ESSAYS ON POST-RETIREMENT FINANCIAL PLANNING AND PENSION POLICY MODELLING IN AUSTRALIA

by

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Thesis
Presented to the Faculty of Business and Economics,
Macquarie University in Partial Fulfillment
of the Requirements for the Degree of

PhD in Economics

Macquarie University, Sydney, Australia

July 2013

Abstract

The problem of population aging prompts the developments of new retirement income products and changes to retirement income systems around the world. In this thesis I develop models to model the financial preferences and behaviour of retired households, and evaluate the impact of various policy changes. The main application focuses on the projection of the future cost of the Age Pension in Australia, taking into account the effect of retirees' financial behaviours and reactions to superannuation and Age Pension policy changes. The models and the system developed in this research can also be applied to the evaluation of retirement income products and other policy changes.

The utility model developed in this thesis offers the first joint consideration of luxury goods (in the form of bequests), housing, 'ultra-necessities' (in the form of a 'subsistence' rate of consumption in retirement) and Australia's Age Pension. The model parameters are calibrated to the ABS data of Household Expenditure Survey and Survey of Income and Housing. The calibrated model reasonably explains the financial behaviour of surveyed Australian households, including the observed concentration of wealth in the family home in Australia, and the slow decumulation of wealth of rich households in retirement.

This thesis then presents long-term projections of the cost of public pensions in Australia, with retirees' financial behaviours modelled with the developed utility model. I assume retirees make financial decisions to maximise their lifetime utilities, and their consumption and asset allocation react to policy changes. I find that the future cost of the Age Pension to be about 13 percent higher than estimated by the Australian Treasury in 2010's Intergenerational Report. As future cohorts retire with more savings, they can allocate more money into owner-occupied properties while preparing for retirement and draw down their savings faster, to optimise their Age Pension entitlements.

Acknowledgments

This thesis is structured according to the guidelines governing Thesis by Publication at Macquarie University. Comprised of four research papers, the first 'Dynamic asset allocation when bequests are luxury goods' is co-authored with Professor Geoffrey Kingston and Dr. Sachi Purcal. I am the principal author of this paper, with my contribution amounting to greater than fifty percent of all work undertaken as per agreement with the co-authors.

This research is sponsored by Capital Market CRC and Rice Warner Actuaries. I thank my supervisor Professor Geoff Kingston, my co-supervisor Dr. Sachi Purcal and my colleagues Michael Rice and Jonathan Ng at Rice Warner Actuaries for their valuable comments and contributions, which made the completion of this thesis possible.

Statement of Originality

The work contained in this thesis has not been previously submitted for a degree or diploma at any other higher education institutions to the best of my knowledge and belief. This thesis contains no material previously published or submitted for publication by another person except where due reference has been made.

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Contents

Chapter 1

Introduction

1.1 Research Background

In recent years, much research and policy development have focused attentively on the problem of population aging around the world. Traditionally, retirees live on defined benefit pensions provided by the government and their employer, and face no financial risks. However, times have changed. According to the 2010 Intergenerational Report (IGR 2010), the aged dependency ratio[1] in Australia is currently 22 percent, but it is forecast to increase to 43 percent by 2051. Changes of this magnitude will inevitably place a severe strain on the public pension system and traditional defined benefit plans.

To deal with this demographic change, in recent years defined contribution plans have gradually replaced defined benefit plans in many countries around the world. Australia has built its three pillar retirement system including:

[1]The ratio of number of people above age 65 to those aged between 15 and 64.

- Superannuation Guarantee (SG), a mandatory contribution (made by employers) which is currently 9 percent of wages,[1] increasing to 9.25 percent from July 2013 and then gradually to 12 percent from July 2019
- Personal Savings, which include voluntary contributions into the superannuation account (many of which involve tax concessions)
- A means-tested Age Pension funded from government's general revenue, the pension commences at age 65 for males and (currently) 64 for females but both increase to age 67 over the next decade[2]

Retirees now face the challenge of managing their defined contribution savings for retirement consumption to deal with various risks. This is becoming increasingly important in Australia as the SG system matures, because more and more Australians are retiring with a large lump sum of superannuation savings but lacking the expertise manage these funds. According to the superannuation Market projections produced by Rice Warner (2012c), in 2012 there were $430 trillion post-retirement assets under management, which is expected to increase by an average of 7.2 percent per annum and will amount to $1,212 trillion in 2027.

Some retirement income products exist in the market (including life annuities, reverse mortgages and various other health and mortality related insurance products) to address the financial and longevity risks faced by retirees. Yaari (1965) demonstrated that in a world without bequest motives and with complete markets, a

[1]SG used to be 3 percent in 1992 and was gradually increased to 9 percent by 2002.
[2]Veterans (pensioners who have been in the armed services) receive identical benefits but are entitled to access them five years earlier than civilians.

household should purchase life annuities with all available wealth. However, in reality, very few retirees voluntarily purchase these products (SoA, 2010; Ganegoda and Bateman, 2008). Research by Dynan et al. (2004) and De Nardi et al. (2010) shows that assets of the elderly decrease slowly if at all, and many die with significant bequests in the form of housing equity and liquid assets. This has generated much interest in utility based models to represent retirees' financial behaviours and preferences.[3]

Much research on this issue has also been conducted in Australia. Current applications of these models in Australia, however, adopt many parameter values from overseas studies.[4] The retirement income system in Australia differs to most other developed economies in that it incorporates complicated means testing rules[5] and covers around 80 percent of elderly households in Australia. Many retirees' financial decisions are affected by the Age Pension means testing rule, especially housing. [6]

[3]Bateman et al. (2007), Yogo (2009) and Ameriks et al. (2010) etc.

[4]Bateman et al. (2007), Kudrna and Woodland, (2009), Cho and Sane, (2009), and Hulley et al. (2012) etc.

[5]Australia's Age Pension payment is subject to two means tests: the assets test and the income test, which reduce the pension entitlement when the income and/or assets of the household increase. The actual pension payment is the lesser of the entitlements under the assets test and income test. The payment is also bounded, such that it cannot be negative or exceed the maximum pension rate.

[6]Homeowners and non-homeowners are treated differently by the assets test. However someone owning a $2,000,000 house is treated the same as someone owning a $200,000 house, creating incentives for the household to allocate a higher amount of wealth in their family home. Current evidence shows that Australian retirees are likely to be overinvested in housing (Cho and Sane, 2013), with ABS data indicating that in Australia about half of the elderly claim to have spare capacity in their homes. Bradbury (2008) shows that home-ownership in Australia post-retirement is greater than in most other developed countries.

The complexity of Australia's retirement system poses significant problems for the government. For example, the retirement system is subject to constant changes for fiscal reasons. However, the three pillars of Australia's retirement system are interrelated and necessitates complex analysis in order to evaluate the impact of a change in policy to the welfare of retirees and the government's budget. Superannuation is complex enough and the Age Pension adds another dimension of difficulty.

IGR (2010) projected the Age Pension payments to rise to about 3.9 percent of Australian GDP in the 2049-50 financial year, while the legislated increase of SG from 9 percent to 12 percent is projected to reduce future Age Pension outlays by $3.8 billion in 2035-2036, with the cumulative total saved for every year from 2012-13 to 2035-36 being $41 billion (ASFA, 2011). According to Rothman (2012), Treasury's projection of future Age Pension payment is less than would result from purely demographic changes, mainly because of the increasing wealth and income of successive cohorts of retirees as Australia's superannuation arrangements mature.

Treasury's projection of future Age Pension payments are based on its RIMGROUP model, which does not model the change of retirees' financial behaviour, especially housing, in response to the changes in personal circumstance or public policy.[7] Currently 75 percent of Australian retirees are home-owners, with the value of owner-occupied housing accounting for about 80 percent of their total wealth. It is also especially important for any Age Pension related modelling, because the value of owner-occupied property is treated leniently by the current means testing rules.

[7]According to Rothman (2012), the RIMGROUP model does not take into account optimal financial behaviours of the underlying population. Instead people's financial decisions such as dissipation rates are set in line with assumptions according to income decile and the level of wealth.

Unless the treatment for owner-occupied properties is adjusted in future policy changes, utility maximizing households are likely to allocate a bigger proportion of their savings into the family home to optimize their Age Pension entitlements. The anticipated reduction in future Age Pension payments due to a wealthier population therefore can be much less than estimated in the IGR (2010).

Projection of the impact of public policy changes taking into account behavioural effect can be achieved with the integration of a utility based model with a policy projection model, which is the main research objective of this thesis. I develop a utility model for the preferences of retirees, and provide the first joint consideration of luxury goods (in the form of bequests), housing, 'ultra-necessities' (in the form of a 'subsistence' rate of consumption in retirement) and public pensions, with the model parameters specifically calibrated for Australian retirees. I then present long-term projections of Age Pension costs in the future, with retirees' financial behaviours modelled with the developed utility model.

1.2 Thesis Structure

This thesis is structured according to Macquarie University's guidelines for Thesis by Publication and is comprised of four research papers:

1. Chapter 2: Ding et al. (2012), "Dynamic asset allocation when bequest are luxury goods".

2. Chapter 3: Ding (2013a), "Australian retirees' choices between consumption, public pension, bequest and housing".

3. Chapter 4: Ding (2013b), "Modelling post-retirement finances in the presence

of a bequest motive, housing and an Age Pension".

4. Chapter 5: Ding (2013c), "Superannuation policies and retiree's economic responses: How much Age Pension?".

These four research papers jointly contribute to fulfill the research objectives of this thesis, which is structured as follows:

1.2.1 Utility models

In recent years, substantial research and attention has focused on studying people's behaviour in the post-retirement phase. Some of the major findings in the previous literature are as follows:

- Various research in Australia and overseas shows that expenditure in the post-retirement phase generally decreases with age (see for example: Bernicke, 2005; Higgins and Roberts, 2011), which can be attributed to declining health as the retiree ages (Yogo, 2011).

- ABS (2011a) data show that there is little difference in the expenditure of different age groups for households in the low to middle wealth bands. This is consistent with the consumption floor argument and Hyperbolic Absolute Risk Aversion (HARA) utility function as presented in Merton (1971), Bateman et al. (2007) and others.

- For many Australian retirees, the family home and contents are their only capital asset (Olsberg and Winter, 2005). Cho and Sane (2013) show that it is optimal for households to over-invest in housing when the value of the family home is exempted from the assets test. It is therefore important to consider jointly family home and Age Pension means testing in Australia.

- Many Australian and international studies find positive or zero saving during retirement is common (see Hulley et al., 2012; Feinstein and Ho, 2000), indicating the importance of bequest motive. ABS (2012a) data show that for households in the middle wealth bands, the majority of their wealth is in the family home. However, the proportion of wealth in the family home then decreases as the households become wealthier. This suggests that saving in liquid wealth only occurs when household wealth is above a certain threshold and can be considered as a luxury good.

Motivated by these findings, Chapter 2 extends the HARA utility model considered in Bateman et al. (2007) to include bequest utility function considered in De Nardi (2004) and Lockwood (2012), which treat bequests as luxury goods. Chapter 2 finds that luxury bequests imply that a higher allocation to risky assets is optimal, compared to the standard case where utility from bequests is treated as having the same power form as utility from non-bequest consumption. There are also systematic effects of age on the optimal allocation, such that expected optimal percentage allocation to equity rises throughout retirement. This result contrasts with the popular strategy adopted by many financial planners that the percentage allocation to equity should decrease with age after retirement.

Ding (2013a) further extended the utility model to offer joint consideration of decreasing consumption, housing, luxury bequests and Age Pension eligibility. A semi-analytical solution to the model is derived in Ding (2013b). Which is shown to be very close to the solution derived using numerical dynamic programming, with clear advantages in computation time.

1.2.2 Parameter Calibration

The solution found in Ding (2013b) allows the utility model in Ding (2013a) to be effectively calibrated to the ABS Household Expenditure Survey and the Survey of Income and Housing (ABS, 2011a), to find the set of parameters that best fit the preferences of current Australian retirees. The result demonstrates that the calibrated model reasonably explains the financial behaviour of surveyed households, including the observed concentration of wealth in the family home in Australia, and the slow decumulation of wealth of rich households in retirement.

Once the utility model is calibrated to ensure the modelled behaviour is similar to the current behaviour of Australian retirees, it can be utilised to analyse the changes in household behaviour given a new investment product or a change in pension policy, by assuming that if the market or policy environment changes, the change of the modelled behaviour is also going to be similar to the actual change.

1.2.3 Age Pension projection

Ding (2013c) presents long-term projections of the cost of public pensions in Australia, taking into account retiree's economic responses by integrating the utility model developed in Ding (2013a) with a population projection model. I estimate that as future cohorts retire with more savings, they can draw down their savings faster and allocate more money into owner-occupied properties to optimize their retirement needs and Age Pension entitlements, and the cost of Age Pension payments in 2035-36 financial year would be about 13 percent higher than estimates according to IGR (2010). In Ding (2013c) I also look at how projected future Age Pension costs are affected by various policy changes, including the legislated increase of superannuation guarantee from 9 percent to 12 percent, the possible changes of including the value of the family home in the assets test, and indexing Age Pension payments to price inflation instead of wage inflation.

These papers are presented in the following chapters of this thesis.

Chapter 2

Dynamic asset allocation when bequests are luxury goods

Abstract

Luxury bequests impart systematic effects of age to an investor's optimal allocation: the expected percentage allocation to equities rises throughout retirement. When bequests are luxuries the marginal utility of bequests declines more slowly than the marginal utility of consumption. This is essentially lower risk aversion. As a retiree approaches death, her expected remaining lifetime utility is increasingly composed of bequest utility, and thus generates progressively lower risk aversion. A retiree responds by increasingly favoring the higher-return risky asset. Compared to standard preferences, luxury bequests elevate a retiree's average exposure to the risky asset, but the difference is small in early retirement.

JEL classification: D14, G11, G23. *Keywords*: bequests, luxury goods, dynamic asset allocation, Merton portfolio problem, European put option, retirement risk zone.

2.1 Introduction

This paper offers the first analysis of the implications for dynamic asset allocation of bequests that are luxury goods.[1] Luxury bequests impart systematic effects of age to an investor's optimal allocation. In particular, one's expected percentage allocation to equity rises with age.[2] By contrast, standard analysis highlights the case of a constant percentage exposure to equity. Sharper still is the contrast between the main result of this paper and a popular rule of thumb that says an investor's percentage allocation to equity should be set at 100 minus her age, even in retirement. Luxury bequests elevate one's average exposure to risky assets, compared to the standard case where utility from bequests is treated as having the same power form as utility from non-bequest consumption. However, a numeric exercise suggests that the difference is small in early retirement, consistent with the notion that a retiree should have an upward-sloping equity-age profile.

[1]Atkinson (1971) and Davies (1982) invoke luxury bequests to explain persistent wealth inequality across generations in the face of regression to the mean in earnings. Menchik (1980) estimates that the elasticity of bequests with respect to lifetime resources is 2.5. Carroll (2002) invokes luxury bequests to explain the high allocation to equity-type assets in the portfolios of the rich. Dynan et al. (2002) and, especially, Lockwood (2011, 2012) point out that luxury bequests and precautionary saving help explain the facts of low voluntary annuitization and low take-up of long-term care insurance, especially by the upper half of the income distribution. They note that slow drawdowns of financial assets by many retirees are consistent with this explanation. Wachter and Yogo (2010) find that the share of risky assets in portfolios tends to rise with the investor's wealth, noting that this is evidence against homothetic preferences such as constant relative risk aversion. De Nardi et al. (2010) and Lockwood (2012) find evidence for luxury bequests based on the Method of Moments.

[2]Merton (1969, fn5) says that if the bequest function is not of power form then "systematic effects of age will appear in the optimal decision-making." Our contribution verifies this observation and builds on Merton's analysis by characterising the systematic effects of age when bequests are luxury goods.

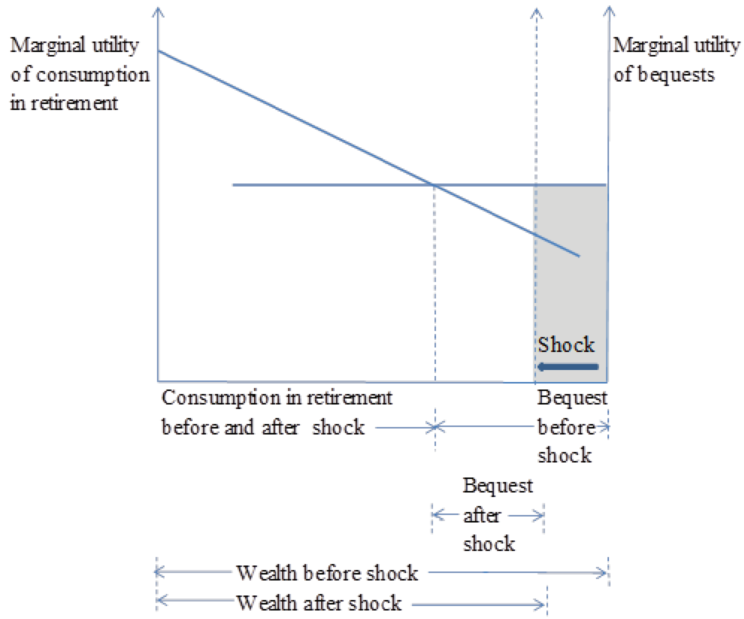

Figure 2.1: Bequests as shock absorbers.

Intuition for an expected high average exposure to investment risk late in life can be conveyed via a diagram for the stylized case when bequests are pure luxury goods. [3] See Figure 1.

The horizontal axis shows wealth taken into retirement, before and after a negative wealth shock. The left-hand vertical axis shows the marginal utility of consumption

[3] An optimum problem that is consistent with Figure 1 is given by

$$\max_{b,c} -\frac{(\bar{c}-c)^2}{2} + \theta b$$

subject to

$$b + c = w$$

and

$$b, c, w > 0.$$

The notation is: b bequest, c consumption, w wealth taken into retirement, $\bar{c}$ satiety level of consumption and θ marginal utility of bequests (assumed constant).

in retirement. The right-hand vertical axis shows the marginal utility of the planned bequest, portrayed in the figure as a pure luxury good in the sense that the marginal utility of the planned bequest is constant, corresponding to a perfectly elastic demand for bequests with respect to wealth. Following a negative wealth shock, the planned bequest drops by an equal amount. This lowers the investor's welfare; the shaded area shows the welfare reduction. But consumption in retirement stays the same.

Figure 1 also helps with intuition in the case of dynamic asset allocation. When bequests are a luxury good the marginal utility of bequests declines more slowly than the marginal utility of consumption. But this is essentially lower risk aversion. As a retiree approaches death, her expected remaining lifetime utility is increasingly composed of bequest utility, and thus generates progressively lower risk aversion. A retiree responds by increasingly favoring the higher-return risky asset.[4]

2.2 Model

We embed a bequest function studied by De Nardi et al. (2010) and Lockwood (2011, 2012), among others, into the portfolio model introduced by Merton (1969, 1971) and amended by Cox and Huang (1989). The investor makes contingent plans for a bequest $b(T)$, consumption rates $c(t)$, and proportionate investments $x(t)$ in risky assets, that maximize expected utility

$$E[\int_0^T e^{-\rho t}\frac{(c(t)-h)^{1-\delta}}{1-\delta}dt + e^{-\rho T}\theta^{\delta}\frac{(\theta a + b(T))^{1-\delta}}{1-\delta}], \tag{2.1}$$

[4]We thank a referee for suggesting this intuition for our main result.

subject to a budget constraint

$$dw(t) = [(x(t)(\alpha - r) + r)w(t) - c(t)]dt + x(t)w(t)\sigma dz(t), \tag{2.2}$$

and initial condition

$$w(0) > \frac{h(1 - e^{-rT})}{r}, \tag{2.3}$$

where the notation is: E expectations operator, T age at death (assumed known), ρ rate of time preference, h nonnegative utility parameter with the interpretation of 'subsistence' or 'protected' or 'habitual' consumption,[5] δ positive utility curvature parameter, $\theta \equiv \phi/(1-\phi)$ transformation of a utility parameter $\phi \in (0,1)$ that has the interpretation of "the marginal propensity to bequeath in a one-period problem of allocating wealth between consumption and an immediate bequest" (Lockwood, 2012, p.6), a nonnegative bequest utility parameter with the interpretation of the "threshold consumption level below which, under conditions of certainty or with full, fair insurance, people do not leave bequests" (Lockwood, 2012, p.7), w wealth, α instantaneous expected return to risky assets, r return to safe assets, σ volatility of risky assets, and dz Wiener increment.

The bequest function in Eq. (2.1) can be related to Fig. 1. The first derivative of

[5] Wachter and Yogo (2010) introduce nonhomothetic preferences by distinguishing between necessities and luxuries in non-bequest consumption, noting but not invoking luxury bequests. We follow Merton (1971) in distinguishing between 'protected' and 'unprotected' consumption. This also generates a high share of risky assets in the portfolios of the rich, but simplifies analysis as it avoids the complication of a relative price between pure necessities and other goods in non-bequest consumption. 'Protected' consumption is a pure necessity as its elasticity of demand with respect to wealth is zero.

the bequest-utility function with respect to the amount of bequest b is

$$\theta^{-\delta}(\theta a + b)^{-\delta} \geq 0, \tag{2.4}$$

and the second derivative is

$$-\delta\theta^{\delta}(\theta a + b)^{-(1+\delta)} \leq 0. \tag{2.5}$$

It follows from Eq. (2.5) that the schedule portraying the marginal utility of bequests in Fig. 1 becomes flatter as the bequest-utility parameter a increases. The schedule is completely flat in the limiting case $a \to \infty$.

2.3 Solution

We solve the problem (2.1)–(2.3) in three steps. Step 1 changes variables and then uses dynamic programming to determine what Cox and Huang (1989) describe as an 'unconstrained policy' for investing wealth. Dynamic programming alone does not solve the problem (2.1)–(2.3), however, as a consequence of the nonnegative bequest shift parameter a in Eq. (2.1).[6] Specifically, a negative bequest could arise if bequests are strong luxuries in the sense that the product term $a\theta$ is high relative to initial wealth net of protected consumption, $w(0) - h(1 - e^{-rT})/r$. Step 2 draws on Cox and Ross (1974) and Cox and Huang (1989) to replicate a European put option that insures against negative bequests. Following Cox and Huang, constrained wealth, i.e., wealth net of the insurance premium, is invested in the unconstrained policy. Step 3 invokes a theorem of Cox and Huang to deduce that we have determined an optimal consumption and investment policy.

Step 1 follows Merton (1971) by changes of variables that map the optimum problem described by Eqs (2.1) and (2.2) into the dynamic-programming problem involving constant relative risk aversion that was posed and solved by Merton (1969), although the changes needed here are more extensive.[7] Define protected wealth

$$\hat{w}(t) \equiv \frac{h}{r}[1 - e^{-r(T-t)}] - \theta a e^{-r(T-t)}, \tag{2.6}$$

[6]Cox and Huang (1989) emphasize this type of problem, as do Sethi et al. (1992). The fact that Eq. (2.1) subtracts (nonnegative) protected consumption h from unprotected consumption, together with the power form of the instantaneous utility function, rules out negative non-bequest consumption. This renders our analysis simpler than Cox and Huang's, which allows negative values of h. Example 8.13 of Karatzas and Shreve (1998, pp. 132–133) solves a problem that looks at first sight to be close to the problem defined by Eqs (2.1)–(2.3) (Karatzas and Shreve use a pure martingale method to solve their problem). But its bequest-function shift parameter is of opposite sign to our shift parameter a, corresponding to the case of bequests that are necessities rather than luxuries. The case of necessities does not need the Cox-Huang complication of an option-type fund to rule out negative bequests.

[7]Our main novelty is the second term on the right-hand side of Eq. (6) below. Ingersoll (1987, p.246) points out in a setting without bequests that Merton's problem readily accommodates non-uniformity in the parameter corresponding to h in our setup.

surplus wealth

$$\tilde{w}(t) \equiv w(t) - \hat{w}(t), \tag{2.7}$$

transformed bequest

$$\tilde{b}(T) \equiv \theta a + b(T), \tag{2.8}$$

surplus consumption

$$\tilde{c}(t) \equiv c(t) - h, \tag{2.9}$$

and surplus investment

$$\tilde{x}(t) \equiv x(t)w(t)/\tilde{w}(t). \tag{2.10}$$

Step 2 ensures $w(T) \geq 0$, i.e.. $\tilde{w}(T) \geq \theta a$. It values and replicates a put option on an 'optimally invested' synthetic security $\widetilde{w}_\lambda(t)$,where the terminology and the subscript follow Cox and Huang (1989). Initial surplus optimally-invested wealth, $\tilde{w}_\lambda(0)$, is just small enough to ensure that sufficient wealth remains to guarantee a nonnegative bequest. Remaining initial surplus wealth, $\tilde{w}(0) - \tilde{w}_\lambda(0)$, is invested in a European put option on optimally-invested wealth[8]. The put's value subsequently time t is given by

$$p(\widetilde{w}_\lambda(t), t) \equiv E_t^Q \max[0, \theta a - \tilde{w}_\lambda(T)], \tag{2.11}$$

where the superscript Q on the right-hand side denotes the value of an expectation taken under the risk-neutral measure.

[8]This option can only be synthetic as it is contingent on the retiree's wealth, it is unlikely to be tradable in practice. Although such product can be of significant interest in the context of uncertain lifetimes, which would introduce the possibility of comparison with Ruin Contingent Life Annuities.

The option is self-funding through time, so that surplus wealth is conserved in the sense

$$\tilde{w}(t) = \tilde{w}_\lambda(t) + p(\tilde{w}_\lambda(t), t). \qquad (2.12)$$

Step 3 notes that Eq. (2.12) in conjunction with our solutions for optimally-invested surplus wealth and the put option fulfills the conditions for Theorem 2.4 of Cox and Huang (1989), thereby establishing that we have located an optimum.

Our main result is this:

Proposition

Optimal surplus consumption in the problem described by Eqs (2.1)–(2.3) is a time-varying fraction of constrained optimally-invested surplus wealth,

$$\tilde{c}^*(t) = \beta(t)\tilde{w}_\lambda(t), \qquad (2.13)$$

and optimal surplus investment is a constant proportion of constrained optimally-invested surplus wealth,

$$\tilde{x}^*(t) = \frac{\alpha - r}{\sigma^2\delta}, \qquad (2.14)$$

where

$$\beta(t) = \left[\frac{1 + (\nu\theta - 1)e^{\nu(t-T)}}{\nu}\right]^{-1} \qquad (2.15)$$

$$\nu \equiv \mu/\delta, \qquad (2.16)$$

and

$$\mu \equiv \rho - (1-\delta)\left[\frac{(\alpha - r)^2}{2\sigma^2\delta} + r\right]. \qquad (2.17)$$

∎

Step 1 finds values of $\tilde{b}(T)$, $\tilde{c}(t)$ and $\tilde{x}(t)$ that maximize

$$E[\int_0^T e^{-\rho t}\frac{\tilde{c}(t)^{1-\delta}}{1-\delta}dt + e^{-\rho T}\theta^{\delta}\frac{\tilde{b}(T)^{1-\delta}}{1-\delta}], \tag{2.18}$$

subject to

$$d\tilde{w}(t) = [(\tilde{x}(t)(\alpha - r) + r)\tilde{w}(t) - \tilde{c}(t)]dt + \tilde{x}(t)\tilde{w}(t)\sigma dz(t). \tag{2.19}$$

At this point we need not specify whether surplus wealth is constrained or unconstrained, i.e., gross or net of the insurance package. Eq. (2.18) is the same as Eq. (2.1) and Eq. (2.19) is the same as Eq. (2.2). Solving the optimum problem described by Eqs (2.1) and (2.2) is therefore the same as solving the optimum problem described by Eqs (2.18) and (2.19). Merton (1969, Sections 3 and 4) solves the latter problem by means of dynamic programming and begins by defining a value function. Our luxury-bequest counterpart is

$$J(\tilde{w}(t), t) \equiv \max_{\tilde{c}(t),\, \tilde{x}(t)} E_t[\int_t^T e^{-\rho s}\frac{\tilde{c}(s)^{1-\delta}}{1-\delta}ds + e^{-\rho T}\theta^{\delta}\frac{\tilde{w}(T)^{1-\delta}}{1-\delta}]. \tag{2.20}$$

Following Merton (1969, Section 4), the associated Hamilton-Jacobi-Bellman equation, upon plugging in the first-order conditions

$$\tilde{c}(t) = (e^{\rho t}J_{\tilde{w}})^{-1/\delta}, \tag{2.21}$$

and

$$\tilde{x}(t) = (-J_{\tilde{w}}/\tilde{w}J_{\tilde{w}\tilde{w}})(\alpha - r)/\sigma^2, \tag{2.22}$$

is given by

$$0 = \frac{\delta}{1-\delta} e^{\frac{-\rho t}{\delta}} \left(J_{\tilde{w}}\right)^{\frac{\delta-1}{\delta}} + J_t + r\tilde{w}J_{\tilde{w}} - \frac{1}{2}\frac{(\alpha-r)^2}{\sigma^2}\frac{J_{\tilde{w}}^2}{J_{\tilde{w}\tilde{w}}}, \tag{2.23}$$

with boundary condition

$$J(\tilde{w}(T), T) = e^{-\rho T}\theta^{\delta}\frac{\tilde{w}(T)^{1-\delta}}{1-\delta}. \tag{2.24}$$

Merton (1969) shows that the solution of Eqs (2.23)–(2.24) is

$$J(\tilde{w}(t), t) = \frac{\beta(t)}{1-\delta} e^{-\rho t}\tilde{w}(t)^{1-\delta}. \tag{2.25}$$

Eqs (2.13) and (2.14) follow from Eqs (2.21), (2.22) and (2.25), with $\tilde{w}(t)$ replaced by $\tilde{w}_{\lambda(t)}$.

Step 2 uses the Black-Scholes-Merton formula, in conjunction with the risk-neutral valuation argument of Cox and Ross (1976), to replicate a put option with strike θa on constrained optimally-invested surplus wealth $\tilde{w}_{\lambda}(t)$. Eqs (2.13), (2.14) and (2.19) show that the stochastic process for constrained optimally-invested wealth has deterministic drift $\tilde{x}^*\alpha + (1-\tilde{x}^*)r - \beta(t)$ and constant volatility $\tilde{x}^*\sigma$:

$$d\tilde{w}_{\lambda}(t) = \left[\tilde{x}^*(\alpha - r) + r - \beta(t)\right]\tilde{w}_{\lambda}(t)dt + \tilde{x}^*\sigma\tilde{w}_{\lambda}(t)dz, \tag{2.26}$$

where $\tilde{x}^*$ is pinned down by Eq. (2.14). Here $\beta(t)$ is like a deterministic (as distinct from fixed) continuous payout from a commodity corresponding to the synthetic risky asset $\tilde{w}_{\lambda}(t)$, analogous to a variable-proportion storage cost. The risk-neutral specialization of the process defined by Eq. (2.26) has instantaneous return r and (constant) instantaneous volatility $\tilde{x}^*\sigma$. Standard theory says that replicating the option specified by Eq. (2.11) with this asset and the safe asset requires going long by an amount

$$\mathrm{N}(-d_2)\theta a e^{-r(T-t)} \tag{2.27}$$

in the safe asset, and short an amount

$$\mathrm{N}(-d_1)\tilde{w}_\lambda(t)e^{-\int_t^T \beta(s)ds} \tag{2.28}$$

in the synthetic risky asset, where

$$d_1 = \frac{\ln\left(\frac{\tilde{w}_\lambda(t)}{\theta a}\right) + \left[r + \frac{(\sigma\tilde{x})^2}{2}\right](T-t) - \int_t^T \beta(s)ds}{\sigma\tilde{x}\sqrt{T-t}}, \tag{2.29}$$

$$d_2 = d_1 - \sigma\tilde{x}\sqrt{T-t}, \tag{2.30}$$

and $\mathrm{N}(\cdot)$ denotes the Normal distribution.[9] Replicating the put with the underlying risky asset at time t therefore requires going short an amount

$$\tilde{x}^*(t)\mathrm{N}(-d_1)\tilde{w}_\lambda(t)\theta a e^{-\int_t^T \beta(s)ds} \tag{2.31}$$

in the underlying risky asset.

Step 3 invokes Theorem 2.4 of Cox and Huang (1989) to deduce that the solution characterized by Eqs (2.12), (2.13), (2.14), (2.27), (2.28), (2.29) and (2.30) is indeed

[9]We have constructed a spreadsheet (available on request) that confirms numerically that Eqs (2.27) to (2.30) do in fact replicate in expectation the required put, and use these data to help draw Fig. 2 below.

an optimum.

2.4 Asset allocation

From Eqs (2.12), (2.14) and (2.31), followed by application of Eqs (2.6) and (2.7) to substitute out $\tilde{w}^*(t)$, the optimal dollar investment $A^*(t)$ in risky assets is

$$\begin{aligned} A^*(t) &= \tilde{x}^*(t)\tilde{w}_\lambda(t) - \tilde{x}^*(t)\mathrm{N}(-d_1)\tilde{w}_\lambda(t)e^{-\int_t^T \beta(s)ds} && (2.32)\\ &= \tilde{x}^*(t)[\tilde{w}^*(t) - p(\tilde{w}_\lambda(t), t) - \mathrm{N}(-d_1)\tilde{w}_\lambda(t)e^{-\int_t^T \beta(s)ds}] && (2.33)\\ &= \left(\frac{\alpha - r}{\delta\sigma^2}\right)[\tilde{w}^*(t) - \mathrm{N}(-d_2)\theta a e^{-r(T-t)}] && (2.34)\\ &= \left(\frac{\alpha - r}{\delta\sigma^2}\right)[w(t) - \frac{h}{r}(1 - e^{-r(T-t)}) \\ &\qquad + \theta a e^{-r(T-t)}(1 - \mathrm{N}(-d_2))]. && (2.35) \end{aligned}$$

Divide through by $w(t)$ to express the optimal proportionate investment $x^*(t)$ in risky assets in terms of the model's state variable and parameters:

$$x^*(t) = \left(\frac{\alpha - r}{\delta\sigma^2}\right)[1 - \frac{h}{rw(t)}(1 - e^{-r(T-t)}) + \frac{\theta a}{w(t)}e^{-r(T-t)}(1 - \mathrm{N}(-d_2)]. \quad (2.36)$$

In contrast to protected consumption without a luxury bequest ($h > 0$ and $a = 0$), luxury bequests ($a > 0$) elevate exposures to risky assets, at any point in time and for a given level of wealth. Like protected consumption, however, luxury bequests impart an upward tilt to the expected trajectory of proportionate investment in risky assets, again for a given level of wealth.

2.5 Numeric illustration

Figure 2 illustrates the effect of luxury bequests on expected asset allocation for a particular initial value of wealth and a particular set of model parameters. The dashed line portrays the standard homothetic case (i.e., protected consumption is zero) where utility from bequests has the same power form as utility from consumption. The expected and actual share of risky assets is 38 per cent of the portfolio. The solid line portrays the case of zero protected consumption along with a positive shift parameter in the function describing utility from bequests (thereby rendering them a luxury good) and a synthetic put option on optimally-invested wealth. The dotted line strips out the effect of our synthetic put on optimally-invested wealth, thereby shedding light on the empirical importance of looking beyond the solution resulting from dynamic programming alone.

At the initial age of 65, and in the case of the solution that rules out negative bequests (i.e., the solution that incorporates a synthetic put option), the expected and actual share of risky assets is 40 percent, or just 2 percentage points higher than in the standard case of a bequest function of power form. So, our example suggests that at the outset of retirement it is not important in practice to account for luxury bequests when allocating assets.

Had we disregarded the need to rule out nonnegative bequests, however, the allocation to risky assets would have been appreciably higher at the outset of retirement. This difference is consistent with the fact that the required synthetic put has considerable time value at the outset of retirement.

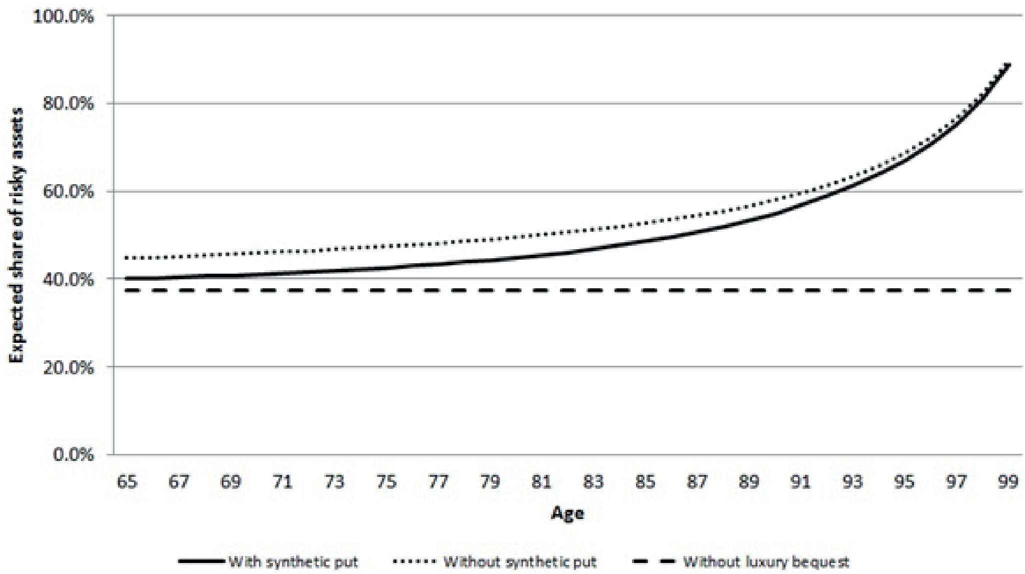

Figure 2.2: Risky assets share with and without luxury bequests. Note that here we have assumed: initial wealth=\$600,000, initial age=65, final age=100, rate of time preference=2% p.a., real interest rate=2% p.a., utility curvature parameter=2, expected return to risky assets=5% p.a., volatility of risky assets =20% p.a., bequest utility parameter = \$20,400, and propensity to bequeath = 0.92. The latter two values are from Lockwood (2012, Table 3). Protected consumption is zero.

At the final age of 100, and in the case of the solution that rules out negative bequests, the expected share of risky assets is 90 per cent, or 52 percentage points higher than it is in the standard case of a bequest function of power form. The expected bequest amount is \$131,933, or 22 percent of the wealth taken into retirement. In our example, then, assuming bequests are luxury goods makes a big difference to asset allocation late in retirement even though the planned bequest is not particularly big. On the other hand, the synthetic put makes scarcely any difference to asset allocation late in retirement, consistent with decay over time in its value.

2.6 Implications for investment advice

The standard case of constant relative risk aversion assumes preferences are homothetic, implying that advice on asset allocation is scalable. Luxury bequests are at odds with the assumption of homothetic preferences, in this way strengthening the case for customised advice. Notably, rich investors with bequest motives may need a more aggressive allocation, in line with Carroll's (2002) observations about actual allocations.

Concerning financial plans for people of middle means, Bengen (2001) and Milevsky and Salisbury (2006) explore numerically the notion of a 'retirement risk zone' whereby allocations at the outset of retirement need to be conservative. Notably, if there is a bear market on the cusp of one's retirement then it is difficult to recoup even if investment markets recover subsequently, as ongoing drawdowns deplete one's remaining wealth. Entering retirement with a high present value of protected consumption relative to wealth is one theoretical justification for a conservative allocation early on.[10]

We showed that luxury bequests justify aggressive allocations later on, in this way strengthening the case for planning an upward tilt in one's percentage exposure to equities throughout retirement.

