Crafting the Hammer:

A Philosophical Examination of Attempts to Capture the Human Capacity for Number

Sumitra Vignaendra

Master of Research Thesis

Department of Philosophy, Macquarie University

October 2017

Supervisor: Associate Professor Richard Menary

TABLE OF CONTENTS

In memory of my parents and grandparents, with love and gratitude.

ACKNOWLEDGMENTS

Deepest thanks to Richard Menary for sharing his considerable knowledge, and for helping me find my own way to this thesis by patiently guiding me through the process. And also for his frank advice given with great kindness.

Heartfelt thanks, too, to Premila Pillay, Graham Hazlitt and Georgina Spilsbury – for the dialogue.

ABSTRACT

This thesis argues that two empirical approaches to capture the human capacity for number are not well-justified and are too narrow, and consequently, are not robust enough to capture this capacity. The two empirical approaches analysed are the nativist approach endorsed by Elizabeth Spelke (e.g. 2011) and the embodied cognition approach formulated by George Lakoff and Rafael Núñez (2000). I argue that the former approach privileges neural modules in the explanation of this capacity without sufficient justification; the latter approach excludes neural circuits from the explanation, also without sufficient justification, as well as fails to provide a plausible evolutionary story to explain the emergence of number in human life. I conclude the thesis by arguing that an empirical approach robust enough to capture the human capacity for number should be informed by theories on niche construction (Sterelny, 2007); mimesis (Tomasello, 1999); the emergence of the modern mind (Donald, 1991); neural reuse (Anderson, 2010) and cognitive integration (Menary, 2007).

INTRODUCTION

What is number? How have humans acquired the capacity for number? How do we obtain an understanding of this? A quick glance at the recent literature on the human capacity for number uncovered the following claims:

- Calculations with numbers are as essential to the human experience as air, and like air, we take number for granted (Budd, 2015).
- There are societies that have survived for centuries that have made no reference to, and apparent use of, the concept of number (Everett, 2017).
- Significant proportions of human populations in North America and Western Europe are innumerate (Peters and Bjalkebring, 2015).
- Number is a naturally-selected intuition shared by humans and some non-human species (Brannon and Merritt, 2011).
- Number is an artefact of human creation; it is a symbol that can only be rendered sensible by other similar symbols (Presmeg et al, 2016).

Taken together, what do these recent and influential claims lead us to think about the human capacity for number? Does the survival of humans depend on understanding and using number? Or is this only the case for those humans who live in cultures shaped by number, that is, industrialised societies? What, however, does this mean for those people in industrialised societies who show little to no capacity for utilising number? Are they at a disadvantage, and if so, what kind of disadvantage? If number is innate to humans – as an intuition that we share with other animals, as is claimed – then how is it that some humans are innumerate, in both industrialised and non-industrialised societies? Further, if number is a biological capacity that is shared by other animals, how is number also a human cultural artefact that is only rendered sensible by other cultural artefacts?

These questions are at the centre of a sub-discipline of cognitive science that is known as 'numerical cognition'. This sub-discipline is empirically driven. That is, the human relationship with number is determined through the empirical examination of behaviour, neural substrate and artefacts such as mathematics textbooks (e.g. Núñez, 2009). As can be determined from a quick glance at the numerical cognition literature, empirical findings have not converged on a single set of agreed claims about the human capacity for number. There are many points of difference. Important amongst these is a disagreement about whether the precursors for number are 'in the brain' (so to speak) or whether number is a wholly cultural creation.

Thesis argument and aim

I suggest that disagreements about the human capacity for number could be explained by the multitude of different tools that are used to capture it. By tools, I mean the research scope, the research question(s), as well as the research methodology. I further suggest that the disparate claims about the human capacity for number are due to some tools not being robust enough to capture this capacity. I refer to these tools as the 'hammer' – in reference to the old adage 'to the person with a hammer, everything looks like a nail'. My argument in this thesis is that some numerical cognition 'hammers' have either been too narrowly conceived and/or the presuppositions that inform them are insufficiently justified.

My thesis aim is to analyse two opposing positions on numerical cognition to demonstrate this point. I have chosen these two positions because of their level of influence on our understanding of the human capacity for number. The first position is the nativist position as it is explicated by the experimental and developmental psychologist, Elizabeth Spelke, a prolific contributor on the topic of numerical cognition, whose research is oft-quoted. She argues (e.g. 2011) that certain naturally-selected neural correlates contain the essential ingredients for number, which are then amplified into what she calls 'abstract number' by natural languages. The main sponsor of the second position – the language-embodied position[1] – is Rafael Núñez, who with linguist, George Lakoff (2000), argues that the human concept of number is culturally shaped by language metaphors, which, in turn, are informed by bodily movements that are shared by, and constrained in the same way for, all humans. I have chosen this second position to analyse because Núñez (2009, 2017) has, and continues, to criticise nativist positions.

The presuppositions that inform Spelke's nativist 'hammer' could be inferred from her preferred methodological approach. Similarly, many of the presuppositions that inform the language-embodied position could be inferred from its critique of other positions, most notably, nativist positions. I will argue that many of the presuppositions of both 'hammers', once made explicit, are not sufficiently justified. Further, the presuppositions of the language-embodied position do not explain its starting premise, namely, that number is shaped by language metaphors, which, in turn, are informed by bodily movements.

I will also argue that both Spelke and the language-embodied position have crafted their 'hammers' too narrowly. Both positions abstract the human capacity for number from the vast array of human cognitive capacities to study it on its own. While Spelke's position at least implicates an evolutionary story, studying number on its own leads her to utilise an evolutionary story that is implausible. The language-embodied position does not include any evolutionary story in its explanation.

[1] Núñez refers to his position as 'embodied cognition'. I have re-named it the 'language-embodied' position to distinguish it from other embodied positions, and to emphasise the role it gives to language metaphors.

Further, both positions only test for the factors that each thinks contribute to the human capacity for number. Spelke's position therefore does not try to determine what role the body plays in the explanation of this capacity, and the language-embodied position does not try to determine what role neural circuits play. I will argue that both omissions are question-begging.

Thesis statement

My first thesis statement is that for a robust 'hammer' to be crafted, the presuppositions that inform it have to be made explicit and suitably justified. My second thesis statement is that in order to theorise about the human capacity for number, one should not craft it too narrowly. This means allowing for the possibility that the body, neural circuits and culture might each play an explanatory role in the human capacity for number. It also means supplying a plausible evolutionary story that accounts for more than just number. Only then can the 'hammer' escape the pejorative claim of the old adage.

In light of these thesis statements, I argue that the following presuppositions best inform the crafting of the 'hammer'. First, that the exponential increase in human sociality put pressure on human societies to find cultural solutions to meet the demands of this exponential growth (Sterelny, 2007, 2014, 2017; Tomasello, 1999, 2008). This, in turn, led to the creation of external cognitive systems to supplement the limitations of biological memory (Donald, 1991, 1993), including – in some human societies – systems that evolved into number systems. The creation and learning of such systems involves the physical manipulation of artefacts (Menary, 2007, 2015) and the re-deployment of ancient brain functions to form new neural coalitions (Anderson, 2010, 2016).

Thesis structure

My thesis comprises four chapters. In Chapter 1, I will provide a brief survey of numerical cognition in order to put the two positions that I analyse in context, and discuss one pragmatic consequence of a well-crafted 'hammer'. In Chapters 2 and 3, I will critique, in turn, Spelke's nativist 'hammer' and the language-embodied 'hammer'. In the Chapter 4, I will justify the presuppositions that I think should inform the crafting of the 'hammer' to capture the human capacity for number, which I listed above.

CHAPTER 1: SETTING THE SCENE

Introduction

All positions on the human capacity for number have to account for how humans come to recognise the properties of number. My first purpose in this chapter is to specify these properties. What is under dispute is not what these properties are, but how an individual comes to learn them. Some positions argue that humans and other animals have a naturally-selected innate capacity to track quantities and magnitudes in the environment, which is then amplified into the properties of number. However, there is disagreement among nativist theories about what factors do this amplification work – whether it is experience alone, natural languages and/or something else. There are, however, theories that argue that the properties of number are wholly cultural creations. Such theories deny that number has innate precursors that are then amplified into the properties of number.

I argue in this thesis that in order to capture the human capacity for number, we have to craft a 'hammer' with enough width, as well as provide sufficient justification for the presuppositions informing this 'hammer'. I analyse two influential numerical cognition positions to demonstrate that they have not done this. To better understand these positions, however, they first need to be considered within the context of numerical cognition more broadly. The second purpose of this chapter is therefore to provide a brief survey of this sub-discipline.

There are pragmatic consequences for correctly capturing the human capacity for number. For example, correctly capturing this capacity could help us define and understand innumeracy, and determine whether and why innumeracy is a problem. The third purpose of this chapter is therefore to discuss what is known about this pragmatic issue, namely that innumeracy is currently understood in vague and multiple ways (Butterworth, 2011; Dowker, 2016; Peters and Bjalkebring, 2015). It is also not clear why innumeracy is a problem, especially given that some human communities do not rely on any explicit number concept, and yet have survived for centuries in their environments (Everett, 2017). My purpose here is not to define innumeracy but instead, simply to raise the point that how the 'hammer' is crafted to capture the human capacity for number also has pragmatic consequences.

This chapter is divided into four parts. In the first part, I will describe the properties of number. In the second part I will provide a brief survey of numerical cognition. In the third part of the chapter, I will discuss innumeracy. In the fourth part, I will clarify my thesis statement with this pragmatic consideration in mind.

1.1 The properties of number

For the purposes of this thesis, I want to distinguish the two different ways that quantity is discussed. In line with De Cruz, Neth and Schlimm (2010), I suggest that when nativist positions speak of approximate quantity discrimination and magnitude, they are speaking of numerosities. This can be distinguished from number, which is abstract and exact. The most basic properties of number are as follows:

- number is discrete;
- numbers are only rendered sensible by other numbers;
- numbers have to be placed in a fixed sequence;
- number systems have regularity; there is an equality of distance between numbers in a number sequence;
- number lines are linear;
- number lines are ordinal;
- numbers can be used to generate other numbers *ad infinitum*; number systems are recursive;
- numbers have cardinality; they do not pick out the properties of objects;
- numbers can therefore be used in a wide variety of tasks, including counting and comparisons; and
- number is expressed in either words or numerical notation. There are a wide variety of numerical notation systems, but the one that is predominantly used throughout the world is the Hindu-Arabic notation system (Chrisomalis, 2004).

De Cruz, Neth and Schlimm suggest that there are different levels of numerical competence. I suggest that the first two levels comprise recognition of the basic properties mentioned above. The third level of competence is to perform basic computations such as addition, subtraction, multiplication and division, to which I will add calculation of ratios and percentages. There are other more advanced properties of number, captured by mathematics. Mathematics can be utilised to generate other ideas, hence its usefulness in, for example, science and engineering. My thesis, however, is not about the capacity for mathematics. Instead, I simply consider how the 'hammer' should be crafted to account for human recognition of the basic properties of number and utilisation of basic arithmetic computations.

1.2 A brief survey of numerical cognition

In this section, I look at a range of positions on the human capacity for number. This survey, however, is not intended to be comprehensive or detailed, but, as mentioned, merely enables the two positions I analyse in Chapters 2 and 3 to be put into context.

There are many different nativist and cultural positions. Importantly, nativist positions do not sit on one end of a continuum with cultural positions on the other. Instead, different positions call on different cultural and/or nativist theories to craft their 'hammer' to capture the human capacity for number. Nativist theories either partially or wholly call on one or the other or both of the following neural theories – the approximate number system (ANS) and the object tracking system (OTS). I will therefore first describe these neural theories. I will then describe cultural positions that do not implicate any neural theories, before introducing the two positions that I will critique in Chapters 2 and 3.

Nativist positions and neural theories

As mentioned, nativist positions on the human capacity for number utilise one or the other, or both, of the following neural theories – Dehaene's (1997, 2011) approximate number system (ANS) thesis and Carey's (2001) object file tracking system (OTS) thesis.

ANS

The ANS theory argues that humans can distinguish among very small quantities (one up to three or four) in what seems like an exact way (subitizing), and distinguish among large quantities in an approximate way (what some call large quantity discrimination or LQD) providing there is sufficient distance among the quantities. Subitizing and LQD together are considered to be 'number sense' (Dehaene, 1997, 2011). Further, it is argued that there is a logarithmic relationship between the physical stimulus and internal representations, such that innate quantities map on to a roughly logarithmic scale. That is, humans have an innate 'number line', with small numbers on the left of the number line, and spaced apart, and larger numbers on the right of the number line and increasingly close to one another (Dehaene, Izard, Spelke and Pica, 2008).

It is argued that the 'number sense' and 'number line' were internalised following millions of years of human evolution in a structured environment (Dehaene, 1997; Shepard, 2001). That is, they were naturally selected. Empirical evidence is offered to demonstrate that 'number sense' and the 'number line' are innate capacities that are also shared by nonhuman animals (Brannon and Merritt, 2011) and are already present in human infants at the time of birth (Spelke, 2011).

A capacity for 'number sense' and the 'number line' has also reportedly been found in humans from cultures that only have a few words for numbers, for example, the Munduruku, an Amazonian Indigene community (Dehaene et al, 2008)[2]. The 'number sense' and 'number line' capacities demonstrated by members of this community is said to mirror those of infant humans in industrialised communities, and adults in western countries who are presented with numbers non-symbolically and under conditions that discourage counting.

Dehaene's (2011) own position about the ANS – a position not shared by all nativists – is that the logarithmic nature of number is partially retained throughout life. He argues that this

[2] Although findings from the Pirahã Indigene community, who have no clear words for number, are mixed (Everett, 2017).

explains why adults in Western countries who are presented with numbers non-symbolically, and under conditions that discourage counting, produce the logarithmic number line and not a linear one. Dehaene argues that the 'number line' is partially extended by formal education into a linear number system or what he calls a discrete number system (DNS), and what I call the properties of number, mentioned at the start of this chapter. Contrary to what Spelke (2011) argues, Dehaene denies that natural languages play any role in extending the 'number line' into the DNS (e.g. Amalric and Dehaene, 2016).

There is said to be empirical evidence that demonstrates that the ANS is neurally-mapped (Dehaene et al, 2004; Butterworth, 1999; Feigenson et al, 2004; Nieder, 2005). That is, single cell and neuroimaging functional studies on monkeys, and some case studies of humans with brain impairment, are said to demonstrate that the ANS is located in the intraparietal sulcus, and also implicates the prefrontal cortex (Piazza, 2011).

OTS

The OTS theory argues that quantity discrimination takes the form of objects that are tracked across space and time (Carey, 2001; Le Corre and Carey, 2007). This tracking is guided by three principles – objects move as bounded wholes, move in connected and unobstructed paths, and do not interact at a distance. These principles allow human infants and animals to perceive object boundaries and predict when objects will move. The number of objects that can be tracked, however, is said to be limited to three or four, and the capacity limit of the OTS is said to be reached in the first year of human life.

