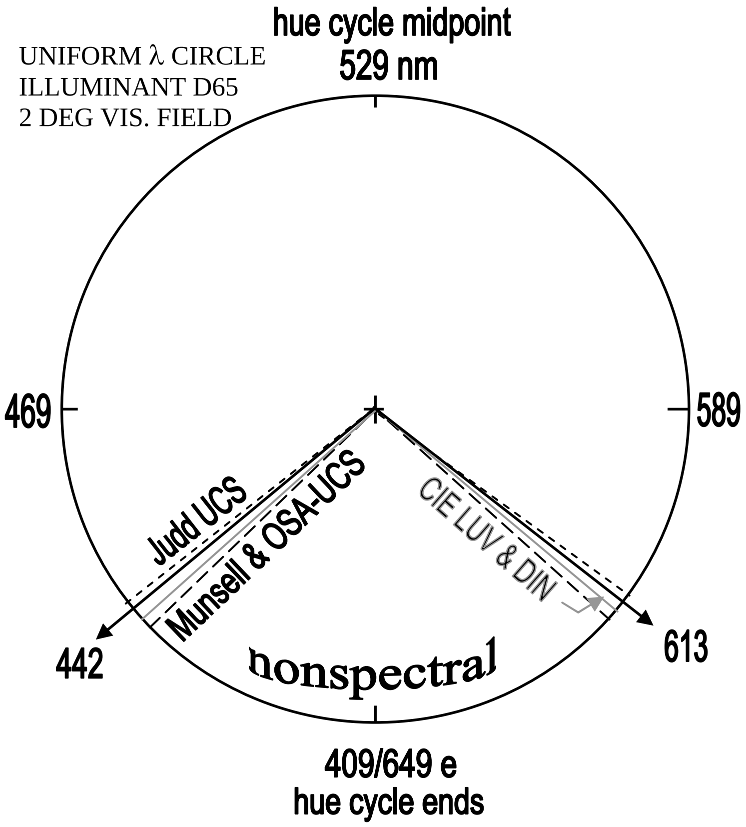

Figure 2. Hue cycle in the (linear) *relative wavelength scale*, showing the 442 and 613 nm boundaries between optimal monochromatic stimuli (spectral hues) and optimal compound stimuli (nonspectral hues). The nonspectral percentage of hue cycle is 29%, very similar to that in Judd's UCS (dotted black line), and similar to DIN and CIE LUV (both about 27%, solid gray line), and to Munsell and the OSA-UCS (both about 26%, dashed black line). The spectral part (442-613 nm) is specified in CIE dominant wavelength, nm, and the nonspectral part from 613 nm through purples to 442 nm is specified in "equivalent wavelength", denoted by "e". See also the same scale in Fig 1.

these hues (each as a sloping straight line) into the color constancy mechanism. These are this paper's two main objectives. The colorimetric accuracy of the above-mentioned unique illuminant-invariant number is able to be checked due to the *complementary linearity law*,[13,14] whereby the complementary wavelengths to a given constant hue (or set of corresponding colors), in two or more illuminants, theoretically represent another constant hue (and thus another unique illuminant-invariant number).

Given that the color-constant hue cycle (comprising the spectral and nonspectral parts) retains invariant ratios across illuminants, the cycle will be a relationally constant structure (say, isomorphic) across illuminants. Isomorphism means the same hues in the same relative positions, though their wavelengths may shift with illuminant. The constant structure may be expected to shift wavelength because illuminants change their spectral power distributions (SPD). For example, the SPD of Daylight illuminant D65 has more radiant power in the short wavelengths than tungsten illuminant A, with its peak power in longer wavelengths.[15] So one may expect the hue cycle to shift to longer wavelength in lower color temperature illuminants such as A, relative to daylight.

WAVELENGTHS OF CONSTANT HUE: FORMULATION

Fig 1 is here described in some detail before proceeding further. The x-axis shows the relative wavelength scale, including dominant wavelength from 442-613 nm and the remainng hue cycle (the nonspectrals) as *equivalent wavelength.* The pair of black lines labeled MCI represent the complementary wavelength pair of *minimum complementary interval* (MCI) for respective illuminants. The MCI pair varies with illuminant, as described in Refs[1,15]. The sloping lines are exactly straight, and parallel or nearly so. The mean of the MCI pair is termed the *hue cycle midpoint* for the respective illuminant.[1] From experimental data, Ref [1] showed that constant hues and their complementary constant hues in this plane plot approximately straight and parallel lines (and thus near-invariant wavelength ratios), whose mean slope is about 87 deg. From these data and the complementary linearity law,[13] it was demonstrated that complementary pairs of near-parallel straight lines (e.g. black lines in Fig 1) represent complementary pairs of constant hues. It was shown that the wavelength ratio of any two constant hues (they may or may not be complementary) is invariant or near-invariant across illuminants. The invariance of ratio is of course independent of the graph used; e.g. the y-axis could be temperature K rather than MK^{-1}, though the lines would in that case be curved.

Given the above structure and adaptive mechanism of spectral constant hues across illuminants, it is required to extend the structure from the known spectral area (black lines) to the nonspectral area and thus complete the hue cycle, for various illuminants. Given (for illuminant D65) the estimated 240 nm hue cycle interval with hue cycle ends at 409 and 649 nm,[10] the hue cycle ends for other illuminants may be estimated by drawing parallel lines to the constant hue lines (black sloping lines), through the known hue cycle ends for D65, as shown in Fig 1 by dashed lines labelled “HC end?”.

Where these lines intersect the horizontal lines representing other illuminants' reciprocal color temperatures, the hue cycle end wavelengths may be read from the x-axis. However, this method is an approximation which depends on the assumed angle of the sloping lines: the angle is known only as 87 deg ±1, and any variation from 87 deg will mean a significant variation of wavelength. Required is a accurate method of calculating (rather than estimating) the hue cycle ends for any CIE Planckian or Daylight illuminant, given the hue cycle interval for D65 is 240 nm or very close to it.[10]

Fig 3 (from Ref[1]) shows a portion of the Fig 1 color space in larger scale to show detail. Unlike Fig 1, it illustrates experimental data (data points "x"). Besides the MCI pair of black lines, four other complementary pairs of black lines, also straight and near-parallel, are shown (as B & Y, C & R, i & ii, m & n). The same five pairs of lines featured in Ref[1] as pairs of constant hue and complementary constant hue, which slope at 87 ±1 deg in the full scale of the graph [where in spatial terms unit x = unit y, i.e. 1 nm of wavelength (x-axis) equals 1 MK^{-1} (y-axis); Figs 1 and 2 are compressed vertically to save space]. Ref[1] showed that only at this angle do complementary pairs of lines plot straight near-parallel lines. The data points "x" on the black lines show wavelengths and complementary wavelengths for the respective illuminants; e.g. consider the black line marked "C". The datapoints show the wavelengths at various illuminants (D40, D50, D65, etc) that fall on the straight sloping line, whose angle (originally 87 deg) was adjusted slightly by trial and error until its complementary wavelengths for the respective illuminants described a precisely straight line, labelled "R" in Fig 3. The resultant pair of lines were 87 deg ±1, and were nearly but not exactly parallel. Further, it may be seen that the angle of the solid black sloping lines is greater (nearer vertical) for shorter wavelengths (see line "B") than the longer wavelengths (see line "R"). Consequently, the required method of calculating such near-parallel lines should be based on some other principle than a fixed angle.

Various possible methods were tested by plotting their predictions, until finding the successful method (below) whose predictions are shown in Fig 3 as the gray dashed lines (data points "o"), which closely align with the slopes of the experimental data (exactly straight black lines, data points "x") to within ±0.5 nm uncertainty over the illuminant color temperature range 4,000-25,000 K (i.e. 40-250 MK^{-1}), representing the entire range of CIE Daylight illuminants.[11] These are extremely accurate

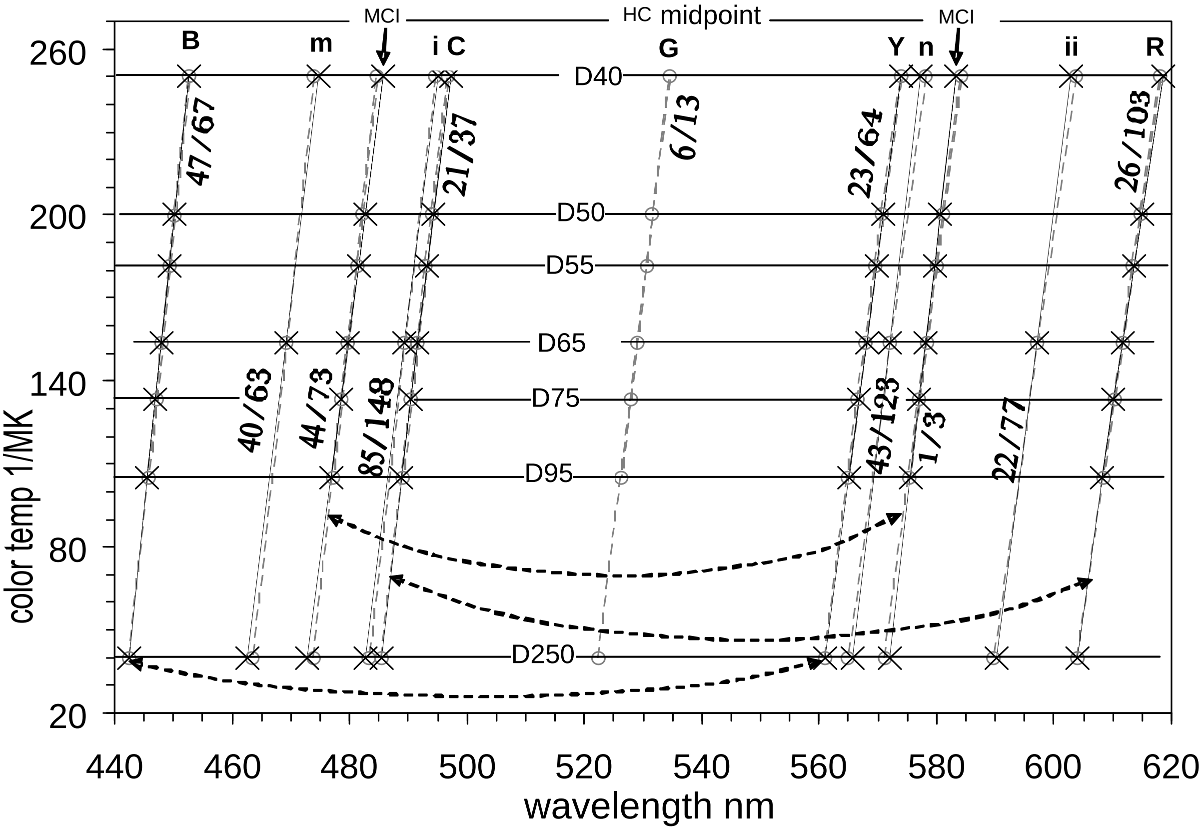

Figure 3. The plane of wavelength and reciprocal illuminant color temperature (MK^{-1}). Seven CIE Daylight illuminants are labeled from D40-D250. Two sloping black lines labeled "MCI" indicate MCI complementary wavelength pairs (data points "x") plotted for respective illuminants; e.g. 479.8 and 578.2 nm for illum D65. Notably they form exactly straight lines. Any parallel line, e.g. solid black line C, has complementary wavelengths (respective to illuminant) that plot data points "x" on another straight and near-parallel line, e.g. solid black line R. Line "N" is complementary to "m", and line "ii" complements "i". The black lines (data points "x") align closely with, and thus can be mathematically defined by, constant interharmonics (labelled, e.g. 40/63), shown as dashed grey lines (data points "o") closely coinciding with the black lines.

predictions.

The successful approach comprises four principles: (1) to use an isomorphic structure in the wavelength (or frequency) dimension, that is, the harmonic period or octave (see Appendix A, Harmonics in Brief); (2) to find the proportion of octave that aligns with the given hue cycle interval (240 nm) for D65; (3) to use that octave proportion to represent the hue cycle interval for all illuminants, as an isomorphic cycle structure; and (4) to center the octave proportion (i.e. hue cycle interval) on the hue cycle midpoint for the respective illuminant. Consequently, the hue cycle interval (thus locked to the hue cycle midpoint) will shift wavelength systematically with illuminant.

The appropriate octave proportion was found (by trial and error) to be 2/3 octave, in logarithmic progression to log base 2 (which incidentally happens to be the same log progression of frequency in the even-temper scale in music), because 2/3 octave, when centered on the hue cycle midpoint (529 nm) for illuminant D65, represents an interval of 240.2 nm. No other progression or log base gives a simple proportion (such as ½, 3/5, 2/3, 5/7, ¾) of octave that aligns so closely with the 240 nm hue cycle interval for illuminant D65. For this log progression, Fig 4 shows hypothetical hue cycle intervals as various octave proportions, extending between the appropriate *interharmonic* ratios representing hypothetical hue cycle ends. (An *interharmonic ratio* is a ratio of octave between the 1st and 2nd harmonics; see Appendix A). E.g. the central 5/7 proportion of an octave ranges from the 1/7 to the 6/7 interharmonic ratio, shown as dash-dot sloping lines (Fig 4). For D65, and centered (by trial and error) on the hue cycle midpoint at 529 nm, this interval measures 257 nm, rather larger than the required 240 nm. In contrast, 3/5 of an octave (between the black dotted sloping lines) measures only 217 nm, rather too small an interval. However, 2/3 or 4/6 of an octave (between the solid black sloping lines) measures from 408.9 nm at the 1/6 interharmonic ratio to 649.1 nm at the 5/6 ratio, giving an interval of 240.2 nm, almost exactly the required 240 nm interval determined in Ref[10]. Hence, log progression to the base 2 is used here for all octave or interharmonic ratios, and the hue cycle is treated as 2/3 of an octave for all illuminants. This practice is a mathematical convenience and does not necessarily mean these relations exist physically in the electromagnetic stimulus.

The relationships between an octave, 2/3 octave, hue cycle interval and midpoint, are schematized in Fig 5 and defined in Appendix B. The octave fulfils the requirement for a relationally constant interval (from any wavelength x to $2x$), without being restricted to a numerically constant wavelength interval. This allows a hue cycle interval to be represented by an octave or a constant proportion thereof in all illuminants and yet shift wavelength or interval with illuminant. The predictions shown in Fig 3 (gray dashed lines, and associated interharmonic ratios, e.g. 47/67) were found iteratively by

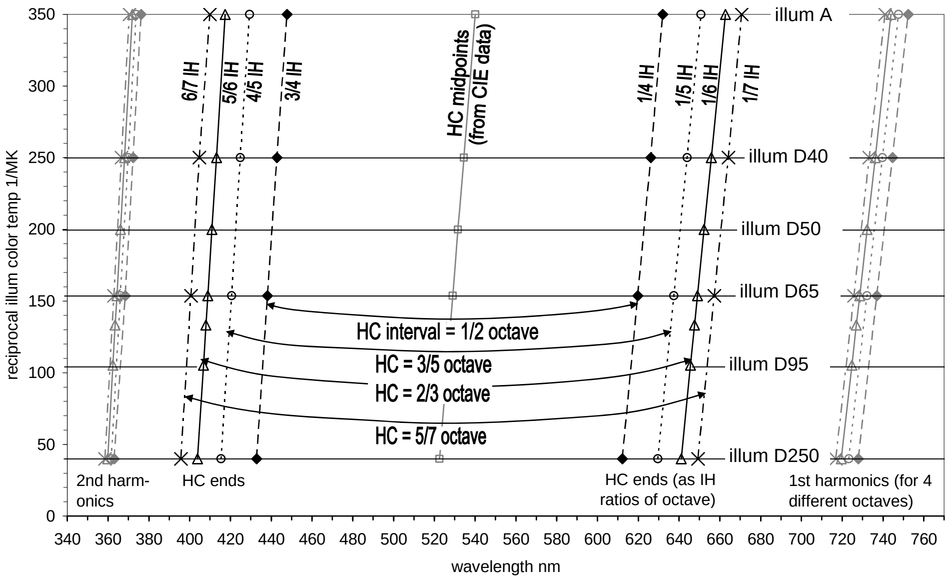

Figure 4. Various octaves (i.e. wavelength interval between 1st and 2nd harmonics) and various ratios of those octaves. Black sloping lines represent ends of hypothetical hue cycles (HC) whose intervals are equivalent to 5/7, 2/3, 3/5, and 1/2 of an octave (see double-ended curved arrows), such that the linear mean of the hue cycle ends aligns with the hue cycle midpoint for the respective illuminant (Table I). Gray lines represent 1st and 2nd harmonics and HC midpoints [colorimetrically defined as mean of the complementary wavelength pair of minimum complementary interval (MCI) for the illuminant].

the constraint of correlating interharmonic ratios with the black lines (data points "x") in Fig 3, such that the interharmonic's wavelengths align most closely with the data points, particularly for D65. The slopes are extremely close to the experimental data (black lines). It's worth noting the slopes are not dependent on the selected progression (log base 2) or the selected octave proportion, but would align with the black lines as interharmonic ratios of any (constant with illuminant) octave proportion, so long as it centers on hue cycle midpoints for respective illuminants.

In summary, the solution specifies a hue cycle which for any CIE illuminant:

(1) occupies 2/3 of a wavelength octave and shares its wavelength scale;

(2) is centered on, and shifts wavelength with, the mean of the MCI wavelength pair in CIE data (i.e. the hue cycle midpoint); and

(3) comprises only optimal color stimuli [i.e. of optimal purity; but because hue cycles of lower purity or chroma comprise the same dominant wavelengths as hue cycles of optimal purity,[16] hue cycles of

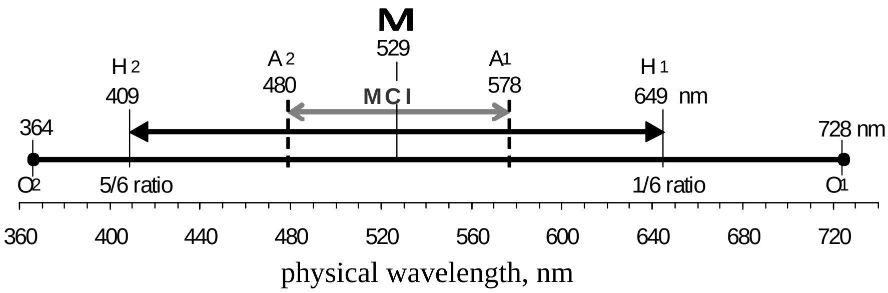

Figure 5. Basic model of hue cycle (black line $H_1 - H_2$), structured on the *minimum complementary interval* (MCI, grey line $A_1 - A_2$) and the octave (black line from $O_1 - O_2$, 1st to 2nd harmonic), for illuminant D65. The common link is hue cycle midpoint M, as the mean of MCI wavelengths 480 and 578 nm, and also the mean of H_1 and H_2 (1/6 and 5/6 interharmonic ratios). As illuminant SPD shifts to longer wavelength, so does the MCI pair, and thus the hue cycle and octave (together with its interharmonic ratios). The *x* axis represents the octave's physical wavelength, same scale as *equivalent wavelength*.

any purity will have the same *relative wavelength* interval for a given illuminant.]

This model of the hue cycle (defined in Appendix B, where it is termed the *color-constant hue cycle*) represents an isomorphic structure which aligns closely with the experimental data (black lines in Fig 3) on spectral constant hues and which predicts constant hues in the nonspectral area. The key of the hue cycle's relationship with the wavelength octave (see Fig 5) is the hue cycle midpoint. The hue cycle, as 2/3 octave, extends from the 1/6 to 5/6 interharmonic ratios, which represent the hue cycle's two ends (at a single magenta hue). By trial-and-error adjustment of the hue cycle end wavelengths (in the physical wavelength scale of Figs 3 or 4, which is the basis of the *relative wavelength* scale for the hue cycle), their mean wavelength coincides with the hue cycle midpoint, which is defined colorimetrically as the mean of the MCI wavelength pair. Hence this hue cycle adapts to illuminant by shifting wavelength with hue cycle midpoint. Main characteristics of the hue cycle structure are in Table I. Because this hue cycle comprises only optimal color stimuli, only the wavelength range 442-613 nm gives monochromatic stimuli and relates directly to CIE dominant wavelength. Other "wavelengths" refer to compound stimuli (nonspectrals) in the equivalent wavelength scale.

Hence the wavelengths of a constant hue (λ_{CH}) may be formulated as the wavelengths (λ_{IH}) of an

Table I. Hue cycle (HC) structure for 8 illuminants from 2,800-25,000 K, with principal complementary wavelength pairs and associated interharmonic ratios (IHs). HC midpoint is mean of the spectral MCI pair (peak of curve B in Fig 6; 2nd & 3rd MCI pairs are peaks of curves A & C). IHs align with wavelength data within the indicated uncertainty, e.g. |±0.3 |, unless italicised. Wavelengths 442-613 nm are monochromatic; other wavelengths denote compound stimuli (nonspectrals). Pairs labelled **B-Y** and **C-R** are half-cycle complementary pairs; **G-M** (not listed) is always a HC end and the HC midpoint. Last row, *Ratio to D65*, denotes the approx ratio of the respective illuminant's wavelengths to those of D65, for any row in the table (the ratios are exact for IH wavelengths).

BASIC DATA	*D250*	*D95*	*D75*	*D65*	*D50*	*D40*	*A*	*C*
chrom coord x	.2499	.2818	.2990	.3127	.3456	.3823	.4476	.3101
chrom coord y	.2548	.2956	.3150	.3291	.3586	.3838	.4074	.3163
color temp K	25000	9500	7504	6504	5003	4000	2856	6740
reciprocal (MK^{-1})	40	105.26	133.26	153.75	199.88	250	350.14	148.37
HC Midpoint	522.51	526.2	527.8	529.0	531.6	534.5	540.1	528.7
6/13 ±0.1	522.6	526.3	527.9	529.1	531.7	534.6	540.2	528.8
HC ends, 5/6	403.89	406.74	407.98	408.9	410.91	413.16	417.48	408.67
and 1/6 IH	641.13	645.66	647.63	649.09	652.28	655.84	662.7	648.73
HC interval	237.24	238.92	239.65	240.19	241.37	242.69	245.2	240.06
Octave ends 2nd	359.82	362.36	363.47	364.29	366.08	368.08	371.93	364.08
& 1st harmonic	719.64	724.73	726.94	728.58	732.16	736.16	743.86	728.17
MAIN COMPLEMENTARY WAVELENGTH PAIRS								
Half Cycle Complementary Pairs								
B-Y pair	442.38	445.6	447.0	448.0	450.25	452.65	457.1	448.8
from CIE data	561.0	565.06	566.82	568.1	570.93	574.0	579.7	568.8
47/67 & both	442.53	445.66	447.02	448.03	450.23	452.69	*457.43*	*447.78*
23/64 IH ±0.2	560.96	564.93	566.65	567.93	570.72	573.84	579.84	*567.65*
C-R pair	485.48	488.8	490.37	491.53	494.24	497.3	502.4	490.4
from CIE data	604.1	608.26	610.2	611.63	614.92	618.6	625	610.4
21/37 & ±0.2	485.58	489.01	490.51	491.61	494.05	*496.75*	*501.9*	*491.34*
26/103 IH ±0.3	604.12	608.40	610.25	611.63	614.64	*618.0*	*624.46*	*611.29*
MCI Complementary Pairs								
Spectral MCI pr	472.86	476.5	478.5	479.8	482.6	485.77	491.5	478.7
from CIE data	572.16	575.5	577.1	578.2	580.6	583.23	588.8	578.7
44/73 & both	*473.88*	477.24	478.69	479.77	482.13	*484.77*	*489.83*	*479.5*
1/3 IHs ±0.5	*571.18*	575.0	576.97	578.27	581.12	*584.29*	*590.40*	*577.95*
2nd MCI pair	414.97	417.19	418.28	419.04	420.65	422.56	426.97	418.54
nonspec-spect	550.97	555.22	557.44	558.9	562.09	565.60	571.71	558.84
75/94 & both	*413.93*	416.71	418.13	419.08	421.14	*423.44*	*427.86*	418.85
13/34 IH ±0.5	*552.1*	555.76	557.7	558.95	561.70	*564.77*	*570.68*	558.65
3rd MCI pair	495.26	498.17	499.59	500.57	502.97	505.64	510.41	500.26
spec-nonspect	628.84	633.45	635.9	637.49	640.68	644.32	651.52	637.16
20/37 & both	494.76	498.07	499.78	500.71	503.37	506.12	*511.41*	500.6
5/26 IHs ±0.5	*629.8*	634.28	636.2	637.65	640.79	644.29	651.05	637.3
Ratio to D65	0.9877	0.9947	0.9977	1.000	1.0049	1.0104	1.021	0.9994

interharmonic ratio (IH) in the color-constant hue cycle across illuminants, as Eqn (1) states:

$$\lambda_{CH} = \lambda_{IH} \qquad (1)$$

Given an interharmonic ratio, say 1/6, the appropriate wavelength for each illuminant (say, D65) is calculated from the 1st harmonic for that illuminant (e.g. 728.58 nm for D65, per Table I) as shown in Appendix A: e.g. 728.58 divided by 6th root of 2 (as 6√2, i.e. 1.1225) = 649.1 nm. The reverse is a longer process: given a wavelength *x* of a constant hue for a given illuminant, the appropriate interharmonic ratio (which gives the same wavelength *x* for the respective illuminant) is determined by iterative process (see Appendix A); having found the interharmonic (representing a constant hue, thus common for all illuminants), the wavelengths for other illuminants are readily calculated from their respective 1st harmonics (listed in Table I).

In Fig 3, the dashed grey lines (data points "o") represent the wavelengths of the interharmonics (labelled) that correlate most closely to the black lines (data points "x"). The fit is excellent over all Daylight illuminants from D50-D100 (±0.5 nm at worst), as shown in Fig 3 and detailed in Table I. Agreement at best is ±0.2 nm over daylight illuminants D50-D100, as exemplified by the 47/67 interharmonic (IH): it aligns with the black line labelled *B* to within 0.2 nm, as shown in Table I. This surprises, considering that the black lines represent CIE complementary pairs derived from color matching functions for only 17 subjects.[15] In some cases, agreement may be improved slightly by using more complex IH ratios; e.g. the 6/13 IH correlates with hue cycle midpoints to within 0.1 nm, but the 121/262 IH correlates even better (to ±0.01 nm).

Table II gives wavelengths and wavelength ratios for complementary pairs of constant hues, and shows their close resemblance to the associated interharmonics (IHs, grey dashed lines), in Fig 3. Note the former give almost invariant ratios, while the latter give exactly invariant ratios across illuminants. In summary, Eqn (1) is remarkably accurate in predicting the wavelengths of a constant hue by formulating the wavelengths of an interharmonic ratio.

As mentioned, the accuracy of interharmonic ratios as unique illuminant-invariant numbers may be checked using the *complementary linearity law*,[13,14] which states that the complementary wavelengths to a set of corresponding hues (say, constant hue *M*) represent another (and complementary) constant hue *N*. Tables I or II give the necessary data to confirm that this is true: e.g. consider the B-Y half-cycle pair of complementaries. In Table I, each row of data is closely matched by an associated interharmonic ratio (47/67 matches the B data, and 23/64 matches the Y data) across the eight illuminants, indicating that the complementary wavelengths to an interharmonic ratio (say, 47/67) represent *both* another (and complementary) constant hue *and* another interharmonic (23/67, or practically so).

Further to formulating constant hue wavelengths (Eqn 1), it is of interest to formulate the angles of constant hue lines in the plane of wavelength and MK^{-1} (e.g. Fig 3). In Fig 3, the IHs (grey dashed lines) are exactly straight but, interestingly, not exactly parallel: they decrease angle slightly with longer wavelength, in agreement with the complementary wavelength data (black lines). Their angle is given by Eqns (2) and (3), where Eqn (2) is the common equation for a straight line. Any point on an IH line in Fig 3, of wavelength x and reciprocal temperature u (the symbol for MK^{-1}), is given by:

$x = u/(\tan\theta) + a$ (2)

where a is x for the respective IH at zero u (= zero MK^{-1}, or infinite K), and where θ is the angle of the IH line from the x-axis. Each IH has a unique angle:

$\tan\theta = u/(x - a)$ (3)

Eqn (2) shows any IH line in Fig 3 or Table II is straight $\pm 0.02^0$, due to the Table's limitation to 2 decimals. The angle of IHs varies linearly with wavelength. E.g. those of 5/6 and 1/6 IHs are 86.02^0 and 87.492^0, mean 86.756, the same as θ for 121/262 (the line of hue cycle midpoints).

In Tables I and II, wavelength ratios between different constant hues (or their associated IHs) are invariant as a *function of illuminant* (e.g. Table I column), and ratios between wavelengths of a given constant hue at different illuminants are invariant as a *function of constant hue* (e.g. Table I row). Consider Table I. Invariant wavelength ratios exist between columns (i.e. wavelengths by illuminant) and between rows (i.e. wavelengths by constant hue or by IH). Exact invariance applies to wavelengths of IHs, and near-invariance applies to the actual complementary wavelength data associated with the IHs. Eqn (5) below is an application of this invariance. For any wavelength λ_1 (say a constant hue in D65) its correlate λ_2 (for the same constant hue) in any other illuminant is found from Eqn (5) using hue cycle midpoints M as a known set of correlates (see Table I). Where the respective midpoints M are known, Eqn (5) is an alternative but less accurate formula than Eqn (1) for constant hue wavelengths.

$M_1 / M_2 = \lambda_1 / \lambda_2$ (4), hence

$\lambda_1 (M_2 / M_1) = \lambda_2$ (5)

where M_1 and λ_1 refer to illuminant 1, and M_2 and the unknown λ_2 refer to illuminant 2. Given a

Table II. Complementary wavelength pairs and ratios for the 4 pairs of lines in Fig 3 (same pairs as Table II in Ref 1), for 7 CIE Daylight illuminants. These plot exactly straight, near-parallel lines in the plane of wavelength and illuminant MK^{-1}. Wavelength ratios between complementary lines vary from near-invariant (for the MCI pair of lines) to precisely invariant for the B & Y pair of lines. Below each line *in italics* is the associated interharmonic's wavelengths and their ratios, which may be seen to be precisely invariant (allowing for the wavelength data listed to only one decimal acuracy). The respective interharmonic ratios are indicated in Fig 3.

