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APPENDIX A

Pilot Study Interview Record Form

Name: ....

School: .... Grade: .... Teacher: ....

Protocols	
1 *Close your eyes. I want you to imagine the numbers from 1 to 100. Can you see a picture of these numbers? Open your eyes. Draw a picture of what you saw.*	
2 Display 8 (5) shells. *How many shells are there here?* Place out 4 (3) shells which are screened from view. *There are 12 (8) shells altogether. How many are hidden?* Smaller numbers given to K and Grade 1 children.	
3 Display a collection of ten counters. *How many shells are there here?* (10) Hide 3 shells. *How many shells are under my hand?*	
4 After establishing how many shells remain when 3 are hidden (from initial collection of 10) - Task 3. *Can you give me any other numbers that add to give 10?*	
5 *Can you count by tens? Count by tens as far as you can.*	
6 *Give the same amount of lollies to each Lego person. We are to use all the lollies.* (12 lollies shared between 3 people).	

7	a) *I am going to give you some shells. I want you to put these shells on the plates so that there is the same number of shells on each plate* (26 shells with 6 plates). *How many shells are on each plate? How many shells are there here altogether? Write down that number.* b) *After putting shells on the plates so that there is the same number of shells on each plate* (26 shells with 6 plates) and establishing that there are 4 shells on each plate (7a): *Count these shells by fours.*	
8	*How many groups of 4? How many left over?* The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many shells you have?* The digit in the tens place is circled and the question repeated.	
9	Show a roll of 10 lollies (transparent). *How many lollies are in this roll?* Show 4 opaque rolls, each containing 10 sweets. *How many lollies are in this roll? How many lollies are in all these rolls?* 1 roll and 5 loose sweets are displayed. *How many lollies?*	
10	4 rolls and 3 loose sweets are displayed. *How many lollies are here altogether?*	
11	4 rolls and 3 loose sweets are displayed (Task 10) *Write down the number* (43). The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many lollies you have?* The digit in the tens place is circled and the question repeated.	
12	*If you added 8 lollies to your collection there, how many lollies would you have altogether?* Assess whether child counts by ones and then trades or uses ten as a unit in the trade.	
13	Show a picture 8 rolls and 6 separate lollies. *How many lollies in this roll? How many lollies altogether?* (Figure A.1)	

14	*Show me five groups of three shells* (jar of shells provided).	
15	Using the display of shells from Task 14. *Count the shells by threes.*	
16	a) *See if you can think aloud as you do this next question.* *There are 12 children with 4 children sitting at each table.* *How many tables are needed?* Give opportunity to draw picture or use material. b) Provide 12 Lego people and 5 trucks made out of Lego blocks. *There are 12 Lego people and some trucks. 4 people go to work in each truck. How many trucks are needed?*	
17	Show a long: *How many does this show?* Show a long and 2 shorts: *How many does this show?* From a set of Dienes block representations of ones (only 40 provided), tens and hundreds the child was asked to: *use these counting blocks to build 52.*	
18	Children who were successful with Task 17 were asked if they could find another way to represent 52. *Can you draw a picture to show how the blocks represent 52.*	
19	52 represented by a non-standard grouping of Dienes blocks (Task 18) *Can you write in numbers how the blocks represent 52.* Digit correspondence task. The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many blocksyou have?* The digit in the tens place is circled and the question repeated.	
20	Tens task - a board to which is affixed a sequence of Dienes longs and shorts is gradually uncovered and each time the cover is pulled back to show more material the child is asked: *how many are there now?* 10, 14, 34, 38, 41, 51, 53, 73	

21	Show a 7 and a 9 pattern board (twos pattern). *How many dots are here?* Show the 7 board. *How many dots are here?* Show the 9 board. *How many dots are there altogether?* (Figure A.2)	
22	3 rolls (opaque coverings) and 7 separate lollies are visible, the child is told 25 lollies are hidden beneath the cloth and asked to find how many lollies there are altogether.	
23	Show 32 represented by three rolls and two separate lollies. Get respondant to close eyes and then hide one roll and five lollies under tin i.e. show 17. *How many lollies are hidden under here?*	
24	Write the number one hundred and three. Write the number one hundred and eleven	
25	Show an array of ten by six planes. *Can you tell me quickly how many planes are here?* (Figure A.3)	
26	Show a roll of ten lollies: *How many lollies are here?* Show a bag of ten rolls: *How many lollies are here?*	
27	Show 2 bags, 1 roll, 4 separate lollies - all transparent coverings. *How many lollies are there altogether? Write down the number of lollies.*	
28	Show 2 bags, 1 roll, 4 separate lollies - all transparent coverings. *How many lollies are there altogether? Write down the number of lollies* (Task 28). Digit correspondence task. The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many lollies you have?* The digits in the tens and hundreds places are circled and the question repeated each time.	
29	Show 3 bags, 12 rolls, 5 separate lollies - all opaque coverings. *How many lollies are there altogether?*	

30	Digit correspondence task for display used in Task 29. The digit in the ones place is circled and the child asked: *Does this part have anything to do with how many lollies you have?* The digits in the tens and hundreds places are circled and the question repeated each time.	
31	Show 143 represented by one bag, four rolls and three separate lollies (opaque coverings). Get respondent to close eyes and then hide 9 rolls, and the 7 lollies under tin i.e. show 46. *How many lollies are hidden under here?*	
32	Child presented with a picture of 143 marks randomly drawn. *Can you tell me quickly how many marks there are drawn there?*	
33	*I am going to do the same thing later with a friend who will be here after you. Could you do something so that, when I show him the sheet, he will be able to tell me very quickly how many marks there are? What did you do? Now can you tell me quickly how many marks there are? How do you know that?*	
34	*Look at what the friend who came before you did* (grouping of groupings is shown). *What do you think of it? Can we see quickly how many marks there are? Suppose you have a younger brother / sister who you are going to help with his/her counting. How would you explain the easy way to count those marks.* (Figure A.4)	
35	Show a label with the expiry date 'use by 01 August' and ask: *What is this number? What does the nought do?* (Figure A.5)	
36	Show a place value chart and place numeral cards for numbers '1' and '13' in the tens and ones places. *What does this mean? What is another name for this number? Is there any other way of writing this?*	

37	Provide a box of unifix cubes. *In our number system, we always make groups of ten. If we made a make-believe number system where all our groupings were based on 5, then we would group these cubes together to make towers of 5. Here are some towers of 5. How many towers would we group together ?* (Show some Dienes blocks. *Here we have some shorts, longs and flats. How many towers do we put together to form a flat?*)	
38	Show the numeral 521 400 written on a card. (Pointing to the digit 2 in 521 400). *What does the 2 stand for?*	
39	Show the numeral 6399. *What number is one more than 6399?.*	
40	*Write any number between 4100 and 4200.*	
41	Show an array of 100×100 dots. *Can you tell me how many dots are here?* (Figure A.6)	
42	Show a hundred square (0 to 99). *Show me how you can get ten more than 36 quickly on the hundreds square.* *Can you show me ten less than 49?* (Figure A.7)	

