

CHILDREN'S UNDERSTANDING OF THE NUMBER SYSTEM

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This thesis represents a major part of the prescribed program of study.

DEDICATION

To my wife Patricia Anne (Dangar) Thomas for her love, support, and confidence in me throughout my professional career.

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Finally, this research would not have been possible without the assistance of the principals, teachers and children in the cooperating schools.

CERTIFICATE

**I hereby certify that this work has not been submitted for a
higher degree to any other university or institution.**

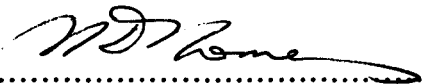

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SUMMARY

The development of numeration and place value knowledge is fundamental to an understanding of mathematical concepts and processes. This study investigates the aspects of developing number knowledge which contribute to the apparent failure of children to make sense of numeration as a system. The strategies that children use in solving numeration tasks involving key elements of counting, grouping, and structuring place value are explored. It is questioned how critical aspects of counting and grouping are related to understanding the base ten structure of the numeration system. The study also describes how children's representations of the counting sequence 1-100 reflect their developing structure of the numeration system.

A broad cross-sectional study of 126 children from Kindergarten to Grade 6 was designed to assess children's acquisition of key elements of counting, grouping/partitioning, regrouping, place value, number sense and structure. Quantitative and qualitative analyses of performance and strategy use were conducted to highlight which aspects were critical in developing numeration knowledge. Further investigation of children's representations of the counting sequence 1-100 is reported and exemplified by a range of pictorial representations drawn from the cross-sectional study and a follow up study of children from Grades 4 to 6.

This study shows that children do not develop sufficient understanding of numeration as a system; what is needed is for children to make the connections between the many different representations for multiunit numbers being used. Key components of counting, partitioning, grouping and number relations, and the visualisation of structure need to be considered more critically. Strategies that children use to solve numeration tasks are too often based on unitary counting and very few Grade 6 children are able to use the holistic strategies that derive from the structure of the number system. Children's representations of counting frequently reflects a lack of structure, grouping is not sufficiently linked to the formation of multiunits, and additive rather than multiplicative relations dominate the interpretation of multidigit numbers. By the end of primary school, most children have not generalised the way the number system is structured. Furthermore, this study highlights that very little progress in understanding the structure of the number system is made during the last three years of their primary school grades.

The analysis of children's visualisations of the counting sequence 1-100 reflect to some extent their developing structure of the numeration system. Examples of imagery provided in this study have given 'snapshots' of the developmental stages that it is suggested children may progress through. It is demonstrated how the process of visualisation can assist in the assessment of children's developing relational understanding of the number.

This study contributes to the developing body of knowledge on children's development of number concepts and relationships. In particular, the study highlights that the teaching of numeration as compartmentalised knowledge restricts the construction of relationships. The teaching of numeration requires a more holistic perspective of what children need to develop in their learning of number. Curriculum and teaching need to reflect the goal of achieving understanding of the structure of the number system through key processes of: counting, grouping, partitioning and regrouping and the formation of multiunit values. Children must be helped to build connections between their intuitive knowledge, various models that might be used and the formal rules of numeration. Children need to build their own mental models as a means of constructing meaning for the number system.