CHAPTER 4

THE PILOT STUDY

The review of the literature on children's development of numeration (Chapter 3) has discussed the contribution of important studies in this field (Bednarz & Janvier, 1988; Cobb & Wheatley, 1988; Denvir & Brown, 1986a, b; Fuson, 1990a, b; Hiebert & Wearne, 1992; Jones et al., 1994; Jones et al., 1996; Kamii, C., 1986, 1989; Kamii, M., 1982; Kamii & DeClark, 1985; Labinowicz, 1985; Ross, 1989a, b; Thompson, 1982a, b). This research has contributed to our understanding of how children use counting, grouping and place value skills, and use this knowledge in multidigit calculations. However, during the early 1990's there were few studies about how children developed an understanding of whole number numeration and place value as part of a system which is consistent and infinitely extendable (Jones et al., 1994; Jones et al., 1996; Sinclair, Garin, & Tieche-Christinat, 1992). As shown in Chapter 1 there was a need to understand how children develop the structure of the numeration system.

Chapter 1 outlined the problem that many children do not recognise that numeration is a structured system and that there is a lack of research which investigates how children develop this structure. At the time of formulating a pilot study it was considered appropriate to investigate how children develop a relational understanding of the numeration system. The research questions formulated for this investigation were to explore some key processes of numeration:

(i) use of equivalent groups in counting;
(ii) the process of regrouping in representations of number, and
(iii) place value and the structure of the numeration system.

Two key questions were raised. When and how does a child begin to understand the structure of the number system? How is structural flexibility developed to allow children to operate meaningfully within the number system?

This chapter describes the pilot study, designed and conducted in 1992, which aimed to identify how young children develop some understanding of the structure of numeration. More specifically, the purpose of the study was to:

(i) trial numeration tasks for a larger investigation;
(ii) trial new tasks specifically designed to examine attributes of 'structure' of numeration not previously investigated;
(iii) trial procedures for task-based interviews, and
(iv) determine the appropriate age range of children to sample in the main investigation.

This pilot study is discussed in four sections outlining the research methodology, results, discussion of results, and limitations and conclusions. The methodology section discusses the cross-sectional design including a description of the sample used, interview tasks, interview procedures and analysis of data. Results, limitations and tentative conclusions of the pilot study are discussed, which give direction for the design of the main study.

4.1 METHODOLOGY

A cross-sectional study was designed and based on constructivist-oriented methodology employed by a range of researchers (Cobb & Wheatley, 1983; Kamii, 1982; Kamii & DeClark, 1985; Reynolds, 1993; Steffe & Cobb, 1988; Thompson, 1982a). The pilot study aimed to explore children's performance and strategy use across a range of numeration tasks suitable for Kindergarten to Grade 4. Clinical interviewing was employed to enable the researcher to establish some insight into why children used particular strategies and to probe children's intuitive mathematical thinking more deeply than other techniques such as paper-and-pencil methods. Task based interviews previously used in many studies of early number learning, enabled researchers to gain a more coherent picture of how number concepts and processes are acquired (Anghileri, 1989; Bednarz & Janvier, 1982; Cobb & Wheatley, 1988; Denvir & Brown, 1986a; Donaldson, 1978; Hart, 1989; Inhelder & Piaget, 1958; Kamii, 1982; Kamii & DeClark, 1985; Labinowicz, 1985; Mulligan, 1991; Ross, 1986; Steffe et al., 1983).

The study is based on a constructivist view of learning and grounded in the work of Piaget (1970). Clinical interviewing is a method of assessment that allows insight to be gained into student's conceptual knowledge and reasoning through encouraging verbalisation and interaction with objects (Labinowicz, 1985; Opper, 1977). Clinical interviewing involves flexible questioning designed to uncover basic features of the student's thinking. The 'talking aloud' procedure can also be used to elicit understanding of student strategies (Ginsburg, Kossan, Schwartz, & Swanson, 1983). The advantages of interviews include the opportunity to delve into students' thinking, representations and reasoning, to better determine their level of understanding, to diagnose misconceptions and to assess their ability to communicate. Clinical interviewing was employed in this study to enable the researcher to investigate the strategies children used in solving critical numeration tasks. This interpretative research approach was used to attempt to make sense of how children acquire understanding, through the analysis of strategies.

Sample

It was considered advantageous to select a cross-sectional sample of children aged 5 to 10 years in order to identify patterns in performance and strategy development and to investigate their understanding of numeration. This is the age range in which children are usually instructed in place value. Thus the study employed clinical interviewing of a cross-sectional sample of 40 children. Four children were selected from each of the grades Kindergarten to Grade 4 from two country NSW Government schools. Both schools were in the same regional town and both catered for a broad socio-economic cross-section of the community. All the subjects had been instructed according to the same mathematics syllabus (NSW Department of Education, 1989), and NSW basic skills tests were completed by all Grade 3 children. The children (16 girls and 24 boys) were selected by the class teachers on the basis of being representative of the spread of achievement levels in mathematics in the class (one chosen as a low achiever, two middle ranking achievers and one as a higher achiever). Teachers made the decisions on the basis of class achievement and, for the Grade 4 children, Basic Skills Test results were used (NSW Department of School Education, 1991) to determine achievement levels. This was considered appropriate for initial pilot work.

Analysis

The numeration tasks were formulated in order to probe children's understanding of the numeration system and elicit aspects of their representations of number that might show their intuitive understandings. Analysis and discussion of results focussed on identifying strategies that children used as well as determining overall performance. The influence of different task variables (i.e. number size, use of concrete, pictorial or symbolic representations, composition of grouped items as exposed or hidden and open or closed tasks) needed to be examined, as it was envisaged that these factors could have an influence on solution processes. Tasks needed to be representative of the range of skills that constitute understanding of the numeration system. Relationships that exist between these skills or concepts would be explored, hopefully leading to the identification of patterns of strategy use for individuals.

4.1.1 Interview tasks

There were four categories of tasks trialled in the pilot study: counting, grouping, regrouping and structure. The task categories were formulated by grouping common characteristics of processes fundamental to an understanding of numeration. It was considered that the category of 'structure' described an area of numeration understanding which had not been previously investigated. The tasks are provided in Tables 4.1 to 4.4 in the categories of counting, grouping, regrouping and extended structure respectively with the tasks numbered in order of presentation at interview. A range of number tasks were included in order to trial which elicited the most critical information from the child's understanding of numeration. On this basis it was anticipated that some tasks would be eliminated from the main study. More explicit descriptions of tasks selected for the main study on the basis of the pilot study are provided in Chapter 5.

4.1.2 Counting tasks

Several counting tasks were used in order to investigate the role of counting as a basis for understanding numeration and to establish links with the framework developed by Les Steffe and colleagues at the University of Georgia (e.g., Steffe & Cobb, 1988; Steffe et al., 1983; Wright, 1990). Children's arithmetical strategies for problems such as missing addend and removed item tasks and the strategies of counting-on, counting-up-to and counting-down-to were investigated. Further, it was important to examine how kindergarten and early first grade children used auditory patterning, counting on and counting back in order to provide the basis for later number work (Thornton, 1989; Wright, 1991b).

Table 4.1 describes the counting tasks (Tasks 2 to 3, 5, 7b and 15). The missing addend and removed item tasks (Steffe & Cobb, 1988; Wright, 1991a) were used to determine whether the children counted the counting acts themselves in subtraction problems, i.e. whether abstract counting was used. Task 5 investigated skip counting by tens beyond one hundred whereas Tasks 7b and 15 investigated the counting of equal groups of items; whether skip, double or multiple counting was used. A grouping task (Task 16) shown in Table 4.2 also explored the use of double counting. Skip counting is the reciting of the count of multiples of a particular number whereas double counting is where there is a simultaneous count of the number of groups. Multiple counting occurs when the total is associated with multiplication.

Table 4.1: Counting Tasks

Counting Tasks	Protocols
2. Missing addend task. Assess whether the counting acts themselves are counted - abstract counting	Display 8 (5) shells. *How many shells are there here?* Place out 4 (3) shells which are screened from view. *There are 12 (8) shells altogether. How many are hidden?*
3. Removed item task, assess whether the counting acts themselves are counted - abstract counting	Display a collection of ten counters. *How many shells are there here?* (10) Hide 3 shells. *How many shells are under my hand?*
5. Skip count, tens	*Count by tens*
7b. Multiple count, fours	*After putting shells on the plates so that there is the same number of shells on each plate* (26 shells with 6 plates) and establishing that there are 4 shells on each plate (7a): *Count these shells by fours.*
15. Multiple count, threes	*Show me five groups of three shells* (jar of shells provided). *Count the shells by threes.*

4.1.3 Grouping Tasks

Table 4.2 describes some selected grouping tasks that are related to multiplication and division, and Table 4.3 describes the grouping tasks related to using ten as an iterable unit. As grouping is a process which is central to the development of an understanding of numeration, partition and quotition tasks (Tasks 6, 7a and 16) and an array task (Task 25) were used to determine the intuitive use children make of grouping in a number of situations. The formation of equivalent groups is an important part of the strategies which are used to solve both partition (sharing) and quotition division word problems (Fischbein, Deri, Nello & Merino, 1985; Kouba, 1989; Mulligan, 1991; Mulligan & Mitchelmore, 1997). Further tasks were used to explore the use of groupings of ten in counting and mental calculations (Tasks 9, 10, 13, 17, and 20).

Table 4.2: Grouping tasks related to multiplication and division

Grouping Tasks	Protocols
6. Partition (Is sharing 1:1 or many:1?)	*Give the same amount of lollies to each Lego person. We are to use all the lollies.* (12 lollies shared between 3 people).
7a. Partition	*I am going to give you some shells. I want you to put these shells on the plates so that there is the same number of shells on each plate* (26 shells with 6 plates). *How many shells are on each plate? How many shells are there here altogether? Write down that number.*
16. Quotition Assess use of double count to keep track of number of groups of 4 eg 1 2 3 4(1), 5 6 7 8(2), 9 10 11 12(3)	a) *See if you can think aloud as you do this next question.* *There are 12 children with 4 children sitting at each table. How many tables are needed?* Give opportunity to draw picture or use material. b) Provide 12 Lego people and 5 trucks made out of Lego blocks. *There are 12 Lego people and some trucks. 4 people go to work in each truck. How many trucks are needed?*
25. Counting objects in an array - assess intuitive use of ten in counting when a ten by six array of pictures is presented	Show an array of ten by six planes. *Can you tell me quickly how many planes are here?*

Partitioning problems (Tasks 6 and 7 shown in Table 4.2) were used to assess whether equivalent grouping is used in the sharing process and whether a collection is partitioned into equivalent groups by one-to-one or many-to-one correspondence. The problems involved situations with remainders (smaller numbers) and without remainders (larger numbers) in order to explore whether equivalence was established in these different situations. An array is an abstract representation of a multiplication situation and so array problems provide another way of using equivalent groups (Outhred, 1993). In Task 25 children were shown an array (ten by six) of pictures of planes and asked how many planes there were altogether. Task 16 was a quotition problem where children had to keep track of the number of groups (double counting). If the children could not solve this word problem they were then given a simpler quotition problem in the context of concrete material (toy people and cars) in order to find whether this task context made any difference to performance.

Table 4.3: Grouping tasks related to ten as a unit

Grouping Tasks	Protocols
9. Counting pregrouped material	Show a roll of 10 lollies (transparent). *How many lollies are in this roll?* Show 4 opaque rolls, each containing 10 sweets. *How many lollies are in this roll? How many lollies are in all these rolls?* 1 roll and 5 loose sweets are displayed. *How many lollies?*
10. Counting pregrouped material	4 rolls and 3 loose sweets are displayed. *How many lollies are here altogether?*
13. Grouping / opaque covering, several groups of ten - assess level of counting, use of ten	Show a picture 8 rolls and 6 separate lollies. *How many lollies in this roll? How many lollies altogether?*
14. Forming groups	*Show me five groups of three shells* (jar of shells provided).
17. Build representation using pregrouped material	Show a long: *How many does this show?* Show a long and 2 shorts: *How many does this show?* From a set of Dienes block representations of ones (only 40 provided), tens and hundreds the child was asked to: *use these counting blocks to build 52.*
20. Uncovering tens task - assess whether child can coordinate counting by tens and ones i.e. ten as an abstract composite unit	Tens task - a board to which is affixed a sequence of Dienes longs and shorts is gradually uncovered and each time the cover is pulled back to show more material the child is asked: *how many are there now?* 10, 14, 34, 38, 41, 51, 53, 73

Tasks 9 and 10 assessed whether groups of ten objects were treated as units and these involved packs of lifesavers (grouped in tens) and individual lifesavers (Denvir & Brown, 1986a). The children established that there were ten lollies in each roll by counting the contents of a transparently wrapped roll and were then asked, in turn, to quantify collections of 15 and 43 lifesavers consisting of standard arrangements of pregrouped rolls (opaque) and individual lollies. An uncovering tens task (Cobb & Wheatley, 1988) was used to assess whether children could coordinate counting by tens and ones (Task 20). The relationship of grouping to recording with numerals was investigated with digit correspondence tasks (Ross, 1990). Task 14 was a simple grouping task and Task 17 required the formation of a concrete representation of 52 using pregrouped materials (Dienes longs and shorts). Tasks 5, 7b and 15 (described earlier in Table 4.1) involved counting multiples of the numbers 10, 4 and 3 respectively and so are also related to grouping experiences.

4.1.4 Regrouping Tasks

Table 4.4 describes the regrouping tasks (Tasks 12, 18, 21, 22, 23, 31 and 36). The operation of regrouping was explored through addition and subtraction tasks. Tasks 12, 22, 23 and 31, which used pregrouped material, assessed whether solution strategies involved counting, adding ones and trading, or holistic strategies (Ross, 1986, 1989a, b; Steffe & Cobb, 1988). Holistic strategies involve breaking numbers up and rearranging parts in ways that make the addition or subtraction easier e.g., bridging tens and hundreds or compensation. Task 21 assessed the possible use of the part-whole relationship for adding two single-digit numbers, when pictorial representations were given (pattern boards with dots in pattern of twos). Other tasks involved the regrouping of concrete representations of number to non-standard forms where more than 9 individual objects were represented (Task 18), and the renaming of numbers with equivalent symbolic names (Task 36).

Table 4.4: Regrouping tasks

Regrouping Tasks	Protocols
12. Regrouping in addition	*If you added 8 lollies to your collection there, how many lollies would you have altogether?* Assess whether child counts by ones and then trades or uses ten as a unit in the trade.
18. Regrouping to form non-standard representation of 52	Children who were successful with Task 17 were asked if they could find another way to represent 52. *Can you draw a picture to show how the blocks represent 52.*
21. Assess use of subitising pattern when adding two single-digit numbers, pictorial representation.	Show a 7 and a 9 pattern board (twos pattern). *How many dots are here?* Show the 7 board. *How many dots are here?* Show the 9 board. *How many dots are there altogether?*
22. Addition of two 2-digit numbers	3 rolls (opaque coverings) and 7 separate lollies are visible, the child is told 25 lollies are hidden beneath the cloth and asked to find how many lollies there are altogether.
23. Missing addend task with 2-digit numbers	Show 32 represented by three rolls and two separate lollies. Get respondent to close eyes and then hide one roll and five lollies under tin i.e. show 17. *How many lollies are hidden under here?*
31. Removed item task	Show 143 represented by one bag, four rolls and three separate lollies (opaque coverings). Get respondent to close eyes and then hide 9 rolls, and the 7 lollies under tin i.e. show 46. *How many lollies are hidden under here?*
36. Renaming a symbolic representation	Show a place value chart and place numeral cards for numbers '1' and '13' in the tens and ones places. *What does this mean? What is another name for this number? Is there any other way of writing this?*

4.1.5 Structure Tasks

The structure of the numeration system was explored through tasks which extended the use of groups, to groups of groups, and the relationships of the system of ones, tens and hundreds. Table 4.5 describes the structure tasks related to groupings, and Table 4.6 describes the structure tasks related to writing and interpretation of numerals.

Structure tasks related to groupings

Tasks 4, 26, 27, and 29 involved the recognition and use of tens and hundreds in a variety of task situations. Task 4 required the recognition of combinations of numbers which give ten. Task 26 required ten groups of ten to be quantified as a hundred. In a further counting task the children were shown two bags, one roll and four individual lifesavers where all the wrappings were transparent. The roll of lifesavers contained 10 lollies and each bag contained 10 rolls. The children were asked to find how many lollies there were altogether (Task 27). A similar task (Task 29) involved three bags, twelve rolls and five individual lifesavers (non-standard representation).

Table 4.5: Structure tasks related to groupings

Structure Tasks	Protocols
1. Mental image of the numbers 1 to 100	Assess whether the mental picture of the numbers 1 to 100 is a single mental number line or a matrix of coordinated horizontal and vertical mental number lines. *Close your eyes. I want you to imagine the numbers from 1 to 100. Can you see a picture of these numbers? Open your eyes. Draw a picture of what you saw.*
4. Combinations to give ten	After establishing how many shells remain when 3 are hidden (from initial collection of 10) - Task 3. *Can you give me any other numbers that add to give 10?*
26. Quantify ten groups of ten	Show a roll of ten lollies: *How many lollies are here?* Show a bag of ten rolls: *How many lollies are here?*
27. Quantify non-standard groupings (transparent)	Show 2 bags, 1 roll, 4 separate lollies - all transparent coverings. *How many lollies are there altogether? Write down the number of lollies.*
29. Quantify non-standard groupings (opaque)	Show 3 bags, 12 rolls, 5 separate lollies - all opaque coverings. *How many lollies are there altogether?*
32. Determine pertinence of grouping in quantifying a collection - pictorial, discrete objects	Child presented with a picture of 143 marks randomly drawn. *Can you tell me quickly how many marks there are drawn there?*
33. Pertinence of grouping - using prompt	*I am going to do the same thing later with a friend who will be here after you. Could you do something so that, when I show him the sheet, he will be able to tell me very quickly how many marks there are? What did you do? Now can you tell me quickly how many marks there are? How do you know that?*
34. Pertinence of grouping - interpretation of circling groups	*Look at what the friend who came before you did* (grouping of groupings is shown). *What do you think of it? Can we see quickly how many marks there are? Suppose you have a younger brother / sister who you are going to help with his/her counting. How would you explain the easy way to count those marks.*
37. Base 5 system - predict size of groupings based on five.	Provide a box of unifix cubes. *In our number system, we always make groups of ten. If we made a make-believe number system where all our groupings were based on 5, then we would group these cubes together to make towers of 5. Here are some towers of 5. How many towers would we group together ?* (Show some Dienes blocks. *Here we have some shorts, longs and flats. How many towers do we put together to form a flat?*)
41. Assess use of groupings of tens, hundreds and thousands when quantifying large collections	Show an array of 100 × 100 dots. *Can you tell me how many dots are here?*
42. Using pattern of tens on the hundred square	Show a hundred square (0 to 99). *Show me how you can get ten more than 36 quickly on the hundreds square. Can you show me ten less than 49?*

Task 32 initially assessed whether children spontaneously used grouping in tens and hundreds to count large collections shown by a pictorial presentation. Tasks 33 and 34 further probed how the collection could be presented in order to facilitate the count and then assessed how the same collection, with the grouping of 'ten groups of ten' shown by circling, was interpreted (Bednarz & Janvier, 1988).

Tasks 41, 37 and 42 explored the use of pattern within the way numbers can be represented, pictorially as an array of dots, as another base, and on a hundred square. Each of these tasks involved aspects of multiplicative thinking. In a counting task (Task 41), children were asked to find how many dots there were in an array of 10 000 (100 by 100), with spacing which separated blocks of one hundred and then of one thousand dots (see Appendix A). Groups of five unifix cubes (towers) were used to explore children's generalisation of the base ten system to a similar system based on a grouping number of five (Task 37). Another task (Task 42) investigated the children's use of the pattern of tens to locate numbers on the hundred square.

Visualisation of the number sequence

An additional structure task (Task 1) involved asking the children to close their eyes and to imagine the numbers from 1 to 100. The idea for this task originated with some earlier classroom work undertaken by the author which attempted to get children to describe what they understood by the pattern of tens in the numbers 1 to 100. It was decided here to make the question more open and so the instruction was given for children to close their eyes and imagine the number sequence. The researcher was uncertain as to what response this task would evoke, although it was intended to assess the structure, or lack of structure, of the mental image that children have of these numbers. The task was carried out at the beginning of the interview so that responses might not be influenced by experiences with other tasks.

Structure tasks related to writing and interpretation of numerals

Table 4.6 describes the structure tasks 8, 11, 19, 24, 28 and 30. The idea of place value is fundamental to the way we structure numbers and so digit correspondence tasks based on the work of Ross (1986, 1990) were used (Tasks 8, 11, 19, 28 and 30). These tasks investigated understanding of the value of digits within numerals in relation to various standard and non-standard concrete representations. There were some tasks that considered the writing and interpretation of numerals (Tasks 24, 35, 38, 39 and 40). Task 24 explored the possible use of combinations of separate symbols for each part of a number name (Bell & Burns, 1981; Fuson, 1990b; Labinowicz, 1985). The use of zero as a place holder (Task 35) based on the work of Sierink and Watson (1990) and the identification of the total value of a digit in a numeral (Task 38) were also investigated. Tasks 39 and 40 considered the ordinal aspects of the counting sequence (Brown, 1981).

Table 4.6: Structure tasks related to writing and interpretation of numerals

Descriptions	Structure Tasks
8. Digit correspondence task with 26 (displayed as 6 groups of 4 and 2 left over)	*How many groups of 4? How many left over?* The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many shells you have?* The digit in the tens place is circled and the question repeated.
11. Digit correspondence task: two-digit numbers	4 rolls and 3 loose sweets are displayed (Task 10) *Write down the number* (43). The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many lollies you have?* The digit in the tens place is circled and the question repeated.
19. Digit correspondence task	52 represented by a non-standard grouping of Dienes blocks (Task 18) *Can you write in numbers how the blocks represent 52.* Digit correspondence task.
24. Writing numerals	Write the number one hundred and three. Write the number one hundred and eleven
28. Digit correspondence task, standard representation	Show 2 bags, 1 roll, 4 separate lollies - all transparent coverings. *How many lollies are there altogether? Write down the number of lollies* (Task 27). Digit correspondence task.
30. Digit correspondence task, non-standard representation	Show 3 bags, 12 rolls, 5 separate lollies - all opaque coverings. *How many lollies are there altogether?* Digit correspondence task.
35. Zero as a place holder	Show a label with the expiry date 'use by 01 August' and ask what is the number and why.
38. Place value	Place value - 521 400 - *The 2 stands for 2....*
39. One more than	The number one more than 6399.
40. Number between	*Write any number between 4100 and 4200.*

The tasks chosen for the pilot study were considered representative of the syllabus content in numeration and place value covered by Kindergarten to Grade 4 children (Mathematics K-6, NSW Department of Education, 1989). To assist the solution process contexts used in the problems were familiar and meaningful for the children.

4.1.6 Interview procedures

The interviews were conducted by the author in a staff office in both schools. After establishing a rapport with the child, the numeration tasks were read aloud by the researcher and supplemented by written, pictorial and concrete material when appropriate. Shells, counters, single lollies and pre-packaged lollies in rolls (10) and bags (100), Dienes blocks (shorts and longs), a place value chart, numeral cards, a picture of a ten by six array of planes, a picture of 143 random marks, and an expiry date label from a cereal pack were made available for various tasks.

Although the tasks were designed to reflect the content of numeration in the categories of counting, grouping, regrouping and structure, they were reordered in what was considered increasing item difficulty, i.e. the order of administration of the tasks is given by their numerical labels. Questioning was discontinued when it became apparent from a number of responses, that a child was unable to respond further. All interviews employed the same tasks, and were administered in the same order indicated by the numerical numbering of items in Tables 4.1 to 4.6. A standard set of initial follow-up questions was asked to elicit each child's explanations about the strategies used. The child was encouraged to 'think aloud' and to record answers with drawings where appropriate. The thinking of the child was probed until the strategy being used was clear or until it was obvious that no further explanation would be forthcoming. Children's explanations and visible strategies were recorded on an interview form (see Appendix A). The interviewer coded each response as correct, incorrect, uncodable or non-attempt at the interview. Interviews ranged in length from 15 minutes for younger children to 45 minutes for the older children. All interviews were audio-taped and this was used for checking the written recordings and coding of responses.

4.1.7 Analysis of Data

The data were analysed for performance and solution strategies for each numeration task by grade level. The analysis provides an overview of the relative difficulty of items and gives an indication of the range of strategies used. The author coded solution strategies according to pre-determined operational definitions described in Appendix C. The solution strategies and definitions of the development of counting were determined from the available research literature (e.g., the stages in the construction and elaboration of the number sequence, Steffe et al., 1983; concepts of ten, Cobb & Wheatley, 1988; Steffe & Cobb, 1988). The responses were coded according to the operational definitions listed in Appendix C. The solution strategies for all numeration tasks were classified by grade level to gain an overview of the most common forms of strategies used. Error strategies were also analysed.

4.2 RESULTS

In this section an analysis of the children's performance on the pilot study tasks is presented. The results for each task category are reported separately and comparisons made across grades. These results have been organised in the categories counting, grouping, regrouping and structure as described in Section 4.1.1, with an additional category of notation and place value. The results of the notation and place value tasks are discussed separately because the responses showed this to be an important distinguishing aspect of structure. Commonalities in strategy use across tasks are described.

4.2.1 Performance and strategy use: Counting

This section reports children's performance and strategy use for five counting tasks. Children's solution strategies for the Counting Tasks 2, 3, 5, 7b and 15 are presented in Table 4.7 and results are further analysed in terms of counting stages (Steffe & Cobb, 1988; Wright, 1991a) in Table 4.8. These tasks assessed how the children used counting skills when presented with simple subtraction problems where concrete material was available. The counting tasks aimed to ascertain the children's level of sophistication of counting ability. These are compared and described in terms of the counting stages.

Table 4.7 shows the number of children using each of the identified solution strategies across grade levels for missing addend and removed item tasks (Tasks 2 and 3). It should be noted that those children whose responses were not successful, either guessed the answer, or were unable to give a response. Performance on Tasks 5, 7b & 15 involving skip counting are also compared.

Table 4.7 Solution strategies for Counting Tasks 2 and 3 and performance on tasks 5, 7b and 15: Number of *correct* responses by strategy use for Grades K-4

Task	Strategy	K	1	2	3	4
2	Counts-on using fingers	0	1	4	1	2
	Counts-on mentally	0	3	2	4	0
	Counts-back mentally	0	0	0	1	0
	Recalls answer	0	1	1	2	6
3	Counts-on using fingers	0	3	1	0	0
	Counts-on mentally	0	0	5	4	3
	Recalls answer	0	2	1	4	5
5	Skip count tens beyond 100	0	2	5	7	8
15	Skip count threes up to 15	0	0	1	3	5
7b	Skip count fours up to 20	0	0	1	3	4

n=8 for each grade level

Table 4.7 shows that unitary counting strategies were used by most children in Grades 1 to 3. Skip counting skills for multiples other than ten (i.e. 3's and 4's) were low across all grade levels (Tasks 5, 7b &15).

As discussed in Chapter 2, the work of Steffe et al. (1983) resulted in a five-stage model of children's counting development (Table 2.1). For the purpose of this study these stages have been reduced to three categories. Children were assigned to one of the three categories on the basis of their responses to the missing addend and removed item tasks. These children demonstrated that they could give the correct answer to at least one of the tasks without the need to use a counting strategy.

The first category of **perceptual / figurative counting** combines Stages 1 and 2 incorporating perceptual, figural, motor and verbal counting types. Abstract unit items are not conceptualised and so children must count-all by ones to find the total of two collections. The second category of **abstract - sequential / progressive integrations** combines Stages 4 and 5 and includes responses from children who count-on or count-back from an abstract unit item when solving number problems, but who do not have the flexibility to separate numbers (abstract unit items) into component abstract unit items (numbers). The third category matches Stage 5 (**abstract - part-whole operations**) and includes children who can use numbers simultaneously as numerical composites and abstract unit items. A child at this stage of part-whole operations has constructed subtraction as the inverse of addition.

Table 4.8 Solution strategies for Counting Tasks 2 and 3: Number of responses according to counting stages for Grades K-4

Response categories	K	1	2	3	4
Perceptual / figurative	8	3	1	0	0
Abstract - sequential / progressive integrations	0	3	6	4	2
Abstract - part-whole operations	0	2	1	4	6

n=8 for each grade level

Table 4.8 compares the number of children by grade level achieving each of the counting stages. It can be seen that all the Kindergarten children (mid year) were perceptual or figurative counters but 38% of children were still not using part-whole operations at Grades 3 and 4. The results also show that the number of children displaying higher stages of counting generally increased from Kindergarten to Grade 4.

4.2.2 Performance and strategy use: Grouping

This section reports the children's performance and strategy use for seven grouping tasks. Table 4.9 shows the number of correct responses for certain strategies used by the children on these tasks. Some tasks assessed understanding of the language of grouping (Task 14) and the use of equivalent groups in partition and quotition problems (Tasks 6 and 16). Children's solution strategies for the Grouping Tasks 10 and 20 are analysed in terms of the conceptualisation of ten as an **abstract composite unit** (Cobb & Wheatley, 1988; Steffe & Cobb, 1988). Tens have a single entity as a unit but children also understand ten as being ten ones. There is no assessment here of whether children understand the increase or decrease in value of a number as they count in mental calculations. Task 25 assesses the intuitive use of tens to quantify the elements in an array.

Table 4.9 shows that Kindergarten and children in Grades 1, 2, 3 had limited skills in the use of equivalent groups when solving partition and grouping problems. At Grade 3 there were few children using equivalent groups to solve the partition problems (38% on Task 6 and 25% on Task 7a), most relying on one-to-one dealing instead.

Most Kindergarten and Grade 1 children (81%) did not understand the instruction to make 5 groups of 3 and none could skip count in threes (shown in Table 4.9). These children had difficulty with the language of grouping as shown by their responses to the instruction to make 5 groups of 3 (Task 14). It was only at Grade 4 that the majority of children used a many-to-one strategy to solve the partition problem (share 26 shells between 6 plates - Task 7).

Table 4.9 Solution strategies for Grouping Tasks: Number of correct responses for Grades K-4

Task	Response categories	K	1	2	3	4
6	Uses equivalent groups to solve a partition problem (small numbers, no remainder)	3	4	4	3	6
7a	Uses equivalent groups to solve a partition problem (larger numbers, remainder)	1	3	4	2	6
14	Makes 5 groups of 3	0	3	6	8	8
16	Uses equivalent groups to solve a quotition problem	1	1	4	6	5
10	Counting pregrouped material (43)					
	Counts in tens and ones with coordination - ten as an abstract composite unit	0	3	6	8	8
20	Uncovering tens task	0	0	2	2	4
	Counts in tens and ones with coordination - ten as an abstract composite unit					
25	Can count objects in an array by tens	0	1	5	6	7

n=8 for each grade level

Most children at Kindergarten and Grade 1 levels were unable to solve the quotition problem (Task 16), and those who were successful needed concrete material to model the problem. Most children in Grades 2, 3 and 4 solved the quotition problem abstractly without concrete material by building-up or building-down groups of four, or using known multiplication or division facts.

When given the task where they were required to count the pregrouped lollies (Task 10), most children in Grades 2 to 4 counted in tens and ones with coordination. These children used ten as an abstract composite unit (Cobb & Wheatley, 1988). Most children up to Grade 4 did not count-on by tens and ones appropriately in the uncovering tens task (Task 20). These children either attempted to count all by ones, restarted each time to count by ones or collected units of same rank, or miscounted tens as ones. Only half of Grade 4 children used ten as an abstract composite unit in solving this task. Most children at Grades 2, 3 and 4 successfully counted by tens or used multiplication to find the number of planes in the array (Task 25).

Considering the results of these tasks overall, there was some increase in performance between Grades 2 and 3 but many children were still not able to make use of, or count in, equivalent groups at Grade 4 level. It should be noted that the associated digit correspondence tasks are reported later in Table 4.12 (Tasks 8, 11 & 19).

4.2.3 Performance and strategy use: Regrouping

Table 4.10 shows the children's solution strategies for seven regrouping tasks. Tasks 12, 22, 23, 21 and 31 were used to assess the use of part-whole operations in mental calculations. Tasks 12, 22, 23 and 31 further explore the level of sophistication of using ten in mental addition and subtraction problems. Cobb and Wheatley (1988) identified two levels of usage of tens in calculating activities. As discussed in Chapter 3 **ten as an abstract**

collectable unit is a single entity that is itself composed of ten ones. Units of ten and one are constructed when meaning is given to each digit in a numeral and then units of the same rank are added or subtracted. In an algorithmic calculation **ten as an iterable unit** is incremented (or decremented) in coordination with unit of ones in an abstract counting activity. Fuson made a similar distinction for multiunit conceptualisations when she identified 'collected multiunit' and 'sequence' methods of multidigit addition and subtraction (Fuson, 1990a, b). Tasks 18 and 36 assess the use of non-standard representations of two-digit numbers (concrete and symbolic respectively).