Illum	**B &**	**Y**	**ratio**	**C &**	**R**	**ratio**	**MCI &**	**MCI**	**ratio**	**i &**	**ii**	**ratio**
D250	442.4	561	0.7886	485.5	604.1	0.8037	472.9	572.2	0.826	482.7	590.3	0.818
IH data	*442.5*	*561.0*	*0.7888*	*485.6*	*604.1*	*0.8038*	*473.9*	*571.2*	*0.8296*	*483.4*	*589.8*	*0.819*
D95	445.6	565.05	0.7886	488.8	608.26	0.8036	476.8	575.6	0.828	486.4	594.7	0.818
IH data	*445.7*	*565.9*	*0,7890*	*489.0*	*608.4*	*0.8037*	*477.2*	*575.2*	*0.8296*	*486.7*	*594.5*	*0.819*
D75	447	566.8	0.7886	490.4	610.2	0.8037	478.5	577.1	0.829	488.1	596.4	0.818
IH data	*447*	*566.7*	*0.7889*	*490.5*	*610.2*	*0.8038*	*478.7*	*577.0*	*0.8296*	*488.2*	*596.3*	*0.819*
D65	448	568.1	0.7886	491.5	611.6	0.8037	479.8	578.2	0.830	489.3	597.2	0.819
IH data	*448.0*	*567.9*	*0.7889*	*491.6*	*611.6*	*0.8038*	*479.8*	*578.3*	*0.8297*	*489.3*	*597.2*	*0.819*
D55	449.35	569.8	0.7886	493.2	613.65	0.8037	481.5	579.7	0.831	491	599.5	0.819
IH data	*449.4*	*569.6*	*0.7889*	*493.15*	*613.5*	*0.8038*	*481.2*	*580.0*	*0.8297*	*490.8*	*599.5*	*0.819*
D50	450.25	570.95	0.7886	494.2	614.9	0.8037	482.6	580.6	0.831	492.15	600.2	0.820
IH data	*450.2*	*570.7*	*0.7889*	*494.05*	*614.65*	*0.8038*	*482.1*	*581.1*	*0.8296*	*491.7*	*600.6*	*0.819*
D40	452.65	574	0.7886	497.3	618.6	0.8038	485.8	583.2	0.833	495.2	603.0	0.821
IH data	*452.7*	*573.8*	*0.7889*	*496.75*	*618.0*	*0.8038*	*484.8*	*584.3*	*0.8297*	*494.7*	*603.6*	*0.819*

wavelength λ_1 for an illuminant, Eqn (5) predicts the correlates (of the same constant hue as λ_1) for any number of other illuminants to ± 1 nm approximately, for central Daylight illuminants D40-95.

With Eqn (1) (or Eqn 5 if necessary) and the Fig 5 model of the hue cycle (defined in Appendix B), one may extend Fig 3 into the nonspectral areas as far as the hue cycle ends, and plot the appropriate "equivalent wavelengths" and draw the appropriate near-parallel lines (one per constant hue).

LINKING "EQUIVALENT WAVELENGTHS" TO CIE COLORIMETRY

Despite the above, the equivalent wavelengths (for nonspectral hues) are unable to identify the stimuli in terms of CIE colorimetry as dominant or complementary wavelengths in the CIE diagram. Obviously the nonspectral hues cannot have dominant wavelengths but only complementary wavelengths; so the nonspectral hues' *equivalent wavelengths* must be specified colorimetrically by their complementary spectral wavelengths. This has already been done for illuminant D65 in Ref[10], as part of developing the hue cycle relative wavelength scale. Given those data for D65, the correlate wavelengths (the same constant hue's wavelengths in other illuminants) can be determined by two methods: First,

Table III. Spectral-nonspectral complementary wavelength pairs for 2 deg visual fields and seven illuminants, A, C, D40, D50, D65, D75, and D95 (bottom right); some spectral-spectral complementary pairs are also listed, in italics. More data are given for common illuminants (D65, D50, C, A). Half-cycle pairs are shown in **bold** and MCI pairs in **underlined** **bold** (spectral pairs in italics). The first and last wavelength pair listed for each illuminant is the 1st hue cycle (HC) end and HC midpoint and the 2nd HC end and HC midpoint. Given the HC intervals for the illuminants (Table III), complementary pairs are ±0.3 nm uncertainty.

Illum D65 pairs		*Illum D50 pairs*		*Illum C pairs*		*Illum A pairs*		*Illum D40 pairs*		*Illum D75 pairs*	
529	**408.9**	**531.6**	**410.91**	**528.7**	**408.87**	**540.1**	**417.5**	**534.5**	**413.16**	**527.8**	**407.98**
532	409.2	535	411.1	533	408.8	544	417.7	539	413.4	533	408.5
535	409.5	538	411.3	537	409.3	548	418	544	413.8	538	409.3
537	409.8	540	411.6	541	410	552	418.3	550	414.7	543	410
541	410.5	543	412.2	545	410.9	556	418.8	555	416.2	548.7	412.5
546	411.8	547	413.2	548	411.9	560	419.6	560	418.4	553	414.8
549	413	551	414.2	551.5	413.4	564	421	**565.6**	**422.55**	**557.45**	**418.3**
553.5	415	555	415.7	555	415.4	568	423.3	568	425.3	561	423.15
556	416.4	559	418.1	557	417.2	570	425	570.7	429.8	563.6	429
558.9	**419**	**562.1**	**420.65**	**558.8**	**418.5**	**571.7**	**426.8**	572.3	435	565.3	435
561.1	421.5	565	424.6	561	421.3	574.1	429.4	*573.5*	*442*	566	439
562.8	424.3	567.2	428.6	562	422.8	575.8	432	***574***	***452.65***	***566.8***	***447***
564.5	428	568.7	432.5	564.4	427	577.7	435	*574.6*	*460*	*567.8*	*455*
565.8	432	569.6	436	565.9	431.2	578.7	438.4	*575.7*	*469*	*569.25*	*463*
566.6	435.5	570.1	439	567.1	435.8	*579.25*	*442*	***583.2***	***485.8***	***577.1***	***478.5***
567.1	438.2	*570.4*	*442*	567.7	439	*579.39*	*447*	*601.9*	*495*	*584.2*	*483.7*
567.6	*442*	*570.7*	*446*	*568.2*	*442*	*579.51*	*451*	*610.4*	*496.5*	*590*	*486.2*
568.1	***448***	***570.9***	***450.2***	***568.8***	***448.8***	***579.7***	***457.1***	**618.65**	**497.3**	***610.2***	***490.4***
568.8	*455*	*571.6*	*458*	*569.4*	*454*	*580*	*463*	624	497.9	618	491.5
570	*462.4*	*572.5*	*464*	*570.4*	*460*	*580.5*	*469*	630	498.8	622	492.4
572.1	*470*	*573.8*	*470*	*573.1*	*470*	*581.35*	*475*	636	500.6	626	493.5
578.2	***479.8***	***580.6***	***482.6***	***578.7***	***478.7***	***588.75***	***491.45***	641	503	629.5	495
585.35	*485*	*585*	*486.25*	*585*	*483.5*	*597*	*497*	**644.3**	**505.6**	**635.9**	**499.6**
590	*487.1*	*595*	*490.8*	*596*	*487.7*	*606*	*499.9*	647	509	638.8	503
600.7	*490*	*605*	*493*	*605*	*489.7*	616	501.6	649.8	513	641.9	507.9
611.6	***491.5***	**614.9**	**494.2**	***610.4***	***490.4***	**625**	**502.4**	651.8	517	644.1	513
616	492.1	617.5	494.45	*614*	*490.9*	628	502.61	653.3	521.5	645.4	517.8
620	492.7	620.2	494.7	617	491.4	631	502.94	654.5	527	646.4	522.4
624	493.4	625	495.4	622.3	492.6	634	503.3	**655.84**	**534.5**	**647.63**	**527.8**
627	494.3	629	496.4	627	494.1	638	504		*Illum*	*D95*	*pairs*
630	495.5	633	497.7	630.5	495.7	642.1	505	**526.2**	**406.74**	***608.3***	***488.8***
633	497.1	635.5	498.9	633.7	497.5	645.7	506.2	530	407.1	*613*	*489.2*
635.5	498.8	638	500.5	635.43	498.9	648.7	508	534	407.6	618	490.1
637.5	**500.6**	**640.7**	**503**	**637.2**	**500.3**	**651.5**	**510.4**	542	409.2	623	491.4
640	503.6	643.8	507	640	503.6	654.3	513.5	550	412.5	627	493
642	506	646.3	511	642.4	507	656.6	517	**555.5**	**417.3**	630.5	495
643	508	648.1	515	643.9	510	658	520	559	422.1	**633.8**	**498.2**
644	509.7	649.4	518	645	512.6	659.8	525	561.3	427	638.1	504
646	515.2	650.3	521	646.1	516	660.85	529	563.7	437	641.4	511
647	518.6	651	524	647.4	521	661.6	533	*564.65*	*442*	642.7	515
648.7	526	651.5	527	648.3	526	662.25	537	***565.1***	***445.6***	644	519
649.1	**529**	**652.28**	**531.6**	**648.73**	**528.77**	**662.7**	**540.1**	***575.6***	***476.8***	**645.66**	**526.2**

from Eqn (1) (or if necessary, Eqn 5). Second, from the method detailed in Ref[10] using a graph of wavelength versus complementary wavelength to determine the "white" curve for each illuminant; reading the x-axis and y-axis coordinates from any point on the white curve gives the complementary wavelength pairs. Fig 6 shows the white curve for five illuminants; note the peak of curves A, B, and

C, represent MCI wavelength pairs for the respective curve (listed in Table I). A list of spectral-nonspectral complementary wavelength pairs, read from Fig 6 or other graphs[10] of its type, is given in Table III for seven illuminants. Other pairs may be found from interpolation, or by reading them as diametrically opposite points in Fig 4B (complementary wavelength circle) of Ref [10].

Below the rows of complementary wavelength pairs in Tables I or II is a pair of rows listing the corresponding wavelengths calculated from the first method, above, i.e. from Eqn (1) and inter-harmonics. Calculations from this method are generally within 0.5 nm of the (spectral) complementary wavelengths to nonspectral "equivalent wavelengths" found from the method shown in Fig 6. The close similarity of results between the two methods supports their accuracy.

Tables IV and V give similar data to Tables I and III but for large visual fields (by using the CIE 1964 instead of 1931 system, see Appendix B), mostly for illuminant D65.

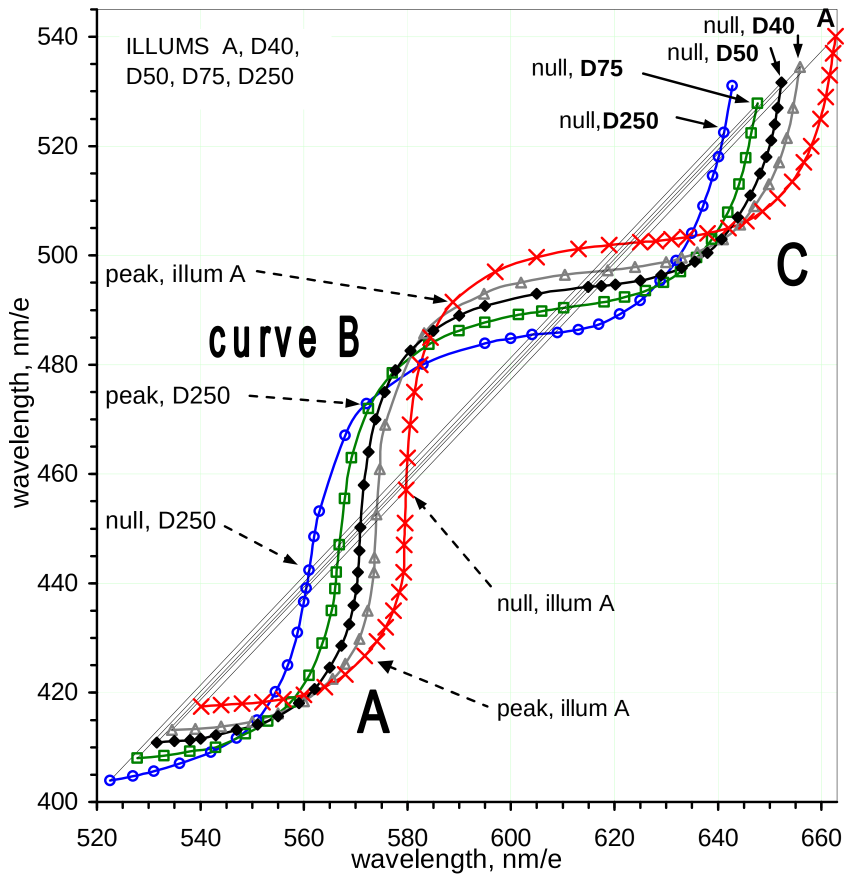

Figure 6. Complementary wavelengths functions for illuminants A, D40, D50, D75, D250 (respective data points red "x", gray triangle, black filled diamond, green square, blue "o"). Diagonals are curve axes for each illuminant from G-M null (i.e. HC midpoint and complementary HC end) to G-M null.

Table IV. Basic hue cycle (HC) data and principal complementary wavelength (λ) pairs for CIE 1964 observer (10 degree visual field), for illuminants D65, D50 and A.

Illuminant	*D65*	*D50*	*A*
HC Midpt	523.05	525.7	534.55
6/13 IH	523.15	525.8	534.6
5/6 and	404.31	406.35	413.16
1/6 IH, HC ends	641.79	645.05	655.84
HC interval	237.48	238.7	242.68
Octave 2nd and	360.196	362.02	368.08
1st harmonics	720.392	724.04	736.16
MCI complementary λ pairs			
Spectral MCI	473	475.6	484
pair (CIE data)	573.1	575.8	585.1
45/74 and	472.6	475.01	482.9
95/288 IH	573.15	576.05	585.7
2nd MCI	413.98	416	422.5
pair	552.98	555.5	565.5
75/94 and	414.37	416.46	423.44
13/34 IH	552.67	555.47	564.8
3rd MCI	494.52	497	505
pair	630.74	633.5	645
39/72 and	494.89	497.4	505.72
5/26 IH	630.49	633.68	644.3
1/2 cycle complementary λ pairs			
B-Y pair	444.7	446.95	456
(from CIE data)	563.44	566.3	577.34
89/128 &	444.9	447.15	454.6
17/48 IH	563.58	566.43	576
C-R pair	484.11	486.56	495.5
(from CIE data)	602.85	605.9	616.84
55/96 &	484.29	486.74	494.9
103/408 IH	602.7	605.75	618

Table V. Spectral-nonspectral complementary wavelength pairs for CIE 1964 observer (10^o field) and illuminant D65. Some spectral-spectral pairs are also listed, in italics. Half-cycle pairs are shown in **bold** and MCI pairs in **underlined** **bold** (spectral pair in italics). The first and last wave-length pair listed is the G-M pair, i.e. HC midpoint and HC end, and the other HC end and HC midpoint.

G-M	**523.05**	**404.31**		*567.6*	*465*
	527	404.5		*570.4*	*470*
	530	404.8	*MCI*	***573.1***	***473***
	533	405.3		*580*	*477.9*
	536	406		*584.6*	*480*
	539	406.7		*591.1*	*482*
	542	407.5		*594.9*	*483*
	545	408.5	*R-C*	***602.85***	***484.11***
	548	410.1		*607*	*484.5*
	551	412.2		*611*	*485*
MCI	**552.96**	**413.98**		615	485.9
	555.35	417		618	486.9
	557.1	420		621	488.1
	558.1	422		624	489.5
	559.3	425		627	491.5
	560.3	428	*MCI*	**630.74**	**494.52**
	561.2	431		633	497.3
	561.9	434		635	500.3
	562.45	437		637.5	505
	563	440.8		638.6	508
B-Y	***563.44***	***444.7***		639.7	511
	564	*450*		640.5	515
	565	*456.4*		641	518
	565.9	*460*	*G-M*	**641.79**	**523.05**

DISCUSSION AND CONCLUSIONS

From Table I, the complete color-constant hue cycle over several illuminants is shown in Fig 7. The cycle is isomorphic, retaining constancy in complementary wavelength structure and in its 2/3 octave interval (between the labelled 1/6 and 5/6 interharmonic ratios) as it shifts beween illuminants. To adapt from one illuminant to another, the isomorphic cycle shifts up or down straight sloping lines to the appropriate illuminant. Fig 7 is a rectilinear representation of the global mechanism of color constancy. It illustrates both an adaptive mechanism and the hue cycle structure over varying illuminant. The hue cycle's isomorphic structure is itself the adaptive mechanism for color constancy.

This is a very different mechanism from previous color constancy models except those related to invariant cone excitation ratios.[5] Nevertheless, the mechanism demands attention due to the remarkable simplicity and symmetry of the mechanism, comprising near-parallel straight lines each representing a constant hue. The simplicity can truly be described as extraordinary and suggests this is an important plane in the visual process.

The slope of the lines (Figs 3 or 7) is 86.75 deg ±0.7, rather than exactly parallel lines. Analysis of Tables I or II, supported by Eqns (2)-(3), shows why: The wavelength ratios between a constant interharmonic and its complementary interharmonic, say the B-Y half-cycle pair, are practically invariant across illuminant (in this case the constant ratio is 0.7886 for either the interharmonics or the associated CIE complementary wavelength pair). Precisely invariant wavelength ratios necessitate the small difference from exactly parallel lines.

Uniform intervals in MK^{-1} are known[17,18] and used (e.g. in television colorimetry) to give perceptibly uniform chromaticity differences in illuminant whites.

Every constant hue (e.g. in Fig 7, the line labeled B, a slightly reddish blue) in any CIE illuminant retains its invariant interharmonic ratio, its invariant wavelength ratios with other constant hues, and its relative position in the hue cycle. This may be shown as angular position (arrowed line) in the hue circle in Fig 8, which shows the complementary wavelength hue cycle for four illuminants (D250, D65, D40, and A) as concentric geometric circles. A constant hue is represented by a constant angle in these (or other) illuminants, providing the top center of each hue circle is the hue cycle midpoint for the respective illuminant and the complementary wavelength distribution is as described in Ref[10].

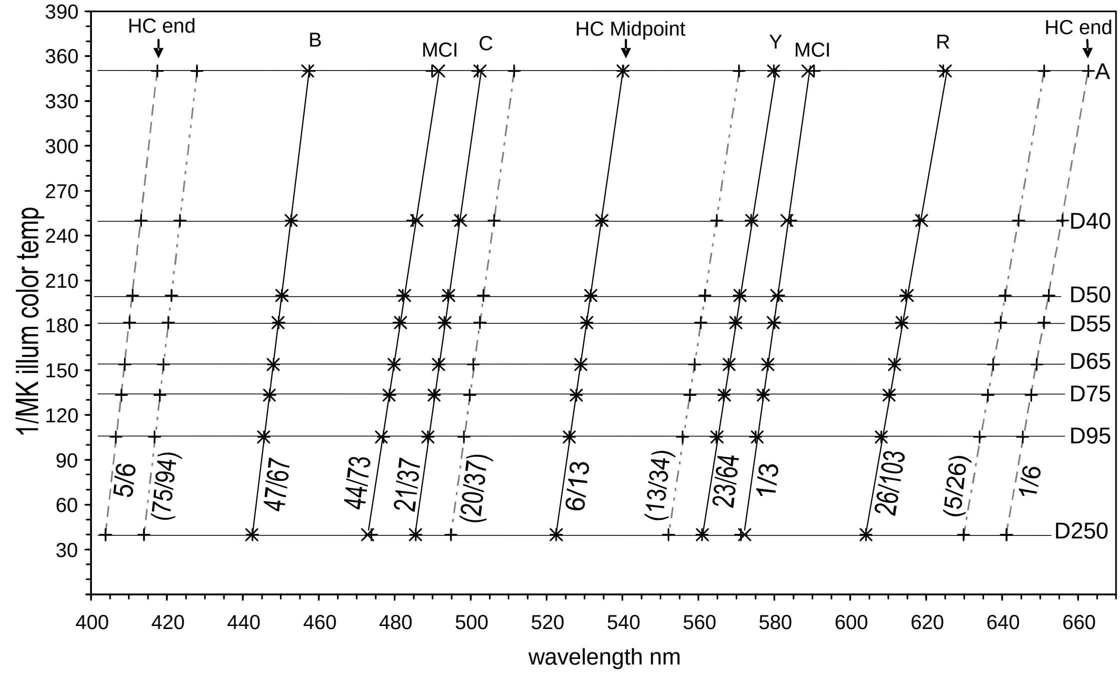

Figure 7. Global mechanism (both spectral and nonspectral) of color constancy, in the plane of wavelength and reciprocal illuminant color temperature (MK^{-1}). The x-axis is physical wavelength in order to represent the octave wavelength scale and interharmonic (IH) ratios, and is numerically the same as the *relative wavelength* scale. Sloping black lines, data points "x", are theoretical constant hues (from Ref.[1]); data points "+" are IH ratios, labelled. Each line (e.g. B) has a complementary line (e.g. Y) whose wavelengths are complementary to those on the first line, for respective illuminants. "Midpoints" are means of MCI wavelength pairs. Hue cycle extends between hue cycle ends (grey dashed lines). Grey dash-dot or dash-dot-dot lines represent two estimated MCI pairs (and IHs in parentheses) for spectral-and-nonspectral complementary pairs.

A model of the nonspectral mechanism and subsequently the global mechanism (comprising spectral and nonspectral hues over the complete hue cycle and all practicable illuminants) of color constancy has been deduced and described (e.g. Figs 7-8). Unlike most previous models, the model does not directly utilize reflectances, depending instead on the wavelength dimension of constant hues in theory and in experimental data.[1]

The color-constant hue cycle is specified as a constant 2/3 log interval of octave, where any interharmonic ratio from 1/6 to 5/6 is an illuminant-invariant number specifying a constant hue (whether spectral or nonspectral). The literature contains many forms of illuminant-invariant numbers for a stimulus,[5, 9,19,20] and the present paper's interharmonic ratio seems as useful a prediction of color constancy as the previous types of numbers. It is intended to develop and test the model in predicting experimental data on three-dimensional corresponding colors (wavelength, purity/chroma, and lightness) in a future paper.

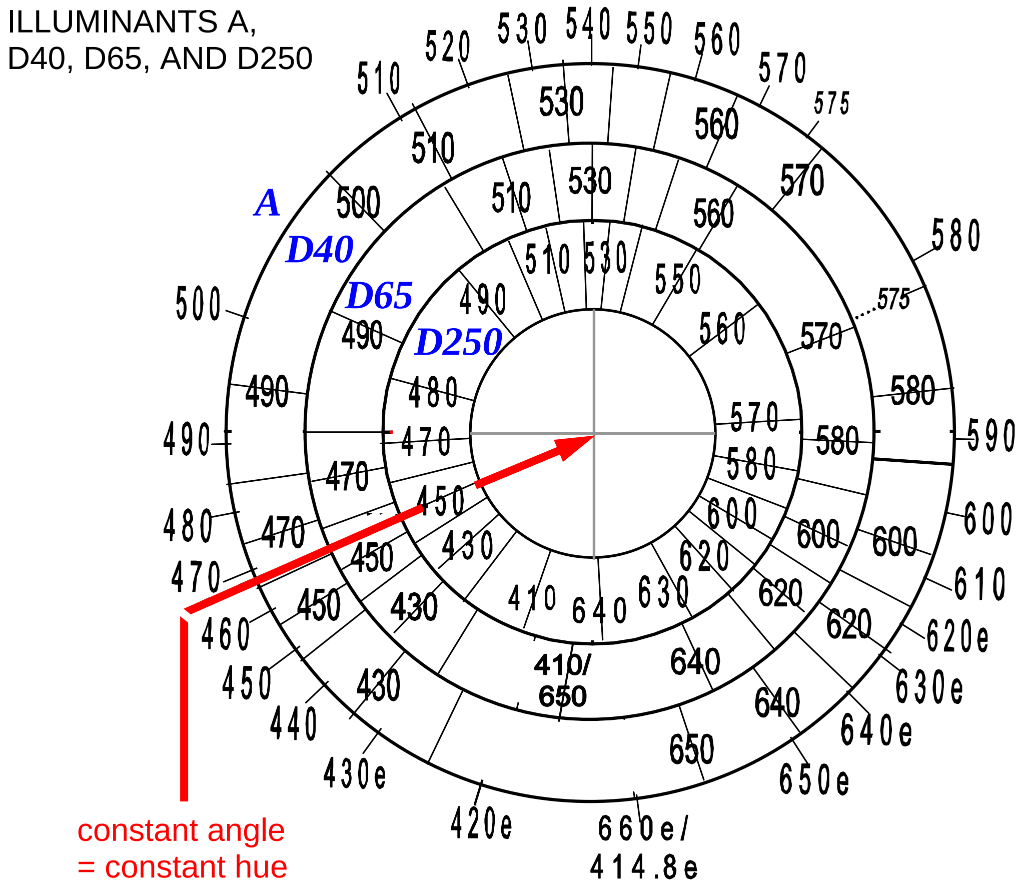

Figure 8. Circular schematic of constant hue as constant angle (e.g. arrowed red line) across four illuminants, derived from Fig 6 (and Fig. 4B of Ref 10). CIE standard illuminants A, D40, D65, and D250 are shown as complementary wavelength circles (i.e. complementary pairs arranged as opposites). Each illuminant circle has its hue cycle midpoint (see Table I) at top center of circle, e.g. 540.1 nm for illuminant A.

APPENDIX A: HARMONICS IN BRIEF

Harmonics are widely employed in mathematics and physics to specify intervals or periods (e.g. spatial or temporal) and their relationships. The term originated with *wave motion harmonics*;[21] these are geometric ratios between frequencies (or wavelengths) of wave motion in all forms of radiant energy, including electromagnetic. Harmonics[22, 23] are defined "as waves superimposed on a fundamental sinusoidal wave of any radiant energy, having a frequency that is a whole multiple of the fundamental frequency." Note the conventional dimension is frequency (e.g. as in radio wave transmission or music) rather than wavelength. Hence in wavelength terms, harmonic wavelengths are submultiples of thefundamental or 1st harmonic, e.g. the 2nd harmonic is 1/2 the fundamental wavelength. The interval from fundamental or 1st harmonic to 2nd harmonic is an octave. Ratios intermediate to these harmonics are termed ratios of octave or *interharmonic ratios*, common in electrical/electronic engineering.[22,23,24] Interharmonics (IH) are specified by the ratio of the interharmonic frequency to the fundamental (or 1st harmonic) frequency, in terms of a selected (linear or geometric) progression. This paper only uses logarithmic ratios (to log base 2, as used also in musicology),[25] e.g. 4/3 or 1.333; in this paper, which deals only with a single octave from 1st to 2nd harmonic, the integer "1" is unnecessary and the IH will be denoted simply by the ratio of an octave, e.g. 1/3 or 0.3333, starting from the fundamental frequency end of the octave. Example (using the well-known 1/12 interval between equal-temper semi-tones in music):[25] a 1/12 IH ratio is 12th root of 2, as $12\sqrt{2}$ or $2^{1/12}$, i.e. 1.0595, in either frequency or wavelength terms. If a wavelength octave is say 720-360 nm, the 1/12 IH's wavelength is 720/1.0595=679 nm, the 2/12th is 679/1.0595=641 nm, etc. The color science convention employs wavelength rather than frequency, and graphs short to long wavelength from left to right; consequently harmonics and interharmonics (since they are frequency-based) will be graphed here from higher to lower (e.g. 2nd to 1st harmonic, or 5/6 to 1/6 IH) from left to right of the x-axis (e.g. Fig 4).

APPENDIX B: BASIC RELATIONS IN THE COLOR-CONSTANT HUE CYCLE

Definitions

Given the white chromaticity w for an illuminant, the four types of wavelengths (A, M, H, and O) shown in the Fig 5 schematic are defined thus:

(1). A_1 and A_2 are the complementary wavelengths, relative to illuminant *w*, whose absolute wavelength difference is the minimum complementary interval (MCI).

(2). *M* is the HC midpoint wavelength, defined colorimetrically as the mean of A_1 and A_2, and theoretically (in this paper) as the mean of H_1 and H_2 (below).

(3). H_1 and H_2 are wavelengths of the HC ends, defined by equations below as 1/6 and 5/6 interharmonic ratios. As HC ends they are coincident at the one hue, a magenta, approximately or (as here assumed for simplicity sake) exactly complementary to the HC midpoint wavelength.

(4). O_1 is the1st harmonic (or start) of a wavelength octave, ie, 1/6 of an octave below H_1, and O_2 ($=2O_1$) is the 2nd harmonic (or end) of the octave.

Fig 5 schematic, above, shows how these basic wavelength types structure the HC from the octave.

Basic Relations

The above wavelengths and relations are formulated in Eqns (6)-(10), and listed in Table I for eight illuminants (for small visual fields, CIE 1931 system), including the wavelength uncertainties (which grow larger as illuminants differ more from D65). Eqns are given below in sequence of calculation, leading to the basic wavelengths for hue cycle and octave. Starting with only colorimetric data, the MCI pair A_1 and A_2 (relative to *w*) are determined from CIE chromaticity diagram or tristimuli. The hue cycle midpoint *M* is defined colorimetrically as the mean wavelength of the MCI pair:

$$M = (A_1 + A_2)/2 \qquad (6)$$

Given *M*, hue cycle ends H_1 and H_2 may be found from Eqns (7) or (8). Table I lists *M* for 8 illuminants. Eqn (7) defines the hue cycle interval *I* between H_1 and H_2, ±0.01 nm uncertainty for all illuminants.

$$M / 2.20243 = I \qquad (7)$$

Constant 2.20243 is empirical. Hue cycle ends *H* are found from *M* and *I*:

$$M - (I/2) = H_1 \quad \text{or} \quad M + (I/2) = H_2 \qquad (8)$$

Given H_1 or H_2 , wavelengths of octave ends are found as:

$$H_1 (2^{-1/6}) = O_1 \quad \text{or} \quad H_2 (2^{1/6}) = O_2 \quad \text{or} \quad 2(O_1) = O_2 \qquad (9)$$

H_1 and H_2 from Eqn (8) are accurate ±0.05 nm and should be adjusted to exact ratio 1.5874 (see Eqn 10) whilst retaining M as their mean, before applying Eqn (9) to determine O_1 and O_2 .

$$H_2 / H_1 = 2^{2/3} = 1.58740 \qquad (10)$$

The above relations also give data for 10 degree visual fields by using the CIE 1964 (rather than 1931) observer and assuming 2/3 octave still specifies the hue circle interval I. (Eqn 7 is less accurate but corrected by Eqns 9-10.) Table IV gives basic hue cycle data and complementary pairs for three illuminants, while Table V gives spectral-nonspectral complementary pairs for illuminant D65.

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COMMUNICATIONS AND COMMENTS

Chromatic Induction: Opponent Color or Complementary Color Process?

Abstract: In present convention, chromatic induction (simultaneous and successive contrast) is usually held to be an opponent color process. Fifty years ago, it was an accepted complementary color process. The latter was never disputed yet apparently overlooked, and is here shown to be the more accurate account by inspecting afterimages and published data on simultaneous and successive hue induction. Col Res Appl, 33, 77–81, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/col.20363

Key words: color appearance; chromatic induction

DEFINITIONS

First, some definitions and descriptions are appropriate since complementary colors and opponent colors are often confused nowadays. Chromatic opponent colors are defined as two opposing pairs of unique hues (or the primary opponent colors), Blue, Yellow and Green, Red. Each primary is determined by its appearance, e.g. B is judged by an observer as pure blue, neither greenish nor reddish.[1,2] Between the four primaries are the four binary opponent colors, e.g. BG is judged as appearing equally blue and green, and its opponent color is YR, judged as equally yellow and red. See Fig. 1(A).

In contrast, complementary colors are determined colorimetrically: complementary colors are defined[3] as a pair of color stimuli (dependent on appropriate wavelength pairs and power/luminance ratios) whose mixture color-matches a given neutral. The mixture may be additive, subtractive, pointillistic, or other process. In contrast, no pair of opponent colors can admix a neutral, in any possible ratio of powers or luminances, except the B-Y pair (see below). In CIE chromaticity diagrams, complementary wavelengths are opposites through the neutral point.

