The Role of Picture Books in Promoting Mathematics Teaching and Learning for Young Children

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Contents

Summary

Picture book experiences in the early years have been found to not only facilitate children's literacy skills but also promote mathematics learning. The inclusion of mathematical ideas and terms (in the text and illustrations of picture books) supports skills acquisition and conceptual understanding, retention of knowledge, and positive attitudes toward mathematics.

Considering the wide range of picture books available on the market, many with rich mathematics learning potential, this study was designed to assist professionals in using picture books to promote mathematics learning and pedagogy.

This research, that used a multi-method approach, provides a broader perspective than do previous similar studies. It was conducted in three phases. Phase 1 comprised a content analysis of 136 books in which three different types of mathematical picture books were identified and classified as perceived, embedded, or explicit. A framework for the selection and evaluation of picture books for mathematical learning was then developed using a modified Delphi process. A survey of 27 professionals evaluated the classification scheme and framework, the results highlighting the value of the framework but the need for professional development for recognising mathematics in real-life contexts.

Interviews with four picture book authors and/or author-illustrators in Phase 2 revealed why and how they incorporate mathematical concepts in their books.

Phase 3 focused on a classroom study of 16 Year 1 children and three teachers using the three different types of mathematical picture books. A content analysis of the text and images of each book indicated many opportunities for rich mathematics learning. A parallel analysis of teachers' and children's talk during shared reading sessions highlighted the significant role of the teacher and that they did not capitalise on the opportunities for mathematics learning. Students' spontaneous representations revealed their cognitive engagement with the mathematics and confirmed the strong influence of the illustrations.

The findings and implications of this research are presented as a *thesis by publication* with publications embedded within the chapters.

Statement of Candidate

I hereby certify that this work entitled *The role of picture books in promoting mathematics teaching and learning for young children* has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree to any other university or institution other than Macquarie University.

I also certify that the thesis is an original piece of research and it has been written by me. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Jennifer Marston (Student ID: 40193020)

Date: ________________________

The research presented in this thesis was approved by Macquarie University Ethics Review Committee (Human Research), (R/No. HSHE28AUG2009-D00022), SERAP (2011073) and Catholic Schools Office, Diocese of Broken Bay (9 August, 2011). See Appendix A.

Acknowledgements

A colleague described completing her PhD as like running a marathon. However, I would equate it more to an obstacle race or course. Working full time for the most part of the thesis, there were times when everything ran smoothly. I collected data, unearthed interesting findings, published an article or completed a chapter, and still managed to be a "good" wife, mum, nanna, daughter, and employee. However, then an obstacle appeared: I had exam results and an article due at the same time; I needed to totally restructure a chapter, or I had to resolve a technical disaster. These slowed down my progress, knocked my confidence, and made the finishing line hard to see.

There are numerous people who I need to thank for both helping and supporting me in this obstacle race. First, thank you to Professor Joanne Mulligan, my supervisor and Assoc/Prof., Marina Papic, my associate supervisor until the final months, and Dr Michael Cavanagh who stepped in as associate supervisor at the very end, who have advised me, read multiple drafts of this thesis and publications, provided valuable feedback, and seen me through to the end of the race. I also acknowledge the editing and formatting assistance of Robert Trevethan in the final stages.

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PREAMBLE

When I was a primary school student and subsequently a primary school teacher there was a strong instrumental approach to teaching that was regimented, consisted of compartmentalised curriculum areas, and did not allow for creative thinking. Creative writing, or composition, was always *Write a story about … and then draw a picture*. My memories of mathematics are pages of calculations, occasional geometry (drawing shapes, using a compass, and measuring angles), and on Fridays we did five problems, always about shopping, for example, *I went shopping and bought … How much did I spend? How much change did I get from … ?* Social studies involved mapping and completing projects about explorers and primary products. I distinctly remember in Year 5 a young new-career teacher introducing reading groups; this was quite different to any previous teaching methods I had yet experienced and his approach sowed the seed for my own teaching career.

When I returned to teaching after taking time to raise my three sons, owning a bookshop, and upgrading my qualifications, including some studies in children's literature, I had a different approach to teaching and learning. I realised it needed to be engaging, relevant, and more hands-on and student oriented than previously. Luckily, pedagogy had changed too in that time and there was a more creative and "relational" attitude to teaching. Professional development sessions concentrated on a more constructivist approach, student involvement, learning for understanding, and integration of curriculum areas. I found myself trying and implementing many of these new strategies which I observed not only improved student learning but also enhanced my teaching and further increased my enjoyment of teaching.

I had always had a passion for picture books and included them in my teaching of literacy and history. However, after reading an article, *Picture Book Maths* which included the sentence, "You know you can think of almost everything as a maths problem" (Steinberger, 1998, p. 22), I saw the possibilities for increased engagement of students in mathematics. Soon after, one teaching experience with a group of kindergarten children and *Alexander's Outing* (Allen, 1993) also demonstrated how students themselves used ideas within the book to create their own mathematical investigations.

The use of picture books within all my classroom teaching ensued. I started by using a variety of picture books as recommended by Griffiths and Clyne (1988) but soon started developing my own units of work involving mathematics and literature (and other curriculum areas). This integrated approach continued when I began teaching primary mathematics education in a tertiary teacher education program. My preservice teacher students were very receptive to this strategy, used it in their professional experience teaching, and commented on the more positive attitude toward mathematics it promoted both in them and their students.

However, many of the books suggested in the available resources were pre 1990 and, although they contained good books and teaching ideas, many settings and themes in the picture books were not current. I knew that many picture books were continually appearing on the market that teachers and my preservice teachers could use to promote quality mathematical learning. I recognised that not all books contained mathematical potential and those containing mathematical opportunities were not always easily recognised. I also hypothesised that there would be different ways the mathematical content was presented that may produce varying responses in young children. My students also encouraged me to provide them with lists of useful books and teaching ideas, but I realised that a criteria for selection and evaluation of all picture books for mathematical teaching and learning opportunities would be a far more helpful and ongoing resource.

One particular book, *Alexander Who Used to be Rich Last Sunday* (Viorst, 1978) also inspired me to think about writing myself. Although it contains some wonderful opportunities for mathematical problem solving, some ideas are no longer relevant, the currency used throughout is not appropriate for Australian students, and the buying power is outdated.

Thus, the idea for this thesis: the development of an instrument for evaluating picture books, interviewing authors about their intentions, researching the effects of using different types of mathematical picture books, and writing resources to aid teachers in the use of picture books for mathematical development.

CHAPTER

1

INTRODUCTION

In an age when technology is emphasised as the way of future education, it may seem unfashionable to promote picture books as a tool for learning. However, educators of young children have recognised that children's literature contributes to their cognitive and affective development (Giorgis & Glazer, 2009). Research in mathematics education involving studies of cognitive engagement and intervention and pedagogical innovations have also demonstrated similar benefits when using picture books in mathematics teaching and learning experiences (e.g., A. Anderson, Anderson, & Shapiro, 2004; Casey, Kersh, & Young, 2004; Van den Heuvel-Panhuizen & Van den Boogaard, 2008). These studies report advantages for children, including improved mathematical performance, a more positive disposition toward mathematics learning, and effective engagement through their spontaneous mathematical utterances. This can occur when children are engaged with books written both with the specific intent of teaching mathematics and also where mathematics is not purposefully included.

As this thesis emphasises the use of picture books to promote mathematics teaching and learning, both mathematics education and children's literature perspectives are taken. In this first chapter, I introduce this thesis by describing the context, definitions, rationale, research problem, and aims. I then present the research questions, an overview of the study in its three phases, and suggest its significance to both pedagogy and further research. I conclude the chapter by outlining the way the thesis is organised.

1.1 Context of the study

In this section I provide background information about picture books and demonstrate the numerous positions that researchers take in defining picture books generally and quality picture books in particular.

1.1.1 What is a picture book?

1.1.1.1 Background

Picture books are a relatively recent development. They first appeared in their commonly perceived form as recently as 1960. They grew out of chapbooks, toy books, and comics. Chapbooks, that included woodcut illustrations, were pocket sized books of 16 pages, popular from the 16th to 19th centuries. Although mostly designed for adults, they consisted of a number of genres, so were also read by children. Lewis (2001) claimed that modern picture books emulate their characteristics. However, the topics and themes of picture books have expanded from a simple illustrated text for young children to sophisticated and complex books with previously forbidden topics suitable for older readers. Postmodern picture books may even include nonlinear or multiple storylines. These changes make them more applicable for use in the school curriculum for both younger and older readers (Martinez & Harmon, 2012).

1.1.1.2 Definitions of picture books

There is some disparity in the definition of a picture book. Some researchers (e.g., Nodelman, 1988) say that they are books where the illustrations play the primary role in telling a story, while others emphasise the importance of the relationship between the text and the illustrations. Some writers have even proposed a difference between picture books (those that tell the story through the pictures) and illustrated books where the "illustrations are extensions of the text and may add to the interpretation of the story" (Martinez & Harmon, 2012, p. 232). Regardless of these distinctions, visual images or illustrations[1] are an important component of picture books. Their role and their interrelationship with the text are discussed in Section 3.1.2[2] as part of the theoretical perspective taken in this study. Several definitions are now provided, while additional ones have been included in some of the publications presented throughout the thesis.

There is also some debate in the literature about how to write/spell the term "picture book": *picturebook, picture-book* or *picture book?* Lewis (2001) and Sipe (1998) prefer the single word, *picturebook,* as it suggests the compound nature of the books themselves. However, throughout this study "picture book" will be used because it reflects the most common usage in Australian children's literature.

Present day picture books, usually comprising 32 pages, have been defined as books that contain multiple visual images and are suitable for very young children. They often contain a simple narrative or descriptive text that is intended to be read aloud and shared between an adult and child or group of children (Muir, 1982; Reeder, 1997). Lewis (2001) used Bader's (1976) definition:

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[1] The terms "visual image", "image", and "illustration" will all be used throughout this thesis to refer to the pictures in a picture book.

[2] This cross-referencing system will be used throughout the thesis, with the first numeral referring to the relevant chapter and the following numerals to the section within the chapter.

> A picturebook is text, illustrations, total design: an item of manufacture and a commercial product; a social, cultural, historical document; and foremost, an experience for a child. As an art form it hinges on the interdependence of pictures and words, on the simultaneous display of two facing pages, and on the drama of the turning pages. On its own terms its possibilities are limitless (Bader, 1976, p. 1 cited in Lewis, 2001, p. 1).

Libby Gleeson (2003), an Australia author of many award-winning picture books, has described a picture book as a story told in words and pictures, with both making an important contribution to the way the story is told and to its meaning. Anstey and Bull (2000) concur by defining a picture book as "where the written text and the illustrative text are in concordance and work independently to produce meaning" (p. 5).

Townsend (1987), defining a picture book, referred solely to the illustrations. However, Reeder (1997) wrote that it is difficult to define a picture book and believed there are three categories of picture books:

(i) those designed for very young children that rely heavily on the pictures to tell the story or describe the scene/situation. These include wordless books, alphabet, counting and concept books and "other books with minimalist text which would be fairly meaningless without the visual accompaniment" (p. 91);

(ii) those designed so that the illustrations and text complement each other in the storytelling, and

(iii) those with text and pictures, where the illustrations do not add to the meaning and play only a "decorative role" (p. 92) and do not promote further thought or discussion.

The Children's Book Council of Australia (CBCA) has been making annual awards since 1945 for children's books, including picture books, and during that time its definition of a picture book has changed. Originally the council described picture books as appropriate for only younger readers, but the age range has now extended to include awards for illustrated books for any age. The council's current definition to be used by those submitting applications for the award is those "in which the author and illustrator achieve artistic and literary unity, or, in wordless picture books, where the story, theme or concept is unified through illustrations" (CBCA, 2012). Recognising that picture books can be "read" by children from birth onward, in this thesis I concentrate on the use of picture books for children from approximately 5 to 8 years of age.

1.1.1.3 What constitutes "quality" in picture books?

Notions of what constitutes "quality" in picture books have also changed over time. Jalongo (2004) pointed out that defining "quality" picture books is difficult. She described picture books that are considered valuable as "cultural artefacts" because they compare favourably with other books of the same genre, the illustrations are by known artists and are recognised as art by the reader, and they have retained their value over time. Some may equate quality with the success of the book, its readability, marketing, and audience appeal (Temple, Martinez, & Naylor, 2002), and therefore include "the classics". The perceived quality of the book, too, may vary over time according to cultural, political, and social changes.

Jalongo (2004) summarised the works of researchers including Lynch-Brown and Tomlinson (1999) and Temple et al. (2002) and concluded that high quality books have:

- memorable, well-portrayed characters;
- a well sequenced and interesting plot;
- authentic and imaginative settings;
- vivid language that "evokes images of actions, and reflects the mood of the story" (p. 43);
- an appropriate and truthful theme that will maintain children's interest;
- quality art and design;
- interplay between pictures and words; and
- appropriateness to students' development and understanding.

Awards are given both in Australia and internationally for what are considered to be quality books. These will be used as a guide in the selection of picture books throughout this research. Despite their definition, the CBCA awards the illustrator rather than the writer of a picture book, as does the prestigious Caldecott Medal[3]. Some art historians believe that "true picture books were solely seen as the creation of the artist" with the text "playing a subsidiary role" (Anstey & Bull, 2000. p. 4). However, surveys of reviews of picture books (Busbin & Steinfirst, 1989) have revealed that only 25% of the reviews focussed on the illustrations of these books, while Rawlins (as cited in Reeder, 1997) suggested it was as low as 10 per cent.

Although I recognise the recent emergence of e-books (and interactive mathematics picture books, e.g., Ginsburg, 2016), only traditional hard copies are examined and referred to in this

[3] See Appendix B.

thesis. The use of e-book versions of picture books would warrant further investigation that is beyond the scope of this thesis.

1.1.2 Defining picture books with mathematical content

Amerikow (2008) asserted that from an early age children are asked to “create pictures in their minds to solve problems and see relationships” (p. 1) and that visual imagery plays an important role in our everyday lives. Therefore, the combining of visual images and narrative, as in a picture book, to promote mathematics learning is appropriate.

Picture books are written in various formats for a variety of reasons. Most books are written to entertain (in Chapter 6 I explore the intent of authors in detail), but many also contain language and content in the text and/or visual images that could promote mathematical learning. Others are written with the express purpose of teaching mathematics. Both types of picture books have been shown to promote mathematical engagement of students (Elia, Van den Heuvel-Panhuizen, & Georgiou, 2010; Young-Loveridge, 2004). Nesmith and Cooper (2010) use the term “mathematics literature” (p. 280) for books with mathematical language in the text and/or mathematical potential in the visual images. In this thesis they will be referred to as *mathematical* picture books.

Shih and Giorgis (2004) suggested that there were three types of mathematical picture books. The first group was where “mathematics is the basis of the story” (p. 328), has the distinct aim of teaching mathematics. In the second group, the mathematics is “embedded” in the story, for example, positional language (inside, between). Therefore, “understanding the mathematics is integral to understanding the story” (p. 329). The third group comprises books in which mathematical questions or investigations arise from the book.

In this thesis I propose three types of mathematical picture books where the mathematical content may be:

(i) *perceived* to be occurring. These books are written to entertain, and the mathematical concepts are unintentional and incidental. Professionals[4] (and students) may identify worthwhile opportunities within the text and visual images for mathematical use;

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[4] Throughout this thesis the term “professionals” refers to academics, teachers, and preservice teachers

(ii) *embedded*. The author's intent is to tell a story that can stand alone from the additional mathematical layer in the text and/or visual images, or

(iii) *explicitly referenced.* The intention of the author is to teach a particular mathematical concept.

These definitions are used throughout the three phases of this thesis. They are evaluated by professionals in Phase 1 (Chapter 5) and discussed and confirmed with picture book authors in Chapter 6 (Phase 2). The use of the three different types of mathematical picture books by teachers and students is reported in Phase 3 (Chapters 7, 8, and 9).

Examples of mathematical picture books

Each type of mathematical picture book is discussed in this section. Examples are provided to demonstrate the different opportunities for mathematical learning. When assessing books for mathematical quality, some researchers also assess the literary quality. As the emphasis in this thesis is on the quality of the mathematical content of picture books, only books that have won awards have been used as examples, thus ensuring their literary quality.

(i) Picture books with perceived mathematical content

At the Beach: Postcards from Crabby Spit (Harvey, 2004) was short-listed by the CBCA in 2004. It is presented as a series of postcards sent by three children to their grandmother while they are on a camping holiday at Crabby Spit. There is no overt reference to mathematics either in the text or in Harvey's unique style of illustration and the author does not intend the book to be used for mathematical purposes (R. Harvey, personal communication, 8 August 2011). Nevertheless, the text and visual images are full of mathematical opportunities including positional, measurement, and money activities. Even the endpapers are maps of Crabby Spit and its environs. Like many picture books that are perceived to contain mathematical content, this book also provides opportunities for integration of the mathematics with other curriculum areas. (See Chapter 10 for more details about the mathematical and cross-curricular opportunities within the book).

(ii) Picture books embedded with mathematical content

There are few examples of picture books with embedded content, although many of the picture books by Mitsumasa Anno could be classified as having mathematics deliberately embedded. These books can be enjoyed without connecting with the mathematics. Some of

Anno's books have no text at all, although he sometimes includes a postscript where he "comments on and expands upon the contents of the book" (Langford, 1994, p. 194). His aim is for the reader to look more carefully at the pictures, and by having the words at the end of the book, readers are seeing before reading—the order being important, just as order is important in mathematics. Anno has a strong belief in the importance of order and pattern in the world, and this is emphasised by the use of the concepts of shape, number, and pattern in his books.

Anno's counting books (e.g., Anno, 1977) are not traditional counting books. Readers need to look carefully for groups of objects that have similar characteristics rather than just groups of the same object. The pictures themselves serve as the story without necessarily engaging with the mathematics. Langford (1994) believed that Anno had a fascination with geometric shapes, using "simple cuboids to complex fluted domes … towers topped with cones, arches varying in span, acute and obtuse angles … striped tiles in regular lines, bricks tessellating and timber frames dividing walls into squares, rectangles and triangles" (p. 201). All these can be enjoyed as part of the illustration or explored more for mathematical qualities.

Graeme Base uses a similar technique in his writing. For example, in *Uno's Garden* (Base, 2006) he used narrative and his unique visual images to deliver an environmental message. He included the additional layer of mathematics to enhance his purpose, which he is adamant was not to "teach" mathematics. Chapter 6 provides further insights into his purpose, and Chapters 7 and 10 contain a more detailed account of the book and the mathematical opportunities it provides. Chapters 8 and 9 include material that demonstrates the book's use in the classroom.

(iii) Explicitly referenced "mathematical" picture books

Counting books are often children's first introduction to numbers or mathematics. Like other types of picture books, counting books can vary in their content and style, i.e., the amount of narrative structure and the way sets of objects or entities are portrayed on each page. Counting books usually encourage the readers to count objects familiar to them in the environment. Some books use the sequence of numerals 0–10 in ascending order, showing the addition of one more object on each page, for example, *My First 1 2 3* (Allen, 2007); others use the reverse counting sequence of the numerals 10–0, as can be seen in *Ten Red Apples* (Hutchins, 2000).

Not only can explicitly referenced picture books present the cardinal numbers, *how many*, they can also encourage familiarity with ordinal numbers, grouping of numbers, number names, and symbols. They may also simultaneously introduce the reader to learning about nonmathematical aspects, including new content knowledge and new and unfamiliar vocabulary (D. Whitin & Wilde, 1992). For example, in a counting book using baby animals, children not only count but learn the names of baby animals and use of singular and plural as in, *one calf, two calves; one lamb, two lambs;* and *one sheep, two sheep.*

Explicit mathematical picture books with a simple storyline are not limited to counting numbers. Some explicit books contain specific mathematical content in a narrative format, either as stand-alone picture books, for example, *If You Were a Triangle* (Aboff, 2010) and *One Hundred Hungry Ants* (Pinczes, 1993) or in series such as *Sir Cumference and the First Round Table* (Neuschwander, 1997). Many of these books were written in response to The National Council of Teachers of Mathematics' (NCTM, 1989, 2000) standards that encouraged the use of picture books in mathematics learning. *Explicit* books are often referred to as trade books, which Schiro (1997, p. viii) described as "any book that has a plot built around a mathematical concept or that directly poses a mathematical problem to its characters or the reader."

Several researchers have challenged the use of explicit mathematical books. Hellwig, Monroe, and Jacobs (2000, p. 139) referred to explicit mathematical picture books as "glorified textbooks". Nesmith and Cooper (2010) went further and warned that the use of explicit books "can be devastating to students" (p. 282) because, according to Nesmith and Cooper, they are written for adults to use for teaching, are often of dubious literary quality, and have no layers that allow students to explore. They concluded that "the books are insulting to children's intelligence, may damage students' motivation and interest in reading and mathematics" and could lead students to believe we are "sugar-coating" mathematics by putting it in a story (p. 282).

One Hundred Hungry Ants (Pinczes, 1993), written in rhyming text, refers to 100 ants heading in a single line to a picnic. One ant realises that travelling in a single line will take a long time so they rearrange first into two, then four, and finally five lines. This book has one intention, to teach the different multiplicative combinations of 100. However, it could be used as a springboard to an exploration of other number combinations.

The majority of picture books written with a mathematical focus (ii and iii above) appear to refer to the concepts of number and shape, e.g., *Anno's Counting Book* (Anno, 1977) and *Shapes, Shapes, Shapes* (Hoban, 1986). Anecdotal evidence suggests that early childhood and primary school educators rely on these counting and concept books that require very basic literacy skills for their interpretation and may trivialise and possibly restrict mathematical learning. Picture books that include other mathematical concepts such as measurement, patterns, and problem solving (e.g., *Amy and Louis* [Gleeson, 2006] and *Ernie Dances to the Didgeridoo* [Lester, 2000]) are less common.

1.2 Rationale: Using picture books in mathematics teaching and learning

The benefits of, and rationale for, using picture books in mathematics teaching and learning are commonly listed in research reports, resource books[5], and journal articles (e.g., Clark, 2007; Van den Heuvel-Panhuizen & Van den Boogaard, 2008). These include giving a common base of experience for the whole class and providing a "graphic illustration of concepts" (Steinberger, 1998, p. 22). Benefits also include presenting familiar problems from a different perspective, offering another strategy when trying to cater for different learning styles, and changing the perception of mathematics, i.e., mathematics is a way of understanding the world around us, not just fractions and division. The opportunity for open-ended activities also allows for students to respond at their own level of competence. Flevares and Schiff (2014) took a different approach by addressing the benefits of using picture books from the angle of the NCTM (1989, 2000) standards. However, there are other strong pedagogical reasons that support a constructivist (or socioconstructivist) approach for adopting picture books for use in mathematics education. These reasons will be discussed in Section 3.1.1.

Although Van den Heuvel-Panhuizen and Elia (2013) date the use of picture books for mathematical learning back to Comenius in 1652, there is little mention again in the literature until the three most recent decades. A. Anderson et al. (2005) claim that this new interest is due to the "increased emphasis on integration in curriculum and instruction" (p. 5).

An integrated or "purposefully connected" (Naylor, 2014) curriculum "involves planning for teaching and learning that draws on two or three subjects within a learning area, or two or three learning areas or subjects" (p. 3) "where clear conceptual link/s" are "needed among the

[5] The terms "resources" and "resource books" are used to refer to books and articles that have been published in professional journals or are available on the Internet and that contain ideas for the use of particular picture books for mathematical learning experiences.

curricular area content descriptions" (p. 9). "Research has consistently shown that students in integrated programs demonstrate academic performance equal to, or better than, students in discipline-based programs" (Drake & Reid, 2010, p. 1). In fact, McKenzie (2002) found that students were able to recall 60% more information within an integrated program (in this case social studies and literature) than did students who did not participate in the trial. Additionally, integration results in students who "are more engaged in school, and less prone to attendance and behaviour problems" (Drake & Reid, 2010, p. 1). Students who are more engaged also perform better (See Section 2.2).

Bosma and De Vries Guth (1995) stated that literature is the "thread that weaves" the integrated curriculum together (p. 7). Children respond well to literature because of a need to connect stories to their own lives (Wolfenbarger & Sipe, 2007). The meaningful and social contexts provided by picture books help children "better understand mathematical ideas and their application to real-world situations" (D. Whitin, 1992, p. 28) and afford opportunities for problem solving and mathematical reasoning (Braddon, Hall, & Taylor, 1993; Schiro, 1997; Welchman-Tischler, 1992, D. Whitin, 1992). Most of this literature is popular belief and does not refer to empirical studies or research evidence. It then appears that children's literature and the shared reading of picture books can be fundamental to mathematical learning and support the constructivist (socioconstructivist) learning theories to be used in this study and outlined in Section 3.1. "There is no more powerful vehicle for meeting new goals in mathematics than the use of children's literature in the classroom" (D. Whitin, 1992 p. 24). If students can see mathematics as part of their everyday lives they will be encouraged to value mathematics more than they would otherwise and become more confident users (NCTM, 1989; D. Whitin, 1992; D. Whitin & Gary, 1994).

Consequently, curriculum bodies and mathematics teacher associations have encouraged cross-curricular integration and the inclusion of numeracy (and literacy) skills within each curriculum area. This is evident in documentation from the Australian National Curriculum (Australian Curriculum Assessment and Reporting Authority [ACARA], 2010), The Early Years Learning Framework (Australian Government, Department of Education, Employment and Workplace Relations [DEEWR], 2009), NCTM (2000), and The Australian Association of Mathematics Teachers (AAMT, as cited in Kemp & Hogan, 2000). ACARA (2012) maintained that a cross-curricular approach "broadens and enriches each student's learning" (p. 22). The Australian National Numeracy Review Report (Commonwealth of Australia, 2008) recommended that since, by definition, "numeracy" requires students to be able to use mathematics in all areas of life, teachers should embed mathematical learning in all disciplines. It points out that "the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment" (p. 7). The elements of "engagement" and "connectedness" espoused above are also promoted in the Quality Teaching Framework (NSW Department of Education and Training, 2003). Journals, too, such as *Teaching Children Mathematics* and *Australian Primary Mathematics Classroom* (APMC), make regular references to, and provide ideas for, the different ways in which picture books can be used as part of an integrated teaching program (e.g., Clarke, 2002).

The problem, however, with an integrated or purposefully connected curriculum is maintaining the integrity of all integrated areas (Perger, 2004). "The key purpose of the curricular area must not be 'watered down' or lost as teaching and learning draws on more than one curricular area" (Naylor, 2014, p. 3). In the case of integrating literature and mathematics, it is also important to be able to emphasise the mathematics without losing the enjoyment or literary quality of the story (Perger, 2011; Shih & Giorgis, 2004).

Stories like those told in picture books, and mathematics, are very similar, as they are both "problem-generated and goal-oriented" (Doxiadis, 2003. p. 14). Therefore their coupling appears sensible and beneficial in many ways. Although I advocate throughout this thesis for picture books to be used in promoting mathematics teaching and learning for young children, I see it as only one strategy of many that teachers can use to meet the mathematics learning needs of their students.

1.3 Research problem

Despite mathematics (and numeracy) being an essential part of everyday life, it is often not a subject enjoyed by many students (Attard, 2015), teachers, or preservice teachers (Young-Loveridge, Bicknell, & Mills, 2012). Many do not have positive attitudes to mathematics or see its relevance in their lives. Low levels of engagement have been shown to limit learning and understanding (Australian Institute for Teaching and School Leadership [AITSL], n.d.; Fredericks, Blumenfeld, & Paris, 2004). Doxiadis (2003) suggested that mathematics educators could overcome the "generalized dislike of mathematics in our culture" through the use of narrative (p. 1). In this case he was not advocating the use of picture books but mathematics learning through the stories of great mathematicians and embedding mathematics in history.

He argued that that mathematics should not be “camouflaged” or “sugar coated” but that it should be put in a more interesting frame of reference.

Most children enjoy a good story, and learning in the context of a story has been shown to be more effective (Casey, Erkut, Ceder, & Young, 2008). Using mathematical picture books appropriately by incorporating constructivist principles rather than an instrumental approach may provide one strategy to overcome this dislike of mathematics in both students and teachers. Using picture books can provide a more engaging environment for quality mathematical learning for all.

However, despite teachers using picture books widely in the teaching of literacy, and encouragement from curriculum bodies, the small amount of research about teacher usage suggests that picture books for mathematics teaching and learning is not commonplace. VerMaas (n.d.) reported that, on average, in primary classes around 10 percent of teachers never use picture books for mathematics learning, 70 per cent of teachers use picture books occasionally (i.e., one a month or less often), and only about 20 percent use them weekly or more often.

This low usage may be for several reasons. First, teachers are not necessarily aware of the benefits of picture books in teaching mathematics. Second, teachers do not know how to choose appropriate books for quality mathematics learning experiences. There are books explicitly written for this purpose, e.g., *One Hundred Hungry Ants* (Pinczes, 1993). Previous research (e.g., A. Anderson et al., 2005; Van den Heuvel-Panhuizen & Van den Boogaard, 2008) has demonstrated that there are other books that also provide potential for classroom use. Although criteria have been developed (e.g., Hunsader, 2004; Schiro, 1997), material in Chapter 2 demonstrates that there are few current guidelines for professionals to source appropriate books for quality learning experiences. Some books written explicitly to teach mathematics are not necessarily accurate or age appropriate, or provide opportunities for problem solving and integration. These books, too, are not inevitably of literary value. Conversely, a book of great literary value may not include any potential for mathematics learning. Additionally, the covers of books may not suggest the mathematical opportunities within. Therefore, teachers need effective and relevant criteria to facilitate selection and evaluation of mathematical picture books.

Third, despite some available resources (e.g., Griffiths & Clyne, 1988; D. Whitin & Whitin, 2009), teachers do not necessarily know how to use the books effectively for the particular mathematical needs of their students. Teachers may also try to impose mathematics on every book, with tenuous links to both the mathematics and their overall program.

Fourth, although different picture books have been shown to elicit different mathematical responses, there has been no formal classification scheme or clear definitions of the different types of mathematical picture books. Nor has any previous description of book types taken into account the intentions of the author. Additionally, there has been no research about a classification's efficacy for identifying the most suitable picture books for mathematical learning or research involving a comparison of the books' use in the classroom.

Fifth, there is also a lack of information about the intentions of authors of picture books, and more so about how and why they intentionally or inadvertently include mathematical ideas within their books. Predominantly, these authors have little understanding of how children learn mathematics.

Finally, there is limited published research about cognitive engagement of young children with mathematical picture books in a naturalistic classroom setting and the role that teachers play in that engagement.

1.4 Aims

The main aim of this research is to promote the use of picture books in mathematics teaching and learning for young children.

This research is significant because its objectives are to:

1. develop and evaluate a classification scheme for different types of mathematical picture books;

2. develop and evaluate a framework for the selection of picture books for quality mathematical learning experiences;

3. investigate the intentions of authors when including mathematical ideas and content in their writing to inform authors and curriculum writers; and

4. ascertain, through a naturalistic classroom study, how children (and teachers) engage mathematically with picture books.

1.5 Research questions

Three questions guided the study:

1. What potential do picture books (of different types) have for promoting mathematical teaching and learning for young children?

2. What are authors' intentions when writing picture books with potential for mathematical learning?

3. How do children and teachers engage with mathematical picture books of different types?

1.6 Overview of the project

An exploratory descriptive study was designed in three phases to evaluate the role of picture books in promoting mathematical teaching and learning in the early years of formal schooling.

Phase 1 of the study highlights the classification of suitable mathematical picture books as *perceived*, *embedded* or *explicit*, and the development and evaluation of a framework[6] for professionals and caregivers when selecting and using mathematical picture books (Chapters 4 and 5).

Phase 2 reveals the intentions of four key authors whose books were identified as including mathematical content either purposefully (*explicit* or *embedded*) or unintentionally (*perceived*) (Chapter 6).

Phase 3 includes a classroom study with 16 six- to seven-year olds (Year 1). This phase first demonstrates how children and teachers engage with three different types of mathematical picture books during shared book reading sessions. Second, it reveals, during a follow-up activity with me, how students responded to each book through spontaneous representations (Chapters 7, 8, and 9).

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[6] The newly developed framework will be subsequently referred to as the framework.

1.7 Significance of the study

The findings of previous studies, detailed in Chapter 2, have indicated that different picture books elicit different responses in young children. However, the different qualities of these books have not been clearly defined. Although these studies demonstrated that picture books do facilitate mathematics learning, their authors have concentrated only on either “trade” (*explicit*) books or *perceived* mathematical picture books. To date there has been no substantial recognition of the third mathematical type that I identified as *embedded* picture books and there has been no comparison of the effects of these three types of books on mathematical learning with young children.

In this thesis I compare the different types of mathematical picture books to determine their influence, particularly their influence on children’s engagement with mathematics. Their effect on teacher practice is also investigated. The results of the study should help to identify particular features of picture books that can support teaching and learning in mathematics.

Frameworks have been developed previously (see Halsey, 2005; Schiro, 1997; Van den Heuvel-Panhuizen & Elia, 2012; D. Whitin & Whitin, 2001, 2004), but the framework devised in this thesis is meant to be more comprehensive in scope. It allows professionals and caregivers to evaluate any book for its mathematical potential according to their curriculum and classroom needs.

The paucity of information about author intent in writing books that particularly include mathematical content prompted the second phase of the study. The information gained here may aid in the future writing of picture books with purposefully included mathematics (either embedded or explicit).

Finally, a third phase was conducted within the classroom setting, where picture books are used most commonly for numeracy (and literacy) skill development. Although there are other studies of shared book reading for mathematics learning, there are few studies that also focus on how teachers take advantage of opportunities for mathematical concept development with the three different types of picturebooks. This study advances the literature by investigating whether opportunities for mathematics learning are realised by the teacher and students.

1.8 Organisation of the thesis

This thesis is written and presented as a "thesis by publication". According to Macquarie University Policy Central:

> A thesis by publication may include relevant papers, including conference presentations, which have been published, accepted, submitted or prepared for publication for which at least half of the research has been undertaken during enrolment. The papers should form a coherent and integrated body of work, which should be focused on a single thesis project or set of related questions or propositions. These papers may be single author or co-authored (Macquarie University, 2014).

It contains some chapters that would traditionally be found in a thesis (i.e., introduction, literature review, theoretical perspective and methodology, and conclusion). Other chapters include eight publications with additional comments to position the publications within the sequence of the study and to promote coherency. Appendices containing more detailed information to add depth and strength to the study are also included.

As each publication requires similar background information, statements of aims, research questions, and methods, some unavoidable repetition has occurred. Some reporting of results, too, was repeated in publications because those publications were intended for different audiences.

Three of the eight publications (one conference paper, one book chapter, three professional journal articles, and three short communications) were coauthored with my principal supervisor (Professor Joanne Mulligan) or other colleagues. Although I consulted regularly with my supervisors about the design and analysis involved in the study, I was primarily responsible for developing and carrying out the surveys and interviews and organising transcription of the interviews and the classroom study. I also recorded, coded, and analysed the data and wrote the first draft of each chapter and publication in the required format. The chapters and publications were then rewritten after consultation with, and suggestions from, my supervisors. The publications were also revised, where necessary, after undergoing the process of peer review.

The following sections contain a brief description of each chapter in this thesis.

Chapter 1 Introduction

In this first chapter I introduce the thesis by providing background information, definitions, a rationale, aims, and the research questions. I also identify the significance of the study and provide an overview of the structure of the study.

Chapter 2 Literature review

This chapter includes a summary of the relevant literature relating to the selection, evaluation, and use of picture books for promoting mathematical teaching and learning.

Chapter 3 Theoretical perspectives and methodology

In this chapter I first describe key theoretical perspectives used throughout this study. I also describe the design and methods used for data collection and analysis. Ethics procedures are also included here.

Chapter 4 Phase 1: The development of the classification scheme and framework

This chapter includes an account of the first stage of Phase 1 of the study: the development of new classification scheme and framework for selecting and evaluating mathematical picture books. The first of three publications introduced the study to a mathematics education research audience.

Publication 1: Marston, J. (2010a). Developing a framework for the selection of picture books to promote early mathematical development. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education*: *Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 2, pp. 383–390). Fremantle, WA: MERGA.

The development of the classification scheme and framework are described for an early childhood audience in the book chapter that forms Publication 2. Examples for use of the framework are also described.

Publication 2: Marston, J., & Mulligan, J. (2012). Using picture books to integrate mathematics in early learning. In P. Whiteman & K. De Gioia (Eds.), *Children and childhoods 1: Perspectives, places and practices* (pp. 209–225). Newcastle upon Tyne: Cambridge Scholars.

The third publication, based on a conference presentation and workshop conducted for practising primary school teachers, demonstrates again the use of the classification scheme and framework in a primary classroom context.

Publication 3: Marston, J. (2011). A framework for teachers to select and evaluate picture books to promote early mathematical development. *Reflections*, *36*(1), 17–19.

A postscript concludes this chapter that specifically contributes to addressing Research Question 1. The potential that picture books (of different types) have for promoting mathematical teaching and learning for young children has been shown through examples aligned to the new framework.

Chapter 5 Phase 1: The evaluation of the classification scheme and framework

In this chapter I provide an account of the second stage of Phase 1 of the study: the evaluation of the framework and scheme for classifying mathematical picture books. I present the methodology, results, analysis, and discussion concerning a survey conducted with professionals about the usefulness of the classification scheme and the framework for pedagogical practice.

Two refereed conference abstracts (Publications 4 and 5) that presented early findings of the survey are included in this chapter.

Publication 4: Marston, J. (2014a). Using picture books to implement the mathematics curriculum: The missed opportunities. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice*. *Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia Incorporated* (p. 759). Sydney, NSW: MERGA.

Publication 5: Marston, J., & Mulligan, J. (2014). Identifying opportunities for mathematics learning in picture books using a framework. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 6, p. 166). Vancouver, Canada: PME.

In this chapter I continue to outline the potential of picture books to provide mathematical learning experiences.

Chapter 6 Phase 2: Authors' views and intentions in writing children's picture books

In this chapter I report Phase 2 of the study. In it I explain the method and analysis of interviews with four leading Australian authors and/or illustrators regarding their writing intention, with particular emphasis on mathematical inclusions within their books. A refereed conference abstract that presented preliminary findings to practising teachers and academics concludes the chapter.

Publication 6: Marston, J. (2012). An analysis of the authors' intentions in developing mathematical picture books. In J. Wright (Ed.), *Proceedings of the Joint International Conference of the Australian Association for Research in Education [AARE] and the Asia Pacific Educational Research Association [APERA]*. Sydney: AARE. Retrieved from http://www.aare.edu.au/publications-database.php/6825/an-analysis-of-the-authors'-intentions-in-developing-mathematical-picture-books

Although this chapter particularly addresses Research Question 2, it also further demonstrates the potential of picture books to facilitate and engage students in mathematical learning (Research Questions 1 and 3).

Chapter 7 Phase 3: Classroom study – Method, and mathematical content analysis

The next three chapters detail the naturalistic classroom study conducted with 16 children in three classrooms across three schools in which I compared the use of the three different types of picture books with young children. In this chapter I report the methods for both data collection and analysis. I also describe the books used and report the results of a mathematical content analysis that was conducted on them.

Chapter 8 Phase 3: Classroom study – Shared book reading sessions

Chapter 8 reports the second stage of the classroom study, the analyses of results of the shared book reading sessions. The chapter's purpose is to review the types, amount, and source of teacher and student mathematical talk in each book. The findings were later compared to the results of the mathematical content analysis conducted in Chapter 7.

Chapter 9 Phase 3: Classroom study – Student representations

In this chapter, the final stage of Phase 3, I report and discuss the findings of the follow-up activity to the shared reading sessions where students responded to each book through drawing and writing. The findings that reveal the content and source of each representation added to the data collected in Chapter 8 to determine the factors that promote mathematical learning through the reading of picture books. A final summary of the findings of Phase 3 concludes the chapter. The combined findings from Chapters 7, 8, and 9 assist in answering Research Questions 1 and 3.

Chapter 10 Implications for teaching, learning, and the curriculum

In this chapter I identify the implications that the findings of the three phases suggest for pedagogy. In it I first highlight the difficulties encountered by classroom teachers in implementing picture books in their mathematics teaching, despite recognising their benefits. I then consider the implications that my findings have for book selection, the role of teachers in selecting and using a mathematical picture book, and how students respond to this teaching strategy.

Two articles in professional journals that demonstrate how picture books can be used in the classroom setting are included in this chapter. The first journal article (Publication 7) demonstrates to classroom teachers the use of the framework to evaluate picture books for mathematical learning experiences. It particularly appraises *Counting on Frank* (Clements, 1990), a commonly used classroom book, and whose content has not been previously questioned. It shows that despite inaccuracies there is still much potential for mathematical problem solving.

Publication 7: Marston, J., Muir, T., & Livy, S. (2013). Can we really count on Frank? Using a framework to select and evaluate picture books for mathematical concept development. *Teaching Children Mathematics, 19*(7), 440–448.

The second article (Publication 8) provides another example of rich mathematical learning experiences using picture books and the framework. This time, three Australian picture books, *At the Beach: Postcards from Crabby Spit* (Harvey, 2004), *How Big is Big?* (Watson, 2009), and *Uno's Garden* (Base, 2006) are used.

Publication 8: Marston, J. (2014b). Identifying and using picture books with quality mathematical content. *Australian Primary Mathematics Classroom*, *19*(1), 14–23.

Chapter 11 Conclusion

In this chapter I first summarise the findings of the three phases of the study. I then outline the limitations and significance of the study and propose ideas for further research in this area. A final remark concludes the thesis.

CHAPTER

2

LITERATURE REVIEW

In this chapter I focus on a review of the key research, primarily from a mathematics education research perspective, that guided the three phases of this thesis. This summary of pertinent research is supported by other significant literature in the following chapters and the publications[1]. The thesis was designed in 2010 based on the literature at the time. However, more recent research has been reviewed in this chapter.

I begin by introducing research about the role of narrative and storytelling in picture books. Second I discuss how cognitive engagement is used as a measure of learning. I then review some studies of cognitive engagement with picture books both from a literacy and mathematical standpoint. Following that, I describe intervention studies focused on children's engagement with picture books in mathematics learning contexts. Studies about the development of frameworks for the selection and use of picture books in educational settings are then reported. Finally, reference is made to literature demonstrating the practical application by educators of picture books in early childhood and school contexts.

2.1 The role of narrative and storytelling in picture books

Storytelling is one way that enables cultures to pass on their traditions and their way of life. In ancient cultures, the storytelling was oral, but modern media, including picture books, also provide these opportunities. A. Anderson, Anderson and Shapiro (2005) note, we must be "sensitive and inclusive of the literary practices of all cultural groups" (p. 6).

D. Whitin and Wilde (1995, p. x) stated that stories are a "fundamental way of making meaning", while to Schiro (2004, p. 49), storytelling or narrative stories are "a primal act of mind". Smith (2014, p. 64) said they "are not a special way of perceiving the world"... "It is the only way." Stories teach the reader right and wrong, and how to interpret the world. Hardy (1977) claimed that we dream, plan, and remember/relive in narrative, and Smith (2014, p. 63) asserted that our "brain is a story-seeking, story-creating instrument". Wells (1986) asserted that narrative is an unconscious mind action, and Smith, (2014), agreed that it is not something taught but just "what everyone does" (p. 64).

[1] Due to the structure of the thesis there is some unavoidable repetition of the literature across the publications and with this review.

Teachers of children in the prior-to-school and school settings have always used stories, particularly in the form of picture books, as part of their literacy programs. These stories and books have been used for enjoyment and to foster a love of reading but also to impart knowledge (nonfiction books) and to serve as a source of social and emotional development such as conflict resolution, cooperation and taking responsibility (Hong, 1996). Children have been found to identify with the characters in books (Keat & Wilburne, 2009; Zazkis & Liljedahl, 2009), and the real-life contexts provided by the picture books help children "make sense of problem situations" (Flevares & Schiff, 2014, p. 4).

A wide body of research (e.g., A. Anderson et al., 2005; Dickinson & Smith, 1994; Dickinson & Tabors, 2001; Giorgis & Glazer, 2009; Shapiro, Anderson, & Anderson, 1997; Whitehurst & Lonigan, 2001) has demonstrated that shared reading either in the school or home environment contributes to children's literacy learning through exposure to rich new words and their meanings, as well as to grammatical, textual, written, and book structures. The inclusion of picture books in literacy programs is therefore well justified. These benefits can be further extended to the use of mathematical picture books for promoting mathematical learning. A. Anderson et al. (2005) claimed that "while literacy has traditionally referred to the ability to encode and decode print, more recent work in multiple literacies (e.g., Cope & Kalantzis, 2000) points to various ways that meaning is coded through different sign systems (including mathematics)" (p. 5).

2.2 Promoting learning through cognitive engagement

According to McLaughlin et al. (2005), "at the most general level, learning occurs through the cognitive engagement of the learner with the appropriate subject matter knowledge" (p. 3). Some key studies about the use of picture books in mathematics learning, particularly those that are outlined in Section 2.4, have used students' utterances as a measure of cognitive engagement. For example, Van den Heuvel-Panhuizen and Van den Boogaard (2008) and Elia, Van den Heuvel-Panhuizen, and Georgiou (2010) analysed children's responses and utterances "as being a sign of mental processing" (Elia et al., 2010, p. 133). See Section 2.3.1. Therefore, as cognitive engagement of students is also used in this study to demonstrate how picture books promote mathematical learning, it is important to define what is meant by "cognitive engagement".

Fredericks, Blumenfeld, and Paris (2004) reviewed several dictionary definitions for "engagement" that include words such as "commitment", "to participate" and "to attract and involve" (p. 60) before stating that there are three types of engagement (that may overlap): behavioural, emotional, and cognitive. The last "draws on the idea of investment; it incorporates thoughtfulness and willingness to exert the effort necessary to comprehend complex ideas and master difficult skills" (p. 60). Although recognising that cognitive engagement is difficult to measure, the authors suggest techniques such as the use of observation to detect, for example, the exchanging of ideas, giving of directions, and justifying of an answer.

More specifically, Helme and Clarke (2001) say that cognitive engagement particularly "involves the thinking that students do while engaged in academic learning tasks" (p. 135), where

> The quality or level of this involvement is generally believed to have a profound effect on learning outcomes, in that students who "really put their minds to it" are much more likely to learn successfully than students whose engagement with the subject matter is low (Helme & Clarke, 2001, p. 133).

High engagement levels have also been linked to higher achievement and persistence in school (Finn & Zimmer, 2012; Frederick et al., 2004) "and it is higher in classrooms with supportive teachers and peers, challenging and authentic tasks, opportunities for choice, and sufficient structure" (Frederick et al., 2004, p. 87). The individual, the learning environment, and the task all influence cognitive engagement (Helme & Clarke, 2001). Likewise, making connections to real life promotes engagement and thus a deeper understanding of the world around us (Bryson, 2014). These strategies all reflect the constructivist (socioconstructivist) approach outlined in Section 3.1.1. Cordova and Lepper (1996, as cited in Casey, Erkut, Ceder, & Young, (2008, p. 30) assert "the motivational benefits of embedding mathematics content within a meaningful context" such as a picture book.

In the case of picture books in general, Moschovaki and Meadows (2005a; 2005b) described cognitive engagement as "the thinking skills that a participant activates in order to understand the text and successfully participate during group discussions" (2005a, Section 1, para. 1). As the "child becomes more involved, he or she participates more actively", thus prompting "the development of thinking skills that help children engage successfully in abstract thinking" (Moschovaki & Meadows, 2005b, Section 1, para. 2).

Moschovaki and Meadows described their approach to children's responsiveness in shared book reading in the following way:

> Children's spontaneous comments and questions indicate their focus of attention during book reading and reveal the thinking skills that children apply during text comprehension. A detailed analysis of their spontaneous participation can shed light on features of the book that have attracted the children's attention; it can also reveal how such features assist their ability to comprehend the text. Children's spontaneous questions can also throw light on the way they try to fill their knowledge gap by eliciting information from the adult who is reading (Moschovaki & Meadows, 2005a, Section 1, para. 2).

2.3 Studies of cognitive engagement using picture books

Moschovaki and Meadows (2005a, 2005b) examined young children's spontaneous participation (2005a) and cognitive engagement during classroom book reading (2005b). Although this study focused on literacy and text comprehension, many of the findings are also applicable to children's cognitive engagement with mathematical picture books.

Moschovaki and Meadows' study was conducted in Greece with 20 teachers and classes comprising 10 to 20 children aged between 3.5 and 5.5 years. The teachers read four picture books (fiction and non-fiction)[2] in random order "to assure that there was no difference between books caused by children's increasing experience of being read to" (Moschovaki & Meadows, 2005a, Section 3, para, 2).

The researchers then investigated children's utterances as they responded to the story read by their teacher according to the different types of books (2005a) as well as text genre and story format" (2005b). In this study children's behaviour was coded according to several categories including one referring to the cognitive engagement. Here all utterances were recoded according to levels of cognitive demand (low, medium, and high). Each category contained multiple subcategories, most of which related to literacy skills. The subcategory of *labelling* "naming objects, describing pictures, identifying features abstractions of physical properties such as color, size" (2005b, Section 3, para, 8) related to responses to the illustrations. The researchers also analysed the role of the teacher according to the genre of the book.

[2] The four books were each placed into a different category: information book with limited expository text: *The Four Elements* (Rius & Parramon, 1992); information book: *Life Under Earth* (Rius & Parramon, 1994); contemporary fiction: *Winnie the Witch* (Paul & Thomas, 1990); and traditional fairy tale style narrative: *The Three Little Wolves* (Trivizas, 1993).

The findings confirmed previous research (including that of Beals, DeTemple, & Dickinson, 1994; Donaldson, 1978; and Wells, 1986) about children's language and literacy development. First they verified that children were meaning makers who found interpretation of text more difficult without illustrations but that this need decreased with age. Second, spontaneous comments from children increased with added familiarity with the book. Differences also appeared in student responses according to text genre and story format. The authors realised this could be influenced by individual responses of the reader, so they analysed only comments initiated by the children. Spontaneous student questions also revealed children trying to fill gaps in their knowledge. Fiction appeared to initiate an "aesthetic stance" that evidenced "emotional engagement ... through their personal responses, reliving the story through recall" (Moschovaki & Meadows, 2005a, Section 5, para 8), and rhyming and singing. Information books initiated an "efferent stance focusing on topic knowledge" (Moschovaki & Meadows, 2005a, Section 5, para 8). However, the researchers felt that references to, and comments about, the illustrations were not "due to the particular text genre or story format, but rather the fascinating illustrations that accompanied the text" (Moschovaki & Meadows, 2005a, Section 5, para 10).

Not only did the children in Moschovaki and Meadows' study react differently to each genre, their responses also changed according to the teacher's presentation style that itself was also influenced by the genre of the book. In narrative/story books the teachers' emphasis was more on understanding and retelling the storyline i.e., comprehension of the text. In expository/information books the teachers concentrated on vocabulary and concept or knowledge building; they would even stop and demonstrate these throughout. Here there was little review of the book or importance placed on comprehension and characterisation, and reference was often made to the illustrations. This was consistent with previous findings from Mason, Peterman, and Kerr (1989), who also found that teachers' presentation differed according to a book's genre.

The illustrations (that were included in their subcategory of labeling) attracted most attention, but were only considered to demonstrate low cognitive engagement. However, information books were found to be of higher cognitive demand than were narratives where high cognitive engagement is considered "particularly useful for the development of children's representational abilities" (Moschovaki & Meadows, 2005b, Section 6, para., 2). They also found that the children wanted to relate their own experiences to the book's topic and events (medium cognitive engagement), thus linking new experiences to prior knowledge as espoused by constructivist theory.

Additionally, teachers were found to talk 59% of the time, while the children spoke 41% of the reading discussion time. Therefore "children's cognitive engagement" depended "on the teachers' choice of cognitive strategies" (Moschovaki & Meadows, 2005b, Section 4, para., 2).

The findings of Moschovaki and Meadows demonstrated that picture books engage children at different levels. The results indicate that the level of cognitive engagement depends on the book's genre, the teacher's presentation of the genre, and teacher questioning skills. However, the analysis did not address the way that the children engaged with mathematical ideas, the focus of the next section.

2.4 Studies on the use of picture books to engage young children in mathematics learning

In this section, I provide an overview of the key studies focusing on the use of picture books to engage young children in mathematics learning across a range of contexts. As well as the pertinent findings reported here, some studies also included findings about other aspects such as the effect of participants' cultural background. These findings are not discussed as they are beyond the scope this thesis.

The focus of studies by A. Anderson, J. Anderson, and Shapiro (together and individually) has been on the mathematical cognitive engagement of young children as they were read to by their parents. In an early study, Shapiro, Anderson, and Anderson (1997) focused on "diversity in parental storybook reading (p. 47). Twelve children, all aged 4 years , were read two books[3] by their mother in one sitting to explore the different "types of parental interaction" (p. 50). Well-known books were chosen because "the style and quality of illustrations are similar and the content and illustrations have the potential for the development of mathematical concepts such as size, shape, number, counting and estimation" (p. 51).

Shapiro et al. (1997) video-recorded all sessions and then coded all verbal and nonverbal events between parent and child into four categories: attention to print, attention to illustrations, attention to mathematics, and attention to knowledge development. Although there were differences between the responses from the mothers across and within categories,

[3] *Swimmy* (Lionni, 1991) and *Mr. McMouse* (Lionni, 1992).

the analyses indicated the illustrations to be the predominant source of verbal and nonverbal events. A lack of attention to text by parents and children was also noted.

Despite the books having similar potential mathematically, in the attention to mathematics category only one book produced discussion about size, and there were surprisingly few references to counting by the participants for both books. This led the researchers to conclude that "different books have different potential as contexts for mediating mathematics" (p. 54).

In another study A. Anderson (1997) explored "the verbal interactions between parents and children as they engaged in activities in the home that had potential to aid the children's mathematical learning" (A. Anderson, 1997, pp. 486—487). Twenty one middle-class parents and their 4-year-old children used a kit of "multilink blocks, a child's book[4], blank paper, and preschool worksheets" (p. 484). Parents were told to use the materials across "four separate 15-minute episodes over 2 days" (p. 484) in whatever way they wished. All sessions were audio recorded by the parents for later analysis. On completion of the tasks, parents were involved in a 5-minute "exit interview".

Coding of 15 mathematical events ensued. "Any time a child or parent used a mathematical term or engaged in a mathematical process, this was deemed a 'mathematical event' and was then coded according to the specific mathematics present in the dialogue" (A. Anderson, 1997, p. 289). Overall, the most commonly used mathematical area was number, particularly counting, followed by geometry and measurement.

During the book reading session, too, dialogue related to counting dominated and comparison of size followed. Much of this dialogue related to only one particular illustration. Number, in particular counting, is usually the most common mathematical concept found in young children's picture books along with geometric shapes and one-to-one correspondence (Tischler, 1988). In an earlier paper, A. Anderson (1996, as cited in A. Anderson et al., 2005, p. 21) stated "that parents of young children tend to equate mathematics with number and/or counting". As she also found differences in results in all areas across the study, A. Anderson (1997) surmised that we should "consider the learner, the more proficient other, the physical materials, and the co-constructed environment as interdependent components of the context" (p. 509).

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[4] *One Snowy Night* (Butterworth, 1989)

A. Anderson et al. (2004) then studied 21 parents reading only one book[5] to their 4-year-old child either at day care or home, with the instruction "Share the book as you normally would with your child". The book was chosen because of “a cumulative story structure that has the potential to elicit mathematical discourse” (p. 11). All sessions were audio-recorded. Discourse or conversational turns as described by Sinclair and Coulthard (1975) were used for analysis (see Section 7.4.2).

The results of the discussions of a focus group of 4 dyads were reported. The mathematical concept of number (counting, subitising, addition, and subtraction) was again most evident throughout the discussions to varying degrees across families. The concept of size was also present. However, in this study there was an emphasis on who initiated the mathematical discourse. This, too, varied across families. In two families the parent dominated all mathematical discourse, in one family the child also contributed, and in another dyad the child initiated the discussion. Discourse also unfolded in different ways (e.g., through direct knowledge questioning, predictions, etc.) that also varied by parent.

A. Anderson et al. (2004) drew several conclusions from their research. The first was that when parents read together and “attend to the mathematics, they do so in the context of meaning making” where the “mathematical discourse appears not to be contrived, it is connected with the story and illustrations” (p. 29). Second, although much literature suggests that picture books are a springboard to mathematical learning, this study also indicated that parents and children “co-construct” (p. 29) the mathematical concepts. Third, the results were varied as each reading brought its own background, routines, and interactions. Finally, the researchers recommended that teachers, researchers, and curriculum writers “continue to explore ways of embedding mathematical discourse into storybook reading in integral ways” (p. 29).

