Optimisation of dynamic toll-pricing schemes, using VISSIM

By

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Typeset in $\mathbb{A}T_{\mathbb{E}}X\,2_{\mathcal{E}}.$

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Aya Alwan

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Abstract

This study initiates work toward developing a new dynamic toll pricing scheme on road networks using simulation. This MRes project is a pilot study aimed at laying the groundwork for a doctoral project. The planned doctoral work will include developing a new approach that can practically contribute in mitigating traffic congestion and maximizing traffic throughput and toll revenue. The simulation software package VISSIM will be used to test the reliability of this approach. A major task in this study will be the development of an external module, using the technical computing package Matlab, through the VISSIM-COM interface. The results of the data analysis and simulation are expected to show that toll lanes operated under such a scheme provide a noticeable time saving while maintaining speed at the minimum limit specified for optimal performance. As work toward the development of such a scheme within the MRes study period, I have done three things:

- 1. Preparing a literature review on similar previous approaches.
- Obtaining the relevant traffic data on Sydney's M5 Freeway, the selected study area.
- 3. Planning and initial development of the methodology that will be adopted in this study, including the required models and algorithms necessary to accomplish the study objectives.

Contents

Acknowledgements iv								iv							
A	bstra	ct													\mathbf{v}
1	Intr	oduction	uction								1				
	1.1	Background										•			1
	1.2	Study objec	tives									•			3
	1.3	Thesis struc	ture				• • •			•		•			4
2	Lite	erature review						5							
3	\mathbf{Pro}	oblem statement								39					
	3.1	Poisson proc	ess									•			42
	3.2	Directed gra	cted graphs							42					
		3.2.1 Exam	nple									•			43
	3.3	Problem for	lem formulation							43					
		3.3.1 Arriv	vals									•			43
		3.3.2 Utili	ty function	n on tol	l (ma	nage	ed) a	and	gen	era	l lar	e			44
		3.3.3 Thro	ughput ar	nd reven	ue.							•			45
		3.3.4 Max	mizing rev	venue a	t any	tim	et.					•			45
		3.3.5 Max	mizing th	roughpu	ıt at	any	tim	et.				•			46

		3.3.6	The maximisation formula for both toll revenue and traf-							
			fic throughput	46						
4	Met	hodolo	ogy	47						
	4.1	VISSI	M simulation software	47						
		4.1.1	Simulation software	47						
		4.1.2	Study area	48						
	4.2	VISSI	M module	50						
		4.2.1	Toll pricing calculation module	50						
		4.2.2	Managed Lane Module	50						
		4.2.3	Driver route choice model	51						
		4.2.4	VISSIM vehicular module	52						
	4.3	Extern	al Module for dynamic toll pricing calculations	52						
	4.4	Extern	al module dynamic toll calculation process	53						
	4.5	Feedba	ack control mechanism	54						
	4.6	The w	willingness to pay on toll lanes							
	4.7	Traffic	c Data collection							
	4.8	Peak h	nour volumes and Peak hour period length	59						
	4.9	Toy M	odel description	59						
5	Data analysis and simulation results 62									
	5.1	Model	ling traffic arrival process	62						
	5.2	Poisso	n process models demonstration	63						
	5.3	Demor	nstration of the Toll setting framework	63						
6	Con	conclusion and recommended future work								
	6.1	Intend	ed doctoral work	68						
Re	eferei	nces		70						

1 Introduction

1.1 Background

Although many studies have investigated traffic congestion pricing in recent years, a unified model for this problem still does not exist. This study comes within the context of developing a new approach for determination of the optimal dynamic tolls pricing schemes with the ultimate goal of reducing traffic congestion and delay and maximizing traffic throughput and revenue. Traffic congestion is a major problem for most road authorities as it undermines peoples quality of life through wasted time, energy, money, as well as associated environmental concerns and safety issues.

Engineering solutions to overcome traffic congestion through building more roads or increasing the capacity of existing highways are not always feasible due to high cost and limited available land. Various operational management traffic policies have been proposed to mitigate traffic congestion at lower cost, such as: reducing demand by imposing bans on heavy trucks for particular hours, discouraging peak-hour travelling, re-timing of traffic lights, metering access to highway and so on as these policies have provided partial solutions but neither has been able to meet the minimum service limits, not to accomplish the objectives set by the road authorities, to mitigate traffic congestion and reduce travel time.

In the last few decades, congestion pricing has emerged as a cost-effective and efficient strategy to mitigate the congestion problem on Freeway roads. The concept of congestion pricing was first introduced in 1920 by Pigou in his famous book Wealth and Welfare [1]. Later, the work on toll pricing strategies was expanded by many researchers to include applying a fixed or variable toll on road users for using a particular lane or roadway segment in an attempt to control travel demand and reduce traffic congestions by encouraging motorists to either switch to alternative routes or change their trip time [2].

In Australia, congestion pricing for roads was found to have the potential to deliver significant benefits to Australian economy [3]. The toll pricing systems implemented on a few Freeway roads such as Sydney Harbour Bridge and Sydney Harbour Tunnel in 2009 represents the first step in validating and testing the benefits introduced by congestion pricing. In comparison to other countries, Australia is still lagging behind international best practice in the field of congestion pricing and management. In the field of congestion pricing and management road tolling in Australia takes two key forms: the first is the cordon pricing, where typically a charge is applied on vehicles to enter a city region such as the CBD, port, or airport. The second form is the corridor pricing, where a charge to use a road or corridor including tolled lanes is applied. The toll pricing mechanisms vary in complexity from simple fixed day/night and peak/off-peak charges to dynamically priced systems based on actual traffic conditions and operating speed. With the remarkable advances in traffic control technologies such as detectors, variable message signs, and electronic toll collection devices, dynamic toll pricing is envisaged to be a reliable strategy for increasing revenue for the road authorities and solving congestion problems with the least cost.

1.2 Study objectives

This study aims at developing a new congestion pricing approach in which the toll is adjusted to accommodate the variations in traffic conditions. The main objectives of this study are the followings:

- 1. To develop a framework to determine optimal dynamic toll pricing strategies through simulation.
- 2. To investigate the effect of road users' income levels on toll route decisions and highlight their effect on the traffic flow characteristics on both managed and general purpose lanes.
- 3. To maximize traffic throughput and revenue while maintaining a minimum desired level of service on managed lanes. The minimum level of service requires the average speed on managed lanes to be greater than specific speed per hour at peak period.
- To maximize the travellers utilities while maintaining a minimum desired level of service on managed lanes.
- 5. To validate and test the developed dynamic toll pricing schemes and models with a focus on maximizing traffic throughput and revenue in microscopic simulation.

While some of the future objective are:

- 1. Demonstrating the toll calculation outputs for the proposed toll pricing schemes under various traffic conditions and other operational constraints.
- 2. Simulating the results performed to test and validate the performance of both toll and general purposes lanes under a set of proposed toll pricing schemes.

3. Accomplish more objectives including the recommended practical steps to put the developed toll pricing schemes into actual implementation.

1.3 Thesis structure

The study will be accomplished over the MRes and doctorate study periods. It will be presented in 5 chapters; the first two chapters are already accomplished within the MRes stage while the remaining three chapters will be expanded and accomplished during the doctorate study.

Chapter 1 includes an introduction to the study. The study objectives are also outlined and detailed in this chapter. Chapter 2 includes the literature review that summarizes the previous studies that have been undertaken with a focus on developing various pricing strategies to solve traffic congestion. A brief review on the static toll pricing research followed by a substantial review of dynamic toll pricing strategies is included in this chapter. Chapter 3 describes the methodology followed in this study and the detailed problem formulation. The methodology includes detailed steps followed in order to accomplish study objectives. It exclusively explains the modules and algorithms used for the development of the optimized toll pricing schemes. It also includes a full description of the built in modules available in VISSIM simulation package and the external modules developed through COM interface using Matlab. A separate section on the study area characteristics such as traffic, geometric configuration and topology in also included in this chapter.

2

Literature review

In the early 1920s, the concept of congestion pricing was first introduced by Pigou who developed the theoretical approach for congestion pricing and introduced this new policy as a measure to reduce traffic congestion on roads [1].

In recent years, and with the remarkable advances in transportation technology, such as traffic signals, variable message signs, loop detectors, traffic data collection equipment and electronic processing devices, many road agencies in many countries around the globe have deployed different types of dynamic toll pricing systems that vary toll rate based on certain traffic conditions [2]. The main purpose of implementing toll system was mainly to recover the construction costs and other costs paid by the government or the private investor and to reduce traffic congestion.

This document reviews the studies and practices in toll pricing and the outcomes resulted from the implementation of these studies. The aim of presenting this literature is basically to broaden our knowledge about toll pricing strategies and to help in fulfilling the objectives of this research.

Definitions:

Headway is the time, in seconds, between two successive vehicles as they pass a point on the roadway, measured from the same common feature of both

vehicles [4].

Vehicle spacing is the average spacing at the minimum queue discharge headway, or maximum queue discharge flow rate (m/veh).

Jam spacing is spacing between vehicles in the queue (m/veh).

VISSIM is a microscopic, time-step and behaviour-based simulation model developed to model urban traffic and public transit operations.

GPLs is general-purpose lanes.

HOTL or HOT is High-Occupancy/Toll Lanes.

HOVL is High-Occupancy Vehicle Lanes.

Level of Service(LOS) is defined as a qualitative measure used to relate the quality of traffic service and usually used to analyse highways by categorizing traffic flow and assigning quality levels of traffic based on performance measure like speed, density.

M/M/1 is queue model used to model traffic flow using exponential arrivals and service time with a single server.

D/D/1 is a queue model in which the incoming stream of vehicles arrive at a fixed deterministic rate and their service time through the intersection is assumed constant for every vehicle.

E-Z Pass is an electronic toll collection system used on most tolled roads, bridges, and tunnels as a means of traffic congestion management.

Timing optimization of signalized intersection using shock wave theory by genetic algorithm

In a recent study aimed at optimizing timing of signalized intersections, the shock wave theory to model the system and optimize the Genetic Algorithm (GA), was implemented using traffic data gathered from three main signalized intersections in Rasht city in the north of Iran [5].

The methodology adopted in this study is based on applying the shock wave

theory through GA optimization method. By using this method, traffic movement at a signalized intersection was analysed by modelling how the queue is formed and dissipated. By determining parameters related to under-saturated or over-saturated traffic stream dynamics along with the design of signal phase timing, the maximum queue length, vehicle delay and wave speed (modelling traffic flow speed) can be predicted or estimated. This methodology was found to be applicable to multiple signalized intersections along a corridor or to all the intersections in an urban road network.

The result of this study showed that the average vehicle delay is reduced using this technique. It was found that reducing green time phase at one of the intersections, from 40 and 43 to 42 and 40, the green time was optimized. Also, the Level of Service(LOS) at each intersection under study was improved.

Finally, applying this method, the average delay time can be reduced by 31% at morning peak, 22% at noon peak and 22% at afternoon peak. It was interesting that the result was obtained even without increasing the cycle.

