

OPTIMAL ANIMAL NUTRITION

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A thesis submitted to Macquarie University
for the degree of
Doctor of Philosophy
Department of Statistics
December 2011



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Typeset in L^AT_EX 2_ε.

JAMES 1: 2-4 (NIV)

Consider it pure joy, my brothers, whenever you face trials of many kinds, because you know that testing of your faith develops perseverance. Perseverance must finish its work, so that you may be mature and complete, not lacking anything.

Abstract

Linear programming has long been used in pig production to generate minimum cost diets that are subject to nutritional constraints. More recently, the development of accurate animal growth models and the development of effective nonlinear optimisation techniques, in conjunction with linear programming, have allowed us to address the bigger problem of building feeding schedules that maximise profitability. These feeding schedules, which we name “optimal feeding schedules”, simultaneously minimise feed costs while maximising gross return.

The random search for the optimal feeding schedule runs over an extremely large space. The optimisation methods used to generate the optimal solution are of critical importance. Software to perform the optimization, named “Bacon Max”, has been developed in Visual C++. The search processes used in Bacon Max to determine the optimum solution for this problem include pure random search, tabu search, Monte Carlo search, ascent search and a genetic algorithm. Among these search processes genetic algorithms were the most successful, despite the long search period needed. This thesis extends this work in four ways.

First (in Chapter 3), an alternative optimisation methodology was developed, which we call the “tailored” method, adapted to the known nature of the objective function, found by examining several random cross sections through a known solution. This revealed a single craggy peak. The tailored method finds the optimum more rapidly than the genetic algorithm. The program for the tailored method is also developed in Visual C++.

Second (in Chapter 4), profitability depends on feed costs and price at slaughter which in turn are subject to variation across time. We show how to include such variation in our modelling and handle it using stochastic programming. The optimal feeding schedule is influenced by pig type, costs and prices, and dietary restraints. The effects are discussed. The Bacon Max software is developed to handle variations of feed cost and price at slaughter using stochastic programming.

Third (in Chapter 5), the methodology is utilized to explore the impact of pig type, costs and dietary restraints on the optimal feeding schedule. At the same time the optimal feeding schedule is compared to the commonly used “feed-to-lean” schedule.

Fourth (in Chapter 6), an application to real data from southern Thailand is presented. The results indicate that profitability can be increased by changing the original feeding schedule to the optimal feeding schedule resulting from Bacon Max.

In summary, results indicate that the optimal feeding schedule from Bacon Max can improve profitability for the producer.

Declaration

I hereby declare that the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a higher degree in any other university or institution.

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Publications

Conference Proceedings

1. “Designing an optimal pig feeding schedule in southern Thailand”, The Proceedings of the 43rd Kasetsart University Annual Conference, Eds. K. Markvichitr et al., Bangkok, Thailand (2005) 96-103. (with G.R. Wood, P.C.H. Morel and Yuthana Siriwananukul)
2. “Global optimisation applied to pig nutrition”, The Proceedings of the International Workshop on Global Optimization, Almeria, Spain (2005) 257-262. (with G.R. Wood and P.C.H. Morel)
3. “Feeding for maximum lean growth does not always maximise profitability: a simulation study”, The Proceedings of the Australian Pig Science Associations Conference, Christchurch, New Zealand (2005), Manipulating Pig Production X, 114 (with P.C.H. Morel and G.R. Wood)
4. “Optimal Animal Nutrition”, The Proceedings of the 21st European Conference on Operational Research (EURO XXI), Iceland (2006) (with G.R. Wood)
5. “Finding optimal feeding schedules in pig production” The Proceedings of Conference on Advances in Global Optimization: Methods and Applications, Mykonos, Greece (2007). (with G.R. Wood, Minqiang Dong and P.C.H. Morel)

Refereed Papers

1. “Designing an optimal pig feeding schedule in southern Thailand”, *Kasetsart Journal: Natural Science*, 39 (5) supplement, (2006) 8-15. (with G.R. Wood, P.C.H. Morel and Yuthana Siritwathananukul)
2. “Effect of genotype, population size and genotype variation on optimal diet determination for growing pigs”, *Acta Horticulturae (ISHS)* 802 (2008) 287-292. (with G.R. Wood and P.C.H. Morel)
3. “Two aspects of optimal diet determination for pig production: efficiency of solution and incorporation of cost variation”, *Journal of Global Optimization*, 43 (2009) 249-261. (with G.R. Wood, Minqiang Dong and P.C.H. Morel)
4. “A new development in pig growth modelling”, *Modelling Nutrient Digestion and Utilisation in Farm Animals*, Eds. D. Sauvant, J. van Milgen, P. Faverdin, N. Friggens, Wageningen Academic Publishers (2010) 83-90. (with P.C.H. Morel, D.L.J. Alexander, R.L. Sherriff and G.R. Wood)
5. “Effect of pig type, ingredient cost and price schedule on dietary nutrient specification for maximum profitability in grower-finisher pig herds: a theoretical approach.” This paper has been submitted to *Livestock Science* (2011) (with P.C.H. Morel and G.R. Wood)

Acknowledgements

I consider myself truly blessed for having had the opportunity to study for a PhD. This experience has edified me in many ways: academically, but also in life. To me the only responsibility that I am accountable for in this life is being faithful with all that Christ has given me. His faithfulness and unfailing love have taught me to trust Him to enable me to finish every good works. I am very grateful for the people He has graciously provided, whose kindness, patience, encouragement and generosity provided direction. I would like to acknowledge,

- The Thai government and Prince of Songkla University for the financial support, and the Office of Educational Affairs staff, for all their assistance.
- I owe my deepest gratitude to my principal supervisor, Professor Graham Wood. Your supervision and support, with patience, consideration, understanding and encouragement through all of my study period have help me to accomplish this work. It is my great honour to learn and work under your caring supervision. My family and I want to thank you sincerely for all your dedication toward this work.
- My co-supervisor, Associate Professor Patrick Morel. Despite our lack of meeting, I am grateful for your prompt and enthusiastic assistance.
- My other co-supervisor, Associate Professor Yuthana Siriwathananukul, for all the data he has collected and made available and all the advices for Chapter 6.
- The Statistics Department of Macquarie University for the opportunity to learn in many other ways, apart from matters immediately related to this PhD.
- My husband, Gary, for your constant companionship. Thank you for helping me understand my purpose and praying that I would be faithful to God.
- My children, Moriah, Zion and Judah for the joy you have been in my journey.
- My family in Thailand, Mum in India, Dad and Auntie Tina in Melbourne for supporting me with love and understanding.
- Friends, for all your encouragement.
- David Alexander, David Bulger, Ming Quiang Dong and Ryan Sherriff for developing Bacon Max and for all the advice received during this research and the subsequent modifications to the software.

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Chapter 1

Introduction

1.1 Brief thesis aims

Linear programming has long been used in pig production to generate minimum cost diets that are subject to nutritional constraints. More recently, the development of accurate animal growth models and the development of effective nonlinear optimisation techniques, in conjunction with linear programming, have allowed us to address the bigger problem of building feeding schedules that maximise profitability. The research work in this thesis continues the development of this area. In particular it includes

- A review of the basic background of linear programming, growth modelling and nonlinear optimisation, the related software, and the landscape of the objective function
- Development of an alternative algorithm to carry out the optimisation
- Accommodation of a changing feed ingredient schedule and price schedule to cover practical aspects of pig production
- Considering the effect of pig parameters, costs and dietary restraints on dietary nutrient specification
- Application of this method and software to an on-farm situation in Thailand
- Associated adaptation of the software in each section.

1.2 General background

1.2.1 Pig industry

Pork has been the most consumed meat in the world in the past two decades. Statistics of world meat consumption (kg/person/year) from the study of the Food and Agricultural Organization of the United Nations (FAO, World agriculture towards 2015/2030) in a report from Alberta Pork (2006) shows that pork consumption is the highest of all meats at 14.6 kg/person/year by comparison to poultry meat consumption (10.2 kg/person/year) and bovine meat consumption (9.8 kg/person/year) in 2000. Further, we learn from Alberta Pork (2006) that the pattern of growth for pork consumption is predicted to steadily rise until 2015. Despite the anticipation of greater demands for poultry meat than pork meat by 2030 a significant proportion of the world's population is still predicted to consume pork meat.

The demand for food has increased due to the rising world population. Moreover, the increased income and spending power of the consumers also influence the demand for meat, especially pork meat. Food production increases to supply the need. Considering the rising demand for pork meat, it is understandable that pig production is the largest meat industry in the world. Demand continues to rise, as can be seen from the data in Table 1.1.

Table 1.1: Annual world pig meat production statistics from the FAO (2011), presented in million metric tons.

Year	Pig meat production (million metric tons)
1970	35.84
1980	52.69
1990	69.92
2000	89.78
2009	106.32

Figure 1.1 (Global Livestock Production and Health Atlas (GLiPHA), 2011) shows a world map of pig population density (livestock units(LU)/square kilometre) in 2007. Countries such as the Netherlands (182.8 LU/sq km), Denmark (154.6 LU/sq km), Vietnam (79.1 LU/sq km), China (15.1 LU/sq km), Canada (6.6 LU/sq km), United

States of America (4.5 LU/sq km), Brazil (4.0 LU/sq km) and most European countries have a high density of pig production, to supply the world demand. The main components across successful countries for pig production are human resources, access to technology, competitive production costs and quality and safety assurance for the consumers (Alberta Pork, 2006). Advances in knowledge and technology have been the most important factor in catering to this growing market.

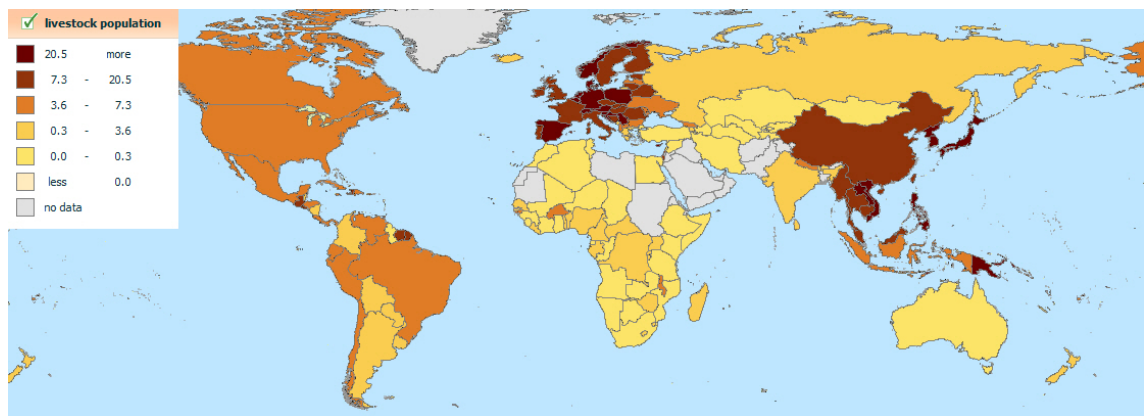


Figure 1.1: World map of pig population density (livestock units(LU)/square kilometre) in 2007 from the Global Livestock Production and Health Atlas (GLiPHA) (2011). The density of pig population is represented using colour, with the darker and lighter red colours signifying the higher and lower density of pig populations respectively.

Efficiency of pig meat production mainly depends on three factors (5M Enterprises Ltd., no date (n.d.)).

1. Environment, size and style of pig farm, farm management strategies and health
2. Feed and nutrition
3. Genetic potential of pig

The first factor is the environment which constitutes climate, temperature and humidity and has an impact on pig growth. Different environments in each country have a distinct impact on pigs and should be managed accordingly. The size and style of the pig farm vary in each country and can change over time. Good farm management, such as ongoing staff training programs, updated market information and farm records, farm hygiene and vaccination (FAO, OIE & World Bank, 2010) are all needed for successful pig production (5M Enterprises Ltd., (n.d.)).

Second factor which is feed and nutrition directly impacts the cost and profitability in pig production. The feed cost is around 55-75% of the total cost of raising a pig from piglet to finisher (Queensland Government, Primary Industries and Fisheries, 2010b).

The last factor is the genetic potential of the pig. A pig's efficiency in converting feed into meat is influenced by the pig breed and pig genotype as much as is influenced by its health and the environment. Understanding the genetic potential of the pig helps the pig producer to provide a suitable feed and management system that maximises pig performance. Market preference influences the lean growth genetic potential in pigs (Coffey, Parker, & Laurent, n.d.; Schinckel, Richert, Clark, Frank, & Turek, 1997). The more modern pig types have high genetic potential to reach full slaughter weight within a short period without losing the carcass quality (Pieterse, Loots, & Viljoen, 2000). Merks (2000) shows that in the last century genetic change improved a pig's average daily gain by 100% with a reduction of backfat thicknesses by 75%.

1.2.2 Size, style of pig farm and farm management

The pig industry has varying farm structures and also variations in scale of farm size. According to the report of Eurostat (2010) (the data is taken from Farm Structure Survey (FSS) (2007)), pigs are recorded in three categories; piglet, breeding sow and fattening pig. In this thesis we consider the fattening pig or grower-finisher pig from the period of life from weaning until slaughter (Agricultural Research Council, 1981). Eurostat (2010) also classified the pig farm by number of sows and fattening pigs into four categories;

1. Small fatteners (no sows and fewer than 10 fattening pigs)
2. Large fatteners (no sows and at least 400 fattening pigs)
3. Large breeders (more than 100 sows and 400 fattening pigs)
4. Other pig farms.

The research in this thesis focuses on farms for fattening pigs, those with fewer than 10 pigs and also those up to 200 pigs. Larger numbers of pigs can be considered, but this demands larger computer processing time. This is the reason why larger numbers of pigs has not been included.

In the past, most pig farms started as small scale farms. This is so in Thailand. Figure 1.2 illustrates an example of a small backyard pig farm in southern Thailand. The photo is taken in 2003. Ten pigs per batch or less than that are taken care of by the owner family. Pigs were fed with the diets that combined herbal plants as medicine and feed additives to produce organic and high quality pork, instead of using high doses of vaccines in the farming process. Table 1.2 compares the number of holdings and number of pigs in Thailand from 1993 to 2003 (the data for 1993 is sourced from the FAO and APHCA (2002) and the data for 2003 from a report in the National Statistics Office (2003), respectively), classified on the basis of the number of pigs per holding. The backyard pig farm was the traditional farm type, along with buffalo, or cattle and poultry. Buffalo or cattle are mainly used for the purpose of labour in the crop field. Pig and poultry supplement income from the rice field or other crops or can be consumed within the family (Tisdell, Murphy, & Kehren, 1998). The pigs are either fed food wastes or by-products from the farm. Table 1.2 shows that the number of holdings which have fewer than 10 pigs roughly halved from 1993 to 2003. On the other hand, the number of holdings which have 500 pigs or more doubled in the same period of time.



Figure 1.2: An example of small pig farm of Mr. Cheewa-isarakul (centre) in southern Thailand (photo taken in 2003). Fewer than 10 pigs were raised at the backyard of the owner's house.

In some countries, small pig farms still carry on with the same principles as traditional farms but have changed to use of modern knowledge and technology. For example, (Eurostat, 2010) Romania, Bulgaria and Lithuania still run small pig farms which have fewer than 10 pigs. Similarly, consumers have also begun to show a keen interest

Table 1.2: Number of holdings rearing pigs in Thailand census in 1993 and 2003 classified by number of pigs per holding. The data is sourced from FAO and APHCA (2002) and the National Statistics Office (2003).

Number of pigs per holding (heads)	1993		2003	
	Number of holdings	Number of pigs	Number of holdings	Number of pigs
Less than 10	471,512	1,343,166	246,552	811,617
10-49	106,163	1,831,254	84,842	1,511,283
50-99	6,853	418,215	5,827	362,403
100-499	5,043	894,132	4,901	1,030,330
500 and over	1,045	1,693,213	2,200	3,466,879
Total	590,616	6,185,953	344,322	7,182,512

in knowing the strategies and conditions employed by farmers who practice pig farming. This further influences trends of pig farming. For example, small scale free-range pig farming is developing where pigs are allowed to move and search for their food freely in the paddock. Also we are witnessing the development of organic pig farming, where no animal health products are used in the process of raising pigs. An example of this (discussed further in Chapter 6) is a small farm in Thailand that uses herbal plants as medicine and feed additives to produce organic and high quality pork.

Many factors are responsible for the growth of the pork industry, such as grain price (price of feed), the high national and international demand for pork meat (Victorian Government, Department of Primary Industries, 2011) and the innovation of farm technology. Many small farms have lost business to larger farms due to their lack of competitiveness in cost of production and product quality (Huynh, Aarnink, Drucker, & Verstegen, 2007). On the other hand, when some smaller pig farm producers work to advance their scientific and technological knowledge and skills of farm managements this results in the positive growth of farm size. These advances not only reduce the cost of production, but also increase the productivity and thus work towards meeting the high demand for pork in the market (Key & McBride, 2008). This exemplifies the trend mentioned in the previous example of Thailand where there is a progressive rise in large scale farms. Huynh et al. (2007) reports the same trend of larger herd size in Vietnam.

The number of large scale pig farms in a limited area of space, is significantly rising, to meet the needs of consumers. The example of a large scale piggery in New Zealand shown in Figure 1.3 is also considered in this thesis (Chapter 5). This is an example of an intensive piggery. A large number of pigs are grown in a limited area, such as a pen or a stall shed. Examples of countries using this type of pig farm are the United States (Britannica, 2009), Canada (The Animal Welfare Foundation of Canada, no date (n.d.)), Denmark and Mexico.

Animal health products are often used in this type of piggery to protect the large number of pigs from diseases that can cause heavy losses. Concerns about such pig farms are numerous. As mentioned earlier, due to growing interest among consumers regarding the outcomes of the pig farming process, consumers prefer the use of organic food that is free from animal health products.

A further concern from both small and large pig farms is the large amount of pig manure which is washed into the environment. This can cause unpleasant odours and the nutrients also cause pollution in the nearby soil and water. Concern regarding nutrient excretion has been included in the extended work from this thesis, as we will discuss in the Conclusion (Chapter 7).



a)



b)

Figure 1.3: An example of large pig farm near Christchurch, New Zealand (photo taken in 2005). A large number of pigs (more than 500) are accommodated in the farm. As can be seen, a large number of pigs are put in the same pen. This type of farm creates concerns for animal welfare, health issues and the environment.

1.2.3 Feed and nutrition

Having considered the importance of the pig industry to the world and the style of pig farms we now move on to discuss the nature of pigs and the management of the pig farm. Pigs have a monogastric digestive system, meaning they have a single stomach. They can eat and digest food such as grain but have a very limited capacity for digesting high fibre food. On the other hand, pigs have an excellent sense of smell. The palatability of food for pigs is important. Diet formulations for pigs have been studied and developed to meet their nutrient requirements. Pigs need a balanced diet to grow and produce the best quality pork meat.

Nutrient requirements for pigs have four components: energy, protein (or amino acids), vitamins and minerals. Lean pork meat is preferable in the market and receives a higher price in return. Although many factors impact upon the profitability of pig production the major components to consider are the cost of production and effective animal nutrition. Feed cost is the major component of total pig farm cost. Feed cost is around 60-65% or up to 70% of the total cost for fattening a pig (Carr & Garth veterinary group, n.d.; Western Australia Government, n.d.). Labour, maintenance and repairs on a pig farm can be around 25% of the total cost. The bulk of the rest of the cost is in animal purchase, electricity, water, medicine, transport and veterinary expenses.

It is for this reason that pig producers consider improving the feed efficiency of prime importance. There are a few ways to accomplish this, such as reducing feed waste, diet formulation, farm management (Sutton, 2008) and use of new pig genotypes (British Pig Executive, 2009). Reducing feed waste controls the feed cost and also attempts to minimise passage of nutrients (from excretion) into the environment. The diet formulation should be adapted to the pig genotype and growth period.

Another way to increase feed efficiency is to consider the feeding system. The feeding system comprises feeding form and method.

Feeding form

There are two types of feeding forms, namely dry feed and liquid feed (DeRouchey & Richert, n.d.). Dry feed is a dry mixture of feed stuff formulated to meet pig nutrient requirements. When using a dry feed it is critical to supply pigs with sufficient water. The benefit of a dry feed is that it is simple and more hygienic for pigs (Elkmann &

Bärlein, 2011) especially when the feed is available to pigs at all times (ad libitum feeding). The excess dust created by this feed type is a limitation raising health risks (for both pigs and workmen) and environmental concerns (Hampshire feeding system, 2011b).

Liquid feed on the other hand, is feed containing approximately 20-30% dry matter and is liquid in form (Livestock knowledge transfer, 2001; Shurson, 2008). Some liquid feeds contain fermented ingredients and lactic acid bacteria that result in improvements in pig digestion and a reduction of undesirable bacteria in pigs (Brooks, Beal, & Niven, 2001; Canibe and Jensen, 2003; Hampshire feeding system, 2011a,b; Livestock knowledge transfer, 2001; Meat and Livestock Commission, 2003). The advantages of liquid feed include an improvement in nutrient utilisation and animal performance, greater flexibility while controlling the feeding program and a reduction in environmental impact (Livestock knowledge transfer, 2001; Shurson, 2008). However the need for a higher control of hygiene to reduce the risk of malfermentation is a disadvantage of this feed type (Meat and Livestock Commission, 2003). Dry feeds are more commonly adopted in the United states, while liquid feeds are more popular in Europe and Canada (DeRouchey & Richert, n.d.).

Feeding method

Commonly used are restricted feeding (controlled feeding) and ad libitum feeding (self feeding) (Agricultural Research Council, 1981).

In the restricted feeding method the quantity of the feed given to the pigs is controlled usually maintaining a level less than the pigs' maximum voluntary intake (Queensland Government, Primary industries and fisheries, 2010a). It results in a lower feed cost, good carcass quality and better pig farm hygiene (EntrePinoys Atbp., 2009). However, a restricted feed method could result in an unequal growth of the herd should some pigs receive insufficient feed. In addition, pigs fed using this feeding method have a lower average daily gain (EntrePinoys Atbp., 2009).

On the other hand, ad libitum feeding provides feed at all times without any restriction (Agricultural Research Council, 1981). A dry feed form is typically used in this feeding method (e.g. in the U.S.) and is known to work well among pigs in a healthy condition and having a high genetic potential (EntrePinoys Atbp., 2009). Every pig is fed until 100% satisfaction with no competition in the herd. An advantage of this feeding

method is a higher average daily gain among pigs. However, disadvantages include a higher feed cost and lower carcass quality (higher backfat thickness). In addition, if pig farms lack an effective feed waste management system this feeding method can cause health risks and digestive problems in pigs (EntrePinoys Atbp., 2009).

There are two feeding systems used in this thesis namely restricted and ad libitum feeding in the form of dry feed based on the practice of pig farms from which our data is collected.

In practice, feeding formulas for fattening pigs consist of two diets, namely grower and finisher diets. According to pig industry terms and definitions from the Queensland Government, Primary Industries and Fisheries (2010b) a grower diet is used for a pig whose live weight is between 20 kg (or weaning) to 70 kg. A finisher diet is used for pigs whose live weight is between 70 to 100 kg (or until slaughter date, which can be varied depending on a country's market demands) (Willis, 2010). However, others prescribe a grower diet for pigs between 20 to 50 kg and finisher diet for pigs between 50 to 90 kg or above (Chiba, 2004; Roesse & Taylor, 2006).

Improving the feed efficiency by trying to reduce feed waste and feed cost has been attempted by many researchers. For example, Willis (2010) suggests use of phase feeding. This allows the diets to change more frequently (possibly even weekly) to match pig nutrient requirements to their growth period. In the study of Edwards (2011), however, there is no significant difference in overall performance between the use of a single diet or various phase feeding diets. The study suggests that the lysine requirements of pig genotypes have to be met in pig diets before it is possible to decide between a single diet and phase feeding.

In this thesis we consider issues that help improve feed efficiency by optimising pig nutrition and minimising feed cost in a manner that is adapted to both pig type and growth period. Farm management, however is not included in this study. Both diet types (grower-finisher and phase feeding) are considered. Chapters 3, 4 and 6 discuss using grower-finisher diets while Chapter 5 considers phase feeding diets (weekly) using a maximum of 15 diets until slaughter.

1.3 Achievements to date

1.3.1 Pig dietary specification and optimisation

We now move to discuss research in animal nutrition. Links between mathematical optimisation and the animal industry have been considered for several decades, especially for animal nutrition. Linear programming (Kall & Wallace, 1994) have been commonly used to calculate an animal diet by generating the minimum feed cost. (The details of linear programming and its application in pig nutrition will be discussed in Chapter 2.) The nutrient requirements for pigs (Agricultural Research Council, 1981; National Research Council, 1998) are used as the constraints in linear programming. For example, Kearney (1971) and the National swine nutrition guide diet formulation evaluation software (U.S. Pork Center of Excellence, 2010) used linear programming to formulate a least cost diet for pigs and also evaluate the adequacy of nutrition in the diets.

The AusPig system (Australian Pork Limited, 2010; Black, Campbell, Williams, James, & Davies, 1986; Menzies, Black, Fleming, & Dean, 1992) has been developed by the Commonwealth Scientific and Industrial Research Organisation (CSIRO) for the Australian pork industry, employing an advanced decision support system. Four components are included in this system: the AusPig growth model, Feedmania, the PIGMAX pig enterprise model and an expert system to interpret the model output. The AusPig growth model simulates pig growth and is able to predict pig performance. AusPig is linked and integrated with Feedmania which calculates the optimal cost diet. Linear programming is also used in parts of the PIGMAX program to predict farm management and marketing strategies to maximise profitability. PIGMAX requires information from AusPig such as feed cost and carcass value to evaluate farm management and marketing strategies, such as utilisation of pig pen space, the profitability of short-term or long-term strategies and the different profitability of male and female pigs.

Another example in Canada (Ferguson, 2008) presents swine simulation software, Watson[®]. This program incorporates a pig growth model developed by Ferguson, Gous, and Emmans (1994) and least cost diet formulation. The optimum feeding strategy and financial strategy can be predicted as the farm production environment is changing. Pig producers can benefit from such models through then making better farm management decisions.

We now consider the bigger problem. Pig producers not only aim to keep the total feed cost low, but also hope to gain maximum profitability from diet formulation. Alexander, Morel, and Wood (2001, 2006) introduce a combination of linear programming, a pig growth model and nonlinear optimisation in order to maximise gross margin in pig production. Figure 1.4 shows the components involved in maximisation of profitability (gross margin per pig place per year) adapted from Alexander et al. (2001). Pig diets are described using three parameters: the proportion of the *ad libitum* digestible energy intake, the lysine to digestible energy ratio (in grams per MegaJoule) and the digestible energy density (in MegaJoules per kilogram). The nature of the objective function for this problem is also examined. The study showed that a genetic algorithm is more successful in identifying the optimal solution than pure random search. Optimisation methods permit improved efficiency in the pig feeding industry.

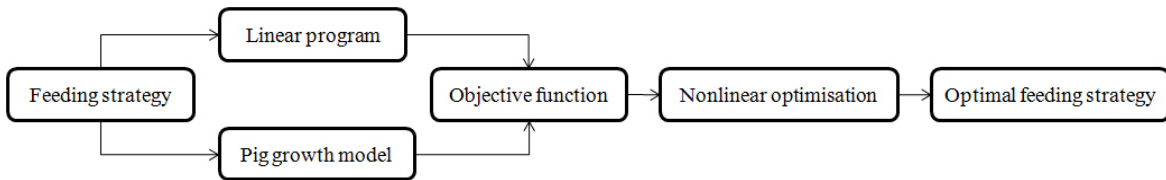


Figure 1.4: The components involved in maximisation of gross margin per pig place per year, adapted from Alexander et al. (2001).

In Jean dit Bailleul, Bernier, van Milgen, Sauvant, and Pomar (2000), a pig growth model (Whittemore, 1983) was developed to determine the feeding management system that maximises net return for two phase feeding, grower-finisher pig production. Four submodels are included: an estimation of the growing pig's maximal energy and protein requirements for the entire growing period, the calculation of the least-cost diet using linear programming that meets nutrient requirements (Agricultural Research Council (1981)), a simulation of the pig growth in terms of protein and fat deposition and the calculation of the net return from the simulated production system. Then, a nonlinear optimisation algorithm (a gradient method (Kuester & Mize, 1973)) is used to find the feeding strategy that maximises the net return (presented in dollars per pig place per year). The optimisation method is based on two parameters, the length of feeding period and the level of satisfaction of maximal protein requirement.

In this thesis, we consider four parameters in the objective function: the proportion of the *ad libitum* digestible energy intake, the lysine to digestible energy ratio, the digestible energy density and the feeding period. Two pig parameters, minimum allowable lipid to protein ratio and maximum daily protein deposition (g/day), characterising genotypes, are also included in the model. The different methods of nonlinear optimisation applied to this unique objective function yield significantly different results, as we will describe in Chapters 2 and 3. The growth model and nonlinear optimisation that considered in this thesis aim to increase the accuracy for finding the optimal feeding strategy. Of importance here is that the objective for pig production should be to determine the optimal feeding strategy, rather than minimum feed cost or maximum pig performance.

Another report, that of Morel and Wood (2005), presents a simulation study to maximise profitability while minimising the nitrogen excretion in pig production. Simulations were conducted using different pig genotypes, various relative economic weightings of profitability, and varying levels of nitrogen excretion for pig production in Switzerland. The results indicate that nitrogen excretion can be reduced and the profitability is increased when using better pig genotypes and the optimal diet.

We carry further in this thesis the integration of linear programming for least cost diets, the pig growth model and nonlinear optimisation to determine the feeding strategy that maximises profitability. The aims for this thesis were listed in Section 1.1.

1.4 Challenges and thesis response

1.4.1 Pig growth model

To better understand the nature of pig growth many pig growth models have been conducted. There are two main purposes for developing a pig growth model. Firstly, it is valuable as a research and education tool and secondly, it predicts the performance of pigs under a range of conditions (de Lange, Marty, Birkett, Morel, & Szkotnicki, 2000). Pig growth models are applied in many ways, such as in pig nutrition, breeding selection and for farm management. One advantage of using a pig growth model is suggested in de Vries and Kanis (1992) where the pig growth model gives better pig production strategies compared with using an economic model. Another example, in Skorupski, Garrick, Blair, and Smith (1995) uses a pig growth model to evaluate the

economic values of traits for pig improvement.

Pig growth models are also used to illustrate nutrient partitioning and growth in pigs. Most recent pig growth models are based on partitioning of energy and amino acid utilization to predict the pig performance. Different approaches to pig growth modelling for various group of pigs have been developed. Some examples of pig growth models are mentioned as follow. Whittemore (1983, 1986) and Whittemore, Green and Knap (2001) present a pig growth model and their applications. A pig growth model of Black et al. (1986) has been used to develop AusPig. A pig growth model of Ferguson et al. (1994) has been used to develop Watson[®]. Another example, by Moughan, Smith, and Pearson (1987), presents a deterministic computer pig growth model for pigs with liveweight in the range of 20-90 kg. Boisen (2000) presents a simple nutrient partitioning model for a growing pig. Schinckel, Li, Einstein, and Miller (2002) develop a simple stochastic compositional pig growth model. Pomar, Harris and Minvielle (1991) develop a computer simulation model for female pig growth, foetal development, milk production, and growth of suckling pigs. The National Research Council (1998) also includes a pig growth model for predicting nutrient requirements and also equations for determining the lean growth rate of pigs. de Lange et al. (2000) however argue that the NRC model has some issues regarding parameters and calculation. These cause difficulty in practical use.

Problems and proposed solutions in the different approaches to pig growth modelling are presented in Emmans and Kyriazakis (1997). Wellock, Emmans, and Kyriazakis (2003) describe a pig growth model that predicts the effects of genotype and the thermal and nutritional environment on food intake, growth and body composition of a growing pig. Another piece of research by Wellock, Emmans, and Kyriazakis (2004) compares the growth functions for pigs and desired criteria for predictors of potential growth such as few parameters and that growth should be seen as a continuous process. The research proposes that the Gompertz function (Gompertz, 1825) is a suitable descriptor of potential growth.

In de Lange (1995) a simplified pig growth model is demonstrated using the basic principles of energy and amino acid partitioning for growth. This model contains all of the important features that should be included in a growth model and predicts pig performance with reasonable accuracy under defined conditions. This pig growth model requires two pig type variables: minimum allowable lipid to protein ratio and

maximum daily protein deposition (g/day) as inputs. These parameters need to be estimated from on-farm conditions and affect the accuracy of the predicted growth model. The specific conditions of pig health and farm environment are captured through these pig parameters. They give more practical results in comparison with using experimental values in ideal conditions (less empirical). Including health and farm environmental factors directly in the model adds to the complexity of the method. Moughan (2003) also describes new directions of simulating the partitioning of pig dietary amino acids. The empirical based growth model is replaced by the less empirical model. The basic principle for modelling is the development of a simple approach involving minimal parameters yet offering greater validity and utility. This is one of the advantages of de Lange's pig growth model (1995) (avoiding over parameterization). The other advantage of de Lange's model is accessibility for public research. For these reasons we used de Lange's growth model in this thesis.

An example of application of a pig growth model in commercial pork production is presented in de Lange et al. (2000). A pig growth model can be a useful tool to improve the efficiency in pig production, but it also requires accurate input information and has some limitations. Detail is explained in Chapter 2 when we use this simple pig growth model.

Another example of new developments in pig growth modelling is presented in Morel (2009). This paper briefly introduces two software products, PorkMaster and BaconMax, using a pig growth model in a commercial environment and in a research, respectively.

Sandberg, Emmans, and Kyriazakis (2005a,b) evaluated various pig growth models based on the rules of partitioning of protein and energy. It considered three variables in the model, namely food, animal and environment variables. Qualitative and quantitative analysis were used in these studies. There is a need for pig growth models to be further developed not only to achieve greater accuracy but also to be more user friendly; especially for their use in commercial pig production. It should be reasonably inexpensive to measure required model inputs and on-farm parameters. Also, they should be simple to run on a computer and give a clear interpretation of model outputs.

We now move from the basic knowledge of pig growth model to the extended optimisation to maximise the profitability that is considered in this thesis. This thesis extends

past work in the four following ways.

1.4.2 Nonlinear optimisation

We study nonlinear optimisation. Nonlinear optimisation is needed to deal with the search for the optimum of the nonlinear objective function, a challenge when the search space is extremely large. A number of such optimisation techniques have been developed, namely the Monte Carlo method (Metropolis, Rosenbluth, Rosenbluth, Teller, & Teller, 1953), the Nelder-Mead simplex algorithm (Nelder & Mead, 1965), ascent search (Eldor & Koppel, 1971), a genetic algorithm (Holland, 1975), simulated annealing (Aarts and Korst, 1989), pure random search (Zabinsky & Smith, 1992) and tabu search (Glover & Laguna, 1997). The genetic algorithm is the most successful approach in this particular problem and is used in this thesis. Details of genetic algorithms are described in Chapter 2.

A first contribution of this thesis is an alternative optimisation method that is able to perform competitively with a genetic algorithm and reduce computer processing time. This detail is described in Chapter 3. The point of this method is that it is adapted to the nature of the objective function.

1.4.3 Cost of production and pig price at slaughter

In reality, pig producers face the challenge of coping with the variation of the feed ingredient cost and pig price at slaughter over time. Decisions concerning feeding and farm management strategies have to be made accordingly (Reese, 2007). A means of addressing the problem of a changing feed ingredient cost and pig price at slaughter would be to reformulate the feed in accordance with the change or by introducing phase feeding (Willis, 2010). A sudden change by reformulation could however affect pig performance (U.S. Pork Center of Excellence, n.d.) and add to the cost of feed replacement (Campos, 2003). Reese (2007) advises that a consideration of feed ingredient prices would better prepare pig farmers against a loss of profitability from pig feed variation. Chavas, Kliebenstein, and Crenshaw (1985) used a differential equation specification to present a production growth model for grower-finisher pigs. The results show that dynamic decisions for pig production are based on knowledge of the growth function. The U.S. Pork Center of Excellence (n.d.) also mentioned that the pig producer should

carefully consider any practices aiming to optimise feed cost and maximise profitability.

Niemi (2006) applied a dynamic programming model for optimising feeding and slaughter decisions for fattening pigs in Finland. The research shows that price changes and changes in slaughter premium can have a large effect on income for pig producers. In this study, the effect of pig genotype on feeding and slaughter patterns is also considered. The results suggest that producers can benefit from improvements in the pig genotype and must adjust feeding and slaughter decisions based on it. Niemi and Sevón-Aimonen (2009) applied optimisation methodology to determine the optimum time for heterogeneous pigs to be delivered to slaughter. The results suggest that by splitting the harvest, that is, by delivering the pigs with a high genetic potential before pigs with a low genetic potential (to avoid the penalty of heavy carcass weight) profitability increases by 5 per pig place per year.

Secondly, this thesis handles the variation across time in the feed ingredient cost and pig price at slaughter in a different way, by using stochastic programming (Chapter 4). The purchased weaner pig cost however is fixed. The other costs, such as that of labour, maintenance and repairs, electricity, water, medicine, transport and veterinary expenses are not included.

1.4.4 Effect of pig type, cost and price change, and dietary restraints on dietary nutrient specification

Many studies show that the genotype has a significant impact on pig growth. Interactions between pig genotype and their nutrition are considered. Interactions between expression of genetic performance potentials and partitioning of energy and protein intake for growth and carcass quality in grower-finisher pigs were studied in de Lange and Coudenys (1996). As mentioned earlier, Niemi (2006) points out that ingredient cost changes and price of pigs at slaughter affect dietary nutrient specification.

Thirdly, we will consider this effect in Chapter 5. Moreover, in response to consumer demand and the higher market price of leaner pork meat, so called “feed-to-lean” growth aiming for leaner pork has been developed. Feed-to-lean growth in pigs can be influenced by both: a) pig genetic improvement and b) farm management strategies (including the feeding schedule) (Schinckel & Einstein, n.d.). The performances of the

optimal feeding schedule and the feed-to-lean schedule are compared in Chapter 5.

1.4.5 On-farm situation

An application of the computer simulation model to real situations also should be considered (Tagliapietra, Ceolin, & Schiavon, 2005). Estimation of pig growth parameters using on-farm data is challenging because of variation due to both data collection and estimation method. The causes of the variation of on-farm data can be many, for example, the method of data collection, the location of the pig farm and the different types of pigs. The methods of estimation of model input parameters also have to be considered due to their impact on model outcome.