A further study by A. Anderson et al. (2005) was conducted, this time with 39 parents and their 4-year-old children, all of whom were in day care. The aim of the new study was to ascertain “how parents and their young children attended to mathematical concepts as they engaged in shared book reading” (p. 5). Parents read the same two picture books as in the Shapiro et al. (1997) study. These books had been shown to include mathematical concepts, although they had produced varied responses from children and their parents. The same

5 *One Snowy Night* (Butterworth, 1989)

instruction was given to the parents as in the A. Anderson et al. (2004) study described previously. Their readings and discussions were videotaped for later analysis.

Although the potential for discussion appeared the same for both books, again there were differing results, with one book (*Swimmy)* achieving three times the number of instances of mathematical discourse—a similar result to the earlier study (Shapiro et al., 1997). However, this time the mathematical concept of size emerged as the most common source of discourse. Number and space were also present in all the discussions as these two concepts were important to the meaning of the story.

In this study, where the analyses focused on the mathematical content, A. Anderson et al. (2005) also found that most of the mathematical discourse between the parent and child generated from both books centred on the illustrations. They also emphasised the importance of the engagement and speculated that young children "would not remain engaged, unless the mathematical talk was meaningful within the context of the shared reading event" (p. 20).

Van den Heuvel-Panhuizen and colleagues have conducted several studies as part of the PICOma (PIcture books and COncept development) project[6], started in 2006, with particular reference to mathematics learning. The project aims to:

> generate more knowledge about the effects of picture books on young children's learning of mathematics. The project's goal is to investigate how picture books can contribute to the development of mathematical concepts in young children, and how the actions of the teacher can strengthen the characteristics of picture books that support learning. (Van den Heuvel-Panhuizen, Van den Boogaard, & Doig, 2009, p. 30).

These studies predominantly focused on reading to individuals or small groups of children by the researchers or teachers using set guidelines. Van den Heuvel-Panhuizen and Van den Boogaard (2008) focused on the "cognitive engagement that is facilitated by the picture books themselves and not on how they are prompted by the reader" (p. 341). They analysed the responses of four 5-year-old children to the reading of one picture book[7]. This book was an award winner for both its literary and illustrative quality. Although the book contained many mathematical concepts, particularly those relating to number, it was not written with the purpose of teaching mathematics.

[6] The project is supported by the Netherlands Organisation of Scientific Research.
[7] *Vifde Zign [Being Fifth]* (Jandl & Jungee, 2000)

Each child was read the book individually by the same reader in their school setting. At the beginning of each session, the reader instructed the child "that the reader would turn the pages" and they were to "tell what happens in the pictures" (p. 353). The text was not read on each page until the child talked about the pictures. Reading guidelines and "reading scenario were developed" to explain "how each page should be presented" (p. 353) and how the reader should react. Therefore all utterances were made by the child without discussion between him/her and the reader. These sessions were video recorded for later analyses.

A grounded theory approach was applied to the analysis of these data, according to three main categories: general qualification of utterances, domain-specific qualification of utterances-number related, and domain-specific qualification of utterances-spatial orientation-related—each with subcategories. Twenty-two types of utterances were found among the total 432 utterances. However, the responses from the four children differed across these categories. With relation to the domain specific responses, 45% were found to be mathematics related, 31% were space orientated (taking a perspective and describing direction), and 14% related to number concepts (mostly "how many"). Of the 61 number-related utterances, 35 involved counting. Utterances were found to be spread almost equally across the book's pages, even the endpapers which are often ignored during reading sessions. However, some pages elicited more domain-specific utterances than did others.

There were also differences between the children with regard to the number of utterances. It was found that the children with the lowest total number of utterances had a higher proportion of utterances that were number related. As the initial instruction to the child by the reader related to the pictures, it seems reasonable to infer that the utterances also related strongly to the illustrations.

The main finding from this study was that 5-year-old children engage with the mathematical ideas in books not specifically written to teach mathematics, even without adult intervention. Van den Heuvel-Panhuizen and Van den Boogaard (2008) believed that this demonstrated the potential for the use of picture books for developing mathematical concepts, but that more evidence was needed. However, they recognised that although there was no verbal interaction between the reader and the child, there was another person present i.e., a "knowledgeable other" (as in constructivist theory, see Section 3.1) using "knowledge-able material" (the picture book) (p. 368).

Van den Heuvel-Panhuizen, Van den Boogaard, and Doig (2009) directed their attention from books with number concepts to three (perceived) books that they believed represented examples of geometry[8], data handling[9], and measurement[10]. It was recognised that some of the mathematical content of the books was beyond the curriculum expectations of the children involved in the study.

In this study, a teacher read and discussed the three books in turn to a group of six 5- and 6-year-old children who had not received any formal reading or mathematics instruction. Reading guidelines were provided for the teacher that included questions about the text and the illustrations and prompts such as requesting that the children respond to the illustrations before the text was read page by page. All sessions were video recorded.

The children's utterances were not analysed as previously. The researchers reported their observations through reflections and "snapshots" of discussion of each session for each book. Again the researchers concluded that "picture books can offer a meaningful context for learning mathematics, and provide an informal basis of experience with mathematical ideas that can be a springboard for more formal levels of understanding" (Van den Heuvel-Panhuizen, Van den Boogaard, & Doig, 2009, p. 37). In this case the children engaged in geometry, data, and measurement concepts. However, another interesting finding was that the children were able to discuss more complex mathematical concepts (e.g., graphs and cross sections) introduced by the author, than would be expected of children of this age and in their curriculum. The group-reading situation, too, engaged the children who interacted with each other to solve problems posed within the books.

Several recommendations were made as a result of this study. The first was that picture books should have the following characteristics: "a good story, engaging for the children, and with the mathematics readily available but not too blatant" (Van den Heuvel-Panhuizen, Van den Boogaard, & Doig, 2009, p. 38). Additionally, children should be exposed to concepts other than counting through picture books. These two ideas are further explored through the PICOma project in the development of their framework, described in Section 2.6. The authors of this study also recommended that teachers take full advantage of the opportunities to develop mathematical concepts through the use of picture books that had often not intentionally been written to teach mathematics.

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8 *O.nee! Pop in de wc...[Oh, No! Doll in the toilet...]* (Huijsing, 2006)
9 *De Verrassing [The Surprise]* (Van Ommen, 2003)
10 *De Prinses met de Lange Haren [The Princess with the Long Hair]* (Van Haeringen, 1999)

In a related study, instead of using perceived books, Elia, Van den Heuvel-Panhuizen, and Georgiou (2010) used a book[11] written explicitly to teach the mathematical concept of counting backwards. As the researchers realised the important role of illustrations, they also chose to specifically investigate the significance of “pictures on children’s spontaneous mathematical cognitive engagement” (p. 275). The study was then based on the “assumption” that images in purposefully written mathematical picture books “can have story-related components and mathematics-related components” (p. 275).

Four children aged 4.5 years from a private kindergarten in Cyprus, who had “received organized instruction in mathematics” (p. 280) were individually read, by the same reader, the picture book. Specific guidelines that included some questions by the reader were provided.

The children’s utterances were analysed in a similar method to that used by Van den Heuvel-Panhuizen and Van den Boogaard (2008). This included the category of general qualification of utterances, but the domain-specific utterances in this study now included number-related, spatial and topographical, and measurement-related utterances. Elia et al. (2010) found that the book elicited children’s mathematical talk that amounted to only 27% of their total utterances. Spatial-topographical utterances (e.g., using words such as “here” and “there”, or a description of the position of an object) were the most common, while measurement-related utterances (size or temporal words) were most rarely used. They inferred that the reason why spatial-topographical talk featured so highly was because “looking at the pictures of a book is a spatial activity for young children … which may evoke mental images and thus prompt relative utterances” (Elia et al., 2010, p. 290). However, they suggested that these explicit mathematical picture books may not be as useful as expected unless there is a teacher to ask the right questions of the student.

In this study that also focused on how the images prompt engagement, Elia et al. (2010) described two types of functions of illustrations: representational and informational. Representational pictures have the mathematics or story information in the text, while informational pictures do not. On this issue, they found that illustrations with a representational function stimulated more mathematical thinking than did those with only an informational function. They then concluded:

[11] *Watch out in the Jungle (Landers, 2005)*. The book was translated into Greek by Stella Zoumba. The English translation of the Greek title is *Six Brave Little monkeys in the Jungle*.

> that combining text and pictures of similar content has a greater power to mathematically engage than combining text and pictures of different content. … Even those pictures whose mathematical components have a representational function but are not congruent with the mathematical content of the text may have the potential to yield stimulating cognitive activity, especially to those children who understand the relationship between the picture and the text (p. 291).

Several gaps in the research about cognitive engagement of children with mathematical picture books have been noted, suggesting the need for further research in naturalistic classroom settings. First, these studies were conducted in different settings (school, home, day care) with research readers, teachers, and parents. Sometimes there was no reader-child discussion; sometimes the reading was directed by guidelines; and sometimes there were no instructions at all, just directions to read the book as the parent/reader/teacher normally would. Second, illustrations have also been noted as a major source of cognitive engagement in many studies, but there is limited research about their significance in shared-book reading of mathematical picture books in a regular classroom setting.

Third, the role of the teacher has not been particularly addressed. A. Anderson et al. (2004) noticed that the parents' use of picture books and sharing of the mathematical concepts was not consistent with the way that teachers use picture books for mathematics learning because of the different context/environment, and that "attempting to directly transpose what happens at home to the school or vice versa, may be difficult and not always desirable" (p. 29). Van den Heuvel-Panhuizen and Van den Boogaard (2008) also recognised the presence of a "knowledgeable other" (p. 368), even when there is no discussion between the reader and child. One aim of the PICO project, mentioned earlier, was "how the actions of the teacher can strengthen the characteristics of picture books that support learning" (Van den Heuvel-Panhuizen et al., 2009, p. 30), and this, too, has not been fully explored. Therefore, more focused studies on how teachers read and discuss picture books to encourage mathematics learning could advance understanding of the learning process (A. Anderson et al., 2005; Moschovaki & Meadows, 2005a, 2005b; Van den Heuvel-Panhuizen & Van den Boogaard, 2008). This research could also inform teaching practice.

A. Anderson et al. (2004) recommended that teachers, researchers, and curriculum writers "continue to explore ways of embedding mathematical discourse into storybook reading in integral ways" (p. 29).

2.5 Intervention studies and pedagogical innovations

Several intervention studies involving mathematical picture books with young children have been conducted over the last 3 decades. These have primarily investigated increased achievement and enjoyment of mathematics by students through the use of picture books. These studies again importantly highlight the benefits of using mathematical picture books for mathematics learning through systematic investigations.

One of the earliest intervention studies was conducted by Jennings, Jennings, Richey, and Dixon-Kraus (1992). They designed an experimental study involving 61 kindergarten children across two elementary schools in the USA. They tested whether reading mathematical picture books to young children increased mathematical achievement on numeracy test scores, mathematical interest, and mathematical language. A control group who used traditional workbooks, counting objects, and pencils, and an experimental group where teachers incorporated 20 mathematical picture books and materials related to the books that would aid mathematical learning, were established at each school. All children were pretested[12] before the 5-month long intervention began. Four teachers, all of whom were interested in the program, led the groups. Teachers of the experimental groups were trained in the use of picture books for teaching the required mathematical concepts.

The language used during the children's subsequent play was recorded by assistants according to mathematical concepts areas (e.g., geometry, number, patterns, and measurement). All play was initiated by the children; the teachers were present only to encourage and respond to children "on request" (Jennings et al., 1992, p. 266). Analysis of the posttest data revealed that all groups showed improvement. However, the degree of improvement was significantly higher for those in the intervention group and was attributed to the Piagetian (Piaget, 1963) teaching strategies (hands-on activities) used. Teachers and parents of the children in the experimental groups also reported increased interest from the children who noticed mathematical concepts in other books and requested rereading of the books used in the experiment. Furthermore, the children were seen "teaching" mathematics to their younger siblings from these books, and were observed transferring their mathematical knowledge and skills to other curriculum areas (e.g., mathematical shapes in art work).

[12] Children were pretested and posttested using Test of Early Mathematics Ability ([TEMA], Ginsburg & Baroody, 1983). The Metropolitan Readiness Test (MRT) was also used toward the end of the study.

In a Korean study of 57 kindergarten children aged 4–6 years, Hong (1996) investigated whether teaching mathematics using literature “can improve young children’s disposition towards mathematics” (p. 488). For this intervention, all children were read 28 books relating to their weekly theme over 7 weeks. A control group was read “ordinary” storybooks followed by activities involving unrelated mathematical materials. The experimental group was read books “with elements that could be developed into mathematics activities or games” (p. 482) followed by a discussion and related mathematical activities. Both groups were then given the opportunity of free play in their own space that contained several areas including a mathematics corner. The materials for the experimental group also related directly to the story content. Similar to the Jennings et al. (1992) study, the teachers were in the play areas, but all play was student initiated.

Pretesting[13] for learning readiness and mathematics achievement of all children revealed no significant difference between the two groups. Contrary to the findings of Jennings et al. (1992), however, posttesting[14] surprisingly showed little difference in mathematics achievement between the two groups. The researchers noted that almost all of the children involved in this study were doing mathematical worksheets, which resembled the posttest questions, at home with their parents and this may have affected the results. It was noted that students did achieve better results on questions involving real-life situations. This implied that the use of picture books seemed appropriate as part of an integrated program as they provide “opportunities for children to express mathematical thoughts and to practice using mathematical language related to the situations in the story, helping them bridge the gap between informal oral language and the formal symbolic code of mathematics” (Hong, 1996, p. 490). Observational data were also collected and analysed to measure children’s disposition to mathematics. “The results showed that more children in the experimental group liked the mathematics corner, chose mathematics tasks, and spent more time in the mathematics corner” (Hong, 1996, p. 477).

A post-study interview with one teacher involved with the experimental group also suggested that she could easily incorporate the use of picture books into her mathematics program to promote an integrated curriculum.

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13 *The Learning Readiness Test* [*LR*] (KEDI, 1989)
14 *The Early Mathematics Achievement Test (EMAT)* revised by Lee (1995)

Halpern (1996) took a different angle to most of the previous studies. Her aim was to examine whether adding explicit mathematical annotations to 'perceived' books enhanced or detracted from the original story, communicated "the mathematical concepts of the story more clearly" (p. 54), and aided in the construction of mathematical knowledge. She chose three commonly used books[15] that she enhanced with number sentences, graphs, and mathematical diagrams where appropriate.

The study involved 209 children in grades 1–3 in two schools in the USA. The children were read either the original or the enhanced book and subsequently asked what they thought the book was about. They were then read the other version, asked again about the storyline, and then asked to choose which book they would prefer to take home. Forty-three adults (parents, teachers, and teacher educators) were also surveyed about their thoughts concerning the books. About 83% of the whole group said they preferred the enhanced version (82% of the children and 84% of the adults). However, adult participants with a mathematics or reading specialist background, and those who enjoyed mathematics or reading, "tended to prefer the original version" (p. 56).

Halpern (1996) resolved the discrepancy by proposing that the annotations did not significantly change the perception of the story line, but communicated the mathematics more clearly and added to student understanding. The adults in the survey believed that the mathematical enhancements encouraged children to see the relationship between reading and mathematics and motivated them in their less preferred mode (reading or mathematics). Therefore, she also found that the annotations made the book more "enjoyable and even fun" (p. 57). Halpern (1996) recognised that all books may not be suitable or produce the same results as those used in her study. However, she suggested that teachers (and children) should not be afraid to add annotations themselves but warned against the overuse of the technique and overloading of individual books with annotations: "a delicate balance must be kept so as not to destroy the literature as a total work" (p. 58).

Picture books and specifically designed games were used in a 2-month program conducted by Young-Loveridge (2004) with 106 children who were 5 years of age in two schools in New Zealand. Investigating the effect of games and books on numeracy levels, she withdrew 23 of the children to form an intervention group who worked in pairs for 30 minutes with a specialist teacher. Each session included a number rhyme, a story involving numbers, and a number

[15] *I'll Teach my Dog 100 Words* (Frith, 1973), *Alexander, Who Used to be Rich last Sunday* (Viorst, 1978), and *The Doorbell Rang* (Hutchins, 1986)

game that involved reading numerals, collecting pieces, and counting and/or moving along a track using one-to-one correspondence. Some sessions concluded with another number rhyme. The remaining children, who acted as the control, continued with their classroom teacher using the Piagetian (Piaget, 1963), New Zealand progam *Beginning School Mathematics (BSM).*

All children's numeracy skills were pretested using individual task-based interviews. "The posttests included the pretest tasks, augmented with more difficult items of the same type, as well as some additional tasks" (Young-Loveridge, 2004, p. 85). Children were again retested 6 and 15 months later. Young-Loveridge, like Jennings et al. (1992), found that an intervention program contributed to improved numeracy levels. Although these lasted for at least the first year after participation in the program, the effects diminished over time. However, after 15 months the intervention group's score was still higher and they were able to perform more complex calculations than were the control group. Young-Loveridge attributed this convergence to the fact that, at the conclusion of the program, intervention children returned to the normal classroom program with the regular classroom teacher where what they had learned in the intervention program was not sustained.

The initial and immediate success of the intervention program was attributed to the children working in pairs with social scaffolding from the specialist teacher (consistent with Vygotsky's notion of scaffolding). In addition, the books again presented real-life contexts, where mathematics was an important component. Furthermore, the stories were accompanied by games that enabled use of manipulatives, and they presented mathematics in a fun way. However, Young-Loveridge (2004) recognised that, as her research was based on both games and books, it was difficult to determine whether one form was more effective than the other or whether it was necessary to use a combination of the two.

Rather than using published picture books, Casey, Kersh, and Young (2004) designed a supplementary program that presented mathematical problems to be solved by prekindergarten to Grade 2 children in the form of sagas. This project was designed around the National Council of Mathematics Principles and Standards (NCTM, 2000) and incorporated all mathematical strands, but most particularly spatial and geometry skills.

Six specifically written and illustrated books[16] were expected to be taught in a "systematic, hierarchical progression" (Casey et al., 2004, p. 168). The content was both embedded within the story as well as explicitly stated. These sagas were "intended to be delivered in a traditional storytelling manner, rather than simply read out loud from the book" (p. 168). This would "allow the storyteller to establish a deeper connection with his or her audience through eye contact, personal expression, and interaction with the audience" (p. 168). Casey et al. believed that this strategy encouraged children to be more involved in the mathematical concepts. Schiro (2004) stated that this approach meant the children were not "outsiders looking in on the world of others" (p. 3), as would be the case in reading a picture book.

Casey et al. (2004) evaluated the use of one saga[17] and found that there was greater improvement in the mathematical skills (pre to post) of the intervention (saga) group compared with those who had not used the story context. They concluded that "to improve the ability to understand and retain mathematical knowledge, it is useful to embed the mathematics in a story context and to develop mathematical concepts through sequenced mathematics problems connected to the storyline" (p. 43). However, to substantiate the benefits of this strategy more research was needed to evaluate other sagas in the series and the long-term benefits of the approach, and to investigate whether this strategy had different effects depending on the child's gender.

Further research (Casey et al., 2008) explored the influence of storytelling on geometry knowledge and skills in girls and boys across two studies. Results of the first study conducted in a lower middle-income suburban situation showed that the girls improved with the intervention (pretest to posttest), but boys improved regardless of which group they were in. As the analyses did not distinguish whether the geometrical activities or the particular choice of story influenced the improvement in the girls particularly, a second study was devised. This time all students, drawn from an urban area with high poverty levels, received either the story and associated activities (intervention group) or only the associated activities. Casey et al., (2008) found those children in the book and activity intervention group made greater improvements (pretest to posttest) than those in the activity only group, and that the girls benefitted more than the boys from any intervention.

Keat and Wilburne (2009) explored how picture books and the characters within them influenced 70 "kindergarten children's mathematical achievement and approaches to

[16] The *Round the Rug Math* series (Casey, 2003.

[17] *Tan and the Shape Changer* (Schiro, Casey, & Anderson, 2002).

mathematics learning" (p. 61). A prestudy meeting between the researchers and three kindergarten teachers interested in quality mathematics teaching developed a 3-week intervention using picture books[18] that depicted the story's characters posing problems about money.

> Data were collected with pre- and post-tests of children's mathematical knowledge of money, surveys of children's attitudes, classroom observations, audio-tapes of story book reading and discussions with children, pre- and post-study interviews with individual teachers, and group discussions with teachers (Keat & Wilburne, 2009, p. 63).

Other data included student work samples, photographs, teacher and researcher journals, and focus group interviews. Similar to Jennings et al. (1992) and Young-Loveridge (2004), posttest results demonstrated an increase in the mean score for knowledge of money concepts. A thematic analysis of transcripts also revealed that the students were "'enthusiastically interested' in the storybook characters and 'actively engaged' in making meaning of the story" (Keat & Wilburne, 2009, p. 66).

A teaching experiment based on constructivist principles and Confrey and Lachance's (2000) model, was developed and used by Worlley and Proctor (2005) with 19 Year 3 students over a 3-week period. The purpose of the experiment was to explore whether children's conceptual understanding of mathematics could be "advanced" using the "mediation tool" of children's literature (Worlley & Proctor, 2005, p. 809).

The books[19] used were purposefully written for teaching multiplication to older students. Lessons were structured around reading and discussion of the books by the teacher and children on multiple occasions. Mathematical activities based on the books then occurred in whole class and small group situations.

Pretesting and posttesting occurred using "audio & video taping, field notes", "interviews" involving 14 questions and "artefact collection" (Worlley & Proctor, 2005, p. 810). Again, improved posttest results led to the conclusion that using a well-structured sequence of learning with links to real life and books, and the use of concrete materials, increased the children's conceptual understanding of multiplication.

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[18] *Benny's Pennies* (Brisson, 1995), *Minnie's Diner: A Multiplying Menu* (Dodds 2004), and *A Chair for my Mother* (Williams, 1982).

[19] Books included *The Squirrels' Store* (Irons & Gardner, 1999), *Polly the Packer* (Irons & Roberts, 1999), and *Shirts and Skirts* (Irons & Reynolds, 1999)

Van den Heuvel-Panhuizen and Elia (2011) investigated the effect of picture books on kindergartners' performance in length measurement through an intervention program as part of the PICO project. Their focus was threefold: "What performance do kindergartners show in length measurement?", "How does the performance in length measurement increase over the kindergarten years?, and "What is the effect of a picture book reading program on kindergartners' performance in length measurement?" (p. 623).

A large study of 308 children from 18 kindergarten classes was conducted. The children's measurement ability was assessed using PICO developed measurement items. The children were then assigned to intervention or control groups, each matched for age, general mathematics ability[20], home language, and socioeconomic status. Children in the intervention groups were read two different picture books[21] that contained measurement concepts (but not purposefully written to teach mathematics) every week for four weeks (*N* = 8 books). To maximise uniformity, all teachers were supplied with a set of reading guidelines, suggested questions they could pose during the reading about the story line, and statements to reinforce ideas and the story. Teachers kept a log of how they read the books. Children in the control group followed their regular program.

Posttesting showed only a "weak but significant effect" (Van den Heuvel-Panhuizen & Elia, 2011, p. 631) from the book reading on students' performance of length measurement. This low result was attributed mainly to the short time span of the intervention, that there was no explicit teaching of measurement, and the provided guidelines may not always have been adhered to, therefore affecting consistency across teachers and schools. Van den Heuvel-Panhuizen and Elia concluded that further research that concentrated on the effects of picture books alone was needed. Other results demonstrated aspects of the development of measurement skills not relevant to this thesis.

Van den Heuvel-Panhuizen and Elia (2013) and Van den Heuvel-Panhuizen, Elia, and Robitzsch (2014a, 2014b) reported on another intervention study. It was developed to investigate how picture books promote learning of number, measurement, and geometry concepts. They also investigated the effect of age, years in kindergarten, gender, mathematics and language ability, home language, and socioeconomic status.

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[20] CITO Ordering Test

[21] For example, *De Lievelingstrui [The Favourite Jumper]* (Veldkamp & Van der Linden, 2001) and *Rosa's Reuze Zonnebloem [Rosa's Giant Sunflower]* (Damon, 1997).

The study involved 384 children (average age 5 years and 4 months) from 18 kindergarten classes in 18 schools. An experimental group of 199 children was read 24 high quality picture books with perceived mathematical content (eight for each of number, measurement, and geometry) selected using the *framework of learning supportive characteristics* (Van den Heuvel-Panhuizen & Elia, 2012; also see Section 2.6 below) over a 3-month period during their regular book reading time. The control group of 185 children was read their regular books that could have included some mathematical content. Pretesting[22] showed no differences in scoring between the two groups.

The teachers of the intervention groups received training including how to use the reading guidelines that had been provided. As these guidelines focused on “each book’s own power to promote the children’s mathematical thinking” (Van den Heuvel-Panhuizen & Elia, 2013, p. 245) they encouraged teachers not to ask too many questions that may “break the flow of the story” (Van den Heuvel-Panhuizen et al., 2014a, p. 8) but to “maintain a reserved attitude” (p. 8). Teacher behaviour such as “(1) asking oneself a question out loud about mathematics, (2), playing dumb, and (3) just showing an inquiring expression” (Van den Heuvel-Panhuizen & Elia, 2013, p. 245) were suggested. Teachers of children in the control groups were not told the purpose of the study until its completion.

Two sessions for each teacher were video recorded during the study and again all teachers kept logs of the reading sessions. Two regression models[23] were used in their analysis “to increase the statistical power” (Van den Heuvel-Panhuizen et al., 2014a, p. 12) that revealed an increased performance of 27% in Model 1 in the experimental group over the control group. Model 2 achieved 22% difference.

The benefits of using picture books for mathematical concept development were found. However, the researchers acknowledged that “the research design did not include opportunities to identify the effective elements of the picture book reading” (Van den Heuvel-Panhuizen et al., 2014a, p. 18). The findings were also considered to be influenced by the motivation of the teachers volunteering for the project, the guidelines that were provided, and teacher presentation style.

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[22] The *PICO* test, containing 42 items addressing arithmetic (number and number relations), measurement (length), and geometry (perspective) was devised for use in this study.

[23] One-way and two-way ANCOVA

The main findings of this review of intervention studies and pedagogical innovations highlight improved understanding and/or an increased positive attitude of children toward mathematics learning, thus strengthening the argument for the use of picture books in mathematics teaching and learning. Several findings from this review of intervention studies supported those in studies of cognitive engagement. First, Hong (1996) suggested that "young children may learn more easily if a task is presented in story format rather than as expository instructions" (p. 479). Second, Keat and Wilburne (2009) implied that the use of "characters in stories may provide young children with the kind of playful learning opportunities that Vygotsky (1978) described as essential to child learning in any content area" (p. 66). Like Moschovaki & Meadows (2005a; 2005b), Keat and Wilburne also found that multiple reading of the books during the sessions benefited student engagement.

Most of the studies reviewed in this section have used specifically selected picture books shown to support mathematical concepts. For picture books to be used beneficially in a classroom setting to promote mathematics learning, the books need to be chosen wisely. In the next section, I review the literature about the development of criteria and frameworks for this purpose.

2.6 Studies on the development of frameworks for the selection and evaluation of picture books for mathematical learning

The standards developed by the National Council for Teachers of Mathematics (NCTM, 1989), in the United States, have encouraged a more constructivist approach to mathematics teaching and learning than did the previous instrumental approach. The standards emphasised hands-on, meaningful, and problem-solving activities rather than isolated skill learning. "The role of written and oral communication in promoting mathematical understanding" (D. Whitin & Whitin, 2004, p. 1) was considered important and included the use of children's literature. When the U.S. state of California developed its mathematics program in 1994, based on these standards, criteria were set for the inclusion of trade picture books. These criteria included relevance to the mathematics concepts, grade appropriateness, suitability for integration across the curriculum, accessibility of resources, and literacy quality (Donahue, 1996). Subsequently, new frameworks and instruments were developed to assist teachers in the selection and evaluation of picture books for developing mathematical concepts in young children.

A chronological review of pertinent research in this field supports the development of a framework for the selection of picture books for mathematics teaching and learning.

Schiro (1997) created a set of mathematical standards and a set of literary standards for assessing the quality of mathematical picture books after finding “no standards that were rigorous or carefully conceived” (p. 2). As there was much written about literary qualities in books already he concentrated his efforts on mathematical standards.

After developing a draft of his assessment instrument, Schiro engaged a study group to use it with children’s mathematical trade books. Participants included nine professionals who were mostly parents of young children as this could provide dual perspectives. The assessment schedule was then revised and a “14 page instrument that contained 13 mathematical standards, each of which was defined by numerous specific criteria” (Schiro, 1997, p. 3) emerged.

After working with other educators Schiro found that he wanted to help them “develop their own critical analytical perspectives with which to view books” (Schiro, 1997, p. 3). He then developed a revised instrument that included both mathematical and literary standards because of his interest in developing integrated programs.

The resultant 11 mathematical standards, with various subcategories specified that the mathematics should be accurate, effectively presented, worthy of being learned, visible, present an appropriate view of mathematics, appropriate to the students intellectually and developmentally, involve the reader, and “provide the information to do mathematics”(Schiro, 1997, p. 77). Additionally, “the book’s story and mathematics should complement each other” (p. 77) and facilitate easy implementation, while the resources needed to “benefit from the book’s mathematics should not be too great” (p77). These standards (along with the 11 literary standards) were then presented as an evaluation form with a 5-point Likert scale and opportunities for reviewers to comment on each standard. Subsequently, Schiro’s (1997) instrument has been widely used and adapted by others when developing new frameworks.

Another consequence of the NCTM’s (1989) recommendations for the use of picture books was a flood of mathematical trade books to address the standards. Some of these books contained “meaningful, engaging experiences for students,” while others were not always of a high literary standard and resembled “glorified textbooks” (Hellwig, Monroe, & Jacobs, 2000,

p. 139). Rather than devising an instrument to assist in finding mathematics in the books, Hellwig et al. (2000) produced a guide to evaluate books “distinctly oriented towards mathematics” (p. 139). Their six-part sliding scale assessed five categories: accuracy, visual and verbal appeal, connections, audience, and the “wow” factor. Elaborating on these categories, they first noted that if there were inaccuracies, teachers should identify and explore these with their students so they too can learn to “spot” errors (p. 140). Second, the illustrations should “correspond with” and “enhance” the text but not “interrupt the flow of reading” (p. 140). Meaningful, real connections that are not contrived were also important as was the appeal to different cultures. They stated that children do not necessarily need to understand all the mathematics the first time that the book is read, but they can build on their knowledge in subsequent readings. Finally, although all books may not have it, the wow factor should motivate and “spark something new in the reader” (p. 142). Above all, they saw the important role of the teacher in devising ways to use picture books “to help children learn concepts meaningfully” (p. 143).

Thatcher (2001) also recognised the benefits of using picture books for mathematical concept development. However, she strongly recommended that these books should be of literary value. Therefore, to facilitate teachers’ evaluation of mathematical picture books for “thoughtful use in the early elementary classroom” (p. 20) Thatcher suggested a simple checklist comprising five questions:

1. Would I read this book to the children even if I weren't choosing it for a math lesson?
2. Does the book stimulate curiosity and a sense of wonder? Are children inspired to do their own investigations?
3. Is the book meaningful to the children? Can they make personal connections?
4. Are the math connections natural?
5. Is the information accurate? (p. 22)

Continuing the theme of literary quality in the books that were to be considered for their mathematical use, D. Whitin and Whitin (2004) developed criteria taken from two perspectives, that of mathematics and language arts. Their four criteria demonstrated the qualities of “good math-related” (p. 2) books, namely:

- "mathematical integrity" (the mathematics is accurate and in "believable contexts" (p. 2), and the "ideas and concepts are accessible to the reader" (p. 10) that are created through real-life situations, the illustrations, and clear explanations;
- "potential for varied response" where "the tone of the book is invitational rather than didactic" (p. 4);
- "an aesthetic dimension" that "heightens the reader's awareness and appreciation of form and design" (p. 4). Additionally, the visual images complement the text; and
- "ethnic, gender, and cultural inclusiveness" (p. 4).

In their report, the authors then related these criteria to the NCTM (2000) standards and demonstrated their application to a range of picture books.

Hunsader (2004) took a different perspective. She believed there was a "lack of systematic assessment of mathematics literature" and that current criteria possessed both "potential and pitfalls" (p. 618). She also questioned whether the lists of recommended picture books provided by many publishers of mathematics curriculum books have been assessed for mathematics and literary quality.

A pilot study (Hunsader, 2004) assessed nine books suited to third grade students against Schiro's (1997) instrument. This led to her reducing Schiro's 11 mathematical and 11 literary criteria to six categories for each to speed up the assessment process and eliminate what she regarded as trivial and repeated or overlapping questions. She did, however, include Schiro's (1997) 5-point scale (five being the best score) that allowed books to be scored relative to each other. The resultant criteria for assessing mathematical content follow.

1. Is the mathematics content (text, computation, scale, vocabulary, and graphics) correct and accurate?
2. Is the book's mathematics content visible and effectively presented?
3. Is the book's mathematics content intellectually and developmentally appropriate for its audience?
4. Does the book facilitate the reader's involvement in, and use and transfer of, its mathematics?
5. Do the mathematics and story complement each other?
6. How great are the resources needed to help the readers benefit from the book's mathematics? (Hunsader, 2004, p. 621).

Hunsader (2004) then used her instrument to assess 75 picture books recommended by two publishers for inclusion in mathematics curricula in the USA. She was very disappointed to find that only 52% from one publisher and 40% from the other were of high enough literary and mathematical standards to be used. A further 17% (Publisher 1) and 8% (Publisher 2) contained a suitable mathematical standard, but insufficient literary quality. Hunsader (2004) claimed that publishers have a responsibility to include only high quality books in their recommendations.

A similar inquiry was conducted by Halsey (2005), this time using 127 books recommended by three publishers of mathematics text books as being suitable for second grade children. Two experts (a university professor and a graduate teaching student) assessed the books with Schiro's (1997) criteria and a Likert scale. However, Halsey added two more criteria: overall literary quality and overall mathematical quality.

The experts first assessed 20 books and checked their interrater reliability before proceeding with the remaining books. Although many books scored well in many or all of the criteria, the majority of books scored below average on at least one criterion. They did find, however, that many books scored below average on the visibility of the mathematics and how effectively the mathematics was presented. In fact, the judges were unsure why some books were included in the lists. Halsey (2005) concluded that more expert teacher knowledge was required when assessing picture books for use in the classroom.

Nesmith and Cooper (2010) further developed Hunsader's (2004) study and used six books with 30 reviewers from varying professional backgrounds. Participants included mathematics professors, mathematics educators, English professors, literacy educators, and third grade teachers. They found differences in evaluations both within and across the groups. Specialised educational background played a part in identifying either mathematical or literary aspects. The classroom teachers were found to have a lower interrater reliability than did the other groups. Their final recommendation was that publishers, curriculum writers, and even teachers, should not rely solely on their own evaluation which may be biased in some way, but that they should use multiple evaluations to produce a more valid rating.

More recently, Van den Heuvel-Panhuizen and Elia (2012) developed and evaluated a two-part framework to assess mathematical opportunities in picture books. They aimed to identify and describe the characteristics in books that contributed to "the initiation and further development of mathematical understanding in young children" (p. 18). Firstly they reviewed the literature from three main areas: the importance of using picture books, previously documented criteria for selecting picture books, and "directions" for using them. Their *learning-supportive characteristics of picturebooks* emerged with two sections. These were Section A: *Supply of learning content* which included *mathematical processes and dispositions*, *mathematical content domains*, and *mathematics-related themes*; and Section B: *Way of presenting mathematical content* that contained *context characteristics* and *form characteristics* (Van den Heuvel-Panhuizen & Elia, 2012).

Van den Heuvel-Panhuizen and Elia (2012) then used a Delphi process (four-round) with seven experts. The first round involved the experts listing their thoughts about learning supportive characteristics for mathematical picture books without referring to the framework. The experts were then requested to use these ideas to add, delete, or amend framework points. The second round involved participants responding to these changes and suggesting any further improvements.

During these rounds it was found that almost all the experts' ideas had already been included in the framework. However, "the experts were more focused on the way of presenting mathematical content" ("degree of connections, the opportunities for engagement and on scope and multiple interpretations") "than on the supply of it" (Van den Heuvel-Panhuizen & Elia, 2012. p. 31). It appeared later that the experts felt that mathematical content was a "self-evident requirement" (p. 31). To avoid perceived overlaps, Section B was revised.

The third round involved using the framework with three books[24]. The experts were asked to evaluate the books first without the framework and then again with it. In some areas, particularly when identifying characteristics relating to the quality of presentation, the number of characteristics identified increased at least sevenfold when using the framework. In the final step of this round participants were asked to provide comments about the Delphi method.

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[24] *De Verrassing* [*The Surprise*] (Van Ommen, 2003), De Lievelingstrui [Favourite Sweater] (Veldkamp, 2001), and Ga je mee? [Let's Go] (Damatons, 2005).

Until Round 4, the experts had not met, but they then convened to discuss the results of the method and were asked to evaluate the importance of each characteristic on a 5-point scale. The most important characteristic identified by these experts was the need for the books to be meaningful to the children.

The framework that resulted from this research, titled *Learning-supportive characteristics of picturebooks for learning mathematics,* was divided into two parts. Part I, Supply of mathematical content, was then divided into three subparts: mathematical processes and dispositions, mathematical content domains, and mathematics-related themes; Part II, Presentation of mathematical content, had two subparts, way of presenting and quality of presentation (Van den Heuvel-Panhuizen & Elia, 2012).

This study, particularly its use of the Delphi process, validated the development of the new framework for the selection and evaluation of mathematical picture books. It also revealed that the framework assisted in the identification of mathematical concepts and "characteristics that go beyond discerning typical content domains" (Van den Heuvel-Panhuizen & Elia, 2012, p. 42) including "mathematical processes and dispositions", "mathematical themes", and "characteristics related to the presentation of the mathematical content" (p. 42). Finally, the authors recommended that further research was needed to investigate literary perspectives and whether "particular characteristics evoke particular learning in children" (p. 43).

An *Early Counting* instrument developed by LeSage (2013) was concerned with evaluating picture books for developing number sense. The instrument was based on information from three sources: the collection of literature on past frameworks, "grade level curriculum expectations from the *Ontario Mathematics Curriculum, Grades 1-8"* (p. 491), and research on the conceptual development of number sense. Three categories emerged: early counting criteria, illustration of counting objects and general appeal, with 13 "prompts" under these categories. The first category was more prescriptive of the mathematical content required in the book than it had been in other frameworks, e.g., counting beyond 10, forward and backward counting, "multiple representations of quantity, including numerals, words and illustrations" (LeSage, 2013, p. 492). The other two categories were based heavily on Schiro's (1997) criteria. It thus appears that this instrument was more suited for use with purposefully written mathematical picture books.

If picture books are to be used for promoting mathematics learning by either teachers or caregivers, there first need to be guidelines to assist in selecting and evaluating the suitability and quality of a book. If inappropriate books are chosen or the mathematical concepts within the book are not correctly identified, mathematics learning opportunities may not be accessed. There are also potential problems that children may become bored and frustrated, lose motivation, develop negative attitudes to mathematics, and may even learn incorrect information (Christy, 2003; Nesmith & Cooper, 2010). However, not all of the above frameworks include aspects of the practical use of picture books in the formal classroom setting. These include teacher presentation and the explicit role played by the text and illustrations. These important aspects are discussed in Chapter 3.

2.7 Literature on classroom application of picture books for mathematics learning

The use of children's literature for promoting mathematics learning has been suggested in teaching practice for decades. There have been texts, resource books, and articles in professional journals highlighting the use of mathematical picture books in the classroom. A summary of some of the available literature promoting classroom application follows.

Beginning in the 1980s, a number of authors (including Braddon, Hall, & Taylor, 1993; Burns, 1992; Griffiths & Clyne, 1988, 1990, 1994; Thiessen 2004; Thiessen & Matthias, 1992; Ward, 2009; D. Whitin & Wilde, 1992, 1995; D. Whitin & Whitin, 2004) have written books providing classroom teaching ideas for specific picture books. Some authors treated the books individually, some classified books according to age suitability, while others used the picture books as part of a cross-curricula theme.

In their first book for teachers, Griffiths and Clyne (1988) introduced educators to the benefits of using picture books in mathematics teaching and learning, instructed them about how to use literature in the classroom, and provided suggestions about assessment. They also provided predominantly counting activities based on individual books, rhymes and poems, and mathematical themes, as well as an annotated list of other mathematical topics. Their second book (Griffiths & Clyne, 1990) was largely focused on counting books, but this time they offered some suggestions for integration with other curriculum areas. The third book (Griffiths & Clyne, 1994) followed a similar theme, but this time emphasised teaching and learning in context.

Phyllis Whitin and David Whitin have collaborated on several books that integrate numeracy and literacy in the classroom. For example, P. Whitin and Whitin (2000) related their experiences in a fourth-grade classroom across 4 years. The book's intention was to demonstrate, with students' work samples, how writing and talking can be integrated into mathematics activities to promote rich learning for all children. D. Whitin and Whitin (2004) not only provided criteria for selection and evaluation of picture books for mathematical learning (Section 2.6.4), they also included practical suggestions for activities related to picture books, with links to the NCTM (2000) principles and standards, and the equivalent English standards. The book also contained student work samples and an annotated list of mathematical picture books.

Ward (2009) produced three resource books for different grade students with literature-based activities that integrated mathematics and other curriculum areas, particularly science, social studies, and visual arts. As well as describing the use of one or two books for different content strands, she provided lists of other related books and websites on each theme. Mathematical activities (across strands) based on the winners of the Children's Book Council of Australia awards have also been published each year since 2005 by McDonald (e.g., McDonald, 2009).

A more recent publication by Janes and Strong (Janes & Strong, 2014), aimed to assist teachers to integrate literature and mathematics through rich and meaningful activities, that linked to the NCTM (2000) standards. The authors first included background about the benefits and strategies for using picture books in the classroom. They then concentrated on children's literature and number sense investigations including counting, place value, and algebraic thinking. Each topic or investigation is based on a particular picture book.

Professional journals have also included articles by both researchers and classroom teachers based on classroom experiences or with suggestions about how to use particular books in the classroom. Some of these journals' articles have incorporated research in their introductions, while others are solely practical application ideas. The journal *Teaching Children Mathematics* (NCTM) has included regular articles about using picture books and highlights this topic through a special section in many editions titled *Links with Literature*. The topics have covered a range of content strands and classroom situations.

Moyer (2000) demonstrated the teaching of division using the explicit book, *A Remainder of One* (Pinczes, 1995). The book presented a "problem-solving context for communicating, understanding, and exploring" (p. 521) partitive division. In another article, Mattone (2007) discussed different representations of patterns explored by students using several appropriate picture books.

Measurement has been explored in several articles. For example, Lubinski and Thiessen (1996) used the book *How Big is a Foot?* (Myller, 1990), and Rozanski, Beckman, and Thompson (2003) made use of *The Grouchy Ladybug* (Carle, 1977). Clarke (2002) based his article on *Alexander's Outing* (Allen, 1992). Geometry or spatial concepts are often highlighted with creative suggestions, for example in publications by Kuhns (2003), Senger, Platte, and Zandt (1997), and Weber (2003). Jenner (2002) described how she used *Selina and the Bear Paw Quilt* (Smucker, 1995) to teach shapes, flipping, and transformations to her class. She also video taped the sessions to reflect on her own teaching, and she observed her students using mathematical language as well as visualising and problem solving during and after the book reading. Jenner (2002) also noticed the opportunities she had missed for mathematical learning. Picture book experiences based on a combination of measurement and geometry were also discussed by Henry (2004) and Reynolds, Cassel, and Lillard (2006).

Several other authors have reported on classroom strategies or particular student groups. A research-based article by Forbringer (2004) demonstrated practically how to differentiate activities using picture books, while Jenner and Anderson (2002) demonstrated how one student with learning difficulties became engaged with mathematics activities when a picture book was used. Huber and Lenhoft (2006) worked on classifying and graphing with early childhood students. The use of multicultural literature has also been addressed by Strutchens (2002).

The journal *Australian Primary Mathematics Classroom* (published by the Australian Association of Mathematics Teachers [AAMT]) has also included articles about action research by teachers who integrated mathematics and children's literature. Goral and Gnaginger (2006) report how they used the same strategy as Casey et al. (2004) who had used mathematical sagas with children. Goral and Gnaginger (2006) told their own story based on place value to a class of first grade students. They found that the students appeared to deepen their understanding through connections to the story and problem solving. Clark (2007) found, after using a variety of picture books over a 3-month period with a Year 5 class, that students were more engaged and had a more positive attitude to mathematics. Geometry (position and location) was the theme when Jenkins (2010) suggested ways to use favourite picture books, while Russo (2016) used the traditional stories of *Three Little Pigs* and *The Big Bad Wolf* and the book *Who Sank the Boat* (Allen, 1982) to investigate equivalence with Year 1 and 2 students. Gough (2014) has also suggested an integration of mathematics and poetry, particularly haiku.

Some authors (e.g., Donaghue, 1996) have provided brief annotated lists of books suitable for use in the classroom. Lists and teaching ideas for many books can now be found on the websites of picture book publishers and authors, and the websites of interested individuals. Social media sites such as *Pinterest* even have a pin board titled “Australian curriculum maths numeracy picture books”.

However, these resources do not necessarily take into account relevant mathematics curricula or the individual needs of each class or students based on research and picture book components (text, images, and structure). Many picture books are published every year. Therefore, guidelines are needed to assist and encourage teachers to select quality mathematical picture books that meet curricula and student needs. Articles by Marston (2010b, 2011, 2012a, 2014b), Marston and Mulligan (2012), and Marston, Muir, and Levy (2013) can be found throughout this thesis (Publications 2, 3, 7, and 8, and Appendix G) with accompanying commentary. These publications have taken into consideration current curricula, pedagogy, and research. They have not relied on the same books as previous articles, as seen in Russo (2016) where the *Three Little Pigs*[25]*, The Big Bad Wolf*[26]*,* and *Who Sank the Boat* (Allen, 1982) are revisited.

2.8 Summary

In this chapter I have provided a review of research supporting the role of picture books for encouraging mathematics learning in young children. I first described the important role storytelling and stories have in developing knowledge and skills in a realistic context and how learning can be promoted through cognitive engagement. Studies involving cognitive engagement and intervention strategies with picture books and young children revealed that:

[25] Traditional fairytale
[26] Traditional fairytale

- using picture books benefits the acquisition of language, literacy, and visual literacy through direct instruction and exposure to literature (e.g., A. Anderson et al., 2005);
- the genre of a book affects the presentation by the reader/teacher and the responses of the children being read to (Moschovaki & Meadows, 2005a; 2005b);
- many picture books contain mathematical concepts and learning opportunities in the text and the visual images (e.g., Shapiro et al., 1997);
- children engage with mathematical concepts both through their own initiation and through discussion with a reader (e.g., Van den Heuvel-Panhuizen & Van den Boogaard, 2008; Van den Heuvel-Panhuizen et al., 2009);
- children engage with mathematical concepts both purposefully (explicit) and unintentionally (perceived) included in picture books (e.g., A. Anderson et al., 2004; Elia et al., 2010; Van den Heuvel-Panhuizen et al.,2009);
- use of picture books promotes better mathematical understanding and achievement levels as well as improved perceptions of and attitudes toward mathematics (e.g., Casey et al., 2004; Van den Heuvel-Panhuizen & Elia, 2011; Young-Loveridge, 2004);
- the same mathematical picture book can produce varied results (A. Anderson et al., 2004; A. Anderson et al., 2005; Van den Heuvel-Panhuizen & Van den Boogaard, 2008)
- different books produce different quantities and types of mathematical responses, thus showing the differing potential of books (A. Anderson et al., 2005); and
- the illustrations in picture books are a dominant source for children's mathematical responses (e.g., Moschovaki & Meadows, 2005a, 2005b; Shapiro et al., 1997; Van den Heuvel-Panhuizen & Van den Boogaard, 2008).

Studies of various measures developed to assist teachers in selecting and evaluating picture books for quality mathematical teaching and learning (such as Schiro, 1997; Van den Heuvel-Panhuizen & Elia, 2012; D. Whitin & Whitin, 2004) indicated that mathematical accuracy and visibility, appropriateness to age and ability, and real-life contexts were important. Additionally, the way the mathematics was presented and different mathematical domains were incorporated. However, these various measures and some resulting frameworks have not necessarily associated their elements with current curriculum expectations or to the important role of the text and the illustrations with various types of books. Embedded mathematical concepts have not been consistently identified by researchers.

Finally, a review of resource books and articles in professional journals that demonstrated the application of picture books in the classroom revealed that many ideas were often based on the same books. These findings again support the need for quality criteria so that teachers can select and evaluate any book for rich teaching and learning experiences.

The review of literature in this chapter supports the research focus described in Chapter 1. It particularly highlights the paucity of research focusing on the cognitive engagement of young children during unscripted shared mathematical picture book readings with their teachers. More research is needed to ascertain the effect of the range of mathematical picture books within the typical classroom context, with particular emphasis on the role of text and illustrations, the teacher, and different book genres.

In the next chapter, I discuss some theoretical perspectives and the important role of images in both picture books and children's learning, and I describe the methods used in the three phases of this thesis.

CHAPTER

3

THEORETICAL PERSPECTIVES AND METHODS

In this chapter I first describe the theoretical perspectives taken throughout the study. I then outline the design and methodological approaches that were employed. As the specific methodology for each phase of the study is provided in the relevant chapter(s), phase(s), or publication(s), a summary of approaches for each phase is included in this chapter. Relevant ethical procedures not detailed in their associated chapters are also included here.

3.1 Theoretical perspectives

The rationale for the use of picture books in promoting mathematical teaching and learning, and the research problem, both described in Chapter 1, together with the review of literature in Chapter 2, reflect the theoretical perspectives taken in this thesis. They also inform the design and interpretation of the results of all three phases of the study.

Research that focused on the use of picture books for mathematical concept development (Section 2.4) by A. Anderson, Anderson, and Shapiro (2005), Elia, Van den Heuvel-Panhuizen and Georgiou (2010), and Van den Heuvel-Panhuizen and Van den Boogaard, (2008), is based primarily on constructivist (including socioconstructivist) as well as cognitive and situated learning theories. These theories form the basis of some current beliefs about how children best learn mathematics.

I also explore picture book use for mathematical learning from another perspective. In constructing my conceptual framework, I have broadened my base and "borrowed from elsewhere" (Maxwell, 2005, p. 35), in this case, the field of children's literature. I therefore draw upon theories concerning the function of visual images and their relationship with the text in picture books.

These perspectives are discussed in the following section with reference to their application to promoting mathematical teaching and learning through the use of picture books.

3.1.1 A constructivist approach to learning

Over the last couple of decades there has been a shift from an instrumental pedagogy where there was little learning for understanding to a relational way of teaching. This shift includes mathematics (see the preamble in the front matter). Constructivists emphasise the importance of the learner's participation in knowledge construction rather than textbook-style learning. "Individuals create their own understandings based upon the interaction of what they already know and believe, and the phenomena or ideas with which they come into contact" (Richardson, 1997, p. 3). In Piagetian pedagogical constructivism (Piaget, 1963), the teacher provides an enquiring and engaging environment that challenges students' thinking processes (Richardson, 1997). Curriculum bodies such as the Australian Curriculum, Assessment and Reporting Authority (ACARA, 2010) and the National Council of Teachers of Mathematics (NCTM, 2000) now advocate a constructivist approach to teaching. Nonetheless, this does not necessarily occur when teachers are faced with the realities of the classroom and a number of internal and external influences (Handal, 2002; Jamieson-Proctor & Byrne, 2000; Speer, 2005).

There are several main principles underlying constructivism. The first is that knowledge is actively constructed by the learner, not passively received from the environment (Steffe & Cobb, as cited in Clements & Battista, 1990). The second is that students can construct new knowledge "by reflecting on their physical and mental actions. Ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge" (Clements & Battista, 1990, p. 34). A third principle, that learning is a social process, is further developed in the socioconstructivist theory of learning espoused by Vygotsky (1978). According to that theory, learning occurs as "a learner interacts with other people, objects, and events in the collaborative environment" (Vygotsky, as cited in Wang, Bruce, & Hughes, 2011, p. 297). In Section 2.2 I reported high levels of cognitive engagement in classrooms using constructivist principles.

Cobb, Wood, and Yackel (1991) asserted that "at risk of over-simplification, an immediate implication of constructivism suggests that mathematics ... should be taught through problem solving" (p.158). However, Cobb (as cited in Clements & Battista, 1990) stated that "taking a constructivist perspective appears to imply two major goals for mathematics instruction" (Clements & Battista, 1990, p. 35). First, the students should increase their knowledge and skills to be able to solve a "wide variety of meaningful problems" (p. 35). Second, "students should become autonomous and self-motivated in their mathematical activity" through "their own explorations, thinking, and participation in discussions" (p. 35).

In the shared book reading[1] situation used in Phase 3 of this study, "learning can occur as a result of mediation by a more proficient other and knowledge can be constructed by the child thorough the processes of accommodation and assimilation" (Shapiro, Anderson, & Anderson, 1997, p. 48). In Vygotsky's view, children or students need to be supported or "scaffolded" in their learning ("zone of proximal development"). Here the sharing of a mathematical picture book provides the social context, while the reader/teacher acts as the mediator to "afford the injection of mathematical ideas in comfortable and unobtrusive ways" (A. Anderson, 1997, p. 509).

Additional to the actual reading of the book where students learn literacy conventions (A. Anderson, Anderson, & Shapiro, 2005) is the student talk or discourse that is central to socioconstructivist theory (A. Anderson et al., 2005). It can be assumed that as book reading and well-structured discussion before, during, and after reading with a teacher or caregiver can assist a child to learn literacy conventions that it also facilitates learning of mathematical conventions (see Section 2.2). Therefore the role of the "constructivist teacher" is to offer "appropriate tasks and opportunities for dialogue" and guide "the focus of students' attention, thus unobtrusively directing their learning" (Bruner, 1986, as cited in Clements & Battista, 1990, p. 35).

Student talk or utterances were used in many of the studies I reviewed in Section 2.4 as a measure of student engagement and understanding. I also use student mathematical talk in Phase 3 to indicate students' cognitive engagement.

3.1.2 A children's literature perspective

Picture books were defined in Section 1.1.1.2 as always containing illustrations, and most often text. The illustrations, either singularly or in conjunction with the text, convey the intended story, information, or message[2]. Mathematical concepts and ideas are found in both the images and text of picture books, and therefore it is important to explain their role and the interrelationship between them.

[1] Although the teaching strategy is known as "shared book reading", the terms "book reading" or "reading" only will be used for the rest of this chapter as equivalents, except in headings.

[2] For the remainder of this thesis, reference is made only to picture books with both text and images.

As with the term "picture book" (see Chapter 1), there is some interchange of terms for the pictorial work within a book. The terms picture, image, illustration, artwork, and graphics are used by a number of authors according to Anstey & Bull (2000) who refer to Cullinan (1989), Huck (1987), Saxby (1997), Sutherland and Arbuhnot (1986), and Townsend (1987). Throughout this study the terms "illustration", "visual image", and "image" will be used interchangeably to denote any pictorial inclusions in a picture book.

"Children are primarily visual learners" (Nesmith & Cooper, 2010, p. 281) and today they are "exposed to various visual media, including television, videogames, and computers" (Brame, 2000, as cited in Costello & Kolodziej, 2006, p. 28) that they are attracted to and motivated by (Callow, 2012). Consequently, they are used to relying on visual images to assist them in learning new content and concepts. Children have also been shown to give a range of simultaneous responses to multiple features in an image, thus proving the power of images in learning situations (Walsh, 2003). High quality picture books that employ visual images and text to collaborate and to convey ideas appear to be ideal instructional facilitators for today's youth (Nesmith & Cooper, 2010; Neal & Moore, 1992). Although there is evidence to support the role of visual images in learning, "other educators insist that illustrations if misused or misinterpreted, may in fact interfere with learning" (Watkins, Miller, & Brubaker, 2004).

3.1.2.1 The role of visual images

There is a large amount of literature about the function of visual images in picture books (see Agosto, 1999; Anstey & Bull, 2000; Kress & van Leeuwin, 1996; Lewis, 2001; Styles & Arizpe, 2001). Visual images are neither an adjunct nor aid to comprehension of the written text, but the visual images and written text together construct the meanings in a picture book. The importance of the visual images for enhancing a reader's understanding and interpretation of the meanings conveyed in the written text of picture books has been investigated and shown to be fundamental. According to Styles and Arizpe (2001), "there are some things that can only be said visually" (p. 278) when creating picture books. A summary of the relevant literature on the role of the images in picture books, traced back to the 1990s, follows.

Kress and van Leeuwin (1996) described three metafunctions (ideational, interpersonal, and textual) of visual images but also emphasised the importance of the visual layout and the balance within the illustrations; balance giving a sense of security, while disordered placement creates a feeling of disquiet or fear. They stated that *visual grammar* is "the way in which depicted people, places and things combine in visual 'statements' of greater or lesser complexity and extension" (p. 1), therefore adding to the meaning of the story. They suggested that with little help children actually seem to develop a surprising ability to use elements of visual grammar.

