Essays on Annuities and their Economic Value for Retirees

by

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Abstract

Faced with an uncertain time of death, retirees may find annuities useful as they can provide a stable lifetime stream of income upon retirement. In reality, around the world, the voluntary annuity take-up rate is low. Researchers suggest this may be due to several possible reasons, such as a strong bequest motive, low financial literacy, a social security system with generous benefits, or the lack of liquidity to prepare for unexpected medical expenses. In this thesis, three research papers are developed, each contributing to the main goal of the thesis, which is to provide readers with a greater understanding of the economic value of annuities for retirees. The first research paper analyses the value for money of two private annuity products in Malaysia, one is sold in 2000 and another in 2012. Two widely known methods used by economists to value annuities, the Money's Worth Ratio and the Annuity Equivalent Wealth, are utilised in this analysis. This is first such study for Malaysia.

The second paper extends the original Annuity Equivalent Wealth method to incorporate several health states that may occur during the life-cycle of annuity buyers. The model allows us to value annuity products with not only the common annuity stream of income but also additional benefits that attach to other health states such as critical illness and total and permanent disability. Another original contribution of this paper is to apply a health state dependent utility function, which values a unit of consumption differently according to the current health state of the consumer. A health utility index taken from health economics studies is used to study the changes of Annuity Equivalent Wealth of annuities under both the health state independent and health state dependent utility function.

The final paper studies the impact of annuitisation in a general equilibrium framework where the economic agents, households, have a bequest motive. The model developed allows households to insure or annuitise optimally over their life-cycle. The model is a simple closed economy model with overlapping generations that consist of a young, middle aged and old generation. The changes in welfare for all generations and the future newborn generation as the economy reaches a new steady-state with the presence of insurance and annuity markets are calculated. Introducing insurance and annuity markets in the economy improves the welfare of the future newborn generation. This is an original contribution, and reverses the finding of a tragedy of annuitisation which arises if no bequest motive is present.

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Finally, I would like to give special thanks to my dearest husband and family members for their unconditional love and support. Without them, the completion of this thesis would not have been possible.

Statement of Originality

This thesis complies with the standard of the Thesis by Publications format which follows the Macquarie University Thesis Submission Guidelines. I hereby certify that this thesis is solely the candidate's own research work and that it has not, nor has any part of it, been submitted for a higher degree to any other university or institution. The sources of information or material where the work of others have been utilised, are acknowledged in the thesis.

Nurin Haniah Asmuni
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Chapter 1

Introduction

1.1 Research Background

Within the context of retirement and during the decumulation phase of retirement savings, annuities have a number of desirable properties. There are various annuity products available in the market. For simplicity, let us consider the most basic annuity plan. A plain vanilla life annuity which starts payment at retirement provides a periodic retirement income, usually monthly, for as long as the annuitant survives. Upon retirement, many retirees have to deal with the problem of how to allocate their retirement wealth to ensure an adequate flow of income in their post-retirement lifetime. Some retirees who choose to invest their retirement savings into other non-annuitised investment vehicles would have exhausted their money because of unexpected poor returns. With annuities, retirees get secured income and protection against such risk[1].

The improvement of human life expectancy has emerged as a major actuarial and financial challenge. The mortality rate in many countries has decreased for many years without showing any sign of abatement. Although past trends show some indication for further changes in mortality rates, the development of future mortality rates remains uncertain (Waegenaere, Melenberg, and Stevens, 2010). This phenomenon has led to nervousness amongst policymakers and has underscored the importance of establishing better retirement schemes to provide some sort of stable retirement income to retirees. Indeed, pension reform is very challenging with the presence of longevity risk which makes the costs of pensions uncertain.

[1]This is true if we ignore the possibility of the insurer's insolvency risk. In practice, such an event might occur for several reasons including poor financial management in the company. Refer to Blake (2001) for the case of an insurer's insolvency.

Annuities have became an interesting topic of policy debate as to whether they can be an effective instrument to manage the risk of longevity of an aging population. Until recently, annuity markets seems almost non-existent in many countries including Malaysia. This is described as the annuity puzzle by economists, for example, Inkmann, Lopes, and Michaelides (2011) and Tanaguchi and Watanabe (2012): where few retirees voluntarily purchase an annuity when it is available in the market – such as the case of annuity experience in Australia (Ganegoda and Bateman, 2008). Despite the good potential benefits associated with an annuity, which may attract future retirees to opt for them as a retirement financing solution, many still have little knowledge about the annuity plan itself. The low financial literacy of current and future retirees is one of the factors that explains the annuity puzzle where the annuitisation rate is very low in the voluntary annuity market (Bateman et al., 2013). Other than that, a strong bequest motive and a social security system with generous payouts may also decrease the optimal annuitisation level for retirees[2].

In contrast, in Chile and Switzerland the annuities take-up rate is high with a significant growing number of policies sold over time[3]. In Chile, for instance, the government introduced an annuity element to the payout phase of its private pension scheme back in 1983. The number of annuity policies grew significantly with 320 000 policies in the market by year 2004 (Rocha and Thorburn, 2007). Given programmed withdrawal and annuities as payout options, almost two-thirds of all retirees do annuitise (James, 2004). According to James, Martinez, and Iglesias (2006), a high rate of annuitisation in Chile is due to guarantees and regulations imposed by government that constrains payout choices. A small lump sum payout option is allowed only under certain restricted circumstances. Joint life annuities are mandatory for married men and men or women with dependants. The life annuity payment is guaranteed by government. All of these factors contribute to the high annuitisation rate in Chile. The annuity experience of Chile shows that for a majority of people in the country, their retirement is funded through annuities. Similarly, Switzerland also has a high annuitisation

[2]Bernheim (1991) shows that people with significant bequest motive would stop shorting their assets into annuities, and instead, they would purchase insurance if their desired bequests substantially exceed their asset holdings. The other possible explanation of low annuitisation rate is a generous amount of pre-annuitised wealth in a form of social security benefit. Turra and Mitchell (2004) calculate the optimal additional annuitisation level to be significantly less than 100% (around -3%-25% of wealth) when retirees already have 75% of pre-annuitised wealth from social security benefit upon retirement.

[3]The Netherlands also has a high annuitisation rate mainly due to government regulations which mandates annuitisation for all three pillars of the pension system of the country (Brown and Nijman, 2011).

rate where only around 10 to 30 percent of all individuals in pension plans cash out their pension. According to Bütler and Staubli (2011), this high annuitisation rate is mainly due to the traditional default option in most pension plans and the high Money's Worth Ratio (MWR) value of Swiss annuities.

Apart from these and other countries, one of Malaysia's neighbour countries, Singapore, has also adopted an annuity in its Central Provident Fund (CPF) scheme recently (Fong, Mitchell, and Koh, 2011). Where appropriate, all governments should consider the annuity as a payout option in reforming their pension schemes.

1.2 Contribution of the Study

This thesis is a combination of three separate body chapters, each written in the form of a paper for publication. Each paper in this thesis contributes to the overall theme of the thesis in a way that it provides a greater understanding of economic value of annuities for retirees. The aim of the thesis is to address the topic of annuity studies by valuing annuities using actuarial and economic analysis methods. The contribution of each research paper in this thesis is outlined below.

1.2.1 Paper 1: Malaysia's Private Annuity Experiment: An Evaluation

Study on annuities on a country by country basis is crucial to a growing literature on annuity markets around the world. Bütler and Staubli (2011) and James (2004) review the annuity market for Switzerland and Chile respectively. Ruscani (2008) and Doyle, Mitchell, and Piggott (2004) study the annuity market of some countries around the world, for example, United Kingdom, United States, Singapore and Australia. This motivates us to study the development of annuity markets in Malaysia, where few scholarly annuity studies exist. In this paper, we discuss the annuity experience and the future of annuity provision in Malaysia. This paper starts with an introduction to the Malaysia's retirement scheme structure, which consists of the Public Pension Fund and the Employees Provident Fund (EPF). Comparison between countries by a measure of retirement income adequacy called the replacement rate is shown as an indicator for the need for pension reform to cater for a better safety net for retirees. The following section provides a detailed description of private annuity products available in the country and the issues and challenges for annuity provision in Malaysia. The history of annuity suspension back in year 2001 is summarised together with several reasons that caused the annuity suspension decision by the Malaysian government during that time. This paper continues further with the economic analysis of the annuities' value for money calculated using two methods, the Money's Worth Ratio and the Annuity Equivalent Wealth. The analysis is conducted both for the suspended annuity scheme, called the EPF annuity scheme, sold in 2000 and the new private annuity product introduced in 2012.

The preliminary findings of this paper have been presented in a Centre for Pensions and Superannuation Seminar, University of New South Wales, Australia, in October 2012. The preliminary analysis assumes that the mortality rate of annuitants is stationary over time and the rate of interest stays at a constant rate over time. Following the suggestion from the experts at the seminar, we extended the analysis by applying the Lee-Carter mortality projection method to the mortality rate of the Malaysian population, as well as incorporating the interest rate term structure, fitted using the Nelson-Siegel method[4].

In this paper, we only analyse the value of the annuity element of the product since the original Annuity Equivalent Wealth method that we used only allows for two states in the life cycle model: alive and dead. We extend the analysis to include *all* benefits covered under the product in Paper 2.

The story of Malaysia's annuity market is important for readers since there are only few papers studying annuities in Malaysia. Thus, this chapter contributes to a better understanding of Malaysia's annuity experience and is a valuable contribution to the literature on international comparitive annuity studies.

1.2.2 Paper 2: Annuity Valuation with a Health State Dependent Utility Function

Paper 2 aims to extend the original Annuity Equivalent Wealth (AEW) method to incorporate several health states involved in the annuity valuation model. The motivation of this paper springs from the features of the private annuity product introduced in Malaysia in 2012 which provides additional benefits on top of the common annuity stream of income. These additional benefits are provided for annuity buyers in the form of a cash value payment for unfortunate events such as death, the diagnosis of critical illnesses and total and permanent disability.

In the consumer's life-cycle model, the alive state is divided into five health states, namely the perfect health state, the very mild, mild, moderate and severe

[4]We would like to thank Dr. Ralph Stevens and all participants of the Centre for Pension and Superannuation Seminar, University of New South Wales for providing valuable comments.

degree of impairment to full health states. As the annuity product also provides benefit for people with total and permanent disability (TPD), we create a parallel world for people with total and permanent disability for each health state except for the perfect health state. Overall, there are ten states in the life cycle model, which consists of a perfect health state, four non-TPD states worse than perfect health, four TPD states worse than perfect health and a dead state. The insurer's annuity model utilised in this paper suits the benefit payments associated with the bundled annuity product in our analysis. There are four states in the bundled annuity model, these are, the healthy state, the state for people who is diagnosed with critical illness, the total and permanent disabled state and the dead state.

In this paper, our AEW analysis uses both the health state independent utility function and the health state dependent utility function. When the health state dependent utility function is used in the analysis, each health state appears in the utility function is assigned with a different weighting value which we refer as the health index of utility. We utilise the concept of health utility index widely used in the area of health economics study, where utility is attached to the health state of individual. In particular, an instrument called the EQ-5D health utility survey is used to elicitate the value of each health states as perceived by individuals. Our key reference for this paper is a study by Levy and Nir (2012) which studies the form of utility function that best describe the health and consumption tradeoff.

There are two principle contributions of this chapter. First, we show the changes of AEW from the annuity element only to the annuity with whole benefits including the additional benefits given the fair premium and market premium of the annuity product. Second, we explore the optimal consumption pattern for consumer when we apply the health state dependent utility function in the model. Then, the AEW results obtained using both the health state independent and health state dependent utility function are compared.

This paper was presented at the 17th International Congress on Insurance: Mathematics and Economics, University of Copenhagen, in July 2013.

Readers would find Paper 2 as a complementary of Paper 1 as Paper 1 reviews the annuity market experience of Malaysia, in particular, the study evaluates the value for money of annuity that is a available in the past (the EPF Annuity Schemes) and annuity product that is currently offered in the market, the New Annuity Product, by comparing the value of annuity element of the product only. On the other hand, Paper 2 provides detailed calculation of value for money of currently available annuity in the Malaysia market. It extends the value for money calculation by incorporating benefits payable in the event of death, total and permanent disability and critical illnesses. Such extensive value for money calculation provides useful information for annuity buyers and insurers.

1.2.3 Paper 3: Ameliorating the Tragedy of Annuitisation: The Case of Altruism

Paper 3 analyses the welfare implications of introducing insurance and annuity markets in an economy where households are altruistic or, in other words, they have a bequest motive. This paper is motivated by the recent research work by Heijdra, Mierau, and Reijnders (2014) which studies the impact of full annuitisation in a general equilibrium framework. They find, interestingly, that future newborn generations are worse off when all individuals fully annuitise their assets after an annuity market is opened in the economy. This result, which they refer to as the tragedy of annuitisation, always holds for the case where accidental bequests were originally transferred to the young generation. For a certain range of plausible values of the intertemporal substitution rate of the household, this result also holds for the case where accidental bequests are wasted by the government in the benchmark steady-state economy before the annuity market is opened. In their model, they assumed that households fully annuitise their assets as households in the economy are non-altruistic. We have seen in the literature, such as in Purcal and Piggott (2008), that the level of bequest motive affects the optimal level of annuitisation for individuals. Thus, our paper aims to extend their study by incorporating a bequest motive in the optimisation problem of the household. Drawing on the US macroeconomy framework cast as an Overlapping Generation General Equilibrium model by Hansen and Imrohoroglu (2008), we modify the features of the model to add a bequest motive for the household as the agent in the economy. We alter the household's expected utility optimisation problem to allow the household to purchase insurance or annuities at the optimal level as in Fischer (1973). Our model is a simple closed economy with three generations, comprising a young, middle aged and old generation. Through our numerical analysis, we observe the welfare changes of households for each generation during the transition from the benchmark steady-state economy without the presence of an insurance (and annuity) market to the new steady-state economy with the presence of an insurance (and annuity) market. We also observe the lifetime expected utility of the future newborn generation after an insurance (and annuity) market is introduced. We show that for the case of altruism, the tragedy of annuitisation is ameliorated as the future newborn generation may be better off as an insurance (and annuity) market is opened in the economy. The findings of this paper were presented at the Ph.D. Workshop Day, organised by the Department of Applied Finance and Actuarial Studies, Macquarie University, in October 2014[5].

1.3 Structure of Thesis

The structure of this thesis follows the Thesis by Publications format as recommended by the Higher Degree Research Department, Faculty of Business and Economics, Macquarie University. This first chapter consists of the introduction to the research study as a whole. There are three research papers which constitute three chapters of the body of this thesis.

1. Chapter 2 - Asmuni and Purcal (2012), "Malaysia's Private Annuity Experiment: An Evaluation".

2. Chapter 3 - Asmuni and Purcal (2013), "Annuity Valuation with Health State Dependent Utility Function".

3. Chapter 4 - Asmuni and Purcal (2014), "Ameliorating the Tragedy of Annuitisation: The Case of Altruism".

The final chapter concludes the thesis and emphasises the key findings of each research paper in the thesis. Limitations and recommendations for future research are also outlined in this last chapter.

[5]We are grateful for the positive feedback and valuable criticism from the paper's discussant, Professor Geoffrey Kingston. He suggests that the analysis may be extended to allow for weak altruism, where households may view bequests as luxuries.

Chapter 2

Paper 1

Malaysia's Private Annuity Experiment: An Evaluation

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Abstract

Mortality improvement in many countries nowadays has drawn policymakers' attention towards providing a financially stable retirement scheme for retirees. In some countries like Chile and Switzerland, annuities are common and a successful product with a good retirement benefit. Private annuities specifically designed for Employees Provident Fund members were also introduced in the Malaysian market in 2000. Despite the high annuitisation rate during that time, this product was suspended by the government a year after. Objections towards the scheme included a belief that insurance companies may profit excessively from the scheme and it provided a lack of protection for contributors' retirement savings. Annuities have been almost non-existent since then. In a recent Malaysian Government Budget an increase in tax relief for income used to purchase annuities seems to promote the development of annuity markets in Malaysia. Until now, there has been a lack of proper analysis in Malaysia to help buyers understand the value of annuities, especially upon retirement. Our study aims to calculate the value for money of Malaysia's private annuities by computing the Money's Worth Ratio (MWR) and the Annuity Equivalent Wealth (AEW) of the annuity component of recent products. This analysis will be used to evaluate whether Malaysian private annuities are worth buying.

Keywords: Malaysian annuity market, value for money, Money's Worth Ratio, Annuity Equivalent Wealth, annuities

2.1 Introduction

Since inception in 1951, the Employees Provident Fund (EPF) as a government-guaranteed institution who manages the retirement fund of private sector employees in Malaysia has been providing reasonable returns to EPF members each year. The investment returns of the EPF average between 4% to 8% per annum over the past 50 years (EPF, 2010). On the other hand, the decumulation phase also requires attention: the risk of outliving ones assets amongst EPF members needs to be managed. This risk is even greater considering the lump sum withdrawal option given to members upon attaining retirement age. According to Thillainathan (2004), there may be a shortage of members' retirement savings since members are allowed to withdraw up to 40% of their savings pre-retirement to finance housing, education and health needs[1].

In the year 2000, a group of private insurance companies named the 'Konsortium Annuiti Malaysia' introduced the EPF Annuity Schemes in an effort to provide a better decumulation scheme for members. Through this scheme, members were allowed to withdraw savings from their EPF account for the purpose of buying an annuity product. The introduction of this scheme has been a controversial issue since it received negative feedback from several entities, including a Malaysian workers' representative body known as the Malaysian Trade Union Congress (MTUC). It was suspected of being an excessively profitable scheme for insurers as it was managed by private insurance companies and not government. Members were exposed to uncertain investment returns and the risk of losing all of their savings if the insurers failed. The scheme could have been more popular if it had been provided by a government funded body or, perhaps, the EPF itself. As a result, this scheme was suspended by government at the end of 2001. Surprisingly, the participation to this scheme was tremendous with total single premium income of over RM4 billion collected from over 200 000 EPF members as at the date that the scheme was suspended (Mohd Kassim, 2003). The discontinuation of the EPF Annuity Schemes has been a huge loss to the insurance industry with a drop of 48.2% in the single premium business — in contrast to the previous years' remarkable growth of 171.7% in 2000 and 131.5% in 2001 (Aziz,

[1]Since 2007, members can only withdraw up to 30% of their EPF savings pre-retirement to reduce the risk of depleting their savings during post-retirement life (EPF, 2014).

2002).

The annuity market since almost shut down with only policyholders who opt to stay in the scheme making up the small number of annuitants in Malaysia. Furthermore, the Central Bank of Malaysia as a financial industry regulator is very strict with the approval of annuity business, which must comply with the high capital requirement following the introduction of the Risk Based Capital requirement in 2007. Recently, the Prime Minister of Malaysia announced an increase in the tax concession on income used to purchase an annuity (in the Malaysian Government Budget of 2012). This incentive seems to promote the development of the annuity market following the approval of the New Annuity Plan (a new private annuity product) few months after the announcement was made. Despite that, due to the controversy surrounding annuities in 2000, it is uncertain whether this new product will receive a positive response from customers.

The main objective of this paper is to provide the value for money analysis of private annuities in Malaysia. Till present, few papers have been written on Malaysia's private annuities. It is the aim of this paper to fill that gap. The structure of this paper is as follows. Section 2.2 describes Malaysia's EPF retirement scheme and examines the adequacy of EPF savings. Section 2.3 discusses the annuity experience of Malaysia, lessons learned from the annuity suspension in 2001 and the future of annuity provision in Malaysia. Methods and models used for annuity value for money analysis, namely the Money's Worth Ratio (MWR) and Annuity Equivalent Wealth (AEW) are explained in Section 2.4. Lastly, section 2.5 contains the results of our analysis and section 2.6 concludes.

2.2 Malaysia's EPF

Annuities fit well with the retirement sector in that they aim to offer a secure stream of income to retirees. Below we provide an overview of the retirement scheme structures in Malaysia. Malaysia currently has two types of retirement schemes. First, the Defined Benefit Plan specifically designed for the government sector employees and is widely known as Public Pension Fund. Second, there is a mandatory Defined Contribution Plan known as Employees Provident Fund (EPF) for private sector and non-pensionable public sector employees. The EPF is also available to the self-employed and housewives on a voluntary basis.

In this paper, we discuss the structure of the EPF retirement fund, with a focus on its decumulation phase. The EPF is a programmed withdrawal scheme where the contributors are allowed to withdraw their savings according to the terms and conditions stated by the EPF administrator. They are also allowed to withdraw all of their savings as a lump sum upon retirement.

Characteristics of Malaysia's EPF scheme include the following:

- As of 2012, the employer's share contribution rate is 13% of monthly wages for employees with wages of MYR5000 and below, and 12% of monthly wages for others. While for employees, the contribution rate is 11% of monthly wages. Refer to Table 2.2.1 for EPF contribution rates since its inception[2].

Table 2.2.1: Historical EPF Contribution Rates from 1952 to January 2012 as a Percentage of Monthly Wages by Employee and Employer

Year	Employee (%)	Employer (%)	Total (%)
1952–June 1975	5	5	10
July 1975–November 1980	6	7	13
December 1980–December 1992	9	11	20
January 1993–December 1995	10	12	22
January 1996–March 2001	11	12	23
April 2001–March 2002	9	12	21
April 2002–May 2003	11	12	23
June 2003–May 2004	9	12	21
June 2004–May 2005	11	12	23
June 2005–December 2008	11	12	23
January 2009–December 2010	8	12	20
January 2011	11	12	23
January 2012	11	12 (13)	23 (24)

Source : EPF (2012a)

- Each member has an individual account which is divided into two, Account 1 and 2.

- Account 1 consists of 70% of total contributions whereas Account 2 consists of 30% of total contributions. Members are given withdrawal options prior to retirement, subject to requirements and approval from the EPF. Table 2.2.2 summarises the various of EPF withdrawal options[3].

[2]Contribution rate vary each year as announced by EPF.

[3]For detailed information regarding EPF withdrawal options and requirements, refer to EPF (2012b).

- Upon retirement, the fund accumulation can be withdrawn in a lump sum or monthly installments or a combination of both.

Table 2.2.2: Types of EPF Withdrawal Options and Types of EPF Account which are allowed to be withdrawn for each Withdrawal Option

Type of withdrawal	Account 1	Account 2
Age 50 Years Withdrawal		X
Age 55 Years Withdrawal	X	X
Withdrawal to Reduce / Redeem Housing Loan		X
Incapacitation Withdrawal	X	X
Leaving Country Withdrawal	X	X
Education Withdrawal		X
Pensionable Employees Withdrawal and Optional Retirees Withdrawal	X	X
Member's Saving Investment Withdrawal	X	
Withdrawal to Purchase a House		X
Withdrawal to Build a House		X
Withdrawal of Savings of More than RM1 Million	X	X
Housing Loan Monthly Installment Withdrawal		X
Death Withdrawal	X	X
Flexible Housing Withdrawal		X
Health Withdrawal		X

Source : EPF (2012b)

2.2.1 Adequacy of EPF Savings

Economically, Malaysia is a developing country and can be classified as a middle income country. Malaysia has been through several stages of demographic transition. In common with other nations, Malaysia enjoys improvements in life expectancy. Life expectancy was 73.4 for 2005–2010, whereas for 2025–2030, it is projected to be 77.1. Based on a current retirement age at 55 and life expectancy of 73.4, retirees have to prepare their savings to cover their living costs for about 18 years in the post-retirement phase.

Lately, the EPF has expressed concern regarding the statutory retirement age for private sector employees in Malaysia. It suggested that the retirement age should be increased from 55 to 60. According to the EPF's Chief Executive Officer, Tan Sri Azlan Zainol, the increasingly long retirement period is one of the factors for the inadequacy of EPF savings (M. Ali, 2011). This concern has recently drawn policymakers' attention resulting in a proposal for a new minimum retirement age for private sector employees of 60, which is expected to be in force in July 2013 (Chin and Dass, 2012).

Many economists use the replacement ratio as an indicator of the adequacy of retirement income. A simple definition of the replacement ratio is a ratio of a person's gross income after retirement over the person's gross income prior retirement. Some authors, such as Pestieau and Stijns (1999) and Holden and Vanderhei (2002), use the average income over the last five years salary as the denominator. On the other hand, Asher (2002) and Ståhlberg et al. (2006) define the denominator as the final last drawn salary. In general, the replacement ratio measures the efficiency of a pension scheme as a retirement income provider for retirees during their retirement phase, replacing earnings or salary as the main source of income prior retirement (OECD, 2007). Table 2.2.3 shows the replacement rate as a percentage of average earnings for selected countries including Malaysia.

Table 2.2.3 shows that Malaysia has a relatively low replacement rate which of 30.4% for men and 27.1% for women. Meeting living costs during retirement with a monthly income of less than half pre-retirement income may prove challenging. Many would have to depend on their family members or the government to support them, especially for the lower income groups. In 2009, Malaysia's neighbour country Singapore adopted several annuity options in its Central Provident Fund (CPF) scheme — called the CPF Life Scheme[4]. In view of this issue, recent effort by the Malaysian government is moving towards providing the third pillar of retirement savings through the approval of Private Retirement Scheme (PRS) and also giving more tax incentives to annuity puchasers. In dealing with ageing issues, the existence of an annuities market, with its ability to provide a life, retirement income to retirees is very useful.

[4]See Fong, Mitchell, and Koh (2011).

Table 2.2.3: Gross Replacement Rates by Individual Earnings (as a Percentage of Average Earning in each Country) for Asia Pacific, South Asia and OECD-Asia Pacific Countries by Gender

Country	Men			Women		
Income Group (% of average earnings)	50	100	200	50	100	200
Asia Pacific						
China	97.9	77.9	67.9	78.5	61.0	52.2
Hong Kong	38.7	34.1	17.0	34.2	32.8	16.4
Indonesia	14.1	14.1	14.1	12.4	12.4	12.4
Malaysia	30.4	30.4	30.4	27.1	27.1	27.1
Philippines	121.8	80.9	60.4	121.8	80.9	60.4
Singapore	12.7	12.7	6.4	11.2	11.2	5.6
Thailand	50.0	50.0	35.1	50.0	50.0	35.1
Vietnam	67.4	67.4	67.4	61.9	61.9	61.9
South Asia						
India	95.2	65.2	49.9	90.9	61.4	46.3
Pakistan	80.0	69.6	34.8	70.0	60.9	30.5
Sri Lanka	48.5	48.5	48.5	30.8	30.8	30.8
OECD Asia-Pacific						
Australia	73.3	47.3	35.4	70.8	44.8	31.8
Canada	76.6	44.4	22.2	76.6	44.4	22.2
Japan	47.9	34.5	27.0	47.9	34.5	27.0
Korea	64.1	42.1	23.9	64.1	42.1	23.9
New Zealand	77.5	38.7	19.4	77.5	38.7	19.4
United States	51.7	39.4	29.7	51.7	39.4	29.7

Source : OECD (2012)

2.3 The Annuity Experience in Malaysia

In 2000, EPF members were allowed to withdraw their savings for the purpose of purchasing an annuity product. This scheme known as the EPF Annuity Schemes comprising the EPF Conventional Annuity Scheme (SAKK) and the EPF Islamic Annuity Scheme (SATK). In this paper, we focus on SAKK as details about the product features and data on annuity prices for SATK have eluded us. The annuity product provided under the SAKK scheme was a type of participating single premium annuity which commenced payment upon retirement at 55. It was provided by a consortium of private insurance companies called the 'Konsortium Anuiti Malaysia'. For each unit purchased under this scheme, the benefits are described by Mohd Kassim (2003) as follows:

- A single life annuity of MYR1200 per year at purchase date plus bonus in addition to the base annuity projected at 2% per annum compound during the deferred period and continuing after annuity payment starts at 55.
- Annuity payment is guaranteed for ten years after the commencement of payment.
- A ten year annuity certain as a death benefit prior to the vesting age of 55.
- A Total and Permanent Disability (TPD) benefit prior to retirement of an immediate annuity for a minimum of ten years commencing from the date of disablement[5].

[5]The sturcture of benefit payments are explained further below. Firstly, in the event of death before age 55, the nominee/beneficiary will receive monthly annuity for the duration of 10 years effective from date of death.

Secondly, in the event of Total Permanent Disability before age 55, members will receive monthly annuity for at least ten years effective from the confirmation date of total permanent disability. If during this period the member passed away, then the nominee/beneficiary will receive balance for the 10 years duration from the confirmation date of total permanent disability. Thirdly, in the event of death after age 55, the nominee/beneficiary will receive monthly annuity for the balance of 10 years guaranteed duration from the date of effective of annuity payment.

Table 2.3.1: Premium Contribution Rates for the EPF Annuity Scheme (SAKK)

Entry Age	Premium Rates (MYR)	
	Males	Females
35	7087	7867
45	11454	12731
55	19152	21288
65	16217	18025

Source: Mohd Kassim (2003)

The single premium contribution rates of the EPF Annuity Scheme can be found in Table 2.3.1. The introduction of this scheme into the market was controversial with negative feedback from several entities, including a Malaysian workers representative body known as the Malaysian Trades Union Congress (MTUC). According to Mohd Kassim (2003), objections made towards this scheme were due to three reasons. First, it was a scheme managed by private insurers and thus suspected of generating excessive profits for insurers. Second, members were not protected from investment risk and could lose their savings. Third, there were instances of product misselling by insurance agents.

At the end of 2001 this scheme was suspended by government due to these objections. Policyholders were given the option to opt out of the scheme and receive a refund of the premium paid. There was a suggestion from the MTUC that the annuity scheme be managed by the EPF instead of private insurers so that policyholders would be more protected. A decision on the re-introducing this annuity scheme has been postponed until further study are made by the EPF — even though the demand for this annuity option was high. Interestingly, at the date the scheme was suspended the total business included 273 392 policies with total single premium income of RM5.1 billion (Tunku Abdullah, 2002).

Since this suspension in 2001, efforts have been made by insurers to reinvigorate the annuity market. The Central Bank of Malaysia is, however, being very strict with product approval. Moreover, with the Risk Based Capital requirement introduced in 2007, annuities have become more expensive to offer in the market. The recent announcement of an increased tax concession on income used to purchase an annuity of up to MYR3000 per annum the Malaysian Government Budget 2012 led to the introduction of a new private annuity plan into the market in the same year (Yee, 2012). This private annuity product which we refer to in our paper as the New Annuity Plan, provides the following benefits:

Table 2.3.2: Illustration of the New Annuity Plan's Premium Term with Associated Annuity Benefit (based on annual premium of MYR3000)

Option A: Retirement Age at 55		
Premium Term	10 years	up to the policy year of retirement age (55/60)
Entry Age	Annuity Income (MYR)	Annuity Income (MYR)
35	4365.00	7747.50
45	3006.00	3259.50
Option B: Retirement Age at 60		
35	7432.50	15052.50
45	5002.50	7327.50

Source: Great Eastern (2012)

- guaranteed yearly annuity income for 10 years or 15 years depending on choice of retirement age of buyers (either at age 55 or age 60). This is a non-participating deferred term annuity plan where benefit of annuity income is only payable for term period of 10 or 15 years upon the survival of the annuitants to age 55 or age 60.

- payment of a policy cash value for unfortunate events such as death, total and permanent disability or diagnosis of critical illness (Angioplasty or other invasive treatments for Major Coronary Artery Disease are not covered). These benefits are payable both prior to and after the vesting age of annuity payment.

Table 2.3.2 illustrates the premium charges and associated annuity benefits of the annuity plan by entry age. Note that the New Annuity Plan is purchased from after-tax savings and not EPF balances. (The EPF Annuity Scheme, on the other hand, was purchased using EPF balances.)

The New Annuity Plan may prove popular. It is a tax-sheltered vehicle for harnessing after-tax savings to provide retirement income and complements existing EPF savings. Moreover, the Malaysian government has also proposed removing an 8% investment income tax on deferred annuity funds. However, the New Annuity Plan faces competition from the Private Retirement Scheme (PRS) — launched in July 2012[6]. PRS is a voluntary long-term investment scheme managed by a non-profit organisation, set up by the government, and known as

[6]Further information on the PRS can be found in the Guidelines on Private Retirement Schemes published by the Securities Commission Malaysia (Securities Commission Malaysia, 2012).

the Private Pension Administrator (Securities Commission Malaysia, 2013). The scheme, also funded by after-tax savings, serves as a complementary voluntary scheme to provide extra retirement income on top of the mandatory EPF savings. Compared to an annuity, PRS has a different framework where investors can choose to invest from a list of selected investment products approved by a regulatory body, the Securities Commission Malaysia. Similarly, this scheme is also entitled to personal tax income relief of up to MYR3000 per annum. Note that the tax income relief for both schemes is only effective for ten years.

The PRS may be more suitable for people who prefer to manage their own retirement funds, having freedom of access to a palette of investment funds. For customers who are looking for guaranteed income without having to worry about investment allocation, the New Annuity Plan may be preferable. In any case, both offer more retirement choices for Malaysians. This is supported by the Life Insurance Association Malaysia who believes that the government's proposal for tax relief on the New Annuity Plan and the PRS is a major boost in retirement planning options (Singh, 2012).

2.4 Methodology

We consider two valuation methods to evaluate the value for money of private annuities in Malaysia. This section explains the concepts underlying each method together with the basic mathematical formulation required for the analysis; further detailed technical work is included in Appendix 2.A.1.

2.4.1 Money's Worth Ratio

Our analysis of the value for money of annuities begins with the construction of the Expected Present Discounted Value (EPDV) formulation of the annuity product. The EPDV will be used to assess the money's worth of the annuity schemes in relation to the initial purchase price, or premium, of an annuity product. This approach is called the Money's Worth Ratio, commonly used by economists around the world for the purpose of valuing annuities. James and Vittas (2000) use the Money's Worth Ratio to analyse the annuity markets in several countries, namely Australia, Canada, Chile, Israel, Singapore, Switzerland and the United Kingdom. Also, Mitchell et al. (1999) use a Money's Worth Ratio computation to value individual life annuities in the United States. The concept of money's worth was introduced in Mitchell et al. (1999) with a term called the expected present discounted value per dollar annuity premium. In a later paper by Knox (2000) the same concept was used, but with a different term called the Money's Worth Ratio and the ratio is then expressed as a percentage. The Money's Worth Ratio provides useful information for both annuity providers and customers to differentiate between ranges of annuity products. As described by Mitchell (2001), the Money's Worth Ratio represents a currency-independent metric for comparing annuity products anytime and anywhere in the world.

There are three important components in the EPDV formulation of an annuity product: the amount of annuity payments, mortality rates and interest rates used as discounting factors. The basic formula for computing the Money's Worth Ratio (MWR) for an annuity product is as follows.

$$MWR = \frac{EPDV}{P} \tag{2.4.1}$$

where

$$EPDV \equiv \text{the Expected Present Discounted Value of an annuity product}$$
$$P \equiv \text{the market premium of an annuity product}$$

The EPDV depends on the structure of the annuity product. Based on the structure of the EPF Annuity Scheme and the New Annuity Plan, the EPDV has three components of benefit, namely the annuity element (both term-certain and life annuity), the death benefit and the additional benefits (consist of benefit payable in the event of total and permanent disability and critical illness). Since the calculation of death benefit and additional benefits in the value for money analysis requires further extensive data (in particular, the cash value information and the total and permanent disability and critical illness rates), we value only the annuity element for both products. Thus, the analysis in this paper shows the value for money of only the annuity element of annuities in Malaysia. We develop the EPDV formulation for the EPF Annuity Scheme as follows[7] — and then apply the same method to derive the EPDV of the New Annuity Plan:

[7] In the formulation of EPDV for the Malaysian annuity products, the choice of limiting age 100 is made because the probability of surviving beyond age 100 is very low based on the Malaysian mortality data. Ganegoda and Bateman (2008) choose 110 as the limiting age for the value for money analysis of annuity products in Australia. Refering to the Australian life expectancy of 82 years and the Malaysian life expectancy of 73 years, the choice of limiting age of 100 is reasonable. We have conducted a sensitivity analysis by extending the limiting age to 120, but this does not have a significant impact on the results. The MWR values are still the same up to two decimal places.

$$V_x(A) = \sum_{j=55-x}^{65-x-1} \frac{A}{\prod_{k=1}^{j}(1+i_k)} + \sum_{j=65-x}^{100-x-1} \frac{A_j P_x}{\prod_{k=1}^{j}(1+i_k)} \tag{2.4.2}$$

where

$V_x(A)$ ≡ the expected present discounted value of a life annuity paying A yearly, purchased by a person aged x. The limiting age is 100.

A ≡ the yearly benefit of a life annuity

${}_jP_x$ ≡ the probability of a person aged x survives to age $x+j$

i_k ≡ the nominal short-term interest rates during the k^{th} period

Based on the MWR formulation, an actuarially fair annuity premium will generate an MWR value of one (Fong, Mitchell, and Koh, 2011). However, in practice, insurers have to include loadings which lead to higher premium charges. These transaction costs cover marketing costs, management costs, insurer's reserves and profits as well as adverse selection costs (Mitchell et al., 1999). The adverse selection cost is taken as the difference between the MWR value calculated using annuitant mortality rates and the MWR value calculated using the population mortality rates, which then expressed as a percentage, as in Fong (2011).

2.4.2 Annuity Equivalent Wealth

The "Equivalent Wealth Valuation", described by Mitchell (2001), is another way of valuing annuity products. It takes into account the insurance value of an annuity product to the consumer which has been neglected in the previous valuation method. Here, the insurance value of annuities can be interpreted as the protection against longevity risk by providing annuitants with a stream of income for as long as they survive. Given an annuity as an option to decumulate retirement wealth, EPF members would be interested in knowing how much he or she should rationally forgo to receive this insurance value. The Annuity Equivalent Wealth (AEW) computation addresses this issue.

The concept of Annuity Equivalent Wealth was used by Kotlikoff and Spivak (1981) to compare the expected utility difference between a consumer with and without access to a perfect annuity market. This method then applied to the individual optimal consumption decision problem by Mitchell et al. (1999) using multi-period additively separable utility functions. One advantage of using this method is that it manages to capture differences risk aversion levels amongst consumers. Generally, ones level of risk aversion affects consumer utility impacting on a consumer's decision to annuitise. For instance, a risk averse consumer will value annuities higher than the value computed using the simple financial of money's worth approach (Mitchell, 2001). Roughly, the AEW measures the ratio of the value of non-annutised assets to the value of annuitised assets producing equivalent utility. Precise details of the AEW are given below.

Consider a representative individual who is assumed to maximise his expected utility function by following an optimal consumption path, C_t. We denote his expected utility function as a value function, V.

$$V = \max_{\{C_t\}} \sum_{t=y-x}^{T-x} \frac{{}_tP_x U(C_t)}{(1+\rho)^t} \tag{2.4.3}$$

where

$$\begin{aligned} C_t &\equiv \text{the optimal consumption at time } t \\ U(C_t) &\equiv \text{the utility function for a consumption of } C_t \\ {}_tP_x &\equiv \text{the probability that a person aged } x \text{ survives to age } x+t \\ T &\equiv \text{the maximum possible lifespan} \\ x &\equiv \text{the person's age at the time of purchase} \\ y &\equiv \text{the person's age when the annuity payment commences} \\ \rho &\equiv \text{the rate of time preference} \end{aligned}$$

The budget constraint for this individual depends on two conditions, first, in the presence of an annuity market, and second, in the absence of an annuity market. Thus, given initial wealth of W_0, and a nominal interest rate (combining a real interest rate of r and an inflation rate of π), the budget constraint is derived below.

