Optimal Asset Allocation for Pension Funds in the Republic of Korea (ROK): A Risk-Adjusted Analysis

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

This thesis examines various asset allocation schemes for retirement pension funds in the Republic of Korea (ROK), particularly constrained and risk-adjusted asset allocation schemes and so-called ‘target risk’ and ‘target return’ schemes. This thesis shows that different asset allocation techniques have a significant impact on the optimal weights of different asset classes such as growth and defensive assets for optimal pension portfolios. The thesis also demonstrates that the proposed optimal asset allocation typically deviates significantly from the actual allocation of the ROK’s pension funds. The thesis also examines the performance of different asset allocation schemes in the presence of skewness and excess kurtosis for returns as well as non-linear dependence across different asset classes. Thus, this thesis considers the entire distribution of the returns that enables us to conduct a risk-adjusted performance analysis. Finally, the author relates the results to recent regulatory changes in the ROK with regards to the asset allocation for pension funds.

Keywords: Pension funds, Optimal asset allocation, Risk-adjusted analysis, Defined Benefit and Defined Contribution plans, Regulatory changes

STATEMENT OF ORIGINALITY

This work has not previously been submitted for a degree or diploma in any university. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Signed: ________________

Date: 23 April 2018

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Soo Heon Kim

LIST OF ABBREVIATIONS

AUM	assets under management
CAPM	capital asset pricing model
CVaR	conditional VaR optimal portfolio for specified target return
CVaR-Skew	CVaR optimal portfolio for specified target return with skewed and leptokurtic returns
DB	defined benefit
DC	defined contribution
DR	downside risk
ERBSA	Employee Retirement Benefit Security Act
EU	European Union
FSS	Financial Supervisory Service
GFC	Global Financial Crisis
IQR	interquartile range
IRP	individual retirement pension
MVO	mean-variance optimisation model
MIVO	minimum variance optimisation
MinVar	Minimum Variance portfolio
MSB	Monetary Stabilization Bond
MSCI AC World	MSCI's all country world index
NPS	National Pension Service
OECD	Organization for Economic Co-operation and Development

MaxSharpe	portfolio that yields a maximum Sharpe ratio
TargetReturn	portfolio with specified Target Return
TargetRisk	portfolio with specified Target Risk
Rogers ICI	Rogers International Commodity Index
ROK	Republic of Korea
SAA	Standard Asset Allocation
UK	United Kingdom
US	United States
VaR	Value at Risk

Chapter 1: Introduction

1.1 Background: the ROK's Current Retirement Pension

The average life expectancy of a Republic of Korea (ROK) citizen has increased from 62.7 years in 1970 to 81.3 years in 2015 (Korea National Statistical Office 2016), 11.3 years more than the world average (United Nations 2013). As part of an effort to prepare for an aging society, the ROK Government enacted the Employee Retirement Benefit Security Act (ERBSA) on 1 December 2005, which required all companies operating in the ROK, regardless of nationality, to provide pensions for their employees by 2022.

A major stipulation under this legislation is that companies operating in the ROK are now prohibited from paying their employees' early redemption for retirement allowance.[1] Kim and Hong (2013) predicted that due to the new regulations the ROK's retirement pension reserve would rapidly increase to approximately 382 trillion won (approximately 382 billion Australian dollars) by 2020 and to 2,122 trillion won (approximately 2,122 billion Australian dollars) by 2050.[2]

Another significant change in the ROK's retirement pension system is that the previous two-tier pension system has been upgraded to a three-tier pension system: social security (first tier), retirement pension (second tier) and personal annuities (third tier). As a result, individual employees are more actively engaged in deciding their own pension plan and companies are required to reserve contributions in the form of cash instead of relying on conventional accounting or bookkeeping practices.

1.2 Features of the ROK's Retirement Pension

Rauh (2006) highlighted that United States (US) companies holding a defined benefit (DB)[3] plan are required to make contributions to their pension funds, with higher contributions required if a pension liability is greater than its asset. Unlike the traditional US DB plan, the ROK's DB plan does not require companies to pay annuity after resignation:

> The level of benefits under subparagraph 4 of Article 13 shall be set in a way that ensures that the amount of lump-sum benefits calculated based on the retirement date of a pension holder is

[1] The ROK Government announced the amendment of the pension regulations on 28 August 2014.
[2] At the time of this thesis, 1 AUD = 1,000 KRW.
[3] Defined benefit (DB) plan: Plan sponsors (usually employers) take a responsibility of investment outcomes while plan participants (usually employees) take a responsibility of investment outcomes in defined contribution (DC) plan.

> equal to or higher than 30 days of the average wages for each year of his/her consecutive service. (ERBSA, Chapter III Defined Benefit Retirement Pension Plan, Article15 (Level of Benefits))

In other words, companies have been responsible only for the retirement allowance generated during the service years of their employees under the ROK's DB plan, which reflected a much shorter term of DB asset management due to the absence of annuity payments that extend beyond an employee's service years.

With regard to defined contribution (DC), the ROK's DC plan guarantees a minimum contribution rate of 8.33%, equal to or more than the amount of a one-month salary relative to the total annual wage of a pension holder:

> An employer who has set up a defined contribution retirement pension plan shall pay in cash contributions amounting to one twelfth or more of the total annual wages of a pension holder into the account of the pension holder under the defined contribution retirement pension plan. (ERBSA, Chapter IV Defined Contribution Retirement Pension Plan, Article20 (Levels of Contributions to Be Borne and Payment, etc., of Contributions))

When it comes to mandatory contributions, the ROK's DC plan is similar to Australia's DC superannuation plan, because it requires companies to provide employees with a pension plan regardless of their voluntary contribution. In contrast, the US 401(k) is designed more flexibly to the extent that companies do not have to cover all employees. Instead, they can offer employees the option to match DC or not. While a mandatory DC plan may be more expensive to companies than 401(k), it offers a sense of security to employees because the benefits are defined by fixed factors (i.e., income) and investment performance regardless of employee contribution. For this reason, it has been suggested that Australia's superannuation approach could serve as a benchmark for the ROK's DC plan as both adhere to the same principle.

According to Willis Towers Watson (2017), the compound annual growth rates of DC reserve is 5.6% while that of DB reserve is 2.6%. This indicates that either more companies have adopted a DC plan than a DB plan, or the portfolio returns of DC plans is higher than that of DB plans, ceteris paribus. Accordingly, companies in the P7 countries[4] tend to favour a DC reserve which will contribute to reducing their pension liability, thus making pension management much simpler.

Under these circumstances, DC participants rightfully began to express increasing concern over their DC portfolios. In response to such concerns, this research will analyse and discuss

[4] Australia, Canada, Japan, Netherlands, Switzerland, the United Kingdom (UK) and the US.

optimal asset allocations in the ROK under the mandatory pension plan with evidential support for both DC participants and DB sponsors. To determine optimal asset allocation strategies, target return and target risk investment strategies will be closely examined in this thesis.

Faced with an aging society, DC participants are required to enter a longer asset management phase. Korniotis and Kumar (2011) reported that older individuals exhibit worse investment performance from direct equity investing. They also found a deteriorating trend in investment performance for individuals residing in the elderly, less educated and low-income categories. Therefore, it is essential to provide an optimal portfolio for DC plans to ensure more DC participants can equally derive benefit from their pension plans regardless of educational background, income status or financial literacy. Possibly such optimal portfolios will provide returns that exceed the rate of salary increase of DB plans.

Moreover, Huberman and Jiang (2006) highlighted that there was an increase in the proportion of the equity of 401(k) account, as the proportion of the relative weight of equity on the mutual fund line-ups that companies offer to their 401(k) participants increases. This implies it is more likely that the proportion of equity inside pension investment line-ups is higher as the equity in portfolios held by mutual funds has also increased.

If more appropriate portfolio construction is possible, the cost that occurs in the difference between salary increase rate and investment performance in DB plan can be reduced, and the total amount of the employees' pension can increase as much as the rise in the investment performance in DC account. The remainder of the thesis is divided into five sections. In Chapter 2, we discuss the research questions that will be examined in the thesis. Chapter 3 review the literature on finding optimal investment strategies using different asset classes, taking into account the performance of an investment portfolio as well as aspects of risk management. In Chapter 4, we examine empirical data, using risk-adjusted analysis with regards to target risk and target returns in addition to conditional value at risk (VaR) analysis. We provide results on the performance of various optimal asset allocation strategies among different investment schemes. Chapter 5 examines the results of ex-post analysis running the rolling window method. Chapter 6 concludes this thesis, providing a summary of the findings and a discussion of the appropriateness of the current investment limit set by the ROK Government. The chapter also suggests future directions for research and recommendations for DB funds and asset management companies.

Chapter 2: Motivations

2.1 Research Questions

This thesis poses three research questions regarding retirement pension funds in the ROK:

1) What is currently the typical asset allocation and performance of pension funds?
2) How much did a recent regulatory change, allowing higher investment into risky assets, impact on the asset allocation and performance of DB and DC plans?
3) What would be possible weights for different asset classes under a better / optimal asset allocation scheme?

Through these questions, this thesis endeavours to provide information that can be used to support practitioners and researchers in the field related to the ROK's pension industry. It is important to provide additional evidence that can be used for developing the most optimal portfolio in terms of risk and return profile of asset classes. While the importance of managing an optimal portfolio has been emphasised since the amendment of ERBSA in 2015, there is little evidence and few case studies to which practitioners can refer to for their portfolio management. As Chapter 3 will discuss, much of the findings in the literature are rather contradictory instead of providing consistent recommendations for portfolio management.

2.2 Statement of the Problem and Need for the Study

Sung, Kim and Choi (2013), when examining the performances of the ROK's pension fund using ex-post deterministic analysis in different scenarios, did not apply an optimisation process. They concluded that an investment product solely comprising stocks would outperform both a fixed income fund and principal and interest guaranteed products. For their analysis they considered the following data: KOSPI 200, KIS Bond Index and Sovereign Bonds. Park, Cheong and Sung (2014) suggested that the propensity of DC participants in selecting investment products can be predicted according to the financial literacy and education level of the pension fund manager. While both studies provide valuable insight relevant to the pension industry, their research is limited because it adopts a restricted approach which merely focuses on proportions in the absence of optimisation which should involve various data sets and asset classes. To the best of my knowledge, the present study is the first to explore optimal asset allocation for the ROK's retirement pension funds by adopting investment indices including the ROK's domestic indices, global indices, a commodity index and cash equivalent. To support companies or pension providers, this research included an actual benchmark and indices currently used for mutual fund management.

For this, finding the optimal proportion from selected investable assets needs to be prioritised. It is important to find optimal asset allocations to growth and conservative assets, such as e.g. equities and bonds. The most optimal weight estimated from each investable asset will contribute to constructing a DB portfolio or determining the weight of balanced style mutual funds (e.g., 60/40 or 70/30), which can also be used for DC plans. However, unconstrained or traditional mean-variance optimisation generates a corner solution problem, which often attributed overwhelmingly high weights to one or two asset classes in the recommended optimal portfolio. To avoid such a problem, this research examines constrained and risk-adjusted approaches under different criteria: target return, target risk and investment constraints in the same asset classes.

Despite the increase in the amount of the retirement pension reserve, in the ROK the income replacement rate remained at 39.3%, much lower than the Organization for Economic Co-operation and Development (OECD) average of 52.9% (OECD 2015). Thus, the issue of asset management in the ROK's pension funds to meet the OECD standard has become a cornerstone for the government, private and public companies, and participants in the pension fund industry. To resolve such disparity, it has been proposed to either raise contribution rates or increase portfolio returns. However, considering that companies are less likely to raise the contribution rates voluntarily, better portfolio management for increased returns seems to be an alternative solution at this point in time. Table 1 shows that more than two thirds of employees remained with DB plan without taking an alternative plan into consideration.

Table 1: Reserve Status of DB, DC and IRP[5]

Classification	DB	DC	IRP
Reserve (AUD, Billion)	99.6	34.2	13.2
Percentage	67.8%	23.3%	9.0%

Source: Financial Supervisory Service (FSS) (2017).

Although the amount of retirement benefit of the ROK's DB plan is simply determined by multiplying the years of service and average wage[6] during the last three months before an employees' retirement. On the other hand, the amount of retirement benefits of a DC plan is mainly determined by the result of the investment performance in the DC account. Most employees prefer DB plans, possibly assuming both schemes might offer similar levels of retirement benefits, while DC plans are riskier due to their dependence on the investment performance. This assumption puts

[5] Individual Retirement Pension (IRP): ROK government renamed the individual retirement account (IRA) as IRP in the year of 2012 when the ERBSA that required employees required to secure their retirement provision until retirement has become effective.
[6] The term 'average wages' means that the average salary for three months immediately before retirement is the basis for calculating retirement wages.

more pressure on companies regarding pension administration, particularly when it comes to recordkeeping and operating a large portfolio. Since the result of operating DB plans portfolios would directly affect the amount of mandatory pension contributions that will be deducted from cash account, portfolio optimisation that would accomplish the required target return in terms of pension liability generated from a DB plan has become a more urgent issue.

For successful (i.e., profitable) investments, companies are required to make careful decisions regarding what to invest in and when to buy or sell assets for DB plans as do individual employees for DC plans. However, Table 2 shows that the ROK's retirement pension assets are tilted towards principal and interest guaranteed products.

Table 2: Reserve status of DB and DC plans in terms of investment style

	DB		DC	
	Amount (AUD, billion)	Proportion (%)	Amount (AUD, billion)	Proportion (%)
Principal and interest guaranteed	94.6	95.0	27.0	78.9
Investment and growth	2.0	2.0	5.7	16.7
Cash equivalent	3.0	3.0	1.5	4.3
Total	99.6	100.0	34.2	100.0

Source: FSS (2017).

Regarding investment product ratio, the ROK's DC plan's proportion of principal and interest guaranteed product is over four times that of investment and growth products. At the same time, the share of investment and growth assets is still substantially higher in DC plans in comparison to DB plans. This supports the notion that DC participants may prefer risky assets to more secure retirement provisions due to longer life expectancy (Lefort & Walker 2002). Although the ERBSA initially allowed DB sponsors to allocate more risky assets in a DB plan than in a DC plan (see Appendix 1), most companies remained with principal and interest guaranteed products for their DB plan. However, as shown in Table 3, DC plans had a superior return relative to DB plans.

Table 3: Annualised performance of DB and DC plans of the ROK's retirement pension fund

Product Classification	Annualised Periods	DB (%)	DC (%)
Portfolio total	5 Years	2.74	3.05
	8 Years	3.12	3.89
Principal and interest guaranteed	5 Years	2.73	3.05
	8 Years	3.05	3.42
Investment product	5 Years	2.89	2.91
	8 Years	4.73	5.19

Source: FSS (2017), Ministry of Employment and Labor (2016).

As shown in Table 2, the extreme concentration on principal and interest guaranteed products indicates that the source of portfolio performance is limited to interest rates. If so, the unsatisfactory returns under low interest rates could have been prevented by portfolio diversification. Table 3 shows the annualised total performances for a DB plan are 2.74% for 5 years and 3.12% for 8 years, while those of a DC plan are 3.05% and 3.89% respectively (11.7% and 24.8% higher than a DB plan). This demonstrates that preference for principal and interest guaranteed product resulted in lower portfolio performances in the decreasing interest rates phase. The performance of funds that concentrate on principal and interest guaranteed products can probably be expected to be even lower in the near future, given the current low interest rate environment.

To address the second research question of this thesis, it must be stated that few changes have been made in asset allocations since the ROK Government's amendment of the pension regulation. Table 2 shows the changes made in the ROK's DB and DC assets in terms of investment style after the amendment of ERBSA on 1 July 2015, while Table 4 shows the investment style prior to the amendment. The difference of the principal and interest guaranteed composition between June 2015 and December 2016 is as little as 2.6% and 1.1% for DB and DC respectively.

Table 4: Reserve status of DB and DC plans in terms of investment style

	DB		DC	
	Amount (AUD, billion)	Proportion (%)	Amount (AUD, billion)	Proportion (%)
Principal and interest guaranteed	73.7	97.6	27.0	77.8
Investment	1.4	1.8	6.4	18.4
Cash equivalent	0.4	0.6	1.3	3.8
Total	75.5	100.0	34.7	100.0

Source: FSS (2016).

As shown in Table 2, the existing conservative DB portfolio heavily depends on principal and interest guaranteed products. The tilted proportion to defensive assets indicates that the recent financial market of low-interest rates prevents the current DB portfolios from reaching their target returns. To solve this problem, another portfolio designed to reduce the gap between the realised investment returns and salary increase rates was suggested to meet the statutory contribution standard.

For a DC plan, the construction of an optimal portfolio as a default investment option is equivalently important. Interestingly, the ratio of DB plan to DC plan is more than two to one in the ROK (see Table 5), which is in stark contrast to the US where the ratio of DB to DC is approximately one to two. It is also notable that the ratio of DC to DB in Australia's superannuation is almost seven to one. This necessitates careful portfolio management of DC plans to secure DC participants' retirement provision. For this reason, there has been made a change that all new default fund contributions must be invested in a MySuper product (Chant, Mohankumar & Warren 2014).

Table 5: DB and DC split

Country	DB	DC
Republic of Korea	74%	26%
Australia	13%	87%
United States	40%	60%

Source: FSS (2017) and Willis Towers Watson (2017).

Prior to implementing a default investment option (similar to MySuper) for the ROK's pension system, diversified asset allocations must be considered—whether it be equity securities, debt securities or other bonds for both DB and DC plans. Tables 2 and 4 suggest that the proportion between growth and defensive assets among retirement schemes saw little to no change even after the ROK Government increased the maximum limit of investments into risky assets from 40% to 70% in 2015. Consequently, we conclude that the new regulation was less effective than anticipated in changing the investment behaviour and asset allocation of pension funds.

To answer the third research question—possible weights of asset classes for a better asset allocation in the ROK's pension fund—optimal asset allocations under different investment criteria will be examined. Results showed that neither DB sponsors nor DC participants changed their asset allocations notably despite opportunities to invest in risky assets under the new regulation. Brinson, Hood and Beebower (1995) investigated actual returns of 91 US corporate pension plans and found that 93.6% of total return variation constitute asset allocations. While this highlighted the importance of asset allocation for pension plans, the previous and current portfolios of the ROK's retirement pension fund accomplishes little towards diversification.

The aforementioned studies, related to asset allocation and glide paths, imply that the previous and current portfolio of the ROK's retirement pension plan, heavily concentrated towards defensive assets, possibly failed to provide investors with higher growth for their pension fund. This trend calls for an investigation into potential better asset allocations for the ROK's retirement pension fund industry. Therefore, this research will examine the performance of various asset allocation schemes, using several major indices for growth and defensive assets.[7]

[7] Data description is in Chapter 4 of this thesis.

Chapter 3: Literature Review

Portfolio optimisation dates back to Markowitz's (1952) and Markowitz and Selection's (1959) classical mean-variance optimisation model (MVO). MVO has been considered a useful tool for investors to find optimal asset weights for the construction of portfolios in terms of expected returns and risk measured by volatility. It was widely used for strategic asset allocation because of its capability to generate optimal combinations with respect to risk-return characteristics of a portfolio. For the construction of optimal portfolios, the following three pivotal steps are suggested: 1) estimating expected returns and volatilities of asset classes, 2) estimating variances and covariances to determine an optimal set of portfolios and 3) comparing differences between investment strategies best suited to the investor's appetites. In the first step of MVO, the most important consideration is expected returns, variances and covariances to calculate the asset weights when it comes to constructing optimal portfolios. This enabled investors to replicate Markowitz's process to construct their own portfolios more focused on maximising returns by increasing risk or decreasing risk by sacrificing potential returns. The complexity lies with estimating expected return, variances and covariance which play a key role in determining asset weights for optimisation.

Sharpe's (1963) single-index model assumes the capital market as an index and that it is the only factor that affects the returns of all individual securities. Sharpe, who extended Markowitz's work on portfolio analysis by estimating the future returns of stocks to determine an optimal set of portfolios, was recognised for his extensions of the Markowitz technique and its application to capital markets. Sharpe proposed the so-called diagonal model that contributed to increasing the possibility of low-cost analysis, and a likelihood in which less parameters are used to generate results similar to those of Markowitz. Although the concept of a single-index model contributed to estimating the returns of securities, it suffers from drawbacks as in reality various factors are expected to affect the performance of asset classes in a dynamic portfolio.

Sharpe (1964) introduced the investment opportunity curve where investable assets locate either the same return with low risk or the same risk with higher return. He also adopted the concept of diversification that allows investors to combine risky assets and riskless asset. Sharpe's (1964) capital asset pricing model (CAPM) estimates the expected return of assets using beta, the sensitivity of an asset's return to market returns.

It is notable that, a few years prior to Sharpe (1963), James and Stein (1961) suggested the concept of grand average to improve the volatility of average when estimating the value of observations. James and Stein (1961) introduced the concept of grand average which refers to an effect that contributes to estimating the mean of observations. For this, they used a shrinkage factor that pushes the mean close to the grand average when it is positive but pulls the mean away when it is negative. That is, the range of the mean of observations decreases, if the value is greater than the grand average, while that of the mean of observations increases, if the value is smaller than the grand average. Thus, positive shrinkage factor enables the reduction of the estimation range without the information of the population. The problem is that as we know it is better to set the negative shrinkage factor as zero to prevent pulling away the mean from the grand average. But if so, the use of the grand average becomes unbeneficial. At this point, it has to be reconsidered that the use of the grand average has actually contributed to estimating the mean of observations, which can be an important indicator of expected returns.

Black and Litterman (1990) suggested the way of estimating expected returns to prevent fluctuations of asset allocation from the process of optimisation. Previous methods (covered above) tend to generate very different optimal weights of asset classes at each time step, as they are very sensitive to changes in input variables. Using MVO, often also yields the problem that the allocation to a specific asset or asset class dominates the portfolio. Consequently, Black and Litterman (1990) allowed investors' views on investable assets and market in terms of estimating expected returns and volatilities. The Black–Litterman model optimises starting from the market portfolio rather than estimating returns and standard deviations from historical data of asset classes. Therefore, the result of asset allocation differs, depending on which index has been chosen as a benchmark for the market.

The methods focused on expected returns failed to provide a proper measure to investors in a period of unprecedented turmoil, the Global Financial Crisis (GFC). Before the GFC, Chopra and Ziemba (1993) had already highlighted the merit of performance of minimum variance optimisation (MIVO), stating that it could be improved when all stocks have the same expected returns. However, the assumption seems to be unrealistic in the real capital market, because it is almost impossible that the expected returns of all stocks are equivalent. Recently, Clarke, de Silva and Thorley (2011) suggested that long-only MIVO optimisation outperformed both the market and a long-short portfolio, where excessive selling and buying on a certain asset are often witnessed. However, MIVO optimisation cannot reach target return to a certain degree since it only considers minimising risk without taking various risky assets into consideration.