According to Anstey and Bull (2000), the role or value of the illustrations has been overlooked by researchers. They argue that great emphasis has been placed on describing "the stages, the skills and processes of reading words" but the same lexicon has not been amassed for the illustrations (p. ix). They believed picture books do not just contain "pictures, but illustrative text, which itself contains the icons, the images, the literacies and the ideologies" (p. ix) that children need to be able to come to terms with in modern times. The "ability to recognize and understand ideas conveyed through visible actions and images", and the inverse, is known as *visual literacy,* (Aanstoos, 2003, p. 189) and is needed by children "in an increasingly globalised, technological age" (Walsh, 2003, p. 123). Hence it is important for teachers who are teaching visual literacy to understand the role of both the illustrations and text themselves and to realise that their interrelationship may differ from book to book (Martinez & Harmon, 2012).

A pilot study by Styles and Arizpe (2001) using in-depth, semi-structured interviews conducted with 84 children aged 4–11 years, but "matched for gender and reading ability" (p. 262) in a range of schools in the United Kingdom explored "the multilayered nature" of the picture book, *Zoo* (Browne, 1992). The researchers' emphasis was on "the high-level cognitive skills involved in reading visual images and links … made between seeing and thinking". Principally, their aim was to "investigate how visual texts are interpreted by children" (p. 261) and "examine the skills children require to deal with complex visual texts" (pp. 261–262). This research however, was not concerned with the acquisition of mathematical concepts.

Children were read the book and participated in group discussions. They were then asked to draw in response to the book reading. Some children were not able to express their feelings well verbally, but their own illustrations revealed both their understanding of the book and their own feelings. Browne, the author, himself had high expectations of his readers and understood that they have a range of ability in both interpreting print and the visual images. He believed children "are capable of more than people think they are … the creative aspect of children's minds is very exciting" (Styles and Arizpe, 2001, p. 265).

Styles and Arizpe (2001) first found that despite some children not being able to read the print of the book, they had developed "impressive capacities for analysing image" (p. 261) and "could make deep and insightful interpretations of visual texts" (p. 266). They believed that even children who were below average readers were able to analyse the images because of the "enabling" (p. 280) Vygotskian environment, i.e., a supportive teacher who used questions that challenged their thinking. One student in the study remarked, "I always remember pictures. I sometime forget words" (Styles & Arizpe, 2001, p. 261), thus demonstrating the power of the illustrations.

Styles and Arizpe (2001) also believed that their finding showed, like those of Kress and van Leeuwen (1996), that making meaning from pictures is a cognitive act. They concluded that the development of visual understanding was linked to increasing age, although it was previously shown that the need for visuals to aid understanding decreases with age.

3.1.2.2 The relationship between text and visual images

Lewis (2001) focused on the nature of the relationship between the visual image and the written text. He also described the key features of the visual image, including line, colour, action and movement, size and location, and symbolism—all of which give the pictures meaning. However, like Kress and van Leeuwen (1996), Lewis also discussed the structural organisation, or grammar, of the image in a picture.

Lewis (2001) explained the varied ways in which words and pictures work together in picture books to "create coherent, intelligible text" (p. 28) as "mutual interanimation" (p. 36). He believed that this relationship is never symmetrical because:

> what the words do to the pictures is not the same as what the pictures do to the words …. The words in a picture book draw attention to the parts of the picture that we should attend to whereas the pictures provide the words with a specificity—colour, shape and form—that they would otherwise lack (p. 35).

Sometimes "the pictures are richer in information than the words and might be said to introduce and develop new themes rather than echo the one introduced by the words" (Lewis, 2001, p. 34). For example, in *Gorilla* (Browne, 1983) the pictures "take us into both Hannah's home and her psyche" (Lewis, 2001, p. 4).

Sipe (1998) used the word "synergy", where the effect of the whole is greater than the sum of the individual parts, to describe the interrelationship of text and image. Others such as Nikolajeva and Scott (2000) have produced a taxonomy of ways in which the text and illustrations interrelate (symmetry, enhancement, counterpoint, and contradiction), while Golden (1990) had a similar five-step rating of the interaction.

Agosto (1999) described books where the text and illustrations are in symmetry or tell the same story as "twice told tales" (p. 267), and he described those where both text and images are required to understand the story as being interdependent. According to Graham (2000, p. 61) a "perfect" picture book used for literacy development should not consist of matching text and images.

> When we look at the pictures in picture books, we're meant not just to do that but also to think about how they relate to the accompanying words and also to the pictures preceding and following them. In other words, we must consider not only their beauty but also how they contribute to our unfolding knowledge of the story.

Kiefer (2008) asserted that because of the interrelationship of the text and images, it is sometimes difficult to decipher the individual input of text and images. Flevares and Schiff (2014), in reviewing the teaching and learning of mathematics with children's literature, argued that "even having attractive, engaging, mathematically relevant illustrations and representations is not enough" (p. 4). The relationship "between the text and images must be coherent" otherwise they "can cause confusion" particularly with spatial and measurement concepts (p. 4).

3.1.3 Summary

In this section I have first discussed the theoretical perspectives that underpin this study. Throughout this thesis there is strong support for a socioconstructive view of mathematics learning through the use of picture books. In the development of the framework in Phase 1, I have considered the opportunities that a picture book provides for students to construct meaning, direct their own learning, and interact with each other and the book's components of text and illustrations. Section 3.1.1 suggested how the design of the shared reading session (in Phase 3, which particularly addresses Research Question 3) promoted learning through the real-life situations provided by picture books. The activities included in Chapters 7 and 8 and Publications 3, 7, and 8 also encourage learning situations that allow students to construct their own knowledge through reflective and investigative practices in social, supportive, and real-world environments.

I have also considered theories about the function of visual images and their relationship with the text in picture books. Picture books contain images and text that individually and together add to the meaning of the story (see Lewis, 2001; Styles & Arizpe, 2001). These components can also contain mathematical concepts and learning opportunities. The function of text and images, and their interrelationship within mathematical picture books, is important for addressing all three research questions. Text, image, and their interrelationship are first considered when developing the framework elements (Phase 1). In Chapter 6 (Phase 2), I report picture book authors and author/illustrators' perceptions of the roles of text and images. I also investigate their role in mathematics teaching and learning, particularly how teachers use images and how students engage with them in their discussion and representations (Phase 3).

3.2 Design and methods

3.2.1 Study design

An overview of the three phases of this study was provided in Chapter 1 (Section 1.6). Each phase required different research methods and data analyses. Within each phase there were also opportunities for a further mix of strategies that will be described in the following sections. Therefore, a multi-method approach was employed, and, as various types of strategies were implemented within that approach, a multiphase design was adopted. This design occurs when a researcher "examines a problem or topic through an iteration of connected quantitative and qualitative studies that are sequentially aligned, with each new approach building on what was learned previously to address a central program objective" (Creswell & Plano Clark, 2011, p. 100). A multi-method, multiphase approach then allowed my individual study components "to address a specific set of research questions that evolve to address a larger program objective" (Creswell & Plano Clark, 2011, p. 100).

Figure 3.1 shows the study design based on Creswell and Plano Clark's (2011) "Flowchart of the Basic Procedures in Implementing a Multiphase Design" (p. 102).

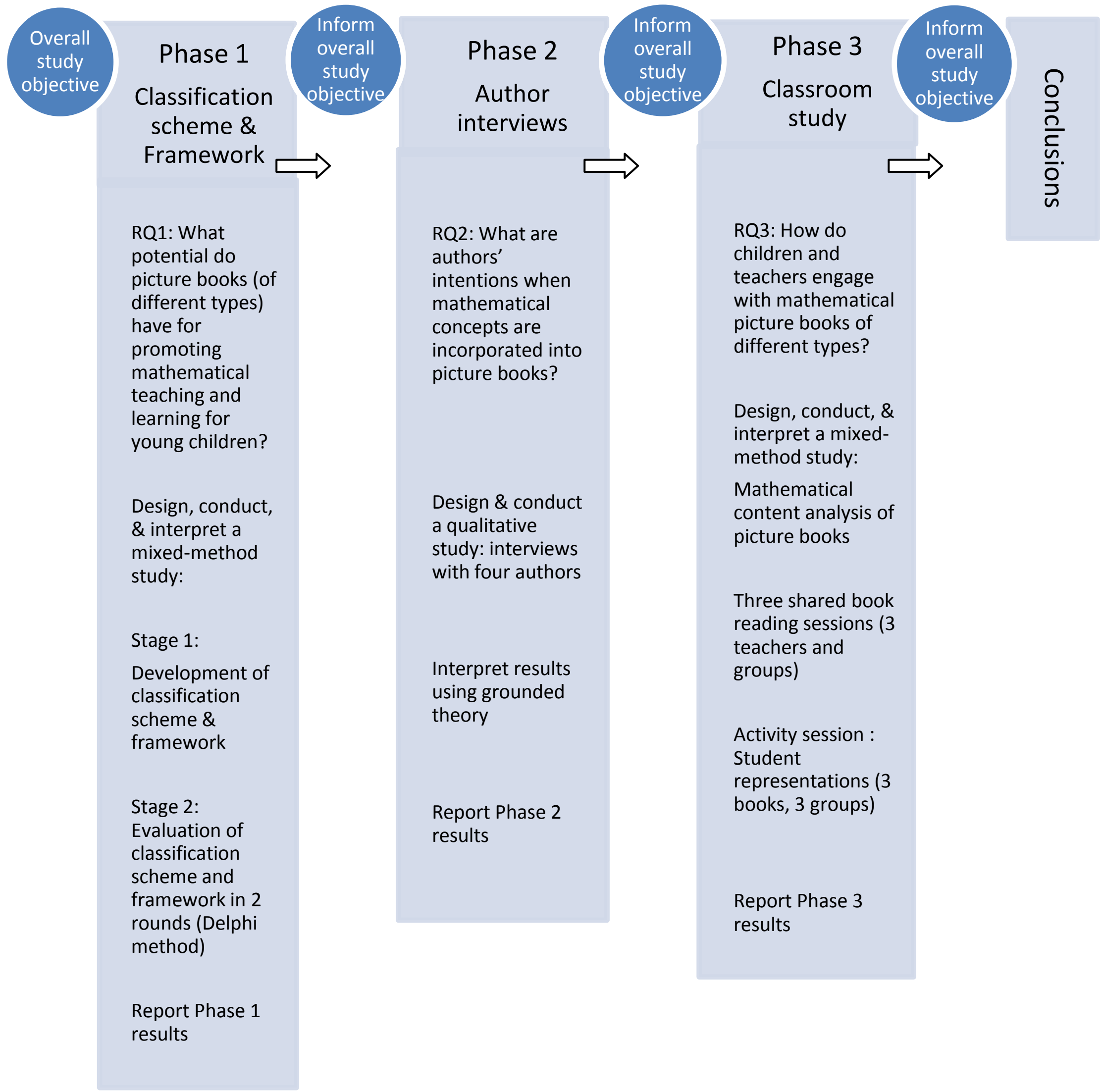

Figure 3.1. Flowchart of multiphase design used in this study, adapted from Creswell and Plano Clark (2011).

3.2.2 Methodological approach

Several methods were used across the three different phases in this study. In all cases, the methods were chosen for their appropriateness to the task and their ability to aid in answering the research questions. I intended them to complement each other as I integrated this thesis and its conclusions.

A detailed description of the methods used in each phase is provided in the relevant chapters. However, in the following sections I provide an overview of the methodological approaches used in each phase and compare them, where appropriate, with those used in previous studies as described in Chapter 2.

3.2.2.1 Phase 1: Development and evaluation of classification scheme and framework

Phase 1 is described in Chapters 4 and 5. In order to develop the classification scheme, I first sourced 136 award-winning picture books from the last decade and conducted an analysis of the content of these books to establish whether, and, if so, how, mathematical concepts were included. I determined there to be three types of mathematical picture books (perceived, explicit, and embedded). Concurrently, in order to develop the draft of the framework, I reviewed previously used criteria (e.g., from Schiro, 1997; Hunsader, 2004) and literature about using picture books and their components (e.g., Lewis, 2001). I also consulted recent curriculum policies and principles. The draft of the framework contained seven categories, each with a series of elements.

I then selected 50 of the original books that contained mathematical content representing the three types of mathematical picture books. A research assistant and I independently classified and scored each book against each element using a 0–5 Likert scale. The results from this scoring were compared with each other. My results were used again later to compare with those collected in the next step: the evaluation surveys of the framework by professionals.

I then used a modified Delphi process to revise and improve the framework. According to Hsu and Sanford (2007, p. 1), "the Delphi technique, mainly developed by Dalkey and Helmer (1963) is a widely used and accepted method for achieving convergence of opinion concerning real-world knowledge solicited from experts within certain topic areas". This method usually involves a "series of rounds of questionnaire surveys ... where information and results are fed back to panel members between each round" (Hanafin, 2004, p. 4). As the Delphi method uses a review process that can be adapted to individual research, it was a suitable approach to take here.

I used two rounds, each of which involved people who could be regarded as experts, but I departed from the standard Delphi procedure in not seeking consensus from them. In Round 1 I trialled the framework through a survey completed by a small group of academics / teacher educators and classroom teachers. Each participant received five picture books and a copy of the survey for each book. The survey contained the same items that I had previously scored my books against, the 0–5 Likert scale, two open-ended questions, and a request to classify the books according to whether they were believed to have perceived, explicit, or embedded content. The responses from the Likert items in this round and the book classifications were set aside for later analysis, while the comments from the participants informed changes to the framework before Round 2. In contrast to the traditional Delphi technique in which the same experts are used in each round, I used a different and larger group of professionals (academics / teacher educators, teachers, and preservice teachers) to repeat the process with the revised survey. This version contained fewer items to be scored on the same Likert scale, the same open-ended questions, and the same book classification. Full details about the modified Delphi process are outlined in Section 5.2.

Data collected from the Likert items in both rounds were analysed separately for each book to enable a comparison of scoring across participant groups and against my scoring. All book classifications were also analysed and compared with my classification. The responses from the open-ended questions were collated, and the similarities and differences between responses within each question were noted for later reporting.

The full procedure for the development of the framework and classification of picture books and the analysis methods for both components, are described in Chapters 4 and 5.

3.2.2.2 Phase 2: Author intent

In the second phase, which is described in Chapter 6, I investigated why and how authors and/or illustrators include mathematical content in their picture books. As the literature had not provided any clues, I again turned to the Delphi technique, but used only one aspect: use of experts. I regarded the most knowledgeable source of information as being the authors themselves, and interviews were conducted as a primary source of data (see Hoepfl, 1997). Semi-structured interviews in which all interviewees were asked the same core questions provided the required information but permitted flexibility so that the interviewees/authors could develop ideas themselves and elaborate where appropriate.

As no other specific studies about author intent in studies of mathematical picture books have been conducted, grounded theory principles were used in the analysis. I adopted the grounded theory approach of Corbin and Strauss (1990) that includes themes, repeating ideas, and relevant raw text in the reporting to develop a theory of author intentions. Detailed descriptions of the methods and analysis used in this phase are included in Chapter 6.

3.2.2.3 Phase 3: Classroom study

Phase 3, described in Chapters 7, 8, and 9, comprises three components: a content analysis of the mathematical content[3] of the books to be used in the reading session, a shared book reading session, and a follow-up activity to the reading session. I provide an overview of each component here, while full details of the methods used in this third phase are included in Chapter 7.

Mathematical content analysis: Chapter 7. I first conducted my own analysis of the mathematical content of the three books that I had selected for investigation. In doing so, I used the statistical component of the data analysis tool, NVivo10 (QSR International, 2014)[4] to determine the percentage of mathematical language in the text of each book. Additionally, I calculated the proportion of each mathematical content strand relative to the total mathematical talk. I also described the mathematical teaching opportunities that I found in the text and images.

Shared book reading sessions: Chapter 8. In these sessions, three books were read to each of three groups of Year 1 classes (6–7-year-olds) by their respective teachers (in three different schools). There have been a number of previous studies in which shared reading of mathematical picture books with young children have been used. Therefore, in the sections that follow below, a comparison of past studies with my study is provided.

Context and sample. Moschovaki and Meadows (2005a; 2005b) used a whole-class setting, and in the study by Van den Heuvel-Panhuizen, Van den Boogaard, and Doig (2009) the teacher read to a group of six 5–6-year-olds in the school gym. I employed a combination of these strategies as the teacher led a small group of students within the classroom setting using a shared book reading strategy.

Procedures. The instructions given to the teachers in my study followed a similar procedure to that of A. Anderson, Anderson, and Shapiro (2004), A. Anderson et al. (2005), and Moschovaki and Meadows (2005a; 2005b) where the readers (parents and teachers) knew the purpose of

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[3] This component will be referred to as "mathematical content analysis" throughout the thesis. It subsumes both the quantitative analysis to determine the proportion of mathematical language in the text of the picture books and the subsequent qualitative analysis of opportunities for mathematical investigations that I found in the text and images.

[4] NVivo10 (QSR International, 2014) will be not be referenced in full repeatedly in this chapter. It will be referred to only as NVivo.

the study and were instructed to "share this book with your child as you normally would" (A. Anderson et al., 2005, p. 8). In other studies, for example by Elia et al. (2010), Van den Heuvel-Panhuizen and Van den Boogaard (2008) and Van den Heuvel-Panhuizen et al. (2009) the researchers had used strict guidelines to maintain consistency throughout the readings.

The naturalistic approach taken in the context and procedures of this study enabled me to ascertain both how teachers focus on mathematical ideas in different types of mathematical picture books and how students respond within the setting that this usually occurs in, namely a classroom with teacher and students.

Similar to all previous studies, all reading sessions were audio recorded (see Chapter 7 for full details) and transcribed for later use.

Analysis. Previous studies of children's mathematical engagement with picture books had included mathematical content strands in their coding but they had coded only student and parent talk. Because I had used a naturalistic study, the dimension of teacher talk was added. I again used the statistical component of NVivo, this time to analyse the teacher and student talk particularly related to mathematical content, as well as curriculum strand. Because other researchers had identified the illustrations as a major source of utterances, I also recoded all mathematical talk according to whether the stimulus was predominantly the text or the images. Importantly, I first discussed and compared the teachers' and students' use of the three different types of mathematical picture books and then the source of their talk.

Activity sessions: Chapter 9. After each reading session, I conducted a follow-up activity in which students provided their spontaneous responses to the book reading in drawings and/or words. These responses were collected for later content analysis using codes similar to those used in the analysis of the talk during the reading session, in this case representations with mathematical content, mathematical content by strand, and the source of the mathematical content. This component had not been included in any of the relevant previous studies of mathematical engagement. However, it is a strategy recommended by, for example, the NCTM (2000) for assessing children's mathematical understanding and it was used by Styles and Arizpe (2001) to enable children in their study to demonstrate their knowledge and feelings.

3.3 Ethical procedures

In accordance with university guidelines, approval was granted by the Ethics Review Committee (Human Research) of Macquarie University for the classroom study: R/No.HSHE28AUG2009-D00022. Amendment approvals were granted on 6 September 2010 for the framework survey and on 24 November 2010 for the author interviews. Approval was also granted by SERAP (2011073) and Catholic Schools Office, Diocese of Broken Bay on 9 August 2011. Copies of all ethics approvals are provided in Appendix A.

The initial application included information and consent forms for principals, teachers, and parents of the students involved, to conduct and record (video and audio) each shared-reading and after-reading activity. The first amendment required approval of information and consent forms for the survey participants, and a copy of the survey. Information and consent forms and copies of the interview questions were required for the author interviews (second amendment). Information and consent forms for all phases are provided in Appendix D.

The privacy of schools and individuals for the classroom study and framework study was preserved at all times. The four authors in Phase 2 consented to quotations from the interview transcripts being included in my thesis All data, including consent forms, surveys, digital images, and soft copies, were stored securely during the course of the project.

The study adhered to the approved guidelines, and a final report was submitted to and approved by Macquarie University Human Research Ethics Committee on 14 October 2014 (Ref: 5200902693).

CHAPTER

4

PHASE 1: DEVELOPMENT OF THE CLASSIFICATION SCHEME AND FRAMEWORK

Picture books containing similar potential for mathematical learning have already been shown to elicit different amounts and types of mathematical responses in young children (Section 2.4). In Section 1.1.2, I identified and defined three types of mathematical picture books that are based on the way the mathematical content is presented within them and the intent of the author. To reiterate, these types were:

(i) *perceived* to be occurring. These books are written to entertain, and the mathematical concepts are unintentional and incidental, for example, *Alexander's Outing* (Allen, 1992). Professionals (and students) may identify worthwhile opportunities within the text and visual images for mathematical use;

(ii) *embedded.* The author's intent is to tell a story that can stand alone from the additional mathematical layer in the text and/or visual images*, for example, the* work of Anno (1997); or

(iii) *explicitly referenced.* The intention of the author is to teach a particular mathematical concept, for example, *Minnie's Diner: A multiplying menu* (Dodds, 2004) or series such as *Sir Cumference* (Neuschwander, 1997).

In Chapter 1, I posed the problem of *there being few gu*idelines for the selection of picture books to facilitate mathematical concept development (Section 1.3). Chapter 2 included details of previous frameworks and criteria used to evaluate picture books for mathematical learning, but their use appeared to be limited particularly to the trade books developed in response of the U.S. National Council of Teachers of Mathematics' (2000) standards. As I intend to explore, in Phase 3 of this study, how children (and teachers) engage with the three different types of mathematical picture books I need to be able to select an appropriate mathematical picture book to represent each of the three types.

In Phase 1 (Chapters 4 and 5), I particularly address Research Question 1: What potential do picture books (of different types) have for promoting mathematical teaching and learning for young children? The potential is found in the different ways mathematical concepts are included in picturebooks and how professionals identify these characteristics.

The focus of this chapter, which is Stage 1 of Phase 1, is to describe:

1. the process of classifying mathematical picture books as perceived, explicit, and embedded, and
2. the development of the framework for selecting and evaluating different types of mathematical picture books.

Three publications comprise the main content of this chapter. These include a paper in the refereed proceedings of a conference, a book chapter, and a paper published in a professional journal. A preamble and critical discussion are included for each publication, and a postscript completes the chapter.

Publication 1: Refereed conference proceedings

Marston, J. (2010a). Developing a framework for the selection of picture books to promote early mathematical development. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education*: *Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 2, pp. 383–390). Fremantle, WA: MERGA.

Publication 2: Book chapter

Marston, J., & Mulligan, J. (2012). Using picture books to integrate mathematics in early learning. In P. Whiteman & K. De Gioia (Eds.*), Children and childhoods 1: Perspectives, places and practices* (pp. 209–225). Newcastle upon Tyne: Cambridge Scholars.

Publication 3: Journal article

Marston, J. (2011). A framework for teachers to select and evaluate picture books to promote early mathematical development. *Reflections*, *36*(1), 17–19.

4.1 Publication 1

The purpose of the first publication was to introduce the study to the mathematics education research audience at the annual conference of the Mathematics Education Research Group of Australasia in Fremantle, Western Australia.

This publication first provided an overview of the aims and methodology of the whole study and the development of a classification scheme and framework for identifying and evaluating mathematical picture books. Second, it provided an account of some previous frameworks or criteria for selecting picture books for mathematical learning—also discussed in Section 2.6. Third, it included the rationale for each category and element within the category, and an overview of the newly devised framework. Because of publication restrictions, some aspects of the classification scheme and framework development were not provided. The following section elaborates on their development.

4.1.1 Classification scheme: Content analysis

The development of the classification scheme for mathematical picture books involved collecting 201[1] books from top-ranking children's book awards in Australia, the USA, and England between 2000 and 2010 (see Appendix B). These included the Children's Book Council of Australia Awards, the Aurealis Awards, the Crichton Awards, the Environment and Western Australian Premier's Awards, the Caldecott Medal (USA), the Boston Globe-Horn Book Awards (USA), and the Kate Greenaway Medal (England). This strategy of using award-winning books to maintain the literary quality was also used by Kesler (2012).

I was able to source 112 of the 201 books (Appendix B) as being representative of perceived and embedded books. In order to ensure sufficient examples for comparison across all three categories, I also sourced the following additional books:

- 10 trade books (explicit) as these were not represented in the awards
- two books awarded post 2010 that I had identified as examples of embedded mathematical picture books but were not represented well among the awards
- 12 popular books (all categories) previously showcased in teacher resource books (e.g., Burns, 1992; Griffiths & Clynne, 1988; Welchman-Tischler, 1996) and

[1] There is a slight discrepancy here of the number of books used in the study when compared with those reported in Publication 1 and Publication 3. Final collation showed overlaps in the books across awards and categories. The number reported here is correct.

journal articles (D. Whitin, 1992). See Appendix B for a list of these additional books.

Of the total of 136 books, 50 were randomly chosen for classification (Appendix B). Of these 34, were classified as perceived, 8 embedded and 8 explicit. The higher percentage of perceived mathematical picture books represents the apparent higher percentage of these books on the market compared with embedded and explicit books. In Publication 1, I reported that an intercoder reliability of .92 was achieved between me and the research assistant. More details are provided in Section 5.3 and Appendix B[2].

4.1.2 Framework development

The literature reviewed for the development of the framework (Halsey, 2005; Hellwig, Monroe, & Jacobs, 2000; Hunsader, 2004; Schiro, 1997; D. Whitin & Whitin, 2004) highlighted aspects that were consistently considered to be important in quality picture books for promoting mathematics teaching and learning. These first indicated that the mathematics needs to be accurate, visible, age-appropriate, and meaningful (i.e., real-life contexts). It should also provide opportunities for reader involvement, varied responses, and problem solving. Additionally, the mathematics should not be didactic or imposed, the presentation should appeal to the reader and promote a positive attitude to mathematics, and the application of the included mathematics needed to be easy to implement. I therefore regarded these characteristics as being necessary within my framework.

Furthermore, I noted the importance of the role of both the text and the images. Advice from curriculum bodies also needed to be heeded, including integration opportunities, inclusiveness of gender and cultural groups, and socioconstructivist pedagogical practices. These ideas were then incorporated into the categories and elements of the new framework.

[2] It should be noted that in these three publications, the word “classification” has been used in reference to both the different types of mathematical picture books and the framework. From this point, the words “classify” and “classification scheme” relate solely to the sorting of mathematical picture books into the three types: perceived, explicit, and embedded.

Pages 75-82 of this thesis have been removed as they contain published material. Please refer to the following citation for details of the article contained in these pages.

Marston, J. (2010). Developing a framework for the selection of picture books to promote early mathematical development. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), Shaping the future of mathematics education: Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia (Vol. 2, pp. 383–390). Fremantle, WA: MERGA.

The presentation[3] of this publication at the conference allowed a fuller description of the seven categories, their definitions, the 39 elements, examples of books containing each element, and the five-level Likert scoring scheme for assessing books. The framework that was circulated during the presentation is provided in Appendix C. Discussion from the audience supported the relevance of the topic and its components, but indicated that the framework was too long for classroom teacher use. In Chapter 5 I discuss the revisions to the framework.

Furthermore, the presentation and publication did not allow for an explanation or examples about how the framework could be applied to picture books. This is explored in Publications 2 and 3.

4.2 Publication 2

The second publication, initially in the form of a conference paper, was presented at the Children and Childhoods Symposium held by the Institute of Early Childhood, Macquarie University, Sydney, in September 2010. It was later published as a chapter of the conference volume. The publication highlighted the importance of this research in the context of early childhood education broadly, and more particularly concerning the integration of numeracy and literacy. This was the sole presentation from a mathematics education perspective at this early childhood conference. Not only did this publication contain a description of the purpose and use of the framework, it also demonstrated its application. Here the classification scheme was first used to select and present pertinent examples of each type of book. Each book was then used as an example to evaluate one framework category. *Grandpa and Thomas* (Allen, 2005), classified as containing perceived content, was used to demonstrate features of the category of curriculum content, policies, and principles. *The Waterhole* (Base, 2001), a book with embedded mathematics, was discussed with regard to the category of integration of mathematics content. *One Hundred Hungry Ants* (Pinczes, 1993), containing explicit content, was evaluated for the category of mathematical content. The suggested activities for each book were based on current socioconstructivist pedagogical practices, e.g., exploring the concept of volume in *Grandpa and Thomas* (2005) through real-life activities and social interaction.

This publication used the revised framework (Chapter 5) with additional coding to show it was the amended version, e.g., APMS3: *Problem posing: text and visual images*.

[3] This paper was presented by the principal supervisor as I was unwell and unable to travel.

Pages 84-100 of this thesis have been removed as they contain published material. Please refer to the following citation for details of the article contained in these pages.

Marston, J., & Mulligan, J. (2012). Using picture books to integrate mathematics in early learning. In P. Whiteman & K. De Gioia (Eds.), Children and childhoods 1: Perspectives, places and practices (pp. 209–225). Newcastle upon Tyne: Cambridge Scholars.

4.3 Publication 3

Publication 3 was based on a presentation and workshop for practising teachers at the Annual Mathematics Association of New South Wales (MANSW) Conference in October, 2010. This presentation and publication took a different approach to Publication 2 as it demonstrated the application of all categories of the framework to one book, as a teacher would when evaluating a picture book for classroom use.

In the workshop I first described the background to the framework, and its structure. However, the main purpose was to allow participants to engage with the framework rather than providing an explanation of it, as in the presentation of Publication 2. I used the picture book, *Amy and Louis* (Gleeson, 2006) to first demonstrate how the book could be evaluated against the framework categories. This modelling was important as not all participants initially "saw" the mathematics. The participants commented in their feedback that they found the ensuing discussion about what mathematics encompasses and how they perceive it in their everyday lives, most beneficial. Once the mathematics was identified, participants were also able to suggest hands-on, real-life activities emanating from the book, and how they could integrate the book with other curriculum areas.

Following the whole-group discussion, participants worked in smaller groups using the framework and a picture book I supplied from the list of 50 used earlier in the classification process. This activity allowed them to exchange ideas and develop their appreciation of how picture books could be used for promoting mathematical learning experiences.

This presentation was later published in the MANSW journal, *Reflections* in 2011. Further examples of the use of both the framework and picture books for mathematical learning can be found in Publication 7 (Marston, Muir, & Levy, 2013) and Publication 8 (Marston, 2014b).

Pages 102-104 of this thesis have been removed as they contain published material. Please refer to the following citation for details of the article contained in these pages.

Marston, J. (2011). A framework for teachers to select and evaluate picture books to promote early mathematical development. *Reflections*, 36(1), 17–19.

4.4 Postscript

These first three publications reveal the new classification scheme and framework. They also demonstrate the framework's application in a range of settings. All publications, and their associated presentations, provided the background literature for the development of the classification scheme and framework. In addition, they have defined mathematical picture books according to the way mathematics is contained within them, and they described the structure and components of the framework. Publications 2 and 3 have also provided examples of how the classification scheme and framework are used. The conference presentation of Publication 3 allowed participants to engage practically with the framework and picture books.

4.5 Conclusion

This chapter has provided a summary of this first stage of Phase 1. There were several informal sources of feedback from participants at workshops, but the classification scheme and the framework required a more systematic evaluation. This is described in Chapter 5.

CHAPTER

5

PHASE 1: EVALUATION OF THE CLASSIFICATION SCHEME AND FRAMEWORK

In Chapter 4, I described the first stage of Phase 1: the development of the classification scheme and framework. In this chapter, I continue to address Research Question 1: What potential do picture books (of different types) have for promoting mathematical teaching and learning for young children?

This chapter provides an account of the second stage of Phase 1 of the study. It comprises a detailed report of the evaluation of the classification scheme and framework. Use of the framework and results of surveys reveal how professionals:

- evaluated picture books against the framework to indicate how well they believed the framework elements were reflected in the book;
- completed written answers in response to questions seeking their views about the effectiveness and use of the framework; and
- classified as perceived, embedded, or explicit, a sample of mathematical picture books.

Two short papers (both refereed conference abstracts), Publication 4 and Publication 5, are included at the end of the chapter. These conference papers highlighted some of the early results of the evaluation of the framework.

Publication 4: Refereed conference abstract

Marston, J. (2014a). Using picture books to implement the mathematics curriculum: The missed opportunities. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice*. *Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia Incorporated* (p. 759). Sydney, NSW: MERGA.

Publication 5: Refereed conference abstract

Marston, J., & Mulligan, J. (2014). Identifying opportunities for mathematics learning in picture books using a framework. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36 (Vol. 6. p. 166).* Vancouver, Canada: PME.

5.1 Preamble

As reported in Publication 1, after the development of the classification scheme and framework, 50 of the 136 books were randomly chosen for their trialling. The research assistant (an experienced classroom teacher and tertiary education tutor) and I then scored each book against the framework categories on a Likert scale. That 0–4 scale measured a book's mathematical content and potential against each framework element. No mathematical content, inaccurate mathematical content, or little mathematical potential scored at the lowest end of the scale. Those books with potential for mathematical learning scored more highly. See Publication 1 and the framework inserted at the conclusion to this chapter for more details.

I also reported in Publication 1 how we classified each of the 50 books as containing perceived, embedded, or explicit mathematical content, achieving an intercoder reliability of .92. Appendix C contains the results of the classification. These results and the data from the evaluation of the books against the framework categories were recorded for later use.

The following sections provide a detailed account of the process of evaluation, the results of the evaluation, and subsequent conclusions.

5.2 Methodological approach

As noted in Chapter 3, a multi-method approach was taken for collecting the data and evaluating both the framework and classification scheme. Qualitative (open-ended responses) and quantitative (Likert scale scores and classifications) data were also collected through use of a survey. I also used a modified Delphi process of two rounds to refine the framework. Instead of an open-ended questionnaire, I first trialled the framework that had already been developed through extensive reference to literature (Chapters 2 and 4), and asked questions of the participants about its use. Hsu and Sandford (2007) consider it an "acceptable and a common modification of the Delphi process format to use a structured questionnaire in Round 1 that is based upon an extensive review of the literature"(p. 2). The benefit of this modification "is that it (a) typically improves the initial round response rate, and (b) provides a solid grounding in previously developed work" (Custer, Scarcella, & Stewart, 1999, p. 1). The feedback from this trial informed the changes for the larger data collection (Round 2). Use of a Delphi process diminished my influence (see Hsu & Sanford, 2007).

5.3 Method

Figure 5.1 provides a summary of the sequence and participants in each part of the development process (Stage 1) and evaluation process (Stage 2) of the framework, the latter of which is described in this section.

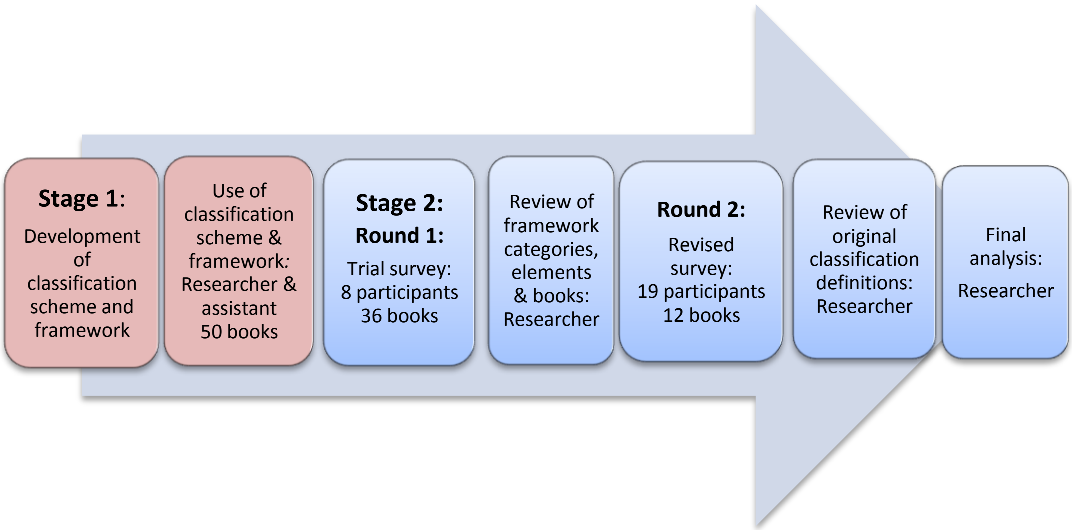

Figure 5.1. Sequence and contents of the development and evaluation process.

5.3.1 Participants

A total of 27 participants completed the survey across the two rounds of survey data collection. They comprised a cross section of professionals from three groups: four academics/teacher educators, 14 primary school teachers, and nine preservice primary teachers. This was a convenience sample. The academics/teacher educators were colleagues of mine whom I approached personally. The teachers were from schools where I had previously conducted either unrelated research projects or professional development. After a telephone conversation with the principals, they organised a time after the school day and a room in which I could work with teacher volunteers. The preservice teachers were students at the university where I worked. Information about the project and a request for volunteers was posted on the electronic blackboard for courses I was not teaching at the time. Preservice teacher volunteers contacted me and we organised a time and place at the university for the survey to be completed. None of the 27 participants had any expertise in using picture books to facilitate mathematics learning. All participants volunteered to take part and their anonymity was assured and preserved. The information and consent forms are provided in Appendix D.

5.3.2 Round 1: The trial

5.3.2.1 The survey[1]

After submitting Publication 1, and prior to its presentation at the conference, a written survey was devised to evaluate the framework. The trial survey (Appendix C), to be completed with a picture book, comprised a copy of the framework (with an example of a book that I deemed, through my earlier scoring reported in Section 5.1, to represent a good model for each element) and an explanation of the Likert scale as outlined in Chapter 4. In addition, there were three questions: two evaluating the framework and one regarding the classification of the book. The classification was the final question, as by this stage the participants were more familiar with the book's content and potential. The three questions were:

1. Would you find this framework a useful tool when selecting picture books to use in the classroom with young children to develop mathematical concepts?
2. Please make any comments regarding the ease of using the framework.
3. Do you consider the picture book to contain perceived, embedded, or explicit mathematical content? (Definitions were provided for each)

5.3.2.2 Picture books

Of the 50 books used in the initial evaluation by the research assistant and me, 36 were selected for the collection of the trial survey data because they had been evaluated as conveying mathematical content, while the remainder contained negligible amounts. I have previously acknowledged that not all books contain mathematical concepts. Of the 36 books, 24 books with perceived content, 6 with embedded content, and 6 with explicit content were chosen. This selection comprised approximately the same proportion of each type of book as was found in the 50 used for the initial classification.

5.3.2.3 Procedure

In this first round, three academics and five teachers from the participant pool described above assessed five picture books using the survey. Each participant was provided with five copies of the survey and a mixed sample of five books (that included at least one copy of each

[1] Before commencing this study, all aspects had been approved by the ethics committee of Macquarie University.

type: perceived, embedded, explicit) from the 36 selected books. Each participant received a different set of five books. Participants were asked to score each book using a separate survey form. Participants were advised at the time of volunteering that the session would take about an hour but that they could stop at any time. Some completed a survey for all five books within an hour. Of the participants who did not finish by that time, some left but others stayed to complete their evaluation of all books

5.3.3 Review of Round 1

The survey responses from this trial were not fully analysed at this point, but were reviewed for ideas that would inform Round 2 of the data collection process. Two main revisions were made, one to the survey itself and one to the books used.

The most common comment from participants, from the written survey and from verbal responses during the workshop (Publication 1), was that there were too many elements in the framework for teachers who were time-poor. Therefore, using this advice, as is customary in a Delphi method, the framework was revised. As a result, some of the elements were combined without removing any of the previous content. For example, there had been two categories of MPS3: Problem posing: text and MPS4: Problem posing: visual images. They were collapsed into a single category of APMS3[2]: Problem posing: text and visual images. The revised framework, found in Appendix C, still contained the original seven categories but now included only 21 elements. The survey questions remained unchanged as they revealed the professionals' classification of the books and their perception of the framework.

I also decided that it would be more beneficial to concentrate on a smaller number of books in depth. Therefore, 12 books were chosen from the 36 used in the trial, and these were distributed to participants in Round 2. This selection was based on a similar ratio of each type of book already used in Round 1. Therefore, there were 6 books that contained perceived content, 3 with embedded content, and 3 with explicit content. Table 5.1 shows the 12 books selected for use. The results reported for the framework evaluation, i.e., the scoring on the Likert scale and Questions 1 and 2 (Section 5.5), are for these final 12 books. The professionals' classifications of the 36 books made during the trial data collection (Round 1) were continued to be used in the reporting to provide additional data and validity.

[2] The code AMPS referred to the new abridged version at the time (and in Publication 2) and distinguished the two versions. As the revised framework superseded the previous framework, all references will now not include the A in the coding.

Table 5.1

Final 12 Books Used in Survey

Type	Author	Title
Perceived	Allen, P.	*Grandpa and Thomas*
	Allen, P.	*The Potato People*
	Lester, A.	*Ernie Dances to the Didgeridoo*
	Gleeson, L.	*Clancy and Millie and the Very Fine House*
	Harvey, R.	*At the Beach: Postcards from Crabby Spit*
	Wild, M.	*Seven More Sleeps*[3]
Embedded	Clement, R.	*Counting on Frank*
	Fox, M.	*Ten Little Fingers*
	Base, G.	*Uno's Garden*
Explicit	Dodds, D.	*Minnie's Diner: A Multiplying Menu*
	Pinczes, E.	*One Hundred Hungry Ants*
	Neuschwander, C.	*Patterns in Peru*

5.3.4 Round 2: The revised framework survey

The same participants were not used for Round 2 as is common in a traditional Delphi method. I used the remainder of the total number of participants, i.e., 19 professionals (one academic/teacher educator, nine primary school teachers and nine preservice primary teachers) to complete the revised survey. I used the same procedure for data collection that had been used when trialling the survey in Round 1, although in this round there were only 12 books. The surveys for the two rounds were completed over approximately three months.

5.3.5 Review of classification scheme

At the same time as I was collecting the survey data I also conducted the interviews with four authors for Phase 2 (see Chapter 6). Not only did the interview data provide pertinent information about author intent but it also informed the review of the classification scheme. Therefore, I reviewed my original classification (see Appendix C). Three changes aiming to develop a more accurate classification scheme were made. The change of classification of *Are We There Yet? A Journey Around Australia* (Lester, 2004) from embedded to perceived and *Uno's Garden* (Base, 2006) from explicit to embedded, were particularly based on advice from

[3] Although the title of the book *Seven More Sleeps* (Wild, 2004), suggest explicit mathematics, I do not believe the book is intended as a counting book, but is meant to be enjoyed for the story alone.

the authors. The third book, *What the Sky Knows* (Bourke, 2005) was changed from embedded to perceived based on my new understanding. On this revision I found an intercoder reliability of .9 (Stage 1 had shown .92) with the research assistant's previous classification. Using my (expert) classification I now found that there were 35 books with perceived content, eight with embedded mathematics, and seven with explicit mathematical content. This revised classification of the books is used for the remainder of the study.

5.4 Analysis

A total of 118 surveys were completed by the 27 participants including four academics (20 surveys), 14 teachers (53 surveys), and nine preservice teachers (45 surveys). As they were completed I coded them according to the group: Academic (A), Teacher (T), and Preservice teacher (PT), and numbered them as received (e.g., A1 = Academic 1, T1 = Teacher 1) in preparation for analysis. These codes are used for reporting throughout this chapter.

As reported in Section 5.3.2.1, the survey responses from the trial had been reviewed only for required changes and then set aside for the overall analysis. After collecting the second set of data I analysed both sets in the following ways:

- The total points on the Likert scale for each category and element of the framework from each survey for each of the 12 designated books were recorded. These were then used to:
  - **i.** compare the mean score of all participants for each category and element of the framework with my (expert) scoring, and
  - ii. examine the range of scores across categories and elements for individual books.
- Comments on Questions 1 and 2 were recorded.
- The classification of all books (Rounds 1 and 2) evaluated by the participants (Question 3) was compared with my revised classification.

5.5 Results

The results from the classification (Survey Question 3) are reported first, followed by the professionals' use of the framework (their scoring on the Likert scale), and finally the professionals' perceptions regarding the framework (Survey Questions 1 and 2). This order facilitates the sequence of reporting and enabled one set of results to inform the next.

5.5.1 Classification scheme

The classification of the books into perceived, embedded, and explicit was initially important to determine whether there were qualitative differences in types of mathematical picture books and to justify the definitions.

Table 5.2 contains a summary of the data, including the number of surveys completed by each participant group (*N* = 118), the number of participants in each group who classified the book in the same way that I did, and the percentage of participants who agreed with my classification in each group.

Table 5.2

Classification of Picture Books by Survey Participants

Participant group	Surveys completed	Same classification as researcher	Percentage agreement
Academic	20	13	65
Teacher	53	27	51
Preservice teacher	45	20	44
Total	118	63	51

Of the 36 books classified, 50% of the books I had regarded as having perceived content were also classified as perceived books by the participants. In the case of embedded books, the agreement was 46%, and the agreement was 54% for the explicit books.

As can be seen from the final column of Table 5.2, the agreement between the participant groups and me appears to increase with levels of mathematical experience.

Appendix C contains the results of the classification of all the books used by the participants.

5.5.2 Professionals' use of the framework to evaluate mathematical picture books

Reporting of scoring on the Likert scale by participants does not evaluate the framework itself, but it shows how effectively the participants used it. The process also assisted them to respond to the first two questions of the survey.

5.5.2.1 *Comparing categories*[4]

The mean score for each category of the framework and for each type of book was first calculated and compared with my scoring.

Figures 5.2, 5.3, and 5.4 provide a summary of these results, showing the mean scores for both the participants and me for all categories of the perceived, embedded, and explicit mathematical picture books respectively. Figure 5.5 compares all three types of book.

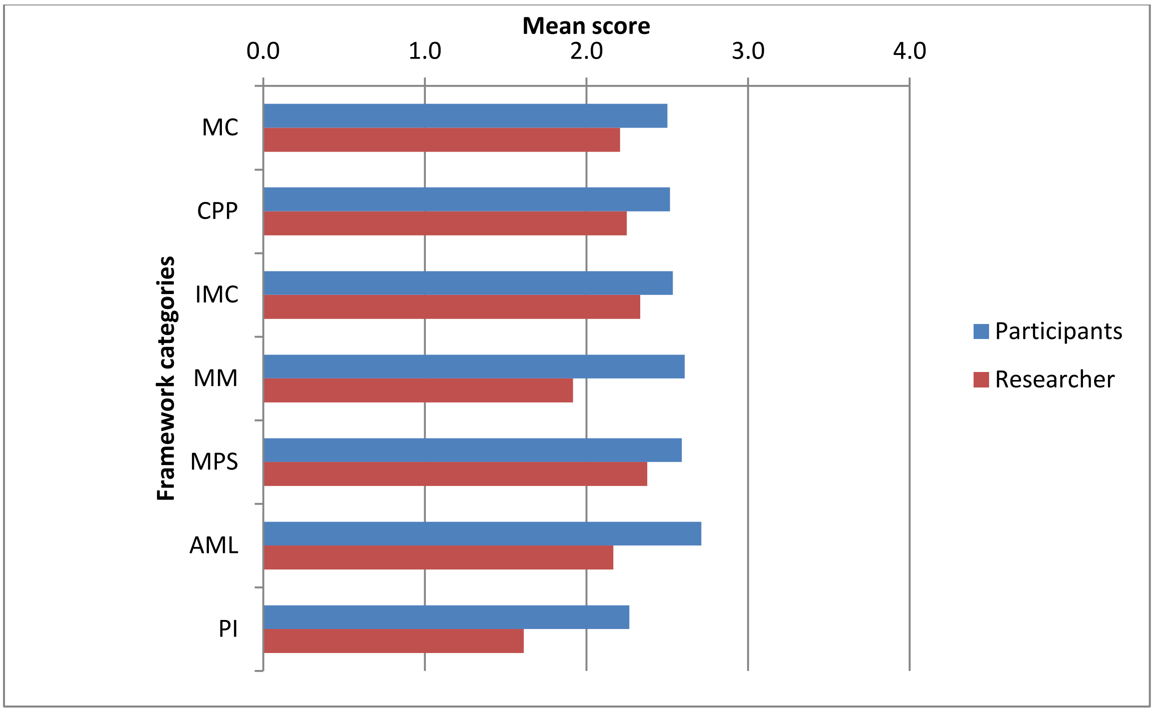

Figure 5.2. Mean scores of the categories of perceived mathematical picture books.

[4] The framework categories are: *Mathematical content* (MC); *Curriculum content, policies, and principles* (CPP); *Integration of mathematics content* (IMC); *Mathematical meaning* (MM); *Mathematical problem solving and reasoning* (MPS); *Affordances for mathematical learning* (AML), and *Pedagogical implementation* (PI).

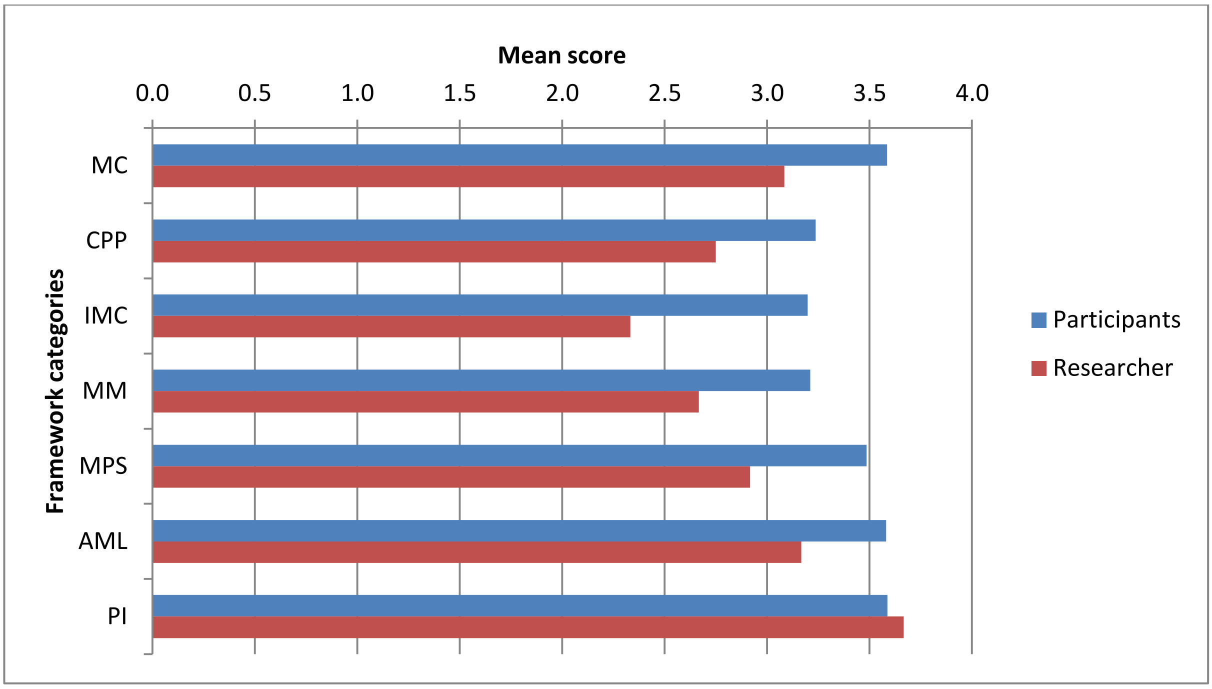

Figure 5.3. Mean scores of the categories of embedded mathematical picture books.

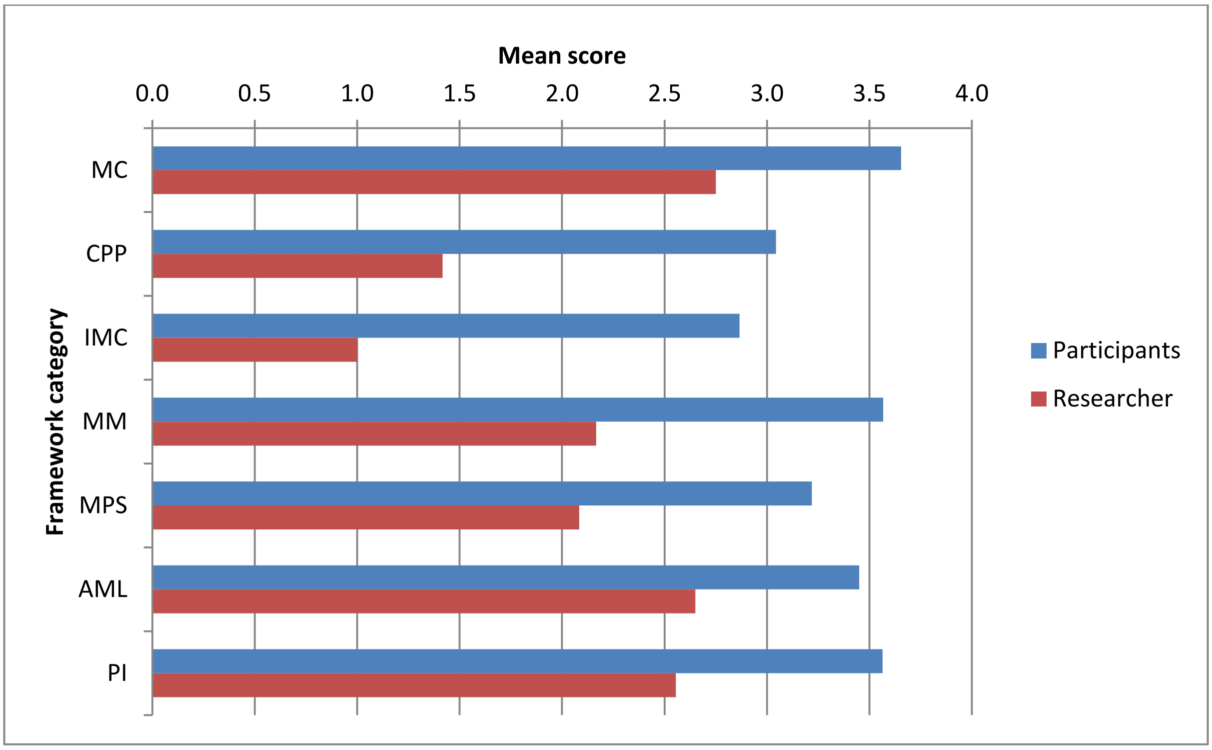

Figure 5.4. Mean scores of the categories of explicit mathematical picture books.

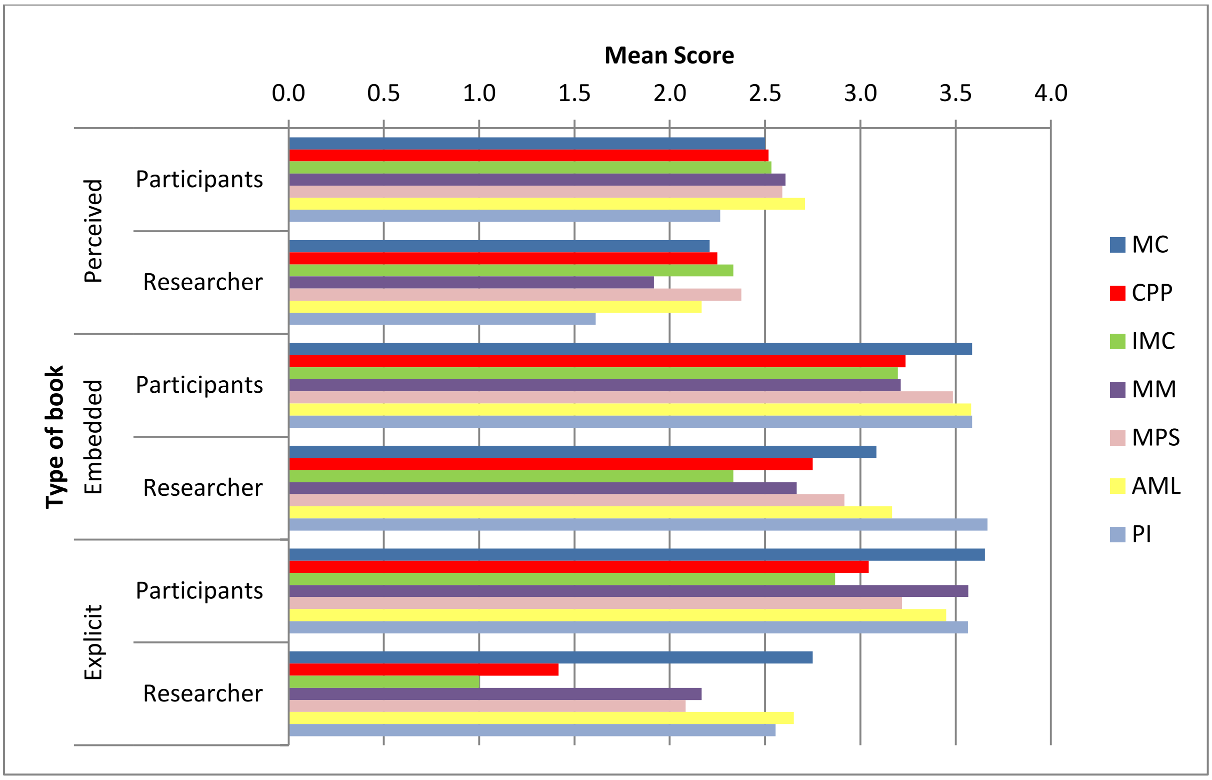

Figure 5.5. Comparison of mean scores for each type of book and framework category as completed by the participants and researcher.