First condition: with an annuity the budget constraint is as follows.

$$W_0 = 0 \tag{2.4.4}$$

$$W_t \geq 0 \tag{2.4.5}$$

$$W_{t+1} = (W_t + A_t - C_t)(1+r)(1+\pi) \tag{2.4.6}$$

Second condition: without an annuity the budget constraint is as follows.

$$W_0 = w \tag{2.4.7}$$

$$W_t \geq 0 \tag{2.4.8}$$

$$W_{t+1} = (W_t - C_t)(1+r)(1+\pi) \tag{2.4.9}$$

In the first condition, the initial wealth after annuitisation is set to be 0 as the consumer consumed all of his initial wealth to purchase annuity with a premium P and in return, receiving A_t of yearly annuity income. We follow the method in Brown (2003) to determine P as follows.

$$P = \sum_{t=y-x}^{T-x} \frac{A_t \cdot_t P_x}{(1+r)^t(1+\pi)^t} \tag{2.4.10}$$

Given a yearly annuity income of A_t, we can determine an actuarially fair price P of an annuity using the above formula. We can also find the market price P' of an annuity which typically allows for loadings, — by multiplying the right hand side of Eq. (2.4.10) by $(1 + l)$ where l is the load factor. From the optimisation problem of Eq. (2.4.3), we find the optimal consumption path for the individual under the first condition. Then, we evaluate the expected utility level, V, associated with that consumption path. Next, we move to the second condition and again solve the optimisation problem — but this time solve it by C_t by finding the initial wealth level w such that the individual has the same level of expected utility V, making him as well off as if he had access to an annuity market.

As in most literature for the AEW analysis, we assume that this individual has a utility function of $U(C_t)$ of form:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2.4.11}$$

where γ is the Arrow-Pratt coefficient of relative risk aversion and $1/\gamma$ is the elasticity of intertemporal substitution in consumption.

Lastly, we compute the ratio required in terms of the initial wealth level for this individual to achieve the same expected utility under both conditions. Here, α is Annuity Equivalent Wealth. It is expressed in terms of ratio of the non-annuitised wealth over the annuitised wealth[8] :

[8]The optimisation problem can be solved using the lagrangian method or dynamic programming techniques, explain further in Appendix 2.A.1. Refer to Mitchell et al. (1999) for the single individual case, Kotlikoff and Spivak (1981) for the case of an annuity in the family setting and

$$\text{Annuity Equivalent Wealth}\,(\alpha) = \frac{\text{non-annuitised wealth }(w)}{\text{annuitised wealth }(P)} \tag{2.4.12}$$

For illustration, let us consider an individual who consumed \$5000 to purchase an annuity (P) in order to achieve his maximum expected utility (V). An α value of 1.5 indicates that he would require 150% of his annuitised wealth or \$7500 of non-annuitised wealth to obtain the same expected utility, V. In this paper, we use the AEW computation to measure whether there is a utility gain if someone makes a purchase under the EPF Annuity Scheme or the New Annuity Plan.

2.4.3 Mortality Projection

The mortality structure used in the analysis is based on the Malaysian Abridged Life Table obtained from the Department of Statistics Malaysia. We fit the central death rate m_x, taken from the Abridged Life Table for year 1991 to 2011 with the Lee-Carter model to find the trend of mortality for Malaysia. Though we are aware of further research works to improve the original Lee-Carter model by the experts in mortality studies, we focus on our objective to find an appropriate structure of Malaysian mortality. Thus, with extensive application of the Lee-Carter model as a base mortality projection model in many literatures, we choose to fit and project the Malaysian mortality rates using the same method. The simple Lee-Carter model that we use has the following form in variables of age x, and year t:

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \tag{2.4.13}$$

where

$$\begin{aligned}
m_{x,t} &\equiv \text{the central death rate at age } x \text{in year } t\\
a_x &\equiv \text{the average age-specific pattern of mortality}\\
b_x &\equiv \text{the age-specific pattern of deviation in mortality as } k_t \text{ varies}\\
k_t &\equiv \text{the mortality index in year } t\\
\varepsilon_{x,t} &\equiv \text{the random error}
\end{aligned}$$

The component of the model that captures the time trend in mortality is k_t,

Brown (2003) for the case of mortality heterogeneity.

which has a linear form in time. The age-specific constant b_t is associated with parameter k_t and it measures the rate of change in mortality profile at each age as k_t changes. Lastly, the age-specific constant a_x is simply the average over time of $\ln(m_{x,t})$. The following normalization of parameters provides us with a unique solution of the model:

$$\begin{aligned} a_x &= \frac{1}{T}\sum_t (\ln m_{x,t}) \\ \sum_x b_x &= 1 \\ \sum_t k_t &= 0 \end{aligned}$$

Solving the model requires a least squares solution of large matrices as we want to fit various years of mortality data. This can be done nicely using the Singular Value Decomposition (SVD) method[9]. This method breaks down a rectangular matrix A into a product of three matrices: an orthogonal matrix U, a diagonal matrix S and a transpose of an orthogonal matrix V, as in this formula:

$$A_{mn} = U_{mm} S_{mn} V_{nn}^T \tag{2.4.14}$$

We follow the steps in Lee and Carter (1992) by first subtracting from the log of age-specific death rates $\ln(m_{x,t})$ constant a_x. With the term $b_x k_t$ on the right hand side of Eq. (2.4.13), we can now proceed with the SVD method where both right hand side of equations Eq. (2.4.13) and Eq. (2.4.14) have to be matched. We use the SVD package in R (R Core Team, 2013) to find the three matrices of U, S and V. Then, we choose the best rank-1 approximation of the input matrix. Based on the normalized formula of b_x which has to be sum to unity, its estimate $\hat{b}_x$ is given by the first column of U divided by the sum of that column. The estimate of $\hat{k}_t$ is given by the first column of V scaled by the first element of S and multiplied by the sum of the first column of U (to cancel out the division in $\hat{b}_x$ estimation). Finally, we project the mortality index k_t for future years using and employing an ARIMA(0,1,0) model which is found to be the best model that fits with the data[10]. Due to a non-uniform sign of parameter b_x, the second stage re-estimation is not advisable to be performed because it will not produce a unique solution (Gerosi and King, 2007). Since this procedure is not a crucial

[9]Further theoretical foundations and applications of the SVD of a matrix can be found in Good (1969).

[10]The estimated parameters for the mortality projection method are reported in the Appendix.

feature of Lee and Carter model, we skip this procedure in this paper such as in Beof (2014) and Giacometti, Ortobelli, and Bertocchi (2009).

Adverse selection is one of the concerns in annuity valuation as people in a very good health state tend to live longer and value an annuity higher than people in a poor health state (Turra and Mitchell, 2004). According to Fong, Mitchell, and Koh (2011), adverse selection cost in annuities can be measured by comparing annuity valuation results using population and annuitant mortality tables. Thus, we also do the valuation analysis using the annuitant mortality table to see whether there is a significant adverse selection effect in Malaysia's private annuities market. Since Malaysia does not have an established database on annuitant mortality, we have to find the best available data to approximate annuitant mortality rates in Malaysia. For the purpose of statutory valuation, insurers use the a(90)m and a(90)f Annuitant Tables from the UK, rated back by two years, which called the Malaysia a(90) Ultimate Male and Female Tables (SOA, 2013).

Similarly, we apply the method used above to project population mortality projection to also project the future mortality rates for annuitants in Malaysia. Due to lack of data availability, we have to used the k_t drift derived from the population mortality table for all projections as we only have one set of annuitant mortality data. Our approximation appears reasonable when compared to the future annuitant mortality rates in the UK (from the IML92 and IFL92 tables) (CMI, 2013). Our fitted mortality rates are shown in Appendix (Figure B.1).

2.4.4 Term Structure of Interest Rates

Our results depend on the assumption of the term structure of interest rates used in the analysis. Most previous literature choose a yield curve fitted from historical data to risk-free discount rates which are generally the yields on government bonds. A second alternative used for comparison by Mitchell et al. (1999) and Rocha and Thorburn (2007) is the interest rate on corporate bonds. In line with these papers, first, we do the analysis using the term structure of interest rates obtained from Malaysian Government Securities (MGS). Next, we compare the results by adding the average difference of the interest rate on MGS and the selected corporate bonds (AAA) to the term structure of MGS interest rates that we obtained previously.

We fit the data of MGS risk-free yields in July 2000 (the date of the EPF Annuity Scheme's introduction) using a non-linear least squares technique. Then, we repeat the same method to a different set of data to match the date of the New

Annuity Plan's introduction in November 2012. All data can be obtained from the Central Bank of Malaysia website (BNM, 2013). The widely used yield curve fitting method, namely the Nelson-Siegel model, is chosen (Nelson and Siegel, 1987). According to Coroneo, Nyholm, and Vidova-Koleva (2008), this model has been used by many Central Banks and public wealth managers because of its simplicity: it fits the data well and it also provides the intuitive interpretation of yields[11]. The Nelson-Siegel yield curve model that we used is a parsimonious three component model which has the following form[12]:

$$y(m) = \beta_0 + (\beta_1 + \beta_2)\left\{\frac{[1 - \exp(-m/\gamma)]}{(m/\gamma)}\right\} - \beta_2[\exp(-m/\gamma)] \qquad (2.4.15)$$

where $y(m)$ is the bond's yield with time to maturity m (which in our study ranges between 1 to 20 years). Term γ is the time constant associated with the equation and the three factors β_0, β_1 and β_2 each contributes to the long term, short term and medium term component of the yield curve respectively.

2.5 Findings

2.5.1 The Money's Worth Ratio Results

As described in Section 2.4, our analysis consists of interpreting value for money metrics for annuities using two valuation methods, namely the Money's Worth Ratio (MWR) and the Annuity Equivalent Wealth (AEW). The analysis of the EPF Annuity Scheme, which was introduced in 2000, is divided into three categories. The product was a participating annuity scheme with a projected 2% compounded bonus per annum prior and after the commencement of annuity payment. However, there is no guarantee that the insurers will always pay the full amount of the bonus every year. Consideration has to be taken for the possibility of no bonus at all or a bonus of less than 2% per annum. Thus, we provide the MWR and the AEW values for three possibilities: without bonus, with 1% compounded bonus per annum and with 2% compounded bonus per annum.

Table 2.5.1, Table 2.5.2 and Table 2.5.3 present the results for the MWR analysis of the annuity element of the EPF Annuity Scheme (SAKK). In addition, we calculate the adverse selection cost of the product by computing the (percentage)

[11] Our fitted yield curve is shown in Appendix (Figure B.2).

[12] See Diebold and Li (2006) for a slightly revised form which is easily interpreted as level, slope and curvature of the yield curve.

Table 2.5.1: Money's Worth Ratios and Adverse Selection (AS) Costs of the EPF Annuity Scheme (SAKK) Basic — without compounded bonus

Entry Age	Male			Female		
	Population	Annuitant	AS (%)	Population	Annuitant	AS (%)
MGS						
35	0.646	0.719	7.3	0.655	0.803	14.8
45	0.702	0.778	7.6	0.711	0.865	15.4
55	0.747	0.819	7.2	0.755	0.907	15.2
65	0.741	0.822	8.1	0.747	0.956	20.9
CB						
35	0.374	0.408	3.4	0.379	0.442	6.3
45	0.490	0.532	4.2	0.495	0.575	8.0
55	0.648	0.699	5.1	0.633	0.727	9.4
65	0.644	0.723	7.9	0.648	0.790	14.2

difference between the MWR value obtained from the annuitant mortality table and its population mortality table equivalent, as in Fong (2011). Comparing results from Table 2.5.1, Table 2.5.2 and Table 2.5.3, we can see that the MWR value are consistently increasing as the bonus payout increases from none to its full bonus amount of 2% per annum for all entry ages. This is expected, given the same annuity market prices P as in Table 2.3.1 (the MWR denominators), but higher annuity payouts (the MWR numerators). Each individual table shows the MWR value calculated using two term structure of interest rates, as explained in Section 2.4.4. The MGS represents the Malaysian Government Securities risk-free interest rates and the CB represents the AAA Corporate Bond which gives the term structure of (risky) bond rates. The higher yield rates for corporate bonds produces lower values for the MWR of the EPF Annuity Scheme, especially when no bonus is paid and the entry age is below 55. On the other hand, if the insurer pays full bonus, the MWR values obtained under the MGS term structure of interest rates are reasonably high, with some values even exceeding unity — which indicates the customer is paying less than the actuarially fair price of the product.

The adverse selection costs of the product ranges from 4 to 30 percent. Similarly, these costs increase as the bonus payout increases, showing that the product favours the group of annuitants with high survival probabilities most when the full bonus amount is paid by insurers. Our results for the adverse selection costs of the EPF Annuity Scheme are quite consistent with the previous literature, which is on average, around 10 to 15 percentage points in Mitchell et al. (1999). However, the cost can be quite high when bonus payments are included.

Table 2.5.2: Money's Worth Ratios and Adverse Selection (AS) Costs of the EPF Annuity Scheme (SAKK) Bonus 1— with 1% per annum compounded bonus

Entry Age	Male			Female		
	Population	Annuitant	AS (%)	Population	Annuitant	AS (%)
MGS						
35	0.861	0.972	11.1	0.874	1.105	23.1
45	0.848	0.952	10.4	0.860	1.078	21.8
55	0.818	0.908	9.0	0.828	1.024	19.6
65	0.794	0.892	9.8	0.801	1.056	25.5
CB						
35	0.494	0.544	7.3	0.500	0.598	9.8
45	0.586	0.642	7.7	0.593	0.705	11.2
55	0.704	0.767	6.7	0.686	0.806	12.0
65	0.686	0.778	8.8	0.691	0.863	17.2

Table 2.5.3: Money's Worth Ratios and Adverse Selection (AS) Costs of the EPF Annuity Scheme (SAKK) Bonus 2 — with 2% per annum compounded bonus

Entry Age	Male			Female		
	Population	Annuitant	AS (%)	Population	Annuitant	AS (%)
MGS						
35	1.150	1.319	16.9	1.171	1.533	36.2
45	1.028	1.171	14.3	1.044	1.355	31.1
55	0.901	1.014	11.3	0.912	1.166	25.4
65	0.853	0.970	11.7	0.861	1.174	31.3
CB						
35	0.653	0.728	10.6	0.663	0.814	15.1
45	0.703	0.780	10.3	0.712	0.869	15.7
55	0.768	0.846	8.3	0.748	0.901	15.3
65	0.732	0.841	10.4	0.738	0.948	21.0

Next, Table 2.5.4 presents the MWR values and the adverse selection cost of the New Annuity Plan which was introduced in 2012. In contrast to results from the EPF Annuity Scheme, the MWR values for female are always higher than male here because we only use one standard annuity price for both genders — the annuity provider charges the same market premium for both genders. Thus, the lower mortality rates of female group gives a higher value of MWR for all entry ages. Besides, the MWR values also slightly higher for payment term 2 compared to payment term 1 since benefit payment is higher when the payment term is longer. Notice that the difference of MWR values under the MGS term structure and the CB term structure for the New Annuity Plan is not as much as in the previous result which supports by the smaller spread of difference between the government bond and the corporate bond interest rates in 2012 as compared to year 2000. Overall, the MWR values of the New Annuity Plan are less than unity for all entry ages, both payment terms and both options A and B. The high capital requirement by the Central Bank of Malaysia for annuity business provider might be the reason for lower than unity MWR values. Appropriate loading factors have to be accounted for when pricing such annuities to ensure the sustainability of the product in the market.

For the EPF Annuity Scheme, with the exception of without bonus MWR values, our result is quite consistent with the MWR values of US annuities (Mitchell et al., 1999) under the government bond term structure of interest rates but lower under the corporate bond term structure of interest rates. For the New Annuity Plan, international comparison is hardly available as most previous literature produce results of MWR values for entry ages of 55 and above. However, if we ignore the entry age and compare the MWR values of the New Annuity Plan with the MWR values of annuities around the world, they are slightly lower. Most MWR values in other countries ranges from 0.8 to 1.1 [13]. Eventhough the value for money of the New Annuity Plan is quite low, the adverse selection effect for the product is very small and consistent with results in Fong (2011) for Singapore annuities.

In considering all the foregoing MWR values an important caveat needs to be drawn to the reader's attention. The MWR analysis in this paper only calculates the dollar value of the annuity element in the numerator — it uses the market premium in the denominator. Since these products also provide other non-annuity benefits (like death benefit, a total and permanent disability or a

[13] Refer to James and Vittas (2000) and Fong, Mitchell, and Koh (2011) for the MWR studies in other countries.

critical illness benefit) the MWR values are not precise, and are lower than the true MWR values. That is, the denominator values are overstated from this pure annuity values; they include other insurance values. For the EPF Annuity Scheme values the degree of understatement will be small (as the expected values of the additional benefits is quite small relative to the annuity values); for the New Annuity Plan the degree of understatement will be larger (as the expected value of additional benefits is larger relative to the annuity values). This calls for deeper research into the (complex) nature of the additional benefits. The AEW values, however, do not suffer from this bias, and it is to their analysis we now turn.

2.5.2 The Annuity Equivalent Wealth Results

The analyses of the Annuity Equivalent Wealth (AEW) in the previous literature assume a fixed real interest rate and inflation rate over time. For instance, Brown (2003) and Fong (2011) used a real interest rate and inflation rate of 3%, where $r = \pi = 3\%$. In our analysis, as we do not have the real interest rates information, we assume to follow the term structure of interest rate that reflects the Malaysian data rather than assuming a fixed rate. According to the Eq. (2.4.10), the total discounted factor in the formulation is the nominal rate of interest so we decide to choose the average nominal risk-free interest rate used in the MWR analysis. We follow their assumption of the rate of time preference, $\rho = 3\%$.

Table 2.5.5, Table 2.5.6 and Table 2.5.7 show the AEW values of the EPF Annuity Scheme (SAKK) for all three possibilities of without bonus, with 1% bonus and with 2% bonus respectively. While Table 2.5.8 and Table 2.5.9 present the AEW values of the New Annuity Plan for both option A and B. For comparison purpose, we choose two different level of risk aversion of $\gamma = 1.5$ and $\gamma = 3$. Since prior studies on consumption such as Laibson, Repetto, and Tobacman (1998) and Brandt and Wang (2003) found that the average risk aversion level parameter is between 1 to 2, the γ values that we choose seem reasonable. The AEW values presented here are calculated based on the actuarially fair annuity price as in most studies.

First, we find that by increasing the risk aversion level from 1.5 to 3, the AEW values for all categories of the New Annuity Plan (Table 2.5.8 and Table 2.5.9) increase by a small amount as the consumer generally values annuity more if he has a high risk aversion level. This is not the case for the EPF Annuity Scheme as this annuity product offers an increase bonus payment on top of the basic annuity income, thus a more risk averse consumer may value it lower as he prefers a smooth

Table 2.5.4: Money's Worth Ratios and Adverse Selection (AS) Costs of the New Annuity Plan

Entry Age	Male						Female					
	Population		Annuitant		AS (%)		Population		Annuitant		AS (%)	
MGS												
Payment term	1	2	1	2	1	2	1	2	1	2	1	2
35	0.727	0.759	0.779	0.805	5.3	4.6	0.793	0.819	0.835	0.858	4.2	3.9
45	0.773	0.777	0.815	0.818	4.2	4.1	0.822	0.826	0.862	0.865	3.9	3.9
CB												
Payment term	1	2	1	2	1	2	1	2	1	2	1	2
35	0.596	0.645	0.639	0.684	4.3	3.9	0.650	0.696	0.684	0.729	3.4	3.3
45	0.693	0.699	0.730	0.735	3.7	3.6	0.736	0.742	0.771	0.776	3.5	3.4

(a) Option A - retirement age at 55

Entry Age	Male						Female					
	Population		Annuitant		AS (%)		Population		Annuitant		AS (%)	
MGS												
Payment term	1	2	1	2	1	2	1	2	1	2	1	2
35	0.710	0.752	0.773	0.805	6.2	5.3	0.791	0.825	0.846	0.875	5.5	5.0
45	0.728	0.754	0.780	0.801	5.2	4.7	0.790	0.812	0.843	0.861	5.2	4.8
CB												
Payment term	1	2	1	2	1	2	1	2	1	2	1	2
35	0.568	0.632	0.617	0.676	4.9	4.5	0.632	0.693	0.676	0.735	4.3	4.2
45	0.634	0.671	0.680	0.713	4.5	4.2	0.689	0.722	0.734	0.765	4.5	4.3

(b) Option B - retirement age at 60

Table 2.5.5: Annuity Equivalent Wealth of the EPF Annuity Scheme (SAKK) Basic - without compounded bonus

Entry Age	Male		Female	
γ	1.5	3.0	1.5	3.0
35	1.299	1.294	1.152	1.142
45	1.298	1.294	1.166	1.157
55	1.288	1.281	1.172	1.163
65	1.456	1.476	1.285	1.284

consumption pattern. Another factor that affects the consumer behaviour here is the higher risk-free interest rate in year 2000 (the average risk-free interest rate is 5.7% in 2000 and 3.5% in 2012). Given a higher rate of interest, a more risk averse consumer would be less willing to forgo current consumption for future consumption, thus value annuity lower than a less risk averse consumer. Only for some category of entry age 65, the AEW is higher for a more risk averse individual (refer to Table 2.5.5, Table 2.5.6 and Table 2.5.7). However, the difference of AEW value calculated using a risk aversion level of 1.5 and of 3, are not significant for all categories. For the New Annuity Plan, the difference in AEW is very small which resulted in equal value up to 3 decimal points if the annuity premium is charged using the actuarially fair price regardless of payment term 1 or 2.

Overall, based on actuarially fair price annuity, our AEW result is consistent with prior studies where both the EPF Annuity Scheme and the New Annuity Plan provide a good value for money for consumers[14]. All values of AEW are greater than 1 which shows that a consumer without access to annuity would need higher initial wealth amount to achieve the same expected utility level as if he had access to annuity by $[AEW(\alpha) - 1](100)\%$. Our result is consistent with previous studies where given an actuarially fair annuity product, consumer would always find full annuitisation as an optimal consumption strategy.

[14]Mitchell et al. (1999) show that the AEW of US annuities ranges from 1.3 to 1.5. Ganegoda and Bateman (2008) calculate the AEW of Australian annuities to be about 1.23 to 1.6.

Table 2.5.6: Annuity Equivalent Wealth of the EPF Annuity Scheme (SAKK) Bonus 1 - with 1% per annum compounded bonus

Entry Age	Male		Female	
γ	1.5	3.0	1.5	3.0
35	1.294	1.293	1.147	1.142
45	1.298	1.293	1.161	1.157
55	1.288	1.287	1.168	1.162
65	1.456	1.476	1.282	1.284

Table 2.5.7: Annuity Equivalent Wealth of the EPF Annuity Scheme (SAKK) Bonus 2 - with 2% per annum compounded bonus

Entry Age	Male		Female	
γ	1.5	3.0	1.5	3.0
35	1.298	1.274	1.151	1.120
45	1.298	1.277	1.165	1.133
55	1.287	1.269	1.172	1.143
65	1.456	1.464	1.285	1.270

Table 2.5.8: Annuity Equivalent Wealth of the New Annuity Plan (Option A - retirement age at 55)

Entry Age	Male		Female	
γ	1.5	3	1.5	3
Payment term 1				
35	1.307	1.333	1.201	1.225
45	1.328	1.356	1.258	1.274
Payment term 2				
35	1.307	1.333	1.201	1.225
45	1.328	1.356	1.258	1.274

Table 2.5.9: Annuity Equivalent Wealth of the New Annuity Plan (Option B - retirement age at 60)

Entry Age	Male		Female	
γ	1.5	3	1.5	3
Payment term 1				
35	1.400	1.433	1.259	1.289
45	1.435	1.470	1.335	1.356
Payment term 2				
35	1.400	1.433	1.259	1.289
45	1.435	1.470	1.335	1.356

2.6 Conclusion

Annuities are less popular in Malaysia compared to other insurance products; the annuity market is very small and almost non-existent in the past several years. Annuitisation rates were quite high in 2000, following the introduction of the EPF Annuity market which allows EPF members to withdraw their EPF savings for the purpose of purchasing annuities. Since the EPF Annuity was not a government funded or backed up scheme, this product was controversial due to the perceived lack of protection of members' retirement savings and uncertain investment returns. Objections by several entities, including a Malaysian workers representative body, to the scheme lead to the suspension of the EPF Annuity Scheme at the end of 2001. Since then, the percentage of new business premiums for annuities has been almost 0% of total new business premiums in the insurance market (Aziz, 2011).

In spite of the high liabilities associated with issuing annuities, they have the attractive feature of providing a secure stream of income for retirees. Thus, governments in several countries still promote annuities as one of the options for retirement income. Singapore's CPF Annuity is the best example of a recent government annuity scheme. It is noteworthy that the price of this annuity really favours buyers. According to Fong (2011), the Singapore CPF Annuity has been designed as a government funded scheme which helps cost saving through economies of scale as retirees tend to choose risk-free retirement funds over riskier products offered by private insurers.

The recent Malaysian government budget of 2012 also supports annuity growth in Malaysia by increasing the tax exemption on income used to purchase annuities. Following that budget announcement, a new annuity plan has been introduced in the market for consumers to enjoy the tax exemption benefits announced by the Malaysian government. The recent controversial history of annuities in Malaysia suggests consumers may be wary.

This paper provides deeper understanding of annuities in Malaysia for consumers by computing the value for money of the annuity element of Malaysia's private annuities using two valuation methods, the Money's Worth Ratios (MWR) and the Annuity Equivalent Wealth (AEW). Assuming a risk-free term structure of interest rates, we found that the suspended EPF Annuity Scheme provides good value for money to consumers when a bonus is included in the annuity benefit.

The suspension of annuities in 2000 has had a great impact on the development of the annuity market in Malaysia. Current stringent regulations require any new annuity product offered in Malaysia to undergo an approval process directed by the Central Bank of Malaysia.

Generally, our findings indicate that the New Annuity Plan introduced in 2012 has slightly lower value for money compared to the values of annuities around the world and the previous EPF Annuity Scheme. This suggests that stricter capital requirements have resulted in more expensive annuities. Furthermore, since the New Annuity Plan was designed as a retirement security product to provide guaranteed annuity income, the insurer would have to charge higher loadings compared to other non-guaranteed insurance products.

Lastly, it is important to point out again that our analysis only treats the annuity element of the product; it does not include the additional benefits provided under both the EPF Annuity Scheme and the New Annuity Plan. Thus, the value for money analysis is only correct for the annuity element of the product. Our calculated metrics ignore the value contribution of additional benefits such as death, total permanent disability and critical illness insurance. Furthermore, our analysis in this paper does not incorporate the tax incentives proposed by the Malaysian government. More extensive analysis requires more data and information on the transition probabilities for different health states incorporated in an extended model we shall pursue in future research.

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2.A Appendix

2.A.1 The AEW Computational Algorithm

Condition 1: With Annuities

We start with a representative consumer who maximises his expected utility by following an optimal consumption path C_t:

$$V = \max_{\{C_t\}} \sum_{t=y-x}^{T-x} \frac{{}_tP_x U(C_t)}{(1+\rho)^t} \tag{2.A.1}$$

with the presence of annuities and given the initial wealth level after annuitisation of $W_0 = w^1$, his budget constraint is subject to:

$$\sum_{t=y-x}^{T-x} ({}_tP_x)C_t(1+i)^{-t} = W_0 + \sum_{t=y-x}^{T-x} ({}_tP_x)A_t(1+i)^{-t} \tag{2.A.2}$$

$$W_t \geq 0 \tag{2.A.3}$$

Note that in Section 2.4.2, our initial wealth level after annuitisation is 0 since we assume the consumer consumed all of his wealth to purchase annuity. Here, we consider a more general consumer problem where we allow for both annuitised and non-annuitised wealth. Thus, our initial wealth level before annuitisation is $(P + w^1)$ where P is the annuity premium. Eq. (2.A.1) can also be solved using a recursive budget constraint of $W_{t+1} = (W_t - C_t + A_t)(1+i)\ \forall t$, which we use to solve the inequality constraint Eq. (2.A.3) under the Lagrangian method[15]. Since we have $W_t \geq 0\ \forall t$, so as W_{t+1} or $(W_t - C_t + A_t)(1+i)$ must be greater or equal to 0 $\forall t$.

The Lagrangian function for this problem is

$$\begin{aligned} L(C_t, \lambda, \mu_t) &= \sum_t P_t U(C_t)\alpha^t + \lambda(W_0 + \sum_t P_t A R^{-t} - \sum_t P_t C_t R^{-t}) \\ &\quad + \sum_t \mu_t[(W_t - C_t + A_t)R] \end{aligned} \tag{2.A.4}$$

where $P_t =_t P_x$, $\alpha = (1+\rho)$ and $R = (1+i)$. Given $U(C_t) = C_t^{1-\gamma}/(1-\gamma)$, the first order conditions of the problem for all t are:

[15] This recursive budget constraint is required to solve the problem using the recursive Bellman equation as in Brown (2003).

$$\begin{aligned}
\frac{\partial L}{\partial C_t} &= P_t C_t^{-\gamma} \alpha^t - \lambda P_t R^{-t} - \mu_t R = 0 \\
\frac{\partial L}{\partial \lambda} &= W_0 + AR^{-t} - C_t R^{-t} = 0 \\
\frac{\partial L}{\partial \mu_t} &\geq 0,\ \mu_t \geq 0,\ \text{with complementary condition of } \mu_t[(W_t - C_t + A_t)R] = 0
\end{aligned}$$

In order to solve the problem, we begin by letting either μ_t or $(W_t - C_t + A_t)R$ in the complementary condition equal to 0.

If $\mu_t = 0$, from the first order condition we get

$$C_t = \frac{\alpha^{\frac{t}{\gamma}} R^{\frac{t}{\gamma}}}{\lambda^{\frac{1}{\gamma}}} \tag{2.A.5}$$

Next, substitute Eq. (2.A.5) into Eq. (2.A.2) and find λ^* such that it satisfies constraint Eq. (2.A.2).

$$\begin{aligned}
\sum_t \frac{P_t \alpha^{\frac{t}{\gamma}} R^{\frac{t-t\gamma}{\gamma}}}{\lambda^{*\frac{1}{\gamma}}} &= W_0 + \sum_t P_t A R^{-t} \\
\lambda^{*\frac{1}{\gamma}} &= \frac{\sum_t P_t \alpha^{\frac{t}{\gamma}} R^{\frac{t-t\gamma}{\gamma}}}{W_0 + \sum_t P_t A R^{-t}} \\
\lambda^* &= \frac{\sum_t P_t^{\gamma} \alpha^t R^{t-t\gamma}}{(W_0 + \sum_t P_t A R^{-t})^{\gamma}}
\end{aligned}$$

Now, we can derive the optimal C_t by substituting λ^* into Eq. (2.A.5) which we called C_t^{1a}:

$$C_t^{1a} = \frac{\alpha^{\frac{t}{\gamma}} R^{\frac{t}{\gamma}} (W_0 + \sum_j P_t A R^{-j})}{\sum_j R^{\frac{j(1-\gamma)}{\gamma}} P_j \alpha^{\frac{j}{\gamma}}}$$

If $(W_t - C_t + A_t)R = 0$, then the optimal C_t can be derived straightaway which we called C_t^{1b} :

$$C_t^{1b} = W_t + A_t$$

Since we have two solutions of C_t for this problem, we have to choose either C_t^{1a} or C_t^{1b} for all time t to find the optimal consumption path of the consumer that maximises his expected utility, say V^1. Note that C_t^{1a} is only feasible if it satisfies $(W_t - C_t^{1a} + A_t)(1+i) \geq 0\ \forall t$. Otherwise, we will choose C_t^{1b} as it always satisfies that condition.

Condition 2: Without Annuities

Here, we begin with the same consumer optimisation problem as in Eq. (2.A.1), but we change the first budget constraint from Eq. (2.A.2) to:

$$\sum_t C_t R^{-t} = W_0 \tag{2.A.6}$$

where $W_0 = w^2$ is the initial non-annuitised wealth. Now, from Eq. (2.A.6) the recursive budget constraint becomes $W_{t+1} = (W_t - C_t)R\ \forall t$. As in the condition 1 problem, we derive the Lagrangian function of condition 2 problem as follows:

$$\begin{aligned} L(C_t, \lambda, \mu_t) &= \sum_t P_t U(C_t)\alpha^t + \lambda(W_0 - \sum_t C_t R^{-t}) \\ &\quad + \sum_t \mu_t[(W_t - C_t)R] \end{aligned} \tag{2.A.7}$$

where $P_t =_t P_x$, $\alpha = (1+\rho)$ and $R = (1+i)$. Given $U(C_t) = C_t^{1-\gamma}/(1-\gamma)$, the first order conditions of the problem for all t are:

$$\begin{aligned} \frac{\partial L}{\partial C_t} &= P_t C_t^{-\gamma}\alpha^t - \lambda R^{-t} - \mu_t R = 0 \\ \frac{\partial L}{\partial \lambda} &= W_0 - C_t R^{-t} = 0 \\ \frac{\partial L}{\partial \mu_t} &\geq 0,\ \mu_t \geq 0,\ \text{with complementary condition of } \mu_t[(W_t - C_t)R] = 0 \end{aligned}$$

In order to solve the problem, we begin by letting either μ_t or $(W_t - C_t)R$ in the complementary condition equal to 0.

If $\mu_t = 0$, from the first order condition we get

$$C_t = \frac{P_t^{\frac{1}{\gamma}} \alpha^{\frac{t}{\gamma}} R^{\frac{t}{\gamma}}}{\lambda^{\frac{1}{\gamma}}} \tag{2.A.8}$$

Next, substitute Eq. (2.A.8) into Eq. (2.A.6) and find λ^* such that it satisfies constraint Eq. (2.A.6).

$$\begin{aligned}
\sum_t \frac{P_t^{\frac{1}{\gamma}} \alpha^{\frac{t}{\gamma}} R^{\frac{t-t\gamma}{\gamma}}}{\lambda^{*\frac{1}{\gamma}}} &= W_0 \\
\lambda^{*\frac{1}{\gamma}} &= \frac{\sum_t P_t^{\frac{1}{\gamma}} \alpha^{\frac{t}{\gamma}} R^{\frac{t-t\gamma}{\gamma}}}{W_0} \\
\lambda^* &= \frac{\sum_t P_t \alpha^t R^{t-t\gamma}}{(W_0)^\gamma}
\end{aligned}$$

Now, we can derive the optimal C_t by substituting λ^* into Eq. (2.A.8) which we called C_t^{2a}[16]:

$$C_t^{2a} = \frac{P_t^{\frac{1}{\gamma}} \alpha^{\frac{t}{\gamma}} R^{\frac{t}{\gamma}} W_0}{\sum_j R^{\frac{j(1-\gamma)}{\gamma}} P_j^{\frac{1}{\gamma}} \alpha^{\frac{j}{\gamma}}}$$

If $(W_t - C_t)R = 0$,then the optimal C_t can be derived straightaway which we called C_t^{2b} :

$$C_t^{2b} = W_t$$

Since we also have two solutions of C_t for this problem, hence, we have to choose either C_t^{2a} or C_t^{2b} for all time t to find the optimal consumption path of the consumer that maximises his expected utility, say V^2. Note that C_t^{2a} is only feasible if it satisfies $(W_t - C_t^{2a})(1 + i) \geq 0 \ \forall t$. In our analysis, for the case of without annuities, C_t^{2a} is always feasible.

AEW Solution

Our objective is to find the AEW(α) value which is the ratio between the initial wealth level in condition 2 (w^2) to the initial wealth level before annuitisation in condition 1 ($P + w^1$), that makes the consumer indifferent in terms of expected utility level. Thus, given V^1 from condition 1, we have to find w^2 that gives us $V^2 = V^1$. Lastly, we can now find the α value using this formula:

$$\alpha = \frac{w^2}{P + w^1} \tag{2.A.9}$$

.

[16]Our derived optimal consumption path has the same form as in Kotlikoff and Spivak (1981).

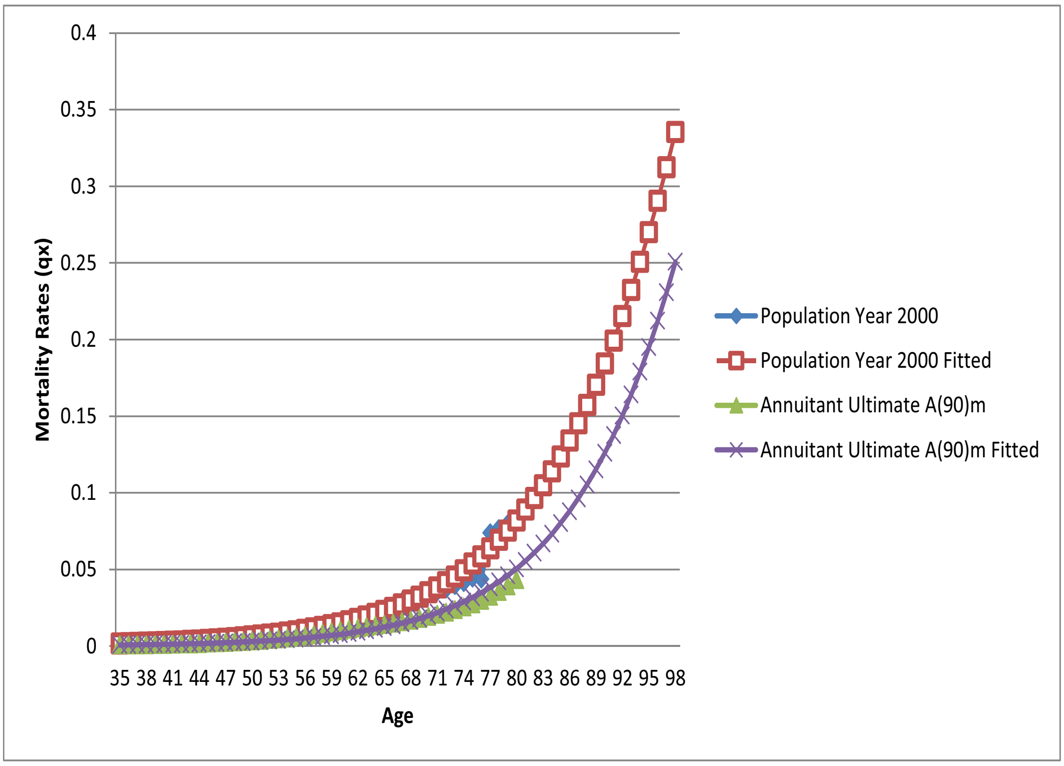
Tables and Figures

Figure B.1 shows the fitted mortality rates for Malaysian males and females based on Malaysian Abridged Life Table obtained from the Department of Statistics Malaysia (population mortality rate year 2000) and Malaysian annuitant mortality rate obtained from Malaysia a(90) Ultimate Male and Female Tables (SOA, 2013). The mortality rates for age 81 to 100 is extrapolated using Gompertz-Makeham formula,

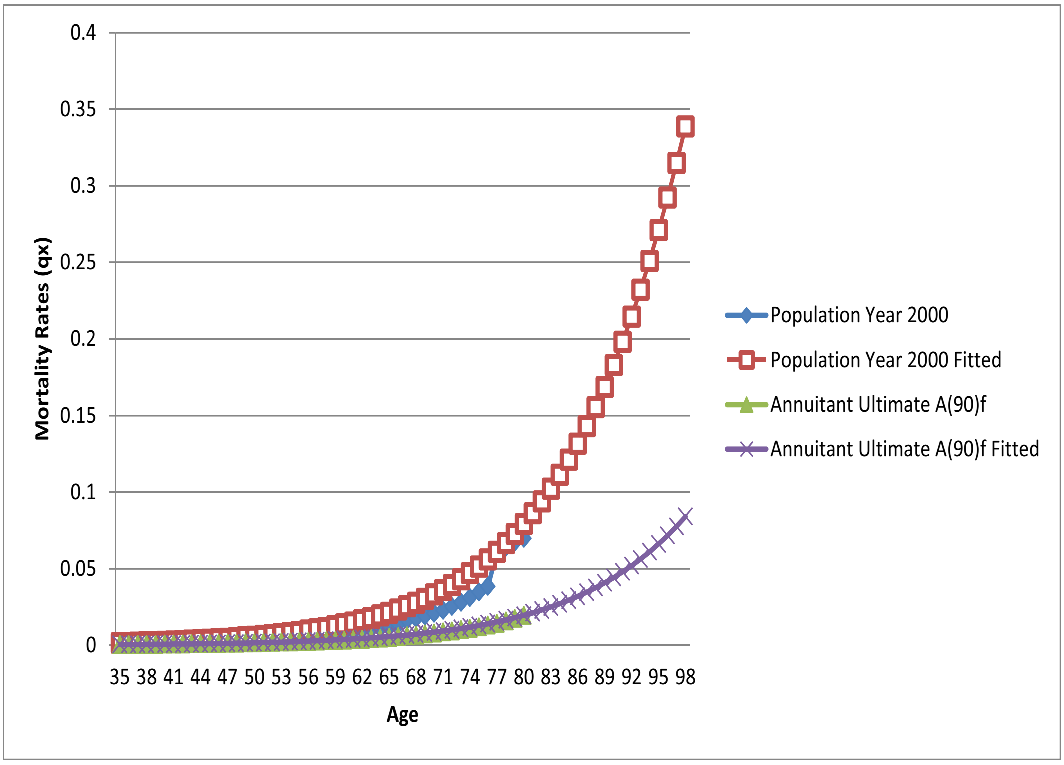
$$\mu_x = a + bc^x \tag{2.A.10}$$

where μ_x is the intensity of mortality, x is the age and a, b and c are parameters. This method is found to be the long time universally used for extrapolating mortality curves (Dotlacilova, Simpach, and Langhamrova (2013) and Bayor and Faber (1983)). Estimated parameters used in the mortality projection of the Malaysian mortality rates are presented in Figure B.2.