Figure 1(A) shows the opponent color primaries and binaries of the Natural Color System (NCS) together with their dominant wavelengths.[4] Superimposed is a complementary colors circle, showing the wavelengths of the 1976 CIE LUV diagram taken at 45° intervals to coincide with the NCS layout. Neither of these systems claims to be uniform hue difference layouts. The only opponent colors that are also a complementary pair (i.e. mixing a neutral) are the B-Y unique hues (and proximal hues, shown in the gray-shaded pair of sectors), as is generally known.[1]

For comparison, Fig. 1(B) shows uniform hue difference circles for two color appearance systems, CIELAB and Munsell. These hue circles are not generally regarded as complementary (the CIELAB layout is theoretically based on orthogonal opponent-color B-Y and G-R axes), but both circles can be seen to be approximately complementary. They are closely complementary (± 3 nm) over all the hue circle except the white sectors, i.e. over 2/3 the Munsell circle and over 3/4 the CIELAB circle. This may surprise some readers.

DISCUSSION

Wu and Wardman's interesting recent article[5] investigates inter alia the effect of simultaneous contrast (i.e. chromatic induction) on hue. They concluded from their results and from previous studies, e.g. Jameson and Hurvich's theory of opponent chromatic induction,[6] that the induced color moves toward the opposite color of the surround (induction) color. They stated several times that their hue shifts agreed with opponent color theory, whilst in fact (see below) demonstrating shifts towards the induction color's complementary rather than opponent color. The distinction seems unnoticed.

Another example: Luo *et al.*'s article on simultaneous contrast[7] states a number of times, in regard to their own results and previous studies (e.g. Ref. 6), that the effect of an induction field is to shift the hue of the test color in the direction of the induction field's opponent color. Nevertheless, their results (e.g. their Fig. 6) indicate the shift direction is opposite the induction color (through the neutral point), hence towards the complementary rather than the opponent. See Fig. 2. A third example: Ref. 8 similarly indicates (in its Fig. 4) that induced color generally shifts towards the induction color's complementary (Y with a G surround is an exception), but again, no mention is made of "complementary."

Figure 2 shows CIELAB space and Ref. 7 test colors, which are very similar to Ref. 5 test colors. Note the direction opposite (i.e. through the neutral point) to a given test color is towards the complementary rather than the opponent color. For example, R is opposite BG (cyan) rather than G, G is opposite RB (magenta) rather than R, and of course B is opposite Y since the BY pair is both complementary and opponent. So when studies, e.g. Ref. 5, describe the change of hue as in the direction "opposite" to the induction color, it indicates a complementary rather than opponent color direction. This is true not only of all CIE color spaces but also of OSA-UCS and Munsell: they are approximately complementary layouts [see Fig. 1(B) above and Fig. A2 of Ref. 9].

Figure 6 of Ref. 7 shows, in CIELAB space, the color shifts when a test color is surrounded by induction colors of the same hue but different lightness. This situation, when induction color is most similar to test color, gives maximum shift of color in hue and colorfulness.[5–7] Though the authors claim the shift direction is opponent to the surround, the shifts are all toward the complementary to the surround (or the test color since both are the same hue); i.e. the R test color shifts toward BG, the G test color shifts towards RB, the B shifts towards the Y, etc (similar to arrows in Fig. 2 above).

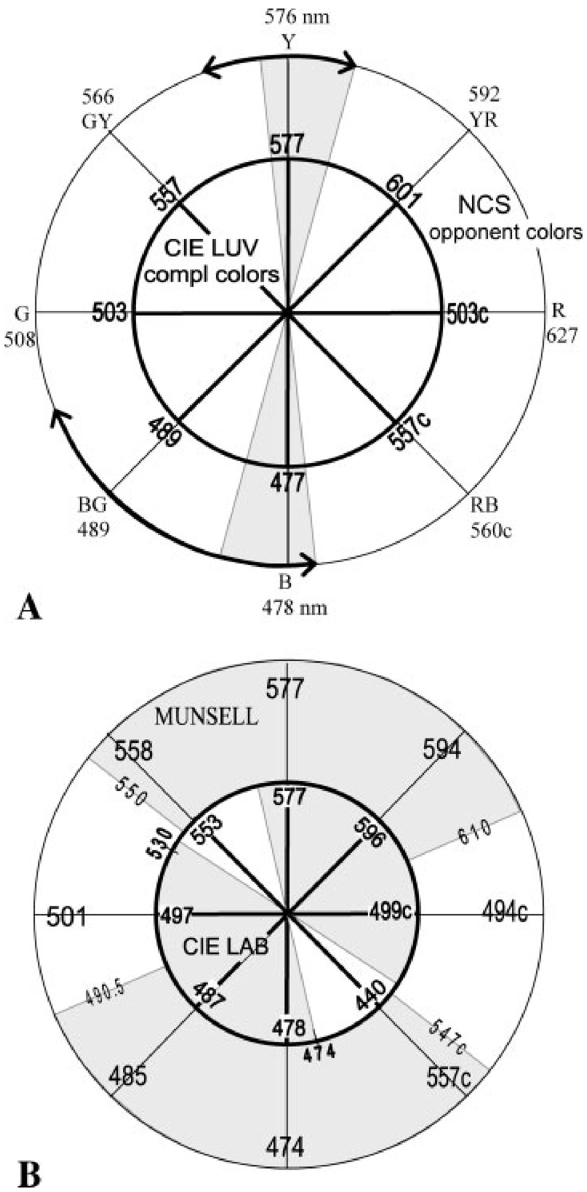

FIG. 1. (A) CIELUV hue circle (wavelengths at 45° angles, circle center at illuminant C point) superimposed on NCS hue circle, normalized on NCS B-Y line; nearest complementary wavelengths are 477 and 577 nm. Arrowed sections on NCS circle show same wavelength distribution (±2 nm) as CIELUV. Gray sectors show where NCS colors are complementary wavelengths (±2 nm); dominant wavelengths (for blackness 00, chromaticness 90) derive from *x, y*, data in Ref. 4. (B) CIELAB hue circle (chroma 50, lightness 50, illum D65) and Munsell hue circle (chroma 10, Value 5, illum C), both circles normalized at 577 nm. Gray sectors show complementary wavelength pairs (±3 nm). Each circle is roughly complementary, excepting white sectors.

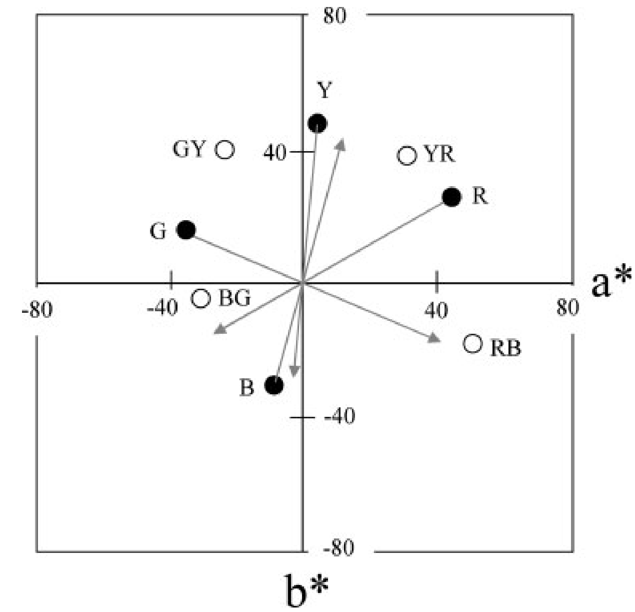

FIG. 2. Adapted from Ref. 7, test colors for Ref. 7 (almost same as Ref. 5) in CIELAB a* b* plane. Unique hues: filled circles. Binary hues: open circles. Opposites (on gray lines through the center) to the test colors are systematically complementary rather than opponent colors. (B and Y are both complementary and opponent.)

Similar examples of obviously complementary chromatic induction, but ascribed to opponent induction, are given in at least two books* published this past decade. These and Refs. 5–7 flie in the face of earlier and widely accepted data[11–14] over the past 150 years showing that chromatic induction is a complementary color process (Ref. 15 gives examples and refs.) Those studies were never disputed or shown to be less accurate accounts of chromatic induction than opponent colors. Hering almost certainly knew Chevreul's famous study[11] of chromatic induction and complementary colors, and while claiming lightness induction was an opponent process he never claimed that of chromatic induction. As a brilliant theorist and observer, he would have had sound reasons. Yet Jameson and Hurvich do so claim,[6] without comparing their theory's performance against the conventional explanation of the time, i.e. complementary colors.

In the past 50 years, most books and articles have referred to chromatic induction as following the opponent

* I wrote to one author,[10] M. Fairchild, offering him a bottle of good Australian wine if he could substantiate that Hering had ever claimed chromatic induction followed opponent, rather than complementary, colors. After research, he kindly corrected his book's 2nd edition (at p 18, but not p 113).[10] The other author recently published a 2nd, corrected, edition without my prompting.

color of the induction color, or as "following opponent color theory." For the 100 years prior to that, books and articles[11–14] referred to chromatic induction as following the complementary color of the induction color. What caused the change? New evidence, or new analyses? No: I am not aware of any data or studies indicating that opponent colors give a better account of chromatic induction than complementary colors. However, in the enthusiasm for opponent color theory, combined with Jameson and Hurvich's article,[6] researchers seemed eager to cite the new theory and forget the previous one based on complementary colors. The latter were apparently out of fashion.

So, which is the more accurate account of chromatic induction: opponent color or complementary color? That is easily settled, below. But first it is informative to discuss the article[6] which initiated the theory of opponent chromatic induction and caused the present confusion. Hurvich and Jameson claimed opponent chromatic induction from data recorded (but not published) in Munsell notation, transformed to CIE tristimuli (which allow infinite variations of chromaticity), and then transformed (at the loss of most chromaticity information) into "chromatic response values of the opponent-color scheme" and published as graphs showing varying amounts of (but not between) four hues only: redness, greenness, yellowness, blueness. Thus the original extensive information on the extent and direction/angle of chromatic induction was deliberately lost by exclusion of all but four hue angles. Why? Because this was not an investigation of chromatic induction in general (from which to determine the best explanation/theory) but only, as the authors state, of induction as it relates to opponent colors, specifically to "assumed physiological mechanisms of opponent neural interactions." This study was surprisingly narrow in its aims and relevance. It makes no mention of "complementary," or of previous studies[11–13] that found chromatic induction (including afterimages) is essentially complementary, despite Wilson and Brocklebank's seminal study published just 6 years earlier[13] and the international repute (in art, industry and science) of Chevreul's detailed and classic study.[11] Jameson and Hurvich, as color scientists, must have known these studies but ignored them. Their study[6] concludes chromatic induction is an opponent color process without considering any other options (e.g. complementary colors) which might give simpler or more accurate accounts. This is unusual (and undesirable) in research articles, and restricts the article's relevance. Readers unaware of the narrow relevance may have been led into thinking the new theory compared with and outperformed previous theory. Its data and conclusions relate only to the role of opponent colors in chromatic induction and tell us nothing of how these experiments relate to previous data.

Other studies naturally followed (due not to logic but to human nature) rather than disputed the famous pair's conclusions[6] and found that induced change in hue "followed opponent color theory," or was in the direction of opponent colors, whilst in fact demonstrating (as described above) in graphs and tables that the direction is more accurately described as complementary than opponent.

Now, to address whether chromatic induction (particularly afterimages, properly *negative* afterimages)[14] is primarily a complementary or opponent color system. Consider the hue cycle in Fig. 3 (note that R, G, and B sections each consist of three subsections so as to show major hue variations). Figure 3 represents the three classic complementary pairs[11–13] R-C, G-M, B-Y, representing prime and antiprime, or RGB and complementary CMY, peaks and troughs of such well known functions as spectral sensitivity, color matching functions, radiant power of colors required to neutralize their complementaries,[3,16] and wavelength discrimination [all of which have RGB peaks and complementary C(M)Y troughs]. In a CIE color mixture diagram, C(M)Y represent small wavelength intervals (each about 15 nm) that complement the rest of the hue cycle, or in other words, they complement (and lie between) three much larger wavelength intervals comprising R, G, or B hues. For example, Cyan (about 480–495 nm) complements (in Illuminant D65) the range of R and reddish hues from 580 to 700 nm and through nonspectrals to 495 c, i.e. about a third of the hue cycle. This demonstrates the trimodality of complementary colors. No less a number than three pairs of complementary intervals (or three peaks and complementary troughs) over the hue cycle can describe complementary color functions.

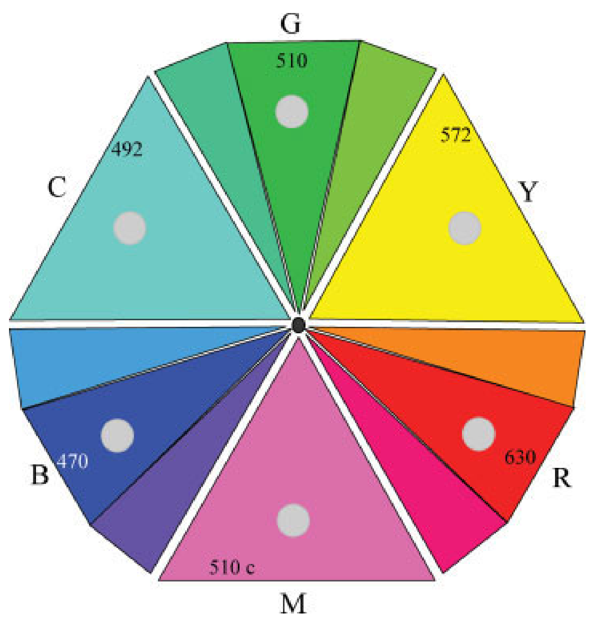

FIG. 3. Trimodal structure (R-C, G-M, B-Y) of complementary colors over the hue circle: RGB are relatively saturated and dark colors, relative to the less saturated and lighter CMY colors. The latter have high relative radiance whereas RGB hues have low relative radiance (required to neutralize complementary colors).[16] R, G, B sections each comprise three subsections, to show hue variations in say R, from yellowish-R to bluish-R. Some approx dominant wavelengths are shown. This figure also illustrates the complementary basis of afterimages. Gaze at the central black dot for 30 s then at white paper to see the (fainter) complementary afterimages.

The important difference between complementary colors and opponent colors is that the former are trimodal functions while opponent colors are bimodal functions (Y, R, peaks, B, G, troughs of chromatic response functions, or B-Y, G-R unique hues). Chromatic induction, e.g. afterimages, is easily shown to be trimodal and thus unable in principle to be an opponent color (bimodal) function.

By gazing at Fig. 3 for 30 s, the color normal reader will find that the afterimage to each of the six (RGB, CMY) sections is the same hue (though fainter) as the opposite and complementary section. For example, the afterimage to the R section (all three subsections) is Cyan. Thus the three sections CMY represent the afterimages produced from the rest of the hue cycle, i.e. the RGB sections. Each of these RGB sections is a large wavelength interval (e.g. say 500–565 nm for G), each producing an afterimage (C, M, or Y) whose range of dominant wavelength (excluding M) is only about 15 nm, e.g. 480–495 nm for C and 565–580 nm for Y. Each of the three subsections, for say R, may be tested to find all their afterimages are Cyan (though each may be a slightly different Cyan). It is clear that successive chromatic induction (e.g. afterimages) is a trimodal function where just three hue areas, CMY, represent the afterimages to the rest of the hue cycle. Hence, (1) chromatic induction is the same trimodal function as complementary colors, and (2) it is not logically possible for chromatic induction (a trimodal function) to be an opponent color process (a bimodal function).

One may solve the problem another way. The dominant wavelengths of some sections in Fig. 2 are labeled, and include the four unique hues RYGB. Remember B and Y are both complementary and opponent colors, so B gives a Y afterimage and vice versa. So the only opponent-color unique hues in contention are R and G. The big question is: do R and G give G and R afterimages, as claimed by the theory[6] of opponent chromatic induction? No, certainly not (see below). Why then did this idea arise? Apparently it was raised (and strangely, widely accepted) so that opponent color theory could be a "theory for everything." However, it is certainly not an adequate theory for chromatic induction: no color normal reader, gazing at Fig. 3 (or part thereof) for 30 s and then gazing at a white sheet of paper, would describe the afterimage to Green as Red, but as RB or Magenta; or the afterimage to Red as Green, but as BG or Cyan; the latter afterimage can no more be described as Green than Blue; it is an equal BG mixture, more or less. One can confidently predict these results for color normal readers, supported by the above discussion, previous experiments over 150 years,[11–15] and the chromatic induction data sets[5,7,8] discussed above which do not mention complementary colors yet demonstrate their governing role in chromatic induction.

It is not accurate to say (as has been said)[10] that chromatic induction can be explained as well by complementary colors as opponent colors. The latter give a simplistic, crude, explanation (e.g. the afterimage to Red is a "greenish" color, or the afterimage to Green is a "reddish" color) which tries to account for a trimodal function by a bimodal function; whereas complementary colors, as a trimodal function, give a much more accurate account. Surely science demands, of two or more competing explanations, the more accurate account?

Afterimages represent successive chromatic induction. Simultaneous chromatic induction works in exactly the same (complementary) hue directions, as shown by the examples in Fig. 2 and discussion of Refs. 5, 7, 8 above, or readers may find their own results by looking at the (all identical) gray circles in Fig. 3. For example, the gray circle in the Cyan triangle appears slightly reddish.

Can anyone doubt that afterimages (successive chromatic induction) are complementary? Afterimages were the first definition of complementary colors until the colorimetric definition.[3] References 13–15 distinguish between "afterimage complementaries" and "additive (color mixture) complementaries"; there are slight differences, e.g. Y's afterimage is a more reddish Blue. However, the differences are mostly caused by the lower saturation of afterimages and consequent Abney effect hue shifts.[9]

Simultaneous induction involves images of different colors falling on adjacent parts of the retinal field, whereas successive induction is thought to involve relative desensitization of cone sensitivities and, when the stimulus ceases, complementary afterimages result from the remaining cone sensitivities.[14] There is no doubt that afterimages are basically complementary. But the question arises: if simultaneous contrast is a different physiological process, does it give a different contrast effect? E.g. a noncomplementary effect? There is no evidence the effects are different.[5–8,11–14] It is not likely the effects could be significantly different, considering that a simultaneous contrast effect at a moment in time is not purely simultaneous contrast but includes some degree of successive contrast effect from observing the focal and surround colors for some period of time during the test, whether it is 0.5 or 20 s (the test viewing time is not restricted)[5,7] just previous to recording the effect. If the effects were significantly different, a simultaneous contrast effect would vary significantly as a function of the viewing time. As Chevreul[11] and Rood[12] noted, the afterimage to a stimulus is present and exerting an influence (on the stimulus and adjacent colors) before the stimulus ceases. Chapter 4 (see pp 115–118) of Ref. 14, written first by S.M. Newhall and reviewed by others of the venerable committee, describe the effects of both simultaneous and successive contrast as approximately complementary, and conclude both effects are essentially the same because, even in simultaneous contrast, "the actual retinal stimulation is … successive because the eyes scan from one color to another during the perception" (p 118)[14]. The above discussion of recent data on simultaneous induction[5,7,8] (e.g. Fig. 2) similarly indicates the effects are complementary, like afterimages.

In conclusion: (1) chromatic induction is trimodal, (2) complementary, but not opponent, colors offer an accurate account of chromatic induction, (3) Ref. 6 theory of "opponent chromatic induction" is inadequate (e.g. in accounting for C afterimage to R, and M afterimage to G) because it is a bimodal process and cannot possibly explain a trimodal function (except simplistically, by combining two opposing pairs C-R and G-M as one pair R-G).

Reference 6 is valuable in many ways but it has caused considerable confusion, due partly to its authors' fame. The authors never claimed "opponent chromatic induction" performed better than the existing theory[11–13] of complementary chromatic induction, but the inference is there: otherwise, readers may ask, why was the article published? The average reader would hardly expect an underperforming theory to be published (as it was). Because of the popularity of opponent color theory and the fame of the authors, it seems most readers simply assumed opponent colors had replaced complementary colors.

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Camera Color Analysis Gamut

Abstract: Michael Brill has opened a valuable discussion about camera gamuts and Roy Berns has questioned the use of the word "gamut" in that proposal. We would like to stress the importance of associating a gamut with an imaging output device and would thus question the use of the concept proposed for assessing the capabilities of an input device. © 2007 Wiley Periodicals, Inc. Col Res Appl, 33, 81–82, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/col.20376

Key words: color gamut; color imaging; image rendering; digital camera

Michael Brill, in his letter,[1] has opened a valuable discussion about camera gamuts. We would however, like Roy Berns in his letter,[2] question the use of the word "gamut" in the definition that he proposes. In imaging, it is the display device that has the gamut that is of practical importance. In the case of additive displays, this is usually specified by the chromaticities of its primaries and colors presented to that device, whose chromaticities fall outside of that gamut, will not be displayed accurately.

To illustrate the difficulties that Michael Brill addresses, it might perhaps be helpful to consider a hypothetical case where a camera has just three monochromatic spectral sensitivities. The spectral power distributions (A, in Michael Brill's analysis) then effectively consist of different amounts of light of these three wavelengths, the camera (B in his analysis) has sensitivities to these three wavelengths only, and the algorithm (C in his analysis) maps camera values to XYZ triplets. The camera color analysis gamut is then the set of XYZ triplets for each of which there is an SPD in A that, when acquired by camera B produces camera values that, when processed by algorithm C, produces that XYZ triplet. The camera color analysis gamut in this case is therefore the gamut corresponding to various amounts of the three monochromatic wavelengths.

But is this useful? The gamut of colors displayed by a reproduction system using such a camera will depend on the display device. If the display device primaries are monochromatic lights of the same three wavelengths, then colorimetric accuracy of the system can be achieved for scenes illuminated by light sources consisting only of light of the three wavelengths. But when normal illuminants are used with usual object colors, inaccuracies will occur, and the interest then lies in the extent to which the camera sensitivities depart from a linear combination of color matching functions. This is a camera analysis problem but seems to us to relate to metamerism rather than to a gamut.

Another case that can be considered in this context is a camera that has as its spectral sensitivities (B) a linear combination of color matching functions. The spectral power distributions (A) can then be any real distributions, and the processing algorithm (C) is then the matrix that converts the camera sensitivities to the XYZ color matching functions. The camera color analysis gamut in this case is therefore the gamut corresponding to all colors within the spectral locus and the purple boundary. Colorimetric accuracy of the system is then achieved for all original colors that lie within the gamut of the display device. In this case, there is no metamerism problem and the only issue in practice is the gamut of the display. Once again, is the concept of camera color analysis gamut useful?

It seems to us that the property of cameras that is important is the degree of departure from a set of color matching functions, and a quantitative means of assessing this would be useful.

It should also be noted that the image processing that takes place subsequent to image capture takes a similar form irrespective of whether the image comes from a camera, a scanner or is computer generated, for example. Thus the additional terms "scanner color analysis gamut," and "image color analysis gamut" might need appropriate definition. But this is not necessary if it is recognized that the gamut is not a function of the input device but of the restrictions imposed by an output device.

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Try Camera Gamut Again: Not for Size, but for Camera and Profile Evaluation

Abstract: Hunt and Pointer call into question the utility of the camera color gamut (at least by my definition). Such doubt may be partly due to the connection of gamut with a size metric. Quite apart from gamut size, the sets that define my gamut definition are essential to the evaluation of digital cameras in conjunction with their profiles. The output-device gamut, emphasized by Hunt and Pointer, is also important, but appears at another stage of color management.

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Key words: camera; color gamut

Hunt and Pointer continue what should become a fruitful discussion about camera color gamuts, and more broadly about how cameras fit into digital color-management. They correctly affirm that the concept of gamut is straightforward only for output devices. Also, they use my definition of camera color gamut correctly in the two simple cases they describe.

For those particular cases, I agree that the concept would not be very useful. But I think others have found the concept generally useful, as attested by published studies by, e.g., Martinez-Verdú *et al.*[1] and by Holm.[2] These investigators use the term (or "camera color analysis gamut") in the sense of my definition,[3] although they do not write an explicit definition.

Now, I admit that my definition of camera color gamut is not particularly simple. Even small perturbations of Hunt and Pointer's special cases lead to complications. For example, suppose ingredient A of my definition restricts the input spectral-power distributions to reflected lights (reflectance between 0 and 1) under a single illuminant. That would impose limits on the gamut in their first example, and could supersede the gamut restrictions of the output device. And for real cameras, for which camera-to-human metamerism always enters the mix, other subtleties emerge. For example, the extremal reflectances of the camera object-color solid may not be the usual Schrödinger-Ostwald reflectances (0 or 1 at each wavelength with at most two transitions between them).[4]

One aspect of my definition need not be an obstacle or a problem, but requires some explanation to defuse Berns's[5] uneasiness with statements like "my gamut is bigger than your gamut." My definition does not imply a size metric. Indeed, if it did, one could use ingredient C (the mapping of camera values to XYZ) to meaninglessly make the gamut as big as desired.

Of course, the set comprising the camera color gamut will be "doing its job" only insofar as the the XYZs determined by ingredient C actually represent the XYZ values that are in the scene acquired by the camera. This quality is not at all conveyed by a "size" of the camera color gamut. It is also not entirely determined by camera-to-human metamerism, which Hunt and Pointer claim is the property of cameras to be characterized.

The ICC[6] accommodates ingredient C in the profile of a camera. The goodness of this profile depends on more than just camera-to-human metamerism. An average ΔE value over a set of test colors might be a figure-of-merit for the combination of camera and profile, and of course such a ΔE value would degrade if an output device were included with its own imprecision and gamut constraints. However, evaluating the whole system of camera and profile as it interacts with real inputs is important to color management, and what we have called the camera color gamut defines the ingredients critical to such evaluation.

I also invite to this discussion parties who have found useful a different definition of camera color gamut: Morovic and Morovic,[7] for example, define the gamut of an input-medium as "the range of color stimuli across which their responses show differences." How these differences show up in the object-color solid, as described by their article, would be interesting to understand on a definitional level.

One more thought on the letter by Hunt and Pointer: Once "camera color gamut" has been defined, the extension to "scanner color gamut" seems obvious and a new term is not needed.

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MEETING REPORT

7th International Symposium of Slovenian Colorists Association: Color of National Symbols

On behalf of the Slovenian Colorists Association, the international symposium entitled Color of National Symbols was held in Ljubljana, Slovenia from 15th to 17th of June, 2006. The topic of this international meeting dealt with the color of national symbols, a topic that is relevant and requires special attention. The legislation that regulates the selection and the quality of color is quite loose. The regulations offer no exact definition of color because they do not consider the difference in materials (paper, polymers, metal, etc.) and do not determine the color differences. When the color is a public matter (state or national), it must not be related left to individual opinion, but should be specified by statute. Our state Republic of Slovenia has in its Law about coat-of-arms, flag, and hymn of Republic of Slovenia (Official gazette No. 67–27th of October 1994) regulated the geometric, art and color rules and also time, manner, place, and purpose of usage. The knowledge of experts from many countries, exchanged at this meeting, will help us to recognize the depth and the importance of this problem. The reproduction of national symbols on different materials demands a thorough interdisciplinary knowledge of color, design, technology, and quality. This is the reason why the exchange of the opinions among scientists, researchers, and experts was so important. The 2-day meeting has brought us many new ideas, and thus enriched the Slovenian and the international treasury of knowledge. At the meeting we did not want to encroach on design of flags, this is left to the competence of ethnologists, historians, heraldists, and designers. We want only to expose the problem of definition and assurance of the law of prescribed symbol colors. Slovenian law regulates the colors with SCODIC marks and the lack of the law is in the definition of permitted color differences respectively, tolerances, which can be defined only with the numerical color evaluation. The problem is even greater, if the law doesn't consider different materials. Indicated Slovenian problems are no exception. It was most interesting to hear about similar dilemmas in other countries.

The scientific program was divided into three sections: (1) Color and legal regulation of national symbols, (2) Materials and production processes, and (3) Color science, colorimetric theory, and numerical evaluation of color. Table I lists all the presented lectures.

Simultaneously, the exhibition entitled Color of National Symbols took place. The students of the Textile Department (course graphic technology) of the Faculty of Natural Sciences of University of Ljubljana under the mentorship of Sabina Bračko, Ph.D., presented the development of Slovenian national symbols and their color evaluation. Symbols are present in different situations and periods of our life. Their colors are impressed in our memory and many times are the symbols recognized just because of colors. Inconsistency use leads to unpredictable situations. The exact definitions of colors are important and also the assurance of their reproduction of good quality on different substrates in defined color tolerances. On posters, the students defined the colors of Slovenian flag in coat of arms with the most used color spaces and systems as the CIELAB; Munsell and Natural color systems. Many problems namely originate from the fact that the SCODIC system, which defines the colors in Slovenian law are not generally used. The students presented also the historical overview of Slovenian symbols through the centuries, from Austrian monarchy till the idea of Pan-Slovenian movement and new state of Slovenia 1991. The symbolic meaning of some flags were presented with examples of same colors in different environments with opposite meanings. The proposal of new colors of Slovenian flag with the purpose of better recognition was presented.

At the symposium an exhibition of color devices was also held. With the poster represented itself also Center for Dyeing and Color from the University of Maribor, Faculty of Mechanical Engineering. The Center for Dye-

TABLE I. Presented lectures.

Color and legal regulation of national symbols	**Color science, colorimetric theory, and numerical evaluation of color**
• Colors of the national symbols *Vojko Pogačar, Andrej Skrbinek, Danijel Zimšek, Igor Nahtigal* • Psychology of color and our academician Trstenjak *Maks Tušak* • Color characteristics of the Slovenian flag first flown in 1918 *Branko Neral, Sonja Šostar Turk, Mirjana Čokl* • Relationship of Lithuanian national flag and national colors *Rimvydas Milašius, Vytautas Mičašius, Ingrida Zdanavičiute, Agne Danelevičiute* • Significance od symbols in Romanian space *Angela Dorogan, Eftalea Carpus* • Contribution to the color semiiotics of heraldry *Francis Edeline* • Legal definition of color of Slovenian flag *Daniel Zimšek, Darko Golob* • Colors of Slovenian flag from the Official gazette (Uradni list) to every day public use *Gorazd Golob, Grega Kukovica, Jure Ahtik* • Is it time for heraldry drama? *Vojko Pogačar, Andrej Skrbinek* **Materials and production processes** • Weather resistant coatings for national symbols *Matjaž Kunaver, Marta Klanjšek Gunde, Nataša Barle, Andrijana Sever Škapin, Vilma Ducman* • Disperse dyes-dyes for printing and dyeing of polyester flags *Alenka Majcen Le Marechal, Boštjan Križanec* • Computer match prediction of colors for slovene national flag *Darko Golob, Darinka Fakin, Danijel Zimšek* • Advanced materials for national flags *Nika Veronovski, Majda Sfiligoj Smole, Tatjana Kreže, Aleksandra Lobnik* • Construction of flag fabrics *Polona Dobnik Dubrovski* • Reproduction of national flags within printing *Marko Kumar* • Ink jet printing of textiles *Branko Neral, Darko Štanc, Sonja Šostar Turk*	• National flags, as a test object in visual research *Lucia R. Ronchi* • A survey on the colors of national symbols amongst people doing color research *Manuel Melgosa* • Natural color cycling model as a ground for modern vexillology *Vojko Pogačar* • Pluralism of color *Andrej Skrbinek* • Disadvantages of flags color definition using subjective evaluation system *Đurđica Parac – Osterman, Martinia Joaneli* • Actual CIE news *Marta Klanjšek Gunde* • Multi-spectral colorimeter for the evaluation of color and pattern quality of textile images *Ioannis S. Chatzis, Evangelos S. Dermatas* • Measurement of chromatic identity of a lit symbol in the night *Arturo Covitti* • Main measuring settings for proper color matching on different substrates *Peter Simmen, Maxim Siniak* • The influence of nano silver on color difference of cotton fabric dyed with reactive dyes *Marija Gorenšek, Petra Recelj* • Determninig the color of structure in dependence of texture, construction and composition *Helena Gabrijelčič, Krste Dimitrovski* • Color analysis of one-colored fabrics created from two differently colored yarns *Tanja Nuša Kočevar, Ljubica Vračar, Krste Dimitrovski* • Multicolored flag: a multifaceted communication medium *Silvia Rizzo*

ing and Color encompasses work connected with dyeing, color sense, and measurement. It incorporates various branches, such as textiles, papermaking, pigments, architecture, civil engineering, art, psychology, the food-processing industry, medicine, and others.