All figures demonstrate the different mathematical content and opportunities seen by the participants for each type of book. They also indicate that, with the exception of one category, the participants recorded more potential for mathematical learning than I did for all books and categories.

5.5.2.2 Range of responses to individual categories

It became obvious while recording the scoring for each book that there was a noticeable range of the scores from the participants not only between categories of different types of books but also within many of the categories and elements. This is demonstrated in Tables 5.3, 5.4, and 5.5. These tables provide the full set of responses from one of each type of book as an example. I have chosen the book that was evaluated most in each type as the example. These books were *Ernie Dances to the Didgeridoo* (Lester, 2000) for perceived content, *Uno's Garden* (Base, 2006) for embedded content, and *Minnie's Diner: A Multiplying Menu* (Dodds, 2004) for explicit content. These results will be discussed in Section 5.6.2.

Table 5.3

Responses for all Categories and Elements for Ernie Dances to the Didgeridoo *(Perceived Content)*

		R[b]	A4[c]	T2	T5	T11	T12	T13	PT3	PT6	PT7	PT8
MC	1	2	0	2	2	3	2	2	2	0	4	1
	2	3	0	2	1	3	1	2	1	0	1	2
	3	4	0	2	3	4	4	3	2	2	NR[d]	3
	4	2	1	2	2	3	4	2	1	2	3	1
CPP	1	2	0	1	1	1	0	1	1	1	4	1
	2	3	0	1	2	2	4	1	2	1	4	3
	3	2	1	2	4	4	0	3	2	4	3	4
	4	3	1	3	4	3	4	3	3	4	4	2
IMC	1	3	0	1	4	2	1	4	2	2	2	0
	2	3	0	2	4	3	4	4	2	4	3	2
MM	1	3	1	2	4	3	4	2	2	0	4	2
	2	2	0	1	3	3	4	3	2	1	3	3
MPS	1	4	0	1	3	1	2	2	2	2	1	2
	2	4	0	1	3	1	2	2	2	2	2	2
	3	3	1	1	3	2	3	3	2	2	3	2
	4	2	0	1	3	1	1	2	2	1	1	2
AML	1	3	0	1	4	1	4	2	2	1	3	3
	2	3	0	1	4	2	4	2	1	1	3	3
PI	1	3	0	1	2	2	4	1	2	0	2	0
	2	3	1	1	3	2	4	2	2	0	4	1
	3	1	0	1	3	3	4	2	2	0	2	0

[a] Categories are represented by their initials; elements within the category are represented by their number.
[b] R = Researcher,
[c] A = Academic, T = Teacher, PT = Preservice Teacher. (A1 = Academic 1, T5 = Teacher 5 etc.)
[d] NR = No response

Table 5.4

Responses for all Categories and Elements: Uno's Garden *(Embedded Content)*

		R	A4	T2	T5	T9	T11	T12	T13	PT3	PT6	PT7	PT8	PT9
MC	1	4	4	4	4	4	4	4	4	4	4	4	4	4
	2	3	4	4	4	4	4	4	4	4	3	4	4	4
	3	3	2	1	2	4	3	4	4	4	1	4	4	3
	4	4	4	4	4	3	4	4	4	4	4	4	4	4
CPP	1	4	2	1	4	4	3	4	4	4	4	4	4	4
	2	4	3	4	4	3	4	4	4	4	4	4	4	4
	3	4	1	1	4	1	4	3	4	4	3	4	4	4
	4	4	4	2	4	4	4	4	4	4	4	4	4	4
IMC	1	3	1	3	2	4	3	4	4	3	3	4	4	4
	2	4	1	1	4	4	4	4	4	3	4	4	4	3
MM	1	4	1	3	4	3	4	3	4	4	4	4	1	3
	2	4	0	3	4	3	4	4	4	4	4	4	4	4
MPS	1	3	2	2	2	2	4	4	4	4	4	4	2	3
	2	4	3	4	3	3	4	4	4	4	4	4	3	3
	3	4	2	4	4	4	4	4	4	4	4	4	4	4
	4	3	0	3	4	4	4	4	4	4	4	4	1	4
AML	1	4	2	3	4	3	4	4	4	4	3	4	4	4
	2	4	1	3	3	3	4	4	4	4	4	4	4	4
PI	1	4	1	4	4	3	4	4	4	4	4	4	4	3
	2	4	3	4	3	3	4	4	4	4	4	4	4	4
	3	4	3	2	4	4	4	4	4	4	4	4	4	4

Table 5.5

Responses for all Categories and Elements: Minnie's Diner: A Multiplying Menu *(Explicit Content)*

		R	T5	T6	T11	T12	T13	T14	PT4	PT9
MC	1	4	4	3	4	4	4	4	4	4
	2	4	4	3	4	4	4	4	4	4
	3	2	4	3	4	2	4	4	2	4
	4	3	4	3	4	4	4	4	4	4
CPP	1	4	4	3	4	4	4	4	4	4
	2	2	4	2	4	3	4	4	4	4
	3	0	4	1	4	2	3	2	1	1
	4	0	3	1	2	1	3	4	2	1
IMC	1	2	2	1	3	2	3	4	1	4
	2	0	2	2	2	1	4	3	1	3
MM	1	3	4	3	4	2	4	4	3	4
	2	3	4	4	4	4	4	4	4	4
MPS	1	2	4	3	4	2	4	4	3	4
	2	3	4	2	4	2	4	4	3	4
	3	3	4	2	3	2	4	4	2	4
	4	1	4	0	1	2	4	2	0	4
AML	1	2	4	4	3	3	4	4	4	4
	2	4	3	4	3	3	3	3	3	4
PI	1	3	3	4	3	3	4	4	4	4
	2	3	3	4	3	2	4	4	4	4
	3	2	4	3	4	4	4	4	3	4

5.5.3 Professionals' perceptions about the framework

In this section I describe how professionals found the use of the framework (Questions 1 and 2). Although some participants mentioned difficulties in using the framework, comments recorded were predominantly positive. Some participants also offered suggestions for improvement. Many remarked that they found the framework much quicker to complete the more times they used it and became familiar with the different elements.

There were also comments about teachers not having time to spend on a framework such as this. However, one (T6) suggested, "I would probably work on this with my stages[5] at the beginning of a year – choose several books; use the framework, work out where they would fit in to the scope and sequence; trial them and re-assess."

Some participants commented that the framework did not need the examples that were provided, while others felt that "the examples given helped clarify the intent" (A3). Many commented that they would prefer a list of useful books with the syllabus connections for easy reference in the classroom rather than spending the time required to use the framework.

Table 5.6 contains most comments made in response to Question 1 on the survey: Would you find this framework a useful tool when selecting picture books to use in the classroom with young children to develop mathematical concepts? Not all responses are listed as these were repetitious across the books completed by a participant, were pertinent to specific books rather than the survey itself, or were "yes" or "no" responses only. Table 5.7 provides a similar overview of Question 2: Please make any comments regarding the ease of using the framework. The complete set of responses is found in Appendix C.

[5] The Australian Curriculum (2012) organises the content in year levels. However, the individual states of Australia have their own adaptations of the curriculum that may include grouping of year levels into stages. For example, in the *NSW Syllabus for the Australian Curriculum* (2012), Years 1 and 2 form Stage 1.

Table 5.6

Sample of Responses to Question 1

Participant	Comment
A1	As a teacher, I'd want something with less detail. Teachers will be looking for what is the mathematical content, what level is it pitched at, and how engaging it is – is it something they will enjoy? How appropriate it is for other issues (KLAs)? How can I use it?
	A teacher would want to look at the issues and would not want to dig too deep.
A2	Yes – great points to consider in selecting and planning when using literature in mathematics / to support & integrate with mathematics.
T2	A *simpler* version of the framework – but yes it is helpful.
T3	Yes, it does cover a range of ideas - however I wouldn't look at it *regurgarly* [*sic*] as it is too in depth.
T4	Not if I needed to pick one for a maths lesson or concept as it doesn't help identify the strand/s the book will look at.
	I would be disapointed [*sic*] if I spent time doing this survey for a book that is unrelated to maths. Needs to be KLA's specific.
	Yes but needs to show which mathematical concept the book has related to the syllabus otherwise teachers won't use it.
T5	Yes it makes me more aware of trying to find meaningful maths learning opportunities.
T13	Yes, I think this would be useful tool to help develop maths concepts with young children The framework inspires me to think about maths .... The framework made me think a lot more about all of the different types of maths that could potentially be found & used in this book (Clancy and Millie and the very fine house). I would not really have linked much of this book to maths. However, it links really well to many volume/capacity areas and 3D shapes. By reading the framework it got me thinking.
PT1	Yes, this is because it allows the teacher to look of ways of intergrating [*sic*] Maths into other KLA's [sic].
PT2	Yes it helps to know that our thoughts are on the right track, we may think that this book is good to use in our teaching for maths purpose, and we know that someone (expert) agrees with us, we feel more confident about it. It also saves teachers time to use on planning their teaching.
PT4	Yes! I have previously used some of these books to teach maths concepts to my charge, however, it wasn't until I used this framework and really thought about the content that I realised just how many different concepts can be found in the one book, I am now really looking forward to revisiting these.
PT5	While the framework appears useful in order to ANALYSE the capacity of a book to teach mathematical concepts, it would seem too detailed and complex for the SELECTION of a book for classroom use. A more simple rubric/ framework identifying key mathematical concepts in a particular book may be more useful.
PT6	Yes or even just a recommended list of books with reference to which mathematical concepts are embedded in the story.
PT9	Yes, I would as they consist of all the elements a good picture book requires in order to effectively develop mathematical concepts in children.

Table 5.7

Sample of Responses to Question 2

Participant	Comment
A2	Example column is not necessarily helpful; only if you know the books.
	Sometimes the way we see maths is mediated by our past experiences; how we define "maths"; it is just number and not all the other areas. depends on our perception of maths
A3	Yes. The examples given helped clarify the intent. It would be useful to have a section where the perceived Mathematical benefits could be measured. Even done finitely so that the pages or concepts could be counted.
A5	It fostered for me an ability to classify the book and helped me decide if I would use this book in the classroom
	A useful tool. Very explicit and user friendly
	It allowed me to notices [*sic*] errors in the book eg number of plants on the page do not add up to the numerical sequence
T5	I feel challenged to incorporate good quality books into my maths lessons. The framework would definitely help me with book selection.
T9	Yes but a list of books already in existence would be easier.
T10	Framework is a little "wordy" and time consuming for classroom teachers to use on a regular basis.
T11	Very easy to use - clear, concise language use. Very easy to use.
PT4	Framework is easy to understand and is grouped accordingly. The format helped me gather my thoughts collectively and to think beyond my initial thoughts. I also utilised the clear examples that were listed.
PT5	Fairly easy to use to analyse a book – use of examples greatly assist the analysis criteria.
PT7	Yes, but need more explanation (Examples). For me, I had no time to look at the examples of the provided names of the book. But if I was actually teaching, I would find the time to research the names of the books under the Heading of 'Example'. This will also be ideal to gather other books when working on the same content.
	Yes, the second time around
	Yes. The more I read it, the better it is getting for me to understand it.
PT9	I found it very easy to use and practical. It was easy to understand and it was well-structured.
	I found this framework very easy to use and useful as it made me think about factors that I had not considered before such as 'problem solving opportunities' and 'environmental awareness'. It was easy to use and very practical.
PT10	I will use it again

5.6 Discussion

In Sections 5.2 to 5.5, I provided the results of the three components of the survey. The inferences or assumptions that follow can be drawn from these data.

5.6.1 Classification scheme

The participants in this survey demonstrated that they recognised that there are different types of mathematical picture books. Table 5.2 (Section 5.5.1) shows that there was 51 % agreement between the participants and me.

I reported in Table 5.2 how the academics' classification results showed the highest percentage of agreement with me, followed by teachers, and then preservice teachers. The research assistant had worked with me and had had the opportunity to discuss the classification scheme before, and had read background literature, thus being the most informed. As did Nesmith and Cooper (2010), in Section 2.5.4, I found variations across and within groups and also that it appears that a better understanding of mathematics education and/or the particular topic increases the ability to recognise mathematical content and opportunities for mathematical learning and the way this is presented outside traditional mathematical settings. Christy (2003) noted that academics in children's literature, who have great understanding of the textual and visual qualities of a book, usually do not have knowledge of the mathematical concepts and their pedagogical application, and that academics in mathematics education are usually not aware of the range and opportunities in literature. Therefore academics in teacher education programs may not have knowledge and experience of using picture books for mathematics learning. In this case, two of the five academics surveyed were generalist primary education academics, while the other three were specifically involved in mathematics education. All had taught in the primary or secondary classroom.

Christy (2003) also found that, preservice teachers could identify mathematics concepts in textbooks but had difficulty identifying them in real-life situations such as those portrayed in many picture books. This indicates a need for more practical experience to increase their understanding of mathematics. I also found this demonstrated in these survey results and through the earlier comments by participants in the workshop associated with Publication 3 (see Section 4.3).

I found the participants' difficulty in recognising the explicit mathematical picture book (only 54% recognition) surprising as this group of books, particularly the ones used here, often had mathematics in their title, e.g., *One Hundred Hungry Ants* (Pinczes, 1993), *A Place for Zero: A Math Adventure* (LoPresti, 2003), and *Minnie's Diner: A Multiplying Menu* (Dodds, 2004). From my own experience, preservice and classroom teachers who have had any contact with a book with mathematics in the title have expected it to be good for mathematics teaching.

A common feature of all the literature reviewed, and both the Marston (2010a) and Van den Heuvel-Panhuizen and Elia (2012) frameworks, is that the content of a picture book must be meaningful and authentic to the child. The Dutch Realistic Mathematics Education approach, however, points out that these contexts need not necessarily be restricted to the "real-world" but can be "'real' in the students' minds" (Van den Heuvel-Panhuizen, 2003, p. 10). It is interesting, therefore, to note that one respondent suggested that there "may be need [for] another category for 'fantasmagorical' books, e.g., *A Place for Zero* (LoPresti, 2003) and *Who Sank the Boat?* (Allen, 1982). There may be a different sort of approach to evaluating them" (A1). However, if the participant uses fantasmagorical to mean anthropomorphic, these books were represented across all categories.

The low agreement between professionals' and my classification may also be because the definitions of the three books were not supported sufficiently with examples, or that the definitions themselves were too limited. This point will be followed up in publication of the research. It may include literature and guidance for successful teacher use, clearer definitions, and examples for teachers to review.

It was not important at this point of the study for the participants to be able to classify the books correctly, but the fact that participants had difficulty, both with identifying mathematics and then classifying how the mathematics was contained in the books, is further discussed in Chapter 10. If it was to be shown in Phase 3 that different types of mathematical books do elicit different responses, it will benefit teachers to be able to distinguish between them when choosing quality books for mathematics learning.

5.6.2 Professionals' use of the framework to evaluate mathematical picture books

The results of the participants' scoring with the 12 books representing the three different types produced some interesting results. In Figures 5.2, 5.3, and 5.4, I compare the mean scores of participants and the researcher for each framework category for perceived (Figure 5.2), embedded (Figure 5.3) and explicit (Figure 5.4). In Figure 5.5 I provide a summary of the comparison of mean scores for each type of book and framework category. Tables 5.3, 5.4, and 5.5 showed the variation among the responses to individual categories and elements for one of each type of book. Considerable discrepancy was found between the potential for mathematics learning identified by the participants and by the researcher. For all but one framework category (pedagogical implementation: embedded books), participants found more potential for mathematics learning than the researcher (see Figure 5.3). This may be the result of an aggregate score from the participants being compared to an individual score. The lower scoring by the expert researcher could have also been attributed to exacting scoring by an “expert” looking for quality mathematical learning experiences. It may also be because the participants knew they were completing a survey about mathematics in picture books and expected it to be in the books provided and therefore scored the books highly. Additionally, professionals may need more experience both in identifying mathematics in picture books and using mathematical picture books.

In the following sections I discuss the variation in professionals' survey responses for each type of book according to the seven framework categories. I also provide examples from some of the books used and illustrate some of the potential of these books, which, if overlooked, are missed opportunities. Section 5.6.2.7 summarises the survey results for the three book types.

5.6.2.1 Mathematical content

The data in Figures 5.2—5.4 demonstrate that the participants found more mathematical content in the embedded (Figure 5.3, mean = 3.6) and explicit (Figure 5.4, mean = 3.7) than the perceived books (Figure 5.2, mean = 2.5). Scoring in the mathematical content category for the six perceived books was wide-ranging. There appeared to be no consistency in the professionals' identification of the mathematical content. In the case of *Ernie Dances to the Didgeridoo* (Lester, 2000), some participants found little real mathematics, but others rated it highly. Table 5.3 shows scores ranging from 0—4. Another of the books, *At the Beach: Postcards from Crabby Spit* (Harvey, 2004) tells the story of a family holiday to the beach. The mathematical inclusions are visible, accurate, and authentic, hence providing opportunities for the use of mathematical language and investigations. For example, the illustrations used throughout the book include maps and cross sections, and the text refers to tides and Frisbee throwing competitions. These provide opportunities for rich real-life mathematical investigations such as graphing tide heights over a set period (science integration as well), measuring, recording, and comparing how far students can throw a Frisbee or other appropriate object, or constructing a map of a favourite place. See Chapter 6 and Publication 8 for further ideas. However, out of a total possible score of 16 for the category of mathematical content, the scores by participants ranged from 5 to 14.

The mathematics is not as visible in the title, text, and images of another perceived book, *Clancy and Millie and the Very Fine House* (Gleeson, 2009a), but there is potential for discussion and activities (e.g., length and 3D objects). The range of scores here was 6 through to 15. There was no consistency, either, in scoring within the participant groups.

It would have been interesting to have more information from the participants about the mathematics that they actually detected in not just these books, but in the embedded and explicit books. As one participant noted, the identification of mathematical concepts and opportunities provided by a picture book depends on our perception of what mathematics is. "Sometimes the way we see maths is mediated by our past experiences; how we define "maths"*;* that it is not just number but all the other areas also" (A2). Perceived books too have the least amount of obvious mathematics.

In the case of the embedded books, where the mathematics was more visible but not a necessary component of the book, the scores were much closer and consistently high for all three participant groups. Table 5.4 shows the range of scores for *Uno's Garden* (Base 2006). Although there were scores of 1, 2, and 3, the participants predominantly scored 4 for many elements. The embedded books contained realistic situations where the story and sometimes "message" provided the authenticity. The added mathematical layer also provided the content visibility and mathematical language opportunities. However, in *Counting on Frank* (Clements, 1990) there are some inaccuracies not necessarily noticed by the participants, possibly due to the limited time available in this instance. As will be seen in Publication 7 (Marston, Muir, & Levy, 2013), this may not be a disadvantage.

The range of scores was much closer and higher for the three explicit books. In the example provided in Table 5.5 of *Minnie's Diner: A Multiplying Menu* (Dodds, 2004), that focuses on the doubling concept, participants predominantly scored each element of mathematical content a 4, with the occasional 2 and 3. However, although all the titles suggested mathematics, the books concentrated on only one mathematical concept. There are certainly good opportunities there for use of mathematical language, and the content is visible and accurate, but the context is not necessarily authentic or realistic. See Section 7.3.5 for more details.

The quality and quantity of mathematics that participants identified in this category of *Mathematical content* may have influenced their scoring in the other categories.

5.6.2.2 Curriculum content, policies, and principles

With regard to curriculum content, policies, and principles, embedded books (Figure 5.3, mean = 3.2) appear to have more potential than explicit (Figure 5.4, mean = 3.0) and perceived books (Figure 5.2, mean = 2.3). Scoring among the perceived books was again wide-ranging. The example in Table 5.3 shows a range 0—4. In the case of *At the Beach: Postcards from Crabby Spit* (Harvey, 2004), where I had previously identified many mathematical ideas and opportunities to include environmental issues, the category total scoring by participants for this book ranged between 9 and 15, again with no consistency between groups.

Of the three embedded books, there were also differences in scoring for two of the books. Participants predominantly found numerous opportunities for connections to the curriculum (Table 5.4) in *Uno's Garden* (Base, 2006). The mathematical content is only related to number concepts, albeit at different developmental levels, and there is cross-curricular integration potential for discussion about environmental awareness (the main intent of the book). See Chapter 6 and 7, and Publication 8 (Marston, 2014b), for more ideas for applying this book to the mathematics classroom program. The illustrations also make it inclusive of all genders, cultures, and socioeconomic groups.

In the case of the explicit books, the participants' responses were similar for the first two elements, but differed greatly for the second two. See the example given in Table 5.5. Although these books do aim to teach mathematics, the common single concept focus (e.g., doubling in *Minnie's Diner: A Multiplying Menu*, Dodds, 2004) would address only one curriculum outcome. While it is certainly age appropriate, it affords few other curriculum connections.

5.6.2.3 Integration of mathematics content

Participants scored this framework category lower for perceived books (Figure 5.2, mean = 2.5) than for the embedded (Figure 5.3, mean = 3.2) and explicit books (Figure 5.4, mean =2.9). Although the perceived books, e.g., *At the Beach: Postcards from Crabby Spit* (Harvey, 2004) and *Ernie Dances to the Didgeridoo* (Lester, 2000), include multiple mathematics opportunities across several mathematics strands as well as English, and science and technology teaching and learning opportunities, the participants did not appear to identify these.

Most surprisingly, scoring on *Uno's Garden* (Base, 2006), whose main purpose is an environmental message and whose mathematics is embedded, still contained varied responses for cross-curricular integration. Table 5.4 shows a range of 1—4. Conversely, the explicit books were often identified, with a score of 4 as seen in the example in Table 5.5, as having this potential, although I rarely found them myself.

5.6.2.4 Mathematical meaning

The framework category of mathematical meaning emphasises real-life situations and the sequence of the mathematics within the book. Figures 5.2—5.4 show the mean score for this category was highest for the explicit books (Figure 5.4, mean = 3.6), and lowest for the perceived books (Figure 5.2, mean = 2.6). The importance of real-life situations was discussed earlier and many perceived books provide everyday real activities, such as a family holiday and families moving. *At the Beach: Postcards from Crabby Spit* (Harvey, 2004), *Clancy and Millie and the Very Fine House* (Gleeson, 2009a), and *Ernie Dances to the Didgeridoo* (Lester, 2000), are three excellent examples of real-life perceived picture books. However the scoring for authenticity by the participants in the last book ranged from 0—4 (Table 5.3).

I also identified in the analysis that the three embedded books displayed good examples of real life situations. In the case of *Uno's Garden* (Base, 2006) several participants did not see any authenticity (Table 5.4) in the environment message represented through the story and mathematics. *Minnie's Diner: A Multiplying Menu* (Dodds, 2004), that only has some real-life aspects to it, was scored highly by many participants (Table 5.5). Authenticity is not always the case for explicit books as the emphasis is on a mathematical idea. For example, a book like *A Place for Zero* (LoPresti, 2003) does not portray real-life situations.

As perceived mathematical picture books have no mathematical intent, the mathematics is not likely to be sequential, although an experienced teacher could devise quality learning experiences based on the potential for using the mathematical ideas. Despite lack of sequence in *Ernie Dances to the Didgeridoo* (Lester, 2000), several participants scored this element highly (Table 5.3). Embedded books may not always be sequential either, but Base (2006) has considered this and it was identified by most participants (Tables 5.4). Since explicit books are purposeful in their teaching of mathematics, sequence should be important and evident. This was the case with *Minnie's Diner: A Multiplying Menu*, Dodds, (2004) that was scored highly by the participants for this element (Table 5.5).

5.6.2.5 Mathematical problem solving and reasoning

The strand of problem solving and reasoning is included in many mathematics curricula and teachers are aware of the need for problem solving. However, they still have difficulty recognising and implementing rich, engaging problems, and they are not providing students with opportunities for their own investigations (J. Anderson, 2009).

In this study, participants generally saw more problem solving opportunities in the embedded book than the other two types of book (Figures 5.2—5.4). Despite the many opportunities I had identified, and not specifically suggested in the books, many participants did not appear to see the problem-solving possibilities in the perceived books particularly. There was only one book that had any agreed level of opportunities for problem solving among all 12 books. This was *Counting on Frank* (Clements, 1990), an embedded content book, where the main character actually poses mathematical problems throughout the story. If a child identifies with the characters in a book in a realistic situation, it increases their engagement and interest (Keat and Wilburne, 2009). Because the book's character poses and solves mathematical problems, the children are more motivated themselves to engage in similar mathematical problem solving and reasoning activities. However, there were participants who saw no problem solving opportunities in *Uno's Garden* (Base, 2006) despite the introductory rhyme, mathematics equations and the games highlighted at the back of the book (Table 5.4).

5.6.2.6 Affordances for mathematics learning and Pedagogical implementation

Generally, the participants scored the embedded and explicit books highly for affordances for mathematics learning and pedagogical implementation (Figures 5.2—5.4). However there was still disparity of scoring across the categories and elements as demonstrated in the examples provided. Scores ranged from 0—4 (Table 5.3) for *Ernie Dances to the Didgeridoo* (Lester, 2000), 1—4 (Table 5.4) for *Uno's Garden* (Base, 2006), and 2—4 (Table 5.3) for *Minnie's Diner: A Multiplying Menu*, Dodds, (2004) for both categories. Again, if the participants were unable to see the mathematics, they could not say it would have positive mathematical influence on the students/readers or be easy to implement for the teacher.

5.6.2.7 Summary of professionals' perceptions of the framework

The results show that professionals identified less mathematics in all categories for perceived books than in the other two types of mathematical picture books (see Figure 5.5). This seems reasonable because there is no intent to include mathematics in these books and the mathematics is therefore not as obvious. Figures 5.3 and 5.4 reveal that the professionals identified more potential in embedded books than in the explicit books for four categories:

curriculum content, policies and principles (mean = 3.2 vs 3.1), integration of mathematical content (mean = 3.2 vs 2.9), mathematical problem solving and reasoning (mean = 3.5 vs 3.2), and affordances for mathematical learning (mean = 3.6 vs 3.4). Equal opportunities were found for both types of books for pedagogical implementation (mean = 3.6), while more potential was found in the explicit books than embedded books for two categories: mathematical content (mean = 3.7 vs 3.6) and mathematical meaning (mean = 3.6 vs 3.2). This compares to my finding of more potential in all categories of the embedded books as a group. Since explicit books are designed to teach mathematics, this is an interesting finding, and will be discussed in view of the results of the classroom study in Phase 3.

Scoring in the framework category of mathematical content depended on participants' perception of mathematics, i.e., it is not only numbers, counting, and shapes but occurs in many aspects of everyday living. For example, what time do I need to leave home to get to work on time? or in the case of the book *At the Beach: Postcards from Crabby Spit* (Harvey, 2004), how do you get from the family's campsite to the shop? However, without actually questioning the participants about what they were identifying as mathematics and mathematical opportunities it is difficult to give a clear reason for this. This problem will be further discussed in Chapter 10.

The large range of responses across the seven framework categories also seems to imply that professionals have limited experience of mathematical picture books, hence missing many opportunities to use these books for rich mathematical learning. Without this knowledge and the ability to recognise it in other contexts (e.g., picture books) it is also difficult to see the connections to the curriculum. The importance of authentic, real-life mathematics, quality problem-solving opportunities, and an integrated curriculum has been identified.

Participants also found it more difficult to consider the application of mathematics, or use of the mathematical inclusions as a springboard for rich learning experiences, in the perceived books. The more obvious mathematics in embedded and explicit books could facilitate this. J. Anderson (2009) found that rich mathematical problem-solving activities are not being implemented well in the classroom. Again, more detailed data about the actual type of mathematics and problem-solving opportunities the participants saw, and how they would apply them, is needed to draw any informed conclusions or make appropriate inferences. Nevertheless, without a good understanding of mathematics and ways to implement problem solving and reasoning within the classroom, opportunities to use picture books for quality mathematical learning and as part of an integrated program will be lost. These implications will be further discussed in Chapter 10.

The survey instrument itself may not have allowed participants to demonstrate their full understanding and therefore influenced the scope and depth of the responses to the survey. Although some participants still found the refined framework time consuming, the overall response was positive. Like Van den Heuvel-Panhuizen and Elia's (2012) respondents, they still reported that it opened their eyes both to the opportunities for using picture books for promoting mathematical teaching and learning and to how using the framework enabled them to identify significant components within the books. One teacher reported: "[It] inspires me to think about maths ... [it] made me think a lot more about all of the different types of maths that could potentially be found & used in this book" (T13). Despite the positive comments, further refinement may be required in future research.

Many participants suggested that a list of new books for teacher use would be beneficial. Examples of books were provided in the framework, and may provide a starting point for teachers. However, this new framework was intended to provide professionals with a way of evaluating all types of new books themselves. This would enable them to meet the particular needs of their students as new books became available, which a list would not do.

Finally, most comments made by the participants appear to indicate that the objective of providing a useful set of criteria for selecting and evaluating picture books for mathematical teaching and learning potential has been achieved. The results found in this evaluation of the framework will be further discussed in Phase 3 (within Chapters 7, 8, and 9) where each of the three types of book will be used in the classroom by teachers and students.

5.7 Publications 4 and 5

Publications 4 and 5 provided the first reports of the analysis of this survey to mathematics education research audiences at two conferences in July 2014. They updated the progress of the classification scheme and framework evaluation reported in the earlier MERGA publication, identified as Publication 1. In these publications I used early analysis to report several aspects of the data.

The refereed conference abstract, Publication 4, is reproduced below. The purpose of this report was to highlight the finding that unless teachers themselves have a good understanding of what mathematics might be, they are not going to be able to identify it well in picture books. (This was similar to the comments of the presentation of Publication 3). During the presentation of Publication 4, I used results from three of the seven categories of the framework as examples. These categories were mathematical content; curriculum content, policies and principles; and mathematical problem solving and reasoning.

I concluded that, without this understanding, teachers will not maximise opportunities to connect mathematics effectively to the curriculum and their teaching program, or apply mathematics in problem-solving situations.

Using Picture Books to Implement the Mathematics Curriculum: The Missed Opportunities

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Picture books have been shown to provide opportunities for developing mathematical concepts in young children. Twenty-seven professionals (academics, teachers and preservice teachers) completed 118 evaluations of 36 mathematical picture books for opportunities of mathematical concept development using a seven category likert scale.. This presentation highlights the range of scores in identifying mathematical content, connections to the curriculum and application to problem solving. It appears that without a good understanding of mathematics and ways to implement problem solving within the classroom, opportunities to use picture books for rich mathematical learning experiences are lost.

The refereed conference abstract (Publication 5)[6], was the first presentation about this study to an international audience at the 38th conference of the International Group for the Psychology of Mathematics Education [PME] and the 36th conference of the North American chapter of the Psychology of Mathematics Education [PME-NA]. The intention of this presentation was to provide an overview of the survey data with particular emphasis on the results indicating how my participants used the classification scheme and framework. Two conclusions were highlighted at the conference. The first was that the results appeared to demonstrate that a better understanding of mathematics / mathematics education increases the ability to recognise mathematical content and opportunities for mathematical learning, particularly in real-life situations. The second conclusion was that, regardless of a book's classification, using the framework assisted in identifying mathematical concepts within the books.

[6] Presented by the principal supervisor.

IDENTIFYING OPPORTUNITIES FOR MATHEMATICS LEARNING IN PICTURE BOOKS USING A FRAMEWORK

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Recent research documents the benefits of using picture books to promote mathematics learning (Anderson, Anderson & Shapiro, 2005; Van den Heuvel-Panhuizen and Elia, 2012). Although there are pedagogical approaches for using picture books (Whitin & Whitin, 2009) there are few guidelines to facilitate selection and evaluation of books that may best afford opportunities for mathematics learning. A framework was devised as part of an exploratory descriptive study in three phases to evaluate the role of picture books in facilitating mathematical development in the early years of formal schooling. Three questions guided the study:

1. What potential do picture books (of different types) have for facilitating the development of mathematical concepts in young children?
2. What are authors' intentions when mathematical concepts are incorporated into picture books?
3. How do picture books facilitate children's engagement with mathematical concepts?

Participants (27 novice and experienced educators) completed a written survey which included classifying supplied picture books as having perceived, explicit or embedded content; rating them according to the seven main framework categories; and answering an evaluative question. A total of 118 surveys were completed.

Although there were discrepancies in the classification of the books between participant groups and expert researcher, the results confirmed that there are qualitative differences in types of *mathematical* picture books. It may also appear that a better understanding of mathematics education increases the ability to recognise mathematical content and opportunities for mathematical learning. Participants also found that the framework aided the identification of the mathematical concepts and opportunities for mathematics learning regardless of their classification.

References

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Whitin, D. J. & Whitin, P. (2009) Legs and more Legs. *Teaching Children Mathematics 16* (2), 80-87.

5.8 Postscript

In Phase 1 (Chapters 4 and 5), to aid the selection of appropriate books for the main classroom study, I have first defined three different types of mathematical picture books and confirmed the classifications with a survey of professionals. Second I have also developed and evaluated a framework that facilitates the selection and evaluation of quality mathematical picture books for rich mathematical learning.

I have also demonstrated that mathematical concepts are included in many picture books, and in different ways, making them suitable for use in a classroom mathematics program. This addresses my first research question: What potential do picture books (of different types) have for promoting mathematical teaching and learning for young children? Participants in the survey and the workshop associated with Publication 3 realised the opportunities for the use of picture books in promoting mathematics teaching and learning. However, the problem of identifying mathematics in everyday life, and particularly in picture books, arose. It appears that the more experience in mathematics education, the better that recognition.

It was unclear at this stage of the study if it is actually important to be able to accurately identify the different types of mathematical picture books. This may be confirmed through the author interviews (Phase 2) and the classroom study (Phase 3). However, it is important that professionals realise that there is mathematical potential in different types of books, not just those that overtly display it. One participant commented that "the title of this book and the cover do not necessarily give the clue that it would be such a useful book to teach mathematics" (T5)—as is often the case with books with perceived or embedded content.

I have also shown that a framework is not only needed to aid selection and evaluation but that survey participants recognised the value of it as an organising tool to aid in the identification of books with mathematical learning potential. However, the diverse evaluation across all framework categories for all books indicates that teachers need further support through professional development, discussion with colleagues, and resources from an expert to recognise these mathematical opportunities for classroom use. This will be further discussed in Chapter 10.

The new framework has also gone further than did previous frameworks and lists of criteria. It is more applicable to a range of books with mathematical learning possibilities than are the previous criteria reviewed in Section 2.6 that appeared to address only trade books. It enables any book to be evaluated for its potential for promoting mathematical teaching and learning experiences. The relevance of the categories of the framework to current pedagogy is borne out by the framework of learning-supportive characteristics of picturebooks for learning mathematics devised by Van den Heuvel-Panhuizen and Elia (2012). Significantly, they suggest that further study of their framework now needs to be undertaken with classroom teachers.

This phase has expressly contributed to Research Question1. Further chapters will continue to address Research Question 1. However, the development of the classification scheme and framework also informed the next two phases of the research reported in this thesis, but more specifically facilitated the choice of books representative of each type of mathematical picture books for the classroom study (Phase 3 and Research Question 3). Whether it is the different types of mathematical picture books, or just different books that produce these diverse responses, may be further determined in Phase 3.

In preparing for what subsequently became Phase 3, I realised that an alternative, and valuable, perspective about the classifications could be gleaned through interviewing authors of mathematical picture books to identify their actual intentions. Therefore, Phase 2: *Authors' views and intentions in writing children's picture books*, in Chapter 6, which was not originally planned, evolved. Authors' reasons for writing generally, and for including mathematical ideas specifically, are investigated to address Research Question 2. Although the framework itself is not used specifically for Phase 2, its background and structure provided an underlying influence when I developed the interview questions to be used with authors.

CHAPTER

6

PHASE 2: AUTHORS' VIEWS AND INTENTIONS IN WRITING CHILDREN'S PICTURE BOOKS

Joy for others and for me

(G. Base, personal communication, 3 April 2011)

In this chapter I report on Phase 2 of the research: an analysis of the views, intentions, and perceptions of picture book authors and/or author/illustrators about the role of mathematical concepts in their writing. In doing so, I present the research involving semi-structured interviews held with four award-winning authors. Their picture books contain mathematical content that I classified as belonging to two of the three types of mathematical picture books: perceived and embedded (see Section 1.1.2 for definitions)[1].

A naturalistic classroom study was planned in which small groups of Year 1 children and their teacher were engaged in the shared reading of the three different types of mathematical picture books: perceived, embedded, and explicit. Before commencing the classroom study, the picture books needed to be carefully selected. Many mathematical picture books had already been identified and their classification evaluated through a survey of professionals. This occurred in Phase 1: Chapters 4 and 5. However, I also considered it beneficial to the study to determine whether authors intended to include mathematical concepts because their insights would further validate my definitions and provide valuable information about why and how they did this.

The author-interview data are used to confirm the definitions and classification of mathematical picture books developed in Phase 1. The data are also used to support the selection of mathematical picture books for the classroom study in Phase 3.

This phase of the study is intended to address Research Question 2: What are authors' intentions when writing picture books with potential for mathematical learning? This chapter therefore adds a different perspective to other studies about the role of picture books in

[1] Some correspondence with authors' pre and post the interviews have also been included.

mathematical concept development because in it I explore why and how mathematical content is represented in picture books from the authors' viewpoint.

Publication 6 is included in the chapter as this refereed conference presentation allowed for dissemination of these data to a wide audience. The full analysis and discussion of results is presented in this chapter.

6.1 The role of author and illustrator intent in writing picture books

In Chapter 2, I reviewed some of the research about children's literature and how picture books are structured. In Section 3.1.2, I highlighted theories about the function of the text and visual images, and the interaction between them.

Research about the intent of authors generally, but particularly in children's mathematical picture books, is scarce. The intentions of the author and illustrator can determine not only the content of text and illustrations but also the book's presentation. In Sections 6.1.1 and 6.1.2, I consider both author intent and the relationship between the author and illustrator as a prelude to Phase 2.

6.1.1 Author intent in picture books

Author intent has been described as "recognising that a text is not neutral and that the author writes to convince the reader of a certain viewpoint" (Haydey, Kostiuk, & Phillips, 2007, p. 19). Some research deals specifically with the concept of metafiction[2]. In the case of children's literature, metafiction focuses more on the types of strategies authors can use to help child readers develop various literacy skills. The strategies include ones that try to teach children about textuality, ways for them to become critical readers, and how stories and pictures work (McCallum, 1996; Nodelman, 2000). Flanagan (personal communication, 12 November 2012) stated that "whether authors of children's books intend it or not, children's literature always has an educative or socialising function".

In the present study, what is referred to when the words "author intent" are used is not how the language is used to present a viewpoint, but rather why the authors actually write,

[2] Metafiction is a term given to fictional writing that self-consciously and systematically draws attention to itself as an artifact in order to pose questions about the relationship between fiction and reality (Waugh, 1984).

and more particularly how and why they include mathematical ideas and language in their writing. It is recognised that some of the books used throughout the study do include "messages" i.e., that the author has a particular reason for writing. For example, in *Uno's Garden* (Base, 2006), as highlighted throughout this chapter and in Section 7.3, there is an environmental message. In *Ten Little Fingers and Ten Little Toes* (Fox, 2008), children hear that no matter where in the world babies are born, they are all essentially the same physically. These messages are the main purpose of the books and may provide opportunities for integration to explore the literary techniques and the social issue being raised. However, both books also include the setting for the additional layer of embedded mathematics that can be explored.

On his website, Australian writer and illustrator of picture books, Shaun Tan includes an essay he wrote for an unnamed conference. He described why he writes, illustrates, and interrelates the text and images in his books, but says that if he stopped to think why he wrote and illustrated, it would actually distract him from his creativity. He believes "the artists' responsibility lies first and foremost with the work itself, trusting that it will invite the attention of others by the force of its conviction" (Tan, 2004).

6.1.2 The author-illustrator relationship

Additional to their literature about the role of images in picture books (Section 3.1.2), Anstey and Bull (2000) discussed the process of the writing, illustration, and publication of picture books. It is most common that, once writers have their manuscript accepted by a publisher, they have very little contact with the illustrator and designer of the final picture book. In fact, it is the policy of many publishers to keep writer and illustrator apart (Gleeson, personal communication, 3 February 2011). This, of course, does not occur when the writer and illustrator are one and the same. There are some instances where there is collaboration from the inception of the book between the writer and illustrator, for example, Gary Crew and Gregory Rogers in the books *Tracks* (Crew, 1992) and *Lucy's Bay* (Crew, 1992). Libby Gleeson[3] is an established and awarded writer who chooses her own illustrators, with whom she works closely. Anstey and Bull (2000) argued that such collaboration produces a different type of picture book and believe that in these cases "the illustrative text constructs discourses of its own which take the narrative in the written text in new but related directions" (p. 173). Therefore, collaboratively-written books "push the boundaries of picture books in new

[3] A full description of Libby's Gleeson and her work is provided in Appendix E.

directions" (p. 173). In books with little or no collaboration, the illustrations are more likely to mirror the written text as, for example, in *Minnie's Diner: A Multiplying Menu* (Dodds, 2004).

A different angle to author-illustrator collaboration is taken by Martinez and Harmon (2012). They investigated whether the different elements of a book (e.g., plot and characterisation) are developed differently if the same person is both author and illustrator, or if the author and illustrator are different people. Although not important to this study, they found that the only difference was in books for younger children where the "pictures played a greater role in developing character when the book was written and illustrated by the same person" (p. 336).

The following sections describe the interview process and reveal the opinions and perceptions of four authors about why and how they write, and particularly why and how they include mathematical concepts in their books.

6.2 Background to the interviewees and interviews

In the classification of picture books in Phase 1, five prominent children's picture book authors had been recognised as consistently including perceived or embedded mathematical concepts (at different levels) in their books. Two writers of mathematically-explicit books had also been specifically identified.

Originally, my intention was to contact these seven authors to make preliminary enquiries about their writing intentions. They were therefore sent an individualised letter through their publisher or an email if an email address was provided on their website. See Appendix E for an example of the letter/email that first explained the purpose of the study. I also enquired whether:

- their intention was solely to entertain young children through their writing,
- the opportunities for mathematics learning (seen by teachers) were incidental or intended/deliberate,
- mathematics was an important consideration in their writing and visual images, and
- the interaction between the text and visual images was important.

All five authors of books with perceived or embedded content replied, and their responses were very insightful; no response was received from the two writers of books identified as containing explicit mathematical content. Four of the authors who responded showed genuine interest in the study, so it was deemed opportune to explore these four authors' intentions and perceptions further through personal interviews. This provided a new perspective to support the broader aims of my research. Not only would the interviews address a new research question (Research Question 2), but they would further assist in addressing Research Questions 1 and 3. Therefore, these interviews were integrated into the study as Phase 2 and the classroom study became Phase 3. Macquarie University ethics approval was sought and approved for the follow-up semiformal personal or phone interviews (Appendix A). Information and consent forms (see Appendix D) were also obtained from the authors according to Macquarie University regulations.

6.3 Methods

This section contains information about the methods that were used in this phase.

6.3.1 Participants

The four authors who agreed[4] to be interviewed for Phase 2 were Graeme Base (GB), Libby Gleeson (LG), Roland Harvey (RH), and Alison Lester (AL)[5]. All of these authors except Libby Gleeson also illustrate most of their own books. Roland Harvey and Alison Lester sometimes also collaborate on books, both together and with other authors.

6.3.2 Approach

This component of the study used qualitative research methods to collect and interpret the data. Qualitative research is beneficial here because it aligns with the methodological approach explained in Section 3.3 and taken through this thesis. Qualitative research can be "multimethod in focus, involving an interpretive, naturalistic approach to its subject matter" (Denzin & Lincoln, 1998, p. 3). Researchers can then "deploy a wide range of interconnected methods, hoping always to get a better fix on the subject at hand" (Krathwohl, 1998, p. 232). A "qualitative hypothesis-generating research" (Auerbach & Silverstein, 2003. p. 8) design was used "to gather descriptive data in the subjects' own words" (Bogdan & Biklen, 2007, p. 103) and "develop hypotheses by listening to what the research participants say" (Auerbach & Silverstein, 2003, p. 7). As there are no other similar studies to guide my approach, I decided

[4] All authors gave permission to have quotations from the interview transcript used and acknowledged.
[5] These initials will be used to reference author comments throughout this chapter.

to use grounded theory techniques, a type of qualitative method because they are particularly useful where little research is previously available.

Grounded theory originally emerged from sociologists Glaser and Strauss (1967). It "derives its name from the fact that theoretical coding allows you to ground your hypotheses in what your research participants say" (Auerbach & Silverstein, 2003, p. 7) and "has its roots in *pragmatism*" (Denscombe, 2010, p. 109, italics in original). Grounded theory methods "consist of systematic inductive guidelines for collecting and analysing data to build middle-range theoretical frameworks that explain the collected data" (Charmaz, 2006, p. 509) and use "two basic principles: (1) questioning rather than measuring, and (2) generating hypotheses using theoretical coding" (Auerbach & Silverstein, 2003, p. 7).

The grounded theory process also allows the researcher to take expert participants through a series of questions in a narrative interview (Auerbach & Silverstein, 2003), so was also ideal for this study. The prescribed and systematic coding process of the grounded theory approach of Corbin and Strauss (1990) that includes themes, repeating ideas, relevant text, and raw text in the reporting was particularly suitable for analysing the interviews.

6.3.3 Interview schedule

A list of 12 core interview questions that addressed the purpose of the study was compiled. They were based on:

- the authors' general writing intentions;
- the authors' reasons for including mathematical concepts;
- the definitions of mathematical picture books (to confirm whether the authors classified mathematical picture books in a similar way, and to ascertain the authors' classification of their own books);
- the authors' ideas about the role of visual images in picture books (to determine whether there was a similarity between the literature and the beliefs and practice of authors and illustrators), and
- relevant elements of the framework.

The core questions asked of each author are listed in Table 6.1.

Table 6.1

Common Interview Questions

IQ	Question
1	What is your intention in your writing?
2	What do you see as the role of the text?
3	What do you see as the role of the illustrations / visual images?
4	Do you see the text, or the illustrations, as the most important component of the book?
5	How important is the interaction of the text and illustrations to you?
6	What level of interaction (symmetry through to contradiction) do you expect between the text and illustrations?
7	Do you have preconceived ideas of the illustrations? Do you talk with the illustrator about your overall expectations/intentions of the final product? OR Is this easier to achieve being both writer and illustrator?
8	Using my definition of different types of mathematics picture books, do you classify your books as explicit in their mathematical content or having the mathematics embedded?
9	How do you decide what mathematics you are going to include, whether you are going to include more than one mathematical concept, and the level of the mathematics content?
10	Is it your intention for teachers to use the books to develop ideas/concepts beyond the pure enjoyment of the books?
11	Is this important to you?
12	Tell me about how you see some of your individual books being used by children, parents, and teachers?

I then compiled an account of each of the authors and their work, recorded in Appendix E, following:

- reference to literature about them and their writing;
- a review of information on their individual websites, and
- a content analysis of each author's books.

Subsequently, additional questions were devised that were suitable for each author. These were inserted among the core questions at the appropriate point in each interview schedule (see Appendix E). The method of reviewing information about each author to "suggest important topics" and taking "the research participants through their history with the phenomenon in question" used the narrative interview procedure advocated by Auerbach and Silverstein (2003, p. 16).

I considered it important to allow opportunities for the participants to suggest or introduce other topics throughout the interviews and for further questions to arise during the course of the interviews. Therefore, each interview was to be semi-structured. Auerbach and Silverstein (2003) note that the questions themselves are not important as long as they allow the participants to talk about what matters to them, then "what they say will shed light on your research concerns" (p. 16).

6.3.4 Interview procedures

The list of relevant questions was sent to each author prior to the interview to enable them time to consider their responses. I conducted all interviews. Libby Gleeson and Alison Lester were interviewed in person and Graeme Base and Roland Harvey by telephone as they both lived in another Australian state to me. The interviews were held between February and August 2011 and ranged in length from 30 minutes to nearly an hour, depending on the communicative disposition of the interviewee. Each interview was audio recorded, with the consent of the interviewee, on a Sony IC recorder. This allowed me to concentrate on the discussion. Not all interview questions were asked specifically (as they may have already been addressed previously during the interview), and some questions were not asked in the anticipated sequence if it appeared appropriate to ask them earlier in the interview.

6.4 Analysis and identification of the themes

6.4.1 Analysis process

The recordings were transcribed by an independent transcribing service in October 2011. Qualitative research methods were then used to analyse and interpret the "interviews in order to discover meaningful patterns descriptive of a particular phenomenon" (Auerbach & Silverstein, 2003, p. 3). According to Auerbach and Silverstein (2003), "there is no right way to interpret the data/interview transcripts" (p. 32), so in order to provide the widest perspective of the authors' comments and make the most accurate conclusions, the interview data were interpreted in several ways.

As it was found that the questions and discussions had not necessarily progressed in the sequence of the interview schedule, the transcripts were reviewed and each response was categorised according to the question number. The answers for each author to each question are recorded in Appendix E.

To ascertain relevance to the study, the questions and their responses were also assessed according to how they aided in answering the research questions and their relevance to the framework categories (see Appendix E).

On reviewing these analyses of the data, it became apparent that the authors' responses and the questions themselves could be grouped into several themes. A grounded theory method, where recurring ideas, themes, and relevant text are identified in data (in this case interview transcripts) to address the research question was also used.

6.4.2 Theme development

I analysed the transcripts, line by line, for recurring ideas in the data. Nine main ideas or groups of ideas were identified initially. Table 6.2 lists these and their frequency, indicating the number of times the idea arose across all four interviews rather than the amount of time spent by each author discussing the idea.

These recurring ideas were reviewed to develop themes. In some cases the ideas were simply renamed as a theme. For example, *Text and visual images (role and interaction)* became *Organisational structure*. In the case of *Author intent*, where the idea appeared to have two distinct parts, the category was separated into the discrete and important themes of *Author general intent* and *Mathematical intent.* Alternatively, where the content of two ideas overlapped they were combined, e.g., *Picture book structure* and *Patterns in text, visual images, and format* became *Conceptual structure.* The theme *Stories in our lives* was included within the theme *Author general intent* where it seemed most suitable.

Table 6.2

Frequency of Recurring Ideas

Recurring idea	No. of occurrences
Text and visual images (role and interaction)	13
Author intent	11
Framework categories or elements	10
Embedding ideas in picture books	9
Picture book structure	8
Embedding mathematics in text and visual images	7
Patterns in text, visual images, and format	4
Using picture books in mathematics learning	3
Stories in our lives	1

Table 6.3 contains the resultant themes that evolved. It also demonstrates how each theme is related to the interview questions, research questions, and framework categories. For example, the theme *Organisational structure* (how these authors viewed the role of the text and visual images) was derived by the analysis of responses to Questions 2–7. The discussion of this theme also contributed to answering Research Questions 1 and 2 and had particular relevance to the framework categories of mathematical content, mathematical meaning, and affordances for mathematical learning. These relationships will be included in the discussions in Sections 6.4 and 6.5.

I decided to report the interview data according to these themes using theoretical narrative (Auerbach & Silverstein, 2003, p. 41). To support the findings and validate the analysis, the actual words of the interviewees (relevant text) are featured in the discussion. Relevance to the research questions is also noted in the discussion.

Table 6.3

Themes and Their Relationship to the Research Questions and Framework

	Theme	Core interview question	Research question	Framework categories[a]
1	General author intent	1	2	
2	Mathematical intent	8, 9, 10	1, 2, 3	MC, CPP, MM, PI, AML
3	Organisational structure	2, 3, 4, 5, 6, 7	1, 2, 3	MC, MM, AML
4	Conceptual structure	7	1, 2, 3	MC, AML, PI
5	Embedment[6] within picture books	1	1, 2	
6	Mathematics embedment within picture books	8, 9	1, 2, 3	MC, MM, IMC MPS, AML, PI
7	Framework links	10, 12	2	MC, CPP, PI
8	Using picture books for mathematics learning	10, 11, 12	1, 3	

[a] Key: MC = *Mathematical content*; CPP = *Curriculum content, policies and principles*; IMC = *Integration of mathematics content*; MM = *Mathematical* meaning; MPS = *Mathematical problem solving and* reasoning; AML = *Affordances for mathematical learning*; PI = *Pedagogical implementation.*

The themes of *Mathematical intent, Embedment within picture books,* and *Mathematics embedment within picture books* may appear to overlap. However, *Mathematical intent* relates specifically to the reasons authors gave for including mathematics in their books; *Embedment within picture books* relates specifically to the authors' general views on embedding material other than mathematical content in their books[7], whether they think it a positive or responsible strategy, and, if so, how they put it into effect; and *Mathematics embedment within picture books* focuses on how the authors specifically embed mathematical content.

[6] *Embedment* is the noun form of "embed …verb … to fix firmly in a surrounding mass" https://www.macquariedictionary.com.au/features/word/search/?word=embedment&search_word_type=Dictionary

[7] For example, Base discussed how he shows, through his illustrations, that frogs are the first thing to disappear when water becomes scarce.

6.5 Results and discussion: Analysis of themes

6.5.1 General author intent

The four authors each had different reasons for writing, some of which were more defined than were others. Analogous to the definition of a picture book by Bader (1976) noted in Section 1.1.1.2, Base and Gleeson identified the picture book as an art form and discussed their personal satisfaction when creating it. According to Libby Gleeson, "It's both exciting, as a practitioner, to do it but I find it very satisfying as an art form. A beautiful picture book, to me, is one of the most … it's a wonderful way to tell a story." More specifically, she emphasised that there is no pedagogical reason for writing, saying, "Occasionally I'm just simply hooked by an idea and I want to explore that idea and see what can come of that". However, Graeme Base spoke simply of his intent as "joy for others and for me".

Alison Lester did not give a precise reason for writing, saying, "It is different for different books". In contrast, Roland Harvey explained that when he began writing it was for a very definite reason: "It was to excite the kids or make them more interested in history and show some relevance of different events." This connects strongly with the general and mathematical embedment themes that are discussed below. He admitted that he had become even "more directed" lately to embedding: "There's more intent to embed information but also I am increasingly interested in doing specifically information books, but make them really interesting because there's so much richness there."

The importance of story in our lives (as described in Section 2.2) was introduced by Gleeson: "I'm a great believer that stories are absolutely fundamental and that we are hardwired for it", thus providing another reason for her writing.

One issue regarding writing intentions was raised by Base who recounted how some publishers want authors to clearly reveal their purposes for each book to the reader.

> Sometimes I have fights with my publishers, more often the American publishers than the Australian, where the concern is that they want to make sure that the kid understands exactly what the intention is and what has happened, whereas I quite like—I liked it as a kid and I think everyone likes—the idea of getting to the end with a couple of "What? How did that come about?" or "What was quite going on there?" And then they go, "Oh I get it, I see now". I don't want to necessarily lay it all out on the table at the beginning. I like there to be surprises and things which you do have to work a little bit at to fully understand when the book is over.

The analysis of these responses provided insight into the authors' intent in writing: for pleasure for both themselves and their readers, motivating children to engage in books even before they can read, and to impart knowledge in an interesting way. This discussion proved effective as a precursor to the questions about their intent to include mathematical ideas.

6.5.2 Mathematical intent

The responses revealed that three authors had no intention to "teach" mathematics through the inclusion of mathematics in their writing. This is exemplified in the following statements: "I'm not a teacher" (GB); "They're certainly not to teach maths" (LG); and "I don't think mathematically" (AL). Harvey was the exception. Having previously acknowledged his intention to embed history, he added: "I can say it is my intention to teach etc., but as I am no mathematician I really wouldn't know whether I succeed or not."

Base did acknowledge the purposeful inclusion of mathematics in some of his books; there is even an appendix or endnote explaining the different mathematical "games" in *Uno's Garden* (Base, 2006).[8] However, he stated that these inclusions are intended to add layers and depth to his writing: "Maths in itself is not a message; it was supporting my message" and "is presented really as a game."

However, despite recognising that the structure of *The Waterhole* (Base, 2001) is like a counting book, Base did not want his books to be seen as mathematics books. When it was published in the USA, he strongly opposed the publisher's desire to attach a sticker calling it a "fun math counting book":

> It's not what I'm about; if the book had been about teaching maths, I mean firstly I wouldn't have written it. It wouldn't have been something that was in my soul to create and to put those years and creativity into, because that wasn't important to me. … This is not a maths book. It's not primarily there to do that job.

Gleeson and Lester were both quite surprised when I showed them many of the opportunities for mathematical concept development in their books. However, after some discussion they both commented about how they saw the mathematics as being simple or underlying mathematics around them in their everyday lives. Libby Gleeson asserted that:

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[8] A full description of *Uno's Garden* (Base, 2006) is provided in Chapter 7.