However, through the process of bootstrapping, the OTS is said to be amplified into the number properties that I described earlier (Carey, 2009). The metaphor that Carey uses to describe the bootstrapping process is the metaphor of extension (think of a ladder) from a foundation or base. That is, when children are taught the first few number names in the natural language of their community – in English, 'one', two', 'three', 'four' – they map these names on to objects they have been naturally tracking via the OTS. Once mapped, children slowly build an exact count system that extends beyond the first four quantities or OTS limit. In the building of this count system, they learn to put numbers into sequence (ordinality), which, over time, they learn to add to (recursiveness). They also acquire the following insights: count words do not pick out the properties of an object or name objects like other words in the natural language do (cardinality); it does not matter in which order one counts a set of objects; and many different sets lend themselves to be counted (e.g. toes, stuffed toys, dogs).

The neural correlates for the OTS are less clear than those for the ANS, but the two systems appear neurally distinct (Piazza, 2011).

There is disagreement among different nativist positions about whether the ANS or OTS or some combination of the two provides the foundation for number (Piazza, 2011). Butterworth (2011) claims that neither the ANS nor the OTS on its own can account for dyscalculia (which is a disability with number that I will discuss in greater detail in the third section of this chapter). His nativist position is one of a few that builds on neural theories that extend beyond the ANS and OTS.

Cultural positions on number

There are positions, however, that do not make any reference to neural correlates for number and, in fact, appear to eschew all neural explanations. I will call these positions wholly cultural positions. One such position comes from situated cognition, and has informed some empirical research on mathematical learning (Weber, Walkington and McGalliard, 2015). The argument here is that number and number calculation is a social practice within a community of practitioners who give this practice meaning (Lave, 1988). Outside this community of practitioners, the capacity for number is not only meaningless, it is also impossible. New initiates into this community do not accrue knowledge so much as become familiar with the norms and practices around number (Weber et al, 2015). The more an individual becomes familiar with these norms and practices, the better able she will be to generalise these practices to a wide variety of contexts where number is also meaningful (Greeno, 1997)[3].

Another theory that has recently gained influence in mathematical pedagogy draws on semiotics (Presmeg et al, 2016). It explains calculating with numbers almost entirely in terms of using symbols. That is, numbers are symbols and these symbols refer to something (Moreno-Armella and Sriraman, 2005). What they refer to, however, is arbitrary. Nevertheless, it is something that is understood by a community of users of this symbol system. The meaning of each symbol is given by the network of relationships among the symbols. As the individual's understanding of this network improves, individual symbols change their meaning for her – they become more transparent and nuanced for her. Correctly learning how to calculate with symbols therefore involves understanding the network of symbols. With increased practice, she is exposed to different aspects of the symbol system and how individual symbols might creatively be combined according to the norms of the community.

Now that I have provided a brief but far from comprehensive survey of numerical cognition, I can turn to the two opposing positions that I will analyse in detail in Chapters 2 and 3. By suggesting that these two positions are opposing, I do not mean to suggest that they debate one another. Spelke (e.g. 2011) makes no reference to the language-embodied position. The language-embodied position (Lakoff and Núñez, 2000), however, does take issue with nativist positions, but makes no specific reference to Spelke's nativist position.

---

[3] I do agree with this position's claims about norms and generalisability (as will be demonstrated in Chapter 4); however, I think other aspects of this position lack merit. I similarly think that the following position from semiotics is also problematic in parts. Given these positions exert less of an influence on our understanding of numerical cognition than the two positions that I analyse for this thesis, I will reserve my critique of these and other positions for a larger project on number cognition, outlined in the Thesis Conclusion.

Spelke's nativist position

The starting premise of this position is that number is ubiquitous, universal and unique (Spelke, 2011). It is considered to be ubiquitous and universal because number is implicated in the most complex human activities, like the building of bridges, washing machines and scientific theories, to the most quotidian human tasks such as baking, buying a train ticket, and annually declaring one's income tax. As such, Spelke argues that humans not only have an affinity with number, we also rely on it quite heavily. She argues that number is unique because, while most human concepts are shaped by experience, number is too abstract a concept to be produced by experience alone. She argues that the evidence for this is as follows: our experiences are finite, and yet number is infinite; most concepts apply to some things but not others, yet number applies to everything. She considers these features of number to be puzzling.

To get at this puzzle, Spelke argues that a multi-pronged approach has been designed, which has generated converging evidence that demonstrates that humans have an innate predisposition for 'number':

- Comparative studies of humans and non-human animals demonstrate that many species, including humans, have ANS and OTS capacities, which, in turn, demonstrates that these capacities have been naturally selected.
- Comparative studies with humans of different ages – from birth to old age – and from different cultures, demonstrate that ANS and OTS capacities are present at birth and do not diminish over age and educational attainment.

I will critically analyse Spelke's presuppositions in Chapter 2.

The language-embodied position

This position argues that number is culturally constructed (e.g. Núñez, Edwards and Matos, 1999; Lakoff and Núñez, 2000; Núñez, 2009, 2017). This is a two-part claim. The first part is that number does not exist in the world independently of human thought. The second part is that the ANS and OTS cannot be scaled up into number. Instead, number is an abstract concept, and this concept is given by the common metaphors in natural human languages. The latter, in turn, are the surface manifestations of human thought, which is informed by our shared biological experiences in the world. That is, we form thoughts based on our movements in the world – which are shared among all humans and constrained in the same ways for all humans by biology. These thoughts generate metaphors in the natural language, which are roughly shared across different human languages, and shape our concept of number and number calculation. This accounts for why number calculations are non-arbitrary and constant across human time and cultures.

Núñez (2011b) argues that uncovering the human capacity for number involves measuring speech-gesture coproduction during number use that are not monitored by the agent, analysing body movements during number communication, and recording the amount and type of allusions to image-schematic structures in the course of inferential mathematical tasks.

I will critically analyse the presuppositions of this position in Chapter 3.

1.3 One pragmatic consequence of a well-crafted 'hammer'

One pragmatic consequence of correctly understanding the human capacity of number is that it helps us define and understand innumeracy and determine whether and why it is a problem. Dowker and Cohen Kadosh (2015) cite concerns about innumeracy as one of the main drivers of the exponential increase in numerical cognition studies over the last two decades. The proportion of children who experience numerical difficulties in the classroom is said to be too high, as is the proportion of adults who are not able to apply their knowledge of number for the successful completion of everyday tasks. This includes comprehending statistical information that is routinely reported in the daily newspapers (Peters and Bjalkebring, 2015).

No consensus has been reached, however, about what constitutes innumeracy and how it should be captured. Butterworth (2011) argues that dyscalculia (as defined by the United Kingdom Department of Education and Skills, 2001) seems to capture innumeracy – it seems to pick out the same construct as 'mathematical disability', as it is defined in DSM-IV and the 'arithmetic skills disorder' as defined by the International Classification of Diseases (World Health Organisation, 1994). However, Butterworth also demonstrates that the core deficits of dyscalculia are still unclear. Deficits take different forms in different patients who are thought to have the problem and the neural basis of this problem cannot be clearly identified.

Currently, the assessment instruments designed to capture innumeracy are too blunt. They seem to also capture mathematical anxiety (Dowker, 2016). Mathematical anxiety is said to disrupt performance on tasks involving numbers – either by causing people to avoid number tasks or by overloading and disrupting working memory during number tasks. So some but not all forms of mathematical anxiety seem to overlap with innumeracy. There is, however, no *a priori* theory to explain this – in part because there is no clear definition of innumeracy.

Peters et al (2006) did attempt to generate a definition of numeracy from findings from ethnographic studies, which reported that less numerate people (among college students in Western Europe and North America), when presented with newspaper reports of probabilities, were unable to interpret these probabilities correctly because they paid too much attention to irrelevant affective considerations. However, there is no agreement that innumeracy comprises, or is limited to, the understanding and use of probabilistic and mathematical concepts. Further, these concepts are not clearly specified for empirical study.

A historical analysis of the term also does not yield a clear definition. The term numeracy seems to have made its first appearance in the "Crowther Report" (HMSO, 1959). It was said to be the minimum knowledge of mathematics and scientific subjects that a person should possess to be considered educated; and the ability to think quantitatively and to avoid statistical fallacies. This definition of the term is considered to be too vague to be of any pragmatic use (Withnall, 1995). Since 1959, numeracy has been referred to in at least three different ways in both school curricula and in discussions about citizens' minimum civic responsibilities (e.g. Willis, 1998), and this continues today (e.g. australiancurriculum.edu.au/f-10-curriculum/general-capabilities/numeracy; nationalnumeracy.org.uk). The first way equates numeracy with mathematical concepts and procedures; the second with performance on quotidian tasks that involve numbers; and the third with problem-solving using logic. It is unclear what these three ways of conceiving numeracy have in common. Further, they have not been operationalised for empirical study.

More problematically, capturing the causes of deficiencies in any or all of these capacities is difficult. The tools available for capturing the physiological changes to the brain and body during experimental tasks are, at this stage, too blunt (Dowker, 2016). Many studies have to rely on self-report; however, people's explanations of their own performance have been found to be unreliable. Peters and Bjalkebring (2015) found that people are not even able to estimate their performance on number tasks, let alone explain them. For example, men are more likely to overestimate their performance scores and women to underestimate them.

In summary, there is no clear definition of innumeracy, no clear idea about what the correlates of innumeracy are, and no means to capture and explain innumeracy and its correlates. I argue that the first step to understanding the pragmatic problem of innumeracy is to provide a clear understanding of the human capacity for number, which means crafting the 'hammer' well to capture it.

Nativist 'hammers' do not appear to shed light on innumeracy. This is because nativists assume that the ANS, the OTS, or some combination of the two, is responsible for humans having a number capacity. However, number deficits have not been found to clearly implicate the neural correlates associated with the ANS or OTS (Butterworth, 2011). Nor can innumeracy be easily explained by how the ANS or the OTS is amplified into number properties. This is because it has yet to be adequately explained how we came to have the capacities responsible for this amplification. For example, Carey's (2001) bootstrapping thesis suggests that number words are responsible for the amplification of the OTS into number; however, she does not adequately explain how we came to have number words.

Wholly cultural 'hammers' also do not seem to be able to assist with pragmatic issues such as innumeracy. As Pantsar (2014) argues, the properties of number and basic arithmetic can never be completely conventional. Something is required to give number its objective foundation – to explain why number is consistently understood across different times and places where number has been present. Wholly cultural theories do not provide this objective foundation, nor do they explain under what conditions groups and individuals will fail to have a capacity for number despite this objective foundation.

1.4 Expanding on the two thesis statements

I argue that finding this objective foundation, as well as explaining deficits despite this objective foundation, requires crafting a 'hammer' with sufficient width and justification. Both nativist and cultural positions fail to do this. Nativist positions craft the 'hammer' without any reference to bodily movements, thereby precluding these movements from playing an explanatory role in the human capacity for number. I will demonstrate why this is a problem in Chapter 2 in relation to Spelke's nativist position. Cultural positions craft the 'hammer' by leaving the neural circuits out of the explanatory picture without justification, and by not providing an evolutionary story to explain the emergence of number in human history. I will demonstrate why this is a problem in Chapter 3 in relation to language-embodied position.

In Chapter 4, I will argue that a 'hammer' robust enough to capture the human capacity for number has to be crafted with an eye to the emergence of the modern human mind, and cannot afford to ignore the potential explanatory role neural circuits, the body and culture might each play. I will argue that the following presuppositions will ensure that the crafted 'hammer' is robust: Following Sterelny (2007, 2014, 2017) and Tomasello (1999, 2008), I argue that the exponential increase in human sociality drove human communities to expand their cognitive repertoire. This included creating what Donald (1991, 1993) calls external cognitive systems, of which number systems were some examples. Following Menary (2007, 2015), I argue that learning, and expanding on, such number systems involves the physical manipulation of artefacts, which is only made possible by redeploying ancient brain functions to create new neural coalitions (Anderson, 2010, 2016; Anderson and Penner-Wilger, 2013).

I also demonstrate in Chapter 4 how such a 'hammer' might help us make deductions about a pragmatic issue such as innumeracy, namely, identify the forms it could take and speculate about its causes.

Conclusion

In this chapter, I outlined what the properties of number are and provided a brief survey of numerical cognition. Specifically, I discussed the key neural explanations utilised by nativist 'hammers' – the ANS and OTS – and cultural positions on number that eschew these neural explanations. I also discussed how a well-crafted 'hammer' could have positive pragmatic consequences, for example, by explaining innumeracy.

In Chapter 2, I will discuss why Spelke's nativist 'hammer' falls short of capturing the human capacity for number. In Chapter 3, I do the same with the language-embodied 'hammer'. In Chapter 4, I justify the presuppositions that create a well-crafted 'hammer'.

CHAPTER 2: SPELKE'S NATIVIST POSITION

Introduction

My aim in this chapter is to analyse the presuppositions that inform the nativist 'hammer' that is endorsed by Spelke (2011) to capture the human capacity for number. While Spelke has not made all her presuppositions explicit, they can be inferred from her preferred research approach. Her presuppositions appear to be that the human capacity for number is a product of specialised modules, and these modules are shaped by natural selection, and amplified into number by natural languages. More specifically, her presuppositions are as follows:

a) The ANS (comprising subitizing, LQD and the 'number line' as described in Chapter 1) and the OTS are the neural ingredients for number.
b) One way to demonstrate the innateness of these capacities is via converging evidence from multiple sources.
c) Humans share neural and biological similarities with some non-human animals as a consequence of shared ancestry (homology), and shared features with unrelated species as a result of similar evolutionary pressures (convergent evolution). If other species show evidence of having ANS capacities, and some of OTS capacities, then this contributes to converging evidence that the ANS and OTS are innate capacities that were naturally selected.
d) Newly born and young human infants have not had the benefit of cultural education. If infants show evidence of having ANS and OTS capacities, then this contributes to converging evidence that the ANS and OTS are innate human capacities.
e) Human culture is diverse, and this is reflected in differences in language, cultural artefacts and access to cultural education about numbers. If adult humans from multiple and varied cultures all show evidence of having ANS and OTS capacities despite these cultural differences, then this contributes to converging evidence that the ANS and OTS are innate human capacities.
f) Primate studies reveal that the ANS and the OTS can be mapped on to neural substrate. Given neural homology between humans and other primates, this contributes to evidence that the ANS and OTS are neural modules. Nevertheless, Spelke does think more evidence needs to be generated – namely comparisons across cognition, action, brain systems, neurons and genes – in order to probe the systems on which our numerical concepts depend.
g) Natural languages convert the ANS and OTS into the number properties mentioned in Chapter 1 – via bootstrapping, which was also described in Chapter 1.

