Pricing of European Options using Empirical Characteristic Functions

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Abstract

Pricing problems of financial derivatives are among the most important ones in Quantitative Finance. Since 1973 when a Nobel prize winning model was introduced by Black, Merton and Scholes the Brownian Motion (BM) process gained huge attention of professionals. It is now known, however, that stock market log-returns do not follow the very popular BM process. Derivative pricing models which are based on more general Lévy processes tend to perform better.

Carr & Madan (1999) and Lewis (2001) (CML) developed a method for vanilla options valuation based on a characteristic function of asset log-returns assuming that they follow a Lévy process. Assuming that at least part of the problem is in adequate modeling of the distribution of log-returns of the underlying price process, we use instead a nonparametric approach in the CML formula and replaced the unknown characteristic function with its empirical version, the Empirical Characteristic Functions (ECF). We consider four modifications of this model based on the ECF. The first modification requires only historical log-returns of the underlying price process. The other three modifications of the model need, in addition, a calibration based on historical option prices. We compare their performance based on the historical data of the DAX index and on ODAX options written on the index between the 1st of June 2006 and the 17th of May 2007. The resulting pricing errors show that one of our models performs, at least in the cases considered in the project, better than the Carr & Madan (1999) model based on calibration of a parametric Lévy model, called a VG model.

Our study seems to confirm a necessity of using implied parameters, apart from an adequate modeling of the probability distribution of the asset log-returns. It indicates that to precisely reproduce behaviour of the real option prices yet other factors like stochastic volatility need to be included in the option pricing model. Fortunately the discrepancies between our model and real option prices are reduced by introducing the implied

parameters which seem to be easily modeled and forecasted using a mixture of regression and time series models. Such approach is computationaly less expensive than the explicit modeling of the stochastic volatility like in the Heston (1993) model and its modifications.

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Certificate

I certify that the work in this thesis is original and has not been submitted for a higher degree to any other university or institution.

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$\big(\Omega, \mathcal{F}, \mathbf{P}\big)$ - probability space,

$\mathbf{P}, \mathbf{Q}$ - probability measures,

$\mathcal{F}$ - σ-field,

$(\mathbf{F}_t)_{t \in [0,T]}$ - an increasing family of sub σ-fields of $\mathcal{F}$,

$\mathbb{N}$ - set of positive integers,

$E^{\mathbf{P}}X$ - expected value of r.v. X with respect to measure $\mathbf{P}$,

EX - expected value of r.v. X,

$E(X|Y)$ - expected value of r.v. X, given r.v. Y,

$\mathbf{L}_X(dx)$ - Lévy measure of r.v. X,

$l_X(x)$ - Lévy density function of r.v. X,

X_t - stochastic process,

$\phi^{\mathbf{P}}_{X_t}(u)$ - characteristic function of X_t, under measure $\mathbf{P}$,

$\phi_{X_t}(u)$ - characteristic function of X_t,

$\psi_{X_t}(u)$ - cumulant function of X_t,

A_X - analyticity strip of a characteristic function of r.v. X,

$\hat{\phi}_n(u)$ - Empirical Characteristic Function (ECF),

$C(t, T, K)$ - model price of Call option at time t, maturity at time T and strike price K,

$\bar{C}(0, T, K)$ - historical price of Call option at time zero, maturity T and strike price K,

$\hat{C}_n(t, T, K; w, p)$ - ECF model price of Call option at time t, maturity at time T, strike price K and parameters w, p,

$H(x)$ - payoff function of European option,

w - Mean Martingale Correcting Term (MMCT),

$\hat{w}_n$ - empirical MMCT,

w^*_n - implied MMCT,

w^f_n - forecasted implied w^*_n,

p^*_n - implied number of days to option expiry,

p^f_n - forecasted implied p^*_n,

Δ^*_n - implied time length of increments,

$o(\cdot)$ - little o,

$\mathbf{1}_D(x)$ - indicator function of a set D.