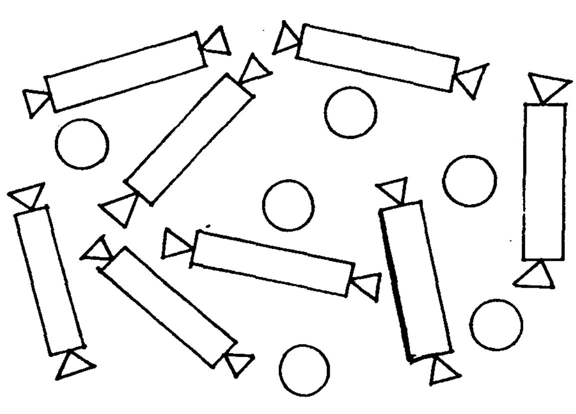

Figure A.1: Rolls and lollies for Task13

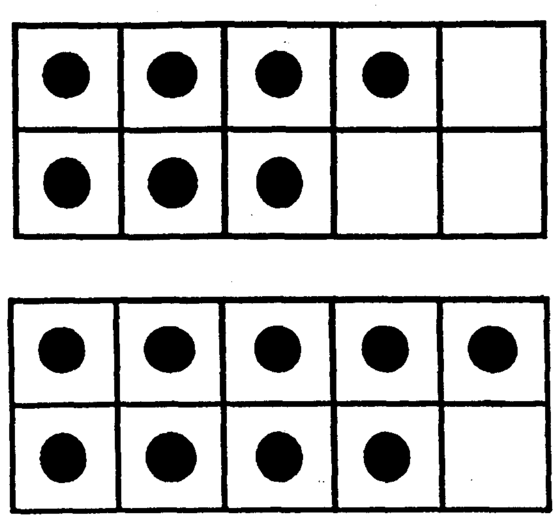

Figure A.2: 7 and 9 pattern boards for Task 21

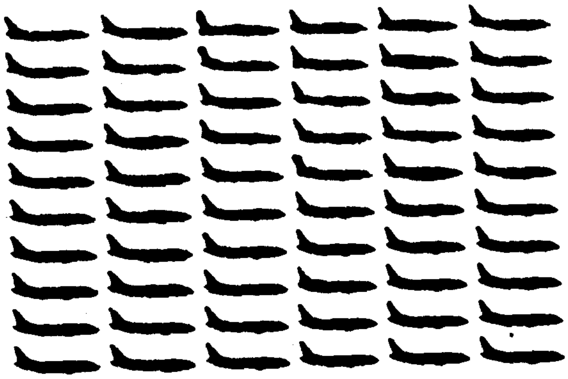

Figure A.3: Array of ten by six planes for Task 25

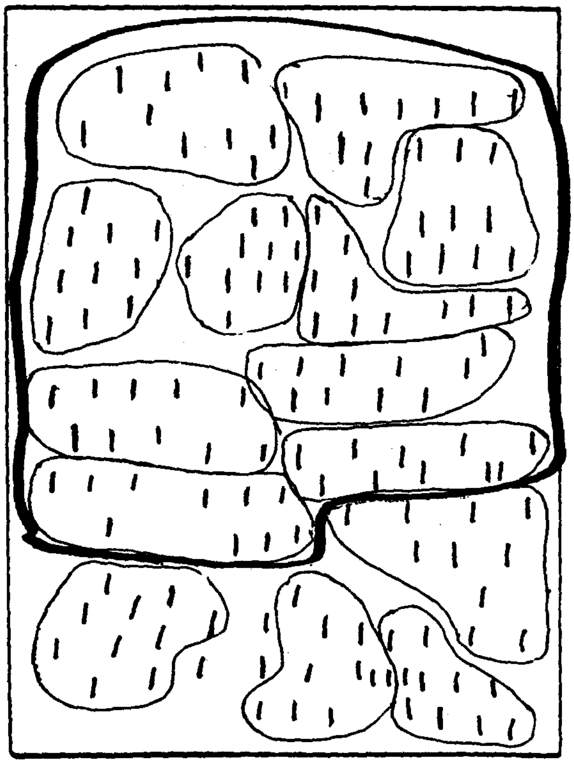

Figure A.4: Picture of 143 marks with groupings shown for Task 34 (Bednarz & Janvier, 1988, p. 310)

Figure A.5: Label with expiry date for Task 35

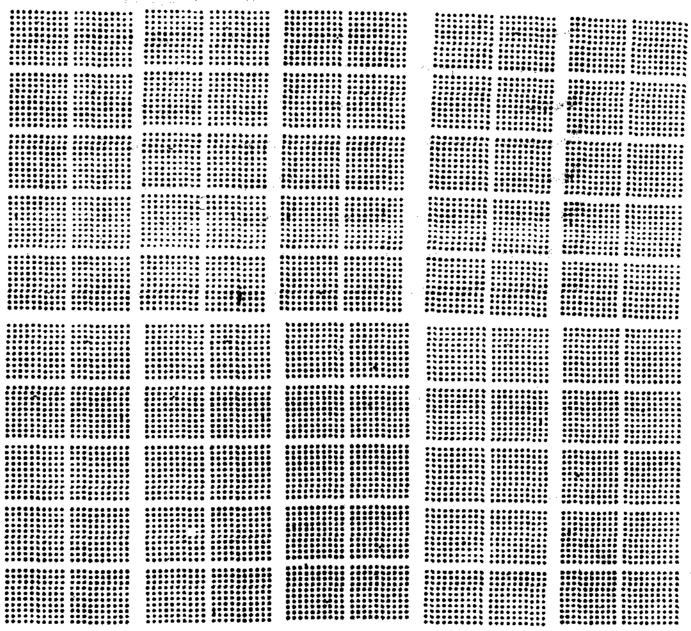

Figure A.6: Array of 10 000 dots for Task 41

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	~~36~~	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Figure A.7: Hundred Square for Task 42

APPENDIX B

MAIN STUDY RESULTS

Counting tasks
C1 Addition task
5 shells are hidden under one container and 7 shells hidden under another.
There are 5 shells here and 7 shells here.
How many shells are there altogether? (shells available if required)

Table B.1: Task Response Categories by Grade Level

	Grade		
Response Categories	n=18 K	n=22 1	n=18 2
0 Guess or no answer	15	5	1
11 Counting all concrete	0	6	3
12 Counting all abstract	1	2	1
2 Counting-on from 5 concretely	0	3	4
3 Counting-on from 7 concretely	1	1	3
4 Counting-on from smaller no. / abstract	0	2	0
5 Counting-on from larger no. / abstract	0	1	3
6 Relate to known fact (doubles), builds ten or known addition fact.	1	2	3

C2 Missing addend task. Display 8 shells. *How many shells are there here?*
Place out 4 shells which are screened from view. *There are 12 shells altogether. How many are hidden?*

Table B.2: Task Response Categories by Grade Level

	Grade		
Response Categories	n=18 K	n=22 1	n=18 2
0 Guess or no answer	15	8	
1 Counting all	1	1	
2 Counting-on / concrete (fingers)	1	5	7
3 Counting-on / abstract	1	5	4
4 Builds ten, known addition fact or relates to known fact		3	7

C3 Removed item task
Display a collection of ten shells. *How many shells are there here?*
Hide 3 shells. *How many shells are under my hand?*

Table B.3: Task Response Categories by Grade Level

	Grade		
Response Categories	n=18 K	n=22 1	n=18 2
0 Guess or no answer	15	8	
1 Counting all	1		
2 Counting-on / concrete (fingers)	1	7	7
3 Counting-on / abstract		3	4
4 Known addition fact (builds ten).	1	4	7

Number sense task
N1 *Can you give me any other numbers that add to give 10?*

Table B.4: Task Response Categories by Grade Level

	Grade		
Response Categories	n=18 K	n=22 1	n=18 2
0 Guess or no answer.	14	9	1
1 Calculates at least two further combinations.	2	5	5
2 Recalls at least two further combinations.	2	8	12

Grouping partitioning tasks - partitioning into equivalent groups
G1 *Give the same amount of lollies to each Lego person. We are to use all the lollies.* (12 lollies shared between 3 people).
G2 *How many lollies did each Lego person get?*

Table B.5: Task Response Categories by Grade Level

	Grade		
Response Descriptions	n=18 K	1	2
0 Unsuccessful	2		
1 One to one dealing	12		
2 Many to one dealing	4		
1 Counts each group	6		
2 Counts only one group	6		
3 Knows answer	6		

G3 *I am going to give you some shells.*
I want you to put these shells in the plates so that there is the same number of shells on each plate (26 shells with 6 plates).