The fourth contribution of this thesis is that on-farm pig growth model parameters are estimated using different methods. Optimal animal nutrition is studied for pig producers in Thailand. A pig growth model for Thailand, based on a simple pig growth model (de Lange, 1995), has been developed, aiming to maximise gross returns for pig producers in Thailand (Siriwathananukul, 2000). Estimation of on-farm pig parameters is also examined and presented in detail in Chapter 6.

1.5 Summary

This thesis focuses on the combining of linear programming, an accurate growth model and nonlinear optimization methods in order to maximise market profitability for pig producers. Figure 1.5 illustrates the work done in this thesis and the order of its presentation.

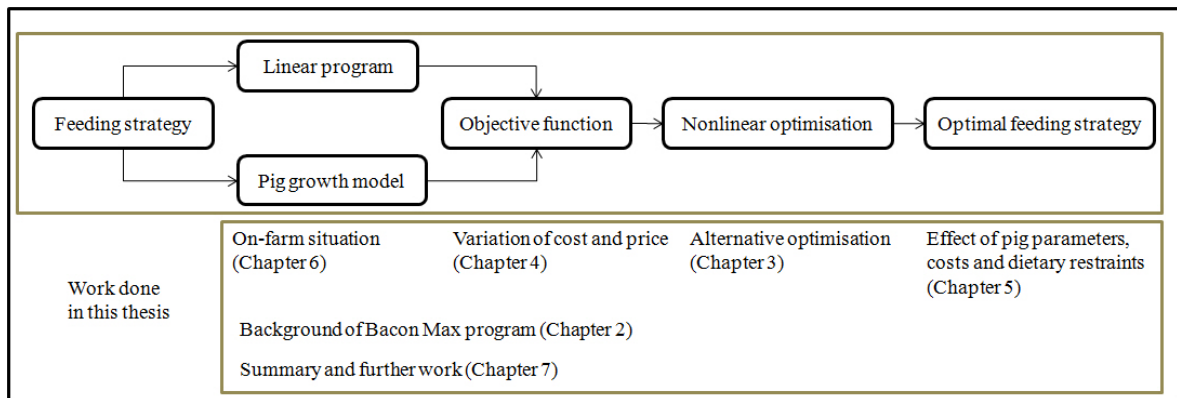


Figure 1.5: A summary of the contributions in this thesis.

We conclude with a synopsis of the content of the coming chapters.

- Chapter 2 A description of the program “Bacon Max”, which is based on the idea of including linear programming to estimate minimum feed cost, a simple pig growth model and nonlinear optimisation. It is written in Visual C++. The landscape of the objective function surface is also explored. The nonlinear optimisation method used in Bacon Max and the genetic algorithm are explained, followed by subsequent discussion of Bacon Max. Covariance of pig type parameters has been added to the Bacon Max program.
- Chapter 3 Development of an alternative optimisation methodology. A new search algorithm called the “Tailored method” with its program written in Visual C++ is introduced.
- Chapter 4 Accommodation of variation in feed ingredient schedule and price schedule. Stochasticity is added to Bacon Max program.
- Chapter 5 Exploring the effect of pig parameters, costs and dietary restraints on dietary nutrient specification.
- Chapter 6 Application of Bacon Max for southern Thailand data including the estimation of on-farm parameters.
- Chapter 7 Final conclusion and further work summarised.

Chapter 2

Optimisation of profitability - the basics

Efficient pig meat production is of critical importance on our increasingly finite planet. Statistics from the Food and Agricultural Organization of the United Nations (FAO) in 2009 inform us of the significantly greater annual world production rates of pig meat (106.07 million metric tons) over other meats such as chicken (79.59 million metric tons), cattle (61.83 million metric tons) and sheep (8.11 million metric tons). With the increasing global human population presenting greater demands for more pig meat produce it is of critical importance to offer pig farmers an efficient pig meat production method that generates maximum profitability. Three major factors that have an impact on the profitability of pig meat production are considered in this thesis. The first being feed cost, the second, pig breed and genotype and finally pig market price at slaughter.

For many decades, linear programming alone has been used to determine pig diets with minimum ingredient cost, based on a range of feedstuffs, their cost, their composition and dietary constraints. With the advent of pig growth models and nonlinear optimisation algorithms, it is now possible to extend this traditional use of optimisation to determine a feeding schedule which maximises profitability, rather than considering only minimisation of feed cost by linear programming.

The format of this chapter is as follows. In the next section well established methods for finding minimum cost diets using linear programming are discussed. Then, the advent of pig growth models is described. Next, nonlinear optimisation methods are discussed in Section 3. Finally, the problem and objective function are explained in Section 4.

2.1 Linear programming

Linear programming (LP) is a mathematical method for finding the optimum (minimum or maximum) of a linear function subject to linear constraints. The linear function to be maximised or minimised is termed the objective function and the conditions that need to be met are linear constraints which can be presented as below (Kall & Wallace, 1994):

$$\begin{array}{ll}
 \text{Objective function} & \text{minimise: } c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
 \text{Linear constraints} & \text{subject to: } \begin{array}{ll} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \vdots & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ x_1, x_2, \dots, x_n & \geq 0 \end{array}
 \end{array}$$

where x_i are the decision variables and c_i, a_{ij} and b_j are the coefficients that have fixed known real values.

In our problem,

x_i is the amount of each ingredient (kg) for example, barley, fish meal and tryptophan, in 1kg of a pig diet, for $i = 1, \dots, n$ where n is the number of feed ingredients that are available for the pig diet.

c_i is the cost (\$) for 1kg of feed ingredient i , as in the example shown in Table 2.1.

a_{ij} is the nutrient (energy, protein, vitamins and minerals) (kg) in 1kg of feed ingredient i appropriate to constraint $j = 1, \dots, m$ where m is the number of constraints, discussed in Section 2.1.2.

b_j is the requirement or limitation for constraint j .

Minimisation of cost per kg of feed has been traditionally set as the objective function for finding the pig diet formulation as we will show in Section 2.1.1. All animals need the basic nutrients, energy and protein, for maintenance of the normal body processes, for growth and reproduction. Small amounts of vitamins, minerals, fibre and water are also needed. The linear constraints are set by the requirements for basic nutrients for pig growth, as we will discuss in Section 2.1.2.

2.1.1 Objective function for LP - feed cost per kg

An example of feed ingredients and their associated costs are shown in Table 2.1; these are used in linear programming to find the least cost pig diet.

Table 2.1: A sample of New Zealand feed ingredient costs in September 2007.

Ingredient	Barley	Fish meal	...	Tryptophan
	x_1	x_2	...	x_n
ingredient cost	0.42	2.3	...	20
(\$/kg)	c_1	c_2	...	c_n

From these feed ingredient cost, the objective function for a least cost diet is

$$0.42x_1 + 2.3x_2 + \dots + 20x_n$$

where x_i is the amount of feed ingredient i (kg/kg) in a pig diet.

2.1.2 Constraints - via nutrients in feed ingredients

As already mentioned, energy, protein, vitamins and minerals are required nutrients for a pig to maintain body processes, to grow and to reproduce as shown in Figure 2.1. Limitations on the amount that can be used in a pig diet for each feed ingredient also have to be considered. Each constraint for the linear program is now described.

Energy

Energy content in feed ingredients can be classified by gross energy (GE), digestible energy (DE), metabolizable energy (ME) and net energy (NE) National Research Council (1998) measured in MegaJoules (MJ), as shown in Figure 2.2.

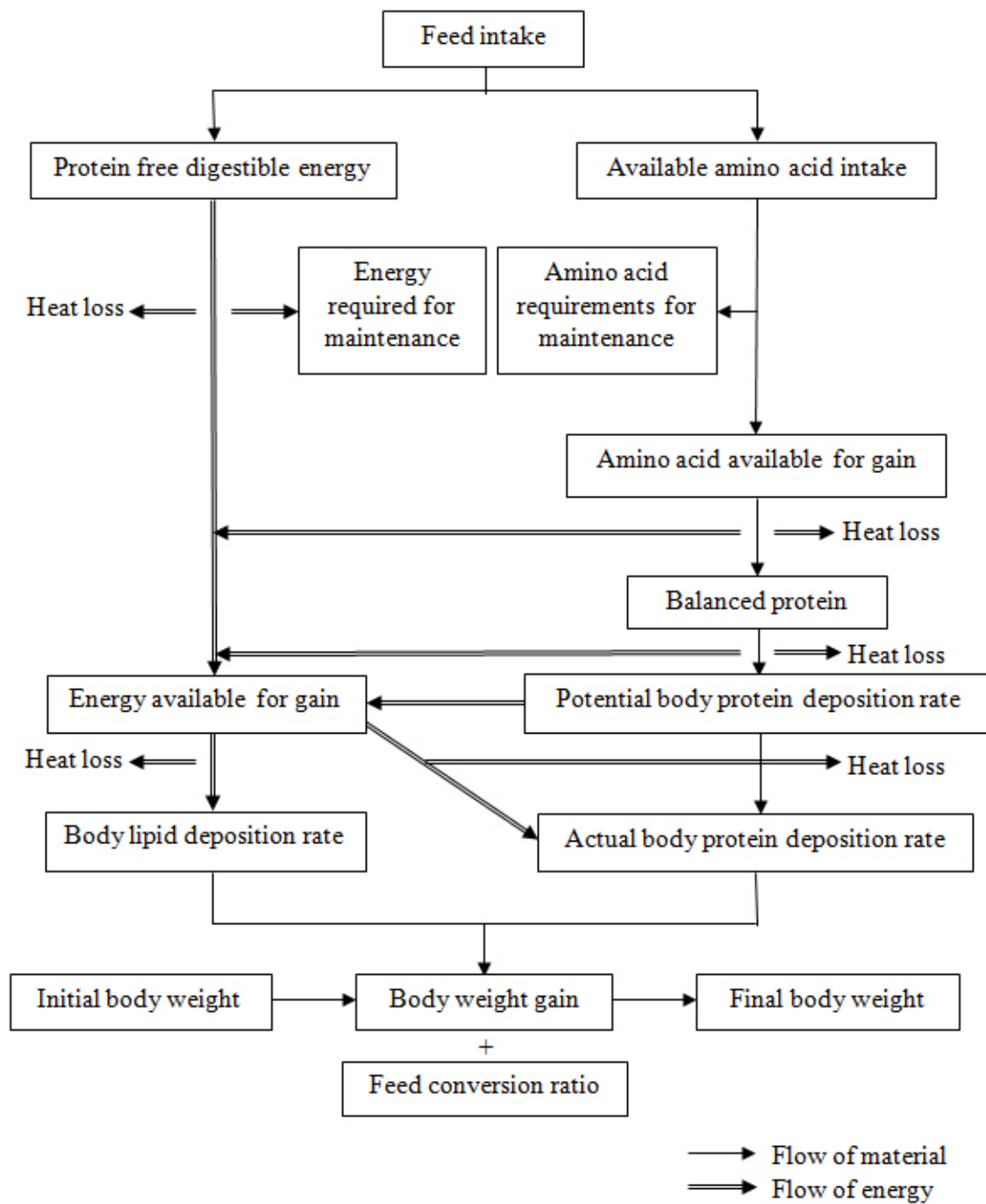


Figure 2.1: Partitioning of energy and protein from feed intake for pig maintenance and growth, adapted from de Lange (1995).

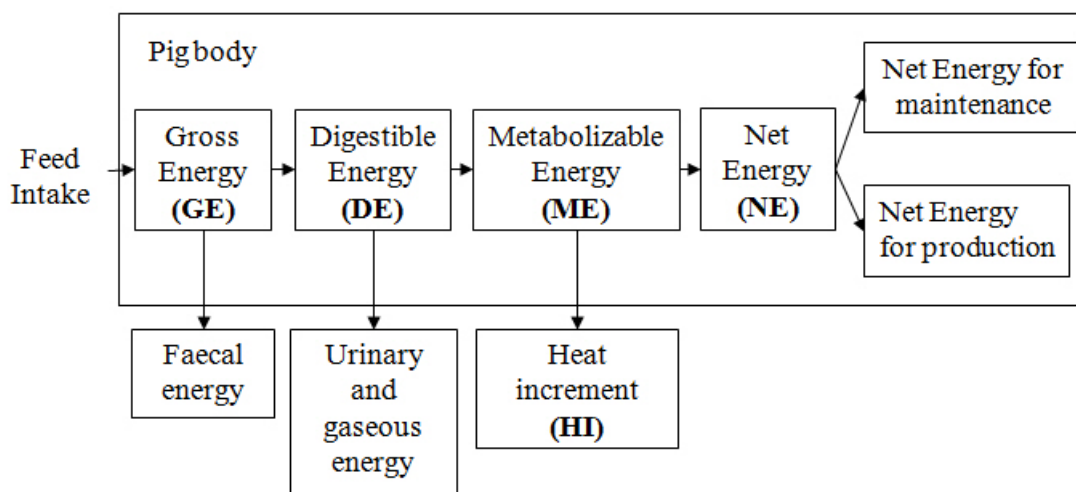


Figure 2.2: Energy breakdown of feed intake into gross energy, digestible energy, metabolizable energy and net energy for pig maintenance and production. Some of the energy is lost in the form of faecal energy, urinary and gaseous energy and heat increment.

Gross energy (GE) is the energy or the amount of heat that is released from the burning process of the specified quantity of feed ingredients (initially at 25°C in a bomb calorimeter after the products have returned to a temperature of 25°C. A bomb calorimeter is a laboratory device used to measure the heat of combustion). The gross energy depends on the proportion of carbohydrate, fat and protein in the feed ingredients.

Digestible energy (DE) is the energy remaining from GE after subtracting faecal energy. Not all energy in the feed ingredient can be digested and be available for use by the pig. Some of the energy will be lost as gas and in faeces. DE is preferred in describing the energy requirements of swine and the energy content of swine feeds, because DE is easy to calculate and more accurate. In addition, DE values are available for most of the commonly used feeds (National Research Council, 1998). So, DE density (MJ/kg) for feed ingredients has been used in the linear program in this study as shown in Table 2.2.

Metabolizable energy (ME) is the digestible energy less the energy in urine and gaseous products of digestion. Net energy (NE) is the final energy after loss of the heat increment from ME. This energy will be use for two purposes, maintenance and production.

Table 2.2: Digestible energy density (MJ/kg) for feed ingredients, using New Zealand data.

Feed ingredient	Barley	Fish meal	...	Tryptophan
	x_1	x_2	...	x_n
DE density (MJ/kg)	13.2	17.14	...	27.7
	a_{11}	a_{12}	...	a_{1n}

In this study, total DE density in a pig diet is a critical parameter in the feeding schedule. We will denote DE density in a pig diet as parameter d from now on. That is,

d = total digestible energy density in a pig diet, in MegaJoules per kilogram ; d has range of 12 - 17 MJ/kg for grower to finisher pigs.

Then, the energy constraint in the linear program is

$$13.2x_1 + 17.14x_2 + \dots + 27.7x_n = d$$

Amino acids and proteins

A protein is a chain of amino acids. That is the reason why we often present a protein in terms of amino acids. Amino acids are the major component for building pig muscle and for body growth. There are two type of amino acids: essential and non-essential, or indispensable and dispensable amino acids. The essential amino acids either cannot be synthesised, or cannot be synthesised at a sufficient rate, for the pig body to permit optimal growth or reproduction. On the other hand, non-essential amino acids can be synthesised in the body. This is the reason that essential amino acids have always been included in a pig diet, to keep them at a sufficient balanced level for maintenance of the pig body, for growth and for reproduction. Lysine in particular is known as the first limiting amino acid for a pig. With insufficient lysine in the pig diet, protein synthesis in the body will malfunction. Methionine, cystine, threonine, tryptophan and isoleucine are also required as essential amino acids in a pig diet (in a particular ratio to lysine, providing so-called “ideal balanced” protein). The ideal balanced amino acids from the Agricultural Research Council (1981) are shown in Table 2.3 and the amino acids in the feed ingredients are shown in Table 2.4; these figures are used to create the constraints for protein and amino acid requirements in the linear program.

Table 2.3: Recommended amounts and associated ratios required for balanced amino acids in pig diets.

Amino acid	ARC recommended amounts of amino acids (g/kg protein)	Amino acid to lysine ratio
Lysine	70	1
Methionine	17.5	0.25
Methionine and cystine	35	0.5
Threonine	42	0.6
Tryptophan	10	0.143
Isoleucine	38	0.543

Table 2.4: Amino acid densities found in feed ingredients used in New Zealand.

Amino acid	Feed ingredient			
	Barley	Fish meal	...	Tryptophan
	x_1	x_2	...	x_n
Lysine	3.19	46.01	...	0
Methionine	1.6	17.25	...	0
Methionine and Cystine	2.85	20.18	...	0
Threonine	3	22.81	...	0
Tryptophan	0.98	5.33	...	980
Isoleucine	3.54	24.55	...	0

Then from Table 2.3 and Table 2.4, the LP constraints for balanced amino acids are

$$\begin{array}{llll}
 \text{Lysine} & 3.19x_1 + 46.01x_2 + \dots + 0x_n & = & dr \\
 \text{Methionine} & 1.6x_1 + 17.25x_2 + \dots + 0x_n & \geq & 0.25dr \\
 \text{Methionine and Cystine} & 2.85x_1 + 20.18x_2 + \dots + 0x_n & \geq & 0.5dr \\
 \text{Threonine} & 3x_1 + 22.81x_2 + \dots + 0x_n & \geq & 0.6dr \\
 \text{Tryptophan} & 0.98x_1 + 5.33x_2 + \dots + 980x_n & \geq & 0.143dr \\
 \text{Isoleucine} & 3.54x_1 + 24.55x_2 + \dots + 0x_n & \geq & 0.543dr
 \end{array}$$

where the parameter r is defined as follows,

r = lysine to digestible energy ratio, in grams per MegaJoule; r typically ranges between 0.2 - 1.2 g/MJ.

Minerals

Minerals are required in a small amount, but have an important role in maintenance and growth. An insufficient amount of minerals can cause malfunction or disease. On the other hand, overdoses of minerals also can be toxic for a pig. For this reason, constraints for minerals are also included in the linear program. We consider five minerals, calcium (Ca), phosphorus (P), sodium (Na), chloride (Cl) and potassium (K), as shown in Table 2.5.

Table 2.5: Minimum and maximum amount for minerals in pig diets.

Minerals	Minimum amount (g/kg)	Maximum amount (g/kg)
Calcium	8	15
Phosphorus	7	11
Sodium	1	2
Chloride	1	2
Potassium	2	10

The amount of feed ingredients in a pig diet

Each feed ingredient must appear within upper and lower bounds in a pig diet as shown in Table 2.6 in order to meet body requirements for maintenance, reproduction and palatability. Some examples for the causes of the limitations on feed ingredients are high fibre, unbalanced amino acids, smell or expense.

Table 2.6: Minimum and maximum amount for feed ingredients in pig diets.

Feed ingredients	Minimum amount of feed ingredients (g/kg)	Maximum amount of feed ingredients (g/kg)
Barley	0	no limitation
Maize	0	40
Fish meal	0	2
Blood meal	0	5
Soya bean meal	0	30
⋮	⋮	⋮
Tryptophan	0	no limitation

2.1.3 Linear program for minimum cost diet

We showed the objective function in Section 2.1.1 and constraints in Section 2.1.2. Then, the linear program that finds the minimum cost diet is shown as an example in Table 2.7. New Zealand feed ingredient costs in September 2007 are used to set up the objective function and the ingredient nutrients are used in the constraints.

Table 2.7: The linear program for the minimum cost diet using New Zealand feed ingredient costs for September 2007 and their nutrient composition.

Ingredient	Barley x_1		Fish meal x_2		...		Tryptophan x_n		
Minimise cost (\$/kg):	$0.42x_1$	+	$2.3x_2$	+	...	+	$20x_n$		
subject to:									
Digestible energy (MJ/kg)	$13.2x_1$	+	$17.14x_2$	+	...	+	$27.7x_n$	=	d
Lysine (g/kg)	$3.19x_1$	+	$46.01x_2$	+	...	+	0	=	dr
Methionine (g/kg)	$1.6x_1$	+	$17.25x_2$	+	...	+	0	≥	$0.25dr$
Methionine and cystine (g/kg)	$2.85x_1$	+	$20.18x_2$	+	...	+	0	≥	$0.5dr$
Threonine (g/kg)	$3x_1$	+	$22.81x_2$	+	...	+	0	≥	$0.6dr$
Tryptophan (g/kg)	$0.98x_1$	+	$5.33x_2$	+	...	+	$980x_n$	≥	$0.143dr$
Isoleucine (g/kg)	$3.54x_1$	+	$24.55x_2$	+	...	+	0	≥	$0.543dr$
					⋮				
Vitamins and mineral bounds (g/kg)	$0.5x_1$	+	$40x_2$	+	...	+	0	≥	8
	$0.5x_1$	+	$40x_2$	+	...	+	0	≤	15
	$3.1x_1$	+	$25x_2$	+	...	+	0	≥	7
	$3.1x_1$	+	$25x_2$	+	...	+	0	≤	11
					⋮				
Ingredient upper bounds (g/kg)	$100x_1$	+	0	+	...	+	0	≤	100
	0	+	$100x_2$	+	...	+	0	≤	2
	0	+	0	+	...	+	$100x_n$	≤	100
					⋮				
Diet mass (kg/kg)	x_1	+	x_2	+	...	+	x_n	=	1

2.2 The advent of pig growth models

Pig growth models have been developed to predict with reasonable accuracy the performance of pigs based on the diets that are fed and critical pig characteristics which differ across pig genotypes. In our study, we use a simple pig growth model which is described in de Lange (1995) and was shown in Figure 2.1. The pig growth model

describes the energy and amino acid partitioning for pig growth for one instant in time (static), for a one day period and for an individual pig with no variation (so is deterministic). The model includes some empirical variables and can be defined as mechanistic.

Pig characteristics depend on pig genotype, breed, strain and gender. This pig growth model requires two pig type variables: minimum allowable lipid to protein ratio and maximum daily protein deposition (g/day). As described in de Lange (1995), these are

$$\begin{aligned}\text{minLP} &= \text{minimum allowable lipid to protein ratio} \\ \text{Pdmax} &= \text{maximum daily protein deposition (g/day)}\end{aligned}$$

Figure 2.3 a) and b) shows that a minLP of 0.6 and 1.0 represent a high genetic potential and a low genetic potential pig type, respectively. A high genetic potential pig type has a lower lipid to protein ratio and is able to reach Pdmax faster than a low genetic potential pig type. A Pdmax of 120 g/day represents a low genetic potential pig type. On the other hand a Pdmax of 200 g/day represents a high genetic potential pig type. A high genetic potential pig type can deposit protein to build body muscle at a maximum level of 200 g/day compared with a low genetic potential pig type which can deposit protein at only 120 g/day.

2.3 Extended optimisation

As mentioned in Section 2.1, in the past linear programming has been used to find the minimum cost diet. Now the combination of a pig growth model (Section 2.2) and nonlinear optimisation is able to maximise pig profitability. In the next section we will describe the domain, objective function and the nature of the objective function for this research. Then the nonlinear optimisation method will be discussed in Section 2.4.

2.3.1 Domain

An animal “diet”, specifying nutrition for a certain growth period, and required as input to a simple growth model, can be described using only three parameters, p , r and d , defined as follows:

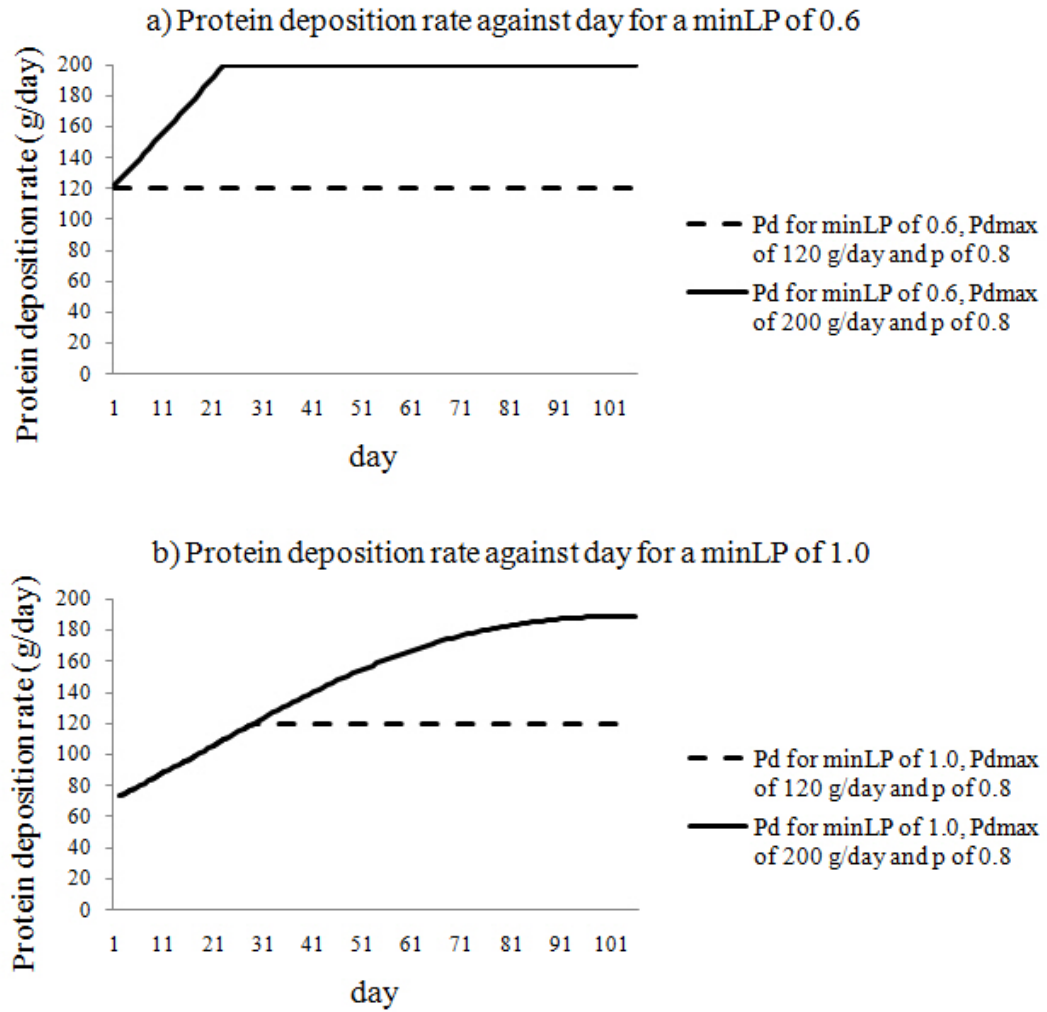


Figure 2.3: Expected protein deposition rate against day from a simple pig growth model for Pdmax of 120 g/day (dashed lines, a low genetic potential pig type) and 200 g/day (solid lines, a high genetic potential pig type) for a) a minLP of 0.6 and b) a minLP of 1.0. All figures are for p of 0.8.

-
- p = proportion of the *ad libitum* digestible energy intake
 r = lysine to digestible energy ratio, in grams per MegaJoule
 d = digestible energy density, in MegaJoules per kilogram

Typical parameter ranges used are $[0.8, 1]$ for p , $[0.2, 1.2]$ for r and $[12, 17]$ for d . The *ad libitum* digestible energy intake is determined by a standard National Research Council (1998) curve, relating digestible energy to live weight (LW) of the animal. Parameter p determines the proportion of that amount to be fed. We mentioned earlier that lysine is an essential amino acid, required for growth, and generally the first amino acid found to be limiting in a diet. For that reason we specify the level of lysine required using parameter r . Finally, the interval constraining energy density d of the diet reflects the range of existing values in the ingredients.

In general, by a “feeding schedule” we refer to a finite sequence of diets, $((p_k, r_k, d_k) : k = 1, \dots, q, \dots, K)$, with the k th diet fed for T_k days where T_k is set at the beginning as an input for the feeding schedule. K is the maximum number of diets for that feeding schedule. Each day within the growth period is represented by the parameter x and generally used to identify the slaughter date (SD). Thus, x is chosen in the range $\{1, 2, \dots, T_{max}\}$, where T_{max} is the maximum limit of the growing period, so $T_{max} = \sum_{k=1}^K T_k$. This total period of T_{max} amply covers the usual time from weaner arrival (it is assumed the producer buys in weaners) to slaughter date.

The feeding periods of T_1, \dots, T_{q-1} are predetermined times (fixed at the outset) with q being the number of diets needed until profitability is maximised ($1 \leq q \leq K$) with T_q is in the range $\{1, 2, \dots, (\text{preset } T_q)\}$.

The feeding schedule that offers maximum profit for given conditions we term the “optimal feeding schedule”. Thus we write a feeding schedule as

$$F = (p_1, r_1, d_1; p_2, r_2, d_2; \dots; p_k, r_k, d_k)$$

Our aim will be to find the optimal feeding schedule and slaughter date, so the domain of the problem is

$$P_1 \times R_1 \times D_1 \times P_2 \times R_2 \times D_2 \times \dots \times P_q \times R_q \times D_q \dots \times P_K \times R_K \times D_K \times \{1, 2, \dots, T_{\max}\}$$

where $P_k = [0.8, 1]$, $R_k = [0.2, 1.2]$ and $D_k = [12, 17]$ for $k = 1, 2, \dots, q, \dots, K$ and $(1 \leq q \leq K)$.

The problem is set in $3K + 1$ dimensional Euclidean space when the slaughter day is allowed to vary. On the other hand, if the slaughter date is fixed, the problem is set in $3K$ dimensional Euclidean space.

In practice on a commercial pig farm today, around the world, pigs are fed two diets (or sometimes three diets), which are growers, and finishers, during the period from 20kg to slaughter; thus $K = 2$. For example, Table 2.8 shows a feeding schedule comprising two diets. The first period (grower) is 35 days and the maximum of the second period (finisher) is 70 days.

Table 2.8: Diets and the number of days that pigs are fed using the k th diet (T_k) for two diets, so $K = 2$. The first period runs from day 1 to 35 and the second period from day 36 to 105.

Number of diets (k)	1	2	Maximum day
	(grower)	(finisher)	(T_{\max})
Number of days that pigs are fed using the k th diet (T_k)	$T_1 = 35$	$T_2 = 70$	$T_1 + T_2 = 105$
Diets	p_1, r_1, d_1	p_2, r_2, d_2	

If the slaughter date is set as day105 then the problem is set in a $2 \times 3 = 6$ dimensional Euclidean space. On the other hand, if the slaughter date is varied in the range $\{1, 2, \dots, T_{\max} = 105\}$ then the search is then set in a $(2 \times 3) + 1 = 7$ dimensional Euclidean space. The feeding period T_2 is in the range $\{1, 2, \dots, 70\}$ days, with T_1 fixed at 35 days. The domain for this two diet feeding schedule is,

$$P_1 \times R_1 \times D_1 \times P_2 \times R_2 \times D_2 \times \{1, 2, \dots, 105\} \text{ where } K = 2$$

With advanced technology, however, it will be possible to change diets more often (weekly or even daily, with phase feeding). Alexander et al. (2006) investigated the effect of changing diets more frequently, on a weekly basis, thus $K = 15$. For example, Table 2.9 shows a feeding schedule comprising 15 diets. Each diet is fed to the pigs for 7 days.

Table 2.9: Diets and the number of days that pigs are fed using the k th diet (T_k) for 15 diets, so $K = 15$. Each diet is fed to the pigs for 7 days up to the maximum period of 105 days.

Number of diets (k)	1	2	...	15	Maximum day (T_{max})
Number of days that pigs are fed using the k th diet (T_k)	$T_1 = 7$	$T_2 = 7$...	$T_{15} = 7$	$\sum_{k=1}^{15} T_k = 105$
Diets	p_1, r_1, d_1	p_2, r_2, d_2	...	p_{15}, r_{15}, d_{15}	

If the slaughter date is set as day 105 then the problem is set in a $15 \times 3 = 45$ dimensional Euclidean space. On the other hand, if the slaughter date is varied in the range $\{1, 2, \dots, 105\}$ then the search is set in a $(15 \times 3) + 1 = 46$ dimensional Euclidean space. For example, if the profit is maximised in the diet 10th, $q = 10$. The feeding period T_{10} is in the range $\{1, 2, \dots, 7\}$ days. The domain for this feeding schedule is,

$$P_1 \times R_1 \times D_1 \times P_2 \times R_2 \times D_2 \times \dots \times P_{15} \times R_{15} \times D_{15} \times \{1, 2, \dots, 105\} \text{ where } K = 15$$

2.3.2 Objective function

The objective function to be maximised is profit, or gross margin per pig, given by

$$g(F) = \max_x g(F, x)$$

where F is a feeding schedule and x the number of days until slaughter and

$$g(F, x) = \text{Gross Return}(F, x) - \text{Feed Cost}(F, x) - \text{Weaner Cost},$$

the gross margin when we feed a pig using feeding schedule F for x days. We write this

as $g(F, x) = GR(F, x) - FC(F, x) - WC$ for notational convenience. Alternatively, we prefer to measure the more practical “gross margin per pig place per year” GMPPY. This is computed as

$$\max_x f(F, x) = \max_x (365/(x + 7))g(F, x)$$

(when there is a seven day turnaround between batches).

The weaner cost is fixed by the price of a pig in the market at 20kg (for example, Chapters 3 and 4 use weaner cost in July 2001, NZ\$70 and Chapter 5 uses weaner cost in September 2007, NZ\$75). Feed cost FC is the minimum feed cost given F (determined using linear programming), for the period of x days; this requires use of a schedule of ingredient costs, as shown before in Table 2.1.

Gross return GR is determined by the backfat thickness and carcass weight of the pig, which in turn are determined by F and x . This requires a schedule of prices, as shown in the example in Table 2.10.

Table 2.10: A New Zealand price schedule giving prices in cents/kg for pigs at slaughter in September 2007.

Backfat (mm)	Carcass weight (kg)										
	35.0 and under	35.1 to 40.0	40.1 to 45.0	45.1 to 50.0	50.1 to 55.0	55.1 to 60.0	60.1 to 65.0	65.1 to 70.0	70.1 to 75.0	75.1 to 80.0	80.1 and over
<6	255	255	255	255	255	255	255	255	255	255	255
6-9	310	310	330	330	320	320	305	305	305	305	300
10-12	310	310	320	320	310	305	305	305	305	305	300
13-15	230	230	230	230	230	260	260	260	260	260	255
16-18	185	185	185	185	185	185	185	185	185	185	185
>18	165	165	165	165	165	165	165	165	165	165	165

An iteration of the routine uses r_k and d_k to complete the right-hand-side constraints in the linear program; the least cost makeup of 1kg of feed for this period is the output. Together with p_k and the standard NRC feed intake curve this allows the feed cost for this k th period to be computed. The amount of balanced amino acid can also be calculated. This, together with the genotype parameters (and at the start the initial

mass P_0 of protein in the pig) and the growth model, allows us to grow the pig for the k th period. Protein and lipid deposition are recorded. Overall growth allows us to compute $FC(F, x)$ (by summing the individual period feed costs) and $GR(F, x)$ (by referring the configuration of the pig at slaughter date to the price schedule).

Pig genotype parameters in the growth model which thence influence the objective function are $Pdmax$, the maximum daily protein deposition, and $minLP$, the minimum allowable lipid to protein ratio.

Thus the objective function of interest is calculated in two steps:

1. Calculation of $f(F, x)$, the gross margin per pig place per year when feeding schedule F is administered for x days.
2. Determination of the maximum gross margin per pig place per year for feeding schedule F , namely $f(F) = \max_x f(F, x)$.

The process of finding the optimal feeding schedule is shown in Figure 2.4.

Our aim now is to determine the maximum gross margin per pig place per year over all feeding schedules, using nonlinear optimisation, as $\max_F f(F)$.

2.3.3 The nature of the objective function

Next, we move to discuss the nature of the objective function for this research. In Alexander et al. (2006) the nature of the objective function was determined by examining several random cross sections through a known solution. This revealed a single craggy peak earning the description “craggy volcano” for the objective function surface as seen in Figure 2.5. The cross-sections exhibit different slopes and the peaks are not always central. Pig genotype parameters, $Pdmax$ and $minLP$ influence the level (and shape) of the objective function. The discontinuities in the function are attributable to discrete changes in x , the number of days for which the pig is grown, together with passage of the grown pig (based on backfat thickness and carcass weight change) from one cell of the discontinuous price schedule to another.

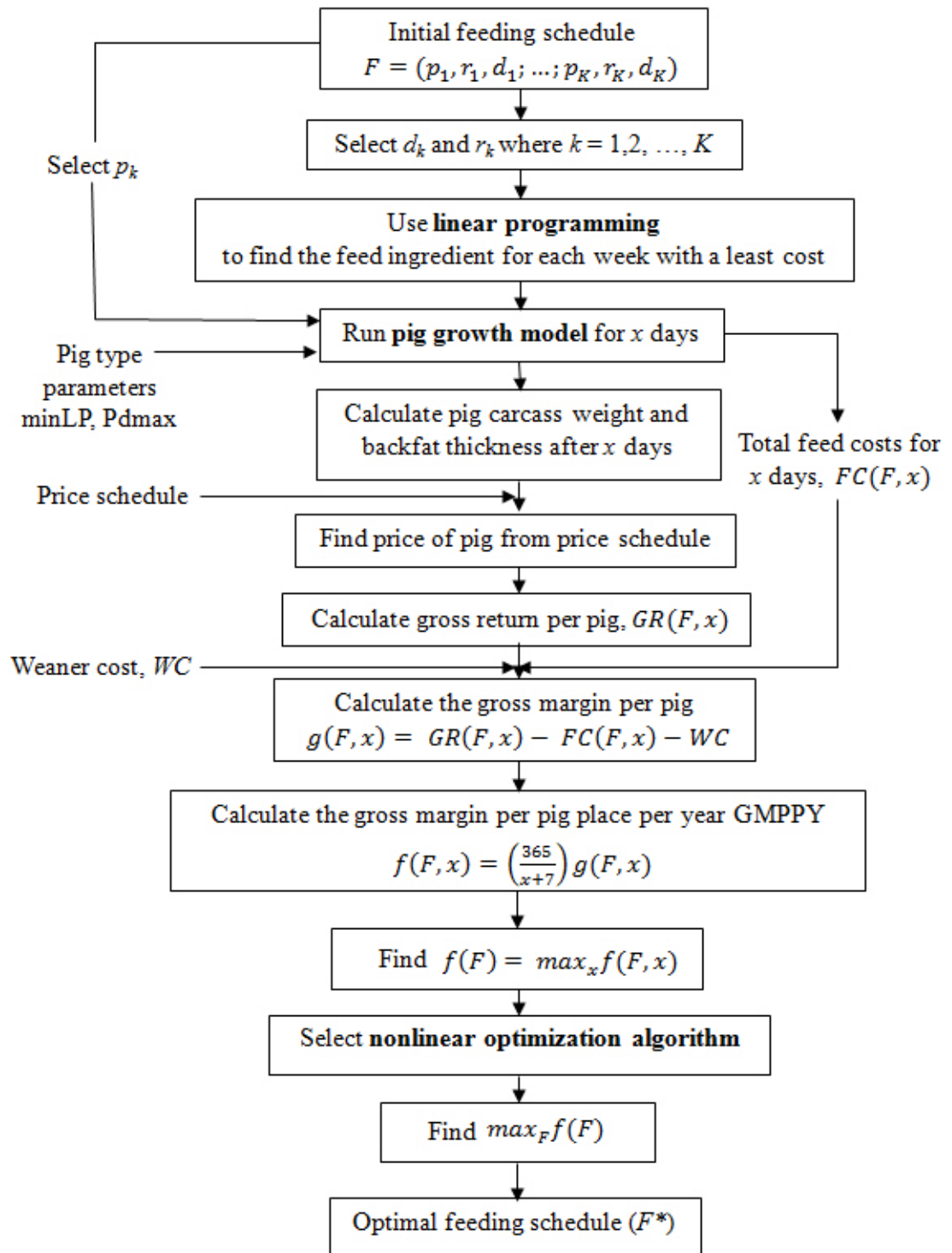


Figure 2.4: The combination of a linear program, a simple pig growth model and nonlinear optimisation for maximisation of pig profitability measured as gross margin per pig place per year GMPPY.