Statistical queuing theory with some applications

Ferreira [6] studied the definition of the single node and networks of queues. Among the interesting applications of statistical queue theory discussed in this paper is traffic intensity parameter. The paper argued that the most important formula in traffic queueing theory is the Cox and Smith formula which takes the form $\rho = \lambda \alpha$, where ρ is the traffic intensity, λ is the arrival rate of the customers (cars) and α is $1/\mu$, where μ is the service rate. Similar to the Cox and Smith formula mentioned above, Little's formula also is an importance result in queueing theory. This formula takes the form of $N = \lambda W$, where N is number of the customers in the system, λ is the arrival rate and W is the mean sojourn time of a customer. The study indicated that when the Cox and Smith formula is used in an M/G/1 queue, the mean waiting time of a customer (car) in the system can be also determined using the following equation: $W_s = \alpha \rho (1 + c_s^2)/2 - 2\rho$ Where C_s is the service time and W_s is mean sojourn time of a customer in the system.

Optimization of signal timing at signalized intersections in Indian scenario

In a study aimed at optimizing signal timing at signalized intersections, a software called VISSIM was used to model part of the road network in India [7]. In order to achieve this objective, five road links, each link of 2 lanes, have been studied and analysed. The inputs used for the simulation include the average speed, vehicular type, geometric features, driver characteristics, signal characteristics and volume. These inputs were used to calculate average travel time, delays and queue length.

The methodology adopted in this study involves calculating the field delays for each system manually. Delays were obtained by tracing individual vehicle trajectories between a point 235 m upstream of the intersection stop line and a point 120 m downstream of the intersection stop line. In this study, the "system" was defined as the roadway segment between these two points.

After measuring the field delays, the same road network was simulated using the Microsimulation software VISSIM. The field delay values were compared to the delays obtained by VISSIM. Inputs variables for VISSIM involved geometric features (number of lanes, lane width, lane change, etc.), driver characteristics (i.e., reaction time, acceleration/deceleration rate), speed, signal characteristics (i.e., green, amber or red times), and vehicular traffic characteristics (i.e., cars, HGV Buses etc.).

Using different green times, delays and queue length parameters were calculated at different values of input variables. The preliminary results showed that delays obtained by simulation were very close to the values of the calculated field delay, with a difference of less than 15%. Therefore, it was decided to perform further analysis.

The analysis showed that different values of speed, vehicular type, traffic volume, and behavioural at different green times have significant impact on the delays. It showed that the increase in traffic volume and speed have a considerable proportional effect on the values of delays, while the change in Heavy Good Vehicles (HGV) percentage, represented by the variable 'Vehicular' has no effect on the delays.

In conclusion, the simulation was considered to give an accurate representation of the existing system, as the differences in delay values obtained from the field data and the simulation results for at different green times were less than 15%. Also, the results were found to be acceptable and valid as it showed that system delays are reduced by 29% at the same cycle.

Application of queuing theory to vehicular traffic at signalized intersection in Kumasi-Ashanti region, Ghana

To investigate the possibility of using queue theory to determine the vehicular traffic flow characteristics and the operational conditions at signalized intersections and optimizing if needed, a study was undertaken in the Kumasi metropolis of Ghana [8]. The objective of this study was to determine how queue theory can be used to reduce traffic congestion and consequently travel time in Kumasi.

An M/M/1 queue theory was used to model vehicular traffic flow. This model was used to determine the best times for the red, amber and green lights so that traffic congestion can be reduced.

The M/M/1 was selected to model traffic flow using exponential arrivals and service time with a single server. Data recorded within the working periods of morning, afternoon and evening were collected and used.

The collected data include average number of arrived cars and the corresponding arrival time in (minutes), average number of serviced cars, and the corresponding service time "in minutes" for each set of cars recorded at each period of time. Using the arrival and serviced counts, traffic intensity was calculated by the formula $\rho = \lambda/\mu$, where μ is the service rate, λ is the arrival rate of the customers and ρ is the traffic intensity.

The computed queuing parameters indicated that traffic operational conditions with one service channel from each route or approach are acceptable, as all the computed values of traffic intensity ρ are less than one. The mean time spent in the queue and the mean time spent in the system were found to be almost negligible.

One of the criticism for this study is the limitation of the scope of work as only one separate intersection was considered. To generalize the queue theory application results, it is may be considered more appropriate to include a set of intersections that are subjected to different traffic conditions and operate under different control systems.

Microscopic simulation and calibration of an integrated Freeway and toll plaza model

Ozbay [9] used Paramics microscopic simulation software to simulate traffic at each toll plaza in New Jersey Turnpike in USA. The model was validated using Vehicle to Vehicle toll data at each toll plaza in the study area.

The main goal of this study was to describe in detail the development, validation and calibration processes of an integrated Freeway and toll plaza model and ultimately to build a valid network model of this facility.

The modelling procedure for the toll plaza simulation at this facility involved updating queue length at each time at each lane, setting out the lane choice decision mechanism for drivers and vehicles and finally establishing the criteria for path-based lane choice. The path based lane choice assumptions include the following: - Vehicles start deciding which path to use only two links before the toll plaza link. - The E-pass vehicles do not make a lane change if the queue lengths of the current lane and the target lane differ by two vehicles or fewer.

The model was then calibrated to reflect the existing conditions at the toll plaza facility. All default model parameters were changed to produce realistic data measured or collected at this facility. The calibration process involved driver behaviour, route choice decision, and validation of traffic volumes in terms of average daily traffic (ADT).

The absolute average relative error for the traffic data at peak hours at AM and PM were 8.9% and 9.1% which are considered acceptable.

The results of the study showed that there are few violations for the pathbased lane choice assumptions in the observed data. Most of the drivers adhered to the path they had been using before entering the e-plaza facility.

The impact of introducing the E-Z Pass toll system on the traffic delay was measured by comparing delay time between both scenarios (i.e., with E-Z Pass and without E-Z Pass). The results indicated that time saving at the toll plaza was about 89% by the introduction of the E-Z Pass for a typical weekday peak.

In fact, this study is not comprehensive and there is a need to introduce other factors to study the effect on the lane or path decision choice such as signposting distance, familiarity of the driver with road network and criteria of queue length for the driver to make shift from his lane to less congested lanes. If these factors were incorporated, then the results will be sound and practical. Further research work is needed on this subject.

Evaluation of toll collection performance using traffic simulation

In a recent study aimed at evaluating intelligent transport systems in Gateway Bridge to the South East Queensland region, which provides an access to Brisbane airport and the port of Brisbane, traffic simulation was used to model current traffic flow conditions in this area [10]. The objectives of this study also included constructing a traffic simulation model of the Gateway toll plaza configuration that can be utilised to test and quantify the impacts of some scenarios aimed at increasing the efficiency of the toll system in the study area.

At the time of the study it was estimated that more than 100,000 vehicles crossed the Gateway Bridge every day. During peak hours, the bridge operated close to capacity. Therefore, more traffic delays were expected.

Among many existing simulation packages, GETRAM software was found to be adequate to do the simulation. This package contains two components: TEDI which is a graphic user interface, and AIMSUN, which can be used to model and simulate traffic results from the model. The model was calibrated to reflect the existing traffic flow so it could give accurate results.

Data used in this study represented the worst-case scenario, that is, the highest traffic flow rates on the bridge. The morning peak period was selected. The model was constructed by replicating the toll plaza and the surrounding aerial image was used as background and lane configuration and speed were established.

The toll plaza system at the Gateway Bridge included 8 manual toll booths, 4 automatic toll booths and 6 E-toll lanes. The toll delay and section time, delay time and system throughput were measured at each toll-booth and couple of scenarios were tested using E toll systems. The overall flow system was run and individual vehicle flow were observed and points of congestion were noted. It was found that increasing the volume of HGV had little impact on the overall performance of the system. Also, the results showed that there was no difference in performance between the E-toll and the automated lanes.

The results indicated also that the current toll system was unable to accommodate or support the expected 2011 traffic flow. Extreme congestion and extended delays were expected at manual and automatic toll lane. Finally, the study found that in order to avoid mass congestion and travel delay, E toll lanes shall be used, to improve the efficiency of the system. The study concludes that implementation of a four- or six-lane E toll system would save half of the travel time.

Multiple intersections traffic signal timing optimization with genetic algorithm

Due to the rapid increase in car use, road networks within urban areas suffer from sometimes severe congestion that usually affect the economic development. Without adopting effective traffic signal timing management to improve the performance of signalized intersections, this problem will persist. To achieve such a goal [11]. introduced a traffic signal timing management system to optimize the green time and discharge traffic effectively without much delay.

The traffic signal timing management system employs a Genetic Algorithm (GA) as an optimization tool. The flexibility of this algorithm helps to optimize the complex and dynamic traffic flow within a specified road network.

In managing traffic at signalized intersections, the traffic signal control system is viewed as a group of service agents, and vehicles represent the customer. The customer is considered as being serviced if they are successfully sent to their destination, and the agent will be paid according to the distance travelled. The time spent serving the vehicles represents the cost for the agent. The above idea can be represented by what is called the fitness function which calculates the profit value which is the difference between the **income** and **cost**. In this case, the optimization scheme at each intersection requires optimizing cycle time, green split, phase sequence and offset considering the delay and fluency.

To test the performance, the proposed GATSTM was implemented at single and multiple signalized intersections by conducting many simulation runs. The results of signal timing generated by GATSTM were compared to signal phase timing calculated using the conventional method. The simulation run at one intersection (4 links, 3 lanes) and the analysis of results showed that values for profit, total number of vehicles served and average speed using GATSTM are much better than values obtained using the conventional fixed time method. The results indicated that GATSTM outperformed the fixed time traffic signal plan and the intersection operated under GATSTM will cause less delay, and more discharged flow at a higher speed.

Implementing GATSTM on intersections with different conditions, such as under saturated and oversaturated traffic networks, gave good results. The control strategy employed intersection by GATSTM at under saturated was successful a dominant signal group with the more traffic demand was given the higher green time. The same results were obtained at an over-saturated intersection where it was found that GATSTM releases the link with lowest traffic demand to clear the waiting vehicles before the link with higher demand is getting crowded. In all cases, GATSTM was found to maximize the use of green time by releasing vehicles at higher flow rate.

Optimal signal timing signalised intersections by global optimization

Solving traffic congestion is in the top of the priorities of all roads agencies. congestion is frequently seen at signalised intersection. Therefore, it is required to collect traffic data and measure signal control parameters at signalised intersections accurately. To achieve this goal, a queueing system has been modelled based on the characteristic of the vehicles behaviour approaching and passing of a signalised intersection [12].

The study approach involved development of three models, each tackling a specific case. The first is associated with a signalised intersection controlled by a pre-timed control system in which the random nature of vehicles' arrival was considered. The second was associated with a semi-actuated control system and the third was associated with a fully actuated control system. The intersection under study was of four traffic streams, each of which is controlled by a two-phase traffic signal.

Pre-timed control, where each signal phase or traffic movement is serviced in a programmed sequence that is repeated throughout the day, is mainly used to estimate the optimal cycle length and green time for phases. Semi actuated control, is a traffic management resource installed mainly at intersections where a main street intersects a secondary street, where the main street is always allocated a minimum green time for at least 7 to 10 'seconds' during a signal cycle. The fully actuated control is similar in definition to the semi-actuated control except that both traffic and pedestrian control systems are installed at all approaches and traffic will always be serviced based only on demand.

Fully actuated control with four traffic streams was considered for this case and tested using the SLCP (which is an abbreviation for Sequential Linear Complementarity [12]) formulation. Using Sequential Linear Complementarity (SLCP) formulation, it was possible to calculate the average red, green and cycle length and estimate the signal timing for every cycle of fully actuated control.

With three traffic streams included in study, the average red, green and cycle length were optimised. The signal timing in each test problem was found to reflect the corresponding traffic flow on each stream.

