Empirical Analysis of Business Cycles using the Unobserved Components Model

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A thesis submitted in fulfilment of the requirements
for the degree of
Doctor of Philosophy

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November 2018

Summary

Timely estimates of the current state of the business cycle in a number of economic indicators are essential in the setting of the stance of macroeconomic policy. There are several methods for extracting estimates of the underlying trends and cycles but results vary considerably by method, and it is sometimes uncertain whether the estimated components actually represent real underlying phenomena. The central challenge of each of the three articles forming the main body of this thesis is finding a meaningful statistical characterisation of the business cycle component in various aggregate macroeconomic variables using an unobserved components model.

The first article demonstrates how a common cyclical component can be extracted from a set of related labour market statistics to make new estimates of Okun coefficients and participation coefficients by age and gender. The estimated coefficients were generally higher than those typically reported in the literature and reveal the characteristic cyclical response of different groups of workers to an economic downturn. The second article sheds new light on previously disparate empirical findings of procyclical or countercyclical productivity, against a theoretical framework in which it is usually held to be procyclical. An extension of the unobserved components model which allowed direct estimates of phase-shift amongst similar cycles was used to explain the apparent countercyclical and lagging behaviour of average labour productivity relative to the output cycle. The third article develops a non-linear version of the model which incorporates the possibility of asymmetric responses to an exogenous recession indicator. Two related questions are considered in the article, namely, whether a non-linear model with asymmetric features provide a better fit to macroeconomic data than a symmetric model, and whether a recession is a real macroeconomic phenomenon, more than simply the mirror image of the expansionary phase of the cycle. Evidence is found in favour of asymmetry.

One important aspect of business cycle research is revealing the characteristic comovement of the cyclical components of several economic aggregate variables in

a multivariate framework. The three articles in this thesis have demonstrated how such a framework can be used in situations where economic policy making will benefit from a better understanding of the cyclical behaviour of key macroeconomic aggregate variables and the interaction between them.

Declaration

This thesis has not been submitted to any other university or institution for a higher degree. The thesis is my own original work. All sources used and assistance received in the preparation of this work have been acknowledged in the text.

Signature: ________________________

Date: ____________________________

Acknowledgements

I would like to thank my supervisors, Professor Lance Fisher and Dr. Ben Wang, for their support and encouragement over the last three years. Their wide knowledge and experience in the field has helped me to frame relevant research questions and to place the work in the context of previous studies. They have reviewed numerous iterations of the empirical analysis and earlier drafts of the articles which comprise the main body of this work, and their helpful suggestions have improved the work and helped me begin to understand the challenge of academic publishing.

I would also like to thank Jonas Mansson, Associate Professor, Linnaeus University, and seminar participants at the Western Economic Association International 14th International Conference for helpful discussions on an earlier draft of the work presented in Chapter 2, and to all the anonymous journal article referees whose constructive feedback has helped to refine the research questions and improve the quality of the work. Special thanks go to my thesis examiners for suggested improvements to the text and for suggested extensions to the research.

Finally, I express my gratitude to my family for their patience and support, giving me the time and space to pursue firstly two masters degrees and this, the final goal.

Contents

Chapter 1

Introduction

1.1 Business cycles

Modern use of the term "business cycle" is often made in the context of aggregate measures of economic activity such as the level of output and employment. Even without formal measurement it is apparent to people engaged in an economy that the level of activity is not constant; there are periods of strong growth when jobs and the income derived from them are plentiful, and periods of decline when prosperity stagnates or falls. In many economies, at least at an anecdotal level, there have been alternating periods of growth and decline which have given rise to the idea that the level of economic activity moves in a recurring but irregular cycle, which overlays a positive long term trend. Burns and Mitchell (1946, p. 3) gave the following working definition of the business cycle:

> Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.

Their study emphasised the role of the many constituent parts which together are the source of aggregate activity. So called "specific cycles" were estimated in a large number of different activities. No individual time series was expected to reveal the business cycle, rather, a study of the specific cycles in the parts was anticipated to reveal useful information about the cyclical characteristics of the whole. A key element of the procedure was the identification of the major turning points, and the classification of distinct phases of each cycle which were expected to follow in orderly progression, albeit with variation in amplitude and duration between one complete cycle and the next.

Lucas (1977) did not support the concept of distinct phases of each cycle as a recurring feature, and preferred a definition of a business cycle as repeated fluctuations of aggregate economic variables about a trend, and held that the important regularities to be understood were the comovements between the cycles in different series. Kydland and Prescott (1990, p. 4) supported a similar definition, measuring "deviations of aggregate real output from trend", and completed the definition by specifying a procedure to identify a smooth trend from a time series. One aspect of the Burns and Mitchell (1946) approach is that it arguably implies the existence of some form of self-sustaining mechanism by which each phase of a cycle gives the impetus for the succeeding phase, and that the completion of one complete business cycle naturally gives rise to the next. Indeed that could imply that the cyclical behaviour itself is a form of equilibrium behaviour. In an alternative view, Kydland and Prescott (1990) highlight other possible mechanisms which could give rise to apparently recurring cycles at a business cycle frequency, such as damped oscillatory responses to shocks in a system which otherwise has a stable equilibrium. In such a system, a succession of shocks through time is what provides the energy for apparent recurring cycles.

1.2 Trend and cycle decomposition

The procedure of Burns and Mitchell (1946) has been superseded by modern econometric techniques for trend and cycle decomposition, which nonetheless tend to retain some element of judgement with regard to parameter setting or component selection. Separating a single time series into trend, cycle, seasonal and irregular components is a filtration exercise which does not have a unique solution. Some common filtration methods used in macroeconomic analysis include the Hodrick and Prescott (1997) filter, which imposes a smooth trend and defines the deviation from trend as being the stationary cycle; the Beveridge and Nelson (1981) decomposition which, for processes which are stationary in first differences, defines the permanent trend as the long run forecast of an ARMA process, yielding a trend which is a random walk and a stationary stochastic residual as the cycle; and the Baxter and King (1999) band-pass filter which identifies the business cycle component of a time series with a filter which only passes through components with periodic fluctuations within a specific range of frequencies[1] while removing components with higher or lower frequencies.

Another methodology used for trend and cycle decomposition, used throughout this thesis, is the unobserved components model[2] of Harvey (1985), and the equivalent multivariate model as specified by Harvey and Koopman (1997). In brief, the model allows a time series to be decomposed into trend, stationary cycle, seasonal and irregular components. The dynamics of the unobserved states are specified as a first order vector autoregression so that the system can be represented in state space and estimated using a Kalman filter. Perceived advantages of the methodology over other possible filtration methods for use in this thesis are flexibility with regard to the specification of the process for the stochastic trend, which may be stationary in

[1] To match the specification and findings of Burns and Mitchell (1946) in relation to the United States economy, the range of 6-32 calendar quarters was applied by Baxter and King.

[2] A detailed specification of the unobserved components model is given in following chapters, in each case with features chosen to suit the particular study.

first differences or second differences; flexibility in a multivariate framework with regard to the inclusion or restriction of components per individual series; and explicit representation of cross-series relationships as required, such as common cycles and correlated innovations across series.

The central challenge of each of the three articles included in this paper is finding a meaningful statistical characterisation of the business cycle in time series of various aggregate macroeconomic variables. Solutions found by the Kalman filter include maximum likelihood estimates of the model parameters, so the estimated trend and cycle components can be interpreted as those which provide the best fit to the empirical data, within constraints. The objective to find the best fit is tempered by the requirement to avoid solutions which are not properly identified or which have no clear economic interpretation. Particularly in the multivariate setting, the most general form of the models which potentially allow correlation between innovations both within and across series are greatly over-parameterised. Judgement is used to restrict parameters so as to generate estimated components which are likely to be properly identified and which are useful for the proposed economic analysis. Particular care has been taken to avoid decompositions where the estimated cyclical components may be an artefact of the filtration process as can happen, for example, with the use of the Hodrick-Prescott filter (Hamilton, 2017).

1.3 Permanent and transitory shocks

An implicit feature of trend and cycle decomposition is that it also provides an estimated separation of permanent and transitory changes to a time series. The trend process is often a random walk (or another non-stationary process with a higher order of integration) so the estimated trend innovations can be interpreted as permanent shocks, whereas innovations to the stationary cyclical component are transitory by construction. There are competing theoretical macroeconomic models, only some of which support the existence of significant stationary shocks to the underlying data generation process. The real business cycle model of Kydland and Prescott (1982) uses permanent exogenous technology shocks as the source of aggregate fluctuations in macroeconomic variables. There are, however, many empirical studies which show that it is difficult to produce the stylised characteristics of macroeconomic series, and the interaction between them, using only technology shocks in the real business cycle model. For example, Galí (1999) finds that an improved empirical fit can be found with a theoretical model which has been augmented with non-technology shocks (such as demand shocks) as well as technology shocks. In this paper, the approach is empirical in nature, and does not seek to explain the source of the business cycle fluctuations. The general specification of the trend and cyclical components potentially allows two polar extremes of, firstly, all of the innovation variance arising only in the trend component or, secondly, only in the stationary component. In almost all cases, foreshadowing later results, the maximum likelihood solutions tend to fall between those extremes and include a mixture of permanent and transitory fluctuations.

1.4 Business cycles in a policy setting

Timely estimates of the current state of the business cycle in a number of aggregate indicators of the level of economic activity are essential to the setting of the stance of macroeconomic policy. Equally important for policymakers is having the ability to differentiate between cyclical fluctuations and changes in the permanent trend. Policy interventions in the nature of short term stimulus or dampening are likely only to affect the transitory cycle, whereas structural reform may be required to affect the long term trend growth rate. The use of empirical business cycle analysis for framing policy responses is not done in isolation of economic theory, although that is not the focus of this thesis. Separate research programs consider important questions such as the underlying source of the fluctuations, such as shocks in technology and aggregate demand, and the mechanisms by which these shocks propagate between economic sectors and give rise to the aggregate behaviour which we characterise as business cycles.

1.5 Organisation of the thesis

The thesis is organised as follows. Chapter 2 considers[3] how the unemployment rate and labour force participation respond to changes in the business cycle. Sensitivities of unemployment and participation by age group and gender are estimated by comparing the amplitude of cyclical responses in the rates to the amplitude of the output gap. In this application, where it is reasonable to conjecture that a large number of labour market series share a common cyclical factor, a multivariate unobserved components model with a common cycle component is estimated. Chapter 3 investigates[4] how the lead or lag in output versus hours worked affects measures of average labour productivity. The numbers of full-time and part-time employees as well as average hours per employee all vary with the business cycle, potentially with phase differences between their cyclical components, which can result in non-intuitive cyclical behaviour in an apparently simple measure of average productivity. The unobserved components model is augmented to incorporate an estimated relative phase-shift amongst the cyclical components so that the behaviour of average productivity can be explained by the behaviour of its components. Chapter 4 develops[5] a non-linear version of the model which incorporates the possibility of asymmetric responses to an exogenous recession indicator. Two related questions are considered in the article, namely, whether a non-linear model with

[3]Chapter 2 is an article published with the following reference details: Evans, A. (2018). Okun coefficients and participation coefficients by age and gender. *IZA Journal of Labor Economics*, 7:5, https://doi.org/10.1186/s40172-018-0065-8.

[4]Chapter 3 is based on an article which has been submitted to an academic journal in May 2018 which is currently under review.

[5]Chapter 4 is based on an article which has been submitted to an academic journal in September 2018 which is currently under review.

asymmetric features provides a better fit to macroeconomic data than a symmetric model, and whether a recession is a real macroeconomic phenomenon, more than simply a major negative cyclical deviation of output below trend. Chapter 5 draws together the findings of the three articles and summarises the overall contribution to a better empirical understanding of business cycles.

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Chapter 2

Okun coefficients and participation coefficients by age and gender

Abstract

Estimates of the Okun coefficient are made for Australian workers grouped by age and gender using an unobserved components model. By analogy we define and estimate a participation coefficient which measures the cyclical response of the labour force participation rate to cyclical output shocks. The trend and cycle decomposition methodology used here leads to higher absolute estimates of the Okun coefficient than those typically found in the literature, although we find a pattern of variation in the coefficient by age and gender which is typical. We also find that, in aggregate, participating males in the middle age groups tend to stay in the labour force throughout the business cycle whereas females of the same age tend to participate procyclically. This has policy implications for attempts to increase the rate of participation of particular groups by age and gender following a cyclical downturn.

2.1 Introduction

One of the most robust historical relationships in macroeconomics has been the negative relationship between unemployment and output growth as described by Okun (1962). This knowledge alone is not useful for guiding policy prescriptions without differentiating between the permanent and transitory components of unemployment and output. Structural or institutional change is necessary to influence the permanent trend whereas short-run policy initiatives are likely to affect only the transitory cycle. The response of cyclical unemployment to cyclical output shocks is of particular interest to policy makers. In this research a trend and cycle decomposition is performed on the unemployment rate and the log of real GDP, and the cyclical components are interpreted as measures of the unemployment gap and output gap respectively. The estimated gaps are used to generate estimates of the Okun coefficient for workers grouped by age and gender.

The behaviour of the labour force participation rate[1] during the business cycle also needs to be considered because, without it, the unemployment rate is an incomplete indicator of the level of unutilised labour. Discouraged workers who transition from unemployment to non-participation during a recession may mask the true number of people who want more work. Participation was identified as a key driver of economic growth, improvement in living standards and community prosperity in the Intergenerational Report (Commonwealth of Australia, 2015). Usually, the long run trends affecting participation are of most interest to policymakers, such as the changing age structure of the population, migration effects and the participation of females and the elderly in the workforce, but cyclical effects also need to be understood. Policy initiatives which are intended to increase participation in a given context need to be designed either to influence the trend or the cyclical component. In this research the focus is the business cycle behaviour of participation. We define a participation coefficient by analogy to the Okun coefficient which measures the cyclical response of the participation rate to output.

In the literature the decomposition of macroeconomic time series is often performed using a Hodrick-Prescott filter (hereafter HP). Shortcomings of the approach are well known, arising from the requirement to choose a value for the smoothness parameter which controls the relative variance of trend and cycle components. The apparent behaviour of the HP cycle may be to some degree an artefact of the filtering process rather than a reflection of characteristics of the true data generating process

[1]The participation rate is the percentage of the civilian population who are in the labour force, which comprises employed and unemployed persons. People who are not working and not actively looking for work are not in the labour force.

(Harvey & Jaeger, 1993). For some critiques of the use of the HP filter for business cycle analysis see Cogley and Nason (1995) and Hamilton (2017). As an alternative we use the structural time series model developed by Harvey (1985) to make maximum likelihood estimates of unobserved trend and cycle components. The key advantage of the unobserved components (UC) model is that the components are estimated using a statistical model rather than being imposed by the structure and parameters of the HP filter. A multivariate model of output, unemployment, participation and total hours worked which incorporates a common business cycle is jointly estimated to extract the trends and cycles. Estimates are made separately by age and gender for Australian data. The first contribution of this paper is a new set of estimated Okun coefficients derived from the relative magnitude of the unemployment and output cycles. The second contribution is a set of participation coefficients which reveal the estimated magnitude of the cyclical response of labour force participation by age and gender to shocks to cyclical output.

In Section 2.2, literature is reviewed which provides theoretical grounds for and empirical description of the business cycle behaviour of unemployment and participation. Section 2.3 describes the labour market data used for the empirical analysis and illustrates some of the salient features of particular age and gender groups. A detailed specification of the empirical model is provided in Section 2.4. Empirical results are given in Section 2.5 and the estimated trend and cycle components are illustrated graphically. Estimates of the Okun and participation coefficients are given by age and gender and a comparison is made with estimates of the former made in the literature. Section 2.6 concludes the paper.

2.2 Theoretical background

2.2.1 Okun's Law

A negative relationship between output and unemployment can be easily motivated by theory, for example, the assumption of a Cobb-Douglas production function with a labour force of fixed size yields an approximately negative linear relationship between log output and the unemployment rate, with sensitivity determined by the output elasticity of labour input. Most studies focus simply on the empirical relationship, such as the relationship between first differences described by Okun (1962):

$$\Delta u_t = \alpha - \beta \Delta y_t + \varepsilon_t, \tag{2.1}$$

where Δu_t is the change in the unemployment rate, Δy_t is percentage change (or log change) in a measure of real output such as real GDP and ε_t is an error term. β is interpreted as the Okun coefficient[2]. A shortcoming of this representation is that it implies that the relationship between Δu_t and Δy_t is purely contemporaneous. The model can be improved easily by adding lagged terms in both variables:

$$\Delta u_t = \alpha + \sum_{i=1}^{p} \gamma_i \Delta u_{t-i} + \sum_{j=0}^{q} \beta_j \Delta y_{t-j} + \varepsilon_t. \tag{2.2}$$

The Okun coefficient can also be interpreted as the long-run impact of Δy_t on Δu_t (also 'long-run multiplier' or 'dynamic beta') which can be derived from the coefficient estimates as

$$\beta = \frac{\sum_{j=0}^{q} \beta_j}{(1 - \sum_{i=1}^{p} \gamma_i)}. \tag{2.3}$$

A difficulty which remains using this approach is that there is no explicit distinction made between temporary and permanent shocks acting through Δy_t. Another

[2]If the empirical relationship between output and unemployment is negative as anticipated then the estimate of β in Equation 2.1 would be a positive number. In the literature the Okun coefficient may be reported as a positive number, and at other times as the negative value $-\beta$.

expression for the relationship between unemployment and output (similar to the 'gap version' described by Okun (1962)) is

$$(u_t - u_t^*) = -\beta (y_t - y_t^*) + \varepsilon_t. \tag{2.4}$$

Different interpretations of y_t^* may apply depending on the context of the research. In Okun's original paper it represented potential output, the amount the economy could produce under conditions of full employment, and the econometric model was used to generate estimates of potential output. In this paper we interpret y_t^* as the (log) natural level of output when the economy is operating at sustainable full capacity. This level will vary though time depending on many factors we have not explicitly modelled, such as the level of demand and institutional constraints. It follows that u_t^* can be interpreted as the equilibrium rate of unemployment which prevails when output is being produced at its natural rate. It must be emphasised that there is a conceptual difference between what is being measured by the parameter β in each of the Equations 2.1 to 2.4, but each of them are sometimes described as 'the Okun coefficient' in the relevant context.

A trend and cycle decomposition of both y_t and u_t will be made in which the estimated trend will be interpreted as the time-varying equilibrium (y_t^* or u_t^*) and the cycle as a measure of the gap represented in Equation 2.4. By construction, the gaps will be stationary with zero mean. The sensitivity of the unemployment gap to the output gap will be interpreted as a measure of the Okun coefficient. Separate estimates of the coefficient will be made by gender and by age bracket.

There is ample evidence that estimates of the Okun coefficient vary across countries which is likely to reflect different institutions, policy settings and cultural differences between them. There are mixed results regarding the stability of estimates through time within country but, on balance, there is evidence that the relationship between output and unemployment may vary due to structural changes which occur over time. Ball, Leigh and Loungani (2012) considered whether Okun's Law was still robust some 50 years after Okun's original paper and found it to be strong and stable in most countries, but with significant variation between countries. They rejected the idea that the relationship has broken down after the most recent recessions leading to claims of so-called jobless recoveries in the United States. Lee (2000) finds statistically significant Okun coefficients across a range of OECD countries but with great variation in magnitude. Some of the variation is attributed to higher rigidity in some of the European labour markets and Japan compared to the United States (see also Nickell (1997)). Lee also finds strong evidence of structural breaks mostly in the early 1970's but which also vary by country. Dixon, Lim and Van Ours (2017) estimated an Okun coefficient using a panel of 20 OECD countries (including Australia) for the period 1985-2013, having controlled for the influence of labour market institutions such as union coverage, unemployment insurance and employment protection legislation. They rejected the hypothesis that the Okun coefficient had remained the same over time in their base model but were able to explain most of the increase using the share of temporary workers in the workforce, amongst other changes to institutional variables. Further cross-country studies of the Okun coefficient can be found in International Monetary Fund (2010) and Moosa (1997).