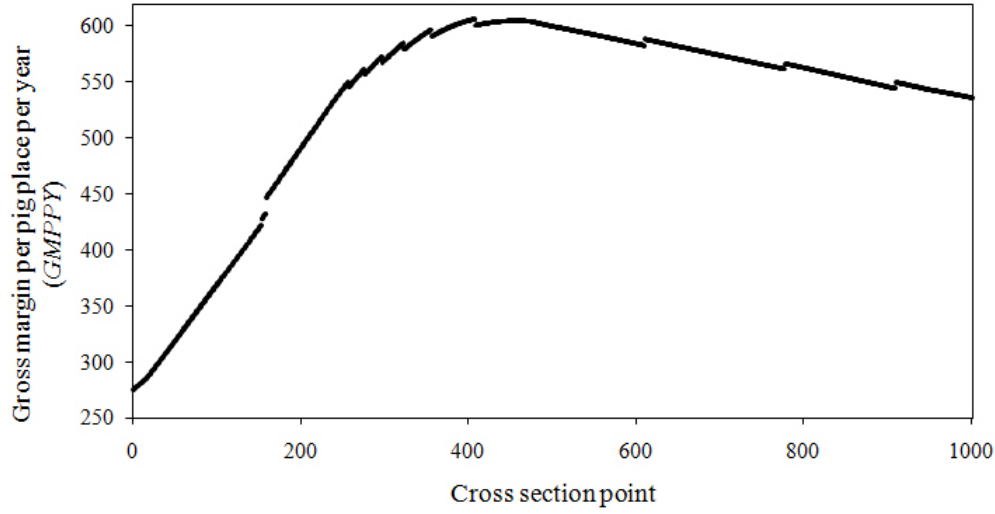


Figure 2.5: Showing a “craggy volcano” crosssection of a GMPPY surface.

2.4 Nonlinear optimisation methods

In the previous section the feeding schedule F involved three parameters, p, r and d . The random search for the optimal feeding schedule covers a large search space, a hypercuboid $\mathbf{R}^{3K} \times \{1, 2, \dots, T_{max}\}$ as shown in Figure 2.6. K represents the maximum number of pig diets (usually $K = 15$) until slaughter date in $\{1, 2, \dots, T_{max}\}$. T_{max} is the maximum limit of the growing period (day).

The probability of the random search to find the peak of this objective function in the domain of $[0,1]$ is shown as

$$\begin{aligned} \text{If 2 dimensions:} \quad & \frac{1}{2} = 0.5 \\ \text{If 3 dimensions:} \quad & \frac{1}{4} = 0.25 \\ \text{If 46 dimensions:} \quad & \frac{1}{2^{3K+1}} = \frac{1}{2^{46}}, \text{ where } 3K + 1 = 46 \end{aligned}$$

The random search for our high dimensions problem, $3K + 1$ dimensional Euclidean space, has very low probability to find the optimal solution. The optimisation methods used to generate the optimal solution are of critical importance. The search processes used in Bacon Max to determine the optimum solution for this problem have included pure random search, tabu search, Monte Carlo search, ascent search and a genetic

algorithm. Among these search processes genetic algorithms were the most successful, despite the long search period needed. We now consider the genetic algorithm and its use in Bacon Max.

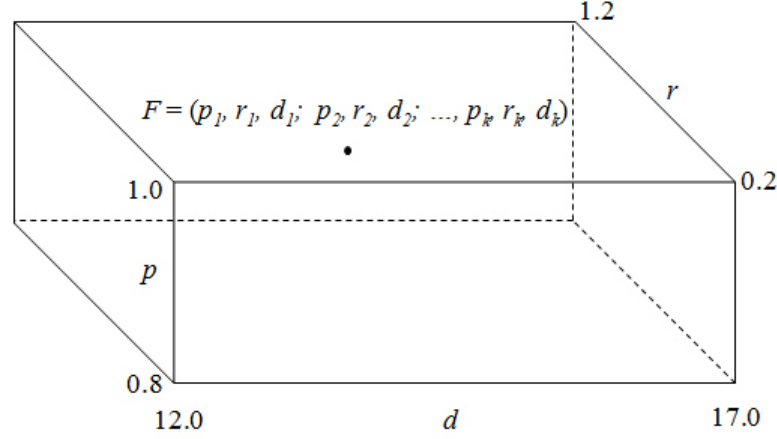


Figure 2.6: An illustration of the search space of the three parameters, p , r and d in the feeding schedule. This cube is multiplied by itself K times to give the full domain.

2.4.1 Genetic Algorithms (GA)

Genetic algorithms were first introduced by Holland (1975). Since then they have become a part of evolutionary computing and evolutionary algorithms, inspired by Darwinian evolution. They further evolved to find optimal solutions for problems using genomes, selection, crossover and mutation. A basic genetic algorithm is now presented (adapted from Obitko (1998)). In the sequel we shall assume that diets are fed weekly (or part-weekly, for the final diet), so that a genome has the form $F = (p_1, r_1, d_1; p_2, r_2, d_2; \dots; p_K, r_K, d_K)$, $k = 1, 2, \dots, K$

1. (Start) Generate a set of g random solutions for the problem, where a genome represents a possible solution. Thus the parent population is a set of g genomes. In our study we start by randomly generating a set of feeding schedules F_1, F_2, \dots, F_g as the parent population, where $F_i = (p_{i1}, r_{i1}, d_{i1}; p_{i2}, r_{i2}, d_{i2}; \dots; p_{iK}, r_{iK}, d_{iK})$, $i = 1, 2, \dots, g$

2. (Fitness) Evaluate the fitness function of each genome in the population. Each feeding schedule in the parent population is used to grow pigs via the growth model and the profitability (GMPPY) calculated. Feeding schedule with higher GMPPY have a higher chance of selection in the next step.

3. (New population) Create a new population by using the concept of reproduction presented below

a) (Selection) Select two parent genomes from the current population according to their fitness (the better the fitness, the greater the chance of selection). [For example, select feeding schedules F_1 and F_2 as those with highest GMPPY in the parent population.]

b) (Crossover or recombination) With set crossover probability, crossover the parents to form new offspring (children). If no crossover is performed, the offspring is an exact copy of parents. [In our example, the parent population is

$$\begin{aligned} F_1 &= (p_{11}, r_{11}, d_{11}; p_{12}, r_{12}, d_{12}; \dots; p_{1K}, r_{1K}, d_{1K}) \text{ and} \\ F_2 &= (p_{21}, r_{21}, d_{21}; p_{22}, r_{22}, d_{22}; \dots; p_{2K}, r_{2K}, d_{2K}) \end{aligned}$$

So if crossover happen at the point between (p_{i1}, r_{i1}, d_{i1}) and (p_{i2}, r_{i2}, d_{i2}) the new offspring (children) are

$$\begin{aligned} F_1^* &= (p_{11}, r_{11}, d_{11}; p_{22}, r_{22}, d_{22}; \dots; p_{2K}, r_{2K}, d_{2K}) \text{ and} \\ F_2^* &= (p_{21}, r_{21}, d_{21}; p_{12}, r_{12}, d_{12}; \dots; p_{1K}, r_{1K}, d_{1K}) \end{aligned}$$

where F_1^* and F_2^* represent the new offspring after crossover.

c) (Mutation) With set mutation probability, mutate new offspring at each locus (position in genome). Mutation provides a minor change to the selected feeding schedules in the hope of improving the objective function value (GMPPY). [In our example,

$$\begin{aligned} F_1^{**} &= (p_{11}^*, r_{11}^*, d_{11}^*; p_{22}, r_{22}, d_{22}; \dots; p_{2K}, r_{2K}, d_{2K}) \text{ and} \\ F_2^{**} &= (p_{21}, r_{21}, d_{21}; p_{12}, r_{12}, d_{12}; \dots; p_{1K}^*, r_{1K}^*, d_{1K}^*) \end{aligned}$$

where $(p_{11}^*, r_{11}^*, d_{11}^*)$ and $(p_{1K}^*, r_{1K}^*, d_{1K}^*)$ represent the mutation position in each offspring feeding schedule. F_1^{**} and F_2^{**} represent the new offspring after crossover and mutation.]

- d) (Accepting) Place new offspring after crossover and mutation, F_1^{**} and F_2^{**} , into the new population
4. (Replace) Use the newly generated population in the algorithm. The new parent population is $F_1^{**}, F_2^{**}, F_3, \dots, F_g$.
5. (Test) Stop if the stopping criterion is met, and return the best solution in the current population. Use a set number of iterations, or run until the best solution (GMPPY) has no change for a given number of iterations as the stopping criterion.
6. (Loop) Go to Step 2, if the stopping criterion is not met

2.4.2 Genetic algorithm in Bacon Max

We now present the Bacon Max genetic algorithm used for this study. Two parameters g and I are used to control the search in the program and need to be set,

$$\begin{aligned} g &= \text{number of genomes in the parent population} \\ I &= \text{number of iterations without change in the objective function} \end{aligned}$$

To begin, set the number of iterations for which the solution must not change as the stopping criteria. The program will run until the stopping criteria is met. The process of the genetic algorithm is illustrated in Figure 2.7.

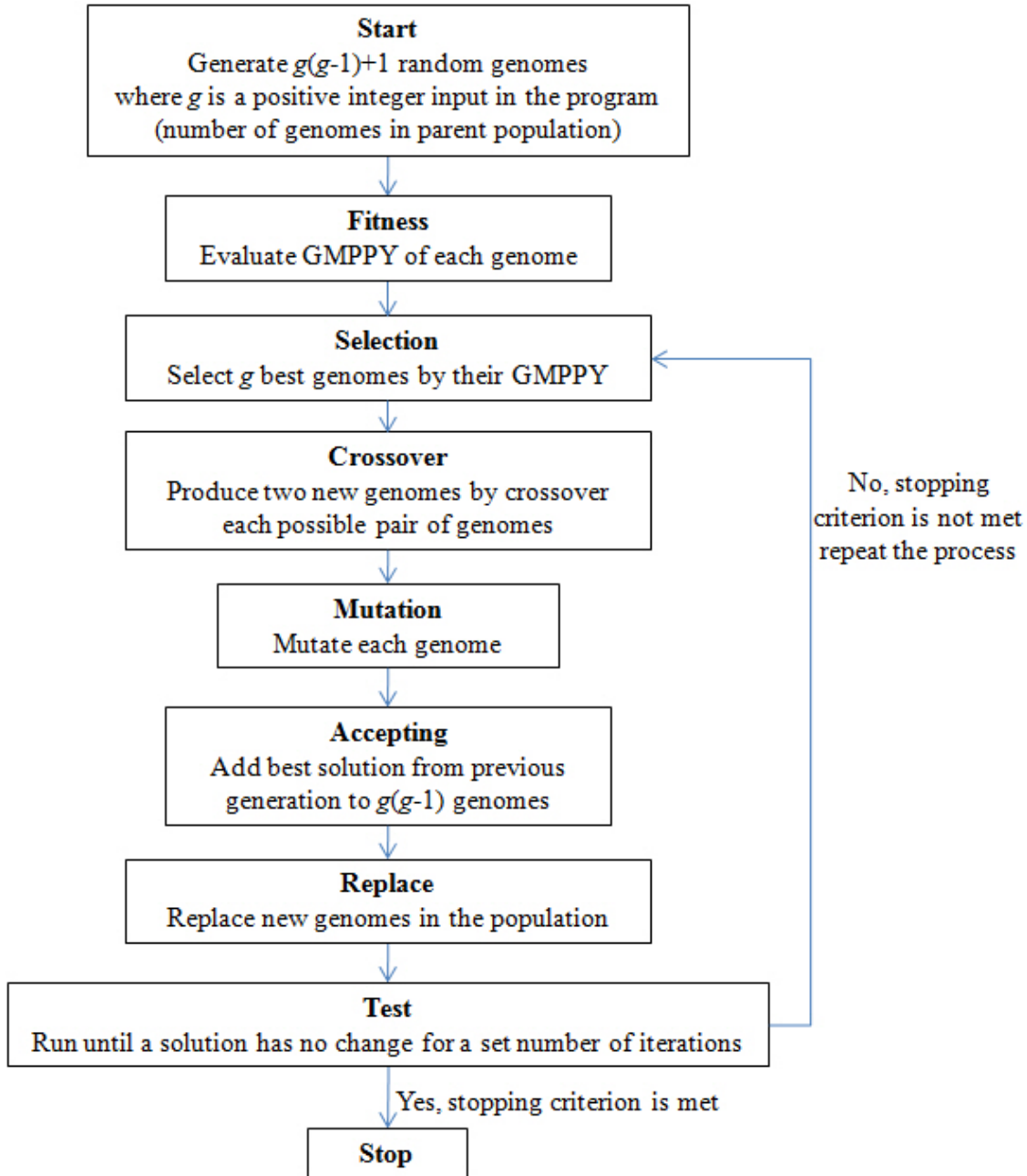


Figure 2.7: An illustration of the genetic algorithm based iterative process used in Bacon Max version 2.1.

2.4.3 Comparison of pure random search and a genetic algorithm

We now compare processor time used by pure random search and by the genetic algorithm. In Alexander et al. (2006), it was shown that the genetic algorithm is more effective than pure random search for finding the optimal solution for our problem, as illustrated in Figure 2.8.

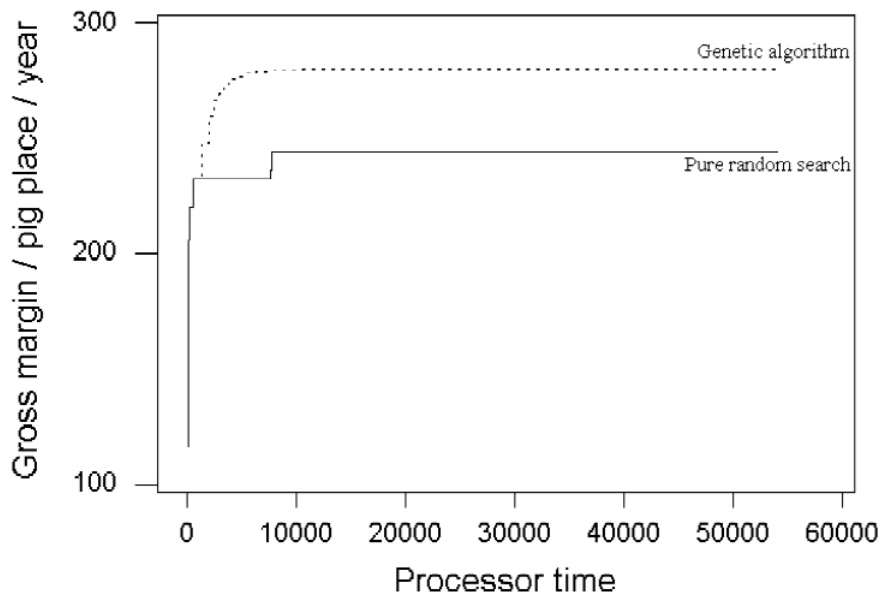


Figure 2.8: Comparison of the processor time (in seconds) between genetic algorithm and pure random search for finding the gross margin per pig place per year in Bacon Max version 2.1 (taken from Alexander et al. (2006)).

2.4.4 Bacon Max version 2.1

We now move on to discuss Bacon Max. Bacon Max, whose opening dialogue boxes are shown in Figure 2.9, was originally developed by Graham Wood, Patrick Morel and a research team at Massey University, New Zealand, to find optimal solution values of the objective function for pig production (gross margin per pig place per year). The simple de Lange pig growth model, linear programming and a genetic algorithm were combined in this program, written in C++.

Bacon Max version 2.1 also includes four Excel files for the input, calculation and output of optimisation results:

1. Input v2.1, (Figure 2.10) Bacon Max has two ways of setting up the input parameters;
 - a) as a single run in the input page; parameters for the optimisation need to be inserted in this page and then “Start” pressed, for a single run.
 - b) as multiple runs by setting up the run file, as illustrated in Figure 2.11; then run the program using “File Run”.
2. Run v2.1 (Figure 2.11) is the input file to set up the parameters for multiple runs.
3. Output v2.1 (Figure 2.12) shows key values calculated during the optimisation, such as combinations of feed ingredients calculated by the linear program, values in the pig growth model for growing pigs until slaughter and the progress of the solution in each iteration.
4. Outsummary v2.1 (Figure 2.13) provides a summary of the outputs of the optimisation.

2.5 Other discussion concerning Bacon Max

2.5.1 Conducting the optimisation for more than a single pig

The number of pigs contributing to the objective function (N) needs to be set up in Bacon Max. In practice, the number of pigs grown exceeds one. An example of results for different numbers of pigs in the objective function (1, 100 and 200) is presented in Figure 2.14. GMPPY for a single pig, 100 pigs and 200 pigs is \$216.6, \$212.3 and \$212.1, respectively indicating that the number of pigs in the objective function influences the resulting GMPPY. As the number of pigs in the objective function increases, GMPPY decreases. In conclusion, feeding the diet which maximises profitability for a single pig to a population of pigs resulted in lower GMPPY. Stochasticity should, however, be included in the model, which we will discuss in the next section.

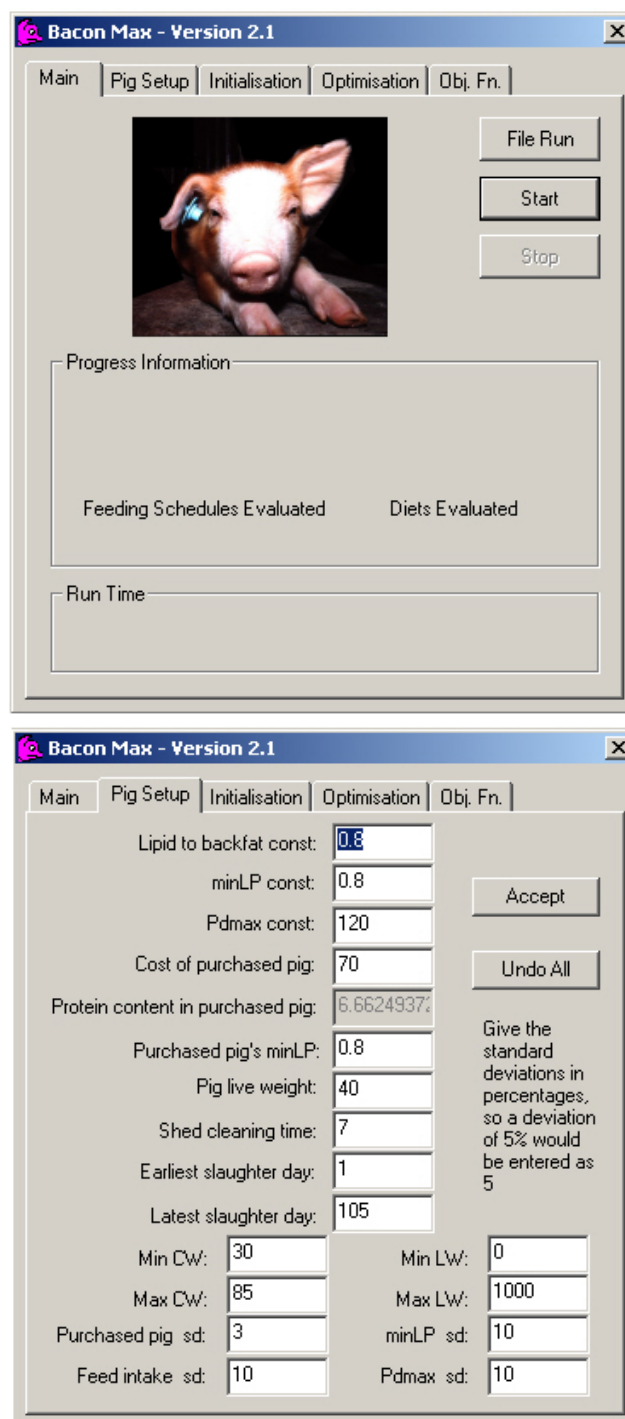


Figure 2.9: Starting page and sample pig setup for input parameters for a single run of Bacon Max version 2.1.

Microsoft Excel - Input v2.1											
File Edit View Insert Format Tools Data Window Help											
Type a question											
100%											
Arial											
10											
B I U											
%											
C4											
2.7											
	A	B	A	B	C	D	E	F	G	H	I
1	AD66		AD66		Broken rice	Blood meal	Broil	Palm Kamal Meal	Copra meal	Soya	
2										bean meal	fish meal
3	COST	Baht/KG	COST	Baht/KG	6.6	0.89	0.26	3.21	0.34	11	18
4	LYSINE	g/kg	LYSINE	g/kg	2.7	72.75	4.49	3.9	1.69	27.4	33.6
5	METHIONINE	g/kg	METHIONINE	g/kg	1.6	10.29	1.95	6.95	2.52	4.44	17.25
6	Meth + Cyst	g/kg	Meth + Cyst	g/kg	3.2	15.54	3.78	4.2	3.53	12	19.7
7	THREONINE	g/kg	THREONINE	g/kg	3.6	36.26	3.23	4.3	2.86	16.6	21.7
8	Tryptophane	g/kg	Tryptophane	g/kg	1.00	6.99	1.26	1.40	0.93	7.10	6.50
9	ISOLEUCINE	g/kg	ISOLEUCINE	g/kg	3.2	3.01	3.59	4.8	3.69	19.9	22.5
10	DE	MM/KG	DE	MM/KG	13.2	18.43	11.54	14.07	12.36	20.01	17.14
11	DP	g/kg	DP	g/kg	79.18	569.46	107.33	246.40	104.54	296.20	520.75
12	CP	g/KG	CP	g/KG	75	920	153.4	150	192	440	550
13											
14	DM	%	DM	%	87.7	91.36	88.1	96.36	91.6	96.76	89.33
15	Weight		Weight		1	1	1	1	1	1	1
16	GE	MM/KG	GE	MM/KG	16.3	22.35	17.6	19.78	20.23	22.84	20.05
17	ASPARTIC ACID	g/kg	ASPARTIC ACID	g/kg	5.65	78.42	8.2	17.48	9.03	32.13	44.59
18	SERINE	g/kg	SERINE	g/kg	3.9	36.71	4.82	9.85	5.17	12.37	19.74
19	GLUTAMIC ACID	g/kg	GLUTAMIC ACID	g/kg	26.78	66.5	25.01	54.4	20.24	53.77	78.42
20	PROLINE	g/kg	PROLINE	g/kg	11.63	25.93	7.61	16.18	1.92	12.97	20.91
21	GLYCINE	g/kg	GLYCINE	g/kg	3.39	26.06	5.85	11.71	3.47	9.59	30.3
22	ALANINE	g/kg	ALANINE	g/kg	3.61	60.38	5.36	11.63	4.72	10.27	35.36
23	VALINE	g/kg	VALINE	g/kg	4.7	61.6	5.31	7.1	6.11	20.6	26.5
24	LEUCINE	g/kg	LEUCINE	g/kg	6.4	99.06	7.13	8.9	7.68	34.2	39.7
25	TYROSINE	g/kg	TYROSINE	g/kg	3.22	22.87	3.15	7.34	2.19	9.32	18.8
26	PHENYLALANINE	g/kg	PHENYLALANINE	g/kg	5.13	56.58	4.37	11.66	5.52	14.68	22.77
27	Phen+Tyr	g/kg	Phen+Tyr	g/kg	7.3	79.45	7.52	10.6	7.71	36.7	39.8
28	HISTIDINE	g/kg	HISTIDINE	g/kg	1.7	48.21	3.48	2.6	2.18	11.7	15.6
29	ARGININE	g/kg	ARGININE	g/kg	4.9	30.43	8.12	15.6	16.94	32.3	32
30	CYSTEINE	g/kg	CYSTEINE	g/kg	1.25	5.25	1.93	3.67	1.01	2.47	2.93
31	CA LOW	g/kg	CA LOW	g/kg	0.3	2.8	1.4	2.8	2.1	2.3	50.2
32	CA HIGH	g/kg	CA HIGH	g/kg	0.3	2.8	1.4	2.8	2.1	2.3	50.2

	A	B	C	D	E	F	G	H	I	J	K	L
1	L9	carcass weight (kg)										
2	cents/kg		35.1	40.1	45.1	50.1	55.1	60.1	65.1	70.1	75.1	80.1
3	backfat (mm)	35	40	45	50	55	60	65	70	75	80	
4	6	185	195	195	195	195	195	195	195	195	195	195
5	9	255	280	290	290	275	265	265	265	265	250	230
6	12	255	280	280	285	270	260	260	265	265	250	230
7	15	225	225	225	225	225	230	230	230	230	220	220
8	18	145	145	145	145	145	145	155	155	155	155	155
9		125	125	125	125	125	125	125	125	125	125	125

	A	B	C	D	E	F	G	H	I
1	117	d	r	p	t				
2	Diet Quad	min	max	min	max	min	max	min	max
3	1	12	17	0.3	1.2	0.8	0.8	7	7
4	2	12	17	0.3	1.2	0.8	0.8	7	7
5	3	12	17	0.3	1.2	0.8	0.8	7	7
6	4	12	17	0.3	1.2	0.8	0.8	7	7
7	5	12	17	0.3	1.2	0.8	0.8	7	7
8	6	12	17	0.3	1.2	0.8	0.8	7	7
9	7	12	17	0.3	1.2	0.8	0.8	7	7
10	8	12	17	0.3	1.2	0.8	0.8	7	7
11	9	12	17	0.3	1.2	0.8	0.8	7	7
12	10	12	17	0.3	1.2	0.8	0.8	7	7
13	11	12	17	0.3	1.2	0.8	0.8	7	7
14	12	12	17	0.3	1.2	0.8	0.8	7	7
15	13	12	17	0.3	1.2	0.8	0.8	7	7
16	14	12	17	0.3	1.2	0.8	0.8	7	7
17	15	12	17	0.3	1.2	0.8	0.8	7	7

Figure 2.10: Input file; feed costs, feed ingredients and their nutrients, price schedule and feeding schedule setup of Bacon Max version 2.1.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Pig Setup																	
2		0.8	0.8	120	70	3.0943	0.8	20	7	1	105	30	85	0	1000	0	0	10
3																		
4																		

Figure 2.11: Run file; the initial setup for input parameters in an Excel file for multiple runs of Bacon Max version 2.1.

Microsoft Excel - Output v2.1

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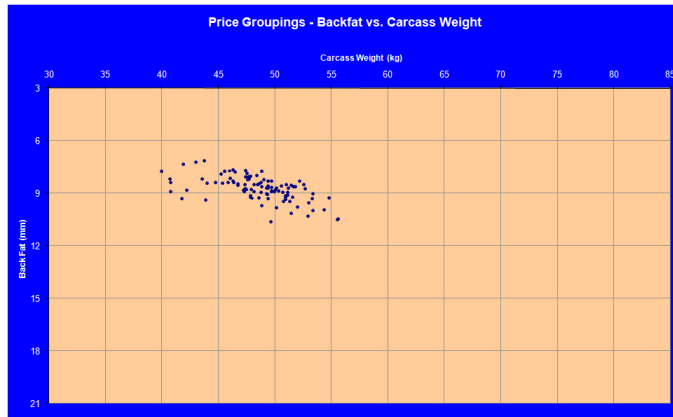
Figure 2.12: Output file; showing the calculation values in the algorithm of Bacon Max version 2.1.

	AR	AS	AT	AU	AV	AW	AX	AY	AZ	BA	BB	BC	BD	BE	BF
1															
2	OUTPUT OUTPUT OUTPUT OUTPUT OUTPUT OUTPUT OUTPUT OUTPUT OUTPUT OUTPUT														
3															
4	Final values														
5	Slaughter day	Live weight	Carcass weight	Back fat	Price/cross	Obj from Pig	Obj from Nit	Obj Fn	Interpretation	FI	DG	FCR	Til FI	Til Feed cost	CP
6	105.000	88.545	67.441	17.669	0.000	-3636.398	0.000	-3636.398	\$/pig place/year	1.864	0.458	4.070	197.630	136.733	32836.416
7															

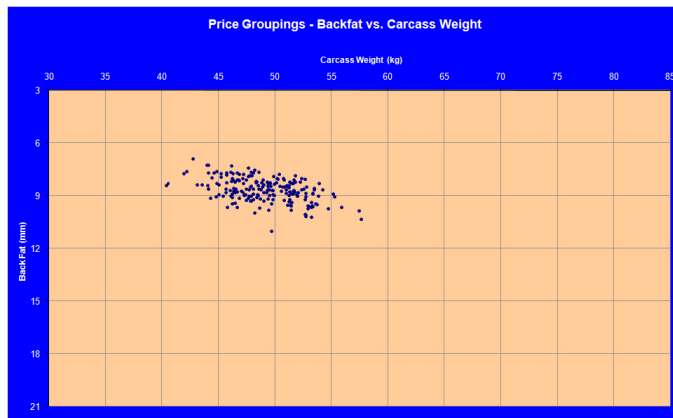
Figure 2.13: Output summary file; this summarises the input parameters and the final output values from the algorithm of Bacon Max version 2.1.



(a) single pig



(b) 100 pigs



(c) 200 pigs

Figure 2.14: The backfat thickness (mm) and carcass weight (kg) resulting from optimal feeding schedules when growing (a) a single pig, (b) 100 pigs and (c) 200 pigs which all outputs include the stochasticity (standard deviation 10% of the mean) on initial protein in purchased pigs P_0 , feed intake, minLP and Pdmax.

2.5.2 Parameter stochasticity

As mentioned in the previous section, for practical reasons we must include more than a single pig in the objective function. At the same time, we include stochasticity on initial protein in purchased pigs P_0 , feed intake, minLP and Pdmax. Values are varied using a standard deviation which is a percentage of the mean, as seen in the starting page of the pig setup (Figure 2.9).

The result of different setup values for varying values of this percentage are shown in Figure 2.15. Adding stochasticity to the parameters in the model via variation in resulting backfat thickness and carcass weight increased. Each pig in the objective function are generated differently when adding the standard deviation 10% of the mean to parameters. Gross return is based on each individual pig performances. This is then adding the variability in gross return and result in GMPPY as shown in Figure 2.16. GMPPY with no parameter variation and with 10% standard deviation was \$249.24 and \$212.13, respectively. The slaughter date remains at 59 days for both no parameter variation and 10% standard deviation, as shown in Figure 2.16. By adding stochasticity to the model parameters, GMPPY decreases when the standard deviations percentage of the parameter mean increases.

A standard deviation of 10% of the means of initial protein in purchased pigs P_0 , feed intake, minLP and Pdmax will be used throughout the remainder of this thesis.

2.5.3 Covariance of pig type parameters

Also for practical reasons, covariance of the pig type parameters, minLP, Pdmax and p proportion of the *ad libitum* digestible energy intake have been included in Bacon Max. Stochasticity is applied to these parameters using the following correlation matrix, shown in Table 2.11 Sherriff (2008):

Table 2.11: Correlation matrix for parameters minLP, Pdmax and p used in Bacon Max program.

	minLP	Pdmax	p
minLP	1		
Pdmax	-0.55	1	
p	0.3	0.25	1

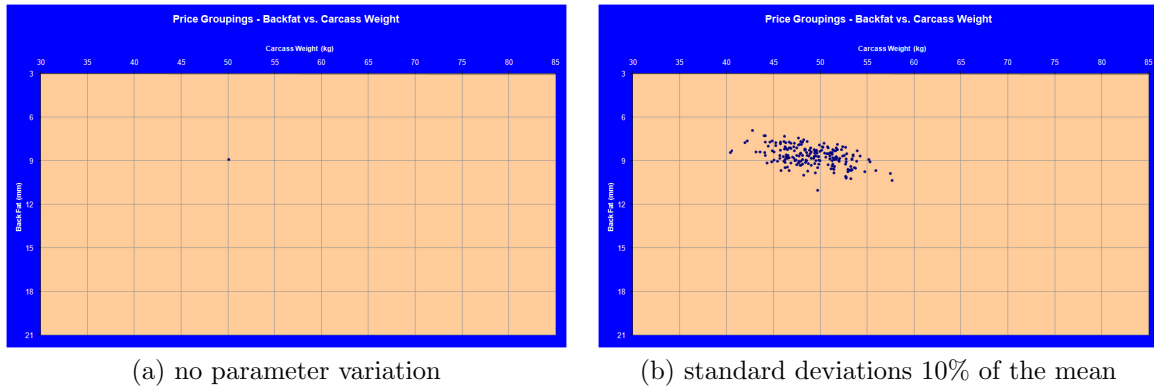


Figure 2.15: The backfat thickness (mm) and carcass weight (kg) resulting from optimal feeding schedules when growing 200 pigs with (a) no parameter variation and (b) standard deviations 10% of the means on initial protein in purchased pigs P_0 , feed intake, minLP and Pdmax.

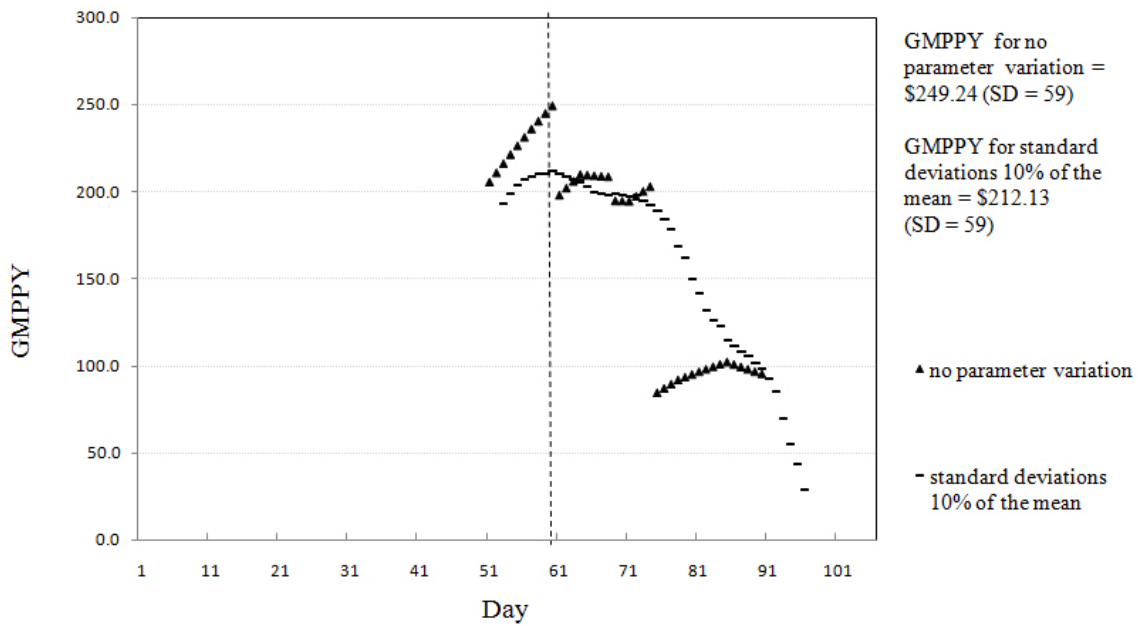


Figure 2.16: GMPPY against time resulting from optimal feeding schedules when growing 200 pigs. This compares between no parameter variation and standard deviations 10% of the mean on initial protein in purchased pigs P_0 , feed intake, minLP and Pdmax. Both result in slaughter at 59 days.

In de Lange (1995), the effects of parameters minLP, Pdmax and p in the pig growth was explained. Firstly, minLP determines the protein deposition rate Pd in the early growing period until a certain point. Then, the determination of Pd will replace by Pdmax for the rest of pig growing period. These explains that minLP and Pdmax are working in the opposite effect. The correlation between minLP and Pdmax in Table 2.11 shows just that, it has moderate negative correlation (-0.55). Secondly, the effect of p on Pd is if p increases, Pd also increases, until it reaches Pdmax. Pd remains at $Pdmax$, even p is continuing to increase. This explains the weak positive correlation (0.25) of p and Pdmax in Table 2.11. Finally, both p and minLP determine Pd at the same time before it reach Pdmax. Once Pd reach Pdmax, p and minLP are stop having any effect on Pd . p has an indirect effect on minLP, but both are determine Pd . For this reason, the correlation between p and minLP in Table 2.11 reflect by weak positive correlation (0.3).

From this correlation matrix (Table 2.11), we now defined parameters minLP, Pdmax and p as x_i where $i = 1, 2, 3$, respectively. For example, correlation of minLP and Pdmax presents as $r(x_1, x_2) = -0.55$.