Empirically, as shown in various studies[11], most households have less exposure to risky assets at older ages. This can be the result of a number of factors, including

[10]Karatzas and Shreve (1998) make this point.
[11]see for e.g. Cile and Milligan (2009), Poterba et. al (2011), Finlay (2012) and Spicer et al. (2013)

public social securities, health declines and the interaction between residential property and bequest motives. These issues will be subjects of further research.

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Chapter 3

Australian retirees' choices between consumption, Age Pension, bequest and housing

Abstract

This paper develops a life-cycle utility model of the preferences of retirees, with joint consideration of bequest motive, housing decision and public pension. The model parameters are calibrated to the ABS data of Household Expenditure Survey and Survey of Income and Housing. The calibrated model reasonably explains the financial behaviour of surveyed households, including the observed concentration of wealth in the family home in Australia, and the slow decumulation of wealth of rich households in retirement.

JEL classification: D14, G11, H55. *Keywords*: life-cycle model, utility, bequests, housing, Age Pension, financial planning, dynamic programming.

3.1 Introduction

This paper develops a life-cycle utility model of the preferences of Australian retirees. I offer the first joint consideration of housing, luxury goods (in the form of bequests), 'ultra-necessities' (in the form of a 'subsistence' rate of consumption in retirement) and public pensions. The model parameters are calibrated to the ABS data of Household Expenditure Survey (HES) and Survey of Income and Housing (SIH) 2009-10.

The calibrated model reasonably explains the financial behaviour of surveyed households. This can be illustrated in Figure 3.1 as an example[1], which plots for couple households the average value of family home as percentage of total wealth, against age and wealth percentiles of the model outputs compared to the ABS data.

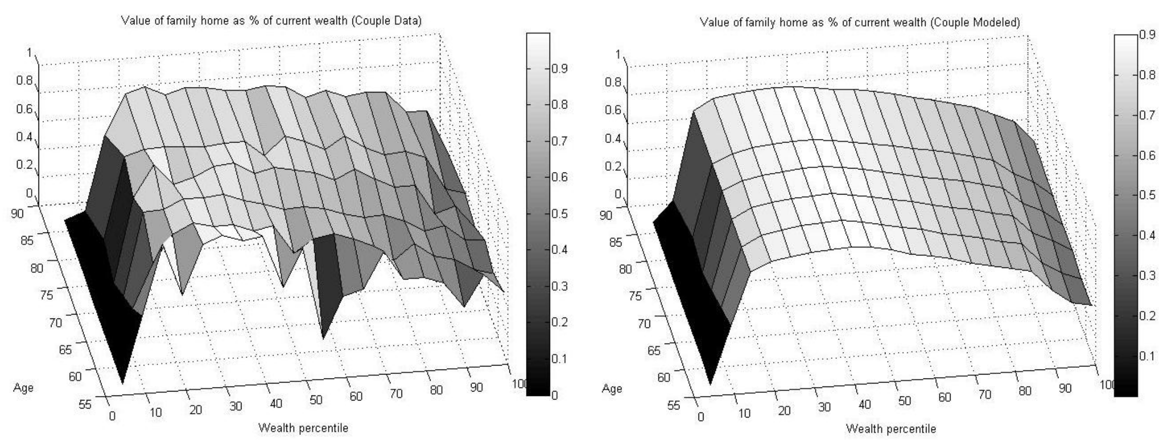

Figure 3.1: % of wealth in family home, Data vs Model output, couple households.

Life-cycle utility helps to model people's financial behaviour.[2] These models can be

[1]see Figure 3.10 in Appendix C for a bigger version of the graph

[2]This dates back to Samuelson (1969), Merton (1971) and Yaari (1965), amongst others.

applied in many practice areas including post-retirement financial planning,[3] pension product design[4] and the evaluation of policy changes.[5] Current applications of these models in Australia, however, adopt many parameter values from overseas studies.[6] The retirement income system in Australia differs from most other developed economies, incorporates complicated means testing rules and covers around 80 percent of elderly households in Australia.

Australia's Age Pension payment is subject to two means tests: the assets test and the income test, which reduce the pension entitlement when the income and/or assets of the household increase. The actual pension payment is the lesser of the entitlements under the assets test and income test. The payment is also bounded, such that it cannot be negative or exceed the maximum pension rate. Many details in the means testing rules can affect retirees' financial decisions. For example, the value of the family home is exempted from the assets assessable under the assets test, creating incentives for the household to allocate a higher amount of wealth in their family home.[7]

[3] One example of professional financial planning software developed using the utility model is ESPlanner (see Koltlikoff, 2008). The program received positive reviews in Turner and Witte (2009).

[4] One example is the Vanguard's Managed Payout Funds based on the research of Americks et al. (2011).

[5] Kudrna and Woodland (2009) analysed the effects of possible Age Pension policy changes with a general equilibrium model including household behaviours; Oliver and Dixon (2010) developed a utility approach to model the behavioural responses to public policy changes for the Australian Treasury.

[6] For example, Bateman et al. (2007), Kudrna and Woodland (2009), Cho and Sane (2013), and Hulley et al. (2012), etc.

[7] Details of Australia's Age Pension system can be found on the Centrelink website: http://www.centrelink.gov.au/internet/internet.nsf/payments/age_pension.htm

This paper extends the Hyperbolic Absolute Risk Aversion utility model considered in Merton (1971), Bateman et al. (2007) and others to include bequest motives (as considered in Lockwood, 2012), utility from housing consumption, and details of Australia's Age Pension means testing. In contrast to previous literature, I calibrate the model parameters to survey data to match the financial preferences of current retirees in Australia.

The calibrated model shows that the financial behaviours of wealthy households and poorer households are very different. The consumption profile of rich retirees departs markedly from consumption smoothing, spending much more on housing and bequests, while their consumption of non-housing, non-bequest goods decreases rapidly with age. They spend similar amounts to poorer households at later ages. In these ways, financial advice is non-scalable, contrary to the relevant prescription of the Constant Relative Risk Aversion utility model.

The findings of this paper also suggest that the high concentration of wealth in the family home in Australia is affected by the publicly provided Age Pension. First, the levels of Age Pension payments are well above the calibrated level of subsistence consumption needs, indicating that households in low wealth bands do not need much wealth outside of their family home to fund their retirement consumption. Second, the Age Pension assets test implies that it is optimal for households in the middle to high wealth bands to allocate wealth in their family home to receive higher Age Pension payments.

3.2 Literature Review

Studying people's behaviour in the post-retirement phase has been the focus of much research in recent years. Some of the major findings in the literature are as follows:

1. **Expenditure in the post-retirement phase generally decreases with age**. Rice and Higgins (2009) suggest that there are three different phases in retirement, with different consumption and health-care expenditure needed in each phase. A US study by Bernicke (2005) identified significant spending declines in every major category except health-care as people age. According to his research , the difference between the expenditure of US people age 65-74 and over 75 is 26.4 percent. Australian research by Clare (2011) and Higgins and Roberts (2011) shows similar declines but of a smaller magnitude. The decline in expenditure can be attributed to declining health (Yogo, 2011) or to fewer resources reserved for consumption at advanced age, due to uncertainty about the possibility of living to a very old age (Milevsky and Huang, 2011).[8]

2. **Most middle and wealthy households decumulate wealth very slowly**. Many Australian and international studies find that positive or zero saving during

[8]These researches did not include long-term-care expenses, which can be significant when people age and moves into an aged-care facility. The data used did not survey people who live in aged-care facilities either. Greenwald (2012) shows that about 60 percent of retirees in US are very or somewhat concerned about having enough money to pay for adequate health-care and long-term care. According to Arthur (2011), the cost of government funded aged-care in Australia can be up to $70,000 per year. Although only 20 percent of retirees use care facilities in Australia, this cost can be considered a significant financial risk for retirees.

retirement is common.[9] Empirical modelling by Hulley et al. (2012) of the data from the Household Income and Labor Dynamics (HILDA) survey, shows that Australian households decumulate slower than is 'optimal'. Although poorer households decumulate at around 5 percent p.a. on average, some wealthy households add around 3 percent p.a. to wealth, even when facing a steeper implicit tax rate due to age-pension means testing.

3. **Concentration of wealth in the family home**.

For many Australian retirees, the family home and contents are their only capital asset and their only income is derived from superannuation, insurance or government pensions (Olsberg and Winter, 2005). Empirical studies across some advanced countries show that the home-ownership, as well as the proportion of household wealth in the family home, is greater than average in Australia (Cho and Sane, 2013).

These characteristics are also found in the ABS data used in this paper, as shown in Section 3. Two factors may contribute to these observations:

- **Bequest motive and precautionary savings**. According to Lawrence and Goodnow (2011), there is evidence of Australian parents' commitment to making bequests to their children. However, it is difficult to distinguish it from the motive of precautionary savings, as in a world of risk, bequests may be accidental instead of intended. According to Olsberg and Winter (2005), the desire to bequeath assets to the next generation seems to be significantly diminishing. When investigating housing arrangements for the elderly, they found that 74 percent of people see the family home as something of value which one can

[9] For detailed references on this topic see Hulley et al. (2012) and Feinstein and Ho (2000).

pass on to ones' family. At the same time, however, 75 percent of people agree that a home can be sold or borrowed against to provide for necessities in old age.

- **Australia's Age Pension**. The Age Pension is a significant part of the retirement income for the majority of Australian retirees. According to Rothman (2012), currently around 80 percent of the Australian population aged above 65 receive a full or part pension. Hulley et al. (2012) show that the Age Pension buffers retired households against shocks to wealth and may influence decumulation and portfolio allocations in retirement. Cho and Sane (2013) show with their model that it is optimal for households to over-invest in housing when the value of the family home is exempted from the assets test.

Many Australian and international studies used life-cycle utility models to model the financial preferences of households post-retirement. For example, Bateman et al. (2007) applied the life cycle model to the post-retirement problem for Australian retirees and illustrated the optimal behaviours assuming a subsistence level of consumption. Lockwood (2011, 2012) investigated the life cycle model treating bequests as luxury goods. Ding et.al. (2012) show that as a luxury good, the planned bequest drops following a negative wealth shock, and optimal allocation into risky assets increases with age in the presence of luxury bequests. Americks (2011) calibrated a utility model to US data and identified separate parameters for bequest and precautionary saving motives.

Life annuities and other insurance products are not considered in this paper. Evans and Sherris (2009) pointed out that there is not enough awareness of longevity protection products amongst Australian retirees.

3.3 Data Analysis

The data used in this paper are the ABS data of HES and SIH. The survey period is 2009 to 2010. I look at people above the age of 55 and not in the labour force. These include survey results of 2,856 single households and 2,652 couple households.[10]

The household characteristics looked at in this paper are as follows: gender; age; total wealth;[11] value of family home net of mortgage; yearly household expenditures;[12] whether any family member has a disability or is a DVA pensioner,[13] and the amount of social security payments entitled.[14]

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[10] This paper excludes samples of all other household types.

[11] Includes the value of family home but excludes the value of vehicle and household contents.

[12] Mortgage repayments are excluded, while both rental expenditure and rental income are included. Note that the expenditure data are only available for 1,861 single households and 1,662 couple households, because everyone included in the HES is also included in the SIH, but not vice-versa. Furthermore, 14 outliers have been excluded and treated as missing data where: 1) the yearly expenditure is less than $3,000; 2) the net yearly expenditure (net of social security income) is greater than half of the total household wealth and three times the maximum Age Pension rate. I believe these households either have unreported wealth or unreported expenditure.

[13] If the person has a disability or is a DVA pensioner, I assume he/she is qualified for the public pension. If not, he/she is assumed to qualify for the public pension from age 65.

[14] For the social security payment, 207 records have been excluded and treated as missing data, where the household does not receive any pension payments while being entitled to significant pension payments according to its reported wealth and expenditure.

3.3.1 Wealth and Expenditure

Table 3.1 summarises the average wealth of Single and Couples households,[15] for different age groups.

Age	Single	Couple
55-59	$320,900	$923,500
60-64	$348,900	$924,100
65-69	$453,900	$742,800
70-74	$353,500	$697,300
75-79	$428,700	$660,200
80+	$447,100	$674,800

Table 3.1: Average total wealth by age group.

There is not an obvious trend for wealth declines during retirement, suggesting a slow rate of decumulation consistent with the findings in Hulley et al. (2012). However significant differences can be observed in wealth before and after age 65 for couple households. This suggests a significant amount of wealth may be dissipated during early retirement, although this observation may be distorted by the cohort effect, if the younger age cohorts are richer.

Table 3.2 shows the average wealth of Single and Couples households by wealth percentiles, we can see that there is significant dispersion and skewness in the distribution of household wealth.

The financial preferences and expenditure between households in different wealth

[15] Value of wealth includes the value of the family home. Couples are placed into age groups according to the age of the older partner.

	Wealth	percentiles								
Age	10th	20th	30th	40th	50th	60th	70th	80th	90th	100th
Single	-3	3	66	208	287	352	422	517	674	1,560
Couple	23	218	322	391	456	530	647	806	1,104	2,787

Table 3.2: Average total wealth by age group.

groups are significantly different, this can be supported by Figure 3.2, which illustrates the pattern of average yearly expenditures by age group and wealth percentiles for couple households.

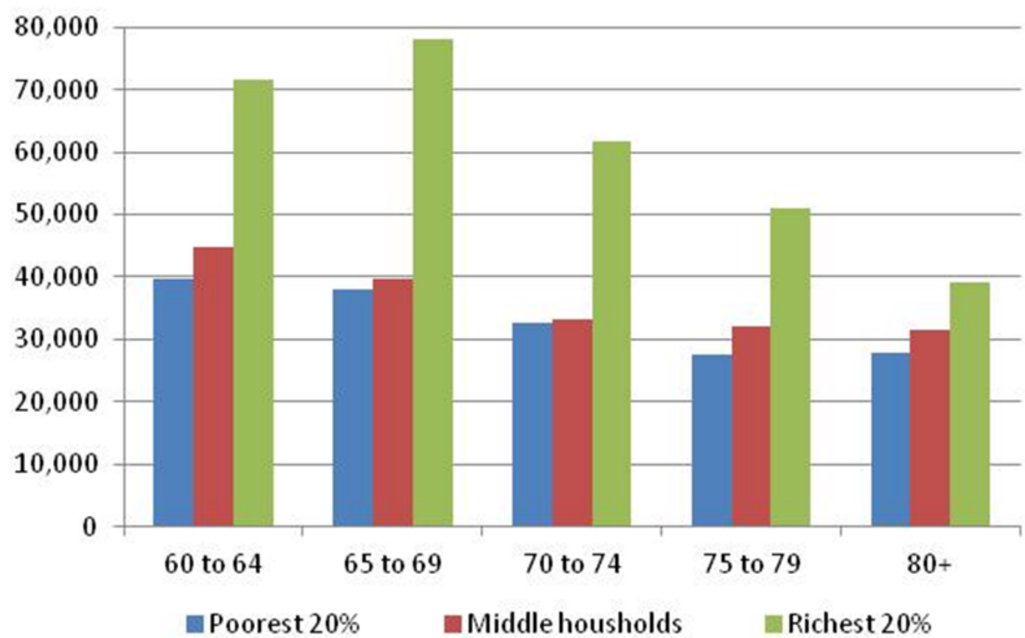

Figure 3.2: Average yearly expenditure ($) by wealth percentiles, Couple data.

The expenditure patterns from Fig 3.2 can be summarised as follows:

- There is not much difference in the expenditure of households with little wealth compared to middle households. This is likely due to the fact that middle households have most of their wealth locked into the family home, hence a large proportion of their consumption comes from the Age Pension, just like households in the low wealth bands.

- There is little difference in the expenditure of different ages groups for households in the low to middle wealth bands. This could be due to a subsistence consumption requirement for all households in all age bands, which is consistent with the consumption floor argument as in Bateman et al. (2007).

- For wealthy households, expenditure clearly decreases as age increases for households in the same wealth band. This supports the assumption that utility from consumption decreases as age increases due to declining health, consistent with Higgins and Robinson (2011), and is evidence for the three phases of retirement as proposed in Rice and Higgins (2009).

- There are definitely savings motives, as the data illustrate that most middle and wealthy households spend much less than they can afford to. This is also illustrated in Figure 3.3, which plots the average yearly expenditure for couple households as a percentage of estimated lifetime wealth, defined as current wealth plus estimated value of future Age Pension entitlements [16]:

Data for single households exhibit the same characteristics, illustrated in Table 3.9 and Fig 3.13 in Appendix C. Figure 3.3 shows that most households are spending less than 4 percent of their estimated lifetime wealth. Research by Bengen (1994)

[16]Estimated lifetime wealth $\bar{W}_n = W_n + e_n * P_n$, where W_n is the current wealth for the nth retiree, e_n is his/her life expectancy (joint life expectancy if couple) and P_n is the maximum Age Pension payment for this Person/Couple (I consider this an appropriate approximation because the time value of money would be approximately offset by the indexation of age pension payments). Table 3.6 and 3.7 in Appendix C gives the average current wealth and the estimated lifetime wealth by age and wealth percentiles.

Figure 3.3: Expenditure as % of estimated lifetime wealth, Couple data.

and Cooley et al. (1998) find that 4 percent is the sustainable level of withdrawal rate, hence most Australian retirees are saving,[17] reinforcing the findings in Hulley et al. (2012). We can also conclude that wealthier households have more wealth saved outside of the family home, as they spend as little as the middle households in percentage terms while allocating a smaller proportion of wealth to the family home.

Figure 3.4 and Figure 3.5 illustrate the average value of the family home as percentage of current wealth for couple and single households by age group and wealth percentiles.

From Figure 3.4 and Figure 3.5 the following can be observed:

- A household is unlikely to be a homeowner if its wealth is less than a certain

[17]Especially if we assume a major proportion of people agree that home equity can be utilized to provide for needs in old age according to Olsberg and Winter (2005).

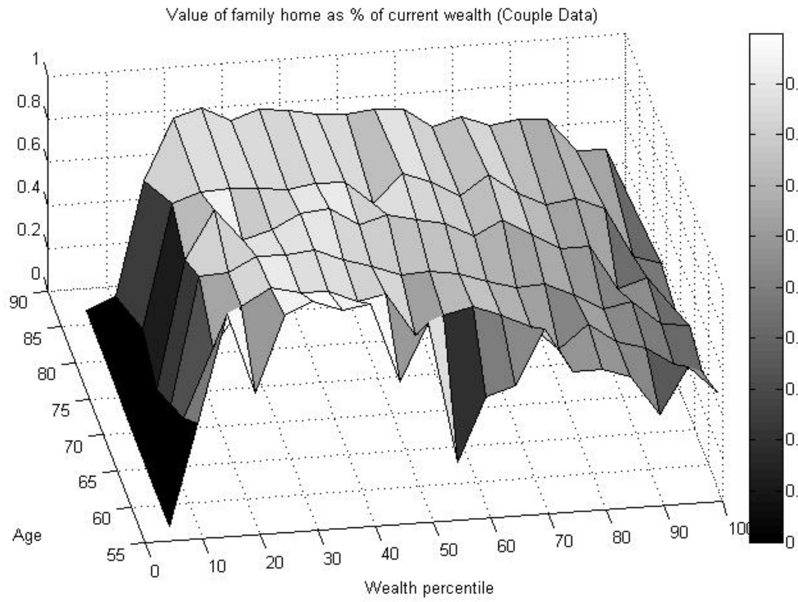

Figure 3.4: % of wealth in family home, Couple data.

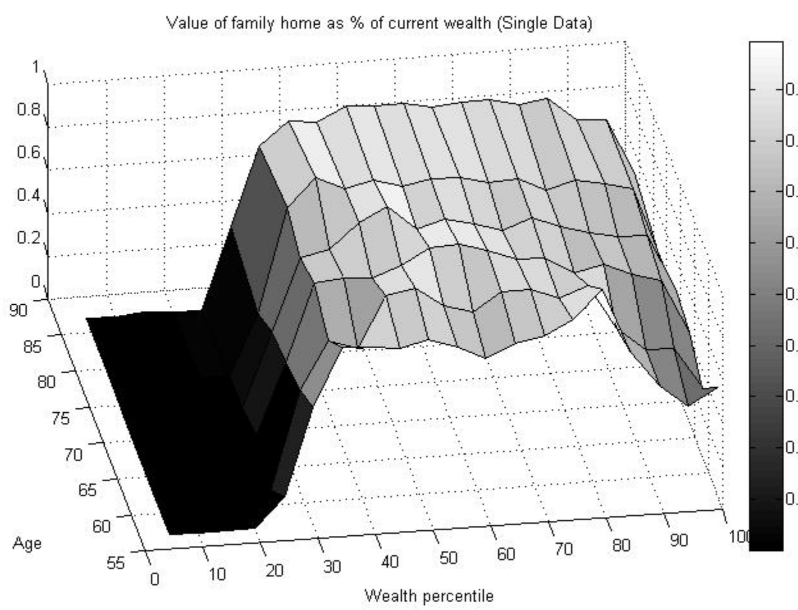

Figure 3.5: % of wealth in family home, Single data.

threshold, for both single and couple households. This may be due to households in the low wealth band possibly experiencing difficulty in obtaining a loan, and are unlikely to be able to afford upfront instalments to purchase a property. It may also be that it may be better for these households to receive government rental assistance instead of owning a house. Data shows that fewer single households are homeowners, which is not surprising as single households have on average, lower wealth than couple households.[18]

- For households in the middle wealth bands, the majority of their wealth is in the family home. This is consistent with previous studies (e.g. Cho and Sane, 2013) on the high wealth weighting towards the family home of Australian households.

- The proportion of wealth in the family home then decreases as the households become wealthier. This suggests that saving in liquid wealth only occurs when household wealth is above a certain threshold, hence can be considered as a luxury good. This is consistent with the assumption of luxury bequests, as in Ding et al. (2012), De Nardi (2004) and Lockwood (2011, 2012).

- The proportion of wealth in the family home is similar between people in different age groups. This suggests the utility from housing is age independent, unlike utility from consumption of nondurables. As a result of declining health, attributable mostly to old age, one spends more time at home.

[18] A simple way to model these factors is to assume a minimum house size as in Cocco (2005), which is the same for both single and couple households.

3.4 Model

Based on the data analysis and related literatures, I propose the following utility model of the post-retirement preferences of Australian households.

Retirees derive utility from consumption of non-housing goods and the flow of services from housing stock, as well as from bequests. Consider a single pensioner at retirement, with H dollars of net wealth allocated in her family home, and makes contingent plans for consumption rate $C(t)$ and proportionate investments $\omega(t)$ in risky assets, up to a certain age T, while aiming to leave a non-housing bequest $B(T)$ at the end of the planning period. Her objective is to choose the value of H and the series $C(t)$ and $\omega(t)$ to maximize the expected sum of utilities:

$$\max E\left[\sum_{t=x}^{T} v^{t-x}\left(F_t\frac{(C_t-\bar{C})^{\gamma}}{\gamma}+\frac{(\psi H)^{\gamma}}{\gamma}\right)+v^{T}\theta^{1-\gamma}\frac{(\theta a+B_T)^{\gamma}}{\gamma}\right], \tag{3.1}$$

subject to the budget constraints,

$$W_{t+1}=[W_t+\mathcal{P}_t-C_t][\omega_t\tilde{z}+(1-\omega_t)R], \tag{3.2}$$

$$W_x=\mathcal{W}_x-H, \tag{3.3}$$

$$0\leq H\leq \max(\mathcal{W}_x-\bar{W},0), \tag{3.4}$$

$$B_T\geq 0. \tag{3.5}$$

The notation is:

- E, expectations operator.
- x, current age of the retiree.

- T, age of the retiree at the end of the planning period. This paper assumes $T = 100$.

- v, utility parameter denoting the retiree's time preference.

- $C_t = D_t + \mathcal{P}_t$, consumption at age t, consists of drawdown from wealth D_t and Age Pension entitlement $\mathcal{P}_t$.

- $\mathcal{P}_t$, Age Pension entitlement at age t, which is a function of drawdown D_t and wealth W_t.[19]

- $\bar{C}$, nonnegative utility parameter with the interpretation of 'subsistence' or 'protected' or 'habitual' consumption.[20]

- γ, utility parameter related to the degree of risk aversion.

- F_t, parameter denoting the state of health, which is used to model the effect of reducing utility gain from consumption as the retiree ages due to declining health.[21] This parameter is approximated with $F_{\bar{x}} = {}_tP_{\bar{x}}$, the probability of

[19]For details on Australia's Age Pension means tests, see Appendix A.

[20]This can be considered a necessity in the sense that its elasticity of demand with respect to wealth is zero. For detail see Bateman et al. (2007).

[21]Various studies (e.g. Bernick, 2005, Yogo, 2009, Higgins and Robinson, 2009, amongst others) point out that retirees' consumption generally decreases with age due to declining health. Survival rates are used here to approximate this decrease of marginal utility of consumption due to declining health when retiree ages. I assume that people are completely healthy before age 55, and their health then declines with age. It is not applied to housing and bequest utilities, as these are assumed to not be affected by health.

someone currently age $\bar{x}$ surviving to age t.[22] This paper uses $\bar{x} = 55$.

- H, value of the family home at retirement. I assume retirees can optimize the value of their family prior to, or at retirement. However, they do not have the option to vary the wealth in their family home after retirement. The value of the property is assumed to increase in line with inflation.

- ψ, utility parameter denoting the retiree's preference for Housing.[23]

- $B_T = W_T + \mathcal{P}_T - C_T$, liquid wealth at the end of period T[24].

- θ, utility parameter denoting the retiree's preference between consumptions

[22] Note that survival probability here is solely used as a proxy for health status, as I assume the retiree makes financial plans up to age T certain. A more realistic assumption is to take into account longevity risk such that the retiree is not certain how long she will live. Unfortunately this assumption cannot be applied in this paper as it requires the mortality rate to be considered in the bequest function, in which case it is difficult to calibrate the model parameters as the model can only be solved numerically.

[23] ψ can be considered as the value of services as a proportion of the housing stock. This setting is similar to Cho and Sane (2013), which also modelled housing utility as additively separable from consumption utilities. Other research including Cocco (2005) and Yogo (2009), modelled housing utility as a multiplicative component, in which case the model cannot be solved analytically.

[24] I model bequest as a luxury good and does not include housing wealth in the bequest utility function, and the investor is assumed to gain utility only from bequest that is 'luxury' (wealth outside of the value of their family home). I realize that family home is a significant part of bequest of Australian households, however, due to its complex nature (as a necessity and a bequest), it cannot be treated the same as other bequests, otherwise the effect of true 'luxury' bequest cannot be estimated due to the dominating value of the family home. How to take into account the bequest utility generated from family home will be subject to future research.

and bequest.[25]

- a, utility parameter denoting how luxury is non-housing bequest.[26].
- W_t, financial wealth (wealth net of the value of family home) at age t.
- $\mathcal{W}_x$, total wealth at age x, including the value of family home.
- $\tilde{z}$, random variable denoting the real return of the risky asset, which is assumed to follow independent and identical log-normal distributions every period, with mean ν and standard deviation σ.
- R, constant real risk free rate of return.
- $\bar{W} = \$40,000$, liquidity constraint; Eq.(3.4) poses the constraint that home-owning households require at least \$40,000 liquid wealth.[27]

[25] The optimal bequest decision in a simpler model included in Americks (2011) Appendix A, is to leave bequest equal to $(c - a)$ per year for θ years (assume C is constant, $\bar{C} = 0$ and no Age Pension). $\theta = \phi/(1 - \phi)$ can also be seen as the transformation of a utility parameter $\phi \in (0, 1)$ that has the interpretation of "the marginal propensity to bequeath in a one-period problem of allocating wealth between consumption and an immediate bequest" (Lockwood, 2012, p.6).

[26] a has the interpretation of the "threshold consumption level below which, under the conditions of certainty or with full, fair insurance, people do not leave bequests" (Lockwood, 2012, p.7), see also Americks (2011) and De Nardi (2004)

[27] The investor does not allocate wealth into owner occupied property that will leave her with less than \$40,000 liquid wealth. This assumption is required for the model to be consistent with the low home-ownership amongst households with low wealth. Without this liquidity constraint, it is optimal for these households to allocate all wealth into the family home, as the Age Pension is enough to cover their consumption requirement. The value of \$40,000 is chosen as the value most consistent with the data of low wealth households. Note this assumption assume the investor can freely borrow against the property, for example, one can allocate \$4,000 into a property worth \$40,000 by borrowing 90%. The reason of imposing the liquidity constrains instead of a minimum house price (as in Cocco (2005) for example) is to be consistent with the data, which shows some very low values allocated to owner occupied properties.

This model can be solved semi-analytically following Chapter 4.[28]

The utility function assumed for couple households is the same as the single households, however, it is reasonable to assume that the health decline of both partners affects the marginal utility of the household. This is supported by the data analysis, which shows significantly higher spending for couple households during the early years of retirement. Hence the survival probability ${}_tP_{\bar{x}}$ is replaced by the joint survival probability ${}_tP_{\bar{x}\cup\bar{x}}$) (the probability that both partners are alive), for couple households, to approximate the effect of lower utility from consumption in later age due to declining health.

Australia's Age Pension means testing treats homeowners and renters differently.[29] To model people's decision about whether to be a homeowner or renter, I assume the following:

- For a renter, the derived utility from housing is also $\frac{(\psi H)^\gamma}{\gamma}$, the same as if she is a homeowner, where H is the optimal value of the family home assuming she chooses to be a homeowner.

- The renter incurs rental expenses equal ϱH, which is a form of subsistence consumption on top of $\bar{C}$. Here $\varrho = 4\%$ is the amount of rent required as proportion of the property value H.

- The renter cannot leave the family home as a bequest, hence her threshold of

[28]The semi-analytical solution makes it feasible to calibrate the model parameters to the ABS data.

[29]Under the Age Pension means testing rules, for every dollar of non-housing wealth above a threshold, the Age Pension entitlement will be reduced by 0.039 dollars. This threshold is lower for homeowners compared to renters.

liquid bequest is lower than θa by the amount of H.

The objective function of a renter can therefore be expressed as:

$$\max E\left[\sum_{t=x}^{T} v^t \left({}_tP_{\bar{x}}\frac{(C_t-\bar{C}-\varrho H)^\gamma}{\gamma}+\frac{(\psi H)^\gamma}{\gamma}\right)+v^T\theta^{1-\gamma}\frac{(\theta a-H+B_T)^\gamma}{\gamma}\right], \quad (3.6)$$

subject to the budget constraints Eq.(3.2) and:

$$W_x = \mathcal{W}_x. \quad (3.7)$$

The solution of Eq.(3.6) is the same as the solution of Eq.(3.1), except with $\bar{C}$ replaced by $\bar{C}+\varrho H$ and θa replaced by $\theta a - H$. To find someone's decision between being a homeowner or a renter, I first find the optimal value of H assuming she is a homeowner, then find the optimal decisions under Eq.(3.6) given H, assuming she is a renter. We then compare the optimal utilities between the case of renting or owing a home[30].

3.5 Calibration

3.5.1 Methodology

We calibrate the model by choosing the utility parameters to minimize the sum of squared error between the data and the model output on: 1) household expenditure in the year of survey; 2) Age Pension received in that year; 3) the value of the family

[30]Note that under the utility function used, this decision is insignificant. The result indicates that the decision to own a home results in better outcome compared to the decision to rent, for nearly every household in the sample.

home. This can be expressed as follows:

$$\min SSE = \sum_{n=1}^{N} \left(\frac{(\hat{C}_n - C_n)}{\bar{W}_n} e_n I_n \right)^2 + \left(\frac{(\hat{\mathcal{P}}_n - \mathcal{P}_n)}{\bar{W}_n} e_n \mathcal{I}_n \right)^2 + \left(\frac{(\hat{H}_n - H_n)}{\bar{W}_n} \right)^2, \tag{3.8}$$

where N is the sample size (2,856 single households and 2,652 couple households). All errors are standardised by the estimated lifetime wealth $\bar{W}_n$. Errors on current expenditure and Age Pension are weighted by expected year of remaining life e_n, and I_n and $\mathcal{I}_n$ are indicators which take value 0 if the expenditure/social security data are missing for this person and 1 otherwise. On average, the three items (consumption, Age Pension and housing) have roughly equal weight in the equation.

Six utility parameters are calibrated separately for single and couple households, namely v, parameter denoting the retiree's time preference; γ, parameter denoting the degree of risk aversion; $\bar{C}$, parameter denoting the consumption floor; ψ, parameter denoting retiree's preference for housing utility; θ, parameter denoting preference for bequest; and a, parameter denoting the degree of non-housing bequest as a luxury good.[31]

Current age x, wealth W_x and disability states are given by the data. Other parameters of the model are economic assumptions given as follows: inflation rate 4.5

[31] The model is calibrated as follows to ensure global optimization: in the first step, four possible values are assigned for each parameter, and I calculate the SSE for all $6^4 = 1,296$ possible set of parameters. in the second step, ten sets of parameters are selected based on the result of the first step. I then further optimise the parameters with the Nelder-Mead Simplex Method, using each of the ten sets of parameters as initial inputs. The result reported here is the set of parameters considered most reasonable after the fine-tuning.

percent;[32] real risk free return is 1% ($R = 1.01$); parameters for risky asset return[33] $\nu = 2.25\%$ and $\sigma = 14\%$; and the Age Pension parameters used[34] are as published by Centrelink in January 2010, reflecting the period of SIH survey.

3.5.2 Results

The optimal set of calibrated parameters are shown in the following table.[35]

	v	γ	$\bar{C}$	ψ	θ	a
Single	0.99	-3	\$10,000	3.2%	21.7	\$14,000
Couple	0.98	-3	\$18,000	4.8%	21.7	\$21,000

Table 3.3: Calibrated utility parameters.

The output of the model compared to the original data is illustrated in Figure 3.1 and Figures 3.11–3.15 in Appendix C, and shows that the calibrated model provides reasonable fits to the data.

It is important to note that I assume a single utility function with a particular set of parameters is applicable for every Australian retiree. This assumes people's behaviour depends only on age, wealth, gender and marital status. In reality no

[32]Australia's Age Pension payment is indexed to the highest of CPI and Pensioners' CPI and Male Full Time Average Earnings, averaging 4.5 percent, hence I set the inflation to 4.5 percent to avoid changing the Age Pension parameters during the calculation. Once the results are calculated, it can be easily adjusted to other inflation rate assumptions.

[33]see Appendix B for details on the assumption of risky and risk-free asset returns.

[34]these rates can be found in Appendix A.

[35]These parameters have been rounded, and some are assumed to be the same for single and couple households, when the calibrated values are very similar.

two people are identical, and the SIH data show that there is a higher level of variability for people in the same age and wealth band. This can be illustrated as in Figures 3.14 and 3.15 in Appendix C, which show that the modelled house values and annual expenditures reasonably fit the data average but only explain a small part of variability of the data. Table 3.4 summarise the statistical measurement of these variabilities, as the mean absolute percentage estimation error (MAPE) for the three data items fitted: the value of the family home, household expenditure, and Age Pension entitlement in the year of survey.

	Family Home	Expenditure	Age Pension
Single	28.5%	43.2%	20.5%
Couple	29.8%	38.2%	26.0%

Table 3.4: Mean absolute percentage error of the model output.

The MAPE can be interpreted as follows: for example, the value of the family home estimated by the model for a single household would, on average, be 28.5 percent different to the true value of the house.

Parameters in Table 3.3 can be explained as follows:

- v implies that Australian households consider receiving \$1 (current dollar) in one year's time to be the same as receiving \$$v$ now. The calibrated value for single households is the same as the real risk free rate. However the value for couple households is higher.[36]

- γ reflects the degree of risk aversion. A high value of γ implies that consumption paths are smoother, and the retiree would allocate less into risky assets.[37] The value of $\gamma = -3$ denotes a relatively high degree of risk aversion, although within the range of values suggested by previous literature.[38]

- $\bar{C}$ denotes the subsistence consumption level. The fitted parameters suggest the level of consumption floors for single and couple households are close to the payment rate of Newstart Allowance in Australia, which seems reasonable as a level of consumption for absolute necessities[39].

- ψ denotes flow of utility (or can be interpreted as a rate of return) from housing stocks; the calibrated parameters suggest that the same value of the house would derive more utility for couples than singles, which seems reasonable as there are two people in a couple household. The calibrated values of 3.2

[36]This may suggest that the time preferences are different for Single and Couple households. However, although mortality rates had been included in the model to approximate the effect of lower utility from consumption in later ages due to declining health, the higher discount rate for couples here may be due to the fact that approximating the effect of declining health for couples, using the joint survival rate as assumed for couples in Section 4.2, is not the best assumption.

[37]In a one-period asset allocation problem, given the return of risky and risk-free assets assumed in this paper, the retiree will allocate 19 percent of wealth to risky assets, compared to 64 percent if their utility function is assumed to be log form (when $\gamma = 0$).

[38]For a discussion on the value of risk aversion parameter, see for e.g. Americks et al. (2011).

[39]We have assumed the household reserve the subsistence level of consumption for every year until age 100. Note this is the minimum consumption level absolutely required, and the household is expected to reserve for it to their maximum possible age, regardless of mortality. Hence the low level of the subsistence consumption estimated seems reasonable in my opinion.

percent and 4.2 percent seem reasonable as it is consistent with the current rental yield of 4 percent in Australia.[40]

- $\theta = \phi/(1-\phi)$, gives $\phi = 0.956$ which denotes the preference between consumption and saving. The presence of a implies that a household is unlikely to have savings in liquid wealth if it cannot afford annual consumption of at least $\bar{C} + a$. The calibrated value of these two parameters are both consistent with previous US studies.[41] The high value of a seems reasonable here as in Australia most bequests are in the form of the family home. Therefore liquid bequests are expected to be luxuries.

The calibrated parameters indicate that there is no specific preference for housing amongst Australian households, since the calibrated flow of utility from housing is consistent with market rental cost. The high concentration of wealth in the family home is most likely affected by the Age Pension. First, the high level of Age Pension payments are well above the calibrated consumption floors, indicating that households in low wealth bands do not need much wealth outside of their family home to fund their retirement consumption. Second, the Age Pension assets test implies that it is optimal for middle to high wealth bands to allocate wealth in their family home to receive higher pension payments.

The calibrated parameters reported are those that provide the best fitted results.

[40] The 4% is calculated as the average weekly housing cost according to ABS 2011 Census Data, divided by the average value of houses according to SIH 2009-2010 (adjusted with inflation to 2011).

[41] According to Lockwood (2012), the literature in the US shows that there is a lack of consensus on the degree of bequest motives with value of ϕ range from 0.88 to 1, and a range from \$5,300 to \$48,400. For example, Lockwood (2012) found $\phi = 0.93$ and $a = \$20,400$, while Americks et al. (2011) suggest $\phi = 0.98$ and $a = \$7,300$.

However these parameters suffer from identification problems similar to the ones reported in Americks et al. (2011). For example, the smoothness of consumption between ages (hence the level of current consumption to be calibrated), can be affected by either v (preference for higher consumption earlier), γ (penalty for higher consumption in a particular period than average) or $\bar{C}$ (the required base level of consumption every period). In fact, the parameters $v = 0.99$ and $\gamma = -2.5$ for couple households, result in very similar model outputs, compared to the results when assuming $v = 0.98$ and $\gamma = -3$, as reported.

The main reason for this problem is due to that the SIH data is a cross-sectional dataset and it is hard to distinguish age and cohort effects. The limitation of using only cross-sectional data for studying retirement decumulation have been noted by previous researches, see for example, Borsch-Supan and Lusardi (2003). Americks et al. (2011) deal with identification problems by incorporating additional survey data, future extensions to this paper can integrate other longitude data and surveys into the study (for example, the HILDA survey).