Table 4.10: Solution strategies for Regrouping Tasks: Number of correct responses for Grades K-4

Task	Response categories	K	1	2	3	4
12.	Uses ten as a unit in trading (43+8)					
	Ten as an abstract collectable unit		0	2	3	4
	Ten as an iterable unit		0	0	1	1
22.	Addition task 37+25					
	Ten as an abstract composite unit		0	0	1	1
	Ten as an abstract collectable unit		1	1	2	5
	Ten as an iterable unit		0	1	1	0
23.	Removed item task 32-17					
	Ten as an iterable unit			0	2	3
18.	Show a non-canonical representation	0	1	3	6	7
36.	Renaming 13 ones as 1 ten and 3 ones			0	3	6
21.	Subitise, partition and combine numbers - compensation	0	0	2	2	5
31.	Removed item task 46+ =143					
	Hundred as an iterable unit			0	0	4

n=8 for each grade level

Table 4.10 indicates that only 50% of Grade 4 children used ten an abstract collectable unit when adding 8 lollies to a pregrouped collection of 43 lollies (Task 12) and only one bridged tens to give the answer. Most children in the earlier grades used unitary counting strategies. Similarly with the addition of 2-digit numbers in Task 22 most children at Grade 4 level used ten as an abstract collectable unit and in the earlier grades unitary counting strategies were most common. Only rarely did a child use ten as an iterable unit in this addition task. An example was the bridging ten / compensation strategy used by Robert in Grade 2 as shown in this excerpt: "Put 3 in to make ten ... then 50, 60 ... then just the two more to give 62".

For the removed item tasks (Tasks 23 and 31) most children were unable to successfully calculate the subtraction. However those successful at Grade 4 level used holistic strategies which involved bridging tens and hundreds or equal adjustments. In subtraction, bridging tens means splitting the subtrahend into two parts so that the first subtraction results in a decade number. Equal adjustments means adding the same number to both the minuend and the subtrahend so that the new subtrahend is a multiple of ten (or a hundred).

There was a steady increase in performance where children were able to give a representation of 52 (with Dienes blocks) that was non-standard (Task 18). The majority of Grade 3 children interpreted 1 ten and 13 ones on a place value chart (Task 36) as 113 whereas most

Grade 4 children gave the correct interpretation of 23. However, performance on Task 21 was poor with most children using unitary counting rather than holistic strategies. It was expected that the visual presentation of the addends (on pattern boards) would have assisted children to subitise the numbers and then, through partitioning and combining part numbers, to use bridging tens or compensation strategies.

Table 4.10 shows that the use of ten as an abstract unit in regrouping was generally not used by children until Grade 4. Even for Task 21, in which the numbers to be added (7 and 9) were presented in a visual manner as arrays of dots, only 25% of Grade 3 children used a holistic strategy, most relying on unitary counting.

4.2.4 Performance and strategy use: Structure

This section reports children's performance and strategy use for the structure tasks in three parts. Children's solution strategies for structure tasks related to writing and interpretation of numerals are described in Table 4.11 and for tasks related to groupings are described in Table 4.12. Table 4.13 shows the number of responses according to identified categories for the visualisation task (Task 1).

Structure tasks related to writing and interpretation of numerals

Table 4.11 shows the children's solution strategies for ten tasks that assess understanding of place value and symbolic representation of multidigit numbers.

Table 4.11: Solution strategies for Notation Tasks: Number of correct responses for Grades K-4

Symbolic Representation and Place Value

Task	Response categories	K	1	2	3	4
11.	Digits interpreted correctly by total values - standard grouping of 43		1	4	5	7
19.	Digits interpreted correctly by total values - non-standard grouping of 52		0	3	2	4
8.	Digits interpreted correctly by total values - non-standard grouping of 26		0	1	1	3
24.	Numeral 103		1	7	8	8
35.	Interpret 01 as one, first		1	5	8	8
28.	Digits interpreted correctly by total values - standard grouping of 214			2	3	7
30.	Digits interpreted correctly by total values - non-standard grouping of 425			0	1	5
38.	Identifies ten thousand place value				1	2
39.	Knows the number one more than 6399			2	5	5
40.	Give number between 4100 and 4200			2	3	6

n=8 for each grade level

Table 4.11 shows that 50% of Grade 2 children correctly interpreted the digits of a 2-digit numeral by their total values (Task 11). For Task 19, where the children were presented with a non-standard representation of '4 longs and 12 shorts' for fifty two, there were fewer children successful with 3 out of 8 Grade 4 children giving the face value of the digits. The majority of Grade 4 children could not correctly interpret the non-standard representation of the number 26 (Task 8) showing that many children did not have a firm understanding of how to use place value even for 2-digit numbers. For this task, children were presented with the non-standard representation of twenty six as '6 groups of 4 and 2 shells left over'. Some children (4 out of 24, Grade 2 to 4 children) attempted to relate the tens digit to the size of the groups, as they pointed to the four shells in a plate. For example they identified "because it is half of four" or "if you added on two it would be four ... that is the number of shells in each plate". Other children (5 out of 24, Grade 2 to 4 children) associated the ones digit with the number of groups and the tens digit with the number of shells left over.

It was found that 38% of Grade 3 children and 88% of Grade 4 children correctly interpreted the digits of the numeral '214' when shown a standard representation of the number (Task 28). For the non-standard representation of '425' (3 bags, 12 rolls and 5 separate lollies) only 1 out of 8 Grade 3, and 5 out of 8 Grade 4 children gave correct interpretations for the digits. In Grades 2 to 4, most children could correctly write the numeral for one hundred and three (Task 24) and interpreted '01' as one or first. Place value identification outside the range 'ones to thousands', was poor (Task 38) for children in all grades. There were two Grade 4 children who did not know the number one more than '6399' and who could not give a number between 4100 and 4200.

Children in Grades K-4 have difficulties with understanding the written system of representing number. Children's understanding of the value of digits in a numeral was shown to be unreliable even for 2-digit numbers (Task 8) and most children showed a lack of knowledge of the naming of the 'ten thousand' place in a numeral (Task 38).

Structure tasks related to groupings

Table 4.12 shows the children's solution strategies for seven tasks that assess understanding of the structure of groupings in the number system.

Table 4.12: Solution strategies for Structure Tasks: Number of correct responses for Grades K-4

Task	Response categories	K	1	2	3	4
4.	Knows combinations which make 10		0	2	4	5
42.	Uses pattern of tens in hundred square -					
	addition		0	4	3	1
	subtraction		0	3	2	1
32.	Spontaneous count by tens			0	0	0
33.	Suggests grouping by tens		1	2	3	2
34.	Recognises and uses groupings of tens and hundreds				1	3
41.	Quantifies large collection by identifying hundreds				1	3
37.	Identify base 5 groupings				0	0

n=8 for each grade level

Table 4.12 shows that although a majority of the children at Grades 3 and 4 knew addition combinations of numbers that make ten (Task 4), ten as an abstract counting unit was seldom used at any grade level. Surprisingly, performance decreased consistently from Grade 2 to Grade 4 on Task 42. Children used unitary counting in this task, rather than using the pattern of tens in the hundred square to add and subtract. No children used grouping by tens to count the number of marks on the picture (Task 32) but a few children demonstrated it as a way of making counting easier for someone else (Task 33). Most Grade 3 and 4 children recognised and used the grouping of ten when it was presented but very few took any notice of the grouping of a hundred shown as ten tens (Task 34). The same few children (Grades 3 and 4) identified the pattern of hundreds, counted to give the pattern of thousands and then counted to successfully give the answer of 10 000 in Task 41.

Overall, there was little indication that children from Kindergarten to Grade 4 level used the structure of the numeration system in their counting or calculations. Unitary counting strategies dominated rather than strategies that related to an understanding of ten or a hundred as iterable units which facilitate quantifying collections and calculating.

Visualisation of the number sequence 1-100

Table 4.13 shows the children's responses for the visualisation task (Task 1). The children were asked to close their eyes and think of the numbers from 1 to 100. They were then asked to talk about what they saw in their mind. The responses were classified into three categories as either pictorial, ikonic or notational. The notational responses were further classified according the form of the mental image. Some of these (random selection of numerals or a single numeral) show no structure whereas others (linear or array) display some structure.

Table 4.13: Solution strategies for Visualisation Task: Number of correct responses for Grades K-4

Visualisation of the number sequence 1-100

Type of representation	1	2	3	4
1. Pictorial: Pictures or marks, no pattern (100 objects, lots of squares).	1	0	1	0
2. Ikonic: Pictures or marks, array or line pattern (100 marks in a line, ten sticks)	1	2	0	0
3. Notational:				
3.1 Random selection of numerals, no pattern in display.	0	1	1	3
3.2 The numeral 100.	1	1	0	0
3.3 Numerals appearing one at a time in order to 100 (flashing).	0	0	0	1
3.4 Numerals in a long number line or a series of number lines for different multiple counting sequences.	0	3	1	2
3.5 Array structure: Numerals in a 10 by 10 grid.	0	0	0	2

n=8 for each grade level

The responses were also analysed for elements of the numeration system. It was noted that the pictorial images did not show any structure but that two of the ikonic images did display elements of structure. Overall approximately one third (11) of the Grade 1 to 4 children did not visualise any mental image, a third (10) visualised a picture with no structure and the remaining third (11) had structure of some kind in their picture. Table 4.13 shows that overall most of the children (76%) who gave a visualisation used notation rather than pictorial or ikonic representations. It should be noted that the categories used here were determined on the basis of the initial responses and at this stage of the pilot work categories were not fully developed. A fuller description of the analysis of visualisation of the number system emerges in Chapter 8.

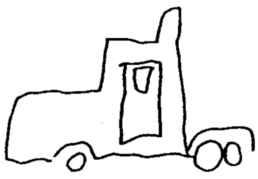

Figure 4.1 Anthony

Figure 4.2 James

Figures 4.1 and 4.2 show the representations given by two Grade 1 children. Anthony drew the truck and explained: "it is how heavy it is ... one hundred ... me and my dad go and we see how much the truck weighs ... he is going to take me somewhere some day ... it is a Camworth ... it goes fast ... it goes a hundred". Anthony associated an image of his Dad's truck with the number 100. This image is highly idiosyncratic, but very meaningful to Anthony. Not only does it relate to the speed that the truck goes but also its mass. Similarly, James focuses on the idea of 'one hundred' rather than the number sequence, just writing the numeral '100'. Lisa (also in Grade 1) describes without drawing her image of a hundred objects ... "all sorts of things ... on the floor".

Figure 4.3 Gary

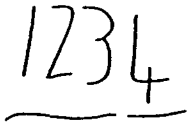

Figure 4.4 Hayley

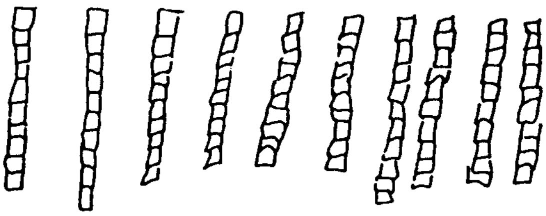

Figure 4.5 Melissa

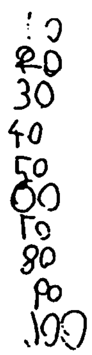

Figure 4.6 Oliver

Figures 4.3 to 4.6 show the representation of the number sequence 1-100 of some of the Grade 2 children. Gary, Hayley and Oliver all drew and described notational characteristics. Hayley only saw the first four numerals in the sequence, whereas Gary saw the first and last number in the sequence. Oliver saw the numerals in a vertical formation, only showing the multiples of ten. Melissa described an ikonic representation of one hundred as groups of ten when drawing ten ten-rods.

0 1 2 3 4 5 6 7 8 9 10

... and on to 100.

Figure 4.7 Michael

1 10 100 90
1 2 3 4 6 9 11 13 20
31 90 100

Figure 4.8 Amber

Michael and Amber in Grade 3 also used notational imagery, both giving some indication of linear structure. Michael counted the first decade of the number sequence by ones, and interestingly, started at zero. Amber gave two sequences of numbers, the first contained the numbers significant to her, 1, 10, 100, and 90. The second sequence contains what appears to be random selection of numbers in numeric order. It is apparent that counting by ones is important for both children and that this skill is still developing. It could be inferred that Amber has a notion that ten and one hundred are significant numbers when counting.

24 89

Figure 4.9 Becky

1 2 3 4 5 6 7 8 9 10
10 20 30 40 50 60 70 80 90
5 10 15 20 25 30 35 40 45 50
55 60 65 70 75 80 85 90 95 100

Figure 4.10 Alex

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32

and so on until 100.

Figure 4.11 Oliver

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

... up to 100.

Figure 4.12 Rebecca

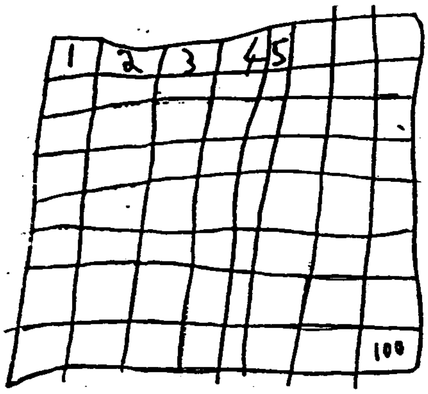

Figure 4.13 Grant

Figures 4.9 to 4.13 show the representations used by some of the Grade 4 children. Becky gave two random numbers whereas Rebecca indicated the imagined number sequence 1-100 writing down only the numbers up to eighteen. Alex gave three sequences, ones to ten, tens to one hundred and fives to one hundred. Both Oliver and Grant gave the numerals in an array configuration. Oliver showed the decades in rows under each other "going onto a hundred". Grant visualised the numbers as a ten by ten array in a grid constructed from intersecting straight lines although it should be noted that he actually drew an 8-by-8 grid.

Responses to the 'visualisation of the numbers 1 to 100' task were classified, both according to a pictorial, ikonic or notational mode of imagery, and to the presence or absence of some element of structure. The results showed that 63% of Grade 2 children 'saw' some element of structure in the number sequence. This can be contrasted with only 13% for Grade 3 children. The grouping structure in the Grade 2 responses is most clearly seen in the mental picture of ten ten-rods as illustrated by Melissa. The most highly developed visualisations of the structure are shown by the pictures illustrated by Grant (Figure 4.13) and Oliver (hundred squares, Figure 4.11), Alex (patterns within the number sequence, Figure 4.10), and David (multiples of five as flashing numerals - illustrated in Figure 8.15, Chapter 8).

4.3 DISCUSSION OF RESULTS

The missing addend and removed item tasks showed that all the Kindergarten and some Grade 1 children were still in the process of constructing the relation 'one more'. These same children solved partition problems by one-to-one, or many-to-one correspondence. Only two of these Kindergarten/Grade 1 children could solve the quotition problem without direct modelling and they used a building-up mental strategy. They were more advanced with skip counting using tens, than with any other number, and this was reflected by skills for counting collections pregrouped as tens and ones. It should be noted that three of the Grade 1 children successfully coordinated the counting in tens and ones of a collection of 43 objects, and two of them regrouped in their mental strategies when adding a single digit number to a two-digit number. At the same time, these children did not use ten as an iterable unit as shown by the uncovering tens task. Further, these children did not subitise ten from visual patterns of two single-digit numbers and did not recognise and use the pattern of tens in the hundreds square.

Most children in Grades 2 and 3 were still reliant on unitary counting in mental computation. A small number of these children though, were shown to be using ten as an iterable unit while others used ten as abstract collectable units. This was reflected by the use of higher order strategies based on ten for solving addition with regrouping tasks and the use of the tens pattern in the hundreds square. Only 25% of the children in Grades 2 and 3 counted-on by tens and ones as appropriate in the uncovering tens task (Task 20). These children used ten as an abstract composite unit in solving this task.

A substantial number of children were still not using part-whole operations at Grades 3 and 4. Steffe and Cobb (1988) point out the implications of this in arguing against the traditional introduction of written algorithms at Grade 3 when they state that "...the decimal system cannot be understood if part-whole operations [Stage 5] are not available" (p. 321). It is interesting to note that for Task 22, none of the Grade 4 children used the higher order strategies involving breaking numbers up into parts, but rather, added units of the same rank (added tens and ones separately). It must be questioned whether this is because of instructional experiences with numeration and algorithms. It would be interesting to know whether many upper primary school children use ten as an iterable unit and so have the flexibility to appropriately use strategies such as bridging, compensation, equal adjustments or relating to operations with powers of ten in mental calculations.

Overall, there appears to be a lack of the use of numeration structure evident in children's strategies; this is shown by their resistance to use the properties of ten in mental calculation. For example, no children in the sample from Grades 1 to 4 spontaneously used equal grouping to find how many marks there were on the card (143 marks altogether) - they either said they did not know, guessed or counted by ones. Only two children recognised the grouping of ten groups of ten, when shown 'what a friend had done to help him/her find quickly how many marks there were', even though the hundred was highlighted with a red circle. Evidence of a lack of equal grouping was shown where most of the Grade 2 and 3 children had poor skip counting skills with numbers other than ten, and again this highlighted the reliance on unitary counting.

There was one Kindergarten child who used equal groups when counting to solve partition and quotition tasks. In contrast, two children used one-to-one dealing to solve the partition problem, and three of the eight children used concrete material or drew pictures to solve the quotition problem. Even with this very small sample size there is evidence of a great range of ability in grouping and regrouping tasks. Children in Kindergarten were building the first level of the number system using the 'one more' relation. The more advanced children in Grades 2 and 3 were constructing a second level of the number system, with important aspects being the ability to partition numbers in many different ways and the use of ten as an iterable unit. The construction of one group of ten out of ten ones, which then becomes a ten unit and then, the use of the two types of units (tens and ones) in regrouping is important for the development of number sense at this level. Many Grade 4 children in this study were still needing to develop this structural flexibility with the use of two-digit numbers. They also showed no indication of coordinating equal groups of tens and ten tens (hundreds); they had not generalised the system of numeration.

How might we hypothesise the development of understanding structure in the numeration system? Young children first generate the numerals by recognising the pattern of using digits in their construction. They also learn through social transmission that the counting words for the teen numbers are not written as they sound. This does not involve an understanding of place value. Place value requires an understanding and integration of both the irregularly value-named system of number words and the positional base-ten system for forming numerals. The base ten place value numeral system requires the mental construction of one group of ten out of ten ones and then the representation by a digit in the "tens place". This involves the construction of a 'second level' to the number system. This second level also involves the idea of multiplication as the groups of ten become the new units.

The exploratory data from the counting, grouping, regrouping, structure and notation tasks collected in this pilot study, gives only an indication of children's understanding of the number system. What this study has shown is that unitary counting, the formation of equal groups in counting, the flexible use of units of ten in mental computations, and multiplicative processes are important elements in children's development of the number system.

Kamii and DeClark (1985) asserted that six and seven year old children are still in the process of constructing the number system with the relation 'one more' and so it might be inferred that they are not ready to fully understand the symbolic notation of two-digit numbers. They suggest it is not possible for a child to construct the 'second level' while the 'first level' of the number system is still being built. Results from this study showed that most Grade 4 children were still not operating efficiently at the 'second level'. There are also indications that development is not uniform and that restricting instruction from one level to the next might not be the best way to assist the development of understanding.

It must be remembered when carrying out any analysis that each child constructs whatever concepts he/she has on the basis of his/her experiences and so any model of a child's thinking must take these experiences into consideration. It appears from the results of this pilot study that, for children up to Grade 4, there has been inadequate development of numeration as a system. The children did not naturally use groups of tens and hundreds, they were restricted in their knowledge of numerals to those represented by conventional grouping arrangements and to the thousands place value, and did not generally use the iterable units of ten and one hundred in their mental calculations. Most children compartmentalised their knowledge, they did not make the necessary connections between ideas that would be required to synthesise their knowledge into a system of relationships.

Previous studies (Bednarz & Janvier, 1988; Denvir & Brown, 1986a, b; Fuson, 1990a, b; Hiebert & Wearne, 1992) focussing on numeration have primarily examined children's counting and grouping strategies, the conceptualisation of tens and hundreds as units (Thompson, 1982a), and have shown the influence of algorithmic instruction on mental calculations (Baroody, 1990; Fuson, 1990b). This pilot study has identified some critical aspects for further investigation; the need to consider carefully children's developing representations for number, and the need for children to make connections between various aspects of number learning. Sinclair, Garin, and Tieche-Christinat (1992) state "we do not possess any hypothesis (or clear ideas) about how a grasp of the structure of our written numerals comes about" (p. 193). This pilot study has made a start to answering this question.

Initial results of the pilot study were reported in a paper presented at the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA) at the University of Western Sydney in July 1992.

4.4 LIMITATIONS AND CONCLUSIONS

The pilot study was limited by the range and size of the sample and so results provide only an indication of the complexity of the process by which children construct their system of numeration. It was also possible that the selection of students by teachers as high and low achievers, and two middle achievers from each class was unreliable and may have resulted in a sample that represented extremes in ability. A larger data base of children's representations would be required in order to carry out a fuller analysis of visualisation.

Task categories, operational definitions, interview procedures and coding of responses have all been refined through the implementation of this pilot study. The categories of counting, grouping and regrouping tasks were expanded to further investigate counting strategies, use of groupings of tens in counting, use of different groupings and the addition of tasks involving regrouping and exchanging from hundreds to tens. A new category of number sense tasks was added to explore the partitioning and use of tens, hundreds and thousands in various problem situations. The category of structure tasks was expanded and separated into place value and structure tasks in order to further probe understanding of the structure of the

the number system. The task categories were organised to reflect the aspects of numeration that children need to develop.

The pilot study data indicated that further research investigating the development of children's understanding of the structure of the numeration system is warranted. Some aspects of children's understanding of numeration were identified but they were found to be limited at Grade 4 level and so further probing of the ways the numeration system is extended would be beneficial. There is evidence of an over reliance on unitary counting strategies through the grades and a developing reliance on learnt algorithmic strategies when doing mental computations at Grade 4. No clear picture emerged of how children generalise the way the numeration system is built because none of the Grade 4 children appeared, from the data collected, to make connections between grouping, regrouping, place value and notation in understanding numeration. There are also indications that children may have more problems than the pilot study showed. Understanding of the extended structure of the number system beyond two-digits (beyond the 'second level') was not sufficiently explored by this pilot study and needs to be the subject of further investigation. There is also a need to find connections between pertinent aspects of children's understanding of numeration.

A larger study, including more explicit tasks related to structure of the numeration system and interviewing children up to the end of the primary school (Grade 6), needs to be undertaken to obtain a more coherent picture of children's understanding of structure of numeration as an extendable system. The visualisation of the number sequence task revealed important attributes of children's representations and raised some interesting and rich results which warrant further investigation. Hence the aspects of imagistic thinking that influence the growth of numerical understandings need to be further investigated. As a result of this aspect of investigation an additional study was conducted with high ability children from Grades 4, 5 and 6 and this will be reported in Chapter 8.

In summary, further research is required to provide a more coherent picture of the development of the system of numeration. Hence the research questions outlined in Chapter 1 examining the development of children's understanding of the numeration system will be addressed through a large exploratory study using clinical interviewing. Associated research carried out since 1992 by Boulton-Lewis (1993a, b, 1996, 1997), Goldin and Kaput (1996), Jones et al. (1994), Jones et al. (1996) and collaborative work by Fuson et al. (1996) will be discussed in relation to the main study. However, methodological aspects of this main investigation were designed in 1993 and are discussed in the following chapter.

CHAPTER 5

THE MAIN STUDY: METHODOLOGY

5.1 INTRODUCTION

This chapter gives an account of the design of the main cross-sectional study, including sampling procedure, a discussion of task-based interviews, a description of the tasks and the clinical interview procedures used to explore the research questions described in Chapter 1. The empirical work focussed on children's construction of a system of numeration and the tasks were designed to expose their knowledge and understanding of counting, grouping, number sense, regrouping, place value and structure. Finally, the procedures used to analyse the data obtained from the interviews are discussed. The research was designed to explore levels of understanding, how the elements of numeration are related to understanding, and how children's representations of number reflect structure in numeration.

Broadly four theoretical perspectives were discussed in Chapter 2; developmental, constructivist, cognitive and representational approaches. At the time of the formulation of the main study in 1993, a number of aspects of these approaches were considered valuable in devising a study to obtain a more coherent picture of children's numeration and place value knowledge. The developmental approach which was used to some extent by Bednarz and Janvier (1982), Denvir and Brown (1986a, b), Kamii (1982), Resnick (1983a, b), and Ross (1986) was central in the planning of this study. Bednarz and Janvier (1982) explored children's use of 'accessible' and 'less accessible' groupings as part of the formulation of a reference framework for generating assessment and learning activities. Denvir and Brown (1986 a, b) identified a learning hierarchy which was used to describe children's current knowledge and to inform the design of teaching activities. Kamii (1990), Kamii and DeClark (1985) and Resnick (1983a, b) adopted a developmental approach, also focussing on the construction of successive systems of multiunits. The developmental approach was to be developed further by Jones, Thornton, and Putt, (1994) and Jones, Thornton, Putt, Hill, Mogill, Rich and Van Zoest (1996) as a framework for multi-digit number understanding. Fuson (1990a) adopted a structural approach in her analysis of conceptual structures for multiunit numbers. All studies explored to some extent the representations of number, using both concrete and symbolic modelling. It was not until 1994 though, after data collection was completed, that an additional analysis based on children's representations of number was more fully incorporated into this study.

Much of the research literature on numeration described was conducted with cross sectional data from selected cohorts or samples of children in the 5 to 10 year age range. While most of these studies provided valuable comparisons across samples to show differences in performance and strategies, the way children developed an understanding of the structure of the number system still needed to be investigated. The way that the aspects of counting, grouping, ordering, partitioning, exchange, regrouping, and place value are related to developing structure still needed to be explored.

The cross-sectional pilot study with Kindergarten to Year 4 children described in Chapter 4, trialed a number of important features for the design of the main study. Procedures for conducting the clinical interviews were also trialed and were considered appropriate for an investigation of this type. In the pilot study, analysis of the clinical interviews with the younger children showed that there were indications of advanced intuitive thinking with some children. It was shown though, that for Grade 4 children, there was very little indication of any understanding of the structure of the numeration system. As a result, it was decided that the main study would need to be extended to Grade 6 level and to include further tasks to investigate explicitly the development of understanding the structure of the numeration system.

In the pilot study, clinical interviewing was used to enable the researcher to establish some insight into why children use particular strategies and to probe children's mathematical thinking more deeply than other techniques such as paper-and-pencil methods. In the main cross-sectional study, it was again considered most appropriate to use clinical interviewing, despite it being time-consuming with a large sample of children. This would enable a knowledge base about the development of numeration and place value to be established. As discussed in Chapter 3, at the time of formulation of the study (1992), numeration research was limited to early number and some aspects of numeration related to 2-digit and 3-digit numbers.

Clinical interviewing of individual children had been used extensively as the preferred method of enquiry in a large number of studies (Anghileri, 1989; Bednarz & Janvier, 1982; Denvir & Brown, 1986a; Hart, 1981; Kamii, 1982; Labinowicz, 1985; Ross, 1986; Sierink & Watson, 1990; Steffe & Cobb, 1988; Wright 1991a). In the constructivist tradition, many researchers investigating the development of mathematical concepts in young children have been focussing their research on the child as an individual, rather than focussing on the teaching method or the teacher. Clinical interviewing techniques have begun to reflect a diminished concern with standardisation in order to obtain a more complete picture of children's developing understanding of mathematical ideas, and the processes used to produce answers (Lesh & Landau, 1983, p. 2). Based on the early work of Piaget (1952), interviewers may structure a specific series of question or mathematical tasks, or conduct the interview in a relatively spontaneous manner. Extensive interviewing used in Bednarz and Janvier's (1982, 1988) research presented individual children with a series of tasks and through observation of their behaviour, and the use of probing questions, interviewers were better able to infer how the child solved the particular problem.

However, some researchers have indicated the difficulties associated with identifying internal cognitive processes. "Most serious is that children's explanations of how they solved the problem may not accurately reflect the processes they actually used to solve it." (Carpenter & Moser, 1983, p. 18). A child may not be able to articulate the process used to solve the problem and may describe another process that was easier to explain. Problems may be too simple for children to be aware of how they solve them. A child could even guess the answer that they think the interviewer is seeking. Furthermore, the problem of 'false negatives', that is concluding that a child does not have certain knowledge that they really do possess, should also be recognised (Brainerd, 1978).

Despite the weaknesses described above, clinical interviewing still provides the most direct measure for investigating the processes children use in solving numeration tasks. For this reason, clinical interviewing is an important feature of the research methodology of this study as outlined in the following section. Furthermore, the clinical interview is clearly a very useful source of rich and detailed information about children's knowledge, misconceptions and idiosyncrasies (Mulhern, 1989). In this study, its application to the teaching/learning situation is critical.

5.2 DESIGN OF CROSS-SECTIONAL STUDY

Steffe and his colleagues (Steffe, 1988; Steffe & Cobb, 1988; Wright, 1994b) extended the work of Piaget by using intensive teaching experiments with small groups of children to detail the development of counting skills. It was decided that, in order to obtain an overall view of how aspects of numeration related to children's understanding of the structure of the numeration system, this study should be a cross-sectional study (Anghileri, 1989; Kamii, 1982) that included children from Kindergarten to Grade 6. In order to carry out clinical interviewing, suitably structured tasks were devised to probe understanding and thus provide a basis for describing individual representations of number in as much detail as possible. This approach was based on the method used by Goldin (1993) and Davis and Maher (1993). The diverse concepts, structures and/or processes that occur inside children's minds were inferred from the observation, classification and description of the various kinds of behaviour displayed. This method is valuable because it provides a means of investigating the interplay between the children's (internal) representations that are inferred by the interviewer and external representations that are constructed during the interviews.

The importance of imagery in the construction of children's understanding of mathematics has been discussed by a number of researchers (Brown & Presmeg, 1993; Brown & Wheatley, 1989; Goldin, 1987, 1988; 1992a, b; Mason, 1992; Presmeg, 1986a, b, 1989). In view of the research literature (discussed in Chapter 3) and following discussions with Gerald Goldin in 1993, it was decided that there should be further investigation of the significance of imagery in the children's understanding of structure in the numeration system. The category system used to analyse the data needed to be elaborated from that used in the pilot study to better describe and understand the imagery used. This chapter will discuss the classification scheme developed to interpret children's representations of number by mode, type of structure and nature of their image developed. There was continuing collaboration with Gerald Goldin about the role of representations in children's numerical processes throughout the course of the study and the analysis and interpretation of children's images of number have been reported at a number of conferences and discussed in published papers (Thomas & Mulligan, 1995; Thomas, Mulligan & Goldin, 1994, 1996). The exploration of children's imagistic representations of the number sequence are discussed in greater detail in Chapter 8.

5.2.1 Sample

A cross-sectional sample of 132 children from Grades K to 6 was randomly selected from six Government schools in the Western Region of New South Wales, Australia. Five of the schools were from three large regional towns and one was the only school in a small rural town. All children were representative of a wide range of socio-economic backgrounds. Teachers were asked to select 4 children by dividing their class lists into four groups and then sending permission letters to the parents of the first child in each group. This was intended to give at least three children from each grade per school, allowing for non-return of permission or the child being absent from school on the interview days. This sample represented a wide range of mathematical abilities.

The composition of the sample in terms of school, grade level, and gender is shown in Table 5.1. The number of classes sampled was 42, the seven grades (K to 6) from each of the six schools. It had been planned to interview three of the four children randomly selected from each class but in some classes teachers believed that all four children should be included in the project and so in some classes three children were selected and in others four children were interviewed.

Table 5.1: Distribution of children in the sample

		School						
		1	2	3	4	5	6	Total
Kindergarten	Girls	2	1	2	2	2	2	11
	Boys	1	2	1	1	1	1	7
Grade 1	Girls	2	2	2	1	2	2	11
	Boys	2	2	1	3	2	1	11
Grade 2	Girls	2	1	2	2	2	2	11
	Boys	1	2	1	1	1	1	7
Grade 3	Girls	1	2	2	2	2	2	11
	Boys	2	1	1	1	2	1	8
Grade 4	Girls	1	2	1	2	1	2	9
	Boys	2	1	2	1	2	1	9
Grade 5	Girls	2	1	1	1	1	1	7
	Boys	1	2	2	2	2	2	11
Grade 6	Girls	2	2	2	1	1	2	10
	Boys	1	1	1	2	2	2	9
Total		22	22	21	22	23	22	132

5.2.2 Classroom instruction

Forty two teachers were surveyed using the questionnaire in Appendix D to ascertain the content and materials being used in the mathematics instruction of the children selected in the sample. All participating teachers followed the official K-6 Mathematics syllabus, which emphasises the acquisition of basic facts and computational skills. A wide variety of concrete materials were used in most classroom instruction.

5.2.3 Interview tasks

There was a range of numeration tasks employed in the study. A total of 89 tasks were incorporated after trialling in the pilot study (described in Chapter 4) with children from Grades K to 4. The tasks were designed to probe understanding of numeration in the key elements of: counting, grouping/partitioning, regrouping, place value, structure of numeration and number sense. The categories of tasks were as following:

- ° counting tasks probed children's counting skills;
- ° grouping and partition tasks investigated children's construction and use of groups in both multiplication and division problem situations;
- ° regrouping tasks focussed upon the use of regrouping in number operations;
- ° place value probed knowledge and understanding of the system of numerals;
- ° structure tasks investigated the multiplicative structure of the numeration system, and
- ° number sense tasks focussed upon children's sense of the way ten and powers of ten are used in the numeration system.

Many of the tasks were refined from those used by previous researchers (Bednarz & Janvier, 1988; Cobb & Wheatley, 1988; Davydov, 1982; Denvir & Brown, 1986a; Labinowicz, 1985; Mulligan, 1992a; Ross, 1986; Steffe & Cobb, 1988; Wright, 1991a).