The comparison of the Sequential Linear Complementarity SLCP, WEB-STER (which is a formula to determine the optimal cycle length and green split at isolated intersections regulated by pre-timed control system [12] and LAN (which is a formulation for determining the optimal cycle length in pre-timed control under saturation conditions developed based on non linear regression analysis [12]) showed that the optimal cycles attained by SLCP for the proposed problems cause the shortest delays.

Over all the three models, the pre-timed control was found to accurately estimate the average cycle length and average green time for signal intersections which can be then used to estimate delay. Finally, it was found that SLCP Algorithm always determines the optimal solution and required reduced computation time.

Effect of flow ratio on signalised intersection control strategies

Oluwatosin [13] aimed at investigating the effect of directional flow ratio on the effectiveness of signal timing, many case studies were carried out in Skudaui town, Johor Malaysia. In these studies, major delays for traffic flows with different directional splits of 20/80, 30/70, 40/60 and 50/50 were computed and thoroughly analysed.

In order to run a signalized intersection there are two strategies: staging is taken as vehicle discharge per arm regardless of the direction of the movements of the vehicle, whereas phasing means movement of vehicle from more than one arm with the overall objective of minimizing conflicts and avoidable delays.

To achieve the objectives of this study, automatic traffic counters were installed at the entry arms or approaches of the intersection for six weeks during peak hours daylight and dry weather conditions to determine variation in traffic volume. The actual green and red times were manually measured. The optimal green time and the delay times were then computed using the appropriate formulas. In addition to the above data, 100 samples for saturated flow at each approach where time for the first three cars recorded were collected.

The data analysis showed that passenger cars are the dominant type of vehicles at all approaches. The computed saturation flow and headway for directional flow for site 1 were found to be 1875 pcu/hr and 1.9 s respectively. The loss time per arm was found to be approximately 2s. Cycle time for the existing traffic condition at split 70/30 was also computed and found to be around to 218s. When the direction flow was changed to 60/40, the optimum cycle time was reduced from 218s to 90s with cycle time reduction of about 59 percentage. Similar results were obtained at other approaches.

The findings of this study showed that cycle time for stage movement is higher than phase movement. It was also found that higher delays may result where directional split ratio is 60/40 or 50/50. Finally, the hypothesis that directional split has an effect on signal timing and control strategies for signalised intersection was supported.

Methods for green times allocation in under saturated signalized intersection

Different methods have been developed by researchers and used to optimize traffic timing and determine the best signal cycle length. in this specific sense, a study was carried out to compare different methods for green timing optimization and green time allocation [14]. The aim of this study was mainly to determine the best method that can be to used to allocate green periods for different movements and combinations of movements for traffic vehicles from intersection approaches. The objective function's dependence on the size of green times and the strategy to obtain an optimal value were also studied.

Four optimization goals were studied. These methods are: equal degrees of

saturation of critical movements, minimum delay per vehicle, equal delay for users, and minimum sum of saturation degree. A simplified case of signalised four-way traffic intersection was considered in this study. The signalized intersection has two phases and only straight transit of vehicles is being allowed.

Considering the delay variable, the analysis showed that the lowest values for delay were obtained using the second method (minimum average delay). Similar results were obtained using the first method (equal saturation degree). The remaining other two methods gave un satisfactory results.

Considering the average duration in relation with flow ratio in one direction obtained in the interval (0.35, 0.45), the analysis showed that all four methods give values between 34 and 34.5s, which means that all optimization methods yielded similar values. Finally, it was found that the first method gives values for flow ratio that are very close to the values obtained by the second method but which was the preferred method as it is the easiest of all four methods to implement.

Simulation optimization: methods and application

In the last few decades, optimization using simulation has received a growing attention in many engineering and science applications. Optimization by simulation is basically a process in which a mathematical model of a system is used for a fixed period with the aim of minimizing resources spent while maximizing the benefits and information gained from simulation experiment. The objectives of this paper is to provide knowledge about the area of simulation optimization including the most common methods and various practical applications [15].

Up to date, there are six major categories of simulation optimization methods. These are:

1. Gradient-Based Search Methods: Methods in this category estimate the

response function gradient to assess the shape of the objective function and employ deterministic mathematical programming techniques. The estimation methods include: Finite Differences method, Likelihood Ratios, Perturbation Analysis and Frequency Domain method. In each method, the inputs variables are analysed differently to obtain an estimate for the gradient of the objective function.

- 2. Stochastic Optimization: Stochastic optimization is the problem of finding a local optimum for an objective function whose values are not known analytically but can be estimated or measured. This type of optimization includes optimization algorithms that are iterative schemes based on gradient estimation.
- 3. Response Surface Methodology (RSM): Response surface methodology is a procedure for fitting a series of regression models to the output variable of a simulation model (by evaluating it at several input variable values) and optimizing the resulting regression function.
- 4. Heuristic Methods: It is a method that can be used to solve problems in a faster manner than computing. It describes a rule or a method that comes from experience. This method is used frequently for simulation optimization.
- 5. A-Teams: An A-team (asynchronous team) is a process that involves combining various problem solving strategies so that they can interact synergistically.
- 6. Statistical Methods: Methods of collecting, summarizing, analysing, and interpreting variable numerical data. These techniques are widely used in the life sciences, in economics, and in agricultural science. They basically require determination of the output variable value at a given input

parameter vector value.

Using the above simulation methods and algorithms, many simulation softwares were developed so that the optimal or near optimal solutions for problems can be found in minutes instead of performing exhaustive examinations of relevant alternatives in days or even months. The area of optimization by simulation is growing at a noticeable rate and currently it is considered and used in many scientific and engineering applications. The simulation optimization methods have been used to different applications with a single objective, applications of multiple criteria and applications with non-parametric objective.

Queue discharge flow and speed model for signalized intersection

Most traffic studies concentrate on the traffic discharge flow rate, without paying attention to the traffic discharge speed. This study comes within the context by tackling this issue and outlining the benefits obtained from considering the traffic flow speed [16].

Data from 18 intersections in Melbourne and Sydney, Australia, have been collected during morning and afternoon peak periods and analysed to describe traffic performance parameters such as queue discharge flow rate (cars per minutes), saturation headway, speed, jam spacing, queue departure, response time, queue departure wave speed and acceleration characteristics.

Results obtained using exponential modelling, which is the type of modelling that describes discharge flow rate and speed relationship, were compared to results obtained using simple modelling, which is based on the use of constant saturation flow and effective green time assumptions.

It was shown in this paper that exponential queue discharge flow rate modelling is more realistic than simple modelling, and accurately estimates delay and queue length and has implication on signal timing optimization. One of the novel contributions of this study is to assume that saturation speed, headway and spacing between vehicles as they pass the stop line remain constant during the saturated part of the green period.

It was found that, using this new method, the flow rate, speed and queue parameters can be observed by microsimulation and consequently, the queuing reasonableness and accuracy can be assessed. The modelling of queue discharge speeds and queue discharge headways allows analysis of many features of adaptive traffic signal control.

It was found that saturation speed, headway and spacing between vehicles do not change as they cross the stop line of the intersection and remain fairly constant. This means that the vehicles do not accelerate when they depart the stop line of the intersection and maintain the same speed until they clear the intersection area.

Based on the outputs of this study, it was observed that there is a need to perform more surveys to calibrate the model to describe traffic flow at paired intersections, CBD type intersection sites, arrow controlled right-turns, analysing of heavy vehicle effects, the effect of jam spacing, jam gap length, and accelerating characteristics at signals. Data on heavy vehicles can be included to investigate the effect of jam spacing and jam gap as well as other flow characteristics, in particular during off-peak time.

Simulation-based multi objective evolutionary approach for traffic signal design

Traffic signal design methods are proposed using simulation-based multi-objective evolutionary approach (MOEA); this innovative approach has an advantage that the numerical simulators and the optimization solvers can be changed and used easily for other applications. The proposed approach is based on using two components, NSGA-II, an optimization solver, and CORSIM, a microscopic traffic simulation tool [17].

A road network of three intersections, with 17 nodes, 30 links and 3 signal intersections were studied. The purpose of this case study was to optimize traffic signal and green time allocation by minimizing total delay and maximizing traffic throughput in the network.

The distribution of the Pareto front, which graphically shows the relationship between total delay measured in sec-veh and throughput traffic volume, was analysed.

The analysis of the above network showed that this approach provides an optimized solution for a set of intersections where the delay is at the minimum level while maximizing traffic volume discharged from the intersections within the same timing plan.

Optimal traffic light control for a single intersection

A study considering a single signalised intersection of two lanes two-way was carried out in order to describe the evolution of queue length with time on each lane. The study aimed also at determining the traffic light switching scheme that optimizes the green time at each approach and minimizes the delay [18].

The researchers were able to compute the optimal traffic light switching scheme that minimizes the average waiting time and the queue length at each lane based on the arrival rate and the maximal departure rate of the vehicles at each lane/approach.

In order to achieve the objectives of the study, a model that describes the system under analysis was constructed. The model considers an intersection that consists of four lanes $(L_1, L_2, L_3 \text{ and } L_4)$. On each corner of the intersection there is a traffic signal $(T_1, T_2, T_3 \text{ and } T_4)$. For simplicity, the model assumes that the traffic light signal is either green or red. The model also

assumes that the average arrival rate of vehicles in L_i is λ_i and the average departure rate of vehicles in L_i is μ_i .

One of the novel contribution of this study that it allows the red-green cycle length to vary from one cycle to another, and the traffic is operated in unsaturated condition where the queue length may be equal to zero during the green time.

The results of this study showed that a model that describes queue length evolution can be derived at an intersection of two-way streets with traffic lights on each corner, and the optimum traffic light switching scheme can be easily determined based on this model.

It was also shown that for an objective function that depends on queue length, the optimal traffic light switching scheme can be computed effectively. Despite the successful development of the model that was used efficiently to determine the optimal switching time scheme, the study did not consider the complete cycle time (i.e., all red phase and an amber phase). In addition, the results of this study may be applicable on a single intersection but it needs to be expanded to a network of intersections.

Modelling overflow queue on urban signalised intersection

Much research effort is paid nowadays to developing more efficient control systems for signalised intersections, to reduce delay and provide good traffic operational conditions, in particular, in urban areas. In this regard, a study was conducted [19], aimed at developing a novel heuristic formula that is able to model and solve the queuing behaviour during an oversaturated period on undersaturated periods. The study aimed at building a framework of a dynamic optimal control policy using microsimulation model by focusing on queuing model instead of a delay model.

The methodology adopted in this study includes generating data using

microscopic simulation technique by modelling the delay using a Markov Chain process. Data on delay calculated using this process were used to calibrate the new model.

Three techniques to simulate traffic performance at intersections were described and used in this study. The first is Macroscopic simulation which deals with traffic flow as a continuous medium, while the second is Microscopic simulation models the elements of traffic flow (i.e., vehicles) as separate entities. The third is Mesoscopic simulation model, which uses probability distributions for the microscopic states. Among these techniques, microscopic simulation is usually the preferred method as it can compute the delay cost for each vehicle.

The analysis of different traffic cases showed that Markov chain processes can provide data comparable with data produced by microsimulation programs and under variable traffic demands. It was also found that Markov chain processes are useful in assessing the role played by the random nature of queue formation and how it affects the delay time at signalised intersection in urban areas. Based on this understanding, a time-dependent queue function could be developed.

Finally, the study showed how the queue can be modelled to give more accurate results than using simple deterministic methods.

Despite the good results obtained in this study, the authors expected that better results could be obtained if cycle length and sub-period length parameters were included.