On the perspective of risk measure for investable assets, Rockafellar and Uryasev (2000) suggested conditional VaR (also known as expected shortfall or expected tail loss) as a tool for risk measurement of a portfolio. Even though Value-at-Risk (VaR) analysis contributes to managing risk, it suffers from the lack of subadditivity and convexity, unlike conditional VaR. Therefore, VaR calculated for a combination of portfolios could possibly be greater than than the sum of VaRs for the individual portfolios (Rockafellar & Uryasev 2000). Unlike VaR, which only considers a specific quantile of the return distribution of the portfolio, conditional VaR takes into account the entire tail of the return distribution to detect downside risk (DR). Conditional VaR can also capture the impact of non-normal distributions, and asset returns that are skewed or have excess-kurtosis. Thus, conditional VaR optimisation might the recommended way to conduct risk-adjusted asset allocation.

With the various efforts of portfolio optimisation and risk measurement, for the pension management, both DB sponsors or DC participants prefer the portfolio with target return and/or target risk to the portfolio without target constraints. In this sense, Basu, Byrne and Drew (2011) examined the effect of setting target rates of return, adjusting the combinations of assets during the simulated working life on DC participants' portfolios. Moreover, when it comes to target risk, investors (individuals or institutions) tend to show more concern about the floor level of risk that they could bear to protect wealth. Thus, the DR of a portfolio as a measure of risk (Harlow 1991; Leal & Mendes 2005; Natarajan, Pachamanova & Sim 2008) has also become a central issue in portfolio optimization and performance evaluation among researchers and practitioners (Harlow 1991; Leal & Mendes 2005; Natarajan, Pachamanova & Sim 2008).

Although Sharpe (1966) introduced the measurement that compares reward to variability (which has been helpful for comparing returns to total risk of a portfolio or asset class to a certain extent), it involves direct comparison of performance between different portfolios and the interpretation of negative features by overlooking the importance of skewness of return distribution. To enhance risk management in consideration of the DR of a portfolio, Sortino and Price (1994) proposed a new measure, the Sortino ratio, which captures DR instead of using variance as a total risk measure. The proposed measure allows for a meaningful performance evaluation of different portfolios also when returns exhibit negative skewness.

Stutzer (2000) suggested the 'Performance Index' as an alternative to performance evaluation under non-normal returns that might be the result of, e.g., economic shocks. He also pointed out that MVO suffers from its inability to calculate the parameters needed to find optimal asset weights. Another problem of MVO is that the standard deviation of returns without penalising is used as a proxy of variance. Since investors prefer the positive returns generating the volatility of right tail to that of left tail, optimisation without penalising the volatility that is generated by the left tail may result in an ill-advised investment. In this sense, investigating volatilities and risk in both tails becomes more important, in particular when the return distribution is asymmetric.

Thus, the non-normality of asset returns may require taking into account also the skewness and kurtosis of the distribution of realised returns as well as their impact on the portfolio performance.

It is important to examine the soundness of optimisation, not only by assessing the mean and the standard deviation of returns, but also using alternative measures such as the maximum drawdown. Leal and Mendes's (2005) maximum drawdown model investigated portfolio risk by measuring and comparing DR in terms of consecutive negative returns and their sum based on simulated optimisations. Additionally, the maximum drawdown period is often used by financial institutions to investigate the longest time period of consecutive negative returns for a portfolio.

Basu and Drew (2009) stressed the importance of contributions in asset allocation under the mandatory pension schemes. Contributions in mandatory pension funds have been accumulated in the employees' pension account during their employment. As a result, increasing inflows of contributions into pension account contributed to making the size of portfolio larger and thus affecting the performance of a portfolio. Basu and Drew (2009) examined the effects of contribution inflows between contrarian lifecycle and conventional lifecycle and then demonstrated the performance of contrarian life cycle strategy is better than that of conventional lifecycle strategy by raising risky assets in asset allocation when plan participants go close to retirement.

Basu and Drew (2010) also showed the adequate involvement of stocks in asset allocation that contributed to both increasing upside potential and reducing downside risk. Since the investment outcomes of default investment option can vary depending on which benchmark each investment strategy follows, it is important to decide which asset classes to be included at the time of constructing a portfolio. However, their research is limited because they mainly discussed the appropriateness of default investment options in DC plans by only using Australian stocks and Australian bonds and bills. To scrutinise optimal portfolios the thesis studies diverse investment schemes by conforming the ROK's local regulations and constraints. It is important to employ different asset classes that may generate different optimisation results, so the scope of investment options can be extended beyond a single market. Employing different asset classes generates different optimisation results, so such an analysis is required to examine optimal portfolios using diverse investment schemes that conform the ROK's local regulations and constraints. To build on this line of work, the thesis includes target risk or target return analysis, by adopting diverse indices as growth assets and defensive assets. The aim is to investigate the impact of different investment schemes on the performance of the portfolio.

Another study on asset allocation that emphasized the importance of DC plans has been conducted by Sialm, Starks and Zhang (2015). It is intriguing to see that they found that DC money is less sticky and more discerning than non-DC money. Such traits in DC money can be best explained by the DC' money moving occurs when plan sponsors usually remove low-performing mutual funds from their DC line-up, replacing them with high-performing mutual funds. Clearly, this change in allocation will eventually affect the performance of the DC account.

Since asset allocation is typically considered as the key determinant of portfolio performance (Brinson, Hood & Beebower 1995), without investigating the optimal asset allocation of mutual funds within DC schemes, it is almost impossible to maintain sustainable DC line-ups. As a result, the importance of strategic asset allocation of mutual funds in DC schemes cannot be ignored. However, Sialm, Starks and Zhang (2015) point out the limited investment options for DC plans in terms of selecting mutual funds. This situation somehow prevents DC participants from making their choices in mutual funds trading, very different to, for example, individual investors who have little limitation for trading. The authors also confirm earlier results by Huberman and Jiang (2006), who suggest that the choice of mutual funds in DC schemes significantly affects the performance.

Mohan and Zhang (2014) further examine the asset allocation of public DB plans, suggesting that underfunded public DB plans have a higher allocation of their reserves to risky assets. This is probably due to the expectation that a successful result from the investment in risky assets might fill the gap between the desirable funded ratio and the pension liability. At the same time, the failure in investment that may be caused by portfolio managers of public DB plans who aggressively accept risky assets can be shifted to taxpayers. Mohan and Zhang (2014) found the tendency of herding behaviour of public DB plans to replicate the Californian Public Employees' Retirement System. They suggest that this is because of the agency problem that might exist among portfolio managers of public DB plans. However, unlike DB plans in the US, the ROK's DB plans are more likely to include a higher share og principal and interest guaranteed products. This shows another form of agency problem in the ROK's DB plans: ROK's plan sponsors tend to avoid their responsibility in case of negative returns even when they occurred due to their aggressive investment choices in risky assets. Thus, it is important to investigate what optimal asset allocations should be constructed for the ROK's DB plans to evade such agency problems, and improving the pension management system.

Basu and Andrews (2014) investigated to what extent the asset allocations of pension funds can be explained by fund returns that shows the relationship between returns and expenses from active management. They concluded that returns of almost two-third of default investment options of Australian superannuation was lower than its benchmarks. Basu and Andrews (2014) insisted that this was because of active management accompanied by unnecessary expenses that are detrimental to DC participants. They underscored the importance of cost efficiency and the scale of economy to enhance the performance of pension funds. However, their research was only limited to the Australian pension market, without taking other forms of pension funds into account such as, for example, university endowments.

Unlike public or private pension funds, it was reported that the asset allocation of university endowment funds is not related to portfolio returns (Brown, Garlappi & Tiu 2010). This implies active management might be superior to asset allocation in endowments. Brown, Garlappi and Tiu (2010) pointed out that although it should be considered as the main factor, asset allocation cannot explain the return variations in the cross section of endowment funds. The most important point of their research is that tactical asset allocation to support active management can better explain the return variations in the cross section of endowment funds than strategic asset allocation. Therefore, endowment managers overall prefer active management to passive management. However, further investigation on the ability of security selection of active fund managers is needed to provide answers to the controversial questions raised by researchers and practitioners for decades.

Recently Salazar et al. (2016) reviewed several factors such as salary, contribution levels, and sequencing risk impacting on the distribution of retirement wealth outcomes for superannuation portfolios. Specifically, the authors focused on the most critical period for securing retirement wealth, which they considered as the last 10 years before retirement. They found the accumulated wealth 10 years prior to retirement linearly affects securing retirees' terminal wealth. Furthermore, they suggest that growth assets and contributions play a key role in accumulating retirement wealth. However, in the paper their discussion is concentrated on estimating terminal wealth, with little investigation of different portfolio optimisation and performance evaluation techniques.

Coleman, Esho, and Wong (2006) suggest that portfolio management is associated with an agency problem. The agency problem for corporate pension funds often occurs when a CFO or finance manager accomplishes little toward maximizing the performance of their pension funds. Most companies in ROK prefer to remain with the principal and interest guaranteed products regardless of their low-interest rate under the assumption that they could avoid the loss in a DB portfolio. This clearly shows the agency problem in pension funds results in more costs when the investment performance cannot match the salary increase. This raised a question that which possible weight for different asset classes under a better / optimal asset allocation scheme contributes to enhancing portfolio returns compared to current pension plans.

Moreover, Shivdasani and Stefanescu (2009) emphasised the importance of portfolio management since the result of portfolio management of pension plans would directly influence the level of cash holdings to the extent of effecting the firm value. The importance of portfolio management for pension funds, which plays a pivotal role in determining a capital structure, cannot be underestimated. Firms are not permitted to access the contribution reserve once they have been deposited into separate pension accounts, whether it to be a DB or DC plan, under the ERBSA. This indicates that pension contributions are considered as an additional payout factor. Poor portfolio performance of pension funds under regular mandatory pension contribution plans may lead firms to experience a cash shortage. This was explained by the Pecking order model, in which raising external funds is regarded as a costly resource of cash in case of insufficient internal cash (Myers & Majluf 1984).

Overall, our review of the literature suggests that far very little research on the role of asset allocation in explaining performance differences between retirement pension funds of the ROK has been conducted. Therefore, this thesis deals with various optimisation schemes among different investable asset classes as well as risk-adjusted analysis to make additional contributions in this area of literature.

Chapter 4: Empirical Analysis

This chapter studies seven different investment schemes to examine optimal asset allocation of the ROK's retirement pension fund: 1) Standard Asset Allocation (SAA), 2) Minimum Variance portfolio (MinVar), 3) a portfolio that yields a maximum Sharpe ratio (MaxSharpe), 4) portfolio with specified Target Risk (TargetRisk), 5) portfolio with specified Target Return (TargetReturn), 6) CVaR optimal portfolio for specified target return (CVaR), and 7) CVaR optimal portfolio for specified target return with skewed and leptokurtic returns (CVaR Skew). Hereby, investment into the following growth and defensive assets is considered: KOSPI 200, S&P 500, Russell 2000, MSCI AC World, Rogers ICI, KIS Bond Index, Monetary Stabilization Bond (MSB) (1-year), Call Rate adding KOSPI 200 and Treasury Bonds (3-year). I consider monthly returns of these different asset classes for the time period February 2001 - December 2017. Note that in the analysis transaction costs and management fees are excluded, while the additional assumption is made that the constructed portfolios do not allow for short sales.

However, to keep the analysis relevant for real world applications, the minimum and maximum weights established by the ROK's pension regulation are considered. Due to the change in the ROK's regulations as can be seen in Appendix 1, which equivalently allowed investment limit to risky assets regardless of DB and DC plan, the result of optimisation became available for both DB plan sponsors and DC participants. When it comes to the difference in the asset allocation between DB and DC plans, Sialm, Starks and Zhang (2015) pointed out that while plan sponsors solely determines the portfolio construction of DB plans, DC participants' portfolio decision can be made by a combination of plan sponsor and individual participants as plan sponsors have the right to decide what to be included in the DC line-ups. Although the result of money allocation into mutual funds between DB and DC plans can be different, the result of the optimal asset allocation of mutual funds can be measured by the same criteria. The change in the 2015 ROK government's regulation, which equally allowed DB and DC plans to invest in risky assets, enables us to simulate optimal asset allocation for the ROK's retirement pension funds whether it to be DB or DC plans.

Section 4.1 discusses the relevance of the considered asset classes and indices. Section 4.2 describes data attributes including the statistics of asset classes, showing which index serves as a proxy for growth and defensive assets. Section 4.3 discusses the composition of an optimal portfolio for the seven investment schemes mentioned above, reviewing optimal solutions for both constrained and unconstrained portfolios. Section 4.4 demonstrates performance analysis of the optimised portfolios derived from Section 4.3.

4.1 The Importance of Using Indices

It was reported that Warren Buffet won his hedge fund bet in the Wall Street Journal on 30 December 2017. For a decade, from 2008 onwards, he had invested in Vanguard S&P 500 index fund, while Ted Seides, former president of Protégé Partners, chose 'fund of funds' rather than the index fund. The annualised performance for the nine years of Buffet's index achieved 7.7%, while that of Ted's five funds was 2.2% (Berkshire Hathaway 2017).[8] Clearly, the index fund would be more beneficial for managing pension funds in a long-term investment horizon.

Table 6 shows that between 2016 and 2017 assets under management (AUM) of passive fund increased by 26.7%, while that of active fund decreased by 19.4%.

Table 6: AUM of active and passive equity fund in the ROK[9]

Management Type	2016		2017		Variation (AUM)
	AUM (USD, Billion)	Performance (%)	AUM (USD, Billion)	Performance	
Active	31	−3.82	25	19.86	−19.4%
Passive	15	8.20	19	31.19	26.7%

Source: FnSpectrum[10].

The table also illustrates that passive funds show a far superior performance in comparison to active fund. Even though active funds still account for a greater portion of investment in the ROK, the fast-growing passive market is an indicator that investors will move their money from active to passive funds. Sharpe (1966) stated the optimal portfolio would be the market portfolio without unsystematic risk and, therefore, he recommends use the market portfolio for benchmarking index funds. In the following, we assume that the considered indices are adequate representatives for investment into different asset classes, such as domestic and international equities or fixed income products. From a fund management perspective, they can also be considered as adequate asset classes for optimising and diversifying portfolios, duplicating the target market at low cost.

The performance of active versus passive fund management has long been debated. This thesis emphasises the importance of investment tools that contributes to minimising human errors, thus, reducing long-term costs for pension funds. Pension funds are normally targeting risk or return to a certain degree rather than advocating passive funds. In this regard, the importance of using indices as asset classes cannot be overlooked in portfolio management. The importance of active management by decomposing the performance of active funds was addressed by Xiong et

[8] Although one year is left on the 10-year bet, Buffet and Ted agreed on the announcement of Buffet's win.
[9] 1 USD = 1,000 KRW. ETFs are included. The AUM of ETFs increased from 20 to 31 billion dollars (55%) for the same period.
[10] Equity Funds over 1 Billion Korean Won as of 2 January 2018 (Master Funds are excluded)

al. (2010). Additionally, Berk and Van Binsbergen (2015) found positive alphas, which explain why investors invest in active funds, and negative alphas, which describe the irrationality of investors by scrutinising all actively managed US mutual funds.

However, we confront the need for optimisation using domestic and global indices in the growing passive market because the sustainability of portfolio performance in the long term might be a superior method of enhancing the risk-return profile of portfolios, in comparison to relying on active managers' intuition. For equity indices, certain industrial sectors such as energy and healthcare and individual country indices other than the US are excluded from the analysis. The effect of the US market in optimisation consisted of large cap using S&P 500 and small cap using Russell 2000—both were examined to see how the indices are differently selected through optimisation to form a portfolio. The MSCI's all country world index (MSCI AC World) is also included to see whether there is a global diversification and if the MSCI AC World is included into the optimal portfolio for the considered optimization schemes. I also include a commodity index, namely the Rogers International Commodity Index (Rogers ICI) as an alternative investment, and to investigate whether commodity markets provide additional investment opportunities for pension funds in the area of growth assets that are different from equities. The details for each index and asset class—eight in total—are provided in the following section.

4.2 Data Description

To decide which indices should be included for analysis, all benchmarks of publicly traded mutual funds in the ROK (3,384 as of December 2017) were investigated. Minor benchmarks that do not have track records matching the analysis period (1 February 2001 to 31 December 2017) were excluded. Additionally, as mentioned in Section 4.1, sector benchmarks are also excluded along with all sectors that do not fulfil the period standard.[11] The period start date was selected because the KIS Bond Index, which plays a key role as a proxy for the performance of fixed income instruments, was launched on this date. The period end date was selected as December 2017 as the most recent available return observation. It is important that all eight asset classes adopted above stem from actual benchmarks targeted by retirement mutual funds. The merit of using actual

[11] Excluded benchmarks are as follows: GBI Global, CSI 500, CSI 300, Hang Seng H, SZSE 100, Bloomberg Barclays Global Aggregate, Bloomberg Barclays Global High Yield, Bloomberg Barclays Emerging Americas, Bloomberg Barclays Global Aggregate Ex-Korea, Bloomberg Barclays US Aggregate, Bloomberg Barclays US High Yield, DWGRTT, FTSE AW ex US, JACI, JP Morgan EMBI Global Diversified Index, JP Morgan GBI-EM Diversified, MSCI World Index, MSCI CHINA A, MSCI Zhong Hua, MSCI AC ASIA ex JAPAN, MSCI Emerging Markets, MSCI Japan Small Cap, MSCI EMU, MSCI ACWI Health Care, MSCI ACWI Energy, MSCI ACWI Financials, MSCI ACWI IT, MSCI ACWI Materials, MSCI BRAZIL, MSCI BRIC, MSCI Europe, MSCI Europe Small Cap, MSCI Germany, MSCI India, MSCI India SMID Cap, MSCI North America, MSCI Japan Value, MSCI Russia, MSCI South East Asia, MSCI USA, MSCI WI Small Cap, MSCI World High Dividend Yield, S&P 100, TOPIX 100, TSE REIT, DWGRTT, Dow Jones Industrial Average, and Dow Jones Euro Stoxx 50.

benchmarks is that the optimised weights of asset classes can be applied to the practical asset allocation of mutual funds. For optimisation, this author investigated mean, standard deviation, covariance, correlation, skewness and kurtosis of the returns as well as the dynamics of each investment scheme reflecting the dependence structure of the eight asset classes.

Since diversification often improves portfolio performance, data of various investment indices including KOSPI 200, S&P 500, Russell 2000, MSCI AC World, Rogers ICI, KIS Bond Index, MSB (1-year) and Call Rate were used as asset classes. Table 7 explains the proxy for each asset classes for the analysis in this research.

Table 7: Asset classes and proxy

Asset Class	Proxy
KOSPI 200	ROK Equity
S&P 500	US Large Cap
RUSSELL 2000	US Small Cap
MSCI AC World	Global Diversification
Rogers ICI	Commodity
KIS Bond Index	ROK Bond
MSB (1-year)	Principal and Interest Guaranteed Product (1-year)
Call Rate	Cash
Treasury Bond (3-year)	Risk Free Asset

Proxies for the KOSPI 200, S&P 500, RUSSELL 2000, MSCI AC World and Rogers ICI were designated as ROK Equity, US Large Cap, US Small Cap, Global Diversification, and Commodity respectively. For defensive assets, proxies for the KIS Bond Index, MSB (1-year) and Call Rate were designated as corporate bonds, short-term bonds and cash, respectively. Additionally, the KOSPI 200 and the Treasury Bond (3-year) were used as benchmarks of the stock market and the risk-free rate.

The KOSPI 200 is a Korean stock market index based on the market capitalisations for 200 of the largest publicly-traded companies in terms of market shares, industry and liquidity. It is considered as one of the most important indicators for the movement of the Korean stock market as a whole. It is used to benchmark the performance of investments and funds.

The S&P 500 is an American stock market index based on the market capitalisations for 500 of the largest companies having common stock listed on the NYSE or NASDAQ. It has long been considered as the best representation of the US stock market, serving as a leading indicator of US equities and large-cap stocks.

In contrast, the Russell 2000 Index is a small-cap stock market index of the bottom 2,000 stocks in the Russell 3000 Index. The Russell 2000 is by far the most common benchmark for mutual funds that identify themselves as ‘small-cap’.

The MSCI ACWI Index, MSCI’s flagship global equity benchmark, is designed to represent the performance of the full opportunity set of large and mid-cap stocks across 23 developed and 24 emerging markets. As of December 2017, it covers more than 2,400 constituents across 11 sectors and approximately 85% of the free float-adjusted market capitalisation in each market (MSCI n.d.).

James B Rogers, Jr designed the Rogers ICI, a composite US dollar-based total return index, in the late 1990s. It was designed to meet the need for consistent investing in a broad based international vehicle. It represents the value of a basket of commodities consumed in the global economy, ranging from agricultural to energy and metal products. The value of this basket is tracked via futures contracts on 37 different exchange-traded physical commodities, quoted in four different currencies (US dollar, UK pound, Euro, Yen) and listed on nine exchanges (Chicago Board of Trade (US), Chicago Mercantile Exchange (US), COMEX (US), ICE Futures Europe (UK), ICE Futures US (US), London Metal Exchange (UK), NYMEX (US), Euronext (European Union (EU)–Paris), and the Tokyo Commodity Exchange (Japan)) in four countries (US, UK, EU and Japan) (Rogers ICI n.d.).

KIS Pricing publishes a bond index called KIS Bond Index which represents the entire Korean Domestic Bond Market. The bond index is computed by including all investable bonds for each category using mark-to-market prices. Depending on the target duration, it can be divided into short-term, mid- and short-term, mid-term, mid- and long-term, and long-term indices.

MSB are discount and coupon instruments with various tenors ranging from 14 days to two years. In the analysis, the MSB (1-year) is employed as it is commonly used as a proxy for term deposits or guaranteed interest contracts. Central bank bonds include monetary stabilisation bonds issued by the Bank of Korea to help absorb liquidity in support of its monetary policy.

The Call Rate is commonly used as a proxy of cash and is the interest rate on a type of short-term loan that banks give to brokers who in turn lend the money to investors to fund margin accounts. For both brokers and investors, this type of loan does not have a set repayment schedule and must be repaid on demand. The length of loans in the call money market are very short, usually lasting no longer than a week and are often used to help banks meet their reserve requirements.