> We aren't talking calculus here. … I find it quite fascinating, the conversation, because when you said there were mathematical concepts I'm thinking, "No, there aren't. I don't put maths in my books." Of course I hadn't thought of it in terms of the conceptual work for very young kids. Frankly, what it suggests to me is that it looks like maths concepts are embedded in our everyday lives because these texts are … Clancy and Millie are not totally realistic in the final phases of them but they're very much rooted in the life experience of small children.

While Gleeson explained that she sees only the mathematical positional language in her books (e.g., under, over, beside, between) as "prepositions", Allen wrote, "You term them mathematical but I think of them as physical". Lester and Harvey commented that they did not see the use of positional language or mathematical contexts as "mathematical" but as a means of giving information in a different way.

I had previously quoted Styles and Arizpe (2001) in Section 2.6.2 who had said of the illustrations that "there are some things that can only be said visually" (p. 278). Harvey expounded on this saying; "If you try to describe a cross section verbally, I don't know if I can do it even, but it's much easier to see what sits on what and what's inside there". Lester's explanation is that:

> When I'm doing a book, I always try to think of the various ways I can make it more interesting. So, doing things like aerial views and cross sections and maps to me are just a natural extension of telling a story ... because it needs to be there as part of the story.

Figure 6.1 provides an example of a cross section and map, the visual form being used by Lester to provide more information for the reader in a clearer way than text would.

Harvey recognised that he included mathematical content as fine details in the illustrations as part of "the big picture" of embedding content (see Section 6.5.5).

> Maths/science is most important to me, but not necessarily in its pure form. I would say that when I see an opportunity, I'll tease out some bits of information not knowing whether a reader will be able to follow but hoping to arouse some extra interest and understanding.

Figure 6.1. Example of a cross section and map used in *Are We There Yet?* (Lester, 2004).

This theme of *Mathematical intent* contributed to answering Research Question 1 as it supports my proposition that mathematical ideas or content can be included in different ways in picture books even though the authors have no intention of teaching maths. The theme also reveals authors' reasons for including mathematics in their writing (Research Question 2) and suggested some ways that picture books can engage children mathematically through the authentic settings of both stories and their contents (Research Question 3).

6.5.3 Organisational structure

This theme emerged from the recurring idea of the role of both the text and visual images and their interaction. The authors all had strong opinions about these.

Gleeson responded that the text is "the main driver of the narrative" and that "if the text can exist on its own then it's not a picture book text but a text of a short story or something". This is very similar to the stance taken by researchers and book organisations when defining picture books, as described in Chapter 1.

Lester talked about writing "as few words as possible … in the most beautiful way". "Sometimes there'll be text that's illustrated", as demonstrated in her series identified in

Appendix E, Table E.5, Group 1, but “other times there’ll be illustration that just stands on its own”. On the other hand, Base, Gleeson, and Harvey believe that the role of the illustrations is only to add to the story, not to repeat what is already in the text. Base emphasised this: “Well, I think the ideal which anyone would aspire to was one where the text and the art are perfectly complementary but don’t overlap into each other’s territory”. Base provided the example of a boy in a jumper: The text can tell the reader that the jumper is warm, but the illustrations will indicate it’s blue. This concurs with Lewis’s (2001) views on the relationship between text and image (Section 2.2.2).

Sometimes, as stated above, the illustrations are used to depict what is too hard to express as text. However, Allen suggested that in many cases the child cannot read the text so “all the clues to understanding can be found in the pictures”. Gleeson extended this further and reinforced another important role of illustrations as the hidden agenda of teaching visual literacy, which was discussed in Section 2.2.1: “I think visual literacy is so important—to teach kids to read images so they don’t just look at the written text but they also look at the images and understand.”

All authors agreed that the text and the illustrations could not be separated. Gleeson, the only author interviewed who does not illustrate her own books, expressed this clearly: “I don’t know that you can separate it completely. … The illustrator is not there to decorate the text. They are there to make the other half of the narrative. It’s very hard to separate out any specific purpose.” This delineation of roles of text and illustrations is probably even harder for author/illustrators because it is an intertwined or “intermarried” (AL) creation, where they write the text consciously (GB), knowing that they can and will add particular detail in the images or reduce the text later to relate the illustrations and text most effectively.

With reference to the importance of the interaction of text and visual images, Harvey had written in his initial correspondence: “That would be a 10 for importance, but in addition I think it is probably better that the reader is left puzzled to some extent that [sic] feel patronised by over-simplification”. During the interview he added:

> It is absolutely totally important because each one … you can do an illustration and the text can be saying something else and you need to reconcile and that makes you think about it all. So the text can give structure or the illustration can provide the structure I think.

Lester said that she did it "in quite an unconscious way, really", while Base contended that the interaction of text should be complementary, as "contradiction I think is hopefully to be avoided. Although, but I like ambiguity and I like questions." On the other hand, Gleeson felt that there is a place for contradiction: "Often [there] is that contradictory/complementary thing where the text might say 'A and B' but there might be a whole 'C and D' part of the story which the images provide."

In their deliberations, both Harvey and Base concluded that the illustrations are the most important component of the book, with Harvey saying:

> The purpose of the artwork is fundamental, essential, isn't it? … I think in the end it's the illustrations that really uplift and capture the imagination of the end user of the book, the reader or the viewer of the book.

The insights revealed by this theme exemplified first the critical individual roles of the text and the visual images in picture books, and second their important interrelationship. The text may relate the story but the illustrations have the potential to enhance the enjoyment and understanding of the story. Without them it would not be a *picture* book. Combined, they create a complete whole.

These responses provide some answers regarding author intentions (Research Question 2), but they also suggest the potential for using the text and illustrations to engage readers and facilitate mathematical development in young children (Research Questions 1 and 3).

6.5.4 Conceptual structure

The theme *Conceptual structure,* which describes how the authors and author/illustrators determine the physical structure or design of their books, arose spontaneously when discussing the role of the text and illustrations discussed in the previous section. All authors, but most particularly the author/illustrators, Base, Harvey, and Lester, appear to have their own distinct "pattern" or way of writing and illustrating their books within the 36 page parameter for all picture books. This design or structure of their books is often mathematically based. These writing, illustrating, or design patterns had become evident during the content analysis of the four authors' books. In Appendix E each author's books are grouped according to the topic, style of book, or intended audience, demonstrating recurring similarities or patterns in these.

The three author/illustrators admitted that they are “visually driven” (GB), but Base continued to explain that it is not possible to start with the illustrations and then construct a story around it.

> It has to be the story first and getting the words on paper and figuring out what’s going to be unfolded on each page. … Of course as an artist, I’ve always got an eye to what it’s going to look like and how I’m going to draw it.

However, because Gleeson saw a separation between text and illustrations, she pointed out that she rarely tells her illustrators what to do as they are expert in their field “unless there’s a deliberate part of the story that I think will come through much better in the images than in the text”. Harvey’s process is to start with the “big picture context” and then he adds the details after. In contrast, Lester said that for her the general design is the most important component where she starts with sketches but “the lay of it, the look of it. ... and to me, that, that's probably a real marriage of the two of them”.

Base also conceded that he likes to have

> it all under one roof, one cranial roof. … There’s also a joy for me in creation and being able to have control over both sides of that equation—of the art and of the writing the words, is a source of immense satisfaction, creative satisfaction.

Base also appeared to be aware of the structure in his books. First, his artistic style is similar in all of his books. He always includes or hides objects and ideas within the complex visual images as one way of adding layers, not necessarily mathematical, as previously described.

> [It] is my love for my detail and rewarding careful study, the fact that there’s 10 ... in a page where it’s about 10, there’s also 10 berries on the vines and 10 leaves on the trees and it goes deeper and deeper and deeper.

Base frequently incorporates patterns within his images. For example, as shown in Figure 6.2, there is intricate patterning in such places as the bridge structure and the spirals of the gondolopes’ horns.

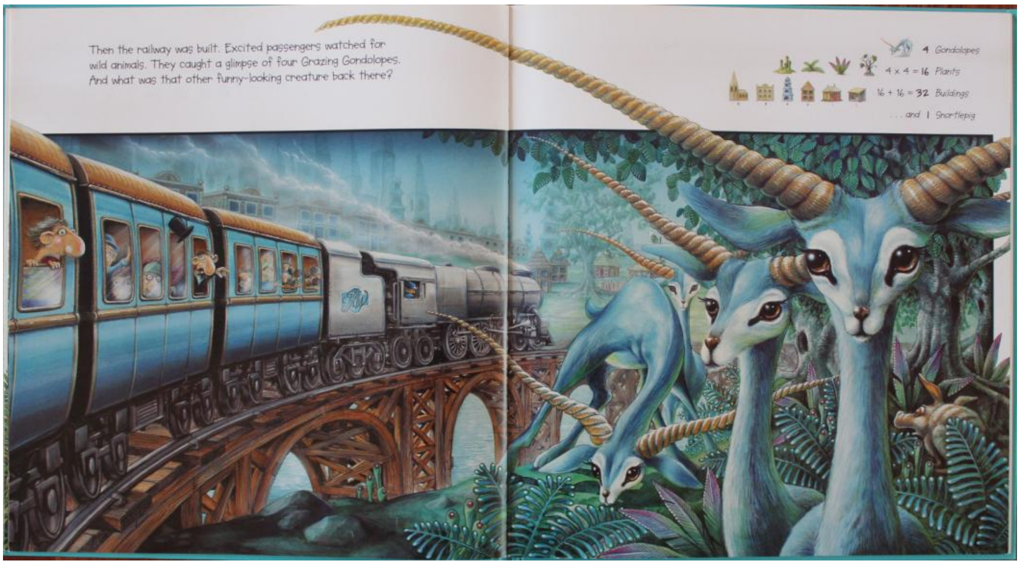


Figure 6.2. Double page spread from *Uno's Garden* (Base, 2006).

Additionally he uses borders around his pages, often with patterns within them, or codes (that are based on patterns) for deciphering and solving. Figure 6.3 is a snapshot of a repetitive border pattern in *The Eleventh Hour: A Curious Mystery* (Base, 1988, unpaged). Here there is a simple repetition using the four card suits around the text, while a more complex four-part pattern is used on the left and right borders of the page. (The other cards on the top and bottom of the page contain hidden clues to solving the book's mystery).

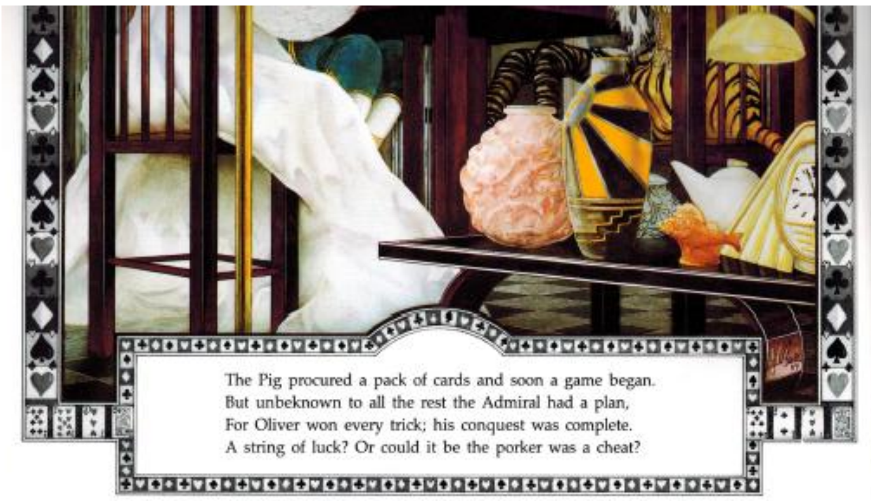


Figure 6.3. Repetitive border pattern in *The Eleventh Hour* (Base, 1988).

Base often writes in verse/rhyme (e.g., in *My Grandma lives in Gooligulch* [1983], at the bottom of Figure 6.3, and in the introductory rhyme to *Uno's Garden* [Base, 2006]) which he recognised has its own mathematical patterns.

> When people say, "Why?" I say, "Because it's easier, it's easier than writing prose", because there is a structure, you've got a certain length line with a certain number of beats in it and a certain number of lines in the verse. It keeps you focused and I find that that discipline is actually an aid to writing, 'cause you know when to stop.

Number patterning, represented both in numerals and visual images also occurs in some of his books. For example, as seen in Figure 6.2, each double-page spread includes several number sentences. These also form number patterns throughout the book. For example, in the first half of the book, the animals decrease by ones, the plants decrease using a square numbers pattern, and the buildings increase using doubling. Figure 6.4 shows a closer snapshot from Figure 6.2 and these number patterns.

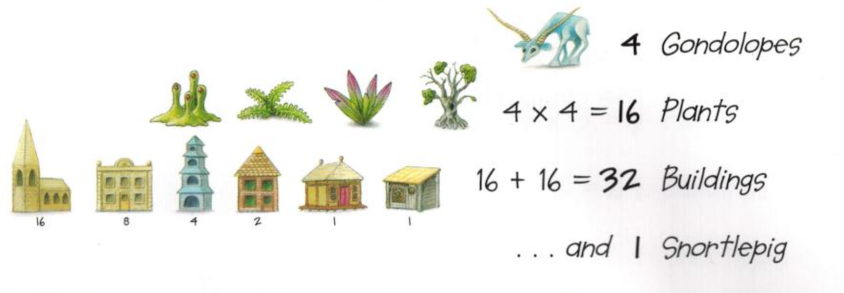


Figure 6.4. Examples of number sentences found on each double page spread from *Uno's Garden* (Base, 2006).

When asked about the recurrence of the number 11 in some of his books he commented that in the case of *"The Eleventh Hour,* it just sounded like a great mystery book title", but in *The Waterhole* (Base, 2001), where the combination of numbers on each double spread comes to 11, it is intended:

> What's appearing and what's disappearing tends to be eleven, but it's really a counting book from one to ten, and one to ten because it is the way that counting books tend to fall in the end. … I think I probably could say that if I did have a lucky number, and I don't, I think there's a very good chance it would be 11. Because I agree with you, I am fond of the number. I couldn't tell you quite why, but yeah it's a goody.

His comment that "counting books tend to always only go to 10" showed how he recognised this pattern within counting books. However, it is interesting that he referred to *The Waterhole* (Base, 2001) here as a counting book whereas in Section 6.4.2 he said the counting aspect is the tool for his message, not the purpose of the book.

Roland Harvey uses the same illustrative style in all of his work, creating a similar and distinctive presentation throughout his books. Many of his books are written as parts of a series that have connected themes, e.g., *In the Bush: Our Holiday at Wombat Flat* (Harvey, 2005) and *In the City: Our Scrapbook of Souvenirs* (Harvey, 2007). See Appendix E, Table E.3, Group 1 for more examples of groups. It appears that this is a formula that works for him. Figure 6.5 is a sample page from his book *In the Bush: Our Holiday at Wombat Flat*. This example depicts the intricate drawings common to all of his books and the inclusion of mathematical ideas. It is akin to the example given for Lester (Figure 6.1), this time showing a cross section of a hut and a diagram/depiction of the asterism (star group) of Orion's Belt and the Dog Star which would be difficult to explain in words in the previous section.

Figure 6.5. Example of a page from *In the Bush: Our Holiday at Wombat Flat* (Harvey, 2005).

Lester's writing and illustrative presentation is different again, and quite intentional. She uses the same structure and techniques repeatedly, commenting that "I think probably for me the most important [sic] for the book is the general design of it: the lay of it, the look of it".

One example that demonstrates patterns at several levels is Lester's five-book series that revolves around seven children and their daily activities. First, the covers of all these books, including *Clive Eats Alligators* (Lester, 1985) and *Rosie Sips Spiders* (Lester, 1988), have the same central oval structure (as seen in Figure 6.6). This design is not exclusive to this series, as she also uses it both for the covers for several other books and within the pages.

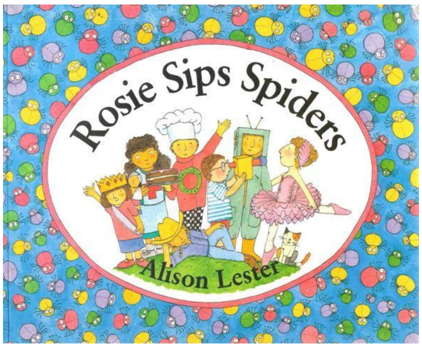


Figure 6.6. Cover of *Rosie Sips Spiders* (Lester, 1988).

Second, each set of four pages (two double-page spreads) form a pattern. Each alternate double-page spread has six of the children, for example, "doing their favourite activity". Figure 6.7 is an example of the "block" picture and text used for each child. Lester described this as:

> just like little images with a caption below them. … Like they're almost laid out like a photo album. I just did it because I liked the look of it. But, I think little kids love them, because they are so simple and easy to understand.

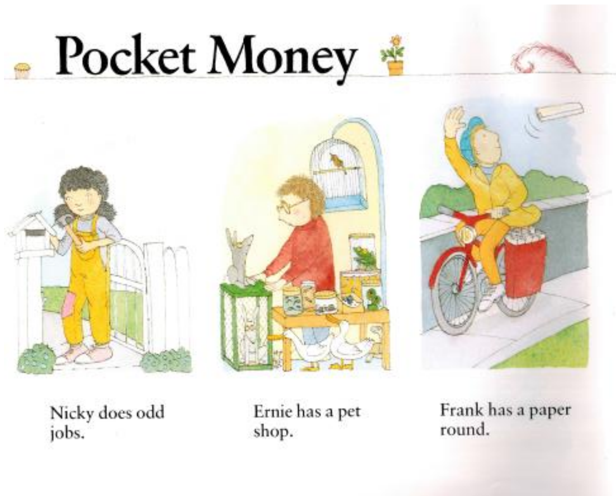


Figure 6.7. Example of the "album" structure in *Tessa Snaps Snakes* (Lester, 1990).

Additionally, across the top of this double-page spread there are always seven illustrated objects: six associated with the children on that page and a seventh to give a clue about what the seventh child may be doing on the next double page. This is demonstrated in Figure 6.7.

Next, the third page highlights items associated with the seventh child's activity, again using the same oval page design as the cover. This illustration sometimes, but not always, includes patterning. Figure 6.8 provides an example of a mathematical cyclic repetition (as described by Papic, 2007) and a symmetrical pattern from *Ernie Dances to the Didgeridoo* (Lester, 2000). The second page of this double-page spread shows the child involved in his/her activity.

Finally, the back cover repeats the oval design inserted with items from the book.

Figure 6.8. Example of a page with patterning in the second pair of spreads in many of Lester's books.

Lester did not see these repetitive design structures as mathematical. "It's really just how I've drawn it so that it looks OK. So there's no logic to it at all other than … I've laid the objects out so that they look nice around the text." However she used mathematical language to describe her method.

> I wish I could say there was; other than, that I tend to like things to be neat and symmetrical. You know, I, I wish I wasn't like that, but I am like that with my work, that I like it all to be sort of tucked in.

However, Lester stated that repetition of her book designs can also motivate children to return to the book, identifying that she does have a sense of pattern but does not realise it is mathematical:

> I've heard, many people have told me that kids have, have memorised, memorised those little captions before they can read, and so they can actually pretend they can read before they actually can, and it gives them great confidence to go further.

This seven-friend story and illustrative design pattern is also used in *Ernie Dances to the Didgeridoo* (Lester, 2000), with one of the original seven friends (Ernie) engaging with six new Australian Indigenous children. Appendix E contains examples of other groups of similar books by Lester.

There is not the same recurrence of design (text and illustrations) in some of Libby Gleeson's books. However, recurrence does happen, particularly when she uses the same illustrator. For instance, when she showed her latest unreleased book[9] to me, I commented that the theme and the illustrative style were very similar to a previous book, *The Great Bear* (Gleeson, 1999). She responded:

> Yes, it's interesting that you say that because somebody said to me, "Oh, this is a completely different book from anything you've ever done before". I said, "No, it's *The Great Bear* all over again." … The theme, well, the content and ideas behind it are very similar.

Another example is the parallel in the storyline and illustrative style of *Amy and Louis* (Gleeson, 2006) and *Clancy and Millie and the Very Fine House* (Gleeson, 2009a). These two books have similar themes (a child moving house), very similar artwork style by the same illustrator (but not the same as in the previous example), and similar incidental perceived mathematical concepts in both text and visual images (e.g., positional words, 3D space, and measurement).

It then appears that not only the actual content of the text and the illustrations, but the way these are presented may have the potential to engage the reader in a general sense (Research Question 3) and may promote mathematics teaching and learning (Research Question 1). With regard to Research Question 2, this theme continues to reinforce how authors include their beliefs and intentions (consciously or not) in their writing.

[9] Subsequently available as *I am Thomas* (Gleeson, 2011).

6.5.5 Embedment within picture books

All authors agreed that, to some extent, they do embed information about various topics and values within many of their books—Gleeson, Harvey, and Lester having also written non-fiction picture books. Harvey explained that he “realised that that’s sort of [the] way you can get a lot of information over and help with the understanding of things if there’s a lot of information presented in an interesting way”. This section, although not pertaining to the inclusion of mathematics, is relevant in that it first shows how authors do embed “information” of different kinds but also the methods that authors use.

Harvey’s earliest work was illustrating books about events in Australian history (e.g., *The Crossing of the Blue Mountains* [Boardman, 1984]). See Appendix E, Table E.3, Group 4. His latest series is about the different environments around Australia (e.g., *To the Top End: Our Trip Across Australia,* [Harvey, 2009]). Here he includes information about that environment, but set in a fictional story style, thus embedding information in “an interesting way” (RH).

Similarly, Lester has deliberately embedded factual information (history, science, and geography) in books such as *Ernie Dances to the Didgeridoo* (Lester, 2000) where she included details about living in the Northern Territory of Australia, and *Are We There Yet? A Journey Around Australia* (Lester, 2004). More recently, she has written some informational books about personal trips in an interesting storybook format, e.g., *One Small Island: The Story of Macquarie Island* (Lester & Tulloch, 2011).

Gleeson has also written several books with embedded information or hidden agendas (see Appendix E, Table E.2, Group 2), one of which is *The Rum Rebellion: The Diary of David Bellamy* (Gleeson, 2001) as part of the *My Story* series and *A Simply Spectacular Hat* (Gleeson, 2009b) for the Cancer Council about sun safety for young children. According to Gleeson, the latter is “very different to my normal work”. When I asked, “So it’s embedded on purpose? It’s almost explicit”, she responded, “Yes, just a slight step back from that”.

Base agreed that he often uses his books (text and illustrations) to convey a message, which is frequently environmental. “I am a human being, I am a father, I’m fifty something now and I’ve got things to say and that’s why the books have the messages that they do.” Base and Gleeson also see a bigger picture of embedding content and values. Gleeson agreed that she is embedding spelling and literacy (and visual literacy) in her books: “Yes, but only in the sense that I consider myself a highly literate person”. She then continued:

> I think you're also embedding a whole moral universe, your values soon come through … which is not to say that you set out to teach values. I'm certainly not doing that but you'd be naïve to think that you failed to do that, just by the very act of creating a story, sub-text and all that.

Base's intentions are a little stronger as his belief in "education by stealth" was mentioned twice during the interview:

> If I do delve into concepts of education just very lightly, I would say that I am attracted to the idea of education by stealth, where kids don't even realise that they've been taught something, that the best way of educating is to entertain. Education by stealth is definitely the way to go. Entertain, and education will follow.

Harvey expressed similar sentiments, stating:

> The embedded is a great way of getting under their guard and demystifying or making it less threatening. … I put it all in there and they can take as much as they are able to out of it.

Base concluded by saying:

> Because it really is a bit of an adage that anything that's educational ... you've really got to be careful, or with a message, you've got to be so subtle and careful not to turn people away by being too enthusiastic about what you're trying to tell them or teach them.

This theme not only continues to describe writers' intentions but it also contributes to an understanding of the potential of picture books to facilitate the development of mathematical concepts in young children.

6.5.6 Mathematics embedment within picture books

The authors in this study were chosen because their books were regarded as containing embedded or perceived mathematical content. The analysis of responses revealed, however, that none of the authors included mathematics with the intent to teach it. As previously noted, despite Base's ability to represent mathematical ideas in text and illustrations, he emphasised:

> I'm terrible at maths; it's not a passion or a specialty in any shape or form. … If the book had been about teaching maths, I mean firstly I wouldn't have written it. It

> wouldn't have been something that was in my soul to create and to put those years and creativity into, because that wasn't important to me.

Base was conscious of the embedding technique, stating:

> I was very aware of *Anno's Counting Book* when I was working on ... well I've known it for years and years, so I know that that was in the back of my mind that that was another example of really a beautiful book about numbers without, I think, hitting you over the head with it all. … In the case of *Uno's Garden*, it was a way of trying to simplify the idea that things get away from you in life.

He provided the example from *The Waterhole* (Base, 2001) where he used the diminishing numbers and shrinking waterhole to embed knowledge:

> because I knew that when a water system is under threat, under stress in the real world, it's the frogs that disappear first. They're like the canary in the coal mine. And I never actually say that anywhere in the book, but that's just an extra thing that eventually somebody, a kid, will tumble to and it will resonate. And I knew that it had value and I knew that it was a really worthwhile layer and that's why I decided to emphasise it on the back to make sure that kids have the fun of going through and picking up on those things.

As experienced writers, these authors know their audience and include features (textual or illustrative) that they believe will enhance the meaning and engagement of their readers. Base believed that he has achieved just the right level of embedment or "osmosis. I'm not trying to teach the concept primarily. I'm getting the kids to go and see what you can find." Talking of *Uno's Garden* (Base, 2006), he added, "If the maths had been any more overt it would have completely skewed the market". This could suggest that he believed there is not a market for picture books that contain overt mathematical content, reflecting the public's perception of mathematics.

As a consequence of the discussion with them, I asked each author about how they would classify the mathematical content they knowingly include, or that I myself had identified, within both the text and the visual images. This would also assist in the verification of my definition and the choice of picture books for Phase 3.

Although Gleeson previously had admitted to "embedding" history, literacy, and sun safety in some of her books, any mathematical content was not intended and she agreed that any mathematical content in her books was therefore perceived.

As Lester believed any mathematical content "needs to be in there" as an "extension" of the story, she indicated her books contained perceived mathematical content as, for example, in the maps in *Are We There Yet?"* (Lester, 2004). See Figure 6.1. *When Frank Was Four* (Lester, 1994) is another book in the "seven friends" series. It gives the appearance of being an explicit mathematical "counting" book, but she stated that the inclusion of counting is incidental and only there because "I know that ... when my kids were little, they loved that counting stuff".

Questioned about why she stopped at "seven" if it is intended as a counting book, and then added a "summary" or supplementary page (Figure 6.7), she responded:

> Any numbers that happen there were really just coincidental. And I had a double, extra double page at the end, and I thought, "Oh, that would be really nice to make a little counting thing that went with all of them". So that's how that came about.

After some discussion she conceded that even this book has perceived mathematical content apart from the end page (Figure 6.9) which contains embedded content.

Figure 6.9. Final double page spread from *When Frank Was Four* (Lester, 1994).

Although Harvey admitted that his aim was to embed information in an interesting way for children and that there are mathematical and scientific elements in his writing, he did not feel the mathematics is specifically embedded. His idea of embedded mathematics appears to be, for example, formulas for wave formations rather than the everyday mathematics that children of the age who would be reading his picture books would explore. When I suggested mathematical opportunities within several of his books such as planning a holiday (e.g., routes, timelines, budgeting, etc.), similar to the storyline of *At the Beach: Postcards from Crabby Spit* (Harvey, 2004), he responded, “I don’t know that I consciously included … these ideas”.

It is possible that his previous training as an architect contributes to his statement that it is “my own way of thinking that led to that”. In the early stages of this study I found it difficult to classify Harvey’s books, wavering between embedded and perceived. However, this discussion confirmed his intentions and my final classification (see Section 5.1) of perceived mathematical content (Appendix C).

Base was very definite in his intentions when including mathematical (and other content, e.g., environmental). It was to embed other layers, including mathematics, within his books.

> I’m not trying to teach the concept primarily. I’m getting the kids to go and see what you can find. And of course they will see the pattern as the forest diminishes and the houses multiply. And that was the simple level that I wanted it to work at.

However, since the interview Base has released *Uno’s Garden Counting Book* (Base, 2015). I therefore contacted him again about his intentions in this new book (G. Base, personal communication, 9 July 2015). He revealed he had “reversioned” his original book into a 1–10 counting book as he is trying to target a younger audience than previously and “because *Uno's Garden* was founded on numbers it seemed natural to make it into a simple counting book for younger readers”.

The layout of this book (200 mm x 200 mm—larger than usual counting book size) uses a flap style. Each page contains the word for the number, the digit, and then a structured arrangement of tiles (subitising) showing a relevant animal (Figure 6.8). Once the flap is opened, the number of animals is depicted in a segment of the relevant page in the original book.

When asked about the design of the pages, such as that shown in Figure 6.10, he responded that, like Lester previously, it “was simply the result of my instinctual guess, as an artist, of how to convey the number most effectively. If it meets with professional, informed approval I am gratified!”

Figure 6.10. Page from *Uno's Garden Counting Book* (Base, 2015).

This new book, as distinct from all previous books of his with mathematical content, can definitely be classified as having explicit mathematical content.

This theme of *Mathematics embedment within picture books* has demonstrated that authors can and do include mathematics, along with other information and values, without necessarily intending to promote or teach mathematics. As Gleeson noted earlier, it is not "pure" mathematics being referred to, but mathematics that is part of everyday life.

The authors' responses regarding the actual classification of their books have corroborated the existence of two of the three types of mathematical picture books: perceived and embedded. The third, explicit mathematical picture books, or "trade books", had already been recognised (Schiro, 1997), but unfortunately could not be explored in this author study.

The theme also suggests that picture books do have potential to facilitate mathematical development while engaging the reader.

6.5.7 Framework links

The analyses of the previous themes and responses indicate links to all framework categories and elements. *Mathematical content* has been shown to be included in the text, visual images, and the design of the books. These can portray mathematics arising in authentic situations of real-life stories. The mathematical content is usually, but not always, age appropriate and can be related to a syllabus outcome, although Base admitted that the mathematics in *Uno's Garden* was designed for an audience across a range of ages.

Three of the four authors explicitly mentioned that they were very conscious of the need to have all content accurate (accuracy is a framework element included in mathematical content). These authors all said that they spend a lot of time researching facts for their books. Base stated clearly the need for “the mushrooms having the right number of spots on each page and the flower having the right number of petals”, and Harvey felt that

> unless it’s obviously a trick or a trap or something which I do a lot, it is important that it is accurate. And the idea, I think there’s a lot … there have been books where the information is not accurate and I think that’s really … it’s almost criminal to do that.

To maintain accuracy and authenticity, Lester consulted with the Indigenous people in Arnhem Land while working on *Ernie Dances to the Didgeridoo* (Lester, 2000). She said:

> I didn’t want it to be a book that a person from the south had gone up and done. And just done it in their way. So I worked very closely with the community and made sure that I got the seasons right and all that sort of stuff. … I used to fax them up the text, and they would correct it and send it back. And so there were lots of things that I would see in a certain way that they saw it in a different way that made me change it. So, it really belonged to that community, rather than it was just my version of it.

Additionally, three of the authors initiated discussion about the issues of gender stereotyping and racial inclusivity (elements of *Curriculum content, policies, and principles)*. Mathematics has been traditionally regarded as a topic dominated by males, so this element was included in the framework to ensure that mathematics is seen to be useful to everyone.

Gleeson raised the issue of gender equality that appeared in one of her early book, *Mum Goes to Work* (Gleeson, 1992) where women are depicted in a range of traditional male and female roles. She states that they are

> going to bring it back in because the woman who is the managing director of that company feels very strongly that it’s one of the few books around that depicts women’s physical labour and mental labour in the workforce which we all know is true.

Harvey commented about “rules” that he has seen enforced in publications, particularly in the USA, regarding the depiction of different races of people. He related an instance where

> I was censored a number of times and actually in *The Eureka Stockade* I had a couple of Aboriginal people, three or four, just standing some distance from the road where the diggers were all walking towards Ballarat and the biggest book club in the country wouldn't buy the book unless I took them out.

The framework category of pedagogical implementation also emerged when authors discussed the importance to them of readers returning to the book. Here, the authors (e.g., Lester, Section 6.4.4) demonstrated their intent to use different strategies with the text and illustrations and include multiple layers (even mathematical) for the reader to come back and explore further.

Base mentioned this specifically:

> The desire for me to, I suppose, have these layers which go deeper and deeper and deeper as far as I can make them, so that kids and their older brothers and sisters and the adults and then the kids when they turn into adults can go back and go, "Oh wow I never really got that" and now they do. So they can be very, very long, slow burn processes and there's always something extra to get out of it.

It is important for the authors that the reader not only enjoys the book the first time, but returns and looks at it from a variety of perspectives: the story, the illustrations, the message, and the different layers (including mathematical). The writing and illustrative styles described in the previous sections assist in achieving this and underlie the whole intent of their writing.

The mathematical content does not necessarily have to be explicit but can be suggested within the components of the book. For example, the book *At the Beach: Postcards from Crabby Spit* (Harvey, 2004) could promote activities around planning a trip or event including mapping, budgeting, and weather which would facilitate mathematical problem solving and reasoning, integration of mathematics content, and other curriculum areas. More examples are provided in Chapter 8.

The intent of these authors when embedding information is to present material (including history and science) in an interesting way, thus promoting a more positive attitude toward a range of specific topics. This has been shown here to occur for mathematics too, and relates to the framework category of affordances for mathematical learning. Although the authors do not intend to teach mathematics through their books, or, for some, even realise that mathematics appears in their books, they recognise they may be providing teachers with a resource that they can implement appropriately for their students (framework category: pedagogical implementation).

6.5.8 Using picture books for mathematics learning

The theme of using picture books for mathematics learning relates to how authors actually perceive the use of their books within a classroom setting for teaching and learning. Gleeson and Lester previously stated that they did not believe they included mathematical concepts in their books and did not see the mathematical possibilities. Both appeared to have a better understanding of this when I pointed out some of the opportunities. In her initial email response Gleeson had stated:

> I am quite self-reflective about my work but I must say that I have never considered embedding mathmatical [sic] concepts in my picture books — which is not to say that there aren't possible opportunities for teaching/learning in the area arising from the texts.

Despite not seeing the potential herself, Gleeson said she had heard “of classes that have done that—seen how far their voices can be heard from *Amy & Louis*”. Lester, too, said that she has heard “people say they’ve used these books. A teacher at Tennant Creek told me once that she used that for maths all the time.” She was also aware that *Ernie Dances to the Didgeridoo* (Lester, 2000) is used by Indigenous communities in Northern Australia because “there are so many books that don’t have any Aboriginal kids”.

Base was quite excited that his books are used in different ways in the classroom by

> inventive, creative teachers [who] can take that source material and use it in so many ways. This is what I love, is that I provide this source material. … It’s incredibly rewarding that that source material may ... hatch this ongoing life and usefulness. … It’s not for me to write plans for how that works, but just to give richness and layering so that it can be used in the future.

In conclusion, and importantly, Gleeson stated that although she is “fascinated” by the idea of using picture books to develop mathematical concepts, she

> would hate to think you’d start out a lesson intent only on teaching maths because, frankly, they’re stories and they’re stories to be enjoyed at the level of story rather than at the level of pedagogy. At the same time, if the discussion that emerges from that story counts as that sort of thing then that’s fine.

This statement implies that Gleeson intends her books to be appreciated primarily for their literary values and that the mathematical aspect is of only secondary importance that should not detract from her original purpose. This was an important point mentioned earlier in Section 1.2 and will be discussed in the closing comments of the thesis.

However, despite these claims of not expecting their books to be used in classrooms or for mathematics lessons, all authors extend themselves into the community by talking about their books in schools, doing workshops based on their books with students, and writing books with students. Gleeson's website even provides a section for teachers, and recently Base's website has an additional section titled *The Graeme Base Educational Suite: Learning Through Literature in the Connected Classroom* (Base, 2015), so his intentions appear to have changed since the interview about the explicit classroom use of his books.

6.6 Conclusion

The analysis of the data presented in this chapter not only revealed why some picture book authors write but it also provided many additional insights. First, it indicated, with relevant text, the reasons why authors write picture books and how they incorporate their ideas through the text and visual images. Second, the analysis provided detailed authors' views about the components of a picture book, its structure, and how authors and illustrators design their own books. These are important considerations when analysing the data in Phase 3.

The authors interviewed represent a small purposive sample from a very large and diverse pool of picture book authors. They offered a range of reasons for writing, and these can even be different from book to book. Some considered the picture book to be an art form; others saw its role as being to entertain. For Base, it was to give "joy for others and for me", while for others it was to provide information in an interesting way. The authors were conscious that even when they did not embed information or a "message", they were including values, morals, and literacy and visual literacy skills that are not intended as pedagogical in nature. As stated in the introduction, these authors realised, "whether authors of children's books intend it or not, children's literature always has an educative or socialising function" (personal communication, V. Flanagan, 12 November 2012). This statement again emphasises the place of picture books in socioconstructivist teaching practice.

The authors in this study were not uniform in their beliefs about the role of the text and visual images. Furthermore, these views were not always consistent with the literature provided in Section 3.1.2. It appears that the authors used their own writing style and illustrative instincts and intuitions and were not concerned with the theoretical aspects of textual and visual roles. They used the illustrations to add detail to the text, give pleasure and visual appreciation, depict ideas difficult to describe in words, embed information, and generally enhanced the picture book. The authors also demonstrated a range of ways of presenting or structuring their book.

Similarly, the mathematical concepts that authors unintentionally or purposefully included both in the text and in visual images were used to enhance the meaning and add depth to the overall work. Van den Heuvel-Panhuizen, Van den Boogaard, and Doig (2009) stated that "picture books authors do not hold to the standard — they do not even know the standards — so they simply include a graph when it fits the story, with surprising consequences" (p. 34). Although it was never the intention of these four authors to know the standards and directly "teach" mathematics, they did not include cross sections or graphs blithely, but used them to illustrate something too hard to describe in words.

One of the limitations of this phase of the study was that the sample did not include the authors of the explicit mathematical picture books because they did not respond to the initial survey and were therefore not able to be included in the formal interview stage. It seems that their intentions are different. They are writing their books to teach mathematical concepts purposefully, although they may have had more complex reasons for their writing that could not be identified.

These interviews have proved that mathematical content is and can be included in the text, visual images, and design of picture books in different ways. I was therefore able to confirm my notion of three different types of mathematical picture books. The discussions also assisted me in clarifying the classification of some books and confirmed my definitions of perceived, embedded, and explicit mathematical picture books. When asked, the authors classified their books according to their views, perceptions, intentions, and the way they included mathematical concepts, and these were consistent with my definitions. This agreement assisted my choice of books for Phase 3, which specifically addresses Research Question 3.

Through the interviews, the authors recognised the potential of both the text and visual images in their books to facilitate the development of mathematical concepts in young children. This opens the way for authors to create picture books that can not only be read for the story, their literary value, and embedded message, but also for mathematical learning purposes. They can include intentionally-placed mathematical concepts and ideas in the language and visual images that may enhance mathematics learning. This will be further discussed in the final chapter.

In Chapters 7, 8, and 9, I provide an account of Phase 3 of this research: the classroom study using the three different types of mathematical picture books with 16 young students. The insights gained from the analysis of responses from these authors supported the selection of the three different types of picture books. The development and discussion of the themes also highlighted aspects of book content and design to be in the analysis of these interactions between the students and the book. The aim of this third phase is to determine how picture books engage students and teachers, and whether one type of mathematical picture book elicits more mathematical responses than does any other (Research Question 3).

6.7 Publication 6: Refereed conference abstract

The core aim of this phase of the thesis was to investigate the intentions and perceptions of authors when they include mathematical concepts in their writing (Research Question 2). Publication 6¸ a refereed conference abstract is an early and abbreviated report of this analysis. This presentation was delivered at the joint International Conference of the Australian Association for Research in Education [AARE] and the Asia Pacific Educational Research Association [APERA] at Sydney University.

Marston, J. (2012b). An analysis of the authors' intentions in developing mathematical picture books. In J. Wright (Ed.), *Proceedings of the Joint International Conference of the Australian Association for Research in Education [AARE] and the Asia Pacific Educational Research Association [APERA].* Sydney: AARE. Retrieved from http://www.aare.edu.au/publications-database.php/6825/an-analysis-of-the-authors-intentions-in-developing-mathematical-picture-books

An analysis of the authors' intentions in developing mathematical picture books

Author: Marston, Jennie

Year: 2012

Type of paper: Abstract refereed

Abstract:

Picture books are used extensively and successfully in literacy programs in prior-to-school and formal school settings. However, there are also many opportunities for their use in the development of mathematical concepts in young children and as part of an integrated curriculum. As part of a study on the role of picture books in the mathematical development of young children, an analysis of 138 children's picture books revealed three types of *mathematical* picture books (explicit, embedded and perceived). *Mathematical* picture books contain mathematical ideas, images and linguistic terms and symbols that may or may not be deliberately included (either explicitly or purposefully embedded within the text and visual images) by the author. Alternatively, mathematics that occurs incidentally may only be perceived by the reader (Marston, 2010).

A framework (Marston, 2010) designed to assist teachers in identifying *mathematical* picture books and evaluating their mathematical content was used to select books for the study to be conducted with three groups of 6 year old students. Fifty books randomly chosen from a larger collection of award winning picture books were categorised for mathematical type (explicit, embedded or perceived) and an intercoder reliability of 0.92 was achieved. Further, it was advantageous to determine the authors' intention to validate the choices made by the researcher.

Interviews with four authors of children's picture books with embedded and perceived mathematical content revealed mixed intentions for their writing. All authors wanted to entertain their reader; those with embedded mathematical content wanted to satisfy a passion but certainly did not write the book with the intention of teaching mathematics. Alternatively, authors of books identified as having perceived mathematical content were often unaware of the mathematical opportunities afforded by their books until these were identified by early childhood professionals, teachers and students.

The interview process also provided opportunities to discuss with authors and author/illustrators the way they compose their text and visual images. Many times they use a particular pattern in the layout of each book or across a series of books; this is also mathematical and the authors were not necessarily aware of this. Importantly, the interviews validated the researcher's categories and choice of the different types of *mathematical* picture books used in the study.

This presentation will not only report and describe writer intentions but will discuss the opportunities authors and illustrators have to include mathematical ideas that may facilitate the development of mathematical concepts in young children.

CHAPTER

7

PHASE 3: THE CLASSROOM STUDY – METHODS AND MATHEMATICAL CONTENT ANALYSIS

In the following three chapters I describe Phase 3 of the study: the methods, analyses, results, and discussion of the classroom study of Year 1 students'[1] cognitive engagement with three mathematical picture books.

Phase 3 is informed by the first two phases in that they contributed to the structure and content of this inquiry. In Phase 1 (Chapters 4 and 5), I created and evaluated a classification scheme and framework for the selection and appraisal of mathematical picture books. The findings supported the classification of the three different types of mathematical picture books and assisted me in my selection of books for Phase 3.

Phase 2 comprised the analysis of semi-structured interviews with four authors of award-winning picture books classified as containing perceived and embedded mathematical content. In this phase (Chapter 6), I documented the intentions of picture book authors particularly when mathematical concepts were included in their writing and illustrations. My purpose was also to seek authors' views of my classification of mathematical picture books and to further support the selection of the picture books for this classroom study.

I designed Phase 3 as a classroom study to address Research Questions 1 and 3:

> What potential do picture books (of different types) have for promoting mathematical teaching and learning for young children?
>
> How do children and teachers engage with mathematical picture books of different types?

[1] Literature referred to in previous chapters predominantly uses the term "children" for the young participants in their studies. As the participants in this study are in their second year of formal schooling, I will refer to them as "students". "Children" will be used as a more general term here.

This third phase comprises two main components: shared book reading[2] sessions and follow-up activity sessions to the shared reading.

In this chapter I first explain the background to, and overall design of, the classroom study. As a foundation to this study, on the basis of the book classification conducted in Chapter 4, I selected three books—one of each type of book—for use in the shared reading and activity sessions. To identify the potential for mathematics learning from each selected book, I then conducted a mathematical content analysis of them. An account of the book selection process and the mathematical content analysis is provided within this chapter, in Section 7.3. Following that, in this chapter, I outline the methods for collecting data and the analysis of the shared book reading and follow-up activity sessions.

In the next chapter, Chapter 8, I describe the first component of the classroom study, the shared book reading sessions. In Chapter 9, I discuss the second component, the activity sessions. General information about the complete classroom study is provided immediately below In Sections 7.1 and 7.2.

7.1 Background

The design, procedures, and methods of analysis for the classroom study were informed by several pertinent studies (A. Anderson, 1997; A. Anderson, & Anderson, 1995; A. Anderson, Anderson, & Shapiro, 2004, 2005; Elia, Van den Heuvel-Panhuizen, & Georgiou, 2010; Moschovaki & Meadows, 2005a, 2005b; Shapiro, Anderson, & Anderson, 1997; Van den Heuvel-Panhuizen & Van den Boogaard, 2008; Van den Heuvel-Panhuizen, Van den Boogaard, & Doig, 2009). In these studies, the researchers explored the effects of picture books on children's cognitive engagement from different perspectives, contexts, and methodologies. Together, they provided a base on which to build my study.

In Sections 2.3 and 2.4, I described how utterances have been analysed effectively in previous research as a measure of engagement during picture book reading. I also adopted this strategy in my study (along with other strategies) by recording and subsequently analysing the teacher and student talk elicited during the book reading sessions. I did this to determine,

[2] Although the teaching strategy is known as "shared book reading", the terms "book reading" or "reading" only will be used for the rest of this chapter except in headings. It was assumed that all teachers were familiar with this teaching strategy.

first, how successfully teachers identify mathematical teaching and learning opportunities and, second, student engagement with the mathematics in picture books.

Although student representations were not collected in the previous studies, I collected these as another source of evidence of students' mathematical thinking. I did this because, according to the National Council of Teachers of Mathematics ([NCTM], 2000, p. 136), young children cannot only ''represent their thoughts about, and understanding of, mathematical ideas through oral and written language" but also through drawings. Representations are also considered to be "an aid supporting reflection and as a means for the communication of mathematical ideas" (Elia, Gagatsis, & Demetriou, 2007, p. 658) and an appropriate indicator of young children's understanding (Department of Education and Early Childhood Development [DEECD], n.d.; McDonough & Cheeseman, 2015; Styles & Arizpe, 2001; Thom & McGarvey, 2015).

7.2 Design

A multi-method approach (see Section 3.2) was employed for data collection and analyses. Data were collected and analysed through three main strategies:

- a mathematical content analysis of the selected books;
- audio recordings and transcripts of the shared book reading sessions involving teachers and students, and
- drawn and written representations collected from the students, and audio recordings and transcripts of the post-activity discussion with each student about their representation(s).

The mathematical content analysis initially involved an analysis of each book's text using NVivo10 (QSR International, 2014)[3] for mathematical versus nonmathematical language. The distribution of the mathematical content in the text was then analysed according to individual mathematical strands (no further analysis of nonmathematical talk occurred). Following the NVivo analysis, I searched for other opportunities for mathematical investigations in each book's text and illustrations. I anticipated that the potential I saw in these books for mathematical learning could subsequently be compared with the teacher and student mathematical talk during reading sessions as well as with the students' representations in the follow-up activity (Chapters 8 and 9).

[3] NVivo10 (QSR International, 2014) will not be referenced in full repeatedly in this chapter. It will be referred to only as Nvivo.

The first component of the classroom study comprised three shared book reading sessions (one session for each type of book) in three different settings (schools)—resulting in a total of nine sessions. In these sessions, the teachers directed the reading of the three selected books to a small group of students. The transcripts of these audio and video recorded sessions were analysed using a similar method to that used in the mathematical content analysis that is described within this current chapter. The transcripts were first analysed to separate teacher talk from student talk. Both teacher talk and student talk were then separately categorised as being mathematical or nonmathematical in content, and no further analysis of nonmathematical talk occurred. The mathematical content in both teacher and student talk was then categorised according to individual mathematical strands. Additionally, all teacher and student mathematical talk was analysed for the source of the talk: text, illustrations, or both. In Section 7.4, I describe the methods of data collection and analysis for this component. In Chapter 8, I report and discuss the findings.

The second component of the study, the activity sessions, were led by me. During these sessions, the students first drew or wrote spontaneous responses to their teacher's reading of each book and then discussed their representation individually with me. In Section 7.4, I also describe the methods of data collection and analysis for this component. In Chapter 9, I report and discuss the students' representations and the accompanying discussions that were then analysed for mathematical features and their source. A summary of the findings of Phase 3 concludes that chapter.

7.3 Mathematical content analysis of the picture books

7.3.1 Background

One purpose of this study was to compare the opportunities for mathematical learning provided by the three different types of mathematical picture books (perceived, embedded, and explicit). Previously, both perceived and explicit mathematical picture books had been used in studies. For example, A. Anderson et al. (2005) had used books with included mathematical ideas, but no specific mathematical intent (perceived); Van den Heuvel-Panhuizen et al. (2009) selected books with explicit geometry, data, and measurement concepts; and Elia et al. (2010) used a book with explicit number content.

Shapiro et al. (1997) used two books where "the style and quality of the illustrations are similar and the content and illustrations have the potential for the development of mathematical concepts" (p. 51). A. Anderson et al. (2005) later used the same two books again as they believed they "held similar potential in terms of the amount and types of mathematical conversation they would engender" (p. 20). Elia et al. (2010) analysed their books' pictures for their function ("story-related" or "mathematics-related" p. 131) as the purpose of their study was to "explore the effects of pictures on children's spontaneous mathematical cognitive engagement" (p. 125).

Like Kesler (2012) and Van den Heuvel-Panhuizen and Van den Boogaard (2008), I used award-winning books so that the literary quality of the book was already assured. From the lists compiled in Phase 1, I selected three books using the criteria developed in my framework (Marston, 2010a) and the information and insights gained about the classifications during the author interviews. I judged these books to be good examples of each type of mathematical content as they displayed many of the framework criteria. The books contained clear and accurate mathematics that matched curriculum expectations for the age group. They also included some problem-solving opportunities, integration opportunities both within mathematics and across other curriculum areas, authentic contexts for the mathematics, a positive view of mathematics, and easy pedagogical implementation. The books selected for the study were *Amy and Louis* (Gleeson, 2006), *Uno's Garden* (Base, 2006), and *Minnie's Diner: A Multiplying Menu* (Dodds, 2004)[4]. Reports of both how *Amy and Louis* and *Uno's Garden* have been assessed against the framework appear later in Publications 7 and 8 respectively. More detailed information about each book's author can also be found in Appendix E.

7.3.2 Procedures and results

The mathematical content analysis was conducted in two ways. First, I conducted an analysis of each book's text to determine the actual proportion of mathematical language and its content strand using NVivo. Following this, I searched for possibilities for mathematical language development and investigations in both the text and visual images.

7.3.2.1 Analysis using NVivo

The use of the framework criteria assisted in assessing the type and quality of the mathematical content of a book. In this study, quantity as well as quality of references to mathematical concepts and language was important. As I was comparing the amount of mathematical language in the text of three books of different lengths, the statistical component of NVivo was used first to determine the actual proportion of mathematical and nonmathematical language in the text of each book. To facilitate this, I broke every part of

[4] In order to facilitate reading, except in headings, for the remainder of the chapter the books will be referred to as *Amy and Louis, Uno's Garden,* and *Minnie's Diner* only, without referencing.

each book's text into the smallest possible unit containing only one mathematical idea. Every word (e.g., high), phrase (e.g., in the sky), or clause (e.g., "make it a double") that suggested a mathematical concept was coded first as mathematical and then recoded into subcategories according to the content strands of mathematics. This approach to coding is consistent with procedures in other studies. For example, A. Anderson et al. (2005) used the content strands of size, number, and shape, and Elia et al. (2010) used the codes of number, spatial and topographical, and measurement. Van den Heuvel-Panhuizen and Van den Boogaard, (2008) analysed their utterances according to "general qualifications" and "domain-specific qualifications" (p. 354), where the latter included number and space content.

The Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2012) has three main strands: number and algebra, measurement and geometry, and statistics and probability. I coded the mathematical content according to the six separate strands of number, algebra, measurement, geometry, statistics, and probability in this mathematical content analysis and later in the analysis of the mathematical talk (Chapter 8) and mathematical representations (Chapter 9). Algebra here was defined in its very simplest terms when patterns were mentioned or when something was the "same" or "different" (Carraher, Schliemann, Brizuela, & Earnest, 2007; Taylor-Cox, 2003). Occasionally, a phrase contained multiple concepts so, as in the study by Van den Heuvel-Panhuizen and Van den Boogaard (2008), it was coded according to its most dominant characteristic.

Figure 7.1 provides a summary of the mathematical content analysis of the text of each book and compares the percentage of mathematical content by strand, and nonmathematical content, of the three different types of picture book. Results are presented as percentages of the total book's text.

The bars in Figure 7.1 indicate that approximately 30% of the content was mathematical and 70% was nonmathematical. This was consistent across all three books. For comparison between strands, language related to number concepts was more evident in the embedded book than in the explicit book where it was the intended focus. Geometry concepts were more prevalent in the explicit book than were number concepts. This was also the case for the perceived book through the wide use of prepositional phrases and clauses in both.

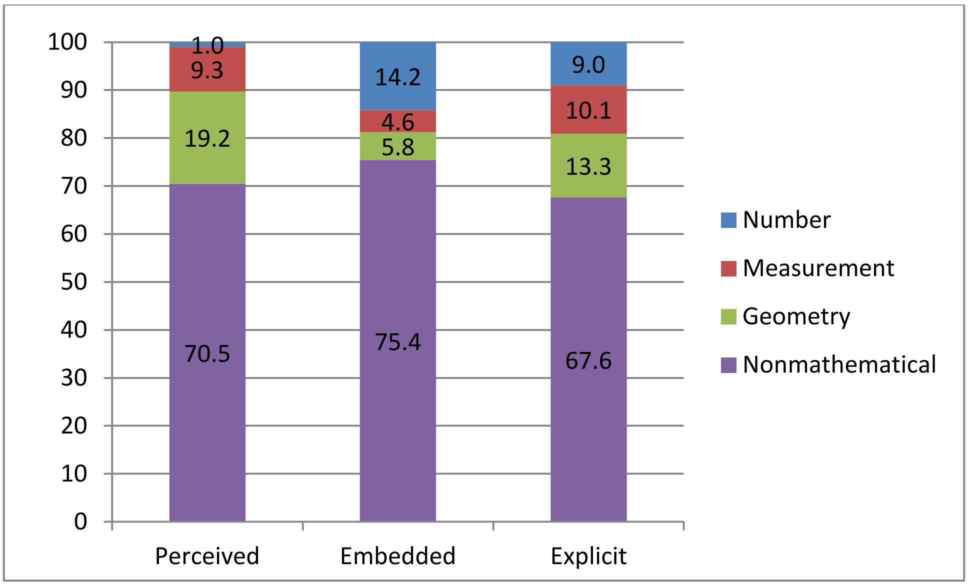


Figure 7.1. Percentage of mathematical content in the text of *Amy and Louis* (perceived), *Uno's Garden* (embedded), and *Minnie's Diner* (explicit).

A detailed discussion of the mathematical language found in each text using NVivo follows in Sections 7.3.2.2, 7.3.3.2, and 7.3.4.2.

7.3.2.2 Analysis for other mathematical opportunities

The books' text and visual images provided opportunities for students to engage in a variety of mathematical learning experiences or investigations that were not explicitly evident in the book. Therefore, as a second step, I reviewed the text and visual images in each book, looking for any potential mathematical teaching and learning opportunities.

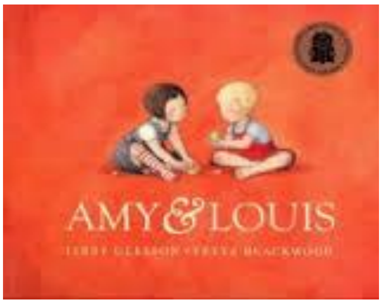
In doing so, I tagged each image or part of an image representing a mathematical idea for its immediate recognisable mathematical content, e.g., specific shapes or potential for positional language. I noted opportunities for concept development and investigations in both the text and illustrations according to my view of the mathematical contexts for learning. The framework's categories and elements that were used to select the books and had been shown to assist in identifying these opportunities (Chapter 5) were again referred to during this process. Identification of opportunities for mathematical investigations can be very subjective, and in this case it was informed by my experience as a mathematics educator and researcher, and as a classroom teacher for many years. Individual interpretations of the mathematical potential of books can be seen in journals such as *Teaching Children Mathematics* with articles such as that by D. Whitin and Whitin (2009). Classroom teachers or other evaluators may find different opportunities for mathematics learning.

In the following sections I describe the results from both the analysis of the text using NVivo and the opportunities for mathematics learning in the text and images for each selected book.