There have also been many studies which consider a potential non-linear relationship between output and unemployment. Cuaresma (2003) specifies a model with a regime-dependent Okun coefficient which allows an asymmetric response of unemployment to output depending on whether the economy is in either of two regimes which correspond approximately with expansion or recession. The absolute sensitivity measured by the Okun coefficient is found to be approximately twice as large when the economy is in recession. Holmes and Silverstone (2006) estimate a model with two forms of asymmetry whereby, in the first case, the Okun coefficient depends on two regimes defined by positive and negative cyclical output and, in the second case, the absolute value of the coefficient depends on the sign of the shock. The authors find evidence of both forms of asymmetry. Lee (2000) finds mixed evidence of asymmetry depending on the specific country. Dixon et al. (2017) are unable to reject the hypothesis of symmetry. Bodman (1998, p. 410) finds evidence of non-linear behaviour in unemployment in Australia, in particular he finds that shocks are more persistent in recessions than expansions, which is broadly suggestive of hysteresis in the labour market.

There are sound reasons to expect different responses to the business cycle amongst different age and gender groups. For example, younger workers are likely to have less experience than older workers and have less employment protection (such as may occur under a temporary contract) and so be more likely to suffer involuntary job loss in a recession. Equally, recent school-leavers are likely to take into account the state of the business cycle when they choose between higher education and joining the labour force (Dellas & Sakellaris, 2003). Both of these factors would contribute to greater cyclicality of youth unemployment. Unemployment for males may be more cyclical than for females because they have higher representation in cyclical activities like building construction (Zanin, 2014). Some females with young children may elect to leave the labour force during an economic downturn, thereby not affecting the official unemployment measure. Lastly, older workers may have a difficult choice to make in a downturn between unemployment and participation given the difficulty they may face re-joining the workforce at a later date, so there may be a more complex interaction between cyclical unemployment and participation for older workers.

In this chapter the focus will be on the differences in coefficients for age and gender groups rather than on possible asymmetry and stability of the coefficients through time. The model will have an inherently linear relationship between output and unemployment shocks. However, the basic framework for estimating the Okun coefficient will be extended to allow joint estimation of the sensitivity of unemployment and labour force participation to the business cycle, to capture any interaction between them during the cycle.

2.2.2 Cyclical participation

Mincer (1966) put forward theories to explain cyclical movement of workers in and out of the labour force. The added-worker effect was used to explain the type of worker who is more likely to join the labour force in an economic downturn to compensate for the potential loss of income by another household member who may become unemployed. Historically, this concept was applied mainly to married women who sought to add to household income to mitigate loss of income by their spouse. The added-worker effect would generate counter-cyclical participation. Mincer also described the discouraged-worker effect which posited that some unemployed workers stop looking for work in an economic downturn (and therefore become non-participants) because they perceive that the probability of finding work is so low it is not worth searching. This same group are likely to re-join the labour force when the economy improves, generating procyclical participation. The added-worker and discouraged-worker effects can coexist and aggregate labour market data tends to reveal only the net contribution of the two effects.

It has been argued by Dixon, Freebairn and Lim (2004) that in relation to the Australian economy it is not credible to try and explain the variation in the participation rate with a sole focus on the movement of discouraged workers between the states of unemployment and non-participation. In the first place, gross flow data[3] reveals that the largest average flows are between non-participation and employment. The magnitude of these flows is almost twice as large as those between non-participation and unemployment (Dixon, Lim & Van Ours, 2015, p. 2530) which

[3]The Australian Bureau of Statistics report monthly gross flow data for the labour market in Catalogue 6202, data cube GM1. The data measures the gross flow of workers in both directions between each of the labour market states of employment, unemployment and non-participation, by comparing the status of matched respondents in a monthly survey which make it possible to determine whether a transition of a worker between states has occurred within the period.

suggests that they are an important part of the overall dynamics of the labour market. Similar observations have been made in other markets including the United States (Blanchard & Diamond, 1990, pp. 91–92). Dixon et al. (2004) find that the main influence on the growth rate of the labour force is the size of the net flow between non-participation and employment. Dixon et al. (2015) argue that it is essential to treat flows to and from non-participation as endogenous within a system also including employment and unemployment flows. Other studies which treat participation as endogenous to examine its behaviour during the business cycle can be found in Darby, Hart and Vecchi (2001), Elsby, Hobijn and Şahin (2015) and an Australian study by Ponomareva and Sheen (2013).

The Intergenerational Report (Commonwealth of Australia, 2015) projects that participation in Australia will decline over the next 40 years due to a changing age demographic which will see a relative decline in age cohorts of people where participation tends to be highest. This is despite an anticipated increase in participation within each cohort. On the other hand, the health of older Australians is improving so that a greater number of them are capable of continuing to work beyond traditional retirement age if circumstances permit. The importance of increasing participation to the Government is reflected in policy measures to support the participation not only of mature-age job seekers but also youth, women and parents (Commonwealth of Australia, 2015, p. 96). Even though the report focusses on long term trends we observe that temporary output shocks can have a long term effect on participation and unemployment if there is hysteresis. Duval, Eris and Furceri (2010) find that severe recessions have a significant and persistent impact on the level of participation in a panel of 30 OECD countries including Australia. They find that aggregate participation may be 1.5-2.5% lower five to eight years after the previous cyclical peak before the commencement of the recession. Persistent or permanent impact on older workers may be explained by irreversible retirement decisions made in response to a recession, which can be influenced by early retirement incentives arising from policy initiatives or those embedded in pension schemes (Duval et al., 2010). Persistent impact on the participation of younger workers may be explained by choices to enrol in longer programs of higher education and training during a downturn. These findings show that it is important for policymakers to consider cyclical shocks to participation as well supporting long term trend growth.

There has been much less empirical analysis of the cyclicality of participation than unemployment in the literature in most countries, including Australia. In macroeconomic studies of the United States, the size of the labour force has often been assumed to be acyclical as discussed in Erceg and Levin (2014), with participation sometimes modelled as a fixed percentage of the civilian population. Benati (2001) found evidence of procyclical participation at a business cycle frequency (a net discouraged-worker effect) in the United States at an aggregate level and for a number of age-sex groups. In many developed countries participation rates declined following the Great Recession and attempts have been made to determine how much of the decline was cyclical and how much reflected a permanent change in labour supply, without universal agreement as to the conclusion. Erceg and Levin (2014) found evidence that cyclical factors accounted for most of the decline since 2007. Van Zandweghe (2012) found that participation was very weakly pro-cyclical from 1948-2011 but that since 2007 the participation rate had become more sensitive to the state of the economy, and that about half of the decline in participation from 2007-2011 could be attributed to cyclical factors.

The empirical relationship between participation and output cycles can be measured using an equation conceptually similar to Equation 2.4 with the unemployment gap replaced by the participation gap. The estimated trend from a decomposition of the participation rate will incorporate all of the permanent influences on the level of participation such as changing demographics and attitudes towards gender and age in the workforce. The cycle component will be interpreted as the participation gap, a transient deviation of the current level from the permanent trend. The sensitivity of the participation gap to the output gap will be estimated by gender and age bracket.

2.3 Australian labour market data

We use seasonally adjusted monthly time series of the unemployment rate (u_t) and labour force participation rate (p_t) for 13 groups of workers: all persons, males, females and each gender separated into five 10-year age brackets. The youngest age bracket is 15-24 years old, followed by 25-34 years old and so on up to 55-64 years old. The characteristic behaviour of unemployment and participation for each age bracket is illustrated in Figure 2.1 from February 1978 to June 2017. For output we use the log of seasonally adjusted real quarterly GDP (g_t) multiplied by 100. We also make use of a seasonally adjusted series of total monthly hours worked in all jobs by all persons, from July 1978. Growth in hours worked over several decades mostly reflects population growth so we use the log of hours worked multiplied by 100 which we denote by h_t. Seasonally adjusted series are used in each case so that any seasonal pattern in the raw series which may appear to be some form of annual cycle does not interfere with the estimation of the cycle component at a business cycle frequency. The relationships between output, unemployment and participation are the main interest in this research, whereas the monthly hours worked series is used primarily to assist in identifying a common cyclical component across the series. The estimation period used for our empirical results will be September 1980 to June 2017.

2.4 Empirical model

A trend and cycle decomposition methodology is required to generate estimates of gaps for output and the labour market series so that it is possible to measure the responsiveness of cyclical unemployment and participation to cyclical output shocks

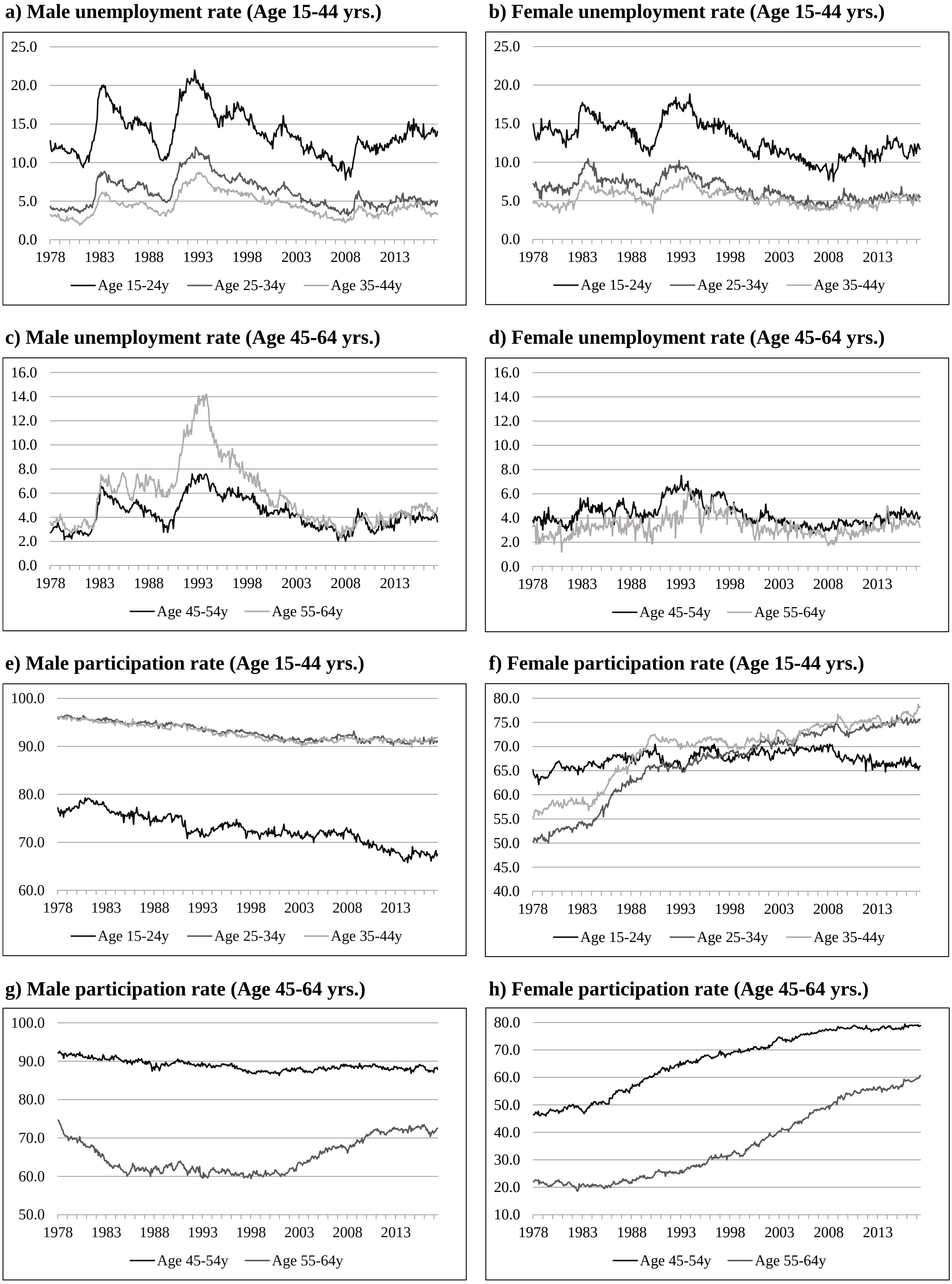

Figure 2.1: **Historic unemployment and participation rates by age bracket.**

in a relationship analogous to the empirical relationship described by Equation 2.4. Studies of Okun's Law in the literature have frequently made use of the HP filter to extract an estimate of the output gap and sometimes also the unemployment gap. A problem with the HP filter for this application is that the chosen smoothness parameter can have a dramatic effect on the relative variance of the trend and cycle components. In this chapter an alternative decomposition methodology is used to generate maximum likelihood estimates of the components without prior restrictions on the relative magnitude of the trend and cycle variances.

2.4.1 Multivariate unobserved components model with a common cycle

The following is a typical specification of an unobserved components model with stochastic trend and cycle components, mostly following the notation of Harvey (1985). The system has a measurement equation by which an observable series is linearly related to a set of unobservable state variables. It also has a set of state or transition equations which determine the evolution of the state variables as first order vector autoregression (VAR) process[4]. The time series model can be represented in so-called state space form so that it can be estimated using the Kalman filter. We will make a joint estimation of a system including output, unemployment, participation and hours worked, but first we present only the parts of the system relating to

[4]The dynamics are not limited to a single lag by this structure since a higher order VAR(p) process can be modelled by a simple recursive scheme.

output in Equation 2.5.

$$\begin{aligned}
g_t &= \tau_{gt} + c_t + \varepsilon_{gt}, & \varepsilon_{gt} &\sim iid(0, {\sigma_{\varepsilon g}}^2),\\
\tau_{gt} &= \tau_{gt-1} + \beta_{gt-1} + \eta_{gt}, & \eta_{gt} &\sim iid(0, {\sigma_{\eta g}}^2),\\
\beta_{gt} &= \beta_{gt-1} + \zeta_{gt}, & \zeta_{gt} &\sim iid(0, {\sigma_{\zeta g}}^2),
\end{aligned} \tag{2.5}$$

$$\begin{bmatrix} c_t \\ c_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} c_{t-1} \\ c_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}, \ \kappa_t, \kappa_t^* \sim iid(0, \sigma_\kappa^2).$$

The trend in log output is represented by τ_{gt}, with slope β_{gt}, and the cycle by c_t. The irregular component ε_{gt} can be interpreted as random noise or as a measurement error. The specification of the trend is very flexible in terms of the types of data generating processes that it can be used to fit. In the most general form both of the variances $\sigma_{\eta g}^2$ and $\sigma_{\zeta g}^2$ are freely estimated and, if both variances are non-zero, the trend would be an integrated process of second order (in the literature this is usually referred to as a local linear trend (LLT) model). If the slope variance $\sigma_{\zeta g}^2$ is restricted to zero then the trend will be a random walk with drift (also known as the local level (LOCL) model). If the level variance $\sigma_{\eta g}^2 = 0$ while $\sigma_{\zeta g}^2 > 0$ then the trend will be 'smooth' (known as the integrated random walk (IRW) model).

The model is estimated separately for each age-gender group (such as males aged 15-24 years). Due to possible substitution in the labour markets between workers of different ages and genders this approach may introduce bias to the estimated coefficients. An all-encompassing model which allows for possible interactions between the groups has been left as an avenue for future research.

2.4.2 Cycle component

The stationary cycle component c_t in Equation 2.5 is generated by a trigonometric stochastic cycle. The component c_t^* is used in the construction of c_t but is not otherwise used in the system. A complete description of the mechanics of the stochastic cycle for interested readers may be found in Harvey (1985) or Pelagatti (2016). One important observation is that the generated cycle will not be a smooth sinusoidal wave since the innovations κ_t and κ_t^* generate random variation in apparent amplitude and frequency. The central frequency is $\lambda \in (0, \pi)$ (with cycle period $= 2\pi/\lambda$), and $\rho \in (0, 1)$ is known as the damping factor since it determines the portion of the previous cyclical value that is propagated into the next period. If $\rho < 1$ then the cycle will be stationary. It can be shown that the cycle has a ARMA(2,1) reduced form, but with some complex cross-parameter restrictions (Harvey, 1985, pp. 219–220). An alternative autoregressive model of the cyclical component could have been used in Equation 2.5 but the reason the stochastic cycle is preferred in economic business cycle analysis is that we can obtain a direct estimate of the period of cycle and of the damping factor which indicates the degree of persistence of cyclical shocks.

Next we add equations for the decomposition of the unemployment rate as shown in Equation 2.6.

$$\begin{aligned}
u_t &= \tau_{ut} + \omega_u c_t + \varepsilon_{ut}, & \varepsilon_{ut} &\sim iid(0, {\sigma_{\varepsilon u}}^2), \\
\tau_{ut} &= \tau_{ut-1} + \beta_{ut-1} + \eta_{ut}, & \eta_{ut} &\sim iid(0, {\sigma_{\eta u}}^2), \\
\beta_{ut} &= \beta_{ut-1} + \zeta_{ut}, & \zeta_{ut} &\sim iid(0, {\sigma_{\zeta u}}^2).
\end{aligned} \tag{2.6}$$

The trend specification is identical to that shown in Equation 2.5 except that the unobserved components and their variances relate particularly to u_t. The common cycle component is evidenced by c_t which appears in both Equations 2.5 and 2.6. The cycle is multiplied by the scaling parameter ω_u in the equation for unemployment, whereas the equivalent parameter in the output equation is normalised to one. The motivation for using a common cycle is two-fold. Firstly, it is an identification strategy, since there are fewer parameters to estimate than a model in which each economic variable has an independent cycle. Secondly, the idea sits neatly inside a macroeconomic framework in which there are one or more unobserved cyclical factors which drive the temporary fluctuations of a large number of indicators of activity, such as aggregate demand, output and production factor inputs. Any aggregate macroeconomic variable is likely to be measured with error or be affected by noise, so there is a perceived advantage in constructing a model which makes a joint estimation of a common unobserved factor from several related observable series. In this analysis the common cycle can be interpreted as a proxy for the business cycle.

To elaborate on the role of the parameter ω_u in the common cycle model, we observe that the unemployment and output cycles are $\omega_u c_t$ and c_t respectively. The cycles are perfectly correlated and the ratio of their variances is ω_u^2. Equivalently, a one unit shock to κ_t will generate a one unit shock to c_t and g_t, and a shock of ω_u to u_t. So ω_u can be interpreted directly as the cyclical response of the unemployment rate to a cyclical shock in log output, which is one measure of the Okun coefficient. To emphasize the connection between the empirical model and theory as described in Section 2, the trend is being interpreted as the equilibrium level and the cycle is being interpreted as the gap, having also removed estimated measurement error.

In a similar fashion we add equations for the decomposition of the participation rate and hours worked to our system each using the same structure to model the trend and each referencing the same common cycle with scaling coefficients ω_p and ω_h respectively (the equations relating to participation and hours are not shown). We can interpret ω_p directly as the cyclical response of the participation rate to a cyclical shock in log output, which we refer to as the participation coefficient.

2.4.3 Correlation between components

In addition to the explicit relation between the variables arising from the common cycle component it is possible that the irregular error components may be correlated. This feature would make the system analogous to a seemingly unrelated regression model in which shared driving factors not explicitly modelled manifest in correlated errors across equations. Restricted and unrestricted correlations between irregular components were investigated and the results suggested none were significantly different to zero except for the correlation between unemployment and participation irregular components ε_u and ε_p. There are strong theoretical grounds to support possible correlation between this pair since measurement errors in one variable may be directly related to measurement errors in the other, for example a person misclassified as non-participating rather than unemployed in the labour market survey at time t will generate an under-estimate of both p_t and u_t. In the models below we allow unrestricted correlation between ε_u and ε_p (labelled $r\varepsilon_{up}$) and we anticipate that the correlation will be positive. All the remaining pairs of irregular components were restricted to be uncorrelated.

Correlations between all the remaining innovations η_t, ζ_t and κ_t were restricted to zero for identification purposes, since preliminary analysis suggested that the model estimation would not converge with freely estimated correlation, which was suggestive of under identification. When the correlation restriction is in place the unobserved components model is not immune from some of the criticisms of the HP filter (Nelson, 1988). For a detailed explanation of identification issues of structural time series models, particularly as it relates to correlation between unobserved components, see Appendix 5.B to Chapter 5. For examples of correlated UC models in the literature, readers can refer to Morley, Nelson and Zivot (2003), Sinclair (2009), Dungey, Jacobs, Tian and Van Norden (2015), Jaeger and Parkinson (1994) and Proietti (2004).