Table 8 shows descriptive statistics of monthly returns for the considered asset classes during the sample period from 1 February 2001 to 31 December 2017. As mentioned earlier, monthly returns are adopted to implement the different optimal asset allocation procedures. For example, the mean returns of 0.0070 of KOSPI 200, 0.0033 of S&P 500, 0.0054 of Russell 2000, 0.0027 of MSCI AC World, 0.0020 of Rogers ICI, 0.0042 of KIS Bond Index, 0.0030 of MSB (1-year) and 0.0026 of Call Rate correspond to average returns of 8.4%, 4.0%, 6.5%, 3.2%, 2.4%, 5.0%, 3.6% and 3.1% per annum, respectively. 'Cumulative' describes the cumulative returns of each asset class for the entire period. The values for skewness and kurtosis obtained from the empirical data in this research for the CVaR Skew analysis are listed in Table 8. Recall that the skewness and kurtosis of the Gaussian distribution are 0 and 3, respectively, such that we find some evidence of skewed and leptokurtic returns. Thus, taking into account skewness and kurtosis of the individual asset classes might lead to different results in the portfolio optimisation exercise.

Considering all data calculated from monthly return series, this author used the monthly values when implementing optimisation including setting target return and target risk. Table 8 provides descriptive statistics for monthly returns of the considered asset classes for the sample period from February 2001 to December 2017.

Standard deviations of asset classes provide us with the clues to risk-return profile when constructing an efficient frontier. KOSPI 200 is the most volatile asset generating the highest monthly mean return. But comparing the mean and standard deviation of S&P 500 and MSCI AC World, we conclude that S&P 500 seems to dominate the MSCI AC World due to its higher mean return and lower volatility throughout the sample period. Similarly, the MSCI AC World seems to dominate the Rogers ICI in terms of its risk-return profile. As such, the optimisation process should find the optimal weight of assets with respect to the risk-return profile.

Figure 1 illustrates the cumulative monthly returns for each asset class and shows how growth assets moved differently in the analysis period (February 2001 to December 2017) which includes turmoil in the market during the GFC. As anticipated, three linear lines represent the performance of the defensive assets, namely the KIS Bond Index, the MSB (1-year) and the Call Rate. Although some significant fluctuations can be seen in the growth assets—KOSPI 200, S&P 500, Russell 2000 and MSCI AC World—generally, cumulative monthly returns tend to be on the rise with fast recovery from the crisis in 2008-2009. It seems that due to the price decrease of raw material, the rise of Rogers ICI may have been prevented.

Table 8: Descriptive statistics for monthly returns of the asset classes (February 2001 to December 2017)

Key Metrics	KOSPI 200	S&P 500	Russell 2000	MSCI AC World	Rogers ICI	KIS Bond Index	MSB 1-Year	Call Rate
Mean	0.0070	0.0033	0.0054	0.0027	0.0020	0.0042	0.0030	0.0026
Median	0.0099	0.0093	0.0132	0.0085	0.0062	0.0041	0.0029	0.0027
Maximum	0.1861	0.1023	0.1426	0.1087	0.1544	0.0460	0.0052	0.0043
Minimum	-0.2352	-0.1856	-0.2345	-0.2220	-0.2862	-0.0157	0.0010	0.0009
Std.Dev.	0.0589	0.0420	0.0553	0.0455	0.0530	0.0065	0.0011	0.0010
Cumulative	143%	67%	111%	55%	40%	85%	60%	52%
Skewness	-0.4	-0.9	-0.8	-1.0	-1.0	1.2	0.0	0.0
Kurtosis	4.2	4.9	4.5	5.8	6.9	11.5	1.9	1.8
Jarque-Bera	17.37	57.50	39.48	101.96	164.69	650.74	10.19	12.02
p	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0061	0.0025
Observations	203	203	203	203	203	203	203	203

Table 9 provides the annual returns for each year of the considered sample period as well as the cumulative return for period February 2001 to December 2017 for each asset class. The table illustrates the poor performance of the Rogers ICI during the 2012 to 2015 period.

Figure 2 indicates that MSB (1-year) and Call Rate are exceptions with regards to the minimum of monthly returns still being positive, i.e. 0.001 (0.1%) and 0.0009 (0.09%), respectively. For all other classes, at least one month with a negative return could be observed. From the onset of the GFC in 2008, Figure 2 shows that the monthly returns of growth assets plummeted, while the KIS Bond Index soared. This can be explained by the graph of MSB (1-year) and Call Rate, as the Bank of Korea lowered the basis interest rate from 5.25% in August 2008 to 2% in February 2009 to aid economic recovery, resulting in the rise of bond prices.

In Figure 3, the Kernel densities of monthly returns for each asset class are provided. For several of the asset classes the returns are clearly non-Gaussian. To test the normality of return series, a Jarque–Bera test was conducted to examine whether the return series can be considered to be from a normal distribution.

Jarque–Bera = $\frac{N}{6}(S^2 + \frac{(K-3)^2}{4})$, where S is the skewness, and K is the kurtosis.

Under the null hypothesis of a normal distribution, the reported probability in Table 8 (p-value) shows that none of the return series is normally distributed. The Jarque–Bera test confirms all null hypotheses are rejected at 1% significance level (equivalent to 99% confidence level). These results recommend that it might make sense to also take the skewness and kurtosis of the returns into account when it comes to portfolio optimisation, incorporating non-normality of asset classes.

The regression line on scatter plots shown in Figure 4 illustrates the intuitive overview of correlation of between the different asset classes.

The detailed mutual strength of correlations in Table 10 describes the 64 pairs of co-movement between designated assets for optimisation.

Table 10: Correlation table of asset classes (February 2001 to December 2017)

The correlation between Call rate (hereafter cash) and MSB (1-year) is quite high (0.96), indicating their returns would behave somewhat similar. Conversely, the correlation between cash and the equity classes is negative (KOSPI 200, –0.10; S&P 500, –0.20; Russell 2000, –0.15; MSCI AC World, –0.19) and the correlation between cash and Rogers ICI is almost zero (0.00). This shows that cash and equity move oppositely in their correlation, while cash and commodity typically exhibit very low correlation. Although both equities and commodities are growth assets, they show different correlation coefficients which enables us to assume that commodities might offer some diversification benefits in portfolio optimization.

The correlation between cash and KIS Bond Index (0.15) is much lower than that of cash and MSB (1-year). This may be due to different combination of bonds: KIS Bond Index are constructed in the combination of all sovereign and corporate bonds over BBB-credit rating (approximate duration of 3 to 4 years) in the ROK while MSB (1-year) only contains sovereign bonds.

The correlations between the KIS Bond Index and growth assets are negative (KOSPI 200, –0.15; S&P 500, –0.13; Russell 2000, –0.13; MSCI AC World, –0.11; Rogers ICI, –0.08). The KIS Bond Index can be employed for constructing optimal portfolio with growth assets showing negative correlation with Rogers ICI (–0.08), unlike MSB (1-year) and cash. A similar pattern is observed for the correlation between equities and MSB (1-year) and the call rate.

It is interesting to observe that correlations between all equity asset classes, are greater than 0.64. At the same time, the correlation between the Rogers ICI and equities is significantly lower (KOSPI 200, 0.37; S&P 500, 0.42; Russell 2000, 0.40; MSCI AC World, 0.54).

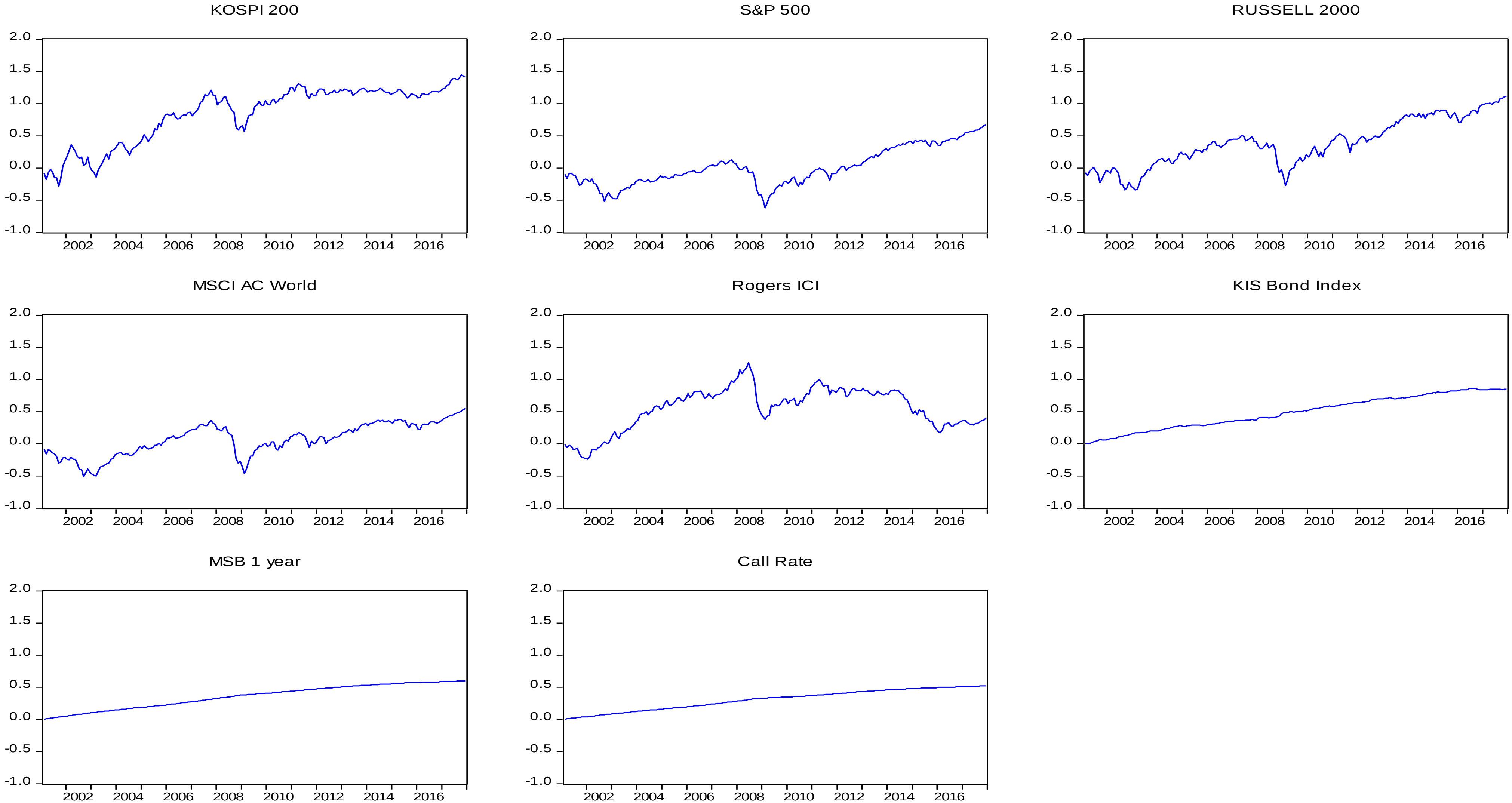

Figure 1: Cumulative monthly returns of asset classes (February 2001 to December 2017)

Table 9: Annual returns and cumulative returns of asset classes (February 2001 to December 2017)

Year	KOSPI 200	S&P 500	RUSSELL 2000	MSCI AC World	Rogers ICI	KIS Bond Index	MSB 1 year	Call Rate
2001	11%	-17%	-4%	-21%	-23%	6%	5%	4%
2002	-9%	-27%	-24%	-23%	29%	8%	5%	4%
2003	28%	23%	37%	27%	28%	5%	4%	4%
2004	9%	9%	16%	12%	19%	8%	4%	4%
2005	43%	3%	3%	8%	18%	1%	4%	3%
2006	4%	13%	16%	17%	3%	6%	5%	4%
2007	26%	3%	-3%	9%	26%	3%	5%	5%
2008	-50%	-49%	-43%	-57%	-53%	10%	5%	5%
2009	42%	21%	22%	27%	23%	4%	3%	2%
2010	20%	12%	23%	10%	17%	7%	3%	2%
2011	-13%	0%	-6%	-10%	-7%	5%	4%	3%
2012	10%	13%	14%	13%	2%	6%	3%	3%
2013	0%	26%	31%	18%	-5%	2%	3%	3%
2014	-8%	11%	3%	2%	-25%	6%	2%	2%
2015	-2%	-1%	-6%	-4%	-30%	4%	2%	2%
2016	8%	9%	18%	5%	13%	2%	1%	1%
2017	22%	18%	12%	20%	5%	1%	2%	1%
Cumulative	143%	67%	111%	55%	40%	85%	60%	52%

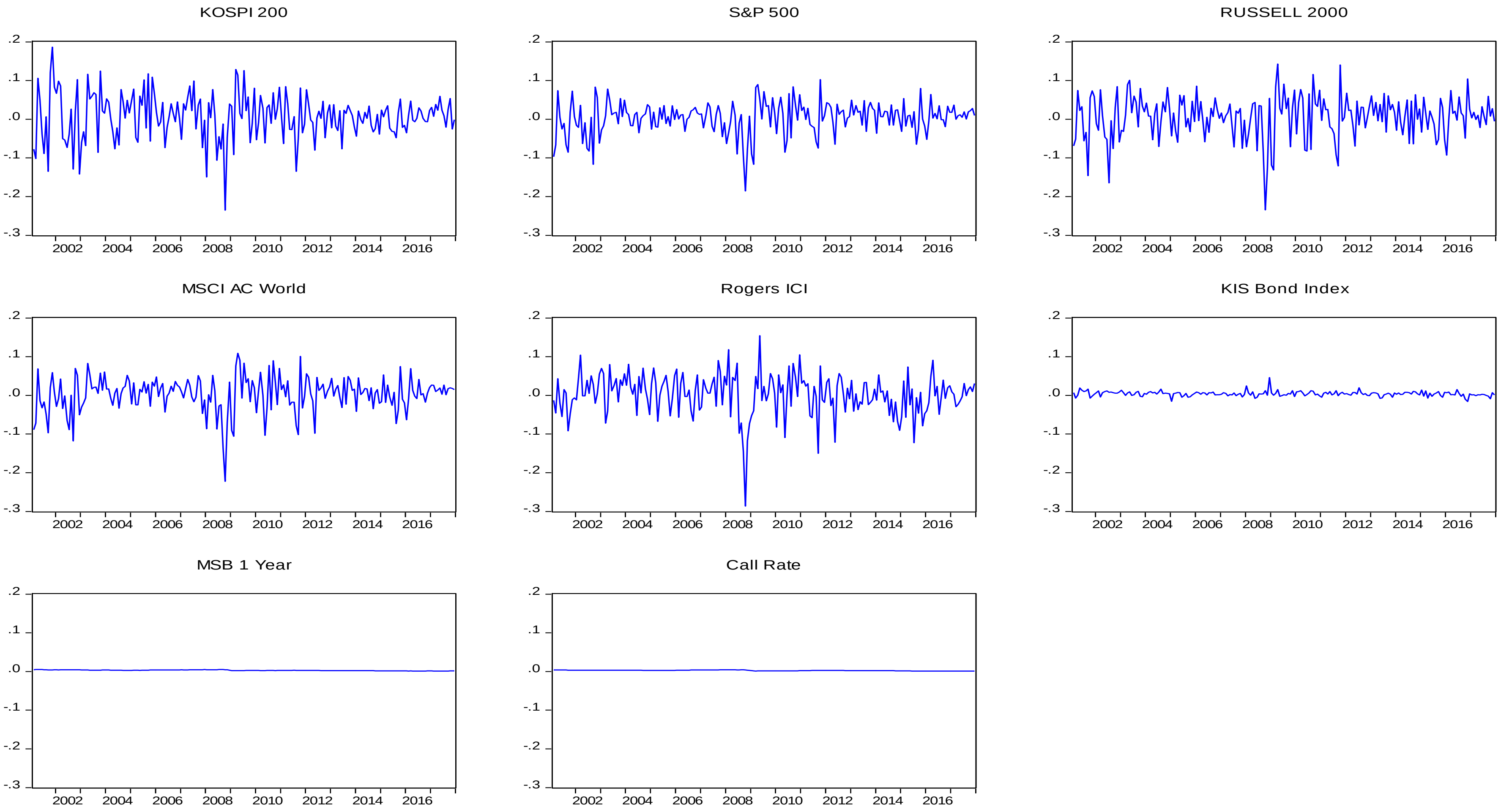

Figure 2: Monthly returns of asset classes (February 2001 to December 2017)

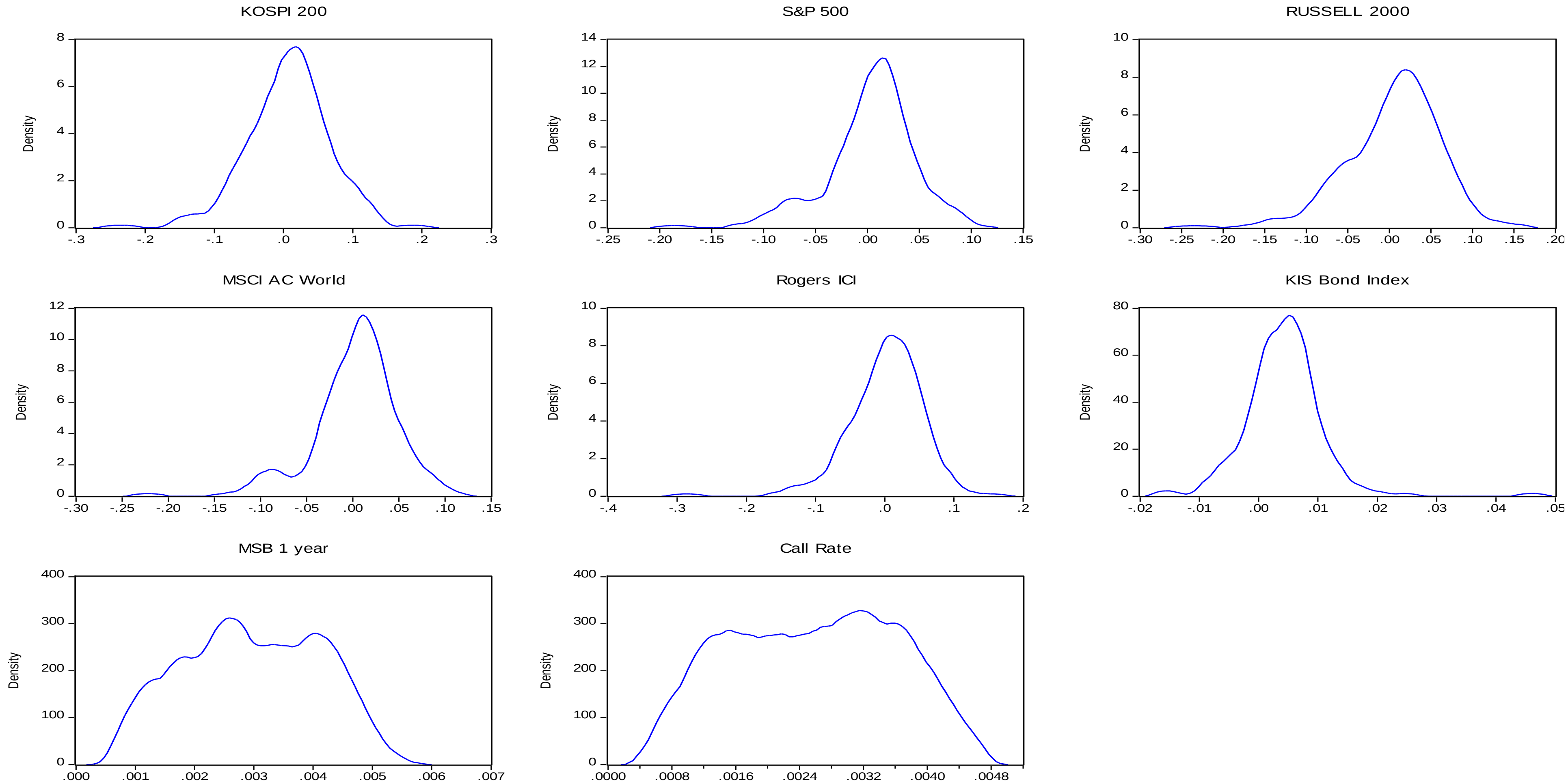

Figure 3 : Distributions of Kernel Density of monthly returns (February 2001 to December 2017)

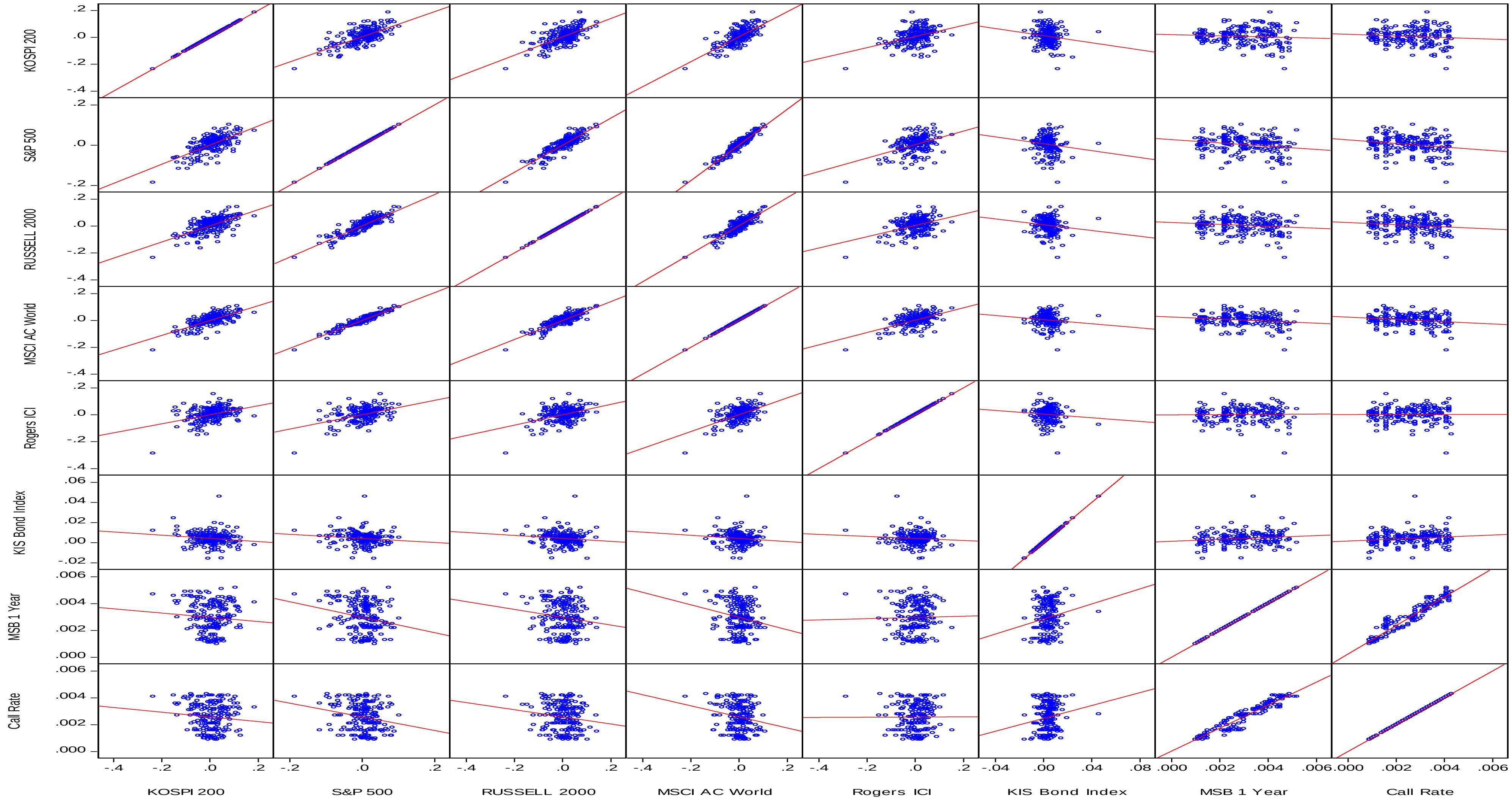

Figure 4: Correlation matrix of asset classes with regression line (February 2001 to December 2017)

Table 10: Correlation table of asset classes (February 2001 to December 2017)

Asset Classes	KOSPI 200	S&P 500	RUSSELL 2000	MSCI AC World	Rogers ICI	KIS Bond Index	MSB 1 year	Call Rate
KOSPI 200	1	0.67	0.64	0.72	0.37	-0.15	-0.08	-0.10
S&P 500	0.67	1	0.88	0.96	0.42	-0.13	-0.22	-0.21
RUSSELL 2000	0.64	0.88	1	0.86	0.40	-0.13	-0.14	-0.14
MSCI AC World	0.72	0.96	0.86	1	0.54	-0.11	-0.19	-0.19
Rogers ICI	0.37	0.42	0.40	0.54	1	-0.08	0.02	0.01
KIS Bond Index	-0.15	-0.13	-0.13	-0.11	-0.08	1	0.16	0.15
MSB 1 year	-0.08	-0.22	-0.14	-0.19	0.02	0.16	1	0.96
Call Rate	-0.10	-0.21	-0.14	-0.19	0.01	0.15	0.96	1

4.3 Portfolio Optimisation

Based on the indices reviewed in Section 4.2, an optimisation analysis was performed by using monthly returns, the standard deviation of returns as well as the correlation between monthly returns of the considered asset classes. Performance was compared by calculating continuously compounded returns and the consistency of correlation was viewed through a variance–covariance matrix. Finally, optimisation was performed by the portfolio optimisation package of the Financial Toolbox provided by MATLAB and Eviews and R to illustrate the return characters and its dependence structure.