The following steps were used to generate a number of pigs N contributing to the objective function:

1. Convert the correlation matrix R as shown in Table 2.11 to a covariance matrix Σ using the set of variances determined by the coefficient of variation (CV).

$$R = \begin{bmatrix} 1 & r(x_1, x_2) & r(x_1, x_3) \\ r(x_2, x_1) & 1 & r(x_2, x_3) \\ r(x_3, x_1) & r(x_3, x_2) & 1 \end{bmatrix}$$

where $\text{cov}(x_i, x_j) = r(x_i, x_j)\sigma_i\sigma_j$ when $i \neq j$, i is the number of column and j is the number of row, $i, j = 1, 2, 3$. Σ is positive definite and we define a_{ji} is the element in row j th and column i th in Σ . When $i = j$, $a_{ji} = a_{ii}$.

We can find $\Sigma = DRD$ where D is a diagonal matrix with the desired variable standard deviations on its diagonal

$$D = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Then

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \text{cov}(x_1, x_2) & \text{cov}(x_1, x_3) \\ \text{cov}(x_2, x_1) & \sigma_2^2 & \text{cov}(x_2, x_3) \\ \text{cov}(x_3, x_1) & \text{cov}(x_3, x_2) & \sigma_3^2 \end{bmatrix}$$

Form the Cholesky factorization of Σ (matrix square root, employing a lower triangular matrix) by

$$\Sigma = LL^T$$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

and

$$L^T = \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

where $l_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2}$ and $l_{ji} = \frac{\left(a_{ji} - \sum_{k=1}^{i-1} l_{jk} l_{ik}\right)}{l_{ii}}$

Then

$$L = \begin{bmatrix} \sigma_1 & 0 & 0 \\ \frac{\text{cov}(x_2, x_1)}{\sigma_1} & \sqrt{\Sigma_{22} - l_{21}^2} & 0 \\ \frac{\text{cov}(x_3, x_1)}{\sigma_1} & \frac{\Sigma_{32} - l_{31}l_{21}}{l_{22}} & \sqrt{\Sigma_{33} - (l_{31}^2 + l_{32}^2)} \end{bmatrix}$$

2. Generate independent standard normal random variables, $A \sim N(0, 1)$. (An $N \times m$ array where N is the number of pigs contributing to the objective function (the number of multivariate samples to be generated) and m is the number of variables (in our case $m = 3$).)
3. Calculate a multivariate random normal variable B ($N \times m$ array) from a distribution with the required correlation matrix, using

$$B = AL^T + \mu$$

where L is the matrix square root (the lower triangular matrix from the Cholesky factorization) of the covariance matrix ($m \times m$ array) and μ is the mean vector of pig parameters ($1 \times m$ array).

4. Repeat Steps 2 and 3 until the parameters for a pig population of size N have been generated.

Numerical example of calculating covariance of pig type parameters

Using the correlation matrix in Table 2.11 and take as a mean vector of minLP, Pdmax and p , $\mu = [0.75, 180, 0.9]$. The standard deviation vector of minLP, Pdmax and p is $[0.075, 18.0, 0.09]$. As mentioned in Section 2.5.2 we bring in stochasticity using standard deviations which are 10% of the mean. The number of pigs contributing to the objective function is $N = 1000$.

The output generated using the correlation matrix from Table 2.11 is presented and Figure 2.17. It shows the scatter plot of the generated values of minLP, Pdmax and p .

$$R = \begin{bmatrix} 1 & -0.534 & 0.349 \\ -0.534 & 1 & 0.244 \\ 0.349 & 0.244 & 1 \end{bmatrix}$$

2.5.4 The connection between standard deviation of Pdmax and resulting standard deviation in algorithm outputs

The aim of this section is to explore the influence of the standard deviation of Pdmax, σ_{Pdmax} on the standard deviation in algorithm outputs GMPPY, σ_{alg} . We need to run the genetic algorithm for sufficiently long so that variation in GMPPY is largely due to variation in Pdmax. We assume that

$$\text{Var}(G_i) = \sigma_{\text{alg}}^2 + \sigma_{\text{Pdmax}}^2$$

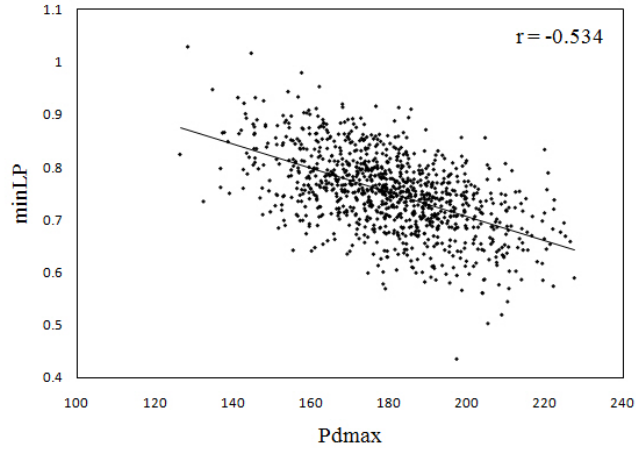
where

$$G_i = \text{GMPPY for pig } i, i = 1, 2, \dots, N$$

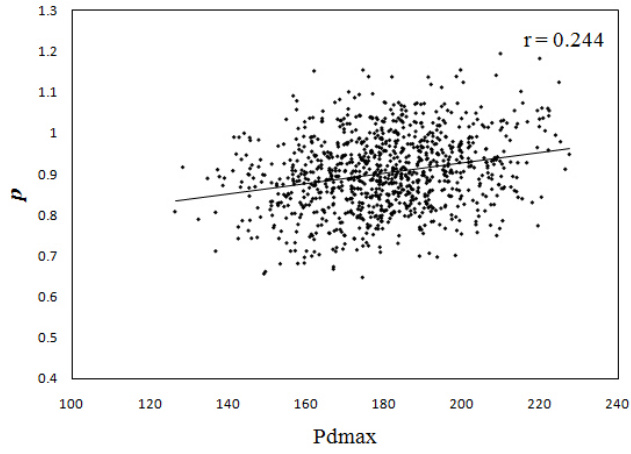
$$N = \text{number of pigs contributing to the objective function}$$

also, let

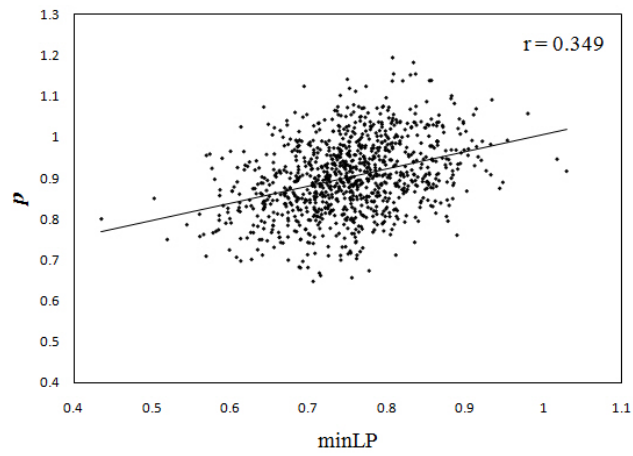
$$\begin{aligned} T &= \text{total GMPPY for every pigs in the objective function (\$),} \\ &= G_1 + G_2 + \dots + G_N \end{aligned}$$



(a) minLP and Pdmax



(b) p and Pdmax



(c) p and minLP

Figure 2.17: Scatter plots of the example generated values using the correlations of Table 2.11, for (a) minLP and Pdmax, (b) p and Pdmax and (c) p and minLP.

Table 2.12 shows the standard deviation of GMPPY in 20 runs for a single pig ($N = 1$) and for $g = 20$ genomes and I (solution repeats) = 10, 20 and 30 using Bacon Max. Values of the standard deviation of Pdmax used were 0%, 5% and 10% (of the mean).

Table 2.12: Standard deviation of objective values GMPPY $\text{Std}(G_i)$ for one pig when $\text{minLP} = 0.8$ and $\text{Pdmax} = 160$ g/day using $(g = 10, I = 10)$, $(g = 20, I = 20)$ and $(g = 30, I = 30)$.

Standard deviation of Pdmax	$(g = 10, I = 10)$	$(g = 20, I = 20)$	$(g = 30, I = 30)$
0	0.34	0.18	0.01
5	7.46	5.31	6.98
10	13.69	13.29	11.66

The first row corresponds to a standard deviation of Pdmax of zero ($\sigma_{\text{Pdmax}} = 0$). For this setting we found the standard deviation of objective values GMPPY, $\text{Std}(G_i)$ is closed to zero (0.01) when we used $g = 30$ genomes and $I = 30$. This means the standard deviation in the algorithm σ_{alg} is also close to zero. From this result, an initial setup of $g = 30$ and $I = 30$ would control the standard deviation due to the variation in the algorithm. Running time with such a setting is very long, however, especially when the number of pigs in the objective function N is large. In the next section we will discuss the effect of setting of parameters g and I in the genetic algorithm.

2.5.5 Tuning the genetic algorithm

In this section, the processor time of genetic algorithm by different setting of parameters g and I are evaluated. The numerical results presented in Table 2.13 allow us to compare the efficiency of the genetic algorithm with the different g and I on the basis of average and standard deviation of GMPPY, average number of feeding schedules evaluated, average and standard deviation of processor time with a ten run trial base. This study was performed on an Inspiron 530 Desktop possessing an Intel[®] Core[™] 2 processor (2.13GHz) with 2 GB of RAM. The operating system was Microsoft Windows Vista Home Premium. Pig genotype parameters, minLP and Pdmax were set to 0.8 and 160 g/day respectively for a 200 pigs in the objective function.

Table 2.13: Numerical results for a comparison of the performance of the genetic algorithm. Ten runs were used in each of the three cases of $(g = 10, I = 10)$, $(g = 20, I = 20)$ and $(g = 30, I = 30)$ for $\text{minLP} = 0.8$, $\text{Pdmax} = 160$ g/day and $p = 0.8$.

Results	$(g = 10, I = 10)$	$(g = 20, I = 20)$	$(g = 30, I = 30)$
Average GMPPY (\$/pig place/year)	213.23	215.24	211.56
Standard deviation of GMPPY	2.98	2.03	2.45
Average number of feed- ing schedules evaluated	6489	28406	89303
Average processor time	1 hrs 3 mins	3 hrs 47 mins	14 hrs 17 mins
Standard deviation of processor time	39 mins	2 hrs 3 mins	6 hrs 27 mins

Table 2.13 shows that the average processor time and number of feeding schedules evaluated increase when g and I are increased. However, the GMPPY of $g = 30, I = 30$ is not better than $g = 10$ and $I = 10$ and $g = 20$ and $I = 20$. As we discussed in the previous section of controlling the standard deviation due to the variation in the algorithm and comparison of the processor time of genetic algorithm. Using of $g = 20$ and $I = 20$ is proved to satisfy the results. For this reason in this study, using $g = 20$ genomes and $I = 20$ these values will be used throughout the remainder of the thesis.

Chapter 3

An alternative optimisation methodology

3.1 Introduction

As discussed in Chapter 2, the search space for the optimal feeding schedule is extremely large and the discontinuous nature of the objective function, the “craggy volcano”, also adds challenge. A number of standard optimisation techniques have been applied to this problem, namely Monte Carlo method (Metropolis et al., 1953), the Nelder-Mead simplex algorithm (Nelder & Mead, 1965), ascent search (Eldor & Koppel, 1971), a genetic algorithm (Holland, 1975; Reeves & Rowe, 2002), simulated annealing (Aarts & Korst, 1989), pure random search (Zabinsky & Smith, 1992) and tabu search (Glover & Laguna, 1997). A genetic algorithm has been the most successful for finding the optimal feeding schedule, although a long search period is needed. It takes up to an hour on a standard computer to satisfactorily solve a typical problem of the type shown in Table 2.13.

3.2 Aim

The purpose of this chapter is to find an alternative optimisation methodology by exploring the nature of the objective function and so formulating an algorithm which is tailored to its form and which moves to the optimum more rapidly than does the genetic algorithm.

We will start by exploring the particular shape of the objective function and then an alternative optimisation method is introduced. After this, numerical results are presented and discussion presented at the end.

3.3 Methods

In this chapter, New Zealand examples were used which employ two diets, namely “grower” and “finisher” diets, with the first growing period of $T_1 = 35$ days and the second period maximum of $T_2 = 70$ days until slaughter ($T_1 + T_2 = T_{\max} = 105$ days). The slaughter date is chosen in the range $\{1, 2, \dots, 105\}$. Thus we write a feeding schedule as

$$F = (p_1, r_1, d_1; p_2, r_2, d_2)$$

where $P_k = [0.8, 1]$, $R_k = [0.2, 1.2]$ and $D_k = [12, 17]$ for $k = 1, 2$.

We pause for some practical supporting comments. On a commercial pig farm today, around the world, pigs are fed two diets (grower and finisher) during the period from weaner pig 20kg to slaughter; this is the situation investigated in this chapter, the next chapter, Chapter 4, and in Chapter 6. With advanced technology, however, it will become possible to change diets more often (weekly or even daily, with phase feeding). In Alexander et al. (2006) and later on in Chapter 5 we investigate the effect of changing diets more frequently, on a weekly basis, as methods exist to do so. In this chapter, the New Zealand situation is used and determines the ingredients, their cost and the price schedule, but not the number of diets fed.

New Zealand data from July 2001 is used in this chapter. The weaner cost is fixed at NZ\$70. A schedule of ingredient costs and a price schedule are shown in Table 3.1 and Table 3.2, respectively.

Table 3.1: New Zealand ingredient schedule from July 2001: this comprises a list of ingredients and the associated costs in \$/kg.

Ingredient	Barley	Blood meal	Soybean meal	...	Tryptophan
Cost (\$/kg)	0.245	0.9	0.84	...	20

Table 3.2: A New Zealand price schedule giving prices in cents/kg for pigs at slaughter in July 2001.

Backfat (mm)	Carcass weight (kg)										
	35.0 and under	35.1 to 40.0	40.1 to 45.0	45.1 to 50.0	50.1 to 55.0	55.1 to 60.0	60.1 to 65.0	65.1 to 70.0	70.1 to 75.0	75.1 to 80.0	80.1 and over
<6	300	300	300	300	300	300	300	300	300	300	300
6-9	360	385	395	395	385	370	370	370	370	365	335
10-12	360	385	385	390	375	370	370	370	370	365	335
13-15	330	330	330	330	330	335	335	335	335	330	305
16-18	260	260	260	260	260	270	270	270	270	270	270
>18	230	230	230	230	230	240	240	240	240	240	240

Using this input, we will find the optimal feeding schedule that maximises GMPPY. First, we move to explore the structure of the objective function.

3.3.1 Exploring the structure of the objective function

In Section 2.3.3 and Section 2.4, the nature of the objective function and the problem of high dimensions were discussed, respectively. We now consider closely the structures of the objective function in directions determined by changing p , r and d through optimal solution, as shown in Figure 3.1, to find an optimisation method adapted to the functional form. The critical observation is that the cross-sections show peaked, discontinuous profiles.

3.3.2 Tailoring a maximisation algorithm to the objective function

In practice, we would like to be able to reduce the long processing time of the genetic algorithm in Bacon Max, with all the variations created by parameter changes. For this reason we have explored the very particular form of the objective function and tailored a method to the finding of the maximum. This method climbs the objective function quickly at the beginning, compared with a genetic algorithm. The method is unashamedly a heuristic, deserving attention thanks to the practical importance of this problem type.

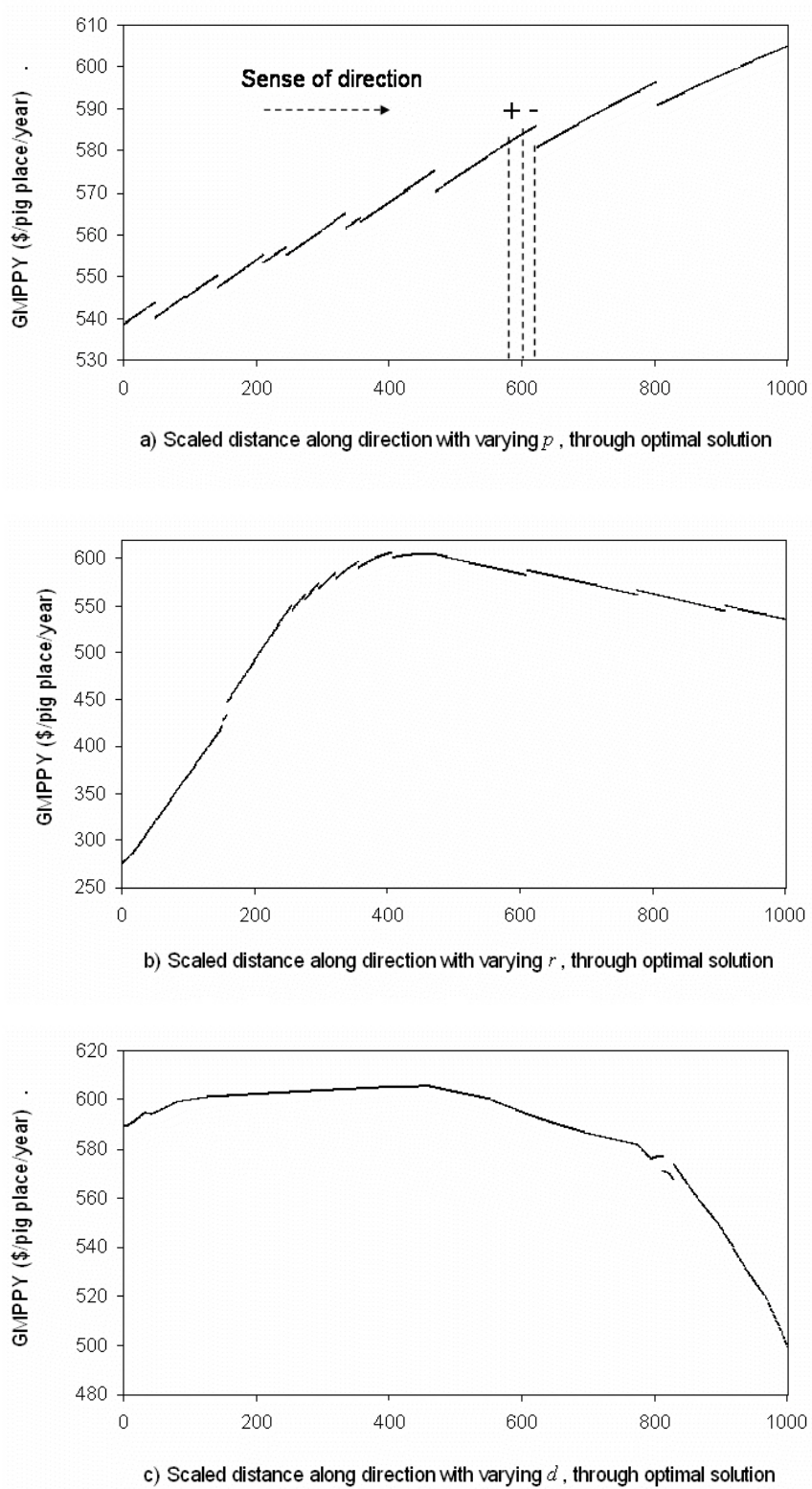


Figure 3.1: Cross-sections types that motivated the look-up table.

We set out to develop a sequential hill-climbing algorithm tailored to the characteristics of this very particular objective function. It can be seen from the sections shown in Figure 3.1 that it is sometimes wise to move downward in the short run, since this can lead to the overall peak; picking the direction in which to move is the challenge. Study of the cross-sections suggests use of comparison of “very close” and “close” function values in order to determine in which direction to step. Experimentation revealed that use of 0.05 and 0.10 of the distance between the current feeding schedule and the edge of the enlarged domain was successful. (The domain is enlarged in order to incorporate reflection into the search, so avoiding jamming in domain corners, as advocated in Romeijn, Zabinsky, Graesser, and Neogi (1999).) The decision regarding sense (forward or backward) along this direction is then made based on the sign and size of the objective function difference between these two points. Details are shown later in Table 3.3. We generate three such random search directions at every iteration, and choose that indicating the largest positive gradient.

After the decision has been made to move in a particular sense (positive or negative) of a given direction, a move is made to the middle of the two calculated points, so 0.075 of the distance from the feeding schedule to the edge of the enlarged domain. In every iteration this step size is reduced by a “shrink factor”, S . This has been tuned and found to work successfully when $S = 0.9 \times e^{-0.001 \times iter}$, where $iter$ is the iteration counter. After many iterations the step size will reduce substantially and allow the algorithm to move close to the edge of the domain.

The three parameters comprising each diet (p , r and d) are on very different scales, since $P_k = [0.8, 1]$, $R_k = [0.2, 1.2]$ and $D_k = [12, 17]$. Standardisation to a unit interval of each parameter ensures that the search spreads over the domain. The enlarged standardised domain is then a product of intervals $[-0.5, 1.5]$ of twice the width.

The stages of this “Tailored method” are now described. For the purposes of description, we use two diets with the feeding periods of a grower diet (T_1) being 35 days and that of a finisher diet (T_2) being 70 days until slaughter ($T_{max} = 105$ days). The slaughter day that maximises the GMPPY is found in the range $\{1, 2, \dots, 105\}$ and so the problem is in seven real dimensions.

3.3.3 Tailored method

1. Generate initial feeding schedule. Generate the initial feeding schedule, on the enlarged domain, using values for each coordinate drawn independently from a uniform distribution on $[-0.5, 1.5]$, giving

$$F'' = (p_1'', r_1'', d_1''; p_2'', r_2'', d_2'')$$

Set $iter = 0$ and $S = 1$.

2. Generate candidates for the next feeding schedule.
 - 2.1 Generate directions for progress. Set $iter = iter + 1$. Randomly draw three directions, V_1 , V_2 and V_3 , with each component of each direction drawn from a standard normal distribution, as in Zabinsky and Wood (2002). Normalise these directions.
 - 2.2 Calculate two nearby feeding schedules for each direction. Calculate the feeding schedules at $0.05S$ and $0.1S$ of the distance from the current feeding schedule to the edge of the enlarged domain in the positive sense of each direction (six points).

To find the feeding schedules at the edge of the domain on the generated random directions, that is

$$F''_{\text{up},i} = F''_i + \beta_i V_i \leq \underline{1}$$

$$F''_{\text{down},i} = F''_i + \gamma_i V_i \geq \underline{0}$$

or

$$F''_{\text{up},i} = (p''_{ij}, r''_{ij}, d''_{ij}) + \beta_{ij} V_{ij}$$

$$F''_{\text{down},i} = (p''_{ij}, r''_{ij}, d''_{ij}) + \gamma_{ij} V_{ij}$$

where i is the number of the generated direction, $i = 1, 2, 3$ and j is the diet number, $j = 1, 2$. The coefficients β_{ij} and γ_{ij} are the distances from the current feeding schedule (j diets) to the edge of the domain on direction i for both sides (up and down), respectively. The coefficients β_{ij} and γ_{ij} can be found in the following way:

$$\beta_{ij} = \left(\left| \frac{1.5 - p''_{ij}}{V_{ij}} \right|, \left| \frac{1.5 - r''_{ij}}{V_{ij}} \right|, \left| \frac{1.5 - d''_{ij}}{V_{ij}} \right| \right)$$

$$\gamma_{ij} = \left(\left| \frac{-0.5 - p''_{ij}}{V_{ij}} \right|, \left| \frac{-0.5 - r''_{ij}}{V_{ij}} \right|, \left| \frac{-0.5 - d''_{ij}}{V_{ij}} \right| \right)$$

These are the distances to move along the generated random direction to reach the edge of the enlarged domain $[-0.5, 1.5]$.

2.3 Reflection into the standardised domain. For each feeding schedule outside the standard domain (including possibly the current one), reflect into the standard domain using

$$y = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \text{ and } x \leq 1 \\ 2 - x, & \text{if } x > 1 \end{cases}$$

where x is a component of F'' and y now constitutes the corresponding component of a standardised feeding schedule F' . An illustration of reflection is shown in Figure 3.2.

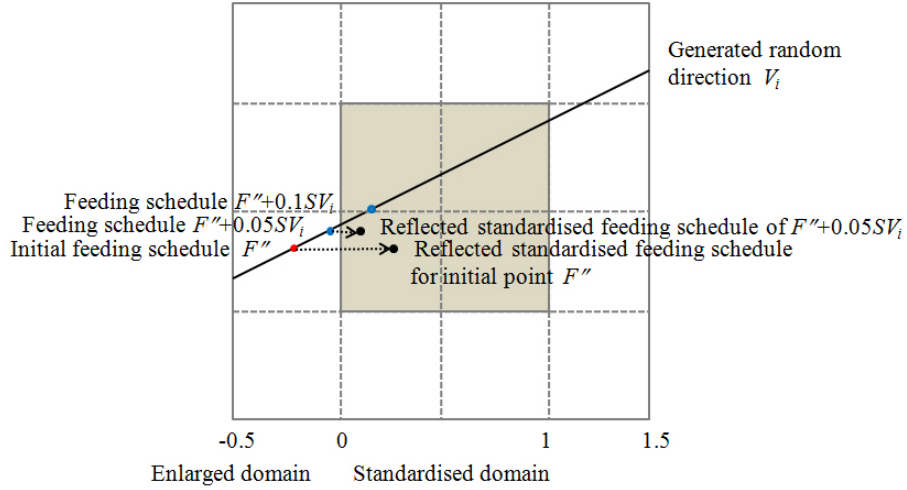


Figure 3.2: An illustration of reflection of a feeding schedule (outside the standard domain) from the enlarged domain $[-0.5, 1.5] \times [-0.5, 1.5]$ to the standard domain $[0, 1] \times [0, 1]$. The red dot presents the initial feeding schedule F'' (see Step 1). The solid line presents the generated random direction V_i , where $i = 1, 2$ or 3 . (see Step 2.1). The blue dots present the two nearby feeding schedule at $0.05S$ and $0.1S$ of the current feeding schedule in the V_i direction (see Step 2.2). The black dots present the reflected feeding schedule from the enlarged domain to the standard domain F' (see Step 2.3); the feeding schedule $F'' + 0.1SV_i$ is already in the standard domain.

-
- 2.4 Back transform all standardised points. Back transform the six feeding schedules and the current feeding schedule from the standardised form after reflection to the original domain, where $P_k = [0.8, 1]$, $R_k = [0.2, 1.2]$ and $D_k = [12, 17]$, via

$$p_k = 0.2p'_k + 0.8, r_k = r'_k + 0.2 \text{ and } d_k = 5d'_k + 12$$

We then have six feeding schedules and the current feeding schedule in the form $F = (p_1, r_1, d_1; p_2, r_2, d_2)$.

3. Calculate objective function values at these feeding schedules.
 - 3.1 Calculate minimum feed cost. Use linear programming to find minimal cost diets for each diet in the six new feeding schedules.
 - 3.2 Grow the pig for x days where x is chosen in the range $\{1, 2, \dots, T_{\max} = 105\}$ using the de Lange's pig growth model (1995) and calculate the backfat thickness (mm) and carcass weight (kg).
 - 3.3 Gross return and gross margin. Find the price of pig at slaughter in the price schedule and calculate the gross margin per pig place per year (GMPPY) for each feeding schedule and each x . Maximise over x and record the objective function value $f(F)$ for each feeding schedule.
4. Choose best direction and sense.
 - 4.1 Calculate objective function changes in the positive sense of each direction. Calculate

$$\Delta F_1 = \text{GMPPY at 0.05 point} - \text{GMPPY at current}$$

$$\Delta F_2 = \text{GMPPY at 0.1 point} - \text{GMPPY at 0.05 point}$$

$$\Delta F_3 = |\text{GMPPY at 0.1 point} - \text{GMPPY at current}|$$
 - 4.2 Choose next direction and sense. Choose the next direction as the steepest, that producing the maximum value of ΔF_3 . Move in the forward or backward sense in this direction, based on the decision criteria in Table 3.3. Figure 3.1 showed varying patterns for the objective function cross sections through the optimal feeding schedule. Table 3.3 responds to this pattern, by providing rules for progress. For example, Figure 3.1a displays a small positive value for ΔF_1 and a larger negative value for ΔF_2 . This is Case 3 in Table 3.3, so we decide to move in the positive sense of this direction.

Table 3.3: This table is used to determine in which sense along the new direction we step. Choose rows, using the signs of ΔF_1 and ΔF_2 and columns two and three (Figure 3.3 is an illustration of the signs). Choose further within rows if necessary using the magnitude of the differences (with experience showing that the dividing point between small S and large L is \$3/pig place/year for New Zealand data). The rightmost column indicates positive sense or negative sense.

Case	Sign		Size		Decision
	ΔF_1	ΔF_2	ΔF_1	ΔF_2	
1	+	+	S/L	S/L	Positive
2	+	-	S	S	Positive
3	+	-	S	L	Positive
4	+	-	L	S	Positive
5	+	-	L	L	Negative
6	-	+	S	S	Positive
7	-	+	S	L	Negative
8	-	+	L	S	Positive
9	-	+	L	L	Positive
10	-	-	S/L	S/L	Negative

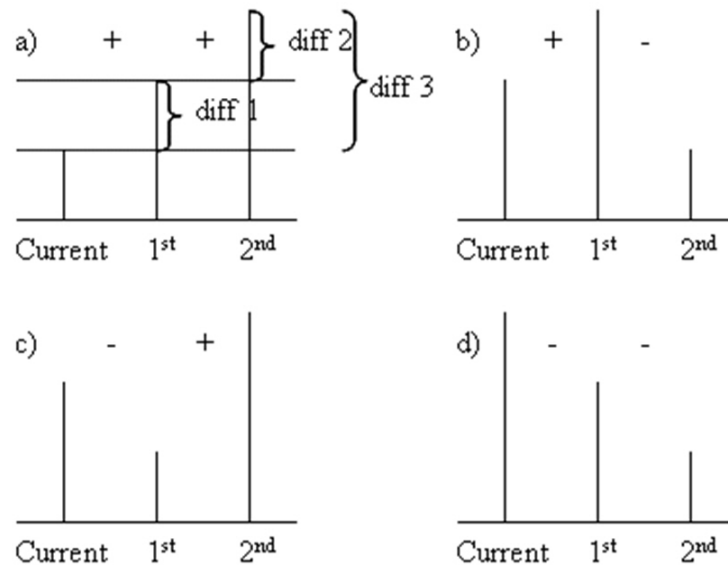


Figure 3.3: An illustration of look-up sign. Figure a) shows sign $(+, +)$ for Case 1 in Table 3.3, b) shows sign $(+, -)$ for Case 2-5, c) shows sign $(-, +)$ for Case 6-9, and d) shows sign $(-, -)$ for Case 10.

-
5. Move to next point. Move in the positive or negative sense on the steepest direction by $0.075S$ of the distance from the initial feeding schedule to the edge of the enlarged domain. This new point becomes the next current feeding schedule, F'' .
 6. Stopping rule. Set $S = 0.9 \times e^{-0.001 \times iter}$. We slow movement by this shrink factor S as the algorithm progresses. This allows the current point to progressively move toward the domain boundary, if necessary. Return to Step 2. (If the objective function does not improve in 10 iterations, return to the previous current feeding schedule and return to Step 2.) Stop after a preset number of iterations (usually 3,000).

3.4 Numerical results

The numerical results, shown in Table 3.4, allow us to compare the efficiency of the genetic algorithm with that of the Tailored method on the basis of average GMPPY, average number of feeding schedules evaluated and average and standard deviation of running time using 10 runs. This study was performed on an Inspiron 6000 laptop possessing an Intel[®] Pentium[®] M processor (1.73GHz) with 504 MB of RAM. The operating system was Microsoft Windows XP Professional. Visual C++ was used in the study of both methods. The genetic algorithm in Bacon Max, however, was processed using a Windows application and the Tailored method was processed using a Console application. Pig genotype parameters, minLP and Pdmax were set to 0.8 and 160 g/day respectively for a single pig in the objective function.

Two situations of live weight at slaughter have been included in this study. Firstly, a live weight at slaughter of 84 – 86kg has been chosen which is consistent with the slaughter weight used in New Zealand. Secondly, a live weight more than 80kg was chosen to examine the efficiency of both methods on a wider range of weight sampling. The price schedule from July 2001 indicated in Table 3.2 has been used for these calculations. The best solution from a genetic algorithm following 20 iterations (predetermined for this study) was a GMPPY of \$578/pig place/year and was chosen as the stopping criteria for the Tailored method in this section.

Table 3.4: Numerical results for a comparison of the performance of the Tailored method and the genetic algorithm. Ten runs were used in each of the four cases. Each run was stopped when an objective function of \$578/pig place/year was reached and unchanged for 20 iterations.

Results	Live weight constraint (kg)			
	84 ≤ LW ≤ 86		LW > 80	
	GA	Tailored	GA	Tailored
Average GMPPY (\$/pig place/year)	578.67	578.58	579.96	579.60
Average number of feeding schedules evaluated	18560	3872	17418	1994
Average running time	90 mins	72 mins	63 mins	38 mins
Standard deviation of running time	55 mins	15 mins	41 mins	10 mins

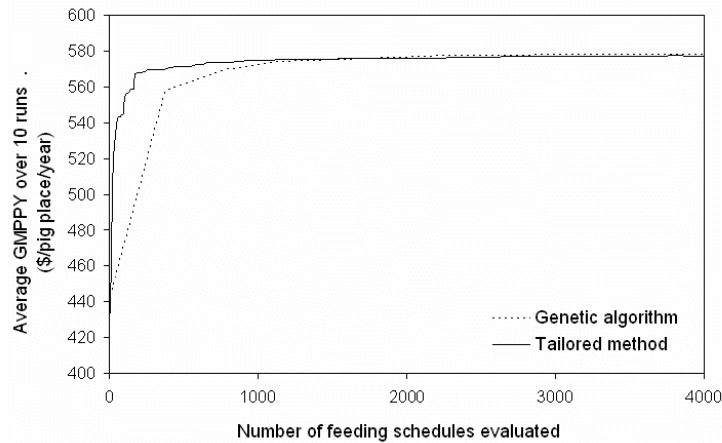


Figure 3.4: A comparison of the performance of the Tailored method and the genetic algorithm for live weight constraint 84 – 86kg.

Figure 3.4 and the results from Table 3.4 provide a typical comparison of the Tailored method with that of the genetic algorithm. An analysis of the results indicates that the Tailored method performs better, using a significantly smaller number of feeding schedules than the genetic algorithm, for both live weight constraints. Further, with regard to the average running time, the Tailored method was found to be slightly faster than the genetic algorithm for live weight constraint 84 – 86kg, but significantly faster when the live weight constraint was more than 80kg. A possible reason for this emerges from an examination of the cross-sections through the optimal value of the objective function under the differing constraints, shown in Figure 3.5.

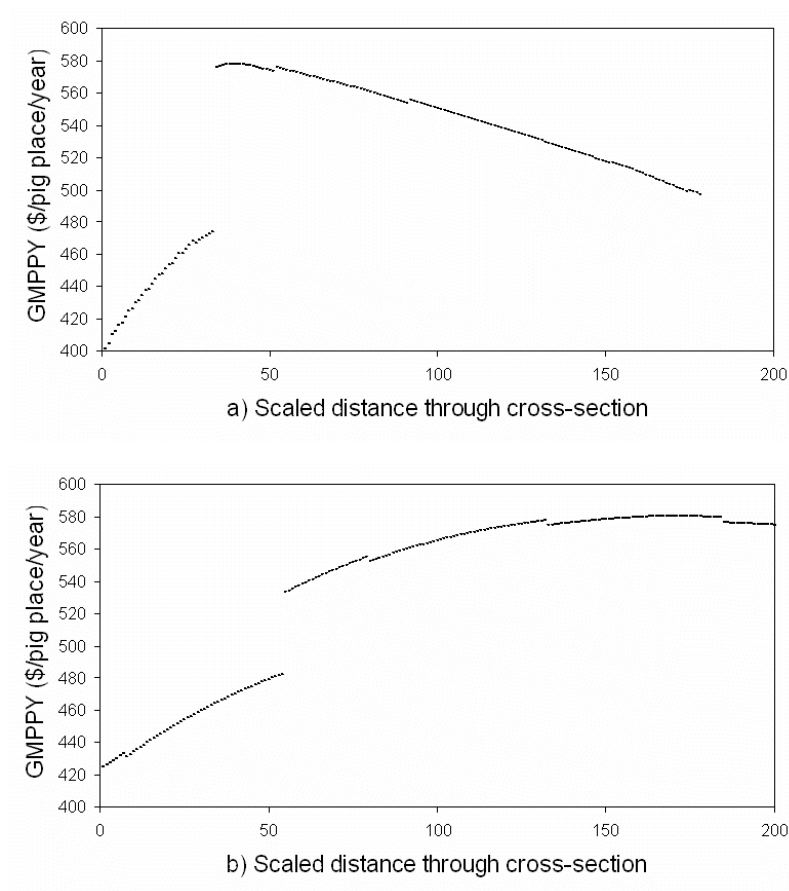


Figure 3.5: Cross-sections through the optimal value of the objective function when a) live weight is constrained to be 84 – 86kg and b) live weight is constrained to be more than 80kg.

For a live weight constraint of 84 – 86kg, the corresponding carcass weight is around 65kg. This yields a relatively narrow objective function peak, as illustrated in Figure 3.5a. On the other hand, for a live weight constraint of more than 80kg, carcass weight is greater than 60kg. This produces a relatively broader peak of the objective function, as seen in Figure 3.5b. For this reason, the Tailored method performs more effectively on the second problem.

3.5 Discussion

We have examined the nature of the objective function, found it to provide a single but very rough peak, and so tailored a heuristic algorithm to its shape. This algorithm

climbs quickly and appears to find better optima than previous methods. We conclude that the Tailored method is more efficient than the genetic algorithm, using fewer feeding schedules evaluations and shorter running times. We have shown that the improvement in performance can depend on the pig genotype parameters and the constraints. The Tailored method performs better when the liveweight constraint was more than 80kg compared with a liveweight constraint of 84-86kg. The performance relates directly to the shape of the objective function. Further investigation of application of the Tailored method to different pig genotypes and various situations still remains.

The usefulness of other optimisation techniques in maximising profitability can also be explored in the future. For example, improving hit and run method (Zabinsky, Smith, McDonald, Romeijn, & Kaufman, 1993), pure adaptive search (Reaume, Edwin, Robert, & Smith, 2001; Zabinsky & Smith, 1992), conjugate gradient method (Shewchuk, 1994), uniform covering by probabilistic rejection methods (Hendrix & Klepper, 2000).

Two feeding periods are the norm in pig production units in countries such as Thailand; however, this can vary as seen in Australia where the norm is four diets. Further, with increased use of computerised feeding on large production units, it will become feasible to change diets more regularly. The methods of this chapter can be applied, but the dimension of the problem will increase. The performance of the Tailored method in such problems remains to be investigated. Inclusion of the Tailored method in the Bacon Max program could also be considered in the future.