The tasks were graded in level of difficulty and different subsets of tasks were given to each grade cohort.

Counting tasks

Table 5.2: Counting Tasks

Counting Tasks	Protocols
1. Addition task	5 shells are hidden under one container and 7 shells hidden under another. *There are 5 shells here and 7 shells here. How many shells are there altogether?* (shells available if required)
2. Missing addend task	Display 8 shells. *How many shells are there here?* Place out 4 shells which are screened from view. *There are 12 shells altogether. How many are hidden?*
3. Removed item task	Display a collection of ten shells. *How many shells are there here?* Hide 3 shells. *How many shells are under my hand?*
4. Skip counting by tens	*Count by tens*
5. Multiple counting groups of three	*Show me six groups of three shells* (jar of shells provided). Cover the shells with a cloth. *Count the shells by threes.* (If unable to count the shells then uncover them and repeat question).
6. Double counting - money	*How many twenty cent coins are there in $1?*
7. Double count	*How many twenty cent coins are there in $3?*
8. Double count	*How many twenty cent coins are there in $10?*

Note: Italics are used to indicate spoken instructions

The counting tasks (1 to 8 shown in Table 5.2) investigated children's performance with arithmetic strategies, rote counting by tens, counting grouped items and calculating the number of coins in an amount of money. The tasks used to elicit arithmetic strategies (Tasks 1 to 3) included counting a collection separated into two parts, with both collections being screened; a missing addend task in which the interviewer screens one part of a collection and tells the child how many in all, and a removed item task where a collection is displayed and some screened items are removed (Steffe & Cobb, 1988; Wright, 1994a). These tasks assess whether the counting acts themselves are counted, whether the children use abstract counting. Task 5 involved assessing how a child used the grouping arrangement she/he had made to count the grouped items when screened. For this counting task, children's responses to counting the hidden grouped shells by threes were coded according to whether rhythmic counting (where all shells had to be counted), double counting (where a tally is kept of the number of threes) or multiplication was used to know the total. Tasks 6 to 8 investigated the use of counting strategies to calculate the number of twenty cent coins in various amounts of money.

Number sense tasks

The number sense tasks (1 to 12 shown in Table 5.3) investigated children's performance with using ten, one hundred and one thousand in various problem situations. The number sense tasks (Tasks 2 to 5, 8 and 9) gave performance on recognition of combinations of numbers which make a hundred and a thousand. For Task 7, the correct responses were categorised as being based on estimation or using mental calculation to explain why the given answer (16 + 9 = 115) was unreasonable.

Table 5.3: Number sense tasks

	Number sense tasks	Protocols
1.	Numbers that add to give ten	After Counting Task 3 where 3 shells are hidden from 10 shells. *Can you give me any other numbers that add to give 10?*
2.	Money - addition to give 100	*What is the change from $1 if something cost 64c?*
3.	Money - addition to give 100	Write down 64c + 36c = $1 *Can you write down 2 other combinations that make $1?*
4.	A metre - addition to give 100	*How many centimetres in 1 metre?* *If you cut 5 cm off a metre length of ribbon, how much ribbon is left?*
5.	A metre - addition to give 100	Write down 0.05 + 0.95 =1 *Can you write down another combination that makes 1 metre?*
6.	Addition - uses ten or counts on from larger number	Show a 6 and a 9 pattern board (twos pattern ten frames). *How many dots are there altogether?*
7.	Estimation - addition	16 + 9 -------- 115 -------- *Is this a reasonable answer? Why?*
8.	Partition one hundred	After finding the number to add to 84 to make 100 using a hundred square (Structure task 6). *Can you give me another combination which makes 100?*
9.	Partition one thousand	*Can you make up any combinations which make 1000?*
10.	Mental addition	What is sixty-five plus sixty-five? Explain how you got that answer.
11.	Compensation tasks - 2-digit numbers	Now think aloud while you do 66+64.
12.	Compensation	Now think aloud while you do 55+75.

Grouping tasks

The grouping tasks (1 to 15 shown in Table 5.4) investigated children's performance with partitioning (Tasks 1 to 4), formation of designated groups (Task 5), quotition (Task 6), the use of different grouping quantities (Task 7), use of prepackaged (tens and ones) materials to show a proportional representation of a 2-digit number (Tasks 8 to 10), intuitive grouping numbers (Task 11), constructing a grouped representation of a 2-digit number (Task 12), counting ones, tens and hundreds in uncovering tasks (Tasks 13 and 15) and counting using the tens structure in an array (Task 14).

Table 5.4: Grouping tasks

Grouping tasks	Protocols
1. Partitioning tasks - partitioning into equivalent groups	*Give the same amount of lollies to each Lego person. We are to use all the lollies.* (12 lollies shared between 3 people).
2. Assess whether a collection can be partitioned into equivalent groups	*How many lollies did each Lego person get?*
3. Partition - directed (larger numbers and remainder)	*I am going to give you some shells.* *I want you to put these shells in the plates so that there is the same number of shells on each plate* (26 shells with 6 plates).
4. Partition word problem	Lauren has planted 20 cabbages. There are 4 equal rows. How many cabbages are in each row?
5. Forming six groups of three	*Show me six groups of three shells* (jar of shells provided). Cover the shells with a cloth.
6. Quotition task - double counting	*See if you can think aloud as you do this next question.* *There are 12 children with 4 children sitting at each table.* *How many tables are needed?* Give opportunity to draw picture. If unsuccessful then ask: *There are 12 Lego people and some trucks. 4 people go in each truck. How many trucks are needed?*
7. Abstract property of quantity task	*You are collecting stickers. You can trade 2 small stickers for a large sticker. How many large stickers are worth the same as 3 large stickers and 4 small stickers?*
8. Groupings of ten	Show a roll of 10 lollies (transparent). *How many lollies are in this roll?* Show an opaque roll, containing 10 sweets. *How many lollies, do you think, are in this roll?* If the response is not ten then say that there are ten lollies in the roll the same as there are in this roll (the transparent roll). 1 roll (opaque) and 5 loose sweets are displayed. *How many lollies?* If successful then the following question is asked.
9. Groupings of ten - larger number	4 rolls (opaque) and 3 loose sweets are displayed. *How many lollies are here altogether?*
10. Using groupings of ten	Count out 37 paddle pop sticks as 3 tens and 7 ones. *Here are 3 bundles of ten sticks and 1, 2, 3, 4, 5, 6, 7 extra sticks.* *How many sticks are there altogether?*
11. Suggest grouping number of ten	From a collection of shells packaged in bags (tens) and as loose shells (ones, only 40 provided). *I have packaged these shells into bags in order to make it easier to count them.* *How many do you think I have put in each bag?*
12. Using pre-grouped material - ones and tens	The child is then asked: *Show me 52 shells.*
13. Ten as an abstract composite unit	Uncovering tens task - a board to which is affixed a sequence of Dienes longs and shorts is gradually uncovered and each time the cover is pulled back to show more material the child is asked: *How many are there now?* 10, 14, 34, 41, 51, 53, 73
14. Use of tens structure in counting	Show an array of ten by six planes. *Can you tell me quickly how many planes are here?*
15. Coordination of counting hundreds, tens and ones	Hundreds task - a board to which is affixed a sequence of Dienes longs and shorts is gradually uncovered and each time the cover is pulled back to show more material the child is asked: *How many are there now?* 100, 120, 40, 200, 6, 104, 30

For Tasks 1 to 3 children's performance on partitioning a collection into equivalent groups was assessed as either one-to-one or many-to-one sharings. Task 3 involved acknowledging a remainder. Tasks 4 and 6 are partition and quotition word problems. In the abstract property of quantity task (Task 7) children needed to count two small stickers as one large sticker and so use a many-as-one coordination scheme (Davydov, 1982; Watanabe, 1995). Grouping Tasks 8, 9 and 10 (Denvir & Brown, 1986a) required children to determine 'how many' were in grouped collections; individual items in the grouped collections were either hidden (Tasks 8 and 9) or visible (Task 10). Tasks 11 and 12 investigated children's performance by suggesting a grouping number and then using groupings of ten items to construct a concrete representation of a 2-digit number (Ross, 1986). Tasks 13 and 15 assessed the coordination of counting by ones, tens and hundreds in uncovering tasks (Cobb & Wheatley, 1988; Labinowicz, 1985). Task 14 assessed the intuitive use of ten in counting when a ten by six array of items is presented.

Regrouping tasks

For the regrouping tasks (1 to 8 and 10 to 12 shown in Table 5.5) responses were coded for mental addition and subtraction problems related to concrete materials. These tasks were adapted from those used by Ross (1986, 1990). Task 9 required the children to use regrouping to quantify a non-standard grouping of concrete materials for a 3-digit number.

Table 5.5: Regrouping tasks

	Regrouping tasks	Protocols
1.	Mental addition	*If you added 8 lollies to your collection there, how many lollies would you have altogether?*
2.	Addition with concrete material	The child is then asked: *Show me 52 shells.* *Use the shells to make the number which is 9 larger.*
3.	Regrouping in addition of single-digit numbers	Show a 6 and a 9 pattern board (twos pattern ten frames). *How many dots are there altogether?*
4.	Addition of two 2-digit numbers	Show 3 rolls (opaque coverings) and 7 separate lollies. *How many lollies are here?* *There are 25 lollies hidden under this tub.* *How many lollies are there altogether?*
5.	Missing addend task	A collection of rolls and separate lollies is visible (e.g. 1 rolls and 8 separate lollies), a child is told how many lollies there are in all e.g. 64, and asked to find how many lollies are hidden.
6.	Written algorithm - use of regrouping in addition	16 + 9 -------- 115 -------- *Is this a reasonable answer? Why?* (Number Sense Task 7) *What should the answer be?*
7.	Addition of 98	Show 245 represented by bags, roll, and separate lollies. *Add to these sweets so that the number of lollies is ten more. How many lollies have we got? Add to these sweets so that the number of lollies is 98 more. How many lollies have we got now?*
8.	Addition of 98 - spike abacus	*Add 98 to your number on the abacus.*
9.	Regrouping - tens and hundreds	Show 3 bags, 12 rolls, 5 separate lollies - all opaque coverings. *How many lollies are there altogether?*
10.	Missing addend task	Show 46 represented by four rolls and six separate lollies (opaque coverings). *There are 143 lollies altogether. How many lollies are hidden under here?*
11.	Missing addend task	Show 122 represented by one bag, two rolls and two lollies (opaque coverings). *There are 300 lollies altogether.* *How many lollies are hidden under here?*
12.	Addition of 98 - non-proportional representation	Show a place value chart with 2 shells in the hundreds place, 3 shells in the tens place and 4 shells in the ones place. *What number does this show and why?* *Add 98 to the number on the place value chart.*

Place value

The place value tasks (1 to 16 shown in Table 5.6) assess the children's ability to write numerals for numbers (Tasks 4 to 9), identify value of places (Tasks 11 and 14), interpret value of digits including zero (Tasks 1 to 3 and 13), identify total values of digits (Tasks 10 and 16) and construct and interpret place value representations (Tasks 12 and 15). The digit correspondence tasks (Tasks 1 and 13) required the children to give interpretations of the digits when shown non-standard representations of 2-digit and 3-digit numbers (Ross, 1990). Tasks 2 and 3 investigated performance on interpreting the meaning of '01' and explaining the role of zero as a place holder (Sierink & Watson, 1990). Tasks 10 and 16 required recognition of the total values of 400 and 80 000 on calculator displays. Analysis of Place Value Task 11 recorded children's performance with recognition of the values of positions of three places to both the left and right of a place value chart (ones, tens and hundreds). Task 15 required children to interpret a non-standard numeric representation ("1" and "13" in the tens and ones places of a place value chart respectively).

Table 5.6: Place value tasks

	Counting Tasks	Protocols
1.	Digit correspondence task with 26 displayed as 6 groups of 4 and 2 left over	*How many groups of 4?* *How many left over?* The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many shells you have?* The digit in the tens place is circled and the question repeated.
2.	Zero as a place holder	Child shown a milk carton with "use by 01 August" stamped on it and asked what number "01" was and why. *What is this number?*
3.	Zero as a place holder	*What does the nought do?*
4.	3-digit numeral	*Write down the number one hundred and three.*
5.	3-digit numeral	*Write down the number one hundred and eleven.*
6.	4-digit numeral	*Write down the number one thousand and eight*
7.	decimal numeral	*Write down the number three tenths*
8.	decimal numeral	*Write down the number fourteen hundredths*
9.	6-digit numeral	*Write down the number six hundred and one thousand and forty*
10.	Total value of digit - hundreds	Show 431 recorded on the screen of a calculator and 31 on a card. *How can you change the calculator number to this number on the card?* (If necessary give instruction to use only one subtraction).
11.	Values of positions on a spike abacus	*Put these labels on the abacus* (ones, tens and hundreds).
12.	Representing a 3-digit number on a spike abacus	*How would you use this abacus to show the number 234 ?*
13.	Digit correspondence task	The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many shells you have?* The digit in the tens place is circled and the question repeated. The digit in the hundreds place is circled and the question repeated (425).
14.	Show a place value chart (ones, tens and hundreds) and digit cards for numbers 0 to 19	Show the '1 card' in each of the three positions to the left and right of the labelled positions and ask for the values.
15.	Renaming numbers	Place the cards "1" and "13" in the tens and ones places respectively and ask for the meaning of this. *What is another name for this number?* *Is there any other way of writing this?*
16.	Total value of digit - ten thousand	Show 286 349 displayed on the screen of a calculator and 206 349 on a card. *How can we change the calculator number to this number on the card?* (If necessary give instruction to use only one subtraction).

Structure tasks

The structure tasks investigated imagery, use of the hundred square, groupings of groupings, and the multiplicative structure and extension of the numeration system. Tasks 1 to 13 are shown in Table 5.7 and Tasks 14 to 26 are shown in Table 5.8.

Table 5.7: Structure tasks 1 to 13

Structure tasks	Protocols
1. Imagery of numbers task	*Close your eyes. I want you to imagine the numbers from 1 to 100. Can you see a picture of these numbers?* *Open your eyes.* *Draw a picture of what you saw.*
2. Ones, tens and hundreds structure	Show a roll of ten lollies: *How many lollies are here?* Show a bag of ten rolls: *How many lollies are here?* Show 2 bags, 4 roll, 5 separate lollies - all **transparent** coverings. *How many lollies are there altogether?* *Write down the number of lollies.*
3. Ones, tens and hundreds structure	Show 245 represented by bags, roll, and separate lollies. *Add to these sweets so that the number of lollies is ten more. How many lollies have we got?*
4. Non-proportional representation of 3-digit number	Coloured counters are assigned values ie a red counter is worth 10 blue counters and a yellow counter is worth 10 red counters. Other coloured counters are available. *These counters are each given different values. If we say that these red counters are each worth 10 blue counters. Each yellow counter is worth 10 red counters.* *If each blue counter is worth 1, use as few of the counters as possible to show the number 246.*
5. Extending the representational model for a 4-digit number	*Explain how you would use counters to show the number 1246.*
6. One hundred as a composite unit	Show a hundred square (0 to 99). *Can you find 84? What do you add onto 84 to make 100.*
7. Use of the hundred square	Show a hundred square (0 to 99). *Show me 36 on this 100 square.* *Show me how you can get ten more than 36 quickly from the hundred square.*
8. Hundred square - subtract ten	*Can you show me ten less than 49?*
9. Hundred square - add nine	*Can you show me nine more than 67?*
10. Groupings of groupings - extending the system	*In our lolly factory we make lollies like this one. We package groups of ten lollies into rolls like these. Then we package ten rolls into a bag like this. When we have lots of bags we want to package them into boxes so that it will be easy to count them. How many bags would you put in a box?* *When we have lots of boxes we want to package them into cases or crates so they can be transported to the shops. How many boxes would you put in a case?*
11. Multiplication of groupings	*How many lollies are there in a box?*
12. Numeral from description of groupings	*If we had 3 lollies, 5 rolls, 7 bags, 1 box and 4 cases, how many lollies would there be altogether?* Show this written.
13. Undoing groupings in subtraction	*If we had a case of lollies and sold 1468, how many lollies would be left?*

Tasks 1 and 18 explore the mental imagery that children describe for the counting sequence. Task 2 assessed use of the ones, tens and hundreds structure through requiring children to quantify a proportionally grouped concrete representation of a 3-digit number. Task 3 assessed the addition of ten to a given grouped collection as counting-on by ones or using place value to know the answer. Tasks 4 and 5 investigated the use of a non-proportional concrete representational system for number.

Tasks 6 to 9 required the children to find the missing addend, and add and subtract ten and nine from numbers on the hundred square (0 to 99 configuration). The Lolly Factory tasks

(Task 10 to 13), which were based on 'The Candy Factory' third grade teaching experiment (Cobb, Yackel & Wood, 1992) investigated the way ten and powers of ten are used in extending the numeration system. Responses to Task 12, where children were asked to determine the number of lollies in a collection of 3 lollies, 5 rolls, 7 bags, 1 box and 4 cases, were coded according to whether they calculated the total as an addition or recognised the total based on the pattern of grouping numbers. Some of the children who calculated the total were only correct up to the 3 or 4 digits.

Table 5.8: Structure tasks 14 to 26

Structure tasks	Protocols
14. Groupings of groupings - determine pertinence of grouping	Child presented with a picture of 144 marks randomly drawn. *Can you guess how many marks there are drawn there?* *I am going to do the same thing later with a friend who will be here after you. Could you do something so that, when I show him /her the sheet, he /she will be able to tell me very quickly how many marks there are?* *What would you do?*
15. Recognise and use groupings	*Look what someone did for you* (grouping of groupings is shown - red circle). *What do you think of it?* *Can you now count how many marks there are?*
16. Use a grouping of groupings.	If the red circle is not mentioned then the following questions are asked. *Can you explain what the red circle does?* *Can you now count a quicker way?*
17. Multiplication on the place value chart	Show a place value chart (ones to hundred millions) with 2 shells in the hundreds place, 3 shells in the tens place and 4 shells in the ones place. *What number does this show and why?* *Use the shells to make the number which is 100 times larger (multiply by 100)*
18. Imagery of numbers task - 1 to 1000	*Close your eyes. I want you to imagine the numbers from 1 to 1000. Can you see a picture of these numbers?* *Open your eyes. Draw a picture of what you saw.*
19. Place values in a base-five system	Provide a box of unifix cubes. *In our number system, we always make groups of ten.* Show some Dienes blocks. *Here we have some shorts, longs and flats.* Discussion of using the blocks to represent the values of the positions of digits in numerals. *This short shows the value of the digits in this position (ones). This long shows the value of the digits in this position (tens). This flat shows the value of the digits in this position (hundreds).* *If we made a make-believe number system where all our groupings were based on 5, then how would we group these cubes?* *Here are some towers of 5. How would we group these towers ?* *How many towers do we put together to form a wall?* *How would the grouping pattern continue?*
20. Rule for extending the system	Show an unlabelled place value chart. *What would be the values of these positions in our new number system?*
21. Working with groupings of groupings - subtraction	*A mum buys lollies for a birthday party. She wraps them in rolls like this one* (showing a sample of a roll) *and puts the roll in bags like this one* (showing a sample of a bag) *in order to give some to the children. She has some lollies over.* *Mum has this many at the beginning* (drawing is shown 234). *She gives away this many* (another drawing is shown 178). *How many bags, rolls and loose lollies does she have left?*
22. Use of groupings of groupings in counting	Show an array of 100 × 100 dots. *Can you tell me how many dots are here?*
23. Extension of the place value system - ten thousand	Show some base-ten numeration blocks. *If this block (a little) has a value of 1. What is the value of this block (long)?* *What is the value of this block (flat)?* *What is the value of this block (block)?* *What is the next value?*
24. Extending the model	If the answer is not correct give the answer as 10 000. *What would a model of 10 000 look like?*
25. Renaming values	*If this block (a little) has a value of 100, what is the value of this block (flat)?*
26. Renaming values	*If this block (big) has a value of 1, what is the value of this long?*

The pertinence of groupings and a grouping of groupings were analysed in Tasks 14 to 16 (Bednarz & Janvier, 1988). Responses for suggesting grouping numbers that could be used (Task 14) were coded as grouping by ten only or grouping ten and ten groups of ten. When asked to use grouping that had been made, the children's responses were coded as recognising and using the groupings by ten only, or recognising the groupings ten and ten groups of ten. Task 17 assessed the use of place value when a number represented by a non-proportional place value model is multiplied by 100. Responses were coded as removing the given shells and placing the correct number of shells in each place value or moving given shells two places to the left on the place value chart. Tasks 19 and 20 involved constructing a grouping system based on powers of five. Responses for suggesting two further grouping numbers (Task 19) were coded as: 10 and 10; 20 and 10, 2 and 10, 5 and 5, or using various other numbers. Task 21 assessed use of the structure of grouped materials, which were described and presented pictorially, in a subtraction task. Responses were coded as separation, aggregation or holistic strategies. For Task 22 children's responses to quantifying the dots in the array were coded according to the strategies of recognising the pattern of hundreds and multiplying by one hundred, multiplying a hundred by a hundred through looking at the whole array, and recognising the pattern of hundreds and then thousands within the array. Tasks 23 to 26 investigated the use of base-ten numeration blocks, with which the children were familiar, to represent numbers involving digits outside the range of ones to thousands. Responses were coded according to values given in Tasks 23, 25 and 26. Responses for a description of a ten thousand block (Task 24) were coded according to the number of big blocks suggested.

5.3 CLINICAL INTERVIEW PROCEDURES

All interviews in the cross-sectional study were conducted by the researcher in October and November, late in the Australian school year. As in the pilot study, the interviews were carried out in a small room separate from the classroom at the schools. The interviewer told the children that he was interested in how they worked out the answers to some questions. Each interview began with a short discussion related to the child's interests in order to establish a rapport with the child. Following the pilot study the interview questions had been clarified and a detailed form prepared to enable the responses to be summarised easily. A full listing of the task protocols in the order in which they were presented for the cross-sectional study are shown in the Main Study Results (Appendix B). The order of presentation of task was the same for all children. Children from each grade level were given a selected group of tasks that were considered appropriate for them. The appropriate starting and finishing points within the sequence of tasks was determined by the researcher on the basis of pilot study results.

The tasks were presented verbally to the children, with concrete or visual material as indicated in the Main Study Results (Appendix B). The tasks were re-read to the child as often as necessary to assist him/her in understanding the intention of the question. Concrete material and pencil and paper were available on the table. The interviewer explained to the child that this material was available for use if required and that he/she was to 'think aloud' as the 'maths activities' were being done. Children were praised for their attempts, but no feedback was given as to the correctness of their responses. When a response was unclear or the child's thinking was not clear, follow up neutral questions were asked by the interviewer such as: "Can you tell me how you did that?"; "Can you describe what you did there?" and "Did you see anything in your mind when you did that?". The interviews were audio-taped and the length of time for each interview ranged from 25 to 65 minutes.

The interview tasks for the pilot study and the rationale for changes made to the pilot study tasks were presented in Chapter 4. The categories of tasks were increased in the main study to include number sense and place value tasks. The most significant change to the interview tasks in the main study, involved addition of tasks that would more thoroughly explore children's understanding of larger numbers and the system of numeration. Many of these tasks investigated structure of the numeration system and are detailed in Tables 5.7 and 5.8.

The procedures for coding responses are described in the next section under the analysis of data.

5.4 ANALYSIS OF DATA

Student responses to the numeration tasks were analysed using both quantitative and qualitative methods. Item response analysis using Student-Problem curve theory (Harnish, 1983, 1987) and the Rasch model (Rasch, 1980) were used to obtain some overall measure of student performance. The analysis was undertaken with responses from children in Grades 4 to 6 on those tasks completed by all of this cohort.

5.4.1 Item response analysis

Harnisch analysis

Student-Problem (S-P) curve theory (Harnisch, 1983, 1987; Harnisch & Linn, 1981; Switzer & Connell, 1990) was used to analyse the overall level of performance of the sample. Harnisch Item Response Patterns were then used to explore student performance data and make comparisons between response patterns of children and between tasks. It was anticipated that response patterns can be inconsistent in that some children may answer some easy items incorrectly while answering other more difficult items correctly. Also, some tasks are answered incorrectly by high scoring children but answered correctly by low scoring children.

A Student-Response (S-P) Chart was constructed for the 38 tasks that were completed by all children in Grades 4 to 6 (n=55). The S-P Chart is a matrix in which students (rows) are arranged from top to bottom in descending order of total test scores and the tasks (columns) are arranged from left to right in ascending order of difficulty.

Rasch analysis

The results of task performance were examined in relation to the Rasch model (Rasch, 1980). Item response theory was used to measure the reliability of the individual tasks in terms of student ability. Analysis was undertaken for 37 of the 38 tasks that were completed by all children in Grades 4 to 6 (n=55). Structure Task 1 was not included because it involved student visualisation and so there was not a sense of correct or incorrect in the analysis of responses. Item calibration results are calculated for the 37 tasks using the measures of: separation index; Cronbach Alpha, logit measure, standard error of logit measure and infit meansquares. The results are reported in terms of the match between model and actual data.

The Harnisch and Rasch analyses will be used to give a broad view of the results. It should be noted that the cohort for this analysis was the children in Grades 4, 5 and 6 and the tasks were those completed by all these children. It would be expected that this group of children has a wide range of performance levels on the numeration tasks and so there should be some divergence between the response patterns.

5.4.2 Analysis of results: performance and strategies

The analysis of results involved classification and coding responses for student performance and strategy use. These results from the clinical interviewing were compared for performance across tasks and grades. The coding of responses was trialled in the pilot study and was devised to indicate correct, incorrect or non-response to the tasks. Strategies used in both incorrect and correct responses to tasks were coded in order to classify thinking patterns that children used. The initial analysis of data was carried out by the researcher in 1993 and further analysis of the visualisation task (Structure Task 1) took place in 1994 after collaboration with Gerald Goldin. The interview protocols and coded responses are provided in Appendix B.

Counting tasks

Table 5.9 shows the classification of the responses to the counting tasks.

Table 5.9: Counting Strategies for Mental Calculations

Strategy	Description
Counting all	Counting each item by ones
Counting on	Counting on from a given number by ones or in equal groups.
Multiple counting	Counting equal groups of items, total related to multiplication
Skip counting	Reciting the count of multiples of a particular number (not intervening numbers)
Rhythmic counting	Counting by ones but in a particular pattern which emphasises the multiples of a particular number.
Double counting	Counting all or skip counting with a simultaneous count of the number of groups at the same time, e.g. "one, two, three (one); ... four, five, six (two); ..."
Relate to known double	Using a known double to find another fact
Builds ten or bridges ten	Partitions one of the numbers so that the other number can be built up (or down) to ten.
Known addition fact	Retrieving an addition fact automatically with no apparent counting.

Grouping tasks

For Task 4, responses to a word problem involving partitioning a collection of items into an array, were coded as: concrete or pictorial modelling; double counting, or relating to known multiplication facts. The quotition task (Task 6) responses were coded according to the use of concrete materials, the drawing of pictorial representations, counting the groups of items on fingers, building up groups mentally (double count), taking away groups mentally (building down) or relating to known multiplication facts.

Responses to Tasks 8 and 9 were coded as guessing, counting each package as one, counting by ones, guessing number in packages, counting each item as one (correct answer), counting in tens and ones without coordination, counting in tens and ones with coordination, counting-on from a multiple of ten by ones or knows answer through subitising quantities of each place value.

Responses to Tasks 13 and 15 were coded as attempts to count by ones, attempts to restart and count by hundreds, tens and ones separately, miscounting-on groupings as ones, miscounting on ones as groupings, attempts to start at ten (or a hundred) and count on by ones, does not count-on but restarts and collects units of the same rank or counts-on by hundreds, tens and ones as appropriate.

Regrouping tasks

The responses to the regrouping tasks were categorised as counting, separation (collecting units of the same rank or written algorithm mentally), aggregation (one number kept whole and the other number separated into place values) or holistic strategies (decomposition and regrouping or compensation). This classification is adapted from that used by Cooper, Heirdsfield and Irons, 1995.

Place value tasks

Responses to the digit correspondence tasks (Tasks 1 and 13) were coded as no meaning given to digits, associated digits incorrectly with groupings presented, interpreted digits by their face values or correctly interpreted digits by their total values. Responses for Task 3 which investigated the interpretation of the meaning of '01' were coded as unsure of meaning, interpreted incorrectly as ten or interpreted correctly as one or first. Interpretations of the non-standard numeric representation in Task 15 were coded as one ten and thirteen ones, 11.3, 113, 14 or correctly as 23.

Structure tasks

Responses to Task 2 which explored the use of ones, tens and hundreds structure were coded as: counting all by tens and ones; counting by hundreds, tens and ones; counting-on from 200 by tens and ones, or subitise the number of hundreds, tens and ones and interpret as correct number.

Responses to Task 4 which required as few coloured counters as possible to show the 3-digit number, were coded as: no response; 246 blue; 20 yellow counters, 4 red and 6 blue counters, or correctly as 2 yellow, 4 red and 6 blue counters. Responses for the explanation of how to represent a 4-digit number (Task 5) were coded as: no response; 100 yellow, 20 red and 46 blue counters; 12 yellow, 4 red and 6 blue counters, or choose a new colour to represent ten yellow counters.

For Tasks 6 and 9, incorrect responses for the missing addend of 84 were coded as: no response; 26 or counting-on by ones to give 15. Correct responses were coded as: counting-on by ones to give 16; adding ten and six to give 16, or knows answer as 16. For the addition and subtraction tasks (Tasks 7-9) correct responses were coded as: counting by ones; mental calculation, or uses pattern of tens on the hundred square.

For the visualisation tasks (Task 1 and 18) the external representations of the counting sequences 1-100 and 1-1000 were categorised and coded according to three dimensions:

(a) the type of imagistic representation identified by the pictorial, ikonic and notational recordings;

(b) the level of creative structural development of the number system, and

(c) evidence of a static or dynamic nature of the image.

Table 5.10 describes the classifications of representation for each of these dimensions by mode, type of structure and nature of image. Although the tasks had been formulated to explore children's possible structure, it was later found that there were similarities to the early work of Galton (1880) as outlined in the literature review in Chapter 3. Section 3.3 discussed the research literature related to children's representations of the numeration system and details the work of Goldin which has had an important impact on the analysis of these tasks.

Table 5.10: Classification of Representations by Mode, Structure and Nature of Image

Mode of Representation
Concrete/pictorial representations: - imagery where the child draws or describes objects that do not have any quantitative relationship to the numbers.
Ikonic representations: - the pictorial imagery relates to a quantity.
Notational: - conventional numerals are used to represent numbers.
Type of Structure
No structure: - objects, pictures or numerals which have no apparent relationship to equal groupings or sequence.
Linear structure: - a linear formation (straight or curved), numbers appearing in sequence.
Emerging structure: - one hundred represented by equal groups of objects, or linear sequence broken into equal segments.
Emerging structure - related in some way to multiplication including multiple count and multiplication grid.
Partial array structure: - objects, pictures or numerals in rows and columns but not a pattern of ten tens.
Array structure: - objects, pictures or numerals in a pattern of ten tens
Nature of the Image
Static: - the image as a fixed representation.
Dynamic: - the image as a representation that is changing and/or moving.

5.4.3 Additional analysis of visualisation task

Chapter 8 will explore the responses of children to the visualisation task (Structure Task 1). The data that is analysed comes from the pilot study, the main cross-sectional study and a further study with a high ability sample consisting of 92 children from Grades 3-6, assessed by teachers for participation in a Program for Gifted and Talented children.

5.5 SUMMARY

In this chapter an overview of the methodology for the main study has been presented. On the basis of previous research and the pilot study, the cross-sectional research provides a broader framework for characterising numeration and may identify why some tasks are more difficult than others. By analysing and classifying solution strategies, some new knowledge can be gained about how the numeration system is acquired and developed. The identification and description of levels in children's solution strategies, may also provide further information about how children develop this understanding and make the connections between counting, grouping, regrouping, place value and structure.

The analysis of the cross-sectional data is described and discussed in Chapters 6 and 7. In Chapter 6, item response analysis for the tasks completed by children in Grades 4 to 6 is reported using Harnisch and Rasch models and an analysis of the children's performance and solution strategies for each of the eighty nine tasks is presented. Chapter 7 discusses aspects of children's understandings of the numeration system, specifically considering the way children recognise place value structure, the way their calculations draw on place value knowledge, the significance of recursive groups of ten and their understanding of the meaning of multiplication.

CHAPTER 6

AN OVERVIEW OF RESULTS: PERFORMANCE AND STRATEGY USE

In the following chapters, Chapters 6, 7 and 8, an analysis of data is presented and discussed using a combination of quantitative and qualitative methods. In Chapter 6 Harnisch S-P Curve and Rasch model analyses are undertaken in order to provide an overview of individual performance by tasks. Second, the main section (6.2) provides a detailed descriptive account of results by task categories. Following this, Chapter 7 will discuss the main findings in relation to other studies. Tasks will be grouped together in ways that enable discussion of various aspects of the children's understanding of the numeration system. The results of all tasks are presented in Appendix B. Additionally, Chapter 8 provides an in-depth descriptive analysis of children's representations of the number sequence 1-100 (Structure Task 1).

Chapter 6 will provide an overview of the results illustrating the performance and strategy use by grade level. First, the general pattern of performance is reported for the thirty seven tasks that were completed by all children in Grades 4, 5 and 6. Item response analyses related to Student-Problem curve theory and the Rasch model were carried out. It was considered important to obtain some statistical analysis of the items to complement the qualitative analysis of student responses in this study. It was only possible to undertake this item response analysis on selections of tasks which were completed by all children in designated grades. Because the focus of the study was on developing structure in the number system it was decided that the most appropriate tasks to analyse in this way would be those completed by children in the upper primary grades.