The allocated asset weights for SAA are used as an initial portfolio. It is derived from the actual asset allocation of the ROK's current pension funds combining DB and DC reserves (FSS 2017). In the following we will concentrate in particular on the difference between the actual asset allocation of pensions funds in the ROK and the suggested optimal allocation for various portfolio optimization schemes. The range of minimum and maximum asset weights are constrained as follows: 1) growth assets from 10% to 70% and 2) defensive assets from 30% to 90%. Their percentages in unconstrained optimisation are consistent with the current regulation of the ROK's pension funds (covered in Chapter 1).

For both unconstrained and constrained optimisation, the following optimisation techniques have been implemented:

1) SAA: no optimisation
2) MinVar:

$$w_i = \frac{\Omega^{-1} * \iota}{\iota' * \Omega^{-1} * \iota}$$

3) MaxSharpe (assuming a risk-free rate 0.0032, equal to the annual return of 3.8% based on Treasury Bond (3-year)):

$$w_i = \frac{w'_p * \mu - r_f}{\sqrt{w'_p * \Omega * w'_p}}$$

4) TargetRisk: portfolio that maximises the expected return for a target variance of E(sigma) = 0.005 (0.5%) monthly (equal to 1.73% per annum)[12]

[12] Annualised standard deviation = monthly value $\times \sqrt{12}$

5) TargetReturn: portfolio that minimises the variance for a target expected return of E(mu) = 0.00375 (0.375%) monthly (equal to 4.5% per annum)
6) CVaR: CVaR minimal portfolio for $VaR_{0.95}$ and expected return of E(mu) = 0.00375 per month
7) CVaR-Skew: CVaR minimal portfolio for $VaR_{0.95}$ and expected return of E(mu) = 0.00375 per month with skewed and leptokurtic returns.

The target values for the variance and expected return set from techniques (4) to (7) can be selected in a flexible way. However, the thesis employed objective target return of National Pension Service (NPS) of ROK. NPS with approximately 600 billion dollars reserve as of December 2017 announces the target portfolio every year. NPS (2016) announced its target return of 4.5% for the year of 2017. The thesis assumed its target return as the standard goal of institutional investors. In this thesis, we examine whether the ROK's current regulation properly uses indices as actual benchmarks and whether other constraints for minimum and maximum weights are within the framework of regulation. When it comes to CVaR and CVaR-Skew analysis, specifying CVaR level (here 0.95) and expected return (or risk) level is required to not get an entire efficient frontier of Mean-CVaR-optimal portfolios. For the CVaR-Skew analysis, the skewness and kurtosis of asset classes are set equal to the empirical values calculated from the data set reported in Table 8.

4.3.1 Unconstrained Optimisation

Unconstrained optimisation was performed to find the maximum weights for both growth and defensive assets under the ROK regulation. For defensive assets, the minimum weight of 5% was set to secure liquidity as set in the pension mutual funds. Figure 5 shows all possible combinations of portfolios in the unconstrained optimisation. While CVaR and TargetReturn have almost the same return and risk on the efficient frontier (95% confidence level, 0.00375 monthly return), CVaR-Skew enhanced its risk-return profile, though the difference between CVaR and CVaR-Skew is small. Figure 5 also shows that SAA deviates significantly from the other six unconstrained investment strategies and the efficient frontier.

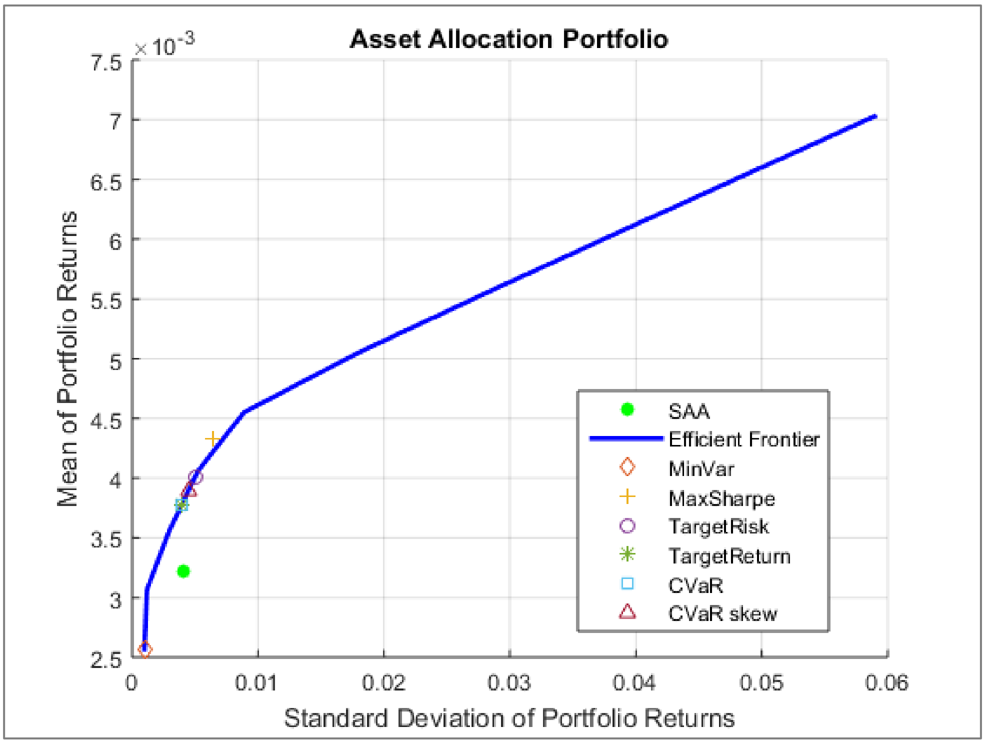

Figure 5: Efficient frontier and location of optimal unconstrained portfolios in the mu-sigma space

Table 11 illustrates how investment schemes allocate their assets upon unconstrained optimisation criteria.

Table 11: Weights for asset classes, fraction of growth and defensive assets and expected volatility and return for optimised unconstrained portfolios

Figure 6 show relative asset allocation of each asset class in terms of different investment schemes for unconstrained optimisation.

In Figure 5, the optimal portfolios derived from different investment techniques without constraints locate on the efficient frontier except for SAA. None of the optimisation methods allocate any proportions to Rogers ICI or MSCI AC World. This is may be due to a relatively high risk-to-reward ratio for KIS Bond Index (mean = 0.0042 per month and standard deviation = 0.0065 per month). MaxSharpe allocates a significant (94.2%) proportion to KIS Bond Index. MaxSharpe enhanced its risk-to-reward ratio by increasing the expected return but maintaining risk at the same level compared to the equivalent combination on the efficient frontier. However, MaxSharpe is the most volatile portfolio with a standard deviation of 0.0065 per month. Although the proportions of KIS Bond Index in TargetRisk, TargetReturn, CVaR and CVaR-Skew vary, even the lowest exceeds 50%. This suggests bonds play a key role in formulating desired portfolio as a tool for limiting DR and targeting mid-risk and mid-return. As expected, the standard deviation of MinVar portfolio is 0.0010, the smallest volatility among investment schemes. Considering that the purpose of MinVar is to minimise the volatility of portfolios, it is unsurprising that the rate of cash allocation is extremely higher (0.991). As a result, the expected return of MinVar remained as low as 0.0020.

Table 11 shows TargetRisk allocates over 70% in KIS Bond Index and 3.5% in KOSPI 200 to construct the portfolio that complies with the target volatility. TargetReturn, CVaR and CVaR-Skew that are all optimised to provide the same expected return (E(m) = 0.00375) per month. TargetReturn and CVaR generate an almost identical result with only a negligible difference in asset allocation; as a result, CVaR accounting for the entire tail risk has made little impact on unconstrained optimisation. The applied level of skewness and kurtosis for CVaR-Skew analysis also has a negligible impact on portfolio. This indicates that it is difficult to conclude that CVaR and CVaR-Skew contributed significantly to enhancing the portfolio efficiency in terms of risk and return profile.

Table 11: Weights for asset classes, fraction of growth and defensive assets and expected volatility and return for optimised unconstrained portfolios

Asset Class	Min Weight	Max Weight	SAA	Min Var	Max Sharpe	E(sigma)=0.005	E(mu)=0.00375	CVaR	CVaR Skew
KOSPI 200	0	0.7	0.068	0.000	0.053	0.035	0.027	0.027	0.025
S&P 500	0	0.7	0.000	0.006	0.000	0.000	0.000	0.000	0.000
RUSSELL 2000	0	0.7	0.000	0.000	0.005	0.006	0.006	0.009	0.000
MSCI AC World	0	0.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ROGERS ICI	0	0.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000
KIS Bond Index	0.05	1	0.000	0.004	0.942	0.734	0.563	0.576	0.678
MSB 1 Year	0.05	1	0.890	0.000	0.000	0.224	0.404	0.388	0.297
Cash	0.05	1	0.042	0.991	0.000	0.000	0.000	0.000	0.000
Growth Assets			0.068	0.006	0.058	0.042	0.033	0.036	0.025
Defensive Assets			0.932	0.994	0.942	0.958	0.967	0.965	0.975
E(sigma)			0.0041	0.0010	0.0065	0.0050	0.0039	0.0039	0.0045
E(mu)			0.0032	0.0026	0.0043	0.0040	0.00375	0.00375	0.00375

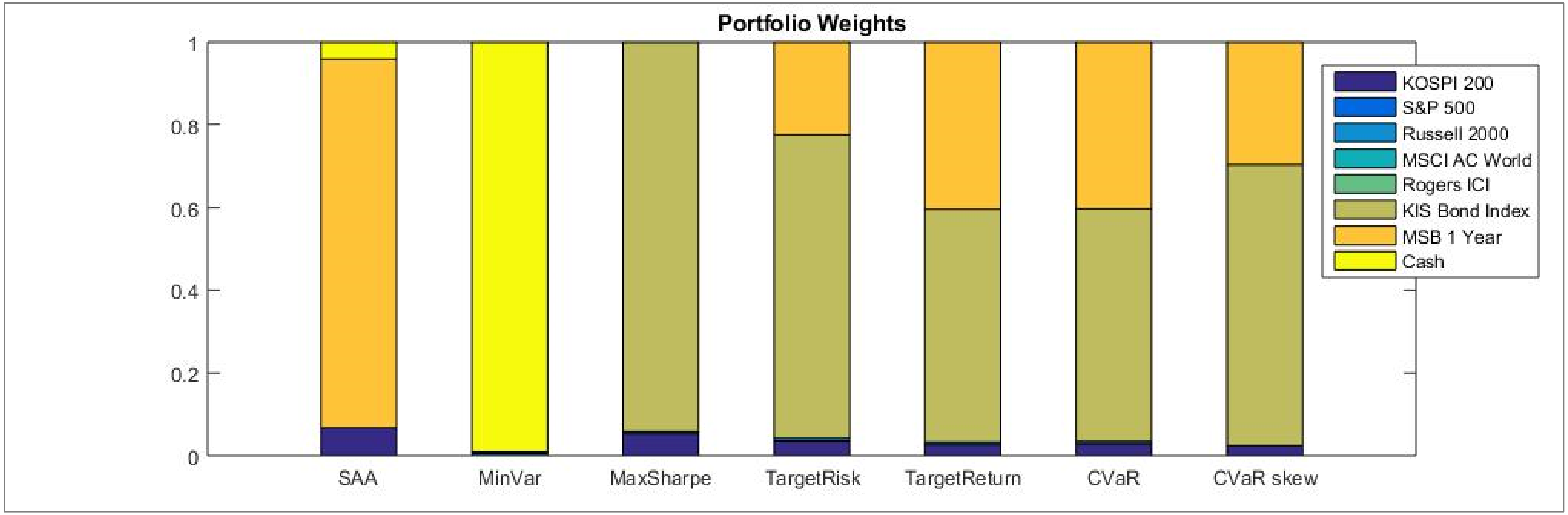

Figure 6: Allocated weights for asset classes for unconstrained portfolio optimisation and seven different optimisation techniques

4.3.2 Constrained Optimisation

As stated in the beginning of the thesis, Appendix 1 shows the current regulation of ROK's pension funds. The regulation sets upper bound for growth assets up to 70% total, and no limits for defensive assets. Thus, if either DB sponsors or DC participants choose to invest in growth assets up to maximum allowance, remaining 30% must invest in defensive assets. Detailed investment constraints for each asset class can be shown in Table 12.

Figure 7 clearly shows the differences between the optimal unconstrained and constrained portfolios. All strategies locate under the efficient frontier when constraints are applied. This is due to the applied additional constraints on asset weights, preventing the seven strategies from reaching their best possible combinations on the efficient frontier.

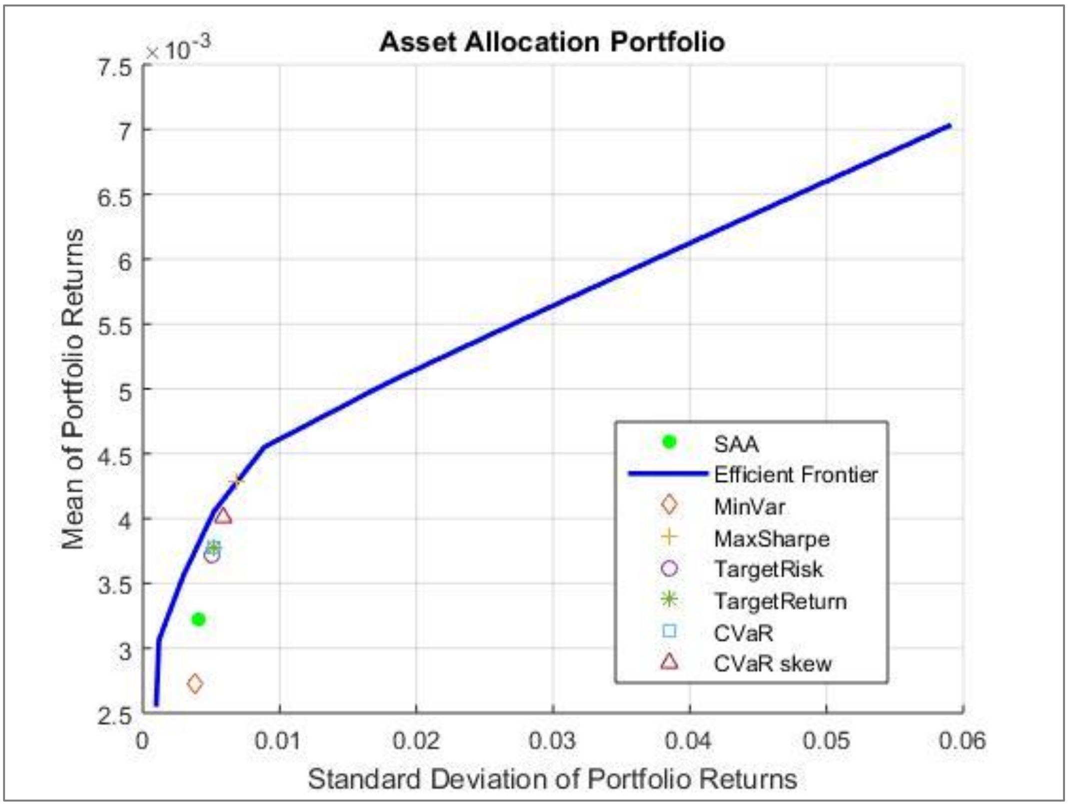

Figure 7: Efficient frontier and location of optimal constrained portfolios in the mu-sigma space

For example, Target Return, and the two CVaR portfolios yield the same expected return as the unconstrained portfolios do having a much higher risk. Also, the constrained Target Risk portfolio provides a much lower monthly return (0.0037) in comparison to the unconstrained Target Risk return (0.0040).

Table 12 illustrates how investment schemes allocate their assets upon constrained optimisation criteria, and Figure 8 shows relative asset allocation of each asset class in terms of different investment schemes for the constrained optimisation.

Table 12: Weights for asset classes, fraction of growth and defensive assets, and expected volatility and return for optimised constrained portfolios

Asset Class	Min Weight	Max Weight	SAA	Min Var	Max Sharpe	E(sigma)=0.005	E(mu)=0.00375	CVaR	CVaR Skew
KOSPI 200	0.1	0.7	0.068	0.001	0.074	0.035	0.037	0.039	0.039
S&P 500	0.1	0.7	0.000	0.072	0.000	0.046	0.042	0.040	0.039
RUSSELL 2000	0.1	0.7	0.000	0.000	0.026	0.003	0.006	0.007	0.003
MSCI AC World	0.1	0.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ROGERS ICI	0.1	0.7	0.000	0.027	0.000	0.016	0.015	0.014	0.020
KIS Bond Index	0.3	0.9	0.000	0.067	0.800	0.520	0.547	0.540	0.746
MSB 1 Year	0.3	0.9	0.890	0.050	0.050	0.330	0.303	0.310	0.105
Cash	0.3	0.9	0.042	0.783	0.050	0.050	0.050	0.050	0.050
Growth Assets			0.068	0.100	0.100	0.100	0.100	0.100	0.100
Defensive Assets			0.932	0.900	0.900	0.900	0.900	0.900	0.900
E(sigma)			0.0041	0.0038	0.0069	0.0050	0.0051	0.0051	0.0059
E(mu)			0.0032	0.0027	0.0043	0.00375	0.00375	0.00375	0.00375

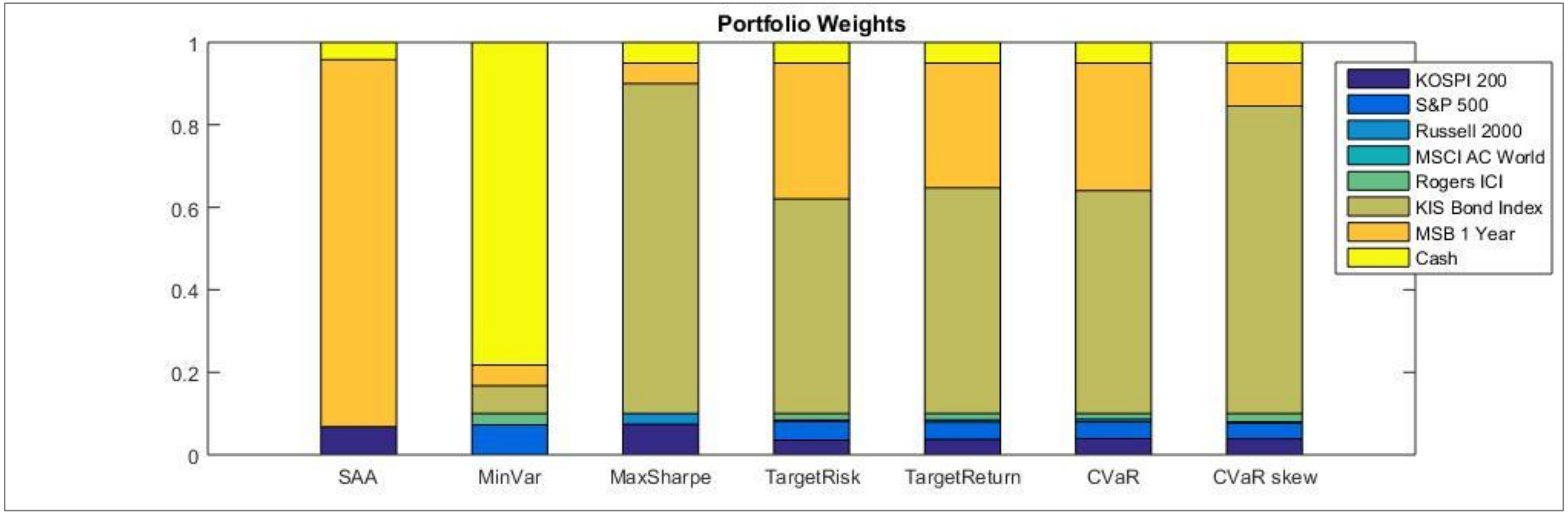

Figure 8: Allocated weights for asset classes for constrained portfolio optimisation and seven different optimisation techniques

The MinVar portfolio allocates 78.3% to the least risky asset class (cash). All the other investment strategies except for SAA allocate 5% to cash, which is equivalent to the minimum set for constraints. From 5% to 10%of cash proportion is favourable in mutual funds to secure liquidity. When maximum weight constraints are set as 70% for growth assets and 90% for defensive assets, the portfolio that maximises Sharpe ratio allocates 80% to KIS Bond Index. MaxSharpe also suggests that KOSPI 200 be allocated 7.4% for portfolio optimisation.