Performance comparison between queueing theoretical optimality and Q-learning approach for intersection traffic signal control

In efforts to solve traffic congestion and reduce vehicle delay, different approaches and models offered by queue theory have been tried and investigated. Researchers were found that different models give different results. In this regard, a comparison study to investigate the performance of traffic signal control using M/M/1 queue (a queueing model that represents the steady state condition for traffic where the vehicle arrivals in each direction are assumed independent and follow Poisson process model and each vehicle is assumed to spend exponentially distributed travel time through the intersection), D/D/1 queue (a queue in which the incoming stream of vehicles arrive at a fixed deterministic rate and their service time through the intersection is assumed constant for every vehicle).and Q learning (which is set of models or approaches that aimed at finding the optimal policy that minimizes the total network delay) approaches were studied [20].

The queueing model proposed in this study includes two separate approaches and single server representing two conflicting traffic flows that enter an isolated intersection from different approaches. In this study, λ_1 denotes the traffic arrival rate for direction 1 and λ_1 denotes traffic arrival rate or direction 2. The model assumes that traffic enters the intersection from both directions and is served by a single server with service rate denoted by μ_1 . The ratio of green time allocated to direction p in a single cycle is denoted by W_p .

In real life, the arrival process may not follow a Poisson process and not be exponentially distributed; therefore, the D/D/1 was used, where the incoming vehicles arrive at fixed time intervals and the service time through the intersection is assumed to be constant for every vehicle.

The Cell Transmission Model CTM was used to update the Q-learning state dynamics as it captures the effect of control actions decided by the Q learning on the flow of vehicles in the system.

MATLAB was used to calculate the optimal split obtained from CTM, Q-Learning, the queueing model M/M/1 and the queueing model D/D/1. The Q-learning was found to improve the green time optimality for the intersection throughput by up to 8.3%, and in jamming condition up to 14.8% in comparison with M/M/1 and D/D/1 models. In addition, it was found that Q-Learning can reduce the average vehicle delay per completed trip by up to 63.4% and 80.7% in comparison with M/M/1 and D/D/1 Models respectively.

Timing over-saturated signals: what can we learn from classic and state-of-the-art signal control models

Large numbers of urban signalised intersections in the United States are suffering from oversaturation the morning and afternoon peak hours. For this type of intersection, theories and signal timing policies are not well established yet. It is well known that optimizing traffic flow through best time for green signal will result in a much better traffic system and decrease delay but there is lack of effective evaluation [21].

The main control objective functions adopted by most researchers to solve traffic congestion problems are the minimum total intersection delay and/or maximum throughput or traffic discharged volumes of the intersection. The first objective function is the most common.

In this study, two typical groups of classical models were tested to investigate the characteristics and applicability of the control policies. The main objective of this analytical study was to evaluate the optimization capability of each model through comparing the green time allocated or phase switching policies under oversaturated conditions. Two main traffic signal optimization models included in Synchro and TRANSYT-7F optimization software were evaluated.

The cumulative curve that represents traffic arrival and departures at an oversaturated approach was investigated and used to develop traffic signal plans that minimize total vehicle delay. Control algorithms of the selected theoretical models such as graphical approach, Pontryagin's control model, discrete minimal delay model and linear programming model were all tried and compared to each other based on the outputs of the analysis.

The analysis revealed that there is a necessity of embedding classical models that have clear phase switching policy, with existing signal timing tools. It was found that Bang-Bang control models, Linear Programming Model and Synchro models perform much better than other models of TRANSYT-7 in optimizing the intersection delay time and queue clearance time.

Finally, a set of guidelines to get the best optimization results at isolated oversaturated intersections was introduced.

Oversaturated Conditions is A signalized intersection becomes oversaturated when the traffic demand rate exceeds capacity, causing upwardly trending queue lengths without equilibrium in one or all of the approaches of the intersection.

A traffic signal split optimization using time-space diagram

In an effort to introduce an innovative approach to solve this problem, a new optimization method was developed to minimize delay per cycle [22]. The main objectives of this study were to develop a dynamic split optimization using time-space diagrams, and to investigate the efficiency of the proposed method by conducting simulation experiments at different traffic demand scenarios.

The novelty of this study is that this method can dynamically adjust splits when queues extend beyond detector locations, so the time-space diagrams using shockwave theory and information from detectors installed upstream of intersection can be used. Using the upstream detectors for constructing the time-space diagram from shockwave theory, the split was optimized by investigating whether a small increase or decrease of the subject phase will reduce the total delay per cycle of the intersection. Simulation experiments were conducted with five traffic demand scenarios, ranging from 1750 veh/h to more than 3000 veh/h on a simple two-phase, two-one-way-street intersection. The first four scenarios were fixed-demand scenarios, while the fifth scenario was a variable-traffic-demand scenario.

The results of this experiment showed that using this method, the splits in all scenarios can be gradually adjusted to converge to the optimal green time allocated based on Webster's method. This method was found to be able to optimize splits which eventually converge to the optimal fixed-time signal settings and the algorithm can correctly adjust splits in response to the change of traffic demand.

The use of KREISIG computer simulation program to optimize signalized roundabout

Different approaches can be used to predict road capacities, queue length and delays at signalised intersections and roundabouts. Such approaches may include empirical models, mathematical models and simulation.

To use simulation in traffic optimization, a study was conducted in Yogyakarta, Indonesia for optimizing the traffic signal at Kleringan roundabout. This study relies on software and outputs from an older study done in Germany using KREISIG computer simulation [23].

This simulation software was originally developed by the author of this paper in Germany to calculate queues and delays and optimize signal timing at roundabouts. The software was calibrated to reflect the behaviour of Indonesian drivers with regards to stopping at signal and stop line approaches.

After software calibration, the software was used to optimise signal timing for all approaching arms. Different green times were tested, and delays and queue lengths were calculated for each arm before signalisation and after signalisation. It was found that if the signalisation option at this roundabout is considered, then delay can be reduced considerably at arms 1 and 4 while discouraging results were obtained for arms 2 and 3.

The study showed that the increase in average delay at 1 and 3 arms in the unsignalized case was high compared with the signalized case. Also, the study showed that KREISIG program has been successfully used to set and optimize the traffic signal at Kleringan roundabout, Yogyakarta, Indonesia.

Optimal discharge speed and queue discharge headway at signalized intersections

Saturation flow is considered one of the most important parameters in signal timing optimization and in the estimation of intersection capacity. Therefore, most road agencies follow the principle of continuous re-timing for the signals at intersections in order to improve traffic flow discharge, reduce delay and eliminate congestion of the national road networks.

In optimization processes, signal cycle length, offset and spilt are considered the main three parameters to be optimized when conducting signal re-timing. This study was aimed at optimizing timing of signalized intersection based on determining the optimum discharge speed so that different trends in saturation headway may take place. The study focused on a busy intersection, Secor Rd/ Monroe in Toledo, Ohio in USA on which the speed limit is 35 mph. Data was collected using an iPhone 5 for video recording the data included twenty queue positions [18].

In this regard, there are always three cases that are considered when studying the relationship between headway and traffic discharge speed. If the discharge speed reached the maximum speed which is at or just below the optimal speed, then the headway remains constant. If the discharge speed continued to increase but never exceeded the optimal speed, then the headway showed a compression trend. Finally, if the discharge speed exceeded the optimal speed, then the headway showed an elongation trend.

To determine which trend the traffic flow takes at this intersection, charts were drawn for queue position related with average headway, queue position with average discharge and average discharge rate with average discharge speed. There was an inverse relation between average discharge speed and flow rate and the average headway.

In this study, it was found that the discharge headway of through lane traffic is of an elongation trend as the relationship between queue position and average headway showed that the headway reached the minimum at a bit after the midpoint of the green time but increased afterwards. The study at this location showed also that the average discharge speed is optimal at the same point.

Traffic congestion evaluation and signal timing optimization based on wireless sensor networks: issues, approaches and simulation

Worldwide, traffic congestion at intersections in urban areas are considered one of the main issues that affect the economic development of the country and level of service of urban residents. It usually create a lot of discomfort, financial loses and poor level of service in transport facilities. This paper introduces a new model and algorithms for traffic monitoring and signal timing optimization that can estimate traffic parameters, estimate a congestion factor to the optimization model of signal timing, report traffic status and finally predict sub-critical state of traffic jams before its formation [24]. This method is mainly based on using wireless sensor networks that collect the data, analyse it, and report on expected traffic jams.

A traffic congestion factor that indicates the general congestion state on the whole road segment on a scale from 0 to 1 was calculated considering an intersection with a four-phase signal. Taking a straight lane and considering traffic flow in two scenarios, one running continuously with no interruption, and the other blocked by red signals at certain times, the congestion factor and cost functions were simulated. the results indicated that congestion factor changes by location and it was found that congestion factor denote the congestion extent of a road segment.

The simulation result shows that this method can improve the spatialtemporal resolution of traffic data monitoring, and can be effectively used to alleviate urban traffic congestion which in turn decreases the average delays and maximum queue length.

One of the shortcoming of this study is that it consider only a simple road segment. More complex road network with all geometric features such as ramps, loops, multi-leg intersections subjected to more complex traffic flow shall be considered in any further research.

Optimal signal control for pre-timed signalised junctions with uncertain traffic: simulation based optimization approach

Most signalized junctions still use pre-timed traffic control. This is because real-time traffic control costs more and needs maintenance, as detectors and processors are needed to calculate the optimal signal timings for each cycle. This paper offer a simulation-based optimization approach to find the optimal green times for pre-timed signal control, in order to minimize the average delay per vehicle [25].

Ten thousand flow profiles were simulated and after that the optimization model was solved to obtain the optimal green times and corresponding delays for each flow profile. The optimal green time was calculated from Gravity Location Model (GLM) (which is an analytic tool widely used for modelling bilateral flows between different geographic entities and measuring the flow from origin i to destination j. In order to minimize the average delay per vehicle, The Bonmin solver [26] was used, taking into consideration the constraints such as cycle length and green times. The Highway Capacity Manual [4] was used to calculate the delay for each movement.

The robustness of the approach is tested using oversaturation and undersaturation cases proposed by [27]. Comparing the average delay for the ten thousand flow profiles using reported green times between this study, the model developed by Yin and the model developed by [28], it was found that this study's model reduces the average delay per vehicle by 4% for oversaturation case, and 3% for the undersaturation case. Also, the model proposed in this study was found to be less time-consuming and can complete the analysis using less than half of the time consumed by the other two approaches.

VISSIM Network Simulation under actuated signal control

In this study, VISSIM is used for a signalised multi-intersection in China to investigate the effectiveness of traffic organization and show the advantages and disadvantages of different actuated-signal controlled-intersection models [29]

The actuated-signal control systems are those systems in which the signal cycle, green time and other parameters are set in advance based on the past traffic conditions and counts. Such systems may cause delay at certain times during the day as they do not respond to changes in traffic demand at different intersection approaches.

To overcome this problem, the variable unit green extension time model was used instead of the fixed unit green extension in order to reduce the wasted green signal time under the actuated signal control with heavy traffic conditions.

The study was applied on 12 signals—individually controlled by the trafficactuated strategy—using three phases. The input parameters for VISSIM were calibrated to reflect traffic flow characteristics and driver behaviour, as these parameters will affect the simulation results.