Chapter 6 presents further analysis of the children's performance and solution strategies for each of the eighty nine tasks which have been organised into six sections corresponding to each of six categories of tasks. These categories, as discussed in Chapter 5, comprise counting, grouping, number sense, regrouping, place value and structure of the number system. The results for each of the tasks are reported separately, with performance and solution strategies being compared across grades. It will be noted that results in this chapter have been presented in graphical form, with the full data set tabulated in Appendix B. Some excerpts from interview transcripts are provided to highlight particular variations in children's responses. Some results highlight the children's performance on particular aspects of numeration, whereas for other tasks an analysis of children's correct and incorrect strategies is reported.

This chapter is organised as follows:

6.1 AN OVERVIEW OF RESULTS

Student-Problem (S-P) curve theory (Harnisch, 1983, 1987; Harnisch & Linn, 1981; Switzer & Connell, 1990) and the Rasch model (Rasch, 1980) were used to investigate the overall level of performance of a cohort of the sample in the content area of numeration and the resulting item response patterns. A Student-Response (S-P) Chart was constructed for the 37 tasks that were completed by all children in Grades 4 to 6 (n=55) and item response patterns investigated. The results were then further examined in relation to the Rasch model.

6.1.1 Student-Problem (S-P) curve analysis

The S-P Chart is a matrix in which students (rows) are arranged from top to bottom in descending order of total test scores and the tasks (columns) are arranged from left to right in ascending order of difficulty. Table 6.1 associates students with (1) their performance score; (2) their Modified Caution Index (MCI) (see Harnisch & Linn, 1981, for the computational formula used) and Modified Caution Signal which is based on their performance level and response pattern and (3) their responses to each task. The second and third columns contain each child's raw and percentage scores. For example, in Table 6.1 Jonathon (Grade 6) has a raw score of 36 and has answered 95% of the tasks correctly.

Table 6.1: Student-Response Chart

	Grade	Score	%	MCI		P14	S2	S3	S7	R6	P2	S8	N7	N10	N4	S9	R9	P13	S6	N8	G15	N5	P15	P16	S11	S14	S23	S12	R7	S24	N11	N9	P3	R10	S25	R11	S22	S26	S21	S10	N12	S13
Jonathon	6	36	95	0.11	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
Dale	6	36	95	0.11	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
Bradley	6	34	89	0.08	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	1	0
Mati	6	34	89	0.13	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
Kerryn	5	34	89	0.04	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0
Andrew	5	34	89	0.04	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0
Jack	5	34	89	0.06	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
Nik	4	33	87	0.02	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0
Jenna	6	32	84	0.02	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0
Mark	6	32	84	0.08	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0
Tony	4	31	82	0.14	A	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	0	0
Kate	6	31	82	0.12	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	0	0	1	0
Michelle	5	30	79	0.13	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	1	1	0	1	0	0	1	1	1	0
Melanie	6	29	76	0.07	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	0	1	1	1	1	1	0	0	0	0
Anna	6	29	76	0.09	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	1	0	0	1	1	1	0	0
Phillip	5	28	74	0.1	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	0	1	1	0	0	0	0	1	0	1	0
Stacey	6	28	74	0.07	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	0	0	0	0	1	0
Kelly	6	28	74	0.11	A	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	0	0	0	0	0
Luke	4	27	71	0.13	A	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	1	1	0	1	0	0	0	0	1	0
Victoria	6	27	71	0.1	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	1	1	1	0	1	0	1	1	0	1	0	0	0
Joel	5	27	71	0.07	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	0	1	0	0	1	0	0	1	0
Ivan	4	27	71	0.03	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	0	0	0	0
Jonathon	5	27	71	0.08	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	0	1	0	1	0	0	0	1	1	0	0	0
Kristy	6	27	71	0.05	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	0	1	1	0	0	0	0
Lisa	4	26	68	0.04	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	1	1	0	1	0	1	0	0	0	0	0	0
Deepack	6	26	68	0.07	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	1	0	0	0	1	0	0	0	0	1	0
Carla	5	26	68	0.04	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0	1	0	1	1	0	0	0	0	0
Tim	5	26	68	0.13	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	0	1	1	0	1	1	0	0	0	1	1	0	1	1	0	0
Christop	4	25	66	0.08	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	1	1	0	1	0	1	0	0	1	0	0	0
Trevor	5	25	66	0.09	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0
Brad	5	25	66	0.08	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0	1	0	0	0	0	0	0	1	0	0
Michael	5	24	63	0.04	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0
Ty	6	24	63	0.09	A	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	1	0	0	1	0	0
Tammy	6	24	63	0.05	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0
Marc	5	23	61	0.07	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	0	1	0	0	0	1	1	1	0	1	0	0	0	0	0	0
Trent	4	23	61	0.03	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0
Rhys	4	23	61	0.15	A	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	0	1	0
Merridy	4	22	58	0.16	A	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1	0	1	1	1	1	1	0	0	1	0	0	0	1	0	0	0	0	1	0	0
Matthew	4	21	55	0.05	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0
Amy	4	21	55	0.04	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
Alison	5	21	55	0.03	A	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Gavin	4	21	55	0.02	A	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Scott	6	20	53	0.03	A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Gemma	4	19	50	0.02	C	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Nicole	4	19	50	0.11	C	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	1	1	0	1	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0
Haley	5	18	47	0.01	C	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Jocelyn	6	18	47	0.06	C	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
Rania	5	17	45	0.07	C	1	1	1	1	1	1	0	1	1	1	1	1	0	1	1	1	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Melissa	5	16	42	0.12	C	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
Brooke	4	16	42	0.13	C	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0
Brian	6	15	39	0.04	C	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Michelle	4	13	34	0	C	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Stacey	5	9	24	0.19	C	1	1	1	0	1	0	0	1	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Melinda	4	9	24	0.13	C	0	1	0	1	0	1	1	0	1	0	1	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Kylie	4	8	21	0.04	C	1	0	1	1	0	1	1	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total						54	54	54	54	53	53	53	52	52	52	52	50	50	47	46	45	41	40	34	34	34	33	32	31	28	27	26	25	24	24	22	20	19	16	14	11	2
Item difficulty level						1.82	1.82	1.82	1.82	3.64	3.64	3.64	5.45	5.45	5.45	5.45	9.09	9.09	14.5	16.4	18.2	25.5	27.3	38.2	38.2	38.2	40	41.8	43.6	49.1	50.9	52.7	54.5	56.4	56.4	60	63.6	65.5	70.9	74.5	80	96.4
MCI						0.04	0	0.04	0.25	0	0.42	0.16	0.13	0.33	0.06	0.3	0.22	0.03	0.09	0.03	0.19	0.14	0.08	0.09	0.07	0.25	0.06	0.03	0.06	0.19	0.05	0.08	0.26	0.02	0.07	0.13	0.04	0.1	0.08	0.29	0.42	0
						Y	Y	Y	Y	Y	Z	Y	Y	Z	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	W	W	W	W	W	W	W	W	W	W	X	W

S - Curve

P - Curve

The fourth column of the S-P Chart contains the Modified Caution Index (MCI). It gives a measure of an individual's response pattern. An MCI value of 0 corresponds to an 'ideal' student, i.e. one whose response pattern is known from their score. MCI's which approach 1 indicate that a student appears to be responding inconsistently, such as missing easy items while answering correctly difficult tasks. In general, one cannot tell whether a child is responding randomly based on the pattern of item responses as there are a variety of factors that could lead to an unusual response pattern. This could include differences in school content coverage, background experiences, motivation, and instructional history. For example, Luke and Kristy both scored 27 (71%) but have different Modified Caution Indices of 0.13 and 0.05 respectively. The greater MCI for Luke indicates that the response pattern was more random and that more caution should be used in interpreting the score 27 for Luke than for Kristy.

The fifth column contains the Modified Caution Signal (MCS) which represents the results of classifying children according to both their total score and their response pattern. Specifically, the MCS refers to each child's classification with respect to test performance (high or low) and MCI (high or low). The Modified Caution Signal (MCS) is an aid for quickly picking out those children with unusual response patterns. An 'A' signifies an adequate total score (>50%) and a consistent response pattern (MCI ≤ 0.3). A 'B' signifies an adequate total score (>50%) but an inconsistent pattern (MCI > 0.3). A 'C' indicates a low total score but a consistent pattern, while a 'D' indicates a low total score and an inconsistent pattern. Sato (1975) suggests that classification in each of these four cells identifies children who are doing everything fine (A), are making careless mistakes (B), are in need of more study or possess sporadic study habits (C), or have insufficient readiness (D). In this sample all children were classified as either signal A or C.

Modified Caution Signals for items are assigned in a similar manner and are shown in the last row of Table 6.1. An item that is relatively hard and has a high MCI was assigned an MCS of 'X'. A hard item with a low MCI was assigned a 'W', an easy item with a low MCI was assigned a 'Y' and an easy item with a high MCI was assigned a 'Z'.

The S-curve on the S-P Chart tells us that Michelle (MCI of 0) has given no incorrect responses to tasks to the left of the S-Curve. Nik and Jenna who both have a MCI of 0.02 have each only given an incorrect response to a task to the left of the S-Curve (easier tasks).

Examination of the pattern of task responses with respect to the P-Curve provides information about each respective task. A task with no incorrect responses above the P-Curve indicates an 'ideal' task i.e. a task failed by no one who got a total score higher than anyone who passed them. Tasks S2, R6, and S13 fit this criteria. Harnisch (1983) suggested that analysis of classifications assigned to tasks can be interpreted according to their difficulty level and modified caution index. Tasks N9, P3, R10, S25, R11, S22, S26, S21, S10 and S13 are difficult items which discriminated high performers from low performers (Signal W). Task N12 with a difficulty rating of 80% and a modified caution index of 0.42 (Signal X) is a task which possibly contains ambiguous phrases or covers unusual content. Tasks P2 and N10 (Signal Z) are tasks which may need revision because of the disparity of student performances. The majority of tasks though are Signal Y (shown in Table 6.1) and are helpful in discriminating among low performers.

It would be expected that there would be some disparity between the S- and P- Curves as children in this sample came from three different grades and so should not be homogeneous in achievement. We would expect such children to be at various levels of understanding of numeration. However the item response analysis discussed here has shown a greater than expected clustering of performances of the children on these tasks demonstrating a general lack of progression of the children over the Grades 4 to 6.

6.1.2 Item response analysis - Rasch model

Item response theory was employed to examine the manner in which the items on the test acted as a cohesive set. The analysis examined the fit of the data to the Rasch model (Rasch, 1980). Item response models assume that both person ability and item difficulty can be measured using the same units. Where this can be demonstrated, the odds of success on each item can be calculated by comparing the estimates of student ability and the item difficulty. Unexpected patterns of response can then be used to identify secondary factors affecting student performance. In this study item response theory was used specifically to measure the reliability of the individual tasks in terms of student ability. The thirty seven tasks that were completed by all children in Grades 4, 5 and 6 were analysed.

Table 6.2: Individual Task Item-Fit Chart

Test of Fit Power EXCELLENT
Analysis Title: ana2

INDIVIDUAL ITEM-FIT

Label		Location	SE	Fit	ChiSq	Probability
Ex001	p14	-4.081	1.15	-0.344	0.164	0.919
Ex002	s2	-4.337	1.28	-0.312	0.342	0.839
Ex003	s3	-4.082	1.15	-0.344	0.164	0.919
Ex004	s7	-4.214	1.21	-0.306	0.241	0.884
Ex005	r6	-3.305	0.86	-0.536	0.417	0.807
Ex006	p2	-2.378	0.65	0.541	12.876	0.000
Ex007	s8	-3.022	0.79	-0.265	0.257	0.876
Ex008	n7	-2.332	0.64	-0.223	0.474	0.784
Ex009	n10	-2.052	0.60	0.016	1.744	0.402
Ex010	n4	-2.672	0.71	-0.507	0.574	0.744
Ex011	s9	-2.105	0.60	0.209	9.168	0.000
Ex012	r9	-1.483	0.52	0.006	0.577	0.743
Ex013	p13	-1.793	0.56	-0.718	1.082	0.571
Ex014	s6	-1.034	0.46	-0.409	0.538	0.758
Ex015	n8	-0.852	0.44	-0.793	0.950	0.612
Ex016	g15	-0.595	0.42	0.095	1.204	0.536
Ex017	n5	0.146	0.37	0.017	2.273	0.303
Ex018	p15	0.221	0.36	0.580	4.318	0.092
Ex019	p16	0.803	0.34	0.206	3.003	0.202
Ex020	s11	0.847	0.34	-0.641	0.547	0.754
Ex021	s14	0.841	0.34	0.886	13.354	0.000
Ex022	s23	1.032	0.33	-0.712	0.707	0.694
Ex023	s12	1.028	0.33	-1.192	4.538	0.079
Ex024	r7	1.234	0.33	-0.601	2.130	0.327
Ex025	s24	1.521	0.33	0.428	2.002	0.351
Ex026	n11	1.614	0.33	-0.379	1.049	0.581
Ex027	n9	1.761	0.33	-0.371	7.496	0.000
Ex028	p3	1.656	0.33	2.120	8.508	0.000
Ex029	r10	2.043	0.33	-1.384	4.424	0.086
Ex030	s25	1.958	0.33	-0.405	2.372	0.287
Ex031	r11	2.163	0.33	0.157	4.437	0.085
Ex032	s22	2.500	0.34	-0.857	1.128	0.557
Ex033	s26	2.510	0.35	0.049	0.198	0.903
Ex034	s21	2.902	0.36	-0.507	1.282	0.514
Ex035	s10	2.892	0.36	1.582	7.337	0.000
Ex036	n12	3.381	0.39	-0.048	2.594	0.254
Ex037	s13	7.284	1.11	-0.407	1.785	0.394

The item calibration results in Table 6.2 are reported by reference to five measures: the logit measure (Location), standard error of the logit (SE), the infit (FIT) mean squares, the Chi Squares and probability ratings. The logit measure is the location on the common metric. This is standardly labelled using 0 as the centre of the scale. In terms of item calibration, the common metric is oriented along item difficulty levels. Thus, positive values like those for Tasks S13 and N12 (7.284 and 3.381 respectively) indicate higher difficulty levels and negative values like those for S2 and S7 (-4.337 and -4.214 respectively) indicate easier items. The standard error associated with each item indicates the precision of the item logit level. The Fit is the index of discrimination for each item estimated within the item response theory. The estimation is directed to the degree of consistency of an item to discriminate levels of student ability. The analysis is reported in terms of the match between model estimated and the actual data. The expected value of the match is 1. The values of Fit for most tasks in this study were under 1 which indicates the match was better than expected.

It can be seen from Table 6.2 that the Fit for most tasks is overfit (i.e., the fit is less than 0.8) which means that the test fits the students well. This normally means that the elements in the sample are clustered and so there is a low generalisability to other populations. In this analysis the sample included children from three grades and so it would have been expected that there would be a range of abilities. The result of the item response analysis of overfit for most tasks is interpreted as indicating a far greater clustering of children's performance on these tasks than would be expected. This demonstrates that there was little difference in achievement levels of children across Grades 4, 5 and 6.

Results of the item response analysis show that the test was well chosen for this sample of children. There was an excellent level of fit to the Rasch model with an item separation index of 0.90 and reliability given by the Cronbach Alpha of 0.90.

The 37 tasks were grouped according to 4 categories (regrouping, number sense, place value and structure tasks) and the average logit difficulty ratings were calculated from the location estimates in Table 6.2. The category of structure tasks had the highest difficulty rating (Logit of 0.384) suggesting that the sample of children need further development of their understanding in this area of numeration. There were a number of the structure tasks (namely items 25, 22, 26, 21, 10 and 13) which had high individual difficulty ratings indicating that the understanding associated with these items was poor. This is important in terms of the purpose of the study, which was to assess the aspects of structure of the numeration system. Thus this analysis supports the assertion that the most difficult aspects of understanding numeration are those relating to working with multiple groupings, relating numeration block representations of multiunit values for large numbers and using the multiplicative property of the number system.

6.2 PERFORMANCE AND STRATEGY USE

6.2.1 Performance and strategy use: Counting

This section will report children's performance and strategy use for the eight counting tasks. It will be recalled from Chapter 5 that these counting tasks assess the children's counting strategies, including skip and double counting, and the application of these to additive and subtractive situations, and to practical problems. It will be made clear that there were wide differences in the performance and strategy use of individual children across tasks. An analysis of children's solution strategies for each task are now reported. The number of children in each grade attempting each task is given in brackets on the graphs. The data illustrated in Figures 6.1 to 6.6 are reported in Tables B.1-3, B.9, B.10 and B.19 respectively in Appendix B.

Figures 6.1, 6.2 and 6.3 illustrate the performance on addition tasks across Grades K, 1 and 2 by solution strategy. A variety of correct counting strategies was used to solve the addition, missing addend and removed item tasks, with four main categories reported here.

Counting Task 1: Addition

This task required children in Kindergarten and Grades 1 and 2 to add two quantities (5 + 7) that were screened from view.

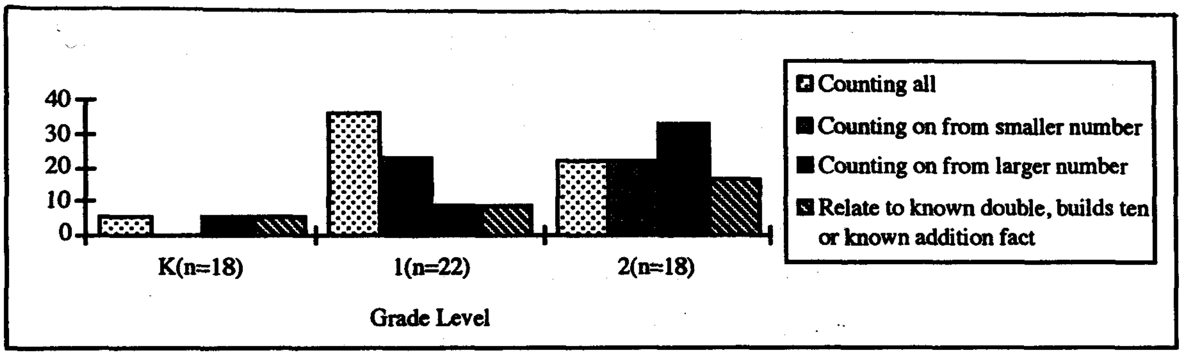

Figure 6.1 Solution strategies for Counting Task 1, Addition: Percentage of sample giving correct responses, by strategy use.

Figure 6.1 shows that the performance on the addition task was very low for Kindergarten children (17%) and high for children in Grades 1 and 2 (77% and 94% respectively). However, many Grade 1 and 2 children used an inefficient counting-all strategy. The counting-all strategy was dominant at Grade 1 (36%) but decreased at Grade 2 (22%). In Grade 1 a larger percentage of children (23%) counted on from the smaller number (the number that had been given first) than those who counted on from the larger number (9%). This preference was reversed in Grade 2.

Counting Tasks 2 and 3: Missing addend and removed item

Task 2 required the children to find the missing addend from two given quantities (12 8), whereas Task 3 required them to find the quantity that had been removed from a known quantity (10 - 7). Performance and strategy use for Task 3 was almost identical to that for Task 2 and so a graph is not shown.

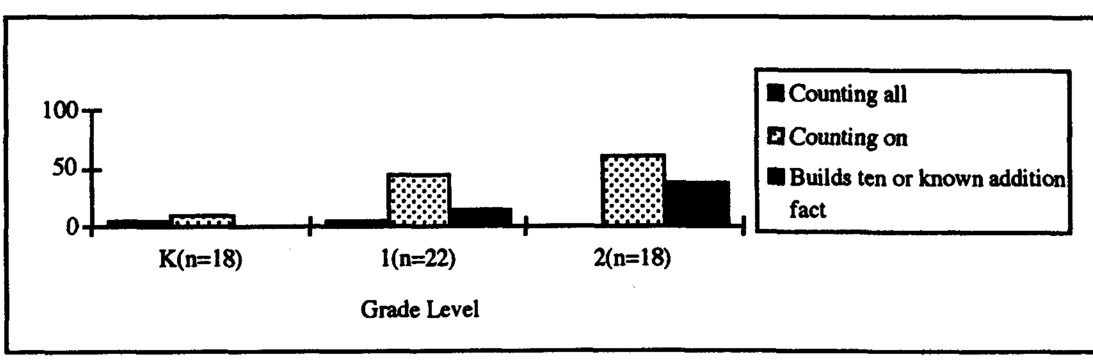

Figure 6.2 Solution strategies for Counting Task 2, Missing Addend: Percentage of sample giving correct responses, by strategy use.

Figure 6.2 shows that performance on the missing addend task (Task 2) was generally low for Kindergarten and Grade 1 children (17% and 64% respectively), but in contrast to the addition task (Task 1) very few children used the inefficient counting-all strategy. With increasing grade level, use of the counting-all strategy was reduced and there was increasing use made of counting on, building ten, and use of known number facts. Note that there was no use made of counting down. There were many Grade 2 children (39%) who used more sophisticated strategies of building ten and known addition facts on both Tasks 2 and 3. However over 60% of second graders were still using elementary counting strategies rather than those based on grouping ten. It should be noted that it is not possible to determine whether the children using known fact strategies were aware of the significance of ten in their calculations.

Counting Tasks 4 and 5: Counting by tens and threes

Tasks 4 and 5 required the children to rote count by tens and to count a collection of screened objects by threes. Skip counting performance on Tasks 4 and 5 is shown in Figure 6.3. Note that Task 4 was given to Grades K-3 only.

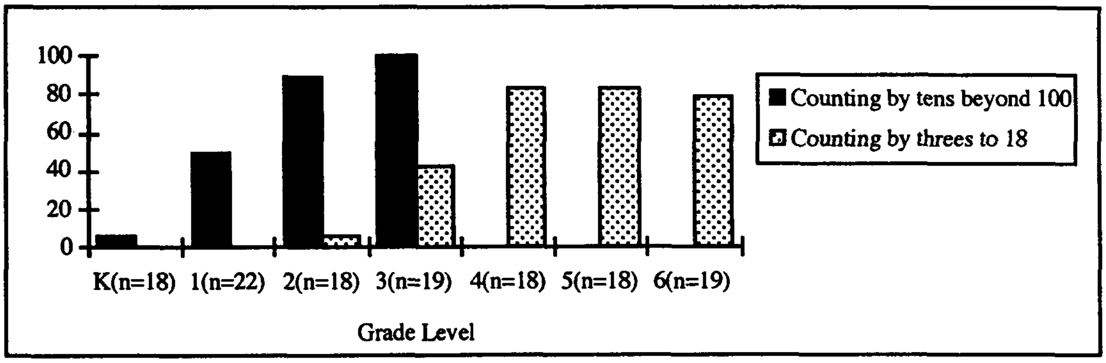

Figure 6.3 Performance on Counting Tasks 4 and 5, skip counting: Percentage of sample giving correct responses, using skip counting.

There was a sharp increase in performance on counting by tens during Grades 1 and 2 and for counting by threes during Grades 3 and 4. However, only 83% and 79% of children in Grades 5 and 6 respectively could skip count by threes to 18 correctly.

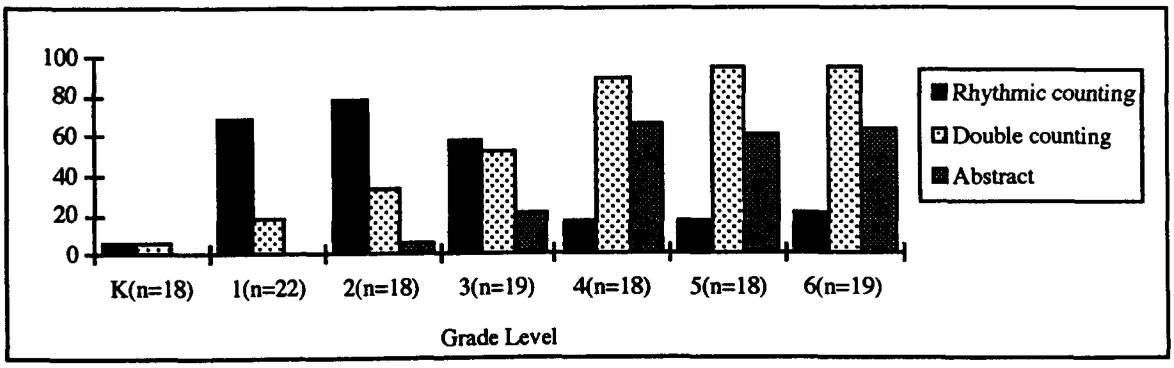

Figure 6.4 Solution strategies for Counting Task 5, Counting in threes: Percentage of sample giving correct responses, by strategy use.

Figure 6.4 shows strategy use by grade level on the 'counting by threes' task. Note some children used more than one strategy. By the end of Grade 1, 68% of children were able to count by threes although many (18%) needed to use their fingers to count on. Rhythmic counting (where the individual shells were counted softly or silently in order to give the rhythm of the pattern of threes) was the dominant strategy used by children in Grades 1, 2 and 3 (68%, 78%, 58% respectively). Double counting, where the child keeps track of the count in threes, emerged as a strategy for many children in Grade 2 (33%) and Grade 3 (52%), and was dominant in Grades 4, 5 and 6 (89%, 94%, 95% respectively). This was characterised by children who needed to keep track of their double count by using fingers or writing tally marks, and those who could keep track mentally. These children were successful in stopping on the count of 18. The skill of double counting at an abstract level, requires the child to keep a mental record of the number of counts. Other children double counted by using knowledge of multiplication to know when to stop the multiple count. This abstract strategy began to be used in Grade 3 and became more apparent in Grades 4, 5 and 6, although about a third of these children still did not use the strategy.

Counting Tasks 6, 7 and 8: Application of counting to money

In these tasks children were asked to calculate the number of twenty cent coins that are equivalent to $1.00, $3.00 and $10.00. Note that the tasks were not given to Grade 6 children.

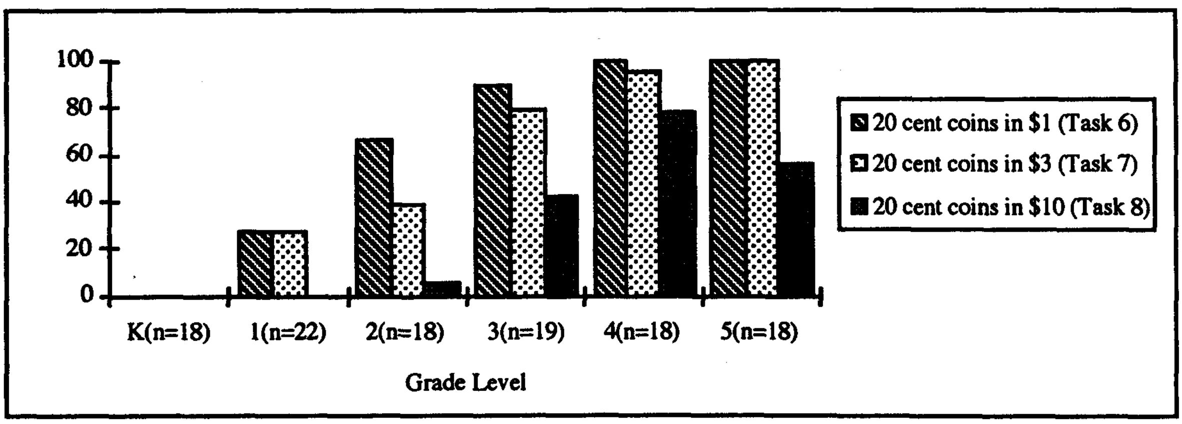

Figure 6.5 Performance on Counting Tasks 6 to 8, Double counting: Percentage of sample, using double counting.

In comparison with Task 5, Figure 6.5 shows that there were higher percentages of children in Grades 1 to 5 who used double counting for Tasks 6 and 7, possibly because it was easier to count in twenties. However, performance on Task 8 was lower; there was a marked decrease in double counting during Grade 5 (only 56% correct). Many children who had been able to correctly calculate the number of twenty cent coins that are equivalent to $1.00 and $3.00 were not able to do the same for $10.00. For example, Carla (Grade 5) very quickly gave the correct answers for Tasks 6 and 7 but when asked for the number of twenty cent coins that are equivalent to $10.00 answered: " ... five thousand ... no ... five hundred ... how do you divide twenty into ten dollars? ... ". Although Carla knew the number of twenty cent coins that are equivalent to $1.00 and $3.00 she was very hesitant and guessed an answer for $10.00, showing no estimation skills and then acknowledged a solution approach but did not have the confidence to carry it through.

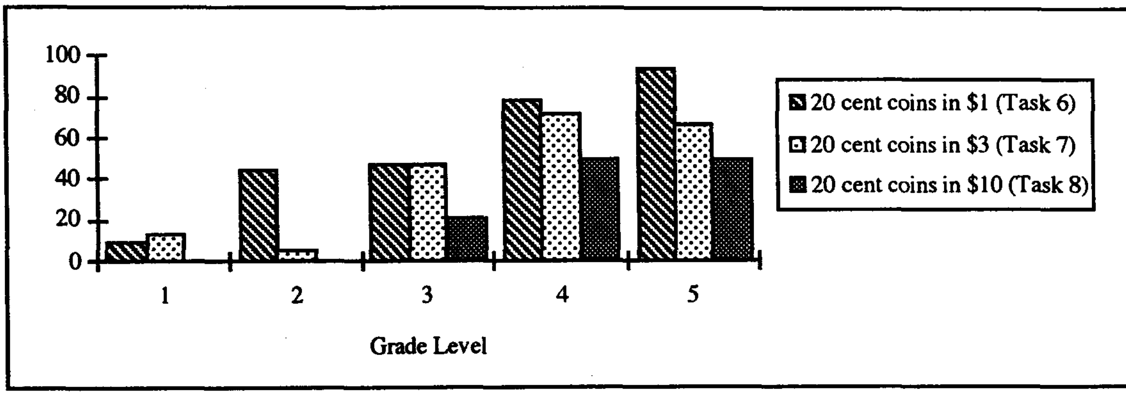

Figure 6.6 Performance on Counting Tasks 6 to 8, Multiple counting: Percentage of sample using multiplication or division to determine the quotient for each task.

The percentage of children who used multiplication or division to solve the money tasks (Tasks 6, 7 and 8) increased from Grades 1 to 4, with some reduction of performance at Grade 5 (Tasks 7 and 8), and was generally higher than the use of multiplication or division in Task 5 [to determine when to stop the multiple count of three]. Figure 6.6 shows that use of multiplication or division peaked at Grade 4 with 72% of the children using their solution from Task 6 to solve Task 7, and 50% using their solution from Task 6 to solve Task 8.

Summary: Counting Tasks 1 to 8

Children's counting skills are critical in their developing understanding of how the numeration system works. The solutions to counting tasks shown here have indicated that some children at the end of their first year of schooling (Kindergarten) are using abstract counting strategies to mentally solve number problems involving addition and subtraction (Tasks 1, 2 and 3). However, the performance levels for children in the higher grades tended to level off or even fall away in some cases. This was evident in the relatively simple task of skip-counting by threes (Figure 6.4), the more sophisticated tasks of using abstract double counting in Task 5 (Figure 6.5), and the calculation of the number of twenty cent coins that are equivalent to $10.00 (Figure 6.6). Performance levels up to Grade 4 appeared very good but results did not suggest similar progress in the later grades.

6.2.2 Performance and strategy use: Number sense tasks

This section will report children's performance and strategy use for the twelve number sense tasks. These number sense tasks assess the use of tens, hundreds and thousands in number tasks (some related to money and measurement contexts) and the use of compensation of ones and tens in mental addition. The data illustrated in Figures 6.7 to 6.10 are reported in Tables B.4, B.20, B.21, B.23 in Appendix B.

Number Sense Tasks 1, 2 and 3: Addition to ten and applications

Task 1 required the children to find numbers that add to give ten. Tasks 2 and 3 required them to find the change from $1.00 after a purchase of 64c and to devise two other combinations that make a dollar. Tasks 4 and 5 required them to calculate the length 5cm less than the measurement 1m and to devise two more combinations that make a metre. It will be noted that Task 1 was only asked at Grades K, 1 and 2 levels, Tasks 2 and 3 were only asked from Grades 3 to 5, and Tasks 4 and 5 only asked from Grades 4 to 6.