Figure B.1: Fitted Population and Annuitant Mortality Rates

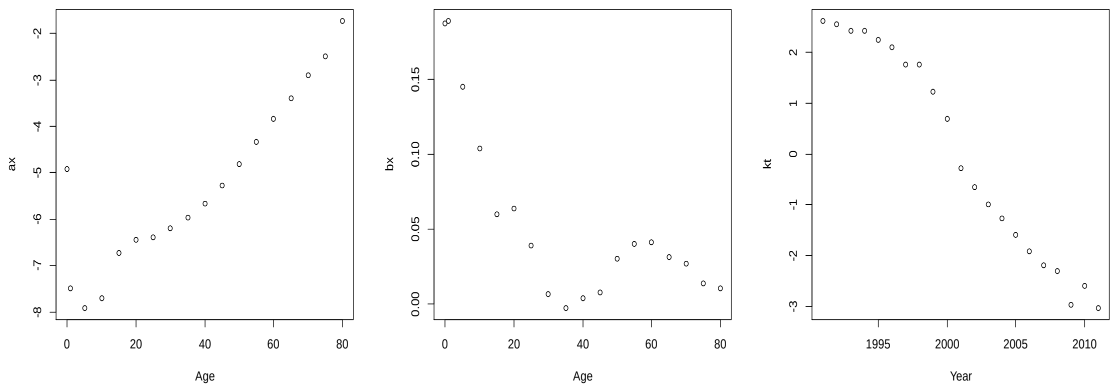


Panel A: Category Male

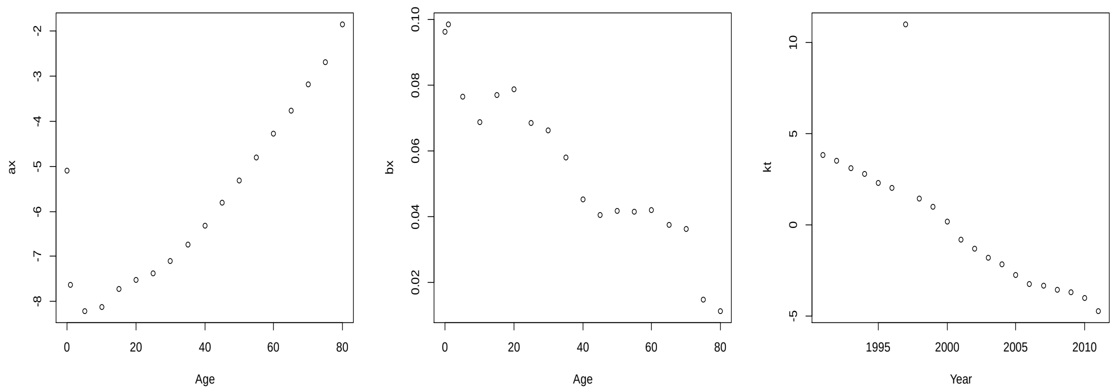


Panel B: Category Female

Figure B.2: Estimated Parameters for the Mortality Projection Method



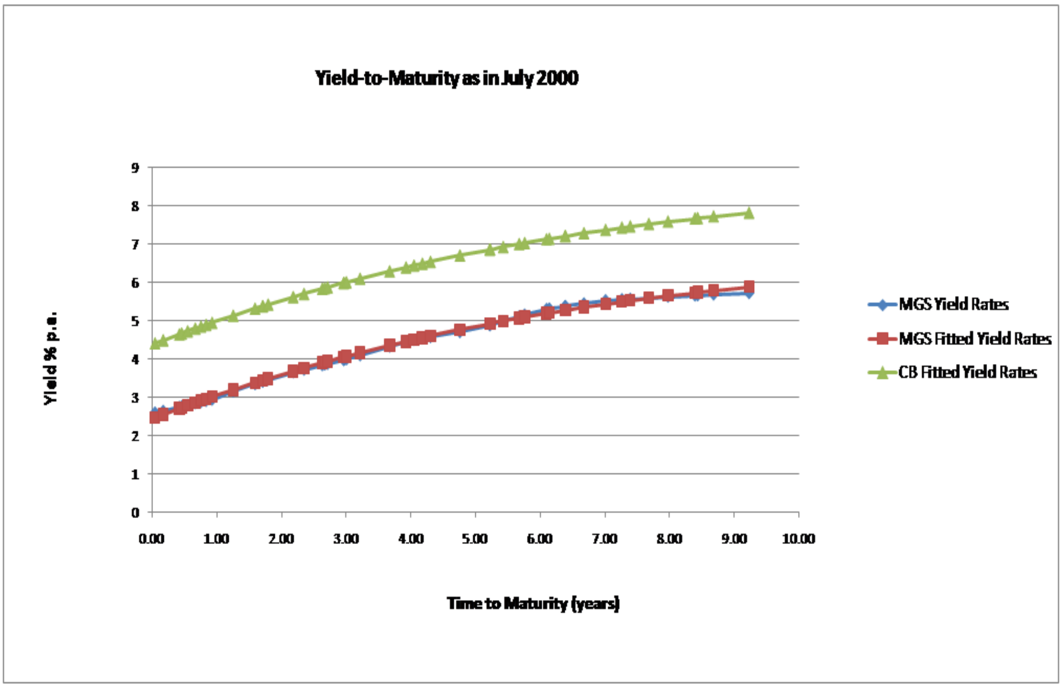
Panel A: Category Male

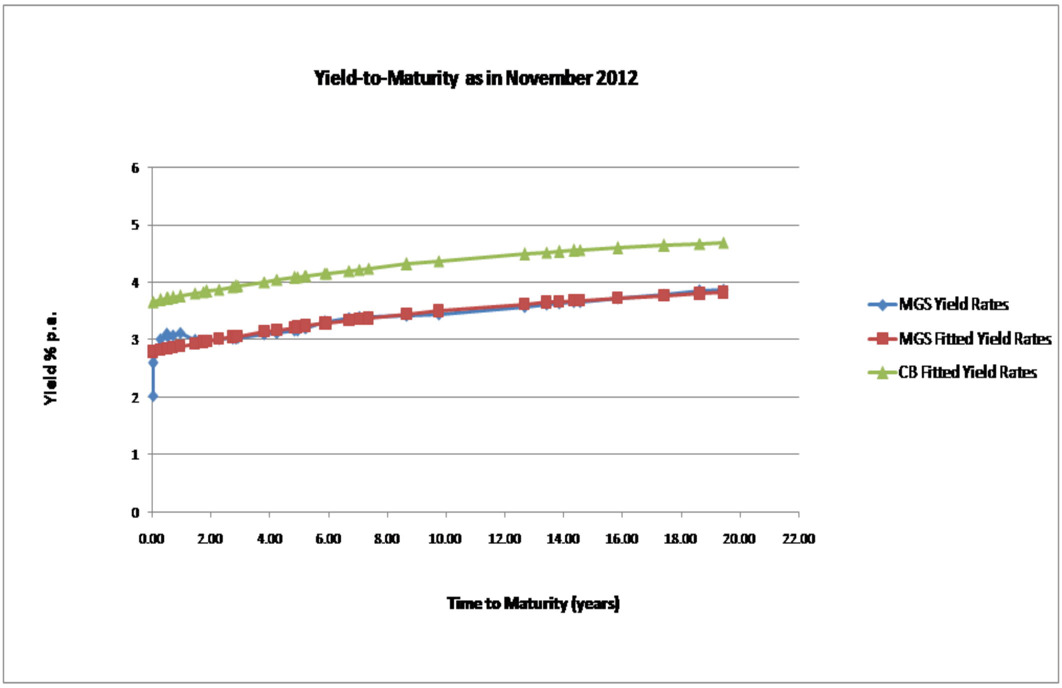


Panel B: Category Female

Figure B.3 shows the fitted yield curve of the Malaysian Government Bond (MGS) yield rates and the AAA Corporate Bond (CB) yield rates obtained from the Central Bank of Malaysia (BNM, 2013).

Figure B.3: Fitted Yield Curve and Malaysian Government Bond (MGS) and AAA Corporate Bond (CB) Yields





Chapter 3

Paper 2

Annuity Valuation with a Health State Dependent Utility Function

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Abstract

Post-retirement, annuities are known as a product that provides a steady stream of income to retirees. Prior studies such as Yaari (1965) and Davidoff, Brown, and Diamond (2005) have shown that consumers without bequest motive, who have access to an actuarially fair annuity product, always find a full annuitisation strategy optimal. These theoretical findings are consistent with studies of the value for money of annuities by a measure called the Annuity Equivalent Wealth (AEW) such as Mitchell et al. (1999) and Brown (2003) which show that a consumer requires a higher amount of non-annuitised wealth to make him as well off as if he has access to annuitised wealth. However, the rate of voluntary annuitisation in many countries is still low and one of the issues with buyers is the lack of liquidity to cover any unexpected medical expenses for retirees. Recent developments in annuity product design in Malaysia includes additional benefits in the form of lump sum payments for unfortunate events such as total and permanent disability, critical illnesses and death. Such benefits enhance the value of the product and it is important, especially for potential annuity buyers, to see how the value for money of annuities changes with these additional benefits. This paper extends the Annuity Equivalent Wealth (AEW) model used by economists to value annuity products by incorporating several health states into the model. The model is developed using a discrete-time multi-state Markovian framework in which national data is used to estimate the transition probabilities into each health state. Instead of using deterministic medical expenditure associated with different health states, we employ a health state dependent utility function which gives a different utility value for a unit of consumption depending on the state of health of the consumer. The concept of health index of utility, a measure of utility associated with health used in the health economics study is applied where each health state is assigned a health index value as a weighting parameter in the utility function. The set of health index values are taken from a survey study Md Yusof, Goh, and Azmi (2012), which utilised the EQ-5D survey instrument developed by the EuroQol Group to generate the health index of utility as perceived by respondents. We compare the results using both the health state independent utility function and the health state dependent utility function. The implication of the health state dependent utility is that, category male consumers value annuity higher if only the annuity stream of income and death benefit are paid, but they value annuity lower when additional benefits associated with total and permanent disability or critical illness are also paid. For female category, the additional benefits have less effect on the value of annuity because the probability of receiving such claim is very low.

Keywords: annuities, Annuity Equivalent Wealth, muti State Markovian model, health state dependent utility, EQ-5D.

3.1 Introduction

The low demand for voluntarily purchased annuities, widely known as annuity puzzle, remains as an issue for annuity providers in many countries. Some efforts to reveal the reason behind this issue have been made, including Inkmann, Lopes, and Michaelides (2011), Purcal and Piggott (2008) and Bateman et al. (2013). The early theoretical work of Yaari (1965), however, supports full annuitisation as an optimal consumption strategy for consumers. And yet, the annuity participation rate is still low in voluntary markets. This poses a challenge for insurers wishing to ensure the survival of annuity products, especially without regulatory supports and government incentives. Recent research work by Peijnenburg, Nijman, and Werker (2013) further explores the potential resolution of the annuity puzzle by considering health cost risk in a consumer's life cycle model. They found that a high amount of out of pocket medical expenses in early retirement could lower the amount of optimal annuitisation for consumers. Their finding is consistent with Turra and Mitchell (2004) and Ameriks et al. (2007).

Unexpected medical costs thus affects consumers decisions to annuitise, making annuities less attractive to buyers. In order to attract potential buyers to purchase annuities, insurers may, however, offer an annuity product with riders or additional benefit options that better suit consumer preferences. In 2012, a commercial annuity product was introduced in Malaysia, which we refer to as the New Annuity Plan, and has the following benefits.

1. Annuity stream of income starting from age 55 up to age 70 or age 60 up to age 70.

2. Lump sum cash value payment in the event of total and permanent disability, diagnosis of critical illnesses (except for angioplasty or other invasive treatments for major coronary artery disease) or death.

A detailed description of this product is given in Asmuni and Purcal (2013)[1]. Considering only the annuity element of the product, that paper found that the market price of this product is quite expensive for buyers. However, we are also aware that our analysis might have underestimated the value of the product since we do not include the additional benefits provided. Hence, this paper aims to extend the traditional Annuity Equivalent Wealth method used in the annuities' value for money analysis by incorporating several health states into the model and thus capturing a value for all the benefits offered by the product. Instead of using a deterministic health cost associated with each health state, as in Turra and Mitchell (2004) and Ameriks et al. (2007), we used a health state dependent utility function to the model which gives a different utility value of a unit of consumption in different health states. Since medical expenses can also be covered by other income such as private health insurance benefits and national health support, the inclusion of deterministic medical expenses into the model would require further consideration of such external amounts. We, however, focus only on our objective which is to find the value for money of the annuity product solely on its own merits. This model is also suitable for international comparison purposes as we have eliminated the aspect of heterogeneous health funding systems in different countries.

[1]Refer to Appendix 3.A.5 for the sample of quotes for the annuity product provided from the insurer.

In addition, we explore the optimal consumption pattern of consumers as their utility is affected by their current health state. Finkelstein, Luttmer, and Notowidigdo (2013) found that the marginal utility of consumption declines as health deteriorates. In comparison with people without chronic diseases, their point estimates show that a one-standard deviation increase in an individual's number of chronic diseases leads to 10–25% decrease in marginal utility. A survey by Levy and Nir (2012) also shows that consumers consume differently depending on their current health state. Hence, the current health state of consumers can have an impact on their consumption patterns and indirectly affects the consumers' perception towards the value of insurance for them. It is the other objective of this paper to investigate the effect of a health state dependent utility function on consumers making annuity and insurance choices. While applications of such utility functions can be found in Levy and Nir (2012) and Peijnenburg, Nijman, and Werker (2013) none of these papers have considered the application to the Annuity Equivalent Wealth (AEW) context.

The paper proceeds as follows. Section 3.2 discusses the concept of utility of health and wealth (or consumption in a multi-period setting) and the foundation of the health index of utility that we utilise in this paper. Section 3.3 explains our methodology in detail, which consists of a combination of two models, namely the multi-state life cycle model and the multi-state annuity model. A desciption of the data used for the analysis is also given. We present our findings in section 3.4 and, lastly, section 3.5 concludes.

3.2 Optimal Consumption with Health State Dependency

Most papers have assumed for a health state *independence* in the life cycle model used to solve for the optimal demand for health related insurance products such as annuities[2]. In contrast, in the health economics area of studies, use is made of a utility function that depends on an individual's current health state. Such a utility function will have implications for the optimal consumption of a person in the life cycle model. Moreover, evidence appears in Bleichrodt and Quiggin (1999), Viscusi and Evans (1990) and Finkelstein, Luttmer, and Notowidigdo (2013) that the marginal utility of wealth increases with health — which implies that there is a tradeoff between health and wealth of an individual. Hence, we

[2]Refer to Kotlikoff and Spivak (1981), Mitchell et al. (1999) and Brown (2003) for the optimal annuitisation problem using a health state *independent* life cycle model.

examine both the health state independent and the health state dependent life cycle models for the purpose of valuing an annuity product and its associated benefits. This section surveys some of the theoretical and empirical work related to the utility of health and wealth. It also elaborates on the measure of health that we apply to the utility function which we call the health index of utility below.

3.2.1 Logarithmic Preference as Utility for Health and Wealth

Levy and Nir (2012) recently consider several possible forms of utility functions that describe people's preference over health and wealth. Note that in their paper, which focuses on wealth as a variable, wealth can be interpreted as consumption in a multi-period setting — this is the approach we adopt below. Since we use a measure of consumption in a multi-period setting below, we proceed with the application of utility to health and consumption, i.e. $U(h_t, c_t)$, where h_t denotes a person's current health state at time t and c_t denotes his consumption level at time t.

Levy and Nir (2012) empirical work shows the consumption pattern of cancer and diabetes patients according to their income and health state. Survey respondents are asked about their willingness to pay for cure by the proportion of consumption that he or she is willing to give up in exchange for the treatment. The current health state of the respondent is also determined by a health index of utility, h-value, which we will explain further in the next subsection. Out of three forms of utility function tested using the survey data, namely the logarithmic function, power function and negative exponential function, the utility function that best decribed the health-consumption tradeoff was logarithmic preferences. Hence, we have adopted the logarithmic preference form as the health state dependent utility function used in the AEW analysis below. This has the following form:

$$U(h_t, c_t) = h_t \cdot \log(ac_t), \tag{3.2.1}$$

where

$$\begin{aligned} h_t &\equiv \text{a measure of health index of utility according to a person's} \\ &\quad\ \text{health state at time } t, \\ c_t &\equiv \text{the consumption level at time } t, \text{ and} \\ a &\equiv \text{a scaling parameter.} \end{aligned}$$

An implication of this utility form's selection for our analysis is that we have to assume the risk aversion level of consumers as unity. This is not unreasonable: unity found to be the average risk aversion level in consumption studies by Laibson, Repetto, and Tobacman (1998).

For the utility function stated above, the condition of $ac_t \geq 1$ must hold so that the utility will always increase with health, otherwise we have $\log(ac_t) < 0$. Note that the scaling parameter is important to ensure the invariance of economic behaviour regardless of the unit of consumption used in the analysis[3]. Since we must have $ac_t \geq 1$, it implies that $c_t \geq 1/a$ where $1/a$ can be interpreted as the minimum consumption level required for existence[4]. For instance, if we impose a minimum consumption level required for existence at a nominal value of \$100, then we have $a = 1/100$. Hence, if someone has a consumption level of say $c_t = \$500$ then the condition will hold — but not for $c_t = \$5$. Also, instead of measuring c_t in unit of dollars, we can convert it into a measure in terms of the minimum consumption level for existence. Then, we have $(1/100).(\$100.c_t) \geq 1$ which indicates that if someone has 1 unit of the minimum consumption level required for existence, $c_t = 1$, it actually implies that he has a consumption level of 100 in dollars unit. This also implies that we can use a utility function of $U(h_t, c_t) = h_t \cdot \log(c_t)$ whenever c_t is measured in units of the minimum consumption level for existence.

3.2.2 Health Index of Utility

One of the contributions of Levy and Nir (2012) is a mathematical proof that shows the basic theoretical foundations for a utility function that describes the health-consumption tradeoff. According to the authors, the definition of the level of health, h_t, is crucial for the formulation of the utility function for health and consumption. Therefore, we review several methods to elicit the value of health as widely used by many scholars in the area of health economics studies, including those mentioned by Levy and Nir (2012).

There are three methods that are frequently used for eliciting the level of health (Brazier, 2007) which we refer to as health index of utility in this paper. The first and simplest method is the Rating Scale (RS) approach where, given several health conditions, the individual is asked to rank them from the most preferred health state to the least preferred health state. The value of h is then determined

[3]Levy and Nir (2012) detail the importance of having a scaling parameter in the utility of health and consumption function.

[4]This is an important feature imposed by Levy and Nir (2012) to ensure that utility always increases with health.

by asking the individual to value those health states according to the scale so that it reflects their preference rank choice. One of the examples for the Rating Scale method is called the Visual Analogue Scale (VAS) as used in the Measurement and Valuation of Health survey in 1995 by the MVH Group, Centre for Health Economics, University of York. All respondents were asked to value 15 health states on a 20 c.m. visual analogue scale on the range of 0 to 100 where 100 is referred to as the best imaginable health state and 0 as the worst imaginable health state[5].

The second method is the Standard Gamble (SG) approach where an individual who is not perfectly healthy is given two alternatives:

1. A choice of treatment that will cure his sickness to the perfect health state with a probability of p, or can cause immediate death with a probability of $(1-p)$.

2. A choice of living in his current health state for certain for the rest of his lifetime.

That individual will have to determine the value of p that will make him indifferent between the two choices. This choice of p by the individual is to be referred as the health index of utility where it has a range of $0 \leq h \leq 1$. Intuitively, a perfectly healthy individual will only be indifferent between the two alternatives if $p = 1$, hence $h = 1$. Whereas a person in a worse health state condition will be indifferent between the two alternatives at $p < 1$, hence $h < 1$. Thus, as the health condition of the person becomes worse, the lower the value of h will be.

The third method is the Time Trade Off (TTO) approach where the health index of utility is determined by asking the individual for the number of years that he is willing to give up out of his expected remaining lifetime to live in the perfect health state for the years left. Levy and Nir (2012) give the formula for eliciting the health index of utility under this approach as $h = 1 - \tau/T$ where τ denotes the number of years that the individual is willing to give up and T denotes the expected number of years of his remaining lifetime. We assume that τ and T are positive integers. Hence, this implies that the health index of utility has a range of $0 \leq h \leq 1$, if the condition of $\tau \leq T$ must hold. The intuition behind the health index of utility under this approach is consistent with other methods where an individual is willing to give up more years out of his lifetime

[5]Refer to MVH (1995) for the transformation formula applied to the health index of utility value so that it is comparable with the other elicitation method used in the survey, called the Time Trade Off (TTO) approach, where full health state has a value of 1 and death has a value of 0. The Time Trade Off (TTO) approach is explained further below.

as health deteriorates resulting in a lower value of health index of utility for a person with a worse health condition.

In contrast to other elicitation methods, there has been a steadily growing debate regarding the TTO approach when health states involve both the better than death (SBD) and the worse than death (SWD) states. For instance, MVH (1995) also allows for a health state worse than death (SWD). There, a respondent is given two alternatives: immediate death, or live in his current health state for $10-x$ years followed by x years of living in the perfect health state and then die. The respondent determines x that will make him indifferent between these two alternatives. This formulation is then applied to elicit the health index of utility for the state worse than death (SWD): $h = -x/(10-x)$.

According to Devlin et al. (2013), the TTO approach for eliciting the SWD is problematic because there is an inconsistency with the TTO approach used to elicit the SBD. The TTO approach that is applied in MVH (1995) to elicit the health index of utility for category health state better than death (SBD) is consistent with the interpretation as in Levy and Nir (2012)[6]. The only difference is instead of using the expected number of years of the respondent's remaining lifetime, T, MVH (1995) asked the respondent for the number of years $10-x$ that he is willing to give up out of 10 years of his lifetime to live the rest of x years in the perfect health state; then, $h = x/10$. Referring to the formulation used to elicit the health index of utility, both the numerator and the denominator will be adjusted according to the respondent's preference for the SWD. However, only the numerator is adjusted for the SBD. Besides that, contrary to the interval scale of the health index of utility for the SBD where $0 \leq h \leq 1$, the lower interval of the health index of utility for the SWD is not bounded by the value of -1 which resulted in an asymmetry between the SBD and the SWD.

Lamers (2007) studies several methods used to transform the health index of utility so that it has a bound of -1, namely monotonic transformation, linear transformation and truncation. However, some reseachers such as Lamers (2007) and Patrick et al. (1994) argue that the health index of utility of the SWD cannot be interpreted as utility score once it is transformed. Recent research work by Devlin et al. (2013) suggests a new TTO approach called the lead time TTO approach to overcome this problem. If the state worse than death (SWD) exists, then this new approach is a potential solution to the elicitation method for the health index of utility of the SWD in the near future.

In this paper, we consider both the non-existence and the existence of the

[6]The existence of health state worse than death (SWD) is not address in Levy and Nir (2012).

state worse than death. We discuss further the implication of allowing the health state worse than death in our analysis in section 3. We do not directly measure the health index of utility in our target group of annuity buyers as this would involve a large survey study, as in Levy and Nir (2012). Instead, we utilise the health index of utility from the published EQ-5D value set for the Malaysian population obtained from a survey studies by Md Yusof, Goh, and Azmi (2012). Like MVH (1995), the EQ-5D is used as a standardised instrument to measure a respondent's health outcome. The EQ-5D is an instrument developed by a group of multidisciplinary researchers (the EuroQol Group) for the purpose of generating a single heath index for health status. The EQ-5D value set is widely used in health care evaluation where the utility index is usually applied to a measure of health effectiveness or cost effectiveness analysis called the Quality Adjusted Life Year (QALY)[7]. We chose the EQ-5D value set elicited using the TTO approach as it is consistent with the theoretical proof of the utility of health and consumption given in Levy and Nir (2012). Since they do not consider for the negative health index of utility resulted from the state worse than death, we show below that the proof by Levy and Nir (2012) also yield the same solution for the negative health index of utility.

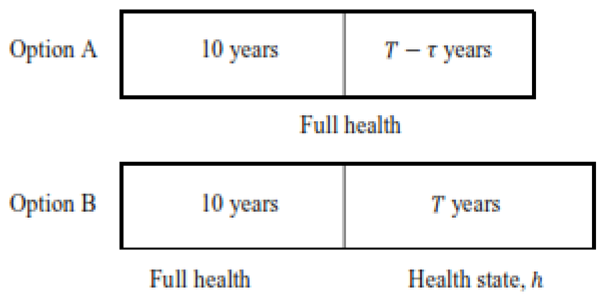
We present the new lead time TTO approach proposed in a paper by Devlin et al. (2013) where both the state better than death and worse than death can be elicited using the same formula which eliminate the inconsistency problem in the original TTO method as in MVH (1995). Under this method, additional years in full health is introduced to Option A and Option B so there will be enough number of years to be traded off in the case where state worse than death exists. Option A and B are illustrated in Figure 3.2.1.

According to the lead time TTO approach, the consumer is indifferent between Option B, living in a current health state h for T years plus 10 years lead time in the full health state or Option A, living in the full health state (where $h = 1$) for $T + 10 - \tau$ years. Then, the h-value can be elicited whenever the consumer is indifferent between Option A and Option B using this formula $[(T + 10 - \tau) - 10]/[(T + 10) - 10]$, where $\tau < T$ for state better than death (SBD) and $T < \tau < T + 10$ for state worse than death (SWD). The total utility of health and consumption for a consumer who lives in a current health state h for T years plus 10 years lead time in the full health state is as below:

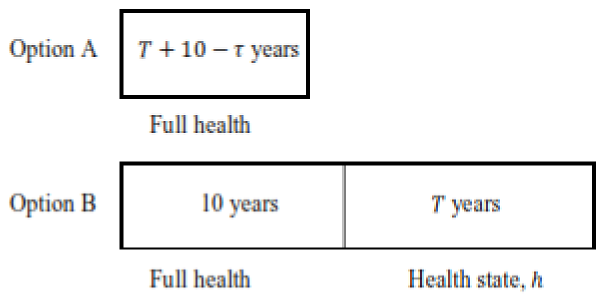
$$\frac{U(1,c)}{(1+\rho)} + ... + \frac{U(1,c)}{(1+\rho)^{10}} + \frac{U(h,c)}{(1+\rho)^{11}} + ... + \frac{U(h,c)}{(1+\rho)^{T+10}} =$$

[7]Further information regarding the QALY can be found in Weinstein, Torrance, and McGuire (2009) and Whitehead and S. Ali (2010).

Figure 3.2.1: The Lead Time Time Trade Off (TTO) Approach



(a) An illustration of the lead time TTO approach for states better than death (SBD)



(b) An illustration of the lead time TTO approach for states worse than death (SWD)

$$\frac{U(1,c)}{\rho}[1 - \frac{1}{(1+\rho)^{10}}] + \frac{U(h,c)}{\rho}[\frac{1}{(1+\rho)^{10}} - \frac{1}{(1+\rho)^{T+10}}]$$

The following equation is for which the consumer who lives in the full health state for $T + 10 - \tau$ years:

$$\frac{U(1,c)}{(1+\rho)} + ... + \frac{U(1,c)}{(1+\rho)^{T+10-\tau}} = \frac{U(1,c)}{\rho}[1 - \frac{1}{(1+\rho)^{T+10-\tau}}]$$

Since the consumer is indifferent between these two states, we obtain the equivalence between two equations above:

$$\frac{U(1,c)}{\rho}[1 - \frac{1}{(1+\rho)^{10}}] + \frac{U(h,c)}{\rho}[\frac{1}{(1+\rho)^{10}} - \frac{1}{(1+\rho)^{T+10}}] = \frac{U(1,c)}{\rho}[1 - \frac{1}{(1+\rho)^{T+10-\tau}}]$$

$$\frac{U(h,c)}{\rho}[\frac{1}{(1+\rho)^{10}} - \frac{1}{(1+\rho)^{T+10}}] = \frac{U(1,c)}{\rho}[\frac{1}{(1+\rho)^{10}} - \frac{1}{(1+\rho)^{T+10-\tau}}]$$

$$U(h,c) = \frac{U(1,c)[1-1/(1+\rho)^{T-\tau}]}{[1-1/(1+\rho)^{T}]}$$

$$U(h,c) = \frac{U(1,c)[(1+\rho)^{T}-(1+\rho)^{\tau}]}{[(1+\rho)^{T}-1]}$$

Then, we follow the last step as in the proof shown in Levy and Nir (2012) by expanding the bracket in the formula above up to the first order in ρ, and we obtain similar result as in their paper:

$$\begin{aligned} U(h,c) &\approxeq U(1,c)[\frac{1+\rho T - 1 - \rho\tau}{1+\rho T - 1}] \\ &\approxeq U(1,c)[1 - \frac{\tau}{T}] \end{aligned} \tag{3.2.2}$$

where $h = 1 - \tau/T$ with $\tau < T$ generates the positive health index of utility while $T < \tau < T + 10$ generates the negative health index of utility[8].

3.3 Extending the Annuity Equivalent Wealth Concept

3.3.1 Review of Studies on AEW

The Annuity Equivalent Wealth measure is widely used by economists in the area of annuity studies as it is a useful tool to value an annuity without ignoring its

[8]This derivation follows from the theoretical work of Levy and Nir (2012) where we can show that there is a relationship between the utility of consumption at health state h, and the utility of consumption at the perfect health state. We apply this relationship in the expected utility function of our Annuity Equivalent Wealth framework by converting the utility of consumption at health state h, into the utility of consumption at the perfect health state using relationship as in equation (3.2.2).

insurance value. As apposed to the Money's Worth Ratio measure, which only takes into account the dollar value of an annuity, the AEW also measures the utility gain from purchasing an annuity product as compared to a person who funds consumption using non-annuitised assets.

The concept was used in Kotlikoff and Spivak (1981) in calculating the utility difference between a consumer with and without access to a perfect annuity market. Later, Mitchell et al. (1999) introduced the term 'wealth equivalence', which was then converted to Annuity Equivalent Wealth (AEW) by Knox (2000) — and the ratio of non-annutised assets to annuitised assets that produces the same expected utility, given two conditions, with or without access to annuity markets[9].

$$\text{Annuity Equivalent Wealth}\,(\alpha) = \frac{\text{non-annuitised wealth }(w)}{\text{annuitised wealth }(P)} \tag{3.3.1}$$

First, the consumer's expected utility is calculated by finding the optimal consumption path given a stream of annuity income from annuitising a wealth level of P. Then, we seek the level of non-annuitised wealth w, that will produce the same expected utility so that the consumer is indifferent between the two conditions, with or without access to annuity market. This measure can be useful for comparing annuity values for different countries. For example, Ganegoda and Bateman (2008) and Brown (2003) used this wealth equivalence concept to estimate the insurance value of annuity products in Australia and U.S respectively.

In past years several extensions have been made to the original AEW measure to allow for certain circumstances. For example, in Turra and Mitchell (2004), out of pocket medical expenses are added to the optimisation problem of the annuitant to allow for the possibility of incuring any medical expenses over the lifetime of consumer. A range of deterministic medical expenses associated with certain health condition is assumed to be paid by consumer, where medical expenses are taken as a proportion of social security income. They find that allowing for medical expenses may reduce the optimal annuitisation level (as a percentage of pension wealth) for people with health problems. In their study, most AEW values are greater than one which indicates that a higher amount of non-annuitised wealth is required to make a person as well off as if she has access to annuitised wealth. This is true apart from for some categories where the consumer has pre-existing annuitised wealth of 75% of his pension wealth.

Fong (2011) studies the impact of risk-based pricing strategy on AEW calcula-

[9] Wealth equivalence measures the annuity value by taking the inverse value of the Annuity Equivalent Wealth measure, that is, the ratio of annuitised wealth over non-annuitised wealth.

tion for an American population categorised by race, gender, history of health and education levels based on the Health and Retirement Study (HRS). The findings show that an annuity is valued less by people with low education and high health risks. Nonetheless, the AEW values shown are still higher than one, which again indicates that even with poor mortality structures, these groups would still benefit from annuitisation. This result conforms to the prior study by Brown (2003) which find that annuitisation is welfare enhancing even for people with higher than average mortality rates, provided that administrative costs are sufficiently low.

In this paper we extend the AEW measure by incorporating several health states that may occur over the life-cycle of a consumer. This is done so as to value the additional benefit payments from a bundled annuity product paying benefits in these health states. In particular, under the New Annuity Plan annuity product offered in Malaysia, there is cash value benefit payment either in the event of total and permanent disability or diagnosis of critical illness. We also conduct the analysis using a health dependent utility function to study the impact of the utility penalty for a health state worse than perfect health on the AEW measure. The extended AEW framework used in this paper is explained further below.

3.3.2 Methodology

In this section, we explain in detail the methodology for the annuity's value for money analysis. The traditional Annuity Equivalent Wealth (AEW) method is extended to incorporate several health states. This extended Annuity Equivalent Wealth model comprises of two models, the life cycle and the annuity model. Section 3.3.3 and 3.3.4 describe each of the models respectively.

We employ the utility of health and consumption in the form of logarithmic preferences in the expected utility maximisation problem of consumers. Similar to the existing AEW method, the consumer aims to maximise his utility over his lifetime subject to his budget constraint. This budget constraint, notably, is determined according to the condition of accessibility to an annuity market. We maximise the consumer's expected utility given a condition with access to an annuity and insurance markets first. Then we seek the level of wealth required under a condition of no access to annuity and insurance markets — and this level of wealth should produce the same expected utility as in the first calculation. The AEW value is then calculated as a ratio of the wealth level required under the second condition over the market price of the bundled annuity product. Below is the utility maximisation problem of a consumer in our model according to the condition of access or not to annuity and insurance markets.

By way of introduction, consider a non-altruistic representative individual who is assumed to maximise his expected utility function by following an optimal nominal consumption path, c_t. We denote his expected utility function as $EU(\mathbf{H}, \mathbf{C})$.

$$EU(\mathbf{H}, \mathbf{C}) = \max_{\{c_t^k\}} \sum_{t=y-x}^{T-x} \frac{[\sum_{k=1}^{n} {}_tP_x^{ik} U(h_t^k, c_t^k)]}{(1+\rho)^t} \tag{3.3.2}$$

where

$$\begin{aligned}
c_t^k &\equiv \text{the consumption in current health state, } k\text{, at time } t \\
h_t^k &\equiv \text{the health index of utility for current health state, } k\text{,at time } t \\
U(h_t^k, c_t^k) &\equiv \text{the utility function of health and consumption} \\
{}_tP_x^{ik} &\equiv \text{the probability that a person aged } x \text{ who is in current health} \\
& \text{ state } i\text{, survives and is in health state } k \text{ at age } x+t \\
T &\equiv \text{the maximum possible lifespan} \\
x &\equiv \text{the person's age at the time of purchase} \\
y &\equiv \text{the person's age when the annuity payment commences} \\
\rho &\equiv \text{the rate of time preference}
\end{aligned}$$

We generalise this problem to the case of altruistic agents below. The budget constraint depends on two conditions: whether annuities are available or not.

First condition: with an annuity the budget constraint is as follows.

$$\begin{aligned}
W_0 &= 0 \\
\sum_{t=y-x}^{T-x} R^{-t}[\sum_k ({}_tP_x^{ik} c_t^k)] &= W_0 + \sum_{t=y-x}^{T-x} R^{-t}[\sum_k ({}_tP_x^{ik} M_t^k)] \\
W_{t+1} &= W_t R + \sum_k {}_1P_{x+t}^{ik} M_t^k - \sum_k {}_1P_{x+t}^{ik} c_t^k
\end{aligned} \tag{3.3.3}$$

where the consumer is given an initial wealth of W_0, and a nominal interest rate consisting of a real interest rate of r and an inflation rate of π, as the interest rate factor $R = (1+r)(1+\pi)$. W_t is denoted as the level of wealth at time t. Note that the initial wealth after annuitisation is set to be 0 as we assume that the consumer consumed all of his initial wealth to purchase the annuity with a premium P and in return, receives M_t^k of the benefit from either the annuity stream payment, or the lump sum cash value payment in the event of death, total and permanent disability or diagnosis of critical illnesses[10].

Since the benefit payment depends on the health state of the consumer where a lump sum cash value payment is paid in the event of total and permanent disability or diagnosis of critical illnesses, a separate optimisation problem has to be solved which only allows the consumer to have access to the cash value benefit payment — and not an income stream — under such health states. Thus, the benefit payment M_t^k may also vary depending on the health state k associated with the benefit payment. We explain further below the full consumer optimisation problem which allows for all benefit payments provided under the New Annuity Plan.

The utility maximisation problem in our paper involved an extended life-cycle model where more than one health state can occur represents by the state $k = 1$ (full health), $k = 2$ (very mild degree of impairment to full health), $k = 3$ (mild degree of impairment to full health), $k = 4$ (moderate degree of impairment to full health) and $k = 5$ (severe degree of impairment to full health). We also allow for a parallel world for people with total and permanent disability (TPD) by the state $k = \bar{2}$ (TPD and very mild degree of impairment), $k = \bar{3}$ (TPD and mild degree of impairment), $k = \bar{4}$ (TPD and moderate degree of impairment) and $k = \bar{5}$ (TPD and severe degree of impairment). We assume that the severe state is for people with critical illness (CI)[11]. The dead state is represented by the state 6[12].