The delay data for the network was determined through VISSIM simulation and showed that single intersection simulation results are different from road network simulation results. using road network simulation results to judge the merits of the control scheme is more convincing. Also, sometimes, the road network simulation results show that the original control scheme of single intersection has quite different design requirements. This indicates that there is no consistency in the results generated by this model when it is applied to a single intersection or a network of intersections.

Dynamic tolling of HOT lanes through simulation of expected traffic conditions

Nikolic [30] developed a dynamic tolling framework that utilizes micro-simulation of short term interval on High Occupancy/Toll Lanes (HOTL) based on expected traffic conditions. The tolling rate schemes can be updated based on future traffic conditions.

High Occupancy/Toll Lanes (HOTL) were introduced recently in the field of traffic management and mainly aimed at reducing the delay on congested roads. They are considered one of the effective means of increasing the effective capacity of a highway corridor.

In order to establish the dynamic tolling framework, certain assumptions and initial values for simulation parameters were set. The initial setting values were established based on the authors' previous experience. The toll rate changing interval and toll rate were also set on selected road sections approximately two kilometres in length. Target performance parameters for this HOTL were set considering speed (with 70 km/h as a target speed) and density measured on these road sections. An algorithm for setting the toll rates dynamically based on traffic conditions and operating speed was developed for each section of HOTL for 5 minute interval. The developed algorithm has the capability to test scenarios under both constant and variable toll rates. For sections with speed above 70 km/h, toll rates for HOTL sections are designed to vary until the maximum decrease in toll rate per interval is reached or the operating speed drops below 70 km/h. For sections operating with speeds below 70 km/h, the algorithm increases the toll rate until either the maximum increase in toll rate per interval is reached or the speed reaches the 70 km/h minimum limit.

The implementation of the above algorithm showed that HOTL utilization increased the conversion rate from HOVL to HOTL at different occupancy at different levels and the proposed dynamic tolling system was successful. Considering traffic speed on HOVL and HOTL versus the GPL's, the results showed that there is improvement in the GPLs speed with the conversion to HOTL lanes, and additional capacity opening up in the HOTLs suggested that higher demand level than those evaluated might be accommodated.

The results showed that maintaining the minimum speed of 70 km/h, which was set as a performance limit, at least 90 percent of time during the peak hours was achieved in most of the cases.

Drivers' willingness-to-pay to reduce travel time: evidence from the San Diego I-15 congestion pricing project

Brownstone [31] conducted a study aimed at estimating how much drivers are willing to pay to avoid traffic congestion on highways and save time. This study took place in Interstate 15 in San Diego. Drivers were asked to choose between three alternative route choices:

- 1. Solo driving on the general purpose lanes,
- 2. Solo driving on the express lanes,

3. Carpooling; based on the estimated traveling time and the given toll price.

The data was collected from a panel survey of drivers who used I-15 in the morning when the express lanes were open. Individual drivers were assumed to have a fixed departure time in response to the change of congestion and toll fee. Over a time period of a month in collecting the survey data, it was found that the drivers are willing to pay roughly AUS \$ 40 for one hour time reduction. The study concluded that the value of roughly AUS \$ 40 was upward biased, because the users on the express lane also had the benefit of safety, since the express lane was separated from other lanes.

Second-best congestion pricing in general static transportation networks with elastic demands

Verhoef [32] conducted a thorough study to evaluate the benefits of implementing the second-best optimal congestion pricing for general static transportation networks where tolls can be charged only on a given subset of links. He assumed the first-best problem, where tolls can be applied on all links, as a special case of the general second-best problem. In this general problem, it was assumed that the road authority apply tolls on certain links so as to maximize the total social welfare, which is defined as total benefits minus total costs subjected to a set of constraints. The maximization problem was considered as a bi-level optimization problem where the regulator aimed at maximizing the total social welfare, given that road users tried to maximize their own benefits as well.

The general second-best problem was defined as a standard non-linear programming problem for interior second-best optimal toll, which was as an optimal toll for which the set of relevant paths does not change due to marginal changes in any of the tolls available. Verhoef demonstrated in this study the theoretical possibilities of the existence of multiple local second-best optima and raised a question about the likeliness of these complications to occur in practical life.

A small representative example network was given to show that interior second-best optimal toll need not always exist and if they exist, need not always be unique. He concluded that it was not a major concern for practical applications since it only can happen if tolls are placed on roads that drivers already rarely use.

A feedback-based dynamic tolling algorithm for high occupancy toll (HOT) lane operations

Zhang [33] investigated the main problems that may degrade the performance of high occupancy tolling (HOT) strategies on managed lanes. In their development of the feedback-based tolling algorithms, for optimizing HOT lane operations, two main problems of dynamic tolling strategies were addressed. The first problem was the severe response delays in traffic system operations caused by under-sensitive tolling algorithm and the second problem was the severe flow-fluctuation on HOT lanes and general purpose lanes (GPL) due to over-sensitivity of imperfect tolling algorithm or strategies.

The proposed algorithm first calculated the optimal flow ratio on HOT lanes based on information of traffic speed and toll-changing patterns using feedback control theory, and the toll rate was then estimated backward using discrete route choice model. The developed model was found to be simple, practical and easy to be implemented. As the built-in modules in VISSIM do not handle HOT lanes operations, an independent HOT lane functional module was developed through VISSIM Component Object Model. (COM) Interface using Microsoft Visual Basic.

To test the proposed dynamic toll algorithm, a VISSIM-based simulation

study on State Route SR 167 in Washington State was applied. The simulation model was well simulated to reproduce the same existing traffic conditions. Results demonstrated that the proposed tolling algorithm performed reasonable well in optimizing overall traffic operations in HOT lanes under different traffic demands scenarios while maintaining travel speed and reliability of High Occupancy vehicles (HOV) to satisfy operational requirements.

Agent-based traffic simulation and traffic signal timing optimization with GPU

Shen [34] conducted a study in which they tried to optimize traffic signal timing by maximizing the number of vehicles leaving network in a specific time. A Multi-Agent System (MAS) model for a road network of four signalized intersections was built, and a Genetic Algorithm (GA) was used to optimize the green time for the traffic signal. To enhance the computing and simulation capabilities, a Graphic Processing Unit (GPU) was introduced into the analysis system along with the Central Processing Unit (CPU). The GPU is a special processing circuit unit designed to offload graphics tasks from the CPU in order to speed up computation and processing work.

A simple network of multi-lane roads subjected to traffic with several phases of traffic lights was analysed using the GPU and CPU. It was assumed that there are vehicles from both directions on this road network and no overtaking or lane changing was considered.

The difference of average time of interactions in Genetic Algorithm with CPU only and that with GPU+CPU was compared and found that both the simulation and the optimization can be accelerated with GPU or CPU as both are successful for parallel computing. This indicate the advantages gained from introducing GPU along with CPU in the optimization and simulation processes.

Optimal toll design problem in dynamic traffic network

Dusica [35] studied the toll design problem under dynamic traffic demand in order to address difficulties and complexities in optimizing the objective function at all network levels. dynamic congestion pricing method with time varying toll over the day was proposed. The dynamic optimal toll problem was presented as a bi-level optimization problem with the road authority setting tolls and drivers responding to the tolls by adjusting decisions of routes and departure time which consequently change the dynamic flow patterns.

The toll pricing problem was formulated as a mathematical program with equilibrium constraints (MPEC) over a network with dynamic traffic demand where only parts of the links were tolled. The proposed model was implemented in three steps:

- 1. loading the dynamic network,
- 2. drivers' route and departure time choice,
- 3. determination of the toll pricing for the selected links.

The study methodology is based on proposing a grid-search algorithm to find the optimal toll that could satisfy the objectives of the road authority while meeting the constraints of traffic assignment. An iterative optimizing procedure was used to determine the optimal toll to maximise the objective function and meets the boundary conditions and constraints set in this problem. Examples were given to demonstrate the application of the proposed dynamic toll scheme. Three different case studies were considered:

- 1. time varying toll with route and departure time,
- 2. route choice only,
- 3. route and departure time choice.

The study showed that pricing may lead to savings in the total travel time on the tolled links when compared to non-toll links. The study also addressed the complexity of modelling and solving varying toll over dynamic traffic network and concluded that it was difficult to find an optimal solution due to the non-linearity and non-convexity of the objective functions.

Distance-based dynamic pricing strategy for managed toll lanes

Yang [36] attempted to maximize traffic throughput and revenue by imposing a toll based on the distance travelled by the vehicle. On these managed or toll lanes, an electronic toll device is available at the entrance and exit of the road lane. The price is usually collected at the exit based on the distance driven. At the approach to these managed lanes, the driver will have to choose between this toll lane and other available non toll links.

If the managed lanes become congested due to a sudden increase in traffic demand, entrance-managed lanes would be temporarily closed until the traffic conditions recovered or the toll would maintain at its maximum. The study adopted a macroscopic traffic flow model in the form of a stochastic partial differential equation model to simulate the traffic evolution. A distance-based simulation algorithm that calculates tolls was adopted in this study to optimize toll prices efficiently in real time.

Li et al. successfully applied a numerical example to demonstrate the process of evolution of traffic flows and determination of the optimal toll rate. They found that this methodology also can fit other traffic flow models. The disadvantage of this process was that drivers will not know the toll price until they are charged.

3

Problem statement

Traffic flow models are used in transportation planning to evaluate the impact of the changes in traffic demand, demography and land use. Drivers' behaviour is introduced into these models through the sequence of modelling steps. Time and cost of travel are the key components for all travel models.

Among the main issues that represent a great concern to road authorities is traffic congestion. The early works on this problem using a managed lanes approach introduced static toll strategies. These strategies are based on fixed toll prices, where all road users pay the same price for using the same road [37]. Many optimal static or constant toll rate strategies were developed assuming time-independent demand and traffic conditions and considering the effect of toll rates on drivers choice between managed lanes and general purpose lanes [38, 39].

In order to seek more flexible traffic management systems, researchers have proposed different approaches and strategies to mitigate congestion taking into consideration the dynamic nature of the traffic flow. Among these solutions is the dynamic toll pricing approach. **Dynamic toll pricing** is defined as a pricing strategy in which toll prices vary over time to match the dynamic nature of traffic conditions while maintaining a minimum level of service in terms of speed and density.

Dynamic traffic assignment models are models that describe timevarying networks and the interaction between traffic flow and drivers' behaviour. They represent the interaction between driver travel choices, traffic flow, time and cost in a coherent manner. Dynamic traffic assignment models have attracted recent attention for traffic congestion management due to their ability to model traffic flow and account for time-varying properties of traffic flow [40].

Dynamic congestion pricing is a complex topic as it includes many variables related to traffic, drivers' behaviour and finally the targeted objective of the road authority. In the last two decades, road authorities have shown great interest in developing effective pricing strategies and policies for traffic congestion relief. Different techniques for predicting the impact of such policies on traffic flow also have improved in recent years. However, these formulations generally lead to extremely complicated solution procedures in traffic analysis, traffic management and traffic pricing schemes. Nevertheless, progress has been made using techniques such as simulation for solving the congestion problem on large networks [41, 42].

Moshe [43] considered that the objective of the combined control-assignment problem is to find a mutually consistent dynamic system-optimal signal setting and dynamic user-optimal traffic flow. The combined control-assignment problem is first formulated as a one-level game: the road authority and the drivers choose their strategies simultaneously. The toll pricing problem may be formulated as a *Stackelberg* or *inverse Stackelberg* game, depending on how the toll varies.