Table B.6: Task Response Categories by Grade Level

	Grade			
Response Descriptions	n=22 1	n=18 2	n=19 3	n=18 4
01 One to one dealing, distributed all objects.	2	1		
02 Many to one dealing, est. too large, unable to solve or distributed all objects.	3		1	
1 One to one dealing, acknowledged the remainder	7	10	9	9
21 Many to one dealing, more than one deal, remainder acknowledged.	7	6	7	7
22 Many to one dealing, used known multiplication or division facts.	3	1	2	2

P1 Place value task
Digit correspondence task with 26 (displayed as 6 groups of 4 and 2 left over).
How many groups of 4?
How many left over?
The digit in the ones place is circled and the child asked:
Does this part have anything to do with how many shells you have?
The digit in the tens place is circled and the question repeated.

Table B.7: Task Response Categories by Grade Level

	Grade				
Response Descriptions	n=18 K	n=22 1	n=18 2	n=19 3	n=18 4
00 No meaning assigned to digits ('don't know', gave an insufficient explanation, unable to write numeral correctly).	11	1			
01 Associates the ones digit with the number of groups and the tens digit with the number of shells left over (the 2 in 26 stands for the two shells and/or the 6 in 26 stands for the six plates).			2	1	1
02 Digits interpreted by their face values.	7	21	13	10	4
1 Digits interpreted correctly by their total values (twenty or two tens and six ones).			3	8	13

G4 Lauren has planted 20 cabbages. There are 4 equal rows. How many cabbages are in each row?

Table B.8: Task Response Categories by Grade Level

	Grade		
Response Descriptions	n=19 3	n=18 4	n=18 5
0 No response	1	2	
1 Concrete / pictorial modelling	1	2	
2 Visualize and partition into 4 equal groups			
3 Double count fours	3	2	
4 Estimate / check	1		
5 Relate to division fact / mult.	13	12	

Skip counting tasks
C4 Count by tens

Table B.9: Task Response Categories by Grade Level

	Grade			
Response Descriptions	n=18 K	n=22 1	n=18 2	n=19 3
0 Not able to count by tens.	15	1		
1 Counts to 90	2	5		
2 Counts to 100		5	2	
3 Counts beyond 100	1	11	16	19

G5 *Show me six groups of three shells* (jar of shells provided).
Cover the shells with a cloth.
C5 Multiple counting groups of three
Count the shells by threes. (If unable to count the shells then uncover them and repeat question).

Table B.10: Task Response Categories by Grade Level

	Grade						
Response Descriptions	n=18 K	n=22 1	n=18 2	n=19 3	n=18 4	n=18 5	n=19 6
Unable to form 6 groups of three							
00 unable to form groups	1	1					
01 made groups of 3	5	1					
02 made group(s) of 6	5	2		1			
03 made 3 groups of 6	2	3					
1 Formed 6 groups of three.	5	15	18	18			
0 Counts by ones or multiple counts, concrete (uncovered).	17	7	3				
1 Rhythmic counting in groups (using fingers).		4	1	1		1	
2 Rhythmic counting in groups beyond 18 (numerical composite), abstract (covered).		7	8	8	2		1
3 Rhythmic counting in groups (numerical composite), abstract (covered) with concrete double count (fingers).	1	4	5	2	1	2	3
4 Double counting, concrete (fingers).				4	3	4	3
5 Double counting (abstract).			1	3	4	1	
6 Multiple count, used multiplication.				1	8	10	12

G6 Grouping Quotition tasks - double counting

a) *See if you can think aloud as you do this next question.*
There are 12 children with 4 children sitting at each table.
How many tables are needed?
Give opportunity to draw picture.
If unsuccessful then ask b).
b) *There are 12 Lego people and some trucks.*
4 people go in each truck.
How many trucks are needed?

Table B.11: Task Response Categories by Grade Level

	Grade				
Response Descriptions	n=18 K	n=22 1	n=18 2	n=19 3	n=18 4
0 Not able to solve problem (meaningful concrete model).	10	10			
11 With materials - concrete	8	4	1		
12 Drawing pictures - concrete		6	9	2	2
13 Counting fingers (marks)- concrete			2	3	2
2 Building up groups of 4 mentally until 12 (double count).		2	3	5	6
3 Taking away groups of 4 - build down.			1		
4 Known multiplication / division facts.			2	9	8

G7 Abstract property of quantity task

You are collecting stickers. You can trade 2 small stickers for a large sticker. How many large stickers are worth the same as 3 large stickers and 4 small stickers?

Table B.12: Task Response Categories by Grade Level

	Grade				
Response Descriptions	n=22 1	n=18 2	n=19 3	n=18 4	n=18 5
00 Not able to solve problem or guessed.	8		3		
01 Deals with only one unit of measure - answer 3 big stickers.		2	1		
02 Deals with only one unit of measure (2 or 7).	9	11	7	5	4
1 Unit of measure is arbitrary - deals with two units similtaneously (5).	4	5	8	13	14

P2/P3 Zero as a place holder
Child shown a milk carton with "use by 01 August" stamped on it (Appendix A, Figure A.5) and asked what number "01" was and why.
What is this number?
What does the nought do?

Table B.13: Task Response Categories by Grade Level

	Grade					
Response Descriptions	n=22 1	n=18 2	n=19 3	n=18 4	n=18 5	n=19 6
01 Unsure of meaning.	7	9			1	
02 Ten.	9		2	1		
1 One, first.	6	9	17	17	17	19
0 No meaning given to zero	21	16	13	10	9	11
1 Meaning as no tens	1	2	6	8	9	8

Structure tasks
S1 Imagery of numbers task
Close your eyes. I want you to imagine the numbers from 1 to 100. Can you see a picture of these numbers?
Open your eyes.
Draw a picture of what you saw.

Table B.14: Task Response Categories by Grade Level

	Grade						
Response Descriptions	n=18 K	n=22 1	n=18 2	n=19 3	n=18 4	n=18 5	n=19 6
0 No picture	9	5		1			
101 Picture ($ sign, house, dog)	2	3	2		1	1	1
102 Pictures or marks - no pattern	1						
103 Numerals - random, 1 &100 or single numeral	5	6		4	5	3	7
203 No structure, dynamic, numeric.		1	1		1		
111 Partially linear, static, concrete.				1	1		
112 Partially linear, static, ikonic.							
113 Numerals - linear, partial sequence	1			2		3	1
213 Partially linear, dynamic, numeric.				1			
121 Linear, static, concrete.						1	
122 Pictures or marks - linear, linear.							
123 Numerals - sequenced in a square / circle pattern, Numerals - linear (1 - 100), static.		2	4	4	4	4	4
223 Linear, dynamic, numeric.					1		
133 Numerals - multiple count (tens, twos), static.			4		1		
233 Numerals moving, multiple count (in a circle, flashing).							
141 Partial array, static, concrete.							
142 Pictures or marks - groups of tens, static, ikonic.							
143 Arrays - pictures or numerals, not 10 by 10, Numerical expression (10 * 10)		5	4	2	3	4	
243 Partial array, dynamic, numeric.							
152 Arrays - pictures, 10 by 10.							
153 Arrays - numerals, 10 by 10.			3	4	1	2	6
253 Arrays, dynamic, numeric.							

Groupings of ten tasks
Show a roll of 10 lollies (transparent).
How many lollies are in this roll?
Show an **opaque** roll, containing 10 sweets.
How many lollies, do you think, are in this roll?
If the response is not ten then say that there are ten lollies in the roll the same as there are in this roll (the transparent roll).
G8 1 roll (opague) and 5 loose sweets are displayed.
How many lollies?
If response involves counting-on from ten in ones then the following question is asked.
G9 4 rolls (opague) and 3 loose sweets are displayed.
How many lollies are here altogether?