The New Annuity Plan is a deferred annuity product with additional benefits in the form of a lump sum cash value payments paid in the event of death, TPD or critical illness whichever occurs first. Hence, we have to first solve the optimisation problem of a consumer given all possible benefits that are payable during the deferral period. Since this annuity product also provides a death benefit payment in the form of a lump sum cash value, we assume that a consumer has a simple bequest function of the same form as the utility function of consumption with a weighting parameter of one, thus allowing for the utility gain from providing this death benefit. The death benefit will only be paid if the annuitant has never claimed a cash value payment.

The maximisation problem of a consumer during the deferral period is shown

[10]A limited optimisation approach has been adopted to avoid further complication in an already difficult model, where we simply assume that the consumer fully annuitised his initial wealth.

[11]This assumption is necessary so as to match the model with the benefits paid under the bundled annuity product. We have examined both our sources for the transition probability to the severe state and probability of critical illness and find that this is not an unreasonable assumption. Discussion of that data sources is below.

[12]Refer to Figure 3.3.1 for a complete life-cycle model of consumer used in this paper.

below. Here, the consumer is expected to maximise his utility by following an optimal consumption path only for health states associated with the benefit payable during the deferral period since this benefit is only paid if a consumer enters into either a severe state (5 and $\bar{5}$) associated with a critical illness condition, or the TPD states ($\bar{2}, \bar{3}, \bar{4}$). Since we assume that the health index of utility does not depend on time in our model, we drop the subscript t for the health index of utility for the rest of the formulation below.

- The consumer optimisation problem during the deferral period is as follows.

$$\begin{aligned} EU(\mathbf{h},\mathbf{C}) &= \max_{\{c_t^k\}} \sum_{t=1}^{T-x} \frac{[\sum_k h^k \cdot \log(c_t^k)_t P_x^{1k}]}{(1+\rho)^t} \qquad (3.3.4) \\ &\quad +\{_{t-1}P_x^{11}(_1P_{x+t-1}^{16}) +_{t-1} P_x^{12}(_1P_{x+t-1}^{26}) +_{t-1} P_x^{13}(_1P_{x+t-1}^{36}) \\ &\quad +_{t-1}P_x^{14}(_1P_{x+t-1}^{46})\}\log(CV_t)\cdot D_t \\ &\quad +\{(_{t-1}P_x^{15} +_{t-1} P_x^{1\bar{5}} +_{t-1} P_x^{1\bar{4}} +_{t-1} P_x^{1\bar{3}} \\ &\quad +_{t-1}P_x^{1\bar{2}})(1.1)_1P_{x+t-1}^{16}\}\log(W_{t+1}) \end{aligned}$$

where k is for all possible values of health state associated with the benefit payment during the deferral period, $k = (5, \bar{5}, \bar{2}, \bar{3}, \bar{4})$ and CV_t is the cash value benefit payment at time t.

The utility function of health and consumption takes a form of $U(h^k, c_t^k) = h^k \cdot \log(c_t^k)$. The $\log(CV_t)$ and the $\log(W_{t+1})$ are utility from bequest functions[13]. D_t is a dummy variable which can take a value of 0 or 1. We set $D_t = 1$ for the deferral period of $(1 \leq t \leq y-1-x)$ and $D_t = 0$ for $t \geq y-x$, as the cash value for death benefit during the post-deferral period will be accounted for in the post-deferral period optimisation problem Eq. (3.3.6) below. The transition probability from the severe state and the TPD states $k = (5, \bar{5}, \bar{2}, \bar{3}, \bar{4})$ to the dead state 6, is assumed to be 10% higher than the annuitant mortality rates[14]. In the presence of insurance markets, a consumer may benefit from the risk pooling mechanism and allocate different consumption levels to different health states. Thus, c_t^k in the above optimisation problem Eq. (3.3.4) and the budget constraint

[13] The bequest function depends on the health state of consumer before he dies. He can only receive the cash value payment for death benefit if he has never claimed for the cash value payment, that is, he stays in health states other than TPD and CI before he dies. This explains the bequest function with a cash value payment CV_t, as an argument. Since this is a deferred annuity product, during the deferral period, the consumer may only hold positive wealth if the consumer claims for the cash value benefit payment in the event of TPD or CI. Thus, if the consumer is in the TPD or severe states before he dies, he will leave his wealth as a bequest. This explains the bequest function with wealth W_{t+1}, as an argument.

[14] This is the assumption used in Gatenby (1991), Haberman, Olivieri, and Pitacco (1997) and Olivieri and Pitacco (2001).

in Eq. (3.3.3) may vary according to state k. Referring to the budget constraint, during the deferral period a consumer can only receive the additional benefit, thus $M_t^k = CV_t$ for states $k = (5, \bar{5}, \bar{2}, \bar{3}, \bar{4})$ and $M_t^k = 0$ for all other states. The probability of receiving benefit is also adjusted where ${}_tP_x^{TPD+CI}$ is the transition probability of the first time entry to states associated with TPD and CI at time t. The above budget constraint in Eq. (3.3.3) is adjusted for time t starting at $t = 1$ (as benefit payment of cash value may be payable a year after the annuity is purchased). The upper value of t for the benefit payment M_t^k is also adjusted to $t = (y-1-x)$ where y is the deferral period (allowing for the benefit payments during the deferral period only). As $W_0 = 0$, we drop the notation W_0 in the formulation below.

- With access to annuity markets, the budget constraint of the consumer during the deferral period is as follows.

$$\begin{aligned} \sum_{t=1}^{T-x} R^{-t}[\sum_k ({}_tP_x^{1k} c_t^k)] &= \sum_{t=1}^{y-1-x} R^{-t}({}_tP_x^{TPD+CI} \cdot CV_t) \\ W_{t+1} &= W_t R + {}_1P_{x+t}^{TPD+CI} \cdot CV_t - \sum_k {}_1P_{x+t}^{ik} c_t^k \end{aligned} \tag{3.3.5}$$

For the post-deferral period, the optimisation problem is done in three steps. We solve a similar form of optimisation problem as to Eq. (3.3.4), first, — given a stream of annuity income payments payable while in health states other than TPD and CI. This income is accessible in all health states since receiving the income while a consumer is healthy now does not prevent him allocating the income to later period consumption in the case if he becomes TPD or critically ill. Thus, the above optimisation problem has to be adjusted to allow for all health states that may occur over a consumer's life-cycle. For a deferral period of y years, the first post-deferral optimisation problem is given as:

- The first step of the consumer optimisation problem during the post-deferral period is as follows.

$$\begin{aligned} EU(\mathbf{h}, \mathbf{C}) = \max_{\{c_t^k\}} & \sum_{t=y-x}^{T-x} \frac{[\sum_k h^k \cdot \log(c_t^k) {}_tP_x^{1k}]}{(1+\rho)^t} \\ & + \{{}_{t-1}P_x^{11}({}_1P_{x+t-1}^{16}) +_{t-1} P_x^{12}({}_1P_{x+t-1}^{26}) +_{t-1} P_x^{13}({}_1P_{x+t-1}^{36}) \\ & +_{t-1}P_x^{14}({}_1P_{x+t-1}^{46})\} \log(W_{t+1} + CV_t) \\ & + \{({}_{t-1}P_x^{15} +_{t-1} P_x^{1\bar{5}} +_{t-1} P_x^{1\bar{4}} +_{t-1} P_x^{1\bar{3}} \\ & +_{t-1}P_x^{1\bar{2}})(1.1) {}_1P_{x+t-1}^{16}\} \log(W_{t+1}) \end{aligned} \tag{3.3.6}$$

where $k = (1,2,3,4,5,\bar{2},\bar{3},\bar{4},\bar{5})$. As the annuity payment has commences during the post-deferral period, the bequest function is also adjusted for health states other than TPD and CI — such that, if the consumer stays in health states other than TPD or CI before he dies, the consumer may hold positive wealth and receive the cash value payment for death benefit when he dies. Thus, $(W_{t+1}+CV_t)$ becomes the argument of the bequest function of the consumer under such states. The budget constraint Eq. (3.3.3) is now adjusted such that M_t^k is only payable for states $k = (1,2,3,4)$. Thus, $M_t^k = A_t$ for states $k = (1,2,3,4)$ where A_t is the annuity income payment at time t and $M_t^k = 0$ for all other states. The probability of receiving benefit is adjusted to ${}_tP_x^{non-(TPD+CI)}$ which is a transition probabilty of moving into states other than TPD and CI at time t. The annuity income payment is only paid up to age 70.

- With access to annuity markets, the budget constraint of the consumer for the first post-deferral optimisation problem is as follows.

$$\begin{aligned} \sum_{t=y-x}^{T-x} R^{-t}[\sum_k ({}_tP_x^{1k} c_t^k)] &= \sum_{t=y-x}^{70-x} R^{-t}({}_tP_x^{non-(TPD+CI)} \cdot A_t) \quad (3.3.7) \\ W_{t+1} &= W_t R + {}_1P_{x+t}^{non-(TPD+CI)} \cdot A_t \\ &\quad - \sum_k {}_1P_{x+t}^{ik} c_t^k \end{aligned}$$

The second step in solving for the post-deferral period is where an optimisation problem is solved given the cash value benefit payable during the post deferral period where it is only accessible by certain states associated with that cash value payment, that is TPD and CI states for post-deferral period up to age 65[15] . Thus, the second post-deferral period optimisation problem up to age 65 has the same form as the deferral period but without a cash value payment for death as this benefit has been included in the first post-deferral period problem.

- The second step of the consumer optimisation problem during the post-deferral period is as follows.

[15]As the TPD benefit is only payable up to age 65, the second post-deferral optimisation problem is solved allowing for the cash value benefit payable for time $t = y - x$ up to $t = 65 - x$. The third post-deferral optimisation problem is solved by allowing for benefit payable post age 65 period which is explained further below.

The coefficient of the bequest function in Eq. (3.3.6) needs to be adjusted for ages 66 to 70 to allow for the TPD benefit payment structure that stops at age 65. It is possible for a consumer to receive a cash value from the death benefit if he has never claimed for the cash value benefit up to age 65 (stays in health states other than TPD and CI), then moves into the TPD state post age 65 and dies.

$$\begin{aligned} EU(\mathbf{h},\mathbf{C}) &= \max_{\{c_t^k\}} \sum_{t=y-x}^{T-x} \frac{[\sum_k h^k \cdot \log(c_t^k)_t P_x^{1k}]}{(1+\rho)^t} \\ &\quad +\{({}_{t-1}P_x^{15} +{}_{t-1} P_x^{1\bar{5}} +{}_{t-1} P_x^{1\bar{4}} +{}_{t-1} P_x^{1\bar{3}} \\ &\quad +{}_{t-1}P_x^{1\bar{2}})(1.1)_1P_{x+t-1}^{16}\} \log(W_{t+1}) \end{aligned} \tag{3.3.8}$$

where $k = (5, \bar{5}, \bar{2}, \bar{3}, \bar{4})$. The budget constraint is now adjusted such that, $M_t^k = CV_t$ and the probability of receiving benefit of ${}_tP_x^{TPD+CI}$.

- With access to annuity markets, the budget constraint of the consumer for the second post-deferral optimisation problem is as follows.

$$\begin{aligned} \sum_{t=y-x}^{T-x} R^{-t}[\sum_k({}_tP_x^{1k}c_t^k)] &= \sum_{t=y-x}^{65-x} R^{-t}({}_tP_x^{TPD+CI} \cdot CV_t) \\ W_{t+1} &= W_tR + {}_1P_{x+t}^{TPD+CI} \cdot CV_t - \sum_k {}_1P_{x+t}^{ik}c_t^k \end{aligned} \tag{3.3.9}$$

Lastly, the third post-deferral period optimisation problem only applies to ages greater than 65 starting at $t = 66 - x$ — where there is a cash value benefit payment for states associated with critical illness (CI) up to age 70. Similar with the second post-deferral period problem, there is no cash value payment associated with death here as it has been included in the first post-deferral period problem.

- The final step of the consumer optimisation problem during the post-deferral period is as follows.

$$\begin{aligned} EU(\mathbf{h},\mathbf{C}) &= \max_{\{c_t^k\}} \sum_{t=66-x}^{T-x} \frac{[\sum_k h^k \cdot \log(c_t^k)_t P_x^{1k}]}{(1+\rho)^t} \\ &\quad +\{({}_{t-1}P_x^{15} +{}_{t-1} P_x^{1\bar{5}})(1.1)_1P_{x+t-1}^{16}\} \log(W_{t+1}) \end{aligned} \tag{3.3.10}$$

where $k = (5, \bar{5})$. For post age 65 period the benefit is given as $M_t^k = CV_t$ and the probability of receiving benefit of ${}_tP_x^{CI}$ which is the transition probability of the first time entry to states associated with CI at time t.

- With access to annuity markets, the budget constraint of the consumer for the final post-deferral optimisation problem is as follows.

$$\begin{aligned} \sum_{t=66-x}^{T-x} R^{-t}[\sum_k({}_tP_x^{1k}c_t^k)] &= \sum_{t=66-x}^{70-x} R^{-t}({}_tP_x^{CI} \cdot CV_t) \\ W_{t+1} &= W_tR + {}_1P_{x+t}^{CI} \cdot CV_t - \sum_k {}_1P_{x+t}^{ik}c_t^k \end{aligned} \tag{3.3.11}$$

Finally, we add all expected utility for both the deferral and the post-deferral period calculated at the purchase time according to the age of purchase of the annuity buyer.

Second condition: In the case where annuity and insurance markets do not exist, a consumer has to allocate a consumption path of $c_t^k = c_t$, ∇k, where the same consumption level c_t is applicable for all health states — as there is no risk pooling mechanism. Thus, a consumer has to allocate the same level of consumption regardless of his health state.

Without an annuity the budget constraint is as follows.

$$\begin{aligned} W_0 &= w \\ \sum_{t=y-x}^{T-x} R^{-t} c_t &= W_0 \\ W_{t+1} &= W_t R - c_t \end{aligned} \tag{3.3.12}$$

where w is the consumer's initial non-annuitised wealth at time 0.

Since consumption during the deferral period is only funded through the additional benefit of a cash value payment in the event of TPD or CI, in the absence of an insurance and annuity market, we only solve for the optimal consumption path of the post-deferral period. We believe that this is a fair comparison as a deferred annuity product is purchased to fund consumption for the post-deferral period. Thus, without the cash value payment during the deferral period, there will be no consumption during the deferral period[16]. The same optimisation problem as in Eq. (3.3.6) is used here with only W_{t+1} as an argument of the bequest function as there is no cash value payment under this condition.

We seek the level of non-annuitised wealth w that produces the same expected utility calculated under the first condition where there is access to annuity and insurance markets. The AEW is then calculated as the ratio of the non-annuitised wealth w over the annuitised wealth P. In this paper, the denominator of AEW, P is calculated as the premium charged for the annuity product. This can be the fair premium (with no loadings) or the market premium (with loadings) as quoted by insurer.

We solve this extended AEW calculation numerically using the GAMS software (Brook, Kendrick, and Meeraus, 1988). Our approach to determining the final AEW value proceeds in "layers". Firstly, we solve just for the annuity element

[16]In reality, this deferral period's consumption has to be funded through other source of income. In this paper, we only consider for solving the consumer's optimal consumption path given the benefits received from the bundled annuity product.

of the product, in the post-deferral period for both with and without access to annuity markets. Then, under the condition of access to annuity markets, we add other additional benefits and the cost paid for these benefits one-by-one for the post-deferral period. Lastly, we add the benefits and the cost of benefits for the deferral period. The changes of AEW values for this step-by-step benefit addition is shown in Appendix 3.A.1 for all categories.

3.3.3 State Worse than Death (SWD)

Since we conduct the AEW analysis using both the health independent and health dependent utility function in this paper, it is important to discuss the role of the health index of utility in solving this optimisation problem. This section explains our approach in dealing with state worse than death (SWD)[17].

If a positive health index of utility is applied, there is no issue to solve the problem. In a survey by Levy and Nir (2012), health states worse than death (SWD) are ignored. Here, we discuss the implication of the negative health index of utility to their survey method of finding the utility function that describes the health and consumption tradeoff. Let us consider that c_t is in a form of minimum consumption level required for existence as in Levy and Nir (2012). In our analysis, without loss of generality we assume the minimum consumption required for existence is $c_t = 1$. According to Levy and Nir (2012) survey, the utility of health and consumption can be written in the form of utility of consumption with medical cost (as a proportion of full consumption) given in Eq. (3.3.13). The left hand side of the equation can be interpreted as the utility without treatment whereas the right hand side of the equation can be interpreted as the utility with treatment but the consumption level after treatment is $c_t(1 - x_t)$.

$$h_t \cdot \log(c_t) = 1 \cdot \log(c_t(1 - x_t)) \tag{3.3.13}$$

where x_t is the maximum proportion of full consumption that the consumer is willing to give up to pay for the medical cost to be cured to the perfect health state, $h_t = 1$. Assuming that the equality holds[18], we can find the amount of x_t that the consumer is willing to give up depending on his current health state using this formula:

[17] The state worse than death exists in the EQ-5D study for UK population MVH (1995), but not in the EQ-5D study for Malaysian population Md Yusof, Goh, and Azmi (2012).

[18] For a range of health index of utility $0 \leq h_t \leq 1$, Levy and Nir (2012) shows that the equality holds for all hypothetical consumption level considered in the survey, subject to small errors.

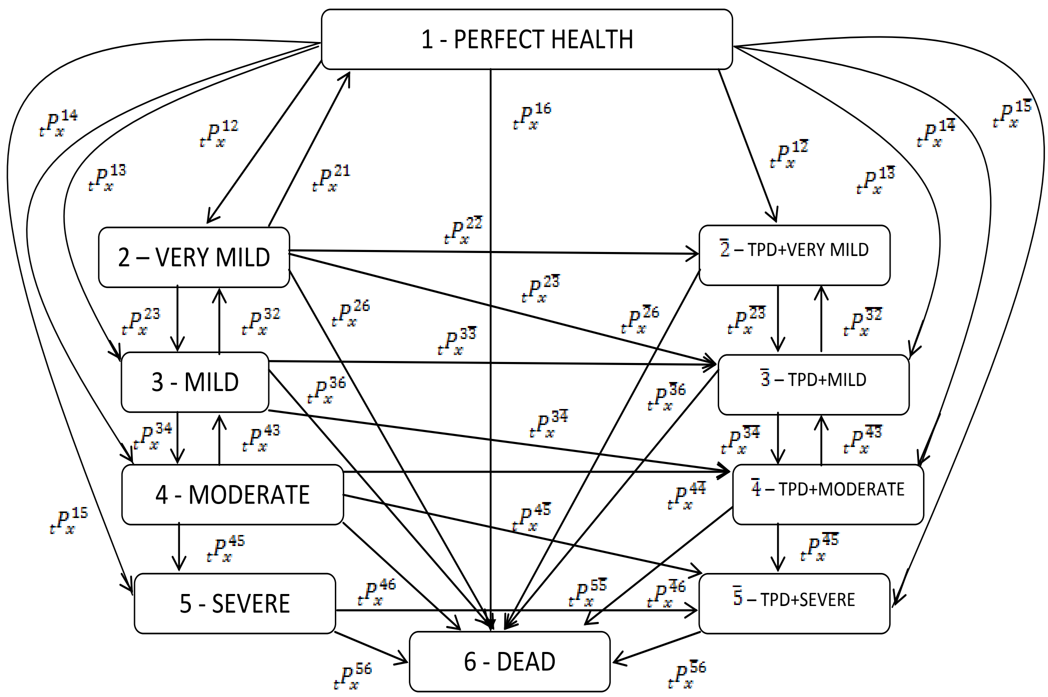
$$x_t = 1 - \frac{1}{(c_t)^{1-h_t}} \tag{3.3.14}$$

According to this formula, if the state worse than death does exist, substituting the negative health index of utility into the formula will give us a consumption level after treatment of $c_t(1 - x_t) < 1$ for $c_t > 1$. This would indicate that such state is so miserable that a person is willing to give up a high proportion of his consumption, to be cured eventhough he is left with less than the minimum consumption level required for existence after treatment $c_t(1 - x_t) < 1$. One can think of a person who does not value his consumption anymore under such state since he would rather die than living in his current health state. This would explains the negative utility resulting from having less than the minimum survival consumption after treatment. Since their study do not consider for the negative health index of utility, we argue that the equality might not hold for the health states worse than death (SWD)[19]. Our approach for dealing with states worse than death (SWD) lies below.

We apply the truncation method where the negative health index of utility is to be truncated to zero, such that, if the SWD state occurs, the consumer is restricted to only give up his consumption until he is left with the minimum consumption level required to survive after the treatment in order to get cured from that miserable health state. Bleichrodt and Quiggin (1999) also consider for states better than death to study the consistency of the life-cycle preferences for health and consumption with cost-effectiveness analysis. They cited a research study by Rosen (1988) which explores further the case where consumption falls below the subsistence level. If such state exists, the consumer will convexify his utility preferences by randomising between death (zero utility) or life utility at a consumption level higher than subsistence level if someone survives. In this case, if we allow for the negative health index of utility, the utility of death (zero utility) is always preferred to the utility of survival after treatment, as the consumption level after treatment is $c_t(1 - x_t) < 1$. Thus, our approach of truncation method is not unreasonable.

[19]There is an ongoing research by health economists such as Lamers (2007), Devlin et al. (2013), and Patrick et al. (1994) in handling the issue of health states worse than death (SWD). None of these study the implication of SWD on the life-cycle preferences for health and consumption. Clearly, further research is required in this topic of health index utility for states worse than death.

Figure 3.3.1: The Annuitant Life Cycle Model



3.3.4 The Life Cycle Model

The first part of the Annuity Equivalent Wealth method involves the expected utility maximisation problem of a consumer subject to his budget constraint given two conditions — with or without access to purchase an annuity. This multi-state life cycle model is developed to study the implication of having a more realistic life cycle model which reflects the cashflows associated with a bundled annuity product, like the New Annuity Plan. Also, if a consumer's utility of consumption depends on his current health state, allowing for more than one health state in the model could have an impact on our annuity valuation analysis. In this paper, we apply a discrete time multi state Markovian framework to the life cycle model of a consumer which consist of five health states and a dead state. Figure 3.3.1 illustrates the model further.

We use a the time-inhomogeneous Markov process in our modeling, where the transition probabilities are is age related and vary over time. For a person aged x at the time of purchasing the annuity, there exists a set of transition matrices consisting of the transition probabilities from state i to j at each time $t = 1, 2, 3, ..., T - x$, where T is the maximum lifetime of the consumer. Note here that t in our model is in units of years. The set of transition matrices for our model are shown in Figure 3.3.2, Figure 3.3.3 and Figure 3.3.4.

Figure 3.3.2: The transition matrix for $t = 1$

i\j	1	2	3	4	5	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	6
1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	$P_{1\bar{2}}$	$P_{1\bar{3}}$	$P_{1\bar{4}}$	$P_{1\bar{5}}$	P_{16}
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
$\bar{2}$	0	0	0	0	0	0	0	0	0	0
$\bar{3}$	0	0	0	0	0	0	0	0	0	0
$\bar{4}$	0	0	0	0	0	0	0	0	0	0
$\bar{5}$	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0

Figure 3.3.3: The transition matrix for $t = 2, 3, ..., T - x - 1$

i\j	1	2	3	4	5	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	6
1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	$P_{1\bar{2}}$	$P_{1\bar{3}}$	$P_{1\bar{4}}$	$P_{1\bar{5}}$	P_{16}
2	P_{21}	P_{22}	P_{23}	0	0	$P_{2\bar{2}}$	$P_{2\bar{3}}$	0	0	P_{26}
3	0	P_{32}	P_{33}	P_{34}	0	0	$P_{3\bar{3}}$	$P_{3\bar{4}}$	0	P_{36}
4	0	0	P_{43}	P_{44}	P_{45}	0	0	$P_{4\bar{4}}$	$P_{4\bar{5}}$	P_{46}
5	0	0	0	0	P_{55}	0	0	0	$P_{5\bar{5}}$	P_{56}
$\bar{2}$	0	0	0	0	0	$P_{\bar{2}\bar{2}}$	$P_{\bar{2}\bar{3}}$	0	0	$P_{\bar{2}6}$
$\bar{3}$	0	0	0	0	0	$P_{\bar{3}\bar{2}}$	$P_{\bar{3}\bar{3}}$	$P_{\bar{3}\bar{4}}$	0	$P_{\bar{3}6}$
$\bar{4}$	0	0	0	0	0	0	$P_{\bar{4}\bar{3}}$	$P_{\bar{4}\bar{4}}$	$P_{\bar{4}\bar{5}}$	$P_{\bar{4}6}$
$\bar{5}$	0	0	0	0	0	0	0	0	$P_{\bar{5}\bar{5}}$	$P_{\bar{5}6}$
6	0	0	0	0	0	0	0	0	0	P_{66}

Figure 3.3.4: The transition matrix for $t = T - x$

i\j	1	2	3	4	5	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	6
1	0	0	0	0	0	0	0	0	0	P_{16}
2	0	0	0	0	0	0	0	0	0	P_{26}
3	0	0	0	0	0	0	0	0	0	P_{36}
4	0	0	0	0	0	0	0	0	0	P_{46}
5	0	0	0	0	0	0	0	0	0	P_{56}
$\bar{2}$	0	0	0	0	0	0	0	0	0	$P_{\bar{2}6}$
$\bar{3}$	0	0	0	0	0	0	0	0	0	$P_{\bar{3}6}$
$\bar{4}$	0	0	0	0	0	0	0	0	0	$P_{\bar{4}6}$
$\bar{5}$	0	0	0	0	0	0	0	0	0	$P_{\bar{5}6}$
6	0	0	0	0	0	0	0	0	0	P_{66}

Given these transition matrices, we solve for ${}_tP_x^{ij}$ for each time t using matrix multiplication. For instance, to solve for ${}_2P_x^{ij}$, the transition matrix for $t = 1$ will be multiplied by the transition matrix for $t = 2$; for ${}_3P_x^{ij}$, we need to multiply ${}_2P_x^{ij}$ by another transition matrix, for $t = 3$, and so on. These multiple matrix multiplication tasks can easily being done using the R software package (R Core Team, 2013), where we use the three dimensional array function as we have three vectors in each transition matrix — a vector for state i, state j and age.

The idea to develop a life cycle model with five health states follows the same concept used in MVH (1995) where respondents were asked to value several EQ-5D health states[20] which have been categorised according to level of health: namely full health together with very mild, mild, moderate and severe impairments to full health. Most of the EQ-5D health states that appear under each category in MVH (1995) match with the list of EQ-5D health states under each category in our model. However, there have been few adjustments to the list of health states for categories very mild, mild, moderate and severe to ensure consistency with the health index of utility values for each category. For instance, there are three EQ-5D health states for the mild category in the MVH (1995) study where the health index of utility given in this study for these states turns out to be too low compared to other EQ-5D health states in that same category. This is not suprising as the categorisation process is done before the respondent values those EQ-5D health states. Since it is important in our model to categorise the EQ-5D health states consistently with their health index of utility value as this will be utilised in the analysis later, we have decided to adjust the category of several inconsistent EQ-5D health states in the MVH (1995) study so that we have an appropriately consistent range of the health index of utility values for each category. We present the adjusted list of EQ-5D health states from the MVH (1995) that we apply in our model in Appendix 3.A.2.

As mentioned in Section 3.2.2, the health index of utility, is obtained from an EQ-5D survey by Md Yusof, Goh, and Azmi (2012). The EQ-5D descriptive system consists of five dimensions: e.i. 1) mobility, 2) self-care, 3) usual activities, 4) pain/discomfort, 5) anxiety/depression. Each of this dimensions has three levels of perceived problem: either no problem, some problem or severe problem. The EQ-5D health state is then defined as a combination of all five dimensions with each combination placed into a classification. For instance, combination 11111 is defined as the full health state where a person has no problem for each

[20]Note that each health state in our life cycle model consists of a list of EQ-5D health states determined using the EQ-5D descriptive system.

dimension. State 33333 is defined as the worst possible EQ-5D health state, where a person indicates severe problems for all dimensions specified in the EQ-5D survey instrument.

We seek for a life cycle model that best reflects the health outcome data that we obtain from the EQ-5D related survey. Since the EQ-5D value set for the Malaysian population does not contain information on prevalence rates associated with the self reported health status of the respondent, we have decided to use prevalence rates associated with the self reported health status for the UK population, which can be found from the Health Survey for England 2006 (A. Ali et al., 2006), hereafter HSE2006. This assumption is reasonable, as a comparison of the UK morbidity rates with the morbidity rates for the Malaysian population obtained from the Malaysian National Health and Morbidity Survey (NHMS) 2006 (Institute for Public Health, 2008), hereafter NHMS2006, reveals only small differences[21].

The prevalence rates data from the HSE 2006 are then converted to transition probabilities as shown in Figure 3.3.1 by following the actuarial method used in Gatenby (1991) to convert the disability prevalence rates to transition probabilities. The derivation for each transition probability is shown in Appendix 3.A.3. Since the HSE 2006 only contains information on the prevalence rates for each state at one point of time, we have to adjust the prevalence rates to allow for the recovery and the deterioration rates that can occur in the model. This is done by adjusting the rates obtained from the HSE 2006 so that it matches the Patient Reported Outcome Measures (PROMS) in England data consisting of the Hospital Episode Statistics England, HES (2011), HES (2012) and HES (2013), from which the recovery and the deterioration prevalence rates can be calculated. The

[21] We find that the average prevalence rate for the three major types of sickness in the UK data is quite consistent with the average prevalence rate of the same type of sickness in the Malaysian data. Those three major types of sickness for adults observed in the HSE 2006 are cardiovascular disease, with a prevalence rate of 13.6% for males and 13% for females, hypertension, with a prevalence rate of 31% for males and 28% for females, and diabetes with a prevalence rate of 5.6% for males and 4.2% for females. These figures produce an average rate of 16.73% and 15.07% for males and females respectively. On the other hand, the Malaysian NHMS 2006 morbidity data shows the prevalence rate for hypertension of 33.3% and 31%, and the prevalence rate for diabetes of 11.9% and 11.3% for males and females respectively. The survey contains detailed information for these two sickness types, but not for cardiovascular disease. We obtain the cardiovascular disease prevalence rates from the same survey but using the prevalence of the chronic illness category comprising all heart related disease, including stroke, with a rate of 1.5% and apply this rate to both genders. The average prevalence rate calculated from all three types of sickness for the Malaysian morbidity data is 15.57% for males and 14.6% for females. Due to a difference of only around 1% in the average prevalence rate for UK and Malaysian data, we have applied the same set of morbidity rates for both populations in our study.

PROMS survey only started several years back in 2009 HES (2011), thus due to limitations of data, we have combined the three years published data available from year 2009 to 2012. For simplicity, several assumptions have been made in the model. The transition from any state worse than that of perfect health is limited to one health state better or worse than the current health state of the consumer. An exception is allowed for the perfect health state where deterioration can occur from this state to any state worse than perfect health, so that the life cycle model features a more realistic consumer's life cycle. In the real world, a consumer can always move to any health state but the available survey data is very limited and does not adequately support such a complicated model. Furthermore, this model is useful to study the impact of health utility on the optimal consumption path of the consumer.

Another important assumption concerns the mortality rates applied to states worse than perfect health and totally and permanently disabled. Since the mortality rate data for these groups is sparse, we follow the assumption made in works of Gatenby (1991), Haberman, Olivieri, and Pitacco (1997) and Olivieri and Pitacco (2001) where the mortality rates for states worse than perfect health associated with disability and critical illness are assumed to be higher by 10%. In addition, the consumer who is not totally and permanently disabled and who enters the severe health state is assumed to be diagnosed with a critical illness and will also be subject to this elevated mortality rates. Thus the mortality rates for states ($\bar{2}$, $\bar{3}$, $\bar{4}$, 5, $\bar{5}$) are 110% of the annuitant mortality rates.

For simplicity, we also assume that the TPD and the severe states are absorbing health states. Thus a consumer may not recover from that state. Upon entry into a severe state the consumer can only stay or move to the dead state[22]. On the other hand, for the TPD states (excepting severe), once entered into a TPD state, a consumer may only make a transition to other TPD states or to the dead state.

3.3.5 The Bundled Annuity Model

The bundled annuity product in our analysis, which we refer to as the New Annuity Plan, also provides additional benefits in the form of a lump sum cash value payment in the event of death, total permanent disability and diagnosis of critical illnesses (except for Angioplasty or other invasive treatments for Major Coronary Artery Disease), in addition to the annuity stream of income payment. The bundled annuity model is developed to suit all benefits provided under the product

[22] Here, we follow models like Olivieri and Pitacco (2001) and Ozkok et al. (2014)

Figure 3.3.5: The Annuity Model

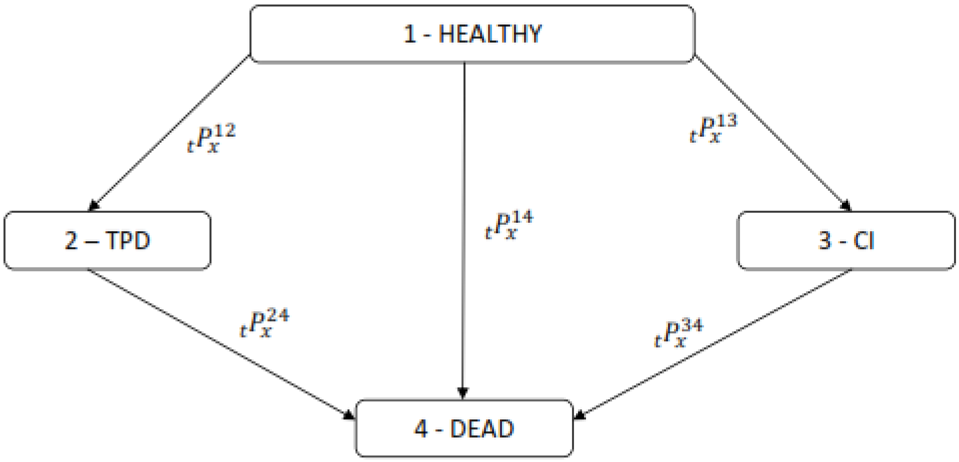


Figure 3.3.6: The transition matrix for $t = 1$

i\j	1	2	3	4
1	P_{11}	P_{12}	P_{13}	P_{14}
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

and so we allow for the healthy state, the total and permanent disability (TPD) state, the critical illness (CI) state and the dead state. Figure 3.3.5 illustrates the annuity model in our analysis further.

We apply a similar procedure here to calculate $_tP_x^{ij}$ as in the life cycle model in subsection 3.3.1, where a set of transition matrices is produced for each time t as shown in Figure 3.3.6, Figure 3.3.7 and Figure 3.3.8.

Figure 3.3.7: The transition matrix for $t = 2, 3, ..., T - x - 1$

i\j	1	2	3	4
1	P_{11}	P_{12}	P_{13}	P_{14}
2	0	P_{22}	0	P_{24}
3	0	0	P_{33}	P_{34}
4	0	0	0	P_{44}

Figure 3.3.8: The transition matrix $t = T - x$

i\j	1	2	3	4
1	0	0	0	P_{14}
2	0	0	0	P_{24}
3	0	0	0	P_{34}
4	0	0	0	P_{44}

Referring to the annuity model above, the expected present value of income received from the bundled annuity product purchased at age x is:

$$EPDV_x = \sum_{t=1}^{\omega-x} R^t [{}_{t-1}P_x^{\bar{1}\bar{1}} P_x^{11} A_t +_{t-1} P_x^{\bar{1}\bar{1}} P_x^{12} CV_t +_{t-1} P_x^{\bar{1}\bar{1}} P_t^{13} CV_t +_{t-1} P_x^{\bar{1}\bar{1}} P_x^{14} CV_t] \tag{3.3.15}$$

where A_t is the yearly annuity stream of income, CV_t is the lump sum cash value payment payable upon death, total permanent disability or diagnosis of critical illnesses, whichever occurs first, and ${}_{t-1}P_x^{\bar{i}\bar{i}}$ is the transition probability of a person age x, stays in state i for $t-1$ years. This expected present value formula will be used to find the Money's Worth Ratio of annuity, which is calculated as $EPDV_x/Premium_x$ — the ratio of an actuarially fair premium ignoring expenses over the market premium with loadings charged by the insurance company.

The prevalence rate for TPD is obtained from the disability data in the Malaysian National Health and Morbidity Survey 2006 (Institute for Public Health, 2008) which contains information on all types of disability prevalence rates for the Malaysian population. Since we only require the prevalence rate for total and permanent disability, we apply the percentage of TPD claims out of total disability claims found from Malaysia's Social Security Organisation Annual Report 2010 (Social Security Organisation, 2010) to the NHMS 2006 total disability prevalence rate.

For the critical illness prevalence rate, the best data available that we can find is – Gen Re's Dread Disease Survey 2004–2008 (Lu and Droste, 2012). In this survey, a number of dread disease types have been identified as the major cause of claims for insurer's dread disease cover, with cancer the leading cause of all. The percentage of cancer claims out of total dread disease claims in Malaysia is shown in the report. We use the cancer prevalence rate obtained from the Malaysia Cancer Statistics 2003–2005 (Lim, Rampal, and Halimah, 2008) and apply the percentage of cancer claims out of total dread disease claims to this cancer prevalence rate to estimate the total prevalence rate of all dread diseases in Malaysia.

We assume that both the TPD and critical illness prevalence rates are stationary over time in our analysis — further data would be required to study the trend of TPD and critical illness prevalence rates of the Malaysian population. Both of these prevalence rates are also be converted to transition probabilities using the actuarial method as in Gatenby (1991). The derivation for each transition probability is shown in Appendix 3.A.4.

3.4 Findings

In this section, we present both results obtained using the health independent and health dependent utility function for the AEW analysis. For the health independence case, the consumer is indifferent between a unit of consumption in the full health state and a unit of consumption in other states worse than full health — that is, we apply $h = 1$ for all health states. Conversely, for the health dependence case, the health index of utility decreases as health deteriorates. As we mentioned in Section 3.2.2, the set of the health index of utility for the Malaysian population can be obtained from the EQ-5D survey by Md Yusof, Goh, and Azmi (2012).

Ideally, the set of the health index of utility should be given on a country basis as the elicited health index of utility may differ according to country. For example, we find a significant differences when we compare the health index of utility in the Malaysian population study by Md Yusof, Goh, and Azmi (2012) to the health index of utility in the UK population study by the MVH (1995) even though the method used is similar for both studies (Szende, Janssen, and Cabasés, 2014). For the exact same health state in the severe category, the health index of utility elicited from the UK population study is a lot lower than the health index of utility elicited from the Malaysian population study. This implies that the UK population's perception towards the severe health state is so poor that they are willing to give up more of their lifetime to be in the full health state. The set of mean health index of utility values that we apply in our model are shown in Table 3.4.1. For each health state category, we calculate the mean health index of utility (hereafter, we refer to this mean value as an h-value) according to the list of EQ-5D health combinations[23] associated with each health state category

[23] An EQ-5D health state consists of five dimensions: e.i. 1) mobility, 2) self-care, 3) usual activities, 4) pain/discomfort, 5) anxiety/depression. Each of this dimensions has three levels of perceived problem: either no problem, some problem or severe problem. The EQ-5D health state is then defined as a combination of all five dimensions with each combination placed into

Table 3.4.1: The mean health index of utility from the EQ-5D Malaysian and UK Population Studies

State	Mean health index of utility	
	Malaysia	UK
Full Health	1.0000	1.0000
Very Mild	0.8030	0.8136
Mild	0.5936	0.6573
Moderate	0.3904	0.3363
Severe	0.2017	0.0629

in our model using this formula[24]:

$$Mean\ (h^k) = \frac{\sum_{j=1}^{n} h_j^k}{n} \tag{3.4.1}$$

where

h_j^k ≡ the health index of utility elicited for the j^{th} EQ-5D health combinations in the list associated with health state category k

n ≡ the total number of EQ-5D health combinations associated to each health state category

3.4.1 The AEW and the MWR Results

We present below the results where the Malaysian health index of utility is employed[25]. Table 3.4.2 shows the AEW results of our analysis for the product

a classification. For instance, combination 11111 is defined as the full health state where a person has no problem for each dimension. State 33333 is defined as the worst possible EQ-5D health state.