3.6 Numerical examples

The following two numerical examples are given to illustrate financial behaviours under the calibrated model:

1. Household A: Single female age 77, with total wealth \$320,000.

2. Household B: Couple, male age 77, female age 72, with total wealth \$1,933,000.

The model output indicates that the optimal allocation of wealth in the family home is $280,000 for household A, 88 percent of her total wealth; and $829,000 for household B, 43 percebt of their total wealth. Figure 3.6 illustrates the consumption path for these two households.

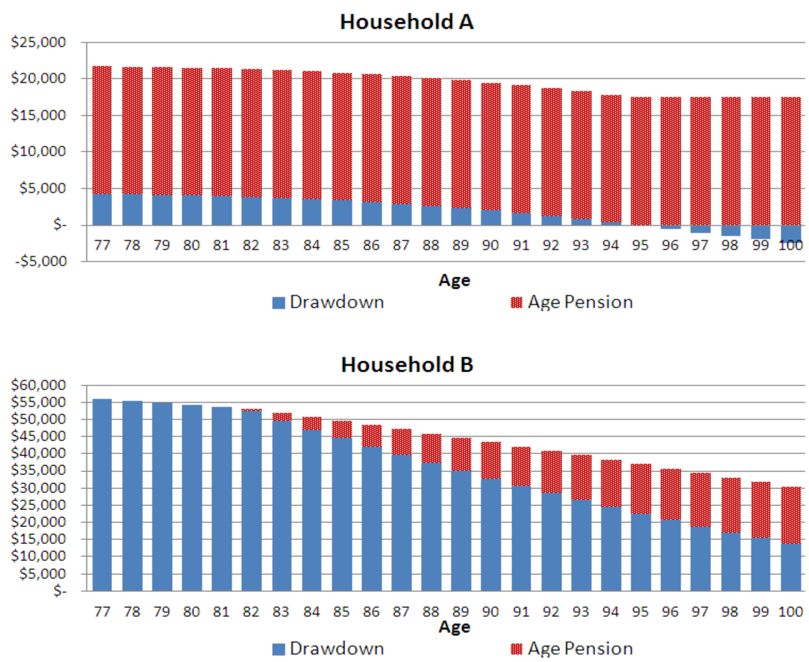

Figure 3.6: Consumption path with Age Pension.

In both households, consumption decreases with age, and the rate of decrease becomes faster at higher ages, presumably due to declining health. Household A is a relatively poorer household and Age Pension payments contribute to a major part of her retirement consumptions. Note that the model implies that at higher ages, Household A would consume less than the Age Pension payments. Household B is a relatively wealthy household, but even at their level of wealth, they would still be eligible for Age Pension payments at higher ages.

Figure 3.7 illustrates the path of financial wealth (net of family home) for these two households.

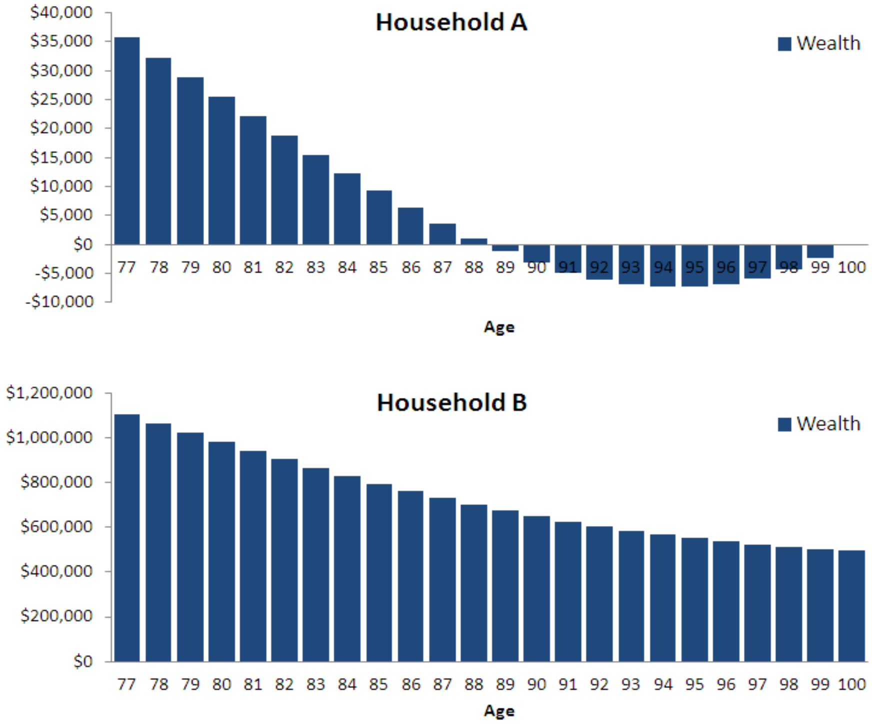

Figure 3.7: Wealth path.

At the end of the planning period (age 100), household A is expected to have no assets outside of the family home, while household B is expected to have $495,600. Note that the wealth of Household A becomes negative at around age 89. This is due to the model only imposing restriction that terminal wealth cannot be negative, yet Household A is allowed to have higher consumption and negative wealth at younger ages, and this shortfall is then made up by consuming less than the Age Pension payments.

Figure 3.8 illustrates the estimated optimal investment path for Household B, as a percentage of financial wealth; for Household A, it is optimal to allocate nearly all liquid wealth into risky assets. This is because the majority of her consumption requirement can be funded by the Age Pension. As it is a risk-free source of income for life, she does not need further risk-free assets.

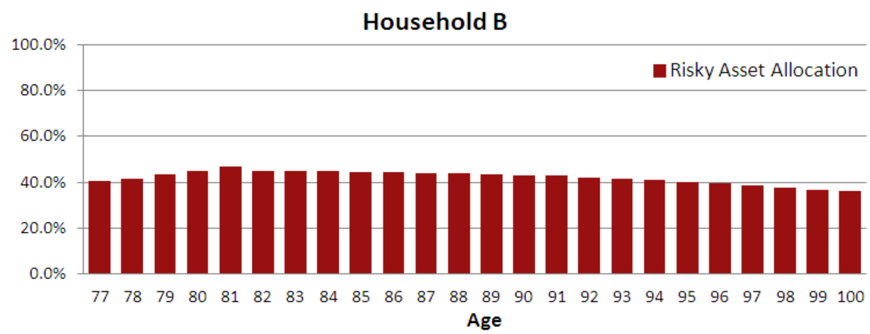

Figure 3.8: optimal path of risky asset allocations.

Figure 3.8 shows that for Household B the optimal allocation in risky assets slightly increases at early ages and then decreases. Two factors contribute to this pattern, with opposite effect. First, the existence of luxury bequests implies a higher allocation into risky assets as the household ages. It provides a buffer for investment risks, and this buffer is more significant at later ages when there are less future consumption requirements to consider.[42] However, at the same time, the existence of the Age Pension implies a lower risky allocation as the household ages. Future Age Pension payments also provide a buffer for investment risks.[43] This buffer diminishes as the household approaches the end of life.

[42]For detail see Chapter 2 of this thesis.

[43]There are two reasons: 1) future Age Pension payments are risk-free hence crowds out allocation in risk-free assets; 2) if the household suffers a lose in their investment, they will be entitled to a higher Age Pension under the assets test.

The financial behaviours of wealthy households and poorer households can be seen to be very different. The consumption profile of rich retirees departs markedly from consumption smoothing, spending much more than poor ones on housing and bequests, while their consumption of non-housing, non-bequest goods decreases rapidly as age increases, and spend a similar amount to poorer households at later ages. In these ways, financial advice is non-scalable, contrary to the relevant prescription of the standard assumptions of Constant Relative Risk Aversion utility model.

3.7 Conclusion

This paper developed a life-cycle utility model of the preferences of Australian retirees. I offered the first joint consideration of housing, luxury goods (in the form of bequest), 'ultra-necessities' (in the form of a 'subsistence' rate of consumption in retirement) and public pensions. In contrast to much of the earlier literature which adopts many parameters values from overseas studies, I calibrate the model parameters to the ABS data of HES and SIH to match the financial preference of current retirees.

The model and calibrated parameters in this paper provide inputs for future research, and have applications in areas including post-retirement financial planning, pension product design and the evaluation of policy changes.

This research has a number of limitations. The calibrated parameters reported are those that provide the best fitted results. However these parameters suffer from identification problems similar to the ones reported in Americks et al. (2011). For example, the smoothness of consumption between ages (hence the level of current consumption to be calibrated), can be affected by either v (preference for higher consumption earlier), γ (penalty for higher consumption in a particular period than average) or $\bar{C}$ (the required base level of consumption every period). In fact, the parameters $v = 0.99$ and $\gamma = -2.5$ for couple households, result in very similar model outputs, compared to the results when assuming $v = 0.98$ and $\gamma = -3$, as reported. The main reason for this problem is due to that the SIH data is a cross-sectional dataset and it is hard to distinguish age/cohort effects. Americks et al. (2011) deal with identification problems by incorporating additional survey data, future extensions to this paper can integrate other longitude data and surveys into the study (the HILDA survey, for example).

I assume a single utility function with a particular set of parameters is applicable for every Australian retiree. This assumes that people's behaviour depends only on age, wealth, gender and marital status. In reality, no two people can be identical, and the SIH data show that there is a higher level of variability for people in the same age and wealth band. Hence the model is suitable to be applied at a macro level, where average results are most relevant. For application in individual financial planning, the model needs to be tailored to the individual's specific preferences. Future research could improve the explanatory power of the model by incorporating other explanatory variables, e.g. education levels.

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Appendix A: Age Pension

Australia's Age Pension is available for Australians age 65 and above.[44]. As at January 2010, the maximum annual age pension payments are $17,456 for singles and $26,099 for couples.

The payment is subject to two means tests: the assets test and the income test. Under the assets test, for every thousand dollars of wealth above a threshold, the maximum pension payments will be reduced by $39 dollars. Similar rules apply to the income test, for every dollar of non-deductable income above a threshold, the maximum pension will be reduced by 50 cents. The actual pension payment is the lesser of the assets test pension and income test pension.

The Age Pension rate used are shown in Table 3.5, these are the rates published by Centrelink as at January 2010.

	Single	Couple
Full Age Pension Rate	$17,456	$26,099
Income Test		
Threshold	$3,692	$6,448
Rate of Reduction	$0.5	$0.5
Asset Test		
Threshold: Homeowners	$178,000	$252,500
Threshold: Non-homeowners	$307,000	$381,500
Rate of Reduction	$0.039	$0.039

Table 3.5: Age Pension parameters as at January 2010.

[44]Age Pension age will increase and reach 67 in 2023. For details of the Age Pension age and transition arrangements see: http://www.humanservices.gov.au/customer/services/centrelink/age-pension

Appendix B: Asset return assumptions

The real return of risky free asset is assumed to be 1% p.a. as the 3 month bank bill rate, adjusted by wage inflation published by the Reserve Bank of Australia (RBA).

The parameters for risky asset return$\nu = 2.25\%$ and $\sigma = 14\%$ are derived from the ASX All Ordinary Index history financial market data. Which is then adjusted by wage inflation statistics published by RBA, and adding 2% dividend and subtracting 1% management fee to obtain real investment return. The following chart graphs the historical distribution of the adjusted investment return of ASX all ordinary index.

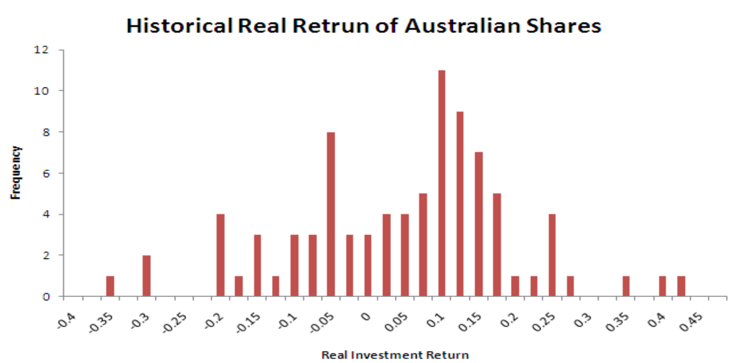

Figure 3.9: Distribution of historical real return of Australian shares

Appendix C: Additional Tables and Figures

Wealth	10th	20th	30th	40th	50th	60th	70th	80th	90th	100th
Couple	23	218	322	391	456	530	647	806	1,104	2,787
Single	-3	3	66	208	287	352	422	517	674	1,600

Table 3.6: Average wealth ($,000) of each wealth percentile for Couple and Single data.

Wealth	10th	20th	30th	40th	50th	60th	70th	80th	90th	100th
Couple										
55-59	679	882	994	1,016	1,102	1,169	1,274	1,449	1,725	3,336
60-64	580	766	857	942	986	1,074	1,181	1,362	1,660	3,206
65-69	474	656	770	836	902	975	1,094	1,256	1,546	3,300
70-74	367	572	670	741	802	876	996	1,153	1,443	3,079
75-79	299	486	587	659	722	794	907	1,069	1,379	3,152
80+	181	377	482	548	615	688	806	995	1,275	3,034
Single										
55-59	484	493	536	669	733	829	894	966	1,130	1,796
60-64	441	420	487	635	712	763	847	925	1,088	1,933
65-69	349	357	411	559	647	703	781	867	1,033	2,050
70-74	270	272	339	475	558	625	692	786	945	1,759
75-79	206	207	271	414	497	561	627	724	882	2,042
80+	120	126	184	330	411	472	546	641	797	1,955

Table 3.7: Average of estimated lifetime wealth ($,000) of each wealth percentile by age.

	Wealth									
Age	10th	20th	30th	40th	50th	60th	70th	80th	90th	100th
55-59	28	19	42	40	38	46	37	53	51	92
60-64	42	38	37	36	34	50	60	51	68	76
65-69	33	43	36	37	43	42	36	46	77	79
70-74	32	33	33	30	30	36	34	35	48	76
75-79	28	27	32	29	33	32	36	30	42	60
80+	29	27	29	28	31	29	33	38	38	40

Table 3.8: Couple data, Expenditure ($,000) by age and wealth percentiles.

	Wealth									
Age	10th	20th	30th	40th	50th	60th	70th	80th	90th	100th
55-59	24	23	42	34	41	46	21	41	17	57
60-64	20	21	25	29	23	21	21	28	26	34
65-69	19	19	21	18	22	24	24	29	28	32
70-74	20	20	24	25	17	24	21	26	25	30
75-79	19	17	20	22	20	19	22	23	26	29
80+	17	16	15	17	17	15	19	18	20	36

Table 3.9: Single data, Expenditure ($,000) by age and wealth percentiles.

Wealth	10th	20th	30th	40th	50th	60th	70th	80th	90th	100th
Couple										
55-59	24%	78%	97%	96%	73%	35%	67%	58%	45%	50%
60-64	16%	74%	89%	84%	74%	74%	67%	52%	52%	39%
65-69	11%	82%	88%	87%	78%	75%	67%	63%	53%	42%
70-74	6%	83%	81%	90%	84%	81%	77%	68%	54%	40%
75-79	12%	84%	86%	85%	80%	85%	80%	71%	67%	39%
80+	3%	73%	87%	87%	83%	84%	76%	73%	66%	43%

Table 3.10: Couple data, Value of family home as % of total wealth, by age and wealth percentiles.

Wealth	10th	20th	30th	40th	50th	60th	70th	80th	90th	100th
Single										
55-59	0%	0%	34%	81%	81%	75%	79%	93%	61%	46%
60-64	0%	0%	23%	65%	88%	78%	85%	82%	58%	43%
65-69	0%	0%	25%	79%	80%	91%	85%	78%	74%	55%
70-74	0%	0%	22%	74%	91%	84%	81%	81%	73%	59%
75-79	0%	2%	17%	86%	88%	87%	86%	82%	81%	58%
80+	0%	1%	19%	80%	87%	90%	87%	87%	82%	62%

Table 3.11: Single data, Value of family home as % of total wealth, by age and wealth percentiles.

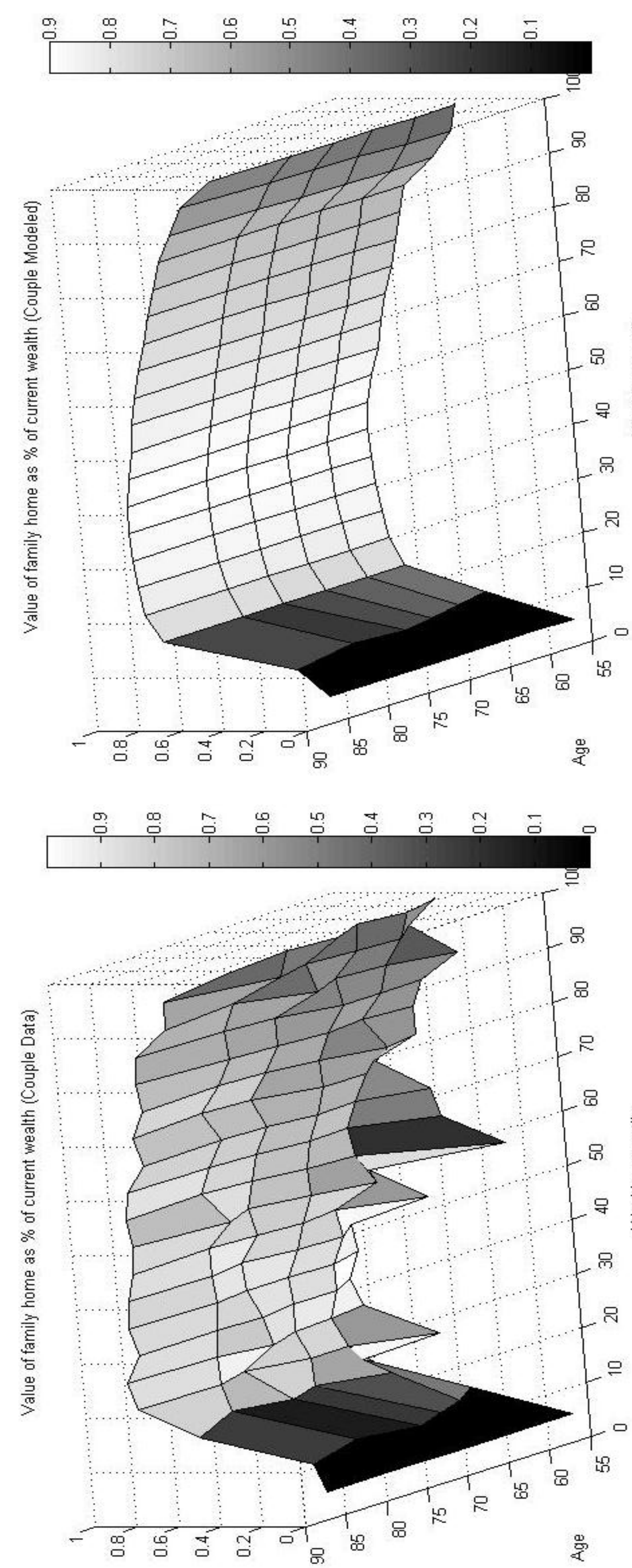

Figure 3.10: % of wealth in family home, Data vs Model output, couple households.

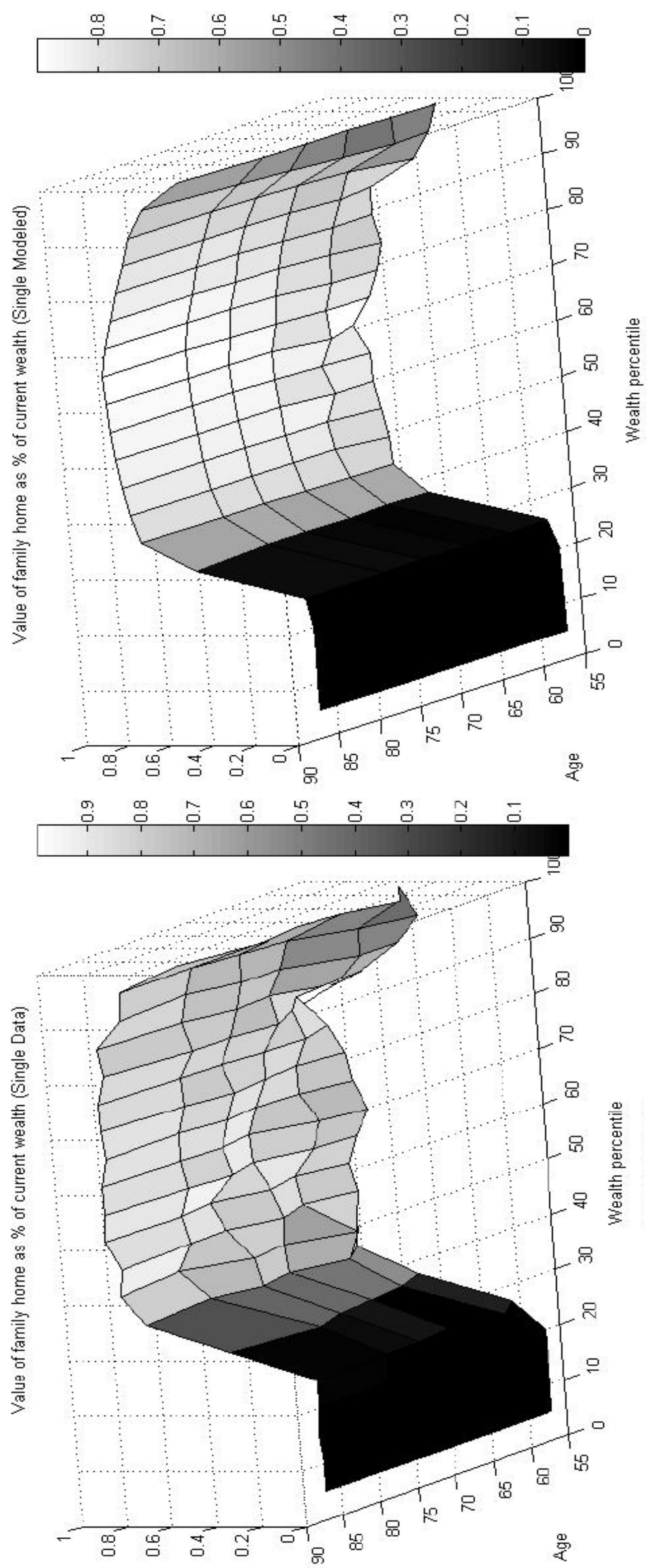

Figure 3.11: % of wealth in family home, Single households, Data vs Model output.

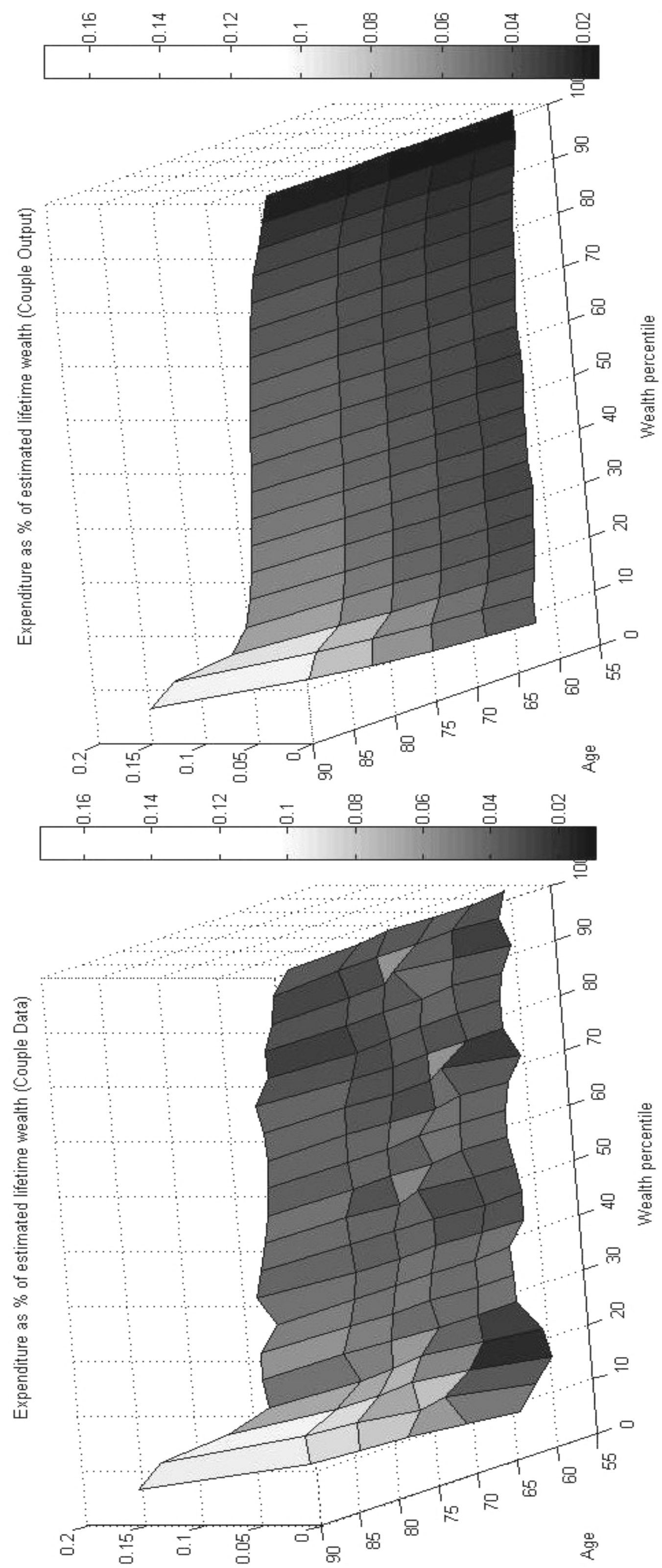

Figure 3.12: Expenditure as % of estimated lifetime wealth, Couple Data vs Model output

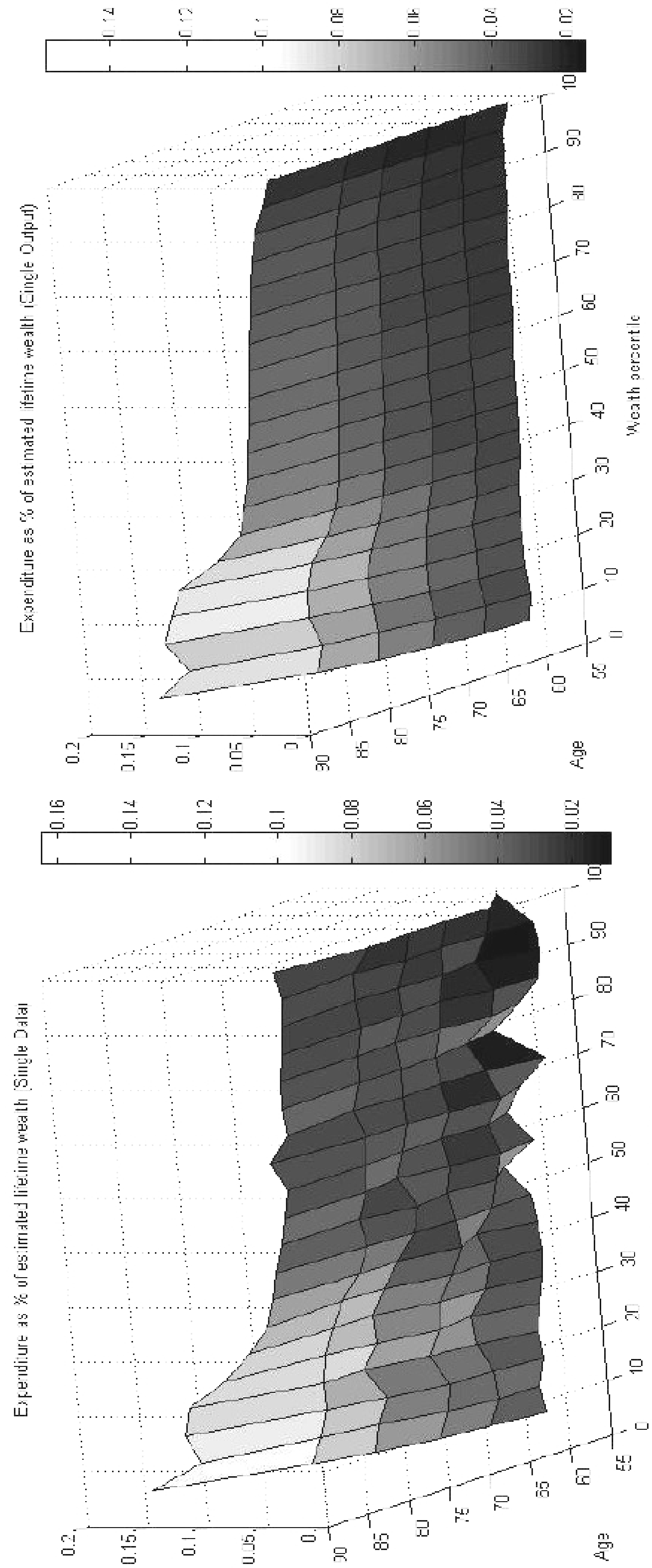

Figure 3.13: Expenditure as % of estimated lifetime wealth, Single Data vs Model output.

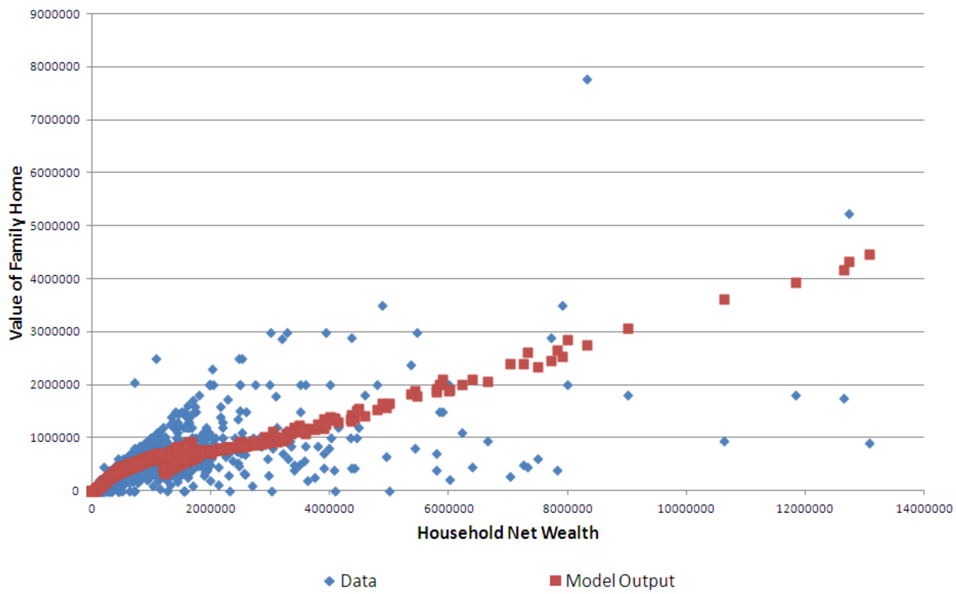

Figure 3.14: Value of family home by Household net wealth, Data and Model outputs, Couple Households.

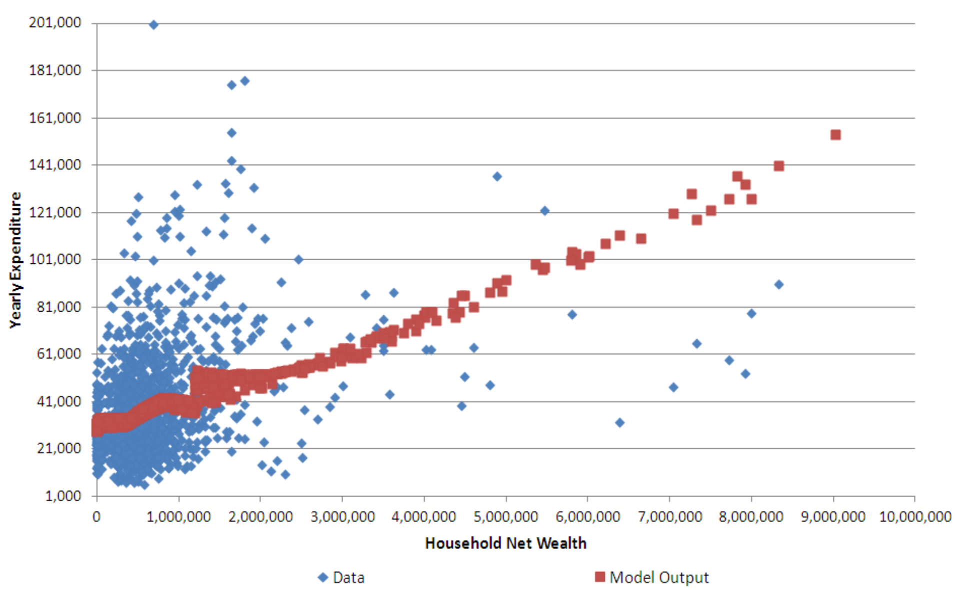

Figure 3.15: Yearly expenditure by Household net wealth, Data and Model outputs, Couple Households.

Chapter 4

Modelling post-retirement finances in the presence of a bequest motive, housing and public pension

Abstract

In this paper I derive semi-analytical solutions to the problem of finding the optimal consumption and asset allocation decisions post-retirement. I assume the preference of retirees follow HARA type of utility function, with joint consideration of a bequest motive, housing and publicly provided Age Pension. The results are close to those derived with numerical dynamic programming, but with a clear advantage in computation time.

JEL classification: D14, G11, H55. *Keywords*: life-cycle model, utility, bequests, housing, Age Pension, financial planning, dynamic programming.

4.1 Introduction

This paper derives semi-analytical solutions to the post-retirement life cycle problem, taking into account a bequest motive, housing and Australia's publicly provided Age Pension. The solution is close to that derived using numerical dynamic programming, but with a clear advantage in computation time. In addition to technical contributions, I enrich the standard retiree preferences. I offer the first joint consideration of luxury goods (the bequest), housing, 'ultra-necessities' (in the form of a 'subsistence' rate of consumption in retirement) and the Age Pension.

Financial economists have addressed the life-cycle problem of financial decision making with dynamic programming technique since Samuelson (1969) and Merton (1971). These models have been widely applied in Australia and overseas to model people's financial behaviours. However, such models typically require numerical methods[1] or significant simplification of the public pension system[2] in order to solve the optimisation problem.

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[1]Kudrna and Woodland (2009), Oliver and Dixon (2010) and Cho and Sane (2013) investigated the retirement problem in Australia with various life-cycle utility models. These research included detailed Age Pension means testing in their model. However, their models could only be solved with numerical methods. Same are the US and UK studies by Sefton et al. (2008), Chai et al. (2011) and Yogo(2011). Due to the long computation time required, numerical methods restrict the applications of these models. Specifically, the parameters of the utility model cannot be easily calibrated from empirical data, and these studies had to rely on much overseas literature for the value of their utility parameters, or calibrate the parameters separately from various sources.

[2]Bateman et al. (2007) solved the life-cycle model for Australian retirees assuming HARA utility function; however they assume a flat pension with no means testing. Americks et al. (2011) calibrated their model parameters to empirical data using maximum likelihood method. However, they adopted simple assumptions concerning people's behaviours and the US social security system.

The Age Pension can be considered a publiclty provided life annuity. It is a significant part of the retirement income for the majority of Australian retirees. According to Rothman (2012), currently around 80 percent of the Australian population aged above 65 receive a full or part pension. It is therefore important to take realistic Age Pension rules into account when modelling the financial behaviour of Australian retirees. However, Age Pension payments are subject to complicated means testing, making an analytical solution hard to find when it is included in a life-cycle model. Although the problem can usually be solved with numerical methods, an analytical solution offers much faster computations and improves efficiency in many situations.[3]

This paper derives semi-analytical solutions to the post-retirement life cycle problem under three sets of assumptions. I first modify the HARA (hyperbolic absolute risk aversion) utility model of Merton (1971), finding a semi-analytical solution when the publicly provided Age Pension in Australia is taken into account. Then I solve the problem with an extended utility function introduced in Chapter 3, which further takes into account bequest motive and housing. And last, I find the solution to the problem under the additional assumption that part of housing assets above a certain threshold are assessable in the Age Pension assets test.

Bateman et al. (2007) suggest that the consumption pattern of rich households are very different to poorer households, and the HARA utility function is favored in comparison to the conventional constant relative risk aversion (CRRA) utility functions. The HARA form of utility functions includes a subsistence consumption

[3]For example, when the model parameters are need to be calibrated to a dataset, the model needs to be repeatedly solved for tens thousands of times to find the most suitable set of parameters. See Chapter 3.

floor and implies that financial advice is non-scalable.

Bequest motives are important in modeling post-retirement behaviours. Many Australian and international studies find that positive and zero saving during retirement is common for wealthy households (see Hulley et al. 2012). According to Lawrence and Goodnow (2011), there is evidence of Australian parents' commitments to making bequests to their children.

Currently 75 percent of Australian retirees are home-owners; the value of owner occupied housing accounts for about 80 percent of their total wealth. It is also important for any Age Pension related modelling, because the value of owner-occupied property is treated leniently by the current means testing rules,[4] and current evidence shows that Australian retirees are likely to be overinvested in housing.[5] If lenient treatment towards owner-occupied properties in the assets tests does lead to over-investment in housing, then it is worth investigating whether economic efficiency can be improved if at least a part of the family home's value is assessed under the assets test.[6]

[4]Currently homeowners and non-homeowners are treated differently by the assets test. However, someone owning a $2,000,000 house is treated the same as someone owning a $200,000 house.

[5]See Cho and Sane (2013), ABS data indicate that in Australia about half of the elderly claim to have spare capacity in their homes. Bradbury (2008) shows that home-ownership in Australia post-retirement is greater than in most other countries.

[6]Henry (2010) proposed to cap the value of homes that qualified for the assets test exemption. The proposal, however, was not adopted by the Australian government.

4.2 Problem Setup

Consider financial planning for a single pensioner who retires at age x with total wealth[7] $\mathcal{W}_x$, and dies at age T, leaving bequest B_T. The pensioner earns no labour income, and consumes out of wealth and Age Pension entitlement $\mathcal{P}_t$. She makes an one-off decision to have H dollars allocated into owner-occupied housing at retirement, and she can allocate her remaining wealth between one risk-free and one risky asset.

The pensioner's problem is to maximise utility over the retirement period by choosing the amount of consumption, C_t, and proportion of wealth to be allocated to risky assets ω_t for every time period from age $t = x$ until age $t = T$. Assume this retiree has separately additive utility functions for the consumption of non-housing goods and the flow of services from housing, as well as from bequest, the problem can be set up as follows:

Problem 1. *Find the value of H and the series $C(t)$ and $\omega(t)$ to maximize the expected sum of utilities:*

$$\max E\left[\sum_{t=x}^{T} U_c(C_t) + U_h(H) + U_b(B_T)\right], \tag{4.1}$$

[7]We ignore possible large wealth shocks at the beginning of retirement, apart from owner occupied property or other investments. We assume the total wealth the retiree carries into retirement is net of any such expenditures (which may include expenditures on a new car, overseas holiday, etc.)

subject to the budget constraints:

$$W_{t+1} = [W_t + \mathcal{P}_t - C_t][\omega_t \tilde{z} + (1 - \omega_t)R], \tag{4.2}$$

$$W_x = \mathcal{W}_x - H, \tag{4.3}$$

$$B_T = W_T + \mathcal{P}_T - C_T \geq 0. \tag{4.4}$$

where consumption $C_t = D_t + \mathcal{P}_t$ consists of drawdown from wealth D_t and Age Pension $\mathcal{P}_t$ [8]; knowing the initial wealth at retirement $\mathcal{W}_x$, the constant real risk free asset return R and the distribution of risky assets returns $\tilde{z}$, which is assumed to be independent and identically distributed.[9]

4.2.1 Australia's Age Pension

Australia's Age Pension is available for Australians aged 65 and above.[10] The payment is subject to two means tests: the assets test and the income test. The actual pension payment is the lesser of the assets test pension and income test pension. The payment is also bounded, such that it cannot be negative or exceed the maximum

[8] D_t can be negative if Age Pension payment is not entirely consumed. Note that Age Pension entitlement $\mathcal{P}_t$, is not a control variable but a function of drawdown D_t, hence we can optimize consumption C_t, by choosing the optimal D_t.

