# Covering Systems

by

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This thesis entitled:

**Covering Systems**

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has been approved for the Department of Mathematics

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Dr. Gerry Myerson

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Date _____

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Thesis directed by Senior Lecturer Dr. Gerry Myerson and Prof. Paul Smith.

**Statement of Candidate**

I certify that the work in this thesis entitled "**Covering Systems**" has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree to any other university or institution other than Macquarie University.

I also certify that the thesis is an original piece of research and it has been written by me. Any help and assistance that I have received in my research work and the preparation of the thesis itself have been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis.

Paul Emanuel (40091686)

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**Summary**

Covering systems were introduced by Paul Erdős [8] in 1950. A covering system is a collection of congruences of the form $x \equiv a_i (\text{mod } m_i)$ whose union is the integers. These can then be specialised to being incongruent (that is, having distinct moduli), or disjoint, in which each integer satisfies exactly one congruence.

This thesis studies incongruent restricted disjoint covering systems (IRDCS), collections of congruence classes which cover a finite interval of the integers disjointly, subject to an additional technical condition. There exist IRDCS of length 11 and all lengths greater than or equal to 17. These IRDCS are used to study questions analogous to those of interest in covering systems. We focus on the following questions.

(1) Can the smallest modulus of some IRDCS be arbitrarily large?

(2) Do there exist IRDCS with all moduli odd?

(3) What is the appropriate two-dimensional generalisation?

This thesis addresses these questions and makes significant headway towards their resolution.

Chapter 5 studies IRDCS with large minimum modulus. We present, amongst other examples, one IRDCS with minimum modulus 50.

In Chapter 6 it is shown that there are IRDCS with only odd moduli. The smallest example is one of length 83. This chapter will present information on all of the known examples of what will be referred to as odd IRDCS.

Finally, in Chapter 7, we extend the definition of IRDCS to two dimensions, determining conditions on the relevant parameters for the existence of such structures. In this chapter we also study some of the structural properties, analogous to those of one-dimensional IRDCS, for these new constructions.

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