Table B.15: Task Response Categories by Grade Level for Tasks G8 and G9

	Grade				
	n=18	n=22	n=18	n=19	n=18
Response Descriptions	K	1	2	3	4
00 Guess or unable to count (15).	2				
01 Counts the package as one.	2				
02 Counts in ones, guessing number in package.	7	2			
1 Counts in ones, correct number in package.	4	2			
2 Counts-on from 10 in ones.	3	11	8	9	
3 Counts 5 - ten and five makes 15		6	10	10	18
00 Guess or unable to count (43).	9	1			
01 Counts singles as tens		2			
02 Counts each package as one.		1			
03 Counts in ones, guessing number in each package.	7	4			
04 Counts in tens and in ones without coordination.		2			
1 Counts in ones, correct number in each package.	1				
2 Counts-on from 10 in ones.	1	5	7	5	9
3 Counts in tens and in ones with coordination.			2	1	
4 Counted-on from 40 by ones		7	9	13	9
5 Uses multiple of tens / knows answer.					

Regrouping
R1 Mental addition
If you added 8 lollies to your collection there, how many lollies would you have altogether?
Assess whether child counts by ones and then trades or uses ten as a unit in the trade.

Table B.16: Task Response Categories by Grade Level for Task R1

	Grade			
	n=22	n=18	n=19	n=18
Response Descriptions	1	2	3	4
0 Counts without coordination, incorrect result.	11	2	1	1
1 Counts-on from 43 by ones.	8	11	6	2
2 Counts-on from 48 by ones.	1	1	2	
3 Adds the ones together and then regroups.	1	1	8	9
41 Adds ten and takes away 2, bridges tens.				1
42 Builds ten, adds-on from 43 / 48.	1	3	2	5

G10 Count out 37 paddle pop sticks as 3 tens and 7 ones.
Here are 3 bundles of ten sticks and 1,2,3,4,5,6,7 extra sticks.
How many sticks are there altogether?
- are the groups of ten used?

Table B.17: Task Response Categories by Grade Level for Task G10

	Grade				
	n=18	n=22	n=18	n=19	n=18
Response Descriptions	K	1	2	3	4
00 Attempted to count all by ones	14	7			
01 Counts ones as tens	1	2			
02 Counts tens as ones	1				
1 Counts tens and ones	2	6	5	1	
2 Counts-on from 30 by ones.		2	1	1	
3 Gives answer.		5	12	17	18

Regrouping tasks - ones and tens
G11 From a collection of shells packaged in bags (tens) and as loose shells (ones, only 40 provided).
I have packaged these shells into bags in order to make it easier to count them.
How many do you think I have put in each bag?
G12 The child is then asked: *show me 52 shells.*
R2 Addition with concrete material
Use the shells to make the number which is 9 larger.
Children who were successful with the above task were asked if they could find another way to show 61.

Table B.18: Task Response Categories by Grade Level for Tasks G11, G12 and R2

Response Descriptions	Grade n=22 1	n=18 2	n=19 3	n=18 4	n=18 5
0 Suggested number other than ten.	18	11	13	8	6
1 Suggested ten.	4	7	6	10	12
00 Unable to show 52	6				
01 Attempt to count separate shells.	8	1	1		
1 Counted 5 bags and 2 shells.	8	17	18	18	18
00 Unable to add 9	7	2			
01 Added 9 bags					1
1 Added 9 shells.	13	13	11	10	5
2 Added 1 bag and took away 1 shell.	2	3	8	8	12
0 Unable to give a second representation.		1	1	2	3
1 Gave a second representation.	2	2	7	6	9

Application tasks

C6 / C7 / C8 *How many twenty cent coins are there in $1?*
How many twenty cent coins are there in $3?
How many twenty cent coins are there in $10?

Table B.19: Task Response Categories by Grade Level for Tasks C6, C7 and C8

Response Descriptions	Grade n=22 1	n=18 2	n=19 3	n=18 4	n=18 5
00 guesses or unable to solve.	14	5	1		
01 multiple count but found difficulty keeping track	2	1	1		
1 Double count to find answer (concrete)	4	4	5	3	
2 Double count to find answer (abstract)			3	1	1
3 Knows answer.	2	8	9	14	17
0 Unable to find answer	16	11	4	1	
1 Double count 20, 40, ... $3 .			1		1
2 Relates to no. in $1, double count (5, 10,15,..).	3	6	5	4	5
3 Uses relationship of quantities (mult., repeated addition).	3	1	5	7	6
4 Knows answer / uses division.			4	6	6
0 Unable to find no. of 20c coins in $10	20	17	11	4	8
1 Relates to no. in $1, double count	2	1	4	5	1
2 Relates to no. in $1, multiplication, knows answer / uses division.			4	9	9

N2 / N3 *What is the change from $1 if something cost 64c?*
Write down 64c + 36c = $1
Can you write down 2 other combinations that make $1?

Table B.20: Task Response Categories by Grade Level for Grades N2 and N3

Response Descriptions	Grade n=18 2	n=19 3	n=18 4	n=18 5
00 guesses or unable to solve	16	10	2	3
01 46c	2	4	4	2
1 36c		5	12	13
0 Unable	18	14	9	7
1 Addition expression for $1		5	9	11

N4 / N5 *How many centimetres in 1 metre?*
If you cut 5 cm off a metre length of ribbon, how much ribbon is left?
Write down 0.05 + 0.95 =1
Can you write down another combination that make 1 metre?

Table B.21: Task Response Categories by Grade Level for Tasks N4 and N5

	Grade		
Response Descriptions	n=18 4	n=18 5	n=19 6
0 Unable	2		1
1 Subtract to give 95cm	16	18	18
0 Unable	5	5	4
1 Decimal number sentence given for combinations of1	13	13	15

Ten as an abstract composite unit
Assess whether child can coordinate counting by tens and ones i.e. ten as an abstract composite unit.
G13 Uncovering tens task - a board to which is affixed a sequence of Dienes longs and shorts is gradually uncovered and each time the cover is pulled back to show more material the child is asked: "how many are there now?"

10, 14, 34, 41, 51, 53, 73

Table B.22: Task Response Categories by Grade Level for Task G13

	Grade			
Response Descriptions	n=22 1	n=18 2	n=19 3	n=18 4
01 Attempts to count all by ones - counts tens by ones (sometimes unsucessfully).	10	4		1
02 Restarts to count-on from ten all by ones or count tens and ones.	1	1	1	
03 Miscounted on tens as ones.				
04 Miscounted on ones as tens.				1
05 Start with ten and count-on by ones.	4			
1 Does not count-on, restarts and collects units of the same rank together.	5	6	8	
2 Counts-on by tens and ones as appropriate.	2	7	10	16

R3 / N6 Show a 6 and a 9 pattern board (Appendix A, Figure A.2).
How many dots are there altogether?

Table B.23: Task Response Categories by Grade Level for R3 and N6

	Grade				
Response Descriptions	n=18 K	n=22 1	n=18 2	n=19 3	n=18 4
0 Unsuccessful		3			
1 Count all by ones.		12	4	1	1
21 Count-on from 6 by ones.		3	5		1
22 Count-on from 9 by ones.		3	4	6	4
23 Count-on from 9 by threes.			1	1	
31 Subitize patterns in both numbers and add by partitioning and combining the numbers - compensation, bridging tens.			3	9	8
32 Add 10, take away one.		1	1	1	3
33 Double and add (or subtract)				1	
34 Knows					1
0 Unsuccessful		21	14	8	6
1 Uses ten pattern.		1	4	11	12

Counting tasks - tens and ones
R4 Addition task
Show 3 rolls and 7 separate lollies.
How many lollies are here?
There are 25 lollies hidden under this tub.
How many lollies are there altogether?