The lectures of the first topic *Color and legal regulation of national symbols* were devoted to historical development respectively, tradition of national symbols, their legal regulation especially, the flag of different countries and the heraldry, the science of coat of arms respectively, science of national symbols. Flags are the most important symbol of different states that represent the national cultural identity and the colors are the most important element of that identity. Reproduction of national symbols in everyday life, especially their colors on different materials, is not simple and even complicated if the law regulation is not exact. The great part of its opus the academician Dr. Anton Trstenjak devoted to the science of colors. The pioneer on this field in Slovenia published in 1950 a phenomelogical analysis of colors. Beside the famous book, *Human and colors*, he published over 40 articles and psychological discussions about the phenomena color.

Following topic, *Materials and production processes,* dealt with the production processes included into production of national symbols on materials, which are used in everyday life. National symbols are displayed on various occasions, in different weather conditions and on different substrates. The influence of weather conditions and direct sunshine to degradation of coatings might lead to changes of color shade in such extent that the national symbols loose their purpose or can not be recognized as such. In the lecture ''Weather resistant coatings for national symbols'' the authors present the developed different coating systems with different pigment combination that would be most resistant to weather conditions and to UV radiation. The lecture ''Disperse dyes, dyes for printing, and dyeing of polyester flags'' was presented by the authors from Faculty of Mechanical Engineering, Textile Department. In the lecture ''Advanced materials for national flags,'' the study to obtain self-cleaning properties of polyester surfaces with nanomodification by TiO_2 nanocoating was presented. But according to the environment, two types of flags are distinguished, indoor and outdoor flags. The main causes of outdoor flag deterioration are wind, sun, and UV rays and the flag should have such construction that will fit end-use demands. The most important are durability, as well as appearance. The ''Reproduction of national flags within printing'' and ''Ink-jet printing of textiles'' were also presented.

The last topic was devoted to the *Science of color, colorimetric theory and numerical evaluation of color*. In the lecture, ''A survey on the colors of national symbols amongst people doing color research'' the author presented the results of the survey, which was carried out amongst color researchers looking to identifying, which are the main symbols where color plays a relevant role and why it is thought that color is important to each one of these symbols. A total of 185 symbols were reported, which were classified into six categories or groups: flags, garments, logos and trademarks, traffic signals, heraldry, and symbols of very different kinds. Color plays an important role in many national and international symbols, and it must be carefully managed in each specific symbol, bearing in mind technical, aesthetical, legal, ethical, and symbolic aspects. A ''natural color cycling model as a ground for modern vexillology'' was presented. A survey of current philosophical discourse in contrast to comparable scientific discourse on color was presented. Two theories were discussed, relational, known also as subjectivistic theory, and realistic theory respectively, eliminativism. But modern evaluation of color can not exist without numerical definitions. The basis of numerical evaluation is the interdisciplinarity of color, which includes physics, chemistry, physiology, and psychology. For this reason CIE standardization is necessary and without it there is no objective evaluation. Also ''disadvantages of flags color definition using subjective evaluation system'' were presented, than the field of measuring devices and measurements conditions and some lectures presented the results of research surveys dealt with the evaluation of color and pattern quality of textile images, ''measurement of chromatic identity of a lit symbol in the night,'' ''measuring settings for proper color matching on different substrates,'' ''the influence of nano silver on color difference of cotton fabric dyed with reactive dyes,'' "determining the color of structures in dependence of texture, construction and composites," ''color analysis of one-colored fabric created from two differently colored yarns'' and ''multicolored flag.''

Two-day meeting has enriched our knowledge about color theory, evaluation of color, production of national symbols and has brought us many novelties and new findings, and thus enrich the Slovenian and the international treasury of knowledge.

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BOOK REVIEWS

Color Influencing Form, by Roy Osborne, Universal Publishers, 2007, 112 pp., $19.95, ISBN 1-58112-542-9

To get a good sense of this book, imagine an experienced design teacher summarizing his lectures on two complex subjects—color and form. Imagine that he is clearly comfortable with the material and can move from subject to subject readily, linking and connecting theory to practical examples in his own work; including such diverse applications as heraldry, painting, and color television. Imagine also that he can refer to the scholarly past of color and design history, sprinkling the discussions with quotes from historical sources. And finally, imagine that he can accomplish all this concisely.

This is Color Influencing Form, Roy Osborne's short—less than 100 pages of text—and simple outline of color theory as it is used in the arts. Intended to be part of a module in an art school program, each chapter, in itself less than 10 pages, contains a summary which could be used for testing as well as developing class assignments. Also included are a final summary checklist, a glossary and bibliography, as well as several pages of black and white illustrations which could be part of classroom direction.

Although there is not anything new here, the organization and text are direct and articulate. Writing without illustrations for theory which is entirely visual is always a challenge. When the assumption is that the readership will be student-level, without previous art or science experience, complex ideas must be simplified and no previous knowledge can be assumed—additional burdens on the author.

Initial chapters deal with traditional color topics such as light, color mixing, vision, and perception. The final six chapters continue the discussion of color theory, but overlay the concepts with traditional design topics including line, form, texture, and composition. So, as the author states in his introduction, this book will give instructors a format to move past teaching color using only simple shapes and formats, and allow them to prepare students to integrate color into their own art or design work.

Because of the book's simplicity, readers more experienced in color and design theory may find themselves frustrated either by errors of omission, or by words used differently than they are accustomed to find. It also does not present some of the newer work on perception and light that has come about as a result of research into the brain and eye relationships. And finally, there is always the decision of what should be presented first in an art class on color; light, color mixing, or perception or some other concept. An instructor wants to attract students' attention, but creativity can best use color theory when the underlying scientific information is understood. So this book presents one—of the many possible—ways to present the wealth of information that leads art students to know how to manipulate color and form to their own creative use. It is, at the end, an introduction.

SANDRA AUSTIN

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Dictionnaire des termes de la COULEUR, by Robert Sève, Michel Indergand and Philippe Lanthony, TERRA ROSSA, 39 route de Lormes, 89200 Avallon, France, 2007, ISBN 978-2-84280-130-4

Although the book is written in French, it is relevant for a publication such as *Color Research and Applica-*

tion to review scientific material from countries all around the world. The first author, Robert Sève is familiar to many readers through his work on whiteness and on gloss. He is the French representative for CIE Division 1 and Associate Editor of *Color Research and Application*. In this dictionary, his contribution is directed toward Colorimetry and Physics. His co-authors have brought alternative points of views. Michel Indergand defines himself as a chromatologist and is a member of the Centre Français de la Couleur. He has specialized in arts and linguistics. Philippe Lanthony is an ophthalmologist, one of the founders of the International Color Vision Society.

The publication is a encyclopedia vocabulary that results from common work and critical exchanges between the authors, so that almost every color word offers the opportunity to be defined in terms of physics and physiology on the one hand, and of history, arts, literature, and applied sciences on the other hand. Besides these points of view, introductory comments to a word are sometimes given and warnings about its possible misuses are added.

As an example, the word "blanc, blanche" that means white, has received an historical introductory comment, and is described in the everyday language as an adjective, as a substantive, and as a term of technical science. Thus, parent adjectives that have been used in the ancient time are glossy, metallic and in latin *albus* and *candidus*. The word has a particular meaning in the expressions white wine or white carnation. It is falsely quoted in advertising, where a laundry detergent washes "whiter than white." A simple but honest explanation is given about white daylight. As a name, several uses are proposed, and even the origin of "Blanche-Neige" "Snow White" in the fairy tale is given. It is followed by a note about the dominant wavelength of acceptable white shades and of possible misuses in place of colorless. Then, a definition in terms of appearance is given, mentioning the work of Hurvich and Jameson, and the corresponding domain of acceptance in colorimetry. In the literature section, a few lines from a poem written in 1972 by Jacques Prévert are quoted. Finally, in the applied section, the perfect diffuser is defined and the use of calibration plates recommended by standards is mentioned.

This is an example among others. In total, 190 terms of color science or of chromatology are explained in 10 chapters. As an appendix, one can find an English–French dictionary of 750 terms, and reverse, translation of 250 abbreviations (CR&A, CAT, LUTCHI … that look exotic to a naïve reader), and a reference list. An inset of 24 color plates illustrates a few definitions.

Who should use the book? First, French color scientists, engineers, and students would benefit from using it, and it can be expected that concepts that are difficult to present in vision and colorimetry could be better understood when analyzed in various contexts. Second, it constitutes a reference for translating from English to French and reverse. Third, because of its crossdisciplinary approach, it will be very helpful to anyone who has to prepare an introductory presentation on a specific color topics. Since I got it, I have already found many responses to my questions.

FRANÇOISE VIÉNOT

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NEWS

Graduate Programs in Color Science at Rochester Institute of Technology

Rochester Institute of Technology (RIT) is seeking outstanding applicants for its M. S. and Ph. D. programs in Color Science. Assistantships and fellowships include full tuition, a 12-month stipend, and an Apple laptop computer. The application due date is January 15, 2008.

Color Science is broadly interdisciplinary, encompassing physics, chemistry, physiology, statistics, computer science, and psychology. This is the only graduate program in the country devoted to this discipline, and it is designed for students whose undergraduate majors are in physics, chemistry, imaging science, computer science, electrical engineering, experimental psychology, physiology, or any discipline pertaining to the quantitative description of color.

The M. S. curriculum includes core courses in color science, electives, and a research project or thesis while the Ph. D. includes additional electives and a research dissertation. Facilities are outstanding and the job placement rate is 100% since the M. S. program began in 1984.

Research at RIT that includes color science is carried out across campus including the Chester F. Carlson Center for Imaging Science that houses the Munsell Color Science Laboratory and the Visual Perception Laboratory, the School of Print Media, the Department of Psychology, the Department of Computer Science, and the Center for Quality and Applied Statistics.

The Munsell Color Science Laboratory (www.mcsl.rit.edu) is the preeminent academic laboratory in the country devoted specifically to color science. Research is currently under way in fundamental color science, color and image appearance models, color-tolerance equations, psychophysics of image quality, effects of size on image appearance, surround adaptation, and low vision among others, spectral-based image capture, archiving, and reproduction of artwork, spectral-based image segmentation and pigment mapping, fluorescence imaging, three-dimensional imaging and rendering of artwork, high dynamic range imaging and rendering, analytical and empirical multi-ink printing models, spectral-based color management, and consumer-digital camera and digital-cinema camera optimization.

To learn more about the color science curricula and admission requirements, visit www.cis.rit.edu or contact the program coordinator, Dr. Roy S. Berns, at berns@cis.rit.edu.

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Complementary Colors: The Structure of Wavelength Discrimination, Uniform Hue, Spectral Sensitivity, Saturation, Chromatic Adaptation and Induction

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Abstract. Complementary colors have long been thought important to color vision due to their ability (as admixed pairs) to extinguish all chromaticity, and to automatically adapt (i.e. wavelength pairs and radiant power ratios) to illuminant. Their role in color matching and chromatic induction is well documented but other roles have not been demonstrated. This paper studies the structure of complementary colors in the wavelength and radiance dimensions over the hue cycle (the nonspectrals are represented by a nominal-wavelength metric). In the wavelength dimension, the basic structure of complementary colors is the complementary intervals ratio *(ratio of a wavelength interval to its complementary interval of 1 nm). The ratio has RGB peaks, complementary CMY troughs, and provides models of chromatic induction, wavelength discrimination, and uniform hue difference in good agreement with data. Novel analyses of six color order/UCS hue circles indicate essential characteristics of a uniform hue scale. In the radiance dimension, basic structure is the* complementary powers ratio *(power of a stimulus required to neutralise its complementary of 1 Watt). The inverse structure has RGB peaks, complementary CMY troughs, and provides models of saturation, spectral sensitivity, and chromatic adaptation to illuminant. The RGB peaks demonstrate spectral sharpening, implying a post-receptoral location in the physiology. The models indicate that complementary colors have a significant role in color appearance besides their well known role in color matching.*

Key words: chromatic adaptation, chromatic induction, color appearance, complementary colors, hue, saturation, wavelength discrimination.

1. INTRODUCTION

Complementary colors are defined[1,2,3] as a pair of color stimuli whose admixture color-matches a selected achromatic color stimulus (usually the light source). Complementary colors depend on appropriate wavelength pairs and radiant power ratios, both of which systematically vary with illuminant.[2,3] Dominant wavelengths and power ratios of complementary colors may be calculated from CIE colorimetry, looked up in tables, or wavelength pairs may be found graphically as opposites through the white point in CIE *Yxy* or CIE *Luv* chromaticity diagrams.[2,3] Unlike opponent colors, where observers estimate their unique hues and other opponent color pairs by appearance, complementary colors are not determined by initial appearance but as pairs which experimentally admix (by appropriate wavelengths and powers) and match a selected white as judged by the observer.

Another perspective on complementary colors is in relation to the conventional two chromatic channels (yellow-blue and red-green) that reflect the processing of cone photoreceptor signals as measured in colour detection thresholds. Complementary colors represent the wavelength compositions of two lights that cancel out the outputs of the two chromatic channels to either light presented alone. The two channels are in a state of continuous chromatic adaptation to illuminant changes. Complementary colors reflect the polarity sensitive nature of the two chromatic channels.

Scientists have researched complementary colors over three centuries, including Newton, Young, Helmholtz, Maxwell, Judd, MacAdam, and Cohen.[4] But as yet complementary colors, and their role in vision, are not well understood other than: (1) their colorimetric quantification in terms of complementary wavelength pairs and relative radiant powers for various illuminants;[1,2, 5,6] (2) their role in basic colorimetry such as color matching and mixture; and (3) their role in chromatic induction, i.e. after images and simultaneous contrast,[7,8] though disputed by some. In contrast, the role of opponent colors in color vision and appearance has been amply described and quantified in the Hering-Hurvich-Jameson theory,[2] and is thought to occur in the later part of the visual process (concerning color sensation) according to Muller and Judd types of zone models of color vision.[2] Such zone models, published in early-to-mid 20th century, ascribe complementary colors to the first zone (retina's receptoral layer, where their role is supposedly limited to color mixture/ matching), corresponding to

Young-Helmholtz trichromatic theory and color matching data. The second or third zone corresponds to Hering's opponent colors theory and concerns color appearance.

However, this exclusion of complementary colors from a role in color appearance seems motivated more by the mid-20th century wave of enthusiasm for opponent colors theory than by physiological or psychophysical evidence, and indeed conflicts with some theory and evidence. For example, Chevreul[9] based his color harmony system (used by such famous artists as Delacroix)[8] on complementary colors, while Ostwald[8] based his uniform hue difference scale on complementary wavelength pairs. Also, it is quite widely known that opposite Munsell Hues, eg, 5R and 5BG, or 5PB and 5Y, are approximately complementary wavelength pairs, as may be seen from Munsell data in the CIE chromaticity diagram [see Fig V(6.6.1) in Ref[2]]. Moreover, complementary colors data are gained from psychophysical color matching experiments where a pair of color stimuli are judged by the observer to admix a selected white. Further, the three RGB peaks of complementary colors' relative radiances (at 445, 532, 606 nm,[1,5,6] shown later below) do not reflect the cone sensitivity peaks (about 445, 530, 565 nm) of the receptor level but instead indicate spectral sharpening (where the R peak of sensitivity shifts from the cone sensitivity peak at 565 nm to about 600 nm),[10] at a post-receptoral stage of vision. Also, some of the physiological data on spectrally-opposed cells (also known as opponent color cells) in the lateral geniculate nucleus (LGN) and the cortex indicate complementary color pairing rather than opponent color pairing, with physiologists describing the opposed pairs in complementary color terms B-Y, G-M, R-C,[11] or as opposites through the white point in chromaticity diagrams (ie, complementaries).[12] In particular, on close inspection of graphs, the supposed red-green opposites are often red-cyan complements; e.g. Svaetichin's famous data (which sparked enthusiasm for opponent color theory) for supposedly R-G opponent responses actually peak at 650 (Red) and 490 nm (Cyan, certainly not Green).[13] In summary, the evidence suggests that complementary colors are not restricted to the receptor layer or the retina but occur also in later stages. This is supported by the models below.

The present paper does not attempt to present a comprehensive theory of the overall role of complementary colors in color vision. Its aim is merely to demonstrate the suitability of complementary colors as math models of several visual functions, and subsequently deduce the role of complementary colors in structuring those visual functions. Any visual function, e.g. wavelength discrimination, incorporates not only colors but also their complementaries. The peaks and troughs of many functions are complementary wavelengths (e.g. RGB peaks and complementary CMY troughs) and in some cases their radiance amounts are reciprocal at complementary wavelengths. This implies that complementary colors have some role in the structure of such functions, possibly in.balancing the opposing or reciprocal quantities in the wavelength and/or radiance dimensions. The balancing is not necessarily explicit in colorimetric calculations but is implicit in the underlying neurophysiological processes. It cannot be denied that the peaks and troughs of many visual functions (e.g. the functions modeled below) are complementary, particularly when the peaks and troughs (separately) of several data sets are averaged. It is time that science asked “Why?”rather than ignore the implications.

This paper describes the two basic structures (both trimodal in RGB) of complementary colors in the wavelength and radiance dimensions, and employs them to model six functions: (a) three in the wavelength dimension: chromatic induction, wavelength discrimination, and uniform hue; and (b) three in the radiance dimension: spectral sensitivity, chromatic adaptation, and saturation.

Complementary color pairs are conventionally defined by wavelength, so their structures similarly need definition in the wavelength dimension (e.g. as the *x*-axis in graphs) which unfortunately is limited physically to spectral hues. Now, nonspectrals are a substantial proportion (about 25-30%) of the hue cycle and should not be ignored if possible. The basic structure in complementary wavelength pairs is shown below to be the ratio of a wavelength interval (nm) to its complementary interval of 1 nm, known as the complementary intervals (CI) ratio. The CI ratio becomes this study’s work horse because it is proportional to many visual functions with RGB peaks, e.g. wavelength discrimination. To utilise the CI ratio in modelling visual functions over the hue cycle, including the nonspectrals, it is desirable to extend the CIE dominant wavelength metric over the nonspectral portion of the hue cycle, as nominal or *equivalent wavelengths*. (“Hue cycle” denotes the complete series of hues, spectral and nonspectral, sometimes arranged as a geometric circle.) By using this *equivalent wavelength* metric as a nominal standard, nonspectral data from various sources can be represented to the same “wavelength” scale and compared. I have used this type of metric in papers since 1999, often calling it an “arbitrary” scaling of the complementary wavelengths (e.g. 560 c) over the nonspectral part of a graph axis. The metric is described briefly in Appendix A.

2. STRUCTURES AND SCALES

Opponent colors are a bimodal system, based on the unique hue pairs R-G and Y-B, and the red-green and yellow-blue chromatic response (or chromatic valence) functions,[2] whose nulls indicate the unique hues BGYR. In contrast, complementary colors are a *trimodal* system [RGB peaks and complementary Cyan, Magenta, Yellow (CMY) troughs] in the wavelength dimension and also in the radiance dimension, as will be detailed below. Hence opponent colors and complementary colors are most suitable as models for quite different (bimodal versus trimodal) visual functions. Though opponent colors have been used to model trimodal RGB functions (e.g. wavelength discrimination),[14] the model is more complex than if complementary colors were used (see section Discussion).

To most readers, a familiar form of complementary wavelengths' trimodality may be the wavelength distribution in CIE chromaticity diagrams.[3] The distribution for CIE LUV is shown in Fig 1, featuring angle per constant wavelength interval (5 nm), characterised by large hue angles in Y and C areas relative to small hue angles in RGB areas. The indicated 5 nm intervals are limited to the wavelength range 442-613 nm, as explained below.

This paper employs object colors and aperture colors. The latter, when taken at constant optimum purity (or optimum relative saturation/chroma/colorfulness) over the hue cycle, are termed *optimal aperture color stimuli*.[5,6] They are effectively two-dimensional (in wavelength and radiance, or hue and brightness) and thus simpler to analyze. Optimal *spectral* aperture color stimuli are monochromatic stimuli in the range 442-613 nm.[5,6] Beyond those limits, the photometric efficiency (in color mixture and in neutralizing complementary colors) of monochromatic or single wavelength stimuli is less than optimum and rapidly decreases to zero at the spectrum extremes, which are invisible. Optimal *nonspectral* aperture color stimuli for the remainder of the hue cycle (nonspectral hues from 613 nm through the purples to 442 nm) are compound colors comprising mixtures of 442+613 nm, i.e. lying on the line 442-613 nm in color space; see Fig 1. This line, and the spectrum limited to 442-613 nm, represents the locus of optimal aperture colors (and of optimum purity). Similar limits and compound colors apply to object colors and the MacAdam limits[2] for luminance Y=10-20 approximately.

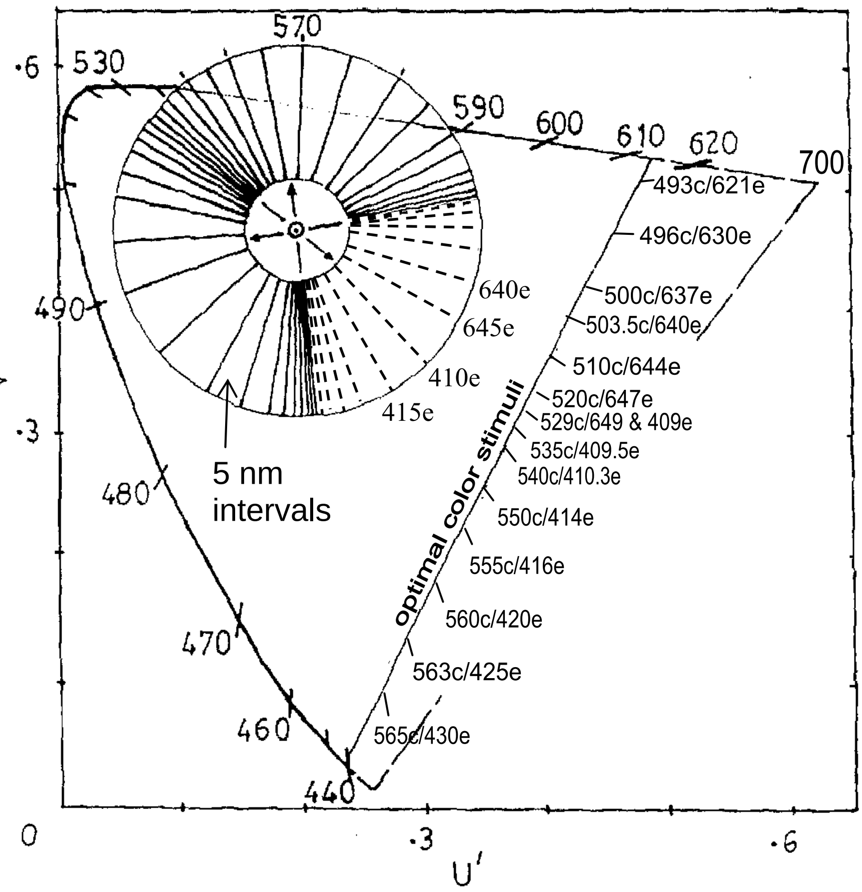


Figure 1. 1976 CIE LUV uniform chromaticity space, showing varying angle of constant 5 nm intervals for the spectral range 440-615 nm (approx limits to monochromatic optimal color stimuli), and illuminant D65. Complementary (opposite) areas of the hue cycle have reciprocal angles per 5 nm interval: RGB hues have small angles while Cyan and Yellow hues have relatively large angles. Nonspectral Magenta has no physical wavelengths but if it had, they would resemble the dotted lines, with large angles as in C and Y areas. These suggest a means of extending dominant wavelength scale over the nonspectrals, in the form of *equivalent wavelength* numbers as labelled (e.g. 410 e, 415 e).

Hence, the hue cycle for optimal aperture color stimuli may be treated as two parts: (1) spectral hues from 442-613 nm, i.e. an interval of 171 nm; and (2) nonspectral hues, whose interval is unknown but can be estimated, for example from color appearance systems, as approximately 29% of the hue cycle (see Appendix A). This gives a hue cycle of some 240 nm including 69 nm *equivalent wavelength* representing the nonspectrals. This nominal wavelength interval for the nonspectrals allows them to be represented together with spectral hues in graphs, as a complete hue cycle in terms of a *relative wavelength metric* in a coherent numerical series. This representation of nonspectrals, as a 69 nm interval representing 29% of the hue cycle, may be an approximation (say ±5 nm uncertainty) but importantly it is a standard to which all nonspectral data may be plotted and compared. Appendix A briefly describes the *relative wavelength metric* and its derivation.

In Fig 1, the Magenta area opposite and complementary to the Green area might be expected to have large hue angles per 5 nm, similar to the Yellow and Cyan areas but, of course, nonspectral hues have no physical wavelength. If they had, they may be expected to look like the dashed lines in Fig 1. The dashed lines represent 5 nm intervals of Appendix A's "equivalent wavelengths" for nonspectrals [from 613 through purple to 442 nm, where 409 and 649 e ("e" for equivalent wavelength) represent the hue cycle ends, complementary to 529 nm]; their spectral complementary wavelengths are also labelled (e.g. 560 c specifies 420 e) so as to identify the "equivalent wavelengths" in CIE colorimetry.

The trimodal (R-C, G-M, B-Y) structure of complementary wavelengths in Fig 1 is illustrated by the three complementary pairs of colors in Fig 2A, with the RGB hues opposite their complementary CMY hues. This figure also demonstrates the complementary structure of *after images* (successive contrast or successive *chromatic induction*). By gazing at Fig 2A for 20-30 seconds, and then shifting the gaze to a sheet of white paper, the after image of each colored triangle will appear as the hue of the opposite (and complementary) triangle in Fig 2A. The after image appears less bright and less saturated and its wavelength may thus evince some hue shift from either or both Bezold-Brucke effect and Abney effect.[15,16] This accounts largely (possibly entirely) for unique yellow's after image (about 475 nm, see Fig 2B and 2C) tending toward reddish blue rather than a pure blue, and for unique blue's after image (about 575 nm) being reddish yellow rather than pure yellow.[7]

For over 150 years, the classic explanation of after images and other forms of chromatic induction such as colored shadows[15,16] has been that they are closely complementary to the inducing stimulus.[7-9] However, in the last few decades at least two (otherwise excellent) color science books[16,17] and several articles (as described in Ref[18]) have attempted to account for chromatic induction (simultaneous and successive contrast) in terms of opponent colors or without any mention of complementary colors. (Both books[16,17] have recently improved this matter in their 2nd editions, mostly ascribing

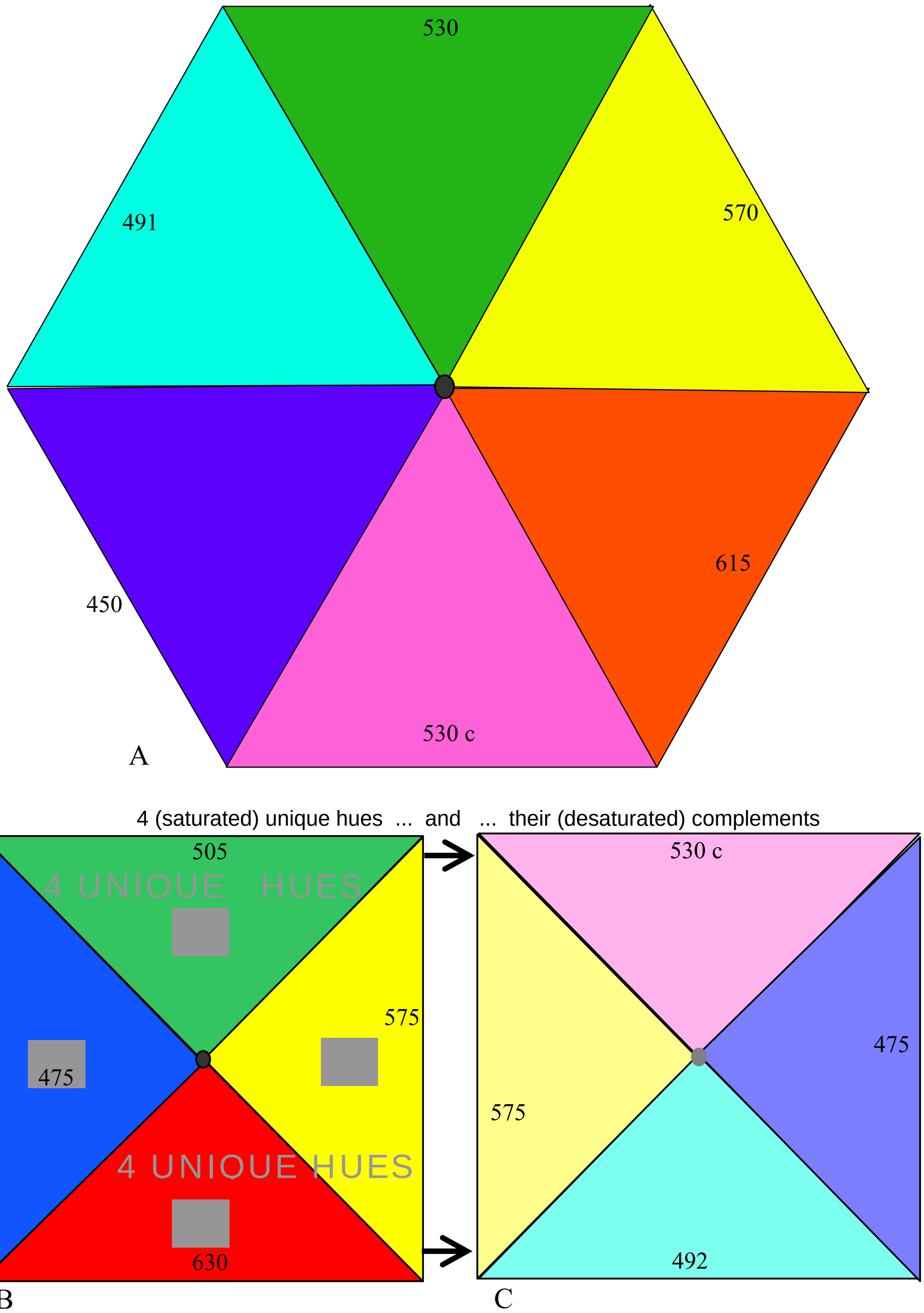


Figure 2. A. Trimodal structure (R-C, G-M, B-Y) of complementary colors: RGB have high *complementary intervals* (CI) ratio (see Fig 3) and low relative radiance (see Fig 5), whereas their complementary CMY hues have low CI ratio and high relative radiance. Approx wavelengths of the CI ratio peaks and troughs for illuminant D65 are shown in black. This figure also illustrates the complementary basis of after images. Gaze at the central black dot for 30 seconds then at white paper to see the (fainter) complementary after images. Fig **B**. Bimodal (B-Y, G-R) opponent color structure. Approximate wavelengths of the 4 unique hues are shown in black. Fig **C**. Shows desaturated complementaries to Fig B. Gazing at black dot in Fig B produces (faint) after images that resemble Fig C (for color normal observers).[7] Note the simultaneous contrast in the gray patches and gray lettering in Fig B.

after images to complementary colors.) Now, unique hues B and Y are not only opponent but also complementary,[18] but unique hues G and R are not complementary. If the reader gazes at Fig 2B for 30 s and then at a white surface, s/he will see after images something like Fig 2C: i.e. the after images to B and Y are Y and B (somewhat desaturated), respectively, but the after images to G and R are certainly not R and G, respectively, but instead are Magenta and Cyan, the complementaries to G and R. Clearly, after images are not opponent colors but complementary colors. Hering himself did not claim chromatic induction, but only light-dark induction, in support of opponent-colors theory. Ref [18] clarifies and reclaims the role of complementary colors in chromatic induction, and the present paper presents complementary-color-based math models of chromatic induction, below.