Chapter 4

Ingredient and price schedules varying with time

4.1 Introduction

Maximising profitability for growing a fattening pig depends on the feed ingredient cost and the pig price at slaughter. So far both feed cost and price schedule have been fixed in our research. In reality, feed costs and price received at slaughter are subject to variation in time as in the example shown in Figure 4.1.

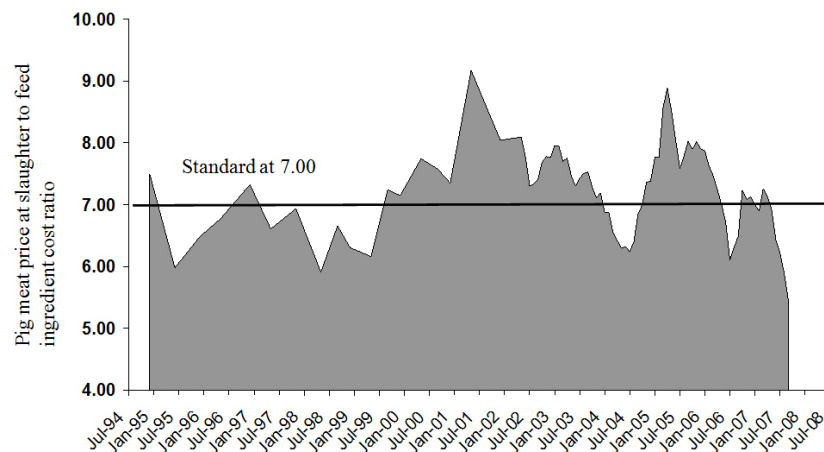


Figure 4.1: An example of the way in which the New Zealand ratio of pig meat price at slaughter (\$/kg) to feed ingredient cost (\$/kg) varies with time, from July 1994 until July 2007.

4.2 Aim

To demonstrate how variation of costs and price received at slaughter can be included in our modelling and handled using stochastic programming.

We begin with an explanation of the method. Then the numerical results are presented and followed by discussion.

4.3 Methods

So far we have regarded the feed costs and price schedule as fixed. In practice, a producer faces uncertainty in both the future cost of ingredients (in the form of the ingredient schedule IS) and the price received for a pig (in the form of the price schedule PS). In this chapter we also considered two diets of grower and finisher. The ingredient schedules IS_1 and IS_2 are presented in Table 4.1 and the price schedule (New Zealand price schedule, in July 2001) for PS_1 in Table 3.2 and PS_2 in Table 4.2. We assume now that we have I feed ingredient schedules IS_i and J price schedules PS_j occurring with probabilities p_i (in any period) and p'_j , for $i = 1, \dots, I$ and $j = 1, \dots, J$ respectively. Note that $FC(F, x)$ is influenced by IS_i and $GR(F, x)$ by PS_j . Note also that we do not change the feed ingredients, but vary only the cost of the feed ingredients. In order to simplify the immediately following presentation, but with no real loss in generality, we assume again that each feeding schedule comprises two diets, so $F = (D_1, D_2)$.

Table 4.1: Ingredient schedules IS_1 and IS_2 used in the chapter: these comprise a list of ingredients and the associated costs in \$/kg. The first ingredient schedule is carbohydrate cheap and protein expensive while the second ingredient schedule is carbohydrate expensive and protein cheap.

Ingredient	Barley	Blood meal	Soybean meal	...	Tryptophan
Costs for IS_1 (\$/kg)	0.2	1.5	1.2	...	20
Costs for IS_2 (\$/kg)	1.0	0.4	0.2	...	20

When finding the first diet, the ingredient schedule is fixed because we know the ingredient cost at that time, but for the second diet, the ingredient schedule will be subject to variation. Thus the producer is faced with a two-stage decision making process as illustrated in Figure 4.2.

Table 4.2: Price schedule 2 (PS_2): a generated price schedule giving prices in cents/kg for pigs at slaughter.

Backfat (mm)	Carcass weight (kg)										
	35.0 and under	35.1 to 40.0	40.1 to 45.0	45.1 to 50.0	50.1 to 55.0	55.1 to 60.0	60.1 to 65.0	65.1 to 70.0	70.1 to 75.0	75.1 to 80.0	80.1 and over
<6	300	300	300	300	300	300	300	300	300	300	300
6-9	335	345	350	365	370	375	380	390	395	405	375
10-12	335	345	350	360	365	370	375	385	390	400	370
13-15	305	330	335	335	335	335	330	330	330	330	330
16-18	270	270	270	270	270	270	260	260	260	260	260
>18	240	240	240	240	240	240	230	230	230	230	230

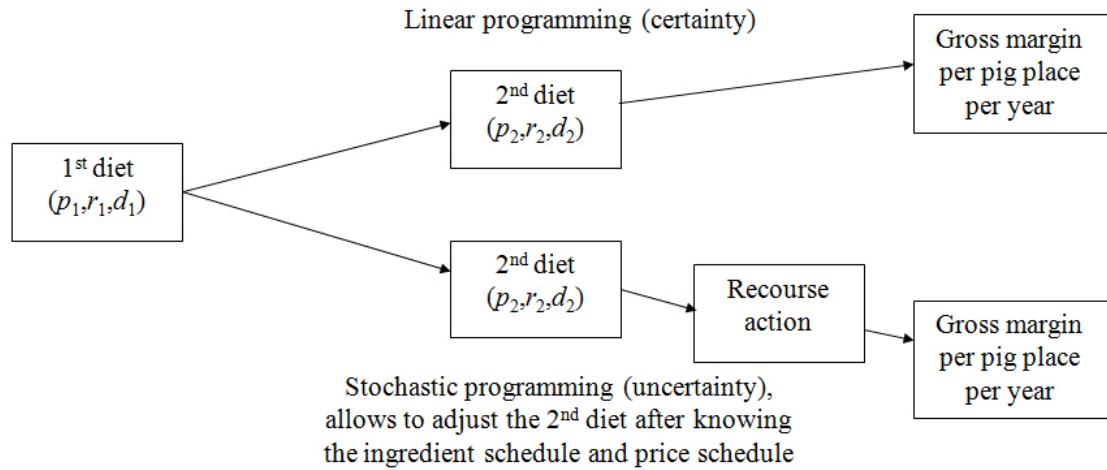


Figure 4.2: An illustration of stochastic programming for pig problem of two diets, $F = (p_1, r_1, d_1; p_2, r_2, d_2)$.

Bellman's principle of optimality makes clear that an optimal strategy must be optimal at each stage (using the outputs from the previous stage); we now separately consider these two stages.

At the outset, a decision must be made about D_1 . This is made facing uncertainty in IS_2 and GR (via the price schedule), so is chosen to be D'_1 , the argument maximising the expected gross margin per pig place per year, through calculation of

$$\max_{D_1, D_2} \max_x c(x) \left\{ \sum_{j=1}^J p'_j GR_j(D_1, D_2; x) - FC_1(D_1; x) - \sum_{i=1}^I p_i FC_{2i}(D_2; x) - WC \right\}$$

where $c(x) = 365/(x+7)$. Here $GR_j(D_1, D_2; x)$ is the gross return using price schedule j and diets D_1 and D_2 for a total of x days, $FC_1(D_1; x)$ is the minimum feed cost using diet D_1 in feed period one for x days and $FC_{2i}(D_2; x)$ is the minimum feed cost using diet D_2 and ingredient schedule i in feed period two, when growth is for a total period of x days.

At the second stage, given diet D'_1 in period one and the now known second period ingredient schedule IS_i , we must choose diet D'_2 and growth period of x days which maximises the revised expected gross margin

$$\max_{D_2} \max_x c(x) \left\{ \sum_{j=1}^J p'_j GR_j(D'_1, D_2; x) - FC_1(D'_1; x) - FC_{2i}(D_2; x) - WC \right\}$$

Note that evaluation of the first stage objective function, for a given F and x , involves $I + 1$ linear programs. Maximisation with respect to x , for fixed F , is carried out pointwise, while maximisation with respect to (D_1, D_2) can be carried out using either a genetic algorithm or the Tailored method. Evaluation of the second stage objective function, for a given $F = (D'_1, D_2)$ and period x , involves just two linear programs; maximisation with respect to x , for fixed $F = (D'_1, D_2)$, is again pointwise and the genetic algorithm or Tailored method used again to find the optimal F .

In the case of $K > 2$ diets, the optimisation proceeds in K stages, with the m th ($m = 1, \dots, K - 1$) stage involving solution of (with notation extending that above in

a ready way)

$$\max_F \max_x c(x) \left\{ \sum_{j=1}^J p'_j GR_j(F; x) - \sum_{k=1}^{m-1} FC_k(D'_k; x) - \sum_{s=m}^K \sum_{i=1}^I p_i FC_{si}(D_s; x) - WC \right\}$$

where $F = (D'_1, \dots, D'_{m-1}, D_m, \dots, D_K)$, and the K th stage solution of

$$\max_{F=(D'_1, \dots, D'_{K-1}, D_K)} \max_x c(x) \left\{ \sum_{j=1}^J p'_j GR_j(F; x) - \sum_{k=1}^{K-1} FC_k(D'_k; x) - FC_K(D_K; x) - WC \right\}$$

4.4 Numerical results

Uncertainty about the price schedule is easily handled, in that the optimisation uses the p'_1 and p'_2 weighted convex combination of schedules P_1 and P_2 respectively. For completeness, in our numerical runs we used two such schedules, those shown in Table 3.2 (PS_1) and Table 4.2 (PS_2), with associated weights $p'_1 = 0.8$ and $p'_2 = 0.2$. As the tailored method is currently unavailable in the Bacon Max software we have chosen to use the available genetic algorithm for the purpose of our study. All optimisations were carried out using a genetic algorithm.

Uncertainty in the ingredient schedule is of much greater interest. Pig producers have to face a two-stage decision making process due to the uncertainty in the ingredient schedule. The optimal diet in the second stage must be chosen using the outputs from the previous stage and after knowing the change in the ingredient schedule. For these we use the ingredient schedules in Table 4.1 with weights of $p_1 = 0.8$ and $p_2 = 0.2$. Optimal feeding schedules (with two diets fed) are shown in Table 4.3 when there is no uncertainty in the diet to be fed for the second period and in Table 4.4 when the digestible energy cheap ingredient schedule IS_1 is far more likely ($p_1 = 0.8$) than the protein cheap ingredient schedule ($p_2 = 0.2$) in the second feeding period. Pig genotype parameters of $\text{minLP} = 0.8$ and $\text{Pdmax} = 160$ g/day are used.

Some comments about the results are now provided. In Table 4.3, cheap energy (carbohydrate) via use of IS_1 in the first period allows the proportion of the NRC standard used in the second period to drop to 0.83 (for IS_1 and IS_2), whereas it remains higher at 0.87 and 0.88 (for IS_1 and IS_2 respectively) if energy is expensive (use of IS_2) in the first period. No matter which ingredient schedule is used in the first period, the move to cheaper protein, from IS_1 to IS_2 , in the second period causes r_2 to increase,

as expected. In Table 4.4, first period diets, for a given ingredient schedule in the first period, do not vary with second period ingredient schedule, again as expected. The first comment made concerning Table 4.3, regarding p , still largely stands, although it is moderated due to the uncertainty in the second ingredient schedule. The most notable change from the deterministic to stochastic result tables is the larger r_2 value when IS_2 is used in the first feeding period and IS_1 in the second. Here the possibility of cheap protein did not eventuate, so more must be taken in the second feeding period.

Table 4.3: Deterministic results: optimal feeding schedules when there is no uncertainty about the ingredient schedule in the second feeding period. Entries are averages over ten runs.

Optimal feeding schedules			2nd period			
			IS_1		IS_2	
			D_1	D_2	D_1	D_2
1st period	IS_1	p	0.95	0.83	0.94	0.83
		r	0.64	0.50	0.65	0.52
		d	12.10	14.40	12.10	13.82
	IS_2	p	1.00	0.87	1.00	0.88
		r	0.98	0.91	0.99	1.01
		d	15.02	14.80	15.01	15.02

Table 4.4: Stochastic results: optimal feeding schedules when there is uncertainty about the ingredient schedule in the second feeding period. Entries are averages over ten runs.

Optimal feeding schedules			2nd Period			
			IS_1		IS_2	
			D'_1	D'_2	D'_1	D'_2
1st period	IS_1	p	0.97	0.81	0.97	0.81
		r	0.69	0.52	0.69	0.55
		d	12.48	13.99	12.48	12.31
	IS_2	p	0.98	0.88	0.98	0.88
		r	0.98	0.99	0.98	1.00
		d	15.08	15.00	15.08	15.01

4.5 Discussion

In this chapter we have considered the challenge of including variation in the ingredient schedule and price schedule, and shown how to find optimal feeding schedules under such conditions using stochastic programming.

Many optimisation methods for pig feeding schedules consider the feed cost and price schedule only at the beginning of the estimation, but do not consider the variation of this cost and price over time. This variation may call for a diet reformulation. In this chapter we considered the challenge of including variations in the ingredient and price schedule, and have shown how to find optimal feeding schedules under such conditions using stochastic programming.

We caution that the optimal feeding schedule for a single pig is unrealistic, since in practice many pigs, exhibiting minor variations in genotype and feed intake, are grown on a single feeding schedule. The optimum schedule in such a situation is different from that found for a single pig. Such variation can be incorporated into an objective function for further work, but was not in this chapter, in order to focus on the development addressed.

We conclude by acknowledging that there will always remain scope for improved methodology in this rich application area for optimisation. Other methodology can be applied, for example Niemi (2006) handles the change in feed cost and pig slaughter price using a dynamic programming technique for optimising feeding and slaughter decisions for fattening pigs in Finland.

Chapter 5

Effect of pig type, costs and dietary restraints on dietary nutrient specification

5.1 Introduction

As discussed earlier in Section 2.3 the optimal feeding schedule offers maximum profit for given conditions. This chapter looks at the effects of three factors on the optimal feeding schedule for grower-finisher pig herds: pig type (genotype), feed costs and carcass payment scheme, and dietary restraints (ad libitum and restricted feeding). In addition, feed-to-lean growth and optimal growth schedules are compared.

5.2 Aims

1. To study the effect of different pig types, ingredient costs and price schedules, and dietary constraints on the optimal feeding schedule.
2. To compare feed-to-lean growth and optimal growth schedules.

This chapter is organized as follows. In Section 5.2 we describe the methodology used for identifying the optimal feeding schedule and also describe the factors whose influence on the optimal feeding schedule are studied. In Section 5.3 we then report results, first considering the influence of pig type on optimal dietary nutrition. The effect of a level change of ingredient cost and the effect of a level change of price schedule

on optimal dietary nutrition is then discussed. Next, the effect of dietary restraints on the optimal feeding schedule is examined; this includes a comparison of feed-to-lean and optimal growth feeding schedules. Finally, the opportunity cost relating to use of feed-to-lean and optimal feeding schedules for a range of pig types is presented. Results are discussed in Section 5.4.

5.3 Methods

Our aim is to find the feeding schedule F that produces maximum profit, measured as “maximum gross margin per pig place per year (GMPPY)” for the different pig types, ingredient costs and price schedules, and dietary constraints. A feeding schedule F takes the form of a sequence of diets,

$$F = (p_1, r_1, d_1; p_2, r_2, d_2; \dots; p_n, r_n, d_n),$$

each fed for a fixed period (typically one week) with each diet described by three parameters (p, r, d) where p is in the range 0.5 to 1.0, r is in the range 0.2 to 1.2 g/MJ and d is in the range 12 to 17 MJ/kg.

Gross return is determined by the backfat thickness and carcass weight of the pig at slaughter. This is taken from a price schedule, an example of which is shown in Table 5.1. Feed cost is the sum of weekly feed costs, each determined using a linear program, for the total period of x days (if x is not a multiple of seven, the final period will be less than one week and pro-rated accordingly). The LP requires use of a schedule of ingredient costs; an example is already given briefly in Table 2.1 and given fully in Table 5.2. The weaner cost is fixed at NZ\$75. We use ingredient costs and price schedules from New Zealand (September 2007) with feeding schedules consisting of 15 diets (so $n = 15$).

Then, we measure the gross margin per pig place per year (GMPPY) when we assume a seven day turnaround time between batches. A population of 20 feeding schedules and a stopping rule of 20 iterations with no change in the best value of the objective function has been used to find the optimal feeding schedule for each growing environment. For practical reasons, we grow 200 pigs in the objective function rather than a single pig; optimal feeding schedules for a single pig can be different to those of a large group of pigs whose type parameters vary about set base values as mentioned earlier

Table 5.1: Standard price schedule: a New Zealand price schedule giving price in cents/kg for pigs at slaughter in September 2007.

Backfat (mm)	Carcass weight (kg)										
	35.0 and under	35.1 to 40.0	40.1 to 45.0	45.1 to 50.0	50.1 to 55.0	55.1 to 60.0	60.1 to 65.0	65.1 to 70.0	70.1 to 75.0	75.1 to 80.0	80.1 and over
<6	255	255	255	255	255	255	255	255	255	255	255
6-9	310	310	330	330	320	320	305	305	305	305	300
10-12	310	310	320	320	310	305	305	305	305	305	300
13-15	230	230	230	230	230	260	260	260	260	260	255
16-18	185	185	185	185	185	185	185	185	185	185	185
>18	165	165	165	165	165	165	165	165	165	165	165

Table 5.2: Standard ingredient costs: a New Zealand feed ingredient list, with costs given in cents/kg for September 2007.

Feed ingredient	Cost (cents/kg)	Feed ingredient	Cost (cents/kg)
Barley	42	Wheat	43
Blood meal	116	Whole milk powder	250
Broll	26	Lysine	355
Full fat soybean meal	106	Methionine	550
Imported fish meal	230	Premix	650
Maize	42	Di-calcium phosphate	150
Meat and bone meal	73	Sodium hydrogen phosphate	100
Skimmed milk powder	245	Salt (Sodium chloride)	55
Soybean meal	70	Limestone	4
Soybean oil	150	Threonine	1400
Tallow	80	Tryptophan	2000

in Section 2.5.1. We vary the pigs by generating pig genotype parameters minLP and Pdmax, initial body protein content at 20kg in the purchased pig and feed intake using a standard deviation of 10% of an inputted central value. Finally, the optimal feeding schedules of 10 runs are summarised for each growing environment.

We now discuss the three factors of pig type, costs/prices and dietary constraints, whose influence on dietary nutrient specification will be investigated.

Factors influencing dietary nutrient specification

5.3.1 Pig type

Two genotype factors are considered:

- minLP, the minimum allowable lipid to protein ratio (with values 0.6, 0.8 and 1.0)
- Pdmax, the maximum daily protein deposition (with values 120, 160 and 200 g/day)

Combinations of these factor levels yield nine pig types. Correlation between the parameters, minLP, Pdmax and p is taken into account when generating a set of 200 pigs, using the correlation structure as discussed earlier in Table 2.11 (Sheriff, 2008).

5.3.2 Costs

In order to calculate the gross margin per pig using feeding schedule F for x days, $g(F, x)$, ingredient costs are needed for calculating feed cost and the price schedule is needed for calculating gross return. A feed ingredient list and associated costs (in cents/kg) were shown in Table 5.2.

A change of what we term “level” in ingredient (equivalently, feed) costs is considered in this research. The level change for ingredient costs is generated using multiples of 0.8 and 1.2 of the costs shown in Table 5.2.

Table 5.1 presented a standard price schedule; the price of a pig (in cents/kg) depends on the carcass weight (kg) and backfat thickness (mm). The level changes (lower and higher) for the price schedule are generated from the standard price schedule in the same way as the level changes for ingredient cost; we term these the “deflated” and

“inflated” price schedules, respectively.

A change of what we term “pattern change” in the price schedule is considered also in this section. The pattern change for price schedule is generated from moving the highest price return cell in the standard price schedule (Table 5.1) to the higher carcass weight as show in Table 5.3 which we call “heavy price schedule”.

Table 5.3: Heavy price schedule: generated from the standard price schedule of Table 5.1 by moving the highest price cell to a higher carcass weight. Price is shown in cents/kg for pigs at slaughter.

Backfat (mm)	Carcass weight (kg)										
	35.0 and under	35.1 to 40.0	40.1 to 45.0	45.1 to 50.0	50.1 to 55.0	55.1 to 60.0	60.1 to 65.0	65.1 to 70.0	70.1 to 75.0	75.1 to 80.0	80.1 and over
<6	255	255	255	255	255	255	255	255	255	255	255
6-9	295	295	300	300	305	305	310	330	330	320	300
10-12	290	290	295	295	300	300	305	310	310	310	300
13-15	230	230	230	230	230	260	260	260	260	260	255
16-18	185	185	185	185	185	185	185	185	185	185	185
>18	165	165	165	165	165	165	165	165	165	165	165

5.3.3 Dietary constraints

We use p and r as dietary constraints and study their influence on the optimal feeding schedule specification. At one extreme is the “feed-to-lean” schedule and at the other is the “fully optimal” schedule (in which there are no restrictions on either p or r).

The growth model used calculates, for a given liveweight (LW), the energy required for maintenance and protein deposition of P_{dmax} . If the energy available is less than this amount, then the extent of protein deposition possible is calculated, keeping the pig lipid to protein ratio lean at minLP (hence “feed-to-lean”). If the energy available is more than this amount, protein is deposited at the rate of P_{dmax} and excess energy put on as lipid, so that minLP is exceeded. From this, r can be calculated. We call the resulting feeding schedule the “feed-to-lean (p fixed)” schedule. A second feed-to-lean feeding schedule is found by progressively reducing p so that minLP is never exceeded; associated with this will be a weekly set of r values, differing from those just described. This schedule we term “feed-to-lean (p reducing)”. We can progressively relax these

constraints, releasing r from both feed-to-lean sequences and finally, allowing both p and r to be unconstrained. These five scenarios are pictured in Table 5.4. In practice, we fix p at one of two values, 0.8 or 1.0. When p is reducing, p is allowed to run below these values. When p is free, it is allowed to run between 0.5 and the fixed upper limit under consideration (either 0.8 or 1.0).

We remark that a feed-to-lean restriction results in a single feeding schedule for a period of N days. On the other hand, any of the three systems with an element of optimality involves a search amidst a family of (F, N) pairs.

Table 5.4: The various levels of restriction on p and r for feed-to-lean and optimal feeding schedules. For convenience, we label the combinations with a site number, using these later in the interests of brevity.

		Optimal	
Free		p and r free (Site 9)	
p	Reducing	Feed-to-lean p reducing (Site 5)	Optimal p reducing, r free (Site 6)
	Fixed	Feed-to-lean p fixed (Site 1)	Optimal p fixed, r free (Site 3)
		Fixed	Reducing
		Free	
		r	

We will look at the influence of each factor (pig type, costs/prices and dietary constraints) on the fixed (or optimal) feeding schedule. Each factor most influences some aspect of feeding; that aspect will be studied. The influence of pig type will be assessed using plots of r against time. The influence of ingredient costs and price schedule will be assessed by a comparison of slaughter dates. Finally, the influence of dietary constraints, for fixed pig type and costs/prices, can be assessed using the plot shown in Figure 5.2. The aim of a feeding schedule is to move rapidly along the minLP horizontal to point P and stay there, at the same time using minimising feed cost; this path maximises GR. That it is efficient to move left to right along the minLP line is based on the observation that if $L_i/P_i = \text{minLP}$ (where L_i and P_i are, respectively, the lipid and protein content of the pig on the i th day), then $L_{i+1}/P_{i+1} = (L_i + Ld_i)/(P_i + Pd_i) = \text{minLP}$ provided $Ld_i/Pd_i = \text{minLP}$ (where Ld_i and Pd_i are the lipid and protein deposition on the i th day).

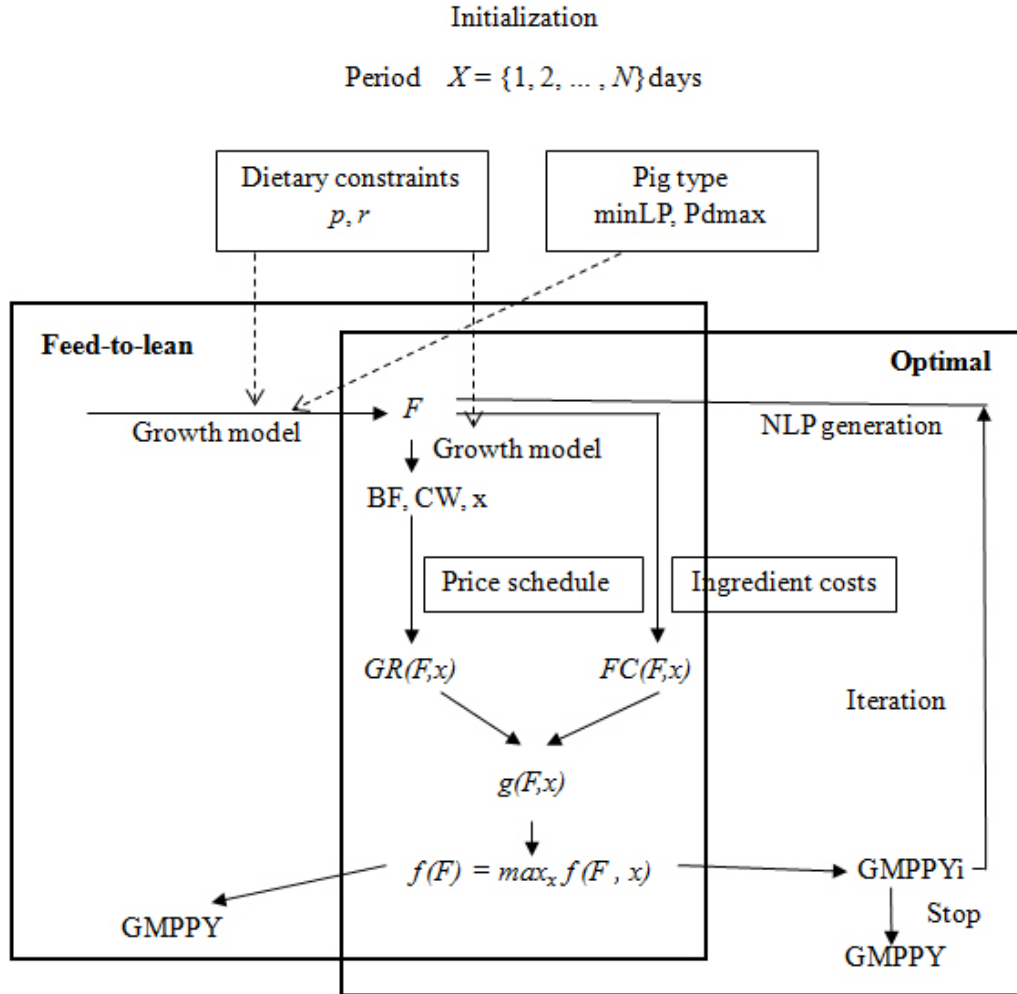


Figure 5.1: The relationship between finding feed-to-lean (large left box) and optimal (large right box) feeding schedules. The influence of the four small-boxed items on the process is studied in this paper.

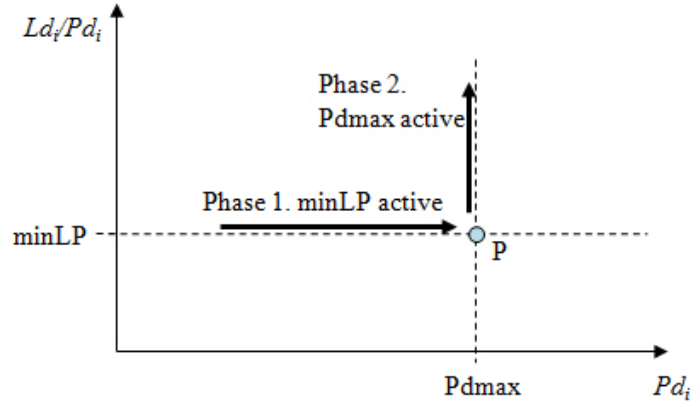


Figure 5.2: Efficient growth follows a trajectory from left to right along the minLP horizontal, then stays at point P, where Pdmax is deposited daily, while keeping the lipid to protein ratio at minLP.

An alternative graphic which monitors how well a dietary restraint works is shown in Fig 5.3. An efficient feeding schedule will have Pd_i rise rapidly to Pdmax, Ld_i/Pd_i stay on the minLP horizontal and the cumulative feed cost curve remain low.

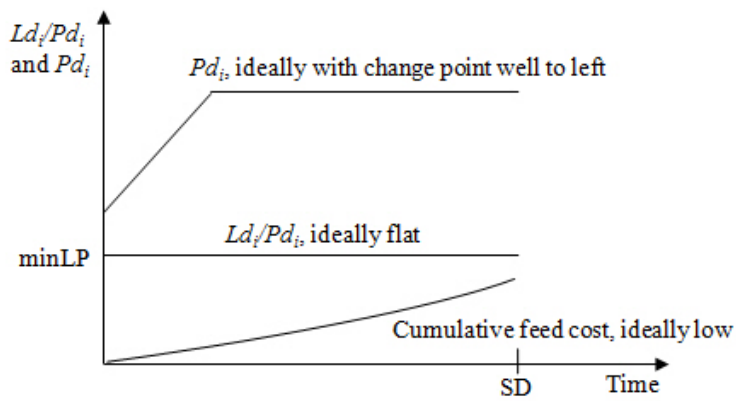


Figure 5.3: Plots of Pd_i , Ld_i/Pd_i and cumulative feed cost against time; SD is the slaughter date.

An illustration that the lysine to digestible energy ratio r curves have two phases which corresponding to the two phases as shown in Figure 5.2; the first phase can be a constant or increasing curve when minLP is active while the second phase is a decreasing curve when Pdmax is active, as shown in Figure 5.4:

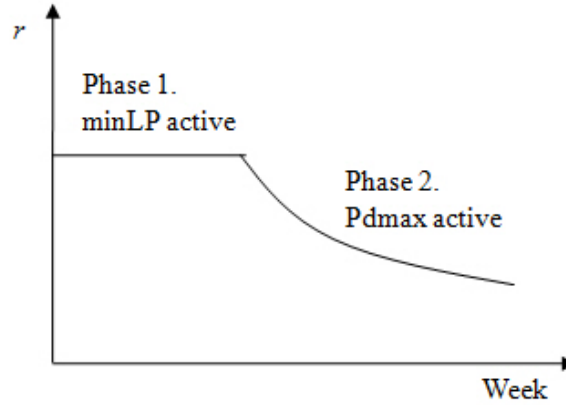


Figure 5.4: Plot of r curve corresponding to minLP and Pdmax. In the initial phase, minLP is active while later Pdmax has greater influence.

5.4 Results

Results are presented in three parts as follows. We begin by considering the influence of pig type (via minLP and Pdmax), on the feed-to-lean and optimal feeding schedules. We then consider the influence of costs and prices and finally, we discuss the influence of dietary constraints on growth.

5.4.1 Influence of pig type

The influence of minLP and Pdmax (using the standard ingredient cost and standard price schedule) on the plots of r against week until slaughter is shown in Figure 5.5 for the p fixed feed-to-lean (Site 1) at $p = 0.8$ and in Figure 5.6 on the average r over 10 runs for the fully optimal feeding schedule (Site 9) where p runs between 0.5 and 0.8. Note that the slaughter dates for both sites varies with minLP and Pdmax; this is made evident by the position of the curve end point in the graphics.

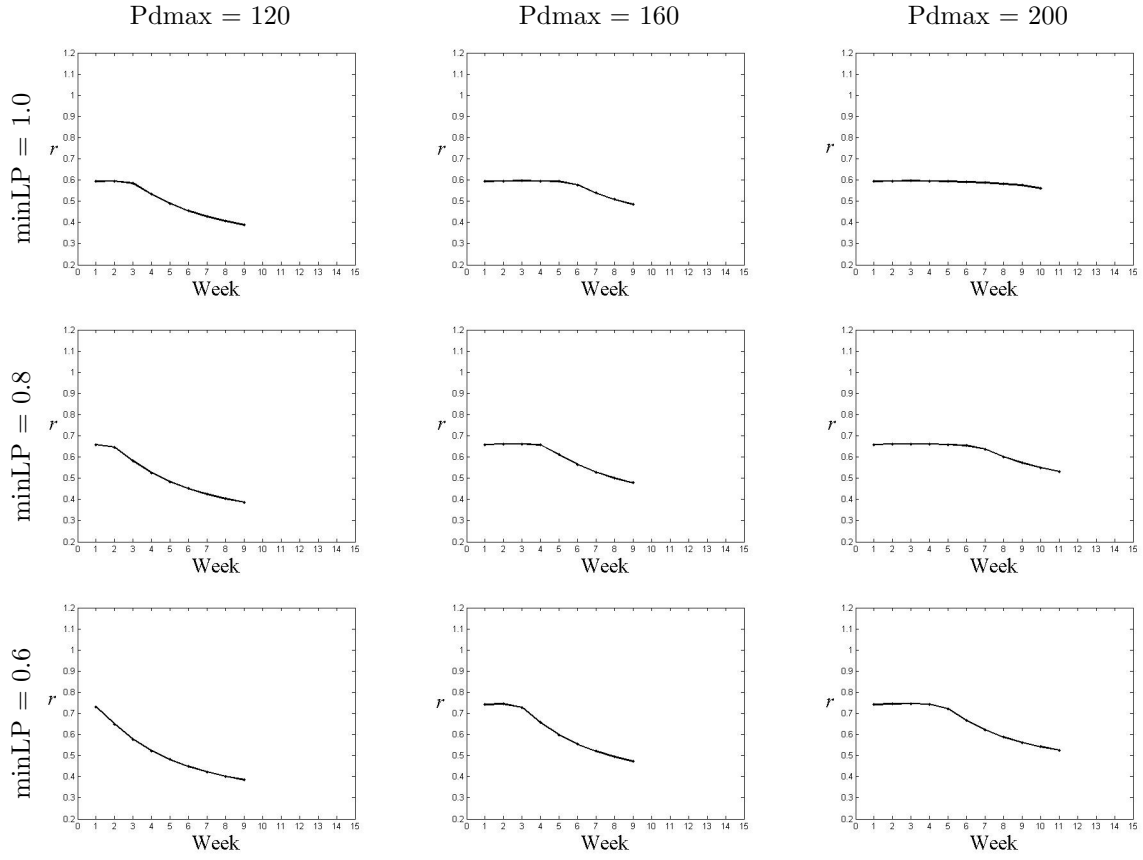


Figure 5.5: A comparison of the r curves of the feed-to-lean feeding schedule, where p fixed at 0.8 as $\min LP$ and $Pdmax$ vary, for the standard ingredient cost and standard price schedule.

It is evident that $\min LP$ and $Pdmax$ significantly influence the r curve. As $\min LP$ decreases (and the genotype improves) the r curves start higher and $\min LP$ is the active constraint for a shorter period. As $Pdmax$ increases (and again, the genotype improves) the r curves start at the same level, $\min LP$ is the active constraint for a longer period and the growth period is longer.

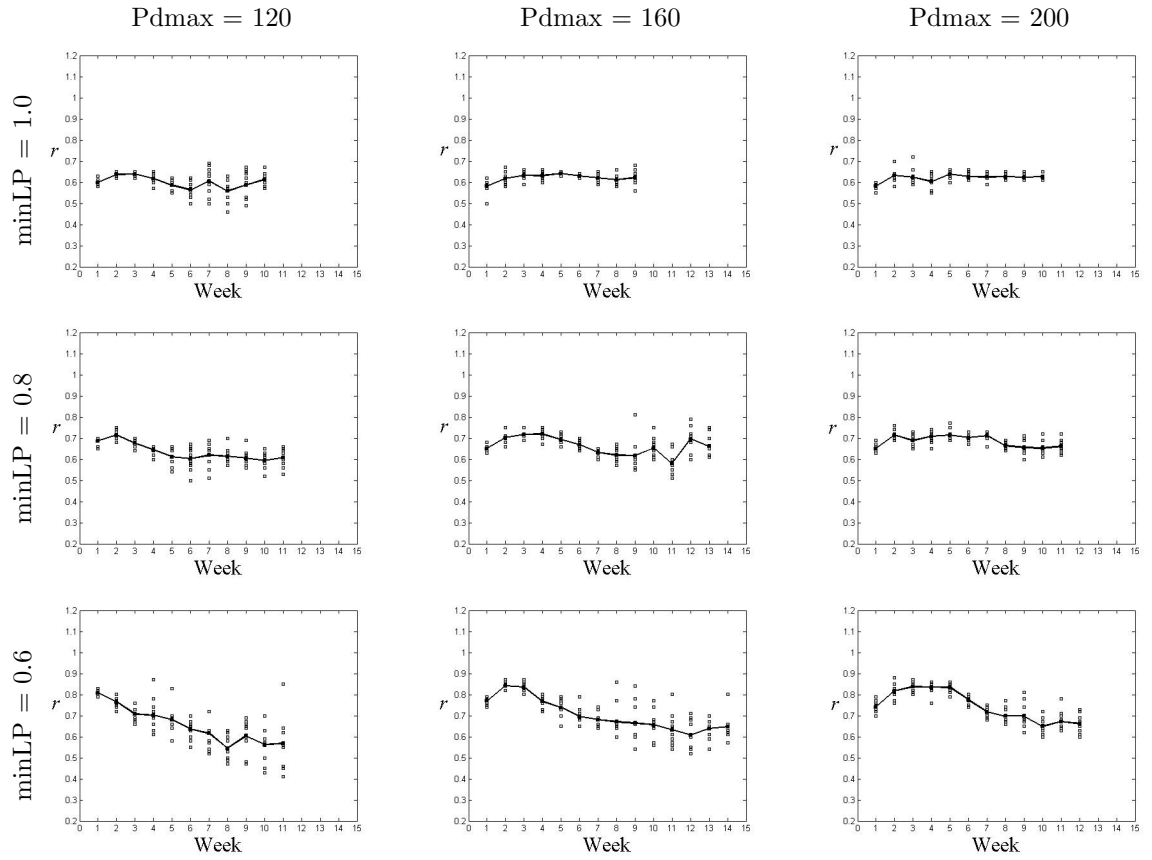


Figure 5.6: A comparison of the r curves using the fully optimal feeding schedule as $\min LP$ and $Pdmax$ vary and p is free to run from 0.5 to 0.8, for the standard ingredient cost and standard price schedule. The optimal feeding schedule curve tracks the average r over 10 runs; individual run values are shown with dots.