My argument in this chapter is that presuppositions (c), (d) and (e) cannot be sustained. As Spelke thinks that (f) requires further explanation than is currently available, I will not address (f) directly. Rather, I will discuss my concerns about (f) in relations to my concerns about (c). Specifically, I do not think the argument for cognitive homology, and explanations for cognitive skills that rest on convergent evolution, can be sustained. I will not discuss (g) in this chapter as this would require a separate thesis; however, if (c), (d) and (e) are called into question, then (g) cannot also be true. Further, I argue that for (c), (d), (e) and (f) to represent convergent evidence (b), an *a priori* explanation has to be provided for what these sources of evidence have in common to potentially stand together as convergent evidence. I will argue that this *a priori* explanation has not been provided. It might be inferred, in part, from Spelke's preferred methodology; however, the inferred explanation cannot be sustained.

I also argue in this chapter that Spelke crafts her 'hammer' too narrowly in two ways – by abstracting number from the vast array of human cognitive capacities to study separately, and by concentrating her explanation on the brain to the exclusion of bodily movements.

This chapter is divided into three parts. In the first part, I will expand on Spelke's position. In the second part, I will demonstrate that Spelke's presuppositions (c)&(f), (d), (e) and (b) cannot be sustained. In the third part of the chapter, I will demonstrate that Spelke's 'hammer' is too narrow.

2.1 Summary of Spelke's nativist position

Spelke's (2011) argues that the ANS and OTS are naturally selected core systems that can be amplified into number by natural languages. Further, each system can be mapped on to neural substrate (Piazza, 2011). That is, behavioural studies with pre-verbal human infants, non-human animals, and adult humans in different cultures, as well as neural studies with primates, provide convergent evidence that these two core systems are innate and naturally-selected.

Neural studies with primates indicate that the neural module for these core systems implicates at least the intraparietal sulcus, in which there are single neurons that are tuned to individual numbers up to four (Nieder, 2011). Comparisons over human development, and across human cultures, demonstrate that the ANS and OTS are present at birth and throughout life and do not depend on educational attainment (Spelke, 2011). Further, comparisons across human groups and other species reveal these two core systems have the following signatures:

- The first signature is that the ability to discriminate one quantity from another depends on the ratio of the two values (e.g. Izard et al, 2009).
- The second signature is that, at any given age, the same ratio limits are present for different types of arrays, whether concrete or abstract, and for different modalities (e.g. Lipton and Spelke, 2003; Wood and Spelke, 2005).
- The third signature is that human and non-human animals do not just discriminate among quantities but can order them to some degree (e.g. Brannon, 2002).
- The fourth signature is that numerical discrimination is impaired or set aside when arrays are presented under conditions that favour the attentive selection and tracking of individual objects (e.g. Feigenson et al, 2002).
- The fifth signature is the ability to spontaneously relate changes in magnitude to other quantitative variables, for example, line length (e.g. de Havia and Spelke, 2010).

Spelke (2011), however, suggests that further work needs to be undertaken. That is, genetic and structural (both neural and cognitive) underpinnings of these core systems need to be clearly identified. Nevertheless, neural evidence is said to corroborate behavioural evidence that there are two distinct core systems on which numerical concepts depend – the ANS and OTS (Piazza, 2011). Further, the two striking limits of these systems – namely, that they are imprecise, and they fail when objects are presented individually and tracked over time and occlusion – are said to be overcome by culture. That is, natural languages make quantity discrimination exact, and extend it beyond subitizing range.

2.2 Analysing Spelke's presuppositions

In this section, I will discuss, in turn, what is problematic about Spelke's presuppositions (c)&(f), (d), (e) and (b). Given the aim of my thesis is to critique the presuppositions that inform the 'hammer', and not the data generated by the 'hammer', I will keep my discussion of empirical findings brief – enough to reinforce my concerns about Spelke's 'hammer'.

Non-human animal studies: (c) and (f)

Spelke's arguments here appear to be as follows: Adult humans possess ANS and OTS capacities. Humans share neural homology with their primate cousins. If primates show evidence of ANS and OTS capacities, this would mean that ANS and OTS capacities are innate to humans. Further, close examination of primate brains will also identify the neural correlates for the ANS and OTS in human brains. If we also find evidence of ANS and OTS capacities in other species unrelated to humans, then we could also conclude that ANS and OTS capacities were the result of environment pressures acting on several species (convergent evolution). That is, we could conclude that the ANS and OTS were naturally-selected.

Further, the best way to determine shared innate capacities among species is to give every species similar quantity tasks to complete – in experimental settings where the influence of extraneous factors are limited.

I will first discuss the problems with the homology and convergent evolution arguments before briefly discussing my concerns with key empirical findings from animal studies.

Convergent Evolution and Homology

The cognitive theory of convergent evolution is as follows. Cognitive processes, as they are expressed behaviourally, are biological adaptations with evolutionary histories (Seed et al, 2009). By mapping similarities and differences in cognitive abilities among unrelated species with similar structures, and/or with different taxa in shared environments, we can uncover what biological characteristics are shared by unrelated groups of organisms who occupied the same environment. We could also conclude that these shared characteristics were a result of evolutionary pressure. Convergent evolution therefore makes it possible to discover the proximate mechanisms that produced similar cognitive outcomes in two or more lineages.

Homology is shared characteristics among species that are inherited from a shared ancestor (Seed and Tomasello, 2010). Homology is said to be potentially useful for extrapolating from findings from one species to understand another, homologous species. Humans share a common biological ancestor with orangutans, gorillas, bonobos and chimpanzees. Homology therefore enables us to generalise experimental findings from non-human primates to humans. This is particularly useful given the invasiveness of neural experiments.

Convergent evolution allows the young from other species to shed light on the early development of humans, which is useful given the time-consuming nature of behavioural studies with human infants. That is, the young of some species might be more experiment-ready than the young of others. With atricial species – such as humans – the young experience a longer ontogeny, during which time they are dependent on caregivers for basic needs. With precocial species – such as domestic chicks – the young are relatively mature and mobile soon after birth or hatching. These chicks therefore make easier test subjects than human infants. Further, birds share some neural homology with humans but no immediate ancestor (Haun et al, 2011). Findings from bird studies might therefore provide the best source of information about humans.

However, there are many problems for establishing shared cognitive capacities using convergent evolution and homology. It requires generating many, many comparative studies across a very large range of species to identify the selective processes that are, for example, common to certain birds and apes, yet which are exclusive to them (Haun et al, 2011). It is also challenge to explain how there might be similarities in cognition between primates and avian species despite divergences in both brain size and large proportions of brain structure. Bees are also said to demonstrate counting skills (Pahl, Si and Zhang, 2013). But how are birds and bees, with smaller brains than humans, tracking approximate quantities in the environment in the same way that humans do? This has yet to be explained. Generalisability from prococial species like birds to humans is therefore premature.

There are also difficulties with demonstrating cognitive homology among primate relatives with whom humans share neural homology. Determining homology in cognitive abilities among genetic relatives requires a particular statistical technique, namely, the phylogenetic comparative method. That is, the analysis of diversity in cognitive ability across species utilises methods that control for the hierarchical relatedness of organisms through the branching process of descent (e.g. Harvey and Pagel, 1991). The methods used to build phylogenetic trees that describe species relationships and track evolutionary changes on these phylogenies rely on inferences from gene sequence data (Felsenstein, 2004; Lemey, Salemi and Vandamme, 2004). However, morphological (Wiens, 2000) and behavioural (Kennedy, Spencer and Gray, 1996) data can, and is often, used. The morphological and behavioural data then inform tree structures, and from these structures, ancestral behavioural, cognitive, morphological and cultural traits are all inferred. These trees also enable the directionality of a trait change, and models of evolution, to be tested.

However, to build phylogenetic trees requires collecting data from different species using experimental methods that are suited to each species equally. Here lies the conundrum for numerical cognition. How do we design studies whose methods suit each species equally given that even species that share neural homology differ in many other ways that impact on experiments? Most notably, other primates do not have language, and therefore cannot be easily directed to perform a cognitive task. We could let our nearest cousins reveal their cognitive abilities in the wild and non-experimentally; however, what criteria do we use to recognise cognitive abilities similar to ours, and how do we control for random noise?

There are deeper problems with relying on convergent evolution and homology than the problems mentioned above. That is, I question the assumption that cognitive processes, as they are expressed behaviourally, are biological adaptations with evolutionary histories. Barrett (2011), for example, argues that cognition might not always be innate but instead, could sometimes emerge from the organism's interaction with its environment. Further, what is innate and what is emergent cannot be inferred by looking at the animal's behaviour alone.

To explain, Barrett argues that different species are designed to respond to the environment in ways that are relevant to them. So, for example, the perceptual systems of each species seem geared to recognise certain things that are salient about the environment *for them*. So even though different species share the same environment, they interact with this environment differently, and this is constrained by their biological capacities, including bodily movement and perceptual view. Human ancestor's bipedalism, for example, enabled them to track their prey with forward-facing eyes and literally by following in their steps. These ancestors appeared to show a tendency to form particular narratives about their prey – for example, where they came from and what they will do next (Jeffares, 2014). This cognitive tendency might have been emergent behaviour contingent on bipedalism.

To understand how this works, it cannot be assumed that perception and cognition are mutually-exclusive, or that the former is passive while the latter is active (Barrett, 2011). Instead, animal brains might operate like tracking devices, where perception enables the organism to take advantage of opportunities in the environment that allow for action. Given animals are active explorers of our environment, it seems highly likely that perception, action and opportunities in the environment engage with each other in a dynamical loop.

So to understand another species, we need to examine the environment from the perspective of their perceptual abilities and biological constraints before looking at what might be happening in their brain. This is something all nativist approaches fail to do. Instead, they consider genes and brains in isolation in order to theorise about evolutionary processes. They ignore the importance of a subtly changing environment, and crucially, the interaction between the environment and genes. That is, they assume that the environment is something static to which genes adapt, which begs the question. As such, they assume behaviour reflects innate cognitive abilities.

I will use an example provided by Barrett (2011) to illustrate why the innateness of a cognitive ability cannot be inferred from an animal's behaviour alone. Consider an ant on the beach that is following a complicated path. Observing this, we might infer that the ant has a naturally-selected complicated map in its brain that is guiding its movements. However, the complexity of the ant's path on a beach might be explained by the complexity of the beach itself and not by the ant's internal state(s). That is, the behaviour we observe might not be pre-programmed but rather, emerge from the ant's interactions with its environment within the limitations of its perceptual capacities.

Given this, if an organism displays a capacity for quantity and magnitude, this might be an emergent behaviour and not a naturally-selected cognitive capacity for quantity. That is, the physical environment might be ordered in patterned ways (within which quantity and magnitude might play an important ordering role), and the sensori-motor system of the organism might pick up these regularities to act in the interests of their survival. That is, quantity- and magnitude-related behaviours emerge from the perception-environment-action loop mentioned earlier. As such, rather than reflecting an innate capacity, quantity tracking might instead reflect a naturally-selected biological capacity to track the environment and respond to it in a flexible way towards survival.

Given each species is physically constructed in a different ways, each species would have different ways of being attuned to the environment and responding to it. Even among species that have the capacity to respond to their environment by tracking quantity, this tracking need not be an inevitable emergent behaviour, even in experimental settings that are structured to elicit this behaviour. This may explain why in experiments with non-human species, methods are unable to consistently elicit the correct quantity responses from non-human animals. I argue that healthy adult humans in numerate societies consistently obtain correct results because learning and practice shape the perceptual-environment-action loop to produce this emergent behaviour. I will discuss this in greater detail in Chapter 4.

Given this, identifying a common cause for similar behaviour that is observed in different species is not straight-forward. If we assume biological homology and/or convergent evolution in advance of eliciting data, then we run the risk of rendering invisible any dissimilarities in responses between species. For example, in quantity and magnitude studies, the difference in findings between healthy adult humans in numerate societies and every other experimental subject, where the former get the task correct 100 per cent of the time, while the latter do not.

Empirical findings from animal studies

In one key experiment, monkeys were presented with different quantities of sweets (apple slices in this instance), which were initially hidden from view (Hauser, Carey and Hauser, 2000)[4]. The monkeys were then allowed to select which hidden treats they wanted. Their choice was between one and two treats, or two and three treats, or one and three treats. They selected the larger quantity over several trials for each monkey. These findings are taken to suggest that rhesus monkeys have an innate capacity to subitize for small numbers; they were not able to distinguish between larger numbers of treats.

What is not always reported in detail in experimental studies is that it takes a period of time to habituate nonhuman animals to experimental conditions, including getting them to a point where they are capable of undertaking the experimental task. I am not suggesting that this information is deliberately omitted. Instead, experimenters do not acknowledge that the experiment-ready processes prime test subjects towards certain emergent behaviours.

More problematically, Núñez (2017) points out that in many animal studies, the performance of animals on experimental tasks are reported as average likelihood scores, which are considered a success if they are greater than chance (e.g. Jordan, Maclean and Brannon, 2008). Yet, as Núñez correctly argues, if quantity tracking and magnitude estimation are innate capacities, non-human species should obtain correct results in every trial, which they do not.

Human infant studies: (d)

Spelke's argument here is as follows: Infants are not exposed to human cultural norms. If infants display evidence of ANS and OTS capacities, then these capacities must be innate. The best way to determine that infants have the same capacities as adults is to give them similar quantity tasks to complete – in experimental settings where the influence of extraneous factors are controlled.

I will first discuss Spelke's claim that infants are not exposed to cultural norms prior to birth, before raising concerns about key empirical findings.

No learning in the womb

The problem with this presupposition is that it is not accompanied by any evidence to demonstrate that the foetus is incapable of learning regular patterns in the mother's environment. There might be some degree of auditory tracking that occurs in the womb. There is also evidence to suggest that the foetus looks at her hands, wiggles her fingers, and engages in repetitive and exploratory behaviour in the womb (Everett, 2017). It would be worth investigating whether some or all of this acts as a priming device for quantity tracking and magnitude discrimination post-birth. If so, then it might be this priming in the womb, and not something innate, which explains post-birth experimental findings.

[4] This was *not* one of the studies for which Hauser was subsequently discredited.

Further, Barrett (2011) argues that human infants might be born with some experience-expectant genetic inheritance, which converts into abilities rapidly after birth. For example, infants are predisposed to recognise important things in the environment, including a caregiver's face and voice – so that they can direct their needs at, and learn from, their caregiver. And caregivers might be directing the infant's attention to quantities from the time of birth – particularly in numerate societies where quantity recognition pervades culture in non-obvious ways (Budd, 2015).

Empirical findings from infant studies

I also question whether infant studies elicit cognitive capacities as opposed to just perceptual capacities.

Spelke (2011) reports that the methodologies adopted for infants studies include measuring preferential looking, looking time habituation, anticipatory head turning and exploratory reaching. This suite of strategies was devised over 30 years ago by Wynn (1992) and were considered semi-revolutionary (Everett, 2017). Unlike with previous experimental methodologies, Wynn's method places no demands on infants to guess the goal of the experiment. Infants also do not have to physically participate or interact with the experimenters in order to complete the task. This method assumes that all humans, including infants, fixate on novel stimuli, which, in infants, takes the form of gaze length and sucking frequency. Given this, an electronically modified pacifier and video capable of tracking gaze are used to capture this information.