But why use wavelength rather than hue angle (eg, CIE LUV), or Munsell Hue number, or any other man-made or colorimetric metric which quantifies the hue cycle in one continuous series? Because, first, they are man-made. Second, of all such metrics, none is unquestionably superior. Third, in all cases except NCS,[17] hue angle metrics claim to be uniform hue scales, which is a function this paper aims to model, and cannot do by means of a metric that is already scaled to uniform hue. The required scale should be neutral in regard to color perception. Wavelength is a neutral scale because it reflects physical measurement. Wavelength is a well known measure in physics whilst hue angle (etc) metrics vary between color spaces, are not measured physical quantities, and are not known or used outside color science. There is little doubt that some wavelength-based metric over the whole hue cycle must arise to break the present hindrance to color science research. There are hundreds of published and unpublished data sets on spectral and nonspectral colors waiting to be graphed to a wavelength-based ratio scale over the hue cycle and at last analyzed (e.g. Figs 9-11 below).

Wavelength Dimension: Complementary Intervals Ratio

The trimodality of complementary wavelengths derives from, and may be formalised as, the ratio of a wavelength interval to its complementary interval, shown in Fig 3 (left *y*-axis). This *complementary intervals ratio* (CI ratio) is independent of chromaticity diagram, as previously noted.[19] Only the filled-diamond data points are known, as only these wavelengths have spectral complementaries. The green area’s CI ratios are unknown because their complementaries are nonspectral purples, but the relative CI ratio is inferred by the angular distribution of wavelength described above (Fig 1) and shown by dashed gray line in Fig 3. This function gives valuable assistance by indicating relative CI ratios, for example, peak ratios in RGB and troughs in the complementary CMY areas. If CI ratio is to be employed as the basic structure of complementary colors, independent of man-made color spaces, a wavelength-based metric is needed to overarch the nonspectrals in order to allow the calculation of CI ratio for both green and nonspectral areas. The required metric is described in Appendix A and in Fig 1, summarised from Ref[20].

The structure of complementary wavelength pairs for any illuminant has a center of symmetry defined by the pair of complementary wavelengths of the *minimum complementary interval* (MCI) for the respective illuminant.[21] The MCI pair varies between illuminants, but for illuminant D65 it is 578.2 and 479.8 nm (each with CI ratio 1:1), and the mean (termed *hue cycle midpoint*) is 529 nm.[20,21] Note 529 nm agrees closely with the 530 nm peak (in G) of the dashed gray line in Fig 3, that it is located in mid-spectrum, and that its complementary (529 c) is located in Fig 1 at about the middle of the nonspectral range, where one would expect to locate the hue cycle ends. In the relative wavelength metric,[20] the hue cycle midpoint is also the mean of the cycle ends, so these are each a half-cycle from the

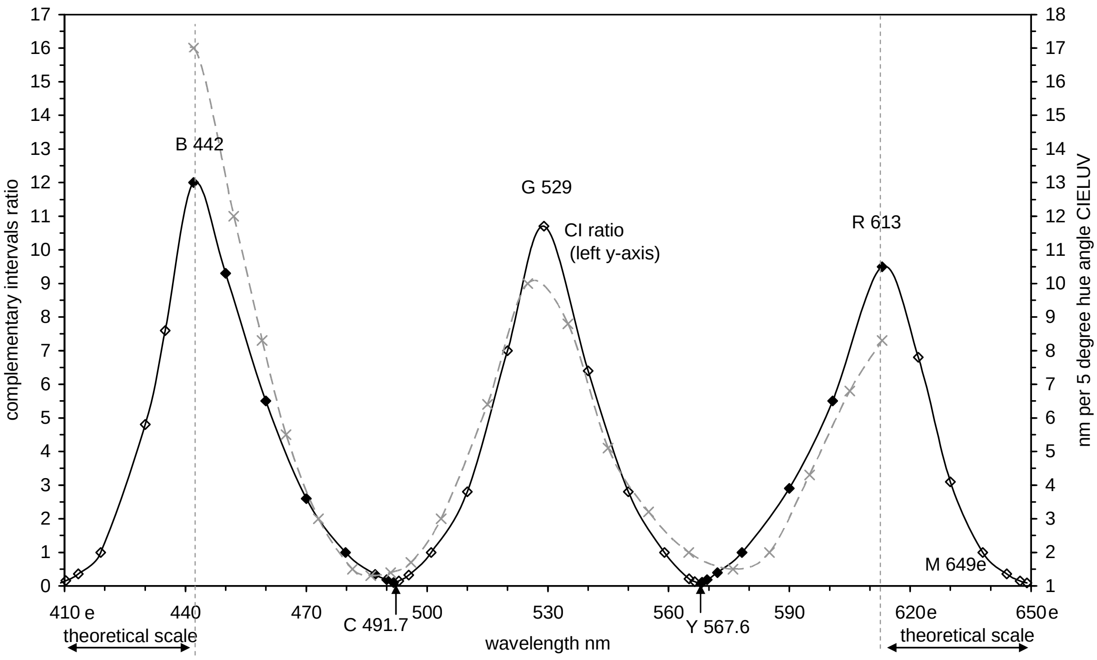


Figure 3. Solid curve (left y-axis): complementary intervals (CI) ratio for illum D65. Solid diamond data points are known; open diamond points are estimates. RGB hues are high CI ratio, and CMY hues are low CI ratio. Dashed gray curve (right y-axis): nm per 5 degree hue angle in the CIE LUV diagram.

midpoint, ie, 120 nm, given the cycle interval of 240 nm/e (Appendix A). Hence the cycle ends are 409 and 649 e, as shown in Fig 3 *x*-axis. From the symmetry of Figs 1 and 3, hue cycle ends are not only equidistant but complementary to the cycle midpoint, and the G peak of CI ratio is about 530 nm, say 529 nm (cycle midpoint), and 10.7 CI ratio (mean of the ratios 12 and 9.5 for B and R peaks).

These details enable a reasonably accurate estimate[20] of the CI ratios of the green and non-spectral curves, shown in Fig 3 (black solid line). Fig 3's *x*-axis shows relative wavelength for the complete hue cycle between hue cycle ends 409 and 649 nm or e (e for "equivalent wavelength"), a 240 nm/e interval. The hue cycle comprises CIE dominant wavelength from 442-613 nm (as optimal monochromatic or near-monochromatic color stimuli) and *equivalent wavelength* for shorter or longer "wavelengths", representing the nonspectrals (as optimal compound color stimuli). To colorimetrically identify the *equivalent wavelengths*, their (spectral) complementary wavelengths must be specified, as described in Ref[20] and listed in Table I (see also Figs 1 and 4B). Table I also lists CI ratios.

Fig 4A illustrates the relative wavelength metric as a *uniform wavelength scale* over the hue cycle (shown here as a geometric circle), including the *equivalent wavelengths* for nonspectrals. In this uniform scale, the only diametrically opposite pairs that are complementary are those with a half-cycle complementary interval (120 nm), ie, 448 & 568 nm, 491.5 & 611.5 nm (both pairs are from CIE data), and 529 nm & 409/649 e. Fig 4B illustrates the *complementary wavelength scale*, where every diametrically opposite pair is a complementary wavelength pair; the distribution of complementary wavelength pairs around the circle is not arbitrary but deduced by methods described in Ref[20] and incorporating the CI ratios listed in Table I and graphed in Fig 3. Some nonspectral *equivalent wavelengths* and their spectral complementary wavelengths are shown in CIE LUV space in Fig 1.

In summary, Fig 4 shows two wavelength scales: a uniform wavelength scale (constant angles give constant wavelength intervals, i.e. 1 degree=1.5 nm) for general use, and a complementary wavelength scale (constant angles give varying wavelength intervals) for special purposes. Though the hue cycle interval, and thus the metric's accuracy, is uncertain (say ±5 nm/e for nonspectrals near the hue cycle ends), the metric is based on logic and reasonable assumptions, and will serve here as a standard *relative wavelength metric* for the hue cycle. Ref[20] gives firther relative wavelength data for other illuminants.

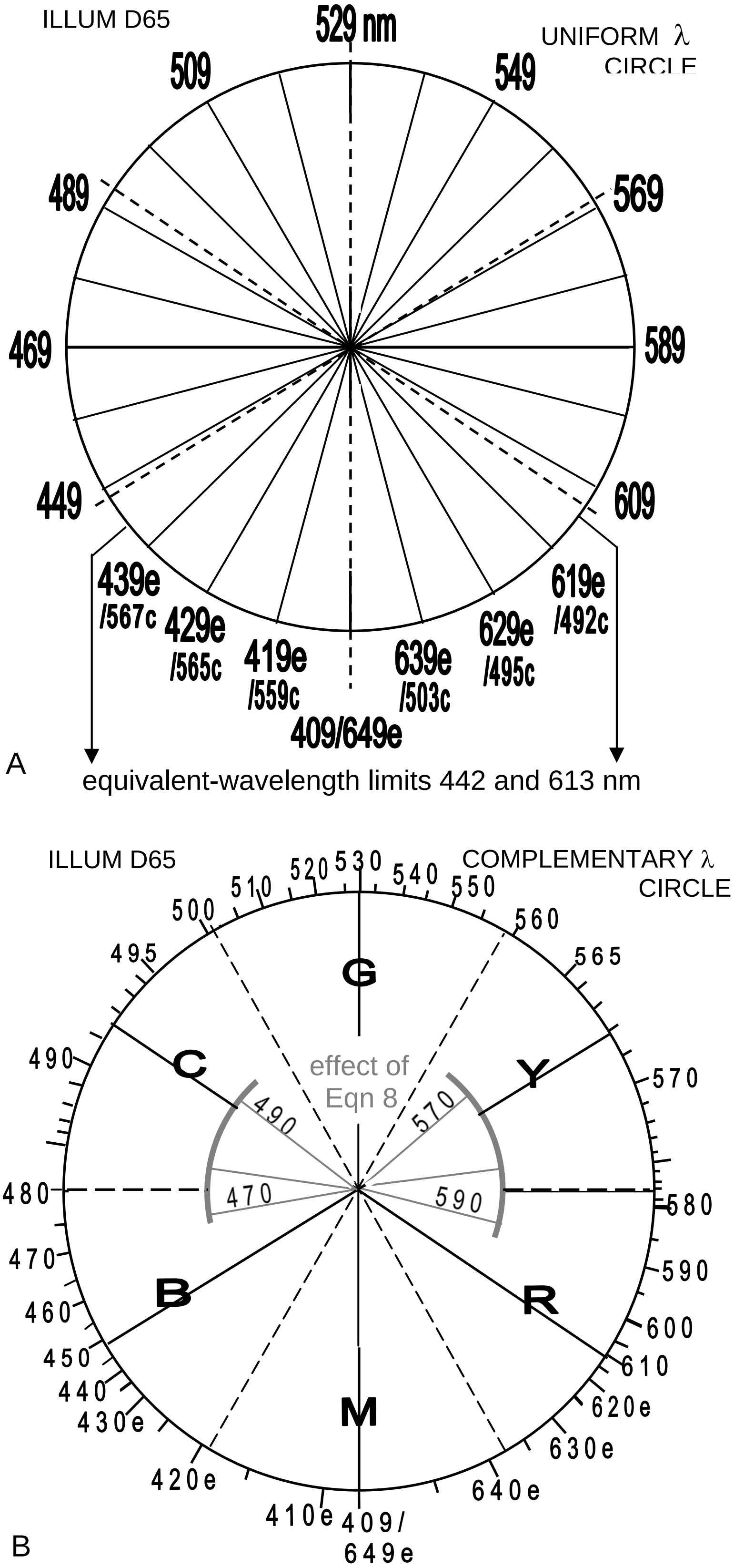


Figure 4. A. Hue circle representing uniform wavelength scale, for a hue cycle of 240 nm/e interval from 409-649 e equivalent wavelength; for Illuminant D65. Hue cycle midpoint 529 nm is top of circle. Dotted lines: the only opposed pairs that are complementary: 448&568 nm, 491.5&611.5 nm, and 529

nm&409/649 e (all half-cycle intervals, 120 nm/e). **B**. Hue circle representing the complementary wavelength scale, from Fig 3 and Ref [20]. Any opposed pair is complementary. 1/2-cycle complementaries 448&568 nm, 491.5&611.5 nm, and 529 nm&409/649 e, are at the same angles/positions as in Fig A. Center: Variation effected by Eqn (8) (see Table I) so as to shift C & Y troughs in Fig 10 from 492 & 568 nm to about 485 & 575 nm as in Munsell, CIE LAB, etc.

Table I. Complementary intervals (CI) ratio for indicated wavelengths (λ) nm or e, for illuminant D65, employed to build models in Figs 4B, 8, 10, 11. Each row shows a pair of complementary wavelengths, with reciprocal CI ratios. Ratio represents 1 nm intervals as either 1:*x* or *x*:1. Unit CI ratios occur at six λs. Max and min CI ratios (shown in bold) occur at three λs and the complementary three λs, respectively. 409 e & 649 e are hue cycle ends (same magenta hue), complementary to 529 nm (hue cycle midpoint). Wavelengths outside the range 442-613 nm are "equivalent λs". Substitute (labeled "subs") ratios are used in Eqn (8) to shift C & Y troughs to 485 and 575 nm for object colors.

λ *nm/e*	*CI ratio*	*subs ratio*	*λc* *nm/e*	*CI ratio*	*subs ratio*
409	**0.094**		**529**	**10.7**	
410.5	0.156		540	6.4	
413.4	0.36		550	2.8	
419	1		559	1	
430	4.8		565.1	0.21	0.5
442	**12**		**567.6**	**0.083**	**0.3**
450	9		568.3	0.11	0.26
460	5.5		569.6	0.18	0.21
470	2.5		572.1	0.4	0.13
476	1.65	1.2	575	0.7	0.09
479.8	1	0.3	578.2	1	0.13
485	0.55	0.1	585.4	2	0.7
487.1	0.35	0.13	590	2.9	1.4
490	0.18	0.18	600.7	5.5	
491.7	**0.1**	**0.2**	**613**	**9.5**	
493	0.143	0.25	622	6.8	
495.5	0.32	0.4	630	3.1	
501	1		638	1	
510	2.8		644	0.36	
520	6.8		647.3	0.143	
529	**10.7**		**649**	**0.094**	

Radiance Dimension: Complementary Powers Ratio

The structure of complementary colors in the wavelength dimension was described above. Here, structure in the radiance (i.e. radiant power, Watts) dimension is described. As MacAdam noted,[1] the luminance of a color relative to its complementary varies over a very wide numerical range, whereas the radiance occupies a small range and is a more symmetrical function. Further, radiance (like wavelength) is a physical quantity, simple to measure accurately, whereas luminance (varying per Watt as a function of wavelength) is a man-made psychophysical metric with problems of nonlinear additivity.

The basic function in the radiance dimension is the power required by a given color to neutralise its complementary color of 1 Watt. This structure is shown in Fig 5 for illuminant D65,[1,5,6] and the

Table II. RGB peaks (wavelength nm) and CMY troughs of Power Ratios (Watts) of monochromatic complementary stimuli (also representing peaks and troughs of spectral sensitivity), with complementary wavelength pairs calculated (as optimally efficient, Watts) rather than found graphically/ approximately from CIE diagram. Data in italics are interpolated (includes all Illum D50). See Figs 6-7.

Illum	*B Peak*	*Y Trough*	*G Peak*	*M Tr'gh*	*R Peak*	*C Trough*
D250	445	561.338	*531.5*	*531.5c*	605	485.566
Watts	0.8966	1.1153	*1.43*	*0.699*	4.239	0.2359
D75	445.25	566.464	*531.5*	*531.5c*	605.75	489.806
Watts	1.4705	0.6800	*1.5*	*0.667*	3.1398	0.3185
D65	445.5	567.837	531.5	531.5c	606	490.877
Watts	1.6714	0.5983	1.531	0.653	2.8992	0.3449
D55	445.5	569.534	531.5	*531.5c*	606.5	492.386
Watts	2.0015	0.4996	*1.55*	*0.645*	2.5865	0.3866
D50	*445.25*	*570.6*	*531.5*	*531.5c*	*606.5*	*493.3*
Watts	*2.27*	*0.4405*	*1.575*	*0.635*	*2.37*	*0.416*
D40	445	573.64	*531.5*	*531.5c*	607	495.953
Watts	3.1369	0.3188	*1.64*	*0.61*	1.921	0.5206
Planckian illuminants:						
C	445	568.42	531.7	531.7c	606	489.86
Watts	1.5421	0.6485	1.633	0.612	2.9138	0.3432
A	445	579.359	532.5	532.5c	608	500.338
Watts	5.7192	0.1748	1.808	0.553	1.2426	0.8048

powers are tabulated in Table II for several illuminants, with improved accuracy relative to previous data.[1,5,6] The Fig 5 curve represents not only the relative power ratios of complementary colors but the *relative radiances* of optimal aperture color stimuli over the hue cycle, such that the admixture of all the colors and their complementaries (of unit W) produces a white, illuminant D65. Evidently RGB colors have low, and CMY colors have high, relative radiances. (By removing the requirement for complementaries at 1W, the square root of the Fig 5 power ratios would provide the same power ratios between complementary color pairs; see Fig 3 in Ref[5], 1978.)

Note the relative radiances (Fig 5) of complementary colors are proportionally reciprocal to the CI ratios. Using the cube root of CI ratios, Eqn (1) gives 1.0 ±0.4 across the hue cycle (1.0 would be exact reciprocity). Where c is CI ratio (Fig 3) and p is power ratio (Fig 5), for illuminant D65:

$$p(c)^{0.333} = 1.0 \pm 0.4 \qquad \text{Eqn (1)}$$

3. SPECTRAL SENSITIVITY AND CHROMATIC ADAPTATION

Imagine the relative radiances of colors in a natural scene uniformly illuminated by daylight, such that any color approximates the appropriate radiance to neutralize its complementary color. For example, blue colors in a scene occur at naturally low radiances relative to yellow colors, which are always relatively high radiances. Otherwise, the “yellow” would appear brown. In contrast, if the blue is a higher than usual radiance, maximum chroma is very much reduced and the scene will lack saturated blues. This may be checked by comparing Munsell chromas for blue hues in Value 4 (where they can be high chroma, e.g. chroma 16) and Value 8 (where chromas for blues are lower than 8 or 10). Hence there is a typical relative radiance for every hue; otherwise, the scene will not appear normal. The Fig 5 curve represents those relative radiances.

The typical relative radiances in Fig 5, varying in radiance across the hue cycle, may be expected to directly relate to human vision’s typical sensitivities to color stimuli over the hue cycle. Blue is normally a low relative radiance so human spectral sensitivity to blue would need to be high, relative to yellow which is normally high relative radiance so spectral sensitivity to it would only need to be low. Hence one would expect that the inverse of the Fig 5 curve would relate to experimental data on spectral sensitivity.

Fig 6 shows the inverted curve, with RGB peaks and CY troughs at wavelengths typical for spectral sensitivity data:[10,22,23,24,25] i.e. 445, 530, 610 nm peaks, and 490, 570 nm troughs. Besides wavelength discrimination, Ref[24] gives spectral sensitivity data for two observers with RGB peaks at 440-460, 530, 600-610 nm, and CY troughs at 480-500 and 570-580 nm, very similar to Ref[23]. Peaks and complementary troughs of these approximate wavelengths are known as primes and antiprimes after Thornton,[22] who argued that spectral sensitivity curves and color-matching functions should have RGB peaks at very similar wavelengths. The curve labeled H&J represents a model of spectral sensitivity.[25] Ref[23] gives sensitivity curves for five observers, two of whom (HL, FM) are shown in Fig 6; the light source was xenon arc. Such individual data typically vary in amplitude, and these two curves vary markedly in the size of the B peak. The color temperature of the light source affects the relative size of the RGB peaks, as shown by the power curve for illuminant D50 (the most balanced power curve of all illuminants). For both illuminants, RGB peaks remain the same wavelengths but B peak rises and R peak falls as color temperature decreases. The variation suggests chromatic adaptation but a greater range of color temperature is needed to demonstrate it.

The Fig 6 model of spectral sensitivity serves also as a model of saturation discrimination

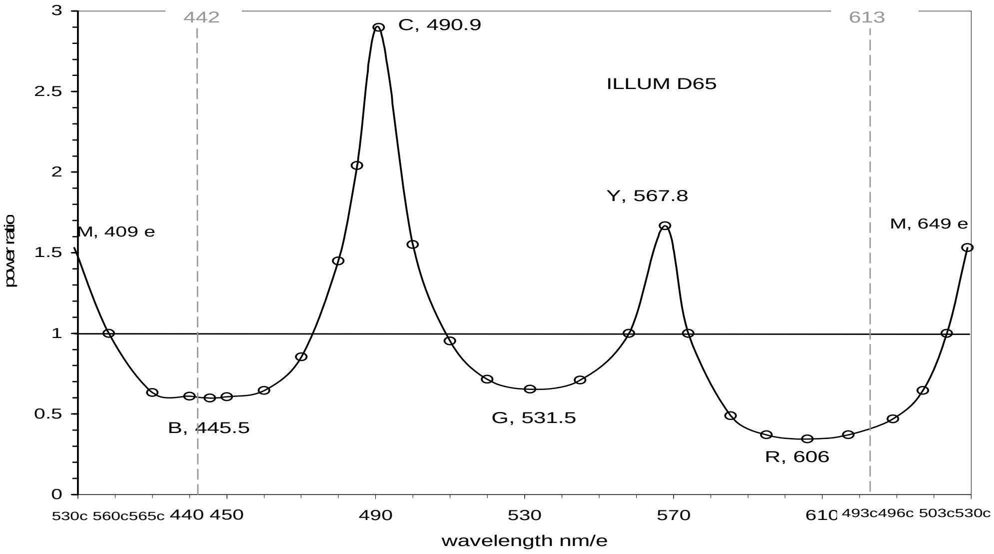


Figure 5. Radiant power required to neutralize unit power of the indicated wavelengths (x-axis), for Illuminant D65. This *complementary powers ratio* is the basic structure of complementary colors in the radiance dimension. See Table II.

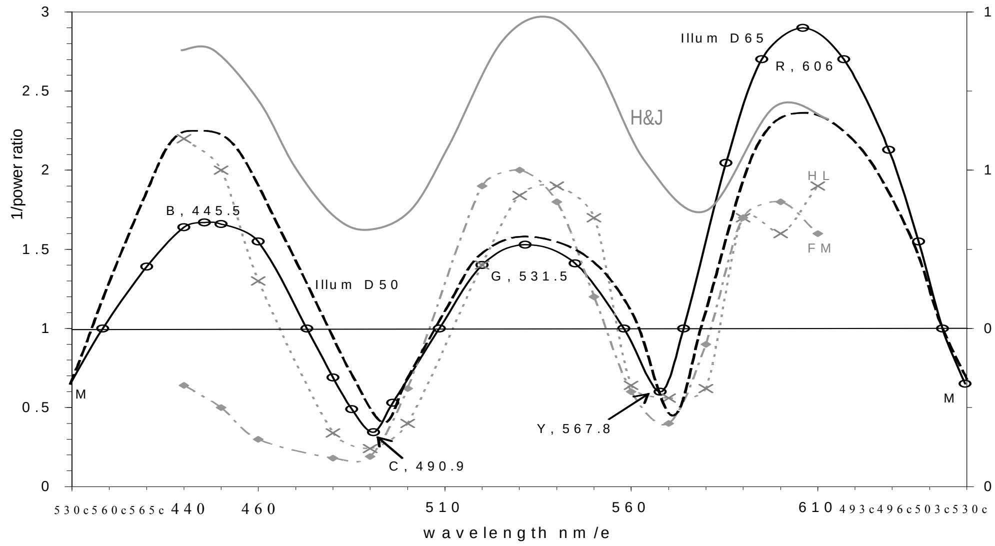


Figure 6. Model of spectral sensitivity for Illum D65 (solid black line, inverse of Fig 5 powers) and Illum D50 (dashed black line). Note RGB peaks and complementary CMY troughs. Gray lines, solid, dotted, and dash-dot: model of spectral sensitivity from Ref[25] labeled H&J, and examples of spectral sensitivity data from Ref[23] for observers HL and FM. The solid black line serves also as the present model of saturation discrimination when measured in radiance units (Fig 12 shows the model for luminance).

The Fig 6 model of spectral sensitivity serves also as a model of saturation discrimination measured in power units, rather than in the usual luminance units, as described in section 7 below.

Fig 7 shows power curves for three very different illuminants: A, D65, and D250. (The latter is at the limit of the CIE recommended range of Daylight illuminants, 4000-25000 K). The color temperatures are 2850 K, 6500 K, 25000 K. Power ratios are listed in Table II. Again, it is clear that B peaks rise and R peaks fall as color temperature K decreases, consistently across the four illuminants shown in Figs 6-7. The CMY troughs vary wavelength widely with illuminant, shifting to longer wavelength as temperature K decreases. However, the RGB peaks remain constant wavelengths 445, 531.5, 606 nm ±2, varying only in size. The structure clearly demonstrates the process of chromatic adaptation, and the spectral-sharpened[10] R peak at 606 nm indicates the process is post-receptoral.

The process of chromatic adaptation is generally thought to be the independent sensitivity regulation of the three cone responsivities, or similar RGB mechanisms, early in the visual process.[2,17] As color temperature decreases, with less power in the short wavelengths and more in the long wavelengths, sensitivity to the former must increase to counter the lack of power there. Responsivity of the RGB mechanisms is thought to rise and fall without changing the peak wavelengths. Recently, it has been accepted the RGB mechanisms are not completely independent,[17] and are probably spectral-sharpened (as in CIECAM97 color appearance model);[26] besides giving improved color constancy,[10] this indicates they are post-receptoral. Such mechanisms have not been identified other than in

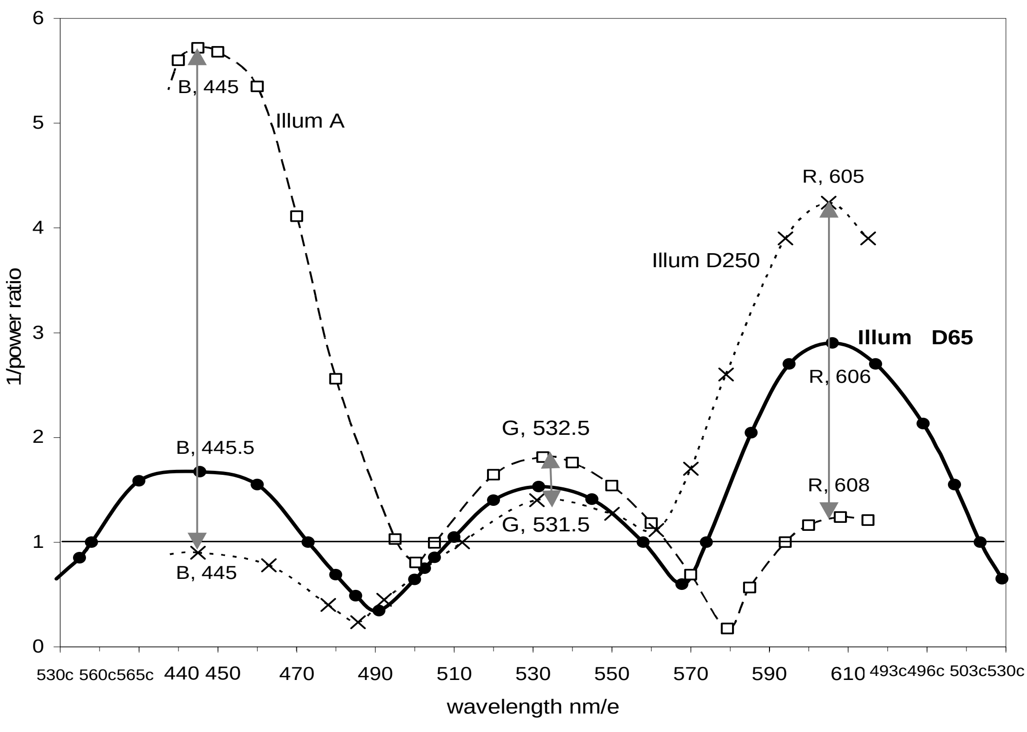


Figure 7. Model of chromatic adaptation. Examples show adaptation of spectral sensitivity to Illuminants D250, D65, A (and D50 in Fig 6). Note RGB peaks are practically constant wavelength, changing only amplitude with illuminant, similar to conventional RGB mechanisms of chromatic adaptation.

chromatic adaptation models, but clearly they must allow for the colorimetric fact that any amplitude change in R, G, or B will mean a reciprocal change in the complementary colors C, M, or Y, because any color has a complementary. That is, the actual mechanisms may be expected to be structured on, or otherwise allow for, complementary colors. The conventional von Kries type RGB mechanisms do not allow for any such colorimetric linkage between complementary peaks and troughs.

The same process of adaptation, and a somewhat similar differentiation of three RGB mechanisms, are evident in Fig 7. It is clearly a chromatic adaptation structure but with an interesting difference. Instead of being three separate RGB curves or mechanisms, Fig 7 structure is a one continuous curve seamlessly joining (at the six nulls, of unit power ratio) three complementary color mechanisms R-C, G-M, B-Y. Does this prohibit it from properly operating chromatic adaptation? Or is it more likely to be an improvement on the conventional von Kries-type of three separate, unconnected, RGB mechanisms? It is worth noting that the three complementary color mechanisms (each a positive curve paired with a separate negative curve, e.g. R-C), though directly linked in colorimetry, operate with some independence. For example, a change of illuminant from D65 to A produces very large changes in the B-Y mechanism, less so in the R-C, and very little in the G-M. Incidentally, the near-constancy of the G-M mechanism in both wavelength and amplitude suggests it contributes to color constancy.