Table B.24: Task Response Categories by Grade Level for Task R4

	Grade			
Response Descriptions	n=18 2	n=19 3	n=18 4	n=18 5
00 Unable or unwilling to calculate total mentally.	9	2	2	2
02 Attempts to collect units of the same rank	2	1		1
03 Attempts to count-on by ones	1			
04 Attempts to count-on from 25 or 37 by tens and ones	1	1		
05 Attempts to break number up into parts and use bridging tens				1
11 Counts-on from 37 by ones			1	
21 Counts-on 5 by ones from 37 and then adds 20.			1	
22 Counts by tens (or collects tens), adds 5 or 7, and then counts-on by ones.		4		1
23 Counts-on from 37 by tens and ones starting in the middle of a decade - ten as an abstract composite unit.	1	2	4	
24 Adds 5 to 37 and then 20 more			1	3
31 Adds ones forming 10 and 2, then adds 2 to 30 and counts-on by tens			1	
32 Adds units of the same rank - abstract collectable units.	2	7	3	7
33 Written algorithm mentally			3	3
4 Breaks up numbers into parts in order to facilitate the addition process, use compensation or bridging tens - a number is a numerical whole and units of tens and ones at the same time.	2	2	2	

R5 Missing addend task
A collection of rolls and separate lollies is visible (e.g. 1 rolls and 8 separate lollies), a child is told how many lollies there are in all e.g. 64, and asked to find how many lollies are hidden.

Table B.25: Task Response Categories by Grade Level for Task R5

	Grade		
Response Descriptions	n=19 3	n=18 4	n=18 5
00 Unable to calculate mentally	5	6	2
01 Attempts written algorithm mentally	1	2	
02 Attempts subtracting tens and then ones	1		3
03 Attempts building-up (tens and part tens)	5	1	1
04 Attempts to build-up, counts by ones	1	1	1
05 Attempt counting back			
1 Written algorithm mentally		2	1
21 Builds-up, mult. of ten or counts by tens and ones	2	1	
22 Builds-up, counts by tens and part tens	2		3
31 Counts-back by ones			
32 Counts-back by tens and ones	1		
33 Counts-back by tens and part tens		4	7
4 Subtract 20, add 2		1	

N7 / R6 Regrouping tasks - ones and tens

```
   16
+   9
--------
  115
--------
```

Is this a reasonable answer? Why?
What should the answer be?

Table B.26: Task Response Categories by Grade Level for Tasks N7 and R6

	Grade				
	n=18	n=19	n=18	n=18	n=19
Response Descriptions	2	3	4	5	6
0 Yes	1		1		
1 Estimation to give no (too big, * instead of +)	17	11	6	10	6
2 Mental calculation to give no		8	9	8	13
0 Attempts to count-on / count all.	3	2			
1 Counts-on to find answer	12	4			1
2 Uses written algorithm mentally	1	9	9	12	16
3 Adds ten, take away one / bridges ten / knows answer	1	4	7	6	2

3-digit numerals

P4/P5/P6/P7/P8/P9 *Write down the number one hundred and three.*
Write down the number one hundred and eleven.
1008
3 tenths
14 hundredths
601 040

Table B.27: Task Response Categories by Grade Level for P4, P5, P6, P7, P8, and P9

	Grade			
	n=18	n=19	n=18	n=18
Wrote numeral correctly	2	3	4	5
P4 103	14	18	18	18
P5 111	11	18	18	18
P6 1008	8	16	16	18
P7 0.3			3	8
P8 0.14			1	6
P9 601 040			6	6

P10 Show 431 recorded on the screen of a calculator and 31 on a card.
How can you change the calculator number to this number on the card?

Table B.28: Task Response Categories by Grade Level for P10

	Grade			
	n=18	n=19	n=18	n=18
Response Descriptions	2	3	4	5
00 Guess or unable	1	1	1	1
01 Subtract 4	9	4	2	2
02 Subtract 40	1	1		
1 Subtract 400	7	13	15	15

G14 Use of tens structure in counting

Assess intuitive use of ten in counting when a ten by six array of pictures is presented.
Show an array of ten by six planes.
Can you tell me quickly how many planes are here?

Table B.29: Task Response Categories by Grade Level for G14

	Grade				
	n=18	n=22	n=18	n=19	n=18
Response Descriptions	K	1	2	3	4
01 Attempts to count all by ones.	18	17	7	1	
02 Attempts to count by sixes.			2		
1 Counts by tens, repeated addition.		5	8	3	4
2 Uses multiplication.		0	1	15	14

S2 Ones, tens and hundreds structure
Show a roll of ten lollies: *How many lollies are here?*
Show a bag of ten rolls: *How many lollies are here?*
Show 2 bags, 4 roll, 5 separate lollies - all **transparent** coverings.
How many lollies are there altogether?
Write down the number of lollies.

Table B.30: Task Response Categories by Grade Level for S2

	Grade			
Response Descriptions	2	3	4	5
00 Attempts to count by ones or no response.	1			
01 Counts tens as hundreds.				
02 Attempts to count tens and ones	2		1	
03 Attempts to count from 200 by ones	3	1		
1 Counts all by tens and ones.				
21 Counts by hundreds, tens and ones	2			
22 Counts-on from 200 by tens and ones.	2	6		3
23 Establishes number of hundreds, tens and ones	4	3	2	4
and immediately gives total.	4	9	15	11

S3/R7 Ones, tens and hundreds structure
Show 245 represented by bags, roll, and separate lollies.
Add to these sweets so that the number of lollies is ten more. How many lollies have we got?
Add to these sweets so that the number of lollies is 98 more. How many lollies have we got now?

Table B.31: Task Response Categories by Grade Level for S3 and R8

	Grade				
Response Descriptions	2	3	4	5	6
00 Unable or adds ten by counting-on by 1's.	7	1	1		
01 Counts all to find ten more.	1				
1 Adds ten.	10	18	17	18	19
00 Unable / guesses / unwilling		9	4	5	2
01 Attempts written algorithm mentally.		1	2	1	2
02 Attempts to start at 240, 245 or 290 and adds or		5	1	1	
counts by tens and ones.					
03 Attempts to start at 245 and bridge 300				1	3
04 Attempts adding units of like rank.			2		
1 Written algorithm mentally			1	1	2
2 Collects units of the same rank			3	1	2
30 Counts in tens and ones			0	0	2
31 Counts-on from 245 by tens and part tens or ones			1	1	
33 Starts at 200, adds 90 or 98, then counts -on by		2	1	3	
tens and part tens or ones.					
40 Starts at 245 and bridges 300 (builds up)		1	2	2	
41 Adds 100, subtracts 10 and then adds 8.		1		1	
42 Adds 100 and subtracts 2.			1	1	5
43 Compensation					1

Structure tasks - non-proportional representations
S4/S5 Coloured counters are assigned values ie a red counter is worth 10 blue counters and a yellow counter is worth 10 red counters. Other coloured counters are available.
These counters are each given different values. If we say that these red counters are each worth 10 blue counters. Each yellow counter is worth 10 red counters.
If each blue counter is worth 1, use as few of the counters as possible to show the number 246.
Explain how you would use counters to show the number 1246.

Table B.32: Task Response Categories by Grade Level for S4 and S5

		Grade
		n=19
Response Descriptions		6
00 Unable		4
01 Showed 246 blue counters		
02 Showed 20y, 4r, and 6b.		3
1 Showed 2y, 4r, and 6b.		12
00 No response		13
01 Showed 100y,20r, and 46b.		1
1 Showed 12y,4r, and 6b.		4
2 Defined a new colour as ten yellows.		1

Place value and regrouping tasks

P11/P12/R8 Spike abacus question.
Put these labels on the abacus (ones, tens and hundreds).
How would you use this abacus to show the number 234 ?
Add 98 to your number on the abacus.