[24] Refer to Appendix 3.A.2 for the full list of EQ-5D health combinations associated with each health state category in the model. The EQ-5D health combinations is categorised according to a range of health utility index values elicited for that particular state. As the health utility index used in our analysis ranges from 0 to 1 such that $h = 1$ is for the perfect health state and $h = 0$ is for the dead state, we divide the range between 0 to 1 equally to four other health states to match our life cycle model where state 2 (very mild degree of impairment) has a range of $0.75 \leq h < 1$, state 3 (mild degree of impairment) has a range of $0.5 \leq h < 0.75$, state 4 (moderate degree of impairment) has a range of $0.25 \leq h < 0.5$, and state 5 (severe degree of impairment) has a range of $0 \leq h < 0.25$. The same mean health index of utility is applied to the TPD parallel world depending on the level of health of the consumer — either very mild, mild, moderate or severe degrees of impairment. In the TPD parallel world the perfect health state is not possible.

[25] The AEW analysis in this paper also applied the same assumption for mortality rates and the term structure of interest rates as in our previous paper Asmuni and Purcal (2013) [Chapter 2 above].

categories purchasing the New Annuity Plan at age 35. Table 3.4.3 shows the AEW results of our analysis for the product categories purchasing the Plan at age 45. For a precise statement of benefits associated with different product categories, see Appendix 3.A.5. The results are calculated using the actuarially fair premium as the denominator of AEW formula, as typically done in the existing literature. We also calculate the AEW values using the market premium (including loadings for expenses and profits) charged by the insurer as the denominator — these are presented in Figure 3.4.1 and Figure 3.4.2.

In each of Table 3.4.2 and Table 3.4.3 we first calculate the AEW values for the New Annuity Plan using both the health independent and health dependent utility function where *only* the annuity element is considered. Then, below, we present AEW values which include the additional benefits (comprising the death, TPD and diagnosis of critical illness) in the analysis. The AEW values for almost all categories and both genders show an improvement when additional benefits are added to the annuity product — except when health dependent utility is used, and then only for the category female age of purchased 45, annuity products (A1) and (A2). We now explain this in more detail.

This is due to a shorter deferral period before annuity payment commences as compared to annuity products (B1) and (B2). During the deferral period, consumers may receive a cash value payment in the event of death, TPD and diagnosis of critical illnesses. Due to a shorter deferral period, the utility gain from these additional benefits is smaller in relative to the increase in the fair price of that benefit. This might be because of the effect of adding the death benefit outweighs the effect of adding the TPD and critical illness benefits. As shown in the Appendix 3.A.1[26], as we add the death benefit only on top of the annuity income, the AEW always decreases. This shows that the fair price of such a benefit is quite expensive relative to the utility gain from it[27].

In contrast, adding the TPD and critical illness benefit only on top of the annuity income always increases the AEW value. In view of this we can see that, for the case of female age 45 purchasers of (A1) and (A2), applying health dependent utility during the deferral period, with a bequest function (associated with the death benefit) with a greater weighting than the utility from health states worse than perfect health, and where these health states (TPD and critical

[26]We show in the Appendix 3.A.1 the changes of AEW value as we add the additional benefits one-by-one, starting with the benefit received during the post-deferral period, and lastly adding together the benefits that may be received during the deferral period.

[27]This result is also parameter dependent. For higher-weighted bequest functions, this result could change.

illness) have very low probabilities of occurrence, the negative impact of the death benefit will dominate the positive effect of the TPD and critical illness benefits.

The weight associated with the bequest function, the probability of death, and the weight associated with utility function for the TPD and critical illness states, the TPD and critical illness rates, are important elements that determine the changes of AEW values as we apply the health state dependent utility model. As explained previously, adding the death benefit will always decrease the AEW value whereas adding the TPD and critical illness benefits will always increase the AEW value. As a result, for the health state dependent utility model, the AEW value will either be higher or lower than the AEW value for the health state independent model depending on the combination of these weghting factors. Since the probability of death and the TPD and critical illness rates for female category are relatively low compared to male, we can see that this explains a very small changes of AEW value as we apply the health dependent utility model for female category.

Overall, given this annuity product, consumers would always be better off if they annuitise their wealth, even when we have taken into account the utility penalty associated with states worse than perfect health. An AEW value higher than one for all categories indicates that a consumer would require a higher amount of wealth in the case where an annuity is not available, to be indifferent in terms of the expected utility level to the case where an annuity is available.

Table 3.4.2: Annuity Equivalent Wealth of the New Annuity Plan (Age 35) using fair premium as the denominator

Category	Male				Female			
	A1	A2	B1	B2	A1	A2	B1	B2
with h-value	1.3026	1.3027	1.3675	1.3674	1.2296	1.2297	1.2725	1.2724
without h-value	1.2867	1.2867	1.3588	1.3587	1.2091	1.2088	1.2552	1.2553
ratio	1.0124	1.0124	1.0064	1.0064	1.0170	1.0173	1.0138	1.0136

(a) Annuity element only

Category	Male				Female			
	A1	A2	B1	B2	A1	A2	B1	B2
with h-value	1.3789	1.4035	1.6056	1.6598	1.2436	1.2533	1.3486	1.3687
without h-value	1.4681	1.5053	1.7351	1.8128	1.2432	1.2552	1.3474	1.3706
ratio	0.9392	0.9324	0.9254	0.9156	1.0003	0.9985	1.0009	0.9986

(b) Annuity with all benefits (annuity income stream, TPD, critical illness and death)

Notes: A - annuity payment commences at age 55, B - annuity payment commences at age 60, 1 - premium payment term of 10 years, 2 - premium payment term up to age 55/60 next birthday

Table 3.4.3: Annuity Equivalent Wealth of the New Annuity Plan for (Age 45) using fair premium as the denominator

Category	Male				Female			
	A1	A2	B1	B2	A1	A2	B1	B2
with h-value	1.3164	1.2803	1.3960	1.3959	1.2545	1.2546	1.3117	1.3117
without h-value	1.2960	1.2606	1.3810	1.3810	1.2311	1.2312	1.2911	1.2911
ratio	1.0157	1.0157	1.0109	1.0108	1.0190	1.0190	1.0159	1.0159

(a) Annuity element only

Category	Male				Female			
	A1	A2	B1	B2	A1	A2	B1	B2
with h-value	1.3319	1.3351	1.5029	1.5298	1.2412	1.2426	1.3385	1.3505
without h-value	1.4119	1.4164	1.6256	1.6635	1.2333	1.2350	1.3292	1.3423
ratio	0.9433	0.9426	0.9245	0.9196	1.0064	1.0062	1.0070	1.0061

(b) Annuity with all benefits (annuity income stream, TPD, critical illness and death)

Notes: A - annuity payment commences at age 55, B - annuity payment commences at age 60, 1 - premium payment term of 10 years, 2 - premium payment term up to age 55/60 next birthday

Refering to Figure 3.4.1 and Figure 3.4.2, we can see the impact of using both health dependent utility and market premium (including loadings) on the AEW values of the annuity. For the categories of males aged 35 and 45, the AEW

Table 3.4.4: The Money's Worth Ratio of the New Annuity Plan with all benefits

MWR	Male				Female			
Category	A1	A2	B1	B2	A1	A2	B1	B2
age 35	0.9570	1.0338	0.9819	1.2048	0.9665	1.0421	0.9957	1.2221
age 45	0.9358	0.9381	0.9400	0.9510	0.9415	0.9435	0.9483	0.9577

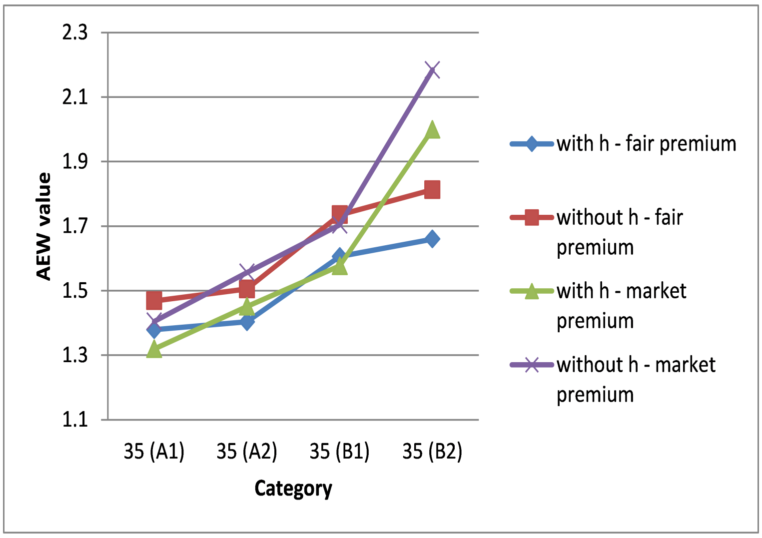
values are affected by both the health dependent utility and loadings factor. On the other hand, for the female categories, the AEW values are affected more by the loadings factor and almost not affected at all by the health dependent utility. As we have discussed earlier, this is mainly due to a lower probability of receiving the TPD and critical illness benefit for this category. In order to understand the impact of the market premium (loadings) factor to this AEW value, we present Table 3.4.4 which shows the Money's Worth Ratio (MWR) of the bundled annuity product. For certain categories (age 35 A2 and B2 for both male and female), as the MWR is greater than 1, the AEW value calculated with a market premium denominator should also be higher than the AEW calculated based on the actuarially fair premium. This is because an MWR value that is greater than one indicates that the premium charged by the insurer is less than the actuarially fair premium for that product.

3.4.2 The Implication of the Health Dependent Utility

In this section, we study the implication of using a health dependent utility function in our analysis. When we employ the health dependent utility function, we find that a consumer who values different health state differently, that is, the health index of utility is lower for states worse than perfect health, will value an annuity higher if he has access to the benefit while he is in the perfect health state and consume more in that state than others. In contrast, adding benefits that may be accessed by the consumer only while he is in states worse than perfect health will reduce the AEW value from the perspective of such a consumer as compared to a person who is indifferent between any health states. This result is clearly shown in Table 3.4.2 and Table 3.4.3 for category male where the ratio for the AEW value with h to the AEW value without h is more than 1 for the annuity element only, and less than 1 for the annuity with all bundled benefits. However, this effect also depends on the probability of receiving a particular benefit. For category female, the probability of receiving the TPD and critical illness benefit is very low which makes the effect of a utility gain from the annuity element greater, thus resulting in this ratio being more than 1 for almost all categories.

Figure 3.4.1: Annuity Equivalent Wealth of the New Annuity Plan with all benefit (Age 35)

(a) AEW for Category Age 35 Male



(b) AEW for Category Age 35 Female

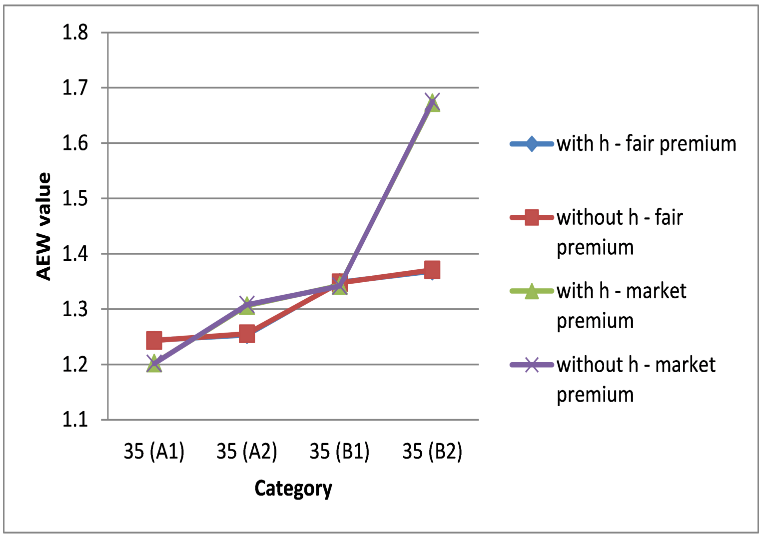
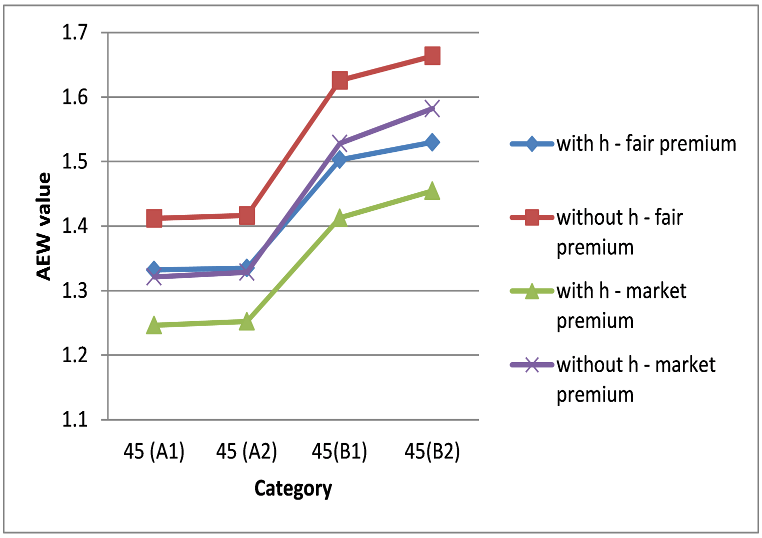
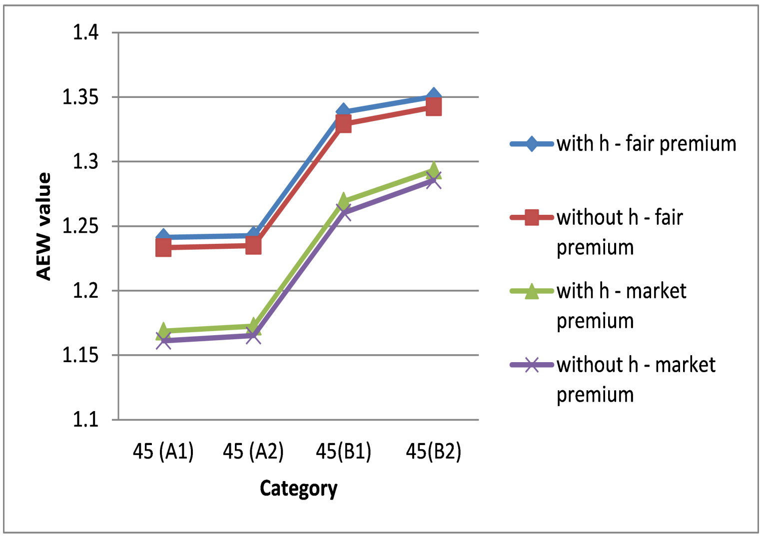


Figure 3.4.2: Annuity Equivalent Wealth of the New Annuity Plan with all benefit (Age 45)

(a) AEW for Category Age 45 Male



(b) AEW for Category Age 45 Female



For comparison purposes, we also apply the UK population mean health index of utility values to study the implication of people's different perception over health states according to different populations. We do our AEW analysis for an annuity with full benefits under the category age 45 (A2) and age 45 (B2) for both male and female. Since the EQ-5D values for UK population elicitation process involved with the state worse than death resulted in a negative health index of utility for the severe health state category, it is important to point out again the method used to treat this negative health index of utility in our analysis. The approach that we apply is the truncation method where all negative health index of utility are truncated at zero[28]. Table 3.4.5 shows the AEW result using the mean health index of utility values for the UK population. When we employ the UK population mean health index of utility values, the AEW value only changes slightly for category age 45 females, but for category age 45 (B2) males, there is a more significant change — the AEW (with health index of utility) is lower by around 11% as compared to the AEW (without health index of utility). For the same category, but using the Malaysian mean health index of utility values, the AEW value decreases by a lower percentage of 8%. This result shows that for a person with a high rates of TPD and critical illness, if he/she values consumption in the states worse than full health very unfavourably such as in the case for UK population (where the mean health index of utility is spread over a wider range from perfect health to severe health state), he/she values such an annuity with additional benefits much lower. Hence, if a consumer's perception towards different health states varies according to country, then the value for money calculated from the AEW analysis of the exact same product offered to two different population is also different.

[28]The other approach is to keep the negative health index of utility as it is and study the implication of allowing this negative health index of utility in the model. However, as shown in Section 3.3.3, allowing for the negative health index of utility indicates that a consumer is willing to give up more consumption which leaves him with less than the minimum consumption required for existence in order to be cured — they would rather die than remain miserable living in their current health state. Thus, we have chosen the first approach in our analysis so that the minimum consumption required for survival requirement for the consumer is satisfied.

Table 3.4.5: Annuity Equivalent Wealth of the New Annuity Plan with all benefits (UK mean h-value but Malaysian transition probabilities)

Category	Male	Female
with h-value	1.3141	1.2365
without h-value	1.4164	1.2350
ratio	0.9278	1.0012

(a) Category age 45 (A2)

Category	Male	Female
with h-value	1.4933	1.3407
without h-value	1.6635	1.3423
ratio	0.8977	0.9988

(b) Category age 45 (B2)

3.5 Conclusion

Recent developments in the annuity market in Malaysia have created a new challenge in determining the optimal annuitisation decision for a consumer given additional benefits are bundled into the annuity stream of income, as compared to a plain vanilla annuity. These additional benefits can be seen as a means by which an insurer can attract customers to purchase an annuity product given an annuity is not so popular in the country. In previous studies where annuity value for money analysis is undertaken, researchers like Mitchell et al. (1999) and Brown (2003) have used a health independent utility function because only survival or death of the consumer matters. However, once additional benefits covering unfortunate events that are related to health states are introduced, such as total and permanent disability and diagnosis of critical illnesses, the broader question concerning the implication of health dependent utility in valuing such annuity products is raised.

In this paper, we firstly show under the traditional health independent utility function, the value for money of annuity increases when additional benefits are included in the annuity product. Second, we present now findings which describe how the consumer's perception towards different health states would affect the value for money of a bundled annuity for the consumer. The concept of a health index of utility is utilised in the model where the utility value of one particular health state can be elicited using several approaches[29]. A utility function of

[29]The three most common approaches used in the health economics literature are the Rating

logarithmic type is applied drawing on the recent work of Levy and Nir (2012), who demonstrated it was the best form of utility function to describe the health and consumption tradeoff. According to our analysis of the extended Annuity Equivalent Wealth concept, comparing between health independent and health dependent utility, we find that the AEW value is higher for a consumer who values different health states differently when the benefit paid is associated with the perfect health state. However, for such consumer, if we add additional benefits that will be paid only in the event of TPD or critical illness (to which he has no access while he is in the perfect health state), the consumer will value these benefits less as compared to a consumer who is indifferent between any health states. This result indicates that for someone who values a unit of consumption in different health states differently, the value of an additional benefit on top of an annuity depends on the health state associated with the payment. In addition, our results imply that if we do not take into account the health utility index value associated with a health state, then a consumer might be undervaluing an annuity product with an annuity income payment, or overvaluing an insurance product with payments associated with health states worse than perfect health.

Health dependent utility is not an unconventional topic in the area of health economics. It has appeared in the literatures for decades and groups of researchers, such as EuroQol and Health Utilities Inc., have formed for the purpose of developing the health index of utility to value the health related quality of life to be used (mostly) in the health care evaluation studies. Here, we have extended the application of the health index of utility to annuity and insurance valuation analysis where people do not only value an annuity based on their survival rate, but it also depends on the health state associated with the benefit payment from the bundled product.

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3.A Appendix

3.A.1 The Summary of Findings of AEW

Below are the summary of findings of AEW analysis as discussed in Section 3.3.2 above.

Figure 3.A.1: Results of AEW for Male age 35

Summary of findings:
Malaysian h-value (Note: This AEW calculation uses the actuarially fair premium as the denominator)
Category Male Age 35
Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category	exp_utility	35 (A1)	exp_utility	35 (A2)	exp_utility	35(B1)	exp_utility	35(B2)
with h	50.762000	1.378862	54.306000	1.403536	40.379000	1.605645	43.689000	1.659761
without h	62.646000	1.468129	67.082000	1.505282	49.780000	1.735056	53.932000	1.812775
ratio		0.939197		0.932408		0.925414		0.915592

Case 1_Annuity payment

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	49.723000	1.302608	53.208000	1.302655	38.999000	1.367490	42.216000	1.367377
without h	60.798000	1.286653	65.107000	1.286731	47.602000	1.358809	51.578000	1.358691
ratio		1.012400		1.012375		1.006389		1.006393

Case 2_Annuity payment + death benefit (post deferral)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	49.871000	1.259594	53.355000	1.259537	39.112000	1.328678	42.328000	1.328380
without h	60.947000	1.238515	65.255000	1.238690	47.715000	1.313954	51.691000	1.313945
ratio		1.017020		1.016830		1.011206		1.010985

Case 3_Annuity payment + CV for TPD and CI (post deferral)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	49.996000	1.344087	53.514000	1.351303	39.261000	1.434037	42.517000	1.446441
without h	61.528000	1.398723	65.916000	1.413630	48.280000	1.517500	52.347000	1.542044
ratio		0.960939		0.955910		0.944999		0.938002

Case 4_Annuity payment + CV for death, TPD and CI (post deferral)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	50.144000	1.300592	53.661000	1.307712	39.374000	1.394083	42.629000	1.405844
without h	61.677000	1.347520	66.064000	1.361916	48.393000	1.468088	52.460000	1.491955
ratio		0.965175		0.960200		0.949591		0.942283

Malaysian h-value (Note: This AEW calculation uses the premium charged by the insurer as the denominator)

Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	50.762000	1.319560	54.306000	1.451012	40.379000	1.576622	43.689000	1.999633
without h	62.646000	1.404988	67.082000	1.556199	49.780000	1.703694	53.932000	2.183980
ratio		0.939197		0.932408		0.925414		0.915592

A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

Figure 3.A.2: Results of AEW for Female age 35

Summary of findings:
Malaysian h-value (Note: This AEW calculation uses the actuarially fair premium as the denominator)
Category Female Age 35
Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category	exp_utility	35 (A1)	exp_utility	35 (A2)	exp_utility	35(B1)	exp_utility	35(B2)
with h	53.478000	1.243639	57.233000	1.253364	43.049000	1.348598	46.604000	1.368694
without h	66.163000	1.243236	70.867000	1.255186	53.111000	1.347426	57.562000	1.370636
ratio		1.000324		0.998548		1.000870		0.998583

Case 1_Annuity payment

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	53.005000	1.229620	56.739000	1.229688	42.374000	1.272484	45.890000	1.272365
without h	65.437000	1.209080	70.099000	1.208834	52.190000	1.255183	56.578000	1.255279
ratio		1.016988		1.017252		1.013784		1.013611

Case 2_Annuity payment + death benefit (post deferral)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	53.096000	1.202890	56.829000	1.202849	42.447000	1.247162	45.963000	1.247022
without h	65.528000	1.179360	70.190000	1.179357	52.264000	1.226988	56.651000	1.226697
ratio		1.019951		1.019919		1.016442		1.016570

Case 3_Annuity payment + CV for TPD and CI (post deferral)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	53.073000	1.239046	56.817000	1.240895	42.437000	1.285831	45.965000	1.288943
without h	65.630000	1.234353	70.316000	1.238010	52.363000	1.287936	56.778000	1.293621
ratio		1.003802		1.002330		0.998365		0.996384

Case 4_Annuity payment + CV for death, TPD and CI (post deferral)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	53.164000	1.212265	56.907000	1.213969	42.510000	1.260309	46.038000	1.263388
without h	65.721000	1.204444	70.407000	1.207939	52.437000	1.258885	56.851000	1.264292
ratio		1.006494		1.004992		1.001131		0.999285

Malaysian h-value (Note: This AEW calculation uses the premium charged by the insurer as the denominator)

Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category		35 (A1)		35 (A2)		35(B1)		35(B2)
with h	53.478000	1.201952	57.233000	1.306112	43.049000	1.342740	46.604000	1.672691
without h	66.163000	1.201563	70.867000	1.308010	53.111000	1.341573	57.562000	1.675065
ratio		1.000324		0.998548		1.000870		0.998583

A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

Figure 3.A.3: Results of AEW for Male age 45

Summary of findings:
Malaysian h-value (Note: This AEW calculation uses the actuarially fair premium as the denominator)
Category Male Age 45
Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	67.726000	1.331943	68.430000	1.335104	52.471000	1.502914	54.931000	1.529829
without h	84.553000	1.411948	85.439000	1.416390	65.702000	1.625631	68.821000	1.663535
ratio		0.943337		0.942611		0.924511		0.919625

Case 1_Annuity payment

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	66.773000	1.316362	67.471000	1.280341	51.176000	1.396001	53.597000	1.395925
without h	82.567000	1.295964	83.438000	1.260589	63.374000	1.380998	66.402000	1.380968
ratio		1.015740		1.015669		1.010864		1.010831

Case 2_Annuity payment + death benefit (post deferral)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	66.935000	1.262597	67.634000	1.230003	51.302000	1.344572	53.722000	1.344288
without h	82.729000	1.238379	83.600000	1.206481	63.500000	1.325007	66.528000	1.324943
ratio		1.019556		1.019496		1.014766		1.014600

Case 3_Annuity payment + CV for TPD and CI (post deferral)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	67.067000	1.343537	67.770000	1.307952	51.454000	1.443917	53.899000	1.449352
without h	83.479000	1.391553	84.364000	1.355551	64.214000	1.519838	67.305000	1.531928
ratio		0.965495		0.964886		0.950047		0.946096

Case 4_Annuity payment + CV for death, TPD and CI (post deferral)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	67.229000	1.289745	67.933000	1.257649	51.580000	1.391663	54.024000	1.396620
without h	83.641000	1.330668	84.526000	1.298300	64.340000	1.459164	67.431000	1.470687
ratio		0.969246		0.968689		0.953740		0.949638

Malaysian h-value (Note: This AEW calculation uses the premium charged by the insurer as the denominator)

Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	67.726000	1.246490	68.430000	1.252406	52.471000	1.412741	54.931000	1.454887
without h	84.553000	1.321361	85.439000	1.328657	65.702000	1.528094	68.821000	1.582044
ratio		0.943337		0.942611		0.924511		0.919625

A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

Figure 3.A.4: Results of AEW for Female age 45

Summary of findings:
Malaysian h-value (Note: This AEW calculation uses the actuarially fair premium as the denominator)
Category Female Age 45
Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	70.568000	1.241199	71.307000	1.242637	55.359000	1.338468	57.976000	1.350488
without h	88.258000	1.233309	89.191000	1.234970	69.269000	1.329192	72.581000	1.342271
ratio		1.006397		1.006208		1.006979		1.006122

Case 1_Annuity payment

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	70.117000	1.254504	70.854000	1.254553	54.675000	1.311663	57.277000	1.311727
without h	87.547000	1.231130	88.476000	1.231166	68.330000	1.291116	71.615000	1.291144
ratio		1.018986		1.018996		1.015914		1.015942

Case 2_Annuity payment + death benefit (post deferral)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	70.230000	1.216917	70.967000	1.224319	54.766000	1.275316	57.367000	1.275125
without h	87.661000	1.191353	88.589000	1.198445	68.420000	1.251746	71.705000	1.251720
ratio		1.021458		1.021590		1.018829		1.018698

Case 3_Annuity payment + CV for TPD and CI (post deferral)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	70.185000	1.260283	70.924000	1.260407	54.737000	1.320929	57.346000	1.322362
without h	87.762000	1.250828	88.695000	1.251282	68.523000	1.317586	71.823000	1.319927
ratio		1.007558		1.007292		1.002537		1.001844

Case 4_Annuity payment + CV for death, TPD and CI (post deferral)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	70.298000	1.222559	71.037000	1.222875	54.828000	1.284506	57.436000	1.285548
without h	87.876000	1.210551	88.808000	1.210829	68.613000	1.277664	71.913000	1.279709
ratio		1.009920		1.009949		1.005356		1.004563

Malaysian h-value (Note: This AEW calculation uses the premium charged by the insurer as the denominator)

Case 5_Annuity with all benefits (annuity income stream, TPD, Critical Illness and death)

Category	exp_utility	45 (A1)	exp_utility	45 (A2)	exp_utility	45(B1)	exp_utility	45(B2)
with h	70.568000	1.168543	71.307000	1.172409	55.359000	1.269212	57.976000	1.293303
without h	88.258000	1.161115	89.191000	1.165176	69.269000	1.260416	72.581000	1.285433
ratio		1.006397		1.006208		1.006979		1.006122

A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

3.A.2 The Set of Health Index of Utility

Below are the list of EQ-5D health combinations and its health index of utility value for the UK and Malaysian populations associated with each health state category in our model, as discussed in Section 3.3.4. For each combination we give its health state category, ranging from full health to severe impairment to full health.

List of Health State	UK health index	Category	Malaysia health index	Category
11111	1	Full Health	1	Full Health
11121	0.796	Very Mild	0.879	Very Mild
11112	0.848	Very Mild	0.852	Very Mild
11211	0.883	Very Mild	0.88	Very Mild
21111	0.85	Very Mild	0.742	Mild
12111	0.815	Very Mild	0.836	Very Mild
11122	0.725	Mild	0.793	Very Mild
11221	0.76	Very Mild	0.826	Very Mild
21121	0.727	Mild	0.795	Very Mild
12121	0.692	Very Mild	0.782	Very Mild
11113	0.414	Moderate	0.731	Mild
11123	0.291	Moderate	0.677	Mild
11212	0.812	Very Mild	0.799	Very Mild
21112	0.779	Very Mild	0.768	Very Mild
12112	0.744	Mild	0.755	Very Mild
11131	0.264	Moderate	0.69	Mild
21211	0.814	Very Mild	0.796	Very Mild
12211	0.779	Very Mild	0.783	Very Mild
22111	0.746	Mild	0.752	Very Mild
13111	0.436	Moderate	0.657	Mild
11311	0.556	Mild	0.695	Mild
11222	0.689	Mild	0.745	Mild
21122	0.656	Mild	0.714	Mild
12122	0.621	Mild	0.701	Mild
21221	0.691	Mild	0.742	Mild
12221	0.656	Mild	0.729	Mild
11213	0.378	Moderate	0.678	Mild
22121	0.623	Mild	0.698	Mild
21113	0.345	Moderate	0.647	Mild
12113	0.31	Moderate	0.75	Very Mild
13121	0.313	Moderate	0.603	Mild
11321	0.433	Moderate	0.641	Mild
31111	0.336	Moderate	0.626	Mild
11223	0.255	Moderate	0.624	Mild
11132	0.193	Severe	0.609	Mild
21123	0.222	Severe	0.593	Mild
12123	0.187	Severe	0.58	Mild
21212	0.743	Mild	0.715	Mild
12212	0.708	Mild	0.697	Mild
11231	0.228	Severe	0.637	Mild
22112	0.675	Mild	0.555	Mild
21131	0.195	Severe	0.606	Mild
12131	0.16	Severe	0.593	Mild
31121	0.213	Severe	0.572	Mild
13112	0.365	Moderate	0.576	Mild
11312	0.485	Moderate	0.614	Mild
22211	0.71	Mild	0.63	Mild
13211	0.4	Moderate	0.604	Mild

List of Health State	UK health index	Category	Malaysia health index	Category
23111	0.367	Moderate	0.657	Mild
21311	0.487	Moderate	0.611	Mild
12311	0.452	Moderate	0.598	Mild
21222	0.62	Mild	0.661	Mild
12222	0.585	Mild	0.648	Mild
22122	0.552	Mild	0.733	Mild
11133	0.028	Severe	0.72	Mild
13122	0.242	Severe	0.522	Mild
11322	0.362	Moderate	0.56	Mild
22221	0.587	Mild	0.645	Mild
21213	0.309	Moderate	0.594	Mild
12213	0.274	Moderate	0.581	Mild
13221	0.277	Moderate	0.55	Mild
31112	0.265	Moderate	0.545	Mild
22113	0.241	Severe	0.55	Mild
23121	0.244	Severe	0.519	Mild
21321	0.364	Moderate	0.557	Mild
13113	0.2	Severe	0.571	Mild
12321	0.329	Moderate	0.544	Mild
31211	0.3	Moderate	0.573	Mild
11313	0.32	Moderate	0.609	Mild
32111	0.232	Severe	0.529	Mild
11232	0.157	Severe	0.556	Mild
21223	0.186	Severe	0.54	Mild
12223	0.151	Severe	0.527	Mild
21132	0.124	Severe	0.519	Mild
12132	0.089	Severe	0.512	Mild
31122	0.142	Severe	0.491	Moderate
22123	0.118	Severe	0.496	Moderate
22212	0.639	Mild	0.615	Mild
21231	0.159	Severe	0.553	Mild
13123	0.077	Severe	0.517	Mild
12231	0.124	Severe	0.54	Mild
31221	0.177	Severe	0.519	Mild
13212	0.329	Moderate	0.523	Mild
11323	0.197	Severe	0.555	Mild
22131	0.091	Severe	0.539	Mild
23112	0.296	Moderate	0.492	Moderate
32121	0.109	Severe	0.666	Mild
31113	0.1	Severe	0.54	Mild
21312	0.416	Moderate	0.53	Mild
13131	0.05	Severe	0.53	Mild
12312	0.381	Moderate	0.517	Mild
11331	0.17	Severe	0.568	Mild
23211	0.331	Moderate	0.52	Mild
22311	0.383	Moderate	0.514	Mild
22222	0.516	Mild	0.564	Mild
13311	0.342	Moderate	0.535	Mild

List of Health State	UK health index	Category	Malaysia health index	Category
11233	-0.008	Severe	0.551	Mild
13222	0.206	Severe	0.469	Moderate
21133	-0.041	Severe	0.52	Mild
23122	0.173	Severe	0.438	Moderate
12133	-0.076	Severe	0.507	Mild
31123	-0.023	Severe	0.486	Moderate
21322	0.293	Moderate	0.476	Moderate
12322	0.258	Moderate	0.463	Moderate
31212	0.229	Severe	0.492	Moderate
22213	0.205	Severe	0.497	Moderate
23221	0.208	Severe	0.466	Moderate
13213	0.164	Severe	0.518	Mild
32112	0.161	Severe	0.448	Moderate
31131	-0.05	Severe	0.499	Moderate
23113	0.131	Severe	0.487	Moderate
22321	0.26	Moderate	0.46	Moderate
21313	0.251	Moderate	0.525	Mild
32211	0.196	Severe	0.476	Moderate
12313	0.216	Severe	0.512	Mild
13321	0.219	Severe	0.481	Moderate
33111	0.122	Severe	0.344	Moderate
31311	0.242	Severe	0.504	Mild
21232	0.088	Severe	0.472	Moderate
12232	0.053	Severe	0.512	Mild
31222	0.106	Severe	0.438	Moderate
22223	0.082	Severe	0.559	Mild
22132	0.02	Severe	0.428	Moderate
13223	0.041	Severe	0.524	Mild
32122	0.038	Severe	0.394	Moderate
13132	-0.021	Severe	0.449	Moderate
11332	0.099	Severe	0.487	Moderate
23123	0.008	Severe	0.433	Moderate
22231	0.055	Severe	0.456	Moderate
23212	0.26	Moderate	0.439	Moderate
21323	0.128	Severe	0.403	Moderate
32221	0.073	Severe	0.422	Moderate
12323	0.093	Severe	0.458	Moderate
31213	0.064	Severe	0.487	Moderate
13231	0.014	Severe	0.477	Moderate
23131	-0.019	Severe	0.446	Moderate
32113	-0.004	Severe	0.443	Moderate
22312	0.312	Moderate	0.433	Moderate
21331	0.101	Severe	0.484	Moderate
33121	-0.001	Severe	0.412	Moderate
12331	0.066	Severe	0.471	Moderate
31321	0.119	Severe	0.45	Moderate
13312	0.271	Moderate	0.454	Moderate
23311	0.273	Moderate	0.451	Moderate

List of Health State	UK health index	Category	Malaysia health index	Category
21233	-0.077	Severe	0.467	Moderate
23222	0.137	Severe	0.385	Moderate
12233	-0.112	Severe	0.454	Moderate
31223	-0.059	Severe	0.433	Moderate
31132	-0.121	Severe	0.418	Moderate
22133	-0.145	Severe	0.423	Moderate
32123	-0.127	Severe	0.389	Moderate
22322	0.189	Severe	0.379	Moderate
13133	-0.186	Severe	0.444	Moderate
32212	0.125	Severe	0.395	Moderate
31231	-0.086	Severe	0.446	Moderate
11333	-0.066	Severe	0.482	Moderate
13322	0.148	Severe	0.4	Moderate
23213	0.095	Severe	0.434	Moderate
32131	-0.154	Severe	0.402	Moderate
33112	0.051	Severe	0.385	Moderate
31312	0.171	Severe	0.423	Moderate
22313	0.147	Severe	0.428	Moderate
23321	0.15	Severe	0.397	Moderate
33211	0.086	Severe	0.413	Moderate
13313	0.106	Severe	0.454	Moderate
32311	0.138	Severe	0.407	Moderate
22232	-0.016	Severe	0.375	Moderate
32222	0.002	Severe	0.341	Moderate
13232	-0.057	Severe	0.396	Moderate
23223	-0.028	Severe	0.38	Moderate
23132	-0.09	Severe	0.365	Moderate
31133	-0.286	Severe	0.413	Moderate
21332	0.03	Severe	0.403	Moderate
33122	-0.072	Severe	0.331	Moderate
12332	-0.005	Severe	0.39	Moderate
31322	0.048	Severe	0.369	Moderate
22323	0.024	Severe	0.374	Moderate
23231	-0.055	Severe	0.393	Moderate
32213	-0.04	Severe	0.39	Moderate
33221	-0.037	Severe	0.359	Moderate
13323	-0.017	Severe	0.511	Mild
22331	-0.003	Severe	0.387	Moderate
33113	-0.114	Severe	0.38	Moderate
23312	0.202	Severe	0.37	Moderate
32321	0.015	Severe	0.353	Moderate
31313	0.006	Severe	0.418	Moderate
13331	-0.044	Severe	0.408	Moderate
31232	-0.157	Severe	0.365	Moderate
22233	-0.181	Severe	0.486	Moderate
32223	-0.163	Severe	0.336	Moderate
13233	-0.222	Severe	0.391	Moderate
32132	-0.225	Severe	0.321	Moderate

List of Health State	UK health index	Category	Malaysia health index	Category
23133	-0.255	Severe	0.36	Moderate
21333	-0.135	Severe	0.467	Moderate
33123	-0.237	Severe	0.326	Moderate
32231	-0.19	Severe	0.349	Moderate
23322	0.079	Severe	0.316	Moderate
12333	-0.17	Severe	0.238	Severe
33212	0.015	Severe	0.263	Moderate
31323	-0.117	Severe	0.364	Moderate
33131	-0.264	Severe	0.339	Moderate
32312	0.067	Severe	0.326	Moderate
31331	-0.144	Severe	0.377	Moderate
23313	0.037	Severe	0.365	Moderate
33311	0.028	Severe	0.344	Moderate
23232	-0.126	Severe	0.312	Moderate
31233	-0.322	Severe	0.36	Moderate
33222	-0.108	Severe	0.278	Moderate
32133	-0.39	Severe	0.316	Moderate
22332	-0.074	Severe	0.306	Moderate
32322	-0.319	Severe	0.272	Moderate
13332	-0.115	Severe	0.327	Moderate
23323	-0.086	Severe	0.311	Moderate
33213	-0.15	Severe	0.337	Moderate
23331	-0.113	Severe	0.324	Moderate
32313	-0.098	Severe	0.321	Moderate
33321	-0.095	Severe	0.29	Moderate
32232	-0.261	Severe	0.268	Moderate
23233	-0.291	Severe	0.307	Moderate
33223	-0.273	Severe	0.273	Moderate
33132	-0.335	Severe	0.258	Moderate
31332	-0.215	Severe	0.296	Moderate
22333	-0.239	Severe	0.374	Moderate
32323	-0.221	Severe	0.267	Moderate
33231	-0.3	Severe	0.286	Moderate
13333	-0.28	Severe	0.322	Moderate
32331	-0.248	Severe	0.28	Moderate
33312	-0.043	Severe	0.263	Moderate
32233	-0.426	Severe	0.263	Moderate
33133	-0.5	Severe	0.253	Moderate
23332	-0.184	Severe	0.243	Severe
31333	-0.38	Severe	0.291	Moderate
33322	-0.166	Severe	0.209	Severe
33313	-0.208	Severe	0.258	Moderate
33232	-0.371	Severe	0.267	Moderate
32332	-0.319	Severe	0.209	Severe
23333	-0.349	Severe	0.238	Severe
33323	-0.331	Severe	0.204	Severe
33331	-0.358	Severe	0.217	Severe
33233	-0.536	Severe	0.2	Severe

List of Health State	UK health index	Category	Malaysia health index	Category
32333	-0.484	Severe	0.194	Severe
33332	-0.429	Severe	0.136	Severe
33333	-0.594	Severe	0.131	Severe

3.A.3 The Transition Probability Derivation

In this section, first we show the derivation of the transition probability applied in our life cycle model. Part of this method is identical to Gatenby (1991) where the method is useful for estimating the transition probability in the case where only prevalence rates data is available. First, we assume that the total and permanent disability (TPD) world is not separated from the non-TPD world, where prevalence rates data for each health state namely 1-full health, 2-very mild, 3-mild, 4-moderate and 5-severe is obtained from the Health Survey for England Report 2006 (HSE2006). Since we try to develop a more realistic life cycle model which involve the deterioration and improvement rates from one state to another, we have combined the HSE2006 with the Hospital Episode Statistics 2009-2012 (HES2009-2012) data that provide useful information regarding the pre-treatment and post-treatment patient's EQ-5D-health state from the Patient Reported Outcome Measures (PROMS) database. We explain below the list of prevalence rates obtain from HSE2006 and HES2009-2012, then we continue with the assumptions and adjustments made to the data for the purpose of deriving the transition probability used in our life cycle model.