7.3.3. Perceived mathematical picture book: *Amy and Louis* (Gleeson, 2006)[5]

7.3.3.1 Book description

Amy and Louis tells the story of two young friends and the things they enjoy doing together. For example, they played in the yard and in the dress up corner, they built towers "as high as the sky"[6] and "saw magical creatures in clouds" and they always used the cry "coo- ee"[7] when they wanted the other to come and play. When Amy moves "half a world away", Louis discovers a way "to rebuild their special bond" (back cover). The realistic illustrations done in watercolour by award winner Freya Blackwood cover the whole of each double-page spread and text is placed on most pages. As stated in Section 2.6.2, Libby Gleeson is able to choose her illustrator. Such close collaboration means that the images and text complement each other. Although the book has many mathematical references in the text, and images as well as other learning opportunities, these are not intended by its author to be used for mathematical purposes (see Chapter 6). See Appendix E for more details about Libby Gleeson and her books.



(Image sourced from *Booktopia*[8], 2014)

7.3.3.2 Analysis of the text using NVivo

The title and cover of *Amy and Louis* do not suggest that this book is rich in mathematical opportunities. Analysis of the text indicated that 29.5% contained mathematical content. This includes words relating predominantly to geometry and measurement concepts, and one reference to number concepts. The larger proportion (70.5%) of text was coded as nonmathematical content. Table 7.1 shows the percentage of the mathematical content in the

[5] Amy and Louis (Gleeson, 2006) is also published as *Half a World Away* (Gleeson, 2007) in the USA. The cover of the book is also different. The Australian publication is used in this study.
[6] Note the pages in picture books are not numbered.
[7] "Coo-ee" is a cry used in Australia to attract attention. It derives from the Australian Aboriginal Dharuk tribe, the original inhabitants of Sydney, Australia (Macquarie Dictionary).
[8] *Booktopia:* An on-line bookstore: www.booktopia.com.au

text by mathematical strand. It should be noted that this is a breakdown of the total mathematical content of 29.5%.

Table 7.1

Distribution of Mathematical Content in the Text by Strand in Amy and Louis

Mathematical strand	Percentage of mathematical content within the text of the book
Number	3.4
Algebra	0.0
Geometry	65.1
Measurement	31.5
Statistics	0.0
Probability	0.0
Total	100.0

The predominant use of mathematical language pertaining to the geometry strand related to positional language (e.g., “across”, “through”). The author, Libby Gleeson (personal communication, 3 February 2011) described these as just “prepositions” and that it was never her intention to include these words for mathematical purposes. The measurement concepts relate to length, size, and time (e.g., “long”, “high as the sky”, “every day”), and the one reference to number is to the fraction, “half”. One phrase (“half a world away”) that contains number, measurement, and geometry concepts was coded as number. These references to mathematical language provide specific opportunities for discussion, e.g., what “half a world away” means, and activities that engage the students, e.g., “put the book on/in the box”, and “draw a tree beside the house”.

7.3.3.3 Analysis for other mathematical opportunities

As noted in Section 7.3.2.2, other mathematical opportunities were available in the text and visual images. The NVivo analysis of the individual words and phrases in the text did not identify the repetitive text structures identified in further analysis of *Amy and Louis.* These included, “When Amy was in the sandpit ...”, “When Louis was in the ...”, and the word “coo-ee”. These repetitions could encourage the idea of patterning, first in language, and then in mathematical situations.

The visual images also suggest opportunities for the development of specific mathematical language. In Figure 7.2 (a double-page spread from *Amy and Louis*), I have first added mathematical words to demonstrate opportunities derived from the visual images for mathematical language use, e.g., “Louis is behind the tree” and “the shape of tent is a triangle”.

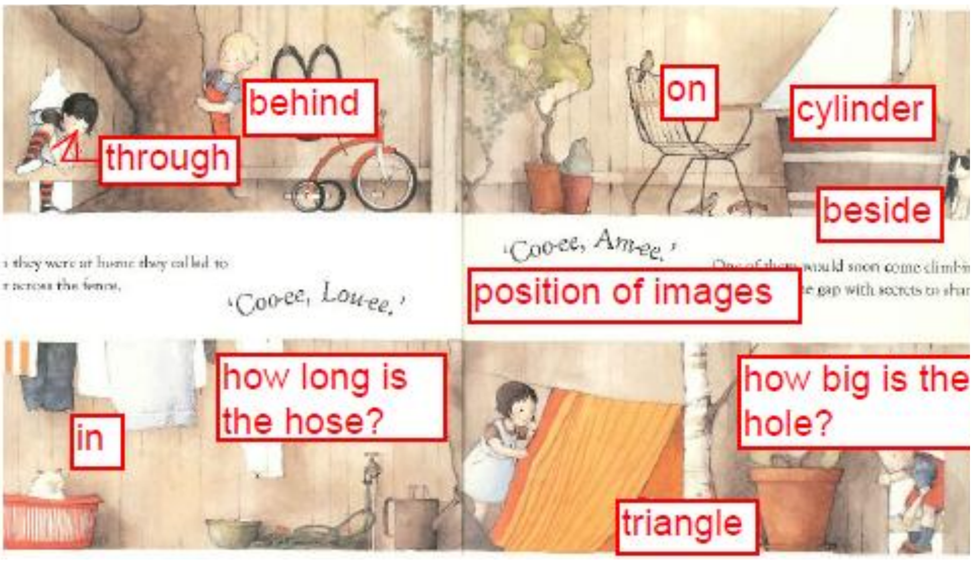


Figure 7.2. Example of one double page spread from *Amy and Louis* “tagged” with opportunities for mathematical language and investigations.

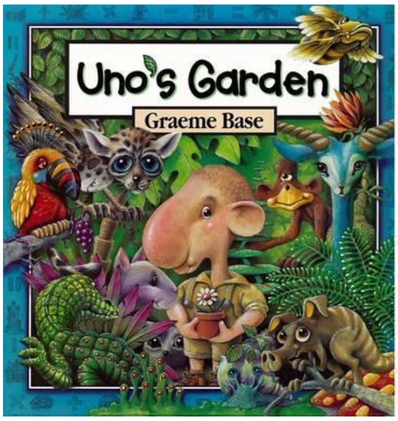
In this figure (Figure 7.2) I also include questions that suggest activities/investigations for the teachers and students to pursue—for example, exploring the size of a hole in the fence needed for Louis to climb through and what other objects could also fit through it, e.g., the cat, the bike, etc. Students could also investigate how to measure the length of a “curved” object like the hose. Another double spread in the book shows a “map’’ of Louis’ town to illustrate how far his call (coo-ee) is travelling. Teachers and students can use this as a springboard to draw maps of their classroom, school, or bedroom, or show how to move from one place to another. Similarly, as students engage in play with three-dimensional (3D) objects (packing and tissue boxes), just as Amy and Louis do, they can learn about their geometric properties in a natural context.

Publication 7 (Marston, Muir, & Levy, 2013) in Chapter 10 highlights numerous mathematical ideas related to length, area, time, volume, two-dimensional (2D) shapes, 3D objects, position, and proportion in *Amy and Louis* that could be explored. Not only are activities for development of individual mathematical strands identified, but so, too, are possibilities for cross-strand mathematics, problem solving, and integration with other curriculum areas. For example, (based on the map picture) there is potential for exploring sound and how far it travels, thus engaging students in scientific concepts. All these examples are authentic activities, part of everyday life, that engage students, promote learning, and provide opportunities for problem posing and solving instigated by both teacher and students.

7.3.4 Embedded mathematical picture book: *Uno's Garden* (Base, 2006)

7.3.4.1 Book description

Uno's Garden is a "storybook, puzzle book and numbers book" (back cover). However, Base's main purpose is to draw attention to the destruction of rainforests and the need to preserve them for future generations. As I reported in Chapter 6, any mathematics included by Base is intended only to add another layer to the book and as the vehicle for telling his message. It is not his objective to teach mathematics. Despite the mathematics equations on each double spread as shown in Figure 6.2 (Section 6.4.4), this book can be read and enjoyed for the story and environmental message alone. It is therefore classified as containing embedded mathematical content. Base illustrates his own books with strong colours, intricate detail, and quirky characters. Therefore he is able to include complementary information in both the text and images as he advocated in Section 6.4.3. See Appendix E for more information about Base and his books.



(Image sourced from *Booktopia*, 2014)

7.3.4.2 Analysis of the text using NVivo

In *Uno's Garden,* mathematical language, numbers, and mathematical symbols are clearly visible. The extra layer to the story features mathematical symbols set faintly in the border of the cover and mathematical equations at the top of each double-page spread replicating the mathematics in the illustrations. An example of this is provided in Figure 7.3. The following explanatory rhyme also introduces the mathematical aspects of the book.

The animals go by one by one
A hundred plants, then there are none
And all the while the buildings double...
This numbers game adds up to trouble

But if you count with utmost care

(And trust me that they are there)

You'll go from ten to nothing, then

The whole way back to ten again! (Base, 2006)

The final page of the book contains a description of the three "mathematics games" (see Section 6.4.4) included in the book. The first 10 prime numbers are hidden in the images throughout the book, providing more advanced mathematical opportunities for teachers and students to explore perhaps for an older age group than in this study.

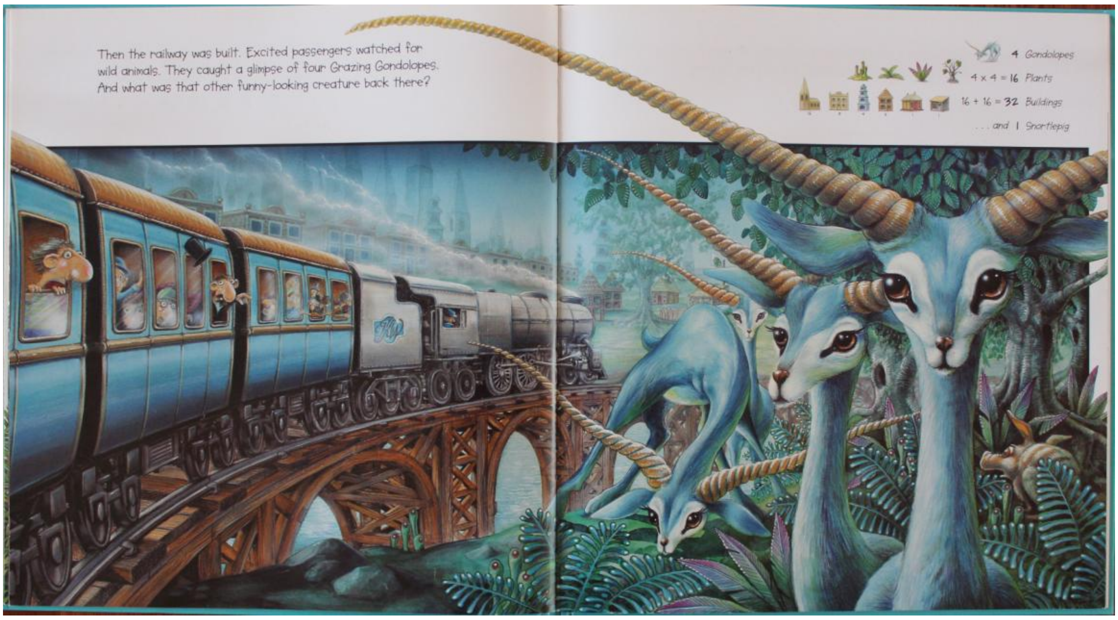


Figure 7.3. Double page spread from *Uno's Garden* showing examples of mathematical content in both text (e.g., number sentences) and illustrations (e.g., spirals).

Figure 7.1 indicated that only 24.6% of the text in this book was identified as being mathematical. Table 7.2 shows the distribution of that total mathematical content by strands. Although this book (classified as having embedded mathematics) consists of nearly double the number of words as *Amy and Louis,* the actual proportion of mathematical content in the text is less than in the perceived example. This mathematical content comprises 58.5% number, 23.2% geometry, and 18.3% measurement concepts. Within the text classed as number, I found ingenious use of mathematical words, e.g., "Uno" and "plus one" (where "and" could have been used), and the numbers that are needed to tell the story. It is difficult to know whether Base realised the mathematical power of some text, e.g., "in perfect balance" when he was showing that the forest and the people now have numbers that can sustain each other.

Although the proportion of mathematical words or phrases used in this book was less than in *Amy and Louis*, this measure does not necessarily represent the extent of mathematical ideas embedded in this book.

Table 7.2

Distribution of Mathematical Content in the Text by Strand in Uno's Garden

Mathematical strand	Percentage of mathematical content within the text of the book
Number[a]	58.5
Algebra	0.0
Geometry	23.2
Measurement	18.3
Statistics	0.0
Probability	0.0
Total	100.0

[a] Many of the references within this code are mathematical equations (e.g., 10 x 10 = 100), and number patterns described in words (e.g., "by one by one") that could be also be coded as algebra. On this occasion, they are coded by the dominant strand of number.

The inclusion of patterning throughout the book was also discussed in Section 6.4.4. That is, the initial rhyming poem, the equations, the decreasing and increasing number sequences, and detailed illustrations showing spirals and bridge supports, e.g., Figure 7.3. However, the rich algebraic content seen through the number patterns of forward and backward counting, doubling, and square numbers, is not represented in the mathematical content analysis as they are recorded within the strand of number.

The remainder of mathematical language is either positional words or phrases (e.g., "there", "back", "amongst the trees", "in his garden"), or words and phrases denoting measurement (e.g., "then", "forever", "bigger", "for many years", "at the very beginning of spring"). These may have been deliberately used or a necessary part of the writing, similar to Gleeson's use.

7.3.4.3 Analysis for other mathematical opportunities

The framework categories and elements were again used to identify mathematical teaching and learning opportunities in the text and images and their appropriateness for students. The concepts included in the mathematical equations, the three different mathematical "games", and the mathematical ideas in the illustrations are suitable for a wide age range of readers and pertain to many mathematics curricula outcomes. Students of the age group used in this study may be familiar with the concepts in the first two equations (multiplication, addition, and doubling) on each double spread (See Figure 7.3) but not necessarily with the size of the numbers included in the book. However, the equations do provide a starting point or springboard for classroom use. Base recognises the range of difficulty in the mathematical ideas (Section 6.5), but stated that his intention was to address a wide span of ages (and abilities) in his audience.

The text particularly promotes counting and multiplication, while the visual images provide opportunities for checking of calculations. However, along with the number and patterning ideas mentioned earlier, the illustrations also include geometry and measurement concepts, such as side and top/aerial views, symmetry in buildings, plants structure, position, time, and length.

The story, the rhyme, and the visual images are all appealing to children and, despite the setting and characters being imaginary, the context of the story is real and engages the reader not only in the story but also in the mathematics, if desired.

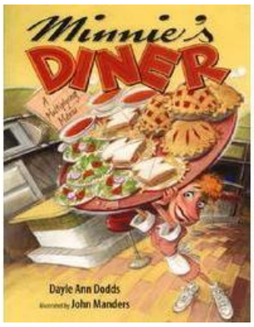
The mathematical games could promote further investigation or problem solving, with suitable guidance. Publication 8, in Chapter 9, demonstrates how the book can be used to develop counting and number sequences, doubling, square numbers, number patterns, and problem solving. The geometry (position) and measurement (size, distance, and time) ideas could promote some discussion as the book is read. The underlying environmental message does allow for integration of activities across other curriculum areas.

7.3.5 Explicit mathematical picture book: *Minnie's Diner: A Multiplying Menu* (Dodds, 2004)

7.3.5.1 Book description

The context of *Minnie's Diner* is rural USA, and the book has the specific intent of teaching the concept of doubling. Written as a rhyming poem, it tells the story of five brothers and their father as they arrive in turn at Minnie's Diner, each requesting double the serve of the previous brother, e.g.,

> "I'll have what Bill has, but make it a double."
> "Of course," said Minnie. "That's no trouble."
> She dashed to the counter and filled up a tray
> piled high with 4 specials for Phill McFay...
> (Dodds, 2004).



(Image sourced from *Amazon,*[9] 2014)

Explicit mathematical books are rare among award-winning books. However, *Minnie's Diner* won the 2007–2008 Young Hoosier Award for best picture book. This book is a clear example of an explicit mathematical picture book with its specific intent to teach the concept of doubling. The artwork by John Manders uses strong colours and sharp lines that replicate the text.

7.3.5.2 Analysis of the text using NVivo

As an explicit mathematical picture book, the mathematics in *Minnie's Diner* is visible and accurate both in the writing and most illustrations. The text and concept are suitable for the intended age group and consistent with school mathematics curricula. Students would find the book engaging with its rhyme and repetitive language and phrases, and it would promote a positive attitude toward learning mathematics

Table 7.3 presents an analysis of the text using *NVivo* showing the percentage of the total mathematical content (32.4%) by strand. It is interesting to note that the text of this explicit book contains only 32.4% mathematical content, not much different from the perceived book, *Amy and Louis,* which had no intended mathematics.

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[9] Amazon: an online bookstore. http://www.amazon.com

Table 7.3

Distribution of Mathematical Content in the Text by Strand in Minnie's Diner: A Multiplying Menu

Mathematical strand	Percentage of mathematical content within the text of the book
Number	27.8
Algebra	0.0
Geometry	41.0
Measurement	31.2
Statistics	0.0
Probability	0.0
Total	100.0

Surprisingly, the focus on number and doubling comprises a little more than a quarter of the mathematical content. As there is much repetitive language or chorus in this book, the actual range of mathematical language is further limited. However, two additional mathematical strands (geometry and measurement) that also featured strongly in the text of *Amy and Louis* and *Uno's Garden* are present within the book. In fact, the book contains more of both geometry (41% of the mathematical language) and measurement (31.2%) concepts than of number concepts (27.8%).

Geometry (mostly position) is included through predominantly prepositional phrases, e.g., "on the farm" and "through the door". The measurement concepts again relate to size and time, e.g., "big" and "one sunny day". However, phrases such as "twice as big as his brother before" that were repeatedly used, and contain measurement concepts, were coded as number because they were referring to the doubling (number) concept. It is difficult to know whether these concepts were purposefully placed within the text to add another dimension (and possible other areas for exploration) or whether they just occur naturally as part of a story. Teachers may focus on the book's main purpose, the idea of doubling, but there are many other possibilities for mathematics learning

7.3.5.3 Analysis for other mathematical opportunities

The visual images in this book are generally a replication of the text and are not purposefully used to portray other ideas (complement the text) as described in Chapter 2. For example, each time another brother arrives and orders double the food of the previous brother, the number of items on their food tray is accurately depicted. However, I found several inaccuracies in the book. First on the cover, there are unequal quantities of food that do not match any calculations in the book. Second, the brothers are supposed to be "twice as big as his brother before". However, although pictures of the brothers increase in size, they do not double either in height or width and there is no consistency in their increase of size throughout the book. Figure 7.4 shows an example of the replication of text in the images but disparity in size of font and brothers. The font also increases in size to emphasise the concept, but again there is no consistency. These inaccuracies could be used to advantage through investigations instigated by the teacher, such as investigating the size each brother should be. See Publication 7, Chapter 10, for more examples of the beneficial use of inaccuracies.

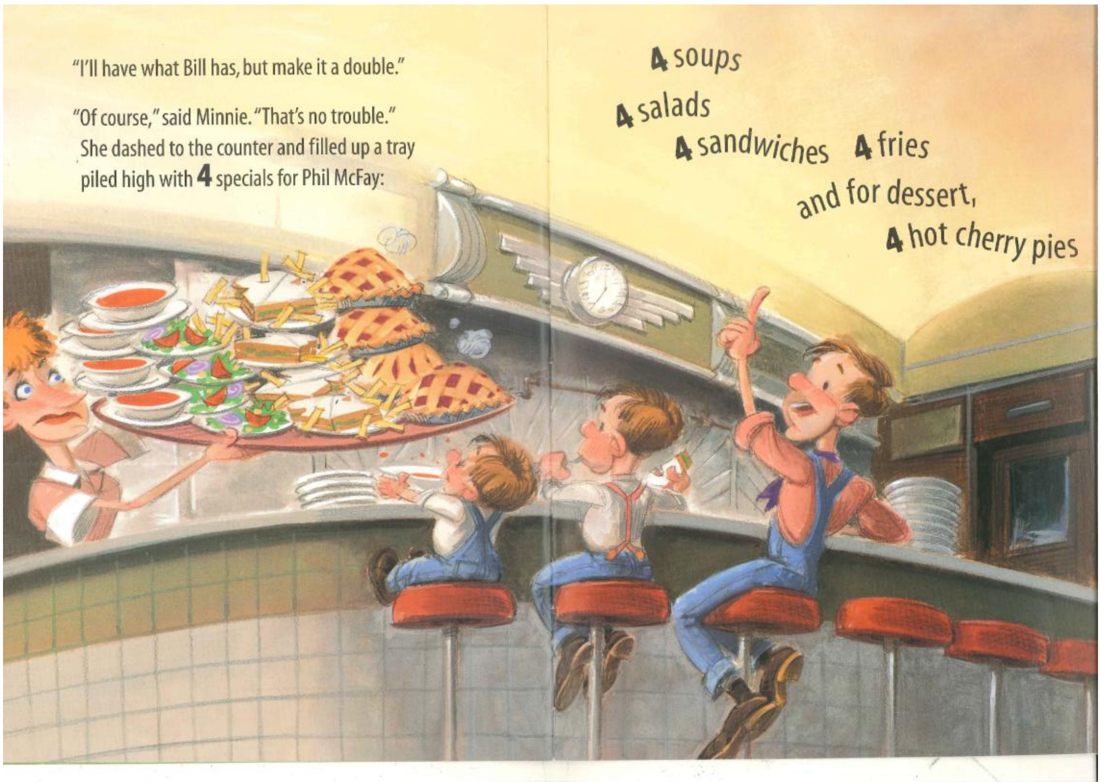

Figure 7.4. Double page spread from *Minnie's Diner: A Multiplying Menu* showing both replications of the text in the visual images and inaccuracies in the size of the brothers.

Within this book there are some good opportunities for use of mathematical language and concept development around doubling, e.g., activities with card and dice games, pattern blocks, number charts, and two-times table connections. The text and illustrations in both *Amy and Louis* and *Uno's Garden* provided other ideas for learning explorations. However, despite the inclusion of the geometric and measurement references here, other opportunities for mathematical investigations in *Minnie's Diner* may be more limited. Students could create menus or shopping lists that required them to double quantities on some or all items or use positional language in a variety of contexts.

7.3.6 Summary: Mathematical content analysis

Despite the different purposes of the writers of each book, the mathematical content analysis of the text of the books using NVivo appears to demonstrate that they have similar potential for mathematical learning. I found about 30% of each book's text contained mathematical terms. Geometry appeared to be the dominant mathematical strand for *Amy and Louis* and *Minnie's Diner,* while *Uno's Garden* contained predominantly number concepts. A further analysis of both the text and illustrations of the books revealed more opportunities for mathematical investigations and integration with other curriculum areas for *Amy and Louis* than for the other two books.

This analysis therefore showed the potential of the books for rich mathematical teaching and learning and justified their use in this research. It is important to note that, although these books represent a good example of each type of mathematical book, other books of the same classification may have different amounts of mathematical content.

The findings of the analysis of the three books' texts using NVivo and the potential I found for mathematical investigation are discussed in reference to the findings in Chapter 8. There I consider differences in teacher and student mathematical talk during the reading sessions and the opportunities that teachers use for mathematical discussion. They are also taken into account when analysing the student representations in Chapter 9.

A more detailed summary of the findings is included in the conclusion to Phase 3 in Chapter 9.

7.4 Methods: Shared book reading and activity sessions

This section comprises a description of the samples used, the procedures employed, and the methods of analysis for the book reading sessions and the drawing and writing activity sessions. The results for these two groups of sessions are reported in Chapters 8 and 9 respectively.

7.4.1 Samples

There were three groups of participants: (i) three primary schools, (ii) three Year 1 teachers, and (iii) 16 Year 1 students across the three schools. These are all fully described below.

7.4.1.1 Schools

Three schools were purposively selected for the study. They represented both Australian school systems (government and independently funded) and a range of socioeconomic contexts. All were coeducational and located in metropolitan Sydney, Australia. The schools were chosen by convenience sampling: one was a school where I had conducted a previous unrelated project, the second was a school whose principal I contacted following the MANSW conference reported in Chapter 4 (Publication 3), and the third was a school with which I had a professional association through the principal. The principals of two government schools and one diocesan school within the Catholic Education Office, Broken Bay, were initially contacted by mail inviting their school to participate in my study. Each letter was accompanied by the relevant university-approved information and consent forms (Appendix C). All principals accepted the invitation.

7.4.1.2 Teachers

In each school the principal organised one Year 1 teacher, each school having more than one class at that year level. In all cases, the principal initially approached one teacher who he/she thought may be interested in the study. The principal supplied the teacher with the information form explaining the purpose of the study as well as the consent form. A self-addressed envelope was also supplied so the teacher could return the signed consent form directly to me, thus avoiding any perceived coercion from the principal. The first teacher asked in each school accepted the invitation. All teachers were females, each with more than 10 years of teaching experience.

7.4.1.3 Students

The teachers were requested to select about six students in their own class who, based on classroom assessment records, were of average ability in numeracy and literacy for Year 1. The assessment records included assessments such as the Schedule for Early Numeracy Assessment ([SENA] NSW Department of Education and Training, 2009b) or other available assessment records. I envisaged that these students may be an already-established mathematics learning group.

Six students in School 1, five in School 2, and six in School 3 were invited to participate by their teachers. The sessions were held toward the end of the second year of formal education for these students, so students were all 6 or 7 years old. The teachers delivered the information forms to the parents/caregivers of the chosen students and collected the signed consent forms. One nominated student at School 1 did not participate in the study for an undisclosed reason. Table 7.4 shows the composition of each group of students: a total of 16 students—6 female, 10 male.

Table 7.4

Sample Composition: Schools and Student Groups

School	Education system	Female students	Male students
1	Government	2	3
2	Government	2	3
3	Independent	2	4

7.4.2 Procedures

There were three sessions in each school that comprised the book reading sessions with the teacher and the drawing and writing activity sessions with me. The sessions were conducted at least 1 week apart at a time convenient for each individual teacher and their class. Each session with the students was expected to last about 50 minutes. All data were collected from the schools during September, October, and November 2011. School holidays and other school events extended the data-collection period.

A meeting was held with each teacher about a fortnight before commencing the data collection to arrange the times and the organisation of the sessions, including the roles and responsibilities of both the teachers and me. At this initial meeting I provided each teacher with the three selected books. I told them that they could decide the order in which they read the books to the group and their approach to each session. This allowed them to prepare and incorporate the books into their program as they considered appropriate. I gave no specific instructions to the teacher for the reading session other than a request that they read the picture books to their group of students and interact with the book in the way they felt was most appropriate. I suggested to the teacher that this would involve asking suitable questions of the students and engaging the students in discussion about the book. I explained they did not need to provide a follow-up activity immediately after the story reading as I would be engaging the students in an activity session.

7.4.2.1 Shared book reading sessions

Each reading session included the teacher and participating students. It occurred within the classroom but separate from the rest of the class who had been provided with independent activities by their teacher. This was sometimes, but not always, mathematics related, and was either independent or group work that was in some cases supervised by a teacher's aide.

7.4.2.2 Activity sessions: Student representations

Following the book reading, the teacher returned to the class while I spent about 30 minutes with the group of participating students. I then investigated students' cognitive engagement and mathematical thinking through their spontaneous written and drawn representations and discussion of these. I first asked the students as a group to "draw a picture or write about what comes into your mind when you think about the story". There was no discussion before students started work in order to enable individual, spontaneous, and unbiased responses.

There were several reasons for me leading this activity. First, early discussions with the participating school principals indicated that the supervision of the rest of the class for more time than was required for reading the picture book would be difficult. Teacher release was financially not an option either. Second, I wanted the drawn responses to be totally spontaneous, without any prompting, and consistent across the groups. Third, it enabled me to question the students about the mathematical content in their representations.

A whole-group discussion about their representations might have meant interjections from other students, and on this occasion I wanted only individual responses. Therefore, as the students completed their representations I provided another short mathematical activity associated with the relevant book (not intended for analysis) to occupy them while I talked with each student individually about their representation. For example, for the session when *Amy and Louis* was read by their teacher, I provided the students with a sheet of paper divided into two columns. In one column they were to draw things they considered "close" and in the other column, things that were "far away. An example of a student's work from this activity is used in Section 8.2 to further demonstrate his understanding.

With the consent of the teachers and the participants' parents/caregivers, all sessions in my study were digitally recorded using a Sony IC audio recorder. In addition, all sessions were video-recorded on an MP4 video recorder. All video recordings were taken from behind the students. As the focus of the study was on the language used by the students that related to mathematics, the video recordings were used as a backup to assist in deciphering inaudible conversation on the audio-recording. Each audio recording was transcribed by a research assistant before coding was commenced.

7.4.3 Methods of analysis

Researchers investigating children's mathematical language during book reading sessions have often recorded the frequency of interaction or events, or utterances. Moschovaki and Meadows (2005a) defined utterances as "phrases that are distinctive in content" (Section 3, para. 3). Similarly, A. Anderson et al. (2004) and A. Anderson et al. (2005) used the conversational or discourse "turn" defined by Sinclair and Coulthard (1975, p. 231 and as cited in A. Anderson et al. 2005, p. 9) as "everything said by one speaker before another began to speak". Van den Heuvel-Panhuizen and Van den Boogaard (2008) used "the smallest possible grammatical part of a response" (p. 354).

The unit of analysis for the teacher and student talk was based on the research by both Moschovaki and Meadows (2005a) and A. Anderson et al. (2004) as I coded complete thoughts (phrases or sentences) as a "turn". This meant that the whole "turn" was coded as mathematical even if only one word in the turn contained mathematical content. Words such as names, fillers (umm, aah, etc.), or repetitive opening words by teachers (e.g., and, so) were not counted in the mathematical code of the "turn".

In all previous studies, the amount of talk was measured by the number of utterances or turns for each category. This meant that a single word or a whole sentence was coded as one utterance. This is a suitable method of comparison when using only one book. However, when I compared mathematical talk for three books of different lengths, I measured the quantity of talk as a percentage of the total talk for each book.

I used NVivo for all coding where all results are given as percentages of the total reading session. I then converted this to percentages of the relevant section where appropriate. Analyses did not include the actual reading of the text, only the transcripts of all teacher and student talk that pertained to the book reading.

Data from the reading and activity sessions were analysed separately in several ways as described in the following sections. The findings of each are reported in Chapters 8 and 9.

7.4.3.1 Primary analysis of teacher and student talk during shared book reading sessions

I first coded the transcripts of all talk from each reading session for each type of mathematical picture book in three steps. Figure 7.5 represents this sequence. A more detailed description of each step follows.

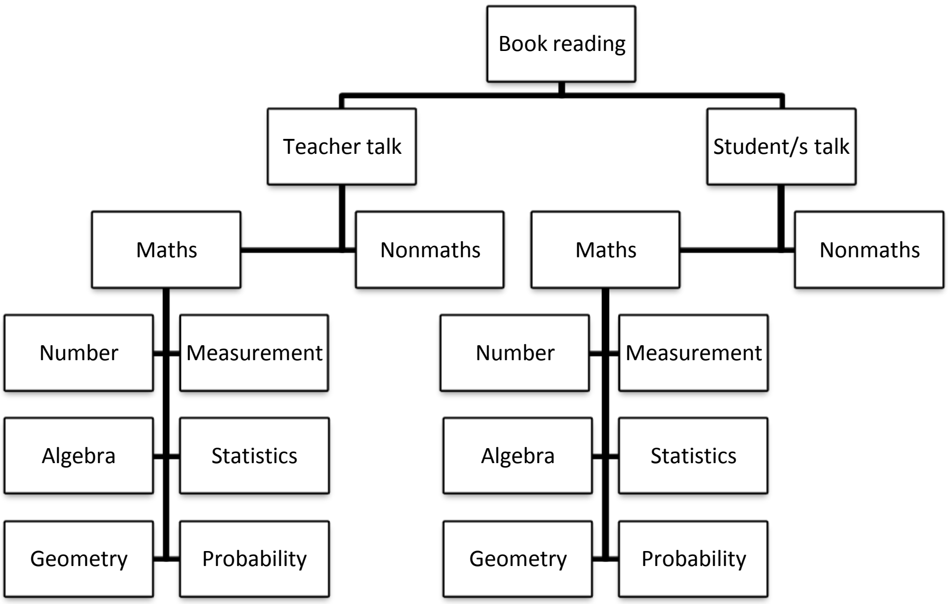

Figure 7.5. Three-step sequence of analysis according to content of talk.

Step 1: Teacher talk versus student talk. In the first step, each book reading session was autocoded[10] in NVivo according to whether the talk emanated from the teacher or from the students. This resulted in separate groupings of teacher talk and student talk for each book.

Step 2: Mathematical versus nonmathematical. In the second step, all teacher talk and all student talk were separately coded as being either mathematical or nonmathematical for each book. For example, the teacher utterances, "How many did his brother have?" (T1[11] during the reading of *Minnie's Diner*) or "Well that's a long way away" (T1 during *Amy and Louis'* reading) were both coded as mathematical, as was the student utterance, "They are telling you that 9 x 9 plants is 81" (S2 during the reading of *Uno's Garden)*. Utterances from the teacher such

[10] Autocoding is a function of NVivo that, in this case, automatically coded appropriately formatted transcripts into teacher talk and student talk.

[11] Coding for teachers and students will be used throughout this chapter, e.g., T1 = Teacher in School 1. S3 = a student in School 3.

as, "So what type of animal do you think they are?" (T3: *Uno's Garden*) or from the students such as, "It was very loud" (S2: *Amy and Louis*) were coded as nonmathematical.

Step 3: Mathematical talk by strand. In the third step, the mathematical content of both the teacher and student talk were separately coded according to the six mathematics strands of number, algebra, geometry, measurement, statistics, and probability. These codes corresponded to those used in the mathematical content analysis of the books (refer to Section 7.3). For example, in the teacher examples given above, the question, "How many did his brother have?" was coded as number; a student's statement "He's got eight pies" (S3) was also coded as number. If more than one strand was represented, only the dominant strand was coded. For example, in the teacher statement, "Well that's a long way away", both measurement and geometry are represented, but only measurement was coded as I believed the emphasis here is on distance rather than position.

Nonmathematical talk that occurred during any of the book readings, whether it was initiated by either teachers or students, was not analysed.

7.4.3.2 Analysis of the source of teacher and student mathematical talk

Previous studies of the mathematical responses of young children to picture books (e.g., A. Anderson et al., 2005; Moschovaki & Meadows, 2005a; Van den Heuvel-Panhuizen & Van den Boogaard, 2008; Yaden, Smolkin, & Conlon, 1989) have shown the strong influence of the illustrations on the children's utterances. In this naturalistic study, there were other influences on teacher talk (e.g., student comments) and student talk (e.g., the teacher and other students). Shapiro et al. (1997) included a specific category for "attention to illustrations" in their analysis. However, in this study I decided to determine the original source of all teacher and student mathematical talk according to the influence of the text, illustrations, or a combination of these.

First, all teacher mathematical talk was recoded using NVivo. For example, the source of the question from Teacher 3 during the reading of *Uno's Garden*, "What can you tell me about the animals on the cover?" would be coded as the illustrations. "So how many do we have on the plate now?" (T2 in *Minnie's Diner*) was prompted by both the text telling the story and the exact depiction in the illustrations.

I had originally proposed that the source of all the students' mathematical talk (discourse turns) would be similarly coded according to text, illustration, or a combination. However there were other influences on the student talk. As will be seen in Chapter 8, the majority of the talk for each book was found to emanate from the teachers and the overall low percentage of student talk was predominantly a response to the teachers' comments or questions. For example, although the student's utterance, "They are telling you that 9 x 9 plants is 81" (S2: *Uno's Garden*) was referring to both the text and illustrations, it was an immediate response to a teacher's question about the book. Therefore, the analysis of student talk is limited because the reporting of the source of student mathematical talk highlights the dominance of teacher talk. A short discussion of the very few spontaneous utterances to the text and illustrations by students is also included in Section 8.2.2.

7.4.3.3 Reliability and validity of coding

Autocoding in NVivo had been used in Step 1 of the primary analysis. To maintain reliability of coding for analysis of both the teacher and student mathematical talk and the source of the utterances, the principal supervisor, associate supervisor, and I each coded a section of one transcript containing 40 "turns". An agreement of .88 was achieved (i.e., we agreed on the coding of 35 of the 40 turns). Discrepancies in this coding were then discussed and resolved. The associate supervisor and I then independently coded clean copies of one of the nine transcripts, this time achieving .96 intercoder agreement. After further discussion of any differences, I coded the remaining transcripts.

7.4.3.4 Analysis of students' written or drawn responses and accompanying discussion: Activity session

All drawn representations in response to each book were coded according to School (1, 2, or 3) and individual Student (A–S) to maintain anonymity, and then digitally scanned. The drawings (and writing if applicable) for each student were then coded in three ways:

1. Each representation was separately coded as containing either mathematical or nonmathematical content. Nonmathematical representations were not further analysed.
2. Each representation with mathematical content was then analysed for its mathematical strands.
3. Each representation with mathematical content was analysed for its source (i.e., text, illustrations, or both).

The discussion from the audiotapes with the students about their representations was transcribed. Pertinent excerpts of transcripts that matched students' mathematical representations were then selected for reporting.

7.5 Summary

In this chapter, I first provided an overview of the design of this classroom study. Second, I provided a rationale for my selection of the three books that were used by the teachers. I described and analysed the content of each book and indicated its mathematical potential for classroom use. Third, I described the methods to be used for the collection and analysis of data for the shared book reading and activity sessions.

The results of the analysis of the teacher and student talk (Chapter 8) and student representations (Chapter 9) follow.

CHAPTER

8

PHASE 3

THE CLASSOOM STUDY – SHARED BOOK READING SESSIONS

In Chapter 7, I outlined the method for the collection and analyses of data for the classroom study. I also described the three books to be used in the classroom study: *Amy and Louis* (Gleeson, 2006), *Uno's Garden* (Base, 2006), and *Minnie's Diner: A Multiplying Menu* (Dodds, 2004)[1], and conducted a mathematical content analysis of them.

In this chapter I describe the analysis of mathematical talk during the shared book reading sessions[2]. Due to what eventuated to be a limited contribution of students to classroom discussion, the main emphasis of the chapter will be on teacher talk. These data are further analysed to highlight differences in mathematical talk in general, and mathematical content strands in particular, between the three types of books (Sections 8.1.1 to 8.1.4). This is followed by a report of the analysis that focused on the source of mathematical talk pertaining to the text, illustrations or a combination of both (Sections 8.2.1 to 8.2.3). A brief summary concludes Section 8.1 and 8.2 of this chapter.

Inferences will be drawn regarding the nature of classroom mathematical discourse, the influence of the type of book on talk, and the use teachers made of the mathematical opportunities those books present.

The next chapter, Chapter 9, contains a discussion of the activity session that followed the shared reading. I describe how students responded spontaneously to the books in the form of their drawn and/or written representations. Examples of the representations and the accompanying discussions with students are included to illustrate the mathematical ideas they identified. A full review of all findings of Phase 3 is found as the conclusion to Chapter 9.

[1] For the remainder of this chapter the books will be referred to *Amy and Louis, Uno's Garden,* and *Minnie's Diner* only, without referencing, except in headings, to facilitate reading.

[2] Although the teaching strategy is known as "shared book reading", the terms "book reading" or "reading" only will be used for the rest of this chapter except in headings.

8.1 Content of talk

As reported in Chapter 7, the main analysis of the book reading sessions was conducted in three steps. Initially, I coded all talk as either teacher talk (T) or student talk (S) (Step 1). All teacher talk and student talk was then coded according to its content: mathematical or nonmathematical (Step 2). All mathematical talk for both teachers and students was further categorised into mathematical content stands (Step 3).

8.1.1 Step 1: Teacher versus student talk

I used the autocoding function in NVivo10 (QSR International, 2014)[3] to code all talk from each reading session as either teacher talk or student talk. Table 8.1 indicates the proportion of teacher and student talk as percentages of the total talk for each school and for each book. The percentages shown here again refer to discussion of each book between the teacher and students as it was read, and they do not include the reading of the text in the book.

Table 8.1

Teacher and Student Talk for all Books and Schools by Percentage

	Amy and Louis		*Uno's Garden*		*Minnie's Diner*		Mean	
School	T	S	T	S	T	S	T	S
1	73.0	27.0	78.8	21.2	72.9	27.1	74.9	25.1
2	80.8	19.2	87.2	12.8	79.2	20.8	82.4	17.6
3	70.1	29.9	65.6	34.4	72.8	27.2	69.5	30.5
Mean	74.6	25.4	77.2	22.8	75.0	25.0	75.6	24.4

Entries in Table 8.1 clearly indicate that, proportionally, teacher talk was about three times as much as the student talk and this was consistent across books, with means of 74.6% (*Amy and Louis*), 77.2% (*Uno's Garden*), and 75% (*Minnie's Diner*). Teacher 1 averaged 74.9% of talk across the books, Teacher 2 accounted for 82.4% on average of the talk for all books, and Teacher 3 averaged 69.5%. The differences in teacher talk between the three teachers may represent the individual teaching styles of the teachers. These results also demonstrate that the percentage of student talk also differed across schools and books.

[3] NVivo10 (QSR International, 2014) will be not be referenced in full repeatedly in this chapter. It will be referred to only as *NVivo*.

In the remainder of this section I initially consider the content of all talk, both mathematical or nonmathematical from both teachers and students, and following that disregard all nonmathematical talk and consider only the content of mathematical talk, first for teachers and then for students.

Because the teachers dominated the talk, the proportion of student mathematical talk was limited, and this was particularly evident when reporting their mathematical talk according to strand. However, it is still important to include the student data to demonstrate their cognitive engagement with the mathematical ideas in the books in response to Research Question 3.

8.1.2 Step 2: Teacher and student talk — Mathematical versus nonmathematical content

All teacher talk and student talk was coded using Nvivo as being either mathematical or nonmathematical for each book. Table 8.2 shows the percentage of teacher and student mathematical and nonmathematical talk categorised for each teacher and each book. The total teacher and student mathematical and the nonmathematical talk for each book are also calculated as means. An example of this coding can be found in Appendix F.

Entries in Table 8.2 first highlight the differences between the mean combined teacher and student mathematical talk for each book as 39.4% (*Amy and Louis*), 64.2% (*Uno's Garden),* and 66.7% (*Minnie's Diner)*. This reveals that the embedded and explicit books (*Uno's Garden* and *Minnie's Diner*) both initiated more mathematical talk than did the perceived book (*Amy and Louis)*. The entries also indicate differences in mathematical talk for each book when comparing teachers and students. These differences in the teacher mathematical talk and student mathematical talk for each book are discussed in the following sections. These results differ from the pattern shown in Table 8.1, where there were few differences in total proportion of talk between the books.

Most nonmathematical talk for all books related to the literacy aspects of the book such as teachers' questioning of the students about their understanding of the book, and how students would react in a similar situation. For example, Teacher 1 reading *Minnie's Diner* asked the students, "What could the boys smell?" as the brothers in the book approached the diner, and a student responded, "The food."

8.1.2.1 Step 2a: Mathematical teacher talk across the books

Analysis of teacher talk revealed that there was approximately the same amount of mathematical content during the reading of *Uno's Garden* (mean 50.1%) and *Minnie's Diner* (mean 47.1%), but that there was much less mathematical content in the reading of *Amy and Louis* (mean 28.1 %).

Table 8.2

Teacher and Student Mathematical Talk for all Books

	Amy and Louis			*Uno's Garden*			*Minnie's Diner*		
	Mathematical Talk			Mathematical Talk			Mathematical Talk		
School	T	S	Total	T	S	Total	T	S	Total
1	39.6	15.3	54.9	60.5	12.0	72.5	53.8	22.3	76.1
2	23.9	9.1	33.0	61.0	10.4	71.4	44.3	16.6	60.9
3	20.8	9.6	30.4	28.8	19.9	48.7	43.3	19.7	63.0
Mean	28.1	11.3	39.4	50.1	14.1	64.2	47.1	19.5	66.7

Differences between teachers. Table 8.2 also shows that Teacher 1 used more mathematical talk than did the other two teachers. Teachers 2 and 3 asked predominantly "who" and "why" questions, but Teacher 1 asked a lot of questions about "when" and "where" things were happening in each book that led to more time (measurement) and place (geometry) talk. The teachers in Schools 2 and 3 used similar amounts of mathematical talk for *Amy and Louis* and *Minnie's Diner,* but very different amounts for *Uno's Garden.*

Opportunities for mathematical learning. Previously, in the content analysis of each book in Chapter 7, I had found (using NVivo) that 29.5% of the text of *Amy and Louis*, 24.6% of the text of *Uno's Garden,* and 32.4% of the text of *Minnie's Diner* was mathematical. Table 8.2 indicates that, for teacher mathematical talk, there were means of 28.1% for *Amy and Louis,* of 50.1% for *Uno's Garden*, and 47.1% for *Minnie's Diner.* The amount of mathematical content in the text cannot be directly compared with the amount of teacher mathematical talk to determine whether teachers are using the books well. First this is because they represent somewhat different domains. Second, the proportion of mathematical content found in each book only relates to the text (Figure 7.1) and does not include a content analysis of the illustrations, or the potential I found for mathematics investigations in the text and images. However, the findings in Table 8.2 do suggest that the amount of mathematical content evidenced in the teachers' talk during shared book reading of a picture book can differ to a greater or lesser extent from the amount indicated by the analysis of the books' text. In the following sections I use the data from the mathematical content analysis and my observations of further activities as indicators of the potential of each book and discuss it against the teachers' mathematical talk.

Much of the teachers' mathematical talk was produced in their general discussion with the students about the books rather than from explicit mathematical teaching using the books' content. For example, they used prepositions and prepositional phrases, e.g., "in America" (S1T) and terms such as "next" and "now" as in, "What do you think is on the next page?"(S3T) and, "Let's see what happens now" (S3T). Teachers may have been unaware of the potential of these words or phrases for mathematics learning. In Chapter 5, I also noted that the survey participants did not always see the potential for mathematical learning when reviewing the books.

However, there were a few instances, in which teachers did highlight when the mathematics arose naturally from the story and illustrations. Here they clearly intended to use mathematical terms to teach mathematical concepts (e.g., doubling, times, count). They also used explicit mathematical language in their instructions to aid the mathematical concept they were developing, for example "count across the decade" (S2T).

In the following sections I provide examples of how the teachers in this classroom study included mathematical content in their talk, I describe opportunities the teachers used for mathematical teaching and learning, and I relate any suggestions the teachers made for further mathematical activities subsequent to reading the book. I also discuss the opportunities for rich mathematical teaching and learning that the teachers appear to have missed.

***Amy and Louis* (Gleeson, 2006).** Although the mathematical content analysis in Section 7.3 had indicated similar potential for the three books, the amount of mathematical teacher talk for the book with perceived mathematical content, *Amy and Louis,* was not as high as in the other two books. Despite knowing my study's purpose, the teachers rarely identified mathematical concepts and opportunities in this book. Teacher 3 even had an English follow-up activity prepared for this book.

As noted above, much of the mathematical talk arose in the teachers' conversation and questions for story understanding rather than in explicit mathematical teaching of concepts that they encountered as they read this book. For example, "Hands up if you have ever laid down on the grass and looked up at the sky? (T3), "So do those two live in the same house do you think?" (T2) and "How do you think that big coo-ee came to Amy all the way in the city?" (T3). The teachers seemed unaware of their own use of these terms such as "looked up" and "same" that could relate to perspective taking, similarity and congruence (sameness) respectively.

However, the following excerpt illustrates one good example from Teacher 1 who identified the potential for mathematics learning provided by the book. It records a discussion between her and the students about what might be considered a short and a long distance. This was in relation to Louis trying to call Amy when she moved across the world. There was obvious intent here to teach the mathematical idea of measurement that may have been previously part of this class's mathematics program.

Teacher: Now when they were across the yard and they called to each other, were they far, far, far away?
Student: No
Teacher: They could still walk to each other. That is called distance. Distance is when someone is a long way away. If they are a long way away it is a big distance. When we are next to each other that is a small distance. You can put your hands together. Do you think that is a distance? (T1)
Student: No.
Teacher: They are close together. What about this?
Student: Small distance
Teacher: Then if your friend is on the other side of the world it is a …
Student: Big distance
Teacher: A very big distance. Just like Amy and Louis.

Teacher 2 also emphasised this page of the book with the students, but not with the same insight. Teacher 2 said, "Look at that, he is trying to get it to go around the world!" She, too, could have taken the discussion further, incorporating the idea of distance.

Teacher 3 intended to engage the students during this next activity, also related to the same page, but it was not from a specific mathematical perspective.

Teacher: How loud do you think that he went? On the count of three you are going to try. I want you to have a try and pretend that you are Louis. So spread your arms as wide as you can. Throw back your head and in your biggest voice you are going to say: 1, 2,
Students: Coo-ee Am-ee.
Teacher: I don't think she would have heard. That was a bit quiet, let's try it again.
Students: Coo-ee Am-ee.
Teacher: I think definitely would have heard that one.
Teacher: Look it is travelling, where is it travelling to? Through the sky. Past the buildings.

This activity could have connected mathematics to the students' everyday lives and arose naturally from the story and illustrations. Without spoiling the storyline, the teacher could have questioned whether in fact the next class, and the people in the playground or the street, could hear their call. Again the emphasis could have been on distance and measuring. The discussions could also have provided further opportunities, at a later point, for integration of science on the topic of sound, and teacher- and student-initiated problem solving. Students could measure and record how far their call could be heard in straight lines and around corners in the playground. These activities could be easily implemented by the teacher in a manner that would continue to engage the students and possibly promote positive mathematical attitudes.

The following exchange from Teacher 2 about another page also illustrated the potential for mathematical discussion within a book with perceived content. It is difficult to discern whether the teacher's questioning was intended to focus on mathematical ideas or whether she was purely asking comprehension questions about the story. However, more explicit attention to the geometric concept of position was possible without affecting the flow of the story.

Teacher: Where was he? He was across the yard wasn't he? Where was Louis when Amy called to each other [sic]? Was she next to him?
Student: No on the other side of the room.

Another example of a simple missed opportunity for teaching and learning, this time of measurement concepts, comes from the book's text: "they built towers as high as the sky" (Gleeson, 2006, unpaged). A query by the teacher at this point such as, "How high do you think the sky is?" could have been appropriate and could have stimulated mathematical discussion. The teacher could then have suggested that she and the class could engage in measurement investigations about the height of real-life objects, including whether the students could build something that reached the sky, at a later point. This is further discussed in Marston (2012: Appendix G). However, no teacher availed herself of this.

***Uno's Garden* (Base, 2006).** There are two important points to note about the way that teachers approached their reading of *Uno's Garden*. First, all teachers read the introduction/explanatory rhyme located on the title page (see Section 7.3.4.2). This could have been overlooked, but the rhyme was an important indicator to the reader to focus on mathematical ideas within the book. However, none of the teachers discussed this rhyme or referred to it later.

Second, Teacher 2 made a clear statement about the purpose of the book before reading it. This strategy was also noted by Moschovaki and Meadows (2005b) when teachers were reading information-style books. Statement of lesson purpose is a teaching technique encouraged by the Quality Teaching Framework (NSW Department of Education and Training [DET], 2003) and was used widely in NSW schools at the time. Teacher 2 also identified that she knew the books contained mathematical concepts and she would be addressing these saying, "We are having a slightly different maths lesson today. We are looking at multiplying, multiplication, and adding and subtracting". She also concluded the session with, "We have just read *Uno's Garden* and saw lots of doubling", again reiterating her purpose.

Similar to the teacher talk in response to *Amy and Louis*, the discussion associated with the book reading of *Uno's Garden* revealed much conversational language with mathematical content. For example, "Remember the very first page when we had no buildings and lots of place" (T2) and "That is like now, it is the beginning of spring" (T1), where the teachers may not have been aware of the mathematical content used. Additionally, much of the teacher mathematical talk was either a reiteration of the text and the equations that were represented, e.g., "and have a look at the buildings. First there was 1, then 2, then 4, then 8, then 16, 32, 64 and now 128 buildings and one snortlepig" (T1) or asking students where the Snortlepig was.

Considering the number of mathematical opportunities that were obvious in the book, there was only a small amount of intentional teaching based on the book's mathematical ideas and equations. Although Teachers 1 and 2 used a higher proportion of mathematical language, this was restricted to simple questions posed by the text with no extended activity.

For example, "10 groups of 10 animals equals ...?" (T1) and "We had a 100. What makes 100? 10 x 10 = 100" (T2).

After Teacher 2 concluded the session she commented, "We ... saw lots of doubling", but only went on to say to the students, "What happens if I give you the number 2? What do we do? 2+2. Do we add the same number?" Here she missed a suitable teaching and learning opportunity. Her intention was to highlight that doubling consists of adding two numbers that are the same or multiplying them by two, but she did not take this far enough to consolidate the concept of doubling or further investigate the patterning opportunities.

Teacher 3 consistently asked the students to count the number of animals, plants, or buildings, for example, the number of moopaloops. This activity appeared very simple for students of this age group as the number never totalled more than 10. However she did use the book reading as an opportunity to emphasise the doubling concept they had been studying, but again she did not extend the ideas sufficiently to establish good understanding:

Teacher: What are they doing with these numbers?
Student: Umm, keep adding up with plusses?
Teacher: And what do we call that? Dan, do you know?
Student: Umm, doing times tables
Teacher: Doing times tables? There's another word.
Student: Umm
Teacher: Can you think of it? We've done it before.
Student: Adding
Teacher: Adding?
Student: Multiply?
Teacher: Multiply.

The small amount of teacher mathematical talk for Teacher 3 when reading *Uno's Garden* can be attributed to her presentation of the book to the students. She had prepared well, placing post-it notes over the answers on several of the equations in the first half of the book so the students had to calculate the answers each time before she revealed the answer. When she was half way through the book she said, "We're just going to read the whole story now. I want you to enjoy the rest of the story". During this reading time there was still some discussion, but not as much as earlier. However, after completing the book and appearing to finish the session, she stated she had an activity for the students and she returned to the book to review some of the calculations. Here there was explicit teaching based on the doubling concept. However, because the criteria set for the teachers was to read and discuss the book but not to do any activity, this mathematical activity was not included within the analyses related to the shared reading sessions.

Teacher 3 may have used this strategy (reading the second half of the book without explicitly referring to the mathematics) to first read the book for its literary value before returning to the book to highlight the mathematics later. This discussion during her activity was almost entirely focussed on mathematical ideas and did not include "how would you feel" questions, typical of narrative book style questioning (Moschovaki & Meadows, 2005b) that she had used previously.

Therefore, in general, apart from using counting and the mathematical language evident in the book, teachers did little to explore the other concepts in the book by involving the students in meaningful discussion. (See Publication 8, Chapter 10 for other uses of this book).

***Minnie's Diner: A Multiplying Menu* (Dodds, 2004).** All teachers used this book to teach the concept of doubling. Like Moschovaki and Meadows (2005a), when the content was explicit, the teachers stopped to explain it. The textual and visual structure of the book also inclined teachers to present the book more in the style of an information book. Additionally, as the visual images reflected the text, there was little else that afforded mathematical discussion.

Again, Teacher 2 started the session by stating her purpose:

> Now we are going to be reading a book today called Minnie's Diner. Now remember we are looking for maths things. As we are going along, if you start to notice something, put your hand up and let me know.

She did not ask questions about the story's plot or characters unless they related to the mathematical aspects.

There was a higher frequency of mathematical terms for this book as teachers used them during a running commentary of the story, reiteration of the storyline, or numerical calculations initiated by the book. These were included presumably to emphasise and improve mathematical understanding for the students. Teachers often restated students' answers to questions they had posed. For example, after one student noted "They are telling you that 9 x 9 plants is 81" (S2), the teacher responded, "Excellent, they are telling you that 9 x 9 plants equals 81" (T2). As with the other two books, there was also much recorded mathematical content in the teacher's conversation with the students, e.g., "So who is she open for, what do you think she's going to expect to happen?" (T1).

There were few examples of explicit teaching, but two of these follow.

Example 1: School 1

Teacher:	What did the little brother get?
Student:	One.
Teacher:	One of each. His next brother came through the door and he wanted double, so what did he get?
Student:	Two.
Teacher:	So what's double one?

Example 2: School 3

The teacher in School 3 had again prepared for the reading of *Minnie's Diner* by supplying each student with cubes and a paper plate so they could replicate the doubling of serves for each brother.

> Well, let's see how clever you are. Let's pretend this is our beautiful cherry pies, our chips. Now choose the one food that you would like, so you're not going to do all the foods. So choose the one and tell me now with this new brother in order so how much do you think he's going to have on his plate, what will Minnie have for him?

Activities such as this during the reading of the book by all teachers would certainly have facilitated the students' understanding of the doubling concept. Further exemplars of meaningful activities for *Minnie's Diner* were detailed in Section 7.3.4.3. They would not have interfered with the telling of the story but engaged the students in meaningful learning in predicting what was to come next.