Table B.33: Task Response Categories by Grade Level for P11, P12 and R8

		Grade
		n=19
Response Descriptions		6
0 Labelled abacus in reverse order		2
1 Labelled abacus correctly		17
1 Recorded 234 on the abacus correctly		19
00 Unable		1
01 Added 7 and 6 to the tens and ones spikes (as many as you can put)		
02 Added 6 and 4 to the tens and ones spikes		1
03 Attempt to add 9 and 8 to the tens and ones (no exchange).		11
1 Added 9 and 8 to the tens and ones and exchange as necessary.		2
2 Added 1 to the hundreds and subtract 2 from the ones spike		4

One hundred as a composite unit

S6/N8/N9 Show a hundred square (0 to 99) (Appendix A, Figure A.7).
Can you find 84?
What do you add onto 84 to make 100.
Can you give me another combination which make 100?
Can you make up any combinations which make 1000?

Table B.34: Task Response Categories by Grade Level for S6, N8 and N9

	Grade				
	n=18	n=19	n=18	n=18	n=19
Response Descriptions	2	3	4	5	6
00 Unsucessfully	10	2	1	2	
01 Gives 26 as answer		1	3		
02 Counts-on by ones using 100 square (15)	2	2	1	1	
1 Counts-on by ones using 100 square (16)	4	8	4	3	4
2 Calculates ten and six from 100 square	2	1	1	1	1
3 Knows answer.		5	8	11	14
0 Unsuccessful	17	9	6	2	1
1 Makes a combination of 100 other than involving zeros or single digit numbers	1	10	12	16	18
0 Unsuccessful	18	17	12	8	10
1 Makes a combination of 1000 other involving zeros or single digit numbers		2	6	10	9

G15 Hundreds task - a board to which is affixed a sequence of Dienes longs and shorts is gradually uncovered and each time the cover is pulled back to show more material the child is asked: "how many are there now?"

100, 120, 40, 200, 6, 104, 30

Table B.35: Task Response Categories by Grade Level for G15

	Grade				
Response Descriptions	n=18 2	n=19 3	n=18 4	n=18 5	n=19 6
00 Unable	5	3			
01 Counts tens as ones		1			
02Attempts to count-on by hundreds, tens and ones.		6	5	1	2
03 Attempts to restart each time	1		1	1	
1 Restarts to count hundreds, tens and/or ones.	5	1		2	
2 Counts-on by hundreds, tens and ones.	2	8	12	14	17

R9 Regrouping - tens and hundreds

Show 3 bags, 12 rolls, 5 separate lollies - all **opaque** coverings.

How many lollies are there altogether?

Digit correspondence task.

Table B.36: Task Response Categories by Grade Level for R9

	Grade			
Response Descriptions	n=19 3	n=18 4	n=18 5	n=19 6
00 Attempts to count by ones or no response.	3			
01 Counts tens as hundreds.	1			
11 Counts all by tens and ones.				
12 Counts-on from 300 by tens and ones.				
13 Establishes number of hundreds, tens and ones and immediately gives total.	15	18	18	19

P13 Digit correspondence task (425)

Table B.37: Task Response Categories by Grade Level for P13

	Grade				
Response Descriptions	n=18 2	n=19 3	n=18 4	n=18 5	n=19 6
00 No meaning assigned to digits ('don't know', gave an insufficient explanation, unable to write numeral correctly).					
01 Digits interpreted by their face values.	9	12	2	3	
02 Digits interpreted correctly by their total values - 4 bags.					
1 Digits interpreted correctly by their total values - four hundred, 4 hundreds, 40 tens.	9	7	16	15	19

Use of the hundred square

Assess how a number's location on the hundred square is found. What use is made of the pattern of tens.

Show a hundred square (0 to 99) (Appendix A, Figure A.7).

Show me 36 on this 100 square.

S7 *Show me how you can get ten more than 36 quickly from the hundred square.*

S8 *Can you show me ten less than 49?*

S9 *Can you show me nine more than 67?*

Table B.38: Task Response Categories by Grade Level for S7, S8 and S9

	Grade				
Response Descriptions	n=18 2	n=19 3	n=18 4	n=18 5	n=19 6
00 Guess or no answer	4			1	
01 Attempts to count-on by ones to get second number (addition).	2				
1 Counts-on by ones to get second number as a result of addition.	2	6		2	
2 Uses pattern of tens (addition).	10	13	18	15	19
00 Guess or no answer	4			1	
01 Attempts to count-back by ones to get second number (subtraction).	2	2		1	
1 Counts-back by ones to get second number as a result of subtraction.	2	5	1	1	
2 Uses pattern of tens (subtraction).	10	12	17	15	19
00 Attempts to count-on by ones	7	3		2	1
01 Attempts to add 10, subtract 1 ("1 down, 1 back")		1			
1 Counts-on by ones	7	8	6	4	2
21 Adds 3 to 70 and then 6 more (bridges ten)			1	2	1
22 Calculates (7 plus 9 and adds to 60)					1
23 Uses pattern of tens, ten more / one less	4	7	11	10	14

Groupings of groupings - extending the system

In our lolly factory we make lollies like this one. We package groups of ten lollies into rolls like these. Then we package ten rolls into a bag like this.

S10 *When we have lots of bags we want to package them into boxes so that it will be easy to count them. How many bags would you put in a box?*

When we have lots of boxes we want to package them into cases or crates so they can be transported to the shops. How many boxes would you put in a case?

S11 *How many lollies are there in a box?*

S12 *If we had 3 lollies, 5 rolls, 7 bags, 1 box and 4 cases, how many lollies would there be altogether?* Show this written.

S13 *If we had a case of lollies and sold 1468, how many lollies would be left?*

Table B.39: Task Response Categories by Grade Level for S10, S11, S12 and S13

	Grade			
Response Descriptions	n=19 3	n=18 4	n=18 5	n=19 6
0 Suggested numbers other than 10	13	11	12	15
1 Suggested 10 bags & 10 boxes	6	4	6	4
00 Unable	5		1	
01 Incorrect no. of lollies in a box	4	9	5	6
1 1000 lollies in a box	10	9	12	13
0 Unable / quesses	15	11	7	5
1 Calculates 753 lollies	3	1	1	1
2 Calculates 1753 lollies	1	1	4	2
3 Calculates 41 753 lollies		1	3	6
4 Recognises 41 753 lollies		4	3	5
1 Mentally finds missing addend.				2

Counting tasks - ones, tens and hundreds

R10 Missing addend task

Show 46 represented by four rolls and six separate lollies (**opaque** coverings).

There are 143 lollies altogether. How many lollies are hidden under here?

Table B.40: Task Response Categories by Grade Level for R10

	Grade		
Response Descriptions	n=18 4	n=18 5	n=19 6
00 No response, only written or unsuccessful.	8	4	5
01 Attempt written algorithm mentally		2	
02 Attempts to build-up, bridge ten/hundred	3		1
03 Attempt to take away	2	1	
04 Attempts comparison (103)		3	2
1 Building up, bridging tens/hundred	2	3	1
2 Takes away to find addend (43 then 3, 6 then 40 or 40 then 3 then 3)	3	4	6
3 Compares 46 (or 146) and 143 (3 less than 100 or add 100 take away 3)		1	4

R11 Missing addend task
Show 122 represented by one bag, two rolls and two lollies (opaque coverings).
There are 300 lollies altogether.
How many lollies are hidden under here?

Table B.41: Task Response Categories by Grade Level for R11

	Grade		
Response Descriptions	n=18 4	n=18 5	n=19 6
00 No response, only written or unsuccessful.	10	2	3
01 Attempt to count-on from 122 by tens and ones, build-up	1	1	
02 Attempts written algorithm mentally		2	1
03 Attempts to take away		1	
04 Attempt to build-up, bridge tens/hundreds	3	5	4
1 Adds, bridges 200 - building up.	4	5	6
2 Takes away to find addend.		2	4
3 Relates to 150 + 150 (compensation)			1

Groupings of groupings
Determine pertinence of grouping and the significance of associated writing.
Responses: no need for grouping - either guess or count by ones; group to count quickly, recount the collection after grouping; use one order of groupings and sees that writing is a code that is directly associated with these groupings; use a grouping of groupings.