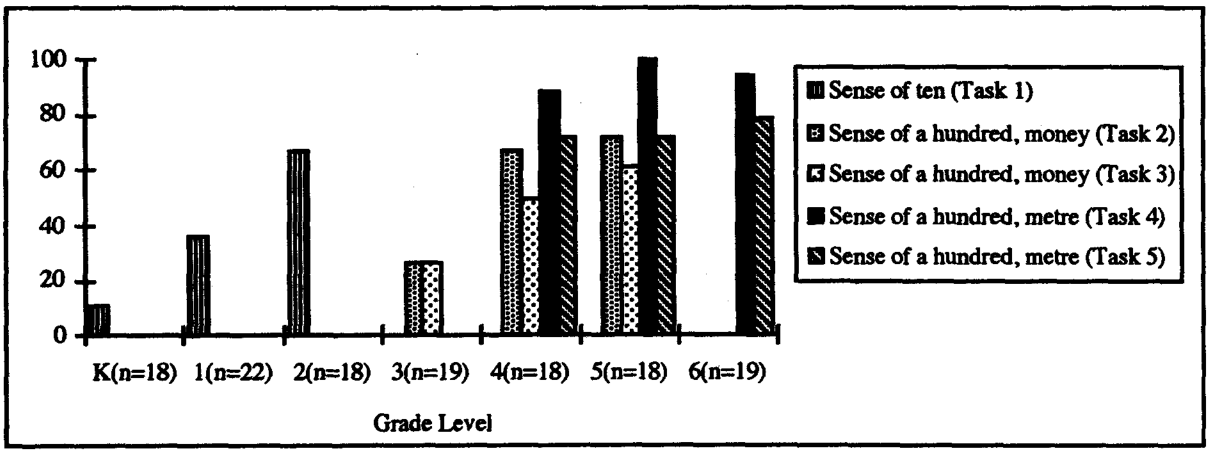

Figure 6.7 Performance on Number Sense Tasks 1 to 5: Percentage of sample partitioning tens and hundreds, on separate tasks.

Figure 6.7 compares performance on Tasks 1 to 5. For Task 1, there was a sharp increase in children's ability to create number combinations that add to ten from Kindergarten to Grade 2 (11% to 66%). There were few children who could create number combinations that add to ten in Kindergarten (11%). At Grade 2, 67% were able to do so (hence many children were still unable to generate simple addition facts to tens). Similarly, the ability to create number combinations that add to one hundred (Task 2) developed markedly from Grades 2 to 4 (0% to 61%) but there was little improvement in Grade 5. At Grade 5, a higher percentage of children decomposed a hundred in the context of length (Tasks 4 and 5) than in the context of money (Tasks 2 and 3). It is considered that performance on Tasks 2, 3 and 5 were poor at Grade 5 level (being below 70%) indicating that many children do not live up to expectations expressed in curricula documents. This illustrates a trend emerging in the results that indicates that children in the higher grades performed below expectation.

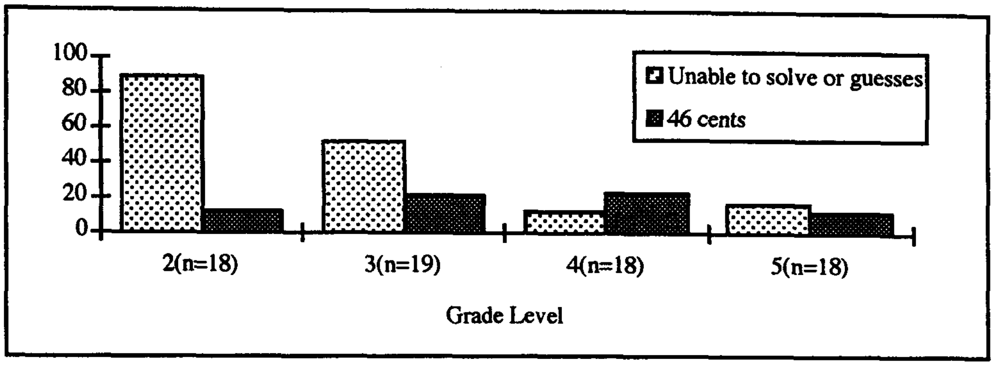

Figure 6.8 Incorrect solution strategies for Number Sense Task 2, Partition \$1: Percentage of sample giving incorrect responses, by strategy use.

Figure 6.8 compares children's incorrect responses to Task 2 (change for 64c from \$1.00). There were a substantial number of children who treated the tens and ones separately and hence gave the answer 46 cents (11%, 21%, 22% and 11% at Grades 2 to 5 respectively).

Overall the performance on money tasks (Tasks 2 and 3) was considered to be poor in terms of what might be expected for a real life context for number.

Number Sense Tasks 6 to 9: Adding using ten frames, estimation, hundred square and one thousand

Task 6 required the children to add using ten frames (6 + 9) whereas Task 7 required them to estimate (16 + 9 = 115 as unreasonable). Tasks 8 and 9 required the children to create number combinations that add to 100 and 1000. The performance on Tasks 6, 7, 8 and 9 are compared in Figure 6.9 and reflect marked differences in difficulty of tasks. Note that Task 6 was given to Grades 1 to 4 and Tasks 7, 8 and 9 were given to Grades 2 to 6.

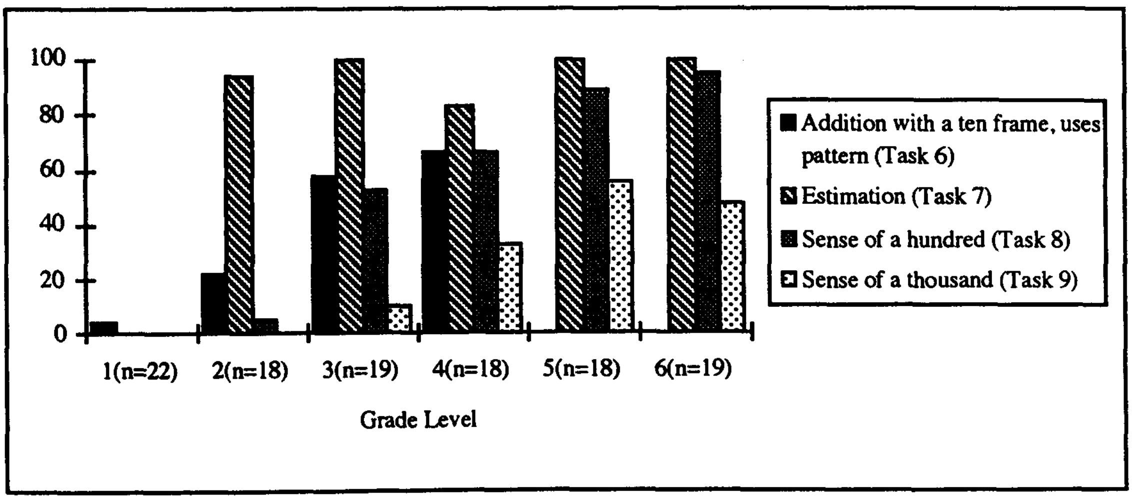

Figure 6.9 Performance on Number Sense Tasks 6, 7, 8 and 9: Percentage of sample giving correct responses, showing number sense.

The percentage of children who correctly used the pattern of ten in Task 6 ranged from a low 5% at Grade 1 to 67% at Grade 4. The majority of Grade 1 children (68%) counted the total number of dots shown, some counting unsuccessfully, others (27%) ignoring the structure of the pattern used a counting-on strategy. Although all children at Grades 2, 3 and 4 successfully calculated the total number of dots, the reliance by many children on unitary counting indicated that the ten frames had not prompted the expected recognition and use of pattern. In contrast, a high percentage of children in Grades 2 to 6 had a sense that the written sum of 16 and 9 (Task 7) was unreasonable. Even the majority of Grade 2 children (94%), who were still developing understanding of place value, identified the given answer of 115 as unreasonable. The reasons given varied from those children who 'sensed' that it was too big, to those children who compared or rounded off numbers to estimate. Others calculated or simply knew the correct answer as a basic fact. The following excerpts illustrate some of the successful reasoning that children used:

Simon (Grade 2):	*16 plus 9 can't be 115 ... it's too big.*
Simone (Grade 3):	*No ... 16 and 9 is not 115 ... 16 plus 9 is 17, 18,* (counted on using fingers) *... 25 ... they added wrong and they can't count ... put them in a special maths class.*
Kelly (Grade 6):	*16 is close to 20 ... 9 is close to 10 ... it should be close to 30.*

An example of incorrect reasoning is provided by:

Merridy (Grade 4):	*Yes ... because 9 and 6 is 15 ... and you just put 1 down and it's 115.*

Figure 6.9 shows that there was a steady increase in performance from Grades 2 to 6 (6% to 95%) for Task 8 (number combinations that add to 100). A similar, but less pronounced increase occurred for Task 9 (number combinations that add to 1000) from Grades 2 to 5 (0% to 56%), but then decreased for Grade 6 children to 47%. The children found working with a thousand (Task 9) much more difficult than with the earlier hundred task (Task 8).

Number Sense Tasks 10, 11 and 12: Mental addition, compensation by ones and tens.
These tasks required the children to calculate 65 + 65 mentally, and to think aloud while calculating 66 + 64 and 55 + 75.

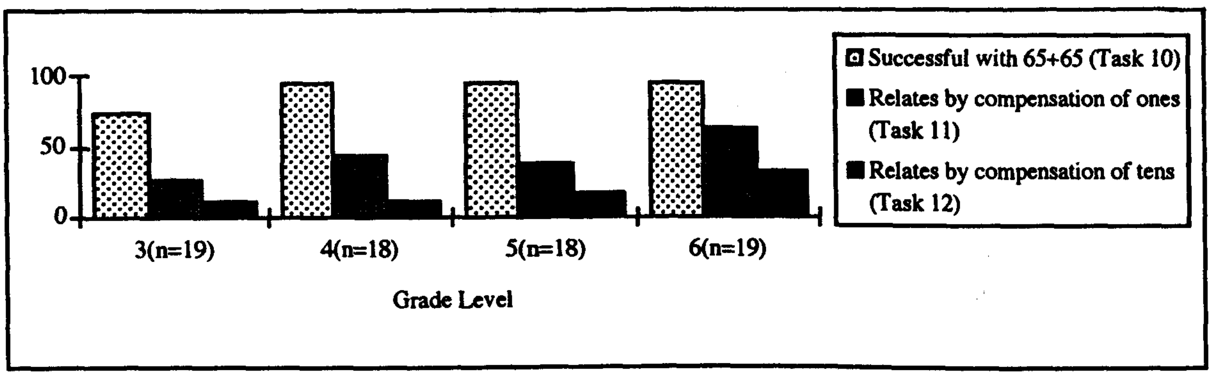

Figure 6.10 Performance on Number Sense Tasks 10, 11 and 12: Percentage of sample giving correct responses, on mental addition and compensation tasks.

Figure 6.10 illustrates mental computation strategies for Tasks 10, 11 and 12. After successfully calculating 65 + 65 mentally, many children were able to complete related calculations through compensation. Only 26% of third graders used compensation by ones to recognise that 66 + 64 gave the same answer as 65 + 65, whereas compensation was used by 63% of sixth graders. As for Task 12, fewer children related 55 + 75 to the earlier calculation of 65 + 65 using compensation by tens (11% and 32% for children in Grades 3 and 6 respectively).

Summary: Number Sense Tasks 1 to 12

Although performance on number sense tasks generally increased as grade level increased, Grade 5 and 6 children's poor performance was consistent with performance on the counting tasks. They were also less successful when tasks with larger numbers were used. It appears that Grade 5 and 6 children also had limited ability to recognise part-whole relationships of the powers of ten (ten, hundred, thousand, ten thousand) and to relate their calculations to these benchmark numbers (Tasks 4 and 9). Although performance on Task 7 was consistently good across the grades (estimation), the ability to break a hundred and a thousand into parts, especially when dealing with money (Tasks 2 and 3), was quite poor. A high reliance on counting strategies rather than the use of pattern and holistic strategies was evident across Grades 1 to 4 (Task 6).

6.2.3 Performance and Strategy Use: Grouping

This section will report children's performance and strategy use for fourteen grouping tasks. It will be recalled that these grouping tasks assess the children's ability to solve partition and quotition problems, deal with different units at the same time, quantify collections from groupings of ones, tens and hundreds, predict ten as a grouping number and use tens structure in an array. The data illustrated in Figures 6.11 to 6.20 are reported in Tables B.5, B.6, B.8, B.10 to 12, B.15, B.17, B.18, B.22, B.29 and B.35 in Appendix B. Task 5 is reported here with Grouping Tasks 8, 9 and 10.

Grouping Tasks 1, 2 and 3: Partitioning into equivalent groups

Kindergarten children were shown 3 Lego people and 12 lollies and asked to give the same number of lollies to each person (Task 1). They were then asked how many lollies each Lego person got (Task 2). The Grade 1 to 4 children were shown 26 shells and asked to put the same number of shells on each of 6 plates (Task 3).

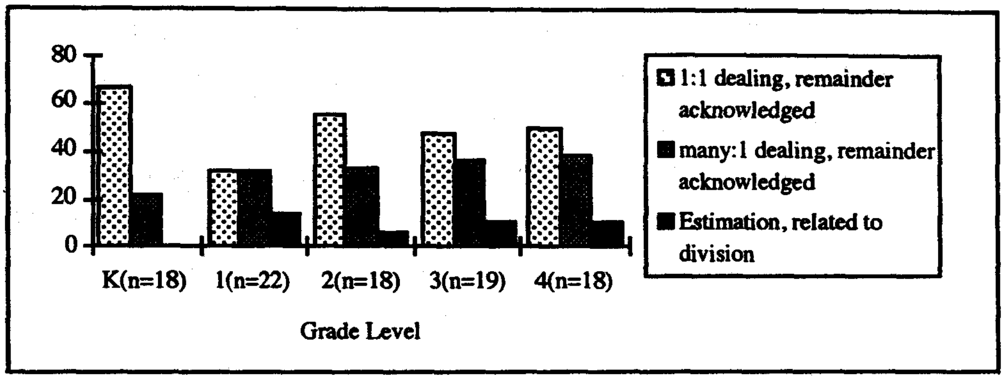

Figure 6.11 Solution strategies for Grouping Tasks 1 and 3, Partition: Percentage of sample using correct responses, by strategy use.

Figure 6.11 shows that a high percentage of the Kindergarten children (89%) successfully shared 12 lollies among the Lego people. Most of these children (67% of the total) used a one-to-one dealing strategy. When the lollies had been physically allocated and the children were asked how many each had received, equal proportions of children (Table B.5) used the following strategies: counting each item in each group; counting only one group, and subitised the number of items in one group (i.e., 4 items) (Task 2 responses are not shown in graph). Task 3, which was completed by the Grade 1 to 4 children, was more difficult as larger numbers were used and a remainder resulted from the sharing process. As expected, there were fewer children successfully using sharing at Grade 1 than with the simpler task at Kindergarten. The percentage of children at Grades 2, 3 and 4 (56%, 47% and 50%) using one-to-one dealing showed no trend to move to one of the other more efficient strategies.

Grouping Task 4: Partition with array

This task required children to solve a partition word problem using an array structure (20 + 4).

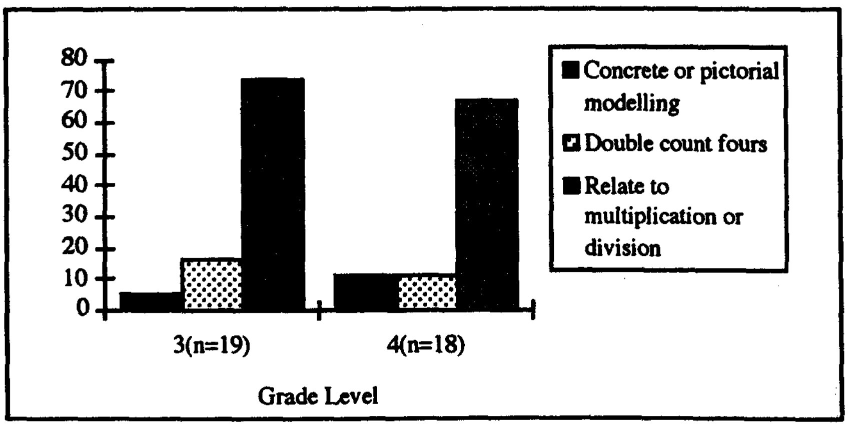

Figure 6.12 Solution strategies for Grouping Task 4, Partition: Percentage of sample giving correct responses, by strategy use.

Similar strategies were used across Grades 3 and 4 for this task (Figure 6.12). Surprisingly some children at both grade levels used a double counting strategy to gain a solution. They counted in fours as if it was a quotition problem. The following excerpts from Grade 3 children illustrate their solution strategies. Warwick, Leigh and Dominque used a double counting strategy as follows:

Warwick: *4, 8, 12, 16, 20, ... 5 in each row.*

Leigh: *4 and 4 is 8 ... another 4 is 12 ... another 4 is 16 ... another 4 is 20 ... 5* (kept count of fours with fingers).

Dominque: *4, 8, 12, 16, 20 ... five ... counted the fours in 20 ... kept track in my head.*

Although Gerlinda used the concrete material available she gave an incorrect response. Using shells she made four rows of 4 and then counted all the shells to get 16. She then made one more row of 4 and gave the answer four. Initially Michael gave the response four, but when prompted to draw a picture he drew a picture of five groups of four tally marks, and responded five.

Some children used known multiplication or division facts based on multiples of 5 such as:

Ben: *5 ... 5, 10, 15, 20.*

Josianne: *5 ... four fives is 20.*

Olivia: *5 ... divide 4 into 20 ... put 20 into 4 equal groups.*

Simone: *three if there were 12 ... no there is 20 ... five.*

Note that Task 5 is discussed with Tasks 8 to 10 on pages 153 and 154.

Grouping Task 6: Quotition word problem (12 ÷ 4)

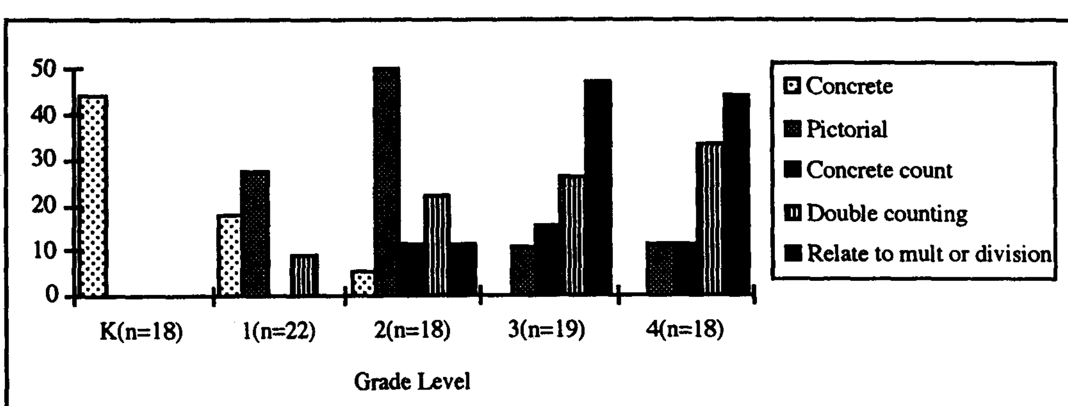

Figure 6.13 Solution strategies for Grouping Task 6, Quotition: Percentage of sample using correct responses, by strategy use.

Figure 6.13 shows that this task was solved successfully by the majority of children in Grades 1 to 4 and almost 50% of kindergarten children. At K-1 level, children relied on the concrete model (12 Lego people and a collection of toy cars). A range of other strategies were employed by children in Grades 1 to 4 including drawing pictures, counting fingers or marks, double counting groups of four (building up and building down) and knowing the answer as a multiplication or division fact. Many Grade 2 children (67%) used concrete or pictorial strategies but fewer did so at Grade 3 and 4 levels (28% and 22% respectively). Double counting in fours steadily increased across the grades, from none at Kindergarten to 33% at Grade 4. Almost all of the children who use double counting built up groups of four mentally until twelve, e.g. Lisa responded "2 fours is eight ... another four is twelve".

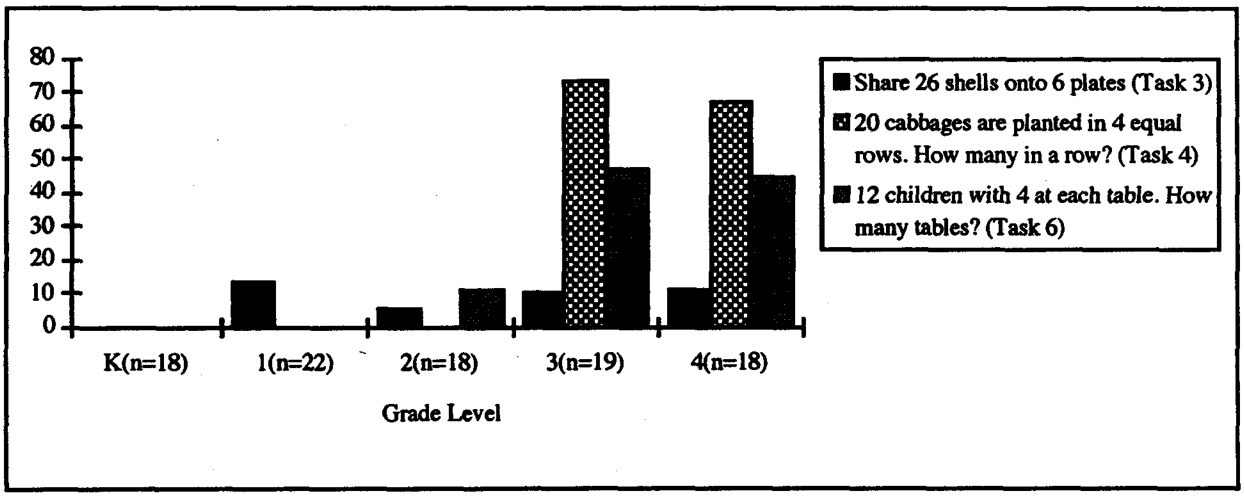

Figure 6.14 A solution strategy used on Grouping Tasks 3, 4 and 6: Percentage of sample, giving responses showing a strategy relating to known multiplication or division facts.

Figure 6.14 compares the relative use of known multiplication or division facts on Tasks 3, 4 and 6. Note that Task 3 was asked from Grades 1 to 4, Task 4 from Grades 3 and 4, and Task 6 from Grades K to 4 The percentage of children who used a known multiplication or division fact remained remarkably low from K through to Grade 2. However at Grade 4, 67% of the children used known multiplication or division facts to solve the partition problem (Task 4) and 44.4% to solve the quotition problem (Task 6). In contrast, only 11% of Grade 4 children used multiplication or division facts to solve Task 3 (fewer than at Grade 1 level), because there was a remainder and simple known facts were not easily recognisable.

Grouping Task 7: Abstract property of quantity using simple ratio with two related units: 2 small stickers can be traded for a large sticker.

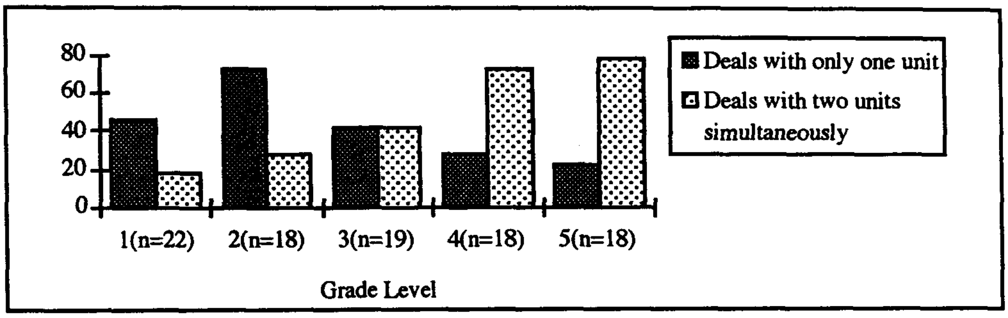

Figure 6.15 Solution strategies for Grouping Task 7: Percentage of sample giving responses, where children dealt with one unit or two units simultaneously.

Task 7 involved quantifying a collection of stickers (3 large stickers and 4 small stickers) according to one of two units of measure (small and large stickers). Figure 6.15 shows that although 18% of children at Grade 1 could deal with the two units, there were 22% who could not do this at Grade 5. Overall, there were 12% of children who were unable to offer any solution and 46% who correctly solved the problem. Many children could only deal with one unit, adding the two unlike units together or trading the small stickers for large stickers but then failing to combine the large stickers. 42% of children gave the incorrect answers of 7 or 2. The solutions to this task show that although there is a developing capacity to deal with different types of grouping units from Grades 1 to 4, there is little additional progress to Grade 5.

Grouping Task 5: Groupings of three

Grouping Tasks 8, 9, 10 and 12: Quantifying from groupings of ten

Tasks 8, 9 and 10 required children to quantify pre-packaged material (15, 43 and 37 respectively) whereas Task 12 required the construction of a grouped representation (52). Note that only Task 12 was asked in Grades 4 and 5.

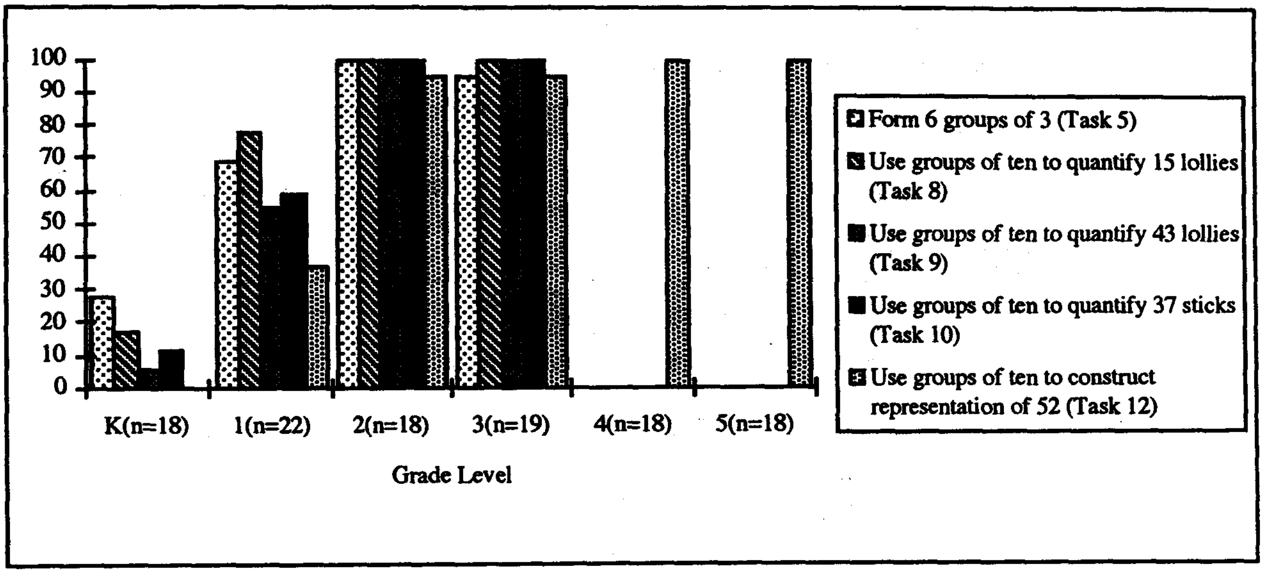

Figure 6.16 Performance on Grouping Tasks 5, 8, 9, 10 and 12: Percentage of sample, using groupings of 3 and 10.

Figure 6.16 compares the performance of children across Grouping Tasks 5, 8, 9, 10 and 12. It can be seen that there is a sharp increase in performance across the grouping tasks from K to Grade 2. The ability to quantify collections of objects pregrouped in tens increased markedly from Kindergarten to Grade 2 level as did the ability to form 6 groups of 3. At Grade 1, it can be seen that the larger the collection to be quantified, the lower the performance. When quantifying the 43 lollies (Task 9), Grade 1 children either used coordinated counting in tens and ones (23%) or subitised the number of tens and ones and used multiplication (32%). Some children (9%) counted the total by ones correctly, but were not deemed to have used the groupings of ten. Strategies used by children which produced incorrect answers included counting packages as ones (5%), counting single lollies as tens (9%), and counting in ones incorrectly (18%). In Task 12, most children could represent the 52 shells with the pregrouped material (prepackaged and loose shells) by the end of Grade 2. It was only at Grade 1 that a significant number (42%) of children attempted to count out separate shells rather than the pregrouped tens and ones to show 52.

Grouping Tasks 11, 13 and 14: Predicting ten as a grouping number, coordinated counting by tens and ones, and use of tens structure in counting

Task 11 required children to predict the grouping number that would make counting easier. Tasks 13 and 14 required them to count, as a sequence of Dienes blocks (longs and shorts) was gradually uncovered (counting sequence 10, 14, 34, 41, 51, 53, 73), and to count the items in a ten by six array of pictures. Note that only Task 11 was asked of Grade 5 children.

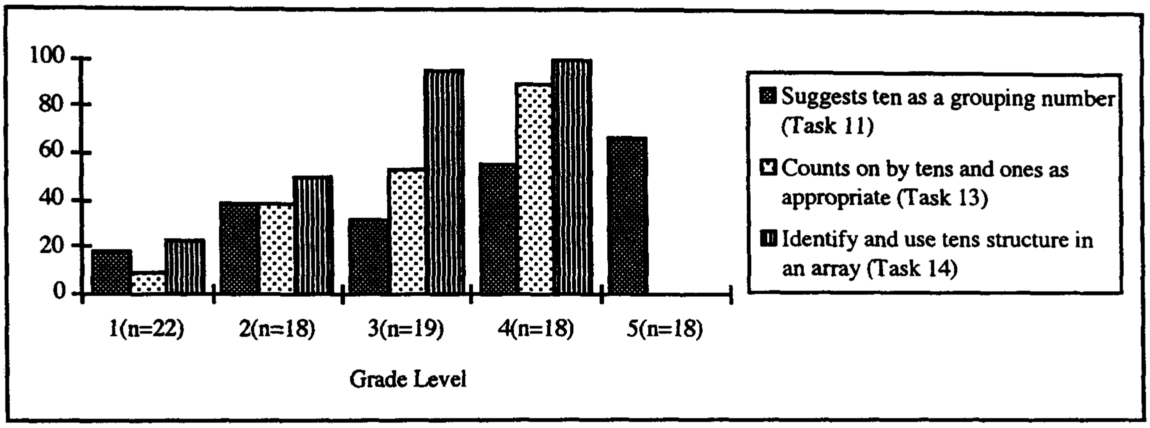

Figure 6.17 Performance on Grouping Tasks 11, 13 and 14: Percentage of sample giving responses, suggesting ten as a grouping number and using ten as an abstract composite unit.

Figure 6.17 shows that less than 40% of the children in Grades 1 to 3 suggested ten as a suitable grouping number to make it easier to count. Even in Grade 5 there were 33% who did not suggest using ten. Alternative answers ranged from groupings in ones to twenties, with the most common being twos, fours and fives. The percentage of children who used ten as an abstract composite unit in the uncovering tens (Task 13) and array (Task 14) tasks increased similarly from Grades 1 to 4 with some divergence of results at Grade 3, where the percentage of children who used the composite units of ten in the array task (95%) was much higher than those who coordinated counting in tens and ones to quantify the uncovered Dienes blocks (53%). In contrast the percentage of children who identified ten as the grouping number, to make the counting of shells easier, was 32% at Grade 3 but this increased to only 67% at Grade 5 level. It appears that children were more likely to identify and use groupings of ten in an array than to coordinate counting in tens and ones.

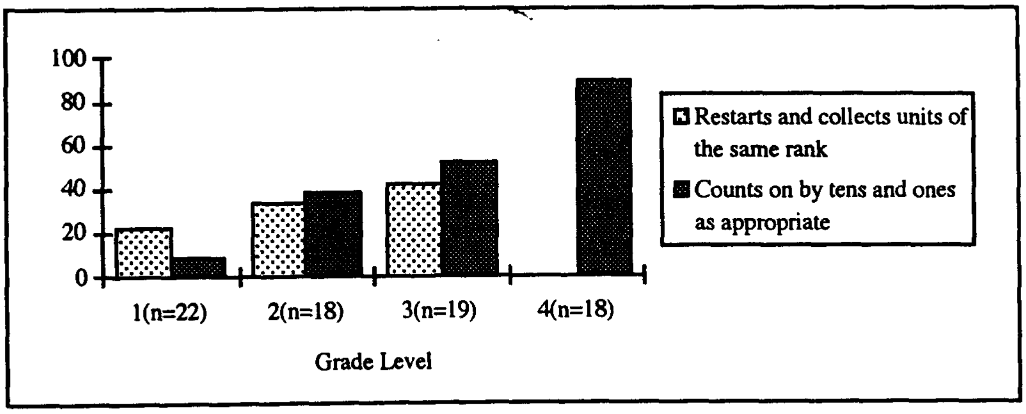

Figure 6.18 Solution strategies for Grouping Task 13, quantifying Dienes blocks: Percentage of sample giving correct responses, by strategy use.

Figure 6.18 shows the performance and strategy use for Task 13 (quantifying Dienes blocks). Children were shown different combinations of Dienes blocks, longs and shorts (to represent tens and ones), in a random order. Each time another combination of tens and ones was uncovered, the children were asked to determine how many the total collection of blocks represented. In this task, the children were required to shift between counting procedures and changing units of tens and ones as presented by the Dienes blocks. It was found that a substantial number of those children who successfully quantified the total at Grades 1, 2 and 3 restarted the count at least once during the uncovering process and collected units of the same rank. For example, Simone (Grade 3) coordinated her count until 34 and then restarted at 30, counting-on by ones the four, seven and two before restarting to count all tens and ones separately. The percentage of children who successfully counted on by tens and ones appropriately steadily increased through the Grades 1 to 4 (9%, 39%, 53% and 89% respectively).