Wynn's methodology is still used today (Everett, 2017). Infants are placed in front of a display case with an opaque screen that can be lifted to block the infant's view of the doll-like displays. There are also gaps on each side of the screen. This allows infants to see that the experimenter's hand is adding or taking away a doll from the display before the screen is raised to reveal the number of dolls in the display. It is said that several experiments demonstrate that infants' register surprise when what they are shown does not correspond to what they saw the experimenter do. For example, the infant sees one of two dolls removed in the gap in the screen. Researchers suggest that she is therefore expecting to see only one doll when the screen is lifted. If, instead, two dolls are present, the infant gazes longer and/or increases her sucking frequency, which is taken to suggest that she is puzzled. These methods are said to generate findings that demonstrate that infants are able to perform rudimentary addition and subtraction. That is, they show evidence of recognising when only one doll should remain in the display and not two, or vice versa.

I argue that the questions Heyes (2014) raises about the interpretation of findings from mind-reading studies could be applied here. Both types of studies use similar methodologies, namely dolls in make-shift theatres, and measurements of sucking frequency and gaze length. In line with Heyes, I argue that it is not clear whether infants are responding to low level novelty in the theatre when dolls are removed or added or to the fact that, for example, there is one doll left when there should be two. That is, sucking frequency and gaze length might have something to do with the number of dolls present or it might be related to the new spatiotemporal relationships between objects in the infants' looking range. Further, this perception of novelty might depend on how the infant was made test ready – that is, how much attention she gave to spatio-temporal features during this test-ready stage, and what was retained in her memory about these spatio-temporal features. Her responses during experimental tasks might therefore reflect how much, and the nature of, spatio-temporal information retained in her memory during the test-ready stage. That is, the findings from these studies are consistent with both the cognitive account Spelke (2011) provides and the perceptual account I have just outlined; however, the perceptual account is the more simple and straight-forward of the two accounts.

Cross-cultural studies: (e)

The presupposition here is that if human culture is diverse – reflected, for example, in the amount of number words in the spoken language – then if ANS and OTS capacities are found evenly across cultures, this contributes to the evidence that the ANS and OTS are innate human capacities.

As with human infant and animal studies, I suggest that cross-cultural experimental findings could be explained differently than how the experimenters account for them. Experimenters emphasise the similarity of the experimental tasks used across different societies, and the proportion of correct responses to these tasks. However, I think the most salient feature of cross-cultural studies, taken together, is that members of non-numerate communities are much less likely to achieve perfect experimental scores than healthy adults from numerate societies (Everett, 2017).

What appears under-reported in studies – perhaps because the experimenters consider it irrelevant – is when and how members of each community track quantities and magnitudes in their quotidian lives. This might explain the mismatch in scores mentioned above. That is, the environments in which non-numerate communities live differ from the ones that numerate communities live in. That is, Indigenous communities in the Amazon, Africa and Australasia do not share the same ecological environments as numerate communities, or, for that matter, each other (Everett, 2017). Ecological features might impact on the amount and type of quantity and magnitude tracking of the environment that occurs in each community, and how this is shaped by culture (that is, how the perceptual-environment-action loop is shaped to produce quantity-related emergent behaviour). This is what cross-cultural experimental findings might reflect. It is known that culturally-different human groups make sense of the experimental tasks in accordance with patterns in their daily lives (Henrich, Heine and Norenzayan, 2010).

What makes me question the interpretation of findings from cross-cultural quantity studies is the following. Everett (2017) reports that his mother attempted to teach Pirahã communities how to count. Given they did not have number words in their vocabulary, Everett's mother borrowed number words from the nearest numerate community. Despite this, this community never learned to count. This could not have been due to any cognitive deficit on their part – their cognitive abilities in other domains ensured their continued survival in their environment over many centuries. Perhaps they do not possess the capacities of ANS and OTS, or they might possess these capacities but number words alone are insufficient for amplifying these capacities into number and counting, or the ANS and OTS are unrelated to number. That is, this anecdote raises questions about the accuracy of one or all of the key explanatory ingredients of Spelke's position on number.

Convergent evidence: (b)

As mentioned, for (c), (d), (e) and (f) to represent convergent evidence (b), then it has to be explained *a priori* what these sources of evidence have in common to potentially stand together as convergent evidence. This *a priori* explanation is not provided; however, one could infer that this explanation is linked, in part, to homology and convergent evolution. That is, different species have environments and/or neural structures in common. Therefore if we submit different species to similar experimental tasks and the responses obtained are roughly the same, this can be taken as evidence of the innateness of these capacities as accounted for by convergent evidence and homology. If I have inferred Spelke's *a priori* explanation correctly, then it cannot be sustained for the reasons I gave earlier. That is, the cognitive responses of different species cannot be accounted for in terms of shared neural structures alone or shared environments. Further, convergent evolution and homology are required to do double duty, namely, justify the 'hammer' and explain the findings generated from this 'hammer'. A circular argument appears to be in play here. That is, factor *x* determines how we craft our 'hammer'; factor *x* explains the findings generated by the 'hammer'.

2.3 The narrowness of Spelke's 'hammer'

I believe Spelke crafts her 'hammer' too narrowly in two ways. First, like other nativists, she assumes that in order to understand the human capacity for number, this phenomenon should be abstracted from the vast array of human cognitive capacities for separate study. This first assumption blinds nativists to all evolutionary explanations for the emergence of number in human society other than natural selection of quantity. I will argue in Chapter 4 that a more plausible explanation for this emergence is the exponential increase to human sociality. This better explains why number did not appear consistently in all human communities (and not at all in some of them) or across the human time scale.

Second, she assumes that to understand cognitive phenomena, we must first examine the brain, implicitly precluding the body from any explanation. This assumption begs the question – no justification is provided for why the brain is given the main role in the explanatory story of numerical cognition. Further, no reason is given for why the body is excluded from this explanatory story or why culture plays a secondary role (namely, natural languages amplify the ANS and OTS into number).

Conclusion

Spelke argues that the ANS and OTS are innate capacities shared by multiple species, which in humans, are amplified into number by natural languages. In this chapter, I raised doubts about whether the ANS and OTS are capacities shared by humans of all ages and cultures and by multiple species. I also raised doubts about the innateness of these capacities even in adult humans in numerate societies who consistently demonstrate this capacity. Instead, the ANS and OTS might be learned and emergent behaviour in these adults – a consequence of the cultural scaffolding of the dynamic relationship between perception, the environment and action as it relates to quantity and magnitude. In this chapter I also argued that Spelke crafts her 'hammer' too narrowly by focussing almost exclusively on the brain without sufficient justification, and by abstracting number for separate study. This closes off alternative, more plausible, explanations for the emergence of number in human life.

CHAPTER 3: THE LANGUAGE EMBODIED POSITION

Introduction

This chapter examines the presuppositions of the language-embodied position. As mentioned in Chapter 1, the name I have given to this position does not correspond to the one adopted by its main proponents. I have coined this new name as the language-embodied position is eager to distinguish itself from other theories of embodied cognition (Núñez et al, 1999). Other theories either suggest that embodiment entails a conscious awareness of one's bodily experience, or that it involves the physical manipulation of tangible objects or the virtual manipulation of graphical images.

By contrast, the main claims of the language-embodied position are as follows (Núñez et al, 1999; Lakoff and Núñez, 2000; Núñez, 2009, 2011a, 2011b, 2017): The human capacity for number and number calculation are shaped in non-arbitrary ways by the possibilities created by human bodily movements, including the constraints of these movements. These movements, in turn, shape our thoughts, and are reflected in our linguistic metaphors. Number – like all human concepts – is built on linguistic metaphors.

Some of the presuppositions of this position have been made explicit and others have to be inferred from its criticism of other positions on number. My argument in this chapter is that some of the position's presuppositions cannot be supported. More worryingly, the starting point of the language-embodied position – that number is given by metaphors in the natural language shaped by human bodily movements – is not explained or justified, even by way of its criticisms of other positions.

The position also crafts the 'hammer' too narrowly – by precluding, without justification, neural circuits from playing a role in the explanation of the human capacity for number, and by not linking this explanation to a plausible evolutionary story about the emergence of number in human life.

This chapter comprises three parts. In the first part, I will summarise the language-embodied position, including describing how the 'hammer' is crafted to capture the human capacity for number. In the second part, I will describe and critically analyse how the language-embodied position characterises what it appears to take as its three main opponents – in order to uncover and critique the implicit presuppositions of this position. In the third part of the chapter, I will demonstrate why the 'hammer' crafted by the language-embodied position is too narrow.

3.1 Summary of the language-embodied position

Embodied cognition theories are quite disparate (Shapiro, 2011). What they do have in common is the view that bodily movements drive cognition and imagination. These positions represent one counterpoint to the dominant view in cognitive science that cognition is the computational processing of internal representations in the head – the body is relegated to the tertiary role of enacting cognitive processes after the fact. Beyond this starting point and shared opponent, embodied theories each implicate a different set of factors to explain how cognition is embodied. The language-embodied position differs starkly from other embodied positions in giving a large explanatory role to linguistic metaphors.

The influence of cognitive linguistics

The language-embodied position on number has been heavily influenced by theories and findings from cognitive linguistics and philosophy of language, most notably Johnson (1987). He argues that human bodily experiences of the world ground our abstract concepts. He explains this using the example of balance. Balance is a constant part of our experience although we are barely aware of this ability. It is an ability that is acquired in infancy during our interactions with the physical world and, importantly, is shared by all humans. Balance is one of many bodily experiences that then inform our abstract concepts, and, because these experiences are shared, they also enable humans to have a shared understanding of these abstract concepts. That is, shared bodily experiences produce shared sense-making. Further, our bodily experiences generate image-schematic structures which are then named in language. Hence sense-making terms such as 'out of balance' and 'too much'.

Johnson argues that image-schemata generated from bodily experiences jointly form the system that organises our experiences of spatial relations. For example, the container schema provides us with the concepts of 'in' and 'out'. Image schemata, however, are not static propositions that characterise abstract relations between symbols and objective reality. Instead, they are the dynamic patterns that order our actions, perceptions and conceptions.

Applying cognitive linguistics to mathematics[5]

The language-embodied position argues that the concept of number develops in three steps, with each step refining the individual's understanding of number (Núñez, 2009). The first step takes us beyond the subitizing range of 'four'. For this step to be achieved, the individual needs to be able to count with her body. This involves the following: placing objects in a sequence; sequentially pairing objects with body parts such as fingers so that she might use gesture to keep track of them; remembering which objects have already been counted and which digits used; detecting that there are no more objects to be counted; and assigning a number to the group that matches the last count word (cardinality). Subitizing and counting, however, only provide the cognitive preconditions for number capacity.

The second step provides the individual with a number capacity by implicating mechanisms for human imagination, namely, conceptual maps (Núñez, 2009). This comprises two capacities. The metaphorical capacity involves conceptualising cardinal numbers and arithmetic operations in terms of basic experiences of various kinds – experiences with

[5] This position does not separate the basic understanding of number from the more advanced understanding of number, namely mathematics.

groups of objects, with part-whole structures of objects, with distances, movement and locations, et cetera. The conceptual blending capacity is the ability to form correspondences across conceptual domains – to put together conceptual metaphors to form complex metaphors.

Since conceptual metaphors preserve inferential organisation, they allow us to ground our understanding of number and arithmetic in prior understanding of extremely commonplace physical activities (Núñez, 2009). This means our most basic mathematical understanding is based on the correlation between the most basic aspects of arithmetic such as subitizing and counting and everyday activities such as collecting objects into groups, taking steps, taking objects apart and putting them back together, et cetera. Thus when we conceptualise numbers as collections, we project the logic of collections on to numbers. In this way, experiences like grouping objects correspond to natural numbers, and give further logical structure to what becomes an expanded notion of number. That is, individuals map from the source domain to the target domain, not just for numbers but for all concepts that we have as humans. The source domain is our bodily experience, the target domain, our conceptual understanding.

It is argued that the inferential organisation of basic arithmetic with natural numbers come from our conceptual metaphors, which in turn, are given meaning by, and are grounded in, our basic bodily experiences (Lakoff and Núñez, 2000). There are four conceptual metaphors – 'arithmetic object collection', 'arithmetic is object construction', 'the measuring stick metaphor', and 'arithmetic as motion along a stick'. These metaphors – at least in their automatic unconscious form – are said to arise naturally from a conflation of experience. That is, humans have an innate capacity to form metaphors based on our experiences in the world.

What is interesting about the first two steps is their remarkable resemblance to the nativist theory generated by Carey (2001). That is, the first two steps resemble the OTS and the bootstrapping process from the OTS into number properties via natural languages, which was described in Chapter 1. However, not only does the language-embodied position not acknowledge Carey's thesis, it argues that nativist theories do not provide a plausible account of the human capacity for number calculation. I will discuss this claim in further detail in the next section of this chapter.

The language-embodied position argues that the third step for the development of number transcends correlational patterns with direct bodily experiences. This step gives us knowledge of the types of numbers that are used by scientists and that are taught in secondary schools, including irrational numbers and imaginary numbers. The third step for developing a number concept relies on the conceptual metaphor. Conceptual metaphors are, for example, able to create something out of nothing, namely, zero. Individuals on their own are not able to generate such concepts; they must be explicitly taught how to use isomorphisms across the four conceptual metaphors to create number concepts such as negative numbers, imaginary numbers and zero.

The empirical project

To test the role bodily movements and language metaphors play in number capacity, Núñez (2011b) suggests moving beyond describing phenomenological experiences – for the mechanics of cognition might be unconscious (remember the example of balance, mentioned earlier). Núñez also suggests that phenomenal experiences lead to the mistaken idea that cognition is embodied in two senses, namely, perception and action on the one hand and on the other, groundedness in the environment. Núñez argues that if this was all it took to produce number, then other primates would also have a capacity for number. After all, they have bodies, bodily experience of space, gravity and motion, and they exhibit social behaviour, emotions and memories that afford similar perceptions, actions and grounding in the physical environment as that of humans. That primates do not have a concept of number is proof that human culture plays a necessary but not sufficient role in human number capacity.

Núñez (2011b) argues that uncovering the human capacity for number requires examining language that is used during number calculation. This includes analysing spoken mathematical reasoning and communication in real time. This, in turn, involves, for example, measuring speech-gesture coproduction in mathematical actions; analysing bodily movements during mathematical communication; and keeping an account of the types and number of allusions to image-schematic structures that are used during mathematical inferential processes.

3.2 Uncovering the position's implicit presuppositions

The purpose of this section is to discuss the presuppositions of the language-embodied position. I argue that many of these presuppositions are contained in its criticism of opposing theories, specifically, Platonist positions, traditional cognition and nativist positions, and situated cognition. In this section, I will critically analyse their criticism of these positions, in turn.