No color or region (e.g. R, G, or B) of the hue cycle can be treated factually as an independent region, since every region has a complementary. It can be said categorically that any radiance or responsivity change will reciprocally affect the complementary region. No region or RGB mechanism can be independent or be treated in isolation. The Fig 5 curve does not just affect a narrow class of colors called complementary colors but corresponds to observed spectral power distribution (SPD) derived from maximum-saturation or Maxwell color matching functions.[2] For example (allowing for its derivation from only one observer and its lack of nonspectral M peak), Wyszecki's SPD curve L(λ) for illuminant D65 closely resembles the Fig 5 curve in its CY peaks and complementary R(G)B troughs [see Fig 5(5.6.5) of Ref[2]]. In summary, (1) the sensitivity curves in Fig 7 demonstrate the central process of chromatic adaptation from one light source to another, and (2) the R-C, G-M, B-Y mechanisms represent a colorimetric improvement to independent RGB von Kries-type mechanisms.

A model of chromatic adaptation should predict spectral sensitivity for any given illuminant. Eqn (2) gives a model of spectral sensitivity derived directly from Fig 6, for a selected illuminant (e.g. D65) and certain experimental parameters.[23] By applying it to other illuminants, Eqn (2) becomes a basic model of chromatic adaptation.

$1/p = 2r$ Eqn (2)

where p is calculated from CIE colorimetry for the respective illuminant,[3] and denotes power ratio (as in y-axis of Fig 5; $1/p$ is shown in Figs 6 and 7), and r denotes spectral sensitivity. The model's simplicity readily allows adjustment of the numerator "2" to suit other parameters.

Fig 7 illustrates how sensitive the complementary color mechanisms R-C, G-M, B-Y are to change of illuminant, and how the mechanisms offer an elegant model of the central mechanism of chromatic adaptation. It is beyond this paper's scope to derive from Fig 7 a specific chromatic adaptation transform.[2,17] There is more to chromatic adaptation than the central structure of the process (Fig 7's mechanism) and it is hoped to treat, and test, the subject further in a later paper.

It is worth noting that, in complementing the hue cycle, RGB hues have high CI ratios (Fig 3) and low power ratios (Fig 5). Low ratio implies greater leverage. Hence RGB colors have greater leverage in the radiance dimension, easily neutralizing CMY colors, whereas CMY colors have greater leverage in the wavelength dimension, complementing larger wavelength intervals. Relative power ratio corresponds roughly to saturation; higher saturation (e.g. RGB) gives lower power ratio (more leverage), while lower saturation (e.g. CMY) gives higher power ratio (less leverage in the radiance dimension but more leverage in wavelength). Hence RGB and CMY hues are opposed, in appearance and psychophysical properties, but counter-balanced in their abilities to neutralize each other.

4. CHROMATIC INDUCTION

This section gives two basic models, based on complementary colors, for chromatic induction. As mentioned in the Introduction, in the past 50 years complementary colors appear to have been replaced (wrongly) by opponent colors as accounting for chromatic induction.[18] Hering claimed light-dark induction, but never chromatic induction, was based on opponent-color processes. However (as described in Ref[18]), Jameson and Hurvich published a study[27] which concluded chromatic induction is an opponent color process, without consideration of or comparison with other options (such as complementary colors) which might give a more accurate account of chromatic induction, and without mentioning *any* of the many previous studies on chromatic induction, all of which (for over 100 years) had reported chromatic induction was effectively a complementary color process.[7-9] Ref[18] further discusses inadequacies in the theory of "opponent chromatic induction."

A general model of chromatic induction is CI ratio as graphed in Fig 3, and illustrated in color by the Fig 2A color-schematic of chromatic induction. In Fig 2A, CI ratios >1.0 represent RGB colors, and CI ratios <1.0 represent the opposite and complementary CMY colors, induced by RGB focal stimuli.

Concerning hue shift of the induced (and desaturated) color, consider unique Yellow (575 nm) and its induced after image (Blue) in Figs 2B and 2C. Note that the induced color (Blue, about 475 nm) is the complementary wavelength of the focal color (Yellow, 575 nm) but its typically low saturation, relative to the unique Blue shown in Fig 2B, has resulted in a hue shift towards reddish-blue, as predicted by the Abney effect.[2,15] This provides a novel and satisfactory account for the commonly made observation[7] that the after image is only approximately complementary to the focal (or inducing) color; in fact, the after image is closely complementary in terms of wavelength, if the Abney (and/or Bezold-Brucke if applicable) hue shift is allowed for.

Eqns (3) and (4) give basic models of chromatic induction. They are simpler (but not necessarily better) than previous models, and their purpose here is not only to predict chromatic induction but also to demonstrate that chromatic induction is a complementary color rather than an opponent color process. Eqns (3) and (4) express in novel terms the direction of hue shifts demonstrated in experimental data over the past 150 years including recent data.[18,28,29] Eqn (3) is a *math* model whilst Fig 2A above is a *color* model (showing the R-C, G-M, B-Y, trimodality of chromatic induction). Eqn (3) applies to *successive contrast* as in after images viewed on achromatic fields, or *simultaneous contrast* as in achromatic samples viewed in highly chromatic inducing surrounds. Where *a* is the hue of the after image to a color stimulus *s*, and *sc* is the complementary hue to stimulus *s*:

$a = sc$ Eqn (3)

$s_i = s\text{-}b$ Eqn (4)

Eqn (4) applies to simultaneous contrast where a color stimulus *s* has a chromatic inducing surround color *b*, and where s_i is the perceived induced color resulting from the induction. The negative sign in *s*-*b* denotes that the color *b* is subtracted (to some degree) from *s*, that is, *s* moves in the direction of the complement to *b*. Two examples: (1): where *s* is a yellow, and surround *b* is a red, *s* moves towards cyan (*b*'s complement) so the resultant s_i appears slightly less saturated. Example (2): as (1) but *s* is a red, similar to but lighter than red surround *b*; again, *s* moves towards cyan (*b*'s complement) so s_i appears much less saturated; because inducing field *b* has greater effect the closer it is to the color of *s*.[18,28,29] (Ref [28] gives a much more refined, though complex, model than Eqn 4.) Because after image *a* is perceived as less bright and saturated than *s*, the color appearance of *a* may be subject to Bezold-Brucke and/or Abney effect hue shifts[14-17] (Ref[15] illustrates each effect and the combined effect).

The after image of Fig 2A agrees with Eqn (3); i.e. the after images are complementary colors, but not opponent-colors (other than the B-Y pair which alone is both an opponent pair and a complementary pair). Fig 2B shows the opponent colors bimodal system of B-Y and G-R orthogonal pairs. If their after images were truly opponent colors, they would be the same hues (allowing for desaturation) but reversed as Y-B and R-G. The Y-B is correct (because this pair is also complementary) but the R-G is not. As readers may check, the after images to G-R do not resemble R-G but are Magenta-Cyan (M-C), as shown at Fig 2C. Again, the after images agree with the Eqn (3) model. As color-normal readers may agree, one cannot properly say red and green are mutual after images.

The gray square patches in Fig 2C, and the gray lettering "4 UNIQUE HUES", are identical gray samples but appear to be (a) darker or lighter than each other, exemplifying light-dark induction, and (b) slightly chromatic. E.g. the gray areas on the cyan triangle appear reddish, and the gray areas on the magenta triangle appear greenish. This exemplifies simultaneous chromatic induction. Chromatic induction is well covered by several references in Agoston's excellent book.[8] Pridmore's recent mini-review of effects[15] includes the effect of varying wavelength on proximal hue (e.g. chromatic induction or simultaneous contrast).

5. WAVELENGTH DISCRIMINATION

Wavelength discrimination will be argued in section 6 Uniform Hue to be the basis of uniform hue difference. Wavelength discrimination in most studies[2] involves judging just-noticeable-differences (JNDs) in hue. Given two halves of a visual field, the observer is required to adjust the wavelength of one half field until a JND (in hue) is perceived. Hence we are really recording hue discrimination (and a degree of saturation discrimination),[25] measured in terms of wavelength (nanometers) per JND. Although the post-receptoral physiology can calculate wavelength from two (or more) cone responsivities,[30] the naive human observer cannot discriminate wavelength by appearance alone, e.g. as short, medium, or long wavelength. Only its color attributes, hue and saturation, can be discriminated. Interestingly, this contrasts with radiant power whose increase or decrease is easily discriminated since it is (nonlinearly) proportional to brightness.

Five black curves in Fig 8 represent experimental data from the indicated studies.[31,32,33,34,35,36] (Refs[34,35] cover different halves of the spectrum so they count here as one study.) The curve labeled "Judd" represents the mean of seven studies involving 20 observers.[33] The *grand mean* (heavy black

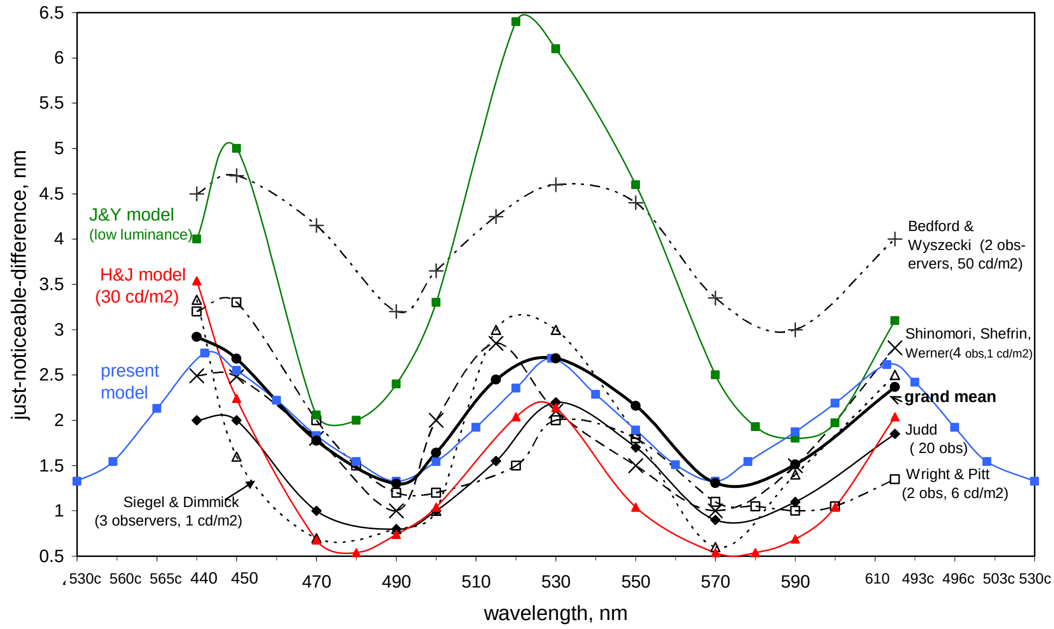


Figure 8. Wavelength discrimination data. Black curves: data from five indicated studies. Judd[33] curve is mean of 7 studies, 20 observers. Heavy black line shows the grand mean. Blue line: present model

of wavelength discrimination. Red line (displaced up 2.5 nm for clarity): Hurvich & Jameson model[25] for mid-luminance, 30 cd/m^2. Green line: Judd & Yonemura model[37] for low luminance, 1 cd/m^2.

curve) is the mean of all these studies, with equal weighting except the Judd study is given twice weight. Eqn (5) expresses a theoretical model (blue line in Fig 8) of wavelength discrimination (largely *hue discrimination)* derived directly from CI ratio. Unlike other models, this eqn allows inclusion of nonspectrals due to the equivalent wavelength metric (Appendix A).

$w = (g+1)^{0.36}+0.26$ Eqn (5)

where w denotes wavelength discrimination in terms of JND (nm) for low-moderate luminance, and g denotes the complementary intervals (CI) ratio. The model agrees well with the grand mean in Fig 8, for which it was designed, and is easily adapted to different luminances or JNDs. To calculate just-noticeable hue difference (Δh) use w in Eqn (5). For a given wavelength, look up in Table I the CI ratio and apply it as g in Eqn (5). Example: find Δh for an orange hue of dominant wavelength 595 nm. CI ratio (g) is 4.0 (interpolated from Table I); hence Δh (or w) = 1.78+0.26 = 2.04 nm.

The red curve in Fig 8 shows a model by Hurvich & Jameson[25] for relatively high luminance (30 cd/m^2); its shape is similar to the present model. The green curve, a model by Judd & Yonemura[37] for low luminance, shows a different shape (possibly due to mesopic response and/or rod intrusion). Its shorter wavelength G peak (about 520 nm) and exaggerated amplitudes are normal for low luminances (<1 cd/m^2).[25] Both models[25,37] resemble the present model's RGB peaks and CY troughs except J&Y's C and Y troughs are 10-20 nm shorter or longer, [37] respectively, than the H&J and present models.

This section has described a model of wavelength discrimination theoretically based on the ability of wavelengths in certain hue areas (e.g. C and Y) to complement relatively larger wavelength intervals (CI ratio, section 1 above). In areas with low CI ratios (e.g. C and Y), wavelength and hue discrimination are very good, and in areas with higher CI ratios (e.g. RGB hues), wavelength and hue discrimination are relatively poor. Hurvich & Jameson[25] draw the same deductions on hue discriminability but explain the mechanism in terms of hue coefficients, which shift rapidly in the C and Y hues. Their explanation and extraordinary insights are supported and amplified by the present account. Their model, which is based on opponent color chromatic responses together with saturation and hue coefficients,[25] is considerably more complex than the present model (Eqn 5, based on CI ratios in Table I) for two reasons: (1) chromatic induction is in the direction of the complementary (rather than opponent) color to the focal color;[7-9,18] and (2) CI ratio is a trimodal function (R-C, G-M, B-Y) as is chromatic induction whereas opponent colors are a quite different, bimodal, function (B-Y and G-R).

6. UNIFORM HUE

Given a model of wavelength discrimination, such as in Fig 8 above, one can predict which hue areas of the hue cycle are good or poor for hue discrimination. CMY areas indicate much smaller JNDs (measured in wavelength nm) than RGB areas and thus better discrimination of hue. If a wavelength metric is used to scale uniform hue difference over a hue circle, wavelength (say, as 5 nm intervals) will be compressed in small angles where hue discrimination is poor (e.g. RGB hues) and expanded where it is good (i.e. CMY hues). The compression/expansion should derive exactly from hue (or wavelength) discrimination. Therefore the above model of wavelength discrimination (Fig 8, blue curve) represents, in principle, a uniform hue scale. The model (Eqn 5) derives directly from CI ratio, so the Fig 4B hue circle in theory may represent a wavelength scale of uniform hue. In other words, wavelength discrimination (in terms of nm per JND) is proportional to uniform hue difference (in terms of nm per hue angle). Hence the general form of Eqn (5), as Eqn (6) below, should also serve to formulate uniform hue.

$u = g^{x} + k$ Eqn (6)

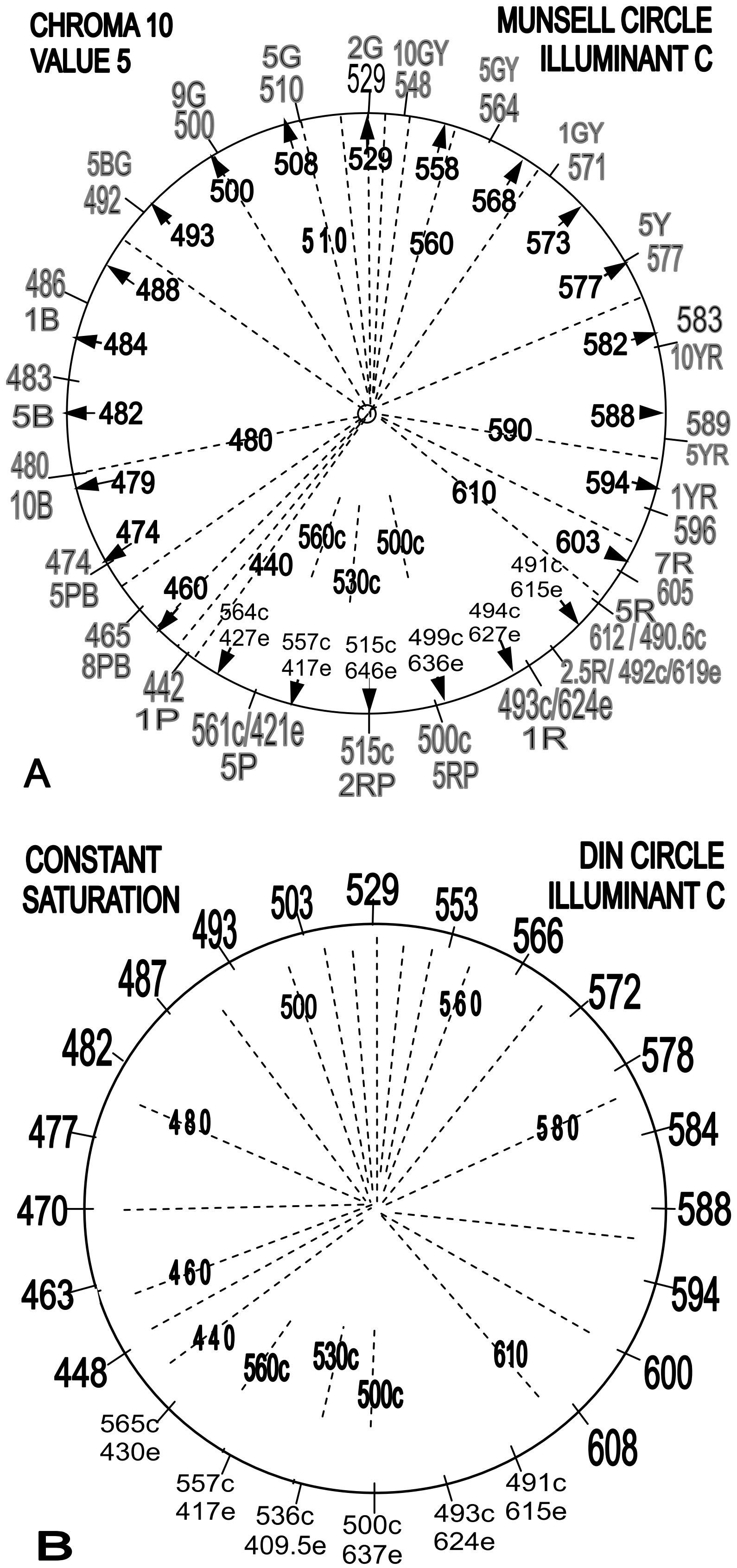
where u denotes uniform hue difference as wavelength nm per constant hue angle, and g (as in Eqn 5) denotes CI ratio; k and exponent x are constants dependent on the constant hue angle and other parameters.

A possible complication arises because wavelength discrimination data represent the perception of not only hue difference but (to some lesser degree) saturation difference,[25] and the latter may be a different curve from hue discrimination. Hence the typical curve of the wavelength discrimination data may represent not purely hue discrimination but some degree of saturation discrimination. However, the present model (Eqn 5) is independent of the possible contamination since it does not derive from wavelength discrimination data but directly from CI ratios, proposed as the structural basis of wavelength discrimination. If the theory is correct, as it seems from the agreement of model (Eqn 5) with data (Fig 8), then the model (as wavelength discrimination JNDs) may be expected to be proportional to uniform hue difference (as Eqn 6).

In section 2 above, the derivation of a wavelength-based metric led to two scales: a uniform wavelength metric (Fig 4A), and a complementary wavelength metric illustrated in Fig 4B as a hue circle. It has been argued previously, for example by Ostwald,[8] that a uniform hue scale requires complementary wavelength distribution, but others (e.g. Kuehni,[16] and Rood[8,16]) have disagreed and/or pointed out that an infinite number of wavelength distributions around a circle can assure complemen tary pairs as opposites, with no indication of which distribution is best. However, the distribution in Fig 4B reflects several novel guides to the correct distribution as described in Ref.[20] This distribution is compared below with several color appearance systems or UCSs and used as a basic (but not final) model of uniform hue difference.

At Fig 9 are five color appearance systems or UCSs: Munsell (full name Munsell Re-notations), the German system DIN,[2,8] the American OSA-UCS,[2,8] CIELAB,[2,4] and Nickerson et al[38,39] (developed by an OSA committee; see also Kuehni[40]). Each hue circle in Fig 9 is divided into 24 equal intervals of 30°, adopted here from the standard commenced by Ostwald[8] and continued in DIN (Fig 9B). Each hue cycle is normalised here by starting with Hue 1 at 529 nm (i.e. the *hue cycle midpoint*, section 2) at top of the circle. Each of the 24 hues is labeled with its wavelength, and dotted radial lines show wavelength at interpolated 10 nm intervals to indicate the wavelength distribution pattern. The OSA uniform chromaticity space (UCS) refers to CIE 1964 (10 deg field), so Fig 9C shows the CIE 1964 wavelengths and, in parentheses, the corresponding CIE 1931 wavelengths. (These are used for Figs 10-11 below, and were converted by overlaying the CIE 1931 x, y chromaticity diagram on the CIE 1964 x_{10}, y_{10} diagram [the coordinates but not the spectrum locus are identical] and finding the CIE 1931 dominant wavelengths that correspond to respective CIE 1964 wavelengths; the former are approximately, from 450-580 nm, some 6 nm longer than the latter).

The Nickerson et al. system reflects substantial research, obtaining almost 57,000 observations (including 14,484 uniform hue observations gained by S.M.Newhall from 102 observers). Yet it was never adopted by Munsell or OSA, when OSA decided to take another direction (towards producing what became OSA-UCS). Wavelength values for 40 and (thence 24) equal intervals shown in Fig 9D derive from CIE $x, y,$ chromaticity coordinates that Kuehni[39] digitised from Judd's paper.[38]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MUNSELL CIRCLE
ILLUMINANT C
5G 510
2G 529
10GY 548
5GY 564
1GY 571
5Y 577
583 10YR
589 5YR
1YR 596
7R 605
5R
612 / 490.6c
2.5R/ 492c/619e
493c/624e 1R
500c 5RP
515c 2RP
561c/421e 5P
442 1P
465 8PB
474 5PB
480 10B
483
5B
486 1B
5BG 492
9G 500
508
529
558
568
573
577
582
588
594
603
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493
488
484
482
479
474
460
510
560
590
480
610
440
560c
530c
500c
491c 615e
494c 627e
499c 636e
515c 646e
557c 417e
564c 427e
A
CONSTANT
SATURATION
DIN CIRCLE
ILLUMINANT C
529
553
566
572
578
584
588
594
600
608
503
493
487
482
477
470
463
448
500
560
480
580
460
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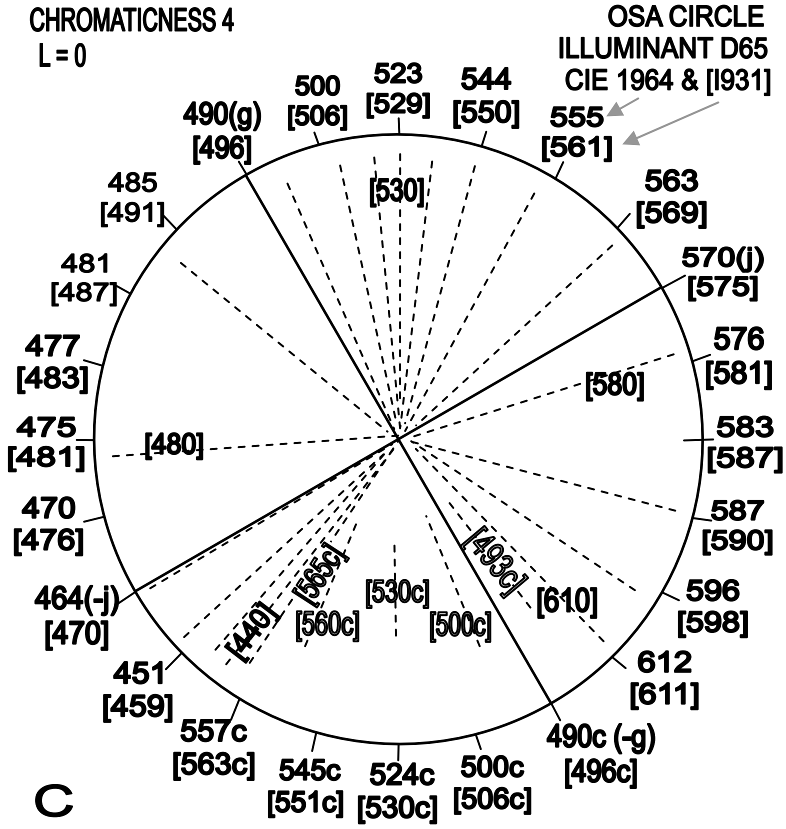
536c 409.5e
500c 637e
493c 624e
491c 615e
B


CHROMATICNESS 4
L = 0
OSA CIRCLE
ILLUMINANT D65
CIE 1964 & [I931]
523 [529]
544 [550]
500 [506]
490(g) [496]
555 [561]
563 [569]
485 [491]
570(j) [575]
481 [487]
576 [581]
477 [483]
583 [587]
475 [481]
587 [590]
470 [476]
596 [598]
464(-j) [470]
612 [611]
451 [459]
490c (-g) [496c]
557c [563c]
545c [551c]
524c [530c]
500c [506c]
[530]
[580]
[480]
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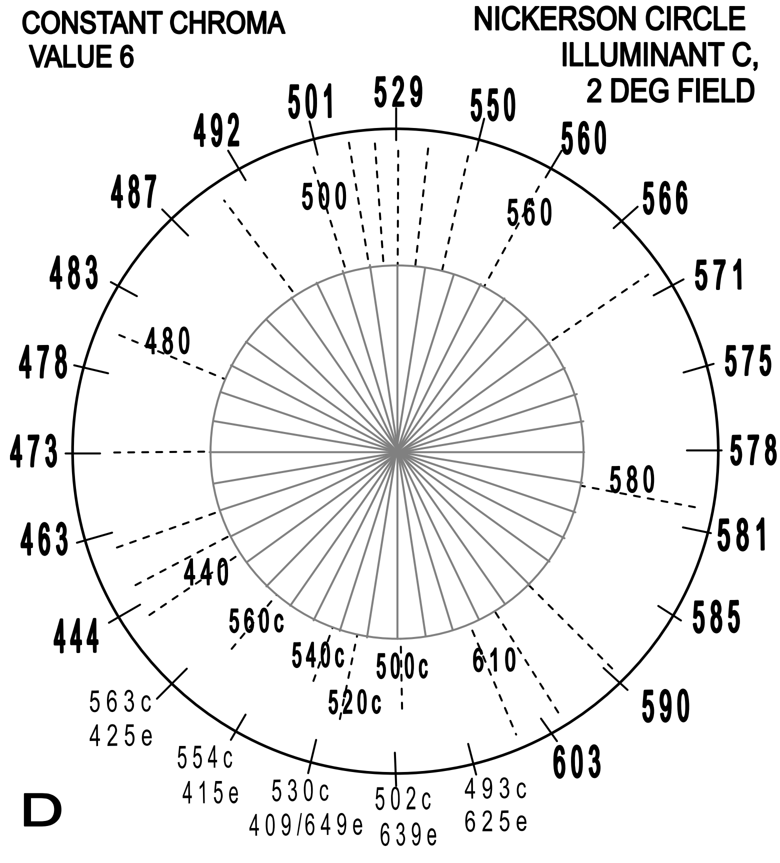
[530c]
[610]
[440]
[560c]
[500c]
C


CONSTANT CHROMA
VALUE 6
NICKERSON CIRCLE
ILLUMINANT C,
2 DEG FIELD
529
501
550
492
560
487
566
483
571
478
575
473
578
463
581
444
585
590
603
500
560
480
580
440
560c
540c
500c
610
520c
563c 425e
554c 415e
530c 409/649e
502c 639e
493c 625e
D

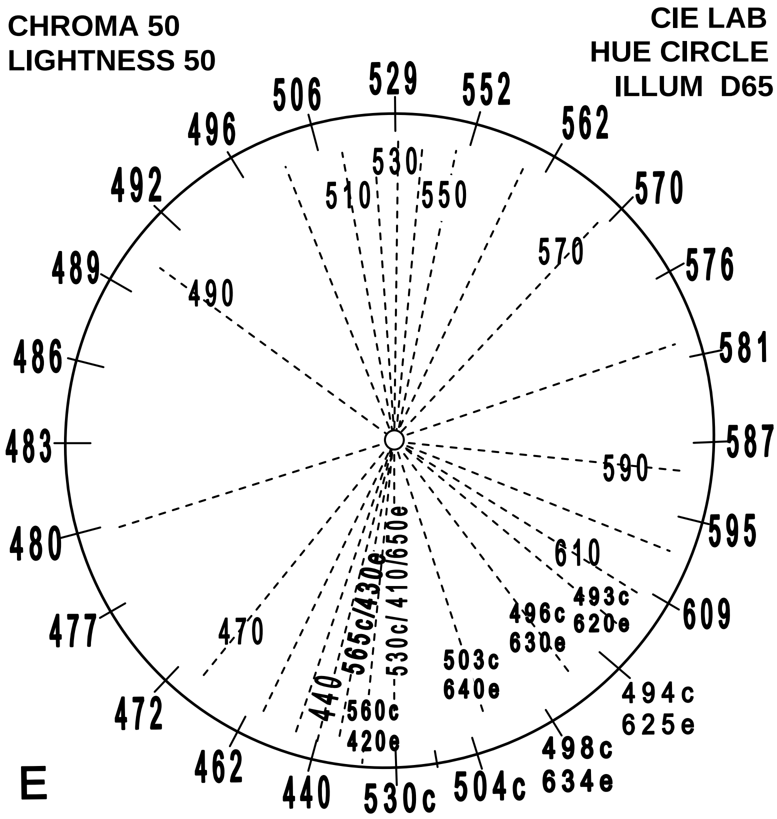


Figure 9. Uniform hue circles for five color order systems, showing dominant wavelengths (for Illuminant D65 or C) at 24 uniform-hue intervals, normalised at 529 nm for Hue 1; nonspectrals are shown by complementary wavelength and also, in most cases, *equivalent wavelength* “e” for Illum C (same ±1nm/e as for Illum D65). **Fig A**. Munsell Hue circle for constant chroma 10 and Value 5. Besides the wavelengths of the 24 uniform hues (arrows), dotted lines show wavelengths at 10 nm (or c) intervals. Some representative Munsell Hues and their wavelengths are labelled in gray. **Fig B**. DIN hue circle, for constant saturation and any lightness level. **Fig C**. OSA-UCS hue circle [1964 CIE 10° observer, wavelengths converted (in parentheses) to CIE 1931 observer] for lightness L=0, and constant chromaticness 4, i.e. j=±4, g=±4. **Fig D**. Nickerson et al[38,39] for constant chroma and Value 6; central area shows the original 40 uniform hues. Note the large angle (80°) for yellows 570-590 nm. **Fig E.** CIE LAB for chroma 50, lightness 50. Note the very large angle (110°) for Blue and Cyan hues 470-490 nm, and the compression of all nonspectrals to 74° (only 21% of hue cycle).