In a Stackelberg game, one player (the leader) moves first, and the other players (the followers) move after observing the action of the leader [44]. This describes the toll pricing problem with fixed or time-varying tolls. In inverse Stackelberg game, the leader announces its strategy as a mapping from the followers decision space into its own decision space. This describes the toll pricing problem with tolls responding to traffic flow in a predictable way. The problem in this case is to find the toll that would minimize the total travel time or maximize the total revenue. Thus, the game is between the road authority, setting toll strategy and the traveller where the road authority is trying to reach its goal, and the travellers, who are attempting to minimize their travel costs [45].

Despite the aforementioned benefits of dynamic pricing, there are still problems associated with the implementation of dynamic toll strategies, such as the risk of inaccurate continuous traffic feedback due to the inhomogeneous arrival process and traffic variation over time which may result in some difficulties in the update process for toll prices. In addition, there could be some difficulties of maintaining minimum speed in some uncontrolled conditions such as accidents and unexpected vehicles incidents. Another problem is related to the availability of sufficient funding and the availability of the technology used for collecting traffic data required for the dynamic toll control and determining traffic demand on a continuous basis.

Recent technological advances in this area have shown the ability to effectively solve and control many of the aforementioned problems, in particular, field data collection and analysis, interfacing and communication between different devices and systems and finally the toll price determination at certain time intervals.

In this study, a new dynamic toll pricing scheme is proposed. This scheme not only considers drivers' choice and speed, but also, it incorporates other important factors such as travel time saving and toll price rates. Also, this study considers drivers' route choice between the toll or general purpose lanes at different income levels for the driver and under various toll pricing objectives of the road operator.

The following subsection describe the characteristics of the Poisson process and the directed graph with explanatory example that explain the directed graph structure and principle. the problem formulation that describe the traffic throughput and revenue optimisation formula and the maximisation function are also included in this chapter.

3.1 Poisson process

This project is a methodological study, in which we try to produce an algorithm to devise an optimal pricing scheme, given a road network and a model of arrivals. In the first stages of developing this algorithm, it is sensible to use a simple arrival model such as a Poisson process.

In analysing road networks, vehicle arrivals are often modelled as Poisson processes to simplify the modelling and traffic data analysis. A Poisson process is a simple and widely used, stochastic random process used for modelling the times at which arrivals enter a system such as road intersection [46].

After development, when this model is applied to specific real-world traffic systems, it is likely that more flexible generalisations of the Poisson process may be required, such as the Cox process.

In traffic, the assumptions that make Poisson arrivals valid are mainly; arrival or events occur independently and at random. This implies that no close upstream traffic signals that may cause non-random arrivals are existed.

Let (T_k) be a sequence of arrival times, let the intensity of vehicle arrivals to be λ , we restrict attention to a time interval[0, T], so that $0 < T_1 < \ldots < T_k < T$, where K is the number of arrivals, Poisson distributed with mean $\int_{t=0}^{T} \lambda(t) dt$.

3.2 Directed graphs

A directed graph (or digraph) is a set of vertices and a collection of directed edges that each connects an ordered pair of vertices. We say that a directed edge points from the first vertex in the pair to the second vertex in the pair.

3.2.1 Example

In formal terms, a directed graph is an ordered pair G = (V, A) where V is a set whose elements are called vertices, nodes, or points; A is a set of ordered pairs of vertices, called arrows, directed edges.

This example considers 3 nodes, and 3 links. The graph shown in Figure 3.1 involves three nodes (1, 2, 3) and three links (1, 2), (1, 3), (2, 3) or we can call it a, b, c. The network consists of two paths. Assume that links a, b and c are all identical, thus path P_2 is more attractive for travellers than path P_1 . In this research we investigate the potential savings in travel time if a toll is imposed to path P_2 only, which means tolling only link c.

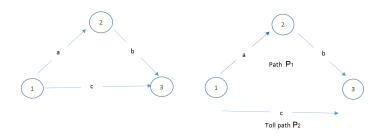


FIGURE 3.1: Network description and route composition-toll and no toll paths

3.3 Problem formulation

3.3.1 Arrivals

To describe the arrival process, the generalised arrival process (GAP) matrix was generated as shown in chapter 5, later this matrix is named as matrix Γ . The results shown in this matrix indicate the random arrival times which are completely independent. The results also indicate that the number of arrivals in a finite interval of length t obeys the Poisson distribution.

3.3.2 Utility function on toll (managed) and general lane

The utility function of driver to chose toll or managed lane can be expressed in terms of total travel time and the toll rate. It takes the following form:

$$U_m(i,t) = -\alpha(i) * c(t) - \beta(i) * T_m(t)$$

Where:

- $\alpha(i)$ denote the rate of change of the utility for a driver *i*;
- c(t) denote the toll rate on the managed lanes at time interval t;
- β(i) denote the rate of change of the drivers' utility for driver i per unit of change of travel time saving;
- $T_m(t)$ denote the travel time on the managed lane at time interval t.

The negative sign of the rate of change of the utility $\alpha(i)$ indicates that when the toll rate **increases**, the utility of the managed lane **decreases**. Similarly, the negative sign of the rate of change of the drivers' utility $\beta(i)$ indicates that as the travel time on the managed lanes **increases**, the utility of the managed lane **decreases**. While the general purpose lane are designed to be toll free. The drivers' utility function can be expressed as follows:

$$U_g(i,t) = -\beta(i) * T_g(t)$$

- β(i) denote the rate of change of the utility for driver i per unit of change of travel time saving;
- $T_g(t)$ denote the travel time on the general purpose lane at time interval t

3.3.3 Throughput and revenue

let *m* denote sets of nodes, *l* denote the tolled links, *i* denote vehicle number i = 1, 2, 3, ..., I. Let v_{mi} be the *m*th node visited by vehicle *i*, let $T(\Gamma, v_{mi}, P_i, i, m)$ be the travel time at which vehicle *i* reaches node *m*. Let C_l be the toll rate at link *l*.

Any variation in the toll rate will affect the drivers' willingness to pay and converge to a new equilibrium after short time. Therefore, let q(C) denote the equilibrium flow entering node m, which should be a function of the toll rate C_l . This function can be written as:

$$q(C) = (q_l, l \in M)^T$$

So, at any time interval t+1, the optimal toll price will be determined based on the prevailing traffic condition and operating speed, let C_l denote the new toll at time t+1; the new toll rate can be calculated as followed:

$$C_i = C_i(t+1) = \max[C_{\min}, \min(C_l(t) + \theta^T q_l, C_{\max})]$$

Let P_i be the probability of route choice,

$$P(i,t) = [e^{U_m(i,t)}] / [e^{U_m(i,t)} + e^{U_g(i,t)}]$$

3.3.4 Maximizing revenue at any time t

In order to maximise the revenue at any time using the toll cost (C) as a decision variable, the following optimisation model is formulated as shown below:

$$\max Z(C) = \sum_{m=1}^{m=M} \sum_{i=1}^{i=I} E(q_l(C)C_l P_m(C, i, t))$$

s.t.

$$S_{lL} < S_l(C) < S_{lu}$$

 $C_{min} \le C_l \le C_{max}$

3.3.5 Maximizing throughput at any time t

In order to maximise the throughput at any time using the traffic flow (q) as a decision variable, the following optimisation model is formulated as shown below:

$$\max Z(q) = \sum_{m=1}^{m=I} \sum_{i=1}^{i=I} E(q_l p_m(i, t) + q_g p_g(i, t))$$

s.t.

$$q_m/q_{\max} \le 1$$

 $q_g/q_{\max} \le 1$

3.3.6 The maximisation formula for both toll revenue and traffic throughput

The following formulation can be used to maximise both the traffic throughput and revenue. In this formula, θ is introduced as a parameter to denote the value of time (VOT) when it is used to maximise the revenue, while it indicates the monetary value of throughput when it is used in the throughput maximisation model. We use θ_1 in the revenue optimisation model, and θ_2 in throughput optimisation model.

$$\max[Z(C) + Z(q)] = \sum_{m=1}^{m=M} \sum_{i=1}^{i=I} \theta_1 E(q_l(C)C_l P_m(C, i, t)) + \sum_{m=1}^{m=I} \sum_{i=1}^{i=I} \theta_2 E(q_l p_m(i, t) + q_g p_g(i, t))$$
(3.1)

s.t.

$$q_m/q_{\rm max} \le 1$$
 $q_g/q_{\rm max} \le 1$

$$S_{lL} < S_l(C) < S_{lu}$$
 $C_{min} \le C_l \le C_{max}$

4 Methodology

This chapter describes the methodology adopted to develop a dynamic toll pricing model that adjusts the toll rates according to traffic conditions so that the traffic flow discharge is maximized and revenue is maximised. We will develop this model using four main elements:

- 1. VISSIM simulation software.
- 2. Driver choice model (logit A).
- 3. External toll lane module for dynamic toll pricing calculation.
- 4. Feedback control mechanism.

The following sections describe the above-mentioned elements in more details:

4.1 VISSIM simulation software

4.1.1 Simulation software

VISSIM is discrete, stochastic, microscopic traffic simulation software that has many built-in modules that can be used to simulate traffic operations on roads under different traffic conditions. The software employs realistic model and has rule-based algorithms for car longitudinal and lateral movements [47].

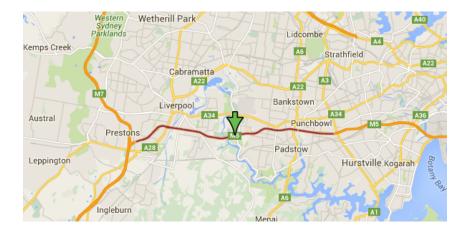
Since it has been commercialised in 1993, VISSIM has gained tremendous attention from various engineering and road authorities for its numerous applications and simulations capabilities. VISSIM is now used by a wide range of engineers and researchers in transport and traffic studies and other engineering projects.

VISSIM was chosen in this study due to its flexibility in modelling any geometric configuration using links and connectors. Also, it has attributes that are related to both driver and vehicle characteristics which enable individual parametrisation. In comparison to other simulation packages such as AIM-SUN, TRANS MODELER, SUMO and many other microsimulation packages, VISSIM is of superior capabilities as it is a multi-modal microscopic simulation package that includes large number of interfaces which provide seamless integration with other traffic management systems using a variety of programming packages such as Matlab, Visual Basic, Java, Python etc.

4.1.2 Study area

The M5 Freeway network located in Sydney, NSW Australia, was selected to be a case study as shown in Figure 4.1. The Freeway segment under study extends from Prestons in the west to Botany Bay in the east with total length of 15km. The road includes two lanes in each side, one with toll and another GPL lane which is toll free, with in and off ramps located at certain distance to allow road users to enter or exit the toll system based on the variable messages revived on traffic density, time saving and speed on the portion of the road ahead.

VISSIM will be used to simulate the response of traffic volumes and density to varying toll pricing rates and speed. To accomplish this task, all in-ramps



and out-ramps and accesses will be identified.

FIGURE 4.1: the location map for M5 Freeway in Sydney, Australia.

VISSIM uses links and link connectors to trace on the background image and establish the Freeway layout including other lateral connections, ramps and intersections as shown in Figure 4.2.

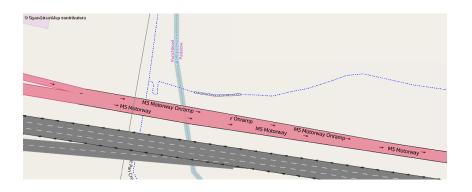


FIGURE 4.2: links and link connectors using VISSIM

The M5 road layout is then drawn on the background image that was generated by downloading the image from Google Maps to match the real road layout. This include the detailed geometric configuration including all in-ramps and out-ramps of the Freeway as shown in Figure 4.3.