Platonist positions

Platonist theories of mathematics claim that number is a feature of the world; mathematics represents timeless eternal objective truths, which provide structure to, and order, the universe (Colyvan, 2012). Lakoff and Núñez (2000) argue that Platonist claims are untenable; however, they accuse Platonists of denying science the role of testing the veracity of their claims. Núñez (2009) describes the Platonist argument as follows. All sciences rely on mathematical methodology to help capture the regularities, laws and phenomena of the physical and biological world. As such, the relationship between science and mathematics is one-way; mathematics comes to science's aide and not vice versa. At most, psychologists and neuroscientists might investigate how people perform mathematically; sociologists and ethnographers, how people practise mathematics; and developmental scientists and educators how children learn mathematics.

Núñez and Lakoff (2000) provide no references for these apparent Platonist claims; however, they argue that cognitive science is able to refute these claims by demonstrating that mathematics is a human invention. However, instead of providing this demonstration, they provide a largely speculative account of how mathematics *could* be conceived as a human invention. Further, I think that their rejection of Platonism is premature – that a thesis cannot be tested via direct empirical means is not sufficient grounds to reject the truth value of its claims.

Traditional cognitive science and nativist positions

This position reserves its greatest criticism for traditional cognitive science, inclusive of nativist positions. Núñez et al (1999) argue that traditional cognitive science mistakenly conceives the individual as a processor of information, and reasoning as the manipulation of arbitrary symbols. They argue that in traditional cognitive science, internal representations are seen to largely correspond to an external reality, and ontological truths are considered to be independent of human understanding. Núñez argues that such a position represents Cartesian Dualism, with the mind seen as an abstract entity separate from, and transcending, the body, and with reason considered to be non-corporeal, timeless and universal.

For Núñez et al (1999), the biggest failing of traditional cognitive science, however, is that it is unable to account for everyday cognitive phenomena such as common sense, a sense of humour and natural language understanding. As such, they think that cognition, as it is characterised by traditional cognitive science, bears little resemblance to real life problem solving. What they mean by this is a little opaque. That is, Núñez et al do not explain how common sense, a sense of humour, and natural language understanding differ from other forms of cognition, nor do they provide an alternative account of these specific capacities.

As for nativist positions on number, Núñez (2009) discusses them as a monolith even as he points to disagreements among nativists about what constitutes subitizing (serial or parallel processing). I think this lack of recognition of different nativist positions sometimes leads Núñez to mischaracterise some nativist positions. This will soon be demonstrated.

Núñez appears to have at least seven overlapping criticisms of nativist positions on number. His first criticism relates to the 'number line' (Núñez, 2009), which was described in Chapter 1. He claims that nativists erroneously suggest that the 'number line' is a product of evolution; however, he argues that number is too recent in human history to have been naturally selected. But Dehaene et al (2008) do not argue that number was a product of evolution, but instead, the precursors for number were naturally-selected. The innate 'number line' that they describe maps approximate quantities and not exact number.

Núñez's second criticism of nativist positions I consider to be the most damning. He argues that there is an over-reliance on data from industrialised societies (Núñez, 2017). Further, where data has been collected from non-industrialised communities, some data remains unanalysed. For example, he claims that Dehaene et al (2008) have only reported unanalysed data in supporting online material (Núñez and Fias, 2015). This unanalysed data, however, reveals that so-called 'uneducated' adults of the Munduruku failed the expected number-to-line mapping – on average, they failed to map the lowest number 'one' with the left endpoint of the presented line segment (Núñez, 2009). And when stimuli were presented in tones, they failed to map them on a line that preserved the fundamental property of order for basic numerosities. Núñez (2009, 2011a) queries how the ANS could be innate given these findings and further, given that number has not appeared in every culture and has not been uniformly present across the human timescale.

Núñez's treatment of nativist theories as a monolith means that he fails to recognise that the last part of this criticism is better directed at some nativists – namely, Dehaene – than others. Both Spelke (2011) and Carey (2001) argue that natural languages – specifically number words – are required to convert approximate quantities into number. This presumably is how they would account for why number was not uniformly present across the human timescale and in every culture. However, as mentioned in Chapter 1, the problem with the bootstrapping thesis is that it does not explain the emergence of number words.

Núñez's (2009) third criticism is that nativists make the implausible claim that the roots of mathematics reside in single neurons. This is a reference to neuronal studies on rhesus monkeys that are said to demonstrate that single neurons encode for cardinality for the first three quantities (one, two, and three) (Nieder, 2011). More specifically, it is suggested that there are neurons that show maximum activity to only one presented quantity, which progressively drops off as the quantity becomes more remote from the preferred number (Nieder et al, 2002; Nieder and Merten, 2007). Further, the widths of the tuning curves increase linearly with the preferred numerosities, and importantly, changes to the physical appearance of displays has been found to have no effect on the activity of these numerosity-selective neurons.

Núñez (2009) argues that it is implausible to think that an extraordinarily small, genetically shaped set of cells could provide the cognitive preconditions for number. He cannot imagine such cells generating the precision, richness and sophistication of number and mathematics. I think this objection is correct; however, his response to this objection is seemingly to eschew giving any role to neural circuitry. But number seems more obviously related to neural circuitry than shared bodily movements. However, rather than this circuitry taking the form of a set of innate modules – like the ANS and OTS – I will instead argue in Chapter 4, following Anderson (2010, 2016), that new neural coalitions are formed out of old brain functions for the purposes of creating and learning number.

Núñez (2017) accepts the claims and findings about the ANS; he is only sceptical about whether the ANS can generate number. Given this, he renames the ANS 'quantical cognition'. His fourth criticism of nativist positions is that they fail to specify a mechanism to explain how 'quantical cognition' relates to numerical cognition. However, it is Núñez who creates this conundrum for the nativists by his renaming of the ANS. Some nativists have already argued that the ANS and OTS can be converted into number via number words (Carey, 2001; Spelke, 2011). As mentioned, what these nativists fail to do is explain the emergence of number words in most human cultures.

Núñez's fifth criticism is that the system being explained by nativists is introduced as part of the explanation. For example, Gallistel, Gelman and Cordes (2006) argue that mental magnitude is number in the brain and Dehaene (2003) argues that the logarithmic line can explain why nature selected an internal slide rule. Núñez (2009) argues that 'number' and 'logarithmic line' are ideas that come from mathematics; they cannot be used to explain how mathematics emerged. I think this is unwarranted quibbling. Gallistel et al and Dehaene are merely borrowing terminology from mathematics to describe experimental phenomena. There is no circularity of argument here.

Núñez's sixth criticism relates to what he considers to be the mistaken teleological approach of the nativist position (2017). That is, he thinks that they explain phenomena in terms of the purposes that they serve today rather than via a causal theory – they take the ubiquity of number to be evidence *ex post facto* that phenomena such as the capacity for number were goals targeted by natural selection. To demonstrate the faultiness of this teleological claim, he uses the analogy of snowboarding. He argues that to claim that 'quantical' cognition is the primitive for number cognition is the equivalent to claiming that crawling, the mastery of balance and limb coordination necessary for bipedal locomotion are the biologically-evolved preconditions for snowboarding. He argues that nativists' faulty argument stems from a failure to acknowledge that number is only ubiquitous in industrialised communities.

Núñez's seventh criticism relates to the over interpretation of animal data, which I found persuasive, as mentioned in Chapter 2.

Situated cognition

The third thesis that the language-embodied position opposes, but only in part, comes from situated cognition. Here, as mentioned in Chapter 1, number is considered to be a practice that is only rendered sensible by a community of practitioners in the social contexts in which number is utilised (Lave and Wenger, 1991). Núñez et al (1999) do agree with situated cognition that number is not explicable in terms of individual or internal cognitive states. They do not think, however, that number could be understood only as inter-individual social practices – for this does not explain what grounds situated learning and cognition, and what makes mutual intelligibility possible. For Lakoff and Núñez (2000), what provides this grounding, and makes number mutually intelligible, are shared bodily experiences.

As mentioned, the link between bodily experiences and number is not obvious. There are better candidates for the objective foundation of number, including, as also mentioned, new neural coalitions that redeploy ancient brain states. The language-embodied position fails to consider such a possibility. It appears to reject all neural theories as a consequence of finding some nativist claims to be faulty.

Analysing the position's presuppositions

The following are the presuppositions that might be inferred from the language-embodied position's refutations of other positions:

- Number cannot be in the world, because such a claim does not lend itself to being scientifically tested.
- Number is not explicable in terms of neurally-mapped preconditions for a few reasons, including that it is difficult to determine how these inexact and basic capacities could scale up to exact and complex number systems.
- Number calculation cannot take the form of the computational manipulation of internal representations as this entails Cartesian Dualism.
- Number is not just inter-individual social practices; such practices need to be grounded in bodily movements for there to be a shared understanding of number.

As demonstrated, the proponents of the language-embodied position sometimes miscast the claims of their opposition, sometimes fail to provide a convincing argument against these claims, and sometimes fail to provide an alternative explanation that is plausible. Given the justification for some of their presuppositions appears to rest on this criticism, this means the justification for some of their presuppositions is weak. Most problematically, their criticism of other positions falls short of justifying why the language-embodied position gives key explanatory roles to either shared bodily experiences or language metaphors.

Their thesis on the three steps for developing a number concept – described in the second part of this chapter – might provide this justification. Here it is argued that for humans to acquire mathematics, what is required are notation systems, as well as high-order mechanisms for human imagination that are not inherently related to numerosities (Núñez, 2009). These higher order mechanisms are culturally-shaped forms of sense-making. This sense-making is the product of the interaction of certain communities of individuals with the appropriate culturally- and historically-shaped phenotype. This phenotype is supported by language, writing systems, artefacts, education, et cetera.

My first concern here is that this presupposition is a mere assertion; it is not demonstrated. My second concern is that this presupposition raises more questions than it answers. The 'high-order mechanism for human imagination' presumably refers to the conceptual metaphors described in the second part of this chapter, which are said to naturally arise. However, it is not obvious how number properties are related to metaphors. That is, even if metaphors do automatically and unconsciously arise naturally from a conflation of experience

– as this position argues – it is still not clear how this leads specifically to the number properties mentioned in Chapter 1.

Further, no explanation is given by the language-embodied position for how notational and writing systems, artefacts and education emerged in human life, and why the human understanding of number is contingent on these cultural phenomena. In addition, no explanation is provided for why these phenomena appeared only in certain human communities – as the presupposition implies – and not in others. Yet this presumably is the position's explanation for the absence of number systems in non-industrialised communities.

3.3 The narrowness of the language-embodied 'hammer'

I argue that the problems with the language-embodied begin with how it crafts the 'hammer', that is, too narrowly. Neural circuits are precluded from playing an explanatory role in the emergence of number. Further, no plausible evolutionary story is provided to account for the emergence of number in human life.

The methodology this position endorses also reflects the unjustified narrowness of this 'hammer'. It suggests that the human capacity for number could be determined through the study of speech-gesture coproduction, bodily movements undertaken during mathematical communication and allusions made to image-schematic structures during mathematical inferential processes. However, it is not clear how such methodologies will demonstrate what I think is one of the strengths of this thesis, namely the idea that the understanding of number is refined as individuals graduate from the small quantity stage to the counting stage to the basic arithmetic stage and finally to the mathematics stage for those who attain this final level. It seems to me that this refining of number understanding might well be accompanied by corresponding neural changes that enable this refining process. However, as mentioned, the language-embodied position closes off the possibility of this explanation by denying a role to neural circuits in the explanatory story of number.

Conclusion

The implicit presuppositions of this position were inferred from its criticism of other positions on number; however, it was demonstrated that some of these criticisms were weak, and convincing counter-arguments were not always provided. Consequently, some of the implicit presuppositions of this position did not seem well justified. This position provides no justification for its explicit presupposition, namely, that bodily movements inform linguistic metaphors which, in turn, generate a concept of number. These problems, in part, appear to be related to the narrowness of the language-embodied 'hammer' – it is not shaped by an evolutionary story that explains the emergence of number in human life, and gives no role to neural circuits in the explanation of number.

CHAPTER 4: ENCULTURATION

Introduction

My argument in this thesis is that to capture the human capacity for number, a robust 'hammer' has to be crafted. This means the presuppositions that inform the 'hammer' have to be made explicit and justified. It also means not crafting too narrow a 'hammer'. I will argue in this chapter that this can be achieved by providing a plausible evolutionary story that accounts for more than just number, and giving a role to neural circuits, the body and culture in the explanatory story. To this end, I argue that the 'hammer' to capture the human capacity for number should be informed by the following presuppositions:

- Number systems were generated in response to pressures created by the complexities of human sociality. That is, as argued by both Sterelny (2007, 2014, 2017) and Tomasello (1999, 2008), the exponential growth in human sociality put pressure on human societies to find a means to keep track of the exponentially increasing number of transactions among individuals and groups – to ensure that these transactions were equitable and did not result in inter- or intra-group conflict. There was also pressure to find resource solutions in response to the depletion of local resources following the growth in efficient cooperation. Number systems were one such solution; however, they were not an inevitable solution.

- Number systems are external cognitive systems. That is, as argued by Donald (1991, 1993), one response to the above-mentioned pressures was to create external memory systems that supplemented biological memory. I argue that examples of such systems included, but were not limited to, early versions of modern number systems. This origin of number is what separates number from natural languages. The distinction between number systems and natural languages also becomes apparent when one considers how each is acquired by individuals, and how number reasoning occurs.

- In the learning of number, new neural coalitions are created from ancient brain functions. That is, following Menary (2007, 2015), I think the learning of number involves the redeployment of older brain functions in line with Anderson's (2010, 2016; Anderson and Penner-Wilger, 2013) neural reuse theory. These older brain functions might include the ANS and OTS[6]; however, if so, it would include more than just this.

- The brain, body and environment come together in the learning of number. That is, I think Cognitive Integration (Menary, 2007, 2015) plausibly explains that learning number is the result of a dynamic feedback loop among the organism's neural processes, its sensorimotor system and the environment.

[6] In Chapter 2, I do question the innateness of these capacities.

In the first part of this chapter, I will justify my four presuppositions – that is, I will craft the 'hammer'. In the second part of the chapter, I will test the robustness of the 'hammer'. One way to test its robustness is to determine if it is possible to make deductions from the 'hammer' about pragmatic problems related to number, for example, innumeracy. Given, however, that the main purpose of my thesis is to justify my presuppositions, and not to test its robustness, the second part of this chapter will not be exhaustive.

4.1 Crafting the 'hammer'

In this part of the chapter, I provide justifications for each of my presuppositions.

Hominin sociality and cooperation

There is no evidence to suggest that number as an exact and abstract concept is shared by other species or by human ancestors (Sterelny and Hiscock, 2014). When did number come about in human history, and why? My first presupposition is that number systems were one of many responses to the pressures created by the exponential increase in human sociality.