Fig 10 shows the wavelength interval per 5 degree hue angle (using the *equivalent wavelength* scale to represent nonspectrals), derived from Fig 9. Fig 10 also shows CIE LUV data (Fig 3) and the wavelength distribution in Fig 4B proposed here as the *base model* (dotted gray line) of uniform hue difference, which will be the basis for the final model below. The base model has similar curves (RGB peaks and CMY troughs), with similar amplitudes, to the color appearance systems in Fig 10 but the base model's C and Y troughs at 492 and 568 nm are at least 5 nm different from most data (troughs about 485 and 575 nm). The reason is uncertain but the difference is significant and needs correction (below). Note the too-small Munsell R peak at 605 nm and lack of a definite M trough, and that the CIE LAB M trough and B peak are both shorter wavelength than most data.

Given constant 5° hue angle, Eqn (6) can be particularized as Eqn (7) below, which closely describes the base model in Fig 10.

$u = g^{0.78} + g^{0.36} + 0.65$ Eqn (7) and (8)

where u denotes uniform hue difference as nm per 5° hue angle, and g denotes CI ratio (Table I). If u is required for some wavelength, look up its CI ratio (g) in Table I and apply it to Eqn (7). For example, 595 nm (orange) has a CI ratio of 4.0 (Table I, interpolated), giving u of 5.28 nm per 5° angle.

Eqn (7) is simple and accurately predicts uniform hue in agreement with data *except* the C and Y troughs, whose correction is the purpose of the final model, expressed as Eqn (8) above. This is identical to Eqn (7) except for two areas 470-495 and 565-595 nm (around the C and Y troughs), where it uses a variation of CI ratios termed *substitute* ratios (see Table I) in order to shift the C and Y troughs to 485 and 575 nm, similar to Munsell, Nickerson, and CIE LAB. To clarify the models: the base model (derived graphically from Fig 4B, the complementary wavelength scale) is the gray line in Fig10. Its math model is Eqn (7) when a formula is required. The final model is Eqn (8), the blue line

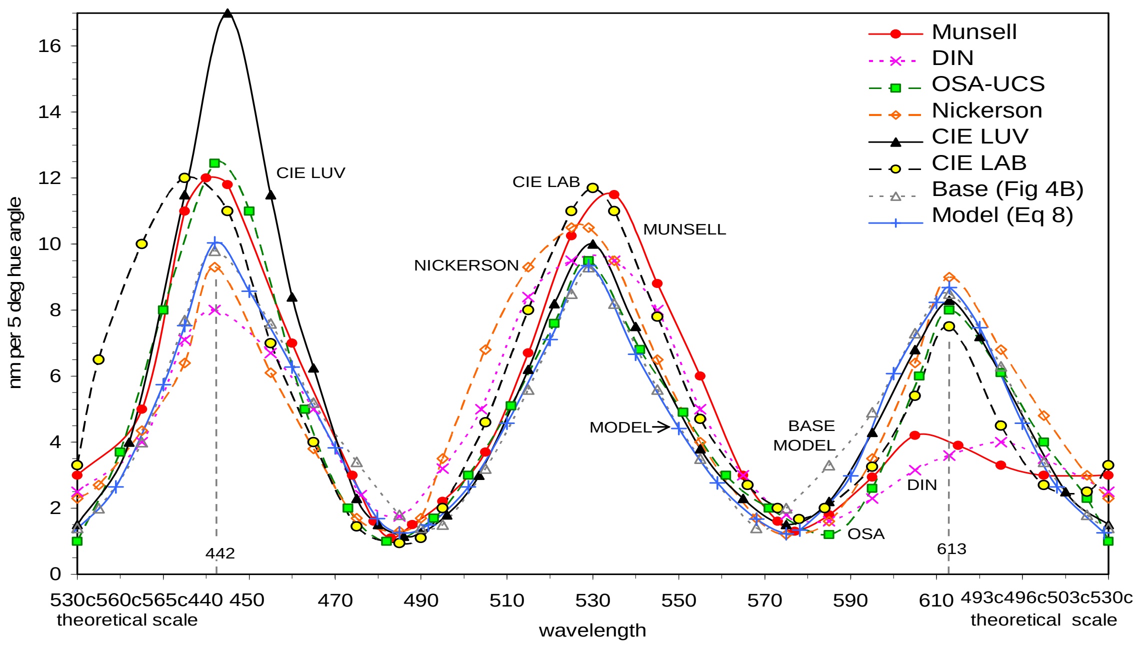


Figure 10. Distribution of wavelength per constant 5° hue angle for 6 color order systems or UCSs

(Munsell, DIN, OSA-UCS, Nickerson,[38] CIE LUV, CIE LAB) plus a base model of uniform hue (gray line, diamonds) representing Fig 4B, and a final model (Eqn 8, blue line, crosses).

in Fig 10. The use of *substitute ratios* makes Eqn (8) less than strictly complementary, and since this model best predicts the data, it indicates uniform hue is basically *but not strictly* complementary.

Fig 11 gives a novel perspective of uniform hue cycles, which is worth detailed description as it provides a new method of analysing uniform hue and wavelength distribution in hue circles, and clarifies how uniform hue derives basically from wavelength discrimination. Fig 11 graphs uniform hue numbers 1-24 to their respective wavelengths, in linear *x*- and *y*-axis scales. Each axis represents one complete hue cycle. Eight hue cycles are represented, comprising Munsell, DIN, Nickerson, OSA-UCS, CIELUV, CIE LAB, the theoretical base model (Fig 4B or Eqn 7), and the empirical final model (Eqn 8). Table III lists each system's 24 uniform hues by wavelength. Notably, the eight curves are generally and similarly trimodal over the hue cycle, and largely remain within the two gray diagonal lines. The trimodal curvature is necessitated by wavelength discrimination: Around C and Y hues (480-495 nm and 565-580 nm), where wavelength discrimination is best (smallest JNDs in Fig 8), the curve may be expected to be nearest to horizontal, and nearest to vertical at RGB where wavelength is poorly discriminated and thus hue difference per wavelength is least. The same relative degrees of discrimination for C,Y, and R, G, B, areas have been reported by Hurvich & Jameson on the basis of hue coefficients.[25] On this basis, Fig 11 labels three expected curve peaks (A, B, C) and three expected troughs (D, E, F). These may be seen to be in general agreement with the data curves.

Hue number is a uniform-interval series but is a non-linear representation of wavelength (see Fig 9), compressed in places (RGB) and expanded in others (CMY). If it were not so, but represented wavelength linearly, the curves in Fig 11 would be straight diagonal lines because the axes $x=y$. The trimodal curvature occurs only to the degree that one axis (hue angle or number on the *x*-axis) represents compressed wavelength in places and expanded wavelength in others. The fundamental reason for the RGB trimodality is wavelength discrimination, as already described.

The Nickerson system is over-extended in the yellow/orange hues (i.e. these occupy too big an angle in Fig 9D), where its curve is too flat, and too compressed (i.e. curve is too steep) in the reds and purples. The over-extended trough covers what should be a trough (labeled F) and a peak (C). Munsell similarly is eccentric in the purple and blue hues where its overly-extended peak covers what should

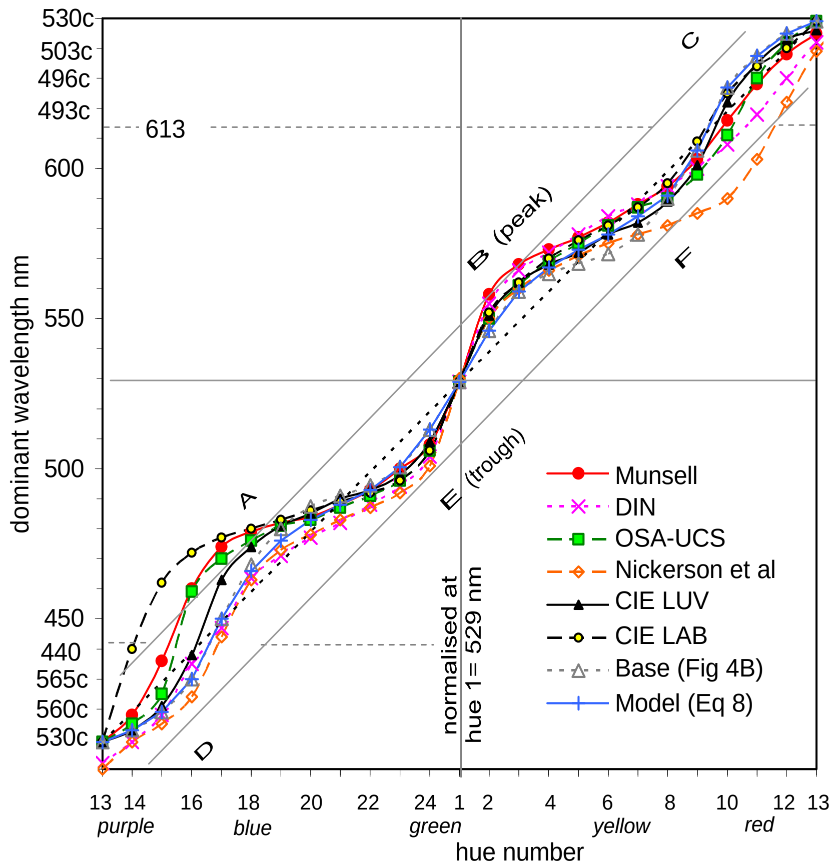


Figure 11. Uniform hue circles for 6 color order systems or UCSs (DIN, Munsell, Nickerson, OSA-UCS, CIE LUV, CIE LAB), a base model, and a final model (Eqn 8), each spaced at 24 uniform hues (see Table III) numbered on the *x*-axis, showing respective dominant wavelengths on the *y*-axis. Each hue circle is normalised at 529 nm for Hue 1. Outside 442-613 nm (see dashed gray horizontal lines), the scale is theoretical (*equivalent wavelength*). Note the different eccentricities of Nickerson, CIE LAB, Munsell, and (to less degree) OSA-UCS (hue angle over-extended in Cyan).

be a trough (D) and a peak (A), and then fails to make a peak at C; in this C area, the Munsell system indicates wavelength discrimination is fairly uniform (45^{o} near-straight line) through reds and magenta (as does DIN). In contrast, OSA, CIE LAB, and CIE LUV indicate discrimination is better for M than R hues (as Fig 8's wavelength discrimination model predicts for nonspectrals). OSA-UCS makes all six peaks and troughs though overdone in A and somewhat underdone in C and D. In the blues around curve A, the CIE LAB curve is even more overdone than Munsell but forms a good peak in the reds at peak C. Table III shows the CIE LAB blue hues from 440-480 nm are lower hue numbers than all other systems, due to being pushed down towards the nonspectrals (see Fig 9E) by hues 470-495 nm being overly extended, occupying 109^{0} or 30% of the total hue circle. In contrast, in Nickerson's circle, the *yellowish* hues 560-590 nm occupy almost 30% of the circle.

Of all the systems in Fig 9, by far the biggest angle for any 10 nm is 53^o for 480-490 nm cyan in CIE LAB (compared to 29^o for 570-580 nm yellow; a remarkable ratio of 1.83). The next biggest, and in stark contrast, is 44^o for 570-580 nm yellow for Nickerson (compared to 31^o for 480-490 nm cyan; ratio 1.4). And the next is 43^o for 480-490 cyan in Munsell (compared to 32^o for 570-580 nm; ratio 1.34). These angles seem overlarge relative to the corresponding angles in yellow (or cyan, for Nickerson), given that cyan and yellow hues are equally well discriminated in most wavelength discrimination data sets (e.g. Fig 8); no wavelength discrimination data set suggests, as does CIE LAB, that cyan is some two times better discriminated than yellow. Not only does the cyan area in CIE LAB seem over-extended but, as a result, the nonspectral area is severely compressed (to 74^o, the smallest of any system in Table III). The same over-compression applies to (CIE LAB-based) LABHNU.[40]

The flaws in Fig 11 curves (showing wavelength distribution over the hue cycle as a whole) are

Table III. 24 uniform-interval hues according to six uniform color spaces (Fig 9), one base model, and one final model (Eqn 8; same as base model except for 10 wavelengths in italics), for illums C or D65. All systems are normalized at 529 nm for Hue 1. Hues outside 440-615 nm are shown as complementary wavelengths, and also (for base and final models, which share the same nonspectrals) as equivalent-wavelengths, e.g. 494.5c = 626e.

	DIN	***Munsell***	***Nickerson***	***OSA-UCS***	***CIELUV***	***CIELAB***	***Base***	***Model***
Hue No	*Wavelength of Hue numbers 1-24 at constant chroma*							
1	529	529	529	529	529	529	529	
2	553	558	550	545	550	552	546	
3	566	568	560	554	561	562	559	
4	572	573	566	562	568	570	565	*567*
5	578	577	571	571	572	576	568.3	*573*
6	584	582	575	577	578	581	571.5	*578*
7	588	588	578	587	582	587	578	*584*
8	594	594	581	590	589	595	590	*591*
9	600	603	585	595	601	609	606	
10	608	615	590	611	493c	494c	494.5c	=626e
11	491.5c	494c	596	494c	498c	498c	500.5c	=637.5e
12	494c	499c	493c	501c	509c	503c	513c	=645e
13	504c	515c	502c	530c	529c	530c	529c	=409e
14	529c	557c	530c	545c	550c	440	546c	=413e
15	557c	564c	554c	555c	561c	462	559c	=419e
16	565c	460	563c	562c	567c	472	565c	=430e
17	448	474	444	464	463	477	450	
18	463	479	463	470	474	480	468	*466*
19	470	482	473	475	481	483	480	*476*
20	477	484	478	477	485	486	487.5	*483*
21	482	488	483	481	490	489	491	*488*
22	487	493	487	485	493	492	494.5	*493*
23	493	500	492	491	498	496	500.5	
24	503	508	501	501	509	506	513	

different from those in Fig 10 (showing wavelength distribution within 5° intervals). Systems that perform well in Fig 10 may not do so in Fig 11. Only the final model performs well in both Figs 10-11, where it is consistently symmetrical and trimodal. The next best are OSA-UCS and surprisingly CIE LUV (though over-compressed around 440-450 nm). Fig 11 offers new ways of analysing uniform hue circles but good examples should not necessarily be highly symmetrical trimodal functions like the base and final models. Possibly cyan hues *are* more discriminable than yellow, as indicated by most uniform hue data sets in Fig 11 (CIE LAB, Munsell, OSA-UCS) though certainly not by wavelength discrimination data (Fig 8), which indicate C and Y hues are equally discriminable.

7. SATURATION DISCRIMINATION

Relative saturation or saturation discrimination ($S\lambda$, same curve) is usually measured in luminance units.[2,25,41,42] Its best known model, by Hurvich & Jameson,[2,25] derives from opponent color chromatic responses and closely models the two gray lines in Fig 12 (experimental data from Purdy for two different luminances, adapted from Ref [25]). This section describes the structural role of complementary colors in saturation discrimination by drawing on previous reports.[5,41] Saturation discrimination of a monochromatic light is usually defined as the inverse just-noticeable colorimetric purity difference from a white at a given luminance.[2,25] When measured in radiance units, the function becomes a typical RGB curve with definite CY troughs and is occasionally related in the literature to spectral sensitivity (always in radiance units) and/or color matching functions.[22,41,42] Usually however, any possible similarity between saturation and spectral sensitivity is obscured by graphing them in different parameters, i.e. luminance and radiance respectively. I commence here with the luminance convention.

In colorimetry, the relative luminance of a color can be computed from certain properties of the CIE 1931 diagram termed mass and *moment* of a color.[1] The inverse ratio of a moment to its complementary moment gives the ratio of CIE luminances required to admix a neutral color.[1] This ratio, as it varies across the spectrum, has been termed relative CIE luminance ($L\lambda$ in Fig 12, dashed black line), and is typified by a narrow peak near 570 nm.[41] Now, it had been noted by Helmholtz, and later developed by Sinden and by Pridmore,[41] that the inverse of relative luminance is perceived relative

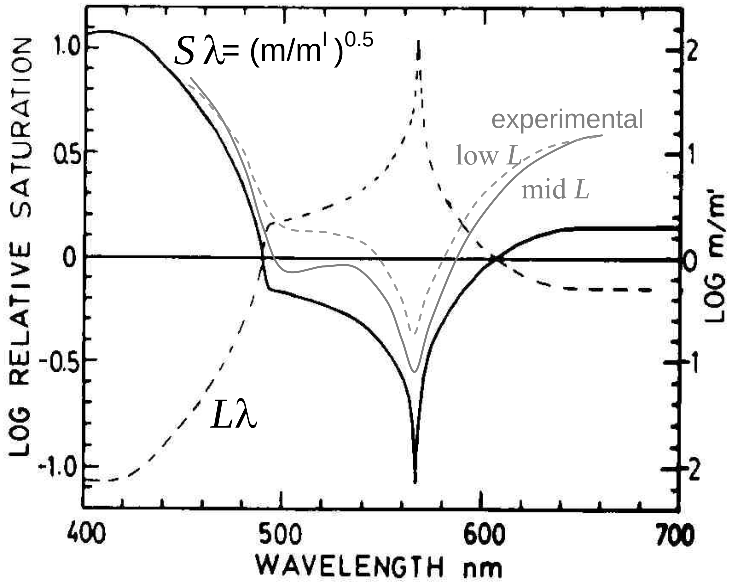


Figure 12. Model of relative saturation $S\lambda$ for Illuminant C, from ratio of moment m to complementary moment m^{l} (right y-axis in log).[1,41] The inverse ratio defines the ratio of their CIE luminances; square root gives relative luminance $L\lambda$ (left y-axis). But the ratio itself $(m/m^{l})^{0.5}$ gives a basic model of relative saturation $S\lambda$ (left y-axis). Gray lines: experimental saturation discrimination data from Fig 6 of Ref [25].

saturation ($S\lambda$ in Fig 12). E.g. 450 nm blue has high perceived saturation and low brightness or luminance, whilst the opposites are true of the complementary 570 nm yellow. The typical $S\lambda$ function in data[2,22,25,41,42] (e.g. gray lines, Fig 12) has B and R peaks, a weak peak in G, and complementary C and Y troughs about 490-500 nm (weak or a mere inflection) and 570 nm (deep trough).

$S\lambda = (m/m^{l})^{0.5}$ Eqn (9)

Eqn (9) (from Ref [41]) gives the present model of saturation discrimination $S\lambda$ measured in luminance units; where m is a color's moment per lumen, and m^{l} is the complementary moment. The ratio of moments is shown in Fig 12 (right y-axis), leading to the $S\lambda$ model (solid black line, to left y-axis). The Fig 12 gray lines can be more closely predicted by variations to Eqn (9).[41]

The model of $S\lambda$ in Fig 12 refers to saturation per lumen, the conventional perspective. But if m per lumen is converted to m per Watt, the function becomes a model of relative *saturation per Watt* ($S\lambda$/W). This function is a typical RGB curve with CY troughs (e.g. chromaticness $C\lambda$ curves in Fig 4 of Ref [42]). This $S\lambda$/W function is factually the same function as the model of *spectral sensitivity* in Fig

6, which therefore models not only spectral sensitivity but saturation discrimination when measured in power units. This relationship between saturation and spectral sensitivity may surprise, yet it is really only to be expected since spectral sensitivity corresponds to saturation rather than hue or brightness. This perspective of relative saturation, as high in RGB and low in CMY, approximates your computer screen when you press the line-color button in Microsoft Drawing Tool (or Color Picker in Adobe Photoshop)[40] and view the "Custom" array of colors across the hue cycle in HSL or HBS mode.[8]

The relationship between relative saturation and relative luminance may be described as both reciprocal and complementary, e.g. the complementary to a color, say blue, has reciprocal saturation and luminance, though only approximately. The exact reciprocity shown in Fig 12 depends on luminance (CIE luminous reflectance Y) being linearly additive, whereas in the real world it is not always so. However, the approximately reciprocal relationship has long (since Helmholtz in 1855)[41] been a sign of the fundamental role of complementary colors in color appearance. The present paper is hopefully one step towards clarifying and formulating that role.

8. DISCUSSION

This study of complementary colors depends on the determination of CI ratios (Fig 3 and Table I), including a wavelength-based metric for the nonspectrals[20] (Appendix A). The nonspectral interval of 240 nm/e, from three methods, is likely accurate to ±5 nm/e, but the metric's accuracy is difficult to measure except from its success in generating values and graph curves whose nonspectral parts fit seamlessly with spectral parts of the hue cycle (in this and previous studies using the metric).[15,43]

It was noted in section 3 Spectral Sensitivity and Chromatic Adaptation, "Hence RGB and CMY hues are opposed, in appearance and psychophysical properties, but precisely balanced in their abilities to neutralize each other." RGB hues are relatively high saturation and low lightness, and have high CI ratio (Fig 3) and low power ratio (Fig 5), whereas CMY hues are relatively low saturation and high lightness, and have low CI ratio and high power ratio. That is, relative CI ratio indicates relative saturation while relative power ratio indicates relative lightness. Further, each of the six hues can be described individually and in contrast to its complementary. In the following, optimal (color stimulus) refers to monochromatic or, in the case of nonspectrals, to optimal compound colors (admixtures of

442 and 613 nm):[5,6] (a) Optimal C is a relatively light and desaturated color, whereas optimal R is relatively dark and saturated; C is a cool color whereas R is a hot color;[8] (b) optimal M is medium lightness and saturation, whereas optimal G is relatively dark and saturated; M is a warm color (think hot pink) while G is a cool color; (c) optimal Y is the lightest color (nearest to white) and least saturated, whereas optimal B is the opposite (nearest to black); Y is a warm color, and B is a cold color.

These distinctions, in psychophysical properties and two of the three color appearance attributes (saturation and lightness), between RGB and CMY hues go some way to explain their opposed nature. The differences support the differences in CI ratio to give a multi-dimensional, trimodal (RGB opposing CMY), structure to wavelength discrimination, spectral sensitivity, and similar RGB-type functions. Hence, to define a pair of colors as complementary means they are opposed in several dimensions: (1) lightness, (2) saturation/ chromaticness, (3) CI ratio, (4) power ratio, (5) warm/ cool colors. But (excepting B and Y) they are not opposed in hue, a unique property of opponent colors.

Although unique hues are systematically opposed in hue and also in warm/cool colors (e.g. Y and R are warm, B and G are cool), they lack other systematically opposed characteristics. Whereas uniques Y and B (which are also complementary) are opposed in relative saturation and lightness, uniques R and G (and some opponent binaries e.g. –B50G and –Y50R) are similar rather than opposed. Opponent colors have no equivalent to CI ratio, and the radiant powers of Y and R chromatic response peaks are not even approximately reciprocal to B and G chromatic response troughs. Though complementary colors require reciprocal amounts to neutralize each other, that is not the case with the infinite number of opponent color pairs (e.g. in the NCS hue circle), excepting the opponent color pair unique B and unique Y (e.g. 475 and 575 nm) because they are also complementary.

In summary, the complementary colors system is a thoroughly and symmetrically opposed system, between RGB peaks and CMY troughs, particularly in the physical dimensions of wavelength and radiance and in the color attributes of lightness and saturation. But they are not opposed in hue appearance (except B and Y). In contrast, the opponent color system is orthogonally opposed in the hue attribute (i.e. opponent unique hues), but not in saturation or lightness, nor (apparently) in the dimensions of wavelength and radiance. Further, whereas the complementary system (Fig 4B or Eqn 7) provides the basis of uniform hue difference, the opponent color system with its orthogonal unique hues does not approach uniform hue: the 90 deg interval between R and B in NCS is far too small according to all color appearance systems claiming uniform hue.[16,40]

In seeking an underlying structure or physiological operation for such RGB functions as spectral sensitivity and uniform hue, the three major options are cone responsivities, opponent colors, and complementary colors. The present models' curves have R peaks about 606-613 nm, clearly indicating spectral sharpening, and cannot be accounted for solely by cone responsivities, whose longest-wavelength peak is about 565-570 nm. Opponent colors are a bimodal system (Y,R peaks, B,G troughs) and do not match the trimodal RGB curve. The function is best and simplest accounted for by complementary colors. These provide not only the appropriate RGB curve but imply an underlying physiological mechanism able to compute wavelengths and powers appropriate to illuminant.

Complementary colors' wavelength pairs and power ratios systematically vary with illuminant, implying this consequently adapts related visual functions (such as the six studied above) to the illuminant. In that case, the sequence of chromatic adaptation would be as follows: (1) cone responsivities adapt to illuminant, (2) the complementary color process, triggered by the cones, adapts to illuminant, and consequently (3) the related visual functions (e.g. wavelength discrimination) adapt to illuminant.

The sequence cannot be in any other order, because the cone responses are necessarily and physiologically first, and the visual functions in Step (3) *all* reflect the structure of complementary colors rather than the reverse. If Step (3) were to precede Step (2) described above, how can one account for the complementary-color structure of the six visual functions? In any case, it is conventionally thought that complementary colors, as innate to the color matching functions, arise in the receptoral layer or at least very early in the retinal process. This would place them in Step (2) as above. However, from the success of the above models, it is deduced that complementary colors are not restricted to an early retinal location but carry into the cortex to structure or otherwise influence color appearance functions. Further, the six visual functions need a neural process able to calculate wavelength and/or radiance, the very capabilities of the complementary colors process.

Section 7 Saturation Discrimination gave an insight into how the physiology employs complementary colors to construct the functions of relative saturation $S\lambda$ and relative luminance $L\lambda$. The physiology seems to compare colors and their complementaries, in quantities such as wavelength and moment.[1] Computing the ratio of moments potentially establishes *S*λ and its inverse *L*λ.

But what about opponent color models of RGB-type functions?[2,25,37] Ref [25] gives a model of spectral sensitivity (there termed 'frequency of seeing'), labeled H&J in Fig 6 above, produced basically by summation of opponent color chromatic response curves. The summation produced a spectral sharpened RGB curve (R peak about 600 nm), i.e. the summation converted opponent color response curves to a typical RGB, complementary-color based, function. This infers the summation may correspond to a neural operation converting one stage of the visual process to another, i.e. from (bimodal) opponent colors to (trimodal) complementary colors. In any case, the RGB curves in Refs[2,25,37] or in the present study are more directly accounted for by complementary colors than by opponent colors.

Complementary colors (and their CI ratios and power ratios) have been shown to provide simple and, in most cases, accurate math models of six color vision functions. Further, as a seventh function (say, "contrast of attributes"), complementary colors (typically R-C, G-M, B-Y) have been shown to represent contrasting attributes [such as warm-cool colors (e.g. R-C), light-dark colors (e.g. Y-B) and saturated-desaturated colors (e.g. R-C)], not only as a pair of colors but within each color (e.g. yellow is bright but desaturated, while blue is dark but saturated), to improve the differentiation, discrimination, and recognition of colors. In summary, complementary colors provide a trimodal, reciprocating (R-C, G-M, B-Y), structural basis for these seven visual functions. The models are not *floating models* (Ref [2], p 584) of purely numerical significance, but imply the complementary (wavelength or power) ratios in Figs 3 and 5 represent plausible physiological functions which differentiate and counter-balance the highs and lows of a given visual function.

In the mid 20th century, complementary colors were relegated to the receptoral layer and the role of color matching.[2,37] But that is contradicted by the evidence above that complementary (wavelength and power) ratios, and models derived therefrom, demonstrate spectral sharpening,[10] implying a post-receptoral location. Hence, it is postulated that complementary colors have a role in color vision beyond the early retina, and particularly in the seven visual functions described above.

Finally, it is worth noting an anonymous referee's comment, that the paper "probably identifies a significant set of new ratio-value constants of the visual response. Importantly, these constants are clearly a related group; they are essentially independent of unit scalar definitions (e.g. wavelength and radiance), and are parallel to the Color Matching Functions because they also describe multidimensionally additive visual responses. The implication is that they might enable a major extension of the affine vector space modeling that underlies the successful predictions of the CIE colorimetric system."

APPENDIX: EQUIVALENT WAVELENGTH FOR NONSPECTRALS

One may reasonably assume that the physiology has a common scale or metric for the hue cycle, for both spectral and nonspectral hues. Otherwise the spectral and the nonspectral scales would be incoherent, whereas we know from color mixture and color appearance systems that they are coherent. This paper is based on aperture color stimuli because they are simply divided into monochromatic (spectral) colors and compound (nonspectral) colors at the common boundaries of 442 and 613 nm (Fig 1).[5,6] At these boundaries, colors change seamlessly from spectral to compound nonspectral.

In contrast, optimal object (or reflectance) colors are more complex due to the lower chromaticness of object colors in the violet area. Hence the line of optimal compound colors is not straight (as in Fig 1) but curves up to omit the violet area, joining (or rather, closely approaching, as near-monochromatic colors) the spectrum locus about 470 nm (rather than 442 nm), for luminance Y=10 as a function of the MacAdam limits for optimal object colors [Fig. 4(3.7) of Ref[2]].

Hence the hue cycle described as spectral from 442-613 nm and nonspectral from 613 through purples to 442 nm may be a simplification for object colors but nevertheless a useful approximation of the hue cycle for both object and aperture colors. The need for a truncated spectrum is to avoid the photometrically inefficient shorter and longer wavelengths particularly near the (invisible) spectrum extremes, just as they are omitted from color appearance systems, e.g. Munsell.

Given the 442-613 nm range of optimal monochromatic aperture-color stimuli,[5,6] an extension of that wavelength scale to the optimal nonspectrals should be computable from known functions that span spectral and nonspectral areas. If, as is thought,[30] the physiology calculates wavelength (by comparing the outputs of two overlapping cone sensitivities) and utilizes wavelength in visual functions, it too needs a coherent extension of wavelength to bridge the nonspectrals. This would require a relationship between nonspectral and spectral hues which adjusts to illuminant. The only apparent such relationship is that of complementary wavelengths. The complementaries to nonspectral hues are spectral wavelengths, which shift systematically with illuminant color temperature. Hence the required hue cycle metric for nonspectrals was derived from complementary wavelength functions as detailed in Ref [20]. The following paragraphs summarise how the nonspectral interval was estimated.[20]

Several methods (three are given below) indicate the total hue cycle interval, including the nonspectrals' equivalent wavelength interval, is about 235-245 nm for illuminant D65. First, the proportion or angle of hue circle occupied by spectral hues relative to nonspectral hues was analyzed, for nine color appearance spaces or uniform chromaticity spaces.[18] As a mean, the spectral hues (442-613 nm) occupied 72.5% of the hue cycle for illuminant D65. Given that the wavelength interval of these spectral hues is 171 nm, then 171 x (1/0.725) gives the total hue cycle interval as 236 nm.

Second, the RGB peaks of the inverse power ratios of complementary colors (Fig 8) are 445.5, 531, 606 nm, for illuminant D65, a mean 80.25 nm interval from R-G and from G-B. Assuming the same interval over the nonspectrals from B-R gives a total (3 x 80.25) 241 nm, in relative wavelength.

Third, in Fig 3 (complementary intervals ratio), the intervals from G peak (assumed at 529 nm) to Y trough (567.6 nm), and from G peak to C trough (491.7 nm), are 38.6 and 37.3 nm respectively. What then would be the complementary intervals, from peak (at B or R) to the common trough in magenta (M, at 529 c)? Analysis of the known complementary pair of intervals from B peak to C trough and from R peak to Y trough implies that a greater range of CI ratios (relative to the complementary interval's CI ratios) gives a bigger wavelength interval than the complementary interval. So the interval from G peak to Y trough (a slightly greater range of CI ratios than the interval from G-C) should be a slightly greater interval (say plus 1 nm) than the complementary interval from B peak to M trough, therefore estimated as 37.5 nm. Similarly, the interval from G peak to C trough should be slightly smaller (say minus 1 nm) than the complementary interval R peak to M trough, thus estimated as 38.5 nm. [If the differences (e.g. plus 1 nm and minus 1 nm) are the same absolute amounts, they cancel out and make no difference to the overall hue cycle interval.] This gives a total hue cycle interval of 247 nm (171+38.5+37.5) in terms of relative wavelength.