[9] Australia's Age Pension payments are indexed half-yearly based on wage growth. Without loss of generality, this paper assumes inflation rate is also based on wage growth, which has the simple interpretation that the pensioner aims to maintain the relative standard of living.

[10] Age Pension age will increase and reach 67 in 2023. For details of the Age Pension age and transition arrangements, see: http://www.humanservices.gov.au/customer/services/centrelink/age-pension

pension rate. The payout function can be written as:

$$\mathcal{P}(t) = f(D_t, W_t) = \max(0, \min(\mathcal{P}_m, \min(\mathcal{P}_a, \mathcal{P}_i))), \tag{4.5}$$

where $\mathcal{P}_m$ denote the annual payment under the maximum pension rate. $\mathcal{P}_a$ and $\mathcal{P}_i$ denote the pension payments determined by the assets test and income test respectively. The payments are subject to the following rules:

$$\mathcal{P}_a(t) = \mathcal{P}_m - (W_t - L_a)\varpi_a, \tag{4.6}$$

$$\mathcal{P}_i(t) = \mathcal{P}_m - (D_t - E_i(t) - L_i)\varpi_i, \tag{4.7}$$

where W_t denotes the pensioner's wealth at age t.[11] L_a denotes the assets test limit and ϖ_a the reduction rate. Hence under the assets test, for every dollar of wealth above L_a, the maximum pension will be reduced by ϖ_a dollars.

Similar rules apply to the income test, where D_t denotes the pensioner's deemed income other than the Age Pension at that time, $E_i(t)$ denotes the amount of pensioner's income that is deductable at age t, L_i the income test limit and ϖ_i the reduction rate. Hence under the income test, for every dollar of non-deductable income above L_i, the maximum pension will be reduced by ϖ_i dollars. In the case that the pensioner's wealth is converted into an allocated pension at retirement, $E_i(t)$ can be calculated as:

$$E_i(t) = \frac{W_x}{e_x}(1 + I)^{x-t},$$

[11]The value of the pensioner's family home is exempted from the assets test, hence is not included in W_t, although homeowner and non-homeowner are subject to different assets test limits.

where x is the pensioner's age at retirement. W_x is the pensioner's wealth put in allocated pension (the purchase price) at that time, e_x denotes the term of the allocated pension,[12] and I the inflation rate. This deduction is calculated at retirement and not indexed to inflation, hence it needs to be deflated when the real value of the deduction is calculated.

4.2.2 Assumptions

This paper investigate Problem 1 under three sets of assumptions:

1. First I ignore the bequest and housing decisions and assume the retiree's preferences are described by the hyperbolic absolute risk aversion (HARA) utility function such that:

$$\begin{aligned} U_c(C_t) &= \sum_{t=x}^{T} v^t \frac{(D_t + \mathcal{P}_t - \bar{C})^\gamma}{\gamma}, \\ U_h(H) &= 0, \\ U_b(B_T) &= 0. \end{aligned} \tag{4.8}$$

   Where the parameter v denoting the retiree's time preference. Note through out this paper I use v^t as substitute for v^{t-x}, this is done to simplify the equations making them easier to understand. This will have an effect on the absolute value of the utility function as instead of discounting utility to the date of retirement, we are discounting the utilities back to the date of birth.

[12]Because the actual term of the allocated pension varies by case, in this paper the term is assumed to be equal to the expected years of remaining life at age x calculated by Centrelink at that time, which is used by Centrelink for life pensions. See Fahcsia(2012).

However as the discount factors cancel out in the equations, this does not affect the optimum solution.

$\bar{C}$ represents a minimum level of consumption that needs to be secured.[13] This assumption is a simple step further than the classic life-cycle problem under HARA utility function[14] by including Australia's Age Pension means testing.

2. Then I consider bequest and housing decisions and assume the retiree's preferences in Problem 1 are described by the following utility functions as suggested in Chapter 3:

$$\begin{aligned} U_c(C_t) &= \sum_{t=x}^{T} v^t {}_tP_{\bar{x}} \frac{(D_t + \mathcal{P}_t - \bar{C})^\gamma}{\gamma}, \\ U_h(H) &= \sum_{t=x}^{T} v^t \frac{(\psi H)^\gamma}{\gamma}, \\ U_b(B_T) &= v^T \theta^{1-\gamma} \frac{(\theta a + B_T)^\gamma}{\gamma}. \end{aligned} \tag{4.9}$$

The notation is:

- ${}_tP_{\bar{x}}$, probability of someone currently age $\bar{x}$ survives to age t, while P_t denotes the probability of someone currently age t to survive one more year.
- H, value of the family home at retirement. I assume retirees can optimise the value of their family prior to or at retirement. However, they do not

[13] Which can be considered a necessity in the sense that its elasticity of demand with respect to wealth is zero. See Bateman et al. (2007).

[14] The solution of which is well known. See for example, Samuelson (1969) and Bateman et al. (2007).

have the option to vary the wealth in their family home after retirement. The value of the property is assumed to increase in line with inflation.

- ψ, utility parameter denoting the retiree's preference for Housing.[15]
- θ, utility parameter denoting the retiree's preference between consumptions and bequest.[16]
- a, utility parameter denoting how luxury is non-housing bequests.[17]

Together, these utility functions describe the preferences of Australian retirees. The detailed explanation of the utility function and its parameters calibrated to Australian data can be found in Chapter 3.

3. Last we consider the case that if the value of owner-occupied housing above a certain threshold L_h is subject to the Age Pension assets test, while the retiree's preferences follow Eq.(4.9). Under this assumption, the Age Pension

[15] ψ can be considered as the value of services as proportion to housing stock. This setting is similar to Cho and Sane (2013), which also modelled housing utility as additively separable from consumption utilities. Other research including Coco (2005) and Yogo (2009), modelled housing utility as a multiplicative component, in which case the model cannot be solved analytically.

[16] The optimal bequest decision in a simpler model included in Americks (2011), is to leave bequest equal to $(c-a)$ per year for θ years (assume C is constant, $\bar{C}=0$ and no Age Pension). $\theta = \phi/(1-\phi)$ can also be seen as transforming a utility parameter $\phi \in (0,1)$ that has the interpretation of "the marginal propensity to bequeath in a one-period problem of allocating wealth between consumption and an immediate bequest" (Lockwood, 2012, p.6).

[17] a has the interpretation of the "threshold consumption level below which, under the conditions of certainty or with full, fair insurance, people do not leave bequests" (Lockwood, 2012, p.7), see also Americks (2011) and De Nardi (2004).

payments entitled under the assets test in Eq.(4.6) is replaced with:

$$\mathcal{P}_a(t) = \mathcal{P}_m - (W_t + max(H - L_h, 0) - L_a)\varpi_a. \quad (4.10)$$

4.3 Solutions

The primary results of this paper show that the optimal consumption and investment decisions can be found as closed form solutions, subject to two approximations.

1. Age Pension means testing goes through four stages sequentially, and in the order of: no pension, assets test pension, income test pension and full pension.
2. The time of the change of the stages can be estimated at the time of planning.

Appendix A shows that these two approximations can be considered very close to reality, and provide a method to estimate the time of the changes of the effective means testing. Numerical examples in Section 5 show that these approximations lead to near identical results compared to solving the problem using numerical methods.

4.3.1 HARA utility

Proposition 1 presents the solution to Problem 1 assuming the HARA utility function Eq.(4.8), Appendices B and C of this paper detail the derivation of the solution.

Proposition 1. *Assuming the retiree's preferences in Problem 1 are described by the utility function Eq.(4.8), consider three time points* $k_1 \leq k_2 \leq k_3$*, where:*

- *k_1 is the age when the pensioner begins receiving assets test pension.*
- *k_2 is the age when the pensioner begins receiving income test pension.*
- *k_3 is the age when the pensioner begins receiving full pension.*

At any time t, define: $\hat{k}_i = \min(\max(k_i, t, A_p), T+1)$ for all $i = 1$ to 3, where A_p is the qualifying age for the Age Pension. The optimal decision rule for drawdown D_t^*, is:

$$D_t^* = \alpha_t \hat{W}_t - \hat{\mathcal{P}}_t,$$

in which:

$$\begin{aligned}
\hat{W}_t =& W_t + \xi_{\hat{k}_1} R^{t-\hat{k}_1} - \bar{C}\frac{R - R^{t-\hat{k}_1+1}}{R-1},\\
\xi_{\hat{k}_1} =& \xi_{\hat{k}_2}\left[R(1-\varpi_a)\right]^{\hat{k}_1-\hat{k}_2} + R(\mathcal{P}_m + L_a\varpi_a - \bar{C})\frac{1-[R(1-\varpi_a)]^{\hat{k}_1-\hat{k}_2}}{R(1-\varpi_a)-1},\\
\xi_{\hat{k}_2} =& \frac{\mathcal{P}_m + L_i\varpi_i - \bar{C}}{1-\varpi_i}\cdot\frac{R - R^{\hat{k}_2-\hat{k}_3+1}}{R-1} + \frac{\frac{W_x}{e_x}\varpi_i}{1-\varpi_i}(1+I)^{x-\hat{k}_2}\frac{R(1+I) - (R(1+I))^{\hat{k}_2-\hat{k}_3+1}}{R(1+I)-1}\\
&+ R^{\hat{k}_2-\hat{k}_3}(\mathcal{P}_m - \bar{C})\frac{R - R^{\hat{k}_3-T}}{R-1}.
\end{aligned}$$

$$\hat{\mathcal{P}}_t = \begin{cases}
-\bar{C} & \textit{for } t < \hat{k}_1;\\
\varpi_a\left(\xi_{\hat{k}_2}\left[R(1-\varpi_a)\right]^{t-\hat{k}_2} + R(\mathcal{P}_m + L_a\varpi_a - \bar{C})\frac{1-[R(1-\varpi_a)]^{t-\hat{k}_2+1}}{R(1-\varpi_a)-1}\right) & \\
+(\mathcal{P}_m + L_a\varpi_a - \bar{C})/(1-\varpi_a) & \textit{for } \hat{k}_1 \le t < \hat{k}_2;\\
(\mathcal{P}_m + (E_i(t) + L_i)\varpi_i - \bar{C})/(1-\varpi_i) & \textit{for } \hat{k}_2 \le t < \hat{k}_3;\\
\mathcal{P}_m - \bar{C} & \textit{for } t \ge \hat{k}_3.
\end{cases}$$

$$\alpha_t = \begin{cases} \hat{\alpha}_t + \varpi_a(1-\hat{\alpha}_t) & \textit{for } \hat{k}_1 \leq t \leq \hat{k}_2 - 1; \\ \hat{\alpha}_t & \textit{for all other } t; \end{cases}$$

and $\hat{\alpha}_t$ can be found recursively such that:

$$\hat{\alpha}_t = \begin{cases} \beta_a \mu \hat{\alpha}_{t+1}/(1+\beta_a \mu \hat{\alpha}_{t+1}) & \textit{for } \hat{k}_1 - 1 \leq t \leq \hat{k}_2 - 2; \\ \beta_i^{-1} \mu \hat{\alpha}_{t+1}/(1+\beta_i^{-1} \mu \hat{\alpha}_{t+1}) & \textit{for } t = \hat{k}_2 - 1 \\ \beta_i \mu \hat{\alpha}_{t+1}/(1+\beta_i \mu \hat{\alpha}_{t+1}) & \textit{for } t = \hat{k}_3 - 1; \\ \mu \hat{\alpha}_{t+1}/(1+\mu \hat{\alpha}_{t+1}) & \textit{for all other } t \end{cases}$$

with the terminal condition $\hat{\alpha}_T = 1$, where:

$$\begin{aligned} \beta_i =& (1-\varpi_i)^{\frac{\gamma}{1-\gamma}}, \\ \beta_a =& (1-\varpi_a)^{\frac{\gamma}{\gamma-1}}, \\ \mu =& \left(vE[(\hat{Z}^*)^\gamma] \right)^{\frac{1}{\gamma-1}}, \\ \hat{Z}^* =& \hat{\omega}^* \tilde{z} + (1-\hat{\omega}^*)R. \end{aligned}$$

The optimal proportion of wealth to be allocated to risky asset at time t after consumption, ω_t^*, can be found as:

$$\omega_t^* = \hat{\omega}^* \cdot \frac{\hat{W}_t - D_t^* - \hat{\mathcal{P}}_t}{W_t - D_t^*},$$

where $\hat{\omega}^*$ is the solution to the following equation:

$$E[(\hat{\omega}\tilde{z} + (1-\hat{\omega})R)^{\gamma-1}(\tilde{z} - R)] = 0.$$

Following Merton (1971), under the assumption that risky asset return $\tilde{z}$ is log-normally distributed with parameters ν and σ, we can write:

$$\hat{\omega}^* = \frac{\nu - \ln(R)}{\sigma^2(1-\gamma)},$$

and

$$\ln(\mu) = \frac{\ln(v^{-1}) - \gamma[\frac{(\nu-\ln(R))^2}{2\sigma^2(1-\gamma)} + \ln(R)]}{1-\gamma}.$$

Appendix A of this paper shows that k_1, k_2 and k_3 can be estimated with the method of forward simulations.

In contrast to the case when age pension is not present, the optimal consumption formula changes at each anticipated date of age pension means-testing changes. The design of age pension means testing tilt the retiree's consumption towards the beginning of retirement, as lower asset and lower consumption at later ages enable the retiree to receive a higher level of pension payments.

The optimal investment decision is also affected, anticipated future age pension payments effectively acting as a safe assets, boosting the retiree's allocation into risky asset at the beginning of retirement. Moreover, the design of asset test act as an effective insurance, in the sense that if the retiree suffers an investment loss, its anticipated future age pension payment increases, this further encourages the retiree to allocate a higher amount of asset in the beginning of retirement.

4.3.2 Bequest and housing

Proposition 2 and Proposition 3 present the solution to Problem 1 assuming the utility functions Eq.(4.9), Appendix D of this paper details the derivation of the solution.

Proposition 2. *Assuming the retiree's preferences are described by the utility functions Eq.(4.9), given the value of the family home H, $W_x = \mathcal{W}_x - H$, the optimal decision rules for drawdown D_t^* can be found as:*

$$D_t^* = \alpha_t \hat{W}_t - \hat{\mathcal{P}}_t,$$

in which:

$$\begin{aligned}
\hat{W}_t =& W_t + \xi_{\hat{k}_1} R^{t-\hat{k}_1} - \bar{C}\frac{R - R^{t-\hat{k}_1+1}}{R-1}, \\
\xi_{\hat{k}_1} =& \xi_{\hat{k}_2}\left[R(1-\varpi_a)\right]^{\hat{k}_1-\hat{k}_2} + R(\mathcal{P}_m + L_a\varpi_a - \bar{C})\frac{1-\left[R(1-\varpi_a)\right]^{\hat{k}_1-\hat{k}_2}}{R(1-\varpi_a)-1}, \\
\xi_{\hat{k}_2} =& \frac{\mathcal{P}_m + L_i\varpi_i - \bar{C}}{1-\varpi_i}\cdot\frac{R - R^{\hat{k}_2-\hat{k}_3+1}}{R-1} + \frac{\frac{W_x}{e_x}\varpi_i}{1-\varpi_i}(1+I)^{x-\hat{k}_2}\frac{R(1+I) - (R(1+I))^{\hat{k}_2-\hat{k}_3+1}}{R(1+I)-1} \\
&+ R^{\hat{k}_2-\hat{k}_3}(Pm - \bar{C})\frac{R - R^{\hat{k}_3-T}}{R-1} + R^{\hat{k}_3-T}\theta a.
\end{aligned}$$

$$\hat{\mathcal{P}}_t = \begin{cases} -\bar{C} & for\ \ t < \hat{k}_1; \\ \varpi_a \left(\xi_{\hat{k}_2} \left[R(1-\varpi_a) \right]^{t-\hat{k}_2} + R(\mathcal{P}_m + L_a\varpi_a - \bar{C}) \frac{1-[R(1-\varpi_a)]^{t-\hat{k}_2+1}}{R(1-\varpi_a)-1} \right) & \\ +(\mathcal{P}_m + L_a\varpi_a - \bar{C})/(1-\varpi_a) & for\ \ \hat{k}_1 \le t < \hat{k}_2; \\ (\mathcal{P}_m + (E_i(t) + L_i)\varpi_i - \bar{C})/(1-\varpi_i) & for\ \ \hat{k}_2 \le t < \hat{k}_3; \\ \mathcal{P}_m - \bar{C} & for\ \ t \ge \hat{k}_3. \end{cases}$$

$$\alpha_t = \begin{cases} \hat{\alpha}_t + \varpi_a(1-\hat{\alpha}_t) & for\ \ \hat{k}_1 \le t \le \hat{k}_2 - 1; \\ \hat{\alpha}_t & for\ all\ other\ \ t; \end{cases}$$

and $\hat{\alpha}_t$ *can be found recursively such that:*

$$\hat{\alpha}_t = \begin{cases} \beta_a\mu_t\hat{\alpha}_{t+1}/(1+\beta_a\mu_t\hat{\alpha}_{t+1}) & for\ \ \hat{k}_1 - 1 \le t \le \hat{k}_2 - 2; \\ \beta_i^{-1}\mu_t\hat{\alpha}_{t+1}/(1+\beta_i^{-1}\mu_t\hat{\alpha}_{t+1}) & for\ \ t = \hat{k}_2 - 1 \\ \beta_i\mu_t\hat{\alpha}_{t+1}/(1+\beta_i\mu_t\hat{\alpha}_{t+1}) & for\ \ t = \hat{k}_3 - 1; \\ \mu_t\hat{\alpha}_{t+1}/(1+\mu_t\hat{\alpha}_{t+1}) & for\ all\ other\ \ t \end{cases}$$

with the terminal condition:

$$\hat{\alpha}_T = \frac{\theta^{-1}({}_TP_{\bar{x}})^{\frac{1}{1-\gamma}}}{1+\theta^{-1}({}_TP_{\bar{x}})^{\frac{1}{1-\gamma}}},$$

where:

$$\beta_i = (1-\varpi_i)^{\frac{\gamma}{1-\gamma}},$$
$$\beta_a = (1-\varpi_a)^{\frac{\gamma}{\gamma-1}},$$
$$\mu_t = \left(P_t v E[(\hat{Z}^*)^\gamma]\right)^{\frac{1}{\gamma-1}},$$
$$\hat{Z}^* = \hat{\omega}^*\tilde{z} + (1-\hat{\omega}^*)R.$$

The optimal proportion of wealth to be allocated to risky assets at time t after consumption, ω_t^*, can be found as:

$$\omega_t^* = \hat{\omega}^* \cdot \frac{\hat{W}_t - D_t^* - \hat{\mathcal{P}}_t}{W_t - D_t^*},$$

where $\hat{\omega}^*$ is the solution to the following equation:

$$E[(\hat{\omega}\tilde{z} + (1-\hat{\omega})R)^{\gamma-1}(\tilde{z} - R)] = 0.$$

Under the assumption that risky assets return $\tilde{z}$ is log-normally distributed with parameters ν and σ, we can write:

$$\hat{\omega}^* = \frac{\nu - \ln(R)}{\sigma^2(1-\gamma)},$$

and

$$\ln(\mu_t) = \frac{\ln((P_t v)^{-1}) - \gamma[\frac{(\nu-\ln(R))^2}{2\sigma^2(1-\gamma)} + \ln(R)]}{1-\gamma}.$$

Compared to Proposition 1, we can see that the luxury bequest enters into the equation in two places, first as a proportional reduction to the amount of consumption parameter $\hat{\alpha}_T$, this has the effect of lowing the anticipated consumption throughout of retirement (saving for the bequest).

Second it has the effect of an addition ($\theta\alpha$) to the amount of anticipated lifetime wealth $\bar{W}$, which is exclusive to the case when bequest is assumed as a luxury good. This slightly offsets the consumption reduction metioned above, and increase the optimal allocation into the risky assets throughout of retirement. Specifically the optimal allocation increases as the retiree become older, as the addition ($\theta\alpha$) component become more significant in $\bar{W}$ as other wealth runs down.

Proposition 3. *The optimal amount of wealth H^* to be allocated into owner-occupied housing is:*

$$H^* = \frac{\vartheta \hat{\mathcal{W}}_x}{1 + (\xi' + 1)\vartheta},$$

with

$$\vartheta = \left[\frac{{}_xP_{\bar{x}}(1 - \varpi_x)^\gamma \alpha_x^{\gamma - 1}(\xi' + 1)}{\psi^\gamma \sum_{s=0}^{T-x} \upsilon^s} \right]^{\frac{1}{\gamma - 1}},$$

$$\xi' = R^{x - \hat{k}_1} \left[R(1 - \varpi_a) \right]^{\hat{k}_1 - \hat{k}_2} \frac{\varpi_i}{e_x(1 - \varpi_i)} (1 + I)^{x - \hat{k}_2} \frac{R(1 + I) - (R(1 + I))^{\hat{k}_2 - \hat{k}_3 + 1}}{R(1 + I) - 1},$$

where $\hat{\mathcal{W}}_x$ is the same function as $\hat{W}_x$, with liquid wealth W_x replaced by total wealth $\mathcal{W}_x$.

Compared to Proposition 2, including housing in the utility function introduce a level reduction to all retirement wealth available for further financial planing. We see that the optimal value of residential property is affected by age pension means testing parameters ϖ_a and ϖ_i, while income testing rules decrease the amount of residential property holdings slightly (as income test deduction amount is dependent on liquid wealth), asset testing rules significantly increase (note $\varpi_a = 0.5$) the optimal amount of wealth allocated into residential property.

4.3.3 Housing in the assets test

The decision rules for the optimal consumption and asset allocation under this assumption follows Proposition 2, except with L_a replaced with $L_a - \max(H - L_h, 0)$. The optimal value of owner-occupied properties is presented in Proposition 4.

Proposition 4. *Assume* $\mathcal{P}_a(t) = \mathcal{P}_m - (W_t + max(H - L_h, 0) - L_a)\varpi_a$*, the optimal amount of wealth* H^* *to be allocated into owner-occupied housing can be found as follows:*

First assume $H^* \leq L_h$*, calculate* H_a^**, which is the same as* H^* *in Proposition 3.*

Second assume $H^* > L_h$*, calculate*

$$H_b^* = \frac{\mathcal{A}(\hat{\mathcal{W}}_x + \mathcal{B}L_h)}{1 + \mathcal{A}\mathcal{B}}$$

where:

$$\begin{aligned}
\mathcal{A} =& \frac{\vartheta_b}{1+(\xi_b'+1)\vartheta_b}, \\
\mathcal{B} =& R^{x-\hat{k}_1} R\varpi_a \frac{1-[R(1-\varpi_a)]^{\hat{k}_1-\hat{k}_2}}{R(1-\varpi_a)-1}, \\
\vartheta_b =& \left[\frac{{}_xP_{\bar{x}}(1-\varpi_x)^\gamma \alpha_x^{\gamma-1}(\xi_b'+1)}{\psi^\gamma \sum_{s=0}^{T-x} \upsilon^s}\right]^{\frac{1}{\gamma-1}}, \\
\xi_b' =& \xi' + \mathcal{B},
\end{aligned}$$

where $\hat{\mathcal{W}}_x$ and ξ' are the same as in Proposition 3.

Then compare the value of H_a^* and H_b^*, and

$$H^* = \begin{cases} H_a^* & \text{if } H_a^* \leq L_h \text{ and } H_b^* \leq L_h; \\ H_b^* & \text{if } H_b^* > L_h \text{ and } H_a^* > L_h; \\ L_h & \text{if } H_a^* > L_h \text{ and } H_b^* \leq L_h; \end{cases}$$

Details of the derivation of the above solution can be found in Appendix D.

We see that this rule do not affects households not wealthier enough to have brought an house above the thresholds. For household wealthier enough, they will need to weight the marginal utility gains from increase the amount allocated to property against the utility loss of reduced future age pension payments.

4.4 Numeric illustrations

4.4.1 HARA utility

First assuming the utility function follows Eq.(4.8), consider financial planning for an investor who retires at age $x = 65$, and plan till final age $T = 100$. Assume inflation rate $I = 4.5$ percent, real interest rate is 1% (hence $R = 101\%$ and rate of time preference $v = (101\%)^{-1}$), utility curvature parameter $\gamma = -2$ and protected consumption $\bar{C} = \$21,930$.[18] I use Age Pension age of $A_p = 67$ and Centrelink (2012) Age Pension rates for single homeowners, which are as follows annually: $\mathcal{P}_m = \$19,643$, $L_a = \$186,750$, $\varpi_a = 0.039$, $L_i = \$3,900$, $\varpi_i = 0.5$ and $e_x = 88$.

Riskless asset only

Following Proposition 1, first assume no risky asset is available. Figure 4.1 illustrates the optimal consumption path for this pensioner including the drawdown and the Age Pension component, assuming she has initial wealth $w_{65} = \$400,000$.

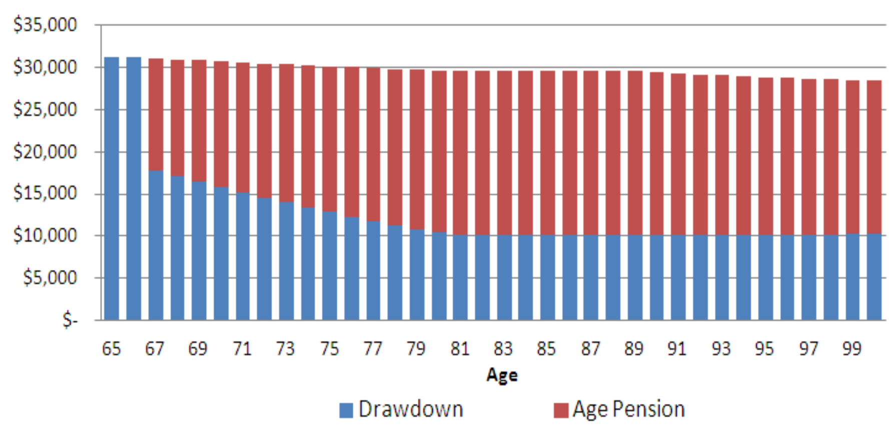

Figure 4.1: Consumption components by age, initial wealth $400,000.

[18] ASFA retirement standard of a modest lifestyle for singles, April 2012, see http://www.superannuation.asn.au/resources/retirement-standard

Figure 4.2 illustrates the decumulation of wealth of this pensioner over retirement.

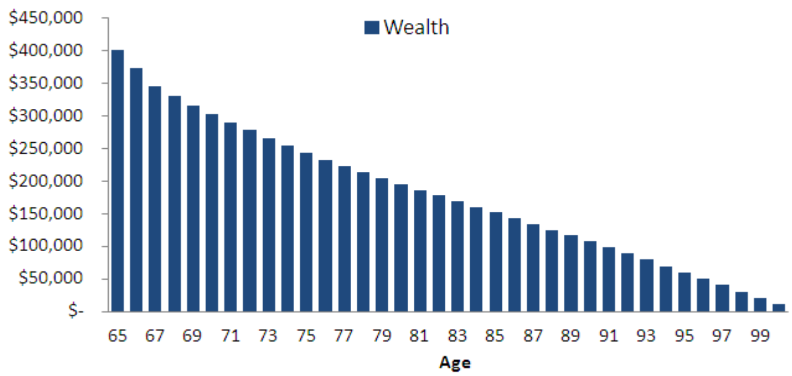

Figure 4.2: Wealth path by age, initial wealth $400,000.

Appendix A illustrates the process of estimating the time of means test changes. The estimated and actual times are: $k_1 = 67$, $k_2 = 81$, $k_3 = 81$. Figure 4.3 gives the consumption components derived using numerical dynamic programming method.[19] We can see that Figure 4.1 is a very good approximation, except that the pensioner is again subject to the income test from age 88 onwards, due to the decrease of the real value of the deductable amount $E_i(t)$.

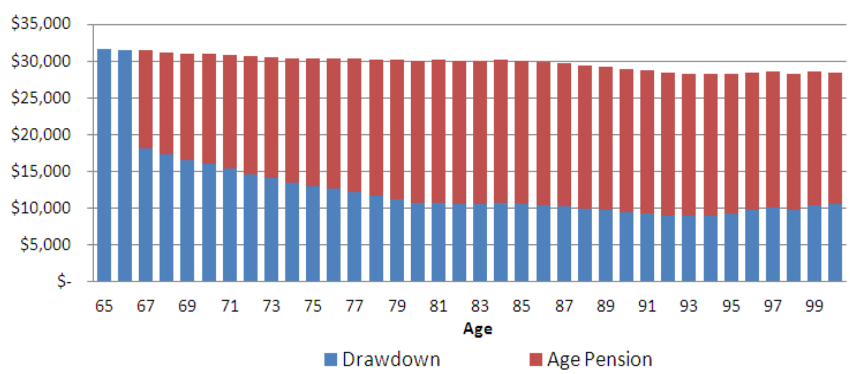

Figure 4.3: Consumption components derived with numerical method.

[19]I used a grid search method, for details see Appendix E.

Figure 4.4 illustrates the optimal consumption path for the pensioner when she has initial wealth $w_{65} = \$1,200,000$.

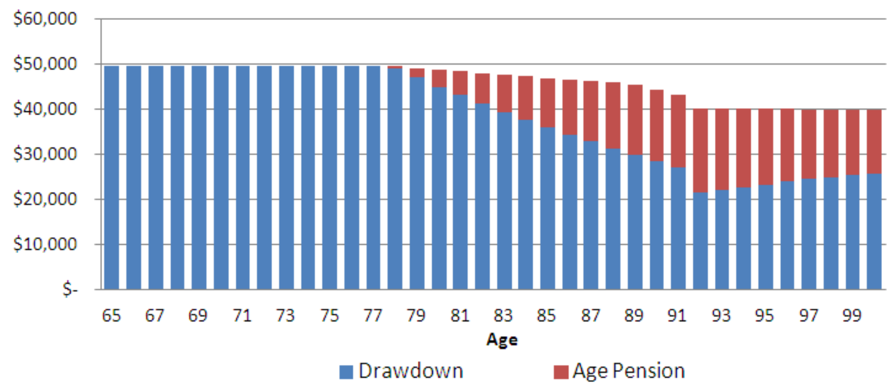

Figure 4.4: Consumption components by age, initial wealth $1,200,000.

In this case the estimated and actual times of means test changes are: $k_1 = 78$, $k_2 = 92$, $k_3 > 100$. From Figure 4.4 it is clearly evident that different behaviours are optimal in different stages of Age Pension payments. Figure 4.5 gives the consumption components derived using numerical dynamic programming method, which in this case, is nearly identical to Figure 4.5.

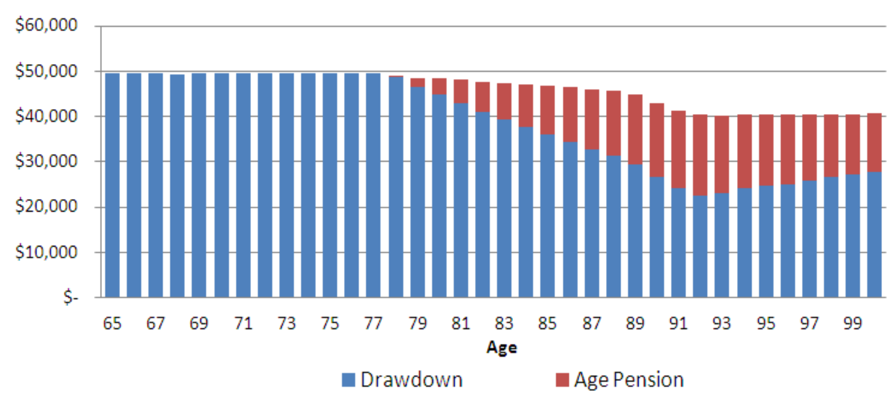

Figure 4.5: Consumption components derived with numerical method.

With risky asset

Assume that a risky asset is available and we have a series of randomly generated investment returns which are log-normally distributed with parameters $\nu = 0.05$ and $\sigma = 0.2$. Figures 4.6–4.8 illustrate for one simulated path of risky asset[20], the consumption components, risky asset allocations and the wealth path for the pensioner when she has initial wealth $w_{65} = \$400,000$.

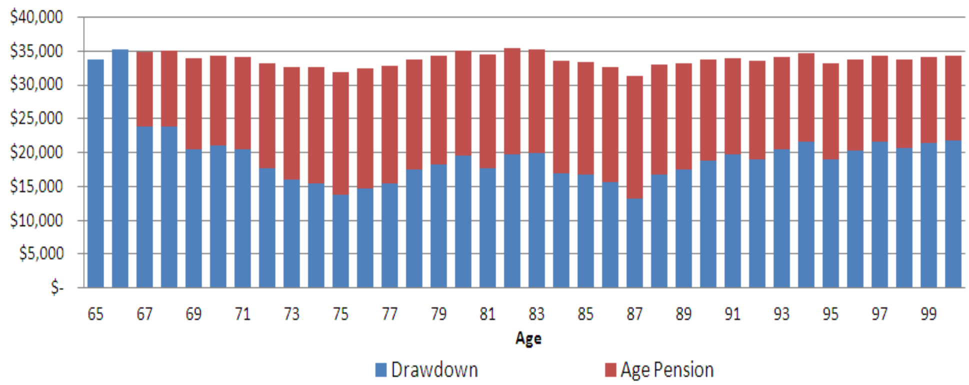

Figure 4.6: Consumption component given random risky asset return.

Figure 4.9 illustrates the result derived with numerical dynamic programming technique. We see that it is very similar to Figure 4.6.

[20]This single simulated path is chosen for no particular reason, except to compare the semi-analytical method proposed in this paper to the numerical method, under the presence of stochastic investment return. If we choose to use an average instead of a single simulated path, the result would look nearly the same as in the previous section.

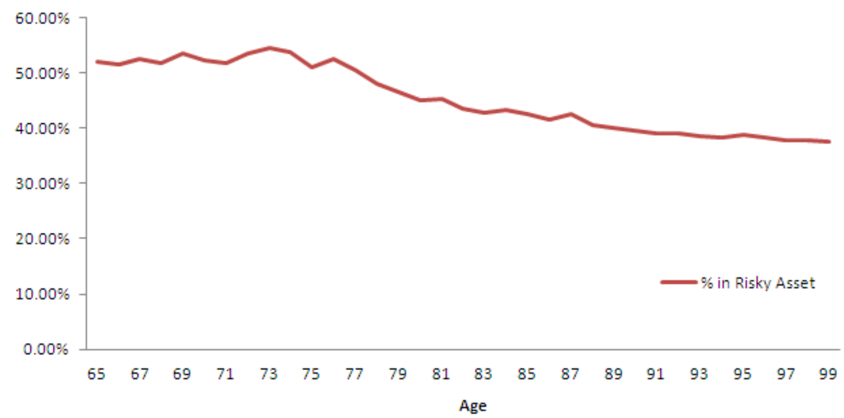

Figure 4.7: Proportion of wealth invested in risky asset.

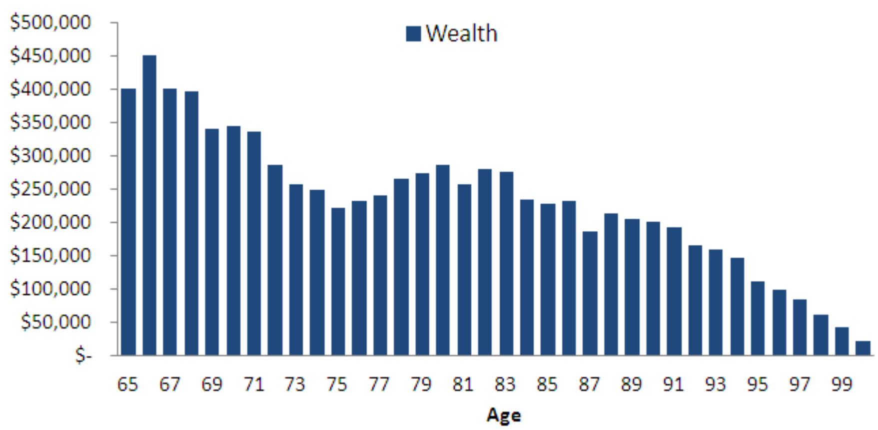

Figure 4.8: Wealth path by age.

4.4.2 Bequest and housing

Now assuming the investor's utility function follows Eq.(4.9), consider financial planning for a pensioner who retires at age $x = 65$, and plan till final age $T = 100$. Assume inflation rate $I = 4.5\%$, real risk free interest rate of 1%, and the following utility parameters.

Figures 4.10–4.12 illustrate the consumption components, risky asset allocations and

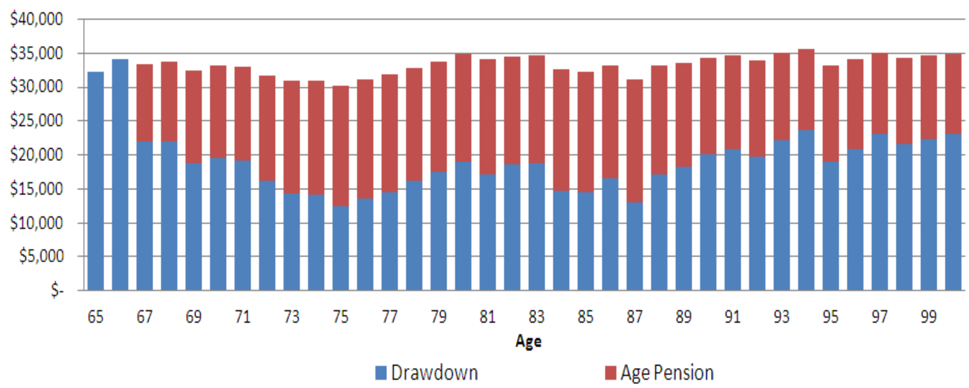

Figure 4.9: Consumption component, numerical method.

	υ	γ	$\bar{C}$	ψ	θ	a
Single	0.99	-3	\$10,000	3.2%	21.7	\$14,000

Table 4.1: Set of utility parameters calibrated in Chapter 3.

the wealth path for the pensioner assume her initial wealth $w_{65} = \$1,200,000$, when the realized real return of risky asset is 2.25 percent every year[21].

This pensioner allocates \$733,400 into family at the beginning of the retirement, and is expected to leave a bequest of \$133,000, if she dies at age 100. In this case the estimated and actual times of means test changes are: $k_1 = 67$, $k_2 = 88$, $k_3 = 88$. Her optimal consumption and risky asset allocation pattern changes when the effective means testing changes at age 67 and 88.

[21] I assume the investor make decision assuming that risky asset return is stochastic, however in this example, only illustrate his retirement outcome when the actual asset return turn out to be 2.25 percent, instead of varying every year. This prevent the consumption and wealth path to be erratically vary over time as in Figures 4.6–4.8, hence better illustrate the effect of bequest and housing on the investor's financial plan, which is the main purpose of this section.