TargetReturn, CVaR and CVaR-Skew are optimised to yield the same expected return (E(mu) = 0.00375) per month. However, CVaR-Skew provides slightly different result, while TargetReturn provides results almost identical to that of CVaR optimization.

Unlike unconstrained optimisation, constrained optimisation suggests that S&P 500 be allocated less than 5% and Rogers ICI less than 2% to meet the purpose of TargetRisk, TargetReturn, CVaR and CVaR-Skew. This indicates that defensive assets are shifted to gross assets to meet the purpose of these four investment strategies under the constraints.

However, the allocation results show that defensive assets decreased overall, but the results differed for KIS Bond Index and MSB (1-year). For example, TargetRisk, TargetReturn and CVaR all decreased in KIS Bond Index and MSB (1-year), while CVaR-Skew shows decreases only in MSB (1-year) but increases in KIS Bond Index. Recall from Table 8 that returns for the KIS Bond Index exhibit positive skewness, i.e. are skewed to the right. Actually, among the considered asset classes the KIS Bond Index is the one, where returns exhibit positive skewness. This would suggest a higher upside potential of returns for this index, but a rather limited downside potential. Therefore, despite the higher level of kurtosis for the KIS Bond Index, the CVaR-Skew optimization suggests to invest a higher share into this index.

Overall, our results suggest that although there is no way to increase returns without taking any risks in portfolios, portfolio returns can be raised up to the extent where we accept risky assets, whether it be growth or defensive assets, on the basis of the empirical dependence structure. It is possible that enlarging the range of growth assets (changes in lower bound and upper bound in growth and defensive assets) may produce portfolios with higher returns. However, what ultimately determines portfolio constraints is an investor’s risk preference or regulation itself.

4.4 Performance Evaluation

In the following the different optimization strategies will be evaluated based on various performance measures. Note that hereby we assume that the weights for each asset class are equal to the proposed asset allocation in the constrained and unconstrained optimization exercise.

Sharpe (1966) introduced the Sharpe index for performance evaluation. This index implies the excess return of assets per unit of risk, especially standard deviation. Risk-free rate of Treasury Bond (3-year) is used to calculate return premium:

$$S_i = \frac{\overline{R_i} - \overline{RFR}}{\sigma_i}$$

A portfolio maximised with the Sharpe ratio generates the highest portfolio return based on the Sharpe index.

The information ratio, which was initially referred to as appraisal ratio by Treynor and Black (1973), differs from the Sharpe ratio in both its numerator and denominator:

$$IR_j = \frac{\overline{R_i} - \overline{R_b}}{\sigma_{ER}}$$

The information ratio uses the standard deviation of return deviation from the benchmark instead of using standard deviation of returns of assets. Namely, σ_{ER} is the standard deviation of the excess return, also called tracking error. Moreover, $\overline{R_b}$, the mean return on the benchmark portfolio, is adopted instead of risk-free asset to see how much a constructed portfolio could outperform its targeted goal. For this analysis, KOSPI 200 was used to measure the performance of optimised portfolios when a proxy of the Korean stock market is benchmarked for the ROK's retirement pension fund.

Jensen (1968) utilised CAPM to investigate portfolio managers' predictive ability in choosing a certain asset to increase the performance of portfolios. Basically, the parameter alpha, also called Jensen's alpha, is abnormal return that can be both positive or negative depending on the consequence of portfolio managers' decision making and operation. Positive alpha suggests superior management ability of funds, whereas negative alpha implies inferior managerial skills. Jensen's index failed to detect diversification ability of managers as done by the Treynor index, where the risk premium in the equation is calculated based on the systematic risk, beta (Reilly & Brown 2011):

$$R_{jt} - RFR_t = \alpha_j + \beta_j[R_{mt} - RFR_t] + e_{jt}$$

Portfolio return of each investment strategy (R_{jt}), KOSPI 200 (R_{mt}), and Treasury Bond (3-year) (RFR_t) were used for Jensen's alpha.

As stated in Chapter 3, Sortino and Price (1994) introduced the measurement that captures DR. The Sortino ratio differs from the Sharpe index in that: 1) τ can be any value a manager wants to adopt (the minimum threshold returns) and 2) DR_i (standard deviation of negative excess returns) is used as the denominator instead of total risk. Using this ratio, by setting τ as the market returns (KOSPI 200), it is possible to investigate how all strategies outperform the market in terms of DR which will contribute to providing the portfolio selection standard when the asymmetry of portfolio returns is witnessed:

$$ST_i = \frac{\overline{R_i} - \tau}{DR_i}$$

Similarly, with the Sharpe ratio and information ratio, a higher Sortino ratio is preferable as it can be an indicator showing how a certain portfolio performs compared to its benchmark by placing bad risk in the denominator slot.

Prior to interpreting the result of optimisation, it must be remembered that all data are based on monthly (not annual) records. Therefore, optimised portfolio performance, TargetReturn and TargetRisk should be interpreted on a monthly basis. Table 13 describes the cumulative performance of the seven investment strategies.

Table 13: Cumulative returns of seven strategies (February 2001 to December 2017)

Cumulative Performance	SAA	MinVar	MaxSharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
Unconstrained	65%	52%	88%	81%	77%	77%	79%
Constrained	65%	55%	87%	76%	77%	77%	81%

Detailed performance analysis is conducted in following sections.

4.4.1 Performance Evaluation for Unconstrained Portfolios

As shown in Figure 9, six investment strategies show negative monthly returns, while MinVar shows slight positive returns.

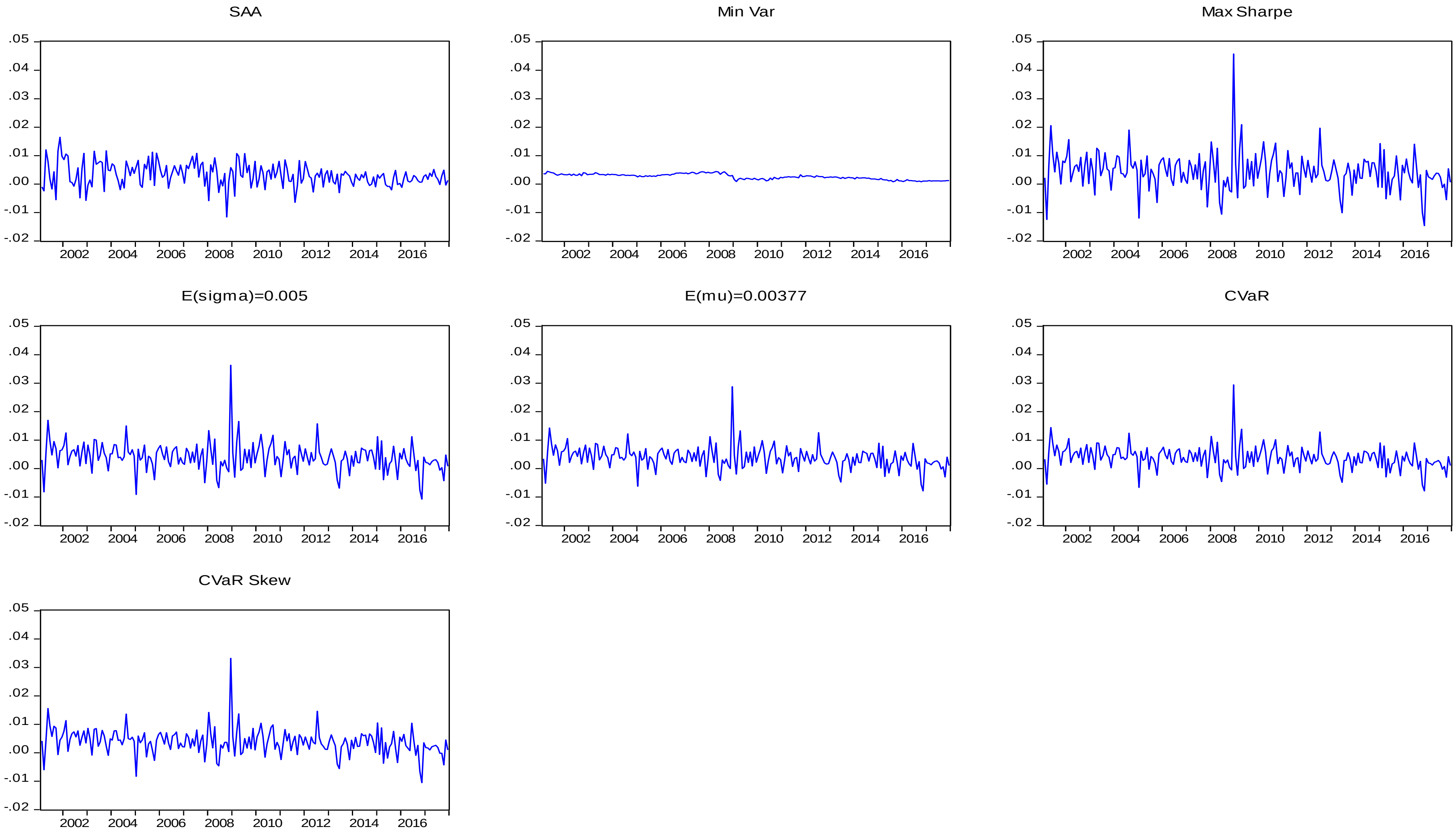

Figure 9: Monthly performance of seven strategies (unconstrained) (February 2001 to December 2017)

This shows how a strategy that only minimises variance succeeded in protecting the principal, although there is a large drop in return around the GFC period. By comparison, SAA, whose priority is securing the principal with 89% allocated to MSB (1-year), failed to secure the principal during the crisis. However, the reachable upside potential of MinVar for return is limited even after the market and the return of asset classes recovered from the crisis. MaxSharpe, TargetRisk, TargetReturn, CVaR and CVaR-Skew in Figure 9 show similar patterns in general, though their peak points differ. This indicates that the peak points can be different depending on the different goals of investment. All the strategies save MinVar focused only on minimising risk and were able to catch up with the returns on designated assets in the time of recovery. SAA, which is a rather naive investment strategy without optimisation, failed to minimise risk or reach investment target. Although MinVar minimises risk to protect portfolios, it may barely reach target return. These unconstrained optimisation results are the justification for targeting risk and targeting return as necessary regardless of setting CVaR or CVaR-Skew.

Figure 10 provides us with relative performances of the seven investment strategies and clearly illustrates the difference in returns during a 17-year period. MinVar (red) is the lowest and SAA (black) the second lowest. TargetRisk (dark blue), TargetReturn (light blue), CVaR (pink) and CVaR-Skew (yellow) are the results of shrinking the bands between maximising return and minimising risk. This implies that it is possible to accomplish the purpose of the portfolio (i.e., TargetRisk and TargetReturn) as well as CVaR and CVaR-Skew, thus, shrinking portfolio dispersion as a by-product.

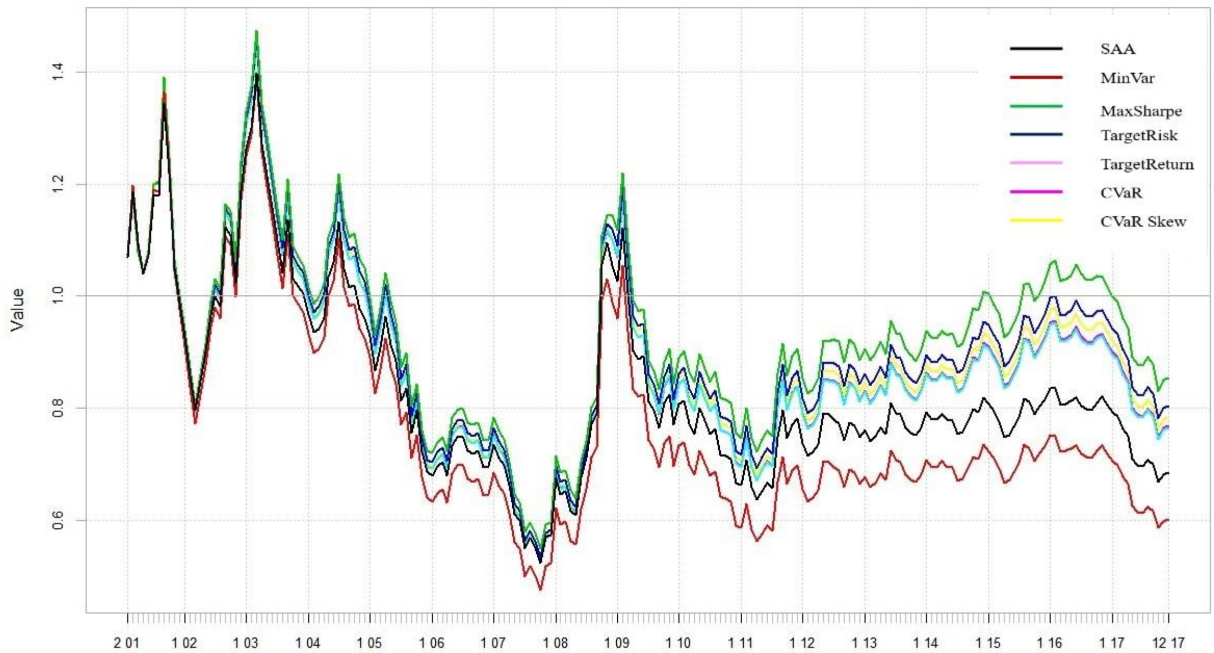

Figure 10: Relative performance of seven investment strategies (unconstrained) (February 2001 to December 17)

Figure 11 illustrates cumulative portfolio performances and the drawdowns for seven investment strategies. MaxSharpe recorded the highest cumulative returns and MinVar recorded the lowest cumulative returns in the analysis period.

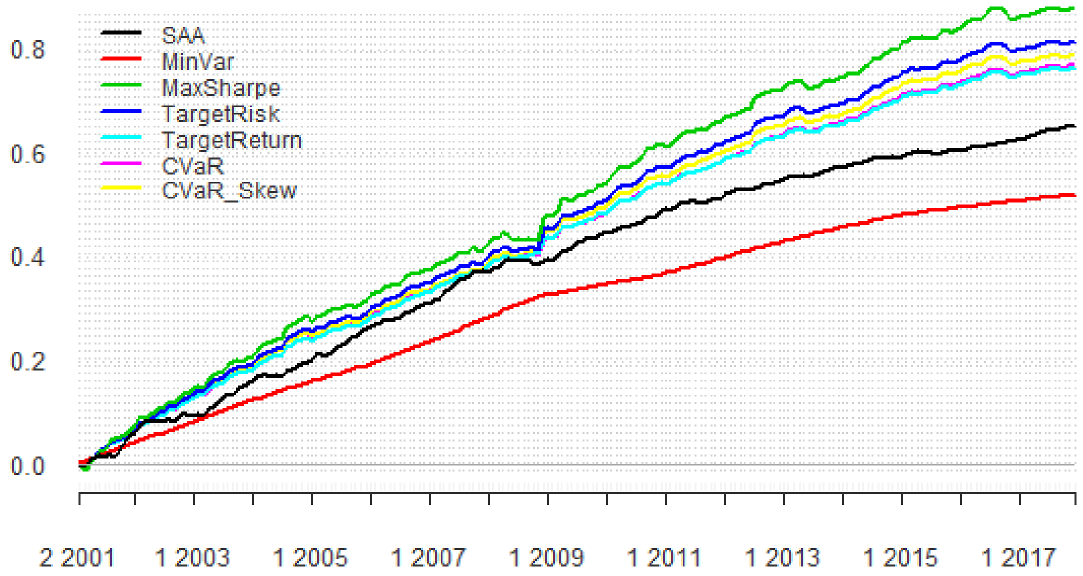

Figure 11: Cumulative performance of seven investment strategies (unconstrained) (February 2001 to December 2017)

However, as shown in Figure 12 and Table 14, drawdown results show the drawback of MaxSharpe—the lowest drawdown of all investment strategies. This implies there is a possibility that investors or portfolio managers may experience a significant drop in return when unexpected turmoil or market downturns occur. Accordingly, when it comes to a long horizon of investment, MaxSharpe may be the best solution for maximising portfolio return of the long-only portfolio without using options or futures.

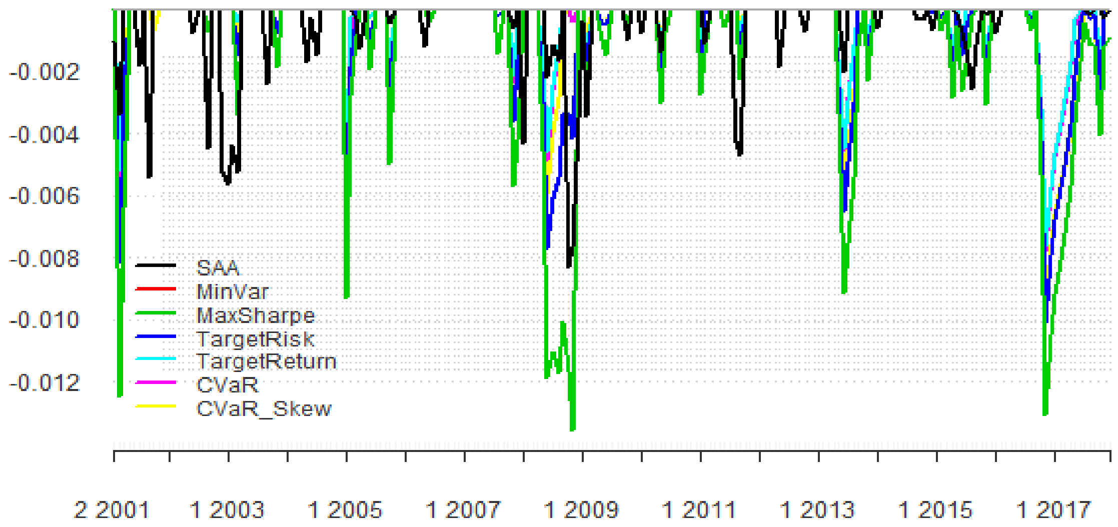

Figure 12: Drawdown for seven investment strategies (unconstrained) (February 2001 to December 2017)

Table 14: Maximum drawdown of seven investment strategies (unconstrained)

Maximum Drawdown	SAA	MinVar	MaxSharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
	-0.0083	0.0000	-0.0136	-0.0101	-0.0077	-0.0078	-0.0096

Table 15 describes the annualised portfolio returns of seven investment strategies for unconstrained optimisation including cumulative and average returns.

Table 15: Annualised portfolio returns of seven investment strategies (unconstrained)

Date	SAA	Min Var	Max Sharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
2001	5.3%	4.0%	6.5%	6.1%	5.8%	5.8%	6.0%
2002	4.2%	4.0%	7.4%	6.9%	6.5%	6.4%	7.0%
2003	6.0%	4.1%	6.7%	6.1%	5.7%	5.9%	5.6%
2004	4.3%	3.7%	8.3%	7.3%	6.5%	6.6%	6.9%
2005	6.6%	3.3%	2.9%	2.9%	3.2%	3.1%	2.7%
2006	4.6%	4.2%	5.7%	5.5%	5.3%	5.4%	5.4%
2007	6.6%	4.7%	4.0%	4.1%	4.4%	4.3%	4.1%
2008	1.5%	4.5%	6.2%	6.2%	6.0%	5.9%	6.9%
2009	5.6%	2.0%	6.3%	5.4%	4.8%	4.9%	4.8%
2010	4.1%	2.1%	7.9%	6.8%	5.9%	6.0%	6.2%
2011	2.4%	3.0%	4.3%	4.2%	4.1%	4.1%	4.4%
2012	3.6%	3.1%	6.0%	5.3%	4.8%	4.9%	5.0%
2013	2.5%	2.7%	2.3%	2.4%	2.5%	2.6%	2.3%
2014	1.7%	2.4%	5.7%	5.0%	4.4%	4.5%	4.9%
2015	1.5%	1.6%	3.8%	3.4%	3.0%	3.0%	3.3%
2016	1.9%	1.3%	2.2%	2.0%	1.9%	1.9%	1.8%
2017	2.9%	1.3%	1.8%	1.7%	1.6%	1.7%	1.4%
Cumulative	65%	52%	88%	81%	77%	77%	79%
Average	3.8%	3.1%	5.2%	4.8%	4.5%	4.5%	4.6%

Figure 13 shows the return distribution of different unconstrained optimisation approaches, comparing the degree of performance dispersion among the seven investment strategies. Figure 13 confirms that MaxSharpe has the largest whiskers, interquartile range (IQR) and outliers (shown as white dots), other than BM70/30, while MinVar has the IQR.