List of prevalence rates obtained from the HSE2006:

$pr_x^1 = l_x^1/l_x^{ALL}$ = the full health state prevalence rate at age x

$pr_x^2 = l_x^2/l_x^{ALL}$ = the very mild state prevalence rate at age x

$pr_x^3 = l_x^3/l_x^{ALL}$ = the mild state prevalence rate at age x

$pr_x^4 = l_x^4/l_x^{ALL}$ = the moderate state prevalence rate at age x

$pr_x^5 = l_x^5/l_x^{ALL}$ = the severe state prevalence rate at age x

where l_x^k is the total number of age x respondents in health state k and l_x^{ALL} is the total number of age x respondents.

List of prevalence rates obtained from the HES2009-2012:

$pr_x^{21} = l_x^{21}/l_x^{ALL}$ =the improvement rate from state 2 to state 1

$pr_x^{32} = l_x^{32}/l_x^{ALL}$ =the improvement rate from state 3 to state 2

$pr_x^{43} = l_x^{43}/l_x^{ALL}$ =the improvement rate from state 4 to state 3

$pr_x^{54} = l_x^{54}/l_x^{ALL}$ =the improvement rate from state 5 to state 4

$pr_x^{23} = l_x^{23}/l_x^{ALL}$ =the deterioration rate from state 2 to state 3

$pr_x^{34} = l_x^{34}/l_x^{ALL}$ =the deterioration rate from state 3 to state 4

$pr_x^{45} = l_x^{45}/l_x^{ALL}$ =the deterioration rate from state 4 to state 5

$pr_x^{22} = l_x^{22}/l_x^{ALL}$ =the rate of staying in state 2

$pr_x^{33} = l_x^{33}/l_x^{ALL}$ =the rate of staying in state 3

$pr_x^{44} = l_x^{44}/l_x^{ALL}$ =the rate of staying in state 4

$pr_x^{55} = l_x^{55}/l_x^{ALL}$ =the rate of staying in state 5 where l_x^{ij} is the total number of age x respondents who is in health state i before treatment and moves to health state j after treatment.

The prevalence rates data obtained from the HES2009-2012 only contains information of patients' self reported health state while the HSE2006 contains information of the whole population self reported health state. Therefore, we have made an assumption that the prevalence rates from state 1-full health (pr_x^{11}, pr_x^{12}, pr_x^{13}, pr_x^{14}, pr_x^{15}) can be obtained by matching these two sets of data such that the following equations hold[30]:

$pr_x^{11} + pr_x^{21} = pr_x^1$

$pr_x^{22} + pr_x^{12} + pr_x^{32} = pr_x^2$

$pr_x^{33} + pr_x^{13} + pr_x^{43} + pr_x^{23} = pr_x^3$

$pr_x^{44} + pr_x^{14} + pr_x^{34} + pr_x^{54} = pr_x^4$

$pr_x^{55} + pr_x^{15} + pr_x^{45} = pr_x^5$

We present the prevalence rates for each health state obtained from the HSE2006 survey data than the HES2009-2012 survey data in Figure 3.A.5 below. The prevalence rates used in our transition probabilities calculation are estimated by matching these two sets of data where linear interpolation technique is used to find the prevalence rates by single age.

For the derivation of transition probabilities, we need the following definitions:

l_x^1 =No. of active lives aged x in full health state-1

l_x^2 =No. of lives aged x in state 2

l_x^3 =No. of lives aged x in state 3

l_x^4 =No. of lives aged x in state 4

l_x^5 =No. of lives aged x in state 5

P_x^{16} =Annuitant life mortality rate at age$x$

P_x^{26} =Mortality rate of lives aged x in state 2, $(1).P_x^{16}$

P_x^{36} =Mortality rate of lives aged x in state 3, $(1).P_x^{16}$

P_x^{46} =Mortality rate of lives aged x in state 4, $(1).P_x^{16}$

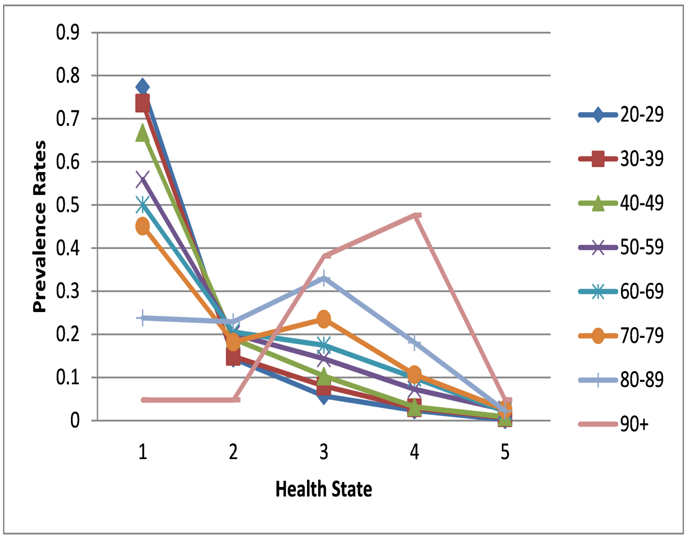
P_x^{56} =Mortality rate of lives aged x in state 5, $(1 + 10\%).P_x^{16}$

P_x^{ij} = The transition probability of a person aged x who is in state i, survives and is in state j at age $x + 1$

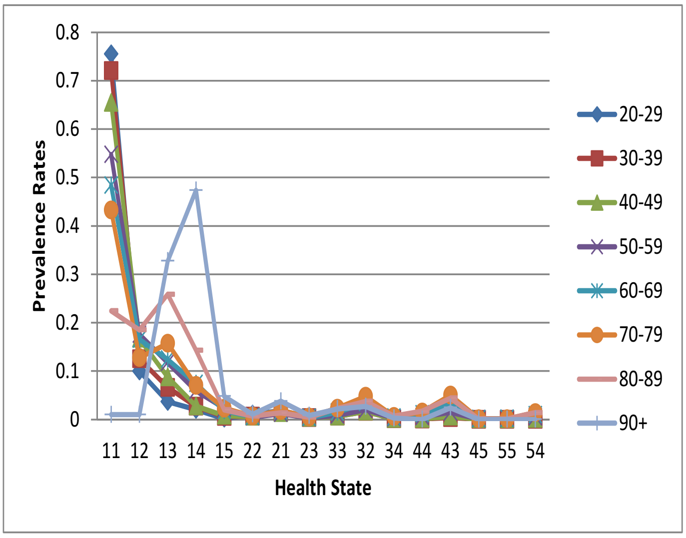
We drop the time t for the notation of P_x^{ij} above where ${}_1P_x^{ij} = P_x^{ij}$ which denotes the transition probability of a person aged x who is in state i, survives and

[30]In the real life cycle of a consumer, this assumption might not be true since there is a possibility that people in either state 2, 3 or even 4 is not a hospital patient. However, for the purpose of estimating the transition probability, due to constraint in the availability of data, we have adopted the assumption that the prevalence rates obtained from the HSE2009-2012 of patients' self reported health state represent the prevalence rates of all population in state 2, 3, 4 and 5.

Figure 3.A.5: The Prevalence Rates data according to Health State (Male)

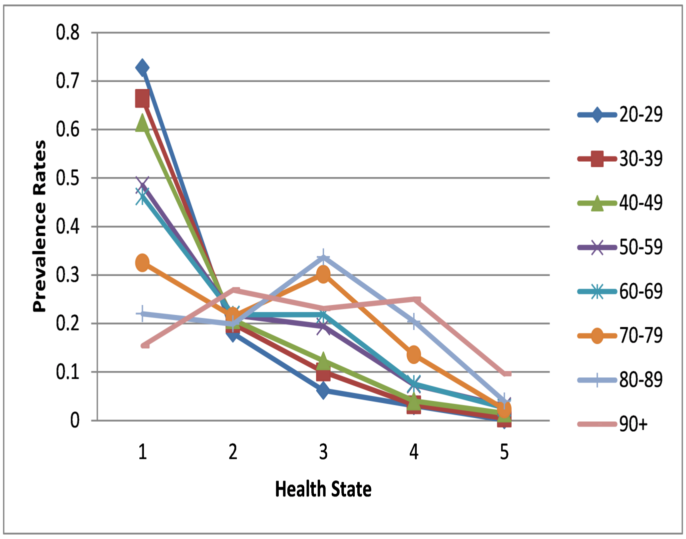


(a) The HSE2006 Survey Data (Male)

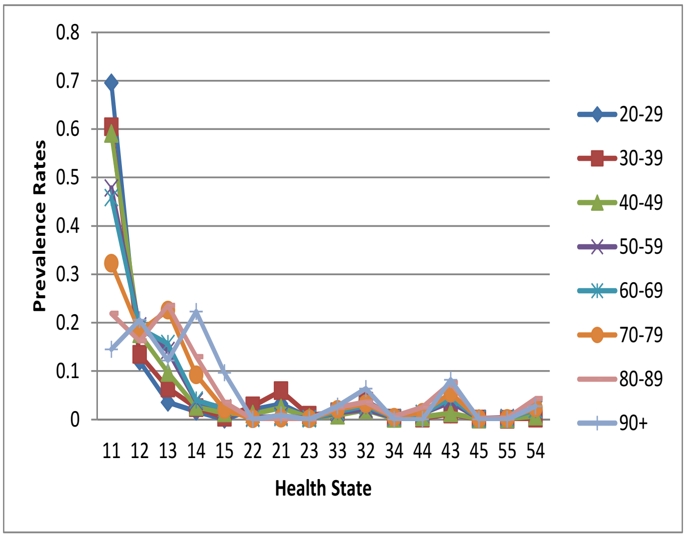


(b) The HES2009-2012 Survey Data (Male)

Figure 3.A.6: The Prevalence Rates data according to Health State (Female)



(a) The HSE2006 Survey Data (Female)



(b) The HES2009-2012 Survey Data (Female) is in state j at age $x+1$. We assume that the annuitant mortality rate for state 1 is also applied for lives in state 2, 3, and 4. This assumption is neccessary to match the life-cycle model and the annuity model used by insurer[31]. Next, we produce the transition probabilities in a form of ratio according to the prevalence rates obtained from the data above where we assume that each transition probability from one particular health state has the same ratio as each prevalence rates from the same health state. Previous studies such as Gatenby (1991) and Haberman, Olivieri, and Pitacco (1997) have assumed for no recoveries and the transition probability from any state to one particular health state worse than it as equal to another. However, we decide not to apply that assumption but to use the ratio obtained from the prevalence rates to produce a set of transition probability that reflects the data set better. Thus, according to our assumption, the following equations hold for state 1:

$$P_x^{11} = (1 - P_x^{16}).(\frac{pr_x^{11}}{pr_x^{11} + pr_x^{12} + pr_x^{13} + pr_x^{14} + pr_x^{15}}) \tag{3.A.1}$$

$$P_x^{12} = (1 - P_x^{16}).(\frac{pr_x^{12}}{pr_x^{11} + pr_x^{12} + pr_x^{13} + pr_x^{14} + pr_x^{15}}) \tag{3.A.2}$$

$$P_x^{13} = (1 - P_x^{16}).(\frac{pr_x^{13}}{pr_x^{11} + pr_x^{12} + pr_x^{13} + pr_x^{14} + pr_x^{15}}) \tag{3.A.3}$$

$$P_x^{14} = (1 - P_x^{16}).(\frac{pr_x^{14}}{pr_x^{11} + pr_x^{12} + pr_x^{13} + pr_x^{14} + pr_x^{15}}) \tag{3.A.4}$$

$$P_x^{15} = (1 - P_x^{16}).(\frac{pr_x^{15}}{pr_x^{11} + pr_x^{12} + pr_x^{13} + pr_x^{14} + pr_x^{15}}) \tag{3.A.5}$$

We can show that this equation holds by converting the probabilities and prevalence rates for the right hand side (RHS) of Eq. (3.A.1) to Eq. (3.A.5) in the form of l_x as in Gatenby (1991). The proof is shown for a transition probability of P_x^{11}.

[31]It is possible to apply a higher mortality rate for state 2, 3 and 4 where information of mortality rate is asymmetry between an insurance company and an annuity purchaser. The annuity purchaser might have private information about their level of health which may reflects their mortality rate. Here, we assume that both insurance company and annuity purchaser have access to the same information of mortality rates from the SOA (2013).

$$\begin{aligned}
\frac{pr_x^{11}}{pr_x^{11}+pr_x^{12}+pr_x^{13}+pr_x^{14}+pr_x^{15}} &= \frac{(l_x^{11}/l_x^{All})}{\left\{\frac{l_x^{11}+l_x^{12}+l_x^{13}+l_x^{14}+l_x^{15}}{l_x^{All}}\right\}} && (3.A.6)\\
(1-P_x^{16}) &= 1-(d_x^1/l_x^1) && (3.A.7)\\
&= \frac{l_x^{11}+l_x^{12}+l_x^{13}+l_x^{14}+l_x^{15}}{l_x^1}
\end{aligned}$$

where d_x^1 is the total number of respondents age x who dies or moves to the dead state 6. The total number of respondents age x who is in state 1, l_x^1 should be equal to the total number of respondents who moves from state $i = 1$ to any state $j = (1, 2, 3, 4, 5, 6)$ that year where $l_x^1 = l_x^{11} + l_x^{12} + l_x^{13} + l_x^{14} + l_x^{15} + d_x^1$. Thus, we substitute Eq. (3.A.6) and Eq. (3.A.7) into Eq. (3.A.1) and we obtain:

$$\begin{aligned}
P_x^{11} &= \frac{l_x^{11}+l_x^{12}+l_x^{13}+l_x^{14}+l_x^{15}}{l_x^1}.\left[\frac{l_x^{11}}{l_x^{11}+l_x^{12}+l_x^{13}+l_x^{14}+l_x^{15}}\right]\\
&= \frac{l_x^{11}}{l_x^1}
\end{aligned}$$

We apply the same assumption as above for health state 2, 3, 4 and 5 accordingly. Next, we need to adjust these single year estimated transition probabilties to create the TPD parallel world where we add one state for each level of health where at the same time, the consumer is total and permanently disabled[32]. For this adjustment, we apply a factor of the TPD incidence rates i_x^{TPD} (Refer to i_x^2 in Eq. (3.A.8)) by age and gender estimated in the annuity model, which will be explained further in the next section. According to this rate factor, a consumer may make a transition to the TPD parallel world where the rate of staying in the current non-TPD state is taken as $(1 - i_x^{TPD})$.

For the severe state 5, we adjust the transition probability for this state to match the transition probability for critical illness in the annuity model explained below[33]. We do this by adjusting the transition probability for state 1 and 4. For

[32]We do not add another state for perfect health with TPD in the model. Since the EQ-5D self-health reported survey include questions regarding mobility and the ability of self care of respondents, we believe that it is reasonable to assume that the perfect health state is only for person without any of this problem.

[33]Both of sources of data for the prevalence rate of the severe state from the HSE2006 (A. Ali et al., 2006) and the prevalence rate of critical illness from the Gen Re's Dread Disease Survey 2004–2008 Lu and Droste (2012) show that the difference of these two rates is not significant with a difference of around (0.007 to 0.02). The available data that we have do not contain information of prevalence rates for people with critical illness according to their state of health. This is one of the limitation of our study where further data is required to estimate the transition probability for critical illness according to the state of health of respondent.

a non-critically ill individual in the life-cycle model, a person can only make a transition to the severe state 5 from state 1 or 4 (Refer to the life-cycle model Figure 3.3.1). We take a ratio of each of the original transition probability from state 1 and 4 to severe state 5, P_x^{15} and P_x^{45}, and multiply them with the transition probability for critical illness estimated from the annuity model i_x^3 (Refer to Eq. (3.A.9)). The formulation is given as follows.

$$\begin{aligned}\hat{P}_x^{15} &= \frac{P_x^{15}}{(P_x^{15}+P_x^{45})}\{i_x^3\}\\ \hat{P}_x^{45} &= \frac{P_x^{45}}{(P_x^{15}+P_x^{45})}\{i_x^3\}\end{aligned}$$

where $\hat{P}_x^{15}$ and $\hat{P}_x^{45}$ are the modified transition probability from state 1 and 4 to severe state 5 respectively. The transition probability from state 1 and 4 to state 4, P_x^{14} and P_x^{44} are adjusted accordingly to reflect this changes. If $\hat{P}_x^{15} < P_x^{15}$, we increase the probability of P_x^{14} such that, this equality holds:

$$P_x^{11} + P_x^{12} + P_x^{13} + \hat{P}_x^{14} + \hat{P}_x^{15} + P_x^{16} = 1$$

where $\hat{P}_x^{14}$ is the modified transition probability from state 1 to state 4. If $\hat{P}_x^{15} > P_x^{15}$, we reduce the probability of P_x^{14} such that, the same equality holds. We do the same method for adjusting P_x^{44}.

For simplicity of the model, we also adjust the transition probability such that there is no recoveries from TPD and severe state (a consumer with critical illness is assumed to be in the severe state)[34]. Finally, we apply this set of transition probabilities calculated for single age above to the transtion matrix for each time t as explained in Section 3.3.1 which is used to calculate the transition probabilities moving into each state at each time t.

3.A.4 The Derivation of Incidence Rates for the Annuity Model

The section explains the derivation of transition probability for the annuity model. This method is identical to method used in Gatenby (1991) for estimating the disability of incidence rates with different level of disability. Let us first define the following notations used in the derivation:

[34] There is hardly statistical data available for the TPD and critical illness recovery rates. Refer to actuarial models like (Olivieri and Pitacco, 2001; Ozkok et al., 2014; Haberman and Pitacco, 1998) which assume no recoveries for such states.

l_x^1 =No. of active (healthy) lives aged x
l_x^2 =No. of lives aged x in state 2 - Total Permanent Disabled (TPD)
l_x^3 =No. of lives aged x in state 3 - Critical Illness (CI)
P_x^{14} =Annuitant life mortality rate at age x
q_x^{All} =All population mortality rate at age x regardless of current health state, assumes to be higher by 5% than the annuitant mortality rate[35], $(1+5\%).P_x^{14}$
P_x^{24} =Mortality rate of lives aged x in state 2, $(1+10\%).P_x^{14}$
P_x^{34} =Mortality rate of lives aged x in state 3, $(1+10\%).P_x^{14}$
$i_x^2(P_x^{12})$ =TPD (state 2) incidence rate at age x
$i_x^3(P_x^{13})$ =CI (state 3) incidence rate at age x
pr_x^2 =TPD (state 2) prevalence rate at age x
pr_x^3 =CI (state 3) prevalence rate at age x

We apply the following relations to derive an equation of the transition probability in the form of prevalence rate:

- $l_x^2 = l_{x-1}^2 + l_{x-1}^1 i_{x-1}^2 - l_{x-1}^2 P_{x-1}^{24}$
- $l_x^3 = l_{x-1}^3 + l_{x-1}^1 i_{x-1}^3 - l_{x-1}^3 P_{x-1}^{34}$

Lastly, rearrange the above equation and we obtain:

$$i_{x-1}^2 = \frac{l_x^2 - l_{x-1}^2(1-P_{x-1}^{24})}{l_{x-1}^{`1}}$$

$$i_{x-1}^3 = \frac{l_x^3 - l_{x-1}^3(1-P_{x-1}^{34})}{l_{x-1}^{1}}$$

$$i_{x-1}^2 = \frac{pr_x^2(1-q_{x-1}^{ALL}) - pr_{x-1}^2(1-P_{x-1}^{24})}{1-pr_{x-1}^2-pr_{x-1}^3} \quad (3.A.8)$$

$$i_{x-1}^3 = \frac{pr_x^3(1-q_{x-1}^{ALL}) - pr_{x-1}^3(1-P_{x-1}^{34})}{1-pr_{x-1}^2-pr_{x-1}^3} \quad (3.A.9)$$

This estimated incidence rates is fitted to the functional form for disability rates as in Hariyanto (2013), Rickayzen and Walsh (2002) and Leung (2004). The functional form is given as:

$$\hat{i}_x^2 = \alpha\left\{(A + \frac{D-A}{1-B^{C-x}}).(1-\frac{1}{3}exp\left[-\left(\frac{x-E}{4}\right)^2\right])\right\}$$

[35]This is the average mortality rates for annuitant and the mortality rates of people with TPD and CI assumed to be 110% of the annuitant mortality rate.

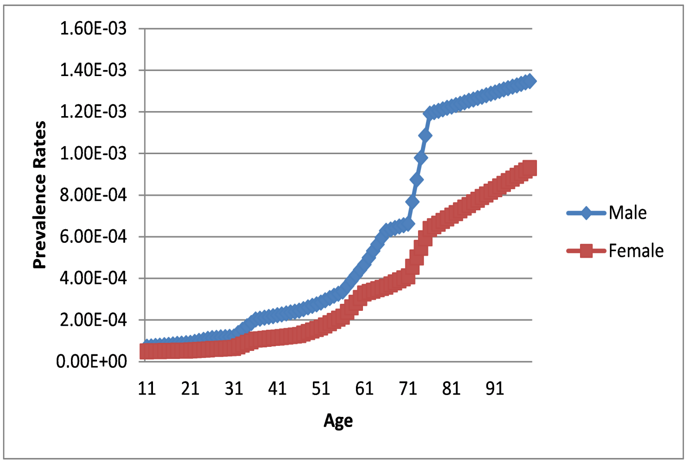
Table 3.A.1: Parameter Values for Estimated Incidence Rates for TPD

Parameter	Male	Female
α	7.8734	6.0000
A	2.1726 E-05	2.1726 E-05
B	0.972174	0.972174
C	137.4952	138.0000
D	-6.935 E-07	-6.935 E-07
E	312.1017	312.1017

The set of parameter used is shown in Table 3.A.1 for both genders. This functional form fitting method fits the data well except for certain age group 70-75 where the incidence rates increase substantially[36].

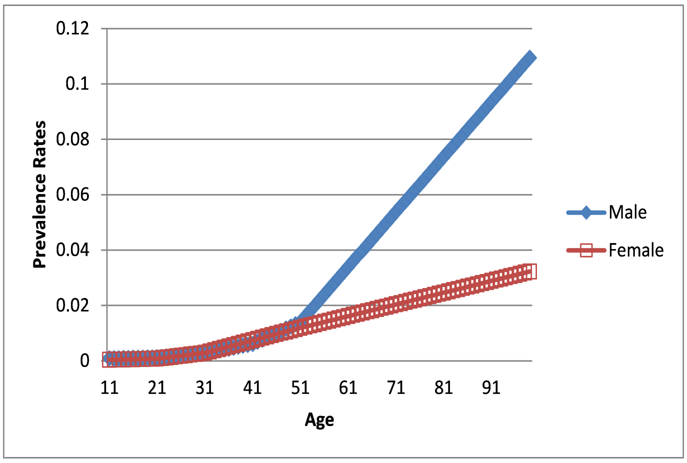
Whereas for the estimated incidence rates of critical illness, the rates are poorly fitted if we used the same functional form. Thus, we follow Gatenby (1991) by using the linear interpolation technique to find the single age incidence rates, and then we apply the spline function to smooth the estimated rates. The prevalence rates for both TPD and critical illness are shown below.

Figure 3.A.7: The Total and Permanent Disability Prevalence Rates by Age



[36]Other smoothing technique such as spline Beer (2011) or graphic method Gatenby (1991) may also be possible.

Figure 3.A.8: The Prevalence Rates for Critical Illness by Age



3.A.5 The Structure of Product Benefit Payments

Below is a sample of the quotes provided by Great Eastern Life Assurance (Malaysia) Berhad giving information on the premium and benefit structure of payments[37]. Quotes for males and females are same.

Figure 3.A.9: Annuity Benefit Payment for Age 35 (A1) for both males and females

Category A1	Age	Attained age by end of yr	Premium 1 (10 years)	Annuity benefit Option A	Death/TPD/CI benefit (Cash Value)
0	35	36	3000		1933
1	36	37	3000		4046
2	37	38	3000		6263
3	38	39	3000		8590
4	39	40	3000		11032
5	40	41	3000		13595
6	41	42	3000		16286
7	42	43	3000		19110
8	43	44	3000		22076
9	44	45	3000		25190
10	45	46			26443
11	46	47			27764
12	47	48			29158
13	48	49			30631
14	49	50			32188
15	50	51			33836
16	51	52			35584
17	52	53			37439
18	53	54			39411
19	54	55			41511
20	55	56		4365	44187
21	56	57		4365	42085
22	57	58		4365	39887
23	58	59		4365	37590
24	59	60		4365	35191
25	60	61		4365	32686
26	61	62		4365	30068
27	62	63		4365	27332
28	63	64		4365	24473
29	64	65		4365	21485
30	65	66		4365	18362
31	66	67		4365	15098
32	67	68		4365	11683
33	68	69		4365	8109
34	69	70		4365	4365

All payments are in terms of Malaysian Ringgit (MYR)
A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

[37] The TPD cash value benefit is only payable up to age 65.

Figure 3.A.10: Annuity Benefit Payment for Age 35 (A2) for both males and females

Category A2	Age	Attained age by end of yr	Premium 2	Annuity benefit Option A	Death/TPD/CI benefit (Cash Value)
0	35	36	3000		2069
1	36	37	3000		4326
2	37	38	3000		6695
3	38	39	3000		9182
4	39	40	3000		11793
5	40	41	3000		14533
6	41	42	3000		17411
7	42	43	3000		20433
8	43	44	3000		23607
9	44	45	3000		26941
10	45	46	3000		30444
11	46	47	3000		34126
12	47	48	3000		37996
13	48	49	3000		42065
14	49	50	3000		46347
15	50	51	3000		50853
16	51	52	3000		55599
17	52	53	3000		60600
18	53	54	3000		65873
19	54	55	3000		71439
20	55	56	3000	7747.5	78093
21	56	57		7747.5	74409
22	57	58		7747.5	70551
23	58	59		7747.5	66512
24	59	60		7747.5	62286
25	60	61		7747.5	57866
26	61	62		7747.5	53246
27	62	63		7747.5	48417
28	63	64		7747.5	43368
29	64	65		7747.5	38088
30	65	66		7747.5	32563
31	66	67		7747.5	26783
32	67	68		7747.5	20731
33	68	69		7747.5	14392
34	69	70		7747.5	7747

All payments are in terms of Malaysian Ringgit (MYR)
A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

Figure 3.A.11: Annuity Payment Benefit for Age 35 (B1) for both males and females

Category B1	Age	Attained age by end of yr	Premium 1 (10 years)	Annuity benefit Option B	Death/TPD/CI benefit (Cash Value)
0	35	36	3000		1861
1	36	37	3000		3899
2	37	38	3000		6037
3	38	39	3000		8281
4	39	40	3000		10636
5	40	41	3000		13108
6	41	42	3000		15703
7	42	43	3000		18426
8	43	44	3000		21286
9	44	45	3000		24288
10	45	46			25494
11	46	47			26764
12	47	48			28102
13	48	49			29514
14	49	50			31003
15	50	51			32577
16	51	52			34240
17	52	53			36001
18	53	54			37867
19	54	55			39847
20	55	56			41950
21	56	57			44189
22	57	58			46576
23	58	59			49125
24	59	60			51851
25	60	61		7432.5	55514
26	61	62		7432.5	51081
27	62	63		7432.5	46449
28	63	64		7432.5	41605
29	64	65		7432.5	36539
30	65	66		7432.5	31239
31	66	67		7432.5	25694
32	67	68		7432.5	19888
33	68	69		7432.5	13807
34	69	70		7432.5	7432

All payments are in terms of Malaysian Ringgit (MYR)
A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

Figure 3.A.12: Annuity Payment Benefit for Age 35 (B2) for both males and females

Category B2	Age	Attained age by end of yr	Premium 2	Annuity benefit Option B	Death/TPD/CI benefit (Cash Value)
0	35	36	3000		2043
1	36	37	3000		4273
2	37	38	3000		6614
3	38	39	3000		9071
4	39	40	3000		11650
5	40	41	3000		14355
6	41	42	3000		17200
7	42	43	3000		20186
8	43	44	3000		23321
9	44	45	3000		26614
10	45	46	3000		30074
11	46	47	3000		33709
12	47	48	3000		37531
13	48	49	3000		41550
14	49	50	3000		45777
15	50	51	3000		50226
16	51	52	3000		54911
17	52	53	3000		59847
18	53	54	3000		65052
19	54	55	3000		70545
20	55	56	3000		76346
21	56	57			82481
22	57	58			88975
23	58	59			95858
24	59	60			103163
25	60	61		15052.5	112428
26	61	62		15052.5	103452
27	62	63		15052.5	94069
28	63	64		15052.5	84260
29	64	65		15052.5	74000
30	65	66		15052.5	63267
31	66	67		15052.5	52036
32	67	68		15052.5	40278
33	68	69		15052.5	27962
34	69	70		15052.5	15052

All payments are in terms of Malaysian Ringgit (MYR)
A - annuity starts at age 55
B - annuity starts at age 60
1 - premium payment term 10 years
2 - premium payment term up to age (55/60) next birthday

Chapter 4

Paper 3

Ameliorating The Tragedy of Annuitisation: The Case of Altruism

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Abstract: Non-altruism is a key feature of an economy where full annuitisation of assets by individuals is optimal. Nonetheless, little is known whether the economy will be better off if all individuals fully annuitise their assets. Recent research work by Heijdra, Mierau, and Reijnders (2014) studies the impact of full annuitisation in a general equilibrium framework and finds a tragedy of annuitisation may arise: future newborn generation are worse off if an annuity market is opened as compared to the previous steady state where an annuity market never existed. This result holds for the case where accidental bequests are transfered to the young generation or even wasted by the government in the previous steady state economy before annuity market introduction. Our study aims to analyse the impact of annuitisation in an economy with the presence of a bequest motive. We show that for the case of altruism, the tragedy of annuitisation is ameliorated and may even be reversed.

Keywords: Tragedy of annuitisation, bequest motive, general equilibrium, annuities, insurance

4.1 Introduction

In the area of annuity studies, much literature has been inspired by the seminal theoretical work by Yaari (1965), which supports annuitisation of assets to produce an optimal life-cycle utility of individuals. More recently, Davidoff, Brown, and Diamond (2005) find that it is extremely hard to achieve optimality with a low annuitisation level within a rational individual life cycle framework. In fact, globally, the voluntary annuity market is quite small. Purcal and Piggott (2008) note that the presence of bequest motives may explains the thin annuity markets as individuals will only annuitise the part of assets that is in excess of their optimal bequest. This indicates that the level of altruism is an important factor in determining the optimal level of annuitisation for consumers — full annuitisation is only optimal if people are non-altruistic.

All of the above study the optimal annuitisation decision of a consumer at the individual level, thus providing little insight of where annuities stand at the whole economy level. Our research is motivated by a recent study of the impact of full annuitisation in a general equilibrium setting by Heijdra, Mierau, and Reijnders (2014). They find that future newborn generation may suffer from a welfare decrease if all individuals fully annuitise their assets. Starting from a steady state economy where accidental bequests are transfered to the young or even wasted by the government, opening an annuity market makes the economy worse off in the long run. For the former case, as accidental bequests are taken away from the young generation, they reduce their savings and this gives a negative impact to economy-wide capital intensity. While for the latter case, opening an annuity market increases the rate of interest since individuals are investing in annuities to get a higher return in the future period. In the long run, this can either induce savings or induce less savings depending on the elasticity of intertemporal substitution rates of the agents in the economy. For a range of plausible values of the elasticity of intertemporal substitution rate, $1/\gamma < 1$, individuals reduce their savings which gives a negative impact to the economy-wide capital intensity level. For both cases, in the long run, opening an annuity market induces capital crowding out, which reduces the steady state welfare of future newborn generation. In the non-altruistic world, full annuitisation may not be socially beneficial which leads to a tragedy of annuitisation.

Our study seeks an answer to whether this tragedy is observed in the presence of a bequest motive. Firstly, we add a bequest motive to the optimisation problem of the household as agent in our economy model. We apply a discrete time consumer optimisation problem developed in Fischer (1973) with consumption and bequest choice as the argument of the function, and allow for insurance and annuity purchases in the budget constraint of the household. The economy is characterised by a simple closed economy in an overlapping generations general equilibrium model, following Auerbach and Kotlikoff (1987), for three generations: young, middle age and old. For our simple model, we find that in the presence of bequest motives, the tragedy of annuitisation is ameliorated and households may even be better off if their assets are partially annuitised at the optimal level. In this paper, we allow for the life insurance purchases by households as there exist an intended bequest component which adds to the existing literature by Heijdra, Mierau, and Reijnders (2014) and Pecchenino and Pollard (1997) where there is no consideration of an intended bequest.

The rest of paper is structured as follows. Section 4.2 describes the model which consists of the characteristics of the household sector, production sector and the goods market equilibrium in the economy. Section 4.3 describes the calibration model and the adjustment of the household optimisation problem to allow for insurance and annuity purchases over the life-cycle. Section 4.4 contains results and discussion. Lastly, section 4.5 concludes.

4.2 Model

The economy model in this study is the overlapping generation general equilibrium (OLG-GE) model based on Auerbach and Kotlikoff (1987). This model has been used extensively in the economic literature. For example, Kudrna and Woodland (2011) study the impact of pension policy changes in the framework of the Australian economy. Heer (2001) also utilised the model to study the wealth distribution of households and the impact of inheritance taxation introduction in the US economy. A model for country comparison study of the impact of ageing populations is also developed using this framework by Merette and Georges (2010). In this paper, we develop a simple OLG-GE model that is quite similar to the closed economy as in Hansen and Imrohoroglu (2008).

4.2.1 Household Sector

Households in this economy model are characterised by the overlapping generations model of three generation ($i = 1$(young), 2(middle age), 3(old)). Each life phase consists of 25 years where a household who is born at age $x = 0$ will reach age 25, 50 and 75 years old if they survive the first, second and third life phase respectively. The young and middle age generation are the working population in the economy, while the old generation are retirees. Households face uncertainty of time of death with a survival probability from birth to the end of life phase i of ${}_ip_x$ that follows the SSA life table for cohort born in 2010 (Bell and Miller, 2005). The population of generation i at one time period is calculated using this formula: $Pop_i = N_t({}_ip_x/(1+n)^{i-1})$ where Pop_i is the population generation i, N_t is the number of individuals born at time t, and n is a constant population growth rate per year. We assume that the population in this model is stable and stationary with no population growth $n = 0$, which follows the model by Heer (2001) where households have a bequest motive[1].

The bequest model in this study follows the one parent one child policy as in Heer (2001). The age gap between parent who leave the bequest and child who inherit the bequest follows the Modigliani (1988) study which is assumed to be 25 years. Generation 1 may die at the beginning of the second life phase and leave a bequest to future period's generation 1 (their new born child), while generation 2 may die at the beginning of the third life phase and leave a bequest to future period's generation 2 (their children who are just entered the second life phase). Nobody survives beyond age 75 which means generation 3 should optimally consume all of their wealth at the end of the third period, unless they have a bequest motive post the final life phase. Since our model only consists of three life phase, there are only two possibilities for the parent to leave his bequest, either at the beginning of period two or at the beginning of period 3. Thus, we assume that households have no bequest motive beyond age 75 and any wealth that is not consumed in the final life phase is considered an accidental bequests[2]. Studies have shown that it is not clear whether bequests are solely

[1]This is a reasonable assumption following the decrease in the world population growth rate which is projected to be 0.33% by year 2050 and moves towards zero the further the projections go (United Nations, 2004). We are aware that a constant population growth rate of 1% is imposed by Hansen and Imrohoroglu (2008) and Heijdra, Mierau, and Reijnders (2014). However, the household optimisation problem in their model assumes the absence of bequest motive. They also assume full or partial annuitisation at a rate set exogenously. In contrast to their paper, we assume the existence of a bequest motive and solve the household optimisation problem for the optimal insurance or annuitisation level for each household. Solving the problem is more complicated with the presence of population growth, since the number of population who leave the bequest and receive the bequest may change over time. As this paper is our first attempt to study the implication of the insurance (and annuity) market introduction in the economy where households are altruistic, we thus, ignore population growth to reduce the complexity of the model. Allowing for positive population growth in the model is a direction for our future research.