Again, $\min LP$ and $Pdmax$ significantly influence the r curve. As $\min LP$ decreases, the r curves start higher, $\min LP$ is the active constraint for a shorter period and the growth period extends. As $Pdmax$ increases, the r curves start at the same level, $\min LP$ remains the active constraint for a longer period and the growth period is longer. The r curve for the optimal feeding schedule is generally higher than the corresponding feed-to-lean r curve.

5.4.2 Influence of costs

Subject to the correctness of the modelling process it is possible to prove the following result about the effect of changes in ingredient cost and price schedule on the slaughter date. A proof of the theorem is provided in the Appendix.

Theorem

Assume that GR and FC are smooth functions of the growing period x .

1. Let the ingredient cost be multiplied by factor α . Then for conditions in which gross margin g is positive, slaughter date decreases if and only if

$$(\alpha - 1)((GR/FC)'(x) + WC * FC'(x)/(FC(x))^2) > 0$$

2. Let the price schedule be multiplied by factor α . Then for conditions in which gross margin g is positive, slaughter date decreases if and only if

$$(\alpha - 1)((GR/FC)'(x) + WC * GR'(x)/(FC(x))^2) < 0$$

We present two immediate corollaries.

Corollary

1. If ingredient cost is multiplied by $\alpha > 1$ and $(GR/FC)'(x) > 0$ and conditions are such that gross margin g is positive, then slaughter date will decrease.
2. If the price schedule is multiplied by factor $\alpha > 1$ and $(GR/FC)'(x) < -WC * GR'(x)/(FC(x))^2$, and conditions are such that gross margin g is positive, then slaughter date will decrease.

We now present the result of runs of the model to illustrate these theoretical results.

Influence of ingredient cost

We consider the influence of level change of the ingredient (or feed) cost on the optimal feeding schedule, using the standard price schedule. The result is shown in Figure 5.7 below.

A level change in ingredient cost appears to very slightly influence the level of the optimal r curve. Evidently the r curves are parallel but at slightly different levels; as ingredient cost increases the level falls. Figure 5.8 shows the effect of ingredient cost change on GMPPY and cumulative feed cost from the example of one run. Note that as ingredient cost rises from 0.8 to 1.0 to 1.2 times the standard cost, the slaughter day decreases from 59 to 58 to 57. Here it will be the case that $(GR/FC)'(x) > 0$, so bearing out the result in i) of the above theorem (since $FC'(x)$ is always positive).

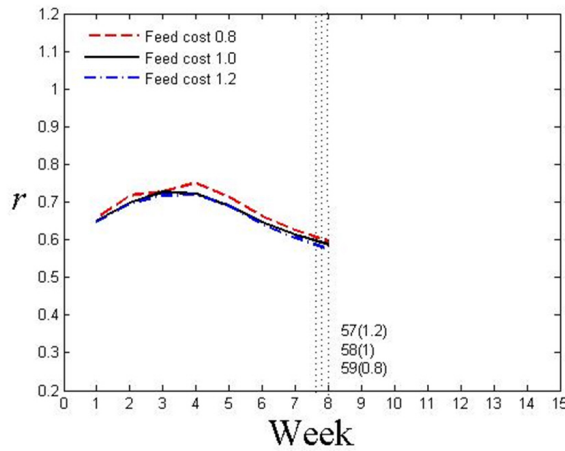


Figure 5.7: Showing r curves, averaged over 10 runs, for level changes in ingredient costs where $\min LP = 0.8$, $Pd_{\max} = 160$ g/day and p fixed at 0.8.

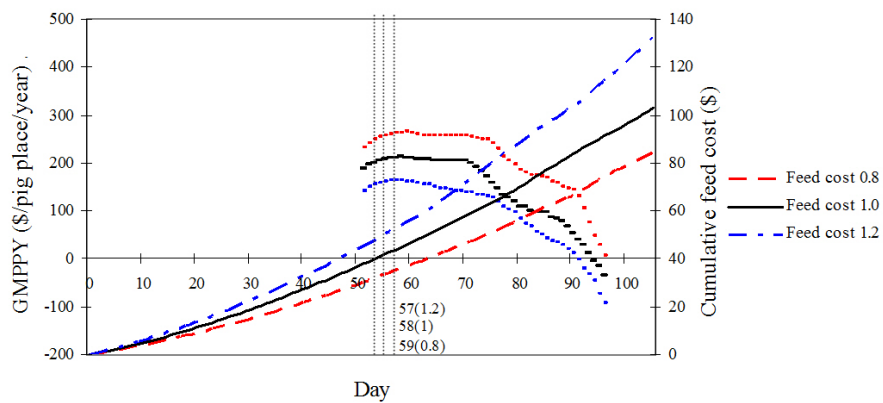


Figure 5.8: GMPPY and cumulative feed cost for different level changes of ingredient cost from an example of one run where $\min LP = 0.8$, $Pd_{\max} = 160$ g/day and p fixed at 0.8.

Influence of price schedule

In this section we consider two types of influence of price schedule, the level change and pattern change on the optimal feeding schedule. Firstly, we present the effect of level change on the r curve and then the effect of pattern change.

Influence of level change in price schedule

In this section we consider the influence of level change in price schedule (deflated, standard and inflated) on the optimal feeding schedule, keeping the standard ingredient cost. The effect of price schedule level change on the r curve is shown in Figure 5.9.

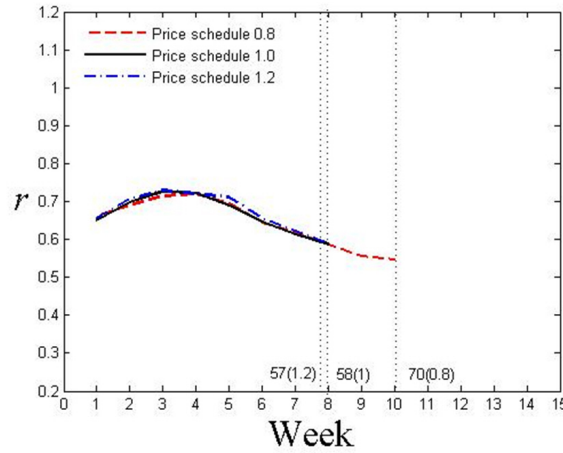


Figure 5.9: Showing r curves, averaged over 10 runs, for the deflated, standard and inflated price schedules where $\text{minLP} = 0.8$, $\text{Pdmax} = 160$ g/day and p fixed at 0.8. Only slaughter date is affected.

Deflated and inflated price schedules do not change the shape of the objective function and so do not alter the optimal feeding schedule. The only difference noticed between the r curves is a change in the slaughter day. This decreases from 70 to 58 to 57 as the level change increases from 0.8 to 1.0 to 1.2. Evidently the conditions described in Corollary 2 hold here.

Less regular changes in ingredient cost or price schedule may cause a considerable change in the optimal feeding schedule. Level changes, however, especially in price schedules are what we normally see.

Figure 5.10 displays the effect of price schedule change on GMPPY and cumulative feed cost.

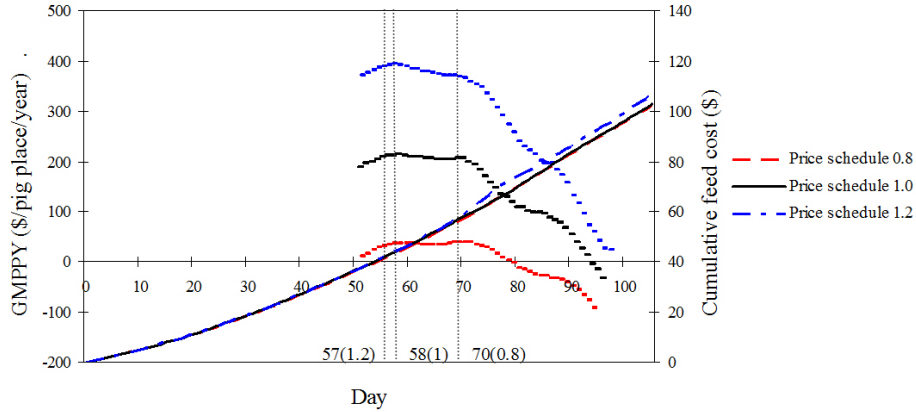


Figure 5.10: GMPPY and cumulative feed cost for level changes in price schedule from an example of one run where $\min LP = 0.8$, $Pd_{\max} = 160$ g/day and p fixed at 0.8.

Influence of pattern change in price schedule

In this section we consider the influence of pattern change in price schedule (standard and heavy) on the optimal feeding schedule, keeping the standard ingredient cost. The effect of price schedule pattern change on the r curve is shown in Figure 5.11.

As discussed before in Section 5.3.1, $\min LP$ and Pd_{\max} significantly influence the r curve. The influence of pattern change in price schedule (standard and heavy) is also different in different pig types. When $Pd_{\max} = 120$ g/day for all $\min LP$ and $p = 0.8$, r curves are significantly altered in that the growing period for the heavy price schedule is much longer than that for the standard price schedule. On the other hand, where $Pd_{\max} = 200$ g/day for all $\min LP$, the pattern change in price schedule has lesser impact on the growth period. The slaughter date for both standard and heavy price schedules are similar. For $\min LP = 0.6$ and for all Pd_{\max} , the level of r curves for the heavy price schedule is higher than for the standard price schedule.

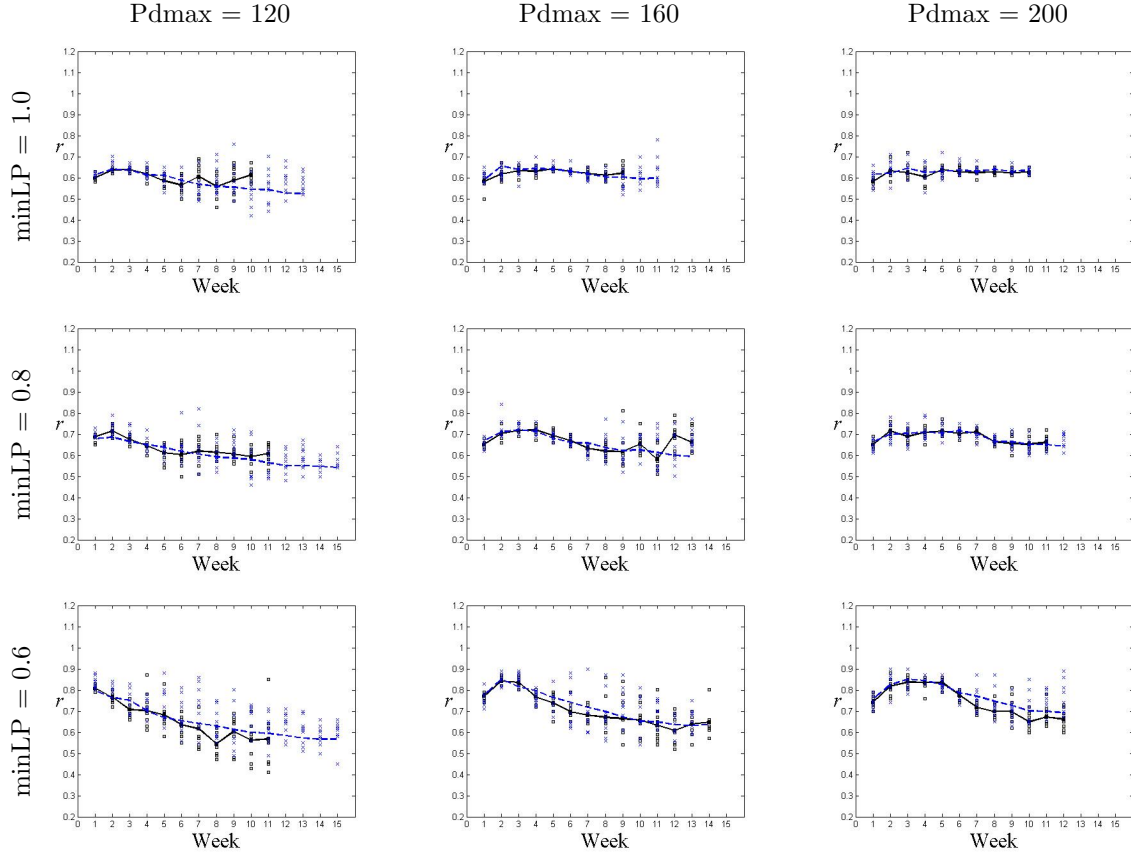


Figure 5.11: A comparison of the r curves using the fully optimal feeding schedule as $\min LP$ and $Pdmax$ vary and p is run from 0.5 to 0.8, for the standard price schedule and heavy price schedule, by keeping the standard ingredient cost. The optimal feeding schedule curve tracks the average r over 10 runs. The solid line and dashed line present the mean of r for the standard and heavy price schedule, respectively. Individual run values are shown; dots and crosses represent r values for the standard and heavy price schedule, respectively.

5.4.3 Influence of dietary constraints

Throughout this section we fix $\text{minLP} = 0.6$, $\text{Pdmax} = 200$ g/day, and $p = 0.8$ and use the standard ingredient cost and standard price schedule. We now present the diagnostic plots described in the Methods section, together with the r -curves, for the five key dietary constraints (the two feed-to-lean strategies (Sites 1 and 5) and three optimal strategies (Sites 3, 6 and 9)). We then draw conclusions.

The upper feed-to-lean plots in Figures 5.12 and 5.13 are distinct in two ways. First, reducing p to keep the lipid to protein ratio at minLP does precisely that - this ratio stays down (at 0.6 in this case). Second, it becomes evident that in order to maximise GMPPY the growing period moves from 77 days to 90 days as also presented later in Table 5.7. The parallel comparison (of Sites 3 and 6 in the lower row) where r is free shows similar results, but the lipid to protein ratio are closer to Pdmax compare with Sites 1 and 5. Finally, the fully optimal solution provides a surprise: the growth period is reduced slightly but we see that GMPPY is maximised by use of a “feast or famine” dietary regime. This optimal feeding schedule allows pigs to grow faster (Site 9, growing period 85 days in this example of a single run) compared with feed-to-lean, p reducing (Site 5, growing period 90 days) with similar carcass weight and backfat thickness at slaughter. These produce higher GMPPY as shown in this example of a single run in Figure 5.14 (\$317/pig place/year) compared with Site 5 (\$298/pig place/year); Figure 5.14 contrasts cumulative feed costs, GMPPY and slaughter date for these two sites.

We conclude this subsection by comparing the r curves for the five sites.

The upper feed-to-lean plots differ in length and amount of lysine fed; in order to maintain the Ld_i/Pd_i ratio at minLP in Site 5 (with p reducing) the diet fed can remain higher in protein. The same comment applies to the comparison of Sites 3 and 6. All optimal curves permit higher ratios of lysine to digestible energy in the early stages of development; the fully optimal curve (Site 9) shows the oscillation towards the end of the feeding period.

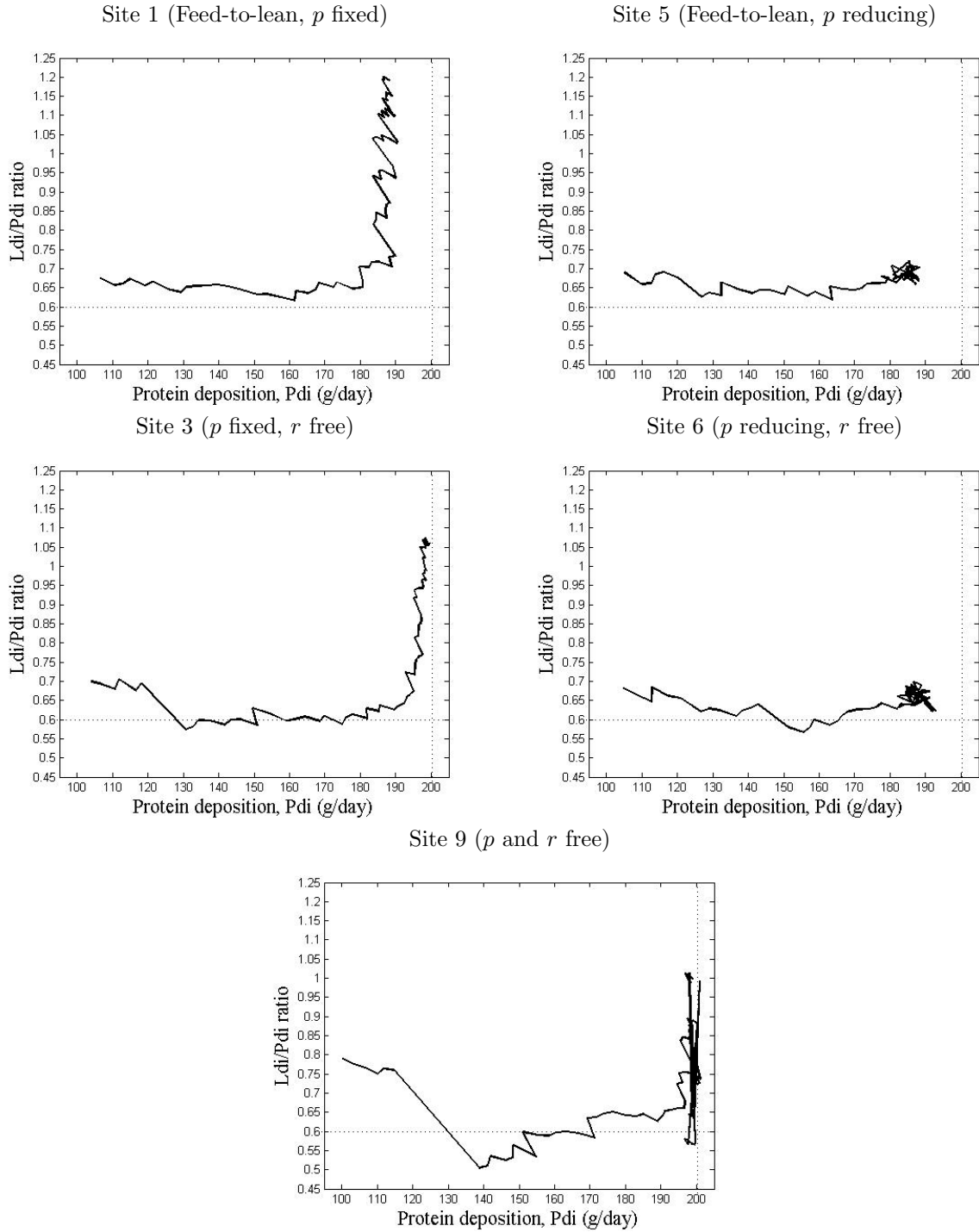


Figure 5.12: Daily lipid deposition to daily protein deposition ratio against daily protein deposition from an example of one run, for each dietary constraints for $\text{minLP} = 0.6$, $\text{Pdmax} = 200$ g/day, and $p = 0.8$ or run below 0.8.

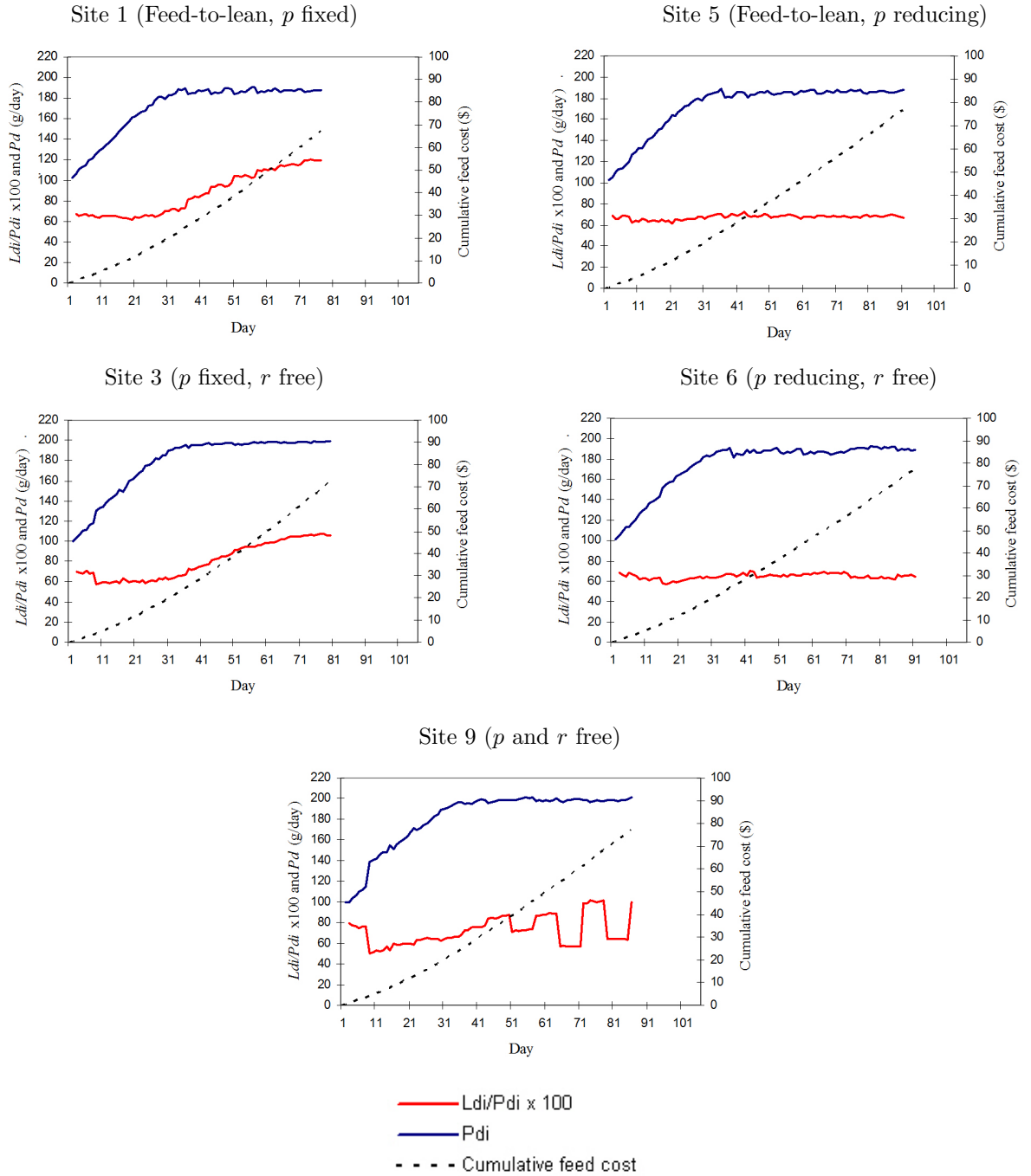


Figure 5.13: Daily lipid deposition to daily protein deposition ratio, daily protein deposition and cumulative feed cost against time from an example of one run. The first row shows the two feed-to-lean dietary restrictions while the second row shows influence on growth of the two partially and the last row shows fully optimal dietary regimes for $\min LP = 0.6$, $Pd_{\max} = 200$ g/day, and $p = 0.8$ or run below 0.8.

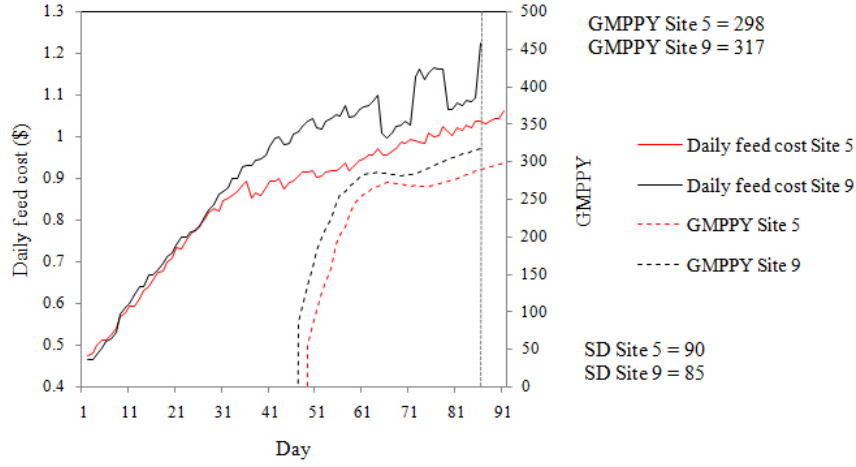


Figure 5.14: Daily feed cost and GMPPY for Site 5 (feed-to-lean, p reducing) and Site 9 (p and r free) against time from an example of one run for $\text{minLP} = 0.6$, $\text{Pdmax} = 200$ g/day, and $p = 0.8$.

5.4.4 The influence of pig type and dietary restraints on GMPPY, carcass weight, backfat thickness and slaughter date.

In this section we present three tables showing in detail, and separately, the influence of pig type and dietary restraints first on GMPPY, then on CW and BF and finally on SD. First, we exhibit, in Table 5.5, the difference (an opportunity cost) between feed-to-lean growth GMPPY and optimal growth GMPPY, for the standard ingredient cost and standard price schedule. These results are an average over 10 runs.

When p is fixed, GMPPY for the optimal feeding schedule (Site 3) is greater than that for the feed-to-lean feeding schedule (Site 1) across all pig types. When p is reducing, GMPPY for the optimal feeding schedule (Site 6) is also greater than for the feed-to-lean feeding schedule (Site 5) across all pig types. The optimal feeding schedule, with p and r free (Site 9), gave the greatest GMPPY for all pig types, except two cases, where $\text{minLP} = 1.0$, $\text{Pdmax} = 200$ g/day and $p = 0.8$ and $\text{minLP} = 0.8$, $\text{Pdmax} = 200$ g/day and $p = 1.0$, when average GMPPY is slightly lower than that at Site 6. Since Site 9 must give a higher GMPPY than Site 6, these two cases reflect the stochastic nature of these values.

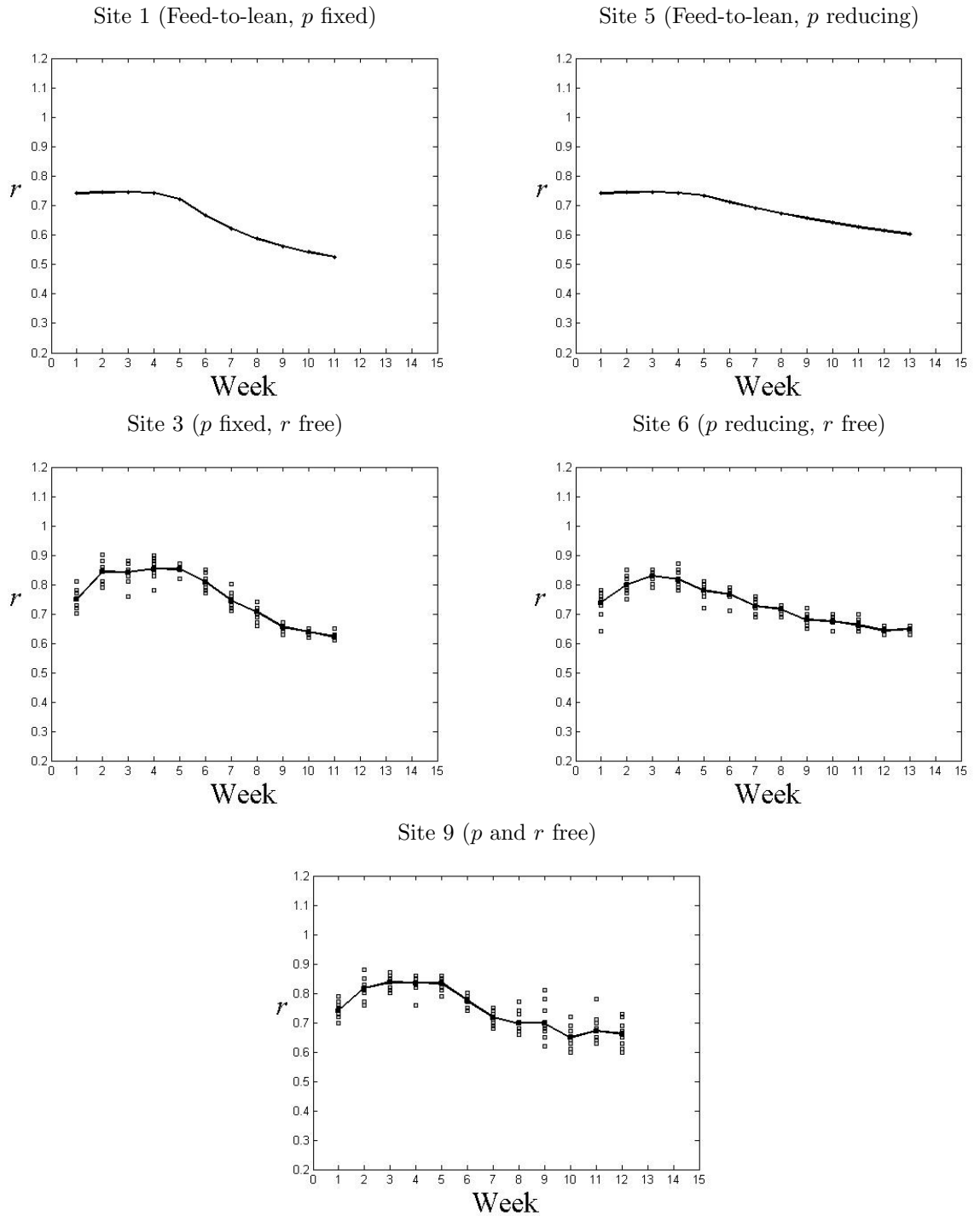


Figure 5.15: Plots of r against time for the two feed-to-lean sites and the average r over 10 runs against time for the three increasingly optimal sites for $\text{minLP} = 0.6$, $\text{Pdmax} = 200$ g/day, and $p = 0.8$ or run below 0.8.

Table 5.5: The mean and standard deviation of GMPPY over 10 runs as pig type and dietary restraints vary, for the standard ingredient cost and standard price schedule.

p	Pdmax	Methods	minLP		
			0.6	0.8	1.0
0.8	120	Feed-to-lean, p fixed	143.87(1.38)	137.01(1.58)	126.23(3.71)
		Feed-to-lean, p reducing	156.98(2.01)	156.17(1.76)	143.27(2.08)
		Optimal, p fixed	151.49(1.70)	146.16(2.27)	134.78(2.62)
		Optimal, p reducing	158.12(1.63)	156.99(2.23)	146.68(2.66)
		Optimal, p free	167.15(2.45)	161.63(1.55)	148.92(1.93)
	160	Feed-to-lean, p fixed	221.57(2.65)	204.72(2.07)	184.50(1.46)
		Feed-to-lean, p reducing	235.46(1.43)	218.92(2.42)	187.51(2.36)
		Optimal, p fixed	230.21(3.74)	212.34(2.26)	192.73(1.82)
		Optimal, p reducing	239.65(1.58)	223.18(3.62)	192.04(2.04)
		Optimal, p free	247.75(2.65)	223.75(3.39)	193.79(2.05)
	200	Feed-to-lean, p fixed	288.98(3.26)	257.22(2.99)	210.45(2.46)
		Feed-to-lean, p reducing	300.72(2.15)	266.08(1.84)	208.81(1.83)
		Optimal, p fixed	303.65(3.31)	266.08(2.87)	213.59(2.64)
		Optimal, p reducing	303.59(2.16)	269.80(2.47)	213.74(2.84)
		Optimal, p free	309.86(2.79)	271.02(2.74)	212.24(2.84)
1.0	120	Feed-to-lean, p fixed	13.92(8.28)	-40.81(7.38)	-68.05(4.16)
		Feed-to-lean, p reducing	156.31(1.18)	157.18(2.10)	147.50(2.08)
		Optimal, p fixed	103.86(4.94)	62.81(8.19)	25.30(8.91)
		Optimal, p reducing	157.98(1.26)	159.65(1.89)	151.80(1.81)
		Optimal, p free	166.12(1.66)	164.33(1.47)	153.52(2.25)
	160	Feed-to-lean, p fixed	208.23(2.80)	197.19(4.73)	179.24(5.46)
		Feed-to-lean, p reducing	241.62(1.89)	228.26(2.88)	213.20(2.48)
		Optimal, p fixed	218.47(3.74)	210.16(4.25)	195.56(4.65)
		Optimal, p reducing	244.38(2.15)	232.65(2.96)	217.78(2.91)
		Optimal, p free	253.27(3.24)	236.05(2.79)	219.56(2.17)
	200	Feed-to-lean, p fixed	282.82(3.41)	265.25(1.95)	250.60(2.48)
		Feed-to-lean, p reducing	319.19(2.85)	294.45(3.58)	260.76(2.67)
		Optimal, p fixed	296.98(4.46)	277.79(2.61)	258.26(2.64)
		Optimal, p reducing	322.65(3.11)	302.71(3.88)	265.46(3.14)
		Optimal, p free	330.82(1.71)	302.00(3.60)	266.02(4.25)

Second, the influence of pig type and dietary restraints on optimal carcass weight and backfat thickness is summarized in Table 5.6.

Table 5.6: The mean carcass weight (kg) and backfat thickness (mm) over 10 runs as pig type and dietary restraints vary, for the standard ingredient cost and standard price schedule.

p	Pdmax	Methods	minLP					
			0.6		0.8		1.0	
			CW	BF	CW	BF	CW	BF
0.8	120	Feed-to-lean, p fixed	47.86	10.50	46.29	10.48	44.91	10.54
		Feed-to-lean, p reducing	56.43	7.35	48.84	7.96	47.28	9.11
		Optimal, p fixed	48.62	10.12	47.51	10.32	45.75	10.35
		Optimal, p reducing	56.29	7.18	49.58	7.89	47.47	8.92
		Optimal, p free	50.70	8.06	49.27	8.18	47.50	9.09
	160	Feed-to-lean, p fixed	48.87	8.32	47.52	8.75	49.14	9.99
		Feed-to-lean, p reducing	74.78	9.79	66.86	10.87	49.17	9.44
		Optimal, p fixed	50.61	8.22	48.13	8.49	48.91	9.52
		Optimal, p reducing	75.53	9.56	67.95	10.72	48.48	9.11
		Optimal, p free	75.53	10.53	67.07	10.78	49.12	9.31
	200	Feed-to-lean, p fixed	69.03	10.94	63.81	10.94	55.91	10.66
		Feed-to-lean, p reducing	76.15	9.95	68.29	11.04	56.52	10.77
		Optimal, p fixed	72.43	10.82	66.36	10.88	55.88	10.44
		Optimal, p reducing	76.08	9.63	69.15	10.92	57.16	10.66
		Optimal, p free	76.15	10.46	68.61	10.87	56.13	10.48
1.0	120	Feed-to-lean, p fixed	43.86	11.89	43.97	12.39	44.11	12.82
		Feed-to-lean, p reducing	56.42	7.38	49.06	7.99	46.96	9.07
		Optimal, p fixed	44.00	11.13	43.73	11.20	43.73	11.42
		Optimal, p reducing	56.56	7.20	49.78	7.88	47.36	8.93
		Optimal, p free	49.68	8.04	48.67	8.18	47.40	9.17
	160	Feed-to-lean, p fixed	47.01	10.57	45.54	10.55	44.22	10.62
		Feed-to-lean, p reducing	76.09	9.98	49.37	8.12	47.27	9.14
		Optimal, p fixed	48.13	10.41	46.67	10.36	45.25	10.42
		Optimal, p reducing	76.20	9.72	51.58	8.22	47.93	9.07
		Optimal, p free	75.52	10.45	49.63	8.24	47.87	9.20
	200	Feed-to-lean, p fixed	46.98	8.67	49.20	9.92	48.10	10.32
		Feed-to-lean, p reducing	76.24	10.04	66.36	10.84	49.65	9.58
		Optimal, p fixed	48.07	8.42	47.25	8.89	48.75	10.08
		Optimal, p reducing	76.07	9.69	68.71	10.85	49.20	9.29
		Optimal, p free	76.12	10.52	67.18	10.76	49.26	9.39

For p fixed, the optimal feeding schedule (Site 3) gives generally higher carcass weight and lower backfat thickness than feed-to-lean (Site 1). Similarly when p is reducing, the optimal feeding schedule (Site 6) gives generally higher carcass weight and lower backfat thickness than the feed-to-lean feeding schedule (Site 5). The optimal feeding schedule when p and r are free (Site 9) gives carcass weight and backfat thickness values lying between those when p is fixed and p is reducing, across all pig types.

Finally, the influence of pig type and dietary constraints on slaughter date are shown in Table 5.7.

The optimal feeding schedule when p is fixed (Site 3), gives similar slaughter dates to those for feed-to-lean (Site 1) across all pig types. Sites 5 and 6 also tend to produce similar slaughter dates but generally larger than those for Sites 1 and 3. The slaughter date for Site 9 is between that for Sites 1 and 3 and Sites 5 and 6, across all pig types.

5.4.5 The influence of price schedule on GMPPY, carcass weight, backfat thickness and slaughter date.

In this section we present three tables of GMPPY, carcass weight and backfat thickness and slaughter date showing in detail to compare between the influence of standard price schedule (Table 5.1) and heavy price schedule (Table 5.3). These results are an average over 10 runs.

For feed-to-lean feeding schedule (Site 1 and 5), GMPPY of heavy price schedule is less than that for the standard price schedule across all pig types. For the optimal feeding schedule (Site 3, 6 and 9), GMPPY for heavy price schedule is greater than that for the standard price schedule, except when $\text{minLP} = 0.8$ and $\text{Pdmax} = 120$ g/day for both p and all pig types when $\text{minLP} = 1.0$ that GMPPY for heavy price schedule is less than that for the standard price schedule.

Second, the influence of pig type and dietary restraints on optimal carcass weight and backfat thickness, for the standard ingredient cost and heavy price schedule, is summarized in Table 5.9.

Most of the carcass weight CW and backfat thickness BF of heavy price schedule are similar or greater than those values in standard price schedule, except all the optimal feeding schedule (Site 3, 6 and 9) when $\text{minLP} = 0.6$ and $\text{Pdmax} = 160$ and 200 g/day for both p (0.8 and 1.0) that CW and BF of heavy price schedule are less than standard price schedule. Notice that in this cases CW and BF for heavy price schedule are keeping at the maximum price return of 330 cent/kg at CW 65 to 70kg and BF 6 to 9 mm.