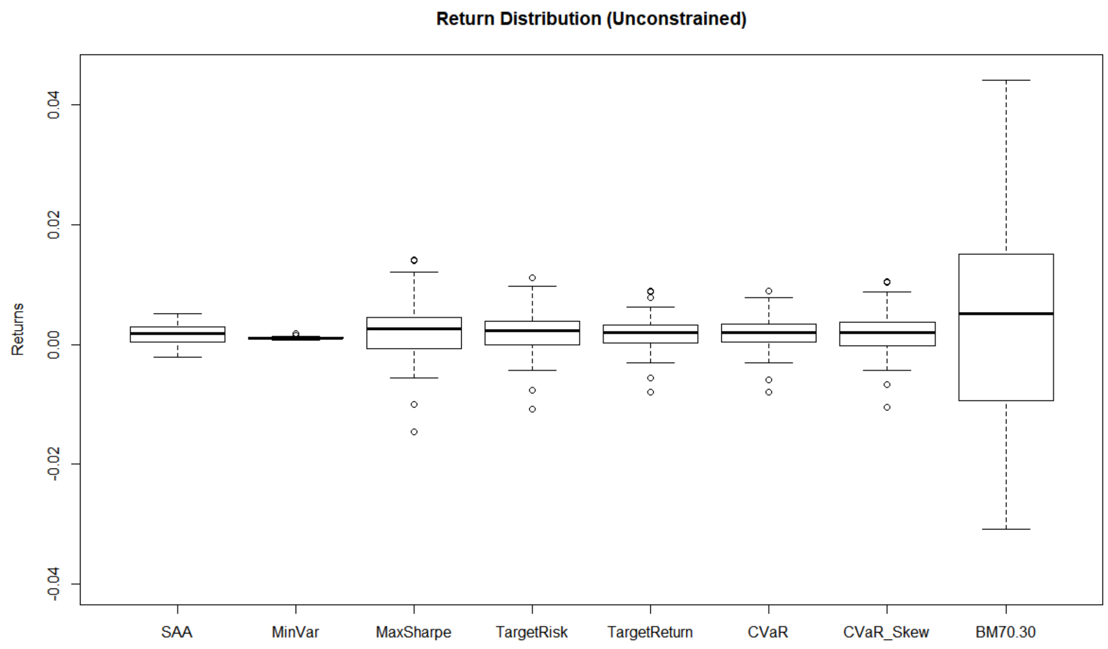

Figure 13: Return distribution for seven investment strategies (unconstrained)

This confirms that MaxSharpe should be accepted for maximising portfolio return, though MaxSharpe may produce a large dispersion which necessarily includes high volatility. In terms of volatility and dispersion of estimated returns, MinVar is the lowest of the seven investment strategies. The return distributions in the box plot are very similar for the pairs TargetRisk and CVaR-Skew and TargetReturn and CVaR. This may be due to CVaR sharing the goal of return with TargetReturn. CVaR-Skew shares the goal of target return with TargetReturn and, additionally, considers empirical skewness and kurtosis in TargetRisk optimisation where those characters of dependence structure may have played a role as a risk factor.

In Table 16, we see the difference in Sharpe ratio between TargetRisk (0.1708) and TargetReturn (0.1569). CVaR enhanced its Sharpe ratio by 1%. This is considered acceptable in this analysis because TargetRisk, E(sigma) = 0.005, locates slightly above the efficient frontier than TargetReturn, E(mu) = 0.00375. However, it is notable that CVaR-Skew increased its Sharpe ratio (0.1627) by 3.7% compared to that of TargetReturn (0.1569). This probably signifies empirical skewness and kurtosis for optimising portfolio contributed to enhancing risk-adjusted performance. The Sharpe ratio of SAA is as low as 0.0153, the second lowest in comparison to other strategies. MinVar recorded a negative Sharpe ratio, indicating that it cannot even outperform risk-free asset in terms of risk-adjusted performance, where both systematic or unsystematic risks are calculated as a denominator in the Sharpe ratio (though it may have the benefit of securing portfolio principal).

Table 16: Performance index for seven investment strategies (unconstrained) (February 2001 to December 2017)

Performance Index	SAA	MinVar	MaxSharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
Sharpe	0.0153	-0.6099	0.1811	0.1708	0.1569	0.1585	0.1627
Information	-0.0662	-0.0731	-0.0444	-0.0495	-0.0535	-0.0531	-0.0510
Tracking Error	0.0543	0.0582	0.0561	0.0568	0.0571	0.0570	0.0575
Alpha	-0.0002	-0.0006	0.0010	0.0007	0.0005	0.0005	0.0007
Sortino	2.3093	Inf	1.7332	2.3375	3.2901	3.1661	2.6738

Information ratio should be interpreted cautiously because, under KOSPI 200, 100% equity was set as its benchmark. The seven investment strategies can be considered as a balanced portfolio as they are not allowed to invest in 100% equity. A balanced portfolio barely beats a full equity portfolio, whether it to be a benchmark or optimised portfolio, in a recovering or booming market. Moreover, the denominator was set as the standard deviation of excess returns. Thus, comparing the performance in terms of information ratio may generate a negative information ratio, which means the inability of active management. However, information ratio, whether it to be positive or negative, still allows us to compare risk-adjusted excess returns of portfolio. For example, the index of the information ratio of MaxSharpe (–0.0444) is the highest among the seven investment strategies, indicating the risk-adjusted performance of MaxSharpe is superior to the other six investment strategies when the denominator was set as a tracking error. Similar to the Sharpe ratio, the information ratio of MinVar is the lowest of the seven investment strategies, confirming that MinVar is not the best option for pursuing returns. However, it may be suitable for minimising risk.

The results of tracking error and alpha index show that tracking error generated from seven investment strategies does not vary significantly over different optimisations. The small value of the alpha index of those strategies also implies passive investment using indices does not target for seeking alphas, which is considered a successful sign of active management, but the alpha index of the seven investment strategies remained small in the analysis. SAA is based on the static and fixed asset allocation of the ROK's current pension fund and, thus, had the lowest tracking error, while the other six investment strategies generated similar tracking errors. SAA and MinVar even recorded negative alphas, though these values were small. This be due to the asset allocation of SAA derived from the ROK's current pension data, or that of MinVar being mainly focused on minimising risk rather than generating alpha. The tracking error and alpha index are limited in the passive strategy, however, active strategy which generally yields higher tracking errors and sometimes succeeds in achieving excess returns when it is believed an active manage has the ability of selecting securities that outperform their benchmarks.

A higher Sortino ratio signifies superiority of performance when taking bad risk as a denominator. The Sortino ratio of TargetReturn (3.2901) and CVaR (3.1661) recorded the highest and second highest values of the seven investment strategies, while MaxSharpe recorded the lowest (1.7332). This cannot be overlooked as it reflects the DR of achievable returns. Thus, we can confirm the importance of targeting return or CVaR when constructing a portfolio in consideration of bad risk. The Sortino ratio of MinVar ('Inf') shows that all returns generated by MinVar were positive from February 2001 to December 2017 and there was no bad risk. Therefore, it is suggested that the Sortino ratio be set as zero for the practical purpose of performance evaluation.

4.4.2 Performance Evaluation for Constrained Portfolios

In comparison with the unconstrained optimisation, shown in Figure 9, constrained optimisation of the seven investment strategies, shown in Figure 14, generated broader band for monthly returns.

As the possible highest peak in each strategy decreases, the lowest bounds of monthly returns also decrease. Specifically, the monthly returns of MinVar in constrained optimisation (Figure 14) differ to those of MinVar in unconstrained optimisation (Figure 9). MinVar was the only strategy that did not generate negative expected returns when unconstrained. However, when constraining on asset weights, MinVar failed to protect principal, especially during the GFC period. This implies that MinVar inevitably limits the upper bound even under unconstrained optimisation which renders it vulnerable to drawdowns in turbulent times.

The DRs in constrained optimisation, except for SAA and MinVar, were enlarged towards the negative monthly return of over –0.01%, while those in unconstrained optimisation were limited to between 0% and –0.01% per month (Figure 9). In contrast, the highest peaks among MaxSharpe, TargetRisk, TargetReturn, CVaR and CVaR-Skew decreased from the higher bound between 0.03% and 0.045% per month to the lower bound between 0.025% and 0.04%. This clearly shows that constrained optimisation failed to depict increasing returns and decreasing risks.

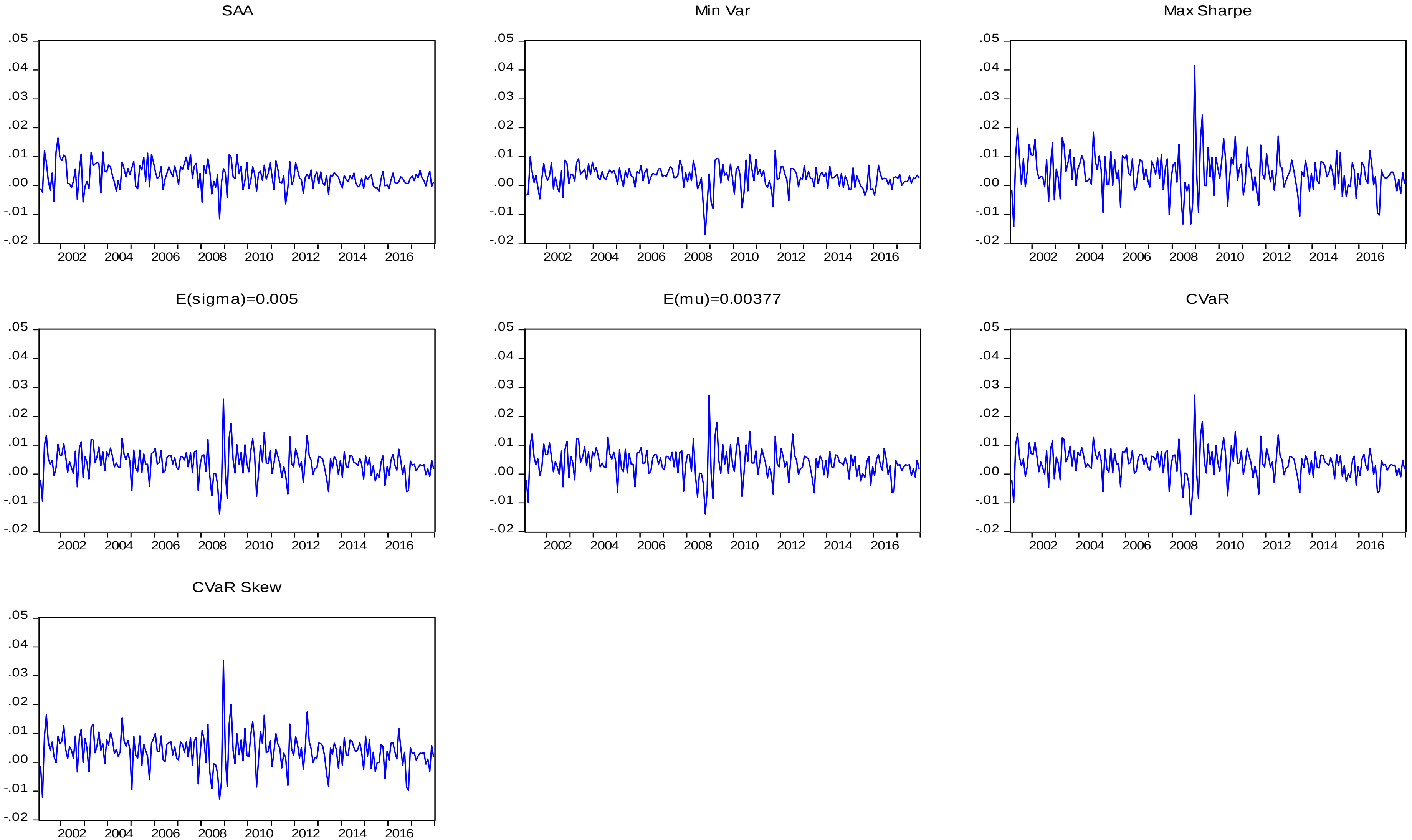

Figure 14: Monthly performance of seven strategies (constrained) (February 2001 to December 2017)

Figure 15 provides us with relative performances of the constrained seven investment strategies. In Figure 10 (unconstrained optimisation), the performance of TargetRisk was higher than CVaR-Skew after the GFC. However, in Figure 15 (constrained optimisation), CVaR-Skew performance after GFC was second followed by TargetRisk and TargetReturn. This implies that the consideration of empirical skewness and kurtosis tend to contribute to increasing portfolio performance when investment constraints are given. MaxSharpe and MinVar positions remained unchanged in constrained and unconstrained optimisation.

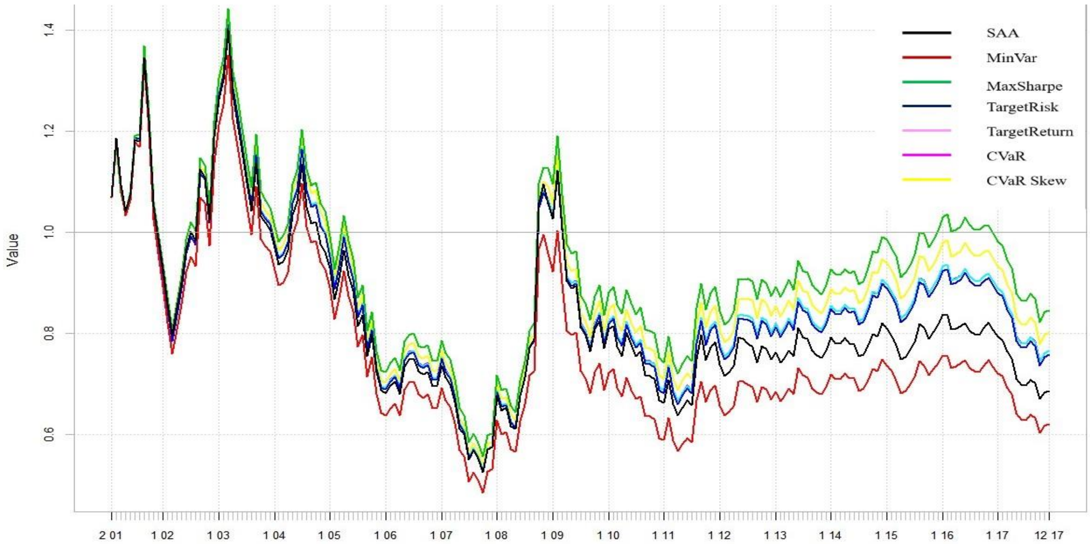

Figure 15: Relative performance of seven investment strategies (unconstrained) (February 2001 to December 17)

Figure 16 shows the cumulative returns of seven investment strategies in constraining asset weights. Cumulative return of CVaR-Skew was second and CVaR-Skew third. TargetRisk, TargetReturn, CVaR and CVaR-Skew tend display similar trends, whether unconstrained or constrained.

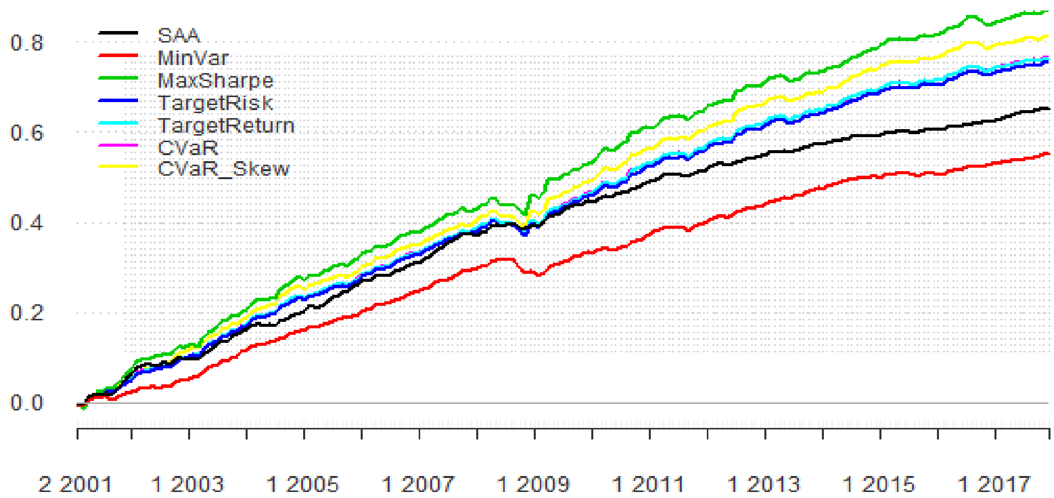

Figure 16: Cumulative performance of seven investment strategies (constrained) (February 2001 to December 2017)

In Figure 17, MinVar showed the lowest drawdown (unlike in Figure 12). This may be due to the difference in asset constitution between unconstrained and constrained optimisation. As shown in Table 11 and Table 12, unconstrained MinVar mainly included cash while constrained MinVar increasingly included risky assets. On average, MinVar had the lowest expected risk, 0.38% per month, among constrained investment strategies. However, constrained MinVar was not able to defend a portfolio during the GFC.

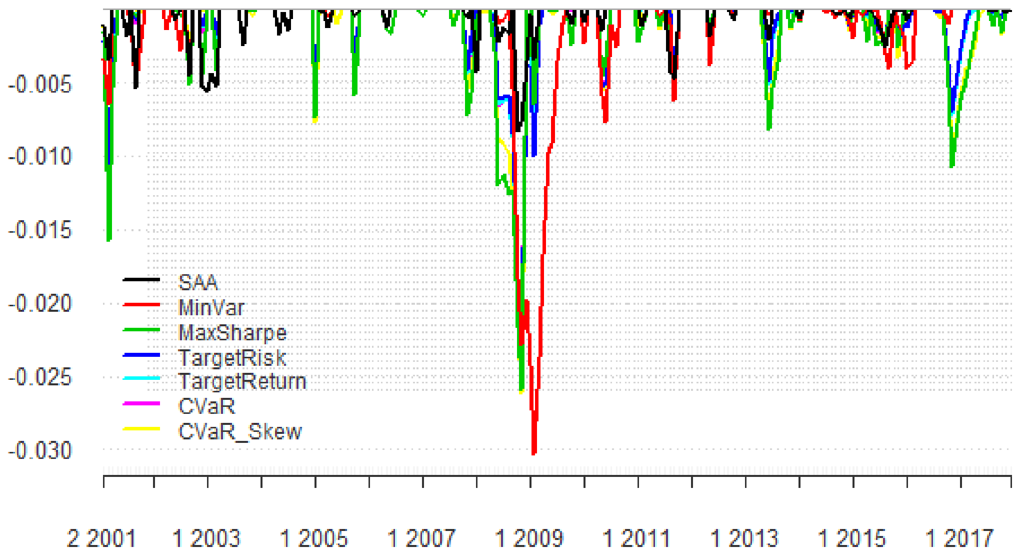

Figure 17: Drawdown for seven investment strategies (constrained) (February 2001 to December 2017)

Table 17 shows the maximum drawdown of the seven investment strategies for the constrained optimisation. We find that the maximum drawdown for the SAA strategy is significantly lower than for all the other strategies, illustrating the additional downside risk of these investment strategies in comparison to the very conservative SAA. Interestingly, most of the six asset allocation strategies also yield very similar results for maximum drawdown. Surprisingly, we observe the highest maximum drawdown for the MinVar strategy during the GFC period.

Table 18 describes the annualised portfolio returns for the seven investment strategies, including cumulative and average returns. For annualised portfolio returns, there is little difference between the results for unconstrained and constrained portfolio optimization, i.e. Table 15 and 18. As expected, MinVar yields the lowest return and standard deviation of returns, while MaxSharpe provides the highest average and cumulative return, but also yields the highest standard deviation.

Table 17: Maximum drawdown of seven investment strategies (constrained)

Maximum Drawdown	SAA	MinVar	MaxSharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
	-0.0083	-0.0303	-0.0259	-0.0224	-0.0227	-0.0226	-0.0261

Table 18: Annualised portfolio returns of seven investment strategies (constrained)

Date	SAA	MinVar	Max Sharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
2001	5.3%	2.1%	6.2%	4.4%	4.5%	4.6%	4.7%
2002	4.2%	3.0%	5.9%	5.2%	5.3%	5.2%	6.2%
2003	6.0%	6.2%	7.7%	7.0%	7.1%	7.1%	7.3%
2004	4.3%	4.8%	8.0%	6.8%	6.9%	6.9%	7.8%
2005	6.6%	3.6%	4.2%	3.8%	3.8%	3.8%	3.2%
2006	4.6%	4.9%	5.7%	5.6%	5.6%	5.6%	5.7%
2007	6.6%	5.1%	4.6%	4.9%	4.9%	4.9%	4.5%
2008	1.5%	-0.3%	3.4%	2.0%	2.2%	2.2%	3.0%
2009	5.6%	4.1%	7.3%	6.2%	6.2%	6.3%	6.5%
2010	4.1%	3.6%	8.0%	6.4%	6.6%	6.6%	7.4%
2011	2.4%	2.7%	3.5%	3.5%	3.5%	3.5%	3.9%
2012	3.6%	3.9%	6.0%	5.1%	5.2%	5.2%	5.7%
2013	2.5%	4.0%	2.9%	3.4%	3.4%	3.4%	3.1%
2014	1.7%	2.5%	4.9%	4.1%	4.2%	4.2%	4.8%
2015	1.5%	0.7%	3.3%	2.2%	2.3%	2.3%	2.7%
2016	1.9%	2.2%	2.6%	2.4%	2.4%	2.4%	2.5%
2017	2.9%	2.5%	2.6%	2.6%	2.6%	2.6%	2.4%
Cumulative	65%	52%	88%	81%	77%	77%	79%
Average	3.8%	3.3%	5.1%	4.4%	4.5%	4.5%	4.8%

Figure 18 shows that the monthly return distributions of the constrained seven investment portfolios generated less outliers than unconstrained strategies. MaxSharpe has the farthest negative outliers from lower whisker without any positive outliers. CVaR-Skew has a similar feature to that of MaxSharpe, unlike in Figure 13 where TargetRisk and CVaR-Skew showed almost the same return distribution. Instead, CVaR, TargetReturn and TargetRisk have similar features in terms of return distribution in constrained optimisation. Apart from SAA, which was not optimised for unconstrained or constrained portfolio, MinVar shares the similarity with the 10/90 benchmark in terms of return distribution. This occurred due to the obligatory asset allocation constrained in growth asset weights regardless of whether the market experiences the ups and downs.

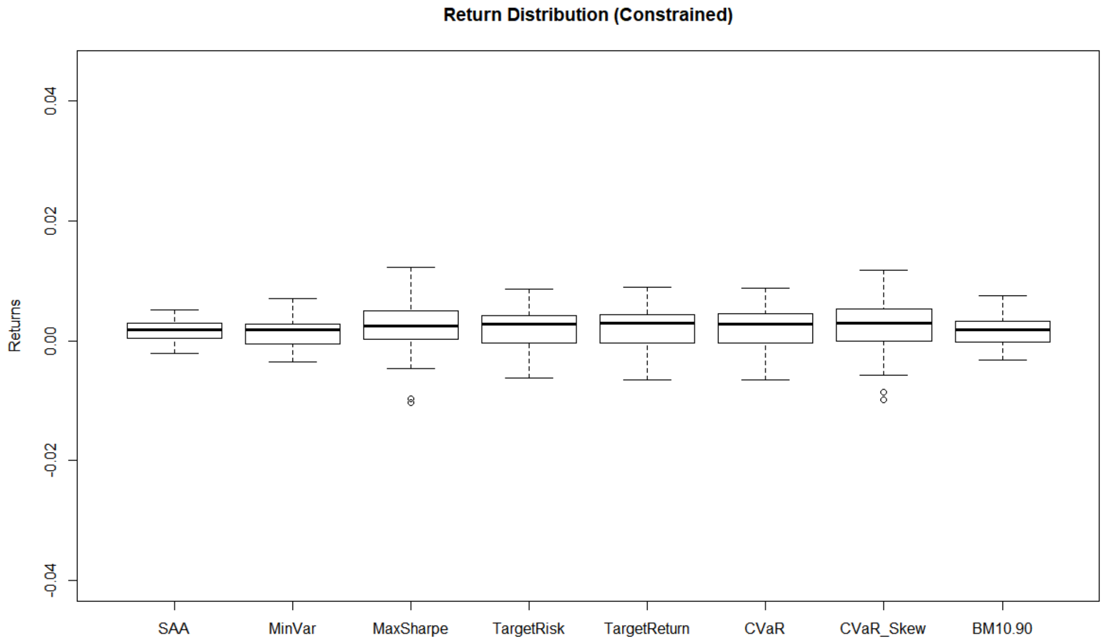

Figure 18: Return distribution for seven investment strategies (constrained)

IQR of constrained return distributions in Figure 18 is much broader compared to the unconstrained return distributions in Figure 13, which implies fat tails in dependence structure. More concentrated and narrow IQR can be interpreted as the enhanced stability of expected return.