[2]Accidental bequests exist in the traditional life cycle model where a household has no intended bequests, but keep their savings due to the need to have precautionary savings or due to annuity market imperfections (Cremer and Pestieau, 2006). Consistent with the one parent one child policy as in Heer (2001) that we adopt in our model, we assume that accidental bequests left by generation 3 at the end of the final life phase will be received by their children (who

intended or consists of accidental bequests and they co-exist in a household's life cycle (Dynan, Skinner, and Zeldes, 2002; Kopczuk, 2013). However, intended bequests are generally present because parents care about the lifetime utility of their children (Cremer and Pestieau, 2006). The middle age generation has the highest income in the economy, and by the time this generation enters the third life phase, they will only live for one more period. This implies that the old generation is no longer dependent on their parents. Thus, the assumption of no bequest motive beyond age 75 is not unreasonable.

For simplicity, we assume that households will survive the first life phase from age 0 to age 25 with a probability of 1. This assumption is also imposed by Heer (2001), which simplifies the bequest model since it eliminates the need for redistribution of accidental bequests left by generation 1 which exist if the asset holding of generation 1 is greater than 0 at the beginning of the period[3]. An individual who is born at time t optimises his lifetime expected utility following the function of two arguments, consumption and bequest over their life-cycle:

$$V = \left\{ \sum_{i=1}^{2} [{}_i p_x U(C_{i,t+i-1}) +_i p_x q_{x+i} v(G_{i+1,t+i})] \right\} +_3 p_x U(C_{3,t+2}) \qquad (4.2.1)$$

The utility function is an additive separable isoelastic utility function of the form:

have just entered the third life phase). In this model, even though the third generation has no bequest motive, they do not consume all of their wealth and thus accidental bequests exist because the total consumption of all generations must be equal to the aggregate consumption in the economy (obtained from the macroeconomic data). In our model, accidental bequests is introduced so that consumption level of the old generation which is solved numerically match with its analytical solution. We formulate the first and the second generation's consumption level and then, solve the third generation's consumption level numerically such that the aggregate consumption in the economy satisfies equation (4.2.11). Heijdra, Mierau, and Reijnders (2014) assume that accidental bequests are collected by the government and redistributed to the young generation only, or the old generation only, or wasted by the government. Hansen and Imrohoroglu (2008) have taken a different approach, where accidental bequests are redistributed equally to all surviving agents.

[3]Even though all newborn generations hold zero wealth at the age of 0, they may receive positive bequests from their parents if their parents die as soon as they were born. Since consumption will only occur at the end of each period, allowing for a probability of dying greater than 0 at the beginning of the first life phase would imply that the newborn generation will leave positive accidental bequests (this is the intended bequests left by their parents).

$$U(C_{i,t}) = (1+\rho)^i \frac{C_{i,t}^{1-\gamma}}{1-\gamma} \quad (4.2.2)$$

$$v(G_{i,t}) = \hat{b}_i \frac{G_{i,t}^{1-\gamma}}{1-\gamma} \quad (4.2.3)$$

where

$C_{i,t} \equiv$ the optimal consumption path of generation i at time t

$G_{i,t} \equiv$ the optimal bequest choice of generation i at time t

$\rho \equiv$ the rate of time preference

$\gamma \equiv$ the coefficient of relative risk aversion

$\hat{b}_i \equiv$ the bequest weighting function of generation i

The households optimise the above value function subject to the following budget constraint:

$$W_{i+1,t+i} = R_{t+i-1}[W_{i,t+i-1} + I_{i,t+i-1} + Y_{i,t+i-1}(p_{x+i}/R_t) - C_{i,t+i-1}/R_{t+i-1}]$$

where R_{t+i-1} is the rate of interest factor $(1+r_{t+i-1})$ at time $t+i-1$, $W_{i,t+i-1}$ is the stock of assets of generation i at time $t+i-1$, and $I_{i,t+i-1}$ is the expected inheritance received by generation i at time $t+i-1$ given as the probability of dying of the parent multiplied with the parent's current assets $q_{x+i}(G_{i+1,t+i-1})$ where $(G_{i+1,t+i-1} = W_{i+1,t+i-1})$[4]. In the stationary state, the amount of inheritance received by each generation from their parents should be the same as the amount of bequest left by them to their children, $(I_{i,t+i-1} = q_{x+i}G_{i+1,t+i})$. We drop the subscript $t=1$ where the probability of a person age $x+i$ dies at the beginning of the following year period is denoted as $p_{x+i} = {}_1p_{x+i}$. For the old

[4]Based on the one parent one child policy, there are two sorts of period one individuals, one who receive bequest as the parent dies and another who do not receive bequest as the parent still alive. Here, we apply the expected inheritance concept which implies that the total bequests leave by one generation are redistributed equally between the whole of the younger generation — that is, all individuals receive the same expected bequests level. This will produce the same consumption level for all individuals in a particular generation, as opposed to a different level of consumption for these two sorts of individuals. However, the aggregate consumption level are equal for both methods of calculation. The aggregate utility of both group of individuals calculated using the average consumption level per capita where we assume that all individuals in the same generation have the same average consumption level as compared to the aggregate utility calculated using a different level of consumption for these two sorts of individuals (as a result of a different bequest level received) only differs slightly — they are the same up to two decimal places. Thus, our results are not affected by the use of expected value in our analysis.

generation (generation 3), the accidental bequests may be positive at the end of the final life phase. We denote this as $G_{3,t+3}$, but households do not value this in their expected utility function. The labour income $Y_{i,t}$ is discounted by the probability of surviving p_{x+i}, which indicates that this is the expected labour income to be received by the household only if he survives to the end of time $t+i-1$ or to the beginning of $t+i$, which is the concept of labour income used in Fischer (1973).

We apply the method as in Georges (2013) to calculate the labor income for the working population young and middle age generation. The labor income for generation i is given by: $Y_{i,t} = w_t LabEP_i$ where w_t is the price of labour at time t, and Lab is the number of non-effective units of labour offered by any individual in the economy calculated as the ratio of the effective number of units of labour available in the economy divided by a weighted earning profile (EP_i) where the weight is the size of each cohort in the economy, $Lab = L_t / \sum_{i=1}^{3}(Pop_i EP_i)$. The earning profile for each generation (taking into account the efficiency and productivity factor as in Hansen and Imrohoroglu (2008)) is estimated econometrically according to parameter values used by Georges (2013) as follows: $EP_i = 1 + 0.25(i) - 0.0285(i^2)$. EP_3 is assumed to be 0 as generation 3 is the retired population in this economy.

4.2.2 Production Sector

The production sector consists of a large number of identical and perfectly competitive firms, where a representative firm has a constant return to scale Cobb-Douglas production function of the following form:

$$Q_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{4.2.4}$$

where Q_t is the total output produced by the firm with two input factors, the capital inputs K_t and labour inputs L_t. The share of capital and labour inputs in production are given by α and $(1-\alpha)$, respectively, where the total share parameter for both inputs equals to 1 which indicates that an increase of 1 unit of inputs has an impact of the same level of increase in the total output. A_t is the total productivity factor which depends on factors of production other than capital and labour inputs. A_t is assumed to be a constant, as in Heijdra, Mierau, and Reijnders (2014) and Heer (2001). Over time, the capital stock depreciates at rate of δ. As the economy reaches a steady state, the aggregate investment only covers the depreciation of capital stock so that the capital stock level remains constant over time and satisfies the condition:

$$K_{t+1} = (1-\delta)K_t + X_t \tag{4.2.5}$$

where X_t is the aggregate investment at time t.

Each firm maximises its profit subject to the cost function:

$$\pi_t = PQ_tQ_t - (Rent_tK_t + w_tL_t) \tag{4.2.6}$$

where

$$\begin{aligned}
\pi_t &\equiv \text{the firm's profit} \\
PQ_t &\equiv \text{the price of goods} \\
Rent_t &\equiv \text{the rental price of capital adjusted} \\
&\ \text{for the depreciation cost } (r_t + \delta) \\
w_t &\equiv \text{the price of labour (wage rate)}
\end{aligned}$$

Solving the above maximisation problem using the lagrangian approach[5], we obtain the first order conditions as follows:

$$K_t = \frac{\alpha PQ_tQ_t}{Rent_t} \tag{4.2.7}$$

$$L_t = \frac{(1-\alpha)PQ_tQ_t}{w_t} \tag{4.2.8}$$

We follow the assumption used in Georges (2013) by assuming that the price of goods in the economy is set as the numeraire, where $PQ_t = \$1$ for all periods. Thus, all other variables are expressed in the units relative to the price of goods in the economy.

4.2.3 Goods Market Equilibrium

The economy model in this study features a closed economy, such as in the Hansen and Imrohoroglu (2008), but without the role of government since we do not consider taxation in this model. The economy finds the rental price of capital after adjustment for depreciation cost $(r_t + \delta)$ and the wage rate w_t that are

[5] Derivation of the first order conditions for the production sector optimisation problem is shown in the Appendix.

determined endogenously by satisfying the market clearing conditions below:

$$K_t = \sum_{i=1}^{3}(Pop_i W_{i,t}) \tag{4.2.9}$$

$$L_t = \sum_{i=1}^{3}(Pop_i LabEP_i) \tag{4.2.10}$$

$$Q_t = \sum_{i=1}^{3}(Pop_i C_{i,t}) + X_t \tag{4.2.11}$$

4.3 Calibration

The model is solved numerically using the GAMS software (Brook, Kendrick, and Meeraus, 1988). The calibration of the model is done by assuming that the economy is at a steady state. Some technology parameters are taken from Hansen and Imrohoroglu (2008), such as the capital share parameter α. Particularly, these parameters reflect the characteristics of the US macroeconomy. We also target the capital-output ratio and the investment-output ratio to be approximately 3.3 and 0.25, as in Hansen and Imrohoroglu (2008). The discount factor for the consumption utility function, which consists of the rate of time preference, the depreciation rate δ and the total productivity factor A_t are close to parameters in Heijdra, Mierau, and Reijnders (2014). The bequest weighting function in the household optimisation problem follows bequest parameter values as in Fischer (1973), where higher weighting is assigned as the family dependency becomes important in life. Table 4.3.1 summarises parameters used in our benchmark calibration model.

4.3.1 Introduction of the Insurance/Annuity Market

We introduce the perfect insurance and annuity market in this model by opening up the market at time period 8. As the market is opened, households may insure or annuitise at the optimal level, which is determined by solving the household optimisation problem as in Eq. (4.2.1), but adjusting the budget constraint to allow for insurance and annuity purchase. At time t, if the household survives to the following period, the next period's stock of wealth is: $W_{i+1,t+i} = [W_{i,t+i-1} + I_{i,t+i-1} + Y_{i,t+i-1}(p_{x+i}/R_{t+i-1}) - C_{i,t+i-1}/R_{t+i-1}](1 - m_{i,t+i-1})R_{t+i-1}$, where $m_{i,t+i-1}$ is the proportion of assets to be insured (or annuitised) by the household — which is simply equal to 0 when the insurance or annuity market does not exist.

Table 4.3.1: Parameter Values for Benchmark Calibration Model

	Notation	Value
Demographics		
survival probability	p_x	SSA, cohort born in 2010
Technology		
capital share parameter	α	0.36 (Hansen and Imrohoroglu, 2008)
depreciation rate	δ	0.07 (Heijdra, Mierau, and Reijnders, 2014)
productivity factor	A_t	2.17 (Heijdra, Mierau, and Reijnders, 2014)
Utility function		
rate of time preferences	ρ	0.02
coefficient of relative risk aversion	γ	3
bequest weighting parameter	$\hat{b_i}$	$\hat{b_2} = 1.01$, $\hat{b_3} = 0.94$ (Fischer, 1973)

In Section 4.2.1, without the insurance/annuity market, the bequest left by a household will be the same as the current stock of wealth of the household.

In the case of insurance and annuity market existence, if the household dies at the beginning of the next period, his bequest level will also include the insurance benefit if he insures (or the annuity premium paid if the household annuitises). Thus, the bequest function is given by the following equation: $G_{i+1,t+i} = [W_{i,t+i-1} + I_{i,t+i-1} + Y_{i,t+i-1}(p_{x+i}/R_{t+i-1}) - C_{i,t+i-1}/R_{t+i-1}][(1-m_{i,t+i-1})R_{t+i-1} + m_{i,t+i-1}Q_{i,t+i-1}]$ where $Q_{i,t+i-1}$ is an actuarially fair single period accumulation on a 1 unit premium paid following the death of the insured — $Q_{i,t+i-1} = R_{t+i-1}/q_{x+i}$ can also be be viewed as the sum insured for a \$1 premium, or even as the amount an annuitant's estate must pay on the death of annuitant in a single period annuity contract paying \$1 at initiation. We illustrate further these payments from the perspective of an insurer below. Figure 4.3.1 illustrates the insurance benefit payment associated with \$1 insurance premium — for a \$1 insurance premium received at time 0, in the event of death of the insured at the end of the period with probability q_{x+i}, the insurer pays a fair single period accumulation of R_{t+i-1}/q_{x+i}. Figure 4.3.2 illustrates the annuity premium payment associated with a contract paying \$1 at initiation — for an annuity contract paying \$1 annuity benefit to an annuitant at time 0, in the event of death of the annuitant at the end of the period with probability q_{x+i}, the insurer receives the annuity premium of R_{t+i-1}/q_{x+i}.

As this optimisation model follows the framework for life insurance purchases as in Fischer (1973), it is not obvious that this framework can also be applied to annuity purchases. Intuitively, negative values of $m_{i,t}$ would indicate the shorting

Figure 4.3.1: Life Insurance Payments (an insurer's perspective)

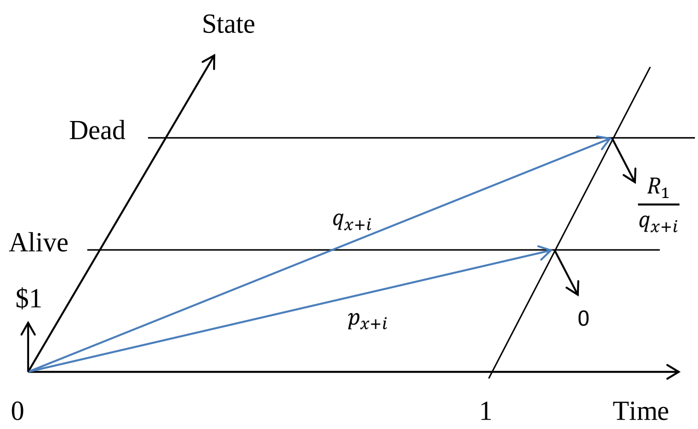
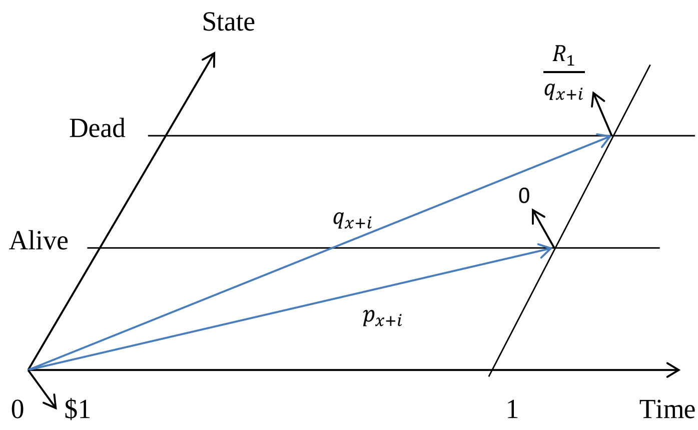


Figure 4.3.2: Annuity Payments (an insurer's perspective)



of life insurance which is the same as purchasing annuities[6]. There is no simultaneous purchase of life insurance and annuity at a time as the parameter value $m_{i,t}$ will either take positive (annuity purchase) or negative (life insurance purchase) value to optimise the household's expected utility. This result is identical to the theoretical work of Fischer (1973), where consumer will only purchase life insurance when the bequest motive is high, resulting in a higher intended bequest than his current wealth. On the other hand, consumer will only purchase annuity when the bequest motive is low, resulting in a lower intended bequest than his current wealth, thus, it would be optimal for the consumer to annuitise his wealth in excess of his intended bequest. Solving the above optimisation problem using the recursive dynamic programming technique leads us to the first order conditions of the household sector[7]:

$$C_{1,t} = \frac{\hat{k}_{1,t}^{-1/\gamma}\left[W_{1,t} + I_{1,t} + I_{2,t+1}\left\{\frac{p_{x+1}}{R_t}\right\} + \left\{I_{3,t+2} - \frac{G_{3,t+3}}{R_{t+2}}\right\}\left\{\frac{2p_{x+1}}{R_t R_{t+1}}\right\}\right]}{(R_t^{-1/\gamma} + \hat{k}_{1,t}^{-1/\gamma}/R_t)} + \frac{\hat{k}_{1,t}^{-1/\gamma}[\{Y_{1,t} + Y_{2,t+1}(2p_{x+1}/R_{t+1})\}(p_{x+1}/R_t)]}{(R_t^{-1/\gamma} + \hat{k}_{1,t}^{-1/\gamma}/R_t)} \tag{4.3.1}$$

$$C_{2,t+1} = \frac{\hat{k}_{2,t+1}^{-1/\gamma}\left[W_{2,t+1} + I_{2,t+1} + \left\{I_{3,t+2} - \frac{G_{3,t+3}}{R_{t+2}}\right\}\left\{\frac{p_{x+2}}{R_{t+1}}\right\} + Y_{2,t+1}\left\{\frac{p_{x+2}}{R_{t+1}}\right\}\right]}{(R_{t+1}^{-1/\gamma} + \hat{k}_{2,t+1}^{-1/\gamma}/R_{t+1})} \tag{4.3.2}$$

$$C_{3,t+2} = R_{t+2}[W_{3,t+2} + I_{3,t+2} - G_{3,t+3}/R_{t+2}] \tag{4.3.3}$$

$$m_{i,t} = \frac{R_t(1-\beta_{i,t})}{R_t + \beta_{i,t}(Q_{i,t} - R_t)} \tag{4.3.4}$$

and where

$$\hat{k}_{2,t+1} = \left(\frac{p_{x+2}R_{t+2}^{1-\gamma}}{(1+\rho)}\right)\{(1-m_{2,t+1})R_{t+1}\}^{1-\gamma} + (q_{x+2}\hat{b}_3)\{(1-m_{2,t+1})R_{t+1} + m_{2,t+1}Q_{t+1}\}^{1-\gamma},$$

$$\hat{k}_{1,t} = \left(\frac{p_{x+1}\sigma_{2,t+1}^{1-\gamma}}{(1+\rho)}\right)\{(1-m_{1,t})R_t\}^{1-\gamma} + (q_{x+1}\hat{b}_2)\{(1-m_{1,t})R_t + m_{1,t}Q_t\}^{1-\gamma},$$

$$\sigma_{2,t+1} = \hat{k}_{2,t+1}^{-1/\gamma}/(R_{t+1}^{-1/\gamma} + \hat{k}_{2,t+1}^{-1/\gamma}/R_{t+1}),$$

$$\beta_{2,t+1} = \left[\frac{Q_{2,t+1} - R_{t+1}}{R_{t+1}} \cdot \frac{q_{x+2}\{\hat{b}_3\}}{p_{x+2}R_{t+2}^{1-\gamma}\{1+\rho\}^{-1}}\right]^{-1/\gamma},$$

$$\beta_{1,t} = \left[\frac{Q_{1,t} - R_t}{R_t} \cdot \frac{q_{x+1}\{\hat{b}_2\}}{p_{x+1}\sigma_{2,t+1}^{1-\gamma}\{1+\rho\}^{-1}}\right]^{-1/\gamma}.$$

[6] Here, without the presence of insurance and annuity markets $m_{i,t} = 0$. With the presence of insurance and annuity markets, $m_{i,t} = 1$ indicates full insurance and $m_{i,t} = 1 - 1/(1 - q_{x+i})$ indicates full annuitisation. For example, if $q_{x+i} = 0.2$ then full annuitisation rate is at $m_{i,t} = -0.25$. Refer to Hansen and Imrohoroglu (2008) for the partial equilibium optimisation problem under the annuity purchase framework.

[7] Full derivation of the first order conditions for the household optimisation problem is shown in the Appendix.

4.4 Findings

Our findings are divided into three subsections. We experiment with the model by considering three cases: the first case is where the insurance (and annuity) market is opened to the young generation only. This scenario is not very realistic but we can think of the scenario that may occur if the sale of insurance is restricted to the low risk group only as insurers are usually selective in offering life insurance. The second case is where the insurance (and annuity) market is opened to the middle age generation only. This scenario is rather more realistic since in the real world, decisions about life insurance and annuity purchases often made by the middle age generation, whereas the young generation who are still studying or just started working are often oblivious to such concerns. The third case is what we call the reality scenario: where the insurance (and annuity) market is opened to all generations. For the old generation, opening the market makes no difference to them in this model, because (one plus) the return on insurance depends on the probability of dying at the beginning of the next period $Q_{i,t} = R_t/q_{x+i}$: here, $q_{x+3} = 1$ so (one plus) the rate of return $Q_{3,t} = R_t$. Realistically, nobody will annuitise given the certainty of dying after the third life phase and, similarly, insurers are reluctant to sell insurance to individuals whose death is certain in the next period of life.

4.4.1 Case 1: Insurance/Annuities only for Young Generation

We conduct the analysis using two categories of altruism given by the values assigned to the bequest weighting function $\hat{b}_t$. According to Eq. (4.3.4), the optimal insurance (or annuitisation) level for an individual in this economy depends on the degree of his bequest motive for the next period. In other words, the young generation decides the proportion of assets to be insured (or annuitised) based on the next period's degree of bequest motive which determines his bequest level if he dies during his middle age life phase. For the low altruistism category $\hat{b}_2 = 0.97^{25}$, the young generation will annuitise at the optimal level of $m_{1,t} = -0.003770$. For the high altruistism category $\hat{b}_2 = 1.01^{25}$ as in the benchmark steady state parameter value, the young generation will insure at the optimal level of $m_{1,t} = 0.009161$. As mentioned in Section 4.3.1, the insurance (and annuity) market is opened at time period 8. For time periods 1 to 7, the economy is in a steady state where the insurance (and annuity) market does not exist. The economy makes a transition from no insurance (and annuity) markets to an economy with insurance (and annuity) markets and reaches the steady state around time period 22. Figure 4.4.1 and Figure 4.4.2 present the results of Case 1 for the low altruistism category and high altruistism category respectively, where panels Con(Gi) represent the consumption levels for generations ($i = 1, 2, 3$) and Beq(Gi) represent the bequest levels for generations ($i = 2, 3$). Recall that generation 1 does not leave any bequest as the probability of surviving the first period is 1.

If the young generation annuitises (refer to Figure 4.4.1), at the shock time the young generation benefit from annuitisation as they enjoy the extra income from the annuity which increases their consumption level (panel Con (G1)), but in the long run, the bequest level left by the middle age generation to the young generation decreases (panel Beq (G2)). This has an impact on the economy-wide capital intensity ($k_t = K_t/L_t$), which falls pushes the rental price of capital r_t up. As we are in a dynamically efficient economy where ($r_t > n$), the decrease in capital intensity decreases the aggregate output level in the economy. As the total output decreases, this has a negative impact to the wage rate and as a result, the future newborn generation suffer a welfare decrease in the long run steady state economy ($\Delta U = -51.231\%$).

If the young generation insure (refer to Figure 4.4.2), at the shock time the young generation suffer the most (panel Con (G1)) as they still receive the same bequest level as in the steady state economy before the insurance (and annuity) market introduction and are also paying for the insurance premium. Their consumption level drops substantially at this point in time. However, in the long run, the bequest level left by the middle age generation to the young generation increases by about 20 units (panel Beq (G2)). This has a positive impact on the economy-wide capital intensity k_t which increases, thus reducing the rental price of capital r_t. Overall, the total output in the economy increases and pushes up the wage rate which then increases the welfare of future newborn generations in the long run steady state economy ($\Delta U = 56.938\%$). As shown in Figure 4.4.2, the positive impact to the new steady state wage rate in the economy also has a positive impact to all generations where consumption and bequest level for all generations increase in the new steady state economy.

4.4.2 Case 2: Insurance/Annuities only for Middle Age Generation

Similarly, the middle age generation decides on the proportion of assets to be insured (or annuitised) based on the next period's degree of bequest motive which

Figure 4.4.1: Case 1: Low Altruism Category

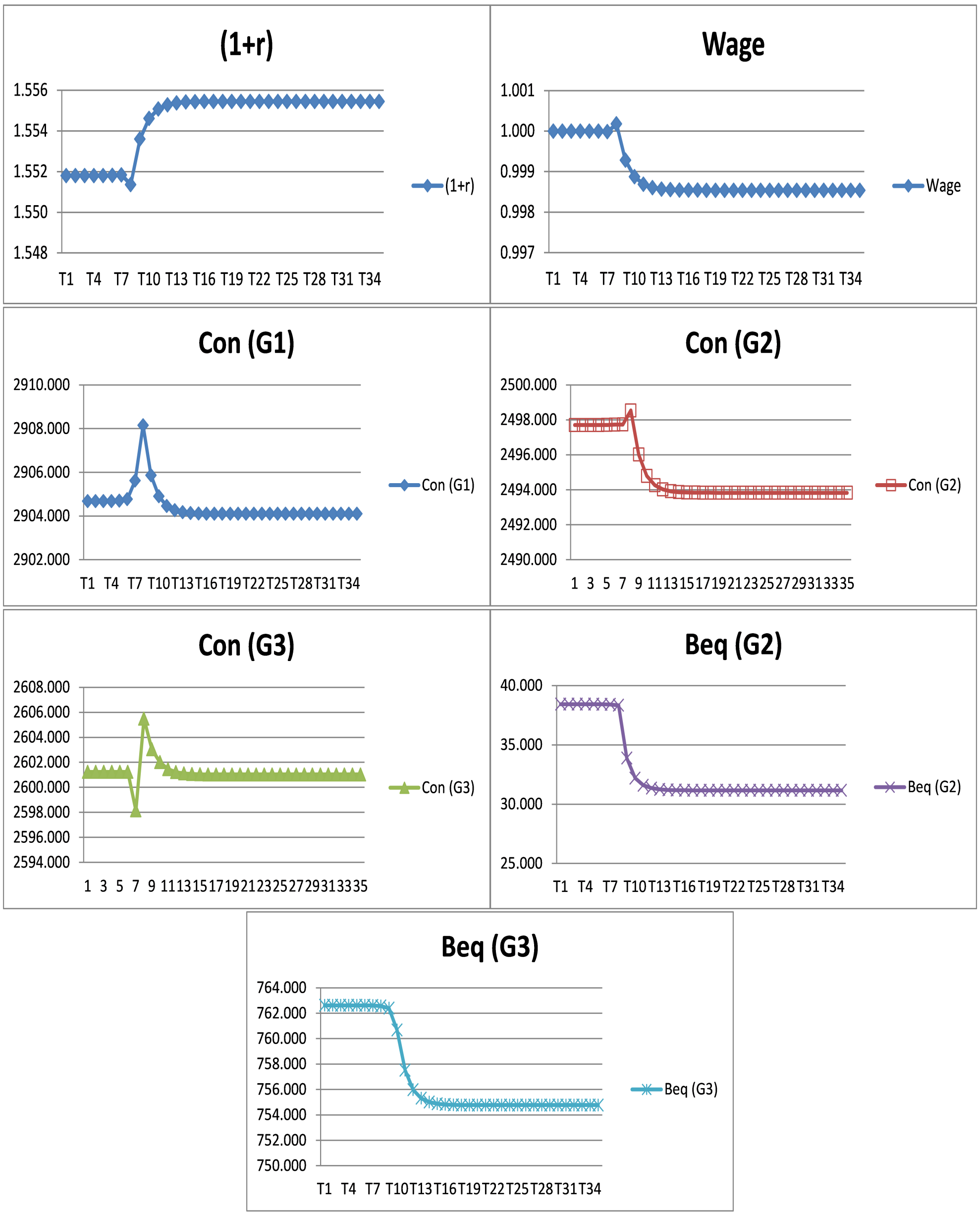
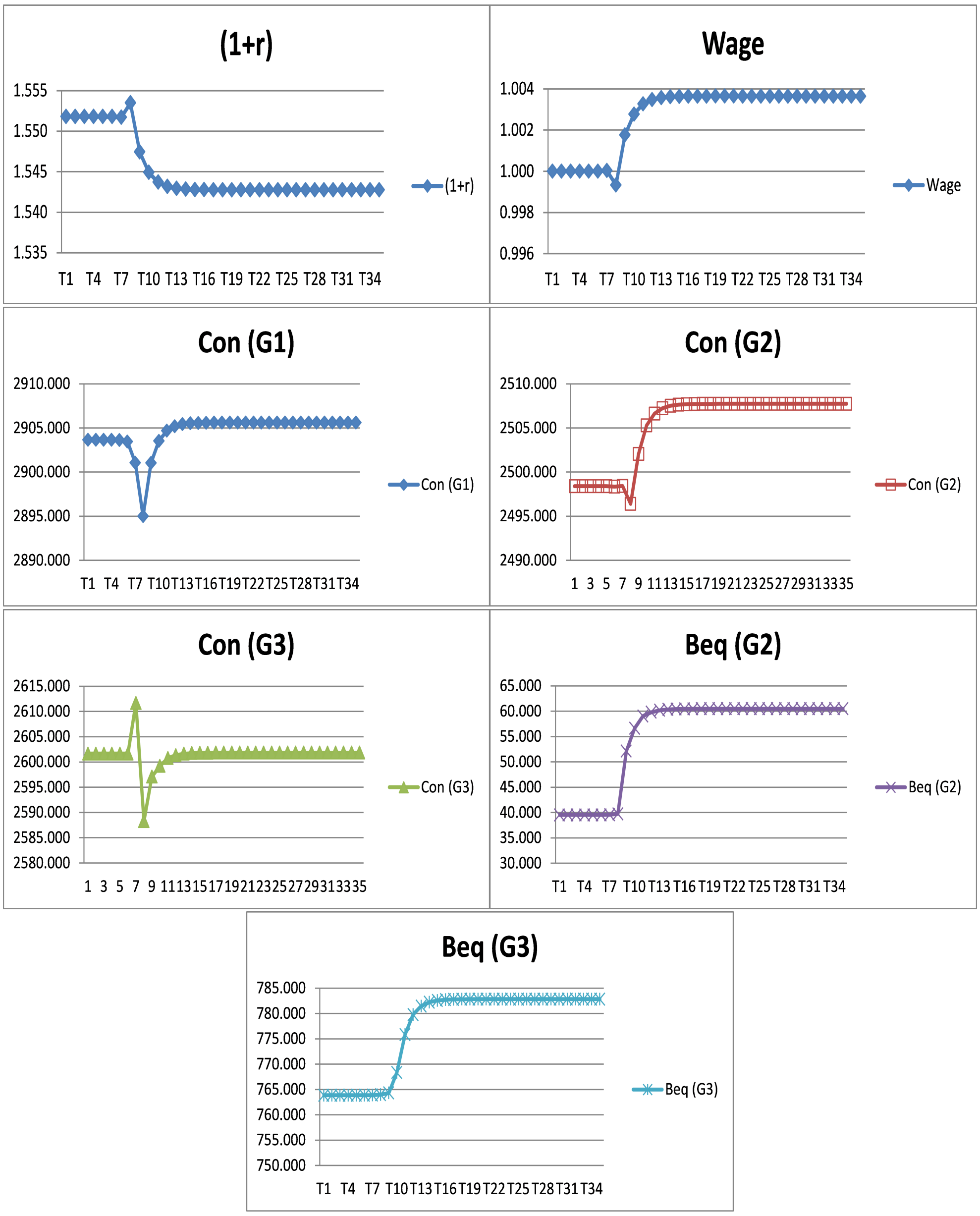


Figure 4.4.2: Case 1: High Altruism Category

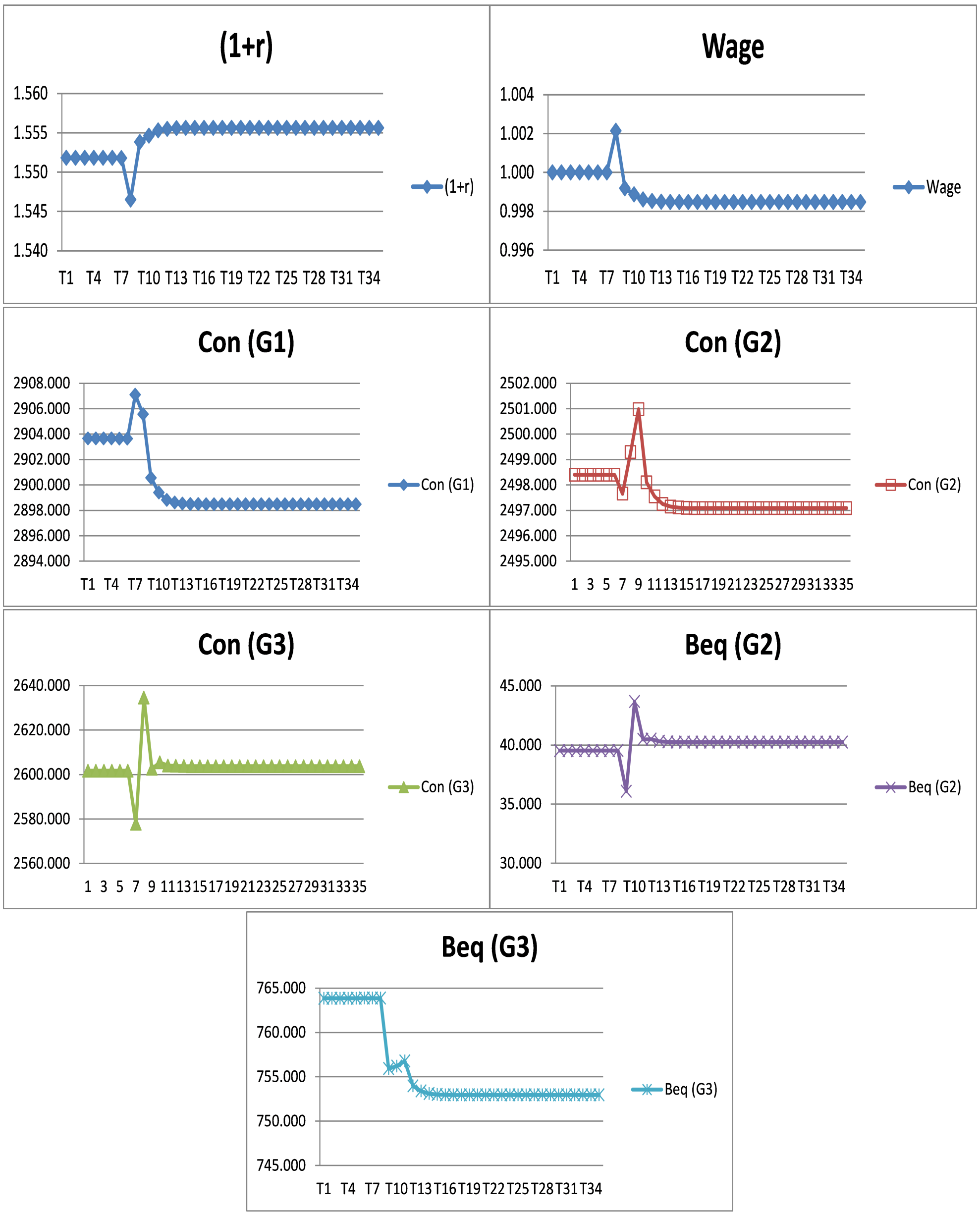


determines the level of bequest if he dies during his old life phase. For this case, we also experiment with the model using two categories of altruism level: high and low. For the low altruistism category $\hat{b}_3 = 0.94^{25}$ as the benchmark steady state parameter value, the middle age generation will annuitise at the optimal level $m_{2,t} = -0.001282$. For the high altruistism category $\hat{b}_3 = 0.953^{25}$, the middle age generation will insure at the optimal level $m_{2,t} = 0.005137$. Figure 4.4.3 and Figure 4.4.4 present the results of Case 2 for the low altruistism category and high altruistis category respectively.

If the middle age generation annuitises (refer to Figure 4.4.3), at the shock time the middle age generation increase their consumption (panel Con (G2)) as they enjoy the extra income from annuities. However, in the long run, the new steady state level of bequest left by the old generation (panel Beq (G3)) reduces as those dying forfeit the capital associated with their annuity premium. This has a negative impact on the economy-wide capital intensity k_t which decreases, thus increasing the rental price of capital r_t. As we have seen before, as the price of capital increases, the new steady state level of the wage rate reduces.

For the case where the middle age generation annuitises, we notice that the changes in welfare for all generations do not solely depend on the level of the new steady state wage rate. It is also important to study the behaviour of households as the interest rate changes. According to Heijdra, Mierau, and Reijnders (2014), as the interest rate increases, for a range of plausible values of the elasticity of intertemporal substitution $1/\gamma < 1$, households who do not value bequests will increase their consumption. In our results, we observe that this is the behaviour of the old generation. As a result of an increase in the rate of interest in the new steady state economy, the consumption level of the old generation increases, thus increases their welfare. However, the behaviour of the young generation is opposite to the behaviour of the old generation. As the young generation values bequests, they will save more as interest rates increase. Thus, the young generation gain utility from leaving a higher bequest in the next period of life even though their consumption level in the new steady state economy decreases. Overall, there is an increase in the lifetime welfare level of future newborn generations in the new steady state economy ($\Delta U = 3.506\%$). This is at the expense of welfare of the middle age generation which reduces by ($\Delta U = -1.375\%$). The behaviour of the middle age generation is similar to the young generation as they also value bequests. They consume less and save more as interest rate increases. However, the bequest level left by them in the next period of life still decreases in the new steady state economy as they also stand to forfeit the capital associated

Figure 4.4.3: Case 2: Low Altruism Category



with their annuity premium should they die.

If the middle age generation insure (refer to Figure 4.4.4), at the shock time the middle age generation reduce their consumption substantially (panel Con (G2)) as they have to pay for the insurance premium. However, the bequest level left by them in the next period of life increases as a result of the insurance payment. This has a positive impact on the economy-wide capital intensity k_t which increases, thus decreasing the rental price of capital r_t. As a result, the new steady state level of wages increases.

Again, we observe that the changes in welfare for all generations for this case do not solely depend on the positive change in the wage rate, but also depend on the behaviour of the household as interest rate changes. Here, the old generations who do not value bequests will consume less as interest rate decreases (panel Con (G3)). A possible reason for this is the need for a precautionary savings. As the value of their savings in the future is less as interest rates decrease, they have to save more to avoid outliving their assets before they die. Thus, their welfare decreases as their consumption level decreases.

For households with bequest motives (young and middle age generation), the behaviour is the opposite to the behaviour of households without bequest motive. They have to make a choice between consuming or saving more as they gain in utility from both. We observe that as the interest rate decreases, the young and middle age generation will consume more (panel Con (G1 $ G2)). This might be due to a decrease in value of their savings in the future, thus, make them value bequests less than the previous steady state level[8]. As the young generation consume more and save less, the bequest levels left by them in the next period of life decreases (panel Beq (G2)). This has a negative impact to their welfare level in the new steady state economy. Meanwhile, for the middle age generation, consuming more and saving less does not reduce their bequest level as they provide an insurance benefit if they die in the next period of life (panel Beq (G3)). Thus, the only generation that gains in utility here is the middle age generation. Overall, the lifetime welfare of future newborn generation decreases ($\Delta U = -15.579\%$).

