

SINGULAR INTEGRALS AND FUNCTION SPACES

By

The Anh Bui

A THESIS SUBMITTED TO MACQUARIE UNIVERSITY
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
DEPARTMENT OF MATHEMATICS
JANUARY 2013



MACQUARIE
UNIVERSITY
FACULTY OF SCIENCE

This thesis entitled:

SINGULAR INTEGRALS AND FUNCTION SPACES

written by The Anh Bui
has been approved for the Department of Mathematics

Xuan Thinh Duong

Paul Smith

Date:_____

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

© The Anh Bui, 2013.

Typeset in L^AT_EX 2_ε.

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree to any other university or institution other than Macquarie University.

I also certify that the thesis is an original piece of research and it has been written by me. Any help and assistance that I have received in my research work and the preparation of the thesis itself has been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis.

The Anh Bui

Abstract

The main aim of this thesis is to study the boundedness of some singular integrals on various function spaces. The main results of this thesis are presented in three parts.

In the first part, two criteria on the L^p -weighted norm inequalities of singular integral operators with non-smooth kernels and the endpoint estimates of the commutators of these operators with BMO functions are obtained. As applications, we first studied the weighted norm inequalities of Riesz transforms associated to Schrödinger operators, Green functions and spectral multipliers and then endpoint estimates of commutators of these singular integrals with BMO functions such as the Riesz transforms, the square functions and the spectral multipliers.

The second part is dedicated to study the Hardy spaces associated to the discrete Laplacians on graphs and applications. Some characterizations of Hardy spaces associated to operators such as the atomic characterization and the square function characterization are obtained. Then we consider the boundedness of singular integrals on these Hardy spaces.

In the third part, we develop the theory of Hardy spaces, RBMO spaces and Calderón-Zygmund operators in the setting of nonhomogeneous spaces. Some important results are addressed in this part such as the Interpolation Theorem between Hardy spaces and RBMO spaces, the boundedness of Calderón-Zygmund operators on Hardy spaces and RBMO spaces and the Calderón-Zygmund decomposition.

Acknowledgements

Firstly, my special thanks goes to my supervisors, Prof. Xuan Thinh Duong and Prof. Paul Smith, for giving me a chance to study harmonic analysis and a great support in all time of research.

My sincere thanks also goes to my friends: Eric Lui, Fu Ken Ly, Ji Li, Galyna Safonova , and Kaiser Lock for their helpful discussion and for all fun we had in more than three years in Macquarie.

I also would like to kindly thank the staff in Department of Mathematics, especially Dr. Fran Griffin and Ms. Christine Hale, for giving me a great environment of research.

Last but not the least, I would like to thank my family: my parents, my parents-in-law, my brothers, and especially my wife for their support and encouragement throughout my life.

Contents

Abstract	v
Acknowledgements	vii
1 Introduction	1
2 Preliminaries	9
2.1 Spaces of homogeneous type and Muckenhoupt weights	9
2.1.1 Spaces of homogeneous type and Hardy spaces	9
2.1.2 Muckenhoupt weights	11
2.2 Holomorphic functional calculus	13
3 Boundedness of some singular integrals with non-smooth kernels	15
3.1 Weighted inequalities of some singular integrals with non-smooth kernels	15
3.1.1 Weighted norm estimates for singular integral operators and their commutators with BMO functions	17
3.1.2 Applications	24
3.2 Endpoints estimates for commutators of singular integrals with non-smooth kernels with BMO functions	35
3.2.1 Hardy spaces associated to operators	37
3.2.2 Boundedness of singular integral operators and their commutators	40
3.2.3 Commutators of BMO functions and Riesz transforms or square functions on doubling manifolds	45
3.2.4 Commutators of BMO functions and Riesz transforms associated with magnetic Schrödinger operators	49
3.2.5 Holomorphic functional calculi and spectral multipliers	56
4 Hardy spaces associated to the discrete Laplacians on graphs and boundedness of singular integrals	65
4.1 Weighted graphs	65
4.1.1 Graphs	65
4.1.2 Assumptions	67
4.1.3 Interior results of singular integrals on graphs and our results .	68
4.2 Hardy spaces associated to the discrete Laplacian	69
4.2.1 Tent spaces on graphs	69
4.2.2 Atomic Hardy spaces	71

4.2.3	Hardy spaces via square functions	72
4.2.4	Characterizations of Hardy spaces $H_L^p(\Gamma)$	73
4.3	The space of functions of Lipschitz type associated to operators	82
4.3.1	Definition of $\mathfrak{L}_L(\alpha, M, \Gamma)$ spaces	82
4.3.2	The spaces $\mathfrak{L}_L(\alpha, M, \Gamma)$ and Carleson measures	84
4.4	Duality of Hardy spaces	87
4.5	Boundedness of some singular integrals on graphs	91
4.5.1	Square functions	91
4.5.2	Spectral multipliers of the discrete Laplacian	93
4.5.3	Riesz transforms	96
5	BMO and Hardy spaces on non-homogeneous spaces	99
5.1	Non-homogeneous spaces, families of doubling balls and singular integrals	100
5.1.1	Non-homogeneous spaces and families of doubling balls	100
5.1.2	Calderón-Zygmund operators	102
5.2	The RBMO spaces	103
5.2.1	Definition of $\text{RBMO}(\mu)$	103
5.2.2	Some characterizations of $\text{RBMO}(\mu)$	104
5.3	Interpolation results	107
5.3.1	The sharp maximal operator	107
5.3.2	An Interpolation Theorem for linear operators	109
5.4	Atomic Hardy spaces and their dual spaces	110
5.4.1	The space $H_{at}^{1,\infty}(\mu)$	110
5.4.2	The space $H_{at}^{1,p}(\mu)$	112
5.5	Calderón-Zygmund decomposition	114
5.5.1	Calderón-Zygmund decomposition	114
5.5.2	The weak $(1, 1)$ boundedness of Calderón-Zygmund operators	119
5.5.3	Cotlar inequality	121
5.6	The boundedness of Calderón-Zygmund operators on RBMO spaces and Hardy spaces	124
5.6.1	The boundedness of Calderón-Zygmund operators from L^∞ to RBMO space	124
5.6.2	The boundedness of Calderón-Zygmund operators on Hardy spaces	126
5.7	Commutators of Calderón-Zygmund operators with RBMO functions	127
	References	133