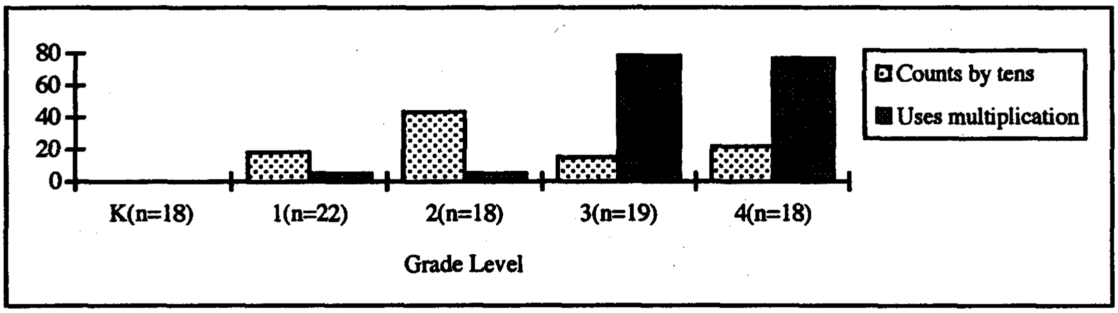

Figure 6.19 Solution strategies for Grouping Task 14: Percentage of sample giving correct responses, by strategy use.

Figure 6.19 shows the strategies children used when quantifying the elements in an array. The percentage of children who identified the tens structure of the array and quantified the elements by counting by tens rose to 44% at Grade 2 and then fell to 16% and 22% at Grades 3 and 4. A higher percentage of children in Grades 3 and 4 used multiplication to calculate totals in the array (78.9% and 77.8% respectively). This increased use of multiplication at Grade 3 is consistent with strategy use on other tasks and is discussed further in Chapter 7.

Grouping Task 15: Uncovering hundreds, tens and ones task

Task 15 required children to count, as a sequence of Dienes blocks (flats, longs and shorts) was gradually uncovered (counting sequence 100, 220, 260, 460, 466, 570, 600).

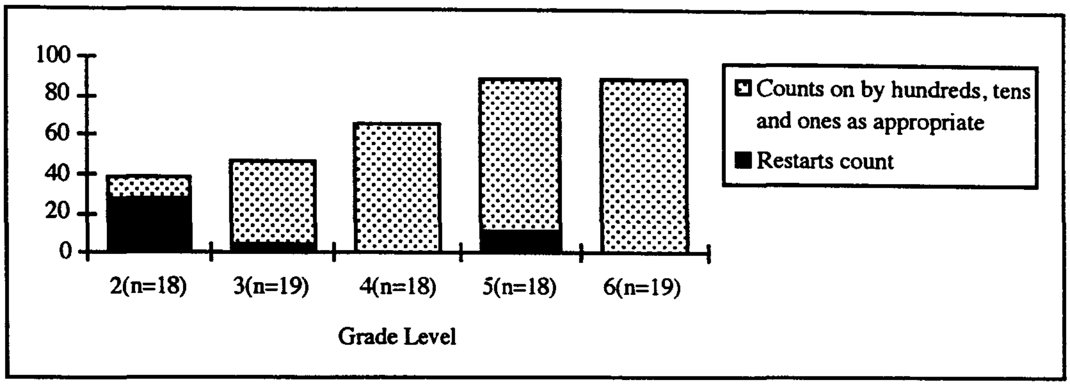

Figure 6.20 Solution strategies for Grouping Task 15: Percentage of correct responses, by strategy use.

Results for Task 15 showed a high level of success across grades, rising steadily from 39% at Grade 2, to 89% and 90% at Grades 5 and 6 respectively. Other than at Grade 2, there were very few children who restarted their count. It is interesting to note that although, as would be expected, there were fewer children successful on this task than with Task 13, a greater percentage of children at Grades 2 to 4 used the groupings provided. There is an indication here that children might benefit from exposure to counting with a greater range of groupings.

Summary: Grouping Tasks 1 to 15

Kindergarten children showed a greater capacity to partition objects into equivalent groups (Tasks 1 and 2) than to solve a concrete quotition problem (Task 6). Rapid progress was made by children from Grades K to 2 in using groups of ten in quantifying or building grouped material (Tasks 8, 9, 10 and 12). Although the use of the tens structure in an array and the coordination of counting by tens and ones steadily increased from Grades 1 to 4, many children did not think of ten as a natural grouping quantity (Tasks 11, 13 and 14). By Grade 4 the majority of children could coordinate counting by hundreds, tens and ones, and at Grade 3 level there was a marked increase in the use of multiplication in solving problems (Tasks 4, 6 and 14). However, it was not until Grade 4 that a majority of children could deal with two related units simultaneously (Task 7).

6.2.4 Performance and strategy use: Regrouping tasks

This section will report children's performance and strategy use for thirteen regrouping tasks. These regrouping tasks assess the children's ability to regroup units of ten and one hundred in addition and subtraction problems and use non-standard representations of numbers. The data illustrated in Figures 6.21 to 6.31 are reported in Tables B.16, B.18, B.23 to 26, B.31, B.33, B.36, B.40, B.41 and B.45 respectively in Appendix B.

Regrouping Task 1: Mental addition (43 + 8) represented by pregrouped material
Refer to Section 5.4 for definitions of strategies.

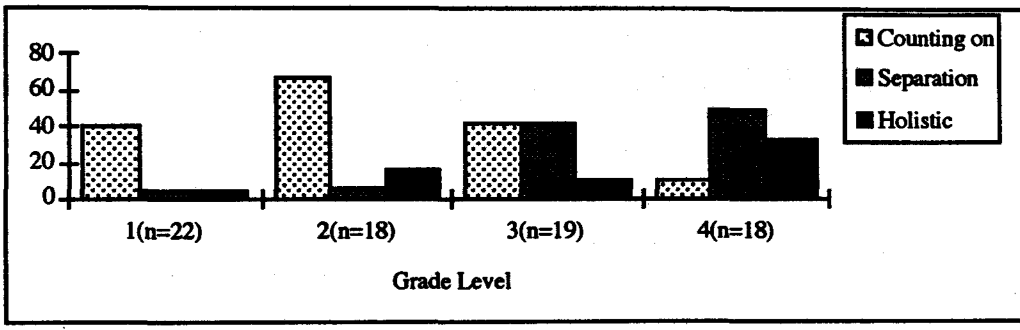

Figure 6.21 Solution strategies for Regrouping Task 1, Addition: Percentage of sample giving correct responses, by strategy use.

The percentage of children who correctly counted on by ones in order to add 8 to 43 was greatest at Grade 2 (67%) and then decreased to 11% at Grade 4. Most of the children counted on from 43, but a few counted on 3 from 48 (11% at Grade 3). The separation strategy, of adding the ones together and then regrouping 11 ones as 1 ten and 1 one, was used by an increasing number of children across the grades (from 5% at Grade 1 to 50% at Grade 4). Holistic strategies involved separating either 8 or 3 into parts so that the next decade of 50 could be bridged, or simply adding 10 and subtracting 2. Holistic strategies, which are more sophisticated, were used by some children in all grades but even at Grade 4 they were used by only 33% of children.

Regrouping Task 2: Addition with concrete material, add 9 to a representation of 52 using bags of 10 shells and single shells

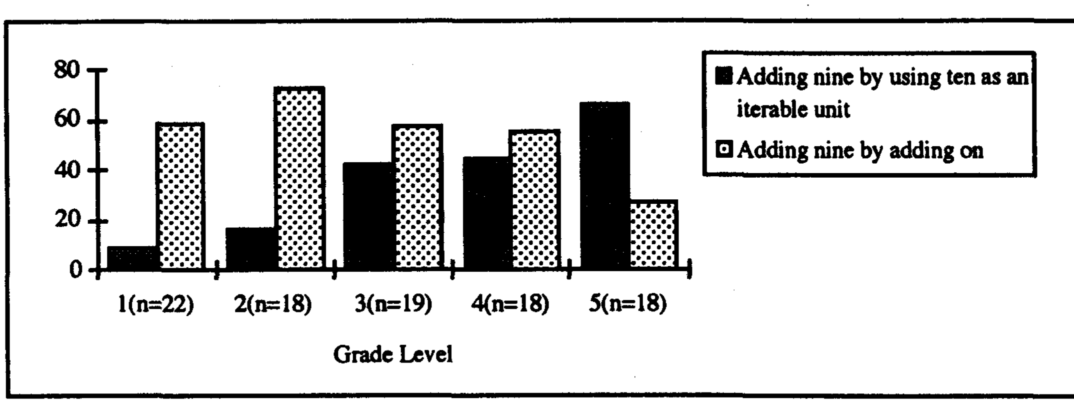

Figure 6.22 Solution strategies for Regrouping Task 2, Addition: Percentage of sample giving correct responses, by strategy use.

Figure 6.22 shows that most children could add on nine shells to their collection of 52, but up to Grade 4 the majority of children used unitary counting, and even at Grade 5 33% did not use the relationship between nine and ten for bridging ten [or adding ten and taking away one].

Regrouping Task 3: Addition using ten frames

Task required the children to add the dots on two ten frames (6 and 9). Refer to section 5.4 for definitions of strategies.

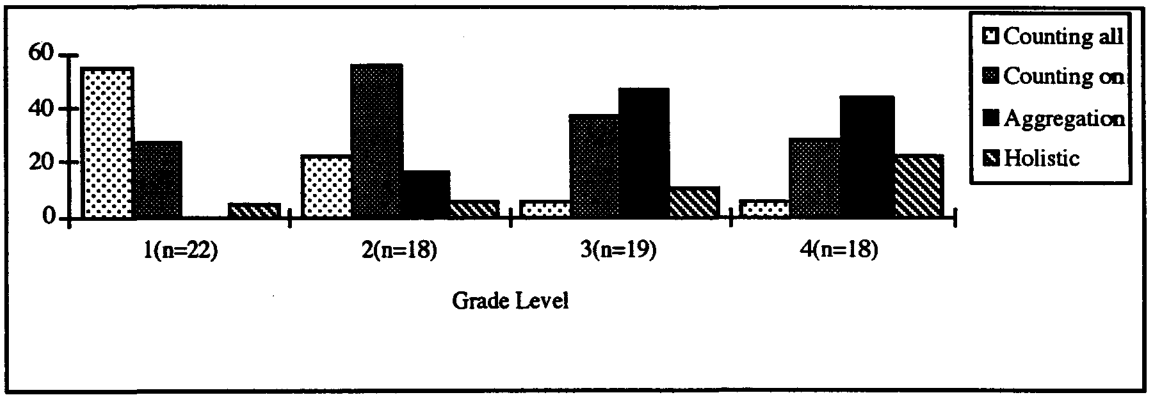

Figure 6.23 Solution strategies for Regrouping Task 3, Addition: Percentage of sample giving correct responses, by strategy use.

Children used a variety of strategies to mentally calculate the addition of 6 and 9 when they had visual representations of the numbers before them (Figure 6.23). Counting all the dots by ones was the predominant strategy for Grade 1 children (55%). This is consistent with earlier results (Counting Task 1) showing a reliance on unitary counting among Grade 1 children. At Grade 2, the strategy of counting on from either 6 or 9 was the dominant strategy (56%). Although the representation of 6 was shown first, half of these children used the more efficient order of counting on from the larger number (9) and of these children one counted on in threes. Surprisingly, there were still many children using counting strategies at Grade 4 with 6% using counting all, and 28% using counting on. The aggregation strategies of subitising numbers from the patterns of dots, and using partitioning and combining (bridging ten) to find the sum, were the dominant strategies in Grades 3 and 4 (47% and 44%) respectively. Use of holistic strategies such as relating to known doubles, or the addition of ten, increased gradually from 5% at Grade 1 to 22% at Grade 4. For example, Simon and Rebecca (Grade 2) used bridging ten (aggregation):

Simon: *9 plus ... like you add 6 ... you add one to nine ... that makes 10 ... and then 5 ... thats 15.*

Rebecca: *15 ... 9 here ... take one away from here* (pointing to the 6) *... that is 5 left.*

Owen (Grade 2) used the strategy of adding ten and subtracting one:

Owen: *15 ... if we wanted 16 we would have to have another one here* (pointing to the 9).

Regrouping Task 4: Addition of two quantities (37+25), the first quantity shown as pregrouped material.

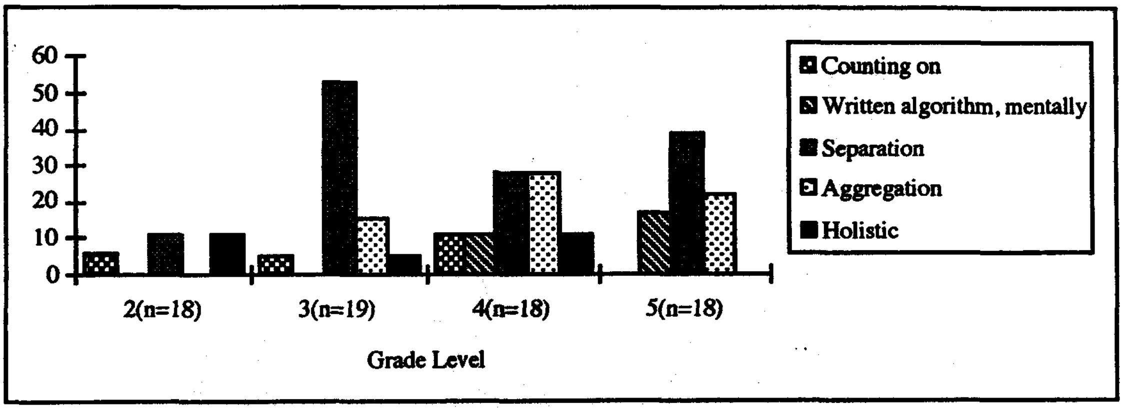

Figure 6.24 Solution strategies for Regrouping Task 4, Addition: Percentage of sample giving correct responses, by strategy use.

Figure 6.24 shows that successful performance on the addition task increased from 28% at Grade 2 level to 89% at Grade 5 and then decreased to 78% at Grade 6 level. Counting strategies which involved counting on by ones, or tens and ones, sometimes in conjunction with addition of parts of the numbers were used by some children in Grades 2, 3 and 4 (6%, 5.3% and 11% respectively). Many children used fingers to double count as is shown by the following excerpts:

Beverley (Grade 2): *37 ... 57 ... 58, 59, 60, 61, 62* (using fingers).

Kristie (Grade 3): *30, 40, 50, 57 ...* (then using fingers) *58, 59, 60, 61, 62.*

Rhys (Grade 4): *37, 38, 39, 40, 41, 42 (fingers) ... 62 (twenty more).*

Merridy (Grade 4): *10, 20, 30, 31, 32, 33, 34, 35, 36, 37 ...* (using fingers) *38, 39, 40, 41, 42, 43, 44, 45. 46, 47,* (starts hands again) *48, 49, 50, 51, 52, 53, 54, 55, 56, 57, ...* (starts hands again) *58, 59, 60, 61, 62.*

Mental calculation using the formal written algorithm was used by 11% of Grade 4 and 17% of Grade 5 children but not at all by Grade 2 or 3 children, e.g.,

Marc (Grade 5): *7 plus 5 is 12 ... put down 2, carry the 1 ... 3 plus 2 is 5 plus 1 ... 62.*

Separation strategies, which involve splitting the numbers according to place value and then adding the parts (right to left or left to right), were used by a large proportion of Grades 3, 4 and 5 children (53%, 28% and 39% respectively). For example Ben and Ryan (Grade 3) used separation strategies; Ben worked right to left whereas Ryan worked left to right.

Ben: *5 and 7 ... 5 and 5 and 2 ... 30 and 20 is 50 ... 62.*

Ryan: *added 30 and 20 ... 50 ... 7 and 5 equal 12 ... carried one over ... 62.*

Michelle (Grade 4) combined separation with counting by tens when responding:

Michelle: *5 and 7 ... 10 and 2 ... 32,42, 52, 62.*

Aggregation strategies, where one number is kept intact and the other number separated into place values before being added to the first number, were used by some children in Grades 3, 4 and 5 (16%, 28% and 22% respectively). For example, in Grade 5, Michael worked right to left and Michelle worked left to right:

Michael:	*37 ... plus 5 is 42 ... plus 20 is 62.*
Michelle:	*37 and 20 ... 57 ... add 5 ... 7 and 3 is ten ... and 2 more is 62.*

Holistic strategies, which involve compensation or partitioning one of the numbers to create convenient ways of combining the numbers, were only used by a few children in Grades 2, 3 and 4 (11%, 5% and 11% respectively). For example:

Josianne (Grade 3):	*take off 2 ... there 5 left and 5 in there makes 10 ... put the 10 there 60 ... 2 left over ... 62.*
Nik (Grade 4):	*37 plus 25 ... chopped off 2 of the 7 ... that's 35 and 25 ... that is 60 ... so 62.*
Jennifer (Grade 2):	*37 ... I need 3 more lollies to make 10 ... 40 ... 22 left over ... 62.*

Task 5 Regrouping: Missing addend (64-18), the first addend shown as pregrouped material.

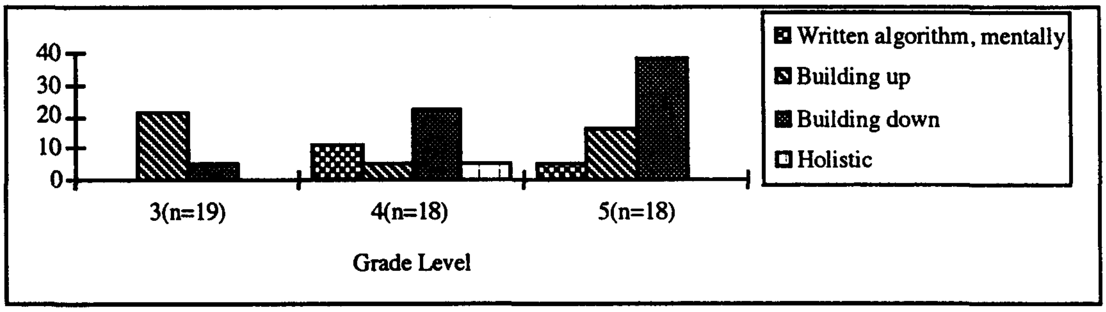

Figure 6.25 Solution strategies for Regrouping Task 5, Missing addend: Percentage of sample giving correct responses, by strategy use.

Performance on the missing addend task increased from 26% at Grade 3 to 61% at Grade 5 level. Figure 6.25 shows that aggregation strategies dominated at all grades with building up being preferred at Grade 3 (21%) and building down preferred at Grades 4 and 5 (22% and 39% respectively). Many of the children using aggregation strategies relied on some counting. The written algorithm was performed mentally by some Grade 4 and 5 children (11% and 6% respectively) and holistic strategies were used by very few children (6% at Grades 4). The following excerpts illustrate some of the successful reasoning that children used. Nichole and Amy (Grade 4) both used a formal algorithm mentally, their explanations reflecting how they had been taught, whereas Josianne (Grade 3) and Michael (Grade 5) used building up strategies.

Nichole (Grade 4):	*64 minus 18. You put 18 under 64, then you can't take away 8, so borrow a ten from the 64, then 14 minus 8 is 6, then you go next to it, 5 take away 1 is 4 ... 46.*
Amy (Grade 4):	*64 minus 18 ... you take off the 6 ... and makes 5 and 14 ... take 8 from 14 ... 6 ... 46.*
Josianne (Grade 3):	*18, 28, 38, 48, 58 ... thats 40* (using fingers for double count) *59, 60, ... 61, 62, 63, 64 ... 46.*
Michael (Grade 5):	*18 there ... put 2 on ... that's 20 ... add 40 ... and 4 ... it is 46.*

Gemma (Grade 4) and Andrew (Grade 5) used building back strategies. Gemma used aggregation left to right whereas Andrew used aggregation right to left:

Gemma (Grade 4):	*Take away 10 from 64 ... 54 ... take away 8 off 54 ... that is 46.*
Andrew (Grade 5):	*64 take away 18 ... take 8 from 64 ... 56 ... take away 10 ... 46.*

Brooke (Grade 4) used the holistic strategy of subtracting 20 and adding 4 when she explained:

Brooke:	*Take 20 away ... 64 ... 44, and then add 2 onto 44, 46.*

Regrouping Task 6: Addition algorithm

Task 6 required children to mentally calculate an addition of two numbers presented in written algorithmic form (16 + 9). This was a follow-up task to Number Sense Task 7 (estimation).

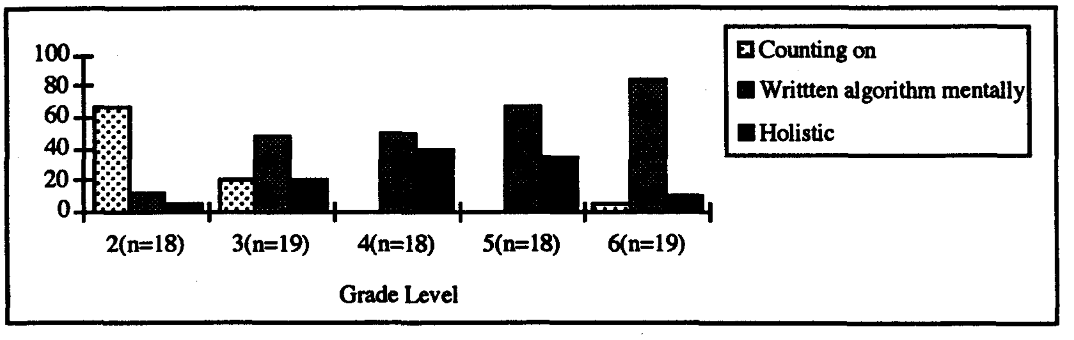

Figure 6.26 Solution strategies for Regrouping Task 6, Addition algorithm: Percentage of sample giving correct responses, by strategy use.

The percentage of children using the counting on strategy decreased from 67% at Grade 2 to 21% at Grade 3 (Figure 6.26). Then no child used this strategy until Grade 6 (5%). The strategy of calculating mentally using the written algorithm was dominant for all grades after Grade 2, rising to 84% at Grade 6. Use of holistic strategies, relating to the addition of ten, or immediate recall, increased to 39% at Grade 4 only to decrease to 11% at Grade 6.

Regrouping Task 7: Addition involving ones, tens and hundreds (245 + 98), the first addend shown as pregrouped material.

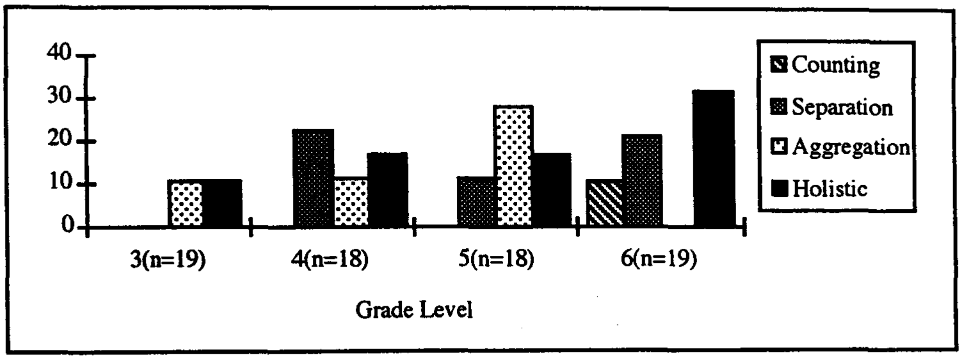

Figure 6.27 Solution strategies for Regrouping Task 7, Addition: Percentage of sample giving correct responses, by strategy use.

Children used a variety of strategies to mentally calculate the addition of 98 lollies to a modelled collection of 245 lollies (Figure 6.27). Counting in tens and ones and separation strategies (where the numbers are separated into place values and collected either from the left or right) were used by many Grade 6 children (11% and 21% respectively). These could be considered 'practice-developed skill strategies' that reflect classroom instruction on place value and algorithms. Aggregation strategies involved one number being kept whole and the other number separated into place values before adding. Use of the aggregation strategy increased over the Grades 3 to 5 (11%, 11% and 28% respectively) but did not exist at Grade 6. Holistic strategies, which involved bridging tens and hundreds, relating to the addition of a hundred, or compensation, were used increasingly (reaching 32% at Grade 6 level). The children in Grade 3 who were successful used aggregation and holistic strategies (11% each). As these children had not had any formal instruction with the addition algorithm involving 3-digit numbers, they were using their own methods. The following excerpts illustrate some of the successful reasoning that children used. Nik (Grade 4) used a separation strategy working from left to right as shown by his explanation:

Nik (Grade 4): *Add 9* (tens) *to the 4* (tens) *... 13* (tens) *... 330 ... 8 to the 5 leaves 13 ... 343.*

Although Ty (Grade 6) used aggregation from left to right, he relied on counting-on by tens and ones to execute his procedure as shown by the following explanation:

Ty (Grade 6): *200 there ... plus 98 ... 298, plus 40 ... 308, 318, 328, 338 ... plus 5* (fingers) *... 338, 339, 340, 341, 342, 343.*

Tim (Grade 5) used aggregation from left to right:

Tim (Grade 5): *200 then 90 ... gives 290 ... left 8 out and put 40 on ... so it is 330 ... then 338 ... then 2 more ... 340 ... then 3 more ... 343.*

Mati and Kate (Grade 6) used holistic strategies, Mati using compensation and Kate using comparison to addition of 100:

Mati (Grade 6): *245 plus 98 is the same as 250 add 93 ... or 243 add 100.*

Kate (Grade 6): *245 plus 100 is 345 ... then you just take away 2.*

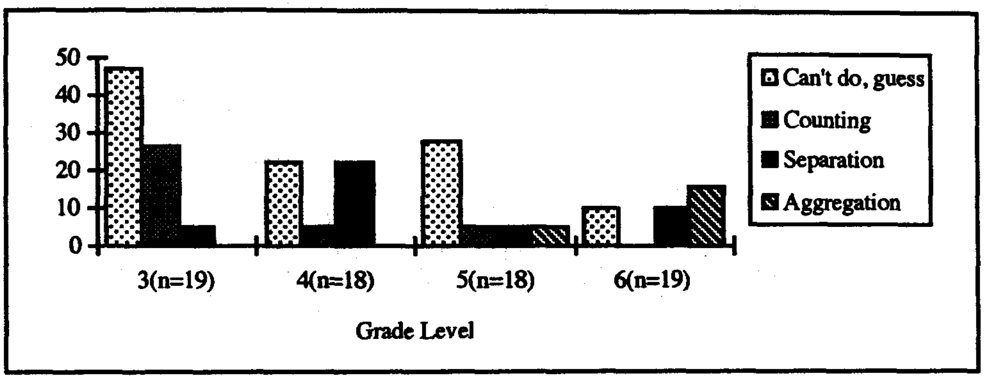

Figure 6.28 Solution strategies for Regrouping Task 7, Addition: Percentage of sample giving incorrect responses, by strategy use.

For the children giving incorrect responses (Figure 6.28), attempting to count on was the most common strategy used by those in Grade 3 whereas separation strategies dominated for Grade 4 children. Grade 6 children who were not successful used both separation and aggregation strategies.

Regrouping Tasks 8 and 12: Addition on a spike abacus and place value chart, (234 + 98), the first quantity was represented on an abacus or place value chart.

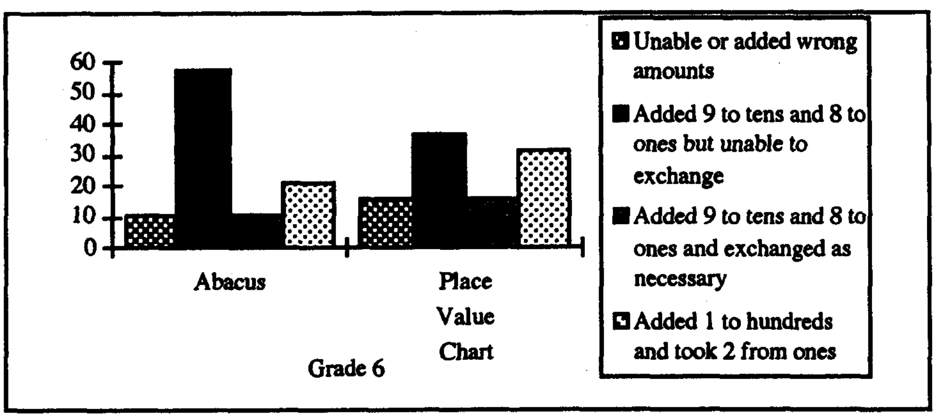

Figure 6.29 Solution strategies for Regrouping Tasks 8 and 12, Addition: Percentage of incorrect and correct responses, by strategy use.

Figure 6.29 compares the strategies used by the Grade 6 children using two non-proportional representations for the first addend (spike abacus and shells on a place value chart). All the children correctly recorded 234 on the spike abacus (using discs) and the place value chart (using shells), 32% of the Grade 6 children successfully recorded the addition on the abacus and 47% successfully recorded the addition on the place value chart. A large proportion of children (58%) added 9 discs to the tens spike and 8 discs to the ones spike but did not carry out the necessary exchanges for '10 ones' and '10 tens'. There were fewer children (37%) who used the same unsuccessful strategy with the place value chart. A small number of children (11% for the abacus and 16% for the place value chart) did carry out these exchanges successfully. The most common successful strategy (21%) for the abacus was to add a disc to the hundreds spike and subtract 2 from the ones spike. Children were more successful when dealing with the place value chart with 32% of children adding a shell to the hundred column and taking 2 shells from the ones column.

Regrouping Task 9: Quantify collection presented by non-stanard representation.
In Regrouping Task 9 where children counted pregrouped material shown in non-standard representation (425 in 3 bags, 12 rolls and 5 separate lollies), 79% of Grade 3 and all children in Grades 4 to 6 established the number of hundreds, tens and ones, and immediately gave the correct total. The upper primary grade children did not have any difficulty interpreting this number representation because it was already grouped.

Regrouping Task 10: Missing addend task (143 - 46).

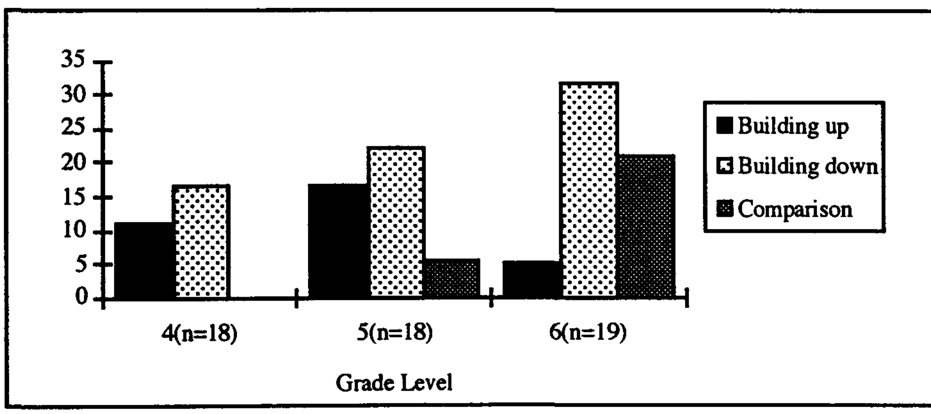

Figure 6.30 Solution strategies for Regrouping Task 10, Missing addend: Percentage of sample giving correct responses, by strategy use.

Figure 6.30 shows that the percentage of children successfully solving the missing addend task increased across Grades 4 to 6 (28% to 58%). Overall building down was the most successful strategy (17%, 22% and 32% respectively) although the holistic strategy of using comparison became more frequent at Grade 6 (21%). The building up strategy was used by children from each grade, being most common in Grade 5 (17%). An example of the holistic strategy that was used by a few children is given by:

Jack (Grade 5): *143 is around 146 ... that's 3 more ... so it is 100 take away 3 ... that's 97.*

In contrast, Tim (Grade 5) also related the missing addend problem to a difference of a hundred but added 3 to make the adjustment rather than subtracting:

Tim (Grade 5): *If it was 43 ... there would be 100 under there ... since that 3 is not counted ... there would be 103.*

Regrouping Task 11: Missing addend (300 - 122).

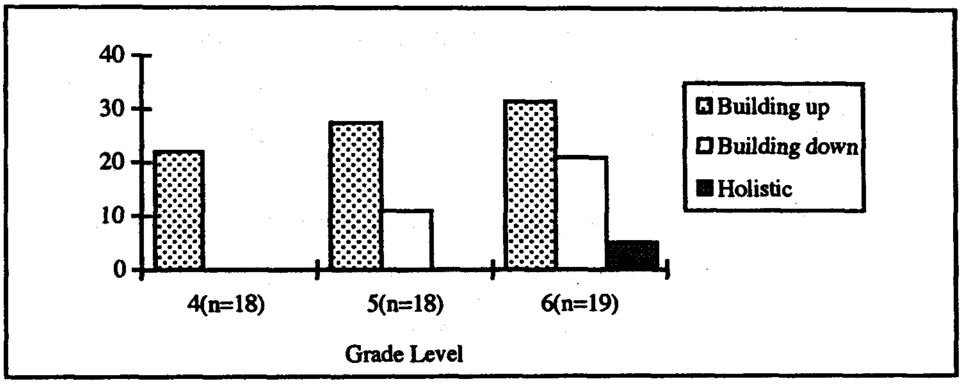

Figure 6.31 Solution strategies for Regrouping Task 11, Missing addend: Percentage of sample giving correct responses, by strategy use.