4.4.3 Case 3: Reality (Insurance/Annuities for all)

For the last experiment, which we refer as the reality case, the level of altruism follows a more realistic bequest motive parameter value as in Fischer (1973),

[8]Refer to a study of Elmendorf (1996) which finds that there is a positive interest elasticity of savings for bequests leaver as they save more when interest rate increases.

Figure 4.4.4: Case 2: High Altruism Category

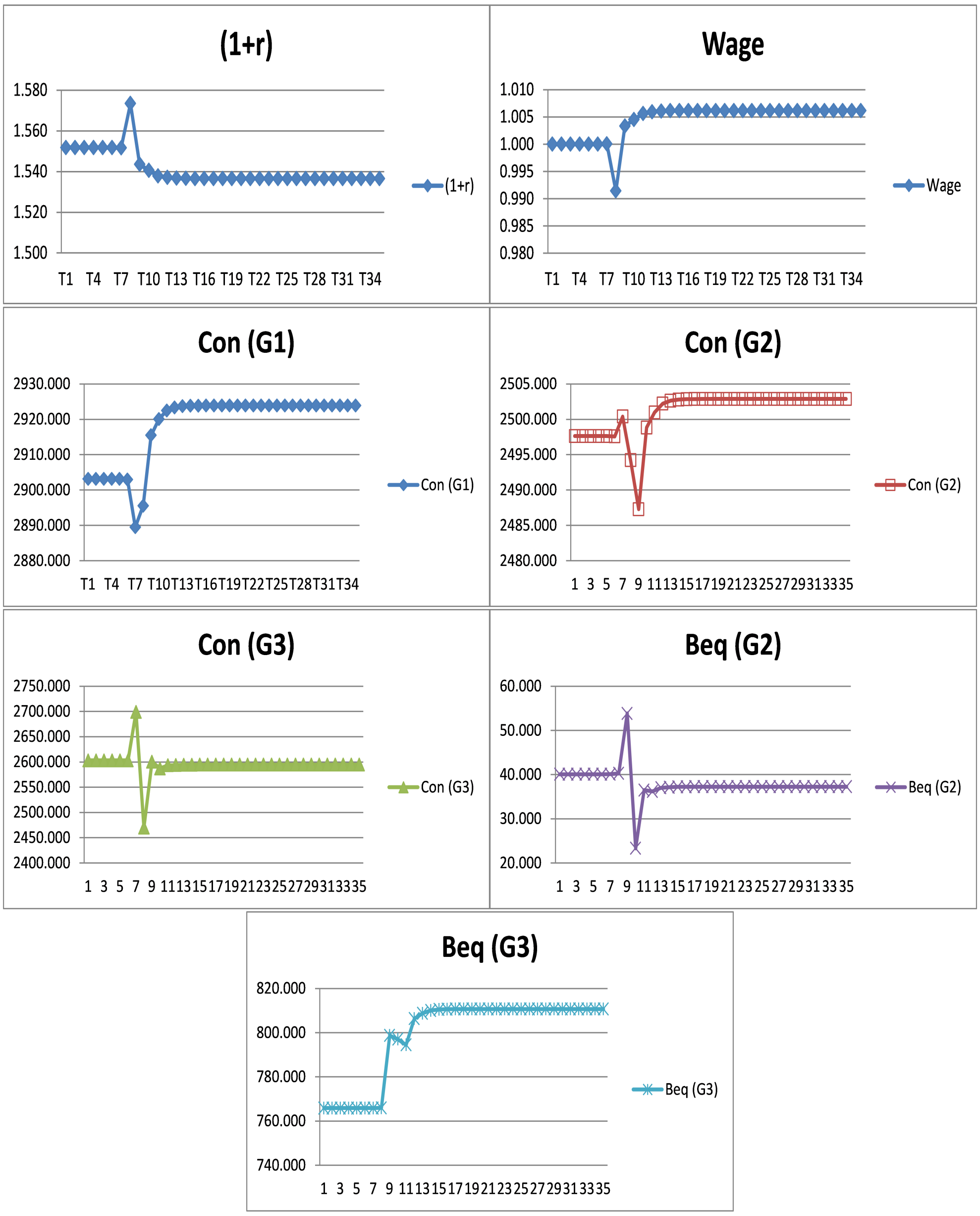


Table 4.4.1: Welfare changes at a new steady state economy

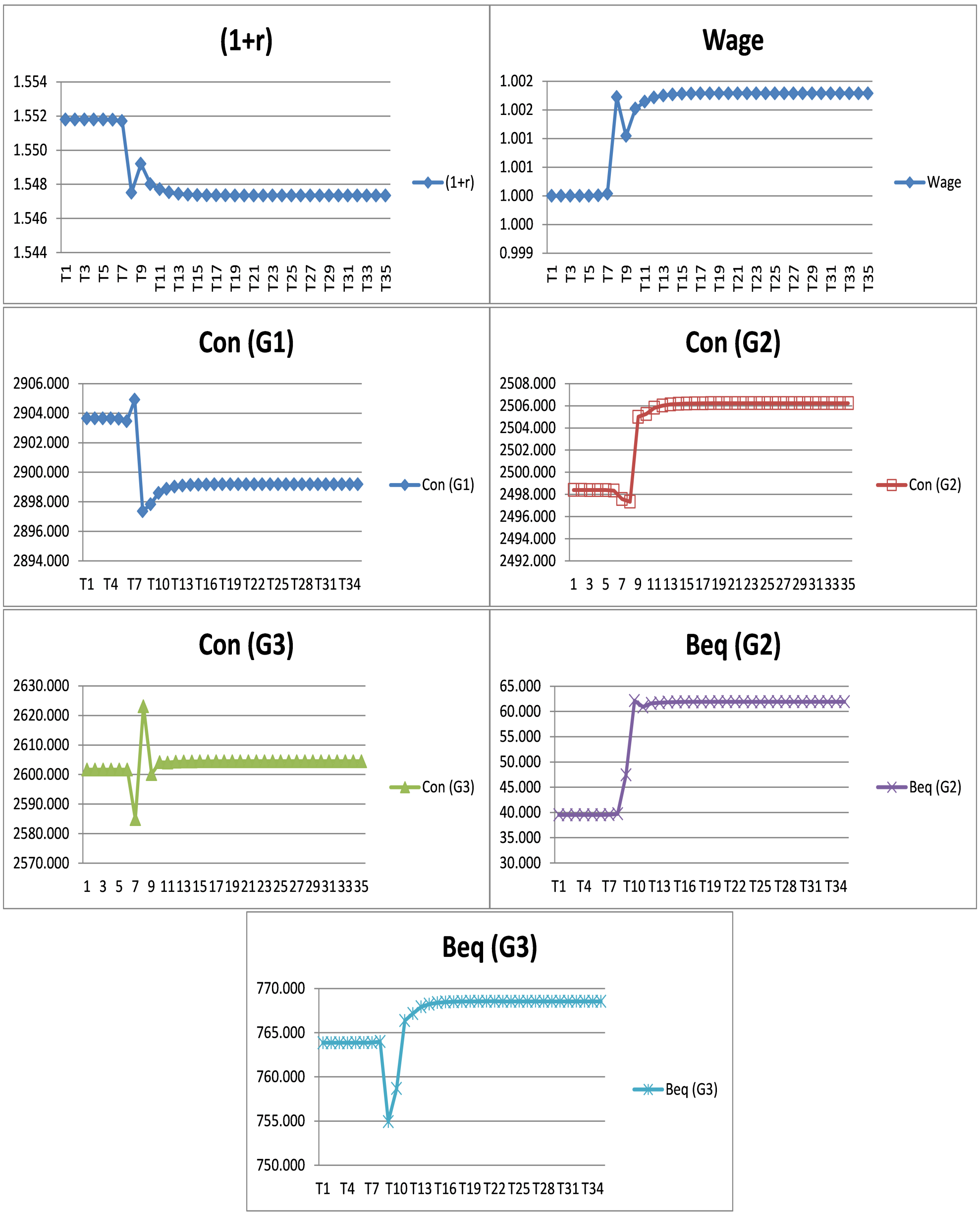
Case	$\Delta U(\%)$			
	G1	G2	G3	All
1 - G1 only annuitise	-51.748	-1.116	-0.016	-51.231
1 - G1 only insure	57.172	2.567	0.014	56.938
2 - G2 only annuitise	3.525	-1.375	0.158	3.506
2 - G2 only insure	-15.672	5.541	-0.656	-15.579
3 - Reality (G1 insure, G2 annuitise)	59.115	0.890	0.217	58.868

with a high value assigned when family dependency is important — which for this economy model is the middle age generation. Thus, the bequest motive parameter is similar to the benchmark steady state economy where $\hat{b}_2 = 1.01^{25}$ and $\hat{b}_3 = 0.94^{25}$. For these bequest motive parameter values, the young generation will insure at optimal level $m_{1,t} = 0.009289$ and the middle age generation will annuitise at optimal level $m_{2,t} = -0.001703$. We present the results of Case 3 in Figure 4.4.5.

Referring back to the findings that we obtained in Case 1 where the young generation insures and in Case 2 where middle age generation annuitises, the combination of these two cases makes the economy better off than any other states considered previously with an increase of welfare for future newborn generation by ($\Delta U = 58.868\%$). This indicates that in the presence of bequest motives, opening up an insurance (and annuity) market may reverse the tragedy of annuitisation as observed in Heijdra, Mierau, and Reijnders (2014). For all cases considered above in our numerical analyses, we observed that the role of intergenerational transfer in the form of bequest from old generation to young generation is important as a welfare boosting factor for the whole economy. Table 4.4.1 summarises the findings from our numerical analyses, which consists of the changes of welfare during the transition from the benchmark steady state economy where insurance (and annuity markets) do not exist to the new steady state economy where insurance (and annuity) markets are opened for each generation together with the overall change in lifetime welfare for the future newborn generation.

According to the summary of welfare changes over the transition to the new steady state economy with insurance (and annuity) markets in Table 4.4.1, the overall future newborn generation welfare (given by the column "All") indicates the expected lifetime utility of future newborn at a new steady state economy which consists of the household's expected utility for all three life phases, young, middle age and old. There are three cases that improve the welfare level of future

Figure 4.4.5: Case 3: G1 - insure, G2 - annuitise



newborn generations: first, when the young generation insures, second, when the middle age generation annuitises, and lastly, when the young generation insures and the middle age generation annuitises. However, if we focus on the welfare changes of each generation individually during this transition, we observe that for some cases, even though the future newborns gain from welfare increase, not all generations benefit from the transition. For example, for the case where only the middle age generation annuitises, while others gain higher utility, the middle age generation suffers a loss in expected utility of 1.375%.

4.4.4 Sensitivity Analysis

In this section, we show the results of our analysis using other combinations of parameter values for $\hat{b}_i$ and γ. For this sensitivity analysis, we show that the choice of parameter value for the elasticity of intertemporal substitution $(1/\gamma)$ is an important factor that affects the behaviour of households towards their consumption and saving decisions. For a sufficiently low level of the elasticity of intertemporal substitution $(1/\gamma = 1/5)$, which indicates that risk aversion levels are high, elements of the tragedy of annuitisation are observed — consistent with the findings of Heijdra, Mierau, and Reijnders (2014). In particular, in Table 4.4.2 we see that annuitisation by the middle age generation (only) leads to a decrease in overall lifetime welfare of future newborn generation ($\triangle U = -5.03\%$). In our earlier results for this case the change was positive ($\triangle U = 3.506\%$).

Further, in Table 4.4.2 we observe that for an economy where households are highly risk averse, there exist general equilibrium repercussions where the lifetime welfare of future newborn decrease if any of generation young or middle aged annuitise (with no presence of insurance). As a result of a decrease in the capital intensity level in the economy, the negative impact of annuitisation on the new steady state level of the wage rate is more severe here. Thus, most generations (young and middle aged) suffer a welfare decrease. Focusing on the welfare changes of each generation individually, the only generation that benefits from annuitisation is the old generation as they consume more when the interest rate increases.

4.5 Conclusion

Following the seminal contribution of Yaari (1965) and together with the contributions of others (Davidoff, Brown, and Diamond, 2005; Mitchell et al., 1999; Brown, 2003), it has been established that the optimality of life-cycle utility

Table 4.4.2: Summary of findings for $(1/\gamma = 1/5)$

Case	m_1	m_2	$(1+r)$	Wage	Con(G1)	Con(G2)	Con(G3)
No insurance/annuity	0	0	1.5518	1.0000	2775.68	2563.47	2679.19
Insurance/annuity to G1 only							
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.91$	0.004132	0	1.5408	1.0045	2781.76	2571.16	2679.23
$\hat{b}_2 = 0.97, \hat{b}_3 = 0.91$	-0.002964	0	1.5596	0.9969	2772.06	2557.55	2678.82
Insurance/annuity to G2 only							
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.93$	0	0.005076	1.5041	1.0197	2822.30	2582.06	2671.25
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.91$	0	-0.001121	1.5627	0.9956	2765.01	2558.96	2681.11
Insurance/annuity for G1 and G2							
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.93$	0.002825	0.004688	1.4982	1.0222	2824.63	2586.59	2671.62
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.91$	0.004522	-0.001669	1.5537	0.9993	2768.54	2566.13	2681.92

Case	Beq(G2)	Beq(G3)	Acc Beq	$\triangle U(All)$	$\triangle U(G1)$	$\triangle U(G2)$	$\triangle U(G3)$
No insurance/annuity	175.96	944.07	2200.42	0	0	0	0
Insurance/annuity to G1 only							
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.91$	202.86	962.22	2213.46	43.37%	43.39%	5.04%	0.01%
$\hat{b}_2 = 0.97, \hat{b}_3 = 0.91$	157.22	930.50	2193.17	-54.72%	-54.78%	-3.87%	-0.02%
Insurance/annuity to G2 only							
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.93$	187.58	1044.84	2312.52	20.94%	20.94%	23.98%	-1.54%
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.91$	173.81	923.63	2174.84	-5.03%	-5.03%	-5.99%	0.29%
Insurance/annuity for G1 and G2							
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.93$	206.86	1053.61	2317.78	46.52%	46.54%	25.76%	-1.48%
$\hat{b}_2 = 1.01, \hat{b}_3 = 0.91$	202.37	935.47	2181.43	42.82%	42.84%	-2.18%	0.41%

for individuals with no bequest motive, is enhanced through annuisation of assets. However, in this world where people are non-altruistic, Heijdra, Mierau, and Reijnders (2014) find that full annuitisation of assets by all agents in the economy will lead to a welfare decrease for future newborn generations — taking into account the impact of annuitisation at the whole economy level. This negative impact (tragedy) always holds for the case where accidental bequests are transfered to young generation at the benchmark steady state economy before an annuity market introduction. Surprisingly, for a range of plausible values of elasticity intertemporal substitution rate, $1/\gamma < 1$, it also holds for the case where accidental bequests are wasted by the government in the economy before an annuity market is opened.

Our study asks the question of whether the tragedy of annuitisation is observed in an economy where people are altruistic. In the presence of bequest motives, households are allowed to take long or short positions in life insurance or, in other words, both life insurance and annuity markets exist. Insurance and annuity market imperfections are allowed to persist: households are not allowed to insure or annuitise during their final life phase. Assuming a benchmark level of elasticity intertemporal substitution rate of $1/\gamma = 1/3$, we find opening up an insurance (and annuity) market increases the welfare of future newborn generation in the long run for realistic levels of bequest motives. In particular, the economy is better off when the young generation insures and the middle age generation annuitises.

Nevertheless, it is still possible to observe "tragic" cases — by imposing a high level of altruism for the old generation which leads to insurance in middle age; by imposing a low level of altruism for the middle age generation which leads to annuitisation by the young. The numerical analysis demonstrates the importance of intergenerational transfers in the form of bequest as an important factor in determining the welfare level of future newborn generations. Also, the direction of welfare changes as a result of an insurance (and annuity) market introduction depend on the behaviour of households towards consumption and savings decision as the price of capital and labour change. The other determinant of households' behaviour toward consumption and savings decision is the degree of the bequest motive. Consistent with Heijdra, Mierau, and Reijnders (2014), we find that households with no bequest motive will consume more as interest rate increases[9]. In contrast, households with a bequest motive react oppositely from households without bequest motive — they will save more as interest rates increase.

[9]See our findings in Section 4.4.2 above.

We also show that elements of the tragedy of annuitisation as in Heijdra, Mierau, and Reijnders (2014) may still be observed when the elasticity of intertemporal substitution is sufficiently low[10]. Our results show that if the rate is at $1/\gamma = 1/5$, the lifetime welfare of future newborn generations decrease as either the young (only) or the middle age (only) generation annuitise. The economy will only be better off if the young (only) or middle age (only) generation insure. The negative impact of annuitisation on the new steady state level of wage rate is more severe, thus reducing the welfare of the young and middle age generations if they annuitise. The only generation that may benefit from annuitisation is the old generation as they consume more when interest rates increase.

Throughout our analysis, we find that the degree of altruism is an important factor in determining the optimal insurance and annuitisation decision of households which, thus, has an impact on the changes in welfare of households as insurance (and annuity) markets are opened. In this paper we have considered a strong level of altruism, where bequests can be interpreted as a necessity for the household as we have assumed that the form of the expected utility function gain from bequest is similar to the functional form for consumption. An interesting future extension to our analysis is to analyse the impact of insurance (and annuity) market introduction where people in the economy are still altruistic but may consider bequest as a luxury good instead of a necessity (Lockwood, 2012). In addition to that, the model used in this paper can be extended to include the government sector with tax element where consideration of an increase in tax concession on income used for annuity purchases can be imposed.

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[10]Here, we only consider for $\gamma = 3$ and $\gamma = 5$, which intends to compare the results of simulation based on a different risk aversion level of consumer, high and low. Other cases where γ less than one which indicates that the consumers are risk lover or γ equal to one which indicates that the consumers are risk neutral may be considered in future research work.

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4.A Appendix

4.A.1 The Household's Optimisation Problem

The optimisation problem of the household sector is solved by deriving the first order conditions of the problem using the backward recursive dynamic programming technique. For the derivation shown below, we have dropped the notation for generation i, and treat the problem similarly for all generation. An individual in the economy who is born at time $T-3$ where $T=3$ is maximum life span of household in the model, maximises his expected lifetime utility as derived below:

$$V = \left\{ \sum_{t=1}^{2} [{}_t p_{T-3} U(C_t) +_t p_{T-3} q_{(T-3)+t} v(G_{t+1})] \right\} +_3 p_{T-3} U(C_3)$$

At time t, if the household survive to the following period, the next period stock of wealth is: $W_{t+1} = [W_t + I_t + Y_t(p_t/R_t) - C_t/R_t](1-m_t)R_t$ where m_t is the proportion of assets to be insured (or annuitised) by the household which is simply equal to 0 when the insurance or annuity market is not exist. The bequest formula is given by the following equation: $G_{t+1} = [W_t + I_t + Y_t(p_t/R_t) - C_t/R_t][(1-m_t)R_t + m_t Q_t]$ where Q_t is an actuarially fair single period accumulation on a 1 unit premium paid following the death of the insured — $Q_{i,t+i-1} = R_{t+i-1}/q_{x+i}$ can also be be viewed as the sum insured for a \$1 premium, or even as the amount an annuitant's estate must pay on the death of annuitant in a single period annuity contract paying \$1 at initiation. For the final life phase, an individual maximises his expected utility without life insurance or annuities purchases.

For the terminal condition, we assume that the household does not receive any income $Y_T = 0$, and there are accidental bequests, G_{T+1}. Since households are not allowed to purchase insurance or annuity during the final life phase, $m_T = 0$. Thus, we obtain:

$$C_T = R_T[W_T + I_T - G_{T+1}/R_T] \tag{4.A.1}$$

.

For the first and the second life phase, an individual maximises his expected utility where life insurance and annuities purchases are available. We follow the step in Fischer (1973) by substituting $W_T = [W_{T-1} + I_{T-1} + Y_{T-1}(p_{T-1}/R_{T-1}) - C_{T-1}/R_{T-1}](1-m_{T-1})R_{T-1}$ into Eq. (4.A.1). Any expected future inheritance is also discounted by (p_t/R_t), the same approach applied to the expected income. The second life phase maximisation problem is derived as the following:

$$\max_{\{C_{T-1},m_{T-1}\}} V_{T-1} = \frac{C_{T-1}^{1-\gamma}}{1-\gamma} + \left\{\frac{p_{T-1}R_T^{1-\gamma}}{(1+\rho)(1-\gamma)}\right\}\Bigg[W_{T-1} + I_{T-1} + \{I_T - G_{T+1}/R_T\}\left\{\frac{p_{T-1}}{R_{T-1}}\right\}$$
$$+Y_{T-1}\left\{\frac{p_{T-1}}{R_{T-1}}\right\} - C_{T-1}\left\{\frac{1}{R_{T-1}}\right\}\Bigg]^{1-\gamma}[(1-m_{T-1})R_{T-1}]^{1-\gamma}$$
$$+\left\{\frac{q_{T-1}\hat{b}_T}{(1-\gamma)}\right\}\Bigg[W_{T-1} + I_{T-1} + \{I_T G_{T+1}/R_T\}\left\{\frac{p_{T-1}}{R_{T-1}}\right\} + Y_{T-1}\left\{\frac{p_{T-1}}{R_{T-1}}\right\}$$
$$-C_{T-1}\left\{\frac{1}{R_{T-1}}\right\}\Bigg]^{1-\gamma}[(1-m_{T-1})R_{T-1} + m_{T-1}Q_{T-1}]^{1-\gamma}$$

We differentiate the above function first with respect to decision variable C_{T-1}.

$$\frac{dV_{T-1}}{dC_{T-1}} = C_{T-1}^{-\gamma} - \left(\frac{1}{R_{T-1}}\right)\cdot\left\{\frac{p_{T-1}R_T^{1-\gamma}}{(1+\rho)}\right\}\Bigg[W_{T-1} + I_{T-1} + \{I_T - G_{T+1}/R_T\}\left\{\frac{p_{T-1}}{R_{T-1}}\right\}$$
$$+Y_{T-1}\left\{\frac{p_{T-1}}{R_{T-1}}\right\} - C_{T-1}\left\{\frac{1}{R_{T-1}}\right\}\Bigg]^{-\gamma}[(1-m_{T-1})R_{T-1}]^{1-\gamma}$$
$$+\left(\frac{1}{R_{T-1}}\right)\cdot\left\{q_{T-1}\hat{b}_T\right\}\Bigg[W_{T-1} + I_{T-1} + \{I_T - G_{T+1}/R_T\}\left\{\frac{p_{T-1}}{R_{T-1}}\right\}$$
$$+Y_{T-1}\left\{\frac{p_{T-1}}{R_{T-1}}\right\} - C_{T-1}\left\{\frac{1}{R_{T-1}}\right\}\Bigg]^{-\gamma}[(1-m_{T-1})R_{T-1} + m_{T-1}Q_{T-1}]^{1-\gamma}$$

Solving the above maximisation function for C_{T-1} we obtain the first order condition as the following:

$$C_{T-1} = \frac{\hat{k}_{T-1}^{-1/\gamma}[W_{T-1} + I_{T-1} + \{I_T - G_{T+1}/R_T\}\left\{\frac{p_{T-1}}{R_{T-1}}\right\} + Y_{T-1}\left\{\frac{p_{T-1}}{R_{T-1}}\right\}]}{(R_{T-1}^{-1/\gamma} + \hat{k}_{T-1}^{-1/\gamma}/R_{T-1})},$$

where

$\hat{k}_{T-1} = (\frac{p_{T-1}R_T^{1-\gamma}}{(1+\rho)})\{(1-m_{T-1})R_{T-1}\}^{1-\gamma} + (q_{T-1}\hat{b}_T)\{(1-m_{T-1})R_{T-1} + m_{T-1}Q_{T-1}\}^{1-\gamma}.$

Next, we solve the problem to obtain the optimal level of insurance (or annuitisation) for the household by finding m_{T-1}:

$$\frac{dV_{T-1}}{dm_{T-1}} = -R_{T-1}\{p_{T-1}R_T^{1-\gamma}/(1+\rho)\}\Bigg[W_{T-1} + I_{T-1} + \{I_T - G_{T+1}/R_T\}\left\{\frac{p_{T-1}}{R_{T-1}}\right\}$$
$$+Y_{T-1}\left\{\frac{p_{T-1}}{R_{T-1}}\right\} - C_{T-1}\left\{\frac{1}{R_{T-1}}\right\}\Bigg]^{1-\gamma}[(1-m_{T-1})R_{T-1}]^{-\gamma}$$
$$+\{q_{T-1}\hat{b}_T\}(Q_{T-1}-R_{T-1})\Bigg[W_{T-1} + I_{T-1} + \{I_T - G_{T+1}/R_T\}\left\{\frac{p_{T-1}}{R_{T-1}}\right\}$$
$$+Y_{T-1}\left\{\frac{p_{T-1}}{R_{T-1}}\right\} - C_{T-1}\left\{\frac{1}{R_{T-1}}\right\}\Bigg]^{1-\gamma}[(1-m_{T-1})R_{T-1} + m_{T-1}Q_{T-1}]^{-\gamma}$$

$$m_{T-1} = \frac{R_{T-1}(1-\beta_{T-1})}{R_{T-1}+\beta_{T-1}(Q_{T-1}-R_{T-1})}, \text{ where } \beta_{T-1} = \left[\frac{Q_{T-1}-R_{T-1}}{R_{T-1}} . \frac{q_{T-1}\{\hat{b}_T\}}{p_{T-1}R_T^{1-\gamma}\{1+\rho\}^{-1}}\right]^{-1/\gamma}.$$

We solve for the first order condition of period $T-2$ similarly as above, which resulted in a similar form but instead we replace $T-1$ with $T-2$, $Y_{T-1}(\frac{p_{T-1}}{R_{T-1}})$ is replaced with the expected current and future income $[Y_{T-2} + Y_{T-1}\{p_{T-1}/R_{T-1}\}](p_{T-2}/R_{T-2})$, and $[I_{T-1} + \{I_T - G_{T+1}/R_T\}\{p_{T-1}/R_{T-1}\}]$ is replaced with the expected current and future inheritance $[I_{T-2} + (I_{T-1} + \{I_T - G_{T+1}/R_T\}\{p_{T-1}/R_{T-1}\})(p_{T-2}/R_{T-2})]$.

Similarly, m_{T-2} has a similar form as m_{T-1} but $T-1$ is replaced with $T-2$, $\hat{b}_T$ is replaced with $\hat{b}_{T-1}$ and R_T is replaced with $\sigma_{T-1} = \hat{k}_{T-1}^{-1/\gamma}/(R_{T-1}^{-1/\gamma} + \hat{k}_{T-1}^{-1/\gamma}/R_{T-1})$. Thus, we obtain:

$$m_{T-2} = \frac{R_{T-2}(1-\beta_{T-2})}{R_{T-2}+\beta_{T-2}(Q_{T-2}-R_{T-2})}, \text{ where } \beta_{T-2} = \left[\frac{Q_{T-2}-R_{T-2}}{R_{T-2}} . \frac{q_{T-2}\{\hat{b}_{T-1}\}}{p_{T-2}\sigma_{T-1}^{1-\gamma}\{1+\rho\}^{-1}}\right]^{-1/\gamma}.$$

4.A.2 The Firm's Optimisation Problem

A representative firm in this model has the constant return to scale Cobb-Douglas production function of $Q_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. At time t, the firm maximises its profit by minimising the cost of production as the following:

$$\pi_t = PQ_tQ_t - (Rent_tK_t + w_tL_t)$$

The lagrangian function of this optimisation problem is derived as:

$L = PQ_tQ_t - (Rent_tK_t + w_tL_t) + \lambda(A_tK_t^{\alpha}L_t^{1-\alpha} - Q)$

We differentiate the function with respect to decision variables and we obtain:

$$\frac{dL}{dQ} = PQ_t = \lambda \tag{4.A.2}$$
$$\frac{dL}{dK_t} = -Rent_t + \lambda(A_t\alpha K_t^{\alpha-1}L_t^{1-\alpha}) = 0 \tag{4.A.3}$$
$$\frac{dL}{dL_t} = -w_t + \lambda[A_t(1-\alpha)L_t^{-\alpha}K_t^{\alpha}] = 0 \tag{4.A.4}$$
$$\frac{dL}{d\lambda} = A_tK_t^{\alpha}L_t^{1-\alpha} = 0 \tag{4.A.5}$$

We substitute Eq. (4.A.2) into Eq. (4.A.3) and Eq. (4.A.4). Then, using the definition of $Q_t = A_tK_t^{\alpha}L_t^{1-\alpha}$, we substitute it into Eq. (4.A.3) and Eq. (4.A.4) and we obtain:

$PQ_t(\alpha Q_t)/K_t = Rent_t$,

$PQ_t(1-\alpha)Q_t/L_t = w_t$.

Solving the above lagrangian function we obtain the first order conditions of the firm's optimisation problem as the following:

$$K_t = \frac{\alpha PQ_tQ_t}{Rent_t}, \; L_t = \frac{(1-\alpha)PQ_tQ_t}{w_t}$$

Chapter 5

Conclusion

Annuities have existed in the economy for centuries, yet currently the world's voluntary annuitisation rate is low. This scenario could be explained by a number of reasons: such as the lack of liquidity to pay for unexpected medical expenses (Turra and Mitchell, 2004; Peijnenburg, Nijman, and Werker, 2013), a high degree of bequest motives of individuals (Purcal and Piggott, 2008; Ameriks et al., 2007), a good social security system with generous payouts (Turra and Mitchell, 2004) and a low financial literacy or engagement in the retirement financial decision by retirees (Bateman et al., 2013; Mackenzie, 2006). These days, retirement is more challenging as populations in many countries are living longer. Thus, it is crucial for retirees to make a wise financial decisions in retirement to avoid the risk of outliving their assets during the post-retirement phase.

Ruscani (2008) addresses the importance of understanding the annuity market on a country by country basis. He suggests that policymaker should not simply refer to other countries' experiences in their policy decision making, but rather should understand the annuity market in the country itself — as customers, suppliers and products may vary from country to country[1]. This thesis explores further the role of annuities as an option for the decumulation phase of a retirement, with particular reference to Malaysia, where annuities have been almost non-existent for years. We study the economic value of annuities for retirees using actuarial and economic models. There are three main topics covered by

[1] James and Vittas (2000) and Doyle, Mitchell, and Piggott (2004) show that the money's worth value of annuities varies according to country. Annuities in some countries, like Switzerland and Singapore, have a money's worth value of higher than one, whereas annuities in other countries, like Australia and the UK, have a money's worth value of less than one. As described by Purcal (2006), the nature of annuity supply may vary from country to country with different challenges faced by annuity providers – for example, the costs of providing such products may vary depending on the availability of financial instruments to hedge the longevity risk associated with annuities.

this thesis; each of these studies are summarised below, together with their main findings.

5.1 Summary and Findings

5.1.1 Paper 1

The first study describes the annuity market experience of Malaysia and the current developments in private annuities in that country. The introduction of the Employees Provident Fund (EPF) linked private annuity schemes was controversial in 2000. At that time a workers' representative body (the Malaysian Trade Union Congress) suspected private insurers of profiting excessively from these annuities. They argued that for such an annuity scheme to be offered, it should be a government backed scheme, so retirees are protected from the risk of losing their retirement funds should investment returns be poor or insurers become insolvent. As a result, the Malaysian government decided to discontinue this annuity scheme and has delayed the decision to re-introduce them until further studies are conducted.

Since the suspension of the EPF annuity schemes in 2001, the number of annuity policies remaining in the market is very low with new business premiums from annuities of almost 0% out of the total new business premiums in the insurance market. Recently, the Malaysian government announced an increase in the tax exemption on income used to purchase an annuity — from MYR1000 to MYR3000 — in the Malaysia's 2012 Government Budget. Later that year, a new private annuity plan was introduced to the market.

In this paper, we perform the analysis of value for money of annuities for both the suspended EPF annuity scheme and the new private annuity plan introduced in 2012. We apply two methods — the Money's Worth Ratio (MWR) and the Annuity Equivalent Wealth (AEW) — to value these annuities.

For the EPF annuity schemes, there is a bonus payment made on top of the base annuity stream of income which is not guaranteed by the annuity provider. Our analysis shows that, using the risk-free rate of return, the EPF annuity schemes provide good value for money for annuity buyers (with a value of MWR and AEW of greater than 1) when full bonus payment is included in the value for money calculations. This indicates that instead of making an excessive profit out of these annuities, insurers are offering a good deal to customers by providing the full bonus payment. However, it is also true that annuity buyers may have to pay for transaction costs or other loadings of up to around 40% if no bonus payment is made by the insurer.

For the new private annuity in 2012, our analysis shows that it is slightly more expensive as compared to the EPF annuity schemes with full bonus payment and annuities around the world — annuity buyers have to pay loadings of up to 20 to 30%. This evidence suggests that annuities offered in the market these days are being charged higher loadings, which is possibly caused by more stringent capital requirements set by the financial regulator for insurers.

Finally, we note that our MWR calculations are based on the actuarially fair premium of the annuity element only, of these products and thus underestimate the MWR value of the product. The new private annuity plan in 2012 also provides additional benefits in the event of death, total and permanent disability and diagnosis of critical illness, and so further research work is required to take into account these benefit payments — which will increase the MWR value of this annuity. We treat this issue further in Paper 2.

5.1.2 Paper 2

Our second study extends the Annuity Equivalent Wealth analysis from the first paper. As the private annuity introduced in 2012 provides not only the common annuity stream of income, but also offers additional benefits in the form of a cash value payment for unfortunate events such as death, the diagnosis of critical illnesses (CI) and total and permanent disability (TPD), we have extended the original Annuity Equivalent Wealth method to incorporate the several health states that may occur over the life cycle of individual. For this extended AEW analysis, we also apply a health state dependent utility function. We assume that consumers may allocate their consumption differently according to their health state at the time of consumption.

We obtain two main findings from the analysis in this paper. First, the value for money calculated by this extended measure of Annuity Equivalent Wealth (AEW) of the bundled annuity product is higher for almost all categories as we add the additional benefits — as compared to the AEW value calculated for just the annuity element of the product. However, this is not the case for certain female products, mainly because of the low probability of receiving the TPD and CI benefit for this group.

Second, as we apply the health state dependent utility function in the presence of insurance markets, and a consumer may allocate different consumption depending on their health state at the time of consumption, the AEW value rises in the case of plain vanilla annuity payments. In contrast, as we add benefit payments associated with health states worse than perfect health (where in particular, the cash value is payable in the event of TPD or CI for this product), the AEW value is lower when the health state dependent utility is used. This indicates including health states affects the value of such products from a consumer's perspective.

In addition, we also find that as perceptions of utility from health often differ by nation, this then impacts on the value of an annuity product. For example, using the Malaysian population's mean health index of utility value, the AEW value for category age 45 (B2) male decreases by 8% (in relation to the AEW calculated without the health index of utility), whereas using health utility values elicited for the UK population, the AEW value for the same category decreases by a higher percentage of 11%. This indicates that if individuals value health states differently, the same annuity product offered to two different populations will also be valued differently.

5.1.3 Paper 3

Lastly, the third paper in this thesis studies the impact of annuitisation in a general equilibrium framework where individuals in the economy are altruistic. The key reference for this study is the recent research work by Heijdra et al. (2014) which studies the impact of full annuitisation at the whole economy level. In the non-altruistic world, it is privately optimal for individuals to fully annuitise their assets. But according to their study, the future newborn generation are worse off as a result of annuity market introduction. There exists a tragedy: there is a negative welfare effect as the annuity market is opened and all individuals in the economy fully annuitise their assets.

For the case where accidental bequests are originally transfered to the young, moving to a new steady-state economy in the presence of annuities reduces the economy-wide capital level as the young generation save less (since accidental bequests are taken away from them). This has a negative impact on the wage rate that clears the price of labour in the economy which then, overall, decreases the welfare of future newborn generations.

For the case where accidental bequests are originally wasted by the government, for a range of plausible values of the intertemporal rate of substitution which is less than 1, as all individuals fully annuitise their assets, the price of capital that clears the market moves towards the rate of return offered by annuities (which is generally higher than the rate of return for non-annuitised assets). For this range of intertemporal rates of substitution, the young generation reduce their savings and increase their consumption as the interest rate increases which then increases the welfare at the shock time for the younger generation. However, in the long run, the capital level in the economy reduces which also has a negative impact to the economy as it decreases the wage rate of households. Over the long run, for both cases, the welfare of future newborn generation decreases.

Following this study, we are interested to see whether the tragedy of annuitisation is also observe in an economy where people are altruistic. We consider three cases. First, we assume that only the young generation may insure or annuitise. Second, only the middle-age generation may insure or annuitise and lastly, all generations may insure or annuitise their assets. For a benchmark level of the elasticity of intertemporal substitution ($1/\gamma = 1/3$), we show that if households in the economy have a bequest motive, the tragedy is ameliorated — future newborn generation benefit from annuitisation. In particular, the welfare of future newborn generation increases as the young generation insure and the middle-age generation annuitise their assets.

It is important to study the behaviour of households towards consumption and saving decisions as the price of capital and labour changes in the economy. For this analysis, households with no bequest motive consume more and save less as interest rate increases, consistent with the findings of Heijdra, Mierau, and Reijnders (2014). In contrast, households with bequest motives consume less and save more as interest rate increases. As there exist an intergenerational transfer in the form of bequest from parent to child in this world where people are altruistic, annuities do not necessarily make the economy worse off. This finding reverses the tragedy of annuitisation which is observed if a bequest motive is not present.

However, if we consider more extreme values of the elasticity of intertemporal substitution ($1/\gamma = 1/5$), we observe that elements of the tragedy of annuitisation appear — the lifetime welfare of future newborn generations decreases if any of the young or middle aged generation annuitise their assets. The negative impact of annuitisation on the new steady state wage rate after the insurance (and annuity) market introduction here is more severe, thus, reducing the welfare of most generations. The only generation that may benefit from annuitisation is the old generation. This is also consistent with the findings of Heijdra, Mierau, and Trimborn (2014) where they find that there exists a partial annuitisation rate which is welfare enhancing. But, if the value of the elasticity of intertemporal substitution is sufficiently low, it is better for the annuity market to remain closed. Overall, we observe that the degree or level of the bequest motive is important to determine the optimal level of insurance or annuitisation for households, which then has an impact on the overall welfare of future newborn generations as the insurance (and annuity) market is opened.

5.2 Limitations and Recommendations for Future Research

There are several limitations of this study which could be improved by future research to enhance the quality of findings of the analysis. These limitations are described below.

- The extended Annuity Equivalent Wealth (AEW) analysis in Paper 2 uses a simple life-cycle model, where a consumer is only allowed to make a transition to a health state that is one state better or one state worse that his current health state. A more realistic life-cycle model would allow for transitions to any state appearing in the model over the life-cycle of the consumer. This requires further survey data to estimate the transition rates allowing for movement to any state from one particular health state.

- The prevalence rates for each health state appearing in the life-cycle model in Paper 2 are assumed to be stationary over time. This assumption is also applied for the prevalence rates for the critical illness and total and permanent disability states in the annuity model. Due to limitations of morbidity survey data available in Malaysia, this assumption has to be imposed. It would be desirable to develop the incidence rates table for these health states which also take into account its trend over time.

- The analysis in Paper 3 used a functional form for bequest utility that has a similar functional form used for utility from consumption, rendering bequests a necessity. The analysis can be extended in the future to allow for weak altruism, that is, bequests may become luxury goods for households in the model, as in Lockwood (2012).

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