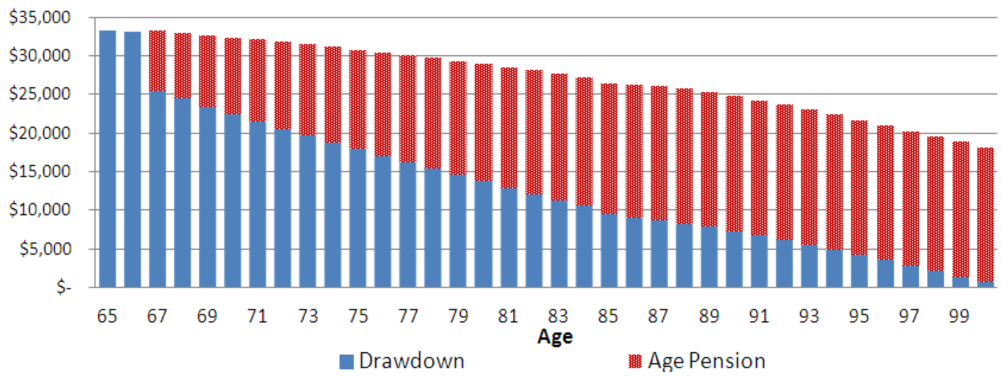

Figure 4.10: Consumption component given random risky asset return.

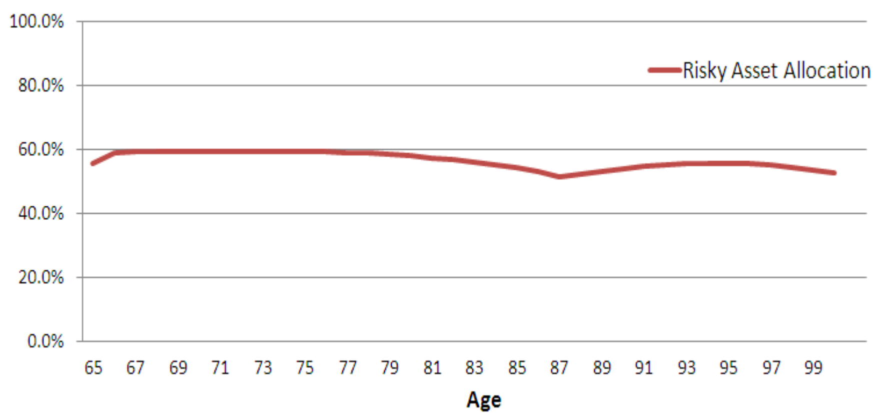

Figure 4.11: Proportion of wealth invested in risky asset.

We see that her consumption decreases with age, as her consumption utility decreases as health declines. This contrasts with Figure 4.4 (in which case the investor's time preference is assumed equal to the rate of investment return). We can also see that her risky asset allocation remains relatively flat throughout retirement, due to two factors with opposite effect as discussed in Chapter 3. The existence of luxury bequests implies a higher allocation into risky asset at later ages, while the existence of the Age Pension implies a lower risky allocation as the household ages.

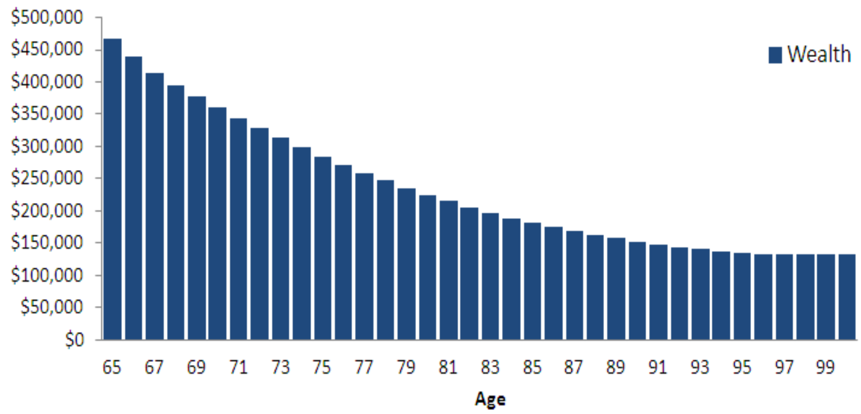

Figure 4.12: Wealth path by age.

The result derived with numerical dynamic programming technique, shows that the optimal asset to be allocated into the family home is around $700,000,[22] and she is expected to leave a bequest of $147,200. Figure 4.13 illustrates the consumption path derived, which is very similar to Figure 4.10.

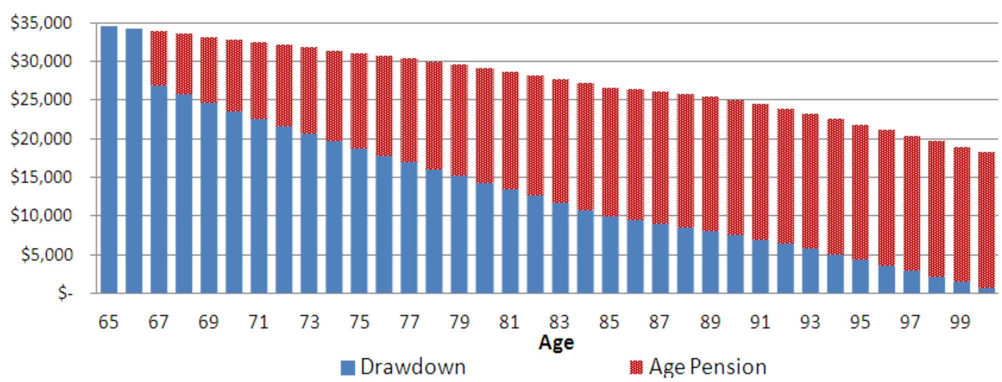

Figure 4.13: Consumption component, numerical method.

[22]Grid search method is used to derive numerical results, and for housing, I assume the grids are of intervals of $50,000.

Housing in the assets test

If we assume the value of the retiree's house in excess of $L_h = \$500,000$, are included as part of assessable assets in the Age Pension assets test. Given the same utility function and parameters for this retiree, the optimal value to be allocated into the family home in this case is found to be \$500,000 (just on the assumed threshold), while she is expected to leave \$183,400 as a bequest at age 100. Figs. 14 and 15 illustrate the consumption components and the wealth path for this pensioner.

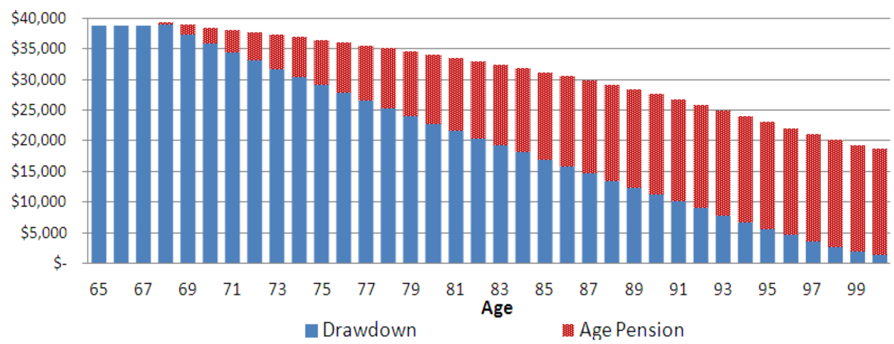

Figure 4.14: Consumption component given random risky asset return.

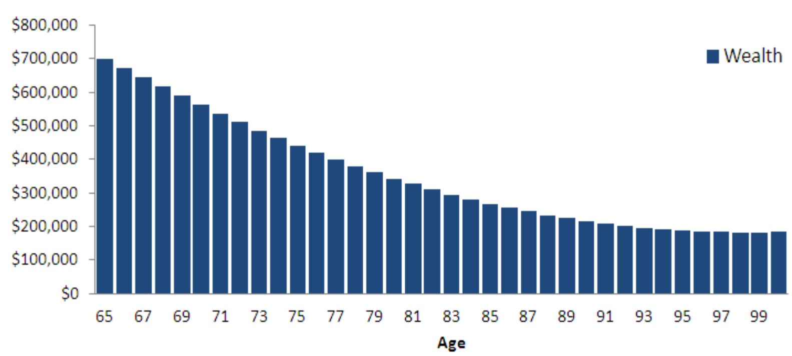

Figure 4.15: Wealth path by age.

Numerical method gives similar results.

4.5 Conclusion

This paper derives semi-analytical solutions to the life-cycle problem under three sets of assumptions. I first modified the HARA utility model studied in Merton (1971), and found a semi-analytical solution when the publicly provided Age Pension in Australia is taken into account. Then I solved the problem with an extended utility function which further takes into account bequest motive and housing. Last, I found the solution to the problem under the additional assumption that part of housing assets above a certain threshold are assessable in the Age Pension assets test.

Previous literature in this area either solved the problem using numerical methods, or required significant simplification of the Age Pension system. While the method presented in this paper provides very close results to the solution derived using numerical dynamic programming, under realistic Age Pension means testings rules, with clear advantages in computation time. This can improve efficiency of many tasks, such as when the utility parameters are to be calibrated to a dataset, as in Chapter 3.

The limitation of this method is that it restricts how the utility function can change to adopt to different assumptions, as closed-form solutions can only be found with specific forms of utility functions (for example, consumption and housing utilities cannot be multiplicative). Future research could improve the flexibility of the utility functions, and extend the method to the public pension system of other countries.

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Appendix A. Discussion of the approximations

The results in this paper are based on two approximations:

1. Age Pension means testing goes through four stages sequentially, and in the order of: no pension, assets test pension, income test pension and full pension.

2. The time of the change of the stages can be estimated at the time of planning.

This section discussed how close these approximations are to the reality and suggests a way of estimating the time of changes of the effective means testing, k_1, k_2 and k_3.

Firstly, without a bequest motive, we can reasonably assume that the pensioner gradually spends down her wealth during retirement, hence $w_{t+1} < w_t$, except in the case of an abnormally high investment return. Secondly, given the utility function we are using, we can reasonably assume that the pensioner's consumption is relatively smooth.

Now from Eqs (4.5), (4.6) and (4.7), it can be shown that, income test pension is effective over assets test pension when

$$(D_t - E_i(t) - L_i)\varpi_i < (W_t - L_a)\varpi_a$$

The right hand side of the inequation is nearly always increasing due to $w_{t+1} < w_t$, while the left hand side is nearly always decreasing (due to the real value of $E_i(t)$ decreasing with time as it is not indexed), it is reasonable to assume income test comes after the assets test, except in the very rare occasion of an abnormally high investment return right after the income test becomes effective, and even in this case, the income test pension and assets test pension would not differ by much.

It is also intuitive that the period of zero pension comes before assets test pension since it is the lower bound. However, it is possible that the pensioner may fall back to income test pension after receiving full pension, due to the fact that the real value of $E_i(t)$ is decreasing. In these cases the pensioner would be receiving near full pension so it would not be far off assuming full pension is entitled.

The validity of the first approximation led to the assumption that there must exist three specific times of changes of the effective means testing, k_1, k_2 and k_3 for a pensioner, given that she behaves optimally. Hence the times k_1, k_2 and k_3 can be estimated as follows:
For Proposition 1:

1. start with arbitrary k_1, k_2 and k_3, simulate the consumption and wealth path of the pensioner following Proposition 1.
2. record the times of means testing changes of this simulation.
3. use the recorded times and run the simulate again, and repeat until the result is satisfactory.

The following illustrates the process of estimating the times of means testing changes of the numeric example in Section 5.1:

First trial: starting with $k_1 = 67$, $k_2 = 67$ and $k_3 = 67$, the resulted consumption path of the simulation looks as follows:

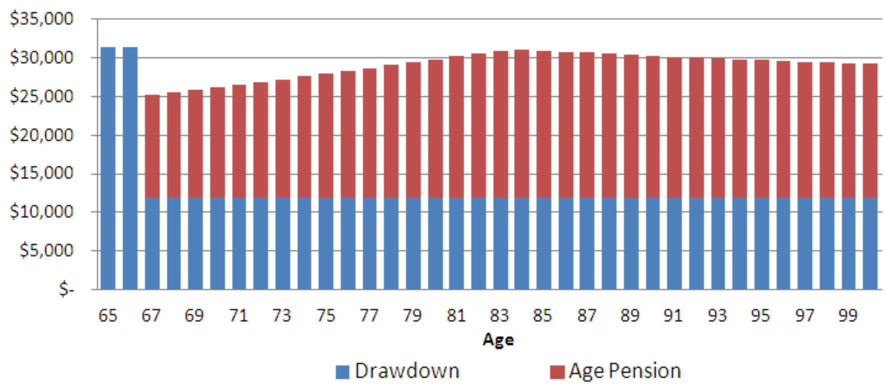

Figure 4.16: initial wealth \$400,000, assume $k_1 = 67$, $k_2 = 67$ and $k_3 = 67$.

Second trial: simulate using $k_1 = 67$, $k_2 = 84$ and $k_3 > 100$ as given in the first trial, the resulted consumption path looks as follows:

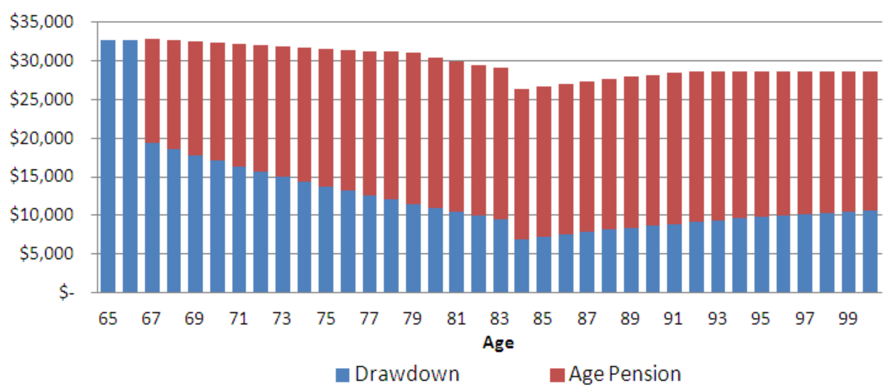

Figure 4.17: initial wealth \$400,000, assume $k_1 = 67$, $k_2 = 84$ and $k_3 > 100$.

Third trial: simulate using $k_1 = 67$, $k_2 = 79$ and $k_3 = 79$ as given in the second trial, the resulted consumption path looks as follows:

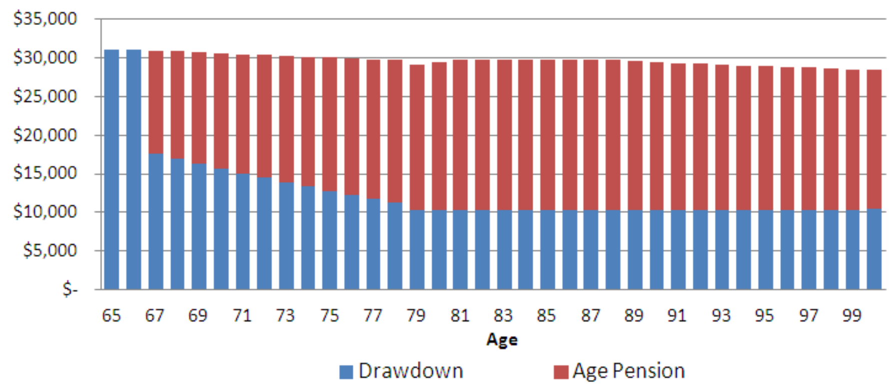

Figure 4.18: initial wealth $400,000, assume $k_1 = 67$, $k_2 = 79$ and $k_3 = 79$.

Fourth trial: simulate using $k_1 = 67$, $k_2 = 81$ and $k_3 = 81$ as given in the third trial, the resulted consumption path looks as follows:

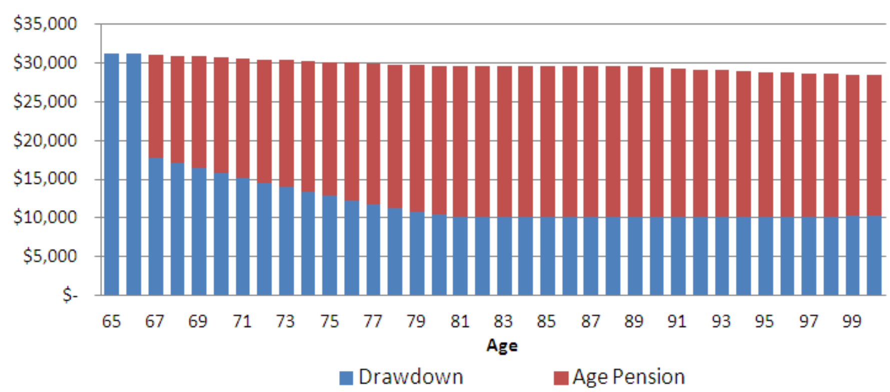

Figure 4.19: initial wealth $400,000, assume $k_1 = 67$, $k_2 = 81$ and $k_3 = 81$.

Any further trials of simulation give the same result, hence we can conclude that $k_1 = 67$, $k_2 = 81$ and $k_3 = 81$ is the solution.

Stoping rules may be designed when applying this method in practice. A suitable stoping rule can be: after n trials, if the $n+1$ trial lead to an expected sum of utility less than or equal to the nth trial, then stop and use the result of the nth trial. Only a small number of trials of simulation are normally needed, for example: the times of means test changes in example in Section 5.2 can be estimated with five trials of simulation.

The task is more difficult to find k_1, k_2 and k_3 in Propositions 2, 3 and 4, when housing decisions are taken into account. This is because the value of houses can significantly affect the effective means testings. For example, if our starting assumption is that the retiree receives a full pension throughout retirement, the optimal value of the house under this assumption would be very big, because the retiree is assumed to receive a large amount of Age Pension and only needs to reserve a small amount of money for consumption. However, we can see that under this asset allocation, the retiree would have little wealth outside of the family home, hence likely to qualify for a full pension, and the resultant k_1, k_2 and k_3 of simulation may be the same as our starting assumption.

This problem is similar to the task of finding the global maximum in the presence of local maximums, and there are several methods that can be applied. One method which is suitable in this case is to set a reasonable initial guess and limiting the step size of the optimisation[23] as follows:

1. Start with arbitrary k_1, k_2 and k_3 and find the value of House according to

[23] This is a typical technic in numerical optimisation and gradient search; it is suitable in this case because we know that the local maximums are located in the two extremes (either too much wealth in the family home or too little), hence if we start the initial value in the middle and prevent the result from swinging to either extreme too quickly, we will most likely end up in the global maximum. Another method that can be used is to trial a range of fixed house values and find the corresponding times k_1, k_2 and k_3, and compare the resultant utility function to determine a suitable pair of starting positions.

Proposition 3 or 4.

2. Limit the value of the House to be no more than $50\% + a$ and no less than $50\% - a$ of initial wealth, and simulate the consumption and wealth path of the pensioner following Proposition 2.

3. Record the times of means testing changes of this simulation.

4. Use the recorded times and run the simulate again, however this time set $a = 2a$, and repeat until the result is satisfactory.

For example, if we set $a = 5$ percent, in the first trial of simulation the value of the house can only be 45 percent to 55 percent of the initial wealth of the retiree, in the second trial 40 percent to 60 percent, and 35 percent to 65 percent in the third trial and so on. After ten trials we will have already obtained reasonable estimates for k_1, k_2 and k_3 that allow the value of the house to be calculated exactly as our formulas. The calculation for the numeric example in Section 4.2 following this process can be illustrated in the following Figures, with step size $a = 3$ percent:

First trial: starting with $k_1 > 100$, $k_2 > 100$ and $k_3 > 100$, and the value of the house restricted to be 47 percent to 53 percent of total wealth ($1,200,000). The optimal value to be allocated into housing is $578,625 and the resulting consumption path of the simulation looks as follows:

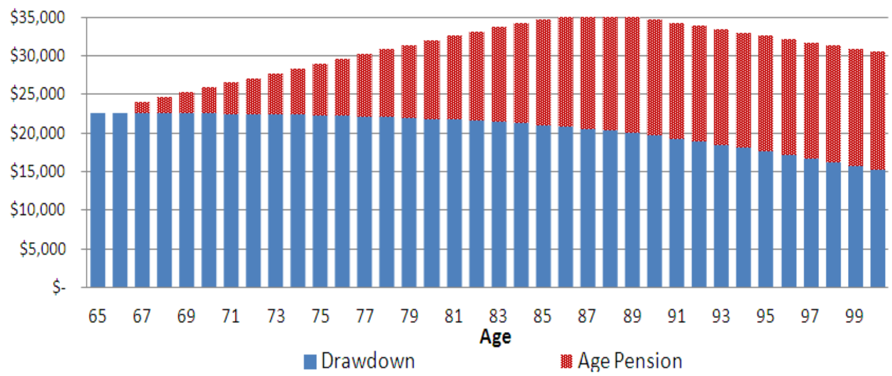

Figure 4.20: initial wealth $400,000, assume $k_1 > 100$, $k_2 > 100$ and $k_3 > 100$.

Second trial: simulate using $k_1 = 67$, $k_2 = 88$ and $k_3 > 100$ as given in the first trial, and the value of house restricted to be 44 percent to 56 percent of total wealth. The optimal value to be allocated into housing is $648,060 and the resulting consumption path of the simulation looks as follows:

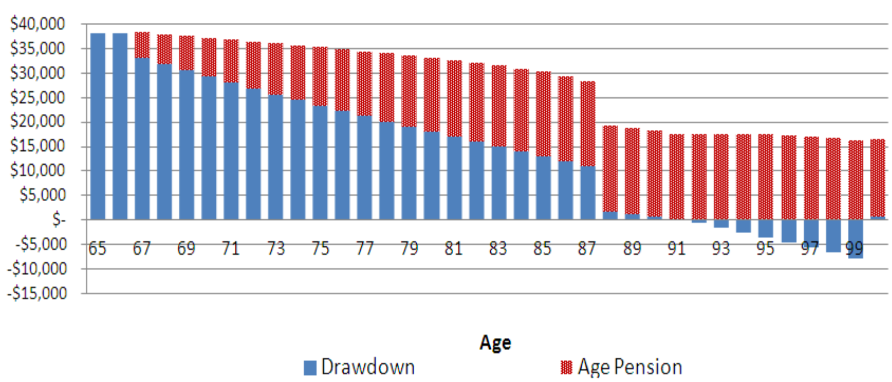

Figure 4.21: initial wealth $400,000, assume $k_1 = 67$, $k_2 = 86$ and $k_3 > 100$.

... Fourth trial: simulate using $k_1 = 67$, $k_2 = 90$ and $k_3 = 90$ as given in the third trial, and the value of the house restricted to be 38 percent to 62 percent of total wealth. The optimal value to be allocated into housing is $719,200 and the resulting consumption path of the simulation looks as follows:

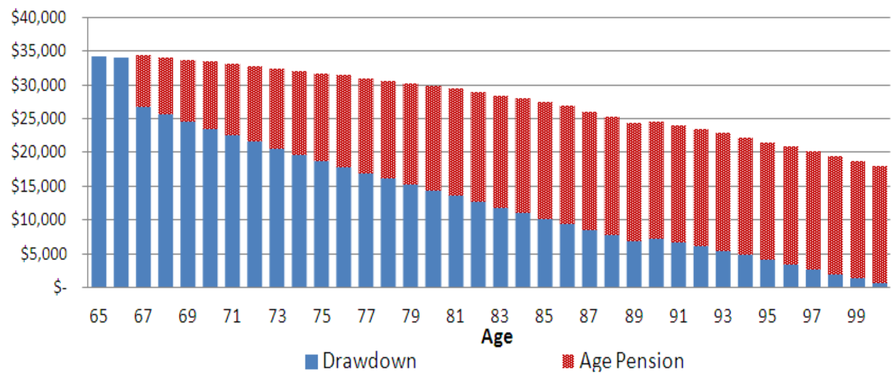

Figure 4.22: initial wealth $400,000, assume $k_1 = 67$, $k_2 = 90$ and $k_3 = 90$.

... Sixth trial: simulate using $k_1 = 67$, $k_2 = 88$ and $k_3 = 88$ as given in the fifth trial, and the value of house restricted to be 32 percent to 68 percent of total wealth. The optimal value to be allocated into housing is $733,400 and the resulting consumption path of the simulation looks as Figure 4.10. Further trials of simulation do not further improve the value of the utility function (in fact, the results then enter into a loop with the results of the seventh trial the same as the fifth and the eighth the same as the sixth, and so on), hence we conclude that what we have is the best result.

Appendix B. Review of classic life-cycle models

To derive Proposition 1, we start by initially ignoring Age Pension, Bequest and Housing, assuming $P(t) = 0$ and revisit some of the solutions of the classical life-cycle models.

B1. CRRA utility

If the investor's preferences are described by $U(C_t) = v^t \frac{C_t^\gamma}{\gamma}$, as for the conventional constant relative risk aversion (CRRA) investor, Eq.(4.1) can be written as

$$\max E\left[\sum_{t=0}^{T} v^t \frac{C_t^\gamma}{\gamma}\right], \tag{4.11}$$

where v^t is the discount rate describing the investor's time preference at age t.

According to Samuelson (1969), the optimal decision rules for consumption and asset allocation, C_t^* and ω_t^* that maximizes Eq.(4.11), assuming the risky asset return $\tilde{z}$ is independent and identically distributed for all t, can be found as:

$$\begin{aligned} C_t^* =& \alpha_t W_t \\ \alpha_t =& \frac{\mu \alpha_{t+1}}{1 + \mu \alpha_{t+1}}, \end{aligned} \tag{4.12}$$

with the terminal condition $\alpha_T = 1$, where $\mu = \left(vE[(\tilde{Z}^*)^\gamma]\right)^{\frac{1}{\gamma-1}}$, and $\tilde{Z}^* = (\omega^* \tilde{z} + (1 - \omega^*)R)$ denote the random investment return at time t with the optimal asset allocation.

The optimal proportion of wealth at time t after consumption to be allocated to risky asset ω^* is the same for all t and can be found as the solution to the following equation:

$$E_t[(\tilde{Z}^*)^{\gamma-1}(\tilde{z}-R)] = 0, \tag{4.13}$$

Proof. The problem can be solved by means of dynamic programming, as at time t, all past utilities have been realised and the decision to be made by the investor would depend only on his current financial position and the future expectations, which can be illustrated through the Bellman Equation of Eq.(4.11).

$$J^*(W,t) = \max E_t\left[\sum_{s=t}^{T} v^s \frac{C_s^\gamma}{\gamma}\right] = \max\left(v^t\frac{C_t^\gamma}{\gamma} + E_t\left[J^*(W,t+1)\right]\right), \tag{4.14}$$

subject to Eq.(4.2).

Following Eq.(4.14) and work recursively: At Time T: $J(W,T) = v^T\frac{C_T^\gamma}{\gamma}$ hence $C_T^* = W_T$;

At Time $T-1$, we need to maximize:

$$\begin{aligned}
J(W,T-1) = &\left(v^{T-1}\frac{C_{T-1}^\gamma}{\gamma} + E_t\left[J^*(W,T)\right]\right)\\
=&E_{T-1}\left[v^{T-1}\frac{C_{T-1}^\gamma}{\gamma} + v^T\frac{W_T^\gamma}{\gamma}\right]\\
=&v^{T-1}\frac{C_{T-1}^\gamma}{\gamma} + E_{T-1}\left[\frac{v^T((W_{T-1}-C_{T-1})\tilde{Z}_{T-1})^\gamma}{\gamma}\right]\\
=&v^{T-1}\frac{C_{T-1}^\gamma}{\gamma} + v^T\frac{(W_{T-1}-C_{T-1})^\gamma}{\gamma}E_{T-1}\left[\tilde{Z}_{T-1}^\gamma\right],
\end{aligned} \tag{4.15}$$

where

$$\tilde{Z}_t = \omega_t \tilde{z}_t + (1 - \omega_t) R. \tag{4.16}$$

Differentiate (4.15) with respect to ω_{T-1} and set to 0 yields:

$$\begin{aligned}
&\frac{d}{d\omega_{T-1}} \ v^{T-1} \frac{C_{T-1}^{\gamma}}{\gamma} + v^T \frac{(W_{T-1} - C_{T-1})^{\gamma}}{\gamma} E_{T-1}[(\omega_{T-1}\tilde{z}_{T-1} + (1-\omega_{T-1})R)^{\gamma}] \\
&\quad \Rightarrow v^T \frac{(\omega_{T-1} - C_{T-1})^{\gamma}}{\gamma} \frac{d}{d\omega_{T-1}} E_{T-1}[(\omega_{T-1}\tilde{z}_{T-1} + (1-\omega_{T-1})R))^{\gamma}] \\
&\quad \Rightarrow E_{T-1}[(\omega_{T-1}^{*}\tilde{z}_{T-1} + (1-\omega_{T-1}^{*})R)^{\gamma-1}(\tilde{z}_{T-1} - R)] = 0. \qquad (4.17)
\end{aligned}$$

The optimal ω_{T-1}^{*} is the solution to (4.17).

Differentiate (4.15) with respect to C_{T-1} and set to 0 yields:

$$\begin{aligned}
&\frac{d}{dC_{T-1}} \ v^{T-1} \frac{C_{T-1}^{\gamma}}{\gamma} + v^T \frac{(W_{T-1} - C_{T-1})^{\gamma}}{\gamma} E_{T-1}[(\tilde{Z}_{T-1}^{*})^{\gamma}] \\
&\quad \Rightarrow v^{T-1}(C_{T-1}^{*})^{\gamma-1} - v^T (W_{T-1} - C_{T-1}^{*})^{\gamma-1} E_{T-1}[(\tilde{Z}_{T-1}^{*})^{\gamma}] = 0 \\
&\quad \Rightarrow C_{T-1}^{*} = \frac{(vE_{T-1}[(\tilde{Z}_{T-1}^{*})^{\gamma}])^{\frac{1}{\gamma-1}} W_{T-1}}{1 + (vE_{T-1}[(\tilde{Z}_{T-1}^{*})^{\gamma}])^{\frac{1}{\gamma-1}}} \\
&\qquad\qquad = W_{T-1} \left(1 + (vE_{T-1}[(\tilde{Z}_{T-1}^{*})^{\gamma}])^{\frac{1}{1-\gamma}}\right)^{-1}, \qquad (4.18)
\end{aligned}$$

where $\tilde{Z}_{T-1}^{*} = \omega_{T-1}^{*}\tilde{z} + (1 - \omega_{T-1}^{*})R$ denotes the random investment return at time $T-1$ with the optimal asset allocation. We see from (4.18) the optimal consumption can be expressed as a proportion of current wealth $C_{T-1}^{*} = \alpha_{T-1} W_{T-1}$, with:

$$\alpha_{T-1} = \left(1 + (vE_{T-1}[(\tilde{Z}_{T-1}^{*})^{\gamma}])^{\frac{1}{1-\gamma}}\right)^{-1}. \tag{4.19}$$

Eq.(4.15) can now be written as:

$$
\begin{aligned}
J^*(W,T-1) =& v^{T-1}\frac{(\alpha_{T-1}W_{T-1})^\gamma}{\gamma} + v^T\frac{((1-\alpha_{T-1})W_{T-1})^\gamma E_{T-1}\left[(\tilde{Z}^*_{T-1})^\gamma\right]}{\gamma}\\
=& v^{T-1}\frac{W^\gamma_{T-1}}{\gamma}\left[\alpha^\gamma_{T-1} + vE_{T-1}[(\tilde{Z}^*_{T-1})^\gamma](1-\alpha_{T-1})^\gamma\right]\\
=& \frac{v^{T-1}W^\gamma_{T-1}}{\gamma}\alpha^{\gamma-1}_{T-1}.
\end{aligned} \tag{4.20}
$$

We can now look at the optimal decision at time T-2, at this time we need to maximize:

$$
\begin{aligned}
J(W,T-2) =& v^{T-2}\frac{C^\gamma_{T-2}}{\gamma} + E_{T-2}\left[J^*(W,T-1)\right]\\
=& v^{T-2}\frac{C^\gamma_{T-2}}{\gamma} + E_{T-2}\left[\frac{(v^{T-1}\alpha^{\gamma-1}_{T-1})W^\gamma_{T-1}}{\gamma}\right]\\
=& v^{T-2}\frac{C^\gamma_{T-2}}{\gamma} + E_{T-2}\left[\frac{(v^{T-1}\alpha^{\gamma-1}_{T-1})((W_{T-2}-C_{T-2})\tilde{Z}_{T-2})^\gamma}{\gamma}\right]\\
=& v^{T-2}\frac{C^\gamma_{T-2}}{\gamma} + v^{T-1}\frac{(W_{T-2}-C_{T-2})^\gamma}{\gamma}E_{T-2}\left[\alpha^{\gamma-1}_{T-1}\tilde{Z}^\gamma_{T-2}\right].
\end{aligned} \tag{4.21}
$$

Eq.(4.21) is identical to Eq.(4.15). Solutions are given similar to Eqs (4.17) and (4.18). It is easy to verify that $J(W,t)$ at each time t is identical to Eqs (4.21) and (4.15). The optimal consumption decisions at any time t can hence be written as

$$
C^*_t = \alpha_t W_t; \qquad \alpha_t = [1 + (vE_t[\alpha^{\gamma-1}_{t+1}(\tilde{Z}^*_{T-1})^\gamma])^{\frac{1}{1-\gamma}}]^{-1}; \qquad \alpha_T = 1, \tag{4.22}
$$

and the optimal portfolio allocation ω^*_t is the solution to:

$$
E_t[(\alpha_{t+1}\tilde{Z}_t)^{\gamma-1}(\tilde{z}_t - R)] = 0. \tag{4.23}
$$

This is the same as the result obtained in Samuelson (1969) and Bateman et al

(2007). When $\tilde{z}_t$ are assumed to be independently and identically distributed. Eq.(4.23) can be written as

$$\alpha_{t+1}^{\gamma-1} E_t[\tilde{Z}^{\gamma-1}(\tilde{z}_t - R)] = 0.$$

This proves Eq.(4.13). We can then write Eq.(4.22) as

$$\alpha_t = \frac{\mu\alpha_{t+1}}{1+\mu\alpha_{t+1}}, \tag{4.24}$$

where $\mu = \left(vE[(\tilde{Z}^*)^\gamma]\right)^{\frac{1}{\gamma-1}}$, with the terminal condition $\alpha_T = 1$. □

B2. HARA utility

If the investor's preferences are described by: $U(C_t) = v^t \frac{(C_t-\bar{C})^\gamma}{\gamma}$, as for the Hyperbolic absolute risk aversion (HARA) utility function, the problem in Eq.(4.1) can be written as

$$\max E\left[\sum_{t=0}^{T} v^t \frac{(C_t - \bar{C})^\gamma}{\gamma}\right], \tag{4.25}$$

where $\bar{C}$ can be considered to be a positive consumption floor.

According to Samuelson (1969), the optimal decision rules for consumption and asset allocation, C_t^* and ω_t^* that maximises Eq.(4.25), can be found as

$$C_t^* = \bar{C} + \hat{C}_t^* \frac{\hat{W}_t}{W_t}, \tag{4.26}$$

where $\hat{C}_t^*$ is the optimal consumption for an investor with the CRRA preference calculated in Eq.(4.12), and

$$\hat{W}_t = W_t - \bar{C}\frac{R - R^{t-T}}{R-1}. \tag{4.27}$$

The optimal proportion of wealth to be allocated to risky asset at time t after consumption, ω_t^*, can be found as

$$\omega_t^* = \hat{\omega}^* \cdot \frac{\hat{W}_t - C_t + \bar{C}}{W_t - C_t}, \tag{4.28}$$

where $\hat{\omega}^*$ is the optimal asset allocation for an investor with the CRRA preference calculated in Eq.(4.13).

Proof. The Bellman Equation of (4.25) can be written as

$$J^*(W,t) = \max E_t\left[\sum_{s=t}^{T} v^s \frac{(C_s - \bar{C})^\gamma}{\gamma}\right] = \max\left(v^t \frac{(C_t - \bar{C})^\gamma}{\gamma} + E_t\left[J^*(W,t+1)\right]\right), \tag{4.29}$$

subject to Eq.(4.2), and the end condition $J(W,T) = v^T \frac{(C_T - \bar{C})^\gamma}{\gamma}$.

Work recursively similar to the CRRA case, at Time T: $C_T^* = W_T$;

At Time $T-1$, we need to maximise:

$$J(W,T-1) = v^{T-1}\frac{(C_{T-1} - \bar{C})^\gamma}{\gamma} + v^T \frac{E_{T-1}\left[([W_{T-1} - C_{T-1}]\tilde{Z}_{T-1} - \bar{C})^\gamma\right]}{\gamma}. \tag{4.30}$$

Define

$$\hat{C}_t = C_t - \bar{C},$$

$$\hat{W}_t = W_t - \bar{C}\sum_{s=t}^{T} R^{t-s}, \tag{4.31}$$

$$\hat{\omega}_t = \omega_t \frac{W_t - C_t}{\hat{W}_t - \hat{C}_t}. \tag{4.32}$$

Eq.(4.30) can then be written as:

$$J(W, T-1) = v^{T-1}\frac{\hat{C}_{T-1}^{\gamma}}{\gamma} + v^{T}\frac{E_{T-1}\left[([\hat{W}_{T-1} - \hat{C}_{T-1}]\hat{Z}_{T-1})^{\gamma}\right]}{\gamma} \tag{4.33}$$

where $\hat{Z}_{T-1} = \hat{\omega}_t \tilde{z}_t + (1 - \hat{\omega}_t)R$. We can see that

$$[W_{T-1} - C_{T-1}]\tilde{Z}_{T-1} - \bar{C} = [W_{T-1} - C_{T-1} - \frac{\bar{C}}{R}]\hat{Z}_{T-1} + \frac{\bar{C}}{R}R - \bar{C}$$

Eq.(4.33) is identical to Eq.(4.15). The solutions for the optimal consumption are given similar to Eq.(4.18)

$$\hat{C}_{T-1}^{*} = \alpha_{T-1}\hat{W}_{T-1}; \qquad \alpha_{T-1} = [1 + (vE_{T-1}[(\hat{Z}_{T-1}^{*})^{\gamma}])^{\frac{1}{1-\gamma}}]^{-1} \tag{4.34}$$

$\hat{\omega}_{T-1}^{*}$ can be found to be the same as given by Eq.(4.13).

Lemma 5 proves that similar results hold true for every time period, and the general results can be written as in Section 3.2.

Lemma 5. *Eq.(4.29) can be written as*

$$J^*(W,t) = \max\left(v^t \frac{\hat{C}_t^\gamma}{\gamma} + E_t\left[J^*(W,t+1)\right]\right), \tag{4.35}$$

subject to

$$\hat{W}_{t+1} = [\hat{W}_t - \hat{C}_t][\hat{\omega}_t \tilde{z} + (1-\hat{\omega}_t)R]. \tag{4.36}$$

Proof. of Lemma 5: Substitute Eqs(4.32) into Eq.(4.2) and (4.29) proves (4.36) and (4.35).