2.4.4 Seasonality

We use seasonally adjusted data for all four of the observable series but still found some residual seasonal patterns in the labour data and found it useful to augment the measurement equations with some lagged first differences of the observable series, principally around 12 and 24 month lags. To increase confidence that the common cycle component has been validly identified it is important to take all possible steps to remove autocorrelation from the estimated residuals.

2.4.5 Estimation method

The structural time series model specified in Equation 2.5 can be represented in state space form so that it can be estimated using the Kalman filter (see Appendix 5.A for a detailed explanation of the state space form). Maximisation of a log likelihood function created from one-step ahead prediction errors and their variances generates maximum likelihood estimates of the model parameters, which are used to generate optimal estimates of the time series of unobserved components. The filtered series provide optimal estimates of a component at time t using only the information available at time t. A smoothing procedure can be applied to the filtered series using all of the information in the sample period to generate so-called smoothed estimates. Whilst smoothed estimates cannot be used for real-time prediction they can be compared with the output from other filtering techniques which use the entire sample period such as the HP filter. For a description of filtering and smoothing algorithms see Commandeur and Koopman (2007, pp. 84–89).

2.4.6 Mixed-frequency data

The labour market series have monthly frequency whereas the GDP series is quarterly. We simply treat the non-calendar-quarter months as missing values in the log GDP series. These are easily handled within the state space framework and estimation by the Kalman filter. The prediction step will make a multi-step prediction over the missing values. The smoother will generate estimates for the missing values, but these would not be available in real time (Harvey, 1985, p. 95). Given that GDP is a quarterly flow variable there is no direct interpretation of the smoothed log GDP values generated for the missing months. In practical terms the sum of monthly log differences will equal the quarterly log difference, by construction. As part of a joint estimation we found that it was useful to include both monthly and quarterly data to identify the common business cycle.

2.5 Results

2.5.1 Trend and cycle decomposition

Three variations of the structure of the trend component were estimated, being LOCL, LLT and IRW as defined earlier. The IRW smooth trend model generated materially lower log likelihood than the other models so it was discarded. The LLT model with stochastic slope generated slightly higher log likelihoods than the LOCL model for both male and female data sets. However, the estimated variance of the slope parameters was not significantly higher than zero. Also, given high standard errors of coefficient estimates and poor convergence, the model was most likely underidentified, i.e. there was not sufficient information in the data to identify both slope and level variances. Accordingly the LOCL model was chosen as the preferred model for the trend. All of the series contained an estimated common cycle component, and all but g_t contained an irregular component. The estimated variance of the irregular innovations $\sigma^2_{\varepsilon g}$ was found not to be significantly higher than zero so it was subsequently restricted to zero.

Transformed variance parameters were estimated so that a positivity constraint could be applied when the parameters were transformed back into natural terms[5]. Similarly a box constraint was applied to estimates of correlation coefficients and the cycle frequency parameter to ensure that the solutions lay within the relevant permissible range[6]. An arbitrary permissible range of 36-144 months was applied

[5]Variance parameters were estimated as the argument of an exponential function to ensure that estimated variance was greater than or equal to zero.

[6]Some parameters such as correlation coefficients were estimated as the argument of a logistic function so that the parameter would be constrained to lie between a specified lower and upper

Table 2.1: **Model estimation**

		Male	Female
Common	Period(mths.)	81.95	105.65
	ρ	0.9887	0.9898
	σ^2_κ	0.0253	0.0272
GDP (g_t)	$\sigma^2_{\varepsilon g}$		
	$\sigma^2_{\tau g}$	0.1262	0.1398
	ω_g	1.0000	1.0000
	Q(24) p.val.*	1.0000	0.9970
Unemployment (u_t)	$\sigma^2_{\varepsilon u}$	0.0067	0.0227
	$\sigma^2_{\tau u}$	0.0145	0.0143
	ω_u	-0.7699	-0.5246
	Q(24) p.val.	0.1360	0.8570
Participation (p_t)	$\sigma^2_{\varepsilon p}$	0.0170	0.0184
	$\sigma^2_{\tau p}$	0.0096	0.0171
	ω_p	0.1764	0.3437
	Q(24) p.val.	0.7360	0.1240
Hours worked (h_t)	$\sigma^2_{\varepsilon h}$	0.1510	0.1421
	$\sigma^2_{\tau h}$	0.0501	0.0089
	ω_h	1.4277	1.9391
	Q(24) p.val.	0.1130	0.1580
Correl. Coeff.	$r\varepsilon_{up}$	0.7400	0.7846

* p-value of the Q-statistic testing for no serial correlation in the one-step ahead prediction errors up to 24 lags.

to the estimated cycle period. A summary of point estimates of key coefficients for males and females (all ages) are shown in Table 2.1. Full estimation results are shown in Appendix 2.A. Graphs of the smoothed components are shown in Figure 2.2.

Table 2.1 shows estimated cycle periods of approximately 82 months and 106 months for males and females respectively, which are a little longer than would typically be associated with the length of the business cycle but which are consistent with the prominent features of the labour data for the sample period, i.e. cursory inspection of the unemployment rate series for all persons reveals local peaks in the rate after sharp economic downturns in 1983, 1992, 2001 and 2009, each eight to nine years apart. The estimated damping coefficients (ρ) are approximately 0.99 which indicates that cyclical shocks are highly persistent.

bound.

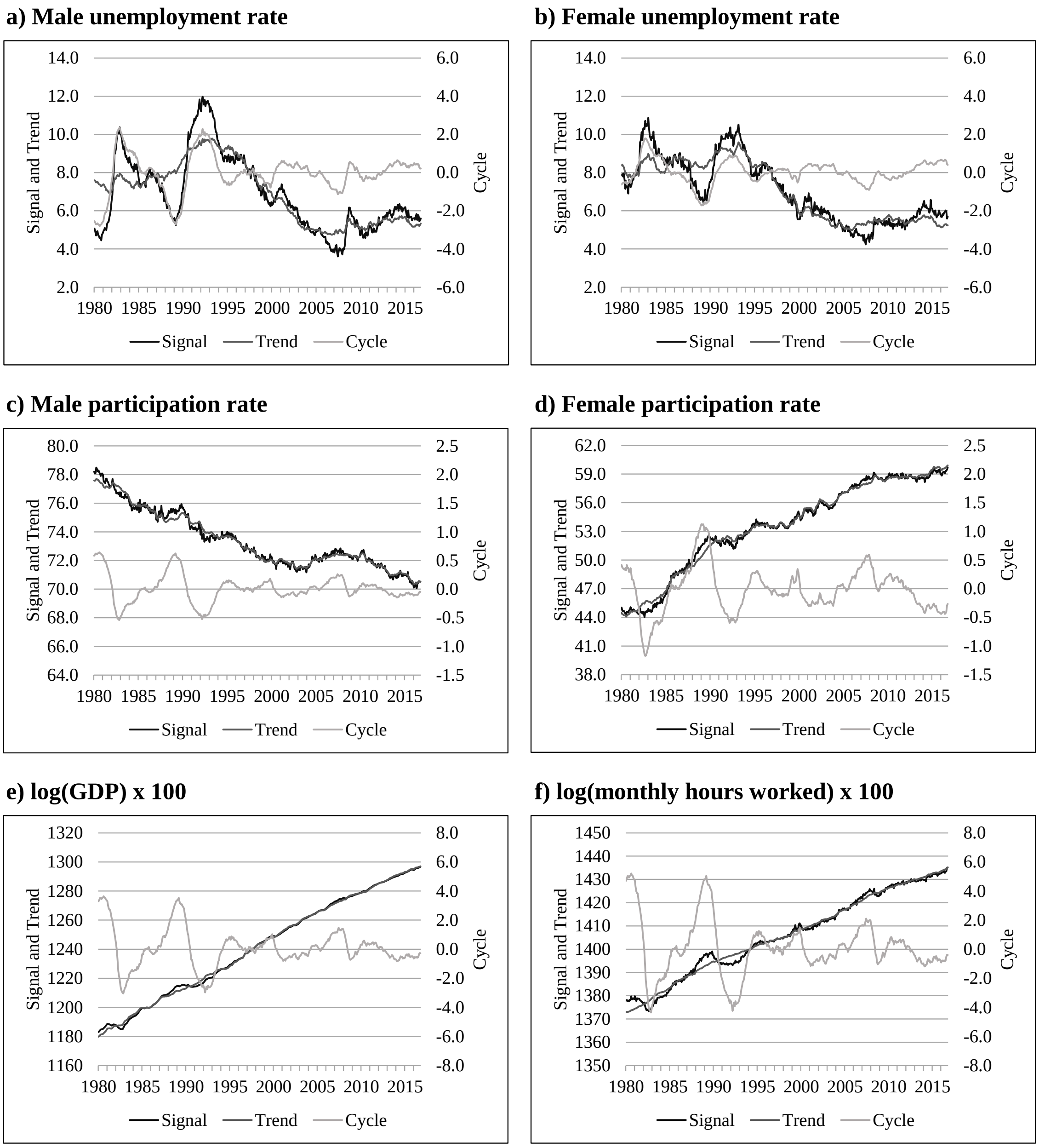

Figure 2.2: **Smoothed trend and cycle components.** Notes: The smoothed components for g_t and h_t in graphs (e) and (f) were those jointly estimated with the male unemployment and participation series.

The model generates joint estimates of the parameters, so estimates of common parameters such as the variance of the cycle innovation σ_κ^2 (in the third row of the table) will vary slightly depending on whether the cycle is estimated jointly with male or female labour data. The magnitude of the output cycle relative to the common cycle ω_g is normalised to one. From the table, the response of male cyclical unemployment to the output cycle is -0.77 whilst the corresponding female response is -0.52. The negative signs of these coefficients indicate that unemployment is counter-cyclical. Hours worked is closely linked to total employment so it was anticipated that h_t would be procyclical as indicated by the positive signs of ω_h. The positive signs of ω_p were also anticipated, indicating that participation is procyclical for males and females. Non-zero estimates of the trend innovation variances ($\sigma^2_{\tau g}$, $\sigma^2_{\tau u}$, $\sigma^2_{\tau p}$ and $\sigma^2_{\tau h}$) indicates that there are stochastic trends which are capturing the permanent shocks to the level of the variables, most notably for output. The positive sign of the estimated correlation coefficient $r\varepsilon_{up}$ for males and females was anticipated due to the theoretical relationship between unemployment and participation described earlier. Full convergence was achieved at the solution for both males and females with no boundary solutions for any of the estimated parameters, which we interpret as indicating that the model is properly identified for these two data sets. The Q-statistics indicate that there is no significant serial correlation remaining in the one-step ahead prediction errors up to 24 lags for any of the signals.

2.5.2 Comparing cycles across age brackets

The same model was estimated for each of 13 data sets, being all persons, all males, all females and then males and females in each of the five age brackets. Figure 2.3 shows plots of smoothed cyclical components for selected age brackets on the same graph. To improve clarity, only the results for the youngest, middle and oldest age brackets are shown since these three series are sufficient to show how the character of some components changes with age. There is strong evidence for both males and females that unemployment is most cyclical for the youngest age bracket, judging by the relative amplitude of the cycle components. This is not surprising since a greater portion of younger people are likely to have casual or temporary work, the availability of which can vary quickly with the level of economic activity. Greater cyclicality of participation for this age group is also anticipated since 15-24 year-olds may often take into account the likely availability of jobs when they choose between joining the labour force and pursuing further education. Observation of the components for females 55-64 years old suggests that both unemployment and participation are less cyclical than for other groups. Whilst there have been stark increases in employment and participation of older females over the last few decades, our results support the understanding that these have been predominantly trend changes rather than cyclical phenomena. On the other hand, for the older male age group both unemployment and participation have prominent cycles which suggests that the level of economic activity is affecting their decision to participate in the labour force and that the type of employment opportunities for them are sensitive to the economic cycle.

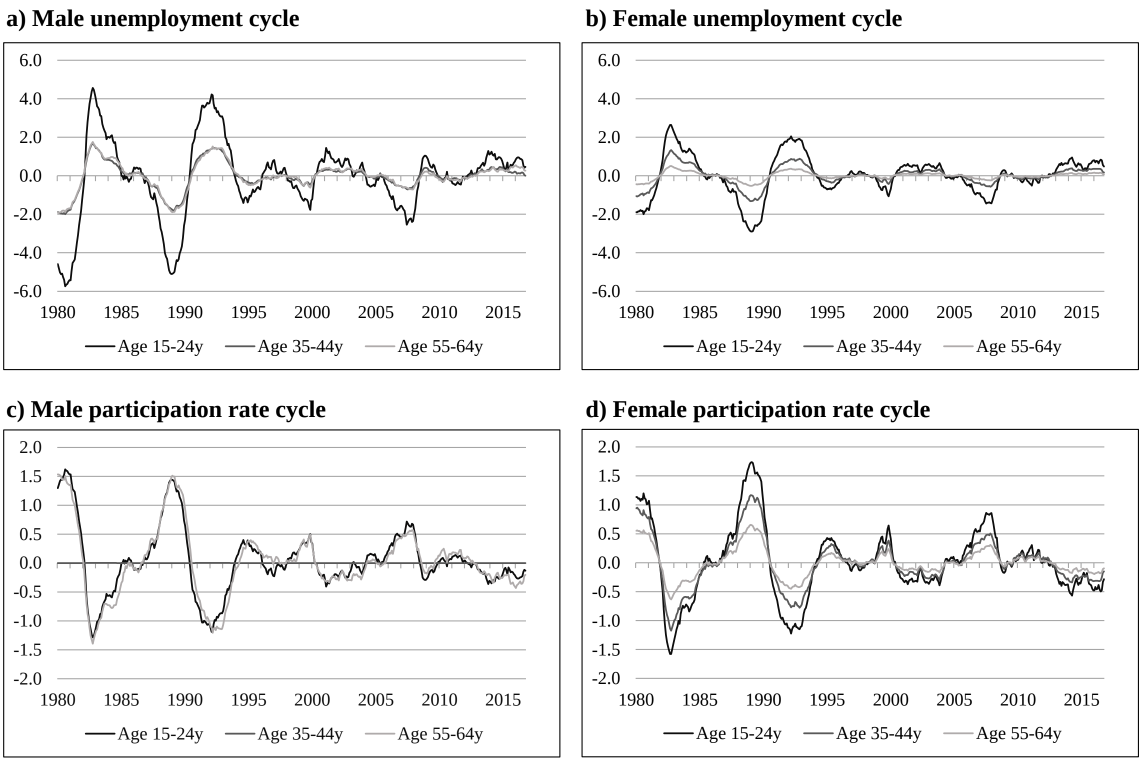

Figure 2.3: **Cycles by age bracket (youngest, middle, oldest).**

The most striking observation in Figure 2.3 is that there is essentially no participation cycle for males in the middle age bracket (the cycle virtually coincides with the horizontal axis) whereas there is a prominent cycles for females of the same age. This can be reconciled with the (arguably old-fashioned) idealised notion of a family unit with a middled-aged male primary wage earner who will typically stay in the labour force throughout the economic cycle, potentially moving only between employment and unemployment. By contrast a middle-aged female from the same family unit may have a higher tendency to move in and out of the labour force depending on the perceived likelihood of getting a job, to contribute secondary income to the family unit when opportunities are available.

2.5.3 Interpretation of coefficient estimates

The amplitude of the unemployment and participation cycles relative to the output cycle can be summarised concisely by interpreting ω_u as the Okun coefficient and ω_p as the participation coefficient as shown in Table 2.2. The first row of the table shows that the estimated signed Okun coefficient for all persons is -0.67 (in the literature this may typically be reported as 0.67 with an assumed but unstated negative relationship). The estimated participation coefficient for all persons is 0.22 (pro-cyclical). The estimated Okun coefficient for all persons in Australia has the same sign but larger magnitude than recent estimates in the literature (for example, 0.54 by Ball et al. (2012)). The pattern in our estimates of higher coefficients for males vs. females of the same age and much higher coefficients for the youngest age group (15-24 years) are typical of the pattern found in the literature (for example, 1.14 for males 15-24 years, 0.76 for females 15-24 years by Dixon et al. (2017) for a panel of 20 OECD countries, and 1.10 for males 15-24 years, 0.61 for females 15-24 years by Zanin (2014) for Australia). The different patterns of cyclicality of participation for males and females are visible in the table. For the middle age brackets 25-54 years the participation coefficients for males are small and not significant, whereas they are positive and significant at 1% for females, which supports our earlier graphical interpretation of the prominence of participation cyclicality for females in the middle age groups. The participation coefficients for the oldest and youngest age groups are also positive and significant except for the oldest female group[7].

Table 2.2: **Estimated Okun and Participation Coefficients by age and gender**

Group	Okun Coeffic.	s.e.	95% Conf.	Particip. Coeffic.	s.e.	95% Conf.
Persons (all age)	-0.67***	0.097	[-0.86, -0.48]	0.22***	0.049	[0.12, 0.31]
Male (all age)	-0.77***	0.106	[-0.98, -0.56]	0.18***	0.044	[0.09, 0.26]
Female (all age)	-0.52***	0.082	[-0.69, -0.36]	0.34***	0.069	[0.21, 0.48]
Male 15-24yrs	-1.44***	0.177	[-1.78, -1.09]	0.41***	0.105	[0.20, 0.61]
Male 25-34yrs	-0.81***	0.122	[-1.05, -0.57]	0.08	0.059	[-0.03, 0.20]
Male 35-44yrs	-0.52***	0.081	[-0.68, -0.36]	0.00	0.048	[-0.09, 0.09]
Male 45-54yrs	-0.50***	0.073	[-0.65, -0.36]	0.12	0.072	[-0.02, 0.26]
Male 55-64yrs	-0.57***	0.105	[-0.78, -0.37]	0.46***	0.142	[0.18, 0.73]
Female 15-24yrs	-0.87***	0.138	[-1.14, -0.60]	0.52***	0.126	[0.27, 0.77]
Female 25-34yrs	-0.56***	0.086	[-0.73, -0.39]	0.30***	0.102	[0.10, 0.50]
Female 35-44yrs	-0.39***	0.078	[-0.54, -0.24]	0.34***	0.097	[0.16, 0.53]
Female 45-54yrs	-0.31***	0.070	[-0.45, -0.18]	0.46***	0.125	[0.22, 0.71]
Female 55-64yrs	-0.16**	0.071	[-0.30, -0.02]	0.20	0.131	[-0.06, 0.45]

Significance at 1%, 5% and 10% indicated by ***, **, * respectively.

This research is useful for policymakers because it highlights the cyclicality of unemployment and participation for specific age and gender groups, and in particular it provides higher estimates of the cyclicality than typically reported in the literature. In addition to their stated focus on the long term trend, the cyclicality of participation may be of concern to the Government because of the risk that a person who leaves the labour force in a cyclical downturn may become a permanent non-participant. The importance of the persistence of participation is explicitly recognised in the Intergenerational Report which stresses the need to "encourage those currently not in the workforce, especially older Australians and women, to enter, re-enter and stay in work"(Commonwealth of Australia, 2015, p. iii). The analysis in this research uses aggregate historic data which naturally includes the impact of any existing policy initiatives, and is not designed to identify the effectiveness of

[7]The higher standard errors of coefficient estimates for the oldest female group is thought to reflect noisier data from the generally small number of participants in this group. The point estimate of the participation coefficient is positive (0.20) but it is not significant.

any specific policy. However, if the Government is concerned about the potential decline in participation in a downturn, particularly amongst certain groups such as females in the middle age brackets, then the results indicate that there is cyclical behaviour which could be the target of further policy support. The types of support suggested in the Intergenerational Report (Commonwealth of Australia, 2015, p. 96) include the provision of flexible and affordable child care and early learning facilities, including fee assistance rebates for parents and prospective parents[8], and the Restart Programme[9] which provides incentives to employers to hire and retain older workers. Policy measures to support participation need to be viewed in conjunction with support for movement of participants into employment rather than unemployment.