Figure 6.31 shows that the percentage of children who were successful on the second missing addend task increased across Grades 4 to 6 (22%, 39% and 58% respectively). The building-up strategy was used most often by children in each of the grades. The use of the building-down strategy increased for Grades 5 and 6 (11% and 21% respectively) and a holistic strategy was only used in Grade 6 (one student). The strategies of building-up can be illustrated by Nik (Grade 4), Michelle (Grade 5) and Jonathon (Grade 5). Nik used a right to left aggregation, showing a good sense of the composition of the number in his explanation:

Nik (Grade 4): *22 plus 8 is another 10 ... I know ... I couldn't give 8 for the next one ... 80 ... I put in a 7 ... then I said one hundred ... 178.*

Michelle used the more common left to right aggregation in her building-up strategy: "

Michelle (Grade 5): *Add 100 to make 222 ... add 70 to get 292 ... 170 ... then 8 to make 300 ... 178.*

Jonathon attempted a building-up strategy with a focus on bridging a hundred but was unsuccessful because he was unable to handle all the information and neglected the extra hundred:

Jonathon (Grade 5): *78 ... I know 20 and 80 is 100 ... so ... if it is over 20 it is 70 ... I took 2 away from 80 and got 78.*

Trevour (Grade 5) used a building strategy with left to right aggregation:

Trevour (Grade 5): *300 take away 122 ... take away 100 ... 200 ... take away 20 ... 180 ... take away 2 ... 178.*

Jenna compared the calculation to a known double and used equal adjustment in her holistic strategy:

Jenna (Grade 6): *122 ... half of 300 is 150 ... take away another 28 ... 150 plus 28 is 178.*

Summary: Regrouping Tasks 1 to 12

Regrouping skills are an integral part of working efficiently with the numeration system. The solutions to regrouping tasks in this study have shown that unitary counting, rather than regrouping is the basis of solution strategies for most children in Grades 1 and 2. Regrouping skills were evident in the responses of Grades 3 to 6 children but many Grade 6 children did not use regrouping. In tasks involving addition of a single digit number (Tasks 1 to 3 and 6), unitary counting strategies were dominant at Grade 1 and 2 levels and still frequent at Grades 3 and 4 (Tasks 1 and 3). For the addition task where children were asked to carry out the addition concretely (Task 2), the corresponding strategy of counting out single items was dominant for Grades 1 to 4 - showing that most of the children intuitively focussed upon individual items in a collection rather than relating the quantity to the groupings of ten provided.

When the children were presented with an addition task written as a vertical algorithm (Task 6), a counting strategy was dominant for Grade 1 children but in Grades 2 to 5 children mostly calculated mentally from the written algorithm. Regrouping objects and renaming numbers are skills required in separation, aggregation and holistic strategies. Less than a quarter of the children in Grades 1 and 2 used these strategies (Tasks 1 to 3). For Grade 5 children there was still a third of the children not using regrouping when adding with concrete material (Task 6), suggesting that children do not develop their strategies for mental calculations from operations with concrete materials.

For Tasks 4, 5, 7, 8 and 10 to 12, involving addition and subtraction using 2 and 3 digit numbers, a variety of mental strategies were used. Addition strategies included separation (including standard algorithmic procedures), aggregation (including counting in tens and ones), and holistic strategies (including compensation and bridging tens and hundreds). Sometimes there were a combination of strategies used for these tasks. Some children used holistic strategies at Grades 2 and 3 levels (Tasks 4 and 7) showing that children can efficiently compute with their own non-standard mental algorithms before instruction with pencil and paper methods. However, there were many children at Grade 6 who could not solve addition and subtraction tasks (42% on Tasks 10 and 11 and 39% on Task 7). Standard algorithmic procedures began to be used in Grade 4 (Tasks 4, 5 and 7) but were not used with the more complex subtraction tasks where aggregation (building up and down) and holistic strategies were used in Grades 4 to 6 (Tasks 11 and 12). For the addition tasks (Tasks 4 and 7) children who used counting by tens and ones, separation or aggregation mental strategies either worked from left to right or right to left. Even though school algorithmic work from Grade 3 has a right to left orientation, almost two thirds of the children in Grades 3 to 6 used left to right procedures. Most Grade 6 children failed to regroup correctly when adding with non-proportional concrete material (68% for Task 8 and 53% for Task 12), which is consistent with low performance on some counting and number sense tasks.

6.2.5 Performance and strategy use: Place value tasks

This section will report children's performance and strategy use for seventeen place value tasks. The place value tasks assess the children's ability to write numerals for numbers, identify value of places in numerals, interpret value of digits including zero, and interpret place value representations. It will be made clear that there were wide differences in the performance and strategy use of individual children across tasks but, as reported in Section 6.1, performance generally increased through the grades. The data illustrated in Figures 6.32 to 6.39 are reported in Tables B.7, B.13, B.27, B.28, B.33. B.37, B.44 and B.46 respectively in Appendix B.

Place Value Task 1: Digit correspondence, 2 digits

This task required children in Kindergarten and Grades 1 to 4 to interpret the digits in a 2-digit numeral from a non-standard representation of the number (26 shells were displayed as 6 groups of 4 shells and 2 left over). For Kindergarten children a smaller number of shells (16) was used.

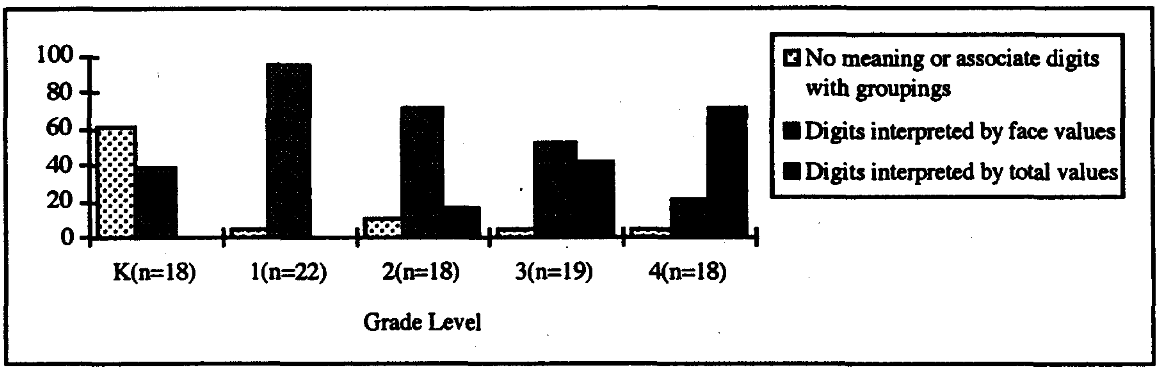

Figure 6.32 Solution strategies for Place Value Task 1, Digit correspondence: Percentage of sample giving correct and incorrect responses, by strategy use.

Figure 6.32 shows the percentage of various interpretations given by the children for the digits in a 2-digit numeral. All but one of the Kindergarten children who were able to write the 2-digit numeral for sixteen interpreted the separate digits by their face values. Bonnie (who was the exception) wrote the numeral '16' but interpreted the '6' as ten and said she did not know what the '1' meant. This meant that no Kindergarten children correctly interpreted the digits by their total values and 39% of the children interpreted the digits by their face value. At Grade 1, most children (96%) used the face value interpretation for the digits in '26'. A small percentage of children at Grades 2, 3 and 4 (11%, 5% and 6% respectively) interpreted the ones digit as the number of groups (6) and the tens digit as the number of shells left over (2). For example, Matthew (Grade 2), who first said he could not count by fours, then proceeded to count the shells in 6 groups of 4 and said that the digit '6' was shown by the 24 shells. He also indicated that the second circled digit (2) was shown by the 2 shells left over. Interpretation of digits by their face values steadily decreased to 22% at Grade 4 whilst the percentage of children who correctly interpreted the digits by their total values increased from 17% at Grade 2 to 72% at Grade 4.

Place Value Tasks 2 and 3: Zero as a place holder, interpret the meaning of the numeral '01' shown on a milk carton ("use by 01 August").

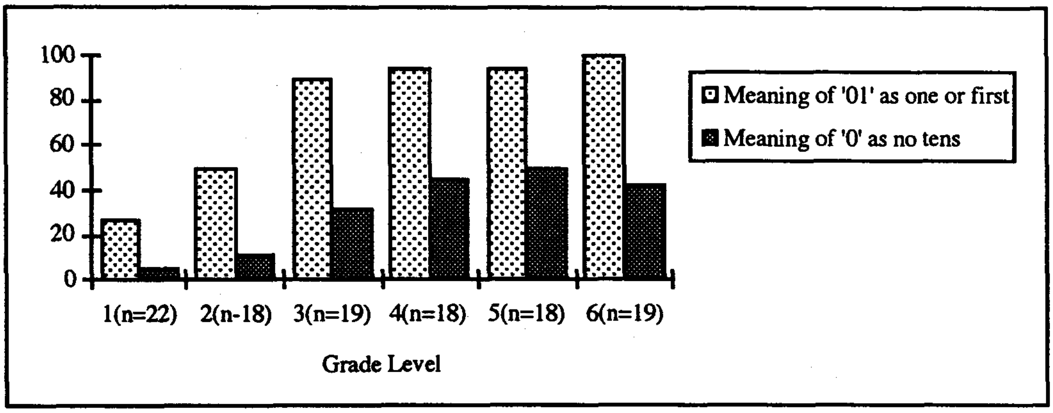

Figure 6.33 Performance on Place Value Tasks 2 and 3, Zero: Percentage of sample giving correct interpretations of numeral '01' and digit '0'.

Three types of answers were given for the interpretation of the numeral '01'. Some children said that '01' was a ten, interpreting the digits but not taking account of their place values. This answer was given by 40% of Grade 1 children, no Grade 2 children, and only 11% and 6% of Grade 3 and 4 children respectively. There were also some children who were unsure of what '01' meant (32% at Grade 1, 50% at Grade 2 and 6% at Grade 5). The percentage of children who successfully identified '01' as one increased from 27% at Grade 1 to 90% at Grade 3. However the percentage of children who could explain the meaning of the zero in the numeral as 'no tens' was significantly lower: 5% at Grade 1, 50% at Grade 5, and 42% at Grade 6. Most of these children simply indicated the meaning was "*no tens*", but a notable exception was Stacey (Grade 6) who explained that the zero "*holds the place so no number goes in*".

Place Value Tasks 4 to 9: writing numerals

In these tasks the children were asked to write down the following numbers: one hundred and three; one hundred and eleven; one thousand and eight; three tenths; fourteen hundredths; and six hundred and one thousand and forty. Note that Tasks 7 to 9 were only asked from Grade 4.

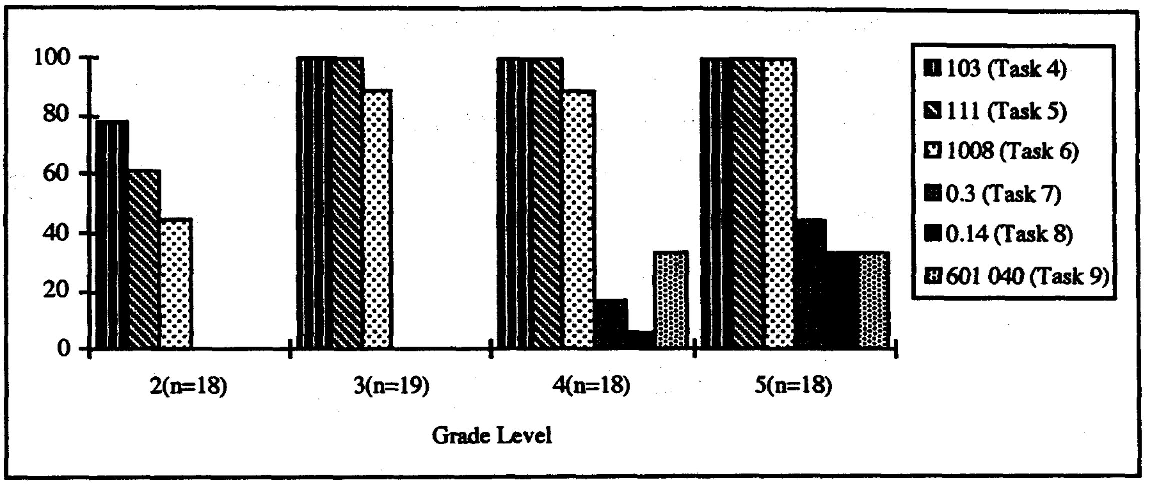

Figure 6.34 Performance on Place Value Tasks 4 to 9, Numerals: Percentage of sample giving correct responses.

Figure 6.34 shows that by Grade 3 most children could write the numerals 103, 111 and 1008 correctly (100%, 100% and 89% respectively). Much more difficulty was experienced by children when asked to write the decimal numerals for the fractions involving tenths and hundredths, and when asked to write a 6-digit number (at Grade 5 there were 44%, 33% and 33% correct for 0.3, 0.14 and 601 040 respectively).

Place Value Task 10: Total value, hundreds

Task 10 required the children to change the number displayed on a calculator screen to a given number by a single subtraction (431 changed to 31).

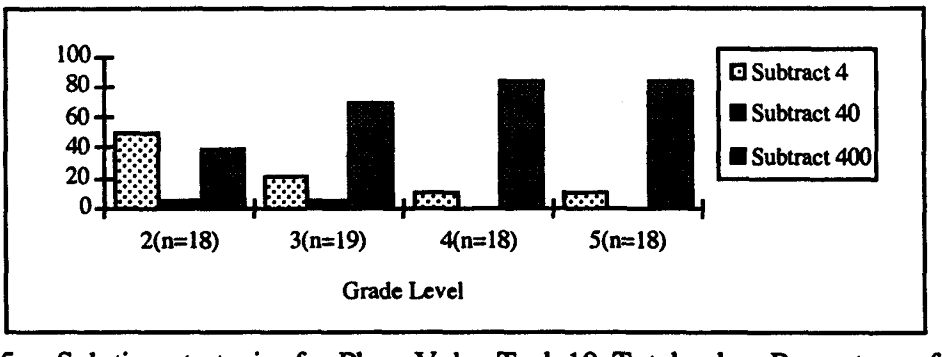

Figure 6.35 Solution strategies for Place Value Task 10, Total value: Percentage of sample giving correct and incorrect strategies.

Figure 6.35 shows that the percentage of children who correctly subtracted 400 to change the calculator display from 431 to 31 increased steadily through Grades 2 to 5 (39% to 83%). At Grade 2 the most common response (50%) was to subtract 4, reflecting a focus on the face value of the digit. This incorrect response was given by children in all grades, but had reduced to 11% by Grade 5. One child in each of Grades 2 and 3 suggested subtracting 40.

Place value tasks 11 and 12: Recording on an abacus

For the tasks requiring the children in Grade 6 to label positions (ones to ten thousands) on an abacus and record a 3-digit number (234), 90% of the children labelled the abacus correctly and 100% recorded the number.

Place Value Task 13: Digit correspondence, 3 digits

This task required children in Grades 2 to 6 to interpret the digits in a 3-digit numeral from a non-standard representation of the number (3 bags, 12 rolls, 5 separate lollies - all opaque coverings).

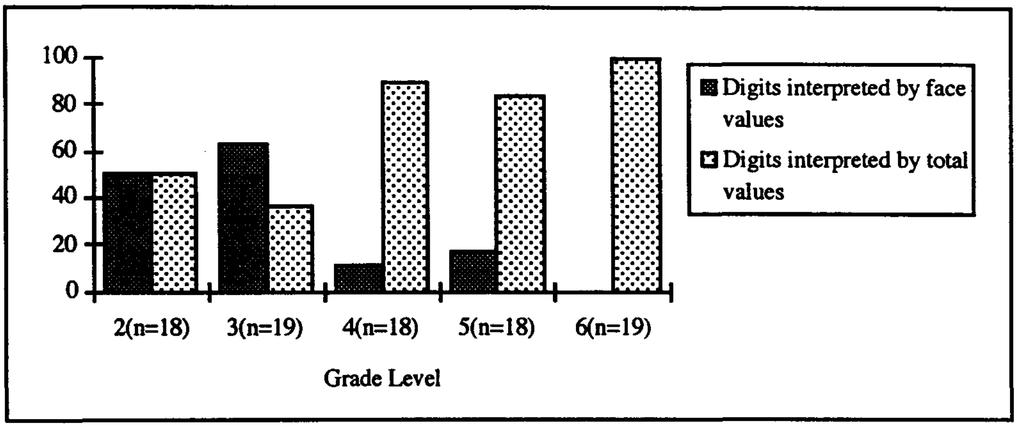

Figure 6.36 Solution strategies for Place Value Task 13, Digit correspondence: Percentage of sample giving correct and incorrect responses, by strategy use.

A substantial percentage of children interpreted the digits by their face values in Grades 2 and 3 (50% and 63% respectively), but in Grades 4 to 6 children predominantly used the correct total value interpretation of the digits (Figure 6.36).

Place Value Task 14: Place value, identify three positions to the left and right of the 'hundreds, tens and ones' positions on a place value chart.

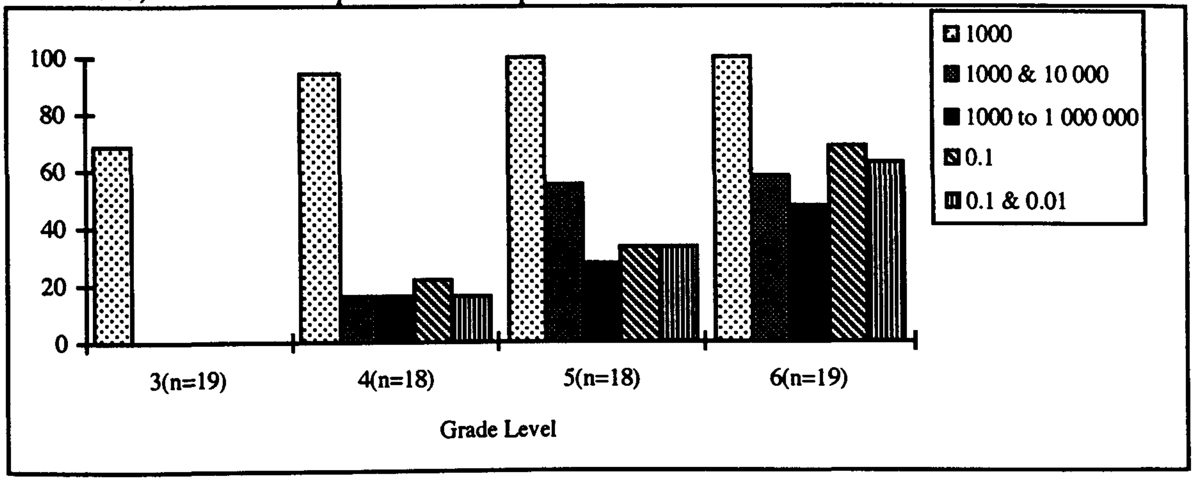

Figure 6.37 Performance on Place Value Task 14: Percentage of sample recognising place value of digits in a numeral.

Figure 6.37 compares performance on identifying place value of positions on a place value chart. There was a high level of performance in identifying the place value of thousands (68%, 94%, 100% and 100% for grades 3, 4, 5 and 6 respectively). Children's recognition of the place values 'ten thousands to millions' was much less successful (17%, 28% and 47% for grades 4, 5 and 6 respectively). Although there was general improvement in recognition of place value across the grades, at Grade 6 there were still 42% who could not identify the ten thousands place value and 32% who could not identify the tenths place value.

Place Value Task 15: Renaming numbers

Task 15 required children to interpret a non-standard numeric representation ("1" in the tens place and "13" in the ones place of a place value chart).

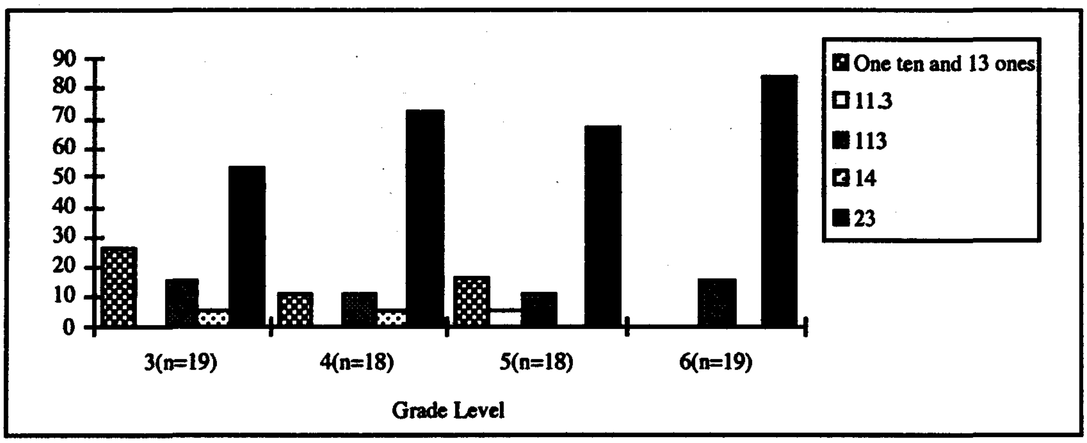

Figure 6.38 Place Value Task 15, Renaming: Percentage of sample giving correct and incorrect interpretations of the place value chart.

Figure 6.38 shows that the percentage of children suggesting correctly that the number could be renamed as 23 increased from 53% at Grade 3 to 84% at Grade 6. At Grades 3, 4 and 5 some children (26%, 11% and 17% respectively) read the number as one ten and thirteen ones but could not give any further meaning or rewrite the number in another way. Merridy (Grade 4) explained:

Merridy: *That can't be a hundred ... because a hundred is there ... so ... I don't know.*

The most common incorrect response was to simply to ignore the designated values of the columns on the place value chart and to read the number as 113. This answer was given by 16%, 11%, 11% and 16% of Grade 3, 4, 5 and 6 children. Other responses, given by a few children, included 11.3 (as a way of avoiding a value in the hundreds position) and 14 (where the digits 1 and 3 are added together).

Place Value Task 16: Total value, ten thousands

Task 16 required the children to change the number displayed on a calculator screen to a given number by a single subtraction (286 349 changed to 206 349).

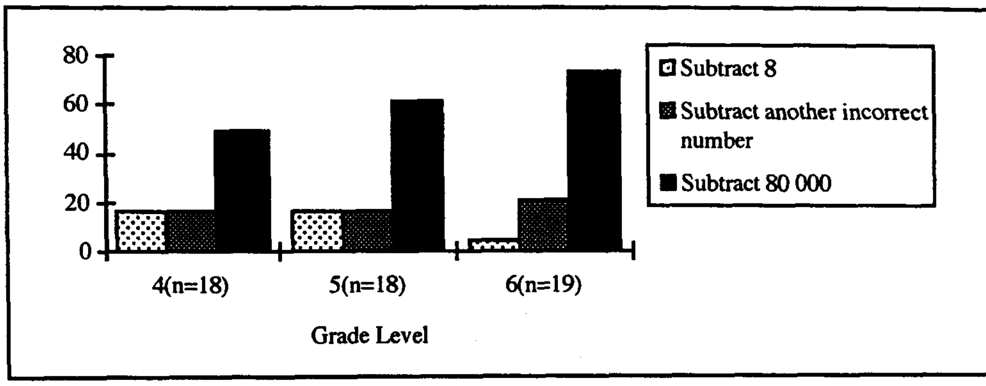

Figure 6.39 Solution strategies for Place Value Task 16, Total value: Percentage of sample giving correct and incorrect strategies.

Figure 6.39 shows that the percentage of children who correctly subtracted 80 000 to change the calculator display from 286 349 to 206 349 increased steadily from Grade 4 to 6 (50% to 74%). At each grade some children suggested subtracting 8, where they focused on the face value (17%, 17% and 5% at Grades 4, 5 and 6 respectively). There were some children who suggested subtracting other numbers such as 80, 800, 8 000, 8 million and 8 billion. Brooke (Grade 4) said "you can't take 8 away" but then was unable to make a further suggestion.

Summary: Place Value Tasks 1 to 16

Understanding place value is an essential part of the ability to use and interpret the notational system of number. The idea of place value develops from grouping experiences and enables the representation of any number with the digits from 0 to 9. Although performance on the place value tasks in this study generally increased as grade level increased, children in Grades 5 and 6 performed poorly on some tasks. It was not until Grade 4 that a majority of children correctly interpreted the digits in 2-digit and 3-digit numbers by their total values, and at Grade 6 there were still 58% of children who did not explain the meaning of zero in a numeral as a place holder. Children in Grade 4 and 5 still had difficulty writing numerals for numbers that involved digits outside the ones to thousands range. There was general improvement in recognition of place value across the grades, but at Grade 6 there were still 42% who could not identify the ten thousands place value (Task 14). The majority of children from Grade 3 correctly used total value (400) to change a calculator display but there were still 17% of Grade 5 children who were not successful (Task 10). When a larger number was used (Task 16) 50% of Grade 4 children correctly used total value (80 000) to change a calculator display but there were still 26% of Grade 6 children who were not successful. A majority of Grade 3 to 6 children correctly interpreted the non-standard numeric representation on Task 15, but there were still 33% of Grade 5 and 16% of Grade 6 children unsuccessful.

6.2.6 Performance and Strategy Use: Structure Tasks

This section will report children's performance and strategy use for twenty six structure tasks. The structure tasks assess the children's ability to identify structure in the number sequence, such as using and extending grouping systems (based on tens and other grouping numbers), using arrays to quantify a large number of items, and calculating with powers of ten. It is evident that there was a diverse range of strategy use across tasks but, as reported in Section 6.1, performance generally increased through the grades with some levelling off (and even decline) in the upper primary grades. The data illustrated in Figures 5.46 to 5.58 are reported in Tables B.14, B.30 to 32, B.34, B.38, B.39, B.42, B.45, B.47 to 51 respectively in Appendix B.

Structure Task 1: Visualisation 1 to 100

Task 1 required the children to imagine the numbers from 1 to 100.

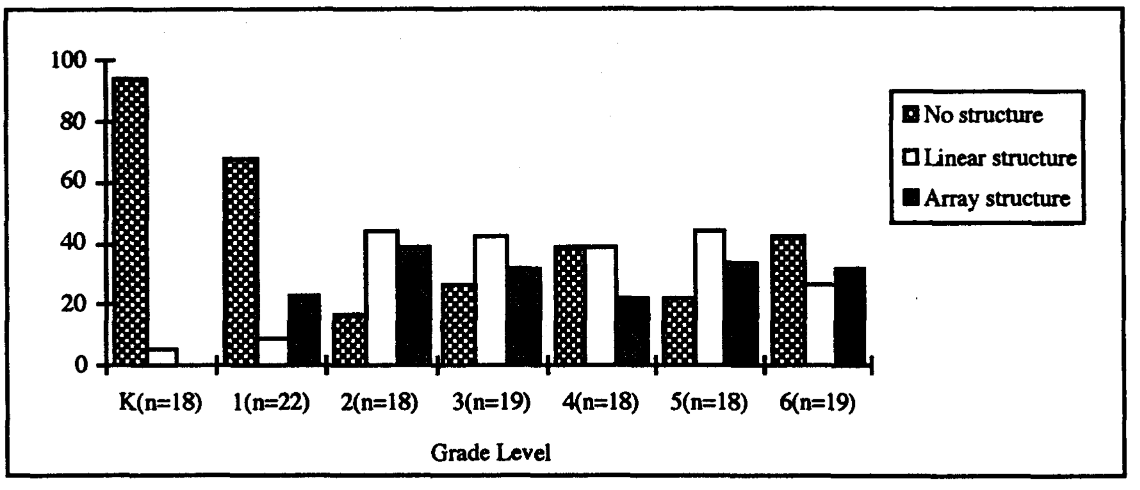

Figure 6.40 Solution strategies for Structure Task 1, Visualisation: Percentage of sample giving responses showing structure in the visualisation of numbers 1 to 100.

It will be recalled that the visualisation task was given to the children early in the order of tasks, before any tasks that used or evoked structured representations of the number sequence. Table 5.10 in Chapter 5 defines the way children's representations are analysed according to three dimensions: mode; type of structure; and nature of the image. Figure 6.40 shows the type of structure used in the children's representations or explorations: (i) no observable structure; (ii) representations with elements of linear structure but no elements of an array structure, and (iii) those with any elements of an array structure (Thomas & Mulligan, 1995). Analysis of the children's drawings and explanations of the numbers one to a hundred showed that by Grade 2 most children had developed either a linear or an array structure; for Grades 3, 4, 5 and 6 there was no significant change in the mix of structures being used. The analysis of the type of image showed that dynamic imagery was used by only 3.8% and notational representations by 78% of the children. Because of the importance of these results to the understanding of the structure of the numeration system, analysis on this task will be reported in depth in Chapter 8.

Structure Task 18 : Visualisation 1 to 1000

Task 18 required the children to imagine the numbers from 1 to 1000.

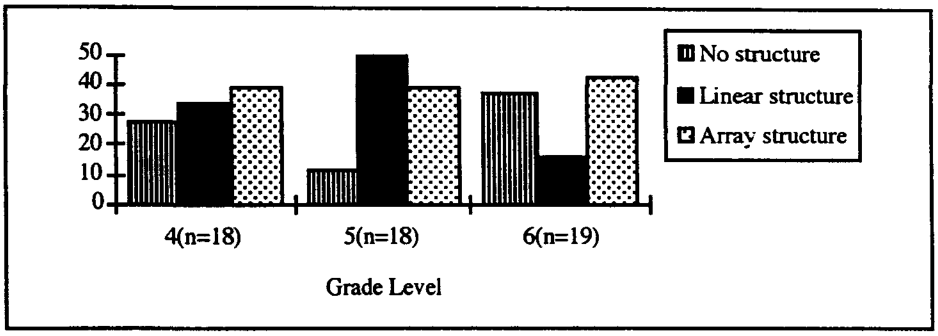

Figure 6.41 Solution strategies for Structure Task 18, Visualisation: Percentage of responses showing structure in the visualisation of the number sequence 1 to 1000 task.

Analysis of the visualisations of the number sequence one to a thousand (Figure 6.41) shows that there was a relatively stable use of an array structure for Grade 4, 5 and 6 (39%, 39% and 42% respectively). Dynamic imagery was used by only 4% and notational representations by 89% of the children. This task will be reported in depth in Chapter 8.

Structure Tasks 2 and 3: Ones, tens and hundreds structure, quantify pregrouped material (2 bags, 4 roll, 5 separate lollies) and add ten without counting.

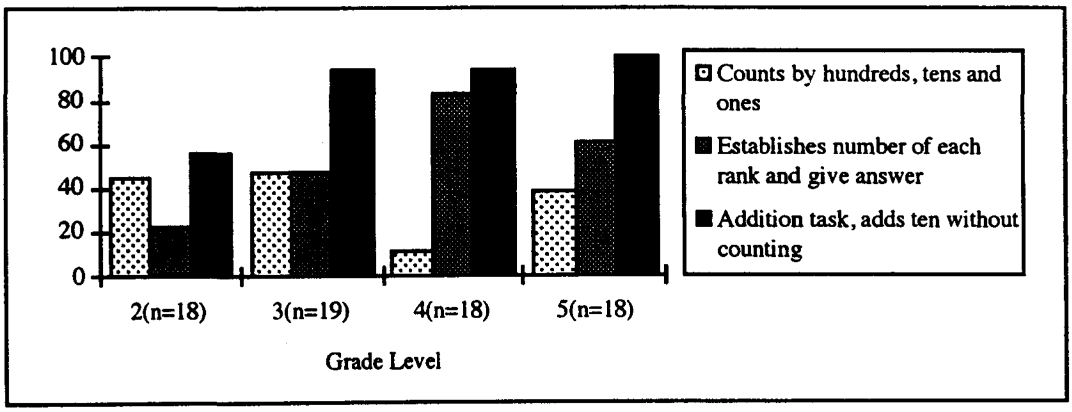

Figure 6.42 Solution strategies for Structure Task 2 and performance on Task 3, Quantify and addition: Percentage of sample that quantified lollies by strategies used and added ten without counting.

Figure 6.42 shows that the percentage of children who counted by hundreds, tens and ones to quantify the number of lollies was reasonably stable across the Grades 2, 3 and 5 (44%, 47% and 39% respectively) but was lower at Grade 4 (11%). At Grade 4 children were likely to establish the number of each rank and give the answer (83%). The ability to recognise, without the need for counting, the number ten more than a given number was high for Grades 3, 4 and 5 (95%, 94% and 100%).

Structure Task 6: One Hundred Square - use a hundred square (0 to 99 configuration) to find what is added to 84 to make 100.

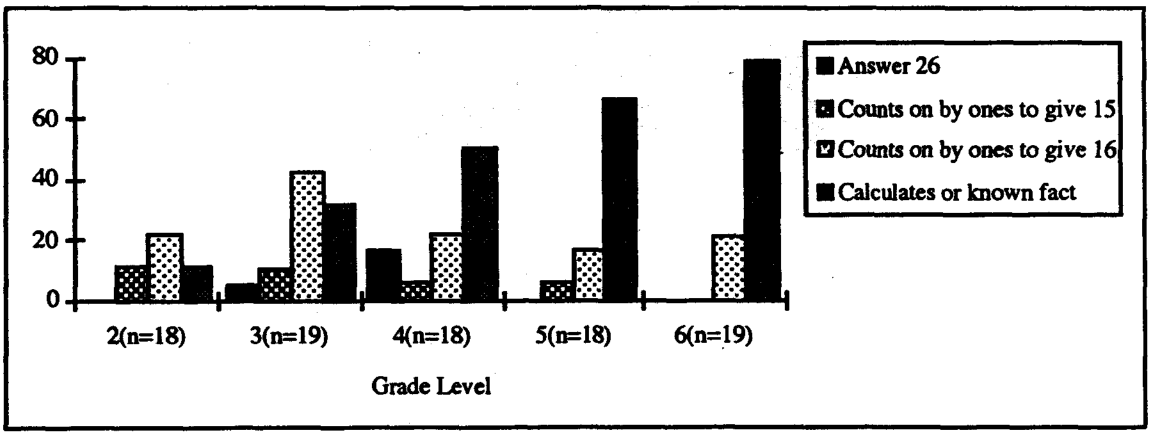

Figure 6.43 Solution strategies for Structure Task 6, Missing addend: Percentage of sample giving correct and incorrect responses by strategies used for addition on the hundred square.