□

□

Appendix C. Derivation of Proposition 1

Take into account the Age Pension $\mathcal{P}_t$, and assume the investor's preferences are described by the HARA utility function. Then the Bellman Equation of Eq.(4.1) can be written as

$$J^*(W,t) = \max E_t\left[\sum_{s=t}^{T} v^s \frac{(D_s + P_s - \bar{C})^\gamma}{\gamma}\right] = \max\left(v^t \frac{(D_t + \mathcal{P}_t - \bar{C})^\gamma}{\gamma} + E_t\left[J^*(W,t+1)\right]\right), \tag{4.37}$$

According to Eqs (4.5),(4.6) and (4.7), Age Pension payment can be classified into four cases:

1. Zero pension: $\mathcal{P}_t = 0$,
2. Full pension: $\mathcal{P}_t = \mathcal{P}_m$,
3. Income test pension: $\mathcal{P}_t = \mathcal{P}_m - (D_t - E_i(t) - L_i)\varpi_i$,
4. Asset test pension: $\mathcal{P}_t = \mathcal{P}_m - (W_t - L_a)\varpi_a$,

Define:

- k_1: the estimated age that the pensioner begins receiving assets test pension
- k_2: the estimated age that the pensioner begins receiving income test pension
- k_3: the estimated age that the pensioner begins receiving full pension

First assume $x < k_1 < k_2 < k_3 < T$, and work recursively,we find that the optimal decision rules for drawdown D_t^*, is of the form

$$D_t^* = \alpha_t \hat{W}_t - \hat{P}_t. \tag{4.38}$$

At time T, without bequest motive $D_T^* = W_T$, hence $\hat{W}_T = W_T$, $\alpha_T = 1$ and $\hat{P}_T = 0$; and we have

Lemma 6. *For $t = k_3$ to $t = T - 1$,*

$$\hat{P}_t = \mathcal{P}_m - \bar{C} \tag{4.39}$$

$$\hat{W}_t = W_t + \hat{P}_t \frac{R - R^{t-T}}{R - 1} \tag{4.40}$$

$$\alpha_t = \frac{\mu\alpha_{t+1}}{1 + \mu\alpha_{t+1}}. \tag{4.41}$$

The optimal proportion of wealth to be allocated to risky asset at time t after consumption, ω_t^*, is found to be the same as in Eq.(4.28).

Proof. The solution follows the HARA case as given in Appendix B, with $\bar{C}$ in Eq.(4.25) replaced by $\bar{C} - \mathcal{P}_m$. □

Lemma 7. *For $t = k_2$ to $t = k_3 - 1$,*

$$\hat{P}_{k_2}(t) = \frac{\mathcal{P}_m + (E_i(t) + L_i)\varpi_i - \bar{C}}{1 - \varpi_i}, \tag{4.42}$$

$$\hat{W}_t = W_t + \sum_{s=t}^{k_3-1} \hat{P}_{k_2}(s) R^{t-s} + R^{t-k_3} \hat{P}_{k_3} \frac{R - R^{k_3-T}}{R - 1}, \tag{4.43}$$

$$\alpha_{k_3-1} = \frac{\beta_i \mu \alpha_{k_3}}{1 + \beta_i \mu \alpha_{k_3}} \tag{4.44}$$

$$\alpha_t = \frac{\mu\alpha_{t+1}}{1 + \mu\alpha_{t+1}} \quad \textit{for} \quad t = k_2 \textit{ to } k_3 - 2 \tag{4.45}$$

where $\beta_i = (1 - \varpi_i)^{\frac{\gamma}{1-\gamma}}$. And ω_t^* can be found same as in Eq.(4.28).

Proof. From Lemma 6, at time k_3: $J^*(W, k_3) = \frac{v^{k_3}\alpha_{k_3}^{\gamma-1}}{\gamma}\hat{W}_{k_3}^{\gamma}$;

At Time $k_3 - 1$, given $P(t) = \mathcal{P}_m - (D_t - E_i(t) - L_i)\varpi_i$, we need to maximise

$$\begin{aligned} J(W, k_3 - 1) =& v^{k_3-1}\frac{(D_{k3-1} + (\mathcal{P}_m - (D_{k3-1} - E_i(t) - L_i)\varpi_i) - \bar{C})^{\gamma}}{\gamma} + v^{k_3}\alpha_{k_3}^{\gamma-1}\frac{E\left[(\hat{W}_{k_3})^{\gamma}\right]}{\gamma} \\ =& \frac{v^{k_3-1}(1-\varpi_i)^{\gamma}}{\gamma}\left(D_{k_3-1} + \frac{\mathcal{P}_m + (E_i(t) + L_i)\varpi_i - \bar{C}}{1-\varpi_i}\right)^{\gamma} \\ &+ \frac{v^{k_3}\alpha_{k_3}^{\gamma-1}}{\gamma}E\left[\left([W_{k_3-1} - D_{k_3-1}]\tilde{Z}_{k_3-1} + \hat{P}_{k_3}\sum_{s=k_3}^{T}R^{k_3-1-s}\right)^{\gamma}\right]. \end{aligned} \tag{4.46}$$

Define

$$\begin{aligned} \hat{D}_t =& D_t + \hat{P}_t(t) = D_t + \frac{\mathcal{P}_m + (E_i(t) + L_i)\varpi_i - \bar{C}}{1-\varpi_i}, \\ \hat{W}_t =& W_t + \sum_{s=t}^{k_3-1}\hat{P}_t(s)R^{t-s} + R^{t-k_3}(\hat{W}_{k_3} - W_{k_3})\sum_{s=k_3}^{T}R^{t-s}, \\ \hat{\omega}_t =& \omega_t\frac{W_t - D_t}{\hat{W}_t - \hat{D}_t}. \end{aligned} \tag{4.47}$$

Eq.(4.46) can then be written as

$$J(W, k_3 - 1) = v^{k_3-1}(1-\varpi_i)^{\gamma}\frac{\hat{D}_{k_3-1}^{\gamma}}{\gamma} + v^{k_3}\alpha_{k_3}^{\gamma-1}\frac{E\left[(\hat{W}_{k_3-1} - \hat{D}_{k_3-1})^{\gamma}\hat{Z}^{\gamma}\right]}{\gamma}. \tag{4.48}$$

The solutions for $\hat{\omega}_{T-1}^*$ can be found to be the same as given by Eq.(4.13). Differentiating Eq.(4.48) with respect to $\hat{D}_{k_3-1}$ and set to 0 yields:

$$0 = v^{k_3-1}(1-\varpi_i)^{\gamma}\hat{D}_{k_3-1}^{\gamma-1} - v^{k_3}\alpha_{k_3}^{\gamma-1}E[(\hat{Z}^*)^{\gamma}](\hat{W}_{k_3-1} - \hat{D}_{k_3-1})^{\gamma-1},$$

$$\Rightarrow \hat{D}^*_{k_2-1} = \frac{\alpha_{k_2}(vE[(\hat{Z}^*)^\gamma](1-\varpi_i)^{-\gamma})^{\frac{1}{\gamma-1}}}{1+\alpha_{k_3}(vE[(\hat{Z}^*)^\gamma](1-\varpi_i)^{-\gamma})^{\frac{1}{\gamma-1}}}\hat{W}_{k_3-1}$$
$$= \alpha_{k_3-1}\hat{W}_{k_3-1}.$$

We can write $\alpha_{k_3-1} = \frac{\mu\beta_i\alpha_{k_3}}{1+\mu\beta_i\alpha_{k_3}}$, where $\beta_i = (1-\varpi_i)^{\frac{\gamma}{1-\gamma}}$. We can also write

$$\begin{aligned} J^*(W, k_3-1) =& v^{k_3-1}(1-\varpi_i)^\gamma \frac{(\alpha_{k_3-1}\hat{W}_{k_3-1})^\gamma}{\gamma} + v^{k_3}\alpha_{k_3}^{\gamma-1}\frac{((1-\alpha_{k_3-1})\hat{W}_{k_3-1})^\gamma E[(\hat{Z}^*)^\gamma]}{\gamma} \\ =& v^{k_3-1}(1-\varpi_i)^\gamma \frac{\hat{W}^\gamma_{k_3-1}}{\gamma}\left[\alpha^\gamma_{k_3-1} + \frac{vE[(\hat{Z}^*)^\gamma]\alpha_{k_3}^{\gamma-1}}{(1-\varpi_i)^\gamma}(1-\alpha_{k_3-1})^\gamma]\right] \\ =& v^{k_3-1}(1-\varpi_i)^\gamma \frac{\hat{W}^\gamma_{k_3-1}}{\gamma}\alpha^{\gamma-1}_{k_3-1}. \end{aligned}$$

We can now look at the optimal decision at time $k_3 - 2$, at this time we need to maximise

$$\begin{aligned} J(W, k_3-2) =& v^{k_3-2}(1-\varpi_i)^\gamma \frac{\hat{D}^\gamma_{k_3-2}}{\gamma} + E\left[v^{k_3-1}(1-\varpi_i)^\gamma \frac{\hat{W}^\gamma_{k_3-1}}{\gamma}\alpha^{\gamma-1}_{k_3-1}\right] \\ =& v^{k_3-2}(1-\varpi_i)^\gamma \frac{\hat{D}^\gamma_{k_3-2}}{\gamma} + v^{k_3-1}(1-\varpi_i)^\gamma \alpha^{\gamma-1}_{k_3-1} E\left[\hat{Z}^\gamma\right]\frac{(\hat{W}_{k_3-2}-\hat{D}_{k_3-2})^\gamma}{\gamma}. \end{aligned} \tag{4.49}$$

The solutions for $\hat{\omega}^*_{k_3-2}$ are still the same. Differentiating Eq.(4.49) with respect to $\hat{D}_{k_3-2}$ and set to 0 yields

$$0 = v^{k_3-2}(1-\varpi_i)^\gamma \hat{D}^{\gamma-1}_{k_3-2} - v^{k_3-1}(1-\varpi_i)^\gamma \alpha^{\gamma-1}_{k_3-1}E[(\hat{Z}^*)^\gamma](\hat{W}_{k_3-2}-\hat{D}_{k_3-2})^{\gamma-1},$$

$$\Rightarrow \hat{D}^*_{k_3-2} = \frac{\alpha_{k_3-1}(vE[(\hat{Z}^*)^\gamma])^{\frac{1}{\gamma-1}}}{1+\alpha_{k_3-1}(vE[(\hat{Z}^*)^\gamma])^{\frac{1}{\gamma-1}}}\hat{W}_{k_3-2}$$
$$= \alpha_{k_3-2}W_{k_3-2},$$

where

$$\alpha_{k_3-2} = \frac{\mu\alpha_{k_3-1}}{1+\mu\alpha_{k_3-1}} \tag{4.50}$$

It is easy to verify that $J(W,t)$ at each previous $t \geq k_2$ is identical to Eq(4.49), and the optimal decisions are as given in Lemma 7. □

Lemma 8. *For $t = k_1$ to $t = k_2 - 1$,*

$$\begin{aligned}\hat{P}_{k_1} =& (\hat{W}_{k_2} - W_{k_2})\varpi_a \left[R(1-\varpi_a)\right]^{t-k_2} + \varpi_a R(\mathcal{P}_m + L_a\varpi_a - \bar{C})\frac{1-[R(1-\varpi_a)]^{t-k_2+1}}{R(1-\varpi_a)-1} \\ &+ (\mathcal{P}_m + L_a\varpi_a - \bar{C})/(1-\varpi_a), \quad (4.51)\\ \hat{W}_t =& W_t + (\hat{W}_{k_2} - W_{k_2})\left[R(1-\varpi_a)\right]^{t-k_2} + R(\mathcal{P}_m + L_a\varpi_a - \bar{C})\frac{1-[R(1-\varpi_a)]^{t-k_2}}{R(1-\varpi_a)-1}, \quad (4.52)\end{aligned}$$

$$\alpha_t = \hat{\alpha}_t + \varpi_a(1-\hat{\alpha}_t) \tag{4.53}$$

$$\hat{\alpha}_{k_2-1} = \frac{\beta_i^{-1}\mu\alpha_{k_2}}{1+\beta_i^{-1}\mu\alpha_{k_2}} \tag{4.54}$$

$$\hat{\alpha}_t = \frac{\beta_a\mu\hat{\alpha}_t}{1+\beta_a\mu\hat{\alpha}_t} \quad for \quad t = k_1 \text{ to } k_2 - 2, \tag{4.55}$$

where $\beta_a = (1-\varpi_a)^{\frac{\gamma}{\gamma-1}}$. And ω_t^* can be found same as in Eq(4.28).

Proof. From Lemma 7, at time k_2: $J^*(W,k_2) = \frac{v^{k_2}(1-\varpi_i)^\gamma \alpha_{k_2}^{\gamma-1}}{\gamma}\hat{W}_{k_2}^\gamma$;

At Time $k_2 - 1$, given $P(t) = \mathcal{P}_m - (W_t - L_a)\varpi_a$, we need to maximise

$$\begin{aligned}
J(W, k_2 - 1) =& v^{k_2-1}\frac{(D_{k_2-1} + (\mathcal{P}_m - (W_{k_2-1} - L_a)\varpi_a) - \bar{C})^\gamma}{\gamma} + v^{k_2}(1-\varpi_i)^\gamma \alpha_{k_2}^{\gamma-1}\frac{E\left[(\hat{W}_{k_3})^\gamma\right]}{\gamma} \\
=& \frac{v^{k_2-1}}{\gamma}\left(D_{k_2-1} - W_{k_2-1}\varpi_a + \mathcal{P}_m + L_a\varpi_a - \bar{C}\right)^\gamma \\
&+ \frac{v^{k_2}(1-\varpi_i)^\gamma \alpha_{k_2}^{\gamma-1}}{\gamma} E\left[\left([W_{k_2-1} - D_{k_2-1}]\tilde{Z}_{k_2-1} + (\hat{W}_{k_2} - W_{k_2})\right)^\gamma\right] \\
=& \frac{v^{k_2-1}}{\gamma}\left(D_{k_2-1} - W_{k_2-1}\varpi_a + \kappa\right)^\gamma \\
&+ \frac{v^{k_2}(1-\varpi_i)^\gamma \alpha_{k_2}^{\gamma-1}}{\gamma} E\left[\left([W_{k_2-1} - D_{k_2-1}]\tilde{Z}_{k_2-1} + \zeta_{k_2-1}\right)^\gamma\right], \quad (4.56)
\end{aligned}$$

where $\kappa = \mathcal{P}_m + L_a\varpi_a - \bar{C}$ and $\zeta_t = \hat{W}_{t+1} - W_{t+1}$, note $\hat{W}_{k_2} - W_{k_2}$ can be found from Eq.(4.47). Define:

$$\begin{aligned}
\hat{D}_t =& D_t + \frac{\kappa + \zeta_t\varpi_a/R}{1-\varpi_a}, \\
\hat{W}_t =& W_t + \frac{\kappa + \zeta_t/R}{1-\varpi_a}, \\
\hat{\omega}_t =& \omega_t\frac{W_t - D_t}{\hat{W}_t - \hat{D}_t}. \quad (4.57)
\end{aligned}$$

Eq.(4.56) can then be written as

$$J(W, k_2 - 1) = v^{k_2-1}\frac{(\hat{D}_{k_2-1} - \hat{W}_t\varpi_a)^\gamma}{\gamma} + v^{k_2}(1-\varpi_i)^\gamma\alpha_{k_2}^{\gamma-1}\frac{E[\hat{Z}^\gamma](\hat{W}_{k_2-1} - \hat{D}_{k_2-1})^\gamma}{\gamma}. \quad (4.58)$$

The solutions for $\hat{\omega}^*_{T-1}$ can be found to be the same as given by Eq.(4.13). Differentiating Eq.(4.58) with respect to $\hat{D}_{k_2-1}$ and set to 0 yields

$$0 = v^{k_2-1}(\hat{D}_{k_2-1} - \hat{W}_{k_2-1}\varpi_a)^{\gamma-1} - v^{k_2}(1-\varpi_i)^\gamma\alpha_{k_2}^{\gamma-1}E[(\hat{Z}^*)^\gamma](\hat{W}_{k_2-1} - \hat{D}_{k_2-1})^{\gamma-1},$$

$$\Rightarrow \hat{D}^*_{k_2-1} = \frac{\alpha_{k_2}\mu\beta_i^{-1} + \varpi_a}{1 + \alpha_{k_2}\mu\beta_i^{-1}}\hat{W}_{k_2-1}$$
$$= \alpha_{k_2-1}\hat{W}_{k_2-1}.$$

We can write $\alpha_{k_2-1} = \frac{\mu\beta_i^{-1}\alpha_{k_2}+\varpi_a}{1+\mu\beta_i^{-1}\alpha_{k_2}}$, and:

$$\begin{aligned} J^*(W, k_2-1) =& v^{k_2-1}\frac{(\alpha_{k_2-1}\hat{W}_{k_2-1} - \hat{W}_{k_2-1}\varpi_a)^\gamma}{\gamma} + v^{k_2}(1-\varpi_i)^\gamma\alpha_{k_2}^{\gamma-1}E[(\hat{Z}^*)^\gamma]\frac{((1-\alpha_{k_2-1})\hat{W}_{k_2-1})^\gamma}{\gamma} \\ =& v^{k_2-1}\frac{\hat{W}^\gamma_{k_2-1}}{\gamma}\left[(\alpha_{k_2-1} - \varpi_a)^\gamma + (\mu\alpha_{k_2}\beta_i^{-1})^{\gamma-1}(1-\alpha_{k_2-1})^\gamma]\right] \\ =& v^{k_2-1}\frac{\hat{W}^\gamma_{k_2-1}}{\gamma}(1-\varpi_a)^\gamma\hat{\alpha}_{k_2-1}^{\gamma-1}. \end{aligned}$$

where $\hat{\alpha}_{k_2-1} = \frac{\mu\beta_i^{-1}\alpha_{k_2}}{1+\mu\beta_i^{-1}\alpha_{k_2}}$.

We can now look at the optimal decision at time $k_2 - 2$, at this time we need to maximise

$$J(W, k_2-2) = v^{k_2-2}\frac{(\hat{D}_{k_2-2} - \hat{W}_{k_2-2}\varpi_a)^\gamma}{\gamma} + v^{k_2-1}(1-\varpi_a)^\gamma\hat{\alpha}_{k_2-1}^{\gamma-1}E[\hat{Z}^\gamma]\frac{(\hat{W}_{k_2-2} - \hat{D}_{k_2-2})^\gamma}{\gamma} \tag{4.59}$$

The solution for $\hat{\omega}^*_{k_2-2}$ is still the same. Differentiating Eq.(4.59) with respect to $\hat{D}_{k_2-2}$ and set to 0 yields:

$$0 = v^{k_2-2}(\hat{D}_{k_2-2} - \hat{W}_{k_2-2}\varpi_a)^{\gamma-1} - v^{k_2-1}(1-\varpi_a)^\gamma\hat{\alpha}_{k_2-1}^{\gamma-1}E[(\hat{Z}^*)^\gamma](\hat{W}_{k_2-2} - \hat{D}_{k_2-2})^{\gamma-1},$$

$$
\begin{aligned}
\Rightarrow \; \hat{D}^*_{k_2-2} =& \frac{\hat{\alpha}_{k_2-1}\mu\beta_a + \varpi_a}{1+\hat{\alpha}_{k_2-1}\mu\beta_a}\hat{W}_{k_2-2} \\
=& \alpha_{k_2-2}W_{k_2-1},
\end{aligned}
$$

where $\beta_a = (1-\varpi_a)^{\frac{\gamma}{\gamma-1}}$. We can write

$$
\begin{aligned}
\alpha_{k_2-2} =& \hat{\alpha}_{k_2-2} + \varpi_a(1-\hat{\alpha}_{k_2-2}) \\
\hat{\alpha}_{k_2-2} =& \frac{\mu\beta_a\hat{\alpha}_{k_2-1}}{1+\mu\beta_a\hat{\alpha}_{k_2-1}}.
\end{aligned} \tag{4.60}
$$

It is easy to verify that $J(W,t)$ at each previous $t \geq k_1$ is identical to Eq.(4.59), and α_t follows the general form given in Eq.(4.53).

Also note that $\zeta_t = \frac{\kappa+\zeta_{t+1}}{1-\varpi_a}$, hence $\hat{D}_t$ and $\hat{W}_t$ can be shown to follow the general form given in Eqs (4.51) and (4.52). □

Lemma 9. *From $t = x$ to $t = k_1 - 1$,*

$$\hat{P}_t = -\bar{C} \tag{4.61}$$

$$\hat{W}_t = W_t - \bar{C}\frac{R - R^{t-k_1+1}}{R-1} + R^{t-k_1}(\hat{W}_{k_1} - W_{k_1}) \tag{4.62}$$

$$\alpha_{k_1-1} = \frac{\beta_a\mu\hat{\alpha}_{k_1}}{1+\beta_a\mu\hat{\alpha}_{k_1}} \tag{4.63}$$

$$\alpha_t = \frac{\mu\alpha_{t+1}}{1+\mu\alpha_{t+1}} \quad \textit{for} \quad t = x \textit{ to } k_1 - 2. \tag{4.64}$$

And ω^*_t can be found to be the same as in Eq.(4.28).

Proof. At time $t = k_1 - 1$, assume Eqs (4.61) and (4.62), we have

$$J(W, k_1 - 1) = v^{k_1-1}\frac{\hat{D}^{\gamma}_{k_1-1}}{\gamma} + v^{k_1}(1-\varpi_a)^{\gamma}\hat{\alpha}^{\gamma-1}_{k_1}\frac{E\left[(\hat{W}_{k_1-1} - \hat{D}_{k_1-1})^{\gamma}\hat{Z}^{\gamma}\right]}{\gamma}. \quad (4.65)$$

Differentiate Eq.(4.65) and solve gives Eq.(4.63). For time $t = x$ to $t = k_1 - 2$, the solution simply follows the HARA case as given in Appendix B.

□

Finally it can be shown that: $\hat{W}_t$ and φ_t in Lemma 9 can be written as in Lemma 8 when $k_1 = t$, can be written as in Lemma 7 when $k_1 = k_2 = t,$, and can be written as in Lemma 6 when $k_1 = k_2 = k_3 = t$.

Therefore, as long as the condition $k_1 \leq k_2 \leq k_3$ holds, the solution for the problem can be summarised as in Proposition 1.

Appendix D. Derivation of Proposition 2, 3 and 4

First assume the value of family home H is given, and $W_x = \mathcal{W}_x - H$, maximising Eq.(4.1) given Eq.(4.9) is the same as

$$\max E\left[\sum_{t=x}^{T} v^t \left({}_tP_{\bar{x}}\frac{(C_t-\bar{C})^\gamma}{\gamma}\right) + v^T\theta^{1-\gamma}\frac{(\theta a + B_T)^\gamma}{\gamma}\right], \tag{4.66}$$

Compared to Eq.(4.8), Eq.(4.66) contain two additional terms. Firstly, ${}_tP_{\bar{x}}$ can be combined with v^t as a time varying interest rate, and the solution easily follows. [24]

The solution to the problem in the presence of the bequest utility function $v^T\theta^{1-\gamma}\frac{(\theta a+B_T)^\gamma}{\gamma}$ follows Chapter 2, and the result can be written as in Proposition 2.

The expected value of Eq.(4.66) at time t under the optimal decisions, can be written as

$$J(W_t,t) = v^t{}_tP_{\bar{x}}(1-\varpi_t)^\gamma\alpha_t^{\gamma-1}\frac{\hat{W}_t^\gamma}{\gamma} \tag{4.67}$$

$$\varpi_t = \begin{cases} \varpi_a & \text{for } \; k_1 \le t < k_2; \\ \varpi_i & \text{for } \; k_2 \le t < k_3; \\ 0 & \text{for all other } \; t; \end{cases}$$

[24] We can rewrite the v^T in bequest function as $v^T{}_TP_{\bar{x}}\frac{1}{{}_TP_{\bar{x}}}$, and $\frac{1}{{}_TP_{\bar{x}}}$ can be treated as a constant. Note that ${}_TP_{\bar{x}}$ is not zero, it is the probability of surviving to the last age T, not the probability of surviving beyond that.

Now consider the pensioner's decision at retirement to allocate H dollars into the family home, given Eq.(4.67) and Eq.(4.3), Eq.(4.1) can be written as:

$$J(\mathcal{W}_x, x) = v^x {}_xP_{\bar{x}}(1-\varpi_x)^\gamma \alpha_x^{\gamma-1} \frac{h(\mathcal{W}_x - H)^\gamma}{\gamma} + \sum_{t=x}^{T} v^t \frac{(\psi H)^\gamma}{\gamma}, \tag{4.68}$$

where $h(W) = \hat{W}$. Differentiating Eq(4.68) we have:

$$v^x {}_xP_{\bar{x}}(1-\varpi_x)^\gamma \alpha_x^{\gamma-1} h(\mathcal{W}_x - H)^{\gamma-1}(-\xi' - 1) + \sum_{t=x}^{T} v^t \psi^\gamma H^{\gamma-1} = 0, \tag{4.69}$$

$$\xi' = R^{x-\hat{k}_1} \left[R(1-\varpi_a)\right]^{\hat{k}_1-\hat{k}_2} \frac{\varpi_i}{e_x(1-\varpi_i)} (1+I)^{x-\hat{k}_2} \frac{R(1+I) - (R(1+I))^{\hat{k}_2-\hat{k}_3+1}}{R(1+I)-1}.$$

By solving Eq.(4.69) we can find the optimal amount of H for single pensioners as in Proposition 3.

Lemma 10. *Given the context of this paper, Eq.(4.68) is a concave function of H and we have a unique solution H^* that maximise equation Eq.(4.68).*

Proof. The second derivative of Eq.(4.68) can be found by differentiating Eq.(4.69), which gives

$$(\gamma-1) v^x {}_xP_{\bar{x}}(1-\varpi_x)^\gamma \alpha_x^{\gamma-1} h(\mathcal{W}_x - H)^{\gamma-2}(-\xi' - 1)^2 + (\gamma-1)\sum_{t=x}^{T} v^t \psi^\gamma H^{\gamma-2}, \tag{4.70}$$

Under the assumptions of this paper Eq.(4.70) is negative for all reasonable value of H, because:

1. The retiree is assumed to be risk-averse hence $\gamma < 1$.

2. The parameters v, ${}_xP_{\bar{x}}$, ϖ_x, α_x and ψ are all positive.

3. The value of $h(\mathcal{W}_x - H)$ is greater than zero for all reasonable value of H, because in the case that the value of H passes the threshold such that $h(\mathcal{W}_x - H)$ become negative, the retiree will not meet her required future consumption floors, and the value of utility function Eq.(4.70) would be negative infinity.

Eq.(4.68) is therefore a concave function of H for all H up to a certain value H_l, and it takes value of $-\infty$ for all $H \geq H_l$. We can therefore find a unique solution H^* that maximises the equation Eq.(4.68). □

Lemma 10 is also important to derive Proposition 4. Under the assumption that the value of owner-occupied housing above a certain threshold L_h is subject to the Age Pension assets test, we have the original assets test threshold L_a replaced by $L_a - \max(H - L_h, 0)$. In this case Eq(4.68) can be seen to be made up of two continuous functions at the range of $H \leq L_h$ (in which case we have $L_a - 0$) and $H > L_h$ (in which case we have $L_a - (H - L_h)$), and both functions are concave following the same proof to Lemma 10.

We can also see that the slope of the utility function at the range of $H > L_h$ must be smaller than the slope of the utility function at the range of $H \leq L_h$, because of the additional $-H$ term in the function. Therefore Eq.(4.68) can only take three possible shapes as illustrated in Figures 4.23; in all cases we can see that Eq.(4.68) is still a concave function and there exists a unique H^* that maximise it as can be found in Proposition 4.

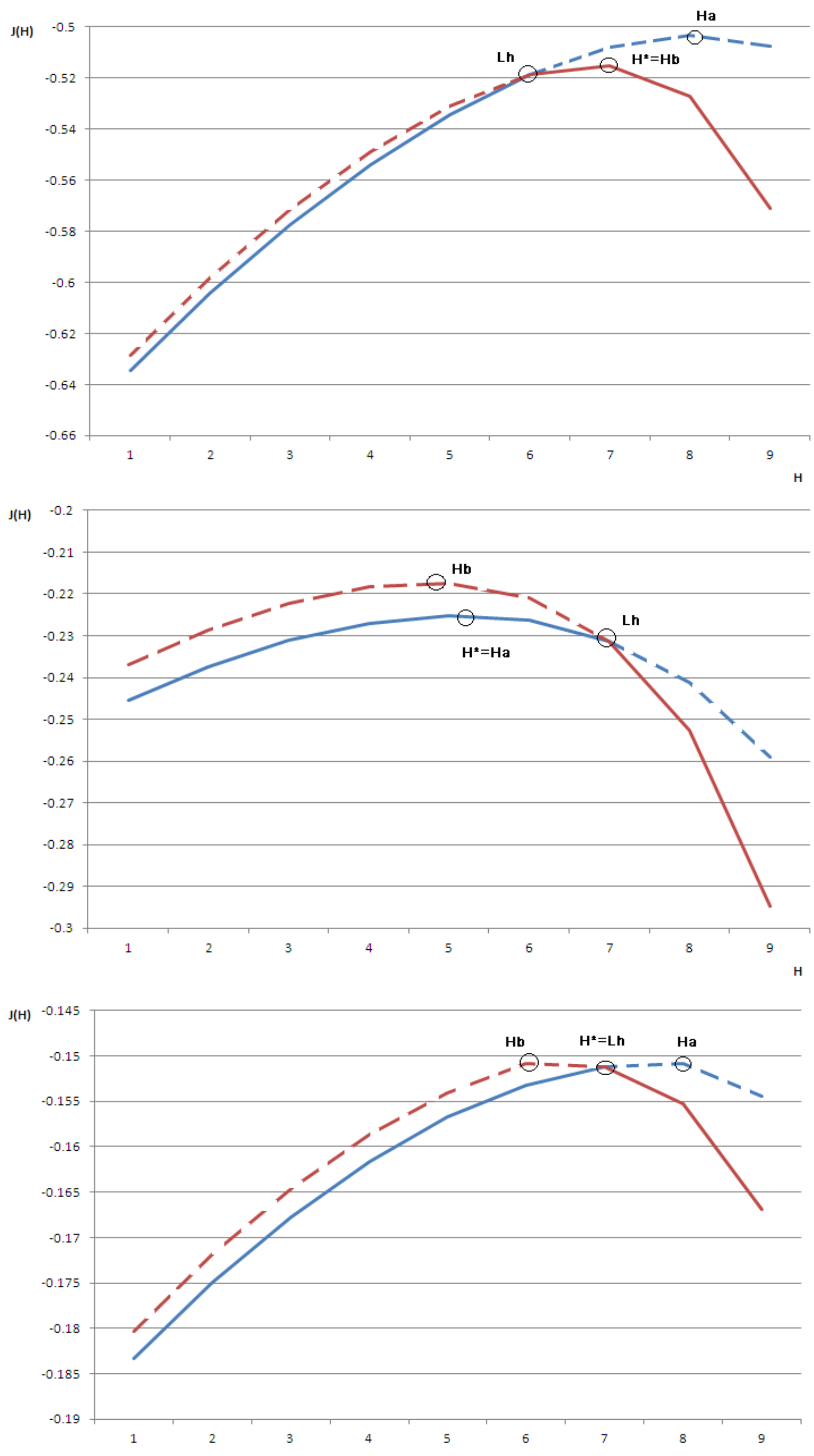

Figure 4.23: Three possible shapes of Eq.(4.68), represented by the solid lines.

One challenge of the model is that we need to ensure $B(T) > 0$, or else a retiree with wealth below a certain threshold may have negative wealth at time T. Chapter 2 dealt with this problem by replicating a put option, the value of which at time T would be $-B(T)$ if $B(T) < 0$ and zero otherwise. The solution however, would be much more complicated with the Age Pension taken into account and requires a numerical method to be implemented in practice. In Chapter 3 of this thesis, the decision is approximated as follows: I calculate the value of consumption assuming $B(T) = 0$ and if it is smaller than our original case, it indicates that in the original case the retiree's saving is negative, we then use the results assuming $B(T) = 0$. This approximation gives very similar results as the method in Chapter 2 in most situations, except a retiree with a moderate level of wealth may allocate more wealth into risky assets than ideal.[25]

[25]Refer to Figure 2.2 in Chapter 2 of this thesis.

Appendix E. Numerical dynamic programming method

We can observe that the investor's state is dependent on two state variables: age and wealth. Noting that at any age, past decisions made are assumed to affect only the initial wealth at that age. The problem can be solved using the grid search method with the following settings:

Setting A: The investor's wealth is discretised into one of N groups denoted by $W(n)$ for $n = 1$ to N. In this paper I set $N = 151$, $W(n) = 2000 * (n-1)^{1.32}$ so the first wealth group denotes wealth less than \$2,000, increasing exponentially to the last wealth group which denotes wealth greater than \$1,500,000. Note this setting poses the constraint that wealth cannot be negative.

Setting B: The distribution of real return of the risky asset $(\tilde{z}|M_t)$ is assumed to be an L point discrete distribution, which can take L total possible values with corresponding probability $P(\tilde{z} = z_l|M_t)$ for $l = 1$ to L.

Setting C: Every period the investor has 500 drawdown choices, denoted by $D(c)$ and 20 asset allocation choices, denoted by $v(d)$. In this paper I set $D(c) = 24000 * (c/100)^{1.32}$ where $c = 0$ to 500, and $v(d) = d/20$ where $d = 0$ to 20, so she can choose to withdraw \$0 to \$200,000 (exponentially increasing) and allocate 0 percent to 100 percent (with 5 percent increment) of remaining wealth into risky assets. Note this setting poses constraints that consumption cannot be greater than wealth and the investor cannot borrow to invest in risky assets.

Setting D: The possible value invested in the family home is discretised to 40 groups from \$0 to \$2,000,000, with interval of \$50,000.

Chapter 5

Superannuation policies and economic responses: How much Age Pension?

Abstract

This paper presents long-term projections of the cost of public pensions in Australia, taking into account retiree's economic responses. I assume retirees make financial decisions to maximise their lifetime utilities, and their consumption and asset allocation react to policy changes. I find that the future cost of the Age Pension is likely to be higher than estimated by Australian Treasury in 2010's Intergenerational Report. As future cohorts retire with more savings, they can allocate more money into owner-occupied properties while preparing for retirement and draw down their savings faster, to optimise their Age Pension entitlements. This paper also examines how projected future costs are affected by various policy changes, including the legislated increase of superannuation guarantee from 9 percent to 12 percent, the possible changes of including the value of the family home in the assets test, and indexing Age Pension payments to price inflation instead of wage inflation.

JEL classification: G11, H31, H55, H68. *Keywords*: Age Pension, national budget, household behavior, financial planning.

5.1 Introduction

Taking retiree's economic responses into account, this paper projects the annual total Age Pension payments to Australian retirees from 2011-12 to 2035-36 financial year. I estimate the cost of Age Pension payments to be $96.4 billion (in 2011 dollars) in the 2035-36 financial year. This is about 13 percent higher than estimates according to Australian Treasury's 2010 Intergenerational Report (IGR 2010), under similar assumptions. Fig 5.1 compares the result of this paper to Rothman (2012). Both estimate that the proportion of part pensioners will increase in the future with a fall in those entitled to the full pension. However I estimate the process will be much slower.

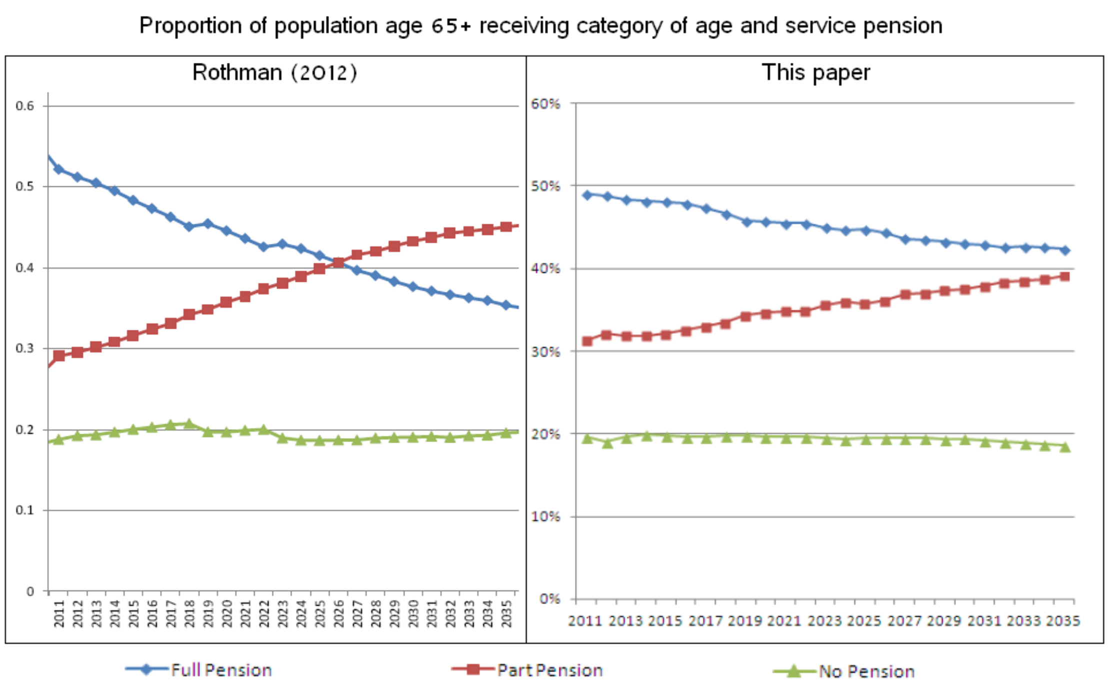

Figure 5.1: Comparison of the projection result to the result in Rothman (2012).

This paper contributes to the area of long-term policy projections while taking into account retiree's economic responses, an area where there are few previous contributions in Australia.

Public policies concerning superannuation and Age Pension in Australia are subject to constant changes for fiscal reasons. The RIMGROUP model has been used extensively by the Australian Treasury to model important public policy changes. According to Rothman (2012), the RIMGROUP model does not take into account optimal financial behaviours of the underlying population. Instead peoples' financial decisions such as dissipation rates are set in line with assumptions according to income decile and the level of wealth. This is similar to the approach adopted in Treasury's 2012 tax expenditure statement, in which the tax expenditure is measured assuming taxpayers' behaviours are unchanged.[1]

Oliver and Dixon (2010) developed a behavioural model add-on for the Australia Treasury's RIMHYPO model, which takes into account utility maximising behaviours of the population, however it is not publicly available and has not been applied to the area of long-term policy projections to my knowledge. Kudrna and Woodland (2008) developed a general equilibrium model with overlapping generations, which illustrates the macroeconomic effect of various Age Pension policy changes. However as a macroeconomic model, it is not designed as the RIMGROUP model to capture the heterogeneity of the population to produce more accurate projections.

[1]See http://www.treasury.gov.au/PublicationsAndMedia/Publications/2013/TES-2012, The Tax Expenditure Framework, Section 2.3.

The Age Pension is a means-tested income support paid to Australian retirees.[2] This benefit is publicly funded from government's general revenue and is now the largest single item of expenditure in the Federal Budget each year. According to Centrelink statistics, in 2011 around 80 percent of Australian population aged above 65 received a full or part pension, with around 1,479,300 people aged 65 or above, received full rate of age/service pension during 2011-12 financial year while 945,100 people received part pension. Total age/service pension payments are estimated at $34.8 billion in the 2011-12 financial year.[3]

In IGR (2010), Australian Treasury projected the Age Pension payments to rise to about 3.9 percent of Australian GDP in the 2049-50 financial year (according to this, the estimated Age Pension payments in the 2035-36 financial year are about $85.4 billion in 2011 dollars).[4] According to Rothman (2012), "*This rise is less than would result from purely demographic changes, mainly because of the increasing wealth and income of successive cohorts of retirees as Australia's superannuation arrangements*

[2]The Age Pension payment is subject to two means tests: an assets test and an income test. The actual pension payment is the lesser of the assets test pension and income test pension. As an example of the means tests, in 2011 a single homeowner with assessable income less than $150 per fortnight, and assets outside the family home that are less than $186,750, is entitled to receive the full pension of $750 per fortnight (including pension supplement). Every dollar of income above the income test threshold reduces the pension entitled by 50 cents per fortnight, and every $1000 of assets above the assets test threshold reduces the amount of pension by $1.50 per fortnight. The value of the family home is exempted from the assessable assets for the assets test, however Non-homeowners are subject to a higher assets test threshold of $321,750.