2.5.4 Robustness

The point estimates of the Okun coefficients made here for Australia and shown in Table 2.2 are somewhat higher (absolute) than other estimates in the literature (for example 0.67 for all persons vs. 0.54 by Ball et al. (2012), 0.40 by Borland (2011) and 0.35 by Lancaster and Tulip (2015)), noting that estimates for Australia are already at the high end of international estimates (for example, most countries in a group of 20 advanced countries had an estimate in the range 0.23-0.54 (Ball et al., 2012) and Dixon et al. (2017) made an estimate of 0.48 for a panel of 20 OECD countries). This could reflect the particular sample period, the decomposition methodology or conceptual differences between what the coefficient measures in different forms of analysis. In this paper the coefficient has a specific meaning which can be interpreted as the ratio of the amplitude of a common business cycle component which is shared

[8]Analysis of the cost effectiveness of subsidised child care is complex, since there is likely to be interaction between tax and welfare policies which may encourage shorter working hours for second income earners within a family unit. Further, subsidised childcare without an activity test may in fact discourage participation (Productivity Commission, 2014, 2).

[9]Details of the Restart Programme of employment assistance can be found on the website of the Australian Government Department of Jobs and Small Business website at https://www.jobs.gov.au/restart-help-employ-mature-workers-0.

by unemployment and output.

To perform a reasonableness test for the particular sample period, we derived another, simpler estimate of the Okun coefficient for all persons by calculating the long run multiplier as defined by Equation 2.3. Using quarterly data from the same sample period of September 1980 to June 2017 and a regression of Δu_t on Δg_t with one lag of Δu and three lags of Δg_t as explanatory variables generated an estimated long run multiplier of -0.48 (see Table 2.3), which is comparable with typical estimates of the average Okun coefficient for an OECD country.

Table 2.3: **Estimation of the long-run multiplier**

Dep. variable Δu	Coeff.	Std. Error	p-value
Constant	0.2725	0.0477	0
$\Delta u(-1)$	0.2560	0.0810	0.0019
Δg	-0.1144	0.0269	0
$\Delta g(-1)$	-0.1409	0.0279	0
$\Delta g(-2)$	-0.0455	0.0298	0.1291
$\Delta g(-3)$	-0.0577	0.0272	0.0356
Long-run multiplier	-0.4820		
LM(4)	5.0408		0.2831
LM(12)	13.8881		0.3079

Breusch-Godf. serial LM(n) test. No serial correl. up to n lags.

Many authors have used a HP filter to extract the cyclical components of one or more elements used in the estimation of the Okun coefficient (for example, Dixon et al. (2017) and Ball et al. (2012)) so we also consider the impact of the chosen model against the HP filter. The empirical model framework in this research can be used to generate an alternative set of estimates of the Okun and participation coefficients consistent with the use of the HP filter for the trend and cycle decomposition. It is well known that the HP filter can be represented in state space form with an underlying process which is the sum of an integrated walk and an irregular component (which is interpreted as the cycle) with a restriction applied to the ratio of the slope and irregular variances which imposes a level of smoothness on the trend (Harvey & Trimbur, 2008). Table 2.4 illustrates the estimated Okun and participation coefficients with a version of our model modified to match the HP specification of the trend and cycle components using a typical value for the smoothness parameter[10]. The Okun and participation coefficients are calculated as the ratio of the standard deviation of the relevant cycle to that of the output cycle. Full results for the model estimation are not shown[11].

Table 2.4: **Okun and Participation Coefficients derived using HP filter specification**

Group	Cycle innovation std. dev. u_t	p_t	g_t	Okun Coeff.	Partic. Coeff.
Persons (all age)	0.568	0.277	1.162	0.49	0.24
Male (all age)	0.669	0.251	1.162	0.58	0.22
Female (all age)	0.466	0.357	1.162	0.40	0.31

This procedure has generated estimates of the Okun coefficients which are comfortably within the range of the estimates of the other cited authors (notwithstanding other significant differences in methodology to those studies cited). This suggests that the particular UC decomposition methodology used in this chapter, with freely estimated variances for trend and cycle components rather than those imposed by the restrictions embedded in a HP specification, is responsible for the higher absolute estimates of the Okun coefficients.

Lastly, it is acknowledged that the point estimates made in this research do not allow for time-variation in the structural relationship between variables, nor in the variances of components, which can lead to biased estimates and misleading inferences (T. Berger, Everaert & Vierke, 2016).

[10]For monthly data a smoothing parameter of 14400 is sometimes suggested for the HP filter, although there is no general agreement as to the best value for this parameter. The parameter setting will generate a relatively smooth trend and allocate deviations from this trend to the cyclical component. A general reference on the issue is Ravn and Uhlig (2002).

[11]The log likelihood at the solution was materially lower than that obtained for the original model, indicating that the smooth trend plus cycle model was a poor fit to the data. A substantial improvement in the log likelihood was seen when the ratio of the variances was freely estimated, but the fit was still materially worse than that for the original model.

2.6 Conclusion

We have used a multivariate unobserved components model to decompose output and three labour markets time series in Australia into trend and cycle components. A key assumption used for identification was that the series share a common business cycle component. The model framework provided a direct estimate of the sensitivity of the labour market series to cyclical output shocks. The results were used to generate estimates of the Okun coefficient and a participation coefficient by age and gender. The estimates of the Okun coefficient are higher (absolute) than those generally reported in the literature which was attributed to the decomposition methodology. The unobserved components model finds maximum likelihood estimates of the components in contrast to more typical cycle extraction using the HP filter. The variation in the absolute values of our Okun coefficients by age and gender tends to follow the pattern found in the literature, with higher values for males than for females, notably higher values for the youngest age group and which tend to decline with age thereafter. Participation is less cyclical than unemployment but the coefficients are positive (it is procyclical) and mostly significant apart from males in the middle age groups. Participation is more cyclical for females than males, particularly in the middle age groups. Taken together these results show that, in aggregate, males in the middle age groups tend to stay in the labour force throughout the business cycle, perhaps moving between employment and unemployment, whereas females of the same age have a higher tendency to move in and out of the labour force procyclically. This has policy implications for attempts to increase the rate of participation of particular groups by gender and age following a cyclical downturn.

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Appendix 2.A Full estimation results

Table 2.5: **Estimation results of LOCL plus common cycle model**

	Male						Female					
Param.	coef.	std. err.	p.val	natural coef.	95% Confid. low	high	coef.	std. err.	p.val	natural coef.	95% Confid. low	high
λ	-1.086	0.195	0.000	0.077	0.068	0.087	-1.983	0.420	0.000	0.059	0.051	0.075
period				81.950	72.087	92.393				105.646	83.437	123.269
ρ	3.005	0.475	0.000	0.989	0.976	0.994	3.140	0.660	0.000	0.990	0.970	0.996
σ^2_κ	-3.677	0.264	0.000	0.025	0.015	0.041	-3.604	0.301	0.000	0.026	0.015	0.047
$\sigma^2_{\varepsilon u}$	-5.005	0.280	0.000	0.007	0.004	0.012	-3.785	0.115	0.000	0.023	0.018	0.029
$\sigma^2_{\tau u}$	-4.232	0.228	0.000	0.015	0.009	0.023	-4.246	0.204	0.000	0.014	0.010	0.022
ω_u	-0.770	0.106	0.000	-0.770	-0.982	-0.557	-0.525	0.082	0.000	-0.525	-0.689	-0.360
$\sigma^2_{\varepsilon p}$	-4.074	0.099	0.000	0.017	0.014	0.021	-3.997	0.146	0.000	0.018	0.014	0.025
$\sigma^2_{\tau p}$	-4.643	0.178	0.000	0.010	0.007	0.014	-4.067	0.171	0.000	0.017	0.012	0.024
ω_p	0.176	0.044	0.000	0.176	0.088	0.265	0.344	0.069	0.000	0.344	0.206	0.482
$\sigma^2_{\varepsilon g}$												
$\sigma^2_{\tau g}$	-2.070	0.123	0.000	0.126	0.099	0.161	-1.968	0.111	0.000	0.140	0.112	0.174
ω_g	1.000						1.000					
$\sigma^2_{\varepsilon h}$	-1.891	0.093	0.000	0.151	0.125	0.182	-1.951	0.097	0.000	0.142	0.117	0.172
$\sigma^2_{\tau h}$	-2.993	0.273	0.000	0.050	0.029	0.087	-4.718	1.260	0.000	0.009	0.001	0.111
ω_h	1.428	0.193	0.000	1.428	1.042	1.813	1.939	0.270	0.000	1.939	1.400	2.479
γ_{u12}	-0.103	0.034	0.002	-0.103	-0.171	-0.035	-0.068	0.034	0.048	-0.068	-0.137	0.001
γ_{u24}	-0.051	0.033	0.123	-0.051	-0.116	0.015	-0.052	0.032	0.106	-0.052	-0.116	0.012
γ_{p12}	-0.035	0.032	0.279	-0.035	-0.100	0.030	-0.009	0.032	0.790	-0.009	-0.073	0.056
γ_{p24}	-0.091	0.028	0.001	-0.091	-0.148	-0.035	-0.038	0.033	0.250	-0.038	-0.105	0.028
γ_{h12}	-0.093	0.040	0.021	-0.093	-0.173	-0.013	-0.099	0.039	0.011	-0.099	-0.177	-0.021
γ_{h24}	-0.208	0.040	0.000	-0.208	-0.288	-0.128	-0.186	0.040	0.000	-0.186	-0.267	-0.106
γ_{h3}	0.068	0.033	0.038	0.068	0.003	0.133	0.079	0.031	0.011	0.079	0.017	0.142
γ_{h5}	0.078	0.032	0.015	0.078	0.014	0.143	0.078	0.034	0.021	0.078	0.010	0.145
γ_{h13}	-0.090	0.039	0.020	-0.090	-0.168	-0.013	-0.093	0.037	0.012	-0.093	-0.167	-0.019
γ_{h25}	-0.168	0.039	0.000	-0.168	-0.246	-0.091	-0.150	0.038	0.000	-0.150	-0.227	-0.073
$r\varepsilon_{up}$	1.901	0.523	0.000	0.740	0.404	0.900	2.114	0.305	0.000	0.785	0.637	0.877
log lik.			-412.44					-525.33				
obs./partial			442	294				442	294			
Convg. status			Convg.	achieved				Convg.	achieved			
no. iterations			74					86				
			u	p	g	h		u	p	g	h	
Q(12) p-val			0.017	0.266	0.976	0.056		0.841	0.269	0.807	0.053	
Q(24) p-val			0.136	0.736	1.000	0.113		0.857	0.124	0.997	0.158	
Q(36) p-val			0.322	0.850	1.000	0.191		0.895	0.191	1.000	0.323	

Notes: The notation γ_{u12} represents the coefficient of the seasonal adjustment factor created using the first difference of u_t lagged 12 months, with corresponding notation for other components and lag lengths (refer to Section 2.4.4). The confid. intervals for parameter estimates shown in natural terms are approx. only.

Appendix 2.B Data sources

Table 2.6: **Data Sources**

Series	Group	Source	Notes
Unemp. rate, Particip. rate.	Persons, males, females.	ABS Catalogue 6202 Table 1	Seas. adj. monthly series. When required, quarterly series created using average of 3 months.
Unemp. rate, Particip. rate.	Males and females in 10-year age groups.	ABS Cat.6291.0.55.001 Data cube LM1.	Original terms. Seas. adj. series were generated using X12-ARIMA.
Gross Domestic Product	-	ABS Catalogue 5206 Table 2	Real, seasonally adjusted. g_t = 100ln(GDP)
Mthly. hours worked in all jobs.	Persons	ABS Catalogue 6202 Table 19	Seas. adj. A dummy variable was used to remove outliers in June 1979 and June 1980. h_t = 100ln(MHW)

Chapter 3

Average labour productivity dynamics over the business cycle

Abstract

This article provides new analysis of the business cycle behaviour of average labour productivity in Australia using a multivariate unobserved components model. Average labour productivity is found to be countercyclical at a business cycle frequency and lagging the cycle in employment by about four quarters. The model is used to determine the cyclical contributions made to productivity at the extensive and intensive margins of labour supply by examining the fluctuations in total hours worked due to both employment and average hours per worker. Full-time employment is a significant procyclical driver of total hours worked whereas part-time employment has been countercyclical since 1997. This suggests that there is some substitution of part-time for full-time workers during economic downturns. Variation at the intensive margin for either full-time or part-time workers does not make a significant contribution to the productivity cycle.

3.1 Introduction

Productivity is one of the key drivers of economic growth because it is not limited by constraints which apply to the growth of labour and capital inputs to production. There is an inextricable link between growth in output per capita and productivity which motivates the desire to sustain growth in productivity in the long term. Increasing the employment rate or participation rate can raise output and income per capita in the short term, but such gains are naturally finite. Raising prosperity through positive growth in income per capita can only be sustained in the long term through productivity improvement, which may arise by means of technological change and more efficient allocation and use of resources.

It is difficult to measure changes in productivity in the short term because the level of capital stock is not directly observable. Labour input is also hard to measure if the real quantity of interest is the amount of effective labour, which takes into account the effort made by workers and the levels of utilisation of labour by firms. Measurement errors tend to confound attempts to measure productivity accurately during periods of rapid economic change. Accordingly, the empirical behaviour of productivity over the course of the business cycle is unclear. Productivity cycles have been studied in the literature for many decades and there are seemingly contradictory findings as to whether productivity is procyclical or countercyclical with respect to output and total hours worked. A few examples of research finding procyclical productivity include Bernanke and Parkinson (1991), Basu and Fernald (2001) and Bhaumik (2011). Countercyclical productivity is found by Estrella (2004), D. Berger (2012), Lazear, Shaw and Stanton (2016) and Mulligan (2011). Basistha (2009) finds that permanent productivity shocks are negatively correlated with cyclical shocks to hours. Panovska (2017) finds evidence that firms have increasingly made use of variation at the intensive margin (average hours per employee) rather than at the extensive margin (number of employees) to manage demand shocks. These disparate findings can be partly reconciled by recognising differences in what is being measured. Measured average labour productivity is not the same as the unobservable labour productivity function, as will be explained further in Section 3.2.

Studies of trend growth rates of productivity tend to look only at the average growth rate between estimated cyclical peaks, but it is also important to try and understand the dynamic behaviour during the cycle, particularly those resulting from rapid changes in economic conditions. The contribution of this research is the provision of new analysis of the business cycle behaviour of average labour productivity in Australia since 1980. Average labour productivity is found to be countercyclical at a business cycle frequency and lagging the cycle in employment by about four quarters. Further insights are given into the cyclical variation in labour input at the extensive margin and at the intensive margin which are important determinants of measured productivity. The empirical results suggest that there is some substitution of part-time employees for full-time employees during cyclical downturns. Part-time employment is found to make a countercyclical contribution to total hours worked at the extensive margin which offsets some of the decline in full-time hours during a downturn.

The rest of the paper is organised as follows. A standard production function is specified in Section 3.2 and the measures of productivity are defined. Previous studies of the dynamics of productivity implied by macroeconomic models are reviewed along with some recent empirical studies of measured productivity during the recent recession in the United States. In Section 3.3 a multivariate unobserved components model is specified for decomposing the output and labour market time series into trend and cycle components. The model incorporates a potential phase-shift between cyclical components so that the effect of leads and lags between output and the labour market can be incorporated in the estimated productivity cycle. Empirical results are presented in Section 3.4 where the estimated cyclical components are graphically illustrated. Section 3.5 discusses the results and concludes.

3.2 Productivity dynamics under macroeconomic models

3.2.1 Production function

The following is a simple Cobb-Douglas production function with an assumption of constant returns to scale:

$$Y_t = M_t K_t^{\alpha} L_t^{1-\alpha} \tag{3.1}$$

The time series M_t defines multifactor productivity (MFP), and Y_t, K_t and L_t are output, capital input and labour input respectively. MFP may be interpreted as an index of the level of technology, or another conceptually similar unobservable function which determines the volume of output that can be produced from given volumes of capital and labour inputs. The effective level of labour input will vary in direct proportion with the average number of hours worked per employee, ceteris paribus, so it is typical to use total hours worked (H_t) rather than the number of employees as the labour input[1]. Elasticity of output with respect to capital and labour inputs are α and $(1 - \alpha)$ respectively. Capital stock is not directly observable so α cannot be estimated directly from Equation 3.1 (in log form) by simple regression. Instead, the typical approach is to make a further assumption that there is competitive equilibrium in both input and output markets so that the output elasticities α and $(1 - \alpha)$ can be equated with the factor income shares of capital and labour respectively, which can be estimated by other means (Zheng, 2005, p. 7). An index of the level of capital services K_t can be derived using the rental price or user cost for each asset type as weights in a Tornqvist index (Zheng, 2005, p. 22). The absolute level of K_t has no meaning but, in log form, the relative

[1] Even after allowing for the actual number of hours worked there remains another important unobserved variable which contributes to the level of effective labour input and that is the amount of effort (per hour) made by employees, which may vary in response to the state of the business cycle. If there is no means of observing or estimating the level of effort then its impact will be absorbed into the estimate of productivity.

change in the factor inputs and in output can be used to derive relative changes in MFP. In Australia, annual estimates are made of MFP and the factor inputs (for example see Productivity Commission (2017)).

An alternative specification of the production function is shown in Equation 3.2 which requires an interpretation that all sources of productivity improvement act through the labour productivity (LP) function A_t.

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \tag{3.2}$$

One attraction of making the assumption of constant returns to scale is that Equation 3.2 can be simply rearranged to express output per unit of labour in terms of A_t and the capital to labour ratio as shown in Equation 3.3.

$$(Y_t/L_t) = (K_t/L_t)^{\alpha} A_t^{1-\alpha} \tag{3.3}$$

Changes in output per unit of labour (Y_t/L_t) are explained in Equation 3.3 by changes in the capital to labour ratio (so called capital deepening) and A_t. Foreshadowing the use of total hours worked as the labour input in the empirical part of this research we define average labour productivity (ALP) as (Y_t/H_t). It is important to observe the distinction between ALP and LP in this framework; ALP is observable whereas LP is not. LP can only be estimated along with estimates of factor income shares and factor inputs. The empirical analysis in this paper will relate to ALP.

3.2.2 Productivity dynamics in the literature

Bernanke and Parkinson (1991, p. 439) argue that "... the procyclical behavior of average labor productivity, also known as short-run increasing returns to labor (SRIRL), has achieved the status of a basic stylised fact of macroeconomics". Three possible explanantions for SRIRL explored by the authors include technology shocks, true increasing returns and labour hoarding. Real business cycle (RBC) models (Kydland & Prescott, 1982) use exogenous technology shocks as the source of aggregate fluctuations in macro variables. If the RBC model was a good description of the true process driving the economy then we should observe strong positive correlation between output and hours worked, and between productivity and hours. This can be explained by the idea that following a positive technology shock there should be a temporary period where hours of work are increased to exploit the more profitable opportunities presented by the improved technology. Labour hoarding during a recession (for example, due to prohibitive restructuring costs or to employment protection legislation) can create the illusion of increasing returns to labour input during a subsequent recovery phase, since the volume of output can be easily increased with better utilisation of existing inputs. Bernanke and Parkinson (1991) use industry level data to test the possible explanations for SRIRL and find moderately favourable support for labour hoarding and argue against the plausibility of technology shocks as the source of procyclicality. Christiano and Eichenbaum (1992) also observed the difficulty of reconciling the measured correlation between productivity and hours worked with the properties implied by the RBC model, but found that the empirical performance of the model was substantially improved by the addition of demand shocks to the model. Galí (1999) finds stark contrast between empirical data and the RBC model but does not argue against procyclical productivity. Galí develops a model with imperfect competition, sticky prices and variable effort in which technology shocks are differentiated from non-technology shocks (such as demand shocks). He finds that variable effort can account for the positive comovement of measured productivity and output expansion due to a demand shock. Basu and Fernald (2001) treat procyclical productivity as a stylised fact and seek to identify the empirical importance of different possible explanations of it using different macroeconomic models. They discount the importance of technology shocks in favour of variable utilisation of inputs and cyclical reallocation between factors with uses with different marginal products, arguing that an economy with strong reallocation effects may exhibit apparent increasing returns to scale.