Modern humans are part of the Great Ape clade (Haun et al, 2011). This clade is relatively recent in human history. While the universe is approximately 14 billion years old and the earth about 4.5 billion years, the emergence of hominids occurred only approximately 65 million years ago, and hominins, including human ancestors, appeared approximately six to seven million years ago (Meredith, 2012). The bipedalism of australopithecine occurred approximately 3.7 million years ago, and hominins with larger brains emerged only over 500, 000 years ago (*Homo erectus* 1.8 million years ago and *Homo heidelbergensis* more than 500,000 years ago). Modern humans are said to have emerged approximately 80-120,000 years ago (Sterelny and Hiscock, 2014).

Human social life is very different to that of our primate cousins' – human groups are socially very large, and even before this, we were dependent on technology, technique and coordinated cooperation (Sterelny and Hiscock, 2014). Early humans also differed from our ape cousins by identifying with their social group in symbolic ways, which we continue to do today. The drivers for this might well have begun with bipedalism, which would have introduced new abilities into the life of australopithecines, including bringing objects to the face for close scrutiny, travelling longer distances to new habitats, and carrying children in front rather than on the back, which, in turn, would have enabled the child to track, and learn from, her parent's movements and interactions as she made them (Jeffares, 2014). Bipedalism would also have enabled hominins to literally follow in each other's footsteps (Shaw-Williams, 2014), make artefacts such as beads (Stiner, 2014), and engage in highly skilled activities, such as knapping, which can only be acquired through apprenticeship (Hiscock, 2014). Bipedalism might therefore have driven new basic cognitive skills and increased sociality. For example, tracking would have led to the ability to form narratives about prey whose tracks were being followed, including where they came from and where they might be going (Jeffares, 2014); the apprenticeship needed for knapping would have created hierarchical social structures and social roles (Stout, 2011; Hiscock, 2014); the making of beads would eventually have led to systems of trade and reciprocity (Stiner, 2014).

The only evidence we have at our disposal to confirm this picture of hominin life are physical artefacts, and only those that survived erosion (Sterelny and Hiscock, 2014). Nevertheless, these archaeological findings do appear to support the thesis that hominin sociality had a long evolutionary history, as did basic cognitive skills like the ones that I mentioned above (Sterelny, 2014).

Nativists (e.g. Klein, 2008, 2009) argue that, over time, these basic cognitive skills amplified into more sophisticated cognitive skills. The nativist argument is as follows. Basic cognitive skills would have increased the fitness for survival for those hominins who possessed these skills, who then would have genetically bestowed these skills to subsequent generations. Over time, these skills would have amplified into the changes in the African archaeology that are said to have occurred 80–120,000 years ago (McBrearty and Brooks, 2000; Henshilwood and Marean, 2003). These changes included artefact diversity, regional differentiation, diversification of habitat and exploited resources, and a dedication to material symbols. These changes are collectively referred to as ‘behavioural modernity’ (Sterelny and Hiscock, 2014).

The nativist account, however, cannot be sustained. As Sterelny (2014) points out, some basic cognitive skills, such as knapping, appeared for a time and then disappeared (Hiscock and O’Connor, 2006), while other cognitive skills, such as mortuary practices, had a patchy and unstable onset (e.g. Pettitt, 2011). Secondly, it is unclear what the connection is between the basic cognitive skills mentioned and the material culture that appeared 80-120,000 years ago (Sterelny, 2014). That is, it is not clear what mechanism would have caused the amplification from one to the other in line with the nativist claim. More worryingly for nativist positions is that they subscribe to the idea of modular theories of mind, with each module mapped on to neural substrate. As, however, Tomasello (1999) and Sterelny (2007) argue, cognitive changes require neural flexibility and not an overly rigid neural structure.

An alternative theory to the nativist one are theses about sociality (Sterelny, 2007, 2014, 2017; Tomasello, 1999, 2008). While there is no direct archaeological evidence for such theses (Klein, 2009, 2013) – or for any thesis for that matter – sociality theses do plausibly explain the recent emergence of behavioural modernity.

Sociality theses suggest that increased sociality drove intra-group complexity, which, in turn, created a need for systems that would keep intra-group conflict to a minimum, preserve inter-generational accumulation of knowledge, and respond to the depletion of resources created by efficient cooperation. The systems created were symbolic in nature and were accompanied by conventions (Sterelny, 2014). I will argue in the next subsection that one such system in some, but not all, human communities was an early version of the number system.

How this might have played out is as follows. Cooperation among hominins would have increased the rate of individual survival (Sterelny, 2007). This is because in forager environments, failure was frequent, unpredictable and thus impacted on survival. By contrast, cooperation would have meant that the individual could benefit from the success of others if her efforts failed, or share her success with others when their efforts failed. Families that cooperated with one another would have procreated together and thereby increased the size of the cooperative. This increased size would have enabled more efficient means of cooperation, such as the creation of technology and shared labour, including, eventually, specialisation. All three would have produced such efficient foraging methods that local resources would soon have depleted; resources would have been harvested faster than they were being replenished. Further, technology and cooperation together would have driven population growth. That is, danger from predators would have been less likely to have been fatal and people would have started to live longer.

Knowledge capital is only useful for the survival of the community, however, if it is retained (Sterelny, 2007). One form of retention is the transmission of skills, technologies, and information about predators and the foraging environments to the next generation in the form of pedagogy. A scaffolded period of ontogeny would therefore have emerged, during which time skills would have been broken down into their component parts and explained to neophytes, component parts converted to crucial routines that could be practised, and trial and error learning and safe exploration made possible. This would have enabled neophytes to grow into productive members of the community, and furnished them with the skills to teach the next generation. That is, humans began to pool their cognitive resources both contemporaneously and over historical time (Tomasello, 1999).

As the transmission of knowledge, the size of social communities, and the creation of technology increased, hominin communities would have evolved from being shaped by the natural ecology to altering their environments in ways that transformed the selective forces acting on it (Sterelny, 2007). That is, ecological complexity would have acted in tandem with social complexity, which would have led to a feedback loop among ecological innovation, social complexity and cultural transmission, which continues to this day.

External memory records

My second presupposition is that in some communities, early forms of number systems was one of many responses to the exponential increase in human sociality. Further, number systems were not given by natural languages. In this subsection, I will justify these claims in turn.

Number as external storage

De Cruz, Neth and Schlimm (2010) argue that theories of what I call number (and they call arithmetic) cannot be meaningfully studied in isolation from the physical, social, historical, psychological and biological context in which they were conceived and applied. If one looks at early forms of number systems, they appear to have served one of two broad purposes. They were used as a record of transactions – a way of determining equitable reciprocation in large and socially-complex in-groups (Everett, 2017). They also appeared to track regularities in the environment, for example, the lunar cycle. One could argue that this tracking might have been undertaken for the purposes of finding or maintaining resources for continued survival. Importantly, as argued by Donald (1991, 1993), such systems did not appear to rely on biological memory, but instead supplemented it. That is, information was offloaded on to an external artefact that could be regularly consulted. External information in the form of markings would have been easier to manipulate than biological memory. Further, the use of external media would have enabled the accumulation of information beyond the scope of individual memory (De Cruz, 2006). As such, without these number systems, transactions might have been forgotten and disputed, some technological knowledge might have been lost, and more sophisticated solutions might never have been found.

Quantity systems emerged independently in different human communities and over different periods (De Cruz, 2008; Everett, 2017). They initially took the form of knots, notches on bone, tokens, body parts, et cetera. Pantsar (2014) argues that numerosities given by the ANS do not capture the defined and precise nature of number systems. Further, De Cruz (2005) argues that number systems do not appear to reference elementary sense data. Even early examples of number systems seem to reflect this precision and estrangement from elementary sense data:

- Segmented Antler carving found in Little Salt Spring, Florida, contains primary and secondary regularities. It is thought to be over 10,000 years old and to have served as a lunar calendar (Gifford and Korski, 2011).
- Engraved Baboon fibula (the Ishango bone) in Lake Edward in The Congo, which is thought to be over 20,000 years old, contains three columns of marks grouped into sets (Pletser and Huylebrouck, 1999).
- A 33,000 year old wolf bone with 55 marks on its side was found in Eastern Czech Republic (Everett, 2017).
- African bone baboon fibula, which is considered to be approximately 44,000 years old (Lebombo bone), was probably used as a lunar calendar (d'Errico et al, 2012).
- The Jarawara Indigene Amazonian groups were considered innumerate for many years; however, tally systems using wood were found in the environments they occupied, with markings in groups of two, three, four and five (Menninger, 1969).
- Engravings were also found in the Amazonian forest carved in the ground in wood. They take the form of enormous geoglyphs dating 2000 years (Parssinen et al, 2009).

Some early number systems still remain today, particularly the use of body parts (Saxe 2012); however, other systems evolved into words in the natural language (Everett, 2017) and complex notation systems (Chrisomalis, 2004). Notational systems appeared in different parts of the world independently of one another. These systems were either additive or pictorial and either had many signs for each power base (cumulative), a single sign for each power base (ciphered) or each power base had two components (multiplicative). In his intra- and inter-exponential analysis of numerical systems, Chrisomalis was able to demonstrate that there are cross-cultural regularities in the way that notation systems were transformed or replaced. What drove these transformations or replacements, however, were politics and/or conventions and not a teleological progression to increasingly better cognitive skills and ideas. That is, cognitive skills were lost, as well as gained, during the transformation or replacement of numerical notation systems.

Number as distinct from natural language(s)

Importantly, number systems do not appear to be related to natural languages. While both are abstract systems, the natural language of the community is picked up inevitably by healthy infants, and without structured learning (Fernández and Smith Cairns, 2011). By contrast, the norms and practices of number systems need to be actively taught in ways that I will describe under the subsection ‘Cognitive Integration’.

Further, number reasoning is language-independent (Dutilh Novaes, 2013). Dutilh Novaes argues that this could be demonstrated as follows. First, number reasoning occurs prior to its expression in public language, with the latter serving only as a medium for communication after the fact. Second, during ontogeny, when children are taught the names and property of number, they take a long time, and repeated instruction, to understand this, despite speaking the vernacular language. Third, the innovation of mathematical ideas usually co-occurs with innovations in numerical notation. Even innovations by people who do not have recourse to the written number systems do not rely on the vernacular language. For example, Bernard Morin was a mathematician blind from birth, who nevertheless was able to solve a mathematical puzzle relating to the inside of a sphere (Jackson, 2002). He claimed to have solved this problem through space-like imagination, where the solution appeared all at once. The vernacular language did not play a role.

Neural reuse

My third presupposition is that even though number systems are one of many culturally generated external cognitive systems, the human creation and learning of these cultural systems involves the redeployment of ancient brain functions. Tooby and Cosmides (1989) maintain that modern human cognitive abilities are too recent in human evolutionary history to be accounted for by cognitive adaptations; it is only in the past several hundred thousand years that these abilities have appeared (Stringer and McKie, 1996). Archaeological evidence suggests that the manipulation of number systems is a recent cognitive ability (Everett, 2017).

Anderson’s (2010, 2016; Anderson and Penner-Wilger, 2013) neural re-use thesis provides us with a means to account for the appearance of recent cognitive skills, including number – namely, the redeployment of ancient brain functions. Anderson’s neural reuse theory argues that local neural circuits are used and reused for diverse purposes in various task domains. Functional differences between task domains is reflected not so much in what neural areas are implicated in supporting domains but rather, in the different patterns of interactions between many of the same elements. Later emerging capacities are supported by a greater number of local circuits more broadly scattered around the brain; the later a capacity emerges, the more potentially useful existing circuitry there will be, and little reason to suppose it will be grouped locally.

Using this theory, it could be argued that the late-emerging capacity for number would co-opt local circuits scattered around the brain that were naturally selected for ancient tasks. Some of these tasks might only have a passing resemblance to number; some might have a resemblance that is not immediately apparent. Anderson and Penner-Wilger (2013) suggest that ancient tasks need to leave some behavioural trace in newer tasks if both implicate a common physical-functional substrate; however, this is difficult to demonstrate with brain changes in evolutionary time. Instead, it might be possible to prove via developmental homology. Anderson and Penner-Wilger suggest searching for behaviours that developmentally co-occur with number learning, such as gesturing and writing. Considerably more work needs to be undertaken in this area. The point to be made, however, is that the innate capacities redeployed during the learning of number need not have a superficial relationship to number. For example, they do not have to be related to quantity tracking or magnitude estimation.

The upshot of the neural re-use theory is that the learning of a new cognitive skill such as number depends on having a plastic brain. I will expand on this point in the next subsection.

Cognitive integration

My fourth presupposition is that the cognitive integration thesis plausibly explains how number could be an emergent cognitive ability.

Tomasello (1999, 2008) argues that the period of ontogeny is a period during which learning occurs through imitation (mimesis). Mimesis is not surface mimicking but instead, reproducing an act that is understood to have been produced intentionally by another person. That is, during ontogeny, social interactions teach children that the actions of others can be motivated by their personal perspective. As such, children are not only taught cultural practices but also social skills so that they know how to exploit the pre-existing cultural resources in order to learn cognitive skills. This begins naturally during infanthood, first as dyadic relationships between the infant and objects that they grasp for and manipulate. This evolves into a triadic relationship when the infant calls the attention of another person to an object – via gestures such as pointing. This leads into imitative learning or the copying of others. Here infants do not just learn from others but also learn through them. That is, they use their social knowledge of others as intentional agents and copy from them in order to produce the same intentional act.

To reinforce what imitation is, Tomasello (1999) contrasts it with what it is not, namely emulation and ontogenetic ritualisation, which are skills that chimpanzees also have.

Emulation involves focusing on the change to environmental events produced by another organism's actions. No perspectival-taking occurs, nor is there recognition that actions are intentional. Ontogenetic ritualisation is a communicatory signal created by two organisms shaping each other's behaviour in repeated instances of a social interaction. For example, signalling what one wants by grabbing at the other, such that the other eventually learns what is wanted before the signal becomes a full grab. These signals are individually invented and ritualised, and so are idiosyncratic to the two people involved. They are not passed down through culture. Imitative learning is not more advanced than emulation or ontogenetic ritualisation; it is just more social. Some gestures now used in imitation might not have begun that way (Tomasello, 2008). For example, pointing might not have always had prosocial motivation; however, following cooperation among humans, it now does.

Menary's (2007, 2015) cognitive integration theory helps us operationalise how imitation can be used to learn about number. This thesis suggests that cognition is the integration of an organism's neural processes with its sensorimotor system and environment, with all three operating in a feedback loop that drives behaviour and further cognition. This then explains why cognitive processes develop idiosyncratically in humans. During ontogeny, infants are introduced to cultural artefacts, practices and norms relating to number, which they learn by using their bodies. They do this by imitating older people in their environments; however, they do this in their own way as each person's body is unique.

That is, in cultures where number is meaningful, older members of the community direct the infant's gaze to objects in the environment that might be put into some order (Menary, 2007, 2015). The ordering process is then matched to a naming convention in the natural language. Older members of the community capitalise on the child's natural inclination to point in order to connect the ordering of objects to the naming conventions. This counting process is broken down into parts and taught separately. The child is encouraged to repeat each part *ad nauseam* until she masters it. As highlighted by Dutilh Novaes (2013), children do this repeatedly before they acquire the cognitive skills of number, counting and number calculation. Each person repeats or practises the steps for each cognitive task differently. This is especially true when more formal scaffolding enters the picture, for example, in the learning of the more advanced properties of number during formal schooling.