[3]2012-13 Commonwealth Budget, Paper No.1, Statement 6.

[4]This is estimated as follows: Firstly Age Pension payments in 2011 are about 2.7 percent of GDP, assuming it increases linearly to 3.9 percent in 2049-50, in 2035-2036 it would be about 3.44 percent. Secondly, GDP as at June 2011 is $1,308 billion, according to intergenerational report real GDP will increase at a rate 2.7 percent p.a, hence in 2035-2036, it is estimated to be $2,480 billion.

mature ... ".[5]

This paper shows that Treasury may have underestimated the future Age Pension payments. While future cohorts of retirees will have more superannuation savings, it is rational for them to allocate more wealth into their family home while preparing for retirement and draw down their savings faster, thereby enabling them to optimise their retirement needs and pension entitlements.

Currently 75 percent of Australian retiree are home-owners. The value of owner-occupied housing accounts for about 80 percent of the total wealth of retired home-owners. It is also especially important for any Age Pension related modelling, because the value of owner-occupied property is treated leniently by the current means testing rules.[6] Evidences show that due to Age Pension means testing, Australian retirees are likely to be overinvested in housing.[7] This paper takes retiree's economic responses into account by assuming people make financial decisions to maximise

[5]In Australia, a policy that made it compulsory for workers to contribute an amount equal to 9 percent of their salary towards their superannuation was introduced in 1992. The SG contribution rate started at 3 percent, has gradually increased nine percent since 1st July 2002, and is legislated to increase to 12 percent in 2019. The rationale behind Australian treasury's projection is easy to understand. On the one hand, future retirees would have higher savings than current retirees, as the superannuation guarantee system matures; on the other hand, Australia's Age Pension payments are means-tested, and someone with higher wealth would receive lower pension payments. Hence the estimated future cost of the Age Pension would be less if the population is wealthier.

[6]Currently homeowners and non-homeowners are treated differently by the assets test, however someone owning a $2,000,000 house is treated the same as someone owning a $200,000 house.

[7]See Cho and Sane (2013), ABS data indicate that in Australia about half of the elderly claim to have spare capacity in their homes. Bradbury (2008) shows that home-ownership in Australia post-retirement is greater than in most other countries.

their lifetime utilities, following the utility model introduced in Chapter 3. I consider jointly retirees' preferences of luxury goods (in the form of bequest), housing, 'ultra-necessities' (in the form of a 'subsistence' rate of consumption in retirement) and the Age Pension. Ding (2013a) finds this utility model reasonably describes the preferences of current Australian retirees.

This paper also look at how projected future Age Pension costs are affected by various policy changes, including the legislated increase of superannuation guarantee from 9 percent to 12 percent, the possible changes of including the value of the family home in the assets test, and indexing Age Pension payments to price inflation instead of wage inflation. The results indicate that policy changes that distort behaviour would not have the best effect of reducing Age Pension payments because people can adjust their financial decisions to optimise their outcome, although these would have other effects such as reducing the value of wealth invested in family homes, which may improve economic efficiency. Actions such as reducing the indexation rate of Age Pension payments, would have a more direct effect on reducing the pension payments because its effect is less affected by change of behaviours. However it also results in greater welfare losses.

5.2 Methodology and Assumptions

Many modelling stages are required for the final results. These are portrayed in Figure 5.2.

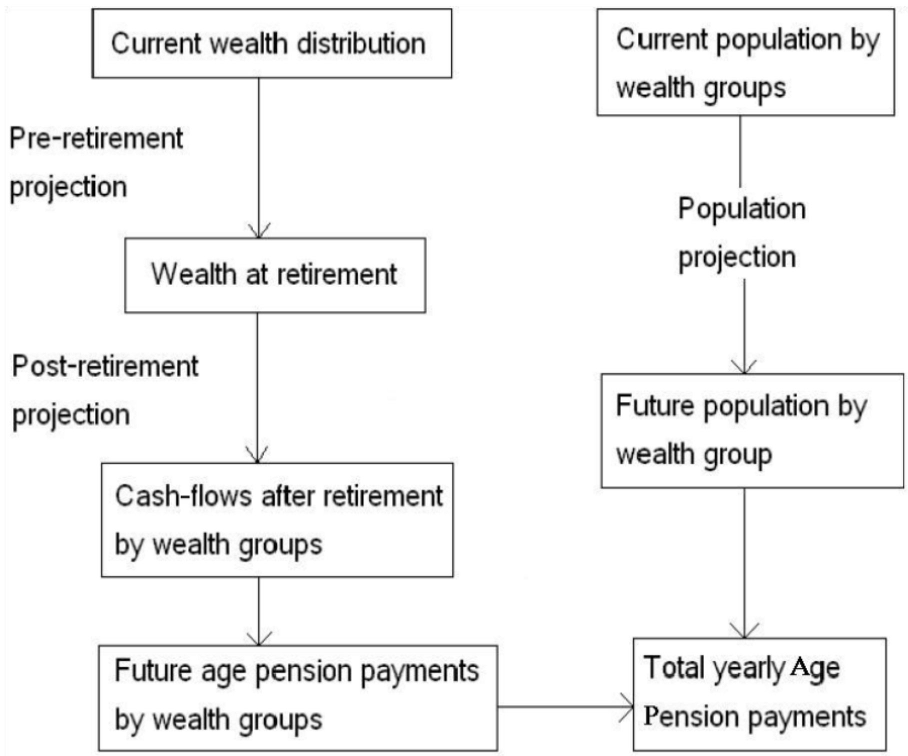

Figure 5.2: Modelling stages to project future Age Pension costs.

The methodologies for each stage of modelling are detailed in the following sections[8].

[8]This research is sponsored by RiceWarner Actuaries, some data and models referenced in this paper are results of RiceWarner research not publicly available.

5.2.1 Current wealth distribution

With references to data from RiceWarner (2011), RiceWarner (2012a), ABS (2011a), ABS (2011b) and APRA (2011). I estimate the total net wealth of Australian population at 30 June 2011 to be $5,970 billion,[9], in which $1,548 billion are superannuation assets,[10] $3,002 billion are owner-occupied properties,[11] and the value of other investments (net of liabilities) amounts to $1,420.[12]

Population statistics as at 30 June 2011 are published in ABS (2011c), which can be summarized as in Table 5.1[13].

'000	35-44	45-54	55-64	65-74	75-84	85-94	95+
Male	1,583	1,516	1,286	826	454	138	8
Female	1,602	1,549	1,311	857	551	245	24

Table 5.1: Number of people by age (,000), as at 30 June 2011.

I then estimate the average value of each asset type per person, breakdown by

[9]This number is very close to the value published in ABS (2011b), which is $6,000 billion.

[10]RiceWarner (2011) estimated a total size of superannuation market of $1,548 billion, including $210 billion unfunded public liabilities.

[11]Estimated with $693 billion total loans to households for owner-occupied properties published in APRA (2011) and the 19 percent average household borrowing ratio calculated from ABS (2011a).

[12]RiceWarner (2012a) estimated $1,943 billion of personal investments, adjustment is then made for financial liabilities other than investment property loans, which is estimated from ABS (2011a), ABS (2011b) and APRA (2011).

[13]This paper only looks at households currently aged 35 or older. This group holds 92.5 percent of Australian's net wealth. Younger households are not expected to reach Age Pension age by year 2035.

age bands and wealth groups, according to RiceWarner (2011) and ABS (2011a).[14] Population in every age band is equally divided into 15 wealth groups (W1 to W15) according to their net wealth, so that each wealth group contains equal proportion of people of that cohort.

The breakdown of average superannuation assets is illustrated as in Table 5.10 in Appendix A, Average value of residential properties and other investments follow similar breakdowns, and are illustrated as in Table 5.11 and Table 5.12.

5.2.2 Pre-retirement Projection

Assuming Males retire at age 65 and Females retire at age 62,[15] I project individual wealth in superannuation, residential property and other investments separately to retirement.

The value of residential property is assumed to increase in line with inflation, while the projection of superannuation savings follows the method used in RiceWarner (2012b), and the projection of other investments follows the method used in Rice-Warner (2012a), Appendix B provide a summary of methodology and assumptions

[14] ABS (2011a) reports personal assets like shares and cash alone with household assets such as the family home. When estimating personal assets distributions in the case of couple households, household assets are equally divided between the couples.

[15] According to ABS (2011d), the average age of people who retired between July 2010 to June 2011 is 62.5 for men and 60.3 for women, while the average age of people who intend to retire are 63.5 for men and 62 for women. The retirement age of 65 and 62 is chosen for the convenience of modelling the couples. I assume couples retire at the same time, with an average age difference of 3 years between them.

adopted in these papers. Total savings are then calculated as the sum of these three components, and interpolated between age bands to individual ages.

The estimates under the two SG scenarios are given in Table 5.13 and Table 5.14 in Appendix A. As expected, increasing SG to 12 percent has a more significant impact on the retirement saving of younger individuals. People who are currently retired are not affected.

I assume that couples retire at the same time, and that their household wealth is estimated as follows: for example, a couple household in wealth group W7 with female aged currently 45, the estimated household wealth at retirement equals the sum of estimated wealth of female aged 45, and male aged 48, both in wealth group W7.[16]

5.2.3 Population Projection

RiceWarner (2011) projected the Australian population up to 30 June 2035. The results are illustrated in Table 5.15 in Appendix A[17].

Research in Rice and Higgins (2012) shows the current proportion of retired in the population aged over 60, broken down by age and gender, these are shown in Table 5.16 in Appendix A.

[16] It is most suitable to assume couple households consist of individuals from the same wealth group, considering that the current personal wealth of the couple wes estimated by dividing household wealth equally between the couple.

[17] This projection gives the estimated number of people of a certain age in a certain year. Note that this table is by actual age, not by current age as in the pre-retirement projection.

From ABS (2011a) and Centrelink data,[18] I estimated the current proportion of retirees who are couples, which are given in Table 5.17 in Appendix A.

Individuals were distributed evenly into the 15 wealth bands. However the distribution is not the same when we look at couple households and single households separately, as people in low wealth bands are more likely to be single than couples. The distribution also changes with age for single households, because when a partner passes away in a couple household, the surviving partner becomes single but with high wealth and should be placed in a higher wealth band. Therefore, I assume the distribution of population by wealth band is age independent for couple households and age dependent for single households. From ABS (2011a), the distributions are estimated as in Table 5.18 in Appendix A.

Assume that the distributions of single/couple, working/retired, and distribution by wealth bands remain constant in the future.[19] Then we can distribute the projected future retired population into 3-D matrixes illustrated as in Figure 5.3:

Four matrixes are constructed for Single Male, Single Female, Coupled Male and Coupled Female.

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[18] Centrelink administrative data on Age Pension recipients June 2011. Data are not published but can be requested from Centrelink.

[19] This may not be the most realistic assumption, especially for the proportion of working/retired. As mortality improves in the future, we would expect people to work till later ages. This is accordingly subject to adjustment in future research.

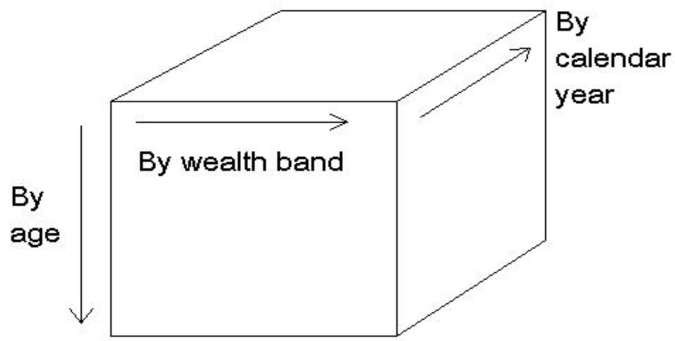

Figure 5.3: Distribution of projected future retired population.

5.2.4 Post-retirement Projection

In recent years, substantial research and attention has focused on peoples' behaviour in the post-retirement phase. Some of the major findings in the previous literature are as follows:

- Research in Australia and overseas shows that expenditure in the post-retirement phase generally decreases with age (see for example: Bernicke 2005 and Higgins and Roberts 2011), which can be due to declining health as the retiree ages (Yogo 2011).

- ABS (2011a) shows that there is not much difference in the expenditure of different ages groups, for households in the low to middle wealth bands. This is consistent with the consumption floor argument and Hyperbolic Absolute Risk Aversion (HARA) utility function as presented in Merton (1971), Bateman et al (2007) and others.

- For many Australian retirees, the family home and contents are their only capital assets (Olsberg and Winter 2005). Cho and Sane (2013) show that it is optimal for households to over-invest in housing when the value of the family home is treated concessionally by the assets test. It is hence important to jointly consider the family home and Age Pension means testing in Australia.

- Many Australian and international studies find that positive or zero saving during retirement is common (see Hulley et al. 2012 and Feinstein and Ho 2000), consistent with a strong bequest motive. ABS (2012a) shows that for households in the middle wealth bands, a majority of their wealth is in the family home, however the proportion of wealth in the family home then decreases as the households become wealthier. This suggests that saving in liquid wealth only happens when household wealth is above a certain threshold, hence can be considered as a luxury good. This is consistent with the assumption of luxury bequests, as in Ding et al. (2012) and Lockwood (2012), among others.

Taking these findings into account, Chapter 3 introduced a utility model that jointly considers retirees' preferences of luxury bequests, housing, subsistence consumption requirement, and the Age Pension. Following this model, this paper assumes retirees derive utility from consumption of non-housing goods and the flow of services from housing stock, as well as from bequests. Households retire with H dollars of net wealth allocated in their family home, and choose the optimal consumption $C(t)$ and investment in risky assets $\omega(t)$, every year until age 100. [20] Their objective is to choose the value of H and the series $C(t)$ and $\omega(t)$ to maximise expected utility

$$\max E\left[\sum_{t=x}^{T} v^{t-x}\left(F_t\frac{(C_t-\bar{C})^{\gamma}}{\gamma}+\frac{(\psi H)^{\gamma}}{\gamma}\right)+v^{T}\theta^{1-\gamma}\frac{(\theta a+B_T)^{\gamma}}{\gamma}\right], \tag{5.1}$$

[20]Age 100 for both partners if it is a couple household.

subject to the budget constraints,

$$W_{t+1} = [W_t + \mathcal{P}_t - C_t][\omega_t \tilde{z} + (1 - \omega_t)R], \tag{5.2}$$

$$W_x = \mathcal{W}_x - H, \tag{5.3}$$

$$0 \leq H_x \leq \max(\mathcal{W}_x - \bar{W}, 0), \tag{5.4}$$

$$B_T \geq 0. \tag{5.5}$$

The notation is:

- E, expectations operator
- x, current age of the retiree
- T, age of the retiree at the end of the planning period, this paper assumes $T = 100$
- v, utility parameter denoting the retiree's time preference
- $C_t = D_t + \mathcal{P}_t$, Consumption at age t, consist of drawdown from wealth D_t and Age Pension entitlement $\mathcal{P}_t$
- $\mathcal{P}_t$, Age Pension entitlement at age t, which is a function of drawdown D_t and wealth W_t[21]
- $\bar{C}$, nonnegative utility parameter with the interpretation of 'subsistence' or 'protected' or 'habitual' consumption[22]
- γ, utility parameter denoting the degree of risk aversion

[21] For detail see Chapter 3

[22] This can be considered a ultra-necessity in the sense that its elasticity of demand with respect to wealth is zero. For details see Bateman *et al.* (2007).

- F_t, parameter denoting the state of health, which is used to model the effect of reducing utility gain from consumption as the retiree ages due to declining health. This parameter is approximated with $F_{\bar{x}} = {}_tP_{\bar{x}}$, probability of someone currently age $\bar{x} = 55$ survives to age t

- H, value of the family home at retirement. I assume retirees can optimise the value of their family prior or at retirement.[23] However, they do not have the option to vary the wealth in their family home after retirement. The value of the property is assumed to increase in line with inflation

- ψ, utility parameter denoting the retiree's preference for Housing, which can be considered as the value of services as proportion to housing stock

- $B_T = W_T + \mathcal{P}_T - C_T$, liquid wealth at the end of period T

- θ, utility parameter denoting the retiree's preference between consumptions and bequest[24]

- a, utility parameter denoting the extent to which non-housing bequest is a luxury good[25]

- W_t, liquid wealth (wealth net of the value of the family home) at age t

[23] At retirement they can use superannuation savings to extend their home, pay off the existing mortgage or buy a new house. They can also plan ahead and buy a more expensive house prior to retirement than their actual needs. According to Cho and Sane (2013), ABS data indicates that in Australia about half of the elderly claim to have spare capacity in their homes.

[24] $\theta = \phi/(1-\phi)$ can also be seen as transforming a utility parameter $\phi \in (0,1)$ that has the interpretation of "the marginal propensity to bequeath in a one-period problem of allocating wealth between consumption and an immediate bequest" (Lockwood, 2012, p.6).

[25] a has the interpretation of the "threshold consumption level below which, under the conditions of certainty or with full, fair insurance, people do not leave bequests" (Lockwood, 2012, p.7).

- $\mathcal{W}_x$, total wealth at age x, including the value of the family home
- $\tilde{z}$, random variable denoting the real return of the risky asset
- R, constant real risk-free rate of return
- $\bar{W} = \$42,750$, liquidity constraint; Eq.(5.4) poses the constraint that home-owning households require at least $40,000 liquid wealth[26]

The parameters used in this paper to project household finances post-retirement follows Chapter 3:[27]

	v	γ	$\bar{C}$	ψ	θ	a
Single	0.99	-3	$10,680	3.2%	21.7	$14,960
Couple	0.98	-3	$19,230	4.8%	21.7	$22,430

Table 5.2: Set of utility parameters used.

More details of household behaviour under this utility function can be found in Chapter 3. The consumption paths are modelled for every single and couple household groups, with household wealth at retirement given in Table 5.13 and 5.14.[28] I

[26]If someone's total wealth is less than $42,750, she does not purchase a house. If someone's total wealth is greater than $40,000, she does not purchase an expensive house that will leave her with less than $42,750 liquid wealth. This assumption is required for the model to be consistent with the low home-ownership amongst households with low wealth, without this liquidity constraint, it is optimal for these households to allocate all wealth into the family home, as Age Pension is enough to cover their consumption requirement. The value of $40,000 is chosen as the value most consistent with the data of low wealth households.

[27]Chapter 3 calibrates the model to ABS (2011a) data of the 2009-2010 financial year. This paper makes the projection using data as at July 2011, hence the parameters are updated. Appendix C provides details of the adjustments.

[28]For males older than 65 and females older than 62 at July 2011, I assume they retire immediately.

record the projected Age Pension payments for each group each year into the future, in the form of 3D matrixes same as the population matrixes described in Figure 5.3.

5.2.5 Age Pension Projections

Multiply the projected Age Pension payments in each group each year by the projected number of retirees in each group each year, and summing the results, we can obtain estimates of future Age Pension payments. We can also estimate the number of full/part pensioners in the future, by identifying which group of people are expected to be full pensioners or part pensioners from the projected Age Pension payments.

The total cost of future Age Pension payments and number of pensioners are then projected and compared under four different scenarios:

1. **Base Projection**: This is the scenario under current superannuation and Age Pension policies, where the legislated increase of SG from 9 percent to 12 percent is incorporated in the assumption.

2. **SG remains at 9 percent**: The legislated increase of Superannuation Guarantee (SG) rate from 9 percent to 12 percent is projected by Australian Treasury to reduce age and service pension outlays by $3.8 billion in 2035-2036, with the cumulative total saved for every year from 2012-13 to 2035-36 being $41 billion.[29] This scenario make the assumption that SG remains at 9 percent in the future and compared to the base projection to estimate the

[29]See ASFA (2011), these numbers are consistent with Rothman (2012), which projected the saving to be about 0.07 percent of GDP in 2035-36.

savings in future pension payments of increasing SG to 12 percent. The result is compared with Treasury's projections.

3. **Consider owner-occupied properties in the assets test**: If lenient treatment towards owner-occupied properties in the assets tests does lead to over-investment in housing, then it is worth investigating whether economic efficiency can be improved if at least a part of the family home's value is assessed under the assets test.[30] Following the methodology in Chapter 4, I look at the change to retirees' behaviour and the future Age Pension payment, if the value of owner-occupied properties above $500,000 is considered part of assessable assets.

4. **Index Age Pension payments to price inflation instead of wage inflation**: Currently the Age Pension payments are adjusted to the higher of the Consumer Price Index (CPI) and Male Average Weekly Total Earnings (MTAWE). Over the last several years MTAWE growth (averaging 4.5 percent p.a.) exceeds CPI growth. This level of indexation is more generous than other social welfare payments, such as the Newstart Allowance. In this scenario, I make the assumption that Age Pension payments and thresholds are indexed at 3 percent per annual instead of 4.5 percent per annum.

[30] Henry (2010) proposed to cap the value of homes that qualified for the assets test exemption; the proposal however, was not adopted by the Australian government.

5.3 Results

5.3.1 Base projection

Table 5.3 and Figure 5.4 illustrate the projection results under the base scenario [31] compared to estimations from IGR (2010).[32]

$billion (2011 dollars)	2011	2015	2020	2025	2030	2035
Base Projection	31.9	39.4	50.7	64.2	79.9	96.4
Estimates from IGR 2010	34.8	40.6	49.1	59.2	71.2	85.4

Table 5.3: Projected total cost of Age Pension in Australia.

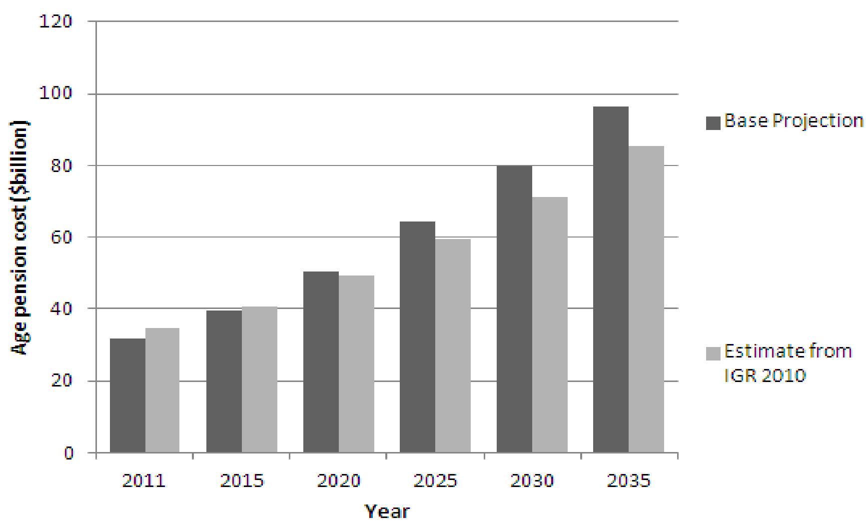

Figure 5.4: Projected total cost of Age Pension in Australia.

[31]The results reported here are after adjustments. For details see Appendix D.

[32]Note that the values estimated from IGR (2010) are higher than the base projection in the early year. This is because the base projection does not include Age Pensioners aged below 65. The numbers are comparable in later years as it is expected that all pensioners would be older than 65 in the future, due to the gradual increase of the Age Pension age to 67.

These results show that when we allow for financial planning behaviours of the underlying population, the total cost of Age Pension payments in year 2035 is estimated to be about 13 percent greater than IGR (2010) estimates. This can also be illustrated with Fig.1, from which we see that the proportion of part pensioners is estimated to increase in the future with a fall in those entitled to the full pension, while the proportion of self-funded retirees remains relatively stable. This result is similar to the estimates in Rothman (2012). However, the change in the proportion of full and part pensioners are estimated to be slower, when endogenous changes in people's financial decisions are taken into account.

5.3.2 SG remains at 9%

Table 5.4 illustrates the projection results under the assumption that SG remains at 9 percent in the future, as compared to the base scenario.

$billion (2011 dollars)	2011	2015	2020	2025	2030	2035
Base Projection	31.9	39.4	50.7	64.2	79.9	96.4
SG remains at 9%	34.8	39.4	50.8	64.5	80.5	97.6

Table 5.4: Projected total cost of Age Pension in Australia.

The savings in future Age Pension payments by increasing the SG to 12 percent is estimated to be $1.2 billion in 2011 dollars, or about $2.3 billion (nominal), markedly lower than the $3.8 billion (nominal) estimated by the Treasury. Similarly the cumulative total saving in future pension payments from 2012-13 to 2035-36 is estimated to be $16.6 billion, less than half as compared to Australian Treasury's estimates of $41 billion.

Table 5.5 gives an example of the difference in consumption and housing decisions for a single household,[33] under the two scenarios.

$ (2011 dollars)	9% SG	12% SG	% change
Estimated wealth at retirement	1,038,600	1,106,100	6.5%
Estimated value of Family Home	540,400	577,100	6.8%
Average annual drawdown	11,800	12,700	7.4%
Average annual Age Pension	26,700	26,400	-1.1%

Table 5.5: Projected post-retirement financial decision changes, single female current age 35 in the 12th wealth band.

We can see that she is expected to have 6.5 percent more savings when we assume the SG increases to 12 percent. Note that the percentage increase of value of her family home (6.8 percent) and annual drawdown (7.4 percent) both exceeded the increase of her retirement savings. These illustrate the behavioural effected of her optimising her drawdown and assets allocation to partly offset the reduction of Age Pension payments due to higher wealth at retirement, as otherwise the percentage increase in the value of her family home and annual drawdown should be less than her wealth increase.[34]

[33] This corresponds to a female household 30-34 in the 14th wealth band in Table 5.13 and 5.14, which is selected as she will be fully affected by the increase of SG from 9 percent to 12 percent, and as a relatively wealthy household, her consumption and housing decision clearly affect her Age Pension payments.

[34] Assuming there is no Age Pension, the percentage increase of her family home and annual drawdown would be exactly the same as the percentage increase of her wealth, under the assumption of CRRA utility function (which assumes financial behaviours are scalable). Under the utility function adopted in this paper, the effect is more complicated. First, due to the existence of subsistence consumption requirement, the percentage increase in consumption is expected to be less than the wealth increase as it contains a fixed base consumption amount. Second, due to the existence of luxury bequests, the percentage increase in housing is also expected to be less than the wealth increase, because non-housing bequests, being luxury goods, should increase faster as wealth increases.

5.3.3 Owner-occupied properties in the assets test

Table 5.6 illustrates the projection results under the assumption that the value of owner-occupied properties in excess of $500,000 as accessible assets under the assets test:

$billion (2011 dollars)	2011	2015	2020	2025	2030	2035
Base Projection	31.9	39.4	50.7	64.2	79.9	96.4
Housing in the assets test	31.3	38.7	49.7	63.1	78.3	93.1

Table 5.6: Projected total cost of Age Pension in Australia.

The savings in future Age Pension costs are limited, which is expected in this situation. It is more viable for a household to invest less in the family home as a loss or reduction of the Age Pension entitlement. Table 5.7 gives an example for the sample household shown in Table5.5, compared to the base scenario.

$ (2011 dollars)	Base scenario	Asset test housing	% change
Estimated value of Family Home	577,100	500,000	- 13.4%
Average annual drawdown	12,700	14,700	15.8%
Average annual Age Pension	26,400	25,500	-3.2%

Table 5.7: Projected post-retirement financial decision changes, single female current age 35 in the 12th wealth band.

It is optimal for this household to investment just $500,000 in the family home to avoid being penalised in the assets test. She can then draw down more money (especially in the early years of retirement) from her savings. First, as she is investing less in the family home, she has more liquid wealth to spend. The second reason is that for having more assets she will receive a lower Age Pension and she will need to withdraw more savings to maintain her living standard. Finally if she draws down her wealth faster in the early years, she will then be qualified for higher Age

Pension later, partly offsetting the effect of higher assessable assets at the beginning.

The example given above illustrates the effect on a household that is most likely to be affected by the policy change; the threshold of $500,000 means that poorer households are less likely to be affected. This can be illustrated as in Fig.5, which shows the estimated proportion of people receiving full/part pension under this scenario, compared to the base scenario.

We can see that if the value of owner-occupied properties in excess of $500,000 as accessible assets under the Age Pension assets test, there would be a lower proportion of part pensioners and a higher proportion of self-funded retirees, while little changes to the proportion of full pensioners. This is because most people who qualify for the full pension are unlikely to invest more than $500,000 in the family home either way and are not affected by the policy.

These results indicate that policy changes that distort behaviour would not have the best effect of reducing Age Pension payments because people can adjust their financial decisions to optimise their outcome, although these would have other effects such as reducing the value of wealth invested in family homes and increasing consumption in retirement, which may improve economic efficiency.

5.3.4 Age Pension payments indexed to price inflation instead of wage inflation

Table 5.8 illustrates the projection results under the assumption that the Age Pension payment is indexed to price inflation (assumed to be 3 percent p.a.) instead of

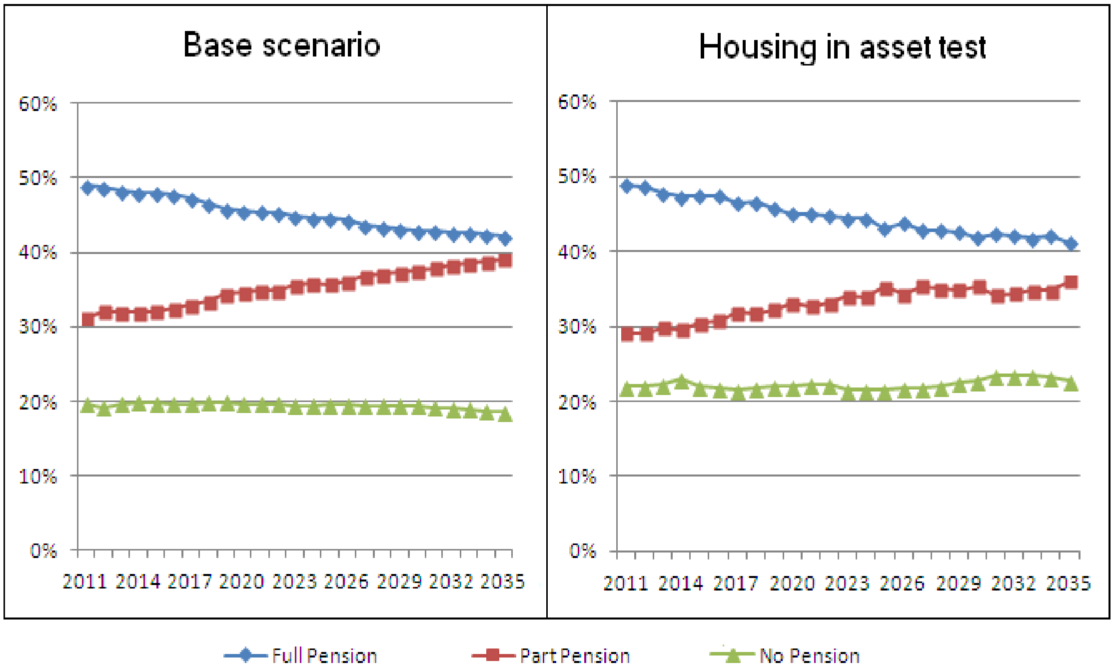

Figure 5.5: Proportion of population age 65+ receiving full/part pension.

wage inflation (4.5 percent p.a.).

$billion (2011 dollars)	2011	2015	2020	2025	2030	2035
Base Projection	31.9	39.4	50.7	64.2	79.9	96.4
Index to CPI	31.3	37.2	43.9	50.9	57.1	61.9

Table 5.8: Projected total cost of Age Pension in Australia.

Figure 5.6 shows the estimated proportion of people receiving full/part pension under this scenario, compared to the base scenario.

We can see the 1.5 percent difference in annual indexation would lead to a significant difference in pension payments in 25 years' time, with marked reduction in the total cost and the proportion of full pensioners. Retirees in this situation cannot easily optimise their pension payment by adjusting their assets allocation. This policy is

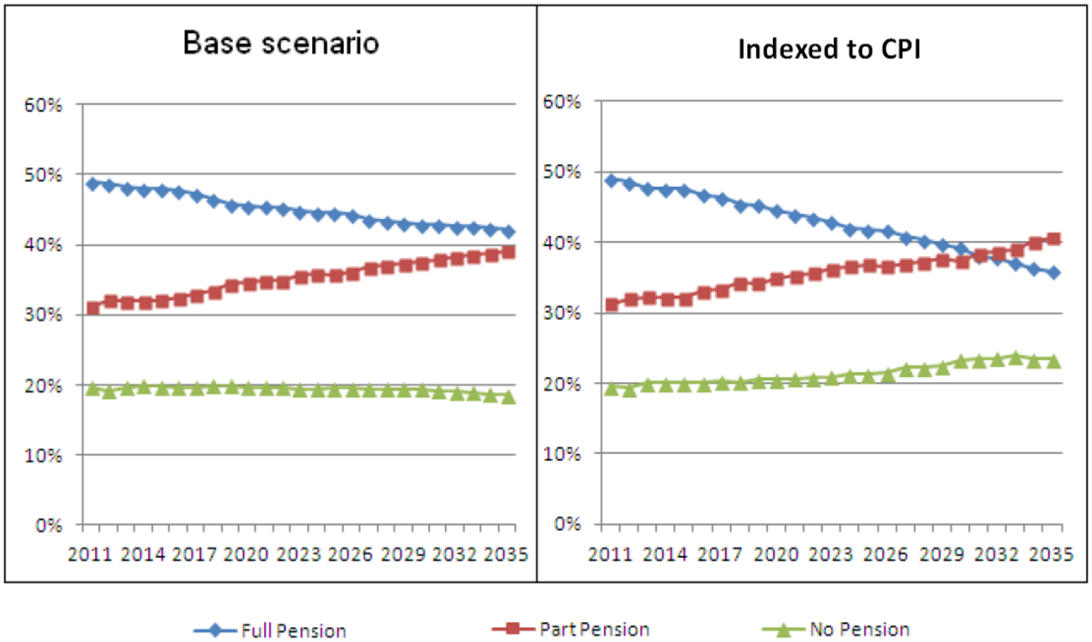

Figure 5.6: Proportion of population age 65+ receiving full/part pension.

more effective in reducing future costs, but also leads to greater welfare losses for the retirees.

5.3.5 Sensitivities

This section present the change in results from base projection under a number of changes to the assumption, this serves to illustrate how sensitive the projections are to changes in various assumptions.

Projections have been undertaken by varying the following assumptions:

1. increase/decrease the wealth at retirement by 10%

2. increase/decrease the absolute value of risk aversion parameter γ by 1

3. increase/decrease the value of ψ, utility parameter denoting the retiree's preference for Housing, by 1%.

The projection results are presented in Table 5.9:

$billion (2011 dollars)	Value in 2035	% difference to Base projection
Base Projection	96.4	
Increase wealth	93.1	-3.4%
Decrease wealth	100.3	4.0%
Increase risk aversion	92.4	-4.2%
Decrease risk aversion	102.2	6.0%
Increase housing utility	99.7	3.4%
Decrease housing utility	92.4	-4.7%

Table 5.9: Projected cost of Age Pension under different assumptions.

The key points we observe from Table 5.9 are:

- The effect of increase/decrease the wealth at retirement results in a change in age pension payments in much lower proportions. This is expected as the retiree is able to allocate an increased/decreased share into owner-occupied property when wealth increases/decreases,this case is similar to the case when we compare 9% to 12% superannuation contributions.

- increase the absolute value of risk aversion parameter, results in retiree to focus more on consumption smoothing, they will spend less during the early years of retirement and allocated less into owner-occupied properties. And increase/decrease the value of housing utility parameter restful in retiree to allocate more/less wealth into owner-occupied properties. However we see that the final age pension payment is not very sensitive to the change of a single assumption.

5.4 Conclusion

This paper presented long-term projections of the cost of Age Pensions in Australia. The results indicate that future costs of the Age Pension are likely to be 13 percent higher than estimated by the Australian Treasury in 2010's Intergenerational Report, when retiree's economic responses are taken into account. This paper also looked at how projected future Age Pension costs are affected by various policy changes, including the legislated increase of superannuation guarantee from 9 percent to 12 percent, the possible changes of including the value of the family home in the assets test, and indexing Age Pension payments to price inflation instead of wage inflation. The methodology developed in this paper can be extended to model the impact of other policy changes.

This research has a number of limitations, specifically:

- Household behaviours in the pre-retirement phase are not modelled. For example, I project the value of house and other personal savings separately and do not take into account interaction between mortgage repayment and other form of savings.
- For example,Rothman (2012)
- Simple assumptions were made regarding couples. For example, They are from similar wealth percentiles, with age difference of 3 years, and retire together.
- I assume that some of the population demographics will remain unchanged in the future, for example, proportion of couple/single by age and proportion of working/retired by age. These however, are more than likely to change with mortality improvements in the future.

- I assume a single utility function with a particular set of parameters is applicable for every Australian retiree. This assumes people's behaviour depends only on age, wealth, gender and marital status, and does not capture the variability in people's behaviours in reality. ABS (2011a) data show that the level of consumption and housing can be very different for people in the same age and wealth band.

Some of the above limitations may be addressed by extending the life-cycle model to pre-retirement, and others maybe improved by incorporating additional data or explanatory variables in the model. These are subject to future research.