Estrella (2004) examines the relationships between the cycles in labour productivity, output and aggregate labour market series in the frequency domain at a business cycle frequency, allowing for phase leads or lags between the series. Estrella finds very limited support for the procyclicality of productivity with output and hours, instead finding that it is almost exactly out of phase with employment, and that productivity growth is in synch with the unemployment rate, arguing by that measure that productivity can be described as being countercyclical. Francis and Ramey (2005) build on the work of Galí (1999) to examine United States data from 1947-2003. They find that technology shocks have a negative impact on hours in the short run and conclude that the data are at odds with RBC theory which expects positive comovement between output, hours and productivity. In more recent research D. Berger (2012) finds that ALP is much less procyclical empirically than it was in the period up to the mid 1980's, arguing that it become acyclical or even countercyclical. Rising labour productivity has been a feature of recent recessions, at least in the United States, along with so-called jobless recoveries. Berger develops a competitive industry model in which firms can selectively dismiss their least productive employees and engage in other restructuring during a recession which increases average productivity. He argues that this is consistent with the emergence of a more efficient firm from the recession which is then better able to expand output without hiring additional workers. Mulligan (2011) makes similar empirical observations for the United States during the 2008-9 recession. Labour hours fell more sharply than output so measured labour productivity increased. This may indicate that the average quality of the workforce had increased or that the apparent increase in labour productivity was due to unobserved increases in other factor inputs. Lazear et al. (2016) used individual worker productivity data at one large firm to try and explain increasing measured productivity during the recent recession.

Possible explanations considered included an increase in the average quality of the workforce and an increase in effort by each remaining worker, both induced by the reduction in outside alternatives available to workers in a recession. The authors found that the primary cause of productivity improvement was that workers worked harder, with effort rising countercyclically in a recession. Deriving high frequency measures of total factor productivity which take into account variable utilisation of both labour and capital inputs is a complex task, as described in Fernald (2014).

This chapter has similar objectives to Estrella (2004) in that it examines the covariation at the business cycle frequency of ALP with output and labour market variables. However, the analysis will be conducted in the time domain rather than in the frequency domain and will be applied to Australian data.

3.3 Empirical model

3.3.1 Data

The following four aggregate variables are used to explore the relationship between output, productivity and the labour market: y_t is log real quarterly GDP, h_t is log total hours worked in all jobs in a calendar quarter, e_t is log total employment and l_t is the log of the size of the labour force[2]. All log values are multiplied by 100 to aid presentation. All of the series have been seasonally adjusted. It is also useful to use the series n_t, log of civilian population, to normalise some results by expressing them in terms of "per civilian", including the labour force participation rate[3]. The sample period for empirical analysis is 1980Q4-2017Q2. See Appendix 3.A for a more complete description of the data and its sources.

[2]The labour force comprises employed and unemployed persons but excludes those who are not looking for work, termed non-participants.

[3]The participation rate is the percentage of the civilian population who are in the labour force. The civilian population comprises civilians of 15 years and over.

3.3.2 Unobserved components model

Let $\boldsymbol{x_t} = (x_{1t}, x_{2t}, \ldots, x_{Nt})'$ represent an $N \times 1$ vector of time series with observations ranging from $1, \ldots, T$. In this application $\boldsymbol{x_t} = (y_t, h_t, e_t, l_t)'$. We specify a basic multivariate form of an uncorrelated unobserved components model developed by Harvey and Koopman (1997). The first equation is the measurement equation which decomposes each element of $\boldsymbol{x_t}$ into trend, cycle and irregular components:

$$x_{it} = \tau_{it} + \psi_{it} + \varepsilon_{it}, \quad \varepsilon_{it} \sim n.i.d.(0, \sigma^2_{\varepsilon i}). \tag{3.4}$$

τ_{it} is a non-stationary trend and ψ_{it} is a stationary cycle. The development of the trend is governed by the state equations

$$\begin{aligned} \tau_{it} &= \tau_{i,t-1} + \beta_{i,t-1} + \eta_{it}, \quad && \eta_{it} \sim n.i.d.(0, \sigma^2_{\eta i}), \\ \beta_{it} &= \beta_{i,t-1} + \zeta_{it}, && \zeta_{it} \sim n.i.d.(0, \sigma^2_{\zeta i}). \end{aligned} \tag{3.5}$$

In the most general form this local linear trend specification allows the level and slope of the trend to show random variation. The specification can incorporate a local level model (LOCL, also known as random walk with drift) by restricting the slope innovation variance $\sigma^2_{\zeta i} = 0$, or an integrated random walk model (IRW, smooth trend) by restricting the level innovation variance $\sigma^2_{\eta i} = 0$. A trigonometric stochastic cycle is used for the cyclical components:

$$\begin{bmatrix} \psi_{it} \\ \psi^*_{it} \end{bmatrix} = \rho_i \begin{bmatrix} \cos(\lambda_i) & \sin(\lambda_i) \\ -\sin(\lambda_i) & \cos(\lambda_i) \end{bmatrix} \begin{bmatrix} \psi_{i,t-1} \\ \psi^*_{i,t-1} \end{bmatrix} + \begin{bmatrix} \kappa_{it} \\ \kappa^*_{it} \end{bmatrix}, \quad \kappa_{it}, \kappa^*_{it} \sim n.i.d.(0, \sigma^2_{\kappa i}) \tag{3.6}$$

If it is plausible on prior grounds that the cycles have similar properties resulting from a common business cycle then the so called similar cycles model (Harvey & Koopman, 1997, p. 272) can be used which restricts the damping factor and cycle frequency to be the same for all cycles. So we set $\rho_i = \rho \in (0,1)$ and $\lambda_i = \lambda \in (0,\pi)$ for all $i \in [1,N]$. The central frequency of the cycle is λ (with cycle period $= 2\pi/\lambda$). The restricted range for ρ ensures that the cycle will be stationary.

3.3.3 Phase-shifted cycle

Whilst the labour market variables may share the same period as the business cycle component of output it is thought that there may be a lag over which output shocks are propagated through the labour market. Following Rünstler (2004) the basic model presented so far can be modified to allow for phase-shift between the different cycles which can be freely estimated. Equation 3.6 is replaced by

$$\begin{bmatrix} \psi_{1t} \\ \psi^*_{1t} \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} \psi_{1,t-1} \\ \psi^*_{1,t-1} \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa^*_t \end{bmatrix}, \ \kappa_t, \kappa^*_t \sim n.i.d.(0, \sigma^2_\kappa) \tag{3.7}$$

and

$$\psi_{it} = \omega_i \left(\cos(\xi_i\lambda)\psi_{1t} + \sin(\xi_i\lambda)\psi^*_{1t}\right), \quad i = 2, \ldots, N. \tag{3.8}$$

Rünstler showed that the parenthesised linear combination of ψ_{1t} and ψ^*_{1t} in Equation 3.8 generates a phase-shifted cycle similar to ψ_{1t}. The phase is shifted by $|\xi_i|$ periods and the shifted cycle leads (lags) ψ_{1t} if ξ_i is positive (negative). The magnitude of ψ_{it} relative to ψ_{1t} is determined by ω_i which is jointly estimated in the model. Observe that shifting the phase of a symmetrical cycle by half of a cycle period would be observationally equivalent to changing the sign of ω_i and making no phase-shift. Accordingly, it is typical to restrict the magnitude of the estimated phase-shift parameter to one quarter of the cycle period on either side of zero shift, i.e. $\xi_i \in (-\pi/2\lambda, +\pi/2\lambda)$. More examples of implementations of phase-shifted cycles in business cycle analysis can be found in Azevedo, Koopman and Rua (2006) and

Koopman and Azevedo (2008).

3.3.4 Uncorrelated components

The most general form of the model may allow for freely estimated correlation between components, such as between trend and cycle innovations. Preliminary analysis suggested that it was not feasible to identify all of the parameters in a model with freely estimated correlations so an orthogonality restriction was applied to the state variable innovations. This is a strong restriction (for example see Morley et al. (2003) and Basistha (2009)), but necessary for identification in this case. For a more detailed consideration of identification issues see Appendix 5.B in Chapter 5.

3.3.5 Secondary cycles and extensions

There are clear advantages for parameter reduction and model identification to be gained from modelling similar or common cyclical elements between series, but the approach does not remove the possibility that each series may contain other idiosyncratic cyclical dynamics. Inadequate modelling of the cyclical components may come out as an autocorrelation problem in the prediction residuals. It is not unusual to find extensions to the similar cycles model in the literature to accommodate differences between cycles. Koopman and Azevedo (2008, pp. 31–32) decompose the cycle into common and idiosyncratic parts and find that the phase-shift applies only to the common part. Koopman and Lucas (2005) use a short cycle and a long cycle in a model of output, business failures and credit spreads and estimate loadings of each element on the two cyclical components. Rünstler and Vlekke (2016) model cycles in business, housing and credit and add an autoregressive distributed lag to the stochastic cycle to make it possible to capture the high level of persistence shown in the housing and credit cycles.

Preliminary analysis of our data suggested that the phase-shifted similar business cycle does a reasonable job of modelling the stationary components but some extensions to the framework were required to improve diagnostic test results for autocorrelation in the prediction errors. Sparing use of dummy variables was made to reduce the impact of some outlying observations. Equation 3.4 was modified to allow an idiosyncratic second stationary component ϕ_{it} for each series and a dummy[4] indicator variable D_{it} with coefficient δ_i as shown below:

$$x_{it} = \tau_{it} + \psi_{it} + \phi_{it} + \delta_i D_{it} + \varepsilon_{it}. \tag{3.9}$$

Equation 3.5 was modified to allow a slope[5] dummy indicator variable S_{it} as shown below[6]:

$$\tau_{it} = \tau_{i,t-1} + \beta_{i,t-1} + \gamma_i S_{it} + \eta_{it}. \tag{3.10}$$

Further details of the secondary cycles and dummy indicator variables are given with the empirical results.

3.3.6 Decomposition of output per civilian

The log of ALP is $(y_t - h_t)$. Insight to the importance of productivity growth can be gained by looking at its contribution to output per civilian[7] using the following decomposition:

$$(y_t - n_t) = (y_t - h_t) + (h_t - e_t) + (e_t - l_t) + (l_t - n_t). \tag{3.11}$$

[4]See Appendix 3.A for a description of the dummy variables.

[5]Observe that Equation 3.10 could be rearranged to show the first difference $\Delta\tau_{it}$ on the left hand side, then it can be seen that the slope of the trend is adjusted by γ_i whilst $S_{it} = 1$.

[6]In a more general form of this model a vector of dummy variables and slope dummy variables could have been used but in this application a single dummy and a single slope dummy variable were found to suffice.

[7]Output per civilian is a slightly less natural measure than output per capita, but it is convenient to use the former because of its relation to the labour force participation rate, which uses the civilian population as its denominator.

The elements above from left to right are (in logs): output per civilian, ALP, hours per employee, employment rate (as a proportion of the labour force) and the participation rate. Rather than analysing only ALP directly we jointly estimate a model of all of the underlying components, including any phase shift between cycles, to analyse their contribution to the whole. In doing so, we can construct the trend and cyclical components of the elements of Equation 3.11 by grouping the relevant estimated non-stationary and stationary components, for example, the cyclical component of ALP can be constructed by subtracting the cyclical components of h_t from those of y_t. The civilian population series n_t is assumed to be fully represented by its trend and has no business cycle component.

3.3.7 The impact of phase-shift on average labour productivity

Prior to looking at the empirical results, it is useful to illustrate the impact that a relative phase-shift can have on a quantity such as ALP derived as the ratio of two other time series. It is immediately evident that if y_t and h_t are endogenously related then the sign of the response of $(y_t - h_t)$ to an output shock will depend on the relative magnitude of the changes in y_t and h_t, and on any lag in the response of h_t to the shock. Looking beyond the simple contemporaneous response, it is much more difficult to anticipate the apparent cyclicality of fluctuations of $(y_t - h_t)$ over the course of the business cycle. For illustration, assume that the fluctuations in h_t emerge as a damped sinusoidal response to a one period shock to y_t using a phase-shifted cycle model of the type set out in Equations 3.7 and 3.8. Consider a scenario in which the cycle has a period of 32 quarters (8 years) and that the cycle in h_t lags the cycle in y_t by 2 periods. Figure 3.1(a) shows the generated productivity cycle if the relative amplitude of cycle h_t to cycle y_t is 0.5. The apparent cycle in productivity looks procyclical with y_t, and *leads* y_t by around 2-3 periods. Note that the modelled phase-shift represents the average phase difference which would be seen between the two cycles in the long-run. It does not mean that there is zero contemporaneous response of h_t to an impulse in y_t. Figure 3.1(b) shows the simulated response if the relative amplitude of the cycles is 1.5. There is a small contemporaneous positive response in productivity but it does not last and, over the course of the business cycle, the cycle in productivity appears to be countercyclical with y_t, and *lags* by about 5 periods. In summary, the phase-shift and the relative

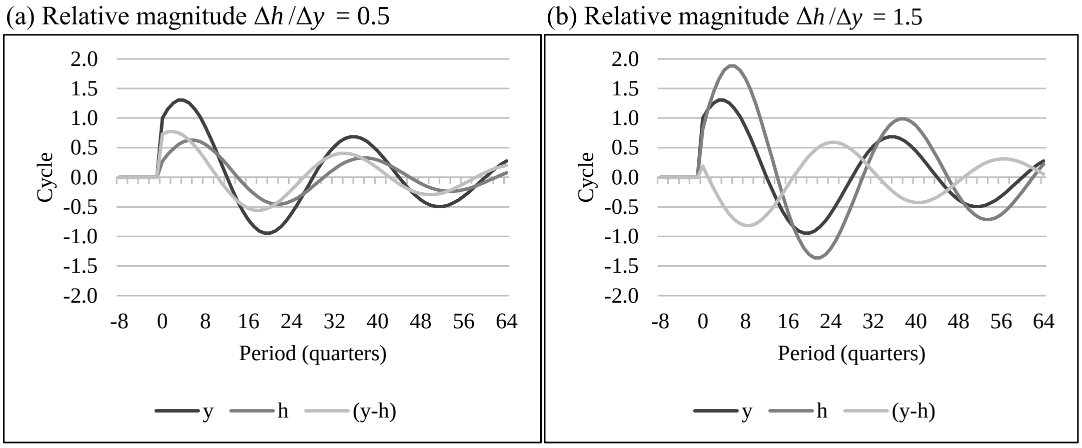

Figure 3.1: **Simulated response of h_t to a one-period shock to y_t, and the implied response of $(y_t - h_t)$.** The cycle in h_t lags y_t by two periods

magnitude of the cycles in y_t and h_t can have a material impact on the *apparent* cyclicality of ALP. If there is no phase-shift between them and the cycle in h_t is bigger than the cycle in y_t then $(y_t - h_t)$ will be countercyclical. If h_t also lags y_t, then $(y_t - h_t)$ will be countercyclical and lagging y_t.

3.3.8 Model estimation

Maximum likelihood estimates of the model parameters were made using a Kalman filter after representing the model in state space form (see Appendix 5.A in Chapter 5 for details of the state space form representation). Many of the model parameters are transformed before estimation to ensure that the estimate will lie within a permissible range after the transformation has been reversed to express the result in natural terms. For example, variances are estimated as the coefficient of an exponential function to ensure that the resulting variance is positive. The cycle frequency λ is estimated within a boxed range using a logistic function to ensure the estimated cycle period will lie within a plausible range for a business cycle, and the phase-shift is estimated within a boxed range to ensure that the shift lies within plus and minus one quarter of a cycle period.

3.4 Results

Parameter estimates in transformed and natural terms are given in Table 3.1. The confidence intervals for the parameters in natural terms are approximate only and have been derived from the boundaries of the transformed parameters created at ± 2 standard errors. Diagnostic tests of the estimation results are discussed in Appendix 3.B.

3.4.1 Trend components

Each of the underlying series y_t, h_t, e_t and l_t required a different combination of trend and irregular components, combined with the cyclical components. The preferred model was chosen considering the goodness of fit, plausibility of parameter estimates and the absence of autocorrelation in the prediction errors. For greater clarity in Table 3.1, the name of the series is used as a label instead of the index number used in Equations. 3.4 to 3.10, for example when $i = 1$ ($x_{1t} = y_t$) the labels $\sigma^2_{\varepsilon 1}$ and τ_{1t} become $\sigma^2_{\varepsilon y}$ and τ_{yt}, and so forth. The trend of y_t is a local level model with significant variance ($\hat{\sigma}^2_{\tau y} = 0.34$) and no irregular component, indicating that a significant portion of variance in y_t can be attributed to permanent level shocks. The trend in h_t is apparently more volatile and noisier than the trend in e_t, so the former was modelled by a local level model with an irregular component, whereas the latter was modelled by a smoother integrated random walk with no irregular component. The trend in the labour force l_t was also modelled as an integrated

Table 3.1: **Estimation results**

	Param.	coef.	std. err.	p.val	natural coef.	95% Confid. low	95% Confid. high	Q(8) (pval)	Q(20) (pval)	H(44) (pval)	JB (pval)
Comm.	λ	-3.052	0.403	0.000	0.172	0.150	0.219				
	per.				36.48	28.75	41.94				
	ρ	1.878	0.659	0.004	0.972	0.926	0.990				
	σ^2_κ	-2.477	0.309	0.000	0.084	0.045	0.156				
y_t	$\sigma^2_{\tau y}$	-1.080	0.161	0.000	0.340	0.246	0.469	10.01	18.84	1.95	3.16
	ω_y				1.000			(0.188)	(0.467)	(0.029)	(0.206)
	ξ_y				0.000						
	δ_y	-0.611	0.857	0.476	-0.611	-2.324	1.103				
	γ_y	-0.404	0.419	0.336	-0.404	-1.242	0.435				
	λ_2	-1.299	0.090	0.000	0.542	0.505	0.584				
	per.				11.59	10.76	12.45				
	ρ_2	2.235	0.472	0.000	0.950	0.891	0.978				
h_t	$\sigma^2_{\tau h}$	-2.613	0.405	0.000	0.073	0.033	0.165	4.43	9.96	1.28	0.45
	$\sigma^2_{\varepsilon h}$	-2.944	0.414	0.000	0.053	0.023	0.120	(0.619)	(0.933)	(0.416)	(0.798)
	ω_h	1.339	0.254	0.000	1.339	0.831	1.847				
	ξ_h	-2.089	0.640	0.001	-6.238	-7.468	-3.072				
	δ_h	-0.980	0.591	0.097	-0.980	-2.162	0.201				
	γ_h	-0.643	0.332	0.053	-0.643	-1.306	0.020				
	ϕ_h	0.200	0.055	0.000	0.200	0.090	0.311				
e_t	$\sigma^2_{\zeta e}$	-8.298	1.381	0.000	0.00025	0.00002	0.00394	5.86	22.75	1.72	0.06
	ω_e	1.167	0.199	0.000	1.167	0.770	1.564	(0.556)	(0.249)	(0.074)	(0.971)
	ξ_e	-2.089	0.640	0.001	-6.238	-7.468	-3.072				
	δ_e	-0.669	1.581	0.672	-0.669	-3.831	2.493				
	γ_e	-0.643	0.336	0.056	-0.643	-1.316	0.030				
	ϕ_e	0.117	0.045	0.010	0.117	0.027	0.207				
l_t	$\sigma^2_{\zeta e}$	-4.121	0.244	0.000	0.016	0.010	0.026	10.54	24.94	1.94	1.26
	$\sigma^2_{\varepsilon l}$	-4.439	0.172	0.000	0.012	0.008	0.017	(0.104)	(0.127)	(0.031)	(0.532)
	ω_l	0.668	0.132	0.000	0.668	0.405	0.932				
	ξ_l	-2.089	0.640	0.001	-6.238	-7.468	-3.072				
	δ_l	-0.769	1.035	0.458	-0.769	-2.839	1.302				
	γ_l	-0.464	0.342	0.175	-0.464	-1.149	0.220				
logl		-402.609									
nobs.		147									
stat.		Converg. achieved									
iter.		427									

Notes: Q(n) is the test statistic for the null hypothesis of zero autocorrelation in one-step ahead prediction errors up to n lags. H(n) is the test statistic for homoscedasticity between the first and final one third (n) of the errors in the sample. JB is the Jarque-Bera test statistic for normality of the errors.

random walk, but it is noisier than e_t and also required an irregular component. All of the estimated trend, slope and irregular innovation variances were significant at the 5% level.