Table 5.7: The mean and standard deviation of slaughter day over 10 runs as pig type and dietary restraints vary, for the standard ingredient cost and standard price schedule.

p	Pdmax	Methods	minLP		
			0.6	0.8	1.0
0.8	120	Feed-to-lean, p fixed	65(1)	63(1)	62(1)
		Feed-to-lean, p reducing	97(1)	77(1)	72(2)
		Optimal, p fixed	65(3)	64(1)	62(1)
		Optimal, p reducing	96(1)	78(2)	72(2)
		Optimal, p free	76(5)	74(1)	70(1)
	160	Feed-to-lean, p fixed	58(1)	58(1)	62(1)
		Feed-to-lean, p reducing	105(0)	90(2)	63(3)
		Optimal, p fixed	59(1)	58(1)	60(2)
		Optimal, p reducing	105(0)	91(1)	61(1)
		Optimal, p free	100(1)	88(2)	62(1)
	200	Feed-to-lean, p fixed	77(1)	73(1)	68(2)
		Feed-to-lean, p reducing	90(0)	80(1)	69(1)
		Optimal, p fixed	79(1)	75(1)	67(4)
		Optimal, p reducing	89(1)	81(1)	68(2)
		Optimal, p free	86(1)	80(1)	68(2)
1.0	120	Feed-to-lean, p fixed	51(1)	51(1)	51(0)
		Feed-to-lean, p reducing	98(1)	77(1)	69(1)
		Optimal, p fixed	50(1)	50(1)	51(1)
		Optimal, p reducing	97(1)	78(2)	69(1)
		Optimal, p free	73(2)	71(2)	67(1)
	160	Feed-to-lean, p fixed	47(1)	46(1)	44(1)
		Feed-to-lean, p reducing	105(0)	59(2)	54(1)
		Optimal, p fixed	47(1)	46(1)	45(1)
		Optimal, p reducing	104(1)	61(2)	54(1)
		Optimal, p free	97(2)	56(1)	53(1)
	200	Feed-to-lean, p fixed	42(0)	45(2)	45(1)
		Feed-to-lean, p reducing	85(1)	70(1)	49(3)
		Optimal, p fixed	42(1)	42(1)	45(1)
		Optimal, p reducing	84(1)	72(1)	48(3)
		Optimal, p free	80(1)	69(1)	47(2)

Table 5.8: The mean and standard deviation of GMPPY over 10 runs as pig type and dietary restraints vary, for the standard ingredient cost and heavy price schedule.

p	Pdmax	Methods	minLP		
			0.6	0.8	1.0
0.8	120	Feed-to-lean, p fixed	96.18(2.99)	83.51(3.60)	69.58(2.56)
		Feed-to-lean, p reducing	145.37(2.28)	145.78(2.51)	120.65(2.10)
		Optimal, p fixed	111.50(4.31)	97.42(3.19)	82.69(3.78)
		Optimal, p reducing	149.20(3.19)	150.93(1.79)	124.34(2.03)
		Optimal, p free	171.23(4.82)	159.18(3.06)	124.68(3.15)
	160	Feed-to-lean, p fixed	199.59(2.03)	178.49(4.55)	149.54(2.93)
		Feed-to-lean, p reducing	261.89(4.00)	227.56(2.12)	170.18(1.34)
		Optimal, p fixed	222.83(3.41)	193.50(3.96)	163.52(2.08)
		Optimal, p reducing	271.06(3.13)	232.67(4.56)	176.58(4.65)
		Optimal, p free	269.93(2.83)	233.65(4.14)	174.04(3.18)
	200	Feed-to-lean, p fixed	299.08(2.28)	259.72(3.64)	195.34(2.84)
		Feed-to-lean, p reducing	323.84(4.59)	276.40(4.18)	195.79(3.52)
		Optimal, p fixed	316.25(3.20)	275.45(5.21)	203.58(4.78)
		Optimal, p reducing	335.79(2.93)	281.67(3.49)	205.84(4.35)
		Optimal, p free	335.45(3.42)	282.47(2.73)	203.63(4.21)
1.0	120	Feed-to-lean, p fixed	-17.68(7.88)	-57.87(6.49)	-77.30(6.04)
		Feed-to-lean, p reducing	145.99(1.92)	148.03(2.18)	125.36(3.14)
		Optimal, p fixed	39.11(7.27)	10.90(8.26)	-24.93(8.75)
		Optimal, p reducing	149.62(3.12)	152.78(2.40)	129.12(2.61)
		Optimal, p free	172.04(2.18)	161.02(3.11)	131.03(3.24)
	160	Feed-to-lean, p fixed	137.57(2.99)	123.81(3.72)	105.03(3.14)
		Feed-to-lean, p reducing	265.34(1.71)	236.97(3.76)	187.16(3.11)
		Optimal, p fixed	161.33(5.43)	144.12(6.12)	121.31(5.22)
		Optimal, p reducing	276.61(2.57)	245.48(3.47)	193.17(3.74)
		Optimal, p free	277.59(2.50)	242.96(5.09)	195.10(2.59)
	200	Feed-to-lean, p fixed	244.86(2.28)	221.35(4.75)	187.62(3.29)
		Feed-to-lean, p reducing	342.21(4.21)	304.87(4.79)	235.84(4.41)
		Optimal, p fixed	269.65(3.51)	239.21(3.26)	206.43(4.13)
		Optimal, p reducing	358.52(4.44)	313.34(2.90)	244.87(3.18)
		Optimal, p free	359.74(3.68)	314.79(2.65)	244.65(4.84)

Table 5.9: The mean carcass weight (kg) and backfat thickness (mm) over 10 runs as pig type and dietary restraints vary, for the standard ingredient cost and heavy price schedule.

p	Pdmax	Methods	minLP					
			0.6		0.8		1.0	
			CW	BF	CW	BF	CW	BF
0.8	120	Feed-to-lean, p fixed	49.09	10.87	47.05	10.71	45.28	10.66
		Feed-to-lean, p reducing	59.97	7.81	62.02	10.12	57.36	11.04
		Optimal, p fixed	51.46	10.90	49.77	10.89	47.09	10.67
		Optimal, p reducing	60.51	7.63	62.67	9.89	58.50	10.99
		Optimal, p free	63.86	8.94	64.82	10.67	57.95	10.97
	160	Feed-to-lean, p fixed	59.70	11.12	56.43	10.99	52.52	10.82
		Feed-to-lean, p reducing	68.98	8.94	67.75	11.00	57.83	11.08
		Optimal, p fixed	62.58	11.01	58.92	10.91	54.91	10.88
		Optimal, p reducing	69.26	8.64	68.53	10.80	59.27	11.09
		Optimal, p free	69.34	8.75	68.46	10.88	59.45	11.15
	200	Feed-to-lean, p fixed	69.10	10.99	64.81	11.14	58.00	11.07
		Feed-to-lean, p reducing	69.45	9.01	67.99	10.99	58.37	11.13
		Optimal, p fixed	72.08	10.80	66.97	10.96	59.68	11.11
		Optimal, p reducing	69.77	8.69	69.41	10.93	60.37	11.20
		Optimal, p free	70.51	8.83	69.41	10.97	60.42	11.19
1.0	120	Feed-to-lean, p fixed	43.82	11.91	43.95	12.39	44.00	12.76
		Feed-to-lean, p reducing	60.09	7.82	62.37	10.18	57.07	11.05
		Optimal, p fixed	43.97	11.13	43.82	11.35	43.77	11.58
		Optimal, p reducing	60.52	7.65	63.02	9.97	57.37	10.81
		Optimal, p free	63.80	8.76	65.07	10.62	57.98	10.95
	160	Feed-to-lean, p fixed	47.74	10.81	46.14	10.75	44.22	10.63
		Feed-to-lean, p reducing	49.83	10.76	48.54	10.84	45.87	10.60
		Optimal, p fixed	48.13	10.41	46.67	10.36	45.25	10.42
		Optimal, p reducing	69.46	8.70	68.47	10.85	58.65	11.03
		Optimal, p free	68.96	8.70	68.01	10.90	59.09	11.12
	200	Feed-to-lean, p fixed	56.06	10.95	52.83	10.91	49.91	10.80
		Feed-to-lean, p reducing	68.59	9.02	67.23	10.99	56.93	11.01
		Optimal, p fixed	58.18	11.02	55.09	10.89	52.23	10.91
		Optimal, p reducing	69.73	8.75	68.74	10.86	59.28	11.15
		Optimal, p free	69.91	8.79	68.70	10.95	59.20	11.10

Finally, the influence of pig type and dietary constraints on slaughter date for the heavy price schedule are shown in Table 5.10.

Table 5.10: The mean and standard deviation of slaughter day over 10 runs as pig type and dietary restraints vary, for the standard ingredient cost and heavy price schedule.

p	Pdmax	Methods	minLP		
			0.6	0.8	1.0
0.8	120	Feed-to-lean, p fixed	67(1)	64(1)	62(1)
		Feed-to-lean, p reducing	105(0)	105(0)	92(2)
		Optimal, p fixed	69(1)	67(1)	64(1)
		Optimal, p reducing	105(0)	105(0)	94(2)
		Optimal, p free	105(0)	105(0)	91(2)
	160	Feed-to-lean, p fixed	73(1)	70(1)	66(1)
		Feed-to-lean, p reducing	96(1)	91(1)	76(1)
		Optimal, p fixed	75(1)	72(1)	69(1)
		Optimal, p reducing	95(1)	92(1)	77(2)
		Optimal, p free	94(1)	90(1)	78(2)
	200	Feed-to-lean, p fixed	77(1)	75(1)	70(1)
		Feed-to-lean, p reducing	81(1)	80(1)	71(1)
		Optimal, p fixed	79(1)	76(1)	72(1)
		Optimal, p reducing	81(1)	81(1)	72(1)
		Optimal, p free	82(1)	81(1)	73(1)
1.0	120	Feed-to-lean, p fixed	51(1)	51(1)	51(1)
		Feed-to-lean, p reducing	105(0)	105(0)	89(2)
		Optimal, p fixed	50(1)	50(1)	50(1)
		Optimal, p reducing	105(0)	105(0)	89(1)
		Optimal, p free	105(0)	105(1)	88(3)
	160	Feed-to-lean, p fixed	48(1)	46(1)	44(1)
		Feed-to-lean, p reducing	93(1)	86(1)	68(2)
		Optimal, p fixed	49(1)	48(1)	45(1)
		Optimal, p reducing	93(1)	87(2)	70(1)
		Optimal, p free	91(1)	85(1)	70(2)
	200	Feed-to-lean, p fixed	51(1)	49(1)	47(1)
		Feed-to-lean, p reducing	75(1)	71(1)	58(1)
		Optimal, p fixed	52(1)	50(1)	48(1)
		Optimal, p reducing	76(1)	72(1)	60(1)
		Optimal, p free	75(1)	71(1)	59(1)

Both the optimal and feed-to-lean feeding schedules for the heavy price schedule, give similar or longer feeding periods to those for standard price schedule across all pig types. Only the slaughter dates for the heavy price schedule for optimal feeding schedules for Sites 3, 6 and 9 for minLP = 0.6, Pdmax = 160 and 200 g/day are shorter than those for the standard price schedule, across all p .

5.5 Discussion

Using empirically determined rules for pig growth and standard mathematical tools this chapter modelled the process of pig production, from weaner to marketplace. This in turn allowed us to study the effects of changes in key inputs (pig type, costs and prices, and dietary restraints) on optimal feeding schedules and GMPPY. As with any modelling, the accuracy of results is only as good as firstly, the accuracy of the model and secondly, our ability to fit an optimal model; here these correspond to the accuracy of the pig growth model and the ability of the GA to maximise GMPPY. The findings in places that are familiar confirm qualitative industry experience, for example, that as P_{dmax} increases, minLP remains the active constraint for longer; the findings here, however, are quantitative. The method, in addition, allows us to probe less familiar situations, for example, revealing that an upward level change in the price schedule can reduce the optimal slaughter date. Within the framework established in this chapter, future improvements in the growth model for example de Lange, Morel, and Birkett (2008) and optimisation techniques, for example in Sirisatien et al. (2009) will improve the accuracy of results.

A world is now open to explore the influence on optimal feeding schedule and GMPPY of many different growth scenarios. Here, for example, we have only considered the effect of level changes in ingredient costs and price schedule. Any pattern change could also be considered. The objective function can be enhanced to consider other costs, such as that of nutrient excretion; this has been worked into the current version of Bacon Max. Beyond this, trials of the optimal diets in pig herds remain to be conducted, to compare observed and predicted outcomes and so lead to improved modelling.

Chapter 6

Formulation of diet for southern Thailand

6.1 Introduction

In previous chapters we have dealt with New Zealand data with its diet type variation (grower, finisher), changing diets every week. In this chapter a particular problem for a pig producer, Mr.Cheewa-isarakul, in southern Thailand, as shown in Figure 6.1, is considered. The aim of this chapter is to use the ideas already presented in this thesis to find an improved feeding schedule for this context. This presents the prior challenge of estimating needed model parameters. The data has been collected by Yuthana Siriwithananukul, a lecturer and research fellow at the Faculty of Natural Resources, Prince of Songkla University, Hat Yai campus.

The pig industry in Thailand is rapidly advancing in the last few decades. Pig diets are now carefully considered by farmers aiming to generate the best quality pork with maximum profitability even in small village farms (backyard holder) as we shall see in this chapter. The number of pigs in the farm are relatively small, ranging from five to ten pigs per batch. Common practice involves two diets, grower and finisher. We will contrast the results of Bacon Max to actual farm based results.



Figure 6.1: Farm of Mr. Cheewa-isarakul (first from the left in Figure a) and centre in Figure b)), in southern Thailand. The data was collected by Yuthana Siriwathananukul, a lecturer and researcher at the Faculty of Natural Resources, Prince of Songkla University, Hat Yai campus (third from the left in a) and first from the left in b)). Observing is Graham Wood (second from the left in a)) in 2003.

6.2 Aim

To explore the practicality of the ideas presented in this thesis and their usefulness in various situations towards improving the feeding schedule.

In the next section we will explain the study method and followed it with the results. Finally, in the discussion and conclusion, strategies for improving profitability by changing the feeding schedule are presented.

6.3 Methods

To begin with we examine the farm data, and so calculate the on-farm proportion p of the ad libitum daily digestible energy intake and estimate the critical growth parameters used in a simple pig growth model (de Lange, 1995): minLP, the minimum allowable lipid to protein ratio and Pdmax, the maximum protein deposition rate. We then use Bacon Max software to determine the optimal feeding schedule that maximises the gross margin per pig place per year for this particular case.

6.3.1 Estimation of the on-farm proportion p of the ad libitum daily digestible energy intake

Ten pigs in the farm were fed using two diets (grower and finisher) in which herbal plants were used as medicine and feed additives used to produce high quality pork. Antibiotics were not used. The first diet was run for 56 days and the second run for 63 days, at which time the pigs were slaughtered. The calculated values of the chemical composition of the two diets are shown in Table 6.1. Pigs of Mr.Cheewa-isarakul in southern Thailand grew from 20kg to 112kg in 119 days as shown in Table 6.2.

Table 6.1: The calculated values of the chemical composition of the two diets from Mr. Cheewa-isarakul's farm, in southern Thailand, collected by Yuthana Siriwithananukul, Prince of Songkla University. Note, these values were not measured chemically.

Chemical composition of the diets	Diet 1	Diet 2
Lysine to digestible energy ratio r (g/MJ)	0.74	0.59
Digestible energy density d (MJ/kg)	14.57	14.65
Crude protein CP (g/kg)	181.3	162.1
Digestible protein DP (g/kg) (estimated at 85%CP)	154.1	137.8
Balanced protein BP (g/kg) (estimated at 85%DP for diet 1 and 76%DP for diet 2)	130.6	104.2
Price (Baht/kg)	8.41	7.92

The p of this producer is calculated using

$$p = \frac{\text{Average southern Thailand DEvi}}{\text{Average theoretical DEvi}}$$

where DEvi is the voluntary daily digestible energy intake (MJ/day). The derivation of the numerator and denominator is now discussed.

i) Average theoretical DEvi

DEvi can be calculated from de Lange (1995)

$$\text{DEvi} = f(\text{LW}) = 55.07 \times \left(1 - e^{(-0.0176\text{LW})}\right)$$

where LW = body liveweight (kg), $-\infty < \text{LW} < \infty$

Table 6.2: Measurement data, LW, daily intake and digestible energy from Mr. Cheewa-isarakul's farm. The first diet was run for 56 days (week 1 – 8) and the second run for 63 days (week 9 – 17), at which time the pigs were slaughtered.

Week	Measurement data	LW (kg)	Daily intake (kg/pig/day)	Digestible energy in diet (MJ/kg)	Digestible energy intake (MJ/day)
Starting	2/6/2002	20.00			
1	02/6/02-08/6/02	24.00	1.40	14.57	20.40
2	09/6/02-15/6/02	27.65	1.58	14.57	23.02
3	16/6/02-22/6/02	30.93	1.70	14.57	24.77
4	23/6/02-29/6/02	35.40	1.81	14.57	26.37
5	30/6/02-06/7/02	40.58	1.85	14.57	26.95
6	07/7/02-13/7/02	46.22	1.91	14.57	27.83
7	14/7/02-20/7/02	52.01	2.01	14.57	29.28
8	21/7/02-27/7/02	57.93	2.11	14.57	30.74
Average					26.17
9	28/7/02-03/8/02	63.81	2.19	14.65	32.07
10	04/8/02-10/8/02	69.60	2.25	14.65	32.95
11	11/8/02-17/8/02	75.50	2.33	14.65	34.12
12	18/8/02-24/8/02	81.51	2.36	14.65	34.56
13	25/8/02-31/8/02	87.58	2.40	14.65	35.15
14	01/9/02-07/9/02	93.68	2.45	14.65	35.88
15	08/9/02-14/9/02	99.85	2.45	14.65	35.88
16	15/9/02-21/9/02	105.97	2.45	14.65	35.88
17	22/9/02-28/9/02	112.08	2.45	14.65	35.88
Average					34.71

The total voluntary daily digestible energy intake of pigs, between LW 20kg and 112kg, can be calculated by integration as the area under this curve (Figure 6.2).

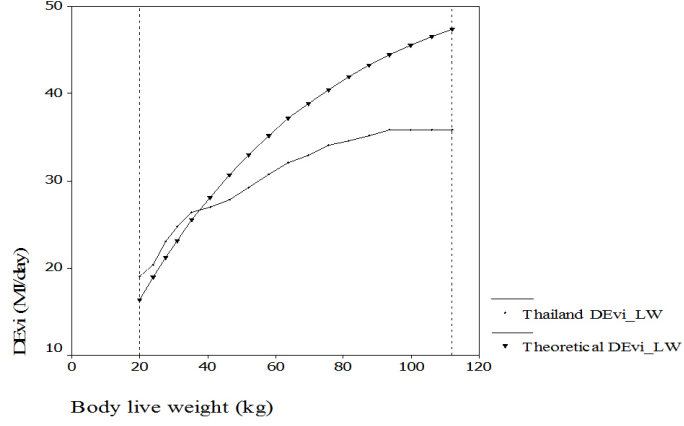


Figure 6.2: Theoretical DEvi (MJ/day) and Thailand DEvi (MJ/day) against LW (kg) for LW from 20kg to 112kg.

a) The first period, LW 20kg to 58kg

$$\text{Total theoretical DEvi} = \int_{20}^{58} f(\text{LW})d\text{LW} = 1019.50 \text{ MJ}$$

Therefore, total theoretical DEvi in this first period is 1019.50 MJ.

The average weight increase of pigs is assumed (in Figure 6.2) to be 1kg/day. According to southern Thailand data the LW of pigs was started from 20kg to 58kg. On average, Thailand pigs grew for 38 days. Then,

$$\text{Average theoretical DEvi} = \frac{1019.50}{38} = 26.83 \text{ MJ/day}$$

b) The second period, LW 58kg to 112kg

$$\text{Total theoretical DEvi} = \int_{58}^{112} f(\text{LW})d\text{LW} = 2282.22 \text{ MJ}$$

Therefore, total theoretical DEvi in this second period is 2282.22 MJ.

The LW of pigs was started from 58kg to 112kg. On average, Thailand pigs

grew for 54 days. Then,

$$\text{Average theoretical DEvi} = \frac{2282.22}{54} = 42.26 \text{ MJ/day}$$

ii) Average southern Thailand DEvi

Table 6.2 shows the data collected in southern Thailand. The average southern Thailand DEvi for the first period of 56 days is 26.17 MJ/day and for the second period of 63 days is 34.71 MJ/day.

Then, the on-farm proportion p of the ad libitum daily digestible energy intake for Thailand data can be calculated as:

$$\begin{aligned} \text{First period; } p_1 &= \frac{26.17 \text{ MJ/day}}{26.83 \text{ MJ/day}} = 0.98 \\ \text{Second period; } p_2 &= \frac{34.71 \text{ MJ/day}}{42.26 \text{ MJ/day}} = 0.82 \end{aligned}$$

In conclusion, the proportion of the ad libitum daily digestible energy intake of Thailand data for the first and second diets are estimated as $p_1 = 0.98$ and $p_2 = 0.82$, respectively. Note that we are making the crude assumption that these proportions remain constant throughout these two periods.

6.3.2 Estimation of on-farm maximum protein deposition rate Pdmax

The special diet used for Pdmax measurement is presented in Table 6.3. The diet was fed to 10 pigs ad libitum and then initial weight, LW and backfat thickness (P2) recorded, measured at 35 days and 82 days during the growing period. Backfat thickness was measured on the left side at the last rib, 6.5 cm from the mid back with a metal ruler (mm). Data is presented in Table 6.4.

Using the mean over 10 runs of data in Table 6.4, a number of approaches were used to estimate Pdmax, described in the next section.

Table 6.3: Feed ingredients for Pdmax measurement.

Feed ingredient	Ingredient/100kg
Broken rice	55.8
Rice bran	10
Palm kernel meal	10
Fish meal (60%CP)	8
Soy bean meal (44%CP)	14.35
Oyster shell	0.55
Di-Calcium Phosphate	0.3
Salt (Nacl)	0.3
Lysine	0.2
Premix	0.5
Total	100.0

Table 6.4: Initial weight, LW and backfat thickness (P2) of 10 pigs which were fed ad libitum using the diet in Table 6.3 measured at 35 days and 82 days into the growing period. The growing period for pig LW from 20kg to 50kg was 35 days and LW between 50kg and 85kg was 47 days.

Pig No.	Initial weight (kg)	LW at 35 days (kg)	LW at 82 days (kg)	P2 at 35 days (mm)	P2 at 82 days (mm)
1	22.5	48.3	86.5	3.71	11.12
2	25.4	51.1	87.7	4.22	10.42
3	26.1	51.6	88.1	4.35	11.92
4	23.3	49.5	85.3	3.81	10.12
5	24.1	48.7	86.4	4.12	10.92
6	22.8	50.1	88.5	5.88	15.12
7	25.5	50.8	89.1	5.45	14.42
8	24.0	49.7	85.0	3.82	9.92
9	23.2	47.6	85.1	4.25	11.92
10	26.4	52.1	89.0	5.90	15.12
Mean	24.33	49.95	87.07	4.55	12.10

i) Estimation of Pdmax by conventional formulas using pig LW and backfat thickness

a) Kerr (1990) as in Siriwananukul (2000)

$$\begin{aligned}\text{Empty whole body weight at 50kg} &= \frac{\text{LW at 50kg}}{1.05} = \frac{49.95}{1.05} \\ &= 47.57\text{kg}\end{aligned}$$

$$\begin{aligned}\text{Empty whole body weight at 85kg} &= \frac{\text{LW at 85kg}}{1.05} = \frac{87.07}{1.05} \\ &= 82.92\text{kg}\end{aligned}$$

$$\text{Total body lipid at 85kg} = \frac{P2}{0.9} = \frac{12.10}{0.9} = 13.44$$

$$\begin{aligned}\text{Fat free body mass at 85kg} &= \text{Empty whole body weight at 85kg} \\ &\quad - \text{Total body lipid} \\ &= 82.92 - 13.44 = 69.48\end{aligned}$$

$$\begin{aligned}\text{Whole body protein content} &= 16.4\% \text{ of Empty whole body weight at 50kg} \\ &= \frac{16.4}{100} \times 47.57 = 7.80\text{kg}\end{aligned}$$

$$\begin{aligned}\text{Protein mass} &= 0.1687 \times (\text{fat free body mass})^{1.063} \\ &= 0.1687 \times (69.48)^{1.063} = 15.30\text{kg}\end{aligned}$$

$$\begin{aligned}\text{Pdmax} &= \frac{\text{Protein mass} - \text{Whole body protein content}}{\text{day taken}} \times 1000 \\ &= \frac{(15.3 - 7.8)}{47} \times 1000 \\ &= 159 \text{ g/day}\end{aligned}$$

By using this method, Pdmax is estimated 159 g/day.

b) Morel (1993)

Total body protein (TBP) while backfat thickness (P2) is measured with ultrasound

$$\begin{aligned}\text{TBP at 50kg} &= 0.59 + (0.190 \times \text{LW}) - (0.227 \times \text{US}) \\ &= 0.59 + (0.190 \times 49.95) - (0.227 \times 4.551) = 9.05\end{aligned}$$

$$\text{TBP at 85kg} = 0.59 + (0.190 \times 87.07) - (0.227 \times 12.10) = 14.39$$

$$\begin{aligned}\text{Pdmax} &= \frac{(\text{TBP at 85kg} - \text{TBP at 50kg})}{\text{number of days on trial}} \\ &= \frac{(14.39 - 9.05)}{47} \times 1000 \\ &= 113.60 \text{ g/day}\end{aligned}$$

By using this method, Pdmax is estimated 113.6 g/day.

c) Wood

$$\begin{aligned}\text{Empty whole body weight at 50kg} &= \frac{\text{LW at 50kg}}{1.05} \\ &= \frac{49.95}{1.05} = 47.57\text{kg}\end{aligned}$$

$$\begin{aligned}\text{Empty whole body weight at 85kg} &= \frac{\text{LW at 85kg}}{1.05} \\ &= \frac{87.07}{1.05} = 82.92\text{kg}\end{aligned}$$

Whole body protein content at 35 days

$$\begin{aligned}&= 16.4\% \text{ of Empty whole body weight at 35 days} \\ &= \frac{16.4}{100} \times 47.57 = 7.80\text{kg}\end{aligned}$$

Whole body protein content at 82 days

$$\begin{aligned}&= 16.4\% \text{ of Empty whole body weight at 82 days} \\ &= \frac{16.4}{100} \times 82.92 = 13.60\text{kg}\end{aligned}$$

$$\begin{aligned}\text{Pdmax} &= \frac{(\text{Whole body protein content at 82 days} - \text{at 35 days})}{\text{day taken}} \times 1000 \\ &= \frac{(13.6 - 7.8)}{47} \times 1000 = 123.4 \text{ g/day}\end{aligned}$$

By using this method, Pdmax is estimated 123.4 g/day.

ii) Estimation of Pdmax by growth curve fitting

In a simple pig growth model (de Lange, 1995), the actual digestible energy intake, and the values of minLP and Pdmax can be used to produce a growth curve. Parameters minLP and Pdmax were adjusted to provide a good fit (lowest root mean square error) between predicted growth curve and the true growth curve. A good fit occurs when minLP = 0.8 and Pdmax = 105 g/day.

A variety of other approaches to estimation of minLP and Pdmax were tried; all produce only “operational” values for these parameters. Determination of true values were hindered by the fact that the diet used on farm appeared to be limiting. This would cause Pdmax, for example, to be estimated as a low figure. For this reason we sought an expert opinion to estimate the on farm Pdmax.

iii) Estimate Pdmax by expert opinion

Guided by the opinion of Dr. Patrick Morel, an animal science specialist, knowledge about performance of the three hybrid cross genotypes used on farm, and indications from our estimation results, we were led to the decision to use a minLP of 0.75 and a Pdmax of 130 g/day. Initial body protein weight in the weaner pig P_0 is set at 3.123kg for 20kg body weight. We use these values of minLP, Pdmax and P_0 to find the optimal feeding schedule in the next section.

6.3.3 Finding the optimal feeding schedule using Bacon Max

We now describe the domain and input to find the optimal solution for this particular problem.

The feeding schedule F for two diets has the form $(p_1, r_1, d_1; p_2, r_2, d_2)$. Typical parameter ranges used are $[0.7, 1]$ for p , $[0.4, 1.0]$ g/MJ for r , $[12, 16]$ MJ/kg for d , and the maximum growth period using two diets is 140 days.

We fixed the weaner cost at 1,000 baht using the market price at that time (March 2003). Feed ingredients and their cost are presented in Table 6.5.

Table 6.5: Thai feed ingredient costs in March 2003.

Feed ingredient	Price (Baht/kg)
Broken rice	6.5
Rice bran	4.91
Palm kernel meal	3.21
Fish meal (60%CP)	18
Soy bean meal (44%CP)	11
Oyster shell	2
Di-Calcium Phosphate	16
Salt (Nacl)	4
Lysine	100
Premix	120

Gross return in Thailand depends on LW (kg) at slaughter (instead of carcass weight as for New Zealand) and backfat thickness (mm), as shown in the typical price schedule (Table 6.6).

Table 6.6: A Thai price schedule, giving prices in baht/kg of pig LW at slaughter date for the farm of Mr.Cheewa-isarakul in March 2003.

Backfat thickness (mm)	LW (kg)			
	< 90	90-100	100-110	> 110
< 10	31	34	35	32
10-13	30	32	33	31
13-16	29	30	31	29
16-18	28	28	28	27

Palatability constraints on feed ingredients are also used in the calculation of the least cost diets. Testing was conducted to determine the best GA parameters, the number of genomes to be used in the parent population g and the number of iterations I that the solution should remain unchanged before stopping. This yielded $g = 20$ and $I = 20$, respectively.

6.4 Results

Optimisation using Bacon Max and the parameters and data so far described that gross margin per pig place per year can be improved by changing the feeding schedule. Table 6.7 gives results with LW at slaughter constrained to 105-120kg and Table 6.8 gives results with no slaughter LW constraint. Existing diets are shown in the centre of the table and the improved diets using Bacon Max are shown for lots of five and ten pigs in the objective function on the right.

Table 6.7: Feeding schedule and pig performance on Mr. Cheewa-isarakul's farm compared with the mean optimal feeding schedule over 10 runs (and standard deviation) using Bacon Max with a LW constraint at slaughter of 105 – 120kg.

Number of pigs in the objective function	Thailand diet		Optimal diet using Bacon Max with LW constraint (105 – 120kg)			
			5		10	
Diets	Diet 1	Diet 2	Diet 1	Diet 2	Diet 1	Diet 2
p	0.98	0.82	0.72 (0.02)	0.70 (0.00)	0.71 (0.01)	0.70 (0.00)
r (g/MJ)	0.74	0.59	0.67 (0.01)	0.54 (0.01)	0.67 (0.00)	0.54 (0.00)
d (MJ/kg)	14.57	14.65	14.00 (0.02)	13.95 (0.00)	14.00 (0.00)	13.95 (0.00)
t (day)	56	63	64(3)	66(4)	65(3)	67(2)
LW at slaughter (kg)		112.08	105.55(0.50)		105.28(0.45)	
Carcass weight (kg)		85.47	81.51(0.45)		81.28(0.41)	
slaughter day		119	123(6)		126(3)	
Average daily gain (kg/day)		0.77	0.70(0.03)		0.68(0.02)	
Feed intake (kg/pig/day)		2.10	1.72(0.03)		1.69(0.02)	
Total feed intake (kg)		250.00	213.36(7.20)		215.85(4.52)	
Feed conversion ratio		2.72	2.47(0.08)		2.51(0.05)	
Backfat thickness (mm)		12.10	13.25(0.64)		13.28(0.44)	
Total feed cost (Baht)		2028.50	1654.14(56.35)		1672.31(33.63)	
GMPPY (Baht/pig place/year)		1367.92	1985.53(487.02)		1791.19(274.43)	
No. of cycles per year		2.89	2.81		2.74	

Table 6.8: Feeding schedule and pig performance on Mr. Cheewa-isarakul's farm compared with the mean optimal feeding schedule over 10 runs (and standard deviation) using Bacon Max with no LW constraint at slaughter.

Number of pigs in the objective function	Thailand diet		Optimal diet using Bacon Max with no LW constraint			
			5		10	
Diets	Diet 1	Diet 2	Diet 1	Diet 2	Diet 1	Diet 2
p	0.98	0.82	0.73 (0.02)	0.70 (0.00)	0.73 (0.02)	0.70 (0.00)
r (g/MJ)	0.74	0.59	0.67 (0.00)	0.55 (0.01)	0.67 (0.00)	0.55 (0.01)
d (MJ/kg)	14.57	14.65	14.00 (0.00)	13.95 (0.00)	14.00 (0.00)	13.95 (0.00)
t (day)	56	63	56(6)	64(5)	56(5)	64(4)
LW at slaughter (kg)		112.08	97.32(2.01)		97.83(2.85)	
Carcass weight (kg)		85.47	74.32(1.74)		74.77(2.45)	
slaughter day		119	113(6)		115(4)	
Average daily gain (kg/day)		0.77	0.69(0.03)		0.68(0.02)	
Feed intake (kg/pig/day)		2.10	1.72(0.03)		1.69(0.02)	
Total feed intake (kg)		250.00	190.42(7.58)		193.35(7.59)	
Feed conversion ratio		2.72	2.44(0.06)		2.46(0.04)	
Backfat thickness (mm)		12.10	12.27(0.35)		12.47(0.44)	
Total feed cost (Baht)		2028.50	1476.62(57.46)		1498.64(59.01)	
GMPPY (Baht/pig place/year)		1367.92	2114.13(229.70)		1976.79(220.91)	
No. of cycles per year		2.89	3.04		2.99	

The results from Table 6.7 and Table 6.8 show that GMPPY using the optimal diets from Bacon Max are greater than for the on-farm diet. GMPPY also increases, more so when there is no slaughter LW restriction than when there is a LW restriction (105-120kg). GMPPY using the optimal diet from Bacon Max for fewer pigs in the objective function, in this case is five pigs, is greater than for 10 pigs.

The results for feeding schedule, p , r and d using the optimal diets and on-farm diets are discussed as follows. Feed intake levels (p) are reduced in the optimal diets compared with on-farm diets. There are similar p values in the optimal diets using different number of pigs in the objective function. The p values for diet 1 of the optimal feeding schedule are slightly higher for no LW restriction compared with having a LW restriction. Similarly lysine to digestible energy levels (r) are reduced in the optimal

diets compared with the on-farm diet. There are no differences between r values of the optimal diets when using different number of pigs in the objective function. Diet 2 for both number of pigs in the objective function have slightly higher r values when no LW restrictions were used. Digestible energy levels (d) are slightly reduced in the optimal diets. There are no differences between d values in the optimal diet using a different number of pigs in the objective function and a different LW restriction.

The optimal diet when there is a LW restriction suggests a longer growing period than the on-farm diet. On the other hand, the optimal diet when there is no LW restriction suggests a shorter growing period than for the on-farm diet. Slaughter day for the optimal diet when there is a LW restriction is longer than when there is no LW restriction. The optimal diet for 10 pigs in the objective function suggests a longer growing period than for five pigs in the objective function for both LW restrictions. Also, the LW at slaughter for the optimal diet for both LW restrictions is lower than on-farm LW at slaughter. The LW at slaughter is greater when there is a LW restriction than when there is no LW restriction. The LW at slaughter for different number of pigs in the objective function is similar for both LW restrictions.

Feed intake and total feed intake using the optimal diet is lower than on-farm diet. The feed intake and total feed intake using the optimal diet when there is a LW restriction is higher than when there is no LW restriction. Feed intake per day using the optimal diet decreases when the number of pigs in the objective function increases. On the contrary, total feed intake increase when the number of pigs in the objective function increases.

Table 6.9 compares the composition of Mr. Cheewa-isarakul's diets on and the least cost diets from the optimal feeding schedule using Bacon Max. Less broken rice is used, and more rice bran and palm kernel meal. These last two ingredients are used at the maximum imposed palatability level of 15 g/kg; fish meal is reduced, being used at the minimum constraint level of 5 g/kg, since the price per kg (18 Baht/kg) of fish meal is higher than for soy bean meal (11 Baht/kg). Soy bean meal use is increased to the maximum constraint level of 17 g/kg. Oyster shell is also increased due to the cheap price per kg. The optimal diets using Bacon Max suggests a cheaper diet compared with Mr.Cheewa-isarakul's diets.

Table 6.9: Diet composition of Mr. Cheewa-isarakul's diets and the average diet composition of the mean optimal diets over 10 runs using Bacon Max.

Feed ingredient	Price (Baht/kg)	Thailand (g/kg) Diet 1	diet Diet 2	Optimal diet using Bacon Max (g/kg)	
				Diet 1	Diet 2
Broken rice	6.5	55.07	56.40	43.50	43.40
Rice bran	4.91	10.00	12.00	15.00	15.00
Palm kernel meal	3.21	10.00	13.00	15.00	15.00
Fish meal (60%CP)	18	8.73	9.00	5.00	5.00
Soy bean meal (44%CP)	11	14.35	8.05	17.00	17.00
Oyster shell	2	0.55	0.45	2.70	2.60
Di-Calcium Phosphate	16	0.30	0.10	0.80	1.10
Salt (Nacl)	4	0.30	0.40	0.30	0.30
Lysine	100	0.20	0.10	0.20	0.00
Premix	120	0.50	0.50	0.50	0.50
Price (Baht/kg)		8.41	7.92	7.86	7.67

Table 6.10: Essential amino acids in Mr. Cheewa-isarakul's diets and the mean essential amino acids over 10 runs from the optimal diets using Bacon Max.

Amino acids	Thailand (g/kg) Diet 1	diet Diet2	Optimal diet using Bacon Max (g/kg)	
			Diet1	Diet 2
Lysine	10.75	8.58	9.38	7.53
Methionine (estimated at 52% of M+C)	3.19	2.96	2.58	2.58
Methionine + Cysteine	6.13	5.70	4.94	4.94
Threonine	7.08	6.35	5.69	5.70
Tryptophan	2.43	2.08	2.04	2.04
Isoleucine	7.46	6.54	6.13	6.14
Digestible Protein	154.10	137.75	151.32	149.17
Crude Protein	181.34	162.06	177.61	175.47

The essential amino acids in diets are calculated and shown in Table 6.10. All essential amino acids in the optimal diets are used less than on-farm values. The amino acid to lysine ratio from the optimal diet is lower than on-farm diet. This results in reducing the feed cost (Baht/kg) as shown in Table 6.9 and lowers the total feed cost (Baht) as shown in Tables 6.7 and 6.8.

The reason for improved profitability is due to the lower feed intake and use of a cheaper diet. When there is no LW constraint, the increased number of cycles per year achieved through a shorter pig growth period also helps to improve profitability.