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Appendix A: Tables and Figures

($Million) Age Band	Current Age	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15	Total
Males																	
25-29	27	566	253	116	214	406	605	818	1,026	1,220	1,303	1,433	1,748	2,013	2,222	1,903	15,847
30-34	32	375	207	365	794	1,263	1,527	1,716	1,951	2,198	2,665	2,882	3,092	3,937	4,630	3,414	31,016
35-39	37	309	343	927	1,695	2,278	2,649	2,981	3,365	3,637	4,375	5,079	5,547	6,721	7,252	5,882	53,040
40-44	42	268	612	1,405	2,377	2,897	3,273	4,038	4,752	5,244	6,220	7,039	8,353	9,758	10,613	8,175	75,024
45-49	47	271	928	1,994	2,842	3,324	3,984	4,673	5,937	6,608	8,256	10,240	12,592	14,910	14,667	9,167	100,390
50-54	52	252	1,131	2,197	2,934	3,479	4,487	5,794	6,977	8,252	10,414	13,497	17,285	18,600	19,593	8,889	123,780
55-59	57	187	1,042	2,071	2,614	3,671	5,379	6,971	8,438	9,926	13,221	17,853	21,173	23,587	23,719	8,593	148,445
60-64	62	179	878	1,665	2,237	3,220	4,952	6,971	8,567	11,302	14,405	18,696	22,301	25,436	28,788	10,732	160,330
65-69	67	21	459	1,006	1,384	1,846	2,652	4,091	5,348	6,525	8,807	12,062	14,850	19,323	24,165	10,728	113,267
70-74	72	6	355	517	946	992	1,352	1,762	2,524	3,444	4,927	7,815	9,908	12,329	18,369	11,922	77,167
75-79	77	0	137	123	241	292	344	550	750	1,148	1,670	2,318	2,841	4,599	6,060	5,016	26,089
80+	85	0	31	112	128	162	236	247	297	480	679	953	1,760	2,077	1,609	2,556	11,328
		2,434	6,376	12,496	18,408	23,829	31,439	40,613	49,932	59,983	76,942	99,868	121,449	143,292	161,685	86,976	**935,723**
Females																	
25-29	27	354	209	92	59	191	410	621	872	958	1,091	1,097	1,255	1,550	1,958	1,388	12,106
30-34	32	328	138	135	420	832	1,090	1,325	1,422	1,631	1,873	2,129	2,329	2,774	3,290	2,473	22,189
35-39	37	277	89	301	788	1,263	1,574	1,715	1,854	2,125	2,701	3,340	3,903	4,427	4,994	3,667	33,016
40-44	42	166	214	535	958	1,418	1,673	1,841	2,129	2,400	3,156	4,195	4,829	5,555	6,790	4,199	40,056
45-49	47	150	364	830	1,231	1,610	1,904	2,144	2,585	3,113	3,885	5,092	5,963	7,106	8,602	5,834	50,410
50-54	52	104	424	983	1,333	1,754	2,109	2,535	3,210	3,964	4,921	6,145	7,776	9,575	10,919	6,654	62,406
55-59	57	86	602	1,374	1,684	1,977	2,385	3,031	4,090	5,188	6,488	8,069	11,013	14,074	14,600	11,150	85,812
60-64	62	67	797	1,692	1,694	1,651	2,061	3,452	5,283	7,127	8,991	12,751	17,183	21,242	23,531	16,809	124,332
65-69	67	14	565	1,120	1,063	909	1,150	2,225	4,087	4,895	6,474	9,294	13,351	16,824	18,151	18,693	98,814
70-74	72	1	176	577	617	533	606	945	1,580	2,157	2,999	4,834	7,593	10,003	11,785	13,179	57,584
75-79	77	0	65	170	214	118	156	145	176	397	616	940	1,294	1,811	2,727	5,248	14,077
80+	85	0	3	26	30	51	33	30	39	55	105	139	191	308	493	1,072	2,574
		1,548	3,644	7,835	10,090	12,306	15,149	20,007	27,327	34,009	43,300	58,025	76,681	95,249	107,841	90,366	**603,376**

Table 5.10: Superannuation assets by age and wealth bands, June 2011

($Million)	Current	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15	Total
Age Band																	
Males	**Age**																
25-29	27	-350	-122	4	28	65	140	268	548	906	1,503	2,368	3,462	4,971	6,911	11,652	32,355
30-34	32	-448	-51	122	168	421	1,051	1,779	2,502	3,471	4,156	5,407	6,577	8,209	10,198	14,050	57,613
35-39	37	-461	97	336	691	1,518	2,925	4,003	5,152	6,365	7,657	9,466	11,132	12,961	15,604	23,480	100,926
40-44	42	-213	123	656	1,391	2,889	4,722	6,012	7,538	9,112	10,545	12,596	14,256	17,157	18,735	31,851	137,369
45-49	47	-118	392	1,401	2,934	4,961	6,847	8,558	10,095	12,136	13,897	15,494	17,101	19,753	22,845	40,986	177,283
50-54	52	-107	564	2,197	4,348	6,453	8,289	9,519	11,332	12,594	14,850	15,911	17,608	20,642	22,921	39,770	186,890
55-59	57	-80	715	3,105	5,507	7,527	8,581	10,299	11,162	12,594	13,791	14,929	16,547	19,251	21,886	36,514	182,328
60-64	62	-82	868	3,627	5,982	7,753	8,850	10,053	11,163	12,144	13,528	14,425	15,386	17,622	20,094	35,034	176,447
65-69	67	-6	728	2,879	4,706	5,738	6,477	7,206	7,748	8,603	9,790	10,501	11,394	12,624	15,341	25,022	128,749
70-74	72	-5	570	2,243	3,506	4,361	4,945	5,375	5,834	6,337	7,117	8,016	8,693	10,090	11,611	18,868	97,560
75-79	77	0	504	1,766	2,726	3,239	3,598	4,053	4,451	4,762	5,354	6,143	6,689	7,876	9,397	12,879	73,438
80+	82	0	944	2,223	3,465	4,250	4,656	5,389	6,118	6,883	7,614	8,354	9,443	11,048	12,757	16,834	99,977
		-1,871	5,332	20,558	35,451	49,173	61,081	72,512	83,643	95,908	109,802	123,610	138,289	162,205	188,298	306,940	**1,450,934**
Females																	
25-29	27	-114	-6	32	9	42	71	238	537	1,088	1,988	2,977	4,201	5,461	6,812	12,963	36,298
30-34	32	-140	0	37	145	376	934	1,741	2,736	4,011	5,047	6,189	7,389	8,940	11,689	16,264	65,356
35-39	37	-121	-5	193	587	1,271	2,447	3,964	5,600	7,032	8,812	10,165	11,920	13,621	17,345	24,515	107,347
40-44	42	-111	108	624	1,408	2,419	3,653	5,204	6,612	8,430	10,292	12,165	13,866	16,070	20,025	29,761	130,525
45-49	47	-65	269	1,216	2,661	3,988	5,474	6,897	8,433	10,069	12,430	13,940	15,713	18,102	22,650	34,668	156,442
50-54	52	-53	295	1,588	3,458	5,106	6,790	8,130	9,461	11,345	13,257	14,820	16,725	18,815	23,566	35,708	169,011
55-59	57	-23	437	2,353	4,584	6,350	8,005	9,309	10,739	12,434	14,047	15,143	16,545	18,912	22,709	34,569	176,116
60-64	62	-16	747	3,170	5,550	7,103	8,659	9,721	10,727	12,275	13,818	14,944	16,222	18,102	22,733	33,647	177,404
65-69	67	-19	662	2,787	4,685	5,867	6,898	7,509	8,267	9,162	10,578	11,724	13,102	14,649	17,247	25,443	138,560
70-74	72	-6	377	1,929	3,420	4,489	5,207	5,862	6,403	7,321	8,114	9,165	10,016	11,585	14,309	19,607	107,798
75-79	77	-5	230	1,337	2,675	3,580	4,286	4,855	5,592	6,234	6,862	7,749	8,844	10,128	12,611	16,460	91,439
80+	82	0	425	1,824	3,993	6,239	7,717	9,204	10,445	12,322	13,606	15,766	17,658	21,328	25,071	33,487	179,084
		-673	3,541	17,090	33,175	46,829	60,140	72,633	85,551	101,724	118,851	134,747	152,201	175,712	216,766	317,090	**1,535,378**

Table 5.11: Residential property holdings by age and wealth bands, June 2011

($Million)	Current	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15	Total
Age Band																	
Males	Age																
25-29	27	-1,544	-586	-115	-128	-128	-47	62	172	377	725	1,145	1,961	3,741	6,534	26,133	38,303
30-34	32	-824	-256	-220	-257	-303	-231	-113	16	252	788	1,484	2,969	4,713	9,351	46,168	63,537
35-39	37	-533	-318	-450	-545	-459	-428	-147	81	721	1,293	2,082	3,642	6,553	13,773	86,790	112,053
40-44	42	-487	-409	-645	-711	-621	-517	-406	-481	249	1,149	2,168	4,216	7,710	16,433	97,422	125,069
45-49	47	-430	-481	-789	-825	-678	-492	-387	-293	331	997	2,569	4,508	8,524	19,429	139,018	171,001
50-54	52	-342	-568	-889	-885	-756	-688	-461	-55	864	1,144	1,943	3,253	7,725	16,982	128,991	156,258
55-59	57	-284	-509	-982	-1,075	-1,461	-1,804	-2,099	-1,469	-584	-1,213	-1,783	-1,185	2,387	11,324	117,072	116,334
60-64	62	-255	-457	-995	-1,260	-1,721	-2,489	-3,352	-3,698	-4,243	-5,334	-6,167	-5,624	-2,905	157	81,900	43,559
65-69	67	-94	-162	-621	-959	-1,208	-1,620	-2,506	-3,140	-3,722	-5,131	-6,872	-7,386	-8,041	-7,138	50,760	2,159
70-74	72	-23	-77	-295	-726	-753	-954	-1,234	-1,772	-2,352	-3,440	-5,476	-6,393	-7,014	-8,830	30,242	-9,097
75-79	77	1	156	110	-82	-73	-94	-224	-297	-497	-670	-1,120	-1,128	-1,634	-230	23,925	18,144
80+	82	16	282	230	113	178	286	325	137	203	141	618	272	1,792	4,592	28,139	37,323
		-4,799	-3,385	-5,660	-7,341	-7,982	-9,078	-10,541	-10,799	-8,401	-9,550	-9,407	-895	23,550	82,375	856,560	**874,644**
Females																	
25-29	27	-1,427	-669	-190	-46	-111	-142	-110	-74	66	155	663	1,497	3,645	6,665	25,881	35,804
30-34	32	-1,225	-324	-104	-233	-316	-249	-195	-47	-77	300	854	2,030	3,927	7,107	37,346	48,794
35-39	37	-1,040	-252	-197	-410	-416	-240	28	238	517	571	1,500	3,067	6,159	9,770	67,651	86,946
40-44	42	-696	-245	-453	-643	-671	-360	11	346	453	676	1,078	3,008	6,795	11,283	71,109	91,691
45-49	47	-425	-314	-684	-895	-697	-376	46	425	658	877	1,785	4,018	7,589	14,250	77,893	104,150
50-54	52	-283	-298	-720	-859	-596	-440	23	407	407	810	1,943	3,651	7,729	12,549	57,106	81,427
55-59	57	-269	-337	-927	-1,018	-901	-964	-773	-631	-911	-754	263	971	2,374	7,830	45,724	49,676
60-64	62	-196	-464	-1,224	-1,220	-977	-1,257	-1,913	-2,775	-3,975	-4,916	-6,367	-7,934	-7,667	-6,028	36,922	-9,992
65-69	67	-88	-231	-788	-792	-617	-834	-1,440	-2,726	-3,059	-4,180	-5,915	-8,397	-9,675	-7,574	19,859	-26,457
70-74	72	-40	72	-235	-399	-333	-387	-533	-972	-1,320	-1,881	-3,171	-5,157	-6,057	-6,136	13,462	-13,085
75-79	77	-26	169	177	20	28	78	238	191	142	90	100	12	220	594	13,095	15,129
80+	82	-15	457	736	698	555	634	751	788	584	716	882	1,454	2,323	3,561	35,849	49,973
		-5,730	-2,435	-4,610	-5,796	-5,053	-4,536	-3,867	-4,829	-6,516	-7,537	-6,384	-1,781	17,363	53,869	501,898	**514,056**

Table 5.12: Other investment holdings by age and wealth bands, June 2011

Age Band	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15
Males															
25-29	184,240	272,581	336,398	364,224	394,327	448,231	486,759	529,888	611,336	682,941	782,735	905,967	1,091,891	1,262,924	2,306,914
30-34	199,709	295,373	369,678	413,012	461,940	535,462	590,348	648,868	747,189	838,738	947,552	1,067,680	1,266,242	1,447,152	2,544,675
35-39	180,853	272,910	353,329	409,703	469,625	553,935	616,594	680,372	776,171	875,258	996,459	1,123,593	1,311,334	1,494,946	2,817,573
40-44	155,320	237,710	319,887	385,770	453,441	540,762	613,241	691,612	788,697	891,462	1,015,528	1,156,941	1,362,863	1,531,361	2,906,765
45-49	123,404	201,517	286,544	357,056	427,412	510,128	583,032	665,163	766,238	879,322	1,007,519	1,164,301	1,369,018	1,543,479	3,118,759
50-54	89,619	162,338	252,558	329,974	404,465	489,112	568,552	650,676	752,536	876,402	1,024,796	1,195,414	1,388,998	1,558,407	2,945,240
55-59	52,034	115,471	213,079	294,374	372,705	459,925	546,649	629,845	726,191	865,027	1,029,007	1,200,201	1,385,091	1,577,951	2,785,771
60-64	20,208	72,988	168,729	249,630	320,741	399,767	487,792	572,819	679,570	806,525	964,740	1,133,550	1,324,766	1,542,461	2,469,833
65-69	-1,025	43,589	134,861	206,567	261,994	317,460	388,021	458,887	532,577	645,025	797,801	949,661	1,173,667	1,527,788	2,305,408
70-74	-445	46,269	126,748	200,250	241,105	280,941	323,646	379,140	445,581	547,367	708,800	859,697	1,060,365	1,465,459	2,325,307
75-79	41	46,627	122,453	180,889	216,131	246,363	281,161	319,008	371,168	436,125	521,578	609,252	795,028	1,052,318	1,977,525
80+	429	54,564	117,028	176,854	213,780	245,126	276,224	315,027	357,330	410,574	465,962	568,996	691,297	832,242	1,673,267
Females															
25-29	236,910	256,284	290,933	294,203	311,412	342,128	390,187	413,146	459,463	542,494	600,241	723,156	838,833	1,021,436	1,682,381
30-34	205,430	220,707	251,231	267,914	299,515	342,834	402,708	435,803	496,504	577,198	638,227	740,832	846,618	1,038,603	1,596,954
35-39	167,893	178,930	213,190	238,601	277,347	328,986	392,279	433,496	492,700	583,367	652,962	759,452	864,632	1,047,094	1,755,164
40-44	143,923	161,213	200,268	231,502	274,332	323,889	385,848	428,757	490,387	583,877	670,172	778,208	898,620	1,105,691	1,856,822
45-49	120,087	139,614	185,239	225,946	271,982	321,947	379,476	424,375	487,488	578,413	661,763	764,481	895,347	1,114,882	1,914,808
50-54	83,261	103,414	153,788	203,916	257,163	311,425	367,267	417,797	485,869	572,251	654,809	775,695	917,159	1,126,185	1,756,131
55-59	46,479	74,266	142,288	202,345	257,703	311,989	370,267	432,994	510,500	597,643	684,627	816,261	992,289	1,171,506	1,784,803
60-64	18,613	60,290	145,176	206,449	251,052	300,217	368,473	444,955	532,919	633,060	765,241	932,245	1,123,766	1,346,941	1,975,923
65-69	-1,607	44,698	132,401	191,581	230,069	272,226	332,212	416,379	483,643	584,955	724,121	904,210	1,107,355	1,297,107	2,087,191
70-74	-1,107	28,357	109,805	173,408	212,339	249,075	289,826	344,979	404,828	481,989	598,402	762,750	940,125	1,175,055	1,917,405
75-79	-1,001	21,484	86,942	155,425	199,704	237,257	270,818	310,215	357,853	411,735	473,918	553,411	662,791	867,596	1,605,979
80+	-246	20,752	67,521	134,318	197,808	247,221	290,101	333,333	381,581	429,712	491,380	567,252	685,031	833,581	1,618,243

Table 5.13: Projected average net wealth at retirement assuming constant 9% SG.

Age Band	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15
Males															
25-29	212,781	312,599	384,828	415,884	448,710	508,341	549,873	595,962	685,059	762,083	869,298	1,000,136	1,197,329	1,373,320	2,497,098
30-34	228,014	335,061	417,709	464,246	515,874	595,077	652,942	714,396	820,304	917,228	1,033,401	1,161,073	1,370,812	1,556,637	2,733,292
35-39	204,169	305,603	392,894	451,907	514,052	603,042	668,155	734,351	836,399	939,914	1,067,176	1,200,525	1,397,472	1,585,134	2,972,945
40-44	172,741	262,135	349,447	417,302	486,635	577,452	651,764	731,942	833,696	939,769	1,068,364	1,214,420	1,427,220	1,598,744	3,022,850
45-49	134,998	217,773	306,218	378,042	449,504	534,546	608,671	692,004	796,186	911,472	1,042,683	1,202,555	1,411,850	1,588,325	3,196,017
50-54	96,000	171,285	263,386	341,524	416,623	502,551	582,662	665,447	769,018	894,096	1,044,148	1,216,467	1,412,571	1,583,087	2,987,759
55-59	53,805	117,953	216,083	297,579	376,078	463,653	550,564	633,943	730,764	869,937	1,034,377	1,206,042	1,391,631	1,584,798	2,797,568
60-64	20,208	72,988	168,729	249,630	320,741	399,767	487,792	572,819	679,570	806,525	964,740	1,133,550	1,324,766	1,542,461	2,469,833
65-69	-1,025	43,589	134,861	206,567	261,994	317,460	388,021	458,887	532,577	645,025	797,801	949,661	1,173,667	1,527,788	2,305,408
70-74	-445	46,269	126,748	200,250	241,105	280,941	323,646	379,140	445,581	547,367	708,800	859,697	1,060,365	1,465,459	2,325,307
75-79	41	46,627	122,453	180,889	216,131	246,363	281,161	319,008	371,168	436,125	521,578	609,252	795,028	1,052,318	1,977,525
80+	429	54,564	117,028	176,854	213,780	245,126	276,224	315,027	357,330	410,574	465,962	568,996	691,297	832,242	1,673,267
Females															
25-29	272,071	293,159	332,444	336,253	354,984	388,474	441,110	464,031	513,184	602,633	662,582	793,781	913,698	1,106,789	1,796,774
30-34	233,237	249,870	284,060	301,170	333,974	379,487	442,981	476,046	538,990	624,759	687,530	796,687	905,826	1,106,106	1,687,423
35-39	188,573	200,619	237,605	263,334	302,974	356,245	422,230	463,425	524,296	618,739	689,628	800,991	908,664	1,097,296	1,822,445
40-44	159,940	178,011	219,178	250,658	294,181	345,001	409,046	451,937	514,860	611,272	698,571	810,381	932,724	1,144,573	1,908,933
45-49	131,316	151,390	198,496	239,375	285,897	336,748	395,738	440,625	504,644	597,618	681,672	787,036	919,256	1,142,140	1,951,340
50-54	89,190	109,632	160,788	211,007	264,510	319,241	375,854	426,378	494,928	582,392	665,321	787,604	929,784	1,140,579	1,775,421
55-59	48,051	75,915	144,144	204,226	259,652	314,061	372,544	435,269	512,902	600,332	687,415	819,419	995,637	1,175,323	1,789,918
60-64	18,613	60,290	145,176	206,449	251,052	300,217	368,473	444,955	532,919	633,060	765,241	932,245	1,123,766	1,346,941	1,975,923
65-69	-1,607	44,698	132,401	191,581	230,069	272,226	332,212	416,379	483,643	584,955	724,121	904,210	1,107,355	1,297,107	2,087,191
70-74	-1,107	28,357	109,805	173,408	212,339	249,075	289,826	344,979	404,828	481,989	598,402	762,750	940,125	1,175,055	1,917,405
75-79	-1,001	21,484	86,942	155,425	199,704	237,257	270,818	310,215	357,853	411,735	473,918	553,411	662,791	867,596	1,605,979
80+	-246	20,752	67,521	134,318	197,808	247,221	290,101	333,333	381,581	429,712	491,380	567,252	685,031	833,581	1,618,243

Table 5.14: Projected average net wealth at retirement assuming SG increase to 12%.

$	2011	...	2022	...	2035
Male					
60	125,821	...	153,262	...	165,751
...	...	...	...	...	...
81	42,466	...	57,311	...	91,274
...	...	...	...	...	...
94	3,426	...	7,440	...	11,815
...	...	...	...	...	...
Female					
...	...	...	...	...	...

Table 5.15: Sample extract of population projection by age.

	Males	Females
60	31.7%	79.9%
61	31.7%	79.9%
62	31.7%	79.9%
63	31.7%	80.1%
64	31.7%	80.1%
65	74.9%	92.2%
66	77.5%	93.0%
67	80.1%	93.8%
68	82.6%	94.5%
69	85.1%	95.3%
70	92.6%	97.7%
71	93.4%	97.9%
72	94.1%	98.1%
73	94.8%	98.4%
74	95.6%	98.6%
75	96.3%	98.8%
76	97.1%	99.1%
77	97.8%	99.3%
78	98.5%	99.5%
79	99.3%	99.8%
80+	100.0%	100.0%

Table 5.16: Proportion of people retired, June 2011.

	Males	Females
60 to 65	75%	62%
66 to 70	75%	60%
71 to 75	74%	54%
76 to 80	72%	44%
81 to 85	67%	28%
86 to 90	56%	13%
91 to 95	39%	5%
96+	31%	2%

Table 5.17: Proportion of retirees who are couples, June 2011.

	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15
Couples															
Male	4.7%	5.0%	6.9%	8.0%	7.8%	7.6%	7.7%	7.2%	6.9%	7.1%	6.5%	6.4%	5.9%	6.3%	6.0%
Female	3.4%	4.2%	7.5%	9.1%	9.1%	8.8%	8.7%	7.9%	7.2%	6.1%	5.7%	5.7%	4.9%	5.1%	6.6%
Singles															
Male															
65 to 69	13.7%	15.2%	7.0%	3.0%	3.7%	3.8%	3.3%	5.4%	6.0%	5.5%	5.3%	6.9%	6.3%	6.8%	7.9%
70 to 74	12.8%	13.2%	6.0%	2.1%	3.4%	2.9%	3.1%	4.5%	5.2%	5.3%	6.8%	7.6%	10.1%	7.9%	9.1%
75 to 79	12.7%	10.3%	4.7%	1.9%	3.1%	2.8%	3.5%	4.3%	4.9%	4.9%	8.3%	7.5%	12.6%	8.4%	10.3%
80 to 84	11.9%	7.8%	3.9%	1.8%	2.8%	3.0%	3.9%	4.3%	5.5%	4.9%	9.2%	8.4%	12.6%	8.9%	11.1%
85 to 89	11.4%	6.3%	3.5%	1.8%	2.7%	3.1%	4.1%	4.3%	5.9%	5.0%	9.7%	8.9%	12.6%	9.2%	11.7%
90 to 94	10.9%	4.7%	3.1%	1.7%	2.5%	3.2%	4.4%	4.2%	6.3%	5.0%	10.2%	9.5%	12.6%	9.5%	12.2%
95+	10.4%	3.2%	2.6%	1.7%	2.3%	3.4%	4.6%	4.2%	6.7%	5.1%	10.8%	10.0%	12.6%	9.8%	12.7%
Female															
60 to 65	13.5%	12.6%	5.8%	1.9%	3.5%	1.6%	5.6%	7.4%	6.0%	8.4%	7.7%	5.8%	6.0%	6.3%	7.9%
65 to 69	12.8%	10.8%	4.7%	3.1%	2.7%	2.9%	4.0%	5.9%	6.7%	7.6%	7.1%	7.8%	8.6%	8.1%	7.1%
70 to 74	12.2%	10.6%	4.9%	3.5%	2.4%	3.2%	3.0%	4.4%	5.9%	7.5%	8.0%	8.8%	9.6%	9.8%	6.4%
75 to 79	10.0%	9.6%	5.9%	3.6%	3.4%	3.9%	3.4%	4.4%	5.5%	7.2%	8.6%	8.6%	9.7%	9.5%	6.7%
80 to 84	7.6%	8.0%	6.6%	4.0%	4.7%	5.0%	4.0%	5.5%	6.1%	6.9%	8.3%	8.1%	9.6%	8.3%	7.3%
85 to 89	6.4%	7.4%	7.1%	4.1%	5.3%	5.4%	4.2%	5.7%	6.0%	6.7%	8.5%	8.0%	9.7%	8.0%	7.6%
90 to 94	5.2%	6.8%	7.5%	4.2%	5.9%	5.8%	4.4%	5.8%	5.9%	6.6%	8.7%	7.8%	9.7%	7.7%	7.8%
95+	4.0%	6.2%	8.0%	4.3%	6.4%	6.3%	4.7%	6.0%	5.9%	6.5%	8.9%	7.7%	9.8%	7.4%	8.0%

Table 5.18: Distribution of Couple and Single households by wealth bands at retirement.

Appendix B: Summary of Projection Methodology

The following summarized the assumption and methodology adopted in RiceWarner (2012a, 2012b).

Future Savings and Contribution Rates

An important component to project the superannuation asset is the roll-up of future contributions. Likely future contributions can be determined by applying contribution rates to the total income in each age/sex/income cohort in the population model. However, for the purposes of this study, we have varied the contribution rate by age only.

Note that Employer Contributions are effectively concessional contributions and include salary sacrifice as well as the Superannuation Guarantee payments. Similarly, Member contributions are all non-concessional contributions including large one-off payments made (e.g. from asset sales). Government co-contributions are made in addition to the Member Contributions. The assumed contributions by age group are as follows.
These contribution rates reflect the fact that individuals closer to retirement tend to contribute more towards superannuation. These individuals have fewer other priorities for their disposable income (such as saving for a car or buying a house) than the younger age groups, and saving for retirement is a more pressing issue.

We consider that the above contribution rates better reflect the ability and propensity of individuals at different ages to make contributions to superannuation. We

Age Group	Employer*	Member
	(%)	
25-29	9.00	0.00
30-34	10.16	0.74
35-39	11.51	1.60
40-44	13.05	2.58
45-49	14.79	3.68
50-54	16.53	4.79
55-59	18.26	5.89
60-64	20.00	7.00
Average	13.98	3.18

Table 5.19: Assumed Contribution Rates,June 2011

note that these contribution rates still produce contribution levels that are broadly consistent with the current contribution levels as published in APRA's Quarterly Superannuation Performance Report dated 30 June 2011 (after allowing for contributions made by high income earners).

Concessional contribution caps were reduced in the May 2009 Federal Budget. From the 2009-10 financial year, the maximum total concessional contributions that persons aged under 50 can make has been halved to $25,000 p.a. (indexed). The existing cap for those aged 50 and over remains at $50,000 but from July 2012 will be reduced to be in line with the prevailing cap for those aged under 50.

Increase of SG from 9% to 12%

We have assumed all future employer contributions will increase from year 2013 as shown in Table 15. We have also assumed that the tabled increases will not impact on our wage inflation assumption of 4.5%. For example, in 2013 wages will increase by 4.5% and the superannuation guarantee will also increase by 0.25%. Note this differ to the assumptions in Rothman (2012).

Year	Increase in Employer contribution
	(%)
2011	0.00
2012	0.00
2013	0.25
2014	0.50
2015	1.00
2016	1.50
2017	2.00
2018	2.50
2019	3.00
After 2019	3.00

Table 5.20: Changes to SG contribution

We note that some people are already contributing more than 12% SG, hence the 3% increase may not have full effect on these people, and the impact of this policy on the savings gap may be overstated. However, it would be difficult to predict people's reaction to the policy, hence we ignore this possibility and illustrate the potential effect of the policy, assuming it will affect everybody equally.

The Co-contribution Scheme

The Government Co-contribution Scheme has been in operation since 1July 2003. In the May 2010 Budget the Government announced that it would scale back the co-contribution scheme.

Statistics released by the former Assistant Treasurer, The Honourable Mal Brough, in February 2005 show that around 450,000 individuals received Co-contribution payments in the 2003-04 income year, 37% of payments were in respect of males, and 63% were in respect of females .A breakdown of Co-contributions by age band was released as follows:

Age Range	Proportion of Co-contribution Payments (%)
Under 21	4.5
21 - 25	6.0
26 - 30	5.4
31 - 35	6.8
36 - 40	9.1
41 - 45	11.5
46 - 50	14.3
51 - 55	15.5
56 - 60	15.1
61 - 65	9.4
66 - 70	2.3
Total	100

Table 5.21: Co-contributions by Age

I have broadly allocated the projected future Co-contribution payments to individual income bands based on the Co-contribution available as well as the ability/propensity to contribute at each income band. I have further allocated the Co-contribution payments by age and sex according to the statistics released by the former Assistant Treasurer.

Note that the ATO taxation statistics for the year to 30 June 2011, indicated that approximately 1.2 million Co-contributions (a take up rate of approximately 9.5% of those eligible to receive a Co-contribution) worth \$700 million were paid (resulting in an average Co-contribution payment of \$610). For the purposes of calculating the value of Co-contributions received I have assumed that this take up rate of 9.5% will continue into the future.

Projection of other investment holdings

The foundations of the projections are:

- The initial personal investments pool, as given in Table 5.12.
- New household savings, including investment earnings, driven in turn by household income and GDP.
- Overall population trends.

Household savings have been determined as a proportion of net household disposable income, as defined by the ABS[35]. This savings ratio in Australia can be tracked historically and in fact has fluctuated significantly. I have adopted an average saving rate of 8.2% of net disposable income, reflecting the trend in the savings ratio over the period 1962 to 2012 which provides a reasonable fit to the observed historical data, albeit that actual savings ratios have departed significantly from the trend line (lower savings ratios) during the period 1992 to 2008.

Note ABS classified mortgage repayments, and contributions to insurance and other financial services (including superannuation), as consumption expenditure[36]. As

[35]see ABS 5204.0 Australian System of National Accounts, Table 7, National Income Account, Current prices; and ABS 5206.0 Australian National Accounts: National Income, Expenditure and Product, Table 30, Key Aggregates and analytical series, Annual.

[36]see ABS Australian System of National Accounts (Concepts, Sources and Methods) 2000, page 461.

such, 'household savings' excludes residential properties and the contribution to superannuation either by individuals or their employers. Hence it can be considered a key driver of future trends in personal investments.

Note this method include realized capital gains as part of disposable income, however, unrealized capital gains are ignored. For this reason I have made an allowance for unrealized capital gains on assets in the investment pool. Based on past experience, this allowance has been assumed to be 1.8% per annum.

The projections in this report assume that the overall personal investments market grows in each future year according to the formula below.

Personal investments pool at start of year

\+ household savings in year

\+ unrealised growth (decline) in personal investment asset values

\+ addition to personal investments pool through population growth in the year

= Personal investments pool at end of year.

Appendix C: Utility parameters

Chapter 3 calibrated the model to data of 2009-2010, and assumes Age Pension rate as at January 2010. This paper makes projection from July 2011 onwards, and the Age Pension rate used is the rate published by Centrelink on September 2011. Table 5.22 lists the difference.[37]

	Single		Couple	
	Chapter 3	Here	Chapter 3	Here
Full Age Pension Rate	\$17,456	19,469	\$26,099	\$29,354
Income Test				
Threshold	\$3,692	\$3,900	\$6,448	\$6,864
Rate of Reduction	\$0.5	\$0.5	\$0.5	\$0.5
Asset Test				
Threshold: Homeowners	\$178,000	\$186,750	\$252,500	\$265,000
Threshold: Non-homeowners	\$307,000	\$321,750	\$381,500	\$400,000
Rate of Reduction	\$0.039	\$0.039	\$0.039	\$0.039

Table 5.22: Updated Age Pension parameters.

Some parameters calibrated in Chapter 3 as shown in Table 5.23, represent a certain consumption level required, and these parameters have been adjusted to Table 5.2, taking into account inflation of 4.5 percent per annum.[38]

	v	γ	$\bar{C}$	ψ	θ	a
Single	0.99	-3	\$10,000	3.2%	21.7	\$14,000
Couple	0.98	-3	\$18,000	4.8%	21.7	\$21,000

Table 5.23: Set of utility parameters calibrated in Chapter 3.

[37] The Age Pension rates are expressed as annual entitlements, income thresholds are expressed as annual income, and the rate of reductions are expressed as the reduction in annual Age Pension rate per dollar over threshold.

[38] Consumption level is inflated to the increase in living standard instead of CPI, and I choose 4.5 percent as it represents the average salary inflation in Australia.

The utility function approximates health statuses with survival probabilities. Furthermore, part of the Age Pension income test deduction is based on the life expectancy at retirement. As we project for the future, change in the survival probability and life expectancy needs to be taken into account to correctly compute the amount of future deductable income for a younger cohort. I have used Institute of Actuaries 2007 life tables together with their mortality improvement factor to estimate the future life expectancies for different cohorts.

Appendix D: Projection Adjustments

The assumptions and methodologies adopted in this paper are subject to a number of limitations. First, I assume a single utility function with a particular set of parameters is applicable for every Australian retiree. This assumes people's behaviour depends only on age, wealth, gender and marital status. Second, Chapter 3 calibrated the utility parameters to data of around 5500 surveyed households, which may not exactly represent the preference of the Australian population as a whole. And last, I make no allowance for some people who are not eligible for the Age Pension due to various reasons (for example, new immigrants need to wait 10 years before becoming eligible to claim the Age Pension), and I do not take into account people who do not claim the Age Pension while eligible. The assumptions in this paper also do not differentiate between those who are eligible to the Age Pension and those eligible to the service pension. The following adjustments are made to account for the these factors:

1. First, ABS (2011a) shows that there are 5.7 percent retired single households

and 4.6 percent of retired couple households in the survey, who do not claim the Age Pension while eligible. Hence I reduce the number of full and part pensioners by 5.7 percent and 4.6 percent of total retired single/couple population. After this adjustment the estimated number of full and part pensioners is 1.66 million and 0.9 million. The total amount of pension payments are adjusted accordingly.

2. Centrelink statistics shows that in 2011, there were about 1.48 million full pensioners and 0.95 million part pensioners above age 65. And I further adjust the number of full/part pensioners to these numbers, and then adjust the total amount of projected pension payments accordingly. After this adjustment the estimated total age/service pension payments in 2011-12 financial year are $35.4 billion for people age more than 65. This number is reasonable compared to the 2012-13 Budget Paper, which estimated that the total amount of age/service pension payments to be $36.7 billion (including people younger than age 65).

3. According to Centrelink statistics, in 2011 there are about 0.17 million full service pensioners and 0.06 million part service pensioners. And I estimated the amount of service pension payments in 2011-12 to be about $3.51 billion. IGR (2010) states that the amount of service pension payments is estimated to remain stable in the future. Hence, I assume the amount of service pension payments to remain at $3.51 billion in real terms, and the amount of future Age Pension payments equal the total pension minus the service pension payments.

Chapter 6

Summary and conclusions

This thesis developed a system of models that can be used to evaluate the impact of superannuation public policy changes in Australia, allowing for behavioural effect. I developed a utility model for the preferences of retirees, and offered the first joint consideration of luxury goods (in the form of bequest), housing, 'ultra-necessities' (in the form of a 'subsistence' rate of consumption in retirement) and public pensions. I calibrated the model specifically for Australian retirees, and presented long-term projections of Age Pension costs in the future, with retirees' financial behaviors modelled with the developed utility model.

6.1 Findings

The findings of this research showed that:

- The bequest motive is important for Australian retirees,[1] and non-housing bequests can be considered as luxury goods.[2] If we ignore Australia's Age

[1]Consistent with Hulley et al. (2012) and Feinstein and Ho (2000).
[2]Consistent with De Nardi (2004) and Lockwood (2012).

Pension, luxury bequests imply that a higher allocation to risky assets is optimal, and the expected optimal percentage allocation to equity rises throughout retirement. The existence of the Age Pension implies even higher allocation to risky assets, while the expected optimal allocation to equity is relatively constant throughout retirement, if we jointly consider luxury bequests and the Age Pension. Either way, these results contrast with the popular strategy that percentage allocation to equity decreases with age after retirement.

- The financial behaviours of wealthy households and poorer households are very different. The consumption profiles of rich retirees depart markedly from consumption smoothing. They spend much more on housing and bequests, while their consumption on non-housing, non-bequest goods decreases rapidly as age increases. They spend similar amounts to poorer households at later ages. In these ways, financial advice is non-scalable, contrary to the relevant prescription following from the standard assumptions of the Constant Relative Risk Aversion utility model.

- The high concentration of wealth in the family home in Australia is likely affected by the publicly provided Age Pension. First, the high level of Age Pension payments are well above the average level of subsistence consumption needs of Australian retirees, indicating that households in low wealth bands do not need much wealth outside of their family home to fund their retirement consumptions. Second, the Age Pension assets test implies that it is optimal for households in the middle to high wealth bands to allocate wealth to their family home in order to receive higher Age Pension payments.[3]

[3]Consistent with Cho and Sane (2013).

Taking these financial behaviours into account, I projected the annual total Age Pension payments to Australian retirees from 2011-12 to the 2035-36 financial year. The cost of Age Pension payments was estimated to be $96.4 billion (in 2011 dollars) in the 2035-36 financial year, about 13 percent higher than estimates in Australian Treasury's 2010 Intergenerational Report (IGR 2010), under similar assumptions. The reason is that as future cohorts retire with more savings, they can draw down their savings faster and allocate more money into owner-occupied properties to optimise their Age Pension entitlements.

I also investigated how projected future Age Pension costs are affected by various policy changes, including the legislated increase of superannuation guarantee from 9 percent to 12 percent, the possible changes of including the value of the family home in the assets test, and indexing Age Pension payments to price inflation instead of wage inflation. The results indicated that policy changes that distort behaviour would not have the best effect of reducing Age Pension payments because people can adjust their financial decisions to optimise their outcome. However these would have other effects, such as reducing the value of wealth invested in family homes, which may improve economic efficiency. Actions such as reducing the indexation rate of Age Pension payment would have a more direct effect on reducing the pension payments because people can do much less about it, but it also results in greater welfare losses.

6.2 Strengths and limitations

The strengths of this research lay in:

- The utility model is developed based on previous literature and data analysis of retiree's behaviours; it takes into account the complex interaction of consumption and housing decision, bequest motive and the Age Pension system; and is realistic enough to capture the main empirical behaviour characteristics of Australian retirees.

- The model has a semi-analytical solution, with clear advantages in computation time over numerical methods.

- The model parameters are calibrated to empirical data, and the modelled behaviours reasonably present the current behaviour of Australian retirees in average.

- I present long-term public policy projections while taking into account retiree's economic responses, an area where there are few previous contributions.

The limitations of this research are:

- Analytical solutions can only be found with specific forms of utility function. Hence there are limitations in how the utility function can be changed to adapt to different assumptions.

- To evaluate the effect of policy changes, I assumed that if the modelled behaviour matches the current behaviour of retirees, then, when pension policy changes, the change in modelled behaviour will be similar to the change in actual behaviour. This assumption may not hold.

- I assumed a single utility function with a particular set of parameters is applicable for every Australian retiree. This assumption suffers from the lack of heterogeneity as peoples' behaviour is assumed to depend only on age, wealth, gender and marital status. In reality, peoples' behaviour can depend on other factors (e.g. education). Hence the calibrated utility model is suitable to be applied at a macro level, where average results are most relevant. For application to individual financial planning, the model needs to be tailored to the individual's specific preferences.

- The utility function used only model household behaviours after retirement. Household behaviours in the pre-retirement phase are not modelled.
- The data used (ABS survey of household expenditure survey and survey of income and housing), consist of around 5,500 households, behaviour of these households may not be totally representative of the entire Australian population.
- The calibrated parameters reported suffer from identification problems.

6.3 Implications for further research

The utility model developed in this thesis with the calibrated parameters reasonably explain the financial behaviour of Australian retirees. This is the first contribution in the area with this level of detail, and provides a valuable reference for further research into retirement problems. The model is expected to have applications in wide areas including post-retirement financial planning, pension product design and the evaluation of pension policy changes in Australia.

As an application of the utility model developed, this thesis presents a methodology for long-term projection of Age Pension costs in Australia, taking into account retiree's economic responses. This methodology can be extended to model the impact of other policy changes.

Future research can also target on improving the model in various areas:

- The calculation method developed can be improved to accommodate more flexible utility functions, or find faster numerical methods to incorporate more flexible assumptions.
- The method can be extended for public pension systems of countries other than Australia.
- Other explanatory variables can be incorporated into the model to better capture the variability in retirees' behaviours.
- Other data and surveys can be integrated into the study (HILDA survey for example), to address the identification problems of calibrated parameters.
- More accurate projection of Age Pension costs (and other policies) can be gained with research on future population demographics. The utility model can also be extended to model pre-retirement saving and investment behaviours.

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