Practice best demonstrates how learning the properties of number implicate reciprocal causal interactions between internal brain states, the sensorimotor system and cultural artefacts in the environment. When in the earliest stages of schooling, children are taught the properties of number, they are taught this by moving their bodies, for example, using their fingers to trace numerals in the air or using a pen and paper to draw them out (Menary, 2015). Because children are not born with the knowledge of what are, after all, cultural symbols, they would have to be redeploying ancient brain functions to form new neural coalitions in the learning of this new cognitive skill. The process of practice therefore transforms the learner; in order to learn number, she creates new neural coalitions out of old brain functions and is thus biologically transformed.

This bodily movement and use of external vehicles is not, as Adams and Aizawa (2008, 2010) suggest, merely the offloading of internal capacities onto external vehicles to make for easier manipulation. Instead, the capacity for number is only achievable through moving bodies in a particular way. That is, the more the child takes opportunities to self-correct her movements (for example, drawing numbers more accurately and in order), the more her understanding of number increases. As such, learning number systems transforms our existing biological and cognitive capacities in ways that our unenculturated brains will not allow (Menary, 2015). Further, as suggested by De Cruz (2007), creating and manipulating symbols in the public space, and not in the head, enables the learner to play with ideas without overloading her biological memory.

To synthesise my position on number and summarise my presuppositions, I tentatively argue that number systems were one set of conventions that emerged in response to the exponential increase in human sociality in some human communities. The parameters of these conventions were determined by the pragmatic purposes that they served, for example, the record-keeping of complex human transactions that the biological brain would have found difficult to retain. The parameters were also determined by the physical constraints of the human body – that is, the bodily movements that humans possessed for learning and using a number system, and the limits to which humans were able to redeploy pre-existing brain regions towards this new task.

Archaeological and anthropological evidence reveals that different number conventions existed in different human communities in different periods of human history since the Late Pleistocene, each seemingly with their own set of pragmatic and physical parameters. Over time, however, one number system came to predominate, and was adopted by most human communities. The parameters of this system were listed in the first chapter of this thesis. One might speculate that this system achieved its dominant position through trade and cultural hegemony; however, it is beyond the scope of this thesis to comment on such a speculation. The point to be made here is that the Cognitive Integration thesis provides us with the means by which to offer an explanation for why, despite its dominant position, some humans do not have an understanding of this number system. That is, the Cognitive Integration thesis provides us with the means by which to gain purchase on pragmatic problems such as innumeracy, which might be explained in terms of cognitive practice. I will discuss this in the next section.

4.2 Testing the robustness of the crafted 'hammer'

In this section, I test the robustness of the proposed "hammer" by determining whether anything meaningful could be deduced from this 'hammer' about innumeracy. In a thesis of this nature and length, I can only provide general-level speculations.

The 'hammer' allows us to define innumeracy in terms of number systems, specifically, whether they are present in the community, whether and how the learning of these systems is scaffolded during ontogeny, whether and how successfully neural coalitions are able to be formed in the individual's brain during this learning, and the type and level of practice in which the individual engages during ontogeny and beyond. I will discuss each of these issues in turn.

Innumerate societies are those societies without a system that contains the properties of number mentioned in Chapter 1. Number systems in modern innumerate societies might have emerged but not survived, or they might never have emerged. In any event, what we can say with certainty is that number systems were not a permanent response to the exponential increase in human sociality in these regions – perhaps because there were no exponential increases of this kind, or because other types of solutions emerged. Are human societies without number systems at a disadvantage? I think 'advantage' can only be determined against survival of a human community. To discuss advantage in any other way without recourse to a justified political thesis is to beg the question. Given this, what could threaten the survival of non-numerate communities are numerate communities that change the ecological landscape that these non-numerate communities live in, or that reduce the chances of subsequent generations learning skills essential for survival in these ecologies.

Within numerate communities, innumeracy can also be defined against the number system. More broadly, innumeracy would be an inability to understand or utilise the number system at its most basic level. That is, the individual is unable to generalise the properties of number and basic calculations to the wide variety of contexts in which it is utilised. Cognitive practice might be at the heart of this problem. That is, either the neural coalitions required for number learning and practice cannot be formed (making innumeracy an 'all' or 'nothing' proposition) or the individual is not engaging in enough of the right kind of number practice (in which case innumeracy is a matter of degree). Let me discuss each of these in turn.

Anderson (2016) suggests that individual regions of the brain participate in multiple functional coalitions for the purposes of forming new cognitive abilities. Such a theory then could be used to speculate about the different forms innumeracy could take. For example, there might be injury from birth in a brain region that in other individuals would be co-opted during the learning of number – making this region unavailable for inclusion in emerging neural partnerships. Other injuries or deficits might not prevent the coalition from happening but instead, may slow the process of building the coalitions. In which case, depending on the clinical details, neighbouring regions might acquire the relevant similar functional bias to serve in the stead of the damaged region or they might not. Alternatively, there might be connectivity disorders that lead to widespread behavioural deficits and developmental delays that also impact on the learning of number systems.

Another problem identified by Anderson (2016) is that regions could become functionally burdened from being incorporated into different neural coalitions. If this is the case, any changes to local structure will impact multiple functional partnerships and therefore be more difficult to implement, given that extant partnerships reinforce existing connectivity. This, for example, might account for why generalising basic number capacities to multiple contexts could decrease with age.

At behavioural level, and following Tomasello (1999), autism might create innumeracy – as it might prevent children from creating and recognising intentional action, which is important for imitative learning. As such, autism might prevent a child from being guided during ontogeny to learn the number system. Also at a behavioural level, individuals without autism might also become innumerate if they are insufficiently engaged in learning the number system, or are reluctant to engage in the constant practice required to master this system for wide application.

So innumeracy could take many forms; however, I suggest that all are explicable against the number system and how the learning of this system is operationalised by the cognitive integration thesis.

Are innumerate people in numerate societies at a disadvantage? At a cognitive level, if the neural re-use thesis is correct, then the neural connectivity that occurs during the learning of number systems will not occur for innumerate people. Learning a number system, however, is not the only way to achieve brain plasticity; the learning of a number of cognitive skills requires new neural coalitions to be formed.

At a political level, if a large proportion of individuals in numerate societies are innumerate, then they will not be able to participate sensibly in civic discussions that rely on applying one's knowledge of the basic properties and calculations of number to a wide variety of contexts. Such a political discussion, however, is outside the scope of this thesis.

Conclusion

My purpose in this chapter was not to provide evidence for the human capacity for number. Instead it was to provide a means to craft the 'hammer' that could later be utilised to generate evidence about the human capacity for number. To craft the 'hammer', I first named the presuppositions that should inform it and then justified these presuppositions. I also briefly demonstrated the robustness of the crafted 'hammer' by making deductions about a pragmatic number-related issue that I raised in Chapter 1, namely, innumeracy. However, another thesis is required to further test the robustness of the crafted 'hammer', and then to utilise it to collect empirical evidence to explain the human capacity for number.

CONCLUSION

In this thesis, I argue that the 'hammers' utilised by two different numerical cognition positions to capture the human capacity for number are not well-justified and are too narrow. Consequently, I argue that they are not robust enough to capture this capacity. The two 'hammers' that I critique are Spelke's nativist 'hammer' (e.g. 2011) and the language-embodied 'hammer' (e.g. Lakoff and Núñez, 2000). My criticism of Spelke's 'hammer', however, could extend to any position that shares the central premise informing this 'hammer', namely that the search for the human capacity for number should begin with the brain, give a secondary role to culture, and implicitly, ignore the body. Similarly, my criticism of the language-embodied 'hammer' could also extend to any embodied and/or cultural position that fails to provide an evolutionary account for the emergence of number, and give no role to neural circuits in the crafting of the 'hammer'.

I end my thesis by suggesting how the 'hammer' to capture the human capacity for number should be crafted, and justify my presuppositions by utilising arguments from the following theses: niche construction (Sterelny, 2007, 2014, 2017); mimesis (Tomasello, 1999, 2008); the emergence of the modern mind (Donald, 1991, 1993); neural reuse (Anderson, 2010, 2016) and cognitive integration (Menary, 2007, 2015).

The justifications for the 'hammer', however, need to be further elaborated and refined. To this end, I recommend three inter-related areas of further study. First, in this thesis, I only suggest how the 'hammer' should be crafted in relation to the human capacity for recognising the basic properties of number and utilising basic arithmetic. The 'hammer', however, should also be crafted to explain advanced number capacities, that is, mathematical capacities. Given the development of mathematics seems to have co-occurred with the development of symbols (Merzbach and Boyer, 2011), and the use of body-parts in some cultures (e.g. Saxe, 2012), semiotic, and cross-cultural embodied, theories should be considered for this purpose.

Second, further work is required to test the robustness of this 'hammer', which is only briefly tested in Chapter 4 in relation to the pragmatic issue of innumeracy. For example, more specific deductions about innumeracy could be generated by using a more refined 'hammer', deductions that could be operationalised and tested empirically.

Third, an empirical program needs to be created – to use the 'hammer' to generate data about the human capacity for number, and to test the deductions that are generated using the 'hammer' about pragmatic problems such as innumeracy. This data, in turn, could be utilised to further refine the 'hammer' for subsequent use in studies on the human capacity for number.

REFERENCES

Adams, F. and Aizawa, K. (2008) *The Bounds of Cognition*, Oxford: Basil Blackwell.

Adams, F. and Aizawa, K. (2010) "The value of cognitivism in thinking about extended cognition" *Phenomenology and the Cognitive Sciences, 9(4)*: 579-603.

Amalric, M. and Dehaene, S. (2016) "Origins of the Brain Networks for Advanced Mathematics in Expert Mathematicians", *Proceedings of the National Academy of Sciences of the United States of America, 113(18)*: 4909-4917.

American Psychiatric Association (1994) *Diagnostic and Statistical Manual of Mental Disorders, 4th Edition*, Washington, DC: American Psychiatric Association.

Anderson, M. L. (2010) "Neural Reuse: A Fundamental Organizational Principle of the Brain", *Behavioral and Brain Sciences, 33*: 245-313.

Anderson, M. L. and Penner-Wilger, M. (2013) "Neural Reuse in the Evolution and Development of the Brain: Evidence for Developmental Homology?", *Developmental Psychobiology, 55*: 42-51.

Anderson, M. L. (2016) "Neural Reuse in the Organization and Development of the Brain", *Developmental Medicine & Child Neurology, 58 (Suppl. 4)*: 3-6.

Australian Curriculum, Assessment and Reporting Authority (ACARA), australiancurriculum.edu.au/f-10-curriculum/general-capabilities/numeracy.

Barrett, L. (2011) *Beyond the Brain: How Body and Environment Shape Animal and Human Minds*, Princeton, NJ: Princeton University Press.

Brannon, E.M. (2002) "The Development of Ordinal Numerical Knowledge in Infancy", *Cognition, 83(3)*: 223-240.

Brannon, E.M. and Merritt, D. (2011) "Evolutionary Foundations of the Approximate Number System" in S. Dehaene and E.M. Brannon (Eds) *Space, Time and Number in the Brain: Searching for the Foundations of Mathematical Thought*, San Diego: Elsevier Academic Press.

Budd, C.J. (2015) "Promoting Maths to the General Public" in R. Cohen Kadosh and A. Dowker *The Oxford Handbook of Numerical Cognition*, Oxford: Oxford University Press.

Butterworth, B. (1999) *The Mathematical Brain*, London: Macmillan.

Butterworth, B. (2011) "Foundational Numerical Capacities and the Origins of Dyscalculia" in S. Dehaene and E.M. Brannon (Eds) *Space, Time and Number in the Brain: Searching for the Foundations of Mathematical Thought*, San Diego : Elsevier Academic Press.

Carey, S. (2001) "Cognitive Foundations of Arithmetic: Evolution and Ontogenesis", *Mind & Language, 16(1)*: 37-55.

Carey, S. (2009) “The Making of Abstract Concepts: A Case Study of Natural Number” in D. Marischal (Ed.), *The Making of Abstract Concepts*, Oxford: Oxford University Press.

Central Advisory Council for Education (1959) *15 to 18*, London: HMSO.

Chrisomalis, S. (2004) “A Cognitive Typology for Numerical Notation”, *Cambridge Archaeological Journal, 14(1)*: 37-52.

Cockcroft Committee (1982) *Mathematics Counts: A Report into the Teaching of Mathematics in Schools*, London: HMSO.

Colyvan, M. (2012) *An Introduction to the Philosophy of Mathematics,* Publisher: Cambridge University Press, Cambridge.

De Cruz, H. (2005) “How do Cultural Numerical Concepts Build upon an Evolved Number Sense” in B.G. Bara, L. Barsalou, M. Bucciarelli (Eds), *Proceedings of the XXVII Annual Conference of the Cognitive Science Society*, Mahwah, NJ: Lawrence Earlbaum.

De Cruz, H. (2006) “Why are Some Numerical Concepts More Successful than Others?: An Evolutionary Perspective on the History of Number Concepts”, *Evolution and Human Behavior, 27*: 306-323.

De Cruz, H. (2007) “How does Complex Mathematical Theory Arise?: Phylogenetic and Cultural Origins of Algebra”, in C. Gershenson, D. Aerts and B. Edmonds (Eds), *Worldviews, Science and Us: Philosophy and Complexity*, New Jersey: World Scientific.

De Cruz, H. (2008) “An Extended Mind Perspective on Natural Number Representation” *Philosophical Psychology, 21*: 475-490.

De Cruz, H., Neth, H., and Schlimm, D. (2010) “The cognitive basis of arithmetic” in B. Loewe and T. Mueller (Eds), *Philosophy of Mathematics: Sociological Aspects and Mathematical Practice*, London: College Publications.

De Havia, M.D. and Spelke, E.S. (2010) “Number-Space Mapping in Human Infants”, *Psychol Sci., 21(5)*: 653-60

Dehaene, S. (1997) *The number sense: How the Mind Creates Mathematics*, New York: Oxford University.

Dehaene, S. (2003) “The neural basis of the Weber-Fechner Law: A Logarithmic Mental Number Line”, *Trends in Cognitive Sciences, 7(4)*: 145-147.

Dehaene, S. , Molko, N., Cohen, L., and Wilson, A.J. (2004) “Arithmetic and the brain”, *Current Opinion in Neurobiology, 14*:218–224.

Dehaene, S., Izard, V., Spelke, E.S., Pica, P. (2008) “Log or linear? Distinct Intuitions of the Number Scale in Western and Amazonian Indigene Cultures, *Science, 320(5880):* 1217-1220.

Dehaene, S. (2011) *The Number Sense: How the Mind Creates Mathematics (2nd edition)*, New York: Oxford University Press.