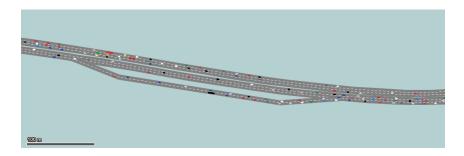


FIGURE 4.3: M5 geometric configuration

For the purpose of the current study, two main built-in modules in VIS-SIM will be used in the analysis of data and optimizing dynamic toll pricing schemes. These modules are the toll-Pricing Calculation Module and the Managed Lanes facilities. Other external module will be developed by the researcher using VISSIM COM programming and the VAB built-in module, which allows the user to control VISSIM's object model directly, using various programming languages such as Visual Basic, Python, Matlab or Java.

The following sections describe each module and the programming requirements, along with the type of data required to run the module.

4.2 VISSIM module

4.2.1 Toll pricing calculation module

VISSIM built-in toll pricing calculation module includes a provision that allows users to utilize average travelling speed on toll lanes and/or travel time saving to control traffic movements and update toll price as can be depicted from the Figure 4.4.

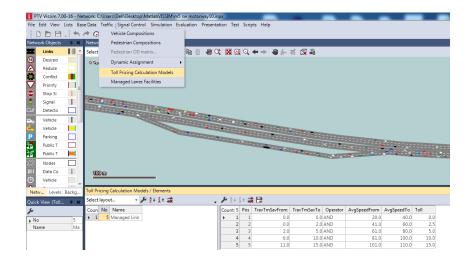


FIGURE 4.4: VISSIMs built-in toll calculation module.

4.2.2 Managed Lane Module

VISSIM's built in managed lane module includes a provision that allows users to utilize toll value on the toll managed lanes and the corresponding time saving to control traffic movement and to update toll price. As can be depicted from Figure 4.5, the user can set up a formula or enter more than one value for time saving and the corresponding toll price to instruct VISSIM how to determine the optimum toll price considering the time saving and the measured operating speed.

4.2.3 Driver route choice model

In order to model drivers' behaviour when choosing whether to drive on the toll lane or on the general purpose lane with no toll, a built-in logit model which models the drivers' decision making is used. The model is represented by the following equation:

$$P(i,t) = [e^{U_m(i,t)}] / [e^{U_m(i,t)} + e^{U_g(i,t)}]$$

• $U_m(i,t)$ denote the utility of choosing the managed lanes for driver i at

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FIGURE 4.5: VISSIMs built-in managed lane module.

interval t;

- U_g(i, t) denote the utility of choosing the general purpose lanes for driver i at interval t;
- P(i, t) denote the probability of choosing the managed lanes by particular driver i at time interval t.

4.2.4 VISSIM vehicular module

VISSIM has a vehicular module that includes a provision for inputting classified traffic types such as trucks, cars, buses etc. Each type has similar geometric and operational characteristics such as number of axles. length, width, and speed distribution.

4.3 External Module for dynamic toll pricing calculations

VISSIM allows general purpose programming language such as Visual Basic, Python, Matlab or Java. The external functional module to be developed in this study will control both dynamic toll lane calculations and simulation, and instruct VISSIM to operate in a certain manner so that the effect of dynamic toll changes on traffic volumes, time saving or delay can be detected.

According to the literature review conducted by the researcher, Matlab is recommended as a first option, as Matlab has many advantages over other programming languages due to the simplicity of the script language and the availability of many built-in mathematical functions. Vectorisation might be another advantage for Matlab, as optimization can be undertaken for operations involving matrices and vectors. These advantages will make it very easy for this external module to integrate well with VISSIMs' main platform through the VISSIM-COM interface [48].

The external dynamic toll calculation module will be designed so that toll price is updated every 5 or 10 minutes (or any other time interval proposed by the road authority). The toll rate will be dependent on the prevailing traffic operation characteristics, such as speed and density on both toll lanes and general purpose lanes, detected from the previous time interval.

4.4 External module dynamic toll calculation process

The following steps describe the process of dynamic toll calculation process as expected to be performed by the developed external module:

- 1. The external toll calculation module is integrated with VISSIM simulation software through VISSIM-COM interface using a script written in Visual Basic, Python, Matlab or Java with priority given to Matlab programming language due to the reason mentioned there before.
- 2. As the external module starts, it calls for VISSIM to run a single step for the toll calculation in the first time interval.
- 3. At the end of the first simulation run, the results on parameters such as average speed and number of vehicles entering/exiting the toll or managed lanes are then exported to the external module from VISSIM database.
- 4. The external module, which includes an algorithm to calculate a new toll rate, will estimate the new toll rate for the next time interval (say 5 minutes).
- 5. The external module sends back the calculation results to the route decision module (Logit module).
- 6. The Logit Model which controls the behaviour of each individual vehicle will then be used to split the traffic flow on the general purpose lanes and toll managed lanes for the next time interval and sends it back to VISSIM simulation model as a new input parameter for the next simulation step.
- 7. The same steps are repeated at the start of every new time interval.

The framework of the proposed model is shown Figure 4.6. The optimization process is performed at the start of each simulation step in order to determine the new toll rate for the subsequent step. A **stepcontrol** is defined as the time between two successive updates which is proposed in this study to be 5 minutes.

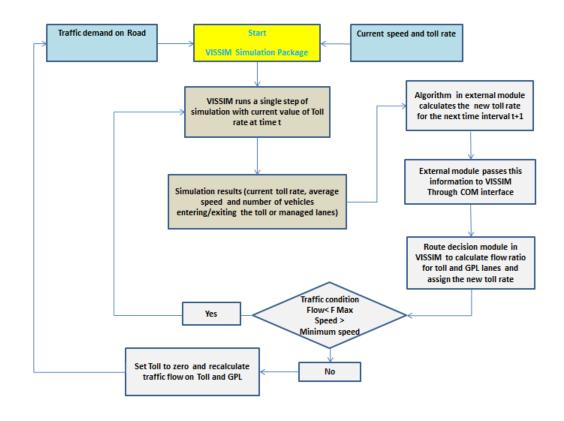


FIGURE 4.6: External module dynamic toll calculation process

4.5 Feedback control mechanism

This process is applicable everywhere in industrial and organization processes. In this study, a feedback control rule or mechanism will be developed in order to calculate the optimal toll rate for each time interval. The aim of this feedback is to maintain an acceptable level of service for the road users and at the same time optimize both traffic throughput toll revenues.

In this study, to develop a feedback control mechanism, the following key performance indicators will be used:

• Average speed on the managed lane. Speed target will be selected in this study as60km/h.

• Travel time saving as a result of using the managed-toll lane in comparison with the general purpose lanes.

$$c(t+1) = \begin{cases} c(t) + \gamma_1(t) * \Delta T(t), & \text{if} S_m(t) > 60 \text{km/h} \\ c(t) + \gamma_2(t) * \Delta T(t), & \text{if} S_m(t) \le 60 \text{km/h} \end{cases}$$

- c(t+1) denote the calculated toll rate for the toll lanes at time interval t+1;
- c(t) denote the current toll rate for the toll lanes at time interval t;
- $\Delta c^*(t)$ denote the change of toll rate;
- γ₁(t) denote the parameter that indicates the change in toll rate per unit
 of travel time saving during the next time interval with speed greater
 than 60km/h;
- γ₂(t) denote the parameter that indicates the change in toll rate per unit
 of travel time saving during the next time interval with speed less than
 60km/h;
- $\Delta T(t)$ denote the travel time saving.

4.6 The willingness to pay on toll lanes

It's strongly believed among the researchers that there is a kind of correlation between the driver income and the willingness to pay for the toll. Therefore it will be necessary to collect the relevant data on the weekly earnings and the yearly income. Figure 4.7 shows the yearly income for the population in Australia. As the road users are considered a representative sample of the population, the data from this figure can be used to evaluate the willingness to pay of every single group considering the yearly income.

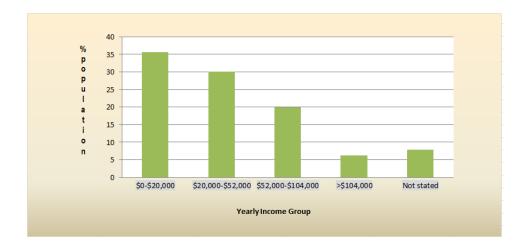


FIGURE 4.7: The income groups for road users in Australia

The value of time (VOT) and the value of toll rate cost (VOTRC) were evaluated based on the income levels for the road users, who are considered as small sample of the population, as shown in Table 4.1. This classification is necessary in order to investigate the probability of taking the toll lanes considering the drivers' willingness to pay, which may be affected by their yearly income.

In this regard it is expected that drivers with high income will give a high value for the time saving and low value for the toll rate cost, while drivers with moderate income will give middle value for the time saving and middle value of the toll rate cost. On the other hand drivers with low income give low value for the time saving and high value for the toll rate cost. However, an additional field survey will be conducted during the PhD study in order to objectively evaluate the willingness to pay and value of time for road users of different income level . The outcome of such survey will provide valuable information on the contribution of such factor on the willingness to pay and the resultant throughput and revenue.

Description	% Population	Road users	Value of time	Value of toll rate cost
		Group	saving (VOTS)	(VOTRC)
\$0-\$20,000	36	1	Low	High
\$20,000-\$52,000	30	2	Moderate	Moderate
\$52,000-\$104,000	20	3	High	Low
>\$104,000	6	4	Very High	Very low
Not stated	8	-	-	-

TABLE 4.1: Value of time and value of toll cost as viewed by Road users income group

4.7 Traffic Data collection

Traffic data from the study area (proposed to be M5 Freeway, Sydney NSW) will be used to study the impact of varying traffic flow on system performance and to predict the variation of traffic flow on both High Occupancy Toll Lanes (HOTL) and General Purpose Lanes (GPL) under various toll pricing scenarios including the zero toll scenario. This data is available on-line from NSW Roads and Maritime websites and is updated automatically on a daily basis using roadside traffic collection devices across the road network of NSW-Australia.

The average daily traffic, estimated in thousands, on the M5, study area was found to be as shown in Figure 4.8. These figures were extracted from a study that estimated the average daily traffic on the major roads in NSW, Australia [49].

Table 4.2 shows the average daily traffic on the M5 Freeway in the year 2016. The up to date traffic data extracted from the on-line traffic viewer indicates that 97% of the vehicle that uses the M5 are cars and only 3% of the traffic is composed of commercial vehicles or trucks. Traffic composition is necessary in this study in order to determine its effect on traffic flow operational conditions.

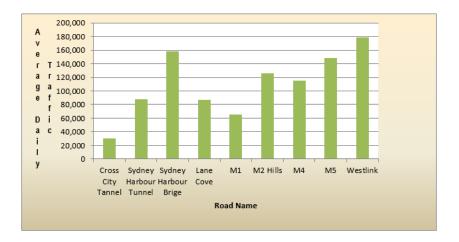


FIGURE 4.8: The average daily traffic on the major roads, including M5, in NSW, Australia

Year	Average Daily Traffic (ADT)	Cars	Commercial	Peak hour volume am	Peak hour volume pm
2016	148, 867	97%	3%	14,800	13,320

TABLE 4.2: Average daily traffic and peak hour volume on M5 Freeway Sydney NSW

4.8 Peak hour volumes and Peak hour period length

In order to optimize the toll pricing over the extreme traffic conditions, it is necessary to collect and analyse peak hour traffic during the day and over the entire week days. The peak hour volumes(PHV), as outlined in Table 4.2, was found to be 13,320 vehicles per hour recorded in the evening and 14,800 vehicles per hour recorded in the morning. Traffic data in terms of average daily traffic (ADT), peak hour volumes (PHV), average travelling speed and traffic intensity is needed to run simulation.