6.5 Discussion

The contribution of this chapter has been to describe a practical approach to the finding of improved feeding schedules using the optimal feeding schedule from Bacon Max. The results show that profitability can be improved by lowering the feed intake and by the use of a cheaper diet. Researchers at Mississippi State University (2010) recommend that diets should be formulated to meet pig requirements with regards to their age and purpose. An oversupply of feed is costly to the pig producer (mentioned earlier) as the feed cost is approximately 55-75% of the total cost. Reformulation of the feeding schedule can reduce feed costs (Willis, 2010). Saskatchewan (2008) shows that the reformulation of the diet could save up to \$2-\$4 per pig sold. The result in this Chapter also shows that the pig grown in a shorter period can improve the profitability.

In addition, this chapter also indicates that the pig genotype parameter, minLP and Pdmax, are measured and calculated using on-farm data and that the farm environment and health condition of pigs are captured through these values.

However, numerical outcomes tabulated should be seen only as indicating directions in which it appears sensible to alter feeding schedules, rather than as exact specifications. A first reason for this is that diets found have depended on estimation of pig type parameters, minLP and Pdmax, and the finding of these has proven difficult here, as existing diets were limiting. A second reason for caution is that only a simple pig growth model was used here improved models are becoming available and could be used in further work. A third reason is current constraint matrix lack of flexibility in the optimal feeding schedule software, as seen in the similarity of the first and second

predicted diets. The constraint matrix used in the current version of the optimisation software cannot vary from diet to diet. Future improvements of the software will remedy this.

Chapter 7

Conclusion

7.1 Overall conclusion

In this thesis, instead of only using linear programming to minimise feed ingredient cost for the pig producers we have used a combination of linear programming, a simple pig growth model and nonlinear optimisation to find the optimal feeding schedule that maximises profitability. “Bacon Max” was originally developed by Alexander, D., Wood, G., Morel, P. and a research team at Massey University. Later, Bacon Max was further developed to include covariance of pig type parameters and stochastic programming, as described in this thesis. Bacon Max has been used as a tool to study and compare results with other methods.

An interesting challenge in this problem had been dealing with the extremely large search space and the difficult nature of the objective function, the “craggy volcano”. Having considered a number of nonlinear optimisation methods a genetic algorithm had proved the most successful approach thus far for finding the optimal feeding schedule (despite its substantial demand on processing time).

In view of this we first developed an alternative “tailored method” that is capable to achieving the same goal within a shorter processor time. This method is adapted specifically to the known nature of this objective function. We have also demonstrated that an improvement in performance can also depend on pig genotype parameters and the constraints.

Second, the variation in feed ingredient cost and price schedule across time were also considered here and handled in the model using stochastic programming. For in prac-

tice, this variation influences profitability.

Third, the effects of three factors: pig type, ingredient cost and price schedule, and dietary restraints on the optimal feeding schedules for grower-finisher pig herds were considered. Results indicate firstly, pig type influences dietary nutrition significantly. Secondly, level changes in ingredient cost have less impact on dietary nutrition compared to level changes in price schedule. Thirdly, pattern changes in price schedule have more impact on dietary nutrition in contrast to level changes. Finally, optimal growth offers greater profit returns than feed-to-lean growth.

Lastly, the optimisation method is applied to a real life situation in southern Thailand. The challenges of estimating the on-farm parameters used in the optimisation method are discussed. The results indicate that pig producer profitability can be improved by adjustment of the feeding schedule using the optimal feeding schedule.

7.2 Discussion and further work

We now very briefly discuss the topics that have been considered in this thesis, followed by considering possibilities for furthering current work.

The diets considered here all arise as least cost diets from a linear program. By entertaining other diets it may be possible to do even better than has been achieved in this thesis.

Improvements in the pig growth model, such as in de Lange, Morel, and Birkett (2008), have increased the accuracy of pig growth estimation and are now available. Improved genetic potential pigs are aiming for lean growth to meet the demand of the market. The body protein deposition (Pd) (see also Birkett and de Lange (2001a,b,c); de Lange, Morel, & Birkett (2003)) of this modern pig is driven by the energy intake for most of their growing period. Pd now is no longer limited by Pdmax or the intake of balanced protein. The model presents the partitioning of retained energy between body protein and lipid deposition in this new condition.

Improved optimisation methodology is still an area that remains wide open to exploration, so moving beyond the genetic algorithm and tailored method that have been

considered in this thesis. We caution that the optimal feeding schedule for a single pig is unrealistic, since in practice many pigs, exhibiting minor variations in genotype and feed intake, are grown on a single feeding schedule. The optimum schedule in such a situation is different from that found for a single pig. Such variation can be incorporated into an objective function for further work, but was not in this thesis, in order to focus on the developments addressed. The performance of the tailored method with multiple pigs in the objective function remains to be investigated. So also the performance of the tailored method when the dimension of the problem (number of diets) is increased remains to be investigated.

The effect of including variation of feed ingredient cost and price schedule on the optimal feeding schedule for more than a single pig also remains to be examined.

The objective function can be enhanced to include other costs for farm management, such as that of labour, maintenance and repairs, electricity, water, medicine, transport and veterinary expenses. Consideration of nutrient excretion also has been worked into the current version of Bacon Max.

The estimation of the on-farm parameters has proven difficult here. Finding accurate parameters is crucial to the result of the optimisation method. Beyond this, trials of the optimal diets in pig herds remain to be conducted, to compare observed and predicted outcomes and so lead to improved modelling.

The work done in this thesis and possible further work is summarised in Table 7.1.

Table 7.1: A summary of work done in this thesis and the possible related further work.

Topic	Thesis	Further work
Linear programming	Used in Bacon Max (Chapter 2)	i) Avoid using linear programming ii) Introduce flexibility in the constraints matrix
Growth model	A simple pig growth model (Chapter 2)	Use more advanced (or accurate) pig growth model
Nonlinear optimisation	Genetic algorithm (Chapter 2) Tailored method (Chapter 3) Program for tailored method in Visual C++	Explore other optimisation methods i) Performance of Tailored method for more than a single pig ii) Performance of Tailored method when the dimension of the problem increases iii) Corporate tailored method in Bacon Max program
Variation in cost and price	Stochastic programming (Chapter 4) An extended Bacon Max program with Stochastic programming	Effect for more than a single pig
Effect of parameters in the model	Effect of pig parameters, costs and dietary restraints (Chapter 5)	Effect of environment and health in using variation maintenance requirement
Objective function	GMPPY with total feed cost and weaner cost (Chapter 2)	Consider nutrient excretion (currently included in the extended work)
Application to real data	Southern Thailand data (Chapter 6)	i) Need accurate estimation for on-farm parameters ii) Trials of the optimal diets in pig herds

Chapter 8

Appendix

8.1 List of symbols and abbreviations used in order of appearance

Symbols and abbreviations Meaning

LP	linear program
x_i	amount of ingredient i (kg) in 1kg of a pig diet
n	number of feed ingredients that are available for the pig diet.
c_i	cost (\$) for 1kg of feed ingredient i
a_{ij}	nutrient (energy, protein, vitamins and minerals) (kg) in 1kg of feed ingredient i appropriate to constraint j
m	number of constraints
b_j	requirement or limitation for constraint j
GE	gross energy
DE	digestible energy
ME	metabolizable energy
NE	net energy
HI	heat increment
minLP	minimum allowable lipid to protein ratio
Pdmax	maximum daily protein deposition (g/day)
p	proportion of the <i>ad libitum</i> digestible energy intake
r	lysine to digestible energy ratio (g/MJ)
d	digestible energy density (MJ/kg)
k	number of diet, $k = 1, 2, \dots, K$

Symbols and abbreviations Meaning

T_k	feeding period (day) using the k th diet
T_{max}	feeding period (day) using diet K which maximises GMPPY
F	feeding schedule containing p, r, d
SD	slaughter date
BF	backfat thickness (mm)
CW	carcass weight (kg)
LW	liveweight (kg)
$g(F)$	$\max_x g(F, x)$
$g(F, x)$	Gross Return(F, x) – Feed Cost(F, x) – Weaner Cost, gross margin when we feed a pig using feeding schedule F for x days
$GR(F, x)$	Gross Return(F, x) (\$)
$FC(F, x)$	Feed Cost(F, x) (\$)
WC	Weaner Cost, price of a pig in the market at 20kg (\$)
$f(F, x)$	$(365/(x + 7))g(F, x)$, gross margin per pig place per year, GMPPY
x	number of days until slaughter
P_0	initial protein in the weaner pig
$f(F)$	$\max_x f(F, x)$, maximum gross margin per pig place per year for feeding schedule F
$\max_F f(F)$	maximum gross margin per pig place per year over all feeding schedules
F^*	optimal feeding schedule
GA	genetic algorithm
g	number of genomes in the parent population in a genetic algorithm
I	number of iterations without change in the objective function
Pd	protein deposition rate
$r(x_i, x_j)$	correlation of x_i and x_j
Σ	covariance matrix
R	correlation matrix
$\text{cov}(x_i, x_j)$	$r(x_i, x_j)\sigma_i\sigma_j$
L	matrix square root (the lower triangular matrix from the Cholesky factorization) of the covariance matrix
σ_{Pdmax}	standard deviation of $Pdmax$

Symbols and Meanings
abbreviations

σ_{alg}	standard deviation in algorithm outputs GMPPY
G_i	GMPPY for pig i (\$)
N	number of pigs in the objective function
T	total GMPPY for every pigs in the objective function (\$)
S	shrink factor, $0.9 \times e^{-0.001 \times \text{iter}}$
iter	the iteration counter
F''	feeding schedule on the enlarged domain $[-0.5, 1.5] \times [-0.5, 1.5]$
V_i	generated random direction
β_{ij}	distance from the current feeding schedule (j diets) to the edge of the domain on direction i for up side
γ_{ij}	distance from the current feeding schedule (j diets) to the edge of the domain on direction i for down side
F'	feeding schedule on the standard domain $[0, 1] \times [0, 1]$
ΔF_1	GMPPY at 0.05 point – GMPPY at current
ΔF_2	GMPPY at 0.1 point – GMPPY at 0.05 point
ΔF_3	GMPPY at 0.1 point – GMPPY at current
PS	price schedule
IS	ingredient schedule
D	diet in the feeding schedule
L_i	lipid content of the pig on the i th day
P_i	protein content of the pig on the i th day
Ld_i	lipid deposition of the pig on the i th day
Pd_i	protein deposition of the pig on the i th day
$DEvi$	voluntary daily digestible energy intake (MJ/day)

8.2 Proof of theorem concerning the effect of changes in ingredient cost and price schedule on the slaughter date

Before we present the proof, we pause to remark that examination of real data does indicate that it can be the case that $(GR/FC)'(x) > 0$. An example is shown in Figure 8.1, where in the critical region (between days 52 and 92) the slope of GR/FC is positive.

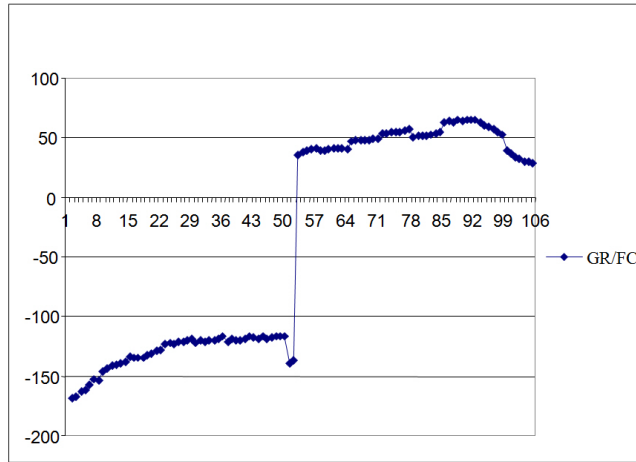


Figure 8.1: An example showing GR/FC and the critical region (between days 52 and 92) where the slope of GR/FC become positive.

Less formally, this condition means that as we move from day x to day $x + 1$, gross return increases by more than feed cost.

Without loss of generality, we now fix the feeding schedule F and suppress it in the notation that follows. Recall that the gross margin per pig g , over a period of x days, is given by

$$g(x) = GR(x) - FC(x) - WC,$$

and the gross margin per pig place per year, when pigs are grown for x days, is

$$f(x) = 365 \cdot g(x)/(x + 7)$$

Suppose that the ingredient cost level is altered by factor $\alpha > 1$. Differentiation of f shows that f is maximised when $g'(x)/g(x) = 1/(x + 7)$; this critical point is a

maximum since the sign of $f'(x)$ is the sign of $(x + 7)g'(x) - g(x)$ and this (from examination of real data) decreases from positive values. Since $1/(x + 7)$ is decreasing, it suffices to examine the behaviour of $g'(x)/g(x)$ as the ingredient cost or price schedule is changed. If the ratio $g'(x)/g(x)$ itself increases, then the point (the slaughter date) where it meets $1/(x + 7)$, will decrease, and vice versa.

i) In the case of an ingredient cost change by factor α , the ratio increases if and only if

$$\frac{(GR'(x) - FC'(x))}{(GR(x) - FC(x) - WC)} < \frac{(GR'(x) - \alpha FC'(x))}{(GR(x) - \alpha FC(x) - WC)}$$

which, with algebraic manipulation, and the assumption that both $GR(x) - FC(x) - WC$ and $GR(x) - \alpha FC(x) - WC$ are positive (reasonable, since we are working in a region where growth is expected to be profitable), to be the inequality

$$(\alpha - 1)(GR'(x)FC(x) - FC'(x)GR(x)) + WC \cdot FC'(x) > 0, \text{ or}$$

$$(\alpha - 1)((GR/FC)'(x) + WC \cdot FC'(x)/(FC(x))^2) > 0$$

as required.

Statement ii) of the result follows in a similar fashion.

References

- Aarts, E., & Korst, J. (1989). *Simulated annealing and Boltzman machines - A stochastic approach to combinatorial optimization and neural computing*. John Wiley & Sons.
- Agricultural Research Council. (1981). *Nutrient requirement of pigs*. Farnham Royal, Slough, UK: Commonwealth Agricultural Bureaux.
- Alberta Pork. (2006). Global pork production: meeting the challenge in a changing world. *Industry report*, 3(1) Feb 2006, Retrieved May 23, 2011, from <http://www.albertapork.com/Uploads/Objects/IndRptFeb2006.pdf>
- Alexander, D.L.J., Morel, P.C.H., & Wood, G.R. (2001). Feeding strategies for maximising gross margin in pig production. *Proceedings of the ORSNZ*, Wellington.
- Alexander, D.L.J., Morel, P.C.H., & Wood, G.R. (2006). Feeding strategies for maximising gross margin in pig production. In: Pinter, J.D. (ed.) *Nonconvex optimization and its applications. Global Optimization: Scientific and Engineering Case Studies*, 85. 33-43. Springer, US.
- The Animal Welfare Foundation of Canada. (n.d.). Farm Animal Welfare. Retrieved June 14, 2011, from <http://www.awfc.ca/english/works/farm.htm>
- Australian Pork Limited. (2010). *AusPig growth model*. Retrieved May 20, 2011, from <http://www.australianpork.com.au/pages/page194.asp>
- Beasley, J.E. (2007). Stochastic programming. Retrieved January 10, 2011, from <http://people.brunel.ac.uk/~mastjjb/jeb/or/sp.html>
- Birkett, S. & de Lange, K. (2001a). Limitations of conventional models and a conceptual framework for a nutrient flow representation of energy utilisation by animals. *British Journal of Nutrition*, 86, 647-659.

-
- Birkett, S. & de Lange, K. (2001b). A computational framework for a nutrient flow representation of energy utilisation by growing monogastric animals. *British Journal of Nutrition*, 86, 661-674.
- Birkett, S. & de Lange, K. (2001c). Calibration of a nutrient flow model of energy utilisation by growing pigs. *British Journal of Nutrition*, 86, 675-689.
- Black, J.L., Campbell, R.G., Williams, I.H., James, K.J., & Davies, G.T. (1986). Simulation of energy and amino acid utilisation in the pig. *Research and Development in Agriculture*, 3, 121145.
- Boisen, S. (2000). A simple nutrient-based production model for the growing pig. Modelling Nutrient Utilization in Farm Animals. 183196. J. P. McNamara, J. France, and D. E. Beever, ed. CAB International, Oxon, U.K.
- Britannica. (2009). Swine flu and factory farms: Fast track to disaster. Retrieved June 10, 2011, from <http://advocacy.britannica.com/blog/advocacy/2009/05/swine-flu-and-factory-farms-fast-track-to-disaster/>
- British Pig Executive (BPEX). (2009). Defining the benefits of new genotypes. Retrieved February 16, 2011, from <http://www.thepigsite.com/articles/2/breeding-and-reproduction/2712/defining-the-benefits-of-new-genotypes>
- Brooks, P.H., Beal, J.D. & Niven, S. (2001). Liquid feeding of pigs: potential for reducing environmental impact and for improving productivity and food safety. *Recent Advances in Animal Nutrition in Australia*, 13, 49-63.
- Campos, M. (2003). *The economics of technologies in Swedish pig production*. (Doctoral thesis. Swedish University of Agricultural Sciences, Uppsala 2003). Retrieved November, 10, 2011, from <http://pub.epsilon.slu.se/459/1/Agraria436.pdf>
- Canibe, N. & Jensen, B.B. (2003). Fermented and nonfermented liquid feed to growing pigs: effect on aspects of gastrointestinal ecology and growth performance. *Journal of Animal Science*, 81(8), 2019-31.
- Carr, J., & Garth veterinary group. (n.d.). *Garth pig stockmanship standards*. Retrieved January 15, 2011, from <http://www.thepigsite.com/stockstds/4/pig-farm-costs-and-example-of-pig-farm-running-costs-in-the-uk>
- Chavas, J.P., Kliebenstein, J. & Crenshaw, T.D. (1985). Modeling dynamic agricultural production response: The case of swine production. *American Journal of Agricultural Economics*, 67(3), 636-646.

- Chiba, L.I. (2004). *Swine production handbook: Grower-finisher pig management*. Retrieved November, 5, 2011, from <http://www.ag.auburn.edu/~chibale/sw08growerfinisherpigs.pdf>
- Coffey, R.D., Parker, G.R. & Laurent, K.M. (n.d.). *Feeding growing-finishing pigs to maximize lean growth rate*. Retrieved November, 8, 2011, from http://www.animalgenome.org/edu/PIH/prod.grow_finish.pdf
- Crenshaw, M.A. (2004). *Suggestions to improve swine feed efficiency*. Animal & Dairy Sciences, Mississippi State University Extension Service, Mississippi State University. Retrieved June 10, 2011, from <http://msucares.com/pubs/infosheets/is1660.pdf>
- de Lange, C.F.M. (1995). *Framework for a simplified model to demonstrate principles of nutrient partitioning for growth in the pig, modelling growth in the pig*. EAAP., Moughan, P.J., Verstegen, M.W.A., Visser-Reyneveld, M.I. (ed.). 78, 71-85.
- de Lange, C.F.M., Morel, P.C.H. & Birkett, S.H. (2003). Modeling chemical and physical body composition of the growing pig. *Journal of Animal Science*, 81(E.Suppl.2), E159E165.
- de Lange, C.F.M., Morel, P.C.H., & Birkett, S.H. (2008). Mathematical representation of the partitioning of retained energy in the growing pig. Chapter 14. pp. 316-338. In. (Ed. J. France and E Kebreab): *Mathematical Modelling in Animal Nutrition*. CABI Publishing, Cambridge, MA 02139, USA. ISBN 9781845933548
- de Lange, C.F.M., Marty, B.J., Birkett, S., Morel, P., & Szkotnicki, B. (2000). Application of pig growth models in commercial pork production. *Canadien Journal of Animal Science*, 81, 1-8.
- de Lange, K., & Coudenys, K. (1996). *Interactions between nutrition and the expression of genetic performance potentials in grower-finisher pigs*. University of Guelph. Proceedings of National Swine Improvement Federation Conference (NSIF) and annual meeting in 1996, 21. Retrieved February 20, 2011, from <http://www.nsif.com/conferences/1996/delange.htm>
- DeRouchey, J.M. & Richert, B.T. (n.d.). *National swine nutrition guide: Feeding systems for swine*. Retrieved November, 5, 2011, from [http://www.usporkcenter.org/UserFiles/file/NSNG/NSNG-Feeding%20Systems%20for%20Swine\(1\).pdf](http://www.usporkcenter.org/UserFiles/file/NSNG/NSNG-Feeding%20Systems%20for%20Swine(1).pdf)

-
- de Vries, A.G., & Kanis, E. (1992). A growth model to estimate economic values for food intake capacity in pigs. *Animal Production*, 55(2), 241-246.
- Edwards, A.C. (2011). Commercial validation of a single diet verses phase feeding in grower-finisher pigs. Pork Co-operative Research Centre (CRC) research summary. Retrieved March 14, 2011, from http://www.porkcrc.com.au/2G-109_Final_Research_Report_110115.pdf
- Eldor, H., & Koppel, L.B. (1971). A generalized approach to the method of steepest ascent. *Operations Research*, 19(7), 1613-1618.
- Elkmann, A. & Bärlein, N. (2011). Dry feeding and liquid feeding in pig rearing. *Pig progress piglet feeding special*, 27, Retrieved November, 6, 2011, from <http://www.bigdutchman.de/en/pig/home/pr-section/press-releases/detailansicht/news/dry-feeding-and-liquid-feeding-in-pig-rearing.html>
- Emmans, G.C., & Kyriazakis, I. (1997). Models of pig growth: problems and proposed solutions. *Livestock Production Science*, 51(1-3), 119-129.
- EntrePinoys Atbp. (2009). *Guide to proper feeding of hog/swine*. Retrieved November, 5, 2011, from <http://www.mixph.com/2009/11/guide-to-proper-feeding-of-hogswine.html>
- Eurostat. (2010). Pol Marquer. Pig farming in the EU, a changing sector. Eurostat statistics in focus, Agriculture and fisheries. 8, 2010. Retrieved May 2, 2011, from http://epp.eurostat.ec.europa.eu/cache/ITY_OFFPUB/KS-SF-10-008/EN/KS-SF-10-008-EN.PDF
- Ferguson, N.S. (2008). Optimum solutions for grower-finisher pigs. *London Swine Conference Facing the New Reality 1-2 April 2008*, 161-166.
- Ferguson, N.S., Gous, R.M., & Emmans, G.C. (1994). Preferred components for the construction of a new simulation model of growth, feed intake and nutrient requirements of growing pigs. *South African Journal of Animal Science*, 24, 10-17.
- 5M Enterprises Ltd. (n.d.). *The PigSite pig health: Efficient production*. Retrieved November 4, 2011, from <http://www.thepigsite.com/pighealth/article/307/efficient-production>
- Food and Agriculture Organization of the United Nations (FAO). (2009). *FAOSTAT-Food and Agricultural commodities production: World + (Total)*. Retrieved June 10, 2011, from <http://faostat.fao.org/site/339/default.aspx>

- Food and Agriculture Organization of the United Nations (FAO). (2011). *FAOSTAT-Agriculture: Livestock primary*. Retrieved June 10, 2011, from <http://faostat.fao.org/site/569/DesktopDefault.aspx?PageID=569#ancor>
- Food and Agriculture Organization of the United Nations (FAO) & Animal Production and Health Commission for Asia and the Pacific (APHCA). (2002). *The livestock industries of Thailand*. Retrieved May 10, 2011, from <ftp://ftp.fao.org/docrep/fao/004/ac797e/ac797e00.pdf>, RAP Publication 2002/23
- Food and Agriculture Organization of the United Nations (FAO), World Organisation for Animal Health (OIE) & World Bank. (2010). Good practices for biosecurity in the pig sector - Issues and options in developing and transition countries. *FAO Animal Production and Health Paper No. 169*. Rome, FAO. Retrieved November, 15, 2011, from <http://www.fao.org/docrep/012/i1435e/i1435e00.pdf>
- Global Livestock Production and Health Atlas (GLiPHA). (2011). *World livestock population*. Retrieved May 20, 2011, from <http://kids.fao.org/glipha/>
- Glover, F., & Laguna, M. (1997). *Tabu Search*. Norwell, MA: Kluwer Academic Publishers.
- Gompertz, B. (1825). On nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. *Philosophical Transactions of the Royal Society*, 115, 513-585.
- Hampshire feeding system. (2011a). *Fermented liquid feeding*. Retrieved November, 6, 2011, from <http://www.hampshirefeedingsystems.com/fermented.html>
- Hampshire feeding system. (2011b). *Why liquid feed?*. Retrieved November, 6, 2011, from <http://www.hampshirefeedingsystems.com/why.html>
- Hendrix, E.M.T., & Klepper, O. (2000). On uniform covering, adaptive random search and raspberries. *Journal of Global Optimization*, 18, 143-163.
- Holland, J.H. (1975). *Adaptation in natural and artificial systems*. Ann Arbor, MI: The University of Michigan Press.
- Huynh, T.T.T., Aarnink, A.J.A., Drucker, A. & Verstegen, M.W.A. (2007). Pig production in Cambodia, Laos, Philippines, and Vietnam: A review. *Asian Journal of Agriculture and Development*, 4(1), 69-90. Retrieved November, 14, 2011, from <http://www.searca.org/ajad/archives/v-03/01-02/>

ajad-v3-n1-n2-huynh-pig-production-in-cambodia-laos-philippines-and-vietnam.pdf

Jean dit Bailleul, P., Bernier, J.F., van Milgen, J., Sauvant, D., & Pomar, C. (2000). The utilization of prediction models to optimize farm animal production systems: The case of a growing pig model. In: McNamara, J.P., France, J., and Beever, D. (eds.) *Modelling nutrient utilization in farm animals*, Wallingford, Oxon, UK, 379-392. CAB International 2000, Wallingford, Oxon, UK.

Kall, P., & Wallace, S.W. (1994). *Stochastic programming*. Chichester, England: John Wiley & Sons.

Kearney, B. (1971). Linear programming and pig feed formulation. *Irish Journal of Agricultural Economics and Rural Sociology*, 3(2), 145-155.

Key, N. & McBride, W.D. (2008). Technology, larger farm size increased productivity on U.S. hog farms. *Amber Waves*, April, 2008. Retrieved November, 14, 2011, from <http://www.ers.usda.gov/AmberWaves/April08/Features/USHogFarms.htm>

Kuester, J.L. and Mize, J.H. (1973). *Optimization techniques with Fortran*. New York, USA: McGraw-Hill.

Livestock knowledge transfer. (2001). *Liquid vs dry feeding*. Retrieved November, 5, 2011, from <http://www.thepigsite.com/articles/?AREA=Nutrition&Display=1065>

Meat and Livestock Commission. (2003). *General guidelines on liquid feeding for pigs*. Retrieved November, 5, 2011, from <http://www.thepigsite.com/articles/?AREA=Nutrition&Display=916>

Menzies, T., Black, J., Fleming, J., & Dean, M. (1992). *An expert system for raising pigs*.

Merks, J.W.M. (2000). One century of genetic changes in pigs and the future needs. In *the challenge of genetic change in animal production. Proceedings of an Occasional Meeting organised by the British Society of Animal Science. BSAS Occasional Publication*, 27, 8-19. Retrieved November, 4, 2011, from <http://bsas.org.uk/downloads/genchan/paper2.pdf>

Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H., & Teller, E. (1953). Equations of state calculations by fast computing machines, *The Journal of Chemical Physics*, 21, 1087-1092.

- Mississippi State University (2010). *Pork production in Mississippi*. Retrieved November 8, 2011, from, <http://msucares.com/livestock/swine/profit.html>
- Morel, P.C.H. (2009). New developments in pig growth modelling. Proceeding of Advancing Pork Production. Retrieved May 25, 2011, from <http://www.nzpork.co.nz/LinkClick.aspx?fileticket=e8jeOF9RofY%3D&tabid=66&mid=758>
- Morel, P.C.H., & Wood, G.R. (2005). Optimisation of nutrient use to maximise profitability and minimise nitrogen excretion in pig meat production systems. *Acta Horticulturae*, 674, 269-275.
- Moughan, P. J. (2003). Simulating the partitioning of dietary amino acids: New directions. *Journal of Animal Science*, 81(14), E60-E67.
- Moughan, P.J., Smith, W.C., & Pearson, G. (1987). Description and validation of a model simulating growth in the pig (20-90 kg liveweight). *New Zealand Journal of Agricultural Research*, 30, 481-489.
- National Research Council. (1998). *Nutrient requirements of swine. 10th revised edition*. Washington, D.C. National Academies Press.
- National Statistics Office. Ministry of Information and Communication Technology. (2003). *2003 Agricultural census whole kingdom*. Retrieved June 10, 2011, from http://service.nso.go.th/nso/nso_center/project/search_center/23project-en.htm
- Nelder, J.A., & Mead, R. (1965). A simplex method for function minimisation. *Computer Journal*, 7, 308-313.
- Niemi, J.K. (2006). *A dynamic programming model for optimising feeding and slaughter decisions regarding fattening pigs*. (Doctoral dissertation). Retrieved April 3, 2011, from <http://www.mtt.fi/afs/pdf/mtt-afs-v15nsuppl1.pdf>
- Niemi, J.K. & Sevón-Aimonen, M. (2009). Economically optimal pig delivery scheduling and the design of meat pricing schemes when pig group is heterogeneous. *Paper presented at Marketing & Trade 17th International Farm Management Congress, Bloomington/Normal, Illinois, USA, Peer Review Paper*, 531-540. Retrieved November, 10, 2011, from http://www.ifmaonline.org/pdf/congress/09_Niemi&Sevon-Aimonen.pdf

-
- Obitko, M. (1998). Genetic algorithm. Retrieved January 5, 2011, from <http://www.obitko.com/tutorials/genetic-algorithms/ga-basic-description.php>
- Philpott, A. (n.d.). *Stochastic programming introduction*. Stochastic Programming Community (COSP). Retrieved January 5, 2011, from <http://stoprog.org>.
- Pieterse, E., Loots, L.P. & Viljoen, J. (2000). The effect of slaughter weight on pig production efficiency. *South African Journal of Animal Science*, 30(Supplement 1), 115-117.
- Pomar, C., Harris, D. L., & Minvielle, F. (1991). Computer simulation model of swine production systems: II. Modeling body composition and weight of female pigs, fetal development, milk production, and growth of suckling pigs. *Journal of Animal Science*, 69, 1489-1502.
- Queensland Government, Primary Industries and Fisheries. (2010a). *Pig industry terms and definitions*. Retrieved November, 4, 2011, from http://www.dpi.qld.gov.au/27_4083.ht
- Queensland Government, Primary Industries and Fisheries. (2010b). *Pig production*. Retrieved November, 4, 2011, from http://www.dpi.qld.gov.au/27_122.htm
- Reaume, D.J., Edwin, H., Robert, R. & Smith, L. (2001). Implementing pure adaptive search for global optimization using Markov chain sampling. *Journal of Global Optimization*, 20(1), 33-47.
- Reese, D.E. (2007). *Swine feed program management tips*. Retrieved November, 11, 2011, from <http://www.thepigsite.com/articles/3/feed-nutrition-and-water/1899/swine-feed-program-management-tips>
- Reeves, R.C., & Rowe, E.J. (2002). *Genetic algorithms-principles and perspectives, A Guide to GA theory*. Massachusetts: Kluwer.
- Roese, G. & Taylor, G. (2006). Basic pig husbandry - grower herd. *Primefact*, 73, Retrieved November, 4, 2011, from http://www.dpi.nsw.gov.au/_data/assets/pdf_file/0019/56152/Basic_pig_husbandry-Grower_herd-_Primefact_73-final.pdf
- Romeijn, H.E., Zabinsky, Z.B., Graesser, D.L., & Neogi, S. (1999). New reflection generator for simulated annealing in mixed-integer/continuous global optimization. *The Journal of Optimization Theory and Applications*, 101(2), 403-427.

- Sandberg, F.B., Emmans, G.C. & Kyriazakis, I. (2005a). Partitioning of limiting protein and energy in the growing pig: description of the problem, possible rules and their qualitative evaluation. *British Journal of Nutrition*, 93, 205-212.
- Sandberg, F.B., Emmans, G.C. & Kyriazakis, I. (2005b). Partitioning of limiting protein and energy in the growing pig: testing quantitative rules against experimental data. *British Journal of Nutrition*, 93, 213-224.
- Saskatchewan (2008). *Feeding tips that benefit the bottom line*. Retrieved November 8, 2011, from <http://www.thepigsite.com/swinenews/16893/feeding-tips-that-benefit-the-bottom-line>
- Schinckel, A.P., & Einstein, M.E. (n.d.). *Concepts of pig growth and composition*. Department of Animal Sciences. Purdue University. Retrieved March 13, 2011, from <http://www.ansc.purdue.edu/swine/porkpage/growth/pubs/aps95-5.htm>
- Schinckel, A.P., Li, N., Einstein, M.E., & Miller, D. (2002). Development of a stochastic pig compositional growth model. Purdue university, *Swine Research Report*, 95-97. Retrieved March 13, 2011, from <http://www.ansc.purdue.edu/swine/swineday/sday02/15.pdf>
- Schinckel, A.P., Preckel, P., Einstien, M.E, Hubbs, T., & Richert, B. (2008). Managing tough times: Impact of increasing feed prices on the optimal market weights and marketing strategy of finishing pigs. *Purdue Extension*. Animal Sciences and Agricultural Economics. Purdue University. Retrieved April 20, 2011, from <http://www.extension.purdue.edu/extmedia/AS/FF-403-W.pdf>
- Schinckel, A.P., Richert, B.T., Clark, L.K., Frank, J.W. & Turek, J.T. (1997). Modeling genetic and environmental effects on pig lean growth. *Proceeding of National Swine Improvement Federation*, 55-74. Retrieved November, 8, 2011, from <http://poultryscience.org/docs/pba/1952-2003/1998/1998%20Schinckel.pdf>
- Sherriff, R.L. (2008). *Use of decision science and genetic optimization to aid selection of genetically superior animals*. (Doctoral dissertation). Massey University, Palmerston North, New Zealand.
- Shewchuk, J.R. (1994). An Introduction to the conjugate gradient method without the agonizing pain. *Technical report CMUCS-TR-94-125*, Carnegie Mellon University. Retrieved November 6, 2011, from <http://www.cs.cmu.edu/~quake-papers/painless-conjugate-gradient.pdf>

-
- Shurson, J. (2008). *What we know about feeding liquid by-products to pigs*. Retrieved November, 4, 2011, from <http://www.ddgs.umn.edu/articles-swine/2008-Shurson-%20What%20we%20know%20about-.pdf>
- Siriwathananukul, Y. (2000). *Project report: maximisation of gross return to the producer in the pig industry in Thailand*.
- Skorupski, M.T., Garrick, D.J., Blair, H.T., & Smith, W.C. (1995). Economic values of traits for pig improvement. I. A simulation model. CSIRO. *Australian Journal of Agricultural Research*, 46(2), 285-303.
- Sutton, A. (2008). Feed Management Practices to Minimize Odors from Swine Operations. College of Agriculture, Purdue University, *Pork Checkoff*. Retrieved June 21, 2011, from http://www.pork.org/filelibrary/Factsheets/Environment/Feed_Mgmt_5.pdf
- Tagliapietra, F., Ceolin, C., & Schiavon, S. (2005). On-farm estimation of pig growth parameters from longitudinal data of live weight and feed consumption and the use of a mathematical model. *Italian Journal of Animal Science*, 4(Suppl 3), 116-118.
- Tisdell, C., Murphy, T., & Kehren, T. (1998). Characteristics of Thailand's commercial pig and poultry industries, with international comparisons. *World Animal Review*, (in press). Retrieved June 5, 2011, from <http://www.fao.org/ag/aga/agap/frg/feedback/war/W6437t/w6437t02.htm>
- U.S. Pork Center of Excellence. (2010). *National swine nutrition guide diet formulation & evaluation software documentation*. Retrieved May 10, 2011, from http://www.usporkcenter.org/UserFiles/file/NSNG/National%20Swine%20Nutrition%20Guide%20Software%20Manual_Final.pdf
- U.S. Pork Center of Excellence. (n.d.). Practical ideas to address high feed and production costs. *Porkcheckoff*. Retrieved November, 11, 2011, from http://www.porkgateway.com/html/spotlight_images/PracticalIdeas to Address HighFeedandProductionCosts.pdf
- Victorian Government, Department of Primary Industries. (2011). *Victoria's pig, goat and deer industry profile winter 2011*. Retrieved November, 14, 2011, from <http://dpi.vic.gov.au/agriculture/animals-and-livestock/victorias-pig-goat-and-deer-industry>

- Wellock, I.J., Emmans, G.C., & Kyriazakis, I. (2003). Modelling the effects of thermal environment and dietary composition on pig performance: Model logic and concepts. *Animal Science*, 77, 255-266.
- Wellock, I.J., Emmans, G.C., & Kyriazakis, I. (2004). Describing and predicting potential growth in the pig. *Animal Science*, 78, 379-388.
- Western Australia Government, Department of Agricultural and Food. (n.d.). *The pig industry in WA*. Retrieved June 6, 2011, from http://www.agric.wa.gov.au/PC_93861.html?s=1393414126
- Whittemore, C.T. (1983). Development of recommended energy and protein allowances for growing pigs. *Agricultural Systems*, 11(3), 159-186.
- Whittemore, C.T. (1986). An approach to pig growth modeling. *Journal of Animal Science*, 63, 615-621.
- Whittemore, C. T., Green, D. M., & Knap, P. W. (2001). Technical review of the energy and protein requirements of growing pigs: Food intake. *Animal Science*, 73, 3-17.
- Willis, S. (2010). *Phase feeding for Pigs*. Primary Industries and Fisheries, Queensland Government. Retrieved January 15, 2011, from http://www.dpi.qld.gov.au/27_11719.htm#Case_study
- Zabinsky, Z.B., & Smith, R.L. (1992). Pure adaptive search in global optimization. *Mathematical Programming*, 53, 323-338.
- Zabinsky, Z.B., Smith, R.L., McDonald, J.F., Romeijn, H.E. & Kaufman, D.E. (1993). Improving hit and run for global optimization. *Journal of Global Optimization*, 3, 171-192.
- Zabinsky, Z.B., & Wood, G.R. (2002). Implementation of stochastic adaptive search with Hit-and-Run as a generator. In: Pardalos, P.M., Romeijn, H.E. (ed.) *Handbook of Global Optimization*, 2, 251-260. Kluwer, London.