The three estimates (236, 241, and 247 nm) give a mean 241.3 nm, say 240 nm in round numbers. Given a hue cycle interval of 240 nm/e, the next problem is to determine the cycle ends (start and finish), or alternatively the cycle midpoint if the latter is defined as the mean of the cycle ends. Hence we need only to determine one (the midpoint or the ends) to find the other. It is shown in Ref[21] in two quite different graphs that the structure of complementary wavelength pairs for any illuminant has a center of symmetry defined by the pair of complementary wavelengths of the *minimum complementary interval* (MCI) for the respective illuminant. The CI ratio for each wavelength of this pair is 1:1. The mean of the MCI pair is termed the *hue cycle midpoint*. The MCI pair varies with illuminant. For illuminant D65 it is 578.2 and 479.8 nm. The mean (529 nm) is hue cycle midpoint, agrees with the 530 nm peak of the gray line in Fig 3, is located in mid-spectrum, and its complementary is about the middle of the nonspectral range (Fig 1), where one would expect to locate the hue cycle ends.

The hue cycle midpoint is defined: (1) colorimetrically as the mean of the MCI wavelength pair, and (2) secondarily as the mean wavelength of the cycle ends.[20] Thus the cycle ends are each a half-cycle (120 nm/e) from the midpoint, i.e. at 409 and 649 e (Fig 3). From the symmetry of Figs 1 and 3, one may assume the hue cycle ends are both equidistant and complementary to the cycle midpoint (Fig 4). The specification of hue cycle ends now allows a good estimate[20] of CI ratios of green and non-spectral curves, shown in Table I. To colorimetrically define the nonspectral *equivalent wavelengths*, their complementary wavelengths must be specified, as in Ref[20] and in Table I and Fig 1 above.

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CONCLUSION

This thesis was instigated by a lack of information in the scientific literature as to the purpose or roles of complementary colours in colour vision. The thesis aim was "to find and describe roles of complementary colours in colour perception. The aim includes the possibility of finding not only minor or discrete roles but the overall role of complementary colours in vision."

Five papers were presented in support of the thesis aim. In the chapter "Introduction," aims were given for each paper, and these will be discussed and conclusions drawn, below. Finally, a general discussion and conclusion will be given for the thesis as a whole.

Paper 1 (*Color constancy from invariant wavelength ratios: I. The empirical spectral mechanism*)

The aim of Paper 1[1] was to demonstrate a spectral mechanism of colour constancy, and the role of complementary colours (specifically complementary wavelengths) in that mechanism. The paper built on the phenomenon, first shown in Ref. 2, of constant hues (including complementary constant hues) plotting approximately straight and parallel lines in the plane of wavelength and reciprocal illuminant colour temperature (MegaKelvin^{-1} or MK^{-1}). Paper 1 proceeded to test and confirm the phenomenon by analysing seven sets of published data on constant hues,[3,4,5,6,7,8,9] over four illuminants. Recalling the complementary linearity law (that the complementary wavelengths to a constant hue represent the complementary constant hue),[2] Paper 1 tested and confirmed that such complementary wavelengths, over respective illuminants, do indeed plot straight and near-parallel lines in the plane of wavelength and MK^{-1}. Since the pair of complementary lines are near-parallel, they are presumably related by a near-invariant wavelength ratio. This was confirmed by experimental data (Table II of Ref. 2).

The paper, in summary, showed that the wavelengths of any constant hue and its complementary constant hue plot straight near-parallel lines (see Figure 1), and that a constant hue and any other constant hue (complementary or not) are related by a near-invariant wavelength ratio, across respective illuminants. Those relationships involve, and are largely dependent on, complementary

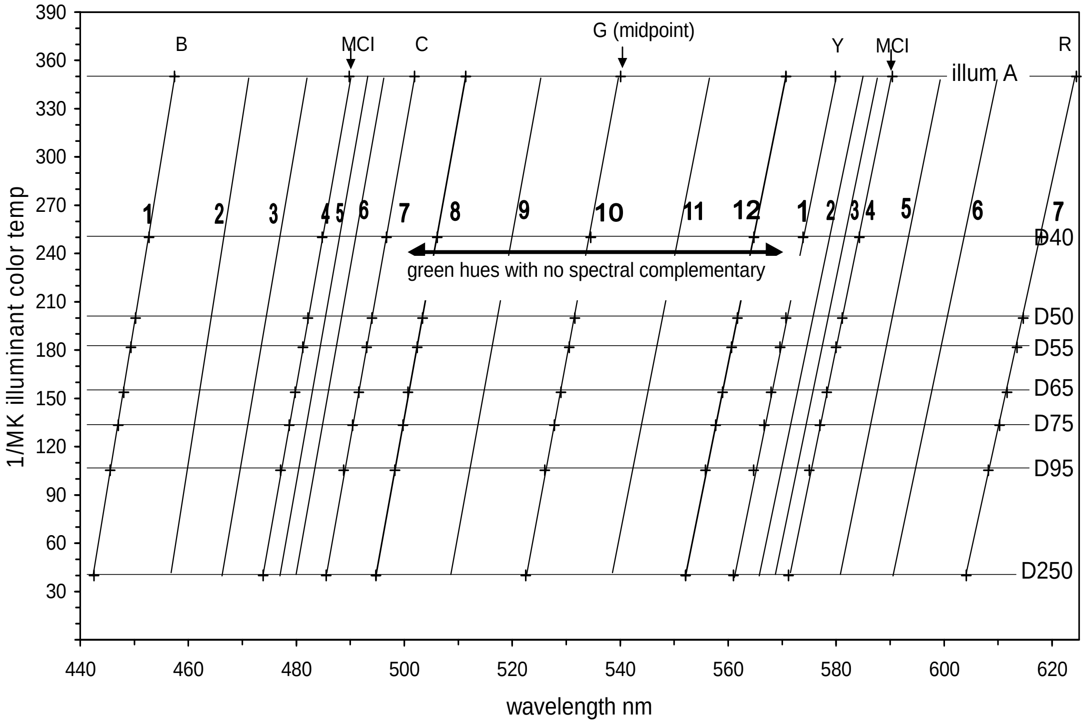

Figure 1. The spectral mechanism of colour constancy. Sloping near-parallel lines (numbered) represent some of the infinite number of spectral constant hues; any pair of lines represent near-invariant wavelength ratios. Each numbered line has a complementary line indicated by the same number, except greenish hues (lines 8-12, arrowed) whose complementaries are nonspectral constant hues (not shown).

wavelength pairs (i.e. on the wavelengths of a constant hue and of its complementary constant hue plotting straight near-parallel lines in the plane of wavelength and MK^{-1}) because such pairs connect the short- and long-wavelength areas (i.e. the bluish and the yellow- or red-ish hues) to either side of the green hues which have no spectral, but only nonspectral, complementary hues.

In conclusion, Paper 1 establishes a role for complementary colours in colour constancy. Specifically, complementary wavelengths have the role of (a) providing the complementary connection between constant hues in the short-wavelength area and the long-wavelength area, in the form of straight near-parallel lines in both areas (see Figure 1), representing near-invariant wavelength ratios, and (b) structuring the mainframe (i.e. all complementary pairs of constant hues) of the symmetrical, linear, colour constancy mechanism (Figure 1).

Whether this structure extends to the midspectral (greenish) hues and nonspectral (purplish) hues is explored in Papers 2 and 3, below. As noted in Paper 1, such a structure and complementary connection may serve two purposes: (a) to complete the colour constancy mechanism, and (b) to complete a wavelength-based hue cycle scale by means of a coherent, wavelength-based, colour constancy mechanism.

Paper 2 (*Relative wavelength metric for the complete hue cycle: Derivation from complementary wavelengths)*

The aim of Paper 2 was to establish a hue cycle structure and wavelength-based hue cycle scale, and to demonstrate the role of complementary wavelengths in that structure and scale.[10] Besides the importance of finding the hue cycle's innate structure, continuous and coherent for both spectral and nonspectral hues, this aim is prerequisite to describing any function over the complete hue cycle (including nonspectrals), to a scale equivalent to dominant wavelength. The scale becomes a basis for Paper 3, concerning colour constancy in nonspectral hues and enabling the latter to be graphed to the same *relative wavelength scale* as spectral hues.

In Paper 2, I proposed a wavelength-based, or *relative wavelength*, scale for the complete hue cycle by extending the wavelength scale of the optimally effective spectrum (442-613 nm, where monochromatic wavelengths are optimal colour stimuli)[11,12,13] into the nonspectrals. The method was to (a) estimate the hue cycle interval in terms of relative wavelength, (b) formulate the structure of complementary wavelength pairs in terms of the ratio of complementary wavelength intervals, and (c) extend the formulated structure into the nonspectrals. This method was based on the symmetrical structure of complementary wavelength pairs first shown in Ref. 2, centering on the complementary wavelength pair of *minimum complementary interval* (for the respective illuminant) and its mean wavelength, termed the hue cycle midpoint. The latter was shown to be the centre of a symmetrical complementary wavelength structure, which was argued to represent a basis of hue cycle structure.

The method gave a uniform *relative wavelength* scale over the complete hue cycle for daylight illuminant D65, including *equivalent wavelengths* for the nonspectral hues. These equivalent wavelengths were identified colorimetrically by specifying their spectral complementary wavelengths. (The corresponding scale and colorimetric specifications for illuminant A are also tabulated in Paper 2, from calculations in Paper 3, so that Paper 2 contains all important information on the relative wavelength metric.) The relative wavelength scales for both illuminants D65 and A were corroborated, to some degree, by calculating the relative wavelength shifts of nonspectral constant hues from illuminant D65 to illuminant A. The shift was shown to be 9.8 nm, very similar to the 9.9 nm shift for spectral hues, thus confirming that the wavelength shift of constant hues, from D65 to A, is approximately uniform over the hue cycle and implying that the *equivalent wavelength* scale is very much the same scale as dominant wavelength for the range 442-613 nm.

Clearly, complementary wavelengths and particularly the ratio of complementary intervals played a large part in determining the relative wavelength metric.

In conclusion, Paper 2 demonstrated the central role of complementary wavelengths in the structure of the hue cycle and in its relative wavelength metric. It is worth mentioning that this role is closely associated with the role of complementary wavelengths in the wavelength-based mechanism of colour constancy, described in Paper 1 (the spectral mechanism) and described further in Paper 3 (the global mechanism) below.

Paper 3 (*Color constancy from invariant wavelength ratios: II. The nonspectral and global mechanisms*)

The aim of Paper 3 was to utilise the relative wavelength scale of Paper 2 to theoretically extend the spectral mechanism of colour constancy (described in Paper 1) into the nonspectrals.[14] This would enable a description of the global mechanism of colour constancy (global in the sense that it covers the total hue cycle and the total practicable range of illuminant colour temperature from 2,800-25,000 K). In doing so, my aim in Paper 3 was also to demonstrate the role of complementary wavelengths in the hue cycle relative wavelength metric and the mechanism of colour constancy.

Paper 3 builds upon Papers 1 and 2 to complete the global mechanism of colour constancy, over the complete hue cycle.[14] It utilises Paper 2's relative wavelength scale for illuminant D65 and tackles the problem of adapting that scale to all the CIE illuminants (from A at 2854 K, to D250 at 25,000 K) that should be covered by a global mechanism of colour constancy. It proposes, as a first step, to systematize the hue cycle interval (which is 240 nm for illuminant D65)[10] as a constant proportion of harmonic period or octave (from 1st to 2nd harmonic) for all illuminants.[15]

The constant proportion or ratio is selected to be 2/3 of the octave interval (i.e. the wavelength interval between hue cycle ends at 1/6 ratio and 5/6 ratio) in logarithmic progression, because the 2/3 interval is 240.2 nm in close agreement with the 240 nm hue cycle interval in illuminant D65. Because the 240 nm hue cycle interval is centered on the hue cycle midpoint for the respective illuminant (e.g. 529 nm for D65), so also is the 2/3 interval of octave; i.e. the mean of the wavelengths of the 1/6 ratio and the 5/6 ratio (representing hue cycle ends) is the hue cycle midpoint (previously determined from colorimetry). The hue cycle interval remains a constant 2/3 octave but shifts wavelength with illuminant; for example, the hue cycle for illuminant A is an interval of 245.2 nm, from hue cycle ends at 417.5 and 662.7 e (where "e" denotes equivalent wavelength) whose mean is the hue cycle midpoint at 540.1 nm.

In this system, the ratios of octave are logarithmic to base 2 (see Paper 3, Appendix A), as is the equal-temper scale in music,[16] for two reasons: Primarily because the base 2 enables the octave range from 1st harmonic (x nm) to 2nd harmonic ($2x$ nm, reflecting the log base 2) to be divided into equal *ratios* (e.g. 1.0595, say 1.06, for each 1/12 ratio of octave, rather than equal wavelength *intervals* per 1/12 octave) which together multiply the total wavelength interval from x nm to $2x$ nm, as follows: x nm times 1.06=1.06 x, times 1.06=1.4 x, times 1.06=1.19 x, ... finally, 1.89 times 1.06=2.0 x. In contrast, an octave in linear progression would use 12 equal wavelength intervals of the 1 x range from x to $2x$ (i.e. 0.083 x), in which the wavelength range of a 2/3 interval of octave would be smaller than the 2/3 range in log progression. Secondly, because the log base 2 produces a 2/3 proportion of octave whose interval is 240 nm (240.2 to be exact) for illuminant D65.

This system was formulated to correlate with the straight near-parallel lines representing constant hues and their complementary constant hues in the spectral mechanism of colour constancy discovered in Paper 1 (see Fig. 1 above). Correlation is remarkably good: these straight lines each align very closely (±0.5 nm at worst) with a ratio of octave (also known as an interharmonic ratio),[17] i.e. a ratio (such as 1/6) between the 1st and 2nd harmonics. Given that constant hues' straight near-parallel lines align with ratios of octave, such ratios (and their corresponding lines) were readily plotted into the nonspectral area, as shown in Fig. 1 of Paper 3, to complete the global mechanism. The problem of colorimetrically identifying the nonspectral equivalent wavelengths, by specifying their spectral complementary wavelengths, was resolved for the respective illuminants by the same method used for D65 (illustrated in Fig. 3 of Paper 1), and these pairs are tabulated in Paper 3.

Paper 3 may be a difficult paper for the reader unfamiliar with harmonics and logarithmic progression. However, the somewhat complex system formulated to theoretically correlate with the sloping lines of constant hues (Figure 1 above) does so with notable accuracy. Clearly, complementary wavelengths played a significant part in Paper 3's completion of the global mechanism.

In conclusion, Paper 3 demonstrates the role of complementary wavelengths in (a) the hue cycle relative wavelength metric, particularly the colorimetric identification of (nonspectral) equivalent wavelengths, and (b) the global mechanism of colour constancy, particularly in plotting the accurate relative wavelengths of spectral-and-nonspectral complementary pairs of constant hues, for respective illuminants.

Here it is worth noting that the roles in (a) and (b) above are not fully understood at this early stage of research, and represent only a start in understanding the complete detailed role of complementary wavelengths in hue cycle structure, hue cycle metric, and colour constancy.

Paper 4 (*Chromatic induction: Opponent color or complementary color process?*)

The aim of Paper 4 was to establish, by analyses of previous published data, that chromatic induction is governed by complementary colours rather than by opponent colours.[18] The latter case has been the convention in colour science (but not art) since about 1960 to the present day.[19,20,21] Until their recent editions (about 2004), two well-known books[22,23] by leading researchers stated that chromatic induction was accounted for by opponent colours rather than complementary colours. Their recent editions generally correct that impression, possibly from the present author's writing to both authors on the subject over recent years since 2001.

Paper 4 commences by defining the differences between complementary and opponent colours. It proceeds to show by (a) analysis of previously published data, and (b) involving the reader in doing some simple experiments in chromatic induction (specifically after images), that chromatic induction is more accurately explained as a complementary colour process than an opponent colour process. The paper also shows that chromatic induction is a trimodal function, where three large wavelength ranges (typified as R, G, B hues) are induced by three relatively narrow wavelength ranges (typified as C, M, Y hues), or the reverse. This agrees with complementary colour functions (e.g. power ratios of a colour and its complementary colour), which typically have three peaks in RGB and three troughs in CMY, but does not agree with opponent colour functions (such as chromatic response functions) which are typically bimodal, i.e. they have only two peaks and two troughs.

The paper notes the main cause of the confusion as to whether chromatic induction is an opponent colour or complementary colour process is Hurvich and Jameson's influential 1961 paper,[19] which described chromatic induction in simplistic terms of opponent colours without considering other explanations (such as complementary colours) which may give a better account.

In conclusion, Paper 4 establishes that chromatic induction is governed by complementary colours rather than by opponent colours. Paper 5, below, gives math models of chromatic induction and several other visual functions.

Paper 5 (*Complementary colors: The structure of wavelength discrimination, uniform hue, spectral sensitivity, saturation, chromatic adaptation and induction*)

The aim of Paper 5 was to demonstrate that complementary colours provide the basic structure of several important visual functions.[24] The paper did this by formulating six mathematical models, one each for wavelength discrimination, uniform hue difference, spectral sensitivity, saturation, chromatic induction, and chromatic adaptation. The six models derive from either the ratio of complementary wavelength intervals or the ratio of complementary powers. The models are psychophysical but the six visual functions they model vary from psychological to psychophysical; e.g. wavelength (or hue) discrimination is psychological, as a matter of perception, as is saturation and chromatic induction. Chromatic adaptation however is an automatic psychophysical process, as is spectral sensitivity; their physiological basis may be the spectrally-opposed single cells described in Ref.[1] and in Figure 4 of the Introduction chapter, above.

Until recently, chromatic adaptation was generally thought to operate as three independent RGB responsivities.[25] But the findings of Paper 5 support a connection between these responsivities, consistent with recent publications.[23,26] My model of chromatic adaptation in Paper 5 is an innovative concept, based on the power ratios of complementary colours. The model or function consists of one continuous curve with RGB peaks, rather than three separate RGB curves as in the von Kries theory.[25] It adapts to change of illuminant by altering the amount but not the wavelength of the RGB peaks, similarly to the von Kries theory.

Paper 5 builds on Paper 4's demonstration that chromatic induction is definitely a complementary colour process. It gives a general model of induction (based on the ratio of complementary intervals) and two formulas for particular conditions.

In summary, the six models accurately predict (or agree with) the data, demonstrating that the six visual functions are basically structured by complementary colours.

As a seventh function, complementary colours (e.g. R-C, G-M, B-Y) were shown to represent contrasting visual perceptions [such as warm-cool colours (e.g. R-C), light-dark colours (e.g. Y-B) and saturated-desaturated colours (e.g. R-C)], not only as a pair of colours but within each colour (e.g. yellow is bright but desaturated, while blue is dark but saturated), to improve the differentiation, discrimination, and recognition of colours. In summary, complementary colours provide a trimodal, reciprocating (R-C, G-M, B-Y), structural basis for the seven described visual functions.

In conclusion, Paper 5 convincingly demonstrates that complementary colours have roles in structuring seven important visual functions. Other functions may be added later, such as colour rendering and colour matching functions.[27,28] These structural roles of complementary colours are, as mentioned, psychophysical though the seven functions vary from psychological to psychophysical. These semantic differences serve to indicate that many important visual functions, including psychological functions like saturation or wavelength discrimination, have a common psychophysical structure or framework based on complementarism.

General Discussion and Conclusion

In the five papers presented in this thesis, I have demonstrated and described some of the roles of complementary colours in colour vision. These roles are likely to represent only some of the possible roles. However, all the described roles excepting that in chromatic induction are new in the literature, and substantially expand scientific knowledge of the purpose and role of complementary colours.

The described roles are summarised in paragraphs 1-5 below, and amount to a total ten discrete roles (including the six math models described in paragraph 4):

1. A role in the psychophysical structuring of the hue cycle, particularly in carrying the wavelength-based structure into the nonspectral hues, in order to complete the hue cycle across all CIE (Daylight and Planckian) illuminants.[2,10] The essential task is for complementary wavelength pairs to connect spectral and nonspectral parts of the hue cycle, or similar widely separated areas of the cycle (such as short- and long-wavelength areas), to form one interconnected coherent cyclic structure.
2. A role (closely related to 1, above) in structuring a psychophysical, wavelength-based, scale over the complete hue cycle,[10] particularly in carrying the spectral dominant wavelength scale into the nonspectral hues, for all CIE illuminants. Such a scale is arguably necessary for (a) the physiology to treat spectral and nonspectral hues in one coherent psychophysical scale, (b) the scale to be based directly on physical quantities, i.e. physical wavelength of physical stimuli, and (c) the forming of hue cycle structure, including nonspectrals, to one coherent scale. The essential task for complementary wavelength pairs is to connect and relate the spectral and nonspectral parts of the hue cycle.
3. A role in structuring a colour constancy mechanism,[1,14] over both spectral and nonspectral hues. The essential task here, again, is for complementary wavelength pairs to connect spectral and nonspectral parts of the mechanism, or similarly separated areas of the mechanism (such as short- and long-wavelength areas), by invariant or almost invariant wavelength ratios. These ratios are expressed as

straight near-parallel lines in the plane of wavelength and reciprocal illuminant colour temperature (see Figure 1 above). These ratios (constant across illuminants) also align very closely with constant ratios of octave (or harmonic period); it is not yet understood if the harmonics relationship is simply a mathematical convenience or has a greater psychophysical significance.

4. Roles in structuring the trimodal (RGB) curve shape of at least six visual functions modelled in Paper 5, i.e. wavelength discrimination, uniform hue, spectral sensitivity, saturation, chromatic adaptation and chromatic induction. The structuring of chromatic adaptation alone is a major task in vision. These six structural roles each derive directly from one of two ratios: The ratio of complementary wavelength intervals, and the ratio of complementary powers.

5. A role in differentiating colours by contrasting their visual perceptions [such as light-dark colours (e.g. Y-B) and saturated-desaturated colours (e.g. R-C)], not only as pairs of colours but within each colour (e.g. yellow is relatively bright but desaturated), to improve the differentiation, discrimination, and recognition of colours.[24] That is, a colour's brightness and saturation attributes are contrasted, so that if one is high the other is low, and thus the colour is more easily recognized and discriminated by observers. These contrasts were initially reported by Helmholtz,[29] and later developed by Sinden (1923)[30] and Pridmore (1990).[31] It has been noted recently by physiologists that some brain areas process colour as a combination of hue and luminance, rather than as segregated hue and luminance circuits.[32,33,34,35] Most hues have an optimal luminance, supporting the idea that some hues combine best with high luminance (e.g. Y) and some with low luminance (e.g. B). Furthermore, as discussed in Paper 5, the colour appearance attributes of brightness and saturation correlate approximately to complementary powers ratio and complementary intervals ratio, respectively.

In conclusion, the thesis has described 10 important roles (varying from major mechanisms such as hue cycle structure to lesser functions such as saturation) played by complementary colours. Including the already known colorimetric role in colour mixture and matching (including the RGB colour matching functions), the total is 11 known roles. There may well be more.

However, these roles give no clear indication of an overall role for complementary colours. The wide variety of the 11 roles makes it difficult to select a particular area of colour vision as the overall role, yet the variety itself may be a sign of that role. Recall that the role of complementary colours has not appeared obvious in some 340 years of documented research, which includes many gifted researchers.[29,36,37,38,39,40,41] Hence, one may reasonably deduce that the overall role is not readily apparent. It may be a hidden, perhaps structural, role. Indeed, the first nine roles (in paragraphs 1-4, above) may be grouped under an overall role of structuring colour mechanisms or functions, that is, in structuring the hue cycle, the wavelength-based scale, the colour constancy mechanism, chromatic adaptation, and various lesser functions. The tenth role (paragraph 5, above) may also be described as structural, in forming colours whose brightness and saturation dimensions are broadly reciprocal, to aid the discrimination and recognition of colours as discussed in the Introduction.

The structure, whether of a major mechanism (Figure 1) or a lesser function, may be described as trimodal, based on RGB as peaks or troughs of response (e.g. spectral sensitivity) or trimodal frequency of occurrence (e.g. ratio of complementary intervals ratio, or constant hue lines per constant wavelength interval; see Figure 1).

Hence, it may be concluded that complementary colours have at least 11 discrete roles in colour vision, with a shared or overall role of structuring fundamental mechanisms/functions to a trimodal RGB framework. But what is the *purpose* of the structuring? Presumably the purpose would be some characteristic or ability well known of complementary colours, though an unexpected purpose is possible. I discuss the structural purpose in the final section of the thesis, below.

Structural Purpose

The thesis has described 11 roles of complementary colours, including the previously known role in colour mixture and matching. The extensive variety of roles implies that complementary colour processes are not restricted to any particular area of colour vision but extend across the visual system, from basic colorimetry (at or near the receptor layer) to colour appearance (in visual cortex).

Furthermore, complementary colours provide structure to the visual system from its very start. The wavelength peaks of the S and L cones about 445 nm and 565-570 nm[22,28,42] are complementary. (Exact CIE complementary pairs in illuminant D65 include 440 and 567 nm, and 448 and 568 nm.)

This means the S and L system can admix white, or in other words, it is in equilibrium. The *b-y* opponent-colour chromatic responses (arising directly from the LMS cone outputs in opponent colours theory)[43,44,45,46] similarly peak about 445 and 568 nm and are complementary. Surprisingly, this direct relationship between S and L cones and *b* and *y* chromatic response curves is not mentioned (to the author's knowledge) in the literature. It is possible that the complementary colours' RGB structure arises from summation of the two opponent colour chromatic response mechanisms *b-y* and *g-r* (note the signs are arbitrary), as first shown (perhaps unintentionally) by Hurvich and Jameson:[45] their special summation produces the RGB-peaked spectral sensitivity function (labeled H&J) shown in Figure 6 of Paper 5. That RGB curve is typical of complementary colours structure.

Eqn 1, below, is an alternative to Hurvich & Jameson's formula.[45]

$\Phi(b+g+y+r) = C$ Eqn 1

where C is the resultant curve from the summation (e.g. gray curve in Figure 6 of Paper 5); and where, in the addition of $b+g+y+r$ curves at every wavelength, symbol Φ denotes an operator whereby the addition of two like signs reverses the sign; e.g. (-0.5)+(-0.3) = 0.8, or 0.5+0.3 = -0.8. This occurs only in the Cyan and Yellow regions, and produces a new peak and complementary trough near 605 nm (Red) and 490 nm (Cyan), similar to the H&J curve in Figure 6 of Paper 5.

Presumably the full complementary colours RGB system is established in the retina, possibly in the horizontal cells immediately following the cones, since spectrally-opposed responses in single horizontal cells in vertebrates display complementary colour pairs R-C, G-M, B-Y in many sets of data.[47,48] Data include Svaetichin's supposed B-Y and R-G responses (actually R-C, about 495 nm),[47] and the triphasic cell responses in G-M,[48] where the (nonspectral) M is admixed from two spectral response peaks in a short- and a long-wavelength. Triphasic cell responses presumably represent the complementary pair G-M, rather than the opponent colour pair G-R (both spectral hues).

In summary, complementary colours' structure appears quite commonly throughout the colour vision system, from basic colorimetry in the cone-receptor layer to colour appearance in the visual cortex, and is evidently a major structural influence in the system. To be so widespread, the structure's purpose may be expected to be very important. So, what is the purpose of the structure?

One of the most important functions in colour vision is chromatic adaptation, the process that gives the perception of colour constancy; that is, an object's colour appears much the same in quite different illuminants whether natural daylight (bluish) or tungsten lamplight (yellowish). The 11 functions described above (including the previously known role in colour mixture and matching) are known to automatically adapt to illuminant,[22,24,28] i.e. they slightly shift wavelengths and amplitudes with illuminant. This presumably is because the RGB peaks and complementary C(M)Y troughs of functions structured by complementary colours are ideally suited to produce white (matching the ambient illuminant) over the spectrum or hue cycle, as discrete pairs of complementary wavelengths or colours. Though this ability is known psychophysically, little is known of its physiology.

However, complementary relations and the complementary nature of most colour-opponent single cells (usually classed as B-Y and R-G opponent-colour responses) have been reported or deduced in retina and cortex in recent years,[49,50,51] as described in the Introduction. For example, Derrington et alia[32] describe some 60 single cells, and Lankheet et alia[52] describe some 220 single cells, i.e. a total of some 280 cells, in the monkey lateral geniculate nucleus, termed either "Y-B" or "R-G" opponent response cells; they are shown in the CIE 1931 colour mixture diagram as opposites through the neutral point, i.e. they are all complementary pairs yellow-blue or red-cyan (although the physiologists do not use the technical colorimetric term "complementary"). Such colour-opponent cells in retina and cortex possibly represent the physiological basis of complementary colours, as discussed in Appendix B of Paper 1. There is no other possible physiological basis known to the author. The ability of complementary pairs to admix the ambient illuminant's white, as judged by a colour-normal observer, and to thus adapt to illuminant, indicates that the typical R-C, G-M, B-Y structure of the visual functions described above is designed to adapt to illuminant. If it instead adapted to some other, perhaps fixed, illuminant, a colour sample could appear so different in different illuminants as to be unrecognisable. That would defeat the purposes of colour vision, one of which is to improve image recognition over that achievable by luminance vision alone.[34,53]

Furthermore, single cell colour-opponent responses are thought by physiologists to closely concern colour constancy (the purpose of chromatic adaptation). For instance, Conway[49] and Gegenfurtner[34] argue that single cells' opposed responses need to be perfectly balanced (spatially and chromatically) to enable colour constancy. To a colorimetrist, such "perfectly balanced" responses must be in equilibrium with the ambient illumination, that is, they must admix or average a white matching the light source, and hence they must be complementary colour pairs. Conway goes so far as to say:[49] "a red-cyan axis might be advantageous because it (and the blue-yellow axis) would be silent to shades of grey. That is, the intersection of the null planes of the two axes would be achromatic." That translates, in colorimetric terms, to each axis comprising a pair of complementary colours, i.e. red-cyan and blue-yellow. This area requires further research but is beyond the present thesis. However, it does support the idea that complementary colours, whether in physiology or in psychophysics, play an essential role in chromatic adaptation.

Therefore, it is deduced that the purpose of the complementary colours trimodal (R-C, G-M, B-Y) structure in functions is to balance the function in equilibrium with the illuminant. That is, every complementary pair, and thus the total function, produces the white of the ambient illuminant. In other words, the function adapts to the illuminant, thus ensuring colour constancy. This in turn indicates that the overall or general role of complementary colours concerns chromatic adaptation. Considering the moderate ambitions of the base studies (Papers 1-5), this is a surprising and important finding. It has considerable implications on chromatic adaptation theory,[23,28] which presently does not mention or utilise complementary colours other than indirectly as part of CIE basic colorimetry.

In conclusion, complementary colours have at least 11 discrete roles in colour perception, with an overall role of colour constancy (i.e. adapting the perception of colour to the ambient illuminant). To do this, complementary colours structure many visual functions to a trimodal R-C, G-M, B-Y (peaks and troughs) framework to assist the functions' chromatic adaptation.

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