In Sydney, there are two peak periods; the morning peak hour period that extends from 7:30 am to 9:30 am and the evening peak hour period that extends from 2:30 pm to 7:00 pm. This information is required to calculate the probability of using toll lanes and general purpose lanes over the peak hour period for various classes of drivers.

4.9 Toy Model description

To demonstrate the randomness of the arrivals using simulation, a toy model that includes a square intersection that dissipates traffic flow into various directions was created using VISSIM as shown Figure 4.9. Four sources and four destination are assumed. Each node was assumed to represent a source and a destination for a two way roads, while vehicle planned to be three types (cars, buses, and heavy vehicles).

The simulation showed that traffic arrivals is distributed into various directions and in both directions in a random pattern and each node was acting as a source and destination at the same time. The arrival times were also random and independent of each other.

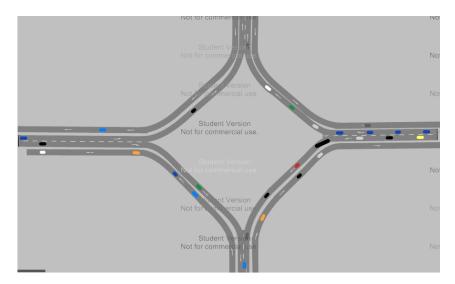


FIGURE 4.9: Traffic flow from sources to destinations as simulated by VISSIM When implementing the proposed toll pricing system in simulation or on

site, detectors or traffic counters will be placed a data collection points at certain locations on both toll lanes and GPL in order to collect real time traffic data or the number of vehicles during the simulation runs. The proposed work mechanism for the toll pricing system is shown in Figure 4.10.



FIGURE 4.10: Toy toll model work mechanism

It is also proposed in this model that toll rates would be updated at 15minute intervals. In order to avoid traffic congestion on toll lanes, a maximum toll rate would be implemented in order to discourage casual use and avoid unnecessary traffic jams on toll lanes in particular during the peak hours periods. Toll rates would change in certain increments with a maximum increase or decrease of a certain limit during any given interval.

During model implementation, a target key performance indicator for the toll lanes is to be set at a minimum speed of 60 km/h, which is proposed in this study. This target speed is to be achieved at least 95% of the time.

This performance indicator is compatible with a toll lane flow rate of approximately 1,500 to 2,250 vehicles per hour per lane which represents the limiting volume that any carriageway lane can absorb before the operating speed begins to deteriorate and traffic congestion starts to form [4]. In Australian

standards, a capacity measure of 1,900 vehicles per hour per lane represents the limiting volume that any carriageway lane can absorb before the operating speed begins to deteriorate below 80 kmph [50].

Therefore, the operating objective used to set toll rates involves achievement of an average operating speed of as close to 60 km/h as possible without dropping below that limit at any time period.

5

Data analysis and simulation results

As stated earlier, a Poisson process, considered one of the most important models in queueing theory, is used to describe vehicles' arrivals at certain points on the roads. Mathematically, the arrival process is described by what a so-called count process N(t). It basically determines the number of arrivals that may occur within a specific time interval, either (0, t) or (t_1, t_2) .

5.1 Modelling traffic arrival process

To describe traffic arrival process, there are many methods that can be used depending on the data and the nature of the dynamic traffic wave movements. Poisson process is often used for modelling count data and it has extensions useful for count models.

Initially, it was decided to use Poisson process due to its flexibility and simplicity of modelling arrivals. However, when this model is applied to specific real-world traffic systems, it is likely that more flexible generalisations of the Poisson process may be required, such as the Cox process.

5.2 Poisson process models demonstration

In order to demonstrates the random arrival process of the traffic flow using Poisson process, a Matlab code was written to describe the generalised arrival process (GAP) as shown in the link http://science.mq.edu.au/~dbulger/ Alwan/ElectronicMaterial.zip. Input variables such as rate change Beta, number of sources for arrivals and number of destination for different types of vehicles were used. The result indicate that the arrival times are random and completely independent. The results also indicate that the number of arrivals in a finite interval of length t obeys the Poisson distribution that takes the following form:

$$P(N(t) = n) = \frac{\lambda^n(t)}{n!(e^-\lambda(t))}$$

5.3 Demonstration of the Toll setting framework

In order to demonstrate the use of the toll lanes framework, a few simulation runs are conducted. As mentioned in the study area section, The M5 Freeway was considered as the pilot study area where the toll lanes facility is to be tested and the response of traffic demand to the set of the boundary conditions and performance key indicators is to be validated.

As shown in Figure 5.1, the toll facility on M5 is a 15 km Freeway of 3 lanes in each section. One lane on each side is dedicated for the experimentation of the toll setting while the remaining two lanes on each side are designated as general purpose lanes. The set-up includes an installed variable message sign (VMS) at 1 km before the entrance to the toll lane at the upstream of this segment. At the downstream end, the toll lane becomes free and therefore, all three lanes are open for free general use with free toll.

The traffic arriving into this segment will be split equally on both the toll and GPL lanes according to the route/lane choice model parameters set up and based on the assumption that when the travel time saving is zero or the travel times on both toll and GPL lanes are the same, then the toll rate is zero.

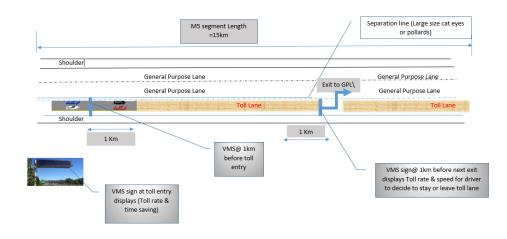


FIGURE 5.1: Toll lane frame work

After establishing the VISSIM-Matlab interfacing, two simulations runs were conducted in order to test and validate the ability of Matlab to control traffic movements in VISSIM simulation framework. The first simulation run was undertaken by writing Matlab code, as shown in the link http://science. mq.edu.au/~dbulger/Alwan/ElectronicMaterial.zip, to determine the arrival counts within a specific time interval on both toll or GPL lanes. The simulation run was designed to push vehicles for one iteration and then stop traffic movement to count the number of arrivals. the simulation run was successful and the arrivals movement in VISSIM were controlled using Matlab code, see Record 1 in the on-line supplementary folder at http://science.

mq.edu.au/~dbulger/Alwan/ElectronicMaterial.zip.

The second simulation run was designed to push cars through Matlab code to VISSIM for multiple iterations and determining the arrival counts in order to simulate dynamic nature of traffic movements by counting the number of vehicles on toll or GPL lanes over more than one time interval so that toll can be adjusted based on traffic intensity so that the toll can be calculated for the next period of time.

The demonstration was successful and it was possible to push cars in more than one time interval and at each interval, the number of cars were counted as shown in Figure 5.2 and Figure 5.3 and also as recoded in the attached video, see Record 2, in the on-line supplementary folder at http://science. mq.edu.au/~dbulger/Alwan/ElectronicMaterial.zip.

Finally, a third simulation run was designed to push cars through Matlab code to VISSIM to split the traffic based on the prevailing operating speed on toll or GPL lanes over more than one time interval so that toll can be adjusted based on traffic intensity so that the toll can be calculated for the next period of time, see Record 3, in the on-line supplementary folder at http://science.mq.edu.au/~dbulger/Alwan/ElectronicMaterial.zip.

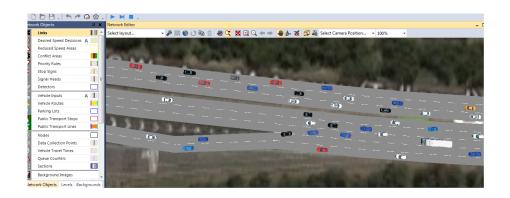


FIGURE 5.2: Traffic assignment between GPL and Managed Lanes

The above two demonstrations represent important achievements towards accomplishing the objectives of this study by controlling the traffic movement in a way that resembles or simulates the actual traffic arrival process and traffic distribution as perceived by drivers and their willingness to pay considering a set of constraints such a toll price, speed etc.

_	
💼 it	1x1 llterator
🕂 ıter	5
🛨 k	100
🛨 res	10
😰 sim	1x1 ISimulation
🛨 vehicleCount	29
🛨 vehicleCountArray	[16,27,29,30,29]
🖻 vehs	1x1 IVehicleContainer
🗊 vis	1x1 VISSIM vissim 700

FIGURE 5.3: Traffic count output as determined through simulation

6 Conclusion and recommended future work

This study is planned to be accomplished in two stages: MRes and Doctorate stages. MRes study mainly include literature review, problem formulation and study methodology. The methodology includes all the modules and algorithms formulated to satisfy the study objectives. Some of the preliminary analysis results for the data collected from the study area with regard to geometric configuration, drivers income levels, peak hour periods, peak hour traffic volumes, and other traffic flow characteristics are included in this study. The outcomes of the preliminary simulation runs performed to establish the communication or the interface between VISSIM simulation package and Matlab showed promising results.

Based on the work accomplished within the time frame of the MRes stage, the following preliminary conclusions can be drawn:

• The proposed methodology in this study develops a new approach to optimize the toll pricing schemes based on dynamic toll pricing principles where the toll price can be adjusted according to prevailing traffic conditions.

- The VISSIM simulation package can be easily interfaced with Matlab and the preliminary runs indicated that the movements of the vehicles (arrival) on both managed and general-purpose lanes can be controlled through Matlab codes subject to a set of rules and constrains of speed and traffic intensity.
- The preliminary runs conducted to establish a successful interface between Matlab and the simulation package VISSIM were found to provide a promising mean to validate theoretical assumptions and optimization modeling through simulation.
- The preliminary simulation runs showed that traffic demand and number of vehicles on both toll lanes and general purpose lanes can be precisely evaluated in any time interval through simulation.

6.1 Intended doctoral work

Please refer to Table 5.1 for the intended work schedule.

- As dynamic toll pricing is a complex topic, it is recommended to expand the scope of work on this topic to include various scenarios related to toll pricing by testing and validating the traffic under performance different tolling scenarios and under various traffic conditions.
- The future work shall also study the impact of applying different toll rates on different vehicles types and evaluate the performance of the traffic demand considering different types of vehicles such as commercial vehicles (trucks) which are usually different characteristics and may require imposing different toll rates to control its usage for the toll lanes.
- Determination the optimal toll scenarios that maximise the revenue and

throughput while maintaining the minimum level of service set in this study.

MRes and PhD Study time line									
	2016 (MRes)		2017 (PhD)		2018 (PhD)		2019(PhD)		
	First semester	Second semester	First semester	Second term	First term	Second term	First term	Second terr	
Literature review (30 papers)									
Abstract and Introduction									
Problem formulation									
Methodology									
Preparation and testing Matlab scripts									
Interfacing Matlab with VISSIM									
Initial runs for VISSIM using Matlab scripts									
Generating traffic demand on both Toll and GPL lanes									
Optimal dynamic toll pricing schemes, writing thesis									
Optimization of both traffic throughput and toll revenue									
Simulation runs for different toll scenarios, writing thesis									
Finishing writing									
-	Accomplished								
	Planned								

TABLE 6.1: MRes and Doctoral study time line

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