S14 Child presented with a picture of 144 marks randomly drawn.
Can you guess how many marks there are drawn there?
I am going to do the same thing later with a friend who will be here after you. Could you do something so that, when I show him /her the sheet, he /she will be able to tell me very quickly how many marks there are?
What would you do?

S15 *Look what someone did for you* (grouping of groupings is shown). *What do you think of it?*
Can you now count how many marks there are? (Appendix A, Figure A.4)

S16 If the red circle is not mentioned then the following questions are asked.
Can you explain what the red circle does?
Can you now count a quicker way?

Table B.42: Task Response Categories by Grade Level for S14, S15 and S16

	Grade				
Response Descriptions	n=18 2	n=19 3	n=18 4	n=18 5	n=19 6
00 No suggestions to help another person, labelling each mark or dividing in half.	10	6	2	3	5
01 Suggests grouping by a number other than ten.	1	3	6	4	1
1 Suggests grouping by tens.	7	10	10	9	12
2 Suggests grouping tens and ten groups of tens.				2	1
00 Does not recognize and use tens when shown what a friend had done.	4	2			
01 Recognizes but does not use successfully the groupings of tens.	5	4		1	
1 Recognizes and uses the groupings of tens.	9	10	15	9	11
2 Recognizes and uses the groupings of tens and hundreds that were made by another person.		3	3	8	8
0 No recognition of meaning	9	6	3	2	
1 Suggests ten groups of ten or 100	9	13	15	16	19

Compensation tasks - 2-digit numbers
N10 What is sixty-five plus sixty-five?
Explain how you got that answer.
N11 Now think aloud while you do 66+64.
N12 Now think aloud while you do 55+75.

Table B.43: Task Response Categories by Grade Level for N10, N11 and N12

	Grade			
Response Descriptions	n=19 3	n=18 4	n=18 5	n=19 6
0 Unable	5	1	1	1
1 Found answer to 65+65	14	17	17	18
1 Relates by compensation of ones.	5	8	7	12
1 Relates by compensation of tens.	2	2	3	6

Renaming numbers
Show a place value chart (ones, tens and hundreds) and digit cards for numbers 0 to 19.
Show the '1 card' in each of the place value positions and ask what is the value of each of the positions.
P14 Show the '1 card' in each of the three positions to the left and right of the labelled positions and ask for the values.

P15 Place the cards "1" and "13" in the tens and ones places and ask for the meaning of this.
What is another name for this number?
Is there any other way of writing this?

Table B.44: Task Response Categories by Grade Level for P14 and P15

	Grade			
Response Descriptions	n=19 3	n=18 4	n=18 5	n=19 6
0 Not able	6	1		
1 Rec. of thousands place.	13	11	6	4
2 Rec. of places from thousands to ten thousands.			3	1
3 Rec. of places from thousands to h. thousands.			1	
4 Rec. of places from thousands to millions.		2	2	1
5 Rec. of places, tenths and thousands.		1		
6 Rec. of places, tenths, and from thousands to millions.				1
7 Rec. of places, hundredths, tenths and thousands.		1		1
8 Rec. of places, hundredths, tenths to ten thousands.		1	2	2
9 Rec. of places to thousandths and to ten thousands.			1	1
10 Rec. of places to hundredths and to millions			2	3
11 Rec. of places to thousandths and to millions		1	1	5
00 One ten and 13 ones, unable to give further meaning.	5	2	3	
01 Meaning as 11.3			1	
02 Meaning as 113.	3	2	2	3
03 Meaning as 14	1	1		
1 Meaning as 23.	10	13	12	16

Show a place value chart (ones to hundred millions) with 2 shells in the hundreds place, 3 shells in the tens place and 4 shells in the ones place.
What number does this show and why?

R12 *Add 98 to the number on the place value chart.*

S17 *Use the shells to make the number which is 100 times larger (multiply by 100).*

Table B.45: Task Response Categories by Grade Level for R12 and S17

	Grade	
Response Descriptions	n=18 5	n=19 6
1 Represents 234 on the place value chart.		19
00 Unable		1
01 Adds 7 and 6 shells to the tens and ones spaces resp. (only ten allowed in each space)		1
02 Makes up to 8 shells in ones and 9 shells in tens place (add 4 and 6 resp)		1
03 Adds 8 shells to the ones place and 9 shells to tens place.		7
1 Adds 8 shells to the ones place, 9 shells to tens place and regroups.		3
2 Adds 1 to tens, takes 2 from ones and adds 1 to hundreds, takes 1from tens.		
3 Adds one shell to hundreds place and takes 2 from ones place.		6
00 Unable or just say add noughts		9
01 Replace the shells with 2, 3 and 4 shells in the thousands, hundreds and tens spaces		
02 Move shells 1 space to the left		1
03 Move shells 3 spaces to the left		1
1 Puts 2, 3 &4 shells on ten thousands, thousands and hundreds places and removes previous shells.		2
2 Moves shells 2 places to left.		6

P16 Numerals - 4-digits and more
Show 286 349 displayed on the screen of a calculator and 206 349 on a card.
How can we change the calculator number to this number on the card?

Table B.46: Task Response Categories by Grade Level for P16

	Grade		
Response Descriptions	n=18 4	n=18 5	n=19 6
00 Not able	3	1	
01 Subtract 8.	3	3	1
02 Subtract another incorrect number.	3	3	4
1 Subtract 80 000.	9	11	14

S18 Imagery of numbers task
Assess whether the mental picture of the numbers 1 to 100 is a single mental number line or a matrix of coordinated horizontal and vertical mental number lines.
Close your eyes. I want you to imagine the numbers from 1 to 1000. Can you see a picture of these numbers?
Open your eyes.
Draw a picture of what you saw.

Table B.47: Task Response Categories by Grade Level for S18

	Grade		
Response Descriptions	n=18 4	n=18 5	n=19 6
0 No picture			1
101 Picture ($ sign, house, dog)			1
102 Pictures or marks - no pattern		1	1
103 Numerals - random, 1, 100 &1000 / single numeral	5	1	5
203 No structure, dynamic, numeric.			
111 Partially linear, static, concrete.			
112 Partially linear, static, ikonic.			
113 Numerals - linear, partial sequence		2	1
213 Partially linear, dynamic, numeric.			1
121 Linear, static, concrete.		1	
122 Pictures or marks - linear, linear.			
123 Numerals - sequenced in a square / circle pattern, Numerals - linear (1 - 1000), static.	3	6	1
223 Linear, dynamic, numeric.	1		
133 Numerals - multiple count (hundreds, tens, twos), static.	2		
233 Numerals moving, multiple count (in a circle, flashing).			
141 Partial array, static, concrete.			
142 Pictures or marks - groups of tens, static, ikonic.			
143 Arrays - pictures or numerals, not 10 by 100, Numerical expression (10 * 100)	1	2	2
243 Partial array, dynamic, numeric.			
152 Arrays - pictures, 10 by 100.	1		
153 Arrays - numerals, 10 by 100.	5	5	6
253 Arrays, dynamic, numeric.			

Two responses missing from Year 4, Bowen PS
One response missing from Year 5, BowenPS

Rules for extending the system
S19 Provide a box of unifix cubes.
In our number system, we always make groups of ten.
Show some Dienes blocks.
Here we have some shorts, longs and flats.
Discussion of using the blocks to represent the values of the positions of digits in numerals.
This short shows the value of the digits in this position (ones). This long shows the value of the digits in this position (tens). This flat shows the value of the digits in this position (hundreds).
If we made a make-believe number system where all our groupings were based on 5, then how would we group these cubes?
Here are some towers of 5.
How would we group these towers ?
How many towers do we put together to form a wall?