Department of Education and Skills (2001) *Guidance to Support Pupils with Dyslexia and Dyscalculia*, London: HMSO.

d'Errico, F., Backwell, L., Villa, P., Degano, I., Lucejko, J.J., Bamford, M.K., Higham, T.F.G., Colombini, M.P. and Beaumont, P.B. (2012) "Early evidence of San material culture represented by organic artifacts from Border Cave, South Africa", *Proceedings of the National Academy of Sciences of the United States of America, 109(33):* 13214-13219.

Donald, M. (1991) *Origins of the Modern Mind: Three Stages in the Evolution of Culture and Cognition*, Cambridge, MA: Harvard University Press.

Donald, M. (1993) "Précis of Origins of the modern mind: Three stages in the evolution of culture and cognition", *Behavioral and Brain Sciences, 16*: 737-791.

Dowker, A. and Cohen Kadosh, R. (2015) "Preface" in R. Cohen Kadosh and A. Dowker (Eds), *The Oxford Handbook of Numerical Cognition*, Oxford: Oxford University Press.

Dowker, A. (2016) "Factors that Influence Improvement in Numeracy, Reading and Comprehension in the Context of a Numeracy Intervention", *Frontiers in Psychology, 7:1929,* doi:10.3389/fpsyg.2016.01929.

Dutilh Novaes, C. (2013) "Mathematical Reasoning and External Symbolic Systems", *Logique & Analyse, 221*: 45-65.

Everett, C. (2017) *Numbers and the Making of Us: Counting and the Course of Human Cultures*, Cambridge, MA: Harvard University Press.

Feigenson, L., Carey, S., and Spelke, E.S. (2002) "Infants' Discrimination of Number vs Continuous Extent", *Cognitive Psychology, 44(1)*: 33-66.

Feigenson, L., Dehaene, S., and Spelke, E.S. (2004) "Core systems of number", *Trends in Cognitive Sciences, 8(7)*: 307-314.

Felsenstein, J. (2004) *Inferring Phylogenies*, Sunderland, MA: Sinauer Associates.

Fernández, E.M. and Smith Cairns, H. (2011) *Fundamentals of Psycholinguistics*, West Sussex: John Wiley & Sons, Ltd.

Gallistel, R., Gelman, R. and Cordes, S (2006) "The Cultural and Evolutionary History of the Real Numbers" in S.C. Levinson and J. Pierre (Eds.), *Evolution and Culture: A Fyssen Foundation Symposium*, Cambridge, MA: MIT Press.

Gifford, J. and Korski, S. (2011) "An Incised Antler Artifact from Little Salt Spring", *The Florida Anthropologist, 64(1)*: 47-52.

Greeno, J.G. (1997) "On Claims that Answer the Wrong Question", *Educational Researcher, 26(1):* 5-17.

Harvey, P.H. and Pagel, M.D. (1991) *The Comparative Method in Evolutionary Biology*, Oxford: Oxford University Press

Haun, D.B.M., Jordan, F.J., Vallortigara, G., Clayton, N.S. (2011) “Origins of Spatial, Temporal and Numerical Cognition: Insights from Comparative Psychology” in S. Dehaene and E.M. Brannon (Eds) *Space, Time and Number in the Brain: Searching for the Foundations of Mathematical Thought,* San Diego: Elsevier Academic Press.

Hauser, M., Carey, S. and Hauser, L. (2000) “Spontaneous Number Representation in Semi–Free Ranging Rhesus Monkeys”, *Proceedings of the Royal Society of London: Biological Science, 267(1445)*: 829–833.

Henrich, J., Heine, S.J. and Norenzayan, A. (2010) “Beyond WEIRD: Towards a Broad-based Behavioral Science”, *Behavioral and Brain Sciences, 33(2-3)*: 111-135.

Henshilwood, C.S. and Marean, C.W. (2003) “The Origin of Modern Human Behavior: Critique of the Models and Their Test Implications”, *Current Anthropology, 44(5)*: 627-651.

Heyes, C. (2014) “False Belief in Infancy: A Fresh Look”, *Developmental Science, 17(5)*: 647-659.

Hiscock, P. and O'Connor, S. (2006) “An Australian Perspective on Modern Behaviour and Artefact Assemblages”, *Before Farming, 2*: 1-10.

Hiscock, P. (2014) “Learning in Lithic Landscapes: A Reconsideration of the Hominid ‘Toolmaking’ Niche”, *Biol. Theory, 9*: 27-41.

Izard, V., Sann, C., Spelke, E.S. and Streri, A. (2009) “Newbord Infants Perceive Abstract Numbers”, *Proceedings of the National Academy of Sciences of the United States of America, 106(25)*: 10382–10385.

Jackson, A. (2002) “The World of Blind Mathematicians”, *Notices of the AMS, 49(10)*: 1246-1251.

Jeffares, B. (2014) “Back to Australopithecus: Utilizing New Theories of Cognition to Understand the Pliocene Hominins”, *Biol. Theory, 9*: 4-15.

Johnson, M., (1987) *The Body in Mind,* Chicago: University of Chicago Press.

Jordan, K.E., Maclean, E.L. and Brannon, E.M. (2000) “Monkeys Match and Tally Quantities Across Senses”, *Cognition, 108(3)*: 617-25.

Kennedy, M., Spencer, H.G. and Gray, R.D (1996) “Hop, Step and Gape: Do the Social Displays of the Pelecaniformes Reflect Phylogeny?”, *Animal Behaviour, 51(2):* 273-291.

Klein, R.G. (2008) “Out of Africa and the evolution of human behaviour”, *Evol. Anthropol. 17*: 267–281.

Klein, R.G. (2009) “Darwin and the recent African origin of modern humans”, *Proceedings of the National Academy of Sciences of the United States of America, 106(38)*: 16007-16009.

Klein, R.G. (2013) “Modern Human Origins”, *General Anthropology, 20 (1):* 1-4.

Lakoff, G. and Núñez, R. (2000) *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being,* New York: Basic Books.

Lave, J. (1988) *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*, New York, NY: Cambridge University Press.

Lave, J. and Wenger, E. (1991) *Situated Learning: Legitimate Peripheral Participation*, New York, NY: Cambridge University Press.

Le Corre, M. and Carey, S. (2007) "One, two, three, four, nothing more: an investigation of the conceptual sources of the verbal counting principles", *Cognition, 105(2)*: 395-438.

Lemey, P., Salemi, M., and Vandamme, A-M. (Eds.) (2004) *The Phylogenetic Handbook: A Practical Approach to Phylogenetic Analysis and Hypothesis Testing*, Cambridge: Cambridge University Press,

Lipton, J.S., and Spelke, E.S. (2003) "Origins of Number Sense: Large Number Discrimination in Human Infants", *Psychol Sci., 14(5)*: 396-401.

McBrearty, S. and Brooks, A.S. (2000) "The revolution that wasn't: a new interpretation of the origin of modern human behavior", *J. Hum. Evol., 39(5)*: 453-563.

Menary, R. (2007) *Cognitive Integration: Mind and Cognition Unbounded*, New York: Palgrave Macmillan.

Menary, R. (2015) "Mathematical Cognition: A Case of Enculturation" in T. Metzinger and J.M. Windt (Eds), *Open MIND:* 25(T), Frankfurt am Main: MIND Group.

Menninger, K. (1969) *Number Words and Number Symbols*, Cambridge, MA: MIT Press.

Meredith, M. (2012) *Born in Africa: The Quest for the Origins of Human Life*, New York, NY: Public Affairs.

Merzbach, U.C. and Boyer, C.B. (2011) *A History of Mathematics (3rd Edition)*, Hoboken, NJ: John Wiley & Sons, Inc.

Moreno-Armella, L and Sriraman, B. (2005) "The Articulation of Symbol and Mediation in Mathematical Education", *ZDM, 37(6)*: 476-486.

National Numeracy, nationalnumeracy.org.uk.

Nieder, A., Freedman, D.J., Miller, E.K. (2002) "Representation of the quantity of visual items in the primate prefrontal cortex", *Science, 297(5587)*: 1708-1711.

Nieder, A. (2005) "Counting on Neurons: The Neurobiology of Numerical Competence", *Nature Reviews Neuroscience, 6*: 177-190.

Nieder, A. and Merten, K. (2007) "A labeled-line code for small and large numerosities in the monkey prefrontal cortex", *J Neurosci., 27(22)*: 5986-93.

Nieder, A. (2011) "The Neural Code for Number" in S. Dehaene and E.M. Brannon (Eds) *Space, Time and Number in the Brain: Searching for the Foundations of Mathematical Thought*, San Diego: Elsevier Academic Press.

Núñez, R., Edwards, L., and Matos, J-F. (1999) "Embodied Cognition as Grounding for Situatedness and Context in Mathematics Education", *Educational Studies in Mathematics, 39 (1-3)*: 45-66.

Núñez, R. (2009) “Numbers and Arithmetic: Neither Hardwired Nor Out There”, *Biological Theory, 4(1):* 68-83.

Núñez, R. (2011a) “No Innate Number Line in the Human Brain”*, Journal of Cross-Cultural Psychology, 42(4):* 651-668.

Núñez, R. (2011b) “On the Science of Embodied Cognition in the 2010s: Research Questions, Appropriate Reductionism and Testable Explanations”*, Journal of Learning Sciences, 1-13.*

Núñez, R. and Fias, W. (2015) “Ancestral Mental Number Lines: What is the Evidence?”, *Cognitive Science,* DOI: 10.1111/cogs.12296.

Núñez, R. (2017) “Is There Really an Evolved Capacity for Number?”, *Trends in Cognitive Sciences, 21(6)*: 409-424.

Pahl M., Si, A. and Zhang, S. (2013) “Numerical Cognition in Bees and Other Insects”, *Frontiers in Psychology, 4: 162*.

Pantsar, M. (2014) “An Empirically Feasible Approach to the Epistemology of Arithmetic”, *Synthese, 191*: 4201-4229.

Parssinen, M., Schaan, D. and Ranzi, A. (2009) “Pre-Columbian Geometric Earthworks in the Upper Purus: A Complex Society in Western Amazonia”, *Antiquity, 83*: 1084-1095.

Peters, E., Västfjäll, D., Slovic, P., Mertz, C.K., Mazzocco, K. and Dickert, S. (2006) “Numeracy and Decision Making”, *Psychological Science, 17(5)*: 407-413.

Peters, E. (2012) “Beyond Comprehension: The Role of Numeracy in Judgments and Decisions”, *Current Directions in Psychological Science, 21(1)*: 31-35.

Peters, E. and Bjalkebring, P. (2015) “Multiple Numerical Competencies: When a Number is Not Just a Number”, *Journal of Personality and Social Psychology, 108(5)*: 802-822.

Pettitt, P. B. (2011) *The Palaeolithic Origins of Human Burial*, London: Routledge.

Piazza, M. (2011) “Neurocognitive Start-Up Tools for Symbolic Number Representations” in S. Dehaene and E.M. Brannon (Eds) *Space, Time and Number in the Brain: Searching for the Foundations of Mathematical Thought,* San Diego: Elsevier Academic Press.

Pletser, V. and Huylebrouck, D. (1999) “The Ishango Artefact: the Missing Base 12 Link”, *Forma, 14*: 339-346.

Presmeg, N., Radford, L., Roth, W-M., Kadunz, G. (2016) *Semiotics in Mathematics Education*, Springer Open.

Saxe, G.B. (2012) *Cultural Development of Mathematical Ideas: Papua New Guinea Studies*, New York, NY: Cambridge University Press.

Seed, A., Emery, N. and Clayton, N. (2009) “Intelligence in Corvids and Apes: A Case of Convergent Evolution”, *Ethology, 15*: 401-420.

Seed, A. and Tomasello, M. (2010) “Primate Cognition”, *Topics in Cognitive Science 2:* 407–419.

Shapiro, L. (2011) *Embodied Cognition*, New York, NY: Routledge.

Shaw-Williams, K. (2014) "The Social Trackways Theory of the Evolution of Human Cognition", *Biol. Theory, 9*: 16-26.

Shepard, R.N. (2001) "Perceptual-cognitive universals as reflections of the world" *Behav Brain Sci., 24(4)*: 581-601.

Spelke, E.S. (2011) "Natural Number and Natural Geometry" in S. Dehaene and E.M. Brannon (Eds) *Space, Time and Number in the Brain: Searching for the Foundations of Mathematical Thought*, San Diego: Elsevier Academic Press.

Sterelny, K. (2007) "Social Intelligence, Human Intelligence and Niche Construction", *Philosophical Transactions of the Royal Society of London Series B, 362(1480)*: 719-730.

Sterelny, K. and Hiscock, P. (2014) "Symbols, Signals and the Archaeological Record", *Biol. Theory, 9*: 1-3.

Sterelny, K. (2014) "A Paleolithic Reciprocation Crisis: Symbols, Signals and Norms", *Biol. Theory, 9*: 65-77.

Sterelny, K. (2017) "Artifacts, Symbols, Thoughts" *Biol. Theory*, https://doi.org/10.1007/s13752-017-0277-3.

Stiner, M. (2014) "Finding a Common Bandwidth: Causes of Convergence and Diversity in Paleolithic Beads", *Biol. Theory, 9*: 51-64.

Stout, D. (2011) "Stone toolmaking and the evolution of human culture and cognition" *Philos. Trans. R. Soc. Lond. B. Biol. Sci., 366(1567)*: 1050–1059.

Stringer, C. and McKie, R. (1996) *African Exodus: The Origins of Modern Humanity*, London: Jonathan Cape Ltd.

Tomasello, M. (1999) "The Human Adaptation for Culture", *Annu. Rev. Anthropol., 28*: 509-529.

Tomasello, M. (2008) *Origins of Human Communication*, Cambridge, MA: The MIT Press.

Tooby, J. and Cosmides, L. (1989) "Evolutionary Psychology and the Generation of Culture, Part I. Theoretical Considerations", *Ethology & Sociobiology, 10*, 51-97.

Weber, E., Walkington, C., and McGalliard, W. (2015) "Expanding Notions of 'Learning Trajectories' in Mathematical Education", *Mathematical Thinking and Learning, 17(4)*: 253-272.

Wiens, J.J. (Ed.) (2000) *Phylogenetic Analysis of Morphological Data*, Washington, DC: Smithsonian Institution Press.

Willis, S. (1998) "Which Numeracy?", *UNICORN, 24(2)*: 33-42.

Withnall, A. (1995) "Towards a Definition of Numeracy", *Adults Learning Mathematics (ALM) Conference*, http://www.alm-online.net/images/ALM/conferences/ALM01/proceedings/ALM01-proceedings-p11-17.pdf.

Wood, J.N and Spelke, E.S. (2005) “Infants’ Enumeration of Actions: Numerical Discrimination and its Signature Limits”, *Dev Sci., 8(2)*:173-81.

World Health Organisation (1994) *International Classification of Diseases, 10th Edition,* Geneva Switzerland: World Health Organisation.

Wynn, K. (1992) “Addition and subtraction by human infants”, *Nature, 358(6389)*: 749-50.