Variation in the trends of the original series are difficult to observe graphically since they tend to be dominated by the overall upward trend, so it is more interesting to consider the *relative* trends representing the elements from the decomposition in Equation 3.11 (such as the trend of ALP, derived from the trends of y_t and h_t), as illustrated in Figure 3.2. The series presented in the figure are the smoothed unobserved components which have been generated from the output of the Kalman filter. The figure demonstrates clearly that almost all of the trend growth in output per civilian since 1980 can be attributed to growth in ALP. Steady growth in labour force participation $(l_t - n_t)$ has also made a noticeable positive contribution, at least until about 2008, but the gains have been offset by an ongoing decline in the average number of hours worked per employee. Changes to the trend employment rate have not made a material contribution over the period. In later sections the cyclical contributions of these elements will be considered.

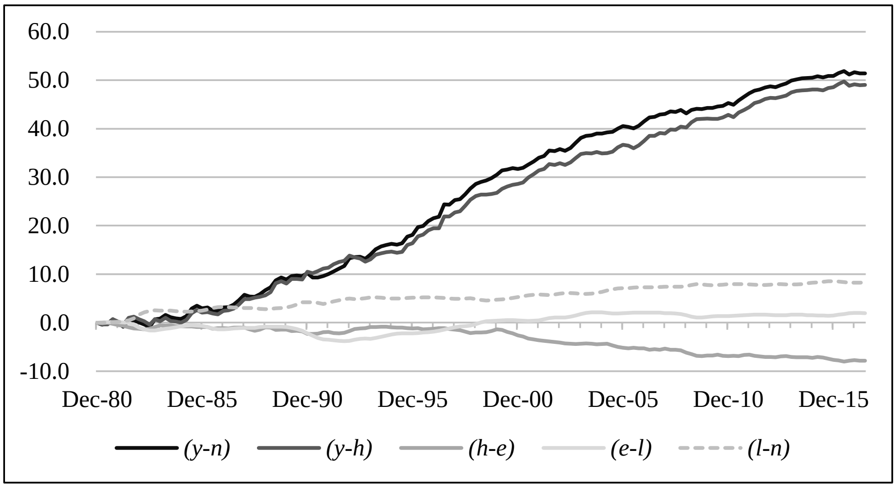

Figure 3.2: **Smoothed relative trend components**

3.4.2 Cycles

The solutions for the stationary components include the main phase-shifted cycle and, in some cases, secondary stationary components. The secondary components were included when required to remove residual autocorrelation and are discussed in detail in Appendix 3.C. The estimated period of the similar cycle is approximately 36.5 quarters (9 years) within a 95% confidence interval of 29-42 quarters. The estimated damping factor is 0.972 which indicates that cyclical shocks are highly persistent. The model estimation converged at the solution, but very slowly, and there were indications that the likelihood function is quite flat in a region where the period of the cycle is large. The author is not aware of any similar multivariate state space model estimates of the average business cycle length in Australia in the literature, but the result fits visibly prominent characteristics of the data, which exhibits significant economic contractions in 1982-83, 1990-91, 2000-01 and 2008-09, each 8-10 years apart. In studies of other economies, Koopman and Lucas (2005) apply a model with two cycles to the United States data and estimate a short cycle period of around 6 years and long cycle of around 11-16 years. Azevedo et al. (2006) use a fixed frequency of 8 years in a band-pass filter which they claim is a typical frequency used for extracting business cycle fluctuations for United States and European data. Rünstler and Vlekke (2016) analyse business, housing and credit cycles in the United States and several European countries and find evidence of short cycles with periods in the range 2.8-8.2 years and long cycles with periods in the range 10.7-18.9 years. Taken together these observations suggest that the estimated cycle period of 9 years is plausible.

The parameters of most interest in Table 3.1 are the relative cycle magnitudes and phase-shifts. In declining order of size the cycle magnitudes are $\hat{\omega}_h = 1.34$, $\hat{\omega}_e = 1.17$ and $\hat{\omega}_l = 0.67$ relative to the output cycle. It was anticipated that the size of the labour force would be less cyclical than the other two series since the participation rate is typically found to be only weakly procyclical (for example, Borland (2011) found that the pattern of variation in the labour force during the economic cycle was quite similar to that in employment, only more muted). There were some indications in preliminary analysis that employment lagged hours slightly and that the labour force lagged employment by a further amount. However, it was not possible to estimate a significant phase-shift independently for each of the labour series so a restriction was imposed requiring a common phase-shift for all three series relative to the cycle in y_t. The estimated shift ($\hat{\xi}_i = -6.2$) indicates that the labour market cycles lag output by about 6 quarters. Recall that the shift represents the average phase difference over the course of the cycle and we may observe some more rapid responses of the labour series to output shocks. There are only a small number of comparable attempts to estimate the phase-shift of labour market series with output in the literature. Rünstler (2004, p. 239) found that total hours lagged the output cycle by 1.5 quarters in the United States from 1952 to 1999. Azevedo et al. (2006) examined data for the Euro area 1986-2002 and estimated that unemployment lagged the GDP cycle by about 16 months (5 quarters), comparable with the estimated phase-shift in this research. Figure 3.3(a)

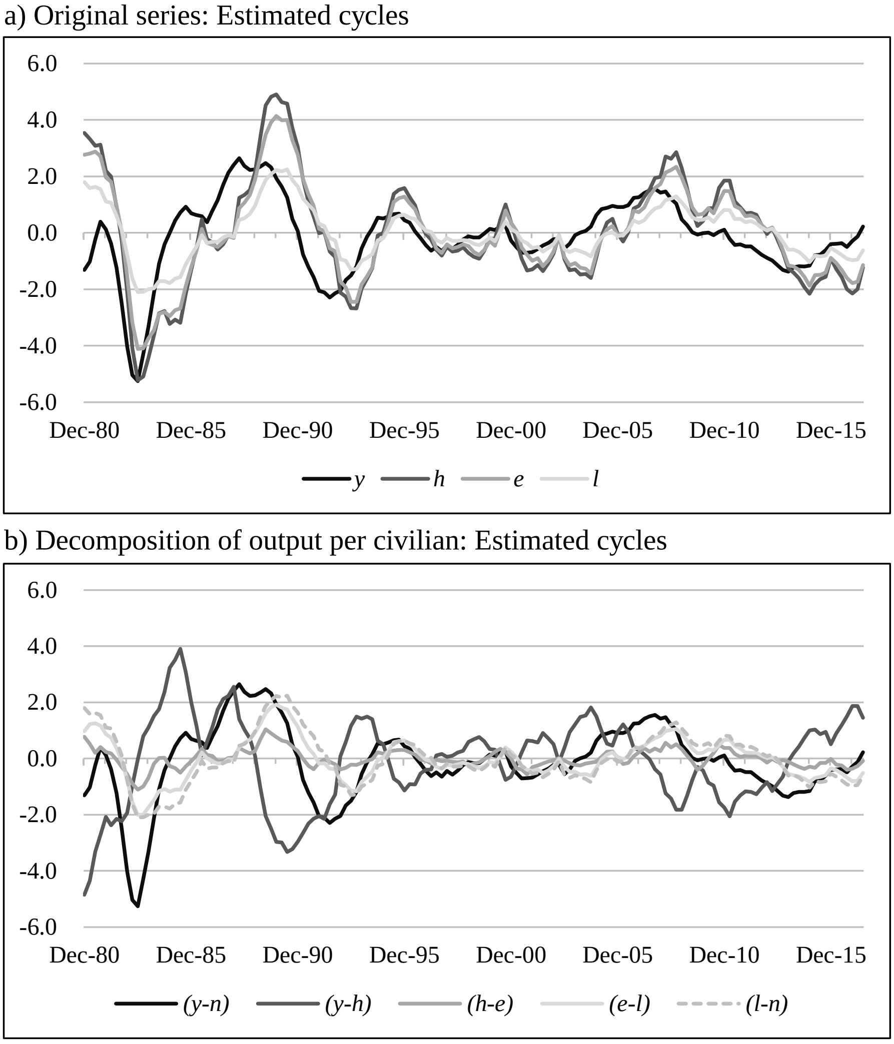

Figure 3.3: **Smoothed cyclical components of the underlying series and the decomposition of output per civilian**

presents the estimated cyclical component for each of the underlying series, being the sum of the phase-shifted similar cycle component ψ_{it} and any idiosyncratic stationary component ϕ_{it}, as set out in Equation 3.9. It is evident from the figure that all of the labour market series are procyclical and lag output. One of the advantages of using the phase-shifted cycles and other idiosyncratic components is that the model can accommodate some temporal changes in the relationship between the various series. For example, during the 1982-83 recession and subsequent recovery it can be seen that the output cycle was sharper than the labour market cycles, since the shock and associated economic reforms took several years to work their way through the labour market. By contrast the labour market cycle initially moved sharply lower during the 2008-09 global financial crisis, whereas output in Australia proved to be quite resilient compared to most developed economies at that time, due in part to its links to the strong Chinese economy and the absence of direct exposure to poorly performing credit securities (Borland, 2011).

3.4.3 Cyclicality of average labour productivity

The relative magnitude of the output and labour market cycles and the phase-shift between them ultimately drive the apparent cycle in ALP and the other elements shown in Equation 3.11, which are illustrated in Figure 3.3(b). These derived cyclical components have been derived by grouping the cyclical components of the underlying series presented in Figure 3.3(a). Cyclical variation in ALP has been the largest contributor to the cycle in output per civilian. The smallest contribution to the cycle in output per civilian has been made at the intensive margin by hours per employee.

There is only a short period when ALP arguably looks procyclical in the figure from about 1983 to 1988, coinciding with the recovery from the 1982-83 recession. In this period cyclical changes in y_t were relatively sharp and large compared to cyclical changes in h_t so ALP tended to move in the same direction as output. That would be considered consistent with the theoretical concept of short run increasing returns to labour. However, for most of the sample period cyclical changes in h_t

tended to be larger than cyclical changes to y_t and lagging it, so ALP arguably looks countercyclical with $(y_t - n_t)$ after 1988 for the reasons explained in Section 3.3.7. In the next section we will make a model estimate of the magnitude and lag of the ALP cycle relative to the business cycle using the output of the first model.

3.4.4 Estimated phase-shift of productivity

Estrella (2004) noted that productivity could be compared with either output or a labour market cycle to characterise it as being procyclical or otherwise with the business cycle (Estrella compared productivity growth with the unemployment rate to characterise the former as countercyclical). ALP and the employment rate $(e_t - l_t)$ are extracted from Figure 3.3(b) and represented in Figure 3.4 to highlight the generally inverse relationship between them. For most of the sample period ALP appears to be countercyclical with and possibly lagging the employment rate. A

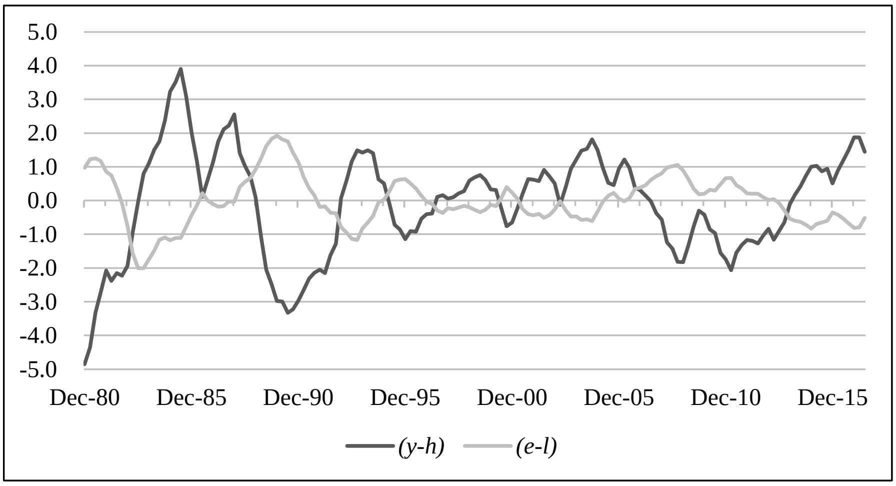

Figure 3.4: **Productivity cycle vs. employment cycle**

direct estimate of the cyclicality of ALP relative to the employment rate was made using the same multivariate model framework specified in Section 3.3 but with only two series, being the previously estimated cyclical components of $(y_t - h_t)$ and $(e_t - l_t)$. Each series was modelled with a cycle and a linear trend[8]. The cycle period was fixed

[8]The purpose of the linear trend is simply to allow for small but non-zero drift of the series over the sample period. In the model framework, we can model a linear trend simply by using the

to the previously estimated value. Similar to the previous estimation, we found it necessary to include secondary stationary components in the measurement equation to remove autocorrelation from the prediction errors. Estimation results are shown in Table 3.2. The estimates indicate that the magnitude of the ALP cycle relative to the employment rate cycle is $\hat{\omega}_{(y-h)} = -2.1$ (the sign indicates countercyclicality) and that ALP lags employment by $|\hat{\xi}_{(y-h)}| = 3.99$ quarters.

Table 3.2: **Magnitude and phase-shift of ALP cycle relative to employment rate cycle**

Param.		coef.	std. err.	p.val	natural coef.	95% Confid. low	95% Confid. high	Q(8) (pval)	Q(20) (pval)	H(47) (pval)	JB (pval)
Comm.	λ				0.172						
	per.				36.48						
	ρ	1.868	0.509	0.000	0.972	0.939	0.988				
	σ^2_κ	-3.911	0.092	0.000	0.020	0.017	0.024				
$(e-l)$	$\omega_{(e-l)}$				1.000			10.45	28.21	2.42	0.53
	$\xi_{(e-l)}$				0.000			(0.235)	(0.105)	(0.003)	(0.768)
	$\phi_{(e-l)}$	0.396	0.042	0.000	0.396	0.312	0.481				
$(y-h)$	$\omega_{(y-h)}$	-2.129	0.060	0.000	-2.129	-2.248	-2.010	12.94	28.51	1.97	1.24
	$\xi_{(y-h)}$	-1.096	0.073	0.000	-3.993	-4.418	-3.536	(0.114)	(0.098)	(0.022)	(0.539)
	$\phi_{(y-h)}$	0.321	0.030	0.000	0.321	0.262	0.381				
logl		50.761									
nobs.		147									
stat.		Converg. achieved									
iter.		59									

Refer to Table 3.1 notes.

3.4.5 Influence of part-time employment on total hours

It is not possible to attribute output separately to full-time and part-time workers but it is still of interest to examine the contribution that each make to cycles in ALP through its denominator. Total hours worked may be managed by employers during the business cycle not only by controlling the proportion of their workforce employed full-time and part-time but also by varying the average hours of work given to them. The trends in employment and average hours are illustrated in Figure 3.5 which show strong growth in part-time relative to full-time employment over the whole sample

LOCL model for the trend and restricting the trend variance to zero.

period[9]. Since the late 1990's this has been coupled with increasing average hours

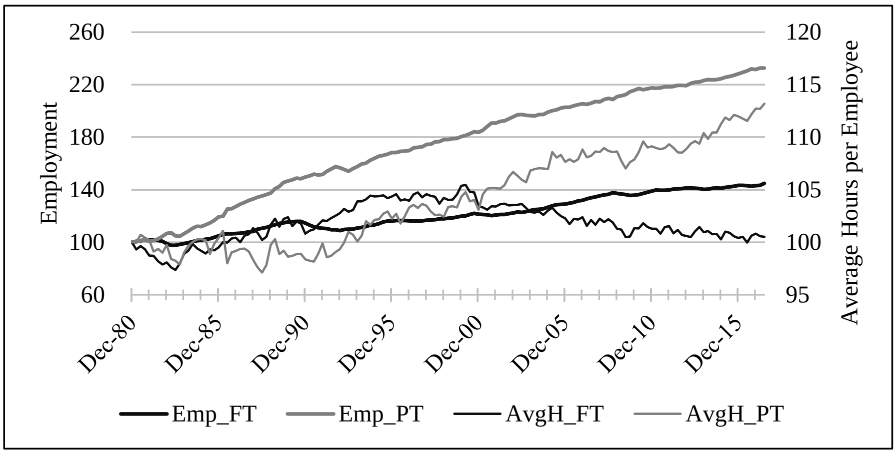

Figure 3.5: **Full-time and part-time employment, and average hours.** All series in logs, with intercepts adjusted to 100

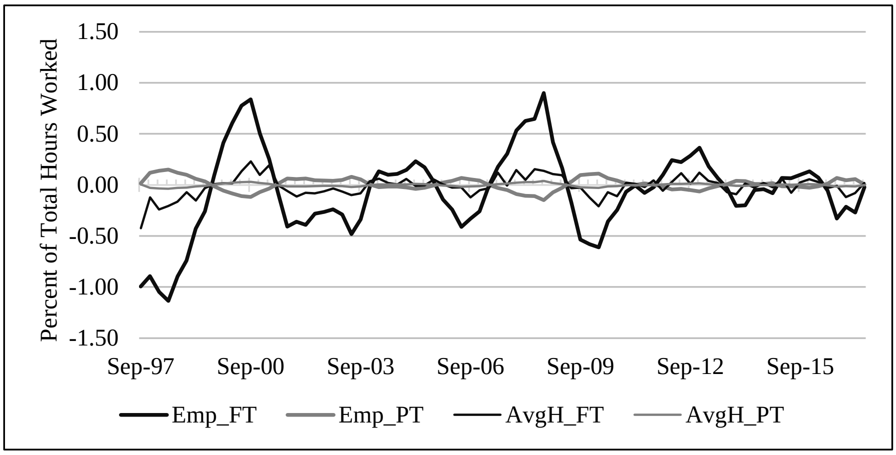

Figure 3.6: **Cyclical contribution to total hours worked**

per part-time employee (AHPT) and falling average hours per full-time employee (AHFT). Part-time employment (EPT) looks less cyclical than full-time employment (EFT). Preliminary analysis suggested that the cyclicality of EPT was not stable for the full sample period, but that since the late 1990's it may have moved

[9]Part-time employment should not be confused with casual employment, which is sometimes blamed for the decline in full-time employment, where casual employment is defined by the absence of leave entitlements. A recent report by the Australian Parliamentary Library (2018) showed that, contrary to some perceptions, casual part-time employment has not grown faster than permanent part-time employment in the last 20 years. Indeed from 1996-2016 the report (and author's calculations) showed that the growth rate of permanent part-time was more than three times higher than that of casual part-time for males and females combined.

countercyclically with the business cycle. The same model framework was used to perform a trend and cycle decomposition of a multivariate system comprising logs of output, EFT, EPT, AHFT and AHPT for the sub-period 1997Q3-2017Q2. Estimation results are shown in Table 3.3. The estimated coefficient ω_{ept} was negative (-1.11, p.value 0.058) indicating that EPT is countercyclical with output and EFT. The point estimates of ω_{ahft} and ω_{ahpt} are positive but are not significant. It is worth noting that the pool of full-time workers is much larger than that of part-time workers so the former is more important than the latter to the cyclicality of total hours. In Figure 3.6 the contribution of each of the four elements to the cyclicality of total

Table 3.3: **Decomposition of full-time and part-time employment and average hours, September 1997 - June 2017**

	Param.	coef.	std. err.	p.val	natural coef.	95% Confid. low	high	Q(8) (pval)	Q(20) (pval)	H(21) (pval)	JB (pval)
Comm.	λ	-1.077	0.492	0.029	0.364	0.234	0.568				
	period				17.28	11.07	26.80				
	ρ	-0.252	1.189	0.832	0.887	0.813	0.977				
	σ^2_κ	-3.262	0.675	0.000	0.038	0.010	0.148				
y_t	$\sigma^2_{\tau y}$	-1.828	0.296	0.000	0.161	0.089	0.291	3.84	13.21	1.33	2.64
	ω_y				1.000			(0.798)	(0.827)	(0.521)	(0.267)
	ξ_y				0.000						
eft_t	$\sigma^2_{\tau eft}$	-1.732	0.301	0.000	0.177	0.097	0.323	11.49	15.04	1.14	0.73
	ω_{eft}	1.073	0.453	0.018	1.073	0.167	1.979	(0.119)	(0.720)	(0.771)	(0.693)
	ξ_{eft}	-0.889	0.432	0.040	-2.505	-4.228	-0.076				
$ahft_t$	$\sigma^2_{\tau ahft}$	-1.816	0.205	0.000	0.163	0.108	0.245	14.92	49.74	1.56	1.16
	ω_{ahft}	0.206	0.287	0.474	0.206	-0.369	0.781	(0.037)	(0.000)	(0.312)	(0.559)
	ξ_{ahft}	-0.889	0.432	0.040	-2.505	-4.228	-0.076				
	ϕ_{ahft}	-0.132	0.085	0.123	-0.132	-0.302	0.039				
ept_t	$\sigma^2_{\tau ept}$	-0.685	0.225	0.002	0.504	0.322	0.790	8.55	30.17	1.71	0.89
	ω_{ept}	-1.107	0.585	0.058	-1.107	-2.276	0.062	(0.287)	(0.050)	(0.224)	(0.642)
	ξ_{ept}	-0.889	0.432	0.040	-2.505	-4.228	-0.076				
$ahpt_t$	$\sigma^2_{\tau ahpt}$	-1.261	0.172	0.000	0.283	0.201	0.399	9.95	21.69	1.35	1.84
	ω_{ahpt}	0.276	0.305	0.366	0.276	-0.334	0.887	(0.191)	(0.300)	(0.495)	(0.399)
	ξ_{ahpt}	-0.889	0.432	0.040	-2.505	-4.228	-0.076				
logl		-426.889									
nobs.		80									
stat.		Converg.	achieved								
iter.		428									

Refer to Table 3.1 notes.

hours are shown on a common scale[10]. Graphically, it can be seen that the cycle in EPT will have a dampening effect on the cycle in EFT but the latter is materially

[10]The method calculating the contribution of each cyclical component to total hours is described in Appendix 3.D

larger. AHFT looks like it makes a procyclical contribution but it is not statistically significant.