8.1.2.2 Step 2b: Mathematical student talk

As noted earlier in Table 8.1, teachers spoke around 75% of the time during each reading session. Of the low percentage of student talk, most talk was answering a teacher's questions, and often these responses were just single words or numbers. There were few spontaneous mathematical comments from the students throughout the reading sessions. This may be the "classroom culture" where they can talk only if they raise their hand or are asked a question.

Teacher 2 had encouraged her students to do this if they noticed anything mathematical before reading *Minnie's Diner*, as reported earlier. Either students did not notice the mathematics or did not show it this way. Many of their responses had no specific mathematical intent either, but, similar to the teachers, mathematical terms were used in the students' general conversation. It should be noted here, too, that although a student response may have been mathematical, it was not necessarily a correct response.

Differences between student groups. Similar to the teacher mathematical talk, there were differences in student mathematical talk across groups for each book. *Minnie's Diner* had the highest mean proportion of all talk (19.5%) that was mathematical. During the discussion of *Uno's Garden,* this was 14.1%, and for *Amy and Louis*, 11.3%. These differences could be due to several reasons including experience of students, differences in ability, and teacher questions and questioning techniques.

The following sections include examples and discussion of the student mathematical talk for each of the three books.

***Amy and Louis* (Gleeson, 2006).** As the amount of teacher mathematical talk was lower for this book, student responses were limited. Much of this was again conversational.

Examples of simple conversational use included "I did it at the beach" (S3), they went "to New York" (S2), "We did not have enough room in our old house" (S1), and, from School 3:

Student: I have read this one before.
Teacher: You have? You read it at home did you?
Student: No we had it in the library last year.

The following two examples demonstrate mathematical language by students that could have elicited further mathematical conversation. The two teachers here did not identify the mathematics suggested by the students, and as a result they missed spontaneous mathematical teaching and learning opportunities. If teachers make use of these opportunities, they provide meaningful situations for student learning.

First, after the teacher had read one double page (Figure 7.2) and shown the accompanying illustration in this book, one student commented spontaneously, "That means they live on the other side of the fence" (S2), thus showing they had understood the position (geometry) concept described. Additionally, the exact placement of Amy and Louis' homes can be deduced by the way the children climb through the fence. The teacher could have developed this idea herself, or asked the child who had called out to explain their reasoning, thus enhancing the meaning of the story using mathematical language. The ideas of other side, next to, and basic mapping could have been pursued further to ensure all students grasped both the story and the mathematics.

Another comment from a student in School 3 was interesting. In the design of the book, the words "Coo-ee Lou-ee" and "Coo-ee Am-ee" are never written straight or parallel with the rest of the text, they are always curved or diagonal, for example:

Coo-ee Aim-ee or Coo-ee Lou-ee

A student noticed this and the following conversation ensued:

Teacher:	What do you notice about "Coo-ee Lou-ee"? She is calling for him. What do you notice about "Coo-ee Lou-ee"?
Student 1:	It is going on the side not straight.
Teacher:	Yes it is on the side, sloping.
Student 2:	It sounds like a bird's call.
Teacher:	Anything else? "Coo-ee Lou-ee".
Student 1	It sounds like a shout.
Teacher:	Do you think they might be rhyming words?

Here the teacher had been looking at the word structures of rhyming words (discussed in Chapter 6 as a form of pattern) while the student had actually identified the position of the words on the page.

Finally, the genre of the book appeared to influence the student talk as found by Moschovaki and Meadows (2005a). Narratives like *Amy and Louis* encourage students to talk about their feelings and relate to their own experiences (socioconstructivist approach). Students gave examples that supported this finding, e.g., "My friend moved to Queensland" (S1). This did not happen with the following two books.

***Uno's Garden* (Base, 2006).** During the reading of *Uno's Garden* much of the mathematical content was single word answers to direct mathematical questions from the teacher. For example, “They are telling you that 9 x 9 plants is 81” (S1), “The buildings are getting more and the plants are getting smaller” (S2) and “16 plus 16 equals 32” (S3).

The following examples highlight two occurrences when student comments could have been pursued further mathematically without spoiling the flow of the story.

Example 1: School 2

Teacher: Now let’s have a look, we have one snortlepig, one leafy tree and one old shack. What do you think is now going to start happening. All the people have left and we are back to one, one and one. What do you think will happen?

Student: I think it will start to go back the other way and there will be more plants and animals.

Here I believe the teacher was taking a more environmental perspective with her question about what was going to happen next (to the forest). The student’s response could then have prompted rich discussion of increasing and decreasing numbers, but this did not happen.

Example 2: School 1

Teacher: Has anyone ever heard of the game *Uno*?

Student: Yes I have and Josh has the game. It has numbers and stuff.

Teacher: Numbers and stuff?

Student: Yeah and you put a card down and you try to look at the colour and if you don’t have the colour you can put the same number down.

Uno is a commonly played commercial mathematical matching and strategy game. A more explicit discussion of the mathematical aspects of the concepts of the game e.g., “same and different” would have been appropriate and beneficial here.

The only other spontaneous student talk that was mathematical during the reading of this book was the students constantly asking, “Where is the snortlepig?”

***Minnie’s Diner: A Multiplying Menu* (Dodds, 2004).** The reading of *Minnie’s Diner* also elicited both explicit mathematical terms and numbers in students’ responses to their teacher’s questions, and in mathematical language within their general conversation. Those in response to the teacher included, “It's littlest to biggest”, (S1) “He's going to get four” (S1), “They are getting bigger” (S2), and “He's going to get three times more food” (S3).

Although there were inaccuracies in the following discussion, the student (S2) has used mathematical language and terms in response to the teacher.

Teacher: It could be. What is the special word for that?
Student: Doubling. It is like 1+1+1+1 is 4.
Teacher: Yes very clever.
Teacher: What does that word mean?
Student: Twice means two, a bit higher.
Teacher: Do we know how much bigger?
Student: 2 times bigger.

The following examples all show unprompted comments during the book reading: “He might not be able to fit through the door” (S1); “It is like *Uno’s Garden* where there is 1, 2, 3, 4, 5, 6” (S2); and “It's in a restaurant. It's in a restaurant” (S3). The first two of these comments required mathematical thinking as opposed to the third that was only conversational. Another student in School 1 commented about *Minnie’s Diner* being “like a counting book”. These comments could have provided good opportunities for the teachers to question the students for their mathematical reasoning and to use rich mathematical conversation. Interestingly, too, a student, rather than the teacher in School 3 jumped up and found a class number chart for the students to look at while they were calculating their doubling. In response, the teacher used the chart briefly, but not as richly as was possible.

Elia, Van den Heuvel-Panhuizen, and Georgiou (2010) found only 27% of the language used by their student participants with the explicit book, *Six Brave Little Monkeys in the Jungle* (O’Leary, 2005), was mathematical. However, in their study they provided specific reading guidelines. They also counted the number of utterances rather than calculated the percentage of all mathematical talk as shown in the present study. Nevertheless, their observation at the time was that these books were not useful without the right questions elicited from the teacher. The findings of my study are consistent with their insights.

8.1.3 Step 3: Mathematical strands

After coding all talk used by both teachers and students as mathematical or nonmathematical, I coded all the mathematical talk according to mathematical strands. Tables 8.3, 8.4, and 8.5 indicate the distribution of the different strands of mathematics for each book. Because the unit of analysis used was the conversational or discourse turn, some turns may have included more than one mathematical strand or concept. For example, “twice as big as his brother” includes both number and measurement concepts, but, as in the mathematical content analysis (Chapter 7), the turn was coded by the dominant number strand. Further examples are provided in the following sections.

Previous researchers (e.g., A. Anderson, Anderson, & Shapiro, 2005; Van den Heuvel-Panhuizen & Van den Boogaard, 2008) found that the language of number and size (often related to the use of adjectives) were the most commonly used by their participants. In this study, although number featured in varied amounts during all book readings, few of the utterances for any of the books involving the language of measurement related to size, as will be seen in the following sections. This was probably due to the influence of the content of the different books used.

***Amy and Louis* (Gleeson, 2006).** The initial mathematical content analysis of the text of *Amy and Louis* revealed about 30% of the book's text used mathematical language. Geometry (particularly positional language) comprised 65.1% of that mathematical content, and measurement 31.5%, and there was little evidence of number concepts (Table 7.1). Table 8.3 shows the distribution by percentage of all teacher mathematical talk and student mathematical talk in each school by strand for *Amy and Louis.* It is interesting to note that the mathematical language of T1 and T2 concentrated on geometry and measurement concepts, but T3's talk included all concepts except statistics.

Table 8.3

Teacher and Student Mathematical Talk by Strand During Reading of Amy and Louis

Strands	T1	S1	T2	S2	T3	S3	Mean T	Mean S
Algebra	0.0	0.0	0.0	0.0	12.5	0.0	4.2	0.0
Geometry	60.9	75.7	90.2	70.7	57.8	91.0	69.6	79.1
Measurement	37.7	24.3	9.8	0.0	17.2	9.0	21.6	11.1
Number	0.0	0.0	0.0	0.0	7.4	0.0	2.5	0.0
Probability	1.4	0.0	0.0	29.3	5.0	0.0	2.1	9.8
Statistics	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Entries in Table 8.3 reveal that references to the language of geometry by both teachers and students were also the most prevalent during the reading session. In fact, as a proportion of their mathematical talk, students (mean 79.1%) used these references more than did teachers (mean 69.6%). This language predominantly comprised positional terms related to prepositions and prepositional phrases, as highlighted in the analysis of the text (see italics). For example, "The clouds are going *across* the ocean" (T2), "Let's see what happens *in* the book", (T1), "My *next door* neighbour, she was (my) best friend, (she) moved *to* America" (S1), and "*In* the city" (S2). Teachers drew attention to geometric shapes only very occasionally, e.g., "They used to look *at* the clouds and make *shapes*" (T1).

The proportion of mathematical language relating to measurement was varied. Words describing length featured more in the language of the teachers than in the language of students. Examples for teachers and students include the previous discussion of distance in School 1, "they are moving a *long way* away" (T2), and "We did not have *enough room*" (S1). Students however used the language of time more than did teachers, for example, "*When* Louie called to Amy" (S3), "*last year*" (S3), and "I have read this one *before*" (S3), and for teachers "it is *night time* for her now" (T1).

There was only a limited reference to number concepts, mostly around the term "*half*", which features in the book, and, "On the count of *three* you are going to try (to call)" (T3). Use of the language of probability was also minimal, with statements such as "*maybe* he will make some new friends. (S2) and "Grandma said *maybe*" (T1).

***Uno's Garden* (Base, 2006).** Table 8.4 indicates the proportions of teacher mathematical talk and student mathematical by strand for *Uno's Garden*. The table demonstrates that the language of number featured highly for both teachers (70.7% mean) and students (68.7%). This had also been the case in the earlier mathematical content analysis of the book, where 58.5% of the mathematical content related to number concepts (Table 7.2). The author, Graeme Base (Chapter 6), also emphasised that the added layer of mathematics referred to number. Entries in Table 8.4 also demonstrate that, although number was always the most dominant strand for teachers and students for each book, there were noticeable differences. The lower response by students in School 1 relative to that used by their teacher may suggest that the mathematical ideas were not conveyed well to these students. However, the students in School 3 appeared to identify these more than did their teacher.

Table 8.4

Teacher and Student Mathematical Talk by Strand During Reading of Uno's Garden

Strand	T1	S1	T2	S2	T3	S3	Mean T	Mean S
Algebra	4.8	3.0	1.6	9.2	1.1	3.3	2.5	5.2
Geometry	9.9	41.9	8.9	2.4	35.4	23.1	18.1	22.4
Measurement	6.3	0.0	5.2	8.4	11.9	2.7	7.8	3.7
Number	78.2	55.1	84.3	80.0	49.7	71.0	70.7	68.7
Probability	0.8	0.0	0.0	0.0	1.9	0.0	0.9	0.0
Statistics	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

As previously stated, much of the teacher talk using the language of number was a reiteration of the text, e.g., "We have 10 moopaloops, 10 x 10 plants, 0 buildings and 1 snortlepig. So 10 lots of 10 is 100 plants" (T1). Additionally there was much talk about counting the characters in each illustration e.g., "Could you count the seven playful puddlebuts?" (T3). The "embedded" equations and not the text (story) was the trigger for most of the number talk and questions to the students, e.g., "4 x 4 is 16: 4 groups of 4. The buildings are getting bigger" (T2). For students, the number language was predominantly simple numerical answers.

The initial mathematical content analysis of the book's text also revealed that the language of geometry and measurement both featured in the mathematical text. The teacher and student mathematical talk also comprised geometry (18.1% for teachers and 22.4% for students), and measurement concepts (7.8% for teachers and 3.7% for students), although this differed across teachers and students. Both teachers and students used these terms predominantly in conversation, e.g., "What can you tell me about the animals on the cover?" (T3) and "He looks like a bunny with little ears" (S3). Frequent use of these terms by a teacher did not necessarily produce the same response in the students. Additionally, the teachers did not take advantage of these conversational opportunities to explore these concepts.

One point of interest for *Uno's Garden* was the terms used by teachers when they read from this book. In the first half of the book, the numbers of plants and animals decrease while the number of buildings increases. In the second half of the book, the opposite happens until an environmental balance is reached. However, the teachers did not always use the correct mathematical terms when describing this to the students. They used language such as, “Some things are going down, some things are going up” (T2), “What is getting bigger?” (T2), “What happened when the buildings kept going higher and higher and higher? (T1), and “What happened to the size of the animals?” (T2). There was also mixed use of terms, as in “The animals are getting smaller, the buildings are getting bigger and the plants are getting less” (T2). Here teachers were referring to the increase or decrease of the numbers of each item, not the size of the animals or buildings. The students then tended to copy that language, e.g., “they (the animals) are getting smaller”. I coded this language as the number strand even though they did not use number terms, as that appeared to be their intention. If I had coded these words individually, rather than as I did as a turn (and its intention), I would have coded terms such as “size” and “bigger” as measurement and this would have produced different results and a much smaller use of the language of number.

***Minnie’s Diner: A Multiplying Menu* (Dodds, 2004).** The mathematical content analysis of the text of *Minnie’s Diner* (Table 7.3) indicated a low proportion of mathematical content. Although the focus of the book was doubling, there were more words and phrases associated with geometry and measurement used in the book than those related to number.

Table 8.5 contains the distribution of teacher mathematical talk and student mathematical talk by percentage for each strand. Here teachers concentrated on the number strand (74.4% mean). There was emphasis on the doubling concept by all teachers, e.g., “So what’s double 2?” (T1), and counting of the food on the trays to check calculations. However, Teacher 1 also included terms such as “count off the decade” and “count on the decade” when she was helping the students to double 16.

Table 8.5

Teacher and Student Mathematical Talk by Strand During Reading of Minnie's Diner

Strand	T1	S1	T2	S2	T3	S3	Mean T	Mean T
Algebra	0.0	3.1	0.0	7.5	6.9	1.0	2.3	3.9
Geometry	20.3	3.3	4.2	0.0	8.1	4.3	10.9	2.5
Measurement	4.8	21.1	9.3	29.5	14.5	21.8	9.5	24.1
Number	69.1	69.8	86.5	63.0	67.6	69.2	74.4	67.3
Probability	5.8	2.7	0.0	0.0	2.9	3.8	2.9	2.2
Statistics	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Student mathematical talk also focussed on the language of number (67.3% mean). Again, many of these were answers to number calculations asked by the teacher. There were also student statements such as "Add up another two that equals four. It's like a counting book" (S1), "I think we will have 60 on the next one" (S2), and "So it's twice as big" (S3).

The total use of language associated with geometry by both teachers and students was low, so there were few examples found that reflected this concept. Teacher 1 frequently used prepositional phrases e.g., "Somebody is still working on the farm".

Students in Schools 1 and 2, particularly, used proportionally more language of measurement than did their teachers as they noticed ideas pertaining to the size of the brothers, e.g., "It's littlest to biggest" (S1), "I think the next one he might not be able to fit through the door (S1)". Despite the higher use of measurement language by Teacher 3 relative to Teachers 1 and 2, from frequent asking of, "What do you think will happen next?", students still responded with similar proportions to the students in Schools 1 and 2. Comments included size and time concepts such as, "He'll get the food like his little brother" (S3).

8.1.4 Summary for content of teacher and student talk

The results indicate that teachers dominated the discussion, speaking on average 75% of the time. Except for a few spontaneous instances, all student talk was in response to teachers' questions.

Teachers and students both used mathematical language throughout the reading of the three books.

More mathematical language was used during the shared book readings of *Uno's Garden* (64.2%) and *Minnie's Diner* (66.7%) than during the reading of *Amy and Louis* (39.4%). This is different to the mathematical content analysis of the text, which showed similar amounts (about 30%) for all books.

Number concepts dominated the teacher and student mathematical talk of *Uno's Garden* and *Minnie's Diner*, while geometry was the dominant strand in *Amy and Louis.* However, much was conversational mathematical language, particularly in *Amy and Louis.* This was also the case in the mathematical content analysis of the text of this book.

Teachers did not appear to notice as many mathematical opportunities as I had highlighted in my analysis of the books, although there were a few good examples of rich teaching. Students did notice some mathematical aspects that were not always used by the teachers.

A more detailed summary and discussion of the findings of this classroom study is found at the conclusion of Chapter 9.

8.2 Source of mathematical talk

An analysis, again using NVivo, was also undertaken to establish the source of teacher and student mathematical talk. As stated in Section 7.4.3.2, the book (text, images, or a combination of both) was the original source of most mathematical talk during the book reading. Although I have analysed teacher mathematical talk for the source using these three categories, it is important to remember that Kiefer (2008) pointed out that because of the interrelationship between the text and illustrations it is sometimes difficult to decipher each component's input. In the following subsections I describe the analysis of the source of the mathematical talk and examine differences between the talk evoked by the three different books.

8.2.1 Source of teacher mathematical talk

Table 8.6 contains the mean percentage for the source of all teacher mathematical talk for the three books. It demonstrates the predominance of the combined text and illustrations as the main source of teacher mathematical talk. The results are similar for A*my and Louis* (52.1%) and *Uno's Garden* (50.3%). The much higher result for *Minnie's Diner* (86.1%) is not unexpected as the text is replicated in the illustrations, and therefore a reference to one is tied to a reference to the other. In *Amy and Louis* and *Uno's Garden,* where images and text complement each other, the text (15.7% and 24.9% respectively) and visual images (32.2% and 19.5% respectively) also play individual roles. The text played a very small role (1%) in *Minnie's Diner*, and images also made a small contribution (12.5%).

Table 8.6

Source of all Teacher Mathematical Talk by Books

Book	Text	Image	Text & image	0ther	Total
Amy & Louis	15.7	32.2	52.1	0.0	100.0
Uno's Garden	24.9	19.5	50.3	5.3	100.0
Minnie's Diner	1.0	12.5	86.1	0.4	100.0

In the following sections, I compare the source of all teacher mathematical talk for the three books.

***Amy and Louis* (Gleeson, 2006).** Table 8.7 contains a breakdown by percentage of the source of all teacher mathematical talk for *Amy and Louis.* Although text and image combined was the main source in all schools, there were variations between the teachers for the importance of each book component (text, image and a combination). For Teacher 1, the combination of text and image was the dominant source and text and image individually played a lesser role. Teacher 3 focussed on the images individually as well as in combination with the text, while Teacher 2 appeared to focus on all components.

Table 8.7

Source of Teacher Mathematical Talk for Amy and Louis

Teacher	Text	Image	Text & image	Other	Total
1	11.3	17.7	71.0	0.0	100.0
2	25.3	37.0	37.8	0.0	100.0
3	10.6	42.0	47.4	0.0	100.0
Mean	15.7	32.2	52.1	0.0	100.0

Utterances such as “Where was he? He was across the yard wasn’t he? Where was Louis when Amy called to each other? Was she next to him?” (T1), “Look at that. He is trying to get it to go around the world! Do you think that it will go across the world?” (T2), and “How do you think that big coo-ee came to Amy came all the way in the city?”(T3) can all be attributed to a combination of text and image.

“There are some interesting words they use here” (T1), are they moving close by? (T2) and “Why do you think that Grandma might be different to mum and dad?” related only to the text, with no information in the illustrations to prompt the teacher.

Finally, “What is out the window?” (T1), “Are there any buildings that might give you a clue as to where they are moving?” (T2), and “It is on the side, sloping” (T3) all referred only to the illustrations.

***Uno’s Garden* (Base, 2006).** The analysis for sources of mathematical talk in Table 8.6 (mean percentages for the source for the three teachers) showed the images and text combined were the dominant source of teacher mathematical talk for this book. However, Table 8.8 indicates that when given a breakdown by percentage of the source of teacher mathematical talk for *Uno’s Garden* only there was a greater variation in their source across teachers.

Table 8.8

Source of Teacher Mathematical Talk in Uno’s Garden

Teacher	Text	Image	Text & image	Other	Total
1	16.0	5.5	75.6	2.9	100.0
2	30.1	10.8	55.8	3.2	100.0
3	28.5	42.3	19.6	9.7	100.0
Mean	24.9	19.5	50.3	5.3	100.0

Entries in Table 8.8 indicate that Teachers 1 and 2 concentrated more on a combination of the text and illustrations and less on text and images separately, while Teacher 3 focussed predominantly on the illustrations. This was from constant asking about where the snortlepig was and how many of each animal the students could see. In a book where both text and illustrations complement each other, all components have evoked mathematical comments. However, in such a visual book it is surprising that the illustrations alone did not play a larger role for all teachers. The differences between teachers may be due to each teacher's perception of the genre of the book and their different teaching styles.

All teachers made mathematical directive utterances during the reading of this book, e.g., "Move up a bit" (T3) so the students could see. These utterances were coded as "other".

***Minnie's Diner: A Multiplying Menu* (Dodds, 2004).** *Minnie's Diner* is an information-style book (explicit) where the text and visual images reflect each other. Table 8.9 contains the distribution by percentage of the source of teacher mathematical talk. It clearly indicates a combination of both of these book components as the foremost source of teacher talk. The teachers used these predominantly to develop mathematical concepts in the students.

Table 8.9

Source of Teacher Mathematical Talk in Minnie's Diner

Teacher	Text	Image	Text & image	Other	Total
1	2.1	30.5	67.4	0.0	100.0
2	0.0	5.4	94.6	0.0	100.0
3	0.8	1.6	96.3	1.3	100.0
Mean	1.0	12.5	86.1	0.4	100.0

Teacher 1 rarely used the images separately in the very visual book, *Uno's Garden*, or in *Amy and Louis* where they complemented the text. However, she focused on them in *Minnie's Diner* more than did the other two teachers. She used the illustrations first to discuss the scene, e.g., "There's some pick-up trucks because this is in America" (pointing to the trucks). Second she (and the students) counted the food on the plate to find the answer to the doubling activity before reading the page with the answer, e.g., "Let's count them. We will have to count by twos. Let's see if we can count the soups by two. Two, four, six, eight, ten. We didn't get the ones underneath" (T1).

8.2.2 Source of student mathematical talk

Previously it was reported that the teachers spoke about 75% of the time during reading sessions (Table 8.1). Therefore student talk was limited. Their talk was predominantly responsive and indicated few spontaneous comments. Although I recognise that the primary source of all talk was the book itself (text, images, and a combination of both), the analysis of the students' talk reinforced the previous finding that their talk was primarily initiated by the teacher.

In the case of *Amy and Louis,* the teacher was the sole influence on student talk. However, there were a few times when students reacted spontaneously (without questions or other statements that would prompt discussion) to the text, images, or both, during the reading *Uno's Garden* and *Minnie's Diner.* All instances occurred when teachers turned a page and a student reacted immediately to the illustration or after the teacher had read the text but had not asked a question. This lack of spontaneity may have been due to classroom culture as mentioned earlier. Although Moschovaki and Meadows (2005a) also recognised the input of the reader, and only counted responses initiated by students, they appear to have recorded a much greater number of spontaneous utterances. Therefore, any comparison is difficult. They had also found that students usually "take the initiative to participate when they can make comments or ask questions related to the illustrations" (2005a, Section 4, para. 1). These differences may be both cultural, as their study was conducted in Greece, and the younger age of the children.

During the reading of *Uno's Garden,* one student in School 1 called out, "There it is" pointing to the snortlepig in the illustration. There were no spontaneous reactions to text or images by students in School 2. For Teacher 3, the quest to find the snortlepig in illustrations encouraged students to call out on several occasions, "There he is". It is surprising that the reactions to the illustrations are so few as they are very vivid and intricate. Further visits to the book may change this. There were also a few references to the text by students in School 3, e.g., "It goes biggest to smallest" and "It's going backwards", while on one occasion all the children impulsively joined in reading the text with the teacher.

The illustrations in *Minnie's Diner* also stimulated five spontaneous comments from students in School 1, e.g., "It's littlest to biggest" and "They had the same faces. They're all the same, it just gets bigger, it just gets bigger and bigger". These were both about different illustrations of the brothers. The comment, "That one's coming off, those two have dropped", was about a large tray of food. The text and images combined prompted comments in all schools (six in School 1, and one each in both School 2 and 3). For example, "That's heaps. Is he going to fit that in? "(S1), "It is like Uno's garden where there is 1, 2, 3, 4, 5, 6 "(S2), and "He's got 8 pies" (S3).

8.2.3 Summary for source of teacher and student mathematical talk

This analysis of the source of mathematical talk has indicated two main findings. First, the teachers used both text and images predominantly (mean 86.1%) and text or images individually to a lesser degree during mathematical talk. This occurred whether the mathematical talk was intended or included inadvertently in discussion with students.

Second, student talk in this study was influenced strongly by the teachers' comments and questions about the book and its components. As noted in Section 8.2.2, there were only a few occasions when students made spontaneous comments about the text, images, or a combination of the two. Therefore this analysis of the source of student mathematical talk did not yield similar results to those of other studies due to different procedures and methods of analysis. For example Shapiro, Anderson, and Anderson (1997) recorded verbal and nonverbal events, and Van den Heuvel-Panhuizen and Van den Boogaard (2008) actually instructed their students to talk about the illustrations (Section 2.3.3).

8.3 Conclusion

In this chapter I have reported the results of the analysis of the shared reading sessions. I first described the distribution of the talk according to teachers and students and the content of that mathematical talk. This was followed by an analysis of the source of both teacher and student mathematical talk. Each section was followed by a brief summary of the findings.

In Chapter 9 I describe the next part of the classroom study: the students' spontaneous responses through representations and discussions. These responses, too, are assessed for their mathematical content and source. A final summary and discussion of findings of Chapters 7, 8, and 9 concludes Phase 3 of the thesis.

CHAPTER

9

PHASE 3:
THE CLASSROOM STUDY – STUDENT REPRESENTATIONS

This chapter represents the third part of Phase 3. In it I first provide descriptions and analyses of the students' representations and the accompanying relevant discussion collected during the activity sessions. This is followed by a discussion of the findings across all of Phase 3.

It will be recalled (see Section 7.4.2.2) that after each shared book reading[1] session, the teacher returned to the rest of her class and I continued working with the small group of students for the activity session. I asked the students to "draw a picture or write about what comes into your mind when you think about the story" for each of the three books. This encouraged a spontaneous response from the students without the influence of the teacher—although I realise that questioning by the teacher during the reading session may also have had some influence. I then interviewed each of the students individually about their representations while the other students were engaged in another activity in the same room. All discussion was audio recorded.

After each session, I first coded each student's representations according to school (School 1, 2, or 3) and student (Student A–S) to preserve anonymity. I also digitally scanned all representations. Later, I analysed each representation first for mathematical content (i.e., words or drawings that contained mathematical concepts). The mathematical content was then coded according to the six mathematics strands of number, algebra, geometry, measurement, statistics, and probability, as was done for the mathematical talk in Chapter 8. I also analysed the students' representations for their source as to whether text, illustrations, or both had inspired the students' writing or drawing.

Audio recordings of the discussions obtained from the nine activity sessions were subsequently transcribed by a research assistant. A full analysis of the transcripts was beyond the scope of this thesis but is subject to postdoctoral analysis. Pertinent excerpts are used for reporting in the following sections.

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[1] Although the teaching strategy is known as "shared book reading", the terms "book reading" or "reading" only will be used for the rest of this chapter as equivalents, except in headings.

9.1 Results of students' representations of shared book reading, and accompanying discussion

In total, 45 representations were provided by the 16 students for the three books [2]. For these representations, 27 students chose to draw only, one provided a written statement only, and 17 chose to draw and write words or numbers. In Table 9.1, I provide a summary of the analysis for mathematical content found in the students' representations for each book by school. The table highlights the number of representations produced by the students for each book. It should be noted that the number of students in each group differed slightly. Twenty-seven of the 45 representations contained mathematical content. The remainder contained no mathematical content, some of which appeared unrelated to the books. Mathematical content was included in 10 of 15 representations in response to *Amy and Louis*, five of 15 for *Uno's Garden,* and 12 of the 15 for *Minnie's Diner*.

The criteria used for identifying mathematics in students' representations in general related to the recognition of mathematics concepts such as number, geometry (position and shapes), and measurement. Positions, shapes, and quantities of different representations of objects are inherent in all children's drawings. Therefore a drawing of a person could be analysed, in other research, as containing shapes such as circles, ovals, and straight lines. In this case, I was looking for mathematical concepts similar to those found in the mathematical content analysis. These included numerical sentences (e.g., 2 x 2 = 4) similar to those used in *Uno's Garden,* or derived from *Minnie's Diner*, maps evidencing students' understanding of position (geometry) and clocks showing time (measurement) for *Amy and Louis,* and different views/perspectives (geometry) such as those in the images of all books. Representations that included a number of the same item in their drawings, for example, two mushrooms, were not coded as mathematical, unless the number of items was relevant to the book, for example, matching the number of brothers in *Minnie's Diner.* Otherwise, all drawings with one or more items would have been regarded as containing number concepts, and thus as being mathematical.

[2] Representations do not total 48 (1 per student per book) due to absence from school: one from School 1 for reading of *Amy and Louis*, one from School 3 for *Uno's Garden,* and one from School 1 for *Minnie's Diner.*

Table 9.1

Number of Mathematical and Nonmathematical Representations for Each Book by Group (School)

	Amy and Louis		*Uno's Garden*		*Minnie's Diner*	
School/ Group	Non-mathematical	Mathematical	Non-mathematical	Mathematical	Non-mathematical	Mathematical
1 (n = 5)	4	1	3	1	3	1
2 (n = 5)	1	4	3	2	0	5
3 (n = 6)	0	5	4	2	0	6
Total (N = 16)	5	10	10	5	3	12

The number of mathematical representations produced by the students also differed across the schools. Students in School 1 produced only three mathematical representations; students in School 2 produced 11; and students in School 3, 13. Several findings are noteworthy. First, the students from School 1 produced so few mathematical representations when previously, during the reading sessions, students from Schools 1 and 3 had produced a similar amount of mathematical talk across the three books. Second, students in School 2 had shown the lowest use of mathematical talk for all three books during the reading sessions, but 41% of the mathematical representations here. Third, Teacher 2 had elicited the highest use of mathematical talk of the three teachers when reading *Uno's Garden* and the students produced their least number of mathematical representations for that book (See Table 8.2).

It is also interesting that, for the reading of *Amy and Louis,* only two of the 10 representations with mathematical content were from the girls. Almost all the female students tended to draw "pretty" pictures unrelated to the mathematical content. In *Uno's Garden,* three of the five mathematical representations were from girls, while for *Minnie's Diner* four of the twelve representations were from girls. Representing the results differently, 37% of all mathematical representations were from females who made up 37.5% of the participants—demonstrating an essentially identical distribution of mathematical representations across genders.

The representations were also analysed for their source. The mathematical features identified in Tables 9.3—9.5 were analysed according to the component of the book that appeared to have inspired the students' representations: text, image, a combination of the two, or other. For example, the maps drawn in response to *Amy and Louis* were prompted only by the images in that book. The category "other" refers to representations that may contain the character/s from the book but bear no connection to the book's purpose or storyline. For example, Student 2F drew Amy from *Amy and Louis* with a crown, two butterflies, and hearts because she thought that doing so looked pretty. The content and focus of the teacher-student discussion during the reading sessions (reported in the previous chapter) that was directed by the teachers, could also have influenced what students drew or wrote and therefore limited determining the extent of the influence of the book alone. This was not considered here (See Section 11.2.5).

Entries in Table 9.2 indicate that the images generated 20 of the 27 mathematical representations; six were inspired by the text and images, and only one by the text alone. Entries in Chapter 8 indicated how it was not possible to discern the source of student mathematical talk in the reading sessions as students were dominated by the teachers. However, the results in Table 9.2 demonstrate that the source of 18 of the 25 (72%) students' mathematical representations was the images. This differs from the teachers, who used a combination of the text and images more often as the source of their mathematical talk.

Table 9.2

Source of Mathematical Representations for Each Book by Group (School)

	Amy and Louis			*Uno's Garden*			*Minnie's Diner*		
School/ Group	Text	Image	Text & image	Text	Image	Text & image	Text	Image	Text & image
1			1		1			1	
2		4		1		1		4	1
3		5						3	3
Total	0	9	1	1	1	1	0	8	4

9.1.1 Analysis of mathematical content by individual book

In the following sections I first provide the descriptive details of all representations for each book (in Tables 9.3, 9.4, and 9.5). A discussion is also provided, along with a total of seven samples of the mathematical drawings (with pertinent commentary) that include examples of mathematical concepts. The remaining drawings with mathematical content were not analysed in depth because of their rudimentary features. Copies of all representations are found in Appendix F. It is interesting to note that although there were more representations with mathematical content created as a result of exposure to *Minnie's Diner,* the majority of examples presented here are in response to the reading of *Amy and Louis*. This predominance will be discussed in the following section.

9.1.1.1 Amy and Louis

Entries in Table 9.3 refer to the content of the representations from the reading of *Amy and Louis.* The table also indicates the source of all representations. Ten of the 15 representations for this book were regarded as containing some mathematical content. Some representations were complete replications of images in the book that contained mathematical ideas. For example, the pictures by Students 3N and 3R have a furniture van with an arrow, which could be demonstrating a directional concept, and 1E's drawing of Amy lying with her hands behind her head could show positional understanding. Alternatively, the representation by Student 1A also has Amy lying with her hands behind her head but he has added components showing perspective and position. Some students showed other perceptive mathematical observations in their maps (geometry, position) and clock faces (measurement of time). Entries in Table 9.3, and the following examples, provide additional details.

The representations that were coded as mathematical were also categorised according to the mathematical content strands used previously for both the mathematical content analysis and the analysis of mathematical talk in the reading session. Of the 10 mathematical representations, five included space/geometry (e.g., position, mapping) concepts, one included measurement-related concepts (height, space), and four included both geometry and measurement. This analysis of mathematical content in the representations reflected the predominance of geometry and measurement that was also found in the content analysis of this book. The use of geometric terms was also high (mean of 79.1% of the total mathematical talk) in the analysis of students' verbal responses during the reading sessions.

Figures 9.1 to 9.4 are representations from four of the students. They are chosen as examples of geometry and measurement concepts that reflect observant features. The drawings are accompanied by the transcript of the associated discussion between me and each student. Three drawings (Figures 9.1, 9.2, and 9.3) are very similar to each other with a strong mapping theme, but they do include differences in the drawn features and the students' descriptions that demonstrate different understandings of mapping. Although the drawings are not accurate maps, they are in the style of the maps in the book and the level expected of students at this stage of schooling.

Table 9.3

Content of Representations in Response to the Reading of Amy and Louis

School	Student Code	M/ F	Non Mathematical (D=Drawing: W=Writing)	Mathematical (D=Drawing: W=Writing)	Strand	Source: T= Text I= Image O=Other
1	1A	M		D: Amy (hands behind her head, as in book image) & Louis outside a house, clouds and plane in the sky	Geometry (position, perspective)	T, I
	1B	F	D: Amy & Louis W:Amy, Louis & "Coo-ee"			T, I
	1C	M	D: Plane			T, I
	1D	M	D: Plane W: Louis & Amy"			T, I
	1E	F	D: Amy & Louis W: (in speech bubbles) "Coo- Amy" & "Coo-Louis"			T, I
2	2F	F	D: Amy with a crown, Louis, 2 butterflies and lots of hearts			O
	2G	M		D: People at a crossing, road, traffic lights, tall buildings, clock face (Fig. 9.1)	Geometry (position) Measurement (time, length)	I
	2H	M		D: Detailed map with tall buildings, clock face with hands, cars (Fig. 9.2)	Geometry (position, perspective) Measurement (time, length)	I
	2J	F		D: Tall building, Amy with hearts around her head, Louis W: "New Youk [sic] known as fairyland"	Measurement	I
	2K	M		D: Map, traffic lights, roundabout, tall buildings, 2 people, clock face with hands (Fig. 9.3)	Geometry (position) Measurement (time, length)	I
3	3M	M		D: Back view of boy looking at clouds (Fig. 9.4)	Geometry (3D shape)	I
	3N	M		D: Furniture van (with arrow as in the image), boy (Louis) and dog outside a house	Geometry (position)	I
	3P	F	Absent			
	3Q	M		D:Amy with hands behind her head (presumably looking up at the sky as in book image)	Geometry (position)	I
	3R	F		D:Furniture van with arrow (as in the image)	Geometry (position)	I
	3S	M		D: Simple map of road, cars, buildings	Geometry (Position) Measurement (length)	I

Example 1

Figure 9.1. Student 2G: Representation including geometric and measurement concepts.

Discussion:

R[3]: Tell me about your picture.

2G[4]**:** It has skyscrapers.

R: What else have you got? Tell me about your towers, your skyscrapers.

2G: One is little, one is medium, and one is big *(indicating second, third, and fourth from left).*

R: How do you know the difference?

2G: Um 'cause that one has less space and that one has more and that one has more (*order as above*).

R: Right. Different sizes. Different space. What about how tall they are? Are they different in how tall they are?

2G: No.

R: Are they the same size, height?

2G: No.

R: Which one is taller? Which one is smallest in height? Which one is medium in height?

2G: This one (*indicating fourth from left).*

R: Is one wider than the other?

2G: Yes. That one is wider *(third from left).*

---

[3] " R" refers to the researcher.

[4] 2G is the code for Student G from School 2.

Student 2G used terms denoting measurement (size) such as "little", "medium", and "big". He also used "less space" and "more" to compare the sizes, but it was not possible to determine whether he meant the area or volume of his buildings. He was perhaps trying to demonstrate how the skyscrapers are different, correctly comparing the height of each. When I asked him to tell me how tall they were, and then which was the tallest, he appeared confused and gave conflicting responses, first saying they are all the same height and then correcting himself. It is interesting to note that he drew the tops of the skyscrapers at the same level. It is not possible to know, without further questioning, whether this reflects poor measurement skills or not aligning the skyscrapers along their baseline. It may have been an attempt to draw in perspective.

Although this representation displays many of the characteristics of a map (streets, buildings, traffic lights, and possibly a roundabout) it could be described more as a side view of a city, similar to the depictions in some pages of the book.

Example 2

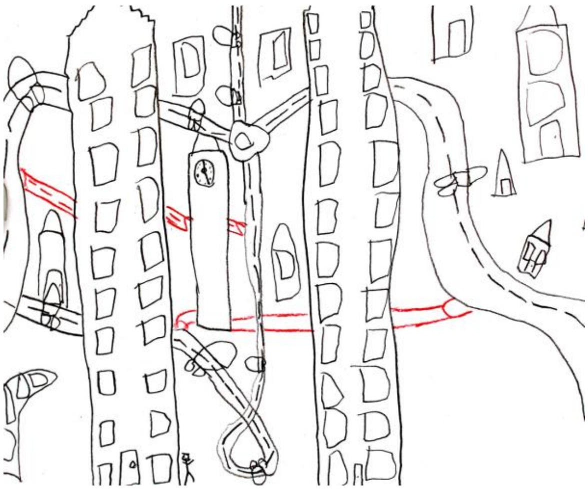

Figure 9.2. Student 2H: Representation of a city.

Discussion:

R: What a wonderful picture! Can you tell me about it?
2H: Skyscrapers and roads.
R: Why did you add the roads in?
2H: There are loads of roads in big cities.
R: What do we call that sort of picture do you know?
2H: umm
R: Have a look at this. What sort of picture might you call that (showing book)? It is a picture of the town. There is a special word that we use to call this. It is a little word. Do you know what it is? It starts with Mm, Aaa

2H:	Next letter?
R:	That would give the whole word.
2H:	Map.
R:	Yours is a bit like a map. Have you seen a map before?
2H:	Yes
R:	Do Mum and Dad use a map to get to places?

Student 2H created a very complicated "map", recognising that maps (and cities) often comprise "loads of roads", intersections, roundabouts, and buildings of different sizes and uses. However, he did not use any mathematical language in the description of his representation, although on reflection my questioning did not focus on mathematical features. His representation could have been in response to several maps and city views within the book. Figure 9.2 also demonstrates the student's understanding of other mathematical concepts, including equal grouping and spacing, symmetry, and perspective.

Example 3

Figure 9.3. Student 2K: Representation of a city.

Discussion:

R:	2K, tell me about yours.
2K:	I drawed [*sic*] a round -about and clock, Amy, a tunnel which takes it here.
R:	Right! So the bottom part is a tunnel. It goes underneath it does it?
2K:	Yes, to a different road.
R:	Yours looks a little bit like this (showing map in book). It has a special name. It is a picture of the town. There is a special word for this. It starts with "Mm. Mm, A". Do you want to tell him 2H?
2H:	Map!
R:	Yes is it like a map?

2K: Yes because it takes you to New York.
R: Right. What sort of things does a map have?
2K: This will be the entry part, this will be the roundabout, and this will be the new tunnel.

Student 2K's map is simpler in construction than 2H's, again clearly showing roads, buildings, a roundabout, and traffic lights. There is evidence of coordination of vertical and horizontal lines (roads). Despite showing these characteristics, his verbal responses do not explicitly address this or any other mathematical concepts. He does, however, say that he was trying to draw different levels within this picture with the addition of a tunnel.

Example 4

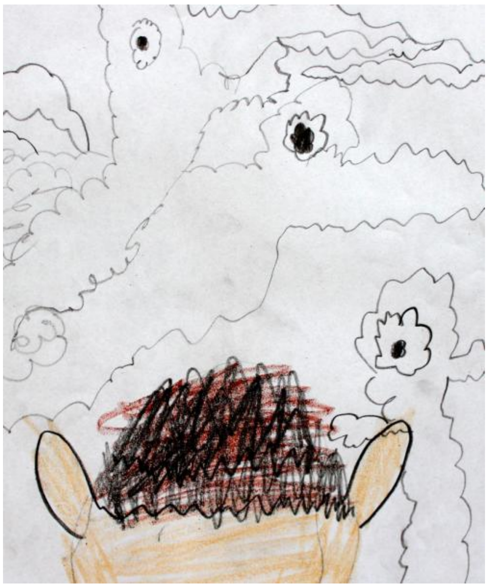

Figure 9.4. Student 3M: Rear view representation.

Discussion

R: 3M tell me about your picture.
3M: This is after Louis called "Co-eee" to Amy and he fell down and saw the clouds.
R: Oh excellent. Are there particular shapes?
3M: Sea horse, sea horse, dragon.
R: Oh great. And we have the back view of his head. So he is looking up at the clouds and we can't see his face. That is excellent. Why did you choose to draw that?
3M: Umm because I really like the sight

With regard to Figure 9.4, Student 3M appears to have only drawn this representation because he liked the clouds in the corresponding illustration in the book. However, he changed the side view depicted in the book to a back view, and drew it from that perspective.

Being able to visualise this perspective is considered far beyond the curriculum expectations for his age group. He further exhibited this ability in Figure 9.5, completed during the additional activity component (explained in Section 7.4.2.2) which was not analysed as part of this study. Here, the students were required "to draw something close/near and something a long way away" (a phrase used in the book). Other students drew simple objects, e.g., their desk for "close" and the local shops or home for "far". However, this student was not only able to represent a globe accurately but was also able to show Australia as "close" on it and the other side of the world as "far away" (a textual, but not visual, reference in the book).

Figure 9.5. Student 3M: Representations of "close" and "far away".

9.1.1.2 Uno's Garden

There were five mathematical representations in responses to *Uno's Garden.* These are summarised in Table 9.4. Two of these included statements (but not drawings)—"the towns are getting bigger" and "the animals get smaller"—reflecting the language (denoting increasing and decreasing numbers) used by the teachers, noted earlier. Student 1B produced her interpretation of the front cover of the book. The drawing was too faint to reproduce and has no mathematical content sourced from the text of the book. However, the student drew a picture with a border similar to that in Base's book. She depicted the same leaf in each corner showing that she had noticed that design on the cover as a pattern. Her reason for drawing it was not mathematical as she explained "because it looks good and I like it". This is an interesting comment as it is a very similar reason to that given by Alison Lester (see Section 6.5.4) when I asked her why she used particular repeating structural patterns in some of her books.

Student 3M again included a globe with just Australia on it in his representation. He also drew arrows pointing to heaven where the animals were and another to the city on the globe/map. Student 3N drew a picture of animals in the jungle with an arrow near the heading "Jungen". Both these representations indicated geometry (position and perspective) concepts.

The remainder of the representations (nonmathematical) were of characters in the book, or drawings unrelated to the book. These may have been attempts to replicate Base's powerful artistic style, and they emphasise the illustrations as the students' main focus.

Table 9.4

Content of Representations in Response to the Reading of Uno's Garden

School	Student Code	M/ F	Non Mathematical (D=Drawing: W=Writing)	Mathematical (D=Drawing: W=Writing)	Strand	Source: T= Text I= Image O=Other
1	1A	M	D: Uno and Uno lookalike			I
	1B	F		D: Front cover with border, leaf in each corner, Uno in centre	(Algebra) Pattern	I
	1C	M	D: Snortlepig, another creature			I
	1D	M	D: Uno and creature			I
	1E	F	Absent			
2	2F	F		D: 4 buildings with windows in each. W: "the towns are getting biger" [sic]. 10 x 10 =100	Number	T, I
	2G	M	D: Tree and sun			O
	2H	M	D: Fish and a tree			O
	2J	F		W: "the animals get smaller". 2 x 2 =4 3 x 3 =9	Number	T
	2K	M	D: Trees and clouds			O
3	3M	M		D & W: Snortlepig, moopaloop and bird. Arrow pointing up to these with word "hevean" [sic] and down to globe like start of map and arrow to it saying 'City"	Geometry (position)	I
	3N	M		D & W: Snorttlepig , tree, mushroom and bird. Arrow pointing down to the word "jungen" [sic].	Geometry (position)	I
	3P	F	D: Uno, snortlepig, sky and tree			I
	3Q	M	D: Uno, mushroom, tent, grass and snortlepig			I
	3R	F	D: Snortlepig face and tree			I
	3S	M	D: Snortlepig and 2 mushrooms			I

9.1.1.3 Minnie's Diner: A Multiplying Menu

Table 9.5 contains entries showing the content (mathematical, nonmathematical and mathematical strands) of all representations for the reading of *Minnie's Diner*. This book elicited more mathematical representations from the students (12 of 15 representations) than did the other two books. All mathematical representations reflected the text and the images that purportedly focussed on the doubling concept but also included other mathematical ideas as reported in the mathematical content analysis (Section 7.3.4). Six students drew their interpretation of the trays of food seen previously in Figure 7.4. Students 3N, 3R, and 3S copied the cross hatching on the pies, and Student 3P included the tiled floor but they probably did not recognise it as two-dimensional (2D) patterning. Four of 15 students used numerical equations within their drawings. For example, Student 2G drew four columns of two items and correctly wrote 2 + 2 + 2 + 2 = 8. Student 2J also drew each of 3 items twice and wrote 2 + 2 + 2 = 6, demonstrating repeated addition.

Nine of the mathematical representations included number and algebraic concepts, two included both number/algebra and geometry, and one included measurement. This predominance of number content is not consistent with the mathematical content analysis that showed number to constitute only 27.8% of the mathematical text and geometry and measurement to be more prevalent. The dominance of representations containing number concepts is to be expected as the main focus of the book is number. Additionally, mathematical language used by students in the reading session included a higher proportion of terms related to number concepts because the teachers themselves also highlighted it.

Student 2K demonstrated doubling by writing 16 + 16 = 32, and then wrote different combinations of 32 (8+24=32, 25+7=32, 8+=24=32 [sic]). These demonstrate a good understanding of combinations for addition but not doubling. Some students also talked about the number of items they drew while they discussed their representations for *Minnie's Diner*. However, it was apparent these were random numbers (e.g., 7, 13, 22) as they had no connection with the numbers used in the story and therefore did not demonstrate relevant mathematical understanding.

Table 9.5

Content of Representations in Response to the Reading of Minnie's Diner

School	Student Code	M/ F	Non Mathematical (D=Drawing: W=Writing)	Mathematical (D=Drawing: W=Writing)	Strand	Source T= Text I= Image O=Other
1	1A	M		D: Back view of 5 brothers getting larger and Minnie. Father coming in door	Measure-ment (length)	I
	1B	F	D: Minnie and food			I
	1C	M	D: Lots of containers with chips in them			I
	1D	M	Absent			
	1E	F	D: Chips, Minnie, pie			I
2	2F	F		D: Large tray with different foods. Some food falling off. W: Numbers, 22, 17, 16, 15 and 7 on or near tray.	Number	I
	2G	M		D: Counter with small tray. 4 x 2 blocks on tray. Man on chair. 2+2+2+2+2=8 above counter. Minnie with tray with 4 x 2 blocks on tray.	Number	I
	2H	M		D: Tray with items in columns (4 x 13, 2 x 14, 2 x 7, 2 x 7, 1 x 7)	Number, Algebra (pattern)	I
	2J	F		D: Minnie with another girl. Counter, tray, 2 x 3 items. W: 2+2+2= 6 foods in box.	Number	I
	2K	M		D: Minnie, large tray with 3 food items W: 16 +16=32, 8+24=32, 25+7=32, 8+=24=32	Number	T,I
3	3M	M		D: 3 trays. 1st tray had one of each item, 2nd tray double. 3rd incomplete.	Number	T, I
	3N	M		D: Top view of a hatched pie (pattern). W: "I thought about the book that it gave more than 32 trays".	Number, Algebra (pattern)	T, I
	3P	F		D: Minnie behind the bench, Back view of one son, 4 empty stools, patterned square tile floor	Geometry (position) Algebra (pattern)	I
	3Q	M		D: Front view of man, Minnie behind the bar, 6 empty stools. W: "I drawed Will and a piye"	Geometry (position) Number	I
	3R	F		D: Cross hatched pie (pattern). W: "I drawed a pie because it was my favriart sort of food that she made and gave the men. In the end she gave the man 32".	Number Algebra (pattern)	T, I
	3S	M		D: Basic cross hatched pie (pattern). W: "I thoughe of song abat pasing".	Algebra (pattern)	I

Three examples follow for *Minnie's Diner*: Figures 9.6 to 9.8. They demonstrate understanding of the mathematical concepts of geometry (position and perspective), number (multiplication/arrays), and measurement.

Example 1

Figure 9.6 is a simple interpretation of one page from *Minnie's Diner* by Student 1A. It demonstrates the occasion when all the brothers arrived at the diner. Although copied from the illustration in the book, it contains several mathematical ideas. First, this drawing is of a back view, containing perspective and positional concepts. However, it is not a change of perspective as in Student 3M's representation (Figure 9.4). When 1A also replicated a scene with perspective and positional ideas for *Amy and Louis,* he added extra but nonmathematical features. Second, it does show the student's recognition of ordering lengths (measurement) as in the book's image, through the increasing size of the five brothers, although the stools they are sitting on are not aligned. However, the drawing does not include any reference to the doubling (number) concepts emphasised by the book and in the book reading discussion with the teacher. Student 1A's only explanation for the drawing was that it showed "when the boss came and they got in trouble".

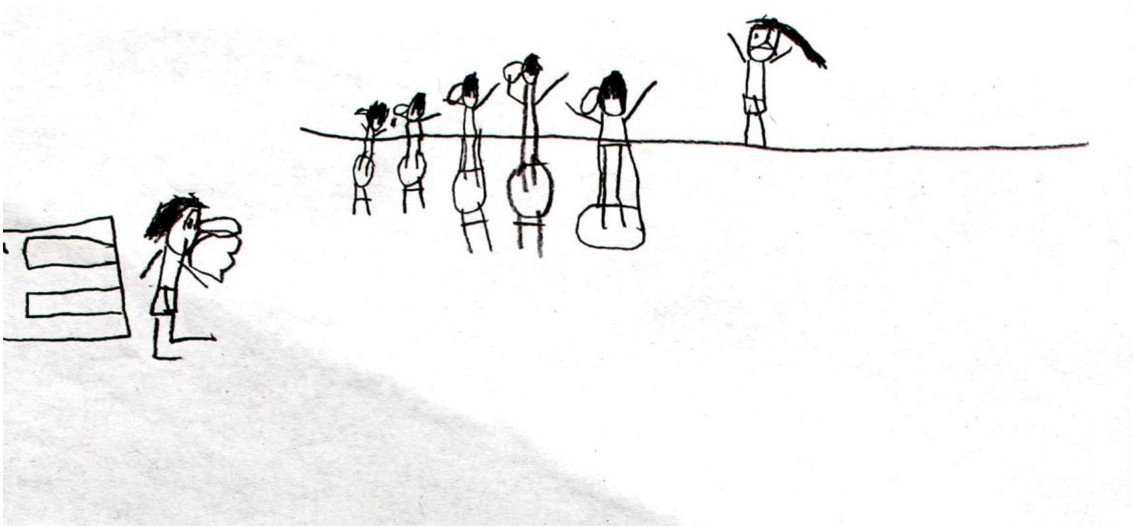

Figure 9.6. Student 1A: Representation showing increasing sizes of the brothers from *Minnie's Diner: A Multiplying Menu.*

Example 2

In Figure 9.7, Student 3M provides an example of doubling. The first tray has one of each item on it, and the second tray has two of everything. His explanation was, "I've done a doubling of all the food. Yeah every time it got bigger, it doubled the food." In this figure, Student 3M has shown that "double" means "twice the items" and the notion of a growing pattern, but it is not possible to ascertain whether he has a sound understanding of the doubling concept.

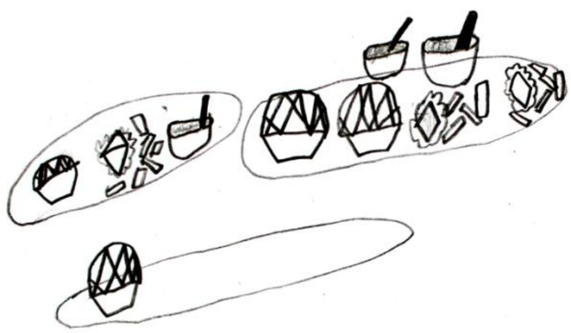

Figure 9.7. Student 3M: Representation in response to *Minnie's Diner: A Multiplying Menu* showing double the quantity of the first tray in the second tray.

Example 3

The final example is provided in Figure 9.8. This representation depicts different types of food on a tray in columns and rows: four columns of 13 items, two columns of 14 items, two columns of seven items, another two columns of seven items, and finally one single column of six. This demonstrates understanding of equal groupings when doubling, array structure, and simple repetition. However, the discussion with Student 2H did not confirm this and he did not relate the use of columns with his intent to draw 32 items. In relation to this, it is important to note that in the book's illustrations the quantities of the different foods on the tray are sometimes depicted in column-like structures (as in Figure 7.4), but sometimes they were only grouped.

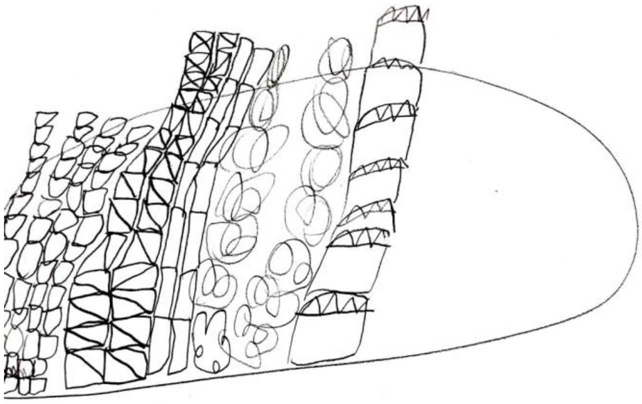

Figure 9.8. Student 2H: Representation in response to *Minnie's Diner: A Multiplying Menu.*

Discussion

R: How are you going? What are you up to?
2H: I have 33.
B: How many do you need to have?
2H: 32.
R: How many have you got there though?
2H: 13
R: 13?
2H: 1,2,3,4,5,6,7,8,9,10,11,12,13.
R: 13, why did you put 13?
2H: I just added one.
R: Oh right, to what?
2H: From 32 I added one.

9.2 Final comments on the analysis of students' representations

The drawn representations and excerpts in response to the readings of *Amy and Louis, Uno's Garden,* and *Minnie's Diner* revealed much about the children's perceptions of mathematical ideas in the books. This was in contrast to when the teacher previously read and discussed each in the shared book session because the teacher dominated the talk and there was limited opportunity to discuss or draw. An important finding was that the images were the dominant source of the mathematical representations.