The hundred square used had the numerals in a 0 to 99 configuration (rather than a 1 to 100 configuration) which made it more difficult to use a counting-on strategy to find the missing addend needed to make 100. Figure 6.43 shows that 42% of Grade 3 children counted by ones successfully on the 100 square to find the number to be added to 84 to make 100, but only 32% could use their knowledge of the pattern of a hundred as ten rows of ten. The ability to demonstrate this understanding increased steadily through Grades 2 to 6 (11% to 79%). The incorrect response "26", where the number identified on the hundreds square (84) was considered as two separate digits, was used by 17% of Grade 4 children. Another incorrect response, "15", arrived at as a result of not accounting for the missing numeral 100, was given by 11% of Grade 2 and 11% of Grade 3 children.

Structure Tasks 7 and 8: One Hundred Square - addition and subtraction of 10, from numbers on the hundred square (0 to 99 configuration).

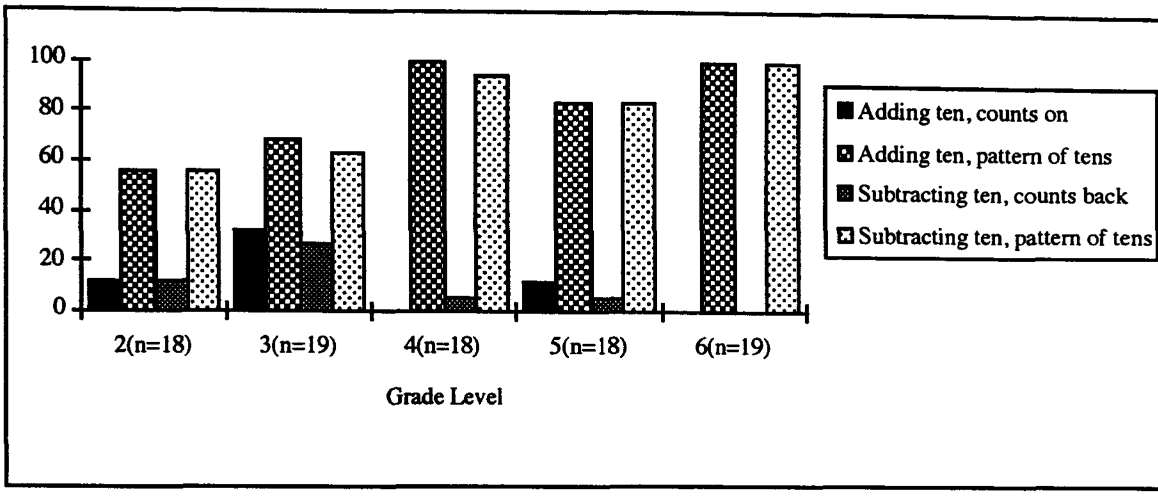

Figure 6.44 Solution strategies for Structure Tasks 7 and 8, Addition and subtraction: Percentage of sample giving correct responses, by strategies used for addition and subtraction of ten on the hundred square.

Figure 6.44 shows that the percentage of children counting on was generally low, the exception being 32% at Grade 3. The majority of children at each grade level used the pattern of tens in the hundred square to quickly identify the number which was 10 more than 36. Again the largest percentage of children who used a counting strategy in order to find the number ten less than 49 was at Grade 3 (26%). The majority of children also used the pattern of tens in the hundred square to quickly identify the number which was 10 less than 49.

Structure Task 9: One Hundred Square - addition of 9.

Task 9 required the children to find nine more than 67 on a hundred square.

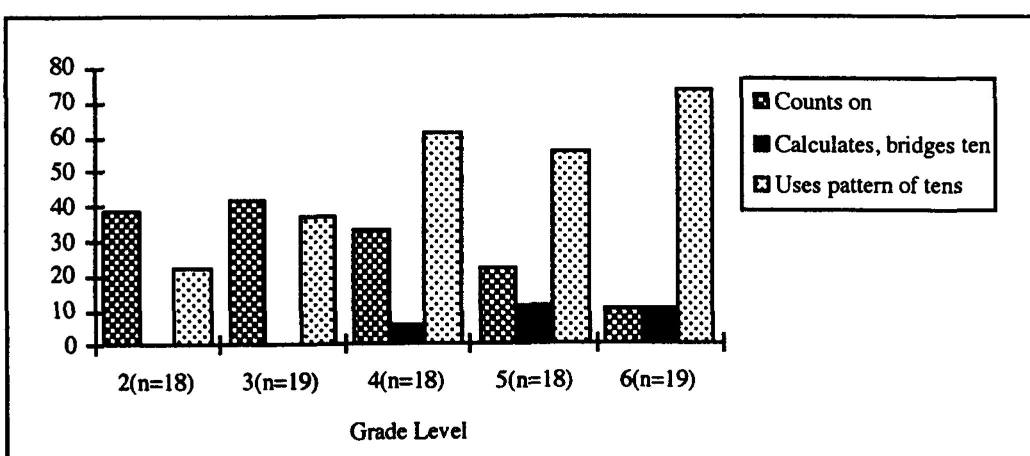

Figure 6.45 Solution strategies for Structure Task 9, Addition: Percentage of correct responses by strategies used for addition of 9 on the hundred square.

Figure 6.45 shows that for the addition of 9 (Task 5) the use of counting on generally decreased as the use of the pattern of numbers in the hundred square increased. The greatest use of the counting-on strategy was by Grade 3 children (42%) whereas the greatest use of the pattern strategy was by Grade 6 children (74%). Some children verbalised their use of the pattern, for example:

Alison (Grade 5): *... ten more is 77 ... 1 less that 10 ... so go back 1 ... 7.*

There was also a small but increasing use of some form of calculating over the Grades 4 to 6 (6% to 11%). For example Tim used the bridging of 70 whereas Victoria talked through a standard algorithm:

Tim (Grade 5): *... 3 on that is 70 ... 6 left ... it is 76.*

Victoria (Grade 6): *... 7 and 9 is 16 ... 60 and 16 is 76.*

Structure Tasks 10 to 13: Lolly factory, groupings of groupings

Tasks 10 to 13 required the children to extend a grouping system for packing lollies (referred to as the "Lolly Factory" by Cobb, 1996). Task 10 asked for the number of bags in a box and boxes in a case. Tasks 11 and 12 asked for the number of lollies in a case and in a collection of 3 lollies, 5 rolls, 7 bags, 1 box and 4 cases. Task 13 asked for the remainder from a case after 1468 lollies are sold.

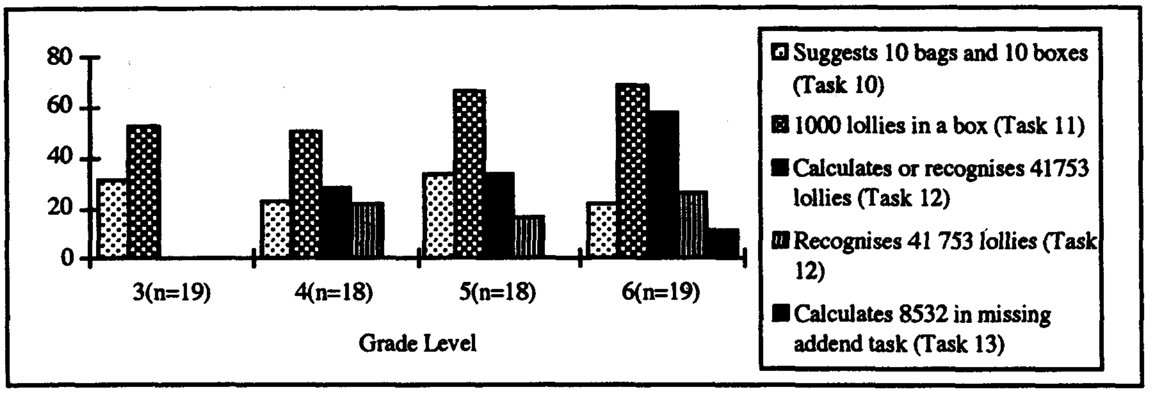

Figure 6.46 Performance on Structure Tasks 10, 11 and 13, and solution strategies for Task 12, Grouping of groupings: Percentage of sample giving correct responses, on Lolly Factory tasks.

Figure 6.46 shows the performance on four grouping tasks: suggesting ten both as a grouping number and as a grouping of groupings number; quantifying the third grouping; quantifying a structured collection (5-digit number), and giving correct responses to a subtraction task (4-digit subtrahend). When asked how they would package bags into boxes, and then boxes into cases, over 60% of children at each grade level did not suggest a consistent grouping of ten (Task 10). When it had been explained that ten was used as the grouping number in each packaging situation, the correct response (1000 lollies) was given by over 50% at each grade level, rising to 69% at Grade 6. When presented with a description of 3 lollies, 5 rolls, 7 bags, 1 box and 4 cases (Task 12) the number of children who were successful (41 753 lollies) increased to 58% at Grade 6. Some correct responses were given automatically indicating that the children could mentally coordinate the arithmetical units of different groupings (22%, 17% and 26% for grades 4, 5 and 6 respectively). Other children (6%, 17% and 32% for grades 4, 5 and 6 respectively) calculated the quantity through writing down the number of lollies in each package and then adding, indicating that they needed to 're-present' the packing activity in order to coordinate arithmetical units (i.e., they needed to imagine each package separately and as they did this represent the packages numerically). Many of the children giving unsuccessful responses (42% at Grade 6 level) attempted to calculate the number of lollies by adding the numbers corresponding to each package. There were only 11% of Grade 6 children who could calculate the number of lollies left after selling 1468 lollies out of a case. Cobb (1992, p.23) discussed the "qualitatively distinct conceptual interpretations" that children gave to similar tasks in a third-grade classroom teaching experiment.

Structure Tasks 14 to 16: Quantify a collection of random marks, suggestions for grouping, grouped marks, and explain grouping of groups.

Tasks 14 to 16 required the children to consider how they would find the number of marks in a picture (144 marks randomly drawn).

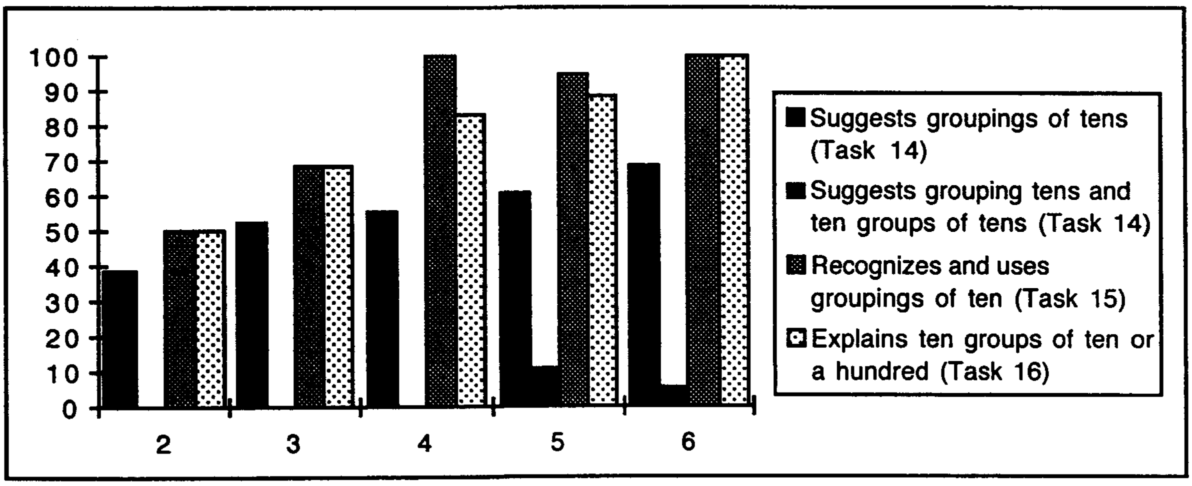

Figure 6.47 Performance on Structure Tasks 14 to 16, Groupings: Percentage of sample suggesting and recognising someone else's groupings of ten as a grouping number and as a grouping of groupings number.

Children were presented with a sheet of paper with a large number of marks drawn on it (Bednarz & Janvier, 1988). When they were asked to do something to the picture that might help someone else count the number of marks, the percentage that grouped the marks in tens by encircling them, increased from 39% at Grade 2 to 69% at Grade 6 (Figure 6.47). Only three children (Grades 5 and 6) suggested also grouping ten of the encircled tens to show a hundred. The children who did not group in tens either grouped in numbers other than ten, divided the whole collection in halves, tried to label each mark or said there was no way to make the counting easier. In a follow-up question, the children were shown a picture of the marks which had been grouped in tens (circled in black) and ten groups of tens (circled in red). When asked to now tell how many marks there were, the number of children who recognised and used the groupings of tens increased from 50% for Grade 2, 68% for Grade 3 and in excess of 90% for Grades 4, 5 and 6. There were no children at Grade 2 who also recognised and used the grouping of groupings shown on the picture. At Grades 3, 4, 5 and 6 there were 16%, 17%, 44% and 42% of children respectively who used the two orders of grouping.

There were some children at Grades 2 and 3 (22% and 11% respectively) who did not see the pertinence of grouping. Even though they had been shown what a 'friend' had done to help them, they still just gave an approximate number or attempted to count one by one. There were others who recognised the groupings of ten but could not successfully use them to quickly give the number of marks (28% Grade 2, 21% Grade 3 and 6% Grade 5). It would appear that many children at Grades 2 and 3 do not have recourse to grouping by tens as a strategy to communicate information about a collection.

Structure Tasks 4 and 5: Non-proportional representations
Structure Task 17: Multiplication by a hundred

Tasks 4 and 5 required the children to represent 246 using coloured counters (a red counter was worth 10 blue counters and a yellow counter was worth 10 red counters) and asked how the system could be extended to show the number 1246.

Task 17 required the children to use the shells on a place value chart (2 shells in the hundreds place, 3 shells in the tens place and 4 shells in the ones place) to make the number 100 times larger.

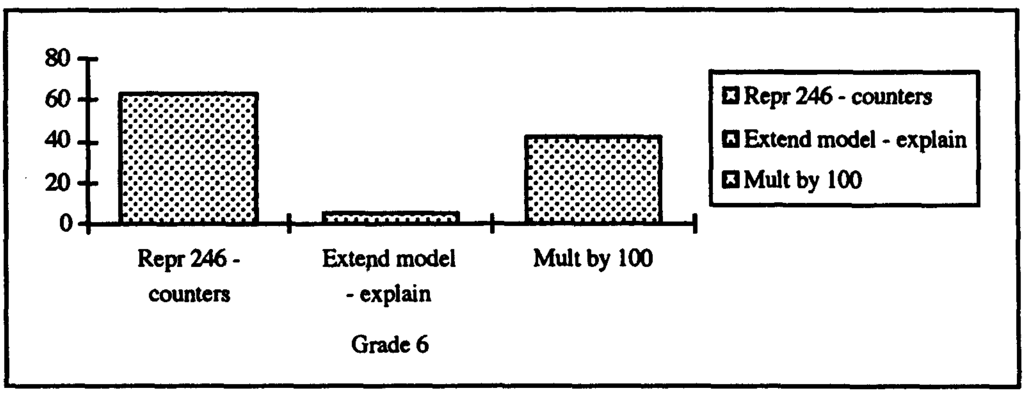

Figure 6.48 Performance on Structure Tasks 4, 5 and 17, Non-proportional representations: Percentage of Grade 6 children giving correct responses.

Figure 6.48 shows that 63% of Grade 6 children correctly recorded the number 246 using the non-proportional representation of coloured counters. When asked to explain what would need to be done to use counters to show a 4-digit number (1246), only 1 student indicated that a new colour would need to be used to represent groupings of 10 of the largest value counters (yellow counters).

After recognising the representation of 234 using shells on a place value chart, 42% of the Grade 6 children successfully used shells to record the number that was 100 times larger (multiply by 100). 11% of the children worked out the answer first and then replaced the counters with new ones in the correct positions for the answer 23 400. For example,

Jenna (Grade 6): ... *twenty three thousand four hundred because the hundred has got 2 noughts and you add them onto the number* (and then put new shells down in the correct positions and removed the old ones).

Other children (32%) just transferred each group of shells 2 places to the left on the place value chart and read the answer from the recording.

Structure Tasks 19 and 20: Rules for extending the system, to predict the groupings in a system of cubes, towers and walls (towers of 5 were constructed) and determine how values are generated.

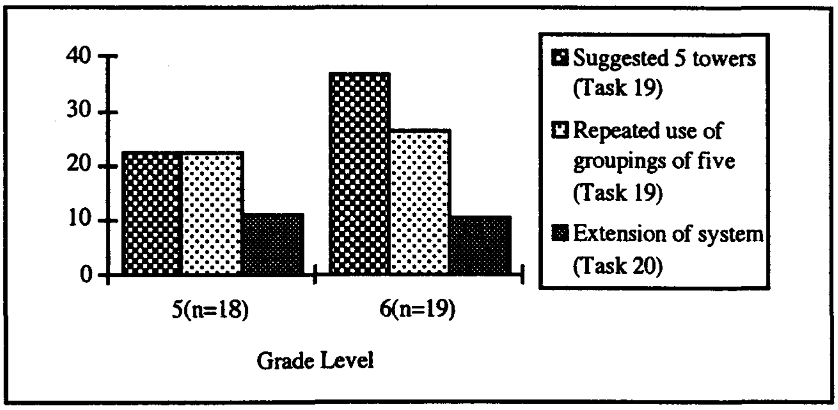

Figure 6.49 Performance on Structure Tasks, 19 and 20, Grouping of groupings: Percentage of sample giving correct responses.

Figure 6.49 shows the percentage of children in Grades 5 and 6 who correctly extended a grouping system based on fives. Although all children agreed that groups of 5 cubes would be combined to form the towers, only 22% of Grade 5 and 37% of Grade 6 children suggested the next groupings to be 5. Other grouping numbers suggested included 2, 3, 4, 10 and 20. At the Grade 5 level, there was a generally even spread of these alternative suggestions but 42% of Grade 6 children suggested groupings of 2. Second groupings of 5 were suggested by only 22% of Grade 5 and 26% of Grade 6 children. A few children (11% for Grade 5 and 11% for Grade 6) also gave some indication that they realised that the groupings of 5 should continue. After suggesting, 5 walls Michelle (Grade 5) said:

Michelle: ... *because we are counting up in fives.*

Similarly Jack (Grade 5) followed his suggestions of 5 with:

Jack: ... *because it is going up by five each time.*

Dale (Grade 6) was the only child who suggested that this gave values in the place value chart (from right to left) of 1, 5, 25, 125, 625 and so on.

Structure Task 21: Working with groupings of groupings

Task 21 requires the children to find the remainder of a pregrouped collection of lollies (pictorial representation) after giving some away (234 - 178)

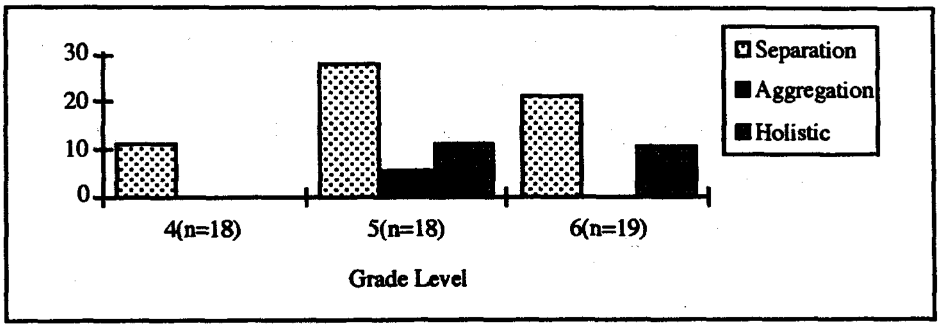

Figure 6.50 Solution strategies for Structure Task 21, Subtraction: Percentage of sample giving correct response, by strategy use.

Figure 6.50 shows the solution strategies for Subtraction Task 21. As the two quantities of lollies in this task were presented as pictures of pregrouped material, many children proceeded from the pictures and mentally opened bags and rolls (43.8% of successful strategies). At Grade 4 the only successful strategy used was separation (11.1%). At Grades 5 and 6 separation strategies were the most successfully used strategies (27.8% and 21.1% respectively) but holistic strategies were also used. The only child to use an aggregation strategy was Phillip (Grade 5) who abstractly worked right to left:

Phillip (Grade 5): *234 minus 8 is 226 ... 226 minus 70 is ... 156 ... 156 minus 100 ... 56.*

The majority of children who successfully used a separation strategy worked from left to right (72.7%), for example:

Kerryn (Grade 5): *Take away 1* (bag) *... 1 left ... 7 rolls take away from 1 bag ... 30 lollies ...* (another 3 rolls) *6 rolls and 4 lollies take away 8 lollies ... 56.*

Anna (Grade 6): *If you had ... can't do it aloud ...* (drew picture showing the bags, rolls and separate lollies) *take 7 rolls away ...* (crossed out 7 rolls leaving 6 rolls) ... *take 8 lollies* (crossed out 8 lollies) ... *56 lollies.*

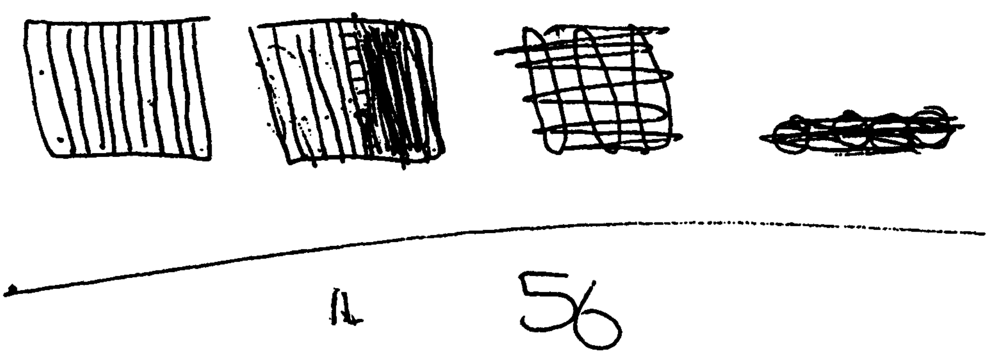

Figure 6.51 Drawing used by Anna to solve Task 21.

The holistic strategies involved bridging, relating to another number (200), and compensation are illustrated by the following excerpts:

Trevor (Grade 5): *178 and 22 is 200 ... 34 ... 56.*

Mark (Grade 6): *234 take away 178 ... take 34 from 234 gives 200 ... 22 onto 178 is 200 ... added 22 and 34 ... 56.*

Jonathon (Grade 6): *234 minus 178 ... 230 minus 170 ... 60 ... minus 4 ... 56* (looking at the pictures).

Some children used holistic or separation strategies but the excessive cognitive load involved with their methods meant that they lost track of information they needed and so did not give the correct answer for the problem. Kelly lost track of the 2 rolls left after unpacking 1 roll:

Kelly (Grade 6): *Take 1 bag away ... 7 rolls taken away from 1 bag ... 3 rolls left ... take 8 lollies from 1 roll ... left 2 lollies ... 6 separate lollies and 3 rolls.*

Kate successfully bridged 180 but then subtracted instead of adding the two components:

Kate (Grade 6): *178 up to 180 ... so 180 to 234 is 54 ... and take 2 away is 52 ... 5 rolls, 2 lollies.*

Deepack (Grade 6) successfully unpacked and subtracted from a bag and a roll, collected together the loose lollies but did not collect together the rolls:

Deepack (Grade 6): *1 bag* (subtracted 1 bag from 2 bags) ... *3 rolls* (subtracted 7 rolls from 1 bag) ... *2 lollies* (subtracted 8 lollies from 1 roll) ... *plus 4 ... 6 lollies.*

Structure Task 22: Quantify an array of dots

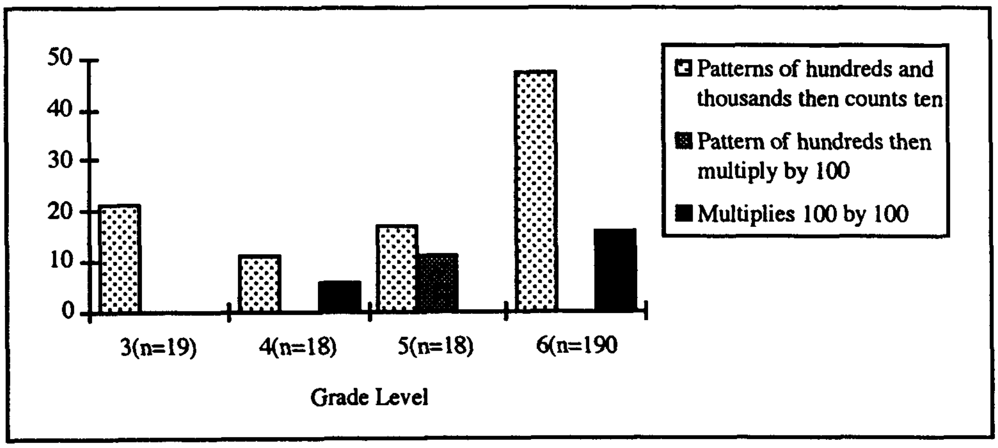

Figure 6.52 Solution strategies for Structure Task 22, Array: Percentage of correct responses by strategies used for quantifying the number of dots in an array of 10 000 dots.

When the children were shown an array of ten thousand dots, which were organised by the use of spaces to separate groupings of a hundred and a thousand, less than a third of those in Grades 3, 4 or 5 could successfully quantify the collection. Figure 6.52 shows that at Grade 6, 37% of the children were still unsuccessful in attempting to use the groupings of tens, hundreds and/or thousands to quantify the collection. The main strategies included:

- determining the pattern of hundreds, counting ten by ten and then multiplying by 100;
- determining the pattern of hundreds and counting by hundreds;
- determining the pattern of hundreds, counting by hundreds to give one thousand and then counting by thousands or identifying ten, and
- multiplying 100 by 100.

There were some children who said they could not find the answer or guessed a number (16% Grade 3, 6% Grades 4 and 5). Only 47% of Grade 3 children recognised the pattern of hundreds. In Grades 4, 5 and 6 most children used ten as the factor of one hundred to calculate or recognise the pattern of hundreds (89%, 89% and 100% respectively) but then could not use this pattern to quantify the whole collection of dots (72%, 61% and 37% respectively). The poor performance on this task could be related to a lack of experience with spatial arrangements of arrays.

Structure Tasks 23 to 26: Extension of the place value system and renaming values for concrete material, 10 000 and 0.001

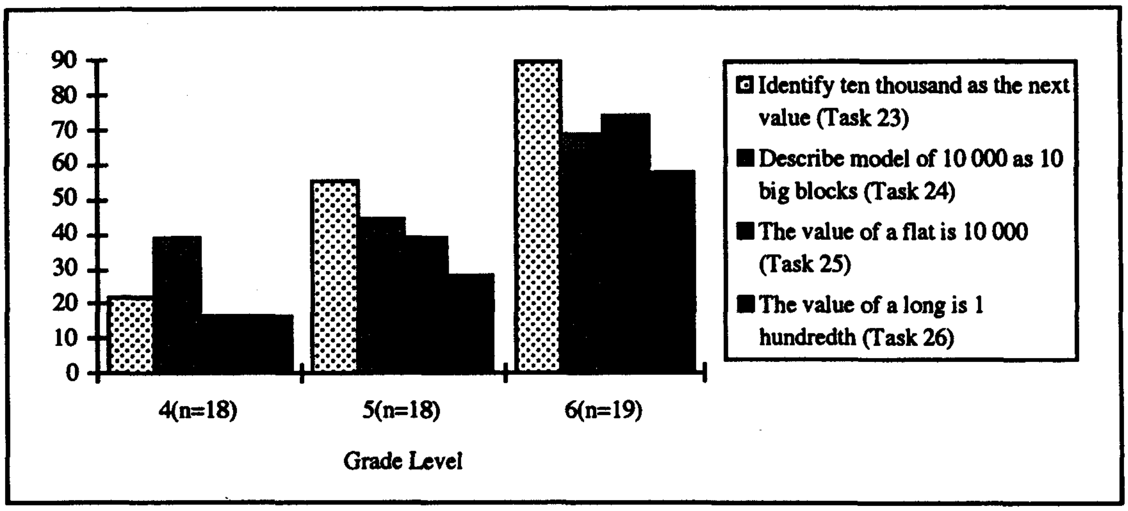

Figure 6.53 Performance on Structure Tasks 23 to 26: Percentage of correct responses on renaming values tasks.

At Grade 4 only 22% of children correctly identified 10 000 as the next value to be represented in the place value system after the ones, tens, hundreds and thousands that are most commonly assigned to the Dienes blocks (Figure 6.53). The percentage of children who correctly performed on this task sharply increased to 90% at Grade 6. When asked to describe a suitable Dienes block model for the number 10 000 there were 39% of Grade 4 children who described a larger block made from 10 big blocks. The percentage of children who correctly performed on this task increased to 68% at Grade 6. Incorrect responses given, were often "2 big blocks" (26%) or "100 big blocks" (7%) but, "4", "12" and "1000" were also suggested. Marc (Grade 5) responded "twice that size" (pointing to the big blocks). Rania (Grade 5) responded "12 together because there is six on this one" (pointing to the 6 faces of the big block).

When asked to assign different values to the Dienes blocks than those normally used in the classroom many children had some difficulty. Of the Grade 6 children 74% correctly gave the value of a flat as 10 000 if a short had a value of 100 but only 58% correctly gave the value of a long as a hundredth if a big block had a value of 1. Incorrect responses for the value of the 'flat', included suggestions of a million, hundred thousand, 1 thousand and 2 thousand. Incorrect suggestions for a long included nothing, minus, 10 thousand, 1 thousand, 1 hundred, ten and several fractions like thirds, tenths.

Summary: Structure Tasks 1 to 26

Response to the structure tasks showed that by the end of Grade 6 the children had a general competence to work with pregrouped concrete material where numbers had 4 digits or less (Tasks 2 and 3) and to operate successfully on the hundreds board (Task 6 to 9). Competence on any tasks which required an understanding of the structure was much lower, for example when larger numbers were worked with mentally and aspects of the recursive nature of the groupings were questioned (Tasks 10 to 16), when tasks required predictions of ways to extend concrete representations to accommodate larger numbers (Tasks 4, 5 and 23 to 26), and when quantifying dots in large arrays (Task 22). Understanding of the relationship of multiplication by powers of ten (and hence division) to the structure of the notational system or application of the multiplicative structure to a different grouping number (Task 19 and 20) was still low at the end of Grade 6. Responses to the structure tasks have shown the lack of relational understanding of the system of numeration of most Grade 6 children and in particular the importance of children developing understanding of the multiplicative and recursive grouping aspects of the system.

6.3 SUMMARY OF PERFORMANCE AND STRATEGY USE

The results presented in this chapter focus on children's development of understanding of the numeration system. This system of number involves both language and notational aspects and is based on the properties of position, a base of ten, addition and multiplication. The foundations of the base ten structured numeration system are the development of early oral counting, the formation of equivalent groups and the beginnings of an understanding of notational place value. When a child mentally computes addition or subtraction tasks, that person has had to use some well-developed concepts of the part-whole relationship of numbers and place value. Competence with the mental manipulation of numbers, taking into account number structure and the ability to be flexible in the use of this structure, is known as number sense. *The results presented in this chapter show that these understandings develop gradually and inconsistently over the Kindergarten to Grade 6 period and that very* *few children have been able to generalise what we will call the multiplicative structure of the system*. There is evidence of the use of abstract counting strategies by some Kindergarten children, but the performance on counting tasks in the upper grades is poor for many children. Although the performance on the estimation task (Task 7) was consistently good across the grades, there was lower than expected performance on many number sense tasks because of a high reliance on counting strategies rather than use of pattern and holistic strategies. Although there was good performance on using grouping in quantifying and building grouped material, there were indications that children did not understand the significance of ten in the number system. This is critical to their further development of understanding and use of the numeration system.

Some children efficiently computed using regrouping, using their own non-standard mental algorithms, before there was instruction on pencil and paper methods. Moreover there were also many children at Grade 6 who could not solve simple addition and subtraction tasks (Tasks 7, 10 and 11). Standard algorithmic procedures began to be used mentally in Grade 4 (Tasks 4, 5 and 7) showing that intuitive procedures were sometimes replaced by learned procedures. Performance on place value tasks which involved zero as a place holder or digits outside the range of ones to thousands was generally poor. Similarly, performance on structure tasks with numbers that involved place values beyond the ones to thousands that are commonly represented by concrete materials (numeration blocks) was also low. This was highlighted by the poor performance over all grades with determining the number of lollies in a pregrouped collection (41 753 lollies) and the small number of children who successfully carried out the mental subtraction of 1468 from 10 000 (Task 13). There were, however, indications that young children's informal modes of representation can show insight into the way children structure the number system. It is also suggested by the results, that older children are often restricted by their experiences with formal written number work.

Chapter 7 will further discuss the significance of these results and make inferences regarding children's development of understanding of the numeration system.