In summary, by far the most significant variation in total hours at a business cycle frequency occurs at the extensive margin due to procyclical EFT, although it is offset to some degree by countercyclical EPT. Variation at the intensive margin does not appear to have an important impact on total hours at a business cycle frequency since neither the cycles in AHFT nor AHPT are significant.

3.5 Conclusion

The cyclicality of productivity is difficult to characterise not least because there are different interpretations of what procyclical productivity means. It can be defined in terms of a positive response of output to a positive shock to unobservable productivity, or as a positive response of measured ALP to an output shock. Lastly, it can be a measure of the behaviour of ALP over the course of a full business cycle. Using the latter measure ALP has been found to be countercyclical and lagging the business cycle, in particular it lags the cycle in the employment rate by about four quarters. A significant driver of this result is found at the extensive margin in the form of strongly procyclical full-time employment which lags the business cycle. Variation at the intensive margin due either to full-time or part-time average hours worked do not make a significant contribution to the productivity cycle.

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Appendix 3.A Data sources

Table 3.4: **Data Sources**

Series	Label	Source	Notes
Expend. on GDP	Y	ABS Cat. 5206 Tab.2 A2304402X	Quart., s.adj. chain vol. measure.
Mthly hours worked			
in all jobs	H	ABS Cat.6202 Tab.19 A84426277X	Mthly.[a], s.adj.
(empl. full-time)	HFT	ABS Cat.6202 Tab.19 A84426278A	
(empl. part-time)	HPT	ABS Cat.6202 Tab.19 A84426279C	
Employed total			
persons	E	ABS Cat.6202 Tab.1 A84423043C	Mthly.[b], s.adj.
(empl. full-time)	EFT	ABS Cat.6202 Tab.1 A84423041X	
(empl. part-time)	EPT	ABS Cat.6202 Tab.1 A84423042A	
Particip. rate	PR	ABS Cat.6202 Tab.1 A84423051C	Mthly.[b], s.adj.
Labour force	L		Derived using PR and N.
Civilian popul.			
aged 15+ years	N	ABS Cat.6202 Tab.1 A84423091W	Mthly.[b,c], original.
GST Dummy	D		2000Q3-Q4=1; zero otherwise.
Slope. Dummy	S		1990Q3-1991Q2=1; zero otherwise.

a Quarterly flows of hours worked were created by aggregating the monthly series.
b Quarterly stocks of employment, participation and population were created by averaging monthly series.
c The original series showed a very small amount of seasonality, which was believed to be spurious, so a seasonally adjusted series was created using TRAMO/SEATS.

Appendix 3.B Diagnostic tests

The four right hand columns of Table 3.1 present diagnostic test results[11] of the one-step ahead prediction errors, separately for each of the four series, generated by the operation of the Kalman filter. The first 15 such errors are excluded from the tests, corresponding with the number of diffuse initial states in the model. The most important test is for autocorrelation in the errors which, if present, would indicate that the modelled cyclical components have not captured all of the autoregressive behaviour. The $Q(n)$ statistic relates to a null of zero autocorrelation up to n lags. The distribution of statistic is approximately chi-squared with degrees of freedom which has been reduced by the number of estimated disturbance variances for the relevant series. The results are all favourable at 8 and 20 lags (2 and 5 years). The $H(44)$ test statistic relates to a null that the error variance is the same for the first and last thirds of the sample period. We use the larger of the variances in the

[11]The diagnostic tests follow the specification of Commandeur and Koopman (2007, pp. 90–93).

numerator and test against a $F(44, 44)$ critical value at 2.5%. The p-values shown in Table 3.1 are for a two-tailed test, which reveal there is heteroscedasticity in the errors for y_t at a significance of 2.9% and in l_t at a significance of 3.1%. This indicates that the standard errors may be understated and that the derived confidence intervals for the true parameter values may be inaccurate. The JB statistic indicates that a normal distribution of the errors for each series cannot be rejected.

Appendix 3.C Secondary cycles and other components

One of the main objectives of this research is to compare the primary phase-shifted cycle across the various series. However, a single cyclical component for each series was not enough to capture adequately all of the stationary autoregressive dynamics for three of the series. Simple solutions corresponding with term ϕ_{it} in Equation 3.9 were found to remove residual autocorrelation from the prediction residuals without detracting from the role of the primary cycle.

For y_t, a second stochastic trigonometric cycle with a shorter period (estimated at 11.6 quarters) was found to be effective (the parameters labelled λ_2 and ρ_2 in Table 3.1 relate to the second cycle in y_t). The estimated innovation variance for this second cycle was very low, so it was restricted to zero, making the cycle deterministic. The interpretation of the secondary cycle is simply that it helps fit the larger and higher frequency components of y_t in the first part of the sample period. A simple stationary autoregressive component was used for h_t and e_t using a lagged three-period difference of the original log series. This is essentially using the lagged differences as an exogenous variable in the measurement equation to help explain changes in the signal. The differences were de-meaned to ensure they had a zero mean, so as not to interfere with the estimated trend. The coefficients of the autoregressive terms were estimated jointly in the system, and estimates of them are represented by ϕ_{it} in the relevant rows in Table 3.1. It seems likely that the lagged difference terms help match short term cyclical dynamics not easily matched by the longer period business cycle component. No secondary cycle was required for l_t.

The slope dummy variable S_{it} in Equation 3.10 was also useful for reducing autocorrelation in prediction errors. An indicator set to one for the period September 1990 to June 1991 (the depths of a severe recession) assisted in modelling high persistence in the slope of e_t and h_t around this time. Lastly, D_{it} in Equation 3.9 was used to indicate the first two quarters after the introduction of the goods and services tax in Australia (introduced 1 July 2000). D_{it} was lagged by one period in all equations except for y_t where there was no lag. This improved diagnostics of the normality of prediction errors. Estimates of coefficients δ_i and γ_i are shown in the table. Observe that the coefficient estimates of the dummy variables are not individually significant but, collectively, they generated a material improvement in diagnostic test results, so they were retained for all of the series.

Figure 3.7 shows the primary (phase-shifted similar cycle) and secondary cyclical components and their sum. The figure illustrates that in each case the contribution of the secondary cyclical component is small, and the total cycle closely resembles the primary cycle as desired.

Appendix 3.D Cyclical contributions of full-time and part-time employment

To determine the partial contribution of each element to the cycle in total hours we proceed as follows. Average hours per full-time employee is $AHFT = HFT/EFT$. Trends and cycles are estimated in logs of $AHFT$ and EFT which are labelled $ahft$ and eft, so $ahft = \tau_{ahft} + \psi_{ahft}$ and $eft = \tau_{eft} + \psi_{eft}$. The decomposition of HFT

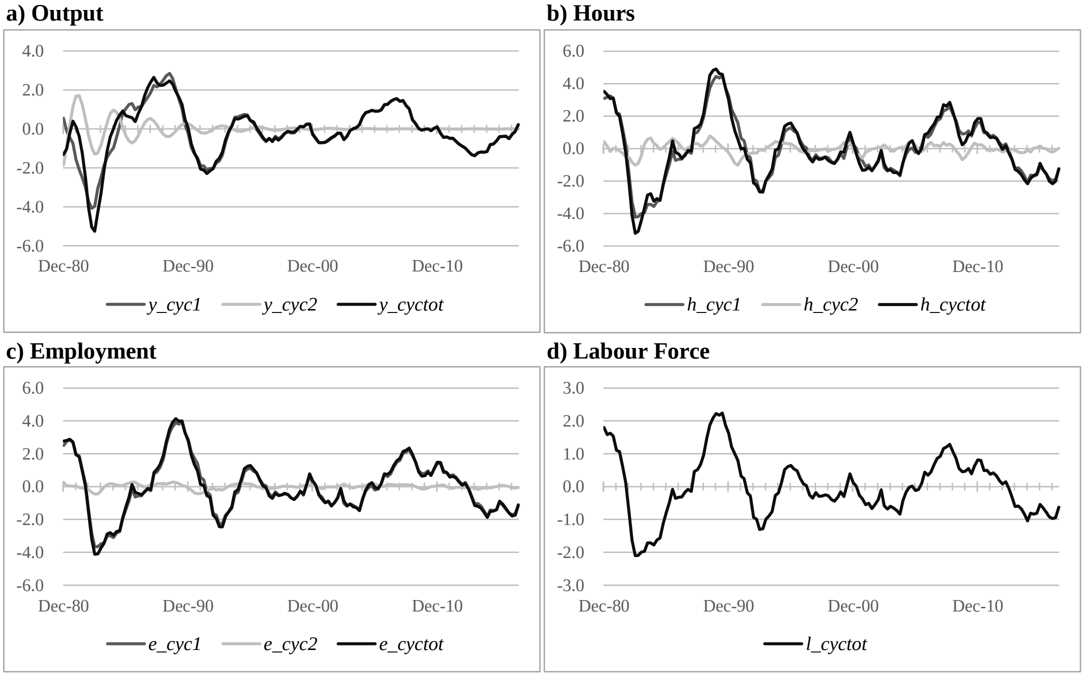

Figure 3.7: **Primary (common) and secondary cycles.** In the figure, "cyc1","cyc2" and "cyctot" represent the primary and secondary cycles and their sum, respectively.

can then be represented via multiplicative factors:

$$
\begin{aligned}
HFT &= e^{ln(EFT \times AHFT)} \\
&= e^{eft+ahft} \\
&= e^{\tau_{eft}+\psi_{eft}+\tau_{ahft}+\psi_{ahft}} \\
&= e^{\tau_{eft}} e^{\psi_{eft}} e^{\tau_{ahft}} e^{\psi_{ahft}}.
\end{aligned}
$$

Since the mean of each of the cyclical components e^{ψ} is one by construction we can define the approximate contribution of the cyclical component of EFT to HFT by

$$e^{\tau_{eft}} \left(e^{\psi_{eft}} - 1\right) e^{\tau_{ahft}} e^{\psi_{ahft}},$$

and the contribution of the cyclical component of AHFT by

$$e^{\tau_{eft}} e^{\psi_{eft}} e^{\tau_{ahft}} \left(e^{\psi_{ahft}} - 1\right).$$

Now, total hours is the sum of full-time and part-time hours so

$$H = e^{\tau_{eft}} e^{\psi_{eft}} e^{\tau_{ahft}} e^{\psi_{ahft}} + e^{\tau_{ept}} e^{\psi_{ept}} e^{\tau_{ahpt}} e^{\psi_{ahpt}},$$

and we can apply the same process described above to calculate the cyclical contributions of AHPT and EPT. All of the cyclical contributions calculated by this method are approximate and will not add to the total cyclical deviation of H from its trend.

Chapter 4

Business cycle asymmetry in a small open economy

Abstract

If the business cycle is a real macroeconomic phenomenon then there should be something fundamentally different about economic behaviour in a recession when compared to normal times. Simply describing any period of below-trend output as a recession would arguably be no more than an exercise in labelling. An asymmetric business cycle may better reflect the different dynamics operating in different economic regimes through features such as deeper or sharper declines in output than subsequent recoveries. One form of asymmetry in output envisaged by Friedman's Plucking Model postulates occasional recessionary downward "plucks" away from an efficient ceiling level, which are subsequently recovered in normal times. In this paper a version of Friedman's Plucking Model is used to test several Australian macroeconomic series for the presence of both transitory and permanent asymmetric responses to recessions, against an alternative with only symmetric responses. The evidence supports a view that there is a fundamental difference in the dynamics of key components of output and the unemployment rate in recessions when compared to normal times.

4.1 Introduction

There are numerous ways in which a macroeconomic time series such as the level of output can be separated into trend and cycle components, sometimes leaving room for doubt as to whether valid identification of the components has been made. The question may be whether the time series is materially different to that which could have been generated by some alternative random process with no cyclical component. Suppose that a zero mean cyclical component has been properly identified and that we choose to interpret it as a proxy for the business cycle, and we label the periods with negative cyclical deviations as recessions (or a similar label such as "economic downturn"). Even under these conditions it can be argued that the identified recessions are neither meaningful nor useful unless they relate to some real macroeconomic phenomenon, under which the behaviour of the economy in recession is fundamentally different to behaviour in so called normal times. It can be argued that asymmetrical behaviour between different phases of the business cycle are in fact its defining characteristics, such as the anecdotal tendency of the rate of output growth to fall more steeply and for a shorter period of time in a recession than its mirror image in the expansionary phase, and for the tendency for the unemployment rate to rise sharply during a recession but afterwards to revert only slowly back towards its presumed natural rate.

DeLong and Summers (1984) argued that there was little evidence of asymmetry in the business cycle for the United States or five other major OECD nations using measures of skewness. Hamilton (1989) developed an effective framework for identifying the existence of distinct phases of economic activity in the form of a regime-switching model in which discrete states could be determined endogenously and growth coefficients could be estimated which depended on the state. In a two-state model, tests for asymmetry could be made by comparing the estimated growth coefficients in the expansionary and recessionary phases. There are many plausible forms of asymmetry, including asymmetry in the sharpness of turning points in the trade cycle which was noted by Keynes (1936). Sichel (1993) illustrated various combinations of steepness and deepness which could characterise departures of a cycle from symmetry. Many authors have considered models with effectively three phases (normal, recession, recovery) with asymmetric characteristics, including Sichel (1994) and Kim, Morley and Piger (2005).

An important area of research concerning the permanent or transitory nature of economic shocks overlaps research into business cycle asymmetry, since the form of asymmetry can be modelled as affecting either the permanent or transitory components of a time series. In this regard there have been mixed findings in the literature, which inevitably depend on the model specification to some degree. Hamilton (1989) found that a typical recession in the United States had a permanent 3% negative impact on the level of GNP. Morley and Piger (2012) argued that the business cycle corresponds primarily to a transitory deviation in the level of economic activity away from the trend. In a multivariate framework with a three-phase characterisation of the business cycle, Kim and Murray (2002) found that a solid majority of the observed variance of a monthly indicator of activity in the United States could be accounted for by the transitory component.

In the 1990's, interest was revived in a model proposed earlier in the 1960's known as Friedman's Plucking Model (FPM) (Friedman, 1969, 1993). This model postulates the existence of a time varying ceiling over the rate of output which would be achieved by current output when resources have been organised and are being used in the most efficient manner. The model allows temporary "plucks" downward away from the ceiling, representing a recessionary period, followed by a recovery phase in which output reverts back towards the ceiling. Analogous models of other macroeconomic variables such as unemployment can be specified. In that example the unemployment rate is postulated to travel along a floor level when unemployment is at equilibrium, with temporary upward spikes in the rate during recessions. The salient feature of this model in the context of this paper is that it favours an asymmetric transitory shock over a symmetric cyclical component. Kim and Nelson (1999) specified a version of FPM which allowed both symmetric and asymmetric fluctuations away from a stochastic trend in both GDP and the unemployment rate and found evidence in favour of the plucking model over symmetric fluctuations. Their model incorporated Markov switching between normal and recessionary regimes and the recessions identified by the model were found to corresponded with NBER reference cycles.

The main contribution of this paper is the specification of a version of the FPM appropriate for a small, open economy using an exogenous recession indicator, which is applied to Australian macroeconomic data. The model is similar to the Kim and Nelson (1999) version of the FPM except that an exogenous indicator replaces the endogenous identification of recessions using a Markov-switching model. Furthermore, the model is specified in a multivariate framework using the unemployment rate as well as components of GDP, which differentiates the approach from that taken in more typical univariate analyses. Insights are given into the channels by which recessionary shocks are transmitted into Australia by examining consumption, investment and net exports rather than just aggregate GDP. Changes in aggregate GDP would otherwise reveal only the net effect of changes in investment and net exports, both of which are volatile but tend to move in opposite directions in an economic downturn.

The empirical findings are significant evidence of asymmetric permanent responses in consumption and unemployment. There is significant evidence of an asymmetric transitory response in investment, and weaker evidence for it in net exports and unemployment. The evidence suggests that there is also a symmetric cyclical component with significant variance for all of the series except consumption. In summary, there is significant evidence of asymmetry but not in the form suggested by the FPM.

The rest of the paper is organised as follows. A review of previous research into business cycle asymmetry is given in Section 4.2 including a look into the nature of Australian responses to global economic downturns. In Section 4.3 a model is specified which allows for the presence of both symmetric and asymmetric cyclical components. The exogenous recession indicator can generate asymmetric permanent responses in the stochastic trend as well as asymmetric transitory changes in the mean of the cyclical component. In Section 4.4 the empirical results are presented and examined with particular emphasis on the significance of the coefficient estimates which would indicate the presence of asymmetry, and which distinguish between permanent and transitory effects. The contribution of each of consumption, investment and net exports to the business cycle in aggregate GDP are also examined, along with an economic interpretation of the findings. Section 4.5 concludes.

4.2 Asymmetric business cycles in the literature

4.2.1 Previous empirical studies

Most empirical studies of business cycle asymmetry have focussed on United States data. Neftçi (1984) found evidence of asymmetry in the business cycle represented by the unemployment rate based on its observed tendency to exhibit sudden jumps in recessions and slower declines during normal times. The two regimes were identified by rising and falling rates respectively and the state indicator modelled as a Markov process. The estimated regime transition probabilities were used to construct empirical tests for the existence of asymmetry. DeLong and Summers (1984) used the skewness of the distribution of growth rates to characterise the series as asymmetric or otherwise, but could not find evidence to support the hypothesis that contractions were shorter and sharper than expansions, although they did find some support for asymmetry in the behaviour of unemployment. Falk (1986) also could not find evidence to support the hypothesis of asymmetry in GNP, investment or productivity. Hamilton (1989) provided strong evidence that the dynamic behaviour of GNP was markedly different in the two phases of economic activity. The discrete-state Markov process was demonstrated to provide a better characterisation of the process than could be achieved using a linear autoregressive model, and the framework became a base for much subsequent research into asymmetric business cycles.