Table 19: Performance index for seven investment strategies (constrained) (February 2001 to December 2017)

Performance Index	SAA	MinVar	MaxSharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
Sharpe	0.0153	-0.1136	0.1625	0.1130	0.1190	0.1194	0.1424
Information	-0.0662	-0.0733	-0.0471	-0.0562	-0.0555	-0.0555	-0.0510
Tracking Error	0.0543	0.0558	0.0539	0.0551	0.0550	0.0549	0.0553
Alpha	-0.0002	-0.0006	0.0008	0.0004	0.0004	0.0004	0.0006
Sortino	2.3093	1.4150	1.5334	1.8248	1.8025	1.8058	1.6232

Table 20 shows that maximum return decreases between unconstrained and constrained optimisation. Overall, the performance index of constrained optimisation has deteriorated. Considering that a higher value of performance index indicates better risk-adjusted performance (with an exception of tracking error in Table 19), the difference in performance index between unconstrained and constrained optimisation provides an indicator of how risk-adjusted performance in constrained optimisation deteriorated. Specifically, the difference of Sharpe and Sortino ratio is notable in the condition where information ratio, tracking error and alpha remained relatively unchanged.

Table 20: Difference in performance index between unconstrained and constrained optimization

Difference in Performance Index	MinVar	MaxSharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
Sharpe	0.4963	-0.0186	-0.0578	-0.0379	-0.0391	-0.0203
Information	-0.0002	-0.0027	-0.0067	-0.0020	-0.0024	0.0000
Tracking Error	-0.0024	-0.0022	-0.0017	-0.0021	-0.0021	-0.0022
Alpha	0.0000	-0.0002	-0.0003	-0.0001	-0.0001	-0.0001
Sortino	1.4150	-0.1998	-0.5127	-1.4876	-1.3603	-1.0506

Sharpe ratio increased by 81.4% in constrained MinVar. This may be due to the result of the unsuccessful mitigation of volatility caused by the constraints on asset weights. Considering the main purpose of MinVar is minimising risk instead of maximising return, it is difficult to assess whether the enhanced Sharpe ratio of constrained MinVar is an indicator for a successful investment strategy. Therefore, we cannot say that constrained MinVar is a better choice than unconstrained MinVar based on a higher Sharpe ratio. The decrease in Sharpe ratio of MaxSharpe was limited to 10.3% with minimum while that of TargetRisk was 33.8% with maximum. The Sharpe ratio of CVaR-Skew only decreased by 12.5%, implying reflecting the dependence structure in constrained optimisation also contributed to portfolio performance in unconstrained optimisation as mentioned earlier. The Sharpe ratio of TargetReturn and CVaR decreased by 24.2% and 24.7% respectively, double that of CVaR-Skew. This shows that MaxSharpe and CVaR-Skew was effective to protect risk-adjusted performance under the situation where asset weights are constrained, although the Sharpe ratio decreased in general.

The Sortino ratio of the seven investment strategies in constrained optimisation decreased in general. Recalling Figure 18, which shows the longer right tail of constrained MaxSharpe, the character of tail dependence of MaxSharpe contributed to minimising the decrease of the Sortino ratio. However, four investment strategies—TargetRisk, TargetReturn, CVaR and CVaR-Skew—allowed us to witness longer left tails. The Sortino ratio decreased between 40% to 50% among these four investment strategies. The difference of the Sortino ratio of MinVar was positive, but this was due to the Sortino ratio of unconstrained MinVar being set to zero.

Chapter 5: Ex-Post Analysis (Out-of-Sample Test)

This Chapter examines the performance of the different allocation strategies in a rolling window out-of-sample setting. The window size for the conducted ex-post analysis is M=72 monthly returns, which corresponds to a six year horizon. First, for each six year window, a correlation matrix is calibrated before applying the various optimization schemes to calculate the optimal asset weights for each period. After that, the out-of-sample returns for the seven investment strategies for both unconstrained and constrained are calculated for month T+1. Rolling the window one month forward is repeated by dropping the previous return and adding a new return until T-1 (T=203, total number of returns of each asset class in the data set). The same criteria for TargetRisk, TargetReturn, CVaR, and CVaR Skew portfolios that were used in Chapter 4 are also applied for optimisation. Appendix 2 shows the monthly performance of each rolling window for the seven investment strategies, taking into account both unconstrained and constrained optimization.

Table 21 reports the out-of-sample mean, annualised mean, standard deviation, annualised standard deviation, minimum monthly returns, and maximum monthly returns for the unconstrained optimisation of the seven investment strategies. The table illustrates that the optimisation based on the rolling window analysis, has typically reduced the average returns for each of the optimization strategies. At the same time, in most cases also the volatility of portfolio returns has decreased in comparison to Table 15 and 18. This indicates that reflecting recent correlations of rolling windows might have contributed to more efficiently reducing the portfolio risk.

Nevertheless, Table 21 shows TargetRisk, TargetReturn portfolio as well as CVaR, and CVaR-Skew which shares the same target return failed to achieve their target goals in an out-of-sample performance analysis. For the unconstrained portfolios, the TargetRisk strategy provides returns with a significantly larger standard deviation than the other strategies. It also provides a monthly standard deviation of returns equal to 0.006 what is slightly higher than the target risk E(sigma)=0.005. At the same time, the strategy is not able to provide returns that are significantly higher than those for TargetReturn, CVaR and CVar-Skew. Similar to the analysis in Section 4, the MaxSharpe strategy yields the highest return, but also yields a standard deviation of returns that is significantly higher than the one for SAA, MinVaR, TargetReturn, CVaR and CVaR-Skew. Finally, in the conducted out-of-sample analysis, MinVaR yields both the lowest return, but also the lowest standard deviation of returns.

Table 21: Performance of rolling window analysis of seven investment strategies (unconstrained)

Unconstrained	SAA	MinVar	Max Sharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
Mean	0.0026	0.0024	0.0035	0.0032	0.0030	0.0030	0.0030
Annualised Mean	0.0313	0.0287	0.0425	0.0389	0.0357	0.0358	0.0356
Standard Deviation	0.0035	0.0010	0.0040	0.0060	0.0023	0.0023	0.0025
Annualised Standard Deviation	0.0121	0.0035	0.0138	0.0209	0.0080	0.0080	0.0086
Min	-0.0116	0.0009	-0.0114	-0.0327	-0.0083	-0.0084	-0.0084
Max	0.0108	0.0049	0.0251	0.0172	0.0097	0.0096	0.0108

Figure 19 illustrates that the return distribution of the rolling window analysis for the unconstrained optimisation is much wider than that of unconstrained optimisation in comparison with Figure 13.

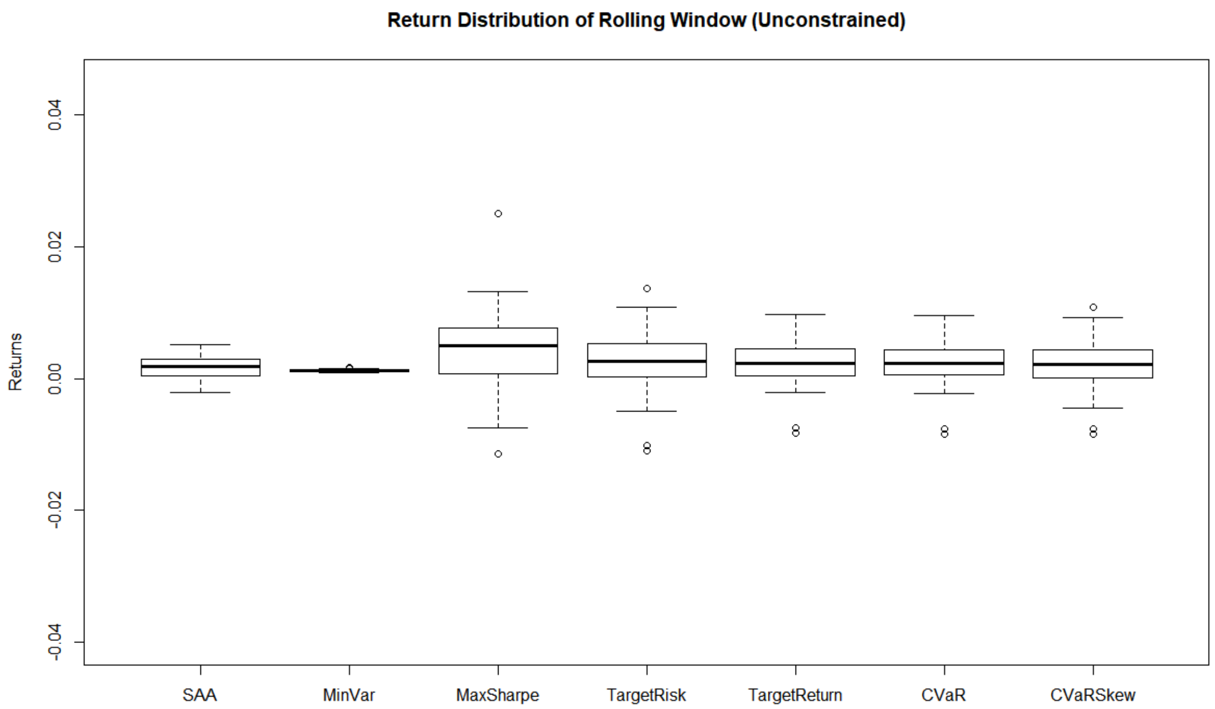

Figure 19: Return distribution of rolling window for seven investment strategies (unconstrained)

Table 22 reports the out-of-sample mean, annualised mean, standard deviation, annualised standard deviation, minimum monthly returns, and maximum monthly returns, when the additional constraints are applied to the seven optimization strategies. Again, the results suggest that both returns and volatility are much lower for the out-of-sample analysis in comparison to the results reported in Table 12 or Table 18. However, similar to the unconstrained out-of-sample results, MaxSharpe yields the highest monthly returns, while SAA and MinVar provide the lowest standard deviation of returns. Again the TargetReturn strategies yield mean returns that are similar to the TargetRisk strategy, but have a significantly lower standard deviation of return. Thus they would clearly suggest a better reward-to-risk ratio.

Table 22: Performance of rolling window analysis of seven investment strategies (constrained)

Mean	SAA	MinVar	Max Sharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
Mean	0.0026	0.0026	0.0040	0.0029	0.0029	0.0030	0.0030
Annualised Mean	0.0313	0.0313	0.0486	0.0352	0.0353	0.0355	0.0361
Standard Deviation	0.0035	0.0042	0.0062	0.0060	0.0045	0.0045	0.0046
Annualised Standard Deviation	0.0121	0.0144	0.0216	0.0207	0.0156	0.0157	0.0158
Min	-0.0116	-0.0174	-0.0210	-0.0327	-0.0174	-0.0178	-0.0182
Max	0.0108	0.0123	0.0212	0.0172	0.0120	0.0120	0.0126

Figure 20 describes the constrained optimisation with limited window size that also broadens the range of return distribution as can be seen in of the unconstrained optimisation Figure 19. In this research, MaxSharpe shows the widest whisker distance with outliers in all optimised strategies. This recommends that investors should take tail values into consideration for maximising portfolio returns.

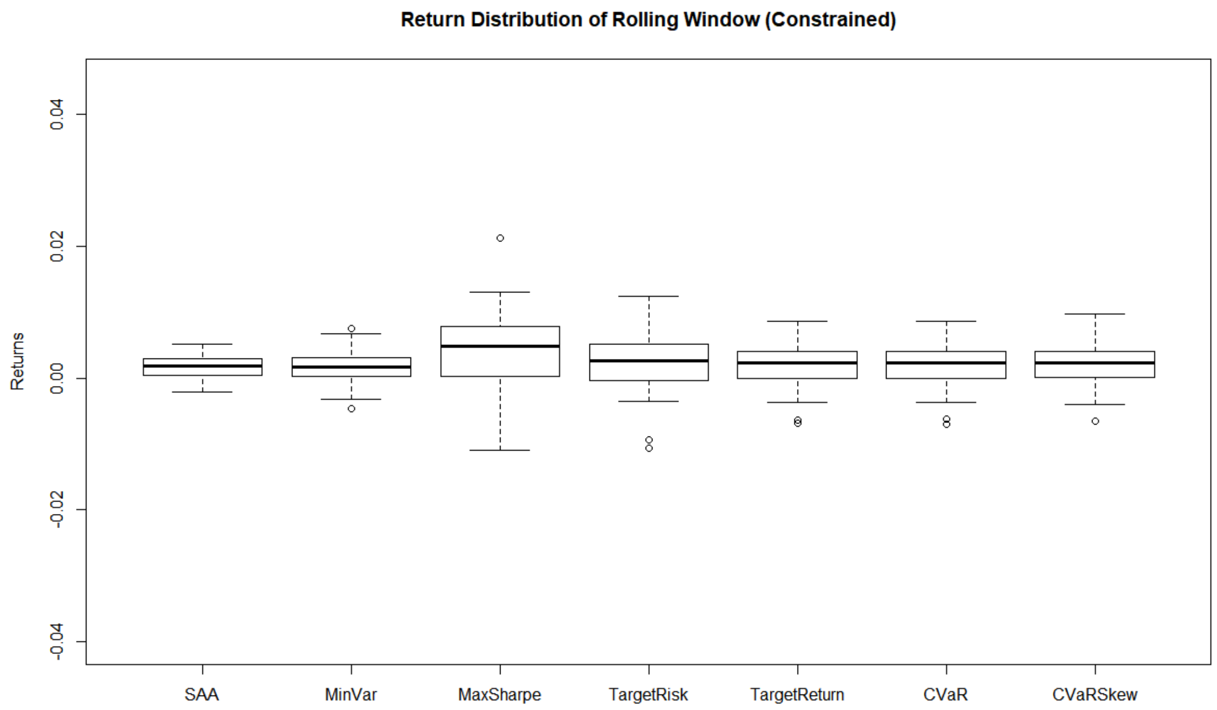

Figure 20: Return distribution of rolling window for seven investment strategies (constrained)

Overall, considering Table 21 and Table 22, we can conclude that both for constrained and unconstrained optimization, in an out-of-sample setting, the target risk portfolio strategy provides a standard deviation of returns that is slightly higher than the initial target of 0.5% monthly. At the same time, also the targe return strategies, fail to provide the target return of 0.375%. This suggests that in an out-of-sample setting, neither the target risk nor target return strategies will necessarily yield their target results. At the same time, both strategies provide an easy-to-interpret strategy for investors.

When it comes to MinVar, Table 22 shows minimising its risk under the constrained investment circumstances is unavailable while the portfolio return of MinVar in Table 21 is similar with that in Table 22. Overall, as could be expected, the risk-return profile appears to be more efficient in unconstrained optimisation. Nonetheless, in reality, setting constraints required by regulations or investment circumstances requires portfolio managers to deviate from optimal investment strategies as suggested by algorithms such as the Mean-Variance optimization. In this sense, target risk or target return strategies might provide interesting alternatives that allow to take into account these additional constraints.

Chapter 6: Conclusion

In recent years, strategic asset allocation has become a major concern in portfolio management of pension funds, endowments, foundations, healthcare institutions and captive insurance companies. Renowned economist Milton Friedman said, ‘There’s no such thing as a free lunch!’—investors often overlook this sentiment due to the asymmetry of risk and return coupled with the desire to have gain and fear of significant losses. From this perspective, this thesis shows that both unconstrained and constrained portfolios cannot enable increased returns without taking additional risk by scrutinising ‘target risk’ and ‘target return’ schemes in addition to other risk-adjusted asset allocation schemes.

This thesis confirmed that optimal asset allocations are impacted by the constraints suggested by the ROK’s pension funds. Overall, unconstrained optimisation for these investments showed better risk-return profiles in comparison to constrained optimisation. However, constrained optimisation failed to achieve more returns when its risk is set at an equal level with unconstrained optimisation, or to minimise risk when its target return is set at an equal level with unconstrained optimisation.

Unconstrained MinVar showed some merit for capital preservation, minimising the risk among the seven unconstrained investment schemes. This was mainly due to heavily tilted allocations towards cash. However, constraining weights on defensive assets accomplished little towards managing risk properly in a constrained MinVar framework, where volatility drastically increased while return remained almost unchanged.

It is notable that TargetRisk limits the lower bound of volatility in terms of portfolio management, which is almost impossible using a strategy that maximizes the Sharp risk-to-reward ratio. Although MaxSharpe contributed to maximising portfolio returns, the strategy also provides a significantly higher standard deviation of portfolio returns—the largest of the seven strategies in both unconstrained and constrained optimisations. A very high allocation to just one asset class was also significant in the MinVar and MaxSharpe strategies, while TargetRisk, TargetReturn, CVaR and CVaR-Skew were relatively well balanced between KIS Bond Index and MSB (1-year).

We propose that ‘target risk’ and ‘target return’ are necessary components when constructing portfolios for pension funds and CVaR and CVaR-Skew should be appropriately applied in the presence of negative skewness and excess kurtosis. Specifically, the results for the conducted ex-post analysis seem to suggest that for the considered period ‘target return’ strategies seem to provide better results than ‘target risk’.

A limitation of this study is that it does not examine portfolio performance over an extended period of time, although the considered sample period included the bursting of the dot-com bubble and the global financial crisis. Considering that it is impossible to know or estimate true parameters, it is unwise to apply a one-size-fits-all solution, based on historical data to a highly dynamic and unpredictable reality. Nevertheless, the risk-adjusted approach used in this thesis serves as a tool to equip portfolio managers of pension funds with a degree of foresight adhering to the purpose of pensions as a long-term investment strategy for retirement.

Another limitation of this study is that the only benchmark for Korean equity employed for performance evaluation is the KOSPI 200. Possibly, the performance evaluation could have been enhanced by adopting diverse angles through various benchmarks for equity investment in the ROK. However, as pointed out in previous research, typically the weights allocated to individual asset classes matter significantly more than the choice of individual stocks or equity investments. Still, actually constructed pension fund portfolios may lead to quite different asset allocation in practice when taking transaction cost, dividends and tax effect into account.

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Appendices

Appendix 1. The Regulation on Supervision of Retirement Pension Plan

<table>
<tr><th colspan="3" rowspan="2">Classification</th><th colspan="2">DB</th><th colspan="2">DC</th></tr>
<tr><th>Previous</th><th>Current</th><th>Previous</th><th>Current</th></tr>
<tr><td colspan="3">Principal guaranteed-type products</td><td>Min 30%</td><td>Min 30%</td><td>Min 60%</td><td>Min 30%</td></tr>
<tr><td colspan="3">Investment-type products</td><td>Max 70%</td><td>Max 70%</td><td>Max 40%</td><td>Max 70%</td></tr>
<tr><td rowspan="8"></td><td colspan="2">Equity Securities
(e.g. Stocks)</td><td>30%</td><td rowspan="7">No investment constraints on individual asset classes</td><td>Prohibited</td><td>Prohibited</td></tr>
<tr><td>Debt Securities</td><td>Domestic investment grade</td><td>100%</td><td>100%</td><td rowspan="5">No investment constraints on individual asset classes</td></tr>
<tr><td>Other bonds</td><td>Below investment grade</td><td>30%</td><td>30%</td></tr>
<tr><td colspan="2">Mutual Funds</td><td>50%</td><td>30 ~ 40%</td></tr>
<tr><td rowspan="2">Equity Linked Securities</td><td>Loss: Max 10%</td><td>100%</td><td>100%</td></tr>
<tr><td>Loss: 10% ~ 40%</td><td>30%</td><td>Prohibited</td></tr>
<tr><td colspan="2">Depositary Receipt</td><td>30%</td><td>Prohibited</td><td>Prohibited</td></tr>
<tr><td colspan="2">Derivatives</td><td>Allowed on hedging purpose only</td><td>Allowed on hedging purpose only</td><td>Allowed on hedging purpose only</td><td>Allowed on hedging purpose only</td></tr>
<tr><td colspan="3">Total</td><td>100%</td><td>100%</td><td>100%</td><td>100%</td></tr>
</table>

Numbers in the shaded boxes show percentage changes occurred in the investment-type products after the implementation of the regulation (1st July 2015). The ROK Government abolished the constraints set up on individual asset classes, while allowing maximum 70% asset allocation in aggregation for investment-type products.
Source: FSS (2016).