S20 *How would the grouping pattern continue?"*
Show an unlabelled place value chart.
What would be the values of these positions in our new number system?

Table B.48: Task Response Categories by Grade Level for S19 and S20

	Grade	
Response Descriptions	n=18 5	n=19 6
01 10 towers /10 walls		2
02 Various numbers given	12	9
03 20 towers / 10 walls or 2 towers (make tens) / 10 walls.	2	3
1 5 towers /5 walls.	4	5
1 Extension of system	2	2

S21 Working with groupings of groupings
A mum buys lollies for a birthday party. She wraps them in rolls like this one (showing a sample of a roll) *and puts the roll in bags like this one* (showing a sample of a bag) *in order to give some to the children. She has some lollies over.*
Mum has this many at the beginning (drawings are shown - 234).
She gives away this many (another drawing is shown 178).
How many bags, rolls and loose lollies does she have left?

Table B.49: Task Response Categories by Grade Level for S21

	Grade		
Response Descriptions	n=18 4	n=18 5	n=19 6
00 Guess / written only / no answer	12	6	6
01 1 bag, no rolls, no lollies		1	
02 Attempt written algorithm in head	1		1
03 Subtracted small number from larger number for each article (ans. 144)		2	
04 Attempt subtraction of bags, rolls and loose lollies with regrouping	3	1	4
05 Attempt building-up			1
06 Attempt compensation.			1
1 Used written algorithm mentally			2
2 Subtract multiples of hundreds, tens and part tens		3	1
3 Proceeded from the picture and mentally opened bags and rolls.	2	3	1
4 Build-up, bridging 200		2	
5 Compare, relate to 200, compensation.			2

S22 Use of groupings of groupings in counting
Assess use of groupings of tens, hundreds and thousands when quantifying large collections.
Show an array of 100×100 dots (Appendix A, Figure A.6).
Can you tell me how many dots are here?

Table B.50: Task Response Categories by Grade Level for S22

	Grade			
Response Descriptions	n=19 3	n=18 4	n=18 5	n=19 6
00 Guesses or unable.	3	1	1	
01 Attempts to count by ones.				
02 Attempt to multiply 1000 by 1000				
03 Counts down and across and attempts to multiply.	5	1	1	
04 Attempts to establish pattern of hundreds but calls them tens.	2			
05 Determines the patterns of hundreds and unsucessfully attempts to count by hundreds.	2	4	1	3
06 Determines the patterns of thousands and unsucessfully attempts to mult. or count by thousands.	3	4	5	2
07 Determines the patterns of hundreds, counts 10 down, 10 across and guesses or attempts to multiply		5	5	2
1 Determines pattern of hundreds, counts 10 by 10 and multiplies to give 10 000			2	
2 Determines the patterns of hundreds and counts by hundreds to give 10 000.				
3 Determines the patterns of hundreds, counts to give the thousand pattern and then counts or mult. to give 10 000.	4	2	3	8
4 Multiplies 100 by 100		1		4

Extension of the place value system
Show some base-ten numeration blocks.
If this block (a little) has a value of 1.
What is the value of this block (long)?
What is the value of this block (flat)?
What is the value of this block (block)?
S23 *What is the next value?*

S24 If the answer is not correct give the answer as 10 000.
What would a model of 10 000 look like?

S25 *If this block (a little) has a value of 100, what is the value of this block (flat)?*
What would the model of 100 000 look like?

S26 *If this block (big) has a value of 1, what is the value of this long?*

Table B.51: Task Response Categories by Grade Level for S23, S24, S25 and S26

	Grade		
Response Descriptions	n=18 4	n=18 5	n=19 6
01 Don't know, billion, five th., etc.	4		
02 100 thousand		1	
03 million	8	7	2
1 ten thousand	6	10	17
00 don't know	4	00	
01 2 big blocks	3	6	5
02 4 big blocks			1
03 12 big blocks		1	
04 1000 big blocks		1	
05 100 big blocks	4		
1 10 big blocks	7	8	13
00 don't know / other number	4	1	1
01 2000		1	
02 1000	2	5	1
03 100 000	1	1	1
04 million	8	3	2
1 10 000	3	7	14
00 No answer / unable	4	1	2
01 Nothing / minus	2	3	1
02 10 000		2	
03 1000	2	2	1
04 100	5	2	2
05 10	1		
06 Another fraction (third, tenth, ...)	1	3	2
1 Hundredth	3	5	11

APPENDIX C

Operational Definitions

Base ten place value: a power of ten is associated with each position in a numeral.

Builds ten or bridges ten: partitions one of the numbers so that the other number can be built up (or down) to ten.

Counting (rational): recitation of number words is coordinated with the conceptual production of an item which may be perceptural, figurative or abstract.

Counting all: counting each item by ones.

Counting on: counting on from a given number by ones or in equal groups.

Double counting: counting all or skip counting with a simultaneous count of the number of groups at the same time, e.g. "one, two, three (one); ... four, five, six (two); ... " or "three (one), six (two), nine (three), ...".

External representations: physically embodied, observable configurations of the number system such as actions, words, pictures and symbols (Goldin & Kaput, 1996).

Imagery: creation of images, classically viewed as internalised perception.

Imaging: construction, representation and transformation of the image (Kosslyn, 1980).

Imagistic: based on mental images.

Internal representations: configurations that are encoded by the human brain and nervous system (mental configurations). Must be inferred from observation.

Multiple counting: counting equal groups of items, total related to multiplication.

Place value: each position in a numeral has a value.

Relate to known double: using a known double to find another fact.

Representations: produced by imagery processes and abstract representational systems.

Rhythmic counting: counting by ones but in a particular pattern which emphasises the multiples of a particular number.

Rote counting: recitation of number words in order.

Skip counting: reciting the count of multiples of a particular number (not intervening numbers).

Subitise: the ability to ascribe a number instantaneously to a pattern of objects.

Ten as an abstract composite unit: A cognitive construct used to characterise a child's view of ten as a single entity and a unit composed of 10 ones.

Ten as a numerical composite: a cognitive construct used to characterise a child's view of ten as a composite of 10 ones, as opposed to one single entity or unit of ten.

Using a known addition fact: retrieving an addition fact automatically with no apparent counting.

Visual imagery: mental images with a strong visual component, internal representations corresponding to diagrams, pictures.

APPENDIX D

Letters of Consent for Research

Letter to Principals

Letter to Teachers

Questionaire for Teachers

Letter to Parents

CHARLES STURT
UNIVERSITY
MITCHELL

7 September 1992

Dear Principal,

I am currently undertaking doctoral research in mathematics education through Macquarie University. This research involves a cross-sectional study that will contribute to our knowledge about how young children understand numeration. It is anticipated that this study will be carried out in Western Region public schools, and Mr Iain McPherson has given his support to the project. I am requesting your co-operation by allowing this research to be conducted in your school.

The first stage of the study would require the teachers to complete the short questionnaire that I have included. Following this, a sample of children from Kindergarten to Year 6 classes will be randomly chosen. I will then interview each child for a period of approximately 40 minutes where they will be asked to solve some mathematical problems. This procedure will only occur with the permission of the class teacher, the parent/s and the child involved. Parents, teachers and children will all be informed about the study and they maintain the right to refuse participation if they wish.

Interviewing will be conducted during fourth term, 1992. It is also essential to interview the teachers regarding content being taught, at the time of the interviews. However, I would like to indicate that the study is in no way assessing the peformance of the class teachers involved. Every effort will be made to interview at times convenient for all those concerned. All information obtained will remain confidential and the anonymity of the school will be preserved. Should you require any further information I am happy to meet with you or provide a detailed account of the study.

If you are willing to allow this research to be conducted in your school I am requesting that you invite the teachers to participate by completing the questionnaire attached. Thank you for your cooperation in this matter

Yours faithfully,

Noel Thomas

Noel Thomas
Lecturer in Mathematics Education