The students' representations also showed, similar to the mathematical content analysis of the three books, that picture books of different types have the potential to elicit mathematical ideas. To varying degrees, the students found and represented mathematical ideas from all three books. In the case of the perceived book, *Amy and Louis,* the students demonstrated mathematical understandings of concepts that were not identified in the book reading session. This was particularly evidenced by the maps drawn by the students. Although there are map-type illustrations throughout the book the teacher did not draw the students' attention to them. This finding is further addressed in Section 9.3.

The embedded book (*Uno's Garden)* elicited fewer spontaneous mathematical representations. Students showed little engagement with the actual mathematics (number sentences and patterns) embedded in the book, but focussed on the content in the illustrations. Almost all students demonstrated some engagement with the mathematics in the explicit book (*Minnie's Diner)*. However, the representations for this book were mostly depictions of the mathematical ideas suggested in the book, or replications of the book's images.

9.3 Summary of findings for Phase 3: The classroom study

The first purpose of Phase 3 (Chapters 7, 8, and 9) was to ascertain the potential for mathematics learning and teaching from different types of mathematical picture books (Research Question 1). Second, I wanted to determine how these books engage teachers and students and whether the different types of mathematical picture books elicited differences in teacher and student mathematical talk (Research Question 3). Finally, I aimed to ascertain the source of all mathematical talk. Within the following subsections I summarise the key findings of this third phase.

9.3.1 Mathematical content analysis

In Chapter 7, I had presented a mathematical content analysis of the three chosen books. The analysis of the text indicated that each type of mathematical picture book had the potential to facilitate the development of mathematical concepts. The text of *Amy and Louis* was found to contain 29.5% mathematical language, *Uno's Garden* 24.6%, and *Minnie's Diner* 32.4%.

Further analysis of the text and images of each book also indicated potential for mathematical concept development through meaningful mathematical investigations. The perceived book, *Amy and Louis,* included opportunities across mathematics strands of geometry (position, 2D and 3D) and measurement (length, time, volume). Additionally, there were multiple cross-curricular opportunities (e.g., science, geography) at a level appropriate to the age of various readers. This book, therefore, appeared to provide opportunities for use within an integrated classroom program where enjoyment of the story, the literary value of the whole book, and the integrity of all integrated areas, including mathematics, would not be lost (see Perger, 2004).

In *Uno's Garden* I found many mathematical opportunities pertaining to the number strand at varying levels of difficulty, and I found patterning in the language and intricate illustrations. The main point of this book is the environmental message, facilitating potential for meaningful integration. The purpose of *Minnie's Diner* was to emphasise one mathematical concept: doubling. Although I found opportunities in this book for the development of mathematical language, I found a greater number of references to both geometry and measurement concepts than the number concept that is emphasised.

9.3.2 Shared book reading sessions

The analyses of the teacher and student talk during the reading sessions and the representations of the students and responses elicited during the activity session indicated different views about the potential of the books.

In this phase, I intended to focus on the mathematical potential for mathematics learning evidenced by students. However, during the shared reading session, I found teachers in all three schools dominated and guided the discussion, speaking on average 75% of the time. Except for a few spontaneous instances, all student talk was in response to teachers' questions. Much of the talk related to text/story recall.

Nevertheless, within their talk, both teachers and students used mathematical language, much of which was conversational. On average, more mathematical talk was used in total during the readings of *Uno's Garden* (64.2%) and *Minnie's Diner* (66.7%) than during the reading of *Amy and Louis* (39.4%). The mathematical content analysis of the text showed comparable amounts (about 30%) for all books.

Similar to the findings of the mathematical content analysis, geometry was the most frequently used concept in *Amy and Louis* by both teachers and students, again predominantly through the use of prepositional phrases. Number was the intended focus concept in *Uno's Garden* and was also the dominant concept during the book reading, but geometry played a lesser role for both teachers and students. However, the opportunities for measurement and geometry learning identified in the mathematical content analysis for *Minnie's Diner* were not realised by the teachers, and the language of number dominated the teacher and student mathematical talk.

Teachers did not appear to notice as many mathematical opportunities as I had in my content analysis of the books. They appeared to identify mathematics more easily when it was explicitly referenced and less so during real-life contexts. There were a few good examples of using the books' rich teaching and learning opportunities. However, the teachers mainly reiterated any obvious mathematical ideas such as the doubling concept in *Minnie's Diner* or the increasing and decreasing of numbers in *Uno's Garden.* None of the teachers connected the idea of doubling across these two books and they did not take the opportunity to reinforce the concept in their discussion. Again, identification of mathematical ideas also differed according to type of book and how explicit the mathematics was within the book (Research Question 1).

Students did identify some mathematical aspects, but these were not always used by the teachers. Nevertheless, despite the large amount of mathematical talk that could be regarded as unidentified or "unelaborated" (A. Anderson, Anderson, & Shapiro , 2005, p. 21), students still benefited from exposure to mathematical terms and language, just as young children develop literacy skills through the use of children's literature (see Chapter 1).

The source of teacher mathematical talk during the book reading was found to be predominantly from the text and illustrations combined across books, although there were again variations across teachers and books. This source was particularly prevalent in *Minnie's Diner,* where the images replicated the text.

On the few occasions when students did respond spontaneously to the illustrations in this study, the students focused on aspects such as finding the snortlepig in *Uno's Garden* and counting the food on the tray in *Minnie's Diner.*

9.3.3 Activity sessions: Student representations

In contrast to what had occurred in the shared book reading sessions, and even if not strongly evident in the associated discussions, students did demonstrate mathematical ideas through their representations. For example, the maps and alternative perspectives (back views) drawn by the students were not discussed by the teachers in the book reading sessions. Styles and

Arizpe (2001) had also found students were often better able to demonstrate their understanding through drawings than through talking.

There were 10 mathematical representations for *Amy and Louis,* five for *Uno's Garden,* and 12 for *Minnie's Diner*. The representations collected in response for *Amy and Louis,* a book that had previously shown the least teacher and student mathematical talk in the reading session, included complex spatial concepts (e.g., maps, perspective) and measurement concepts (length and time). Those for *Uno's Garden* were predominantly simplistic and were replications of the text and illustrations, while representations in response to the reading of *Minnie's Diner* included evidence of number concepts such as repeated addition, multiplication, arrays (rows and columns), and length measurement.

Therefore, when students were given the chance to represent and discuss their ideas without the teacher dominating the discussion, they showed engagement with a range of mathematical ideas in picture books. Sometimes this awareness was beyond age expectations. The follow-up activity therefore supports the belief "that when children are in an inspiring environment with elements that can be mathematized, they inevitably come up with mathematics-related thinking" (Van den Heuvel-Panhuizen & Van den Boogaard, 2008, p. 367). Additionally, 74% of the students' mathematical representations were responses to the illustrations in each book.

Overall, these findings support the socioconstructivist approach to teaching as the students appear to have interacted with the books and each other in an enquiring, supportive, and engaging environment to construct and demonstrate their mathematical ideas. However, the findings have also highlighted the role of the teacher. The teachers both dominated discussion and did not appear to recognise many of the teaching and learning mathematical opportunities. Therefore, Vygotsky's (1978) theory that students need to be supported or scaffolded in their learning by "a more proficient other" (Shapiro, Anderson, & Anderson, 1997, p. 48) was not fully realised.

This classroom study has gone further than previous studies. I compared the use of the three types of picture books in different situations. I first analysed both teacher and student mathematical talk systematically in a shared book reading situation, an approach typical of practice in Australia. Mathematical talk was found to be evident throughout both teacher and student talk for all types of books. Each book had its individual benefits. However the teachers were found to dominate the discussion, using both the text and the images as a stimulus for their talk. I then compared a mathematical content analysis of the books with the mathematical talk during book reading sessions. This demonstrated that the teachers did not take full advantage of the mathematical teaching and learning opportunities within the books. I also compared student engagement with mathematical concepts in the book reading sessions to their spontaneous representations during a follow-up activity. The content of the representations provided new findings that demonstrated that the students identified mathematical opportunities and concepts in the books, not noticed by the teachers. I also found that the images were the predominant source for their representations.

These findings, and the study itself, have limitations. First, the naturalistic setting allowed the teachers to control the talk. Their teaching approach may also have reflected their limited experience with recognising mathematics in picture books for mathematical learning. Finally, teachers had been asked not to continue with an activity beyond the book reading. All teachers incorporated some mathematical discussion during the reading of all books. However, I reported how Teacher 3 changed her strategy half way through this book and then proceeded with an activity related to the book's content that was rich in mathematical talk. If this had occurred for all teachers, it might also have provided more insight into the mathematical potential that they "saw" in the book.

The findings of this phase, as well as the findings from Phases 1 and 2, have implications for mathematics teaching, learning, and the curriculum. I discuss these implications in Chapter 10 and provide examples of the application of shared book reading for mathematics learning through two publications.

CHAPTER

10

IMPLICATIONS FOR TEACHING, LEARNING, AND THE CURRICULUM

In this chapter my purpose is to highlight the implications for teaching, learning, and the curriculum that relate to the use of picture books for supporting mathematical teaching and learning for young children. These implications arise directly from the findings of this three-phase study.

The findings of the study suggest implications in several areas. I first address the ones that apply to teaching and learning. These include the identification of opportunities for mathematics learning provided by picture books, the selection and evaluation of appropriate picture books for use in the classroom, and the important role of the teacher in encouraging students to engage cognitively with picture books through discussion and representation of ideas. Next, I discuss the implications for professional learning, and finally I consider the implications for curriculum, including incorporating an integrated mathematics learning approach.

Two publications detailed below, both in peer-refereed professional mathematics education journals, complement the chapter by providing practical exemplars for professionals about how to integrate mathematical picture books into their teaching.

Publication 7:

Marston, J., Muir, T., & Livy, S. (2013). Can we really count on Frank? Using a framework to select and evaluate picture books for mathematical concept development. *Teaching Children Mathematics, 19*(7) 440–448.

Publication 8:

Marston, J. (2014b). Identifying and using picture books with quality mathematical content. *Australian Primary Mathematics Classroom*, *19*(1), 14–23.

10.1 Implications for teaching and learning

Throughout this thesis I have endorsed the integration of literacy and numeracy to promote rich, realistic learning and engagement in mathematics for young children. This coupling also suggests the potential of mathematical picture books that provide real-life contexts as part of a comprehensive and rich integrated classroom program. Hence, as the framework I developed encourages incorporating mathematics across all curriculum areas, suggestions for integration have also been made where applicable in this thesis. Additionally, I have discussed the benefits of a socioconstructivist approach to teaching and I have used it as the basis for the recommended meaningful learning experiences in Publications 2, 3, 7, and 8, and in Chapters 7 and 8.

10.1.1 Identification of opportunities for mathematics learning in picture books

The evaluation of the framework demonstrated that many teachers do not easily identify mathematical concepts or opportunities for mathematics learning in picture books. The framework survey showed that there were wide variations in what teachers perceived mathematics to be. As discussed previously, mathematics is often considered to be essentially about counting numbers, equations, fractions, shapes, or calculations, for example, rather than something that occurs naturally in our everyday lives. Therefore if that is also the teachers' perception, they will find it difficult to identify anything more than these ideas in picture books. Nor will they be able to create the authentic integrated learning experiences suggested in books such as *Amy and Louis* (Gleeson, 2006) and *Are We There Yet? A Journey Around Australia* (Lester, 2004), for example, planning trips, exploring different ways to measure, or explaining ideas through diagrams (e.g., cross sections or maps).

There was a predominance of teacher mathematical talk during the reading of the three books in the classroom study (Chapter 8). Although the teachers talked approximately 75% of the time, the proportion of mathematical talk was low and mostly conversational in nature. The teachers stopped to explain literary and spelling features but rarely demonstrated that they recognised the potential of the mathematics in the books beyond explicit number concept references in *Minnie's Diner* and *Uno's Garden*, or applied them in their teaching. On many occasions, the teachers did not recognise the mathematics and therefore did not take advantage of the opportunities for mathematics learning such as those demonstrated in the mathematical content analysis (Chapter 7) or the two publications provided in this chapter. This finding was similar to that for participants in the framework survey. If teachers cannot identify mathematical potential readily and with accurate mathematical explanation, teachable moments are lost.

10.1.2 Selecting and evaluating appropriate picture books

Many participants responding to the framework survey found it was too time consuming. Nevertheless, like the participants in the Van den Heuvel-Panhuizen and Elia (2012) study, they commented that having a framework with elements that highlighted the mathematical content and characteristics for conveying mathematical ideas assisted them not only in the selection of appropriate books but assisted in the identification of the mathematics within the books (it "sharpened the eye" [p. 40]) and initiated ideas for classroom investigations. Frequent use of the framework also made it easier and more efficient to use. These findings suggest that it is advantageous for teachers to use frameworks/criteria to identify appropriate books for teaching and learning as well as to identify the mathematical content in those books and create beneficial activities related to that content.

Therefore, teachers first need to know of the existence of criteria and frameworks to help select books. They also need opportunities to develop knowledge and skills to use the framework confidently (See Section 10.2.3) so they can select and evaluate books themselves and not rely on publisher recommendations (Nesmith & Cooper, 2010) that have not always been shown to be reliable (Hunsader, 2004).

During Phase 2 of the study, the authors and author/illustrators explained their various reasons, and the techniques they employ, when intentionally including mathematical concepts in their books. Independent of that, during the classification and evaluation of books in Phase 1, and the mathematical content analysis in Phase 3, I found additional features, not purposefully included by the authors that also provide mathematical learning opportunities in all types of mathematical picture books. These were identified both in the images and the text, and the authors confirmed that they played distinct roles in the meaning of the story. Teachers may be familiar with and understand the role of these in literacy development. However, it is also important for teachers to recognise that the text, images, and the structural design of books may contain mathematical content that is useful for classroom applications.

Originally, I defined the three different types of picture books according to how the mathematical ideas are presented within them, and I later affirmed this through discussion with the authors. After analysing the data from the framework survey, I also suggested that it may not be necessary for teachers to distinguish between perceived, explicit, and embedded mathematical picture books. However, the findings in the classroom study indicated the need for teachers to be skilled in distinguishing the type of book. Mason, Peterman, and Kerr (1989) and Moschovaki and Meadows (2005a) had previously found that teachers take a different presentation stance for narrative books relative to information style books.

Although the books in this study were all considered fictional, I found that the teachers in the classroom study appeared to present each one differently depending on what they saw as the book's focus and genre. *Amy and Louis* (perceived) was read as a narrative with a focus on its literary meaning, while *Minnie's Diner* (explicit) was read for its informational content, with teachers emphasising the mathematical concepts. The teachers appeared to address (to some degree) both the literary and mathematical features of *Uno's Garden,* with its embedded mathematical content. The teacher in School 3 even appeared to change her purpose half-way through the reading of *Uno's Garden,* possibly confused about the book's genre. She seemed unsure whether to focus on the literary aspects or mathematical content. Teachers may be aware of how genre has been found to influence their presentation style and subsequently student learning during literacy sessions. However, they also need to be aware that the type of book they choose for mathematical learning may influence their presentation style and thus the potential for mathematical teaching and learning.

In Phase 3, I reported that no single type of mathematical picture book stood out as containing more potential for mathematical teaching and learning opportunities than the others. I found different benefits and potential in each type of book through the mathematical content analysis, the shared book reading sessions, and the students' representations. In the classroom study, I found that the same book elicited different responses across the schools, possibly due to teacher scaffolding. Student background may also have had a bearing. Teachers need to be aware that although, for example, the perceived book, *Amy and Louis,* elicited strong mathematical representations from the students in this study, this may not be the case for all perceived mathematical picture books or with another group of students. It also appears that different types of books and different books within the same type generate different levels of cognitive engagement.

For a variety of reasons, good book choice has been shown to be important. Whether teachers wish to select books for a particular mathematical concept, e.g., doubling, to promote mathematical problem solving and reasoning across mathematics content strands, or as part of an integrated program, they need to select wisely. The framework, an understanding of the effect of different genres, and the role of text and images in conveying mathematical content should assist both book choice and the creation of real-life classroom activities.

10.1.3 The important role of the teacher

The teacher's task in the classroom study was to read each of the three picture books in a naturalistic manner that included questioning the students about the book and its contents as they read. Consequently, there was no scripted or prescribed way for teachers to read and present to their students, nor was there a consistent set of questions pertaining to each book. Thus comparative data about the student-teacher interactions and the analysis of their talk in each classroom must be viewed in light of the different classroom norms and teacher styles.

I highlighted in Chapter 8 that in this naturalistic setting teachers dominated and guided the discussion. This did not necessarily allow the students to react to the book in the same way as in the reading sessions of previous studies with no reader-child discussion (e.g., in the study by Van den Heuvel-Panhuizen & Van den Boogaard, 2008). This is not to be taken as a criticism of the participating teachers but rather a reflection of the reality of what happens in the classroom. Therefore, this situation again implies the crucial role of the teacher.

In Vygotskian socioconstructivist theory, students require a mediator (teacher) to bring attention to important elements within a book. Although it was reported that children acquire knowledge simply by exposure to picture books (A. Anderson et al., 2005), the "full value of reading can only be realized if the adult and child engage in conversations that support meaning construction" (Beals, DeTemple, & Dickinson, 1994, p. 21). Moschovaki and Meadows (2005b) also found that the quality of discussion of a book affected the "children's cognitive engagement" (Section 5 para. 3). Therefore, the teacher's individual questioning and discussion technique is significant. The literal questions often asked in the classroom study are a significant technique used in shared book reading sessions to consolidate and assess student knowledge. However they do not necessarily help children acquire new mathematical knowledge (Heath, as cited in Moschovaki & Meadows, 2005a) and are deemed of low cognitive engagement. Moschovaki & Meadows (2005b) showed that "children's cognitive engagement depends on teachers' choice of cognitive strategies. In particular, teachers' high cognitive demand participation elicited children's high cognitive demand participation" (Section 5 para 2). Additionally, "the more teachers talk, the less students participate spontaneously" (Moschovaki & Meadows 2005b, Section 4 para. 3) and the lower their cognitive engagement.

Therefore, teachers need to be aware of the strong influence they have on enabling cognitive engagement. Teachers should use a variety of teaching strategies, including picture books for promoting mathematical learning, when devising activities to both cognitively engage students and meet their individual learning needs. If teachers use a socioconstructivist approach, students need "space if they are to elaborate and analyse during group discussions" (Moschovaki & Meadows, 2005b, Section 5 para. 4), and they need opportunities to interact and learn from their peers. This approach allows them to build on past experiences and make sense of new knowledge. Unfortunately, teacher dominance limited this during the shared reading session of the classroom study.

10.1.4 Student cognitive engagement with picture books

The more involved the students are, the more they develop and display their knowledge (Moschovaki & Meadows, 2005a, 2005b). However, the students' responses during the reading session were influenced by several variables. First, the dominance of the teacher and limited involvement of the students did not allow the students to engage effectively in the content of the books. The culture within the classroom of "only speaking when spoken to" could also have influenced the extent of spontaneous response from the students. It may have also been the teacher's presentation of the books and their interpretation of the genre that influenced students' responses.

Although the students in this classroom study identified many mathematical concepts, in each of the explicit, embedded, and perceived mathematical picture books during the shared readings, their strongest and most meaningful responses were revealed in their representations. The analysis of the representations demonstrated that the students cognitively engaged with a range of mathematical concepts predominantly shown in the images (but also the text) that were not enabled by the teachers. Examples included the maps in *Amy and Louis* and increasing size of the brothers in *Minnie's Diner.* This was contrasted with the simple mathematics concepts of the shapes of clouds (*Amy and Louis*) and number of items (*Uno's* Garden) noticed by the teachers. These findings support the theoretical perspective highlighted in Section 3.1.2 concerning the important influence that images and their interrelationship with the text have on children, even without teachers highlighting them.

The way that students demonstrated how they cognitively engage with mathematical picture books suggests several implications for pedagogy. First, teachers need to pay more attention to the content and role of the illustrations, the source of most student engagement. This will enable them to assist students to both use what they see and extend their understanding. Second, teachers should use the content of the text more with annotations such as suggested by Halpern (1996), or attach post-it notes, as demonstrated by Teacher 3 when reading *Uno's Garden.* This may direct students to the text and facilitate rich learning experiences. Third, students require opportunities to explore, investigate, and demonstrate their ability in a variety of ways. The representations that the students made in the activity sessions, and the individual discussions I conducted with each of them in those sessions, revealed a better understanding of the mathematical concepts that are included in the books than the students had previously shown during the shared reading sessions. Therefore, allowing students to respond through drawing and writing (and child-initiated activities) should be employed more often. Similarly, further use of one-to-one discussions to both assess student understanding and extend student thinking, albeit difficult within the classroom setting with many students, is recommended.

Like Van den Heuvel-Panhuizen et al. (2009), I also found that students understand more complex concepts than those specified in curriculum expectations. This needs to be taken into account by teachers who should always consider a variety of rich learning experiences, activities, and assessments that allow students to express their knowledge and skills to their full understanding and potential.

Students in this study have demonstrated that they do cognitively engage with the mathematics in the text but predominantly in the images of picture books in a naturalistic setting. Therefore their use in the classroom is validated for promoting mathematical teaching and learning opportunities. However, the responsibility for student cognitive engagement and learning again focuses on the teacher. Their choice of book; their identification of mathematics potential in the book; their presentation, questioning, and discussion skills; and other teaching strategies all influence student engagement.

10.2 Implications for professional learning

The analyses of the findings of Phases 1 and 3, and in Section 10.1 immediately above, also suggested that teachers might need professional learning in several areas. First, teachers' ability to see mathematics in real-life contexts and picture books was found in Phase 1 to be affected by their own content knowledge of mathematics and their experience of teaching mathematics. This implies that better instruction in this area could be needed in preservice teacher programs and in teacher professional development courses to both improve the understanding of the nature of mathematics and teachers' perception of it.

Second, participants in the framework survey (Phase 1) and in a workshop associated with Publication 3 (see Section 4.3) commented that they benefitted through more reading about the topic, frequent use of the framework, modelling by an expert, and peer discussions. This enabled them to use the framework, identify mathematical ideas, and consider the use of the mathematical content for rich learning and teaching experiences. Encouragement and opportunities for more experience applying the framework should be provided through practically orientated articles in professional journals, preservice teaching programs, practical and online professional development sessions, and websites incorporating videos and/or webinars.

Additionally, the teachers in Phase 3 tended to source their mathematical talk from a combination of text and images (i.e., when the text and image replicated each other) and did not appear to maximise the mathematical opportunities in the individual components. The presentation style of the teachers in this phase may also have affected students' engagement. Preservice teacher programs may already include training about the role of the text and images, the effect the book's genre on the presentation style of the reader, and how to use picture books effectively in the classroom, during courses in literacy development. However, training in these three areas also needs to be extended to using picture books for promoting mathematics teaching and learning.

The implications for the curriculum, including the incorporation of an integrated approach to teaching and learning, follow presentation of the publications, in Section 10.4.

10.3 Publications

This section provides two publications that are designed to meet some of the needs described in the previous sections. They suggest activities that allow teachers to act as the mediators of meaningful activities so that students can explore new concepts, interact with each other, and build on past experiences to develop new knowledge as espoused in Vygotskian socioconstructivist theory. Appendix H contains examples of the other types of professional development that I have been asked to provide as a result of the earlier publications.

10.3.1 Publication 7

Publication 7, *Can we really count on Frank? Using a framework to select and evaluate picture books for mathematical concept development* (Marston, Muir, & Livy, 2013) was authored with two other mathematics education researchers who were using the Marston (2010a) framework with their preservice teacher education students in different Australian universities. They approached me to be the lead author in this joint paper. All authors are academics whose passion is to use real-life and hands-on activities with young children to promote positive experiences during mathematics teaching and learning activities. The resulting manuscript was published in *Teaching Children Mathematics.* This is a widely recognised international professional journal providing practical applications, based on research, for elementary teachers and teacher educators of mathematics (National Council of Teachers of Mathematics [NCTM], 2015). In this publication, we reviewed three picture books against three categories of the framework, i.e., one category for each type of mathematical picture book.

The first aim of the article was to introduce and demonstrate to teachers the use of the framework for identifying mathematical picture books and evaluating their text and images for quality learning experiences. The second aim of the article was to demonstrate to teachers how to incorporate the findings from use of the framework into the classroom setting with practical suggestions and examples using real-life applications. A book does not necessarily have to score highly in every category to be a useful teaching tool. This article is particularly pertinent as it suggests assessing a book against one criterion of the framework that may be relevant to the classroom teaching program, i.e., a teacher may wish to incorporate a book that facilitates good problem-solving investigations.

The first book is *One Is a Snail, Ten Is a Crab* (Sayre & Sayre, 2003). It was regarded as containing explicit mathematical content. This book has featured in other resources including that of D. Whitin & Whitin (2009). In the article, we first reported the practical application of the activities suggested by the text and images in the book with a class of young children. The article also demonstrated to teachers how to extend mathematical problem solving and reasoning (Framework category: MPS) activities beyond the explicit exercises suggested by the book with pictorial examples of the problems posed by the students themselves. Using the students' own representations also demonstrated to teachers the responses possible from students through the use of books such as this.

The title of the article, *Can we really count on Frank?* refers to the second picture book assessed using the framework. *Counting on Frank* (Clement, 1990) is often included in resource books and articles, and it has been widely used by teachers in mathematics lessons over many years. Although there is very noticeable mathematics content throughout the images and text, it is regarded as containing embedded content as the story and images can be enjoyed without actually engaging with the mathematical calculations.

Although there are many problem-solving and reasoning opportunities for teachers posed in the text of *Counting on Frank,* and the illustrations provide ideas for the answer to the question posed, this book was assessed against the framework category of mathematical content. The particular emphasis was on the element pertaining to mathematical accuracy. We revealed two particular inaccuracies between the text and images, but we demonstrated that this may not necessarily mean the book should not be used as it might afford other worthwhile mathematical investigations. For example, one problem posed in the book and discussed in the article, shows the answer both in the text and in the illustration. While the calculation in the text is correct, the illustration of peas filling a room to tabletop level, totally underestimates the actual number of peas required. Therefore, although replication of the text in the visual images is discouraged, accurate portrayal is necessary when used. Teachers could be encouraged to recognise the important role of the text and images, and their interrelationship, when creating rich teaching and learning activities.

The third book evaluated was also used in the classroom study: *Amy and Louis* (Gleeson, 2006), published as *Half a World Away* (Gleeson, 2007) in the United States. This book was assessed against the framework category of integration of mathematical content. In the article, we demonstrated how a teacher can integrate either across mathematical strands or/and across curriculum areas. Specific examples of activities and investigations are first provided for classroom use for across-mathematical strands integration including measurement (length, time, area, and volume) and spatial concepts (two-dimensional [2D] and three-dimensional [3D], position, and proportion). Suggestions, too, are made for across-the-curriculum integration activities including science, geography, and interpersonal relationships. However, as noted in Chapters 7 and 8, there are far more possibilities for rich mathematical learning from both the text and images across all strands of mathematics in this book.

Pages 259-267 of this thesis have been removed as they contain published material.
Please refer to the following citation for details of the article contained in these pages.

Marston, J., Muir, T., & Livy, S. (2013). Can we really count on Frank? Using a framework to select and evaluate picture books for mathematical concept development. *Teaching Children Mathematics,* 19(7) 440–448.

Postscript. In this publication we first provided more examples of how teachers can use the framework to assess and apply books of different types for learning and teaching. The difficulty some teachers have in seeing mathematics as just shapes and numbers is also addressed. We used three books with mathematical content that represent real life or that can be applied to real-life investigations. The ideas provided here may also encourage teachers to look at not just the text, but also at the illustrations in all picture books for mathematical enquiry ideas. Many of the activities suggested here for *Amy and Louis* (Gleeson, 2006) are derived from the visual images.

10.3.2 Publication 8

In Publication 8, I provided more examples for teachers, again demonstrating the use of both the framework and picture books for supporting mathematical teaching and learning. This publication specifically addressed an Australian audience in the Australian Association of Mathematics Teachers (AAMT) professional journal for primary teachers. This time, three Australian books, one of each mathematical type, were assessed against all the categories of the framework with suggestions for associated activities. The books used in this example are *At the Beach: Postcards from Crabby Spit* (Harvey, 2004) as an example of perceived content, *How Big is Big* (Watson, 2009) with explicit content, and *Uno's Garden* (Base, 2006) with embedded content.

Another purpose in this article was to link the suggested activities to *The Australian Curriculum: Mathematics* (ACARA, 2012), thus emphasising their connection to the framework category of curriculum content, policies and principles. This should give confidence to teachers that they are meeting the mathematics curriculum (and other curricula) outcomes while using picture books as a teaching strategy that not only develops mathematical learning, but engages their students in enjoyable and meaningful activities. I also demonstrated again in this publication that a book does not have to score highly in all categories to be a useful classroom resource.

The real-life situation, a school excursion, provided in *How Big is Big* should also inspire teachers to devise their own investigations. This book, created by a teacher and her class, is an excellent example of how we use mathematics in our lives. It clearly shows how a simple everyday occurrence can produce many mathematical concepts and activities. The teacher here has also demonstrated the use of illustrations to both replicate and add meaning to the text. Good interaction (not replication) of the text and illustrations has provided problem-solving opportunities and has also demonstrated mathematical strategies. For example, a tree diagram was used to calculate how many outfits the students would have if they took just three t-shirts and two pairs of shorts on their trip, and a graph is used to show the flavours of pancakes chosen for breakfast.

The mathematical content and the ways in which it has been included in *At the Beach: Postcards from Crabby Spit* and *Uno's Garden,* have been discussed previously in Chapter 6. I have also shown and discussed the application of *Uno's Garden* with students in the classroom study, in Chapters 7, 8, and 9.

Pages 270-279 of this thesis have been removed as they contain published material. Please refer to the following citation for details of the article contained in these pages.

Marston, J. (2014). Identifying and using picture books with quality mathematical content. *Australian Primary Mathematics Classroom*, 19(1), 14–23.

Postscript. The results of the classroom study (Phase 3) highlighted the important role of the teacher and emphasised the need for teachers to understand how they can use a book effectively. Therefore articles such as Publication 8, which first provide practical ideas for classroom activities that are linked to specific curriculum outcomes, enable teachers to develop their understanding of mathematics and how to teach it using the recommended socioconstructivist principles.

As the article was written before the final analysis of both the author interviews and classroom study, Publication 8 does not explicitly discuss the ways authors include mathematics in the text, illustrations, and design structure or the ways students may respond mathematically to books. Publication 8 does, however, lay the foundation of the idea that mathematics can be found in the text and images.

10.4 Implications for curriculum: An integrated mathematics learning approach

The findings of the three phases demonstrate that picture books have a place in the integrated curriculum in promoting mathematics learning experiences. These findings can also inform the work of curriculum bodies and curriculum writers. As stated in Section 1.2, many curriculum bodies and mathematical associations support an integrated program. The NCTM (1989, 2000) included information about the use of picture books in the classroom and examples of books for mathematics teaching when developing their standards.

The Department of Education and Training [DET] and Curriculum K-12 Directorate NSW (2009a) have previously developed curriculum outcomes groups (COGs) as cross disciplinary units of work. However, these COGs contained limited mathematics content and use of picture books. The Australian National Curriculum (Australian Curriculum Assessment and Reporting Authority [ACARA], 2015) recommended integration for all stages through the use of, for example, literature in history. Within all areas of the curriculum there are the general capability links to numeracy and literacy, where numeracy refers to the application of mathematical knowledge to everyday situations. However, there is no reference to the use of picture books for mathematics learning. For example, the numeracy link to the Australian National Curriculum's mathematics outcome *Interpret simple maps of familiar locations and identify the relative positions of key features (ACMMG044)* for Year 2 could recommend use of the book *At the Beach: Postcards from Crabby Spit* (Harvey, 2004). Additionally, there are opportunities for picture books to be included in the proficiency strands of problem solving and reasoning. The recent STEM (**S**cience, **T**echnology, **E**ngineering, and **M**athematics) Project

(Board of Studies Teaching and Educational Standards, NSW, 2016) emphasises the importance of integration, but again contains few instances of picture book use in the mathematics units of work.

Most teachers understand and support the advantages of an integrated curriculum incorporating socioconstructivist principles (Naylor, 2014). Most have also heard about the use of picture books for promoting mathematical teaching and learning. Nevertheless, these strategies are not necessarily implemented by teachers at the coalface (Handal, 2002; Jamieson-Proctor & Byrne, 2000; Speer, 2005). Rich mathematical learning experiences using picture books and fully integrated units of work that maintain the integrity of all areas (see Naylor, 2014; Perger, 2004) do take more time to prepare than do traditional textbook style forms of teaching or discipline based strategies. It also appears easier to use traditional instrumental strategies, thus limiting opportunities for involvement and sustained cognitive engagement.

Many picture books have been written in ways that could promote integrated learning. Curriculum bodies and curriculum writers could first review the wealth of books available on the market and assess these books using frameworks. They could also provide annotated bibliographies and suggestions for teaching and learning experiences, at all stages, that do not depend solely on publishers' recommendations which are not necessarily valid and unbiased (Halsey, 2005; Hunsader, 2004; Nesmith & Cooper, 2010). Curriculum bodies and curriculum writers could then suggest rich learning experiences based on the theoretical principles espoused in Section 3.1.

The publication of mathematical picture books of poor literary value may continue to exist on the market. One way of promoting better quality and more purpose-written books would be to encourage collaborative writing between mathematics educators and picture book authors and illustrators. Together they could create well written, meaningful, accurate, age appropriate books that promote mathematical ideas for teaching and learning. These could allow investigations that incorporate socioconstructivist perspectives and theories on the function of the text and images to cognitively engage the students.

10.5 Summary

In this chapter I have discussed the findings of this study that have implications for teaching, learning, and the curriculum. Two publications emphasised how to advance teachers' understanding and support them in developing both the knowledge framework and pedagogical strategies that facilitate mathematical teaching and learning for young children. These examples first demonstrated what mathematics really is and how to apply mathematics in the classroom through realistic and rich learning experiences. I have also highlighted that teachers need to know the different ways in which mathematics is contained within the text, illustrations, and structure of picture books, and how to use it. If teachers use the ideas within articles such as Publications 7 and 8 with their students it is anticipated that they will gain the confidence to incorporate picture books more widely in their mathematics teaching and learning experiences.

In Chapter 11, I provide a summary of the findings from the three phases of the study, and discuss limitations of the study. I also describe some further implications for research and practice.

CHAPTER

11

CONCLUSION

In this concluding chapter, I first briefly summarise the findings of all three phases that comprise the research within this thesis. These findings were discussed in more detail, where appropriate, within each phase, and they were referred to in Chapter 10 when I considered their implications for teaching and learning. Following the summary of the findings, I outline the limitations of the research and the implications for further research that arise from it. Finally I discuss the significance of the research before providing some concluding comments.

11.1 Summary of the findings

The findings of this research need to be viewed relative to the aims of the research and how those aims were pursued. I intended to investigate the role of picture books in promoting mathematics teaching and learning for young children from several perspectives. I included the views and needs of professionals when developing a classification scheme and framework that can assist them when selecting and evaluating picture books for rich mathematical learning experiences. I also sought the views of authors and author/illustrators of the books to broaden research about the role of text and images, particularly in mathematical picture books, and to support my classification scheme. Furthermore, I used teacher professionals and Year 1 students to demonstrate how they engage with picture books in the classroom. I presented the study in three phases, each responding to individual research questions, the overall purpose, and the theoretical basis of the study. How the research questions have been addressed will be highlighted in this chapter.

The findings of Phase 1 contributed to answering the first research question*:* What potential do picture books (of different types) have for promoting mathematical teaching and learning for young children? I first identified and defined three types of mathematical picture books according to the way the mathematics is included. These types were perceived, embedded, and explicit. The classification scheme was evaluated through a survey with education professionals, and it was found that the more experience participants had in mathematics education, the more consistent they were in their classification with my classification. However, as reported in Chapter 10, recognition of opportunities for mathematics teaching and learning by professionals in picture books is affected by their perception of what constitutes mathematics. The definitions of the three types of picture book were further corroborated through author interviews (Phase 2) about their intent and their strategies for including mathematical concepts particularly.

In Phase 1, I also provided an account of the development and evaluation of a new framework that contained characteristics considered important for selecting and evaluating picture books to promote mathematics teaching and learning. Survey participants also used the framework with a selection of the three different types of picture book. They found it more difficult to identify mathematical opportunities in books classified as perceived, where the mathematics is not as apparent as in the other two types of picture book. The large variation in responses across all types of books and framework categories suggested again the restricted perception that teachers appear to have of mathematics. The findings of the framework survey also indicated that use of picture books for mathematical learning, in particular, is currently limited. Survey participants not only reported this framework as beneficial for identifying suitable books but, like Van den Heuvel-Panhuizen and Elia (2012), they found that it helped them identify the mathematics itself and generate ideas for rich learning experiences. The framework was then used to select books for use in the classroom study (Phase 3). Phase 1 was further supported by Publications 2 and 3 that demonstrated how to identify and apply different types of mathematical picture books for classroom use.

In Phase 2, I interviewed the authors and/or illustrators of books that had been classified as perceived and embedded books, to determine their general writing intentions. Their overriding intent was always to entertain and give joy to their readers. Previous research literature had demonstrated the important role of the illustrations in picture books and their interrelationship with the text. The interviews provided new insight through the authors' personal perceptions that were not necessarily based on theories about textual and visual roles. However, the authors emphasised that these book components should always work together to build the whole story and that the illustrations should not just duplicate the text.

The author interviews unearthed, in particular, how and why mathematical inclusions appear in their text and illustrations. Although the authors were aware that some of their books were being used in schools for other curriculum areas, they did not realise the range of mathematical possibilities that I had identified in Publications 2, 3, 7, and 8. They recognised that they (or their illustrator) incorporated diagrams, maps, and cross sections, for example, in the illustrations but these were used to depict what cannot be described easily in text. They did not acknowledge these or the mathematical words and prepositional phrases as being mathematical or having the potential to promote mathematical thinking. It was also noted that in many cases the design or structure of their books was also mathematical. At the time of the interviews, there was no intention by any author to teach mathematics. The insights gained have answered the second research question: What are authors' intentions when writing picture books with potential for mathematical learning?

The results of the classroom study (Phase 3), albeit with a small sample, demonstrated that all three types of mathematical picture books have the potential to engage students mathematically, and have benefits in different ways. First, the mathematical content analysis of the text of the three books revealed similar quantities of mathematical content within each of them. I found that there was more language of geometry in the perceived book as well as in the explicit book despite the latter's intended focus being doubling. More language of number was found in the embedded book. The qualitative analysis of the text and images of these books then revealed numerous possibilities for mathematical problem-solving investigation and integration both within mathematics strands and across curriculum areas particularly with perceived and embedded content books.

The teachers in the naturalistic classroom setting dominated the talk. They used significantly more mathematical talk while reading the explicit and embedded books than when reading the perceived book. Talk involving number was most prevalent in the embedded and explicit books, while the language of geometry was used more often for the perceived book. The text and illustrations in combination were the predominant source of all teacher mathematical talk. However, importantly the teachers did not appear to notice or explore the rich mathematical teaching and learning opportunities in the text and images and missed teachable moments, as suggested in Chapters 7 and 8.

Previous studies where there was no discussion of the book, or where any teacher/reader talk was scripted, demonstrated that children can initiate mathematical talk. However, in this study because the teachers dominated and guided the talk, I found little student talk. This does not necessarily mean the students in my study did not identify mathematics as much as in previous studies. It likely means that they did not necessarily have the opportunity to demonstrate this during the discussions. Most of their talk was in response to the teacher.

However, the representations completed after the book reading revealed the mathematical ideas students had seen and engaged with in the three books. There were more drawings with mathematical content for the explicit book, mostly based around numbers, but not always the doubling focus of the book. Additionally, the mathematical ideas in the representations for *Minnie's Diner* were not always accurate or showed greater depth of understanding. They were predominantly duplications of the book's illustrations. The few mathematical representations for the embedded book centred on the illustrations and characters in the book. The students responded to the perceived book with intricate maps (geometry) and measurement concepts that the teachers had not mentioned. These drawings, too, were mainly triggered by the illustrations of the book.

The mathematical ideas shown in these drawings suggest, like those obtained in research by Styles and Arizpe (2001), that representations are a valuable way to assess student understanding. McDonough & Cheeseman (2015) adopted the *Impress me* method and group discussion to assess their students' mathematical understanding during a unit of work. Their instruction to the students was "show me on this piece of paper all you know about ... You can write or draw or do both! Take your time and show your ideas and thinking as best you can. I want you to *impress me* with all you know about ..." (p.89). If the method used by McDonough & Cheeseman (2015) had been applied in the present study the students' may have provided fuller and more explicit responses with reference to the mathematical ideas.

Each type of book appears to have its merits. Although professionals (who participated in Phases 1 and 3) and the authors found the mathematics more difficult to identify in perceived books, the students cognitively engaged with it and I found many opportunities for rich activities using scaffolding (Vygotsky, 1978) and integrated learning. Embedded books were shown in the framework survey, the content analysis, and teacher-directed discussions to have potential for mathematical learning. However, the students' cognitive engagement with *Uno's Garden* was not reflected in the representations. This may have been due to that particular book and not necessarily all books with embedded content.

In Chapter 1 (Section 1.1.2), I referred to Hellwig, Monroe, and Jacobs' (2000) comment that picture books with explicit mathematical content are "glorified text books" (p. 139). I also included a warning from Nesmith and Cooper (2010) that explicit books "can be devastating to students" (p. 282) as they are written for adults to use for teaching, are often of dubious literary quality, and have no layers that allow students to explore. I also found during the content analysis of books in Phase 1 that many explicit mathematical picture books were of poor literary quality and did not afford problem-solving or integration potential with other curriculum areas. However, I do not necessarily believe that my findings in Phases 1 or 3 indicate that explicit mathematical picture books are detrimental for students if used appropriately. They may just need good teacher questioning skills as suggested by Elia, Van den Heuvel-Panhuizen, and Georgiou (2010).

The findings within Phase 3 demonstrate that in a naturalistic classroom setting students engage with mathematics through the real-life situations depicted in the text and illustrations of picture books of different types. Chapters 8 and 9 have also highlighted the crucial role of the teacher to exploit the situation. It is the way that teachers view mathematics, use the components of the picture book, and incorporate socioconstructivist principles that is most important when using picture books for mathematical learning. Therefore Phase 3 has particularly contributed to addressing the third research question: How do children and teachers engage with mathematical picture books of different types? but has also demonstrated the potential of picture books used in a naturalistic setting to promote mathematical teaching and learning for young children (the focus of Research Question 1).

11.2 Limitations of the study

The limitations of this research, including the overall scope of the study, the books selected for Phases 1 and 3, the sample size of the groups of participants in all phases, and some design aspects of the naturalistic study of Phase 3 are acknowledged in the following sections.

11.2.1 Scope

The scope of this thesis was broad because it aimed to address several exploratory aspects of this emerging research area. Although the three phases of this study complemented each other and aided in addressing the three research questions, each phase could have been designed to provide more in-depth analysis of the data. Larger and more representative samples would have provided more evidence to answer the research questions and provide opportunities for generalisation of the findings. In summary, these three phases provided initial investigations of a larger and more complex problem of the role of picture books in promoting early mathematics teaching and learning.

In this research I did not take into account influences such as teachers' perceptions of mathematics and their experiences in using picture books for promoting mathematical teaching and learning. The teachers' pedagogical content knowledge was addressed only at a surface level. Furthermore, the classroom study was limited to a small sample of children. The domination of the teachers limited the amount of student talk, particularly mathematical talk.

I identified many opportunities for mathematics teaching and learning through the development of the classification scheme and framework in Phase 1, the mathematical content analysis in Phase 3, and the included publications. However, the design of the study first did not allow for knowing the actual mathematics or the potential the professionals identified in the books in Phase 1. Second, the design did not allow for why professionals had classified books as they did. Third, teachers in Phase 3 did not have the opportunity to demonstrate how they would have further used the books within their classroom program.

The important role of the parent/caregiver reported in Section 2.4 in developing mathematical knowledge was also beyond the scope of the project.

Finally, some limitations are expected given the nature of the classroom study. Most researchers of cognitive engagement not involving parent readers (e.g., Elia et al., 2010; Van den Heuvel-Panhuizen & Van den Boogaard, 2008) used more prescriptive procedures than I did. Although use of a less structured approach in this research had advantages and advanced knowledge, the results are difficult to compare fully with earlier work and generalisations are less possible. These aspects will be further considered in subsequent sections.

11.2.2 Phases 1 and 3: Selection and content analysis of mathematical picture books

The picture books used in Phase 1, for both the classification process and trialling of the framework, and implementation in Phase 3, were primarily chosen from Australian and international book award lists. Although this strategy of using books with literary value has been used by others (e.g., Kesler, 2012) so that the book's literary qualities did not need to be considered, it may have restricted the selection of books. A broader sample of books that included those with high use by teachers may have produced different findings in classifying perceived, embedded, and explicit mathematical content. The percentage of mathematical content within each of the books chosen for the classroom study, Phase 3, was very similar. This may not have occurred with a mix of award and predominantly non-award books found on the market and used in classrooms.

Additionally, although the text of the books was analysed thoroughly, and opportunities were noted for investigations in the images, it was not within the scope of this study to thoroughly analyse each image in the books. Therefore the findings cannot be directly compared with the studies such as that of Elia et al. (2010).

11.2.3 Phase 2: The authors

The authors invited to participate in the study were carefully selected because their books reflected substantial inclusion of mathematical concepts in a variety of ways. These four prominent Australian authors were interviewed because of their interest in the topic and willingness to participate. Thus there was no intent for this sample to be representative of all authors' or author/illustrators' perceptions. Responses from a broader sample may not have resulted in similarly rich and insightful data or such cooperation. It was not possible to obtain a sample of authors of explicit mathematical picture books, so a more complete analysis of the way that mathematical content is included in picture books was not achieved.

The analysis of the author interviews was limited to some extent to the semi-structured questions that I had developed prior to the interview. Further probing questions about the nature of the authors' perception about what was mathematical could have supported their responses. I also noticed as I read the transcripts of the interviews and analysed the results how I had missed opportunities to pursue an idea further or clarify a point. Although I obtained much new and rich information about author intent, and how authors plan, write, and structure their books, these interview data were restricted to a once-off snapshot of their views. Further content analysis of the range of each author's books, longitudinally, would support a more complete analysis.

11.2.4 Phase 3: The teachers

Different findings might have been obtained with a more diverse or larger sample of teachers, or with teachers more experienced in use of picture books for mathematical purposes. The pedagogical approach of each teacher was different and may also have reflected their limited experience with recognising mathematics in picture books for mathematical learning. Van den Heuvel-Panhuizen and Elia (2013) recognised that even though they had provided a script for their teachers, it was difficult to "isolate the influence the characteristics of the book itself from the influence caused by the reading of the book" (p.232). However, providing guidelines for these teachers or an "expert" conducting the session, while not naturalistic, could have provided more consistent results.

11.2.5 Student cognitive engagement

The aim of the classroom study was to record and analyse the student mathematical talk as a measure of cognitive engagement, but this was limited by the teachers' dominance of all talk. Students' responses too may have been different if the book was familiar to them and there were repeated readings. In terms of the study's design, observation of repeated readings of the book on subsequent sessions may have yielded more student responsiveness (and different presentation style of the teachers). Additionally, the design of the study included teacher-student discussion during the book reading sessions. Again teacher dominance made it difficult to determine the full extent of the children's engagement (through their talk and representations) with the book alone.

Another issue was that there was no assessment of the students' literacy or numeracy levels. Although the students selected were expected to represent average ability in numeracy and literacy for this stage of learning, there may have been differences between the groups from each school. Prior knowledge of the book by the students was not ascertained either. Furthermore, the analysis focused on the immediate response of the students and not the long term effect. Students of different abilities and backgrounds might also produce different results.

11.3 Implications for further research

The findings in this study have implications for further research about the role of picture books in the mathematical development of young children. The limitations noted above also indicate the possibilities for future research for each phase of the study including:

Framework

- Further research about the effective use of the framework with a larger and more diverse sample of professionals, including questions about how and why professionals classified and categorise mathematical picture books;
- Evaluation of a wider range of books from the three classifications using the framework;
- A study of what education professionals perceive as mathematics in general;
- A study of the mathematical concepts identified by education professionals;
- An empirically based "controlled" study to ascertain the influence of the framework on selection of quality mathematical picture books. Two groups of participants could first evaluate the same books under two conditions: with and without the framework; and
- A study of the influence of teacher professional development concerning use of the framework for book selection and the identification of mathematical concepts within books.

Author Intent

Exploration using a larger data set (including the authors of explicit mathematical picture books) about the intent of authors of picture books generally, but more particularly when they include mathematical concepts within their books.

Classroom practice

Variations of the classroom study to assess the efficacy of the three types of mathematical picture books could comprise:

- A larger study using multiple books of each category to compare the **i**nfluence of book type;
- A more structured classroom study with protocols for using the books using either the teacher or a researcher, i.e., script of comments and questions to ask during reading session;
- Students' cognitive engagement being measured over several sessions with the same book;
- A larger study that includes a more diverse and representative sample of students in schools;
- A study that includes core opportunities for discussion with students about their representations ;
- An intervention study using the three types of mathematical book;
- A strand-specific intervention study using the three types of mathematical picture books;
- The effect of e-books compared with hard copies, e.g., Can picture books move with technology? and
- Opportunities for teachers to demonstrate how they would have used mathematical picture books beyond the shared reading session.

11.4 Significance

As my purpose in this research was to investigate the role of picture books for promoting mathematical teaching and learning experiences for young children from a number of perspectives, it is significant in several ways. Not only does it confirm and support some previous research, it advances the field by demonstrating that children cognitively engage with the mathematics in picture books when given the opportunity to discuss and represent their ideas individually and in groups.

This study was informed first by the findings of A. Anderson, Anderson, and Shapiro (2005) that different books influence the quantity and type of mathematical utterances from children. Other studies, e.g., those of Elia et al. (2010), Van den Heuvel-Panhuizen and Van den Boogaard (2008), and Van den Heuvel-Panhuizen, Van den Boogaard, and Doig (2009) have recognised and used both purposefully written books and those not intended to teach mathematics, and have revealed various levels of cognitive engagement. I expanded the general descriptions of the book types and defined three types of mathematical picture books according to the intentions of the author and the way in which the mathematics is included. This enabled me to compare children's engagement with the three different types of picture books as suggested by Elia et al. (2010).

Frameworks and criteria described in Chapter 2 were the basis for the framework developed in Phase 1. This new measure was created to enable professionals and caregivers to evaluate any picture book for mathematical learning potential. The framework took account of current pedagogy and research and, although derived from an Australian viewpoint, it is general enough to be internationally applicable. Use of the framework by the survey participants encouraged them to focus attention on the mathematical content and in turn this may have assisted them in developing rich mathematical experiences. A follow-up study of the influence of the framework and the exemplar books in classroom practice could provide evidence of the impact of the framework on teaching and learning. The study's findings have emphasised the need for a better understanding of what mathematics is and how mathematics can be found in picture books.

Importantly, this study has revealed for the first time the intentions of authors when they include mathematical ideas and concepts in picture books. The authors stated that it was never their intent to teach mathematics; their main purpose was to entertain. However, they recognised that mathematical ideas may be included in both the text and images, either unintentionally or purposely. Thus their insights supported the theoretical perspectives proposed about the function of images and text in picture books outlined in Section 3.1.2. These results also suggest that a collaborative approach between experienced writers and mathematics educators could produce high quality mathematical picture books.

Van den Heuvel-Panhuizen and Van den Boogaard (2008) suggested that studies should not be limited to only those that explore the effect of picture books on children without any "probing" by the reader: "The role of the adult reader is equally or may be even more important'' (p. 369). Thus there is a need for further studies into the role of the teacher in using mathematical picture books in the classroom to advance this emerging field (Van den Heuvel-Panhuizen & Elia, 2012).

However of notable significance are the students' representations, not previously explored, that revealed students' high cognitive engagement with the mathematics in the books.

Finally, the study has addressed and extended the premise that picture books promote mathematical cognitive engagement in young children. This has first been demonstrated through the content analysis of picture books and the findings of the framework survey in Phase 1, and the classroom study (Phase 3). However, as well as providing a framework for book selection and evaluation for professionals, I have included practical real-life suggestions to facilitate the use of picture books in publications in primarily professional journals. This standpoint has not been previously taken. These publications are based on the theoretical principles of socioconstructivism and the role of images and text outlined in Section 3.1. They may address the problems of professionals knowing the benefits of picture books in a mathematical setting, their (and their students) perceptions of and attitudes to mathematics, their choice of suitable books, and how to use them effectively in the classroom.

11.5 Final remarks

Previous research has justified the use of picture books for the development of literacy skills, and this study has added to the growing corpus of research that supports their use in mathematics teaching and learning for young children.

There is also growing awareness that traditional approaches to teaching mathematics are not producing positive learning outcomes (Sullivan, 2011; Van den Heuvel-Panhuizen, 2001). A socioconstructivist teaching style is seen as more appropriately meeting the needs of present day society, where individuals need to be able to think for themselves and apply their knowledge to new situations (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2012). Picture books provide the perfect opportunity for promoting mathematics learning using these principles.

Although the use of picture books for mathematical learning within the classroom is still not high (VerMass, n.d.) there appears to be increasing interest in using picture books to promote mathematical teaching and learning from some schools, universities, professional associations, and researchers. Daly and Blakeney-Williams (2015) report widespread use of picture books by teacher educators across all curriculum areas, including mathematics, in New Zealand because of “the strong beliefs in and understanding of the power of picture books as a pedagogical tool with pre-service teachers” (Daly & Blakeney-Williams, 2015, p. 99). They not only saw the power of picture books in changing negative attitudes in some subjects such as mathematics but also found that the text and illustrations “provide an avenue for gaining access to the perspective of the child learner” (Daly & Blakeney-Williams, 2015, p. 99).

Therefore, curriculum bodies, policy makers, teacher professional development courses, and preservice teacher education programs, with particular reference here to Australia, need first to consider the research findings about the benefits to student engagement and understanding of the use of picture books and other types of literature as part of an integrated curriculum. Second, curriculum bodies and curriculum writers need to promote the use of picture books in mathematics teaching and learning for young children. Curriculum writers may also help teachers to identify the characteristics that support mathematics learning and the different responses those characteristics might elicit. Specific examples of the use of picture books in the mathematics curriculum, particularly with links to resources, would aid teachers in classroom application. The crucial role of teachers, already identified, to make links between formal mathematics and real-life application also requires further acknowledgement.

Casey, Kersh, and Young (2004) stated that “to improve the ability to understand and retain mathematical knowledge, it is useful to embed the mathematics in a story context and to develop mathematical concepts through sequenced mathematics problems connected to the storyline” (p. 43) and images of picture books. Consequently, more needs to be done at all levels to promote teaching and learning opportunities with mathematical picture books, of all types, that have been proven to engage young children and improve their understanding and perception of mathematics. Most people enjoy reading or being read a good book/story. Using this strategy is not intended to sugar-coat mathematics but to place it where it is: around us and linked to all aspects of our everyday lives.

Postscript

As a result of the knowledge gained from developing the framework and the interviews with the authors, I wrote and published a picture book with embedded mathematical content. This piece of "creative work" is included here to demonstrate another consequence of my research. However it is not an examinable component.

Sophie's Prize (Marston, 2014) is the story of a young girl who wins $100.00 in a school art competition and needs to decide how she will spend it. The book can be read and enjoyed for the story alone. No amounts of money, other than the initial $100 are mentioned throughout the book. Teachers and parents can then estimate with their students and children, the different combinations of money Sophie could have spent on each item, or they can imagine what they would do if it was them. The book also includes other opportunities for mathematical activities or integration and some of these are suggested in the final spread.

This book is my attempt to write a book with potential for rich mathematical learning experiences. I believe that the embedded content and book address each category and many of the elements of the framework. There is accurate mathematical content with problem solving and reasoning potential at an appropriate curriculum stage, a meaningful real life context that promotes a positive attitude to mathematics, opportunities for integration across other curriculum areas, and easy pedagogical implementation in the classroom setting.

The Illustrations, by Lexie Watt, add to the meaning of the text, and include mathematical concepts and opportunities for problem solving and investigations.

However, it initiates the opposite problem to that posed in Chapters 1 and 6, of experienced and award winning authors writing books of literary value with mathematical inclusions but no mathematics education background. Here the experienced mathematics educator has tried to write a book that may not have literary value. Right back in 1997 Schiro emphasised a need for more quality mathematical picture books that were based on concepts in meaningful, real-life situations. Therefore, a combination of award winning authors and mathematics educators may be needed to provide quality mathematical picture books for students.

The book has been selling in general bookshops and through Mathematical Associations around Australia with repeat orders received. It is also being recommended in teacher training programs in Tasmania and South Australia.

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