Appendix 2. The Monthly Performance of Rolling Window for Seven Investment Strategies

Rolling Window	Unconstrained							Constrained						
	SAA	MinVar	Max Sharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew	SAA	MinVar	Max Sharpe	E(sigma) =0.005	E(mu) =0.00375	CVaR	CVaR Skew
1	0.0003	0.0038	0.0040	-0.0008	0.0040	0.0041	0.0041	0.0003	0.0030	-0.0001	-0.0010	0.0030	0.0028	0.0028
2	0.0066	0.0038	0.0042	0.0092	0.0041	0.0042	0.0042	0.0066	0.0044	0.0081	0.0093	0.0044	0.0048	0.0047
3	0.0054	0.0038	0.0042	0.0064	0.0041	0.0041	0.0041	0.0054	0.0051	0.0060	0.0064	0.0051	0.0051	0.0052
4	0.0077	0.0039	0.0042	0.0025	0.0042	0.0043	0.0042	0.0077	0.0056	0.0033	0.0025	0.0056	0.0053	0.0046
5	0.0098	0.0039	0.0044	0.0069	0.0043	0.0043	0.0043	0.0098	0.0057	0.0063	0.0069	0.0057	0.0055	0.0055
6	0.0055	0.0038	0.0043	0.0037	0.0042	0.0043	0.0043	0.0055	0.0035	0.0042	0.0037	0.0035	0.0035	0.0032
7	0.0108	0.0039	0.0045	0.0133	0.0043	0.0044	0.0044	0.0108	0.0042	0.0098	0.0133	0.0042	0.0045	0.0043
8	0.0024	0.0041	0.0043	-0.0024	0.0043	0.0043	0.0044	0.0024	0.0029	-0.0006	-0.0025	0.0029	0.0026	0.0027
9	0.0067	0.0042	0.0047	0.0101	0.0046	0.0047	0.0045	0.0067	0.0096	0.0101	0.0101	0.0096	0.0097	0.0097
10	0.0077	0.0042	0.0047	0.0116	0.0046	0.0047	0.0046	0.0077	0.0074	0.0094	0.0116	0.0074	0.0076	0.0079
11	-0.0008	0.0041	0.0041	-0.0054	0.0042	0.0042	0.0043	-0.0008	-0.0002	-0.0016	-0.0054	-0.0002	-0.0003	-0.0008
12	0.0042	0.0042	0.0048	0.0078	0.0046	0.0047	0.0047	0.0042	0.0056	0.0063	0.0077	0.0056	0.0059	0.0060
13	-0.0059	0.0041	0.0045	0.0081	0.0044	0.0045	0.0044	-0.0059	0.0023	0.0045	0.0081	0.0023	0.0031	0.0036
14	0.0068	0.0042	0.0043	0.0172	0.0041	0.0042	0.0043	0.0068	0.0066	0.0114	0.0172	0.0066	0.0075	0.0065
15	0.0042	0.0042	0.0040	-0.0036	0.0042	0.0041	0.0041	0.0042	0.0010	-0.0009	-0.0036	0.0010	0.0007	0.0008
16	0.0092	0.0043	0.0045	0.0117	0.0044	0.0045	0.0044	0.0092	0.0086	0.0093	0.0117	0.0086	0.0086	0.0090
17	0.0047	0.0043	0.0043	0.0035	0.0044	0.0044	0.0044	0.0047	0.0059	0.0049	0.0034	0.0059	0.0057	0.0057
18	-0.0030	0.0039	0.0042	0.0072	0.0041	0.0042	0.0043	-0.0030	0.0018	0.0048	0.0072	0.0018	0.0025	0.0016
19	0.0014	0.0049	0.0047	-0.0076	0.0049	0.0048	0.0048	0.0014	-0.0002	-0.0025	-0.0076	-0.0002	-0.0007	-0.0008
20	-0.0007	0.0043	0.0047	-0.0018	0.0047	0.0047	0.0048	-0.0007	0.0025	0.0008	-0.0019	0.0025	0.0021	0.0023
21	0.0037	0.0042	0.0042	-0.0151	0.0044	0.0043	0.0046	0.0037	-0.0070	-0.0075	-0.0151	-0.0070	-0.0072	-0.0072
22	-0.0116	0.0033	0.0032	-0.0327	0.0035	0.0033	0.0041	-0.0116	-0.0174	-0.0210	-0.0327	-0.0174	-0.0178	-0.0182
23	0.0009	0.0032	0.0040	-0.0059	0.0038	0.0040	0.0042	0.0009	-0.0049	-0.0074	-0.0070	-0.0051	-0.0051	-0.0046
24	0.0058	0.0034	0.0041	0.0152	0.0039	0.0040	0.0039	0.0058	0.0052	0.0096	0.0087	0.0056	0.0054	0.0104

25	0.0044	0.0018	0.0021	0.0060	0.0019	0.0020	0.0021	0.0044	-0.0062	0.0042	0.0001	-0.0043	-0.0044	-0.0035
26	-0.0043	0.0014	0.0017	-0.0045	0.0017	0.0017	0.0019	-0.0043	-0.0087	-0.0073	-0.0069	-0.0078	-0.0078	-0.0082
27	0.0107	0.0016	0.0026	0.0112	0.0024	0.0025	0.0024	0.0107	0.0096	0.0161	0.0121	0.0113	0.0113	0.0109
28	0.0097	0.0022	0.0028	0.0150	0.0026	0.0027	0.0025	0.0097	0.0100	0.0182	0.0124	0.0111	0.0111	0.0110
29	0.0030	0.0019	0.0021	0.0008	0.0022	0.0022	0.0021	0.0030	0.0089	0.0030	0.0078	0.0085	0.0086	0.0091
30	0.0023	0.0020	0.0023	0.0009	0.0023	0.0023	0.0023	0.0023	0.0018	0.0006	0.0011	0.0014	0.0014	0.0014
31	0.0107	0.0026	0.0029	0.0096	0.0030	0.0030	0.0030	0.0107	0.0084	0.0118	0.0098	0.0091	0.0092	0.0102
32	0.0040	0.0027	0.0028	0.0029	0.0028	0.0028	0.0027	0.0040	0.0048	0.0028	0.0032	0.0039	0.0039	0.0036
33	0.0066	0.0031	0.0032	0.0072	0.0035	0.0034	0.0032	0.0066	0.0055	0.0082	0.0065	0.0060	0.0060	0.0062
34	-0.0014	0.0028	0.0026	-0.0015	0.0024	0.0024	0.0026	-0.0014	0.0021	-0.0025	0.0007	0.0016	0.0016	0.0009
35	0.0018	0.0030	0.0029	0.0059	0.0032	0.0032	0.0031	0.0018	0.0080	0.0062	0.0061	0.0069	0.0070	0.0068
36	0.0080	0.0029	0.0030	0.0038	0.0030	0.0031	0.0035	0.0080	0.0039	0.0053	0.0053	0.0046	0.0047	0.0047
37	-0.0011	0.0025	0.0026	0.0024	0.0026	0.0026	0.0028	-0.0011	-0.0021	-0.0004	-0.0023	-0.0022	-0.0021	-0.0016
38	0.0018	0.0028	0.0028	0.0061	0.0033	0.0032	0.0032	0.0018	0.0058	0.0067	0.0059	0.0059	0.0059	0.0071
39	0.0064	0.0028	0.0030	0.0106	0.0042	0.0042	0.0040	0.0064	0.0071	0.0146	0.0088	0.0085	0.0084	0.0083
40	0.0040	0.0023	0.0026	0.0058	0.0031	0.0031	0.0030	0.0040	0.0038	0.0079	0.0054	0.0050	0.0050	0.0049
41	-0.0020	0.0018	0.0018	-0.0027	0.0011	0.0012	0.0014	-0.0020	-0.0070	-0.0065	-0.0068	-0.0069	-0.0069	-0.0065
42	0.0043	0.0025	0.0027	0.0040	0.0029	0.0029	0.0031	0.0043	-0.0023	0.0052	0.0016	0.0015	0.0015	0.0028
43	0.0050	0.0029	0.0030	0.0064	0.0037	0.0037	0.0038	0.0050	0.0092	0.0082	0.0085	0.0085	0.0085	0.0083
44	0.0017	0.0027	0.0029	0.0059	0.0034	0.0034	0.0038	0.0017	-0.0021	0.0060	0.0009	0.0004	0.0004	0.0003
45	0.0070	0.0030	0.0033	0.0104	0.0048	0.0049	0.0052	0.0070	0.0106	0.0136	0.0111	0.0111	0.0110	0.0109
46	0.0022	0.0024	0.0023	0.0015	0.0021	0.0021	0.0022	0.0022	0.0058	0.0012	0.0041	0.0043	0.0043	0.0045
47	0.0042	0.0025	0.0027	0.0044	0.0031	0.0031	0.0030	0.0042	0.0021	0.0054	0.0034	0.0033	0.0032	0.0036
48	0.0080	0.0029	0.0030	0.0058	0.0036	0.0036	0.0033	0.0080	0.0094	0.0086	0.0098	0.0098	0.0098	0.0101
49	0.0030	0.0027	0.0025	-0.0001	0.0019	0.0020	0.0017	0.0030	0.0047	-0.0005	0.0033	0.0035	0.0035	0.0028
50	-0.0016	0.0030	0.0027	0.0003	0.0021	0.0021	0.0025	-0.0016	0.0055	-0.0021	0.0026	0.0032	0.0031	0.0032
51	0.0085	0.0033	0.0037	0.0097	0.0053	0.0053	0.0044	0.0085	0.0036	0.0142	0.0073	0.0070	0.0070	0.0066
52	0.0055	0.0032	0.0033	0.0053	0.0037	0.0037	0.0035	0.0055	0.0056	0.0070	0.0061	0.0060	0.0060	0.0060
53	0.0010	0.0030	0.0030	0.0040	0.0033	0.0032	0.0031	0.0010	0.0006	0.0035	0.0008	0.0007	0.0006	0.0007

54	0.0009	0.0028	0.0026	0.0001	0.0022	0.0021	0.0022	0.0009	0.0001	-0.0015	-0.0008	-0.0005	-0.0005	-0.0007
55	0.0034	0.0031	0.0032	0.0038	0.0033	0.0033	0.0033	0.0034	0.0015	0.0041	0.0028	0.0026	0.0026	0.0023
56	-0.0064	0.0027	0.0027	0.0023	0.0026	0.0026	0.0026	-0.0064	-0.0029	-0.0029	-0.0033	-0.0033	-0.0033	-0.0017
57	-0.0007	0.0023	0.0023	-0.0010	0.0016	0.0017	0.0012	-0.0007	-0.0061	-0.0059	-0.0066	-0.0068	-0.0067	-0.0067
58	0.0082	0.0034	0.0034	0.0056	0.0039	0.0039	0.0038	0.0082	0.0123	0.0126	0.0122	0.0120	0.0120	0.0126
59	0.0003	0.0029	0.0031	0.0052	0.0035	0.0035	0.0035	0.0003	0.0017	0.0034	0.0019	0.0020	0.0020	0.0022
60	0.0018	0.0029	0.0028	0.0029	0.0029	0.0029	0.0029	0.0018	0.0028	0.0014	0.0025	0.0021	0.0021	0.0018
61	0.0079	0.0032	0.0033	0.0043	0.0035	0.0035	0.0038	0.0079	0.0072	0.0104	0.0076	0.0083	0.0083	0.0081
62	0.0053	0.0031	0.0030	0.0029	0.0030	0.0030	0.0029	0.0053	0.0066	0.0060	0.0065	0.0064	0.0064	0.0063
63	0.0026	0.0029	0.0027	0.0012	0.0023	0.0023	0.0025	0.0026	0.0044	0.0000	0.0036	0.0024	0.0023	0.0030
64	0.0021	0.0029	0.0032	0.0057	0.0039	0.0039	0.0037	0.0021	0.0019	0.0055	0.0024	0.0032	0.0032	0.0028
65	-0.0028	0.0025	0.0028	0.0047	0.0034	0.0033	0.0035	-0.0028	-0.0050	-0.0018	-0.0046	-0.0040	-0.0040	-0.0038
66	0.0028	0.0028	0.0028	0.0031	0.0029	0.0029	0.0029	0.0028	0.0057	0.0032	0.0055	0.0048	0.0048	0.0048
67	0.0038	0.0029	0.0045	0.0147	0.0073	0.0073	0.0066	0.0038	0.0046	0.0190	0.0057	0.0104	0.0103	0.0105
68	0.0023	0.0025	0.0029	0.0053	0.0035	0.0035	0.0034	0.0023	0.0043	0.0060	0.0045	0.0051	0.0053	0.0051
69	0.0054	0.0025	0.0025	0.0026	0.0025	0.0025	0.0022	0.0054	0.0046	0.0061	0.0046	0.0049	0.0049	0.0050
70	-0.0011	0.0022	0.0023	0.0028	0.0024	0.0024	0.0028	-0.0011	-0.0005	-0.0011	-0.0006	-0.0008	-0.0008	-0.0001
71	0.0033	0.0023	0.0020	0.0010	0.0018	0.0018	0.0018	0.0033	0.0027	0.0018	0.0026	0.0023	0.0023	0.0029
72	0.0047	0.0023	0.0018	0.0006	0.0016	0.0017	0.0016	0.0047	0.0030	0.0030	0.0029	0.0025	0.0026	0.0026
73	0.0006	0.0025	0.0032	0.0044	0.0032	0.0032	0.0038	0.0006	0.0060	0.0031	0.0058	0.0051	0.0050	0.0052
74	0.0047	0.0023	0.0051	0.0059	0.0040	0.0040	0.0039	0.0047	0.0029	0.0086	0.0032	0.0048	0.0050	0.0038
75	0.0008	0.0023	0.0055	0.0045	0.0032	0.0033	0.0033	0.0008	0.0044	0.0035	0.0044	0.0043	0.0044	0.0041
76	0.0000	0.0022	0.0033	0.0029	0.0025	0.0025	0.0024	0.0000	0.0024	0.0004	0.0023	0.0018	0.0019	0.0030
77	0.0035	0.0020	-0.0075	-0.0044	-0.0011	-0.0011	-0.0014	0.0035	0.0036	-0.0044	0.0026	-0.0007	-0.0005	0.0008
78	-0.0031	0.0019	-0.0064	-0.0037	-0.0013	-0.0012	-0.0010	-0.0031	-0.0006	-0.0113	-0.0019	-0.0059	-0.0059	-0.0056
79	0.0036	0.0025	0.0013	0.0016	0.0018	0.0018	0.0019	0.0036	0.0063	0.0057	0.0062	0.0057	0.0057	0.0054
80	0.0031	0.0021	0.0031	0.0027	0.0025	0.0025	0.0026	0.0031	0.0002	-0.0005	0.0001	-0.0006	-0.0006	0.0020
81	0.0045	0.0024	0.0054	0.0044	0.0037	0.0037	0.0035	0.0045	0.0044	0.0107	0.0047	0.0061	0.0063	0.0050
82	0.0036	0.0024	0.0026	0.0025	0.0025	0.0025	0.0026	0.0036	0.0052	0.0047	0.0054	0.0060	0.0061	0.0043

83	0.0028	0.0022	-0.0049	-0.0024	-0.0009	-0.0009	-0.0008	0.0028	0.0039	-0.0003	0.0033	0.0007	0.0007	-0.0022
84	0.0007	0.0022	0.0065	0.0051	0.0042	0.0042	0.0040	0.0007	0.0034	0.0073	0.0041	0.0058	0.0058	0.0038
85	-0.0009	0.0019	0.0022	0.0022	0.0022	0.0021	0.0022	-0.0009	-0.0014	-0.0004	-0.0015	-0.0014	-0.0014	-0.0012
86	0.0035	0.0024	0.0070	0.0055	0.0048	0.0048	0.0049	0.0035	0.0059	0.0098	0.0062	0.0080	0.0080	0.0082
87	0.0020	0.0022	0.0019	0.0021	0.0021	0.0021	0.0021	0.0020	0.0025	0.0012	0.0025	0.0027	0.0027	0.0025
88	0.0013	0.0022	0.0015	0.0019	0.0019	0.0019	0.0018	0.0013	0.0023	-0.0015	0.0023	0.0029	0.0029	0.0028
89	0.0033	0.0022	0.0082	0.0064	0.0058	0.0058	0.0069	0.0033	0.0034	0.0078	0.0047	0.0072	0.0074	0.0090
90	0.0022	0.0022	0.0083	0.0065	0.0055	0.0055	0.0055	0.0022	0.0033	0.0086	0.0042	0.0065	0.0066	0.0074
91	0.0043	0.0019	0.0053	0.0051	0.0041	0.0041	0.0043	0.0043	0.0015	0.0045	0.0021	0.0026	0.0026	0.0034
92	0.0008	0.0019	0.0044	0.0037	0.0032	0.0032	0.0029	0.0008	0.0037	0.0067	0.0046	0.0052	0.0051	0.0044
93	-0.0005	0.0016	0.0094	0.0073	0.0056	0.0057	0.0056	-0.0005	-0.0007	0.0072	0.0013	0.0029	0.0030	0.0022
94	0.0000	0.0016	0.0095	0.0073	0.0055	0.0055	0.0056	0.0000	0.0024	0.0100	0.0047	0.0057	0.0057	0.0035
95	0.0025	0.0016	0.0059	0.0043	0.0034	0.0034	0.0033	0.0025	0.0013	0.0060	0.0046	0.0043	0.0042	0.0027
96	-0.0010	0.0017	0.0003	0.0008	0.0012	0.0012	0.0013	-0.0010	-0.0021	0.0003	0.0005	0.0002	0.0003	-0.0020
97	0.0032	0.0016	0.0100	0.0108	0.0072	0.0072	0.0086	0.0032	-0.0007	0.0087	0.0052	0.0026	0.0025	0.0040
98	0.0020	0.0017	0.0035	0.0017	0.0017	0.0016	0.0005	0.0020	0.0054	0.0038	0.0050	0.0056	0.0056	0.0041
99	0.0030	0.0016	0.0074	0.0075	0.0046	0.0046	0.0058	0.0030	0.0001	0.0069	0.0058	0.0011	0.0013	0.0037
100	0.0037	0.0013	-0.0054	-0.0050	-0.0020	-0.0020	-0.0029	0.0037	0.0033	-0.0050	-0.0035	0.0011	0.0011	0.0010
101	-0.0002	0.0014	0.0063	0.0055	0.0038	0.0038	0.0042	-0.0002	0.0014	0.0061	0.0053	0.0030	0.0029	0.0030
102	-0.0009	0.0013	-0.0047	-0.0034	-0.0015	-0.0014	-0.0014	-0.0009	-0.0012	-0.0045	-0.0035	-0.0020	-0.0020	-0.0021
103	-0.0009	0.0012	0.0056	0.0046	0.0033	0.0033	0.0031	-0.0009	0.0004	0.0055	0.0047	0.0033	0.0032	0.0021
104	-0.0021	0.0012	-0.0024	0.0004	0.0007	0.0009	0.0011	-0.0021	-0.0046	-0.0026	-0.0020	-0.0036	-0.0036	-0.0033
105	0.0023	0.0012	0.0066	0.0073	0.0051	0.0050	0.0059	0.0023	-0.0001	0.0058	0.0043	0.0021	0.0022	0.0034
106	0.0048	0.0012	0.0060	0.0043	0.0032	0.0033	0.0020	0.0048	0.0076	0.0080	0.0082	0.0083	0.0083	0.0075
107	-0.0002	0.0012	-0.0045	-0.0043	-0.0021	-0.0022	-0.0019	-0.0002	-0.0004	-0.0041	-0.0031	-0.0015	-0.0015	-0.0015
108	0.0002	0.0012	0.0050	0.0057	0.0044	0.0043	0.0060	0.0002	-0.0004	0.0046	0.0042	0.0025	0.0025	0.0036
109	-0.0012	0.0012	0.0015	0.0029	0.0024	0.0024	0.0040	-0.0012	-0.0032	0.0004	-0.0003	-0.0016	-0.0017	0.0001
110	0.0019	0.0012	0.0076	0.0078	0.0057	0.0057	0.0059	0.0019	0.0014	0.0070	0.0059	0.0043	0.0043	0.0052
111	0.0043	0.0012	0.0064	0.0049	0.0037	0.0037	0.0033	0.0043	0.0067	0.0076	0.0076	0.0075	0.0075	0.0068

112	0.0010	0.0011	0.0020	0.0020	0.0018	0.0018	0.0017	0.0010	0.0014	0.0020	0.0019	0.0017	0.0017	0.0016
113	0.0007	0.0012	0.0020	0.0016	0.0015	0.0015	0.0013	0.0007	0.0016	0.0022	0.0022	0.0023	0.0022	0.0014
114	0.0012	0.0010	0.0132	0.0137	0.0097	0.0096	0.0108	0.0012	0.0026	0.0125	0.0124	0.0076	0.0076	0.0097
115	0.0030	0.0010	0.0085	0.0074	0.0050	0.0050	0.0047	0.0030	0.0042	0.0082	0.0076	0.0059	0.0059	0.0061
116	0.0024	0.0010	-0.0024	-0.0024	-0.0011	-0.0011	-0.0014	0.0024	0.0010	-0.0022	-0.0022	-0.0007	-0.0007	-0.0005
117	0.0012	0.0011	0.0026	0.0027	0.0022	0.0022	0.0023	0.0012	0.0014	0.0025	0.0027	0.0018	0.0018	0.0018
118	0.0006	0.0009	-0.0114	-0.0110	-0.0075	-0.0076	-0.0077	0.0006	-0.0020	-0.0109	-0.0106	-0.0069	-0.0070	-0.0065
119	0.0007	0.0009	-0.0074	-0.0102	-0.0083	-0.0084	-0.0084	0.0007	0.0014	-0.0065	-0.0094	-0.0063	-0.0062	-0.0039
120	0.0027	0.0010	0.0075	0.0049	0.0047	0.0048	0.0045	0.0027	0.0031	0.0076	0.0048	0.0047	0.0048	0.0047
121	0.0032	0.0009	0.0049	0.0027	0.0026	0.0027	0.0024	0.0032	0.0031	0.0051	0.0027	0.0026	0.0026	0.0028
122	0.0016	0.0010	0.0090	0.0051	0.0048	0.0048	0.0036	0.0016	0.0035	0.0092	0.0051	0.0051	0.0051	0.0051
123	0.0038	0.0011	-0.0005	-0.0003	-0.0002	-0.0002	-0.0003	0.0038	0.0017	-0.0004	-0.0003	-0.0002	-0.0002	0.0000
124	0.0027	0.0011	0.0036	0.0024	0.0024	0.0024	0.0020	0.0027	0.0021	0.0037	0.0024	0.0024	0.0023	0.0025
125	0.0051	0.0012	0.0037	0.0020	0.0020	0.0020	0.0018	0.0051	0.0036	0.0040	0.0020	0.0020	0.0020	0.0020
126	0.0027	0.0011	0.0033	0.0025	0.0026	0.0026	0.0026	0.0027	0.0022	0.0033	0.0025	0.0026	0.0026	0.0026
127	0.0017	0.0011	0.0078	0.0037	0.0038	0.0038	0.0025	0.0017	0.0029	0.0084	0.0037	0.0038	0.0038	0.0034
128	-0.0002	0.0011	0.0001	0.0002	0.0002	0.0002	-0.0001	-0.0002	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002
129	0.0029	0.0013	0.0116	0.0015	0.0015	0.0015	0.0003	0.0029	0.0032	0.0130	0.0015	0.0015	0.0015	0.0005
130	0.0049	0.0014	0.0092	-0.0023	-0.0019	-0.0020	-0.0045	0.0049	0.0041	0.0123	-0.0023	-0.0019	-0.0021	-0.0040
131	-0.0004	0.0014	0.0251	0.0089	0.0087	0.0086	0.0092	-0.0004	0.0024	0.0212	0.0089	0.0087	0.0086	0.0093
132	0.0013	0.0015	0.0091	0.0024	0.0024	0.0024	0.0016	0.0013	0.0021	0.0071	0.0024	0.0024	0.0024	0.0017