The characterisation of shocks as permanent or temporary for particular samples of data can be materially affected by the modelling approach. Clark (1987) found evidence of a large and significant cyclical component in GNP using an uncorrelated unobserved components model, whereas Campbell and Mankiw (1987) found that shocks were largely permanent using an ARMA framework. Morley et al. (2003) found that if correlation between trend and cycle innovations in an unobserved components model (with a symmetric cycle) was freely estimated then it also provided evidence that shocks were mostly permanent. They showed that there is a direct equivalence between the correlated UC model and the Beveridge and Nelson (1981) decomposition, which was already known to attribute almost all of the variance in GDP to the trend component. However, Sinclair (2010) showed that ignoring asymmetry in the model underestimated the role of temporary movements in GDP.

Findings relating to asymmetry will be similarly qualified by the modelling approach, particularly since there are many plausible forms which the asymmetry might take, including Keynes (1936) notion of sharp turning points at the end of recessions, the plucking model (Friedman, 1993), the relationship between the size of contractions relative to preceding expansions, and vice versa (Goodwin & Sweeney, 1993), deepness and steepness in the cycle (Sichel, 1993) and asymmetry in the cycle frequency in different phases (Koopman & Lee, 2005). More than two phases of the cycle are sometimes modelled explicitly, or may be implied by the estimated dynamics, examples of which include a model of a "bounce-back" period after recessions (Kim et al., 2005), and a three-phase model of the business cycle by Sichel (1994). Morley and Piger (2012) compare the performance of a wide set of different models of post-war United States GDP. The set includes linear autoregressive and unobserved components models, and non-linear variations of them which have been augmented to model recessions which could be "L-shaped", "U-shaped", "V-shaped" or which have recovery strength based on recession depth. The authors find empirical support for a non-linear model incorporating asymmetry in the cyclical component, and find that recessions can be characterised as periods of large negative transitory fluctuations in output. However, they acknowledge several close competitors in the model selection process from amongst the linear and non-linear alternatives and suggested that a model averaging process could be appropriate.

There are a small number of papers which apply versions of the FPM to international output data beyond the United States. Mills and Wang (2002) find support for the model in a number of G-7 countries using data from 1950-1999 (the period varies by country), in terms of asymmetric transitory shocks in recessions and largely permanent shocks during normal times. They find mixed results for the existence of the 'ceiling' in output implied by the model. Nadal De Simone and Clarke (2007) find less robust evidence in support of the FPM over an alternative symmetric model for a selection of 12 economies during 1970-2000 (varies by country), although they also find that negative shocks are largely transitory while positive shocks are mostly permanent.

4.2.2 Recessions in a small open economy

There have been numerous studies which have documented the significant effect that shocks to global demand have on the Australian economy. As a small economy it is usually assumed that the level of activity in Australia does not have a corresponding impact on the level of global activity, which is treated as an exogenous variable. Dungey and Pagan (2000, 2009) used a SVAR model with an exogenous block of foreign variables to allow the estimation of responses of domestic variables to foreign shocks. Other studies finding a significant role for foreign demand shocks in include Leu (2011), Buncic and Melecky (2008), Nimark (2009) and Jääskelä and Nimark (2011). The China driven resources boom and the associated increase in commodity prices which commenced in about 2003 is a recent example of how foreign activity can drive the Australian business cycle. The impact of the boom has been observed well beyond the direct impact on the resources sector due in part to indirect exchange rate effects. The dramatic and sustained improvement in Australia's terms of trade during the China boom has supported higher incomes and, in turn, growth in consumption. Negative shocks have also been easily transmitted into Australia, due in part to its reliance on foreign investment and sensitivity to the global level of aggregate demand. Significant domestic responses have been observed to events including the 2008 Global Financial Crisis, the bursting of the internet bubble in 2000 (Garnaut, 2012; Makin, 2010; Rees, Smith & Hall, 2016) and, to a lesser extent, the 1997 Asian financial crisis (Duncan & Yang, 2000; Makin, 1999). Prominent endogenous cycles have also been observed and it has been argued that higher levels of household debt have created higher sensitivity of consumption to fluctuations in interest rates, income and housing prices (Debelle, 2004; Macfarlane, 2003).

Tyers (2014) sets out a theoretical framework for asymmetry in the responses of the Australian economy to boom-bust shocks in the resources sector. The sector may be a minor contributor to GDP in normal years, but it can have a large effect on the broader economy through its effect on the real exchange rate. A commodity resource boom has typically been associated with an increase in the real value of the Australian dollar and its terms of trade. The overall effects of these increases tend to be positive but there are negative impacts for non-resource exporters and also import-competing industries. Sustained changes in relative profitability may lead to reallocation of resources between different sectors of production. The adverse effects of real exchange rate appreciation felt in non-resource production sectors (the so called Dutch Disease) may not be fully reversible if and when the boom turns to bust, leading to asymmetry between the expansion and contraction phases (Corden, 2012; Tyers, 2014). Garnaut (2012) argues that wide cyclical fluctuations exist in minerals and energy prices due to the long lead times between investment and actual production. Since long range forecasting of the level of demand can be highly inaccurate, this can lead to extended periods of under-supply or over-supply. Asymmetric cyclical responses can arise after sharp spikes in demand or investment if they are followed by a much longer period during which actual production comes into line with recurrent demand. Tyers (2014) argues for a form of asymmetry in which booms have a proportionately larger effect on performance than busts and that busts do not place all the gains from a boom at risk, since not all boom shocks are reversible. If this is correct then in the context of the models considered in this paper it would be anticipated that the influence of the China growth boom would be found mostly in the permanent trend, whereas the corresponding decline would have a mostly transitory impact.

There have been few papers which examine Australian data using non-linear models similar to the form considered in this paper. Kim et al. (2005) specify a model with Markov-switching regimes and which feature a "bounce-back" period of higher than normal growth in the immediate aftermath of a recession. If the model fits the data then this would also imply a reduction in the permanent impact of the recession compared to a model without the bounce-back feature. The authors found relatively large bounce-back effects in Australia and the United States and a small permanent effect, but the same result did not hold for Canada and the United Kingdom, using data from 1973-2003. Nadal De Simone and Clarke (2007) found evidence of asymmetry in the depth of recessions for Australia but did not find significant evidence against an alternative hypothesis of a symmetric model. Using non-parametric methods, Razzak (2001) found evidence of deepness but not steepness for Australian recessions.

4.3 Empirical model

4.3.1 Features of the macroeconomic data

A preview of the time series for Australian GDP and the unemployment rate is provided in Figure 4.1. For the purpose of illustration only, the log GDP series shown is the residual after removing a linear time trend equivalent to the average growth rate of about 3.2%p.a. If the FPM was a good model of the true data generation process then the plot of GDP would exhibit downward plucks away from a ceiling level during recessions, and return to it afterwards. At first sight, the theoretical model may be a reasonable match for GDP for the first half of the sample period incorporating the 1982 and 1991 recessions, but the pattern is less clear for the second half. There is a visible decline in the growth rate after 2008, and again after 2012 which is sustained for the remainder of the period. It is ultimately an empirical question whether this decline is better characterised as a transitory downturn or a permanent change to the slope of the trend. It is also notable that Australian output did not suffer the same deep trough during the 2008 Global Financial Crisis that was seen in most industrialised countries. This was due in part to ongoing links to the Chinese economy, but due also to a sudden boost to net exports after the onset of the crisis. Prior levels of excess demand had been feeding into higher imports, but that stopped abruptly at the end of 2008, and exports were helped by a large fall in the value of the Australian dollar against the US dollar (the significant positive spike in net exports during the financial crisis can be seen easily in later Figure 4.3).

The unemployment rate shows the anticipated sharper spikes associated with economic downturns and slower, asymmetric recovery towards a normal rate, but it appears that the floor for the rate may vary significantly over time. The model used in this paper will allow a stochastic trend and cycle with both potentially affected by the recession indicator.

4.3.2 Exogenous recession indicator

One typical definition of a recession is two consecutive quarters of negative growth in real GDP. This is not very useful in the context of this paper since there have been no such observations in Australia since June 1991. Instead, possible exogenous indicators of significant economic downturns will be considered without reference to

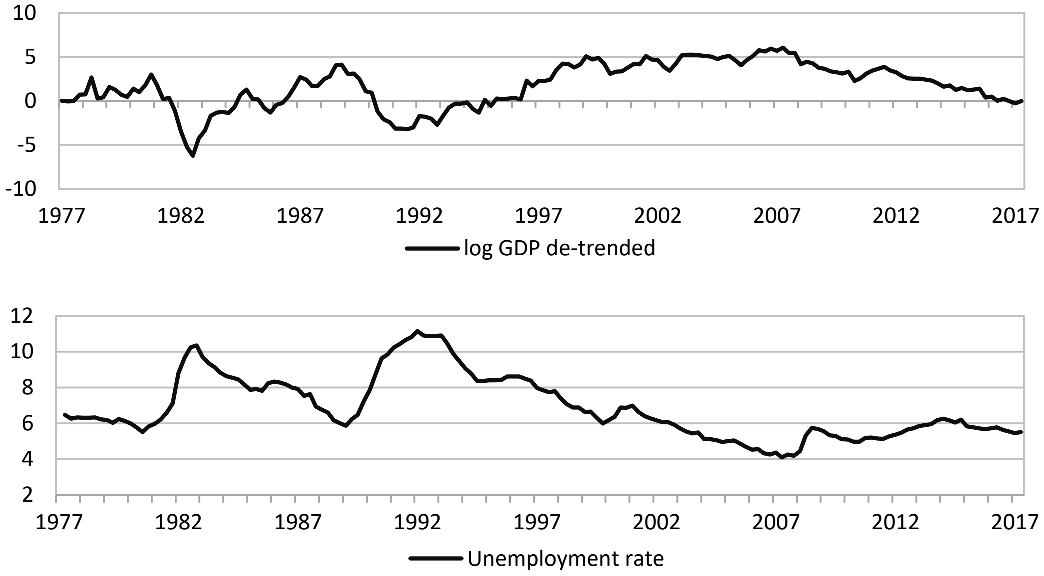

Figure 4.1: **Log GDP and the unemployment rate (Australia).** Log GDP has been linearly detrended (for the graph only).

whether the conventional definition of a recession is satisfied or not[1]. In Figure 4.2, linearly de-trended log GDP for Australia is plotted against possible indicators of a global recession. The NBER recession indicator for the United States is considered a reasonable proxy for a global economic downturn due to the large share of global output contributed by the United States historically and to its broader influence on global activity. Two other indicators are constructed from the OECD composite leading indicators (CLI) for the United States and the OECD group of countries[2] labelled RECUSA and RECOECD respectively.

The CLI is designed to provide early signals of turning points in business cycles showing fluctuation of the economic activity around its long term potential level and has been scaled to have a long term average of 100. In this illustration a threshold for the index has been determined to capture the periods corresponding approximately with the bottom quartile of levels of the index for the USA and the

[1]For ease of terminology the economic downturns indicated will be referred to simply as "recessions".

[2]A description of the OECD Composite Leading Indicator and time series data can be found at https://data.oecd.org/leadind/composite-leading-indicator-cli.htm.

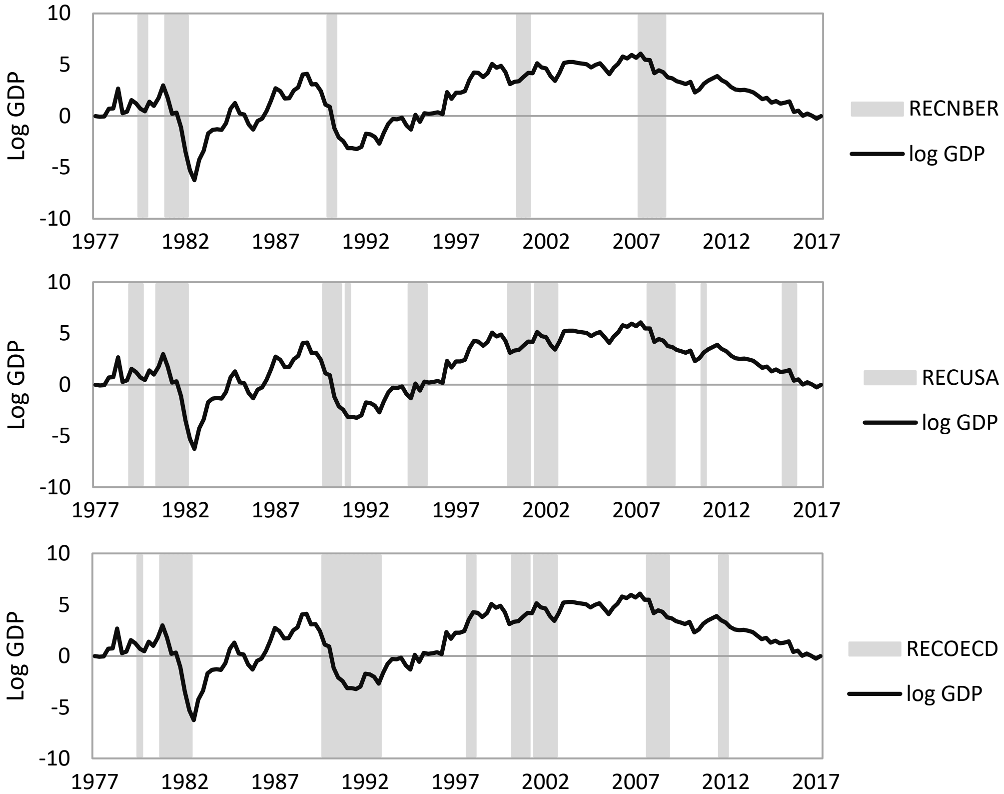

Figure 4.2: **Log GDP Australia (de-trended) vs. exogenous recession indicators.** The recession indicators take zero or one values and are represented by the shaded periods in the figure.

OECD respectively[3]. The periods during which the index is below the threshold are dubbed "recessions". The figure shows that all of the series do a reasonable job of indicating periods of low growth rates in Australian GDP. There is an obvious trade-off to be made between a more sensitive indicator which captures smaller and more frequent global shocks and the number of false positive indications of a recession that it generates. Preliminary analysis suggests that RECOECD is the most useful of the shortlist of three for the empirical analysis in this paper in terms of goodness of fit. The robustness of results to the choice of indicator is considered in Appendix 4.C.

[3]Thresholds of 99.2 and 99.5 for the USA and OECD respectively cut off approximately the bottom quartile of months by level of CLI for the sample period December 1977 to March 2018. The snapshot of the last month in each calendar quarter was used to create a quarterly time series.

4.3.3 GDP components and unemployment

The components of GDP under the expenditure method are consumption, investment and net exports[4]. It will be shown that the three components do not share the same business cycle characteristics, so rather than performing the empirical analysis on the aggregate GDP series, the three components are individually represented in a multivariate system. The unemployment rate is also included in the joint framework so that any asymmetric behaviour in this key cyclical indicator of the labour market can be revealed. Plots of each of these components and the unemployment rate are shown in Figure 4.3 against the RECOECD recession indicator. Prima facie, investment appears to be procyclical with economic activity since it tends to fall during recessions, as anticipated. The exogenous recession indicator appears to provide a reasonable indication of the timing of periods of lower growth rates in investment. Consumption exhibits less prominent business cycle characteristics than other components of GDP, in particular compared to investment (Rees et al., 2016, p. 391), which would be expected in the household component if there were smoothing behaviour as anticipated under the permanent income hypothesis (Friedman, 1957). The unemployment rate appears to be countercyclical as anticipated. Net exports tends to rise in an economic downturn, softening the impact on GDP of the fall in consumption and investment. Increased demand for imported goods tends to accompany periods of rapid growth in domestic demand due in part to Australia's small manufacturing sector. In an economic downturn, especially one accompanying a decline in the resources sector, the fall in demand for imports can be reinforced by the depreciation of the real exchange rate. There will be some substitution of domestically produced goods for previously imported goods. The lower exchange rate may also boost exports. Cole and Nightingale (2016) have estimated that a 10% depreciation of the exchange rate is associated with an increase over two

[4]In this representation of GDP, household and government consumption have been combined, and changes to inventories have been ignored on the basis of their relatively small size.

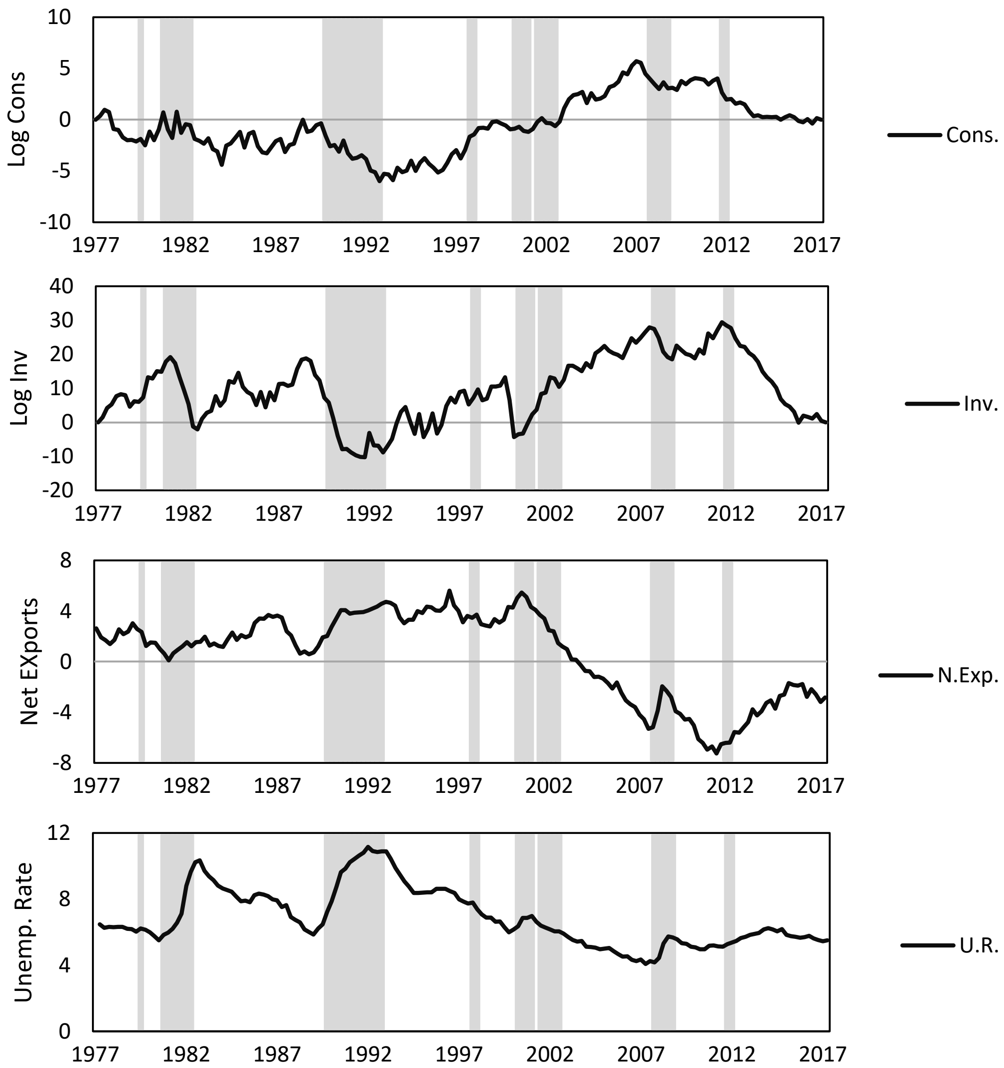

Figure 4.3: **GDP components and unemployment rate.** Log consumption and log investment have been linearly de-trended for display only. Net Exports is the difference between real dollar values of exports and imports and is expressed as percent of GNE. The shaded areas represent the recession indicator RECOECD.

years in exports of 3% and a decrease in imports of about 4%, with a cumulative contribution of about 1.5% to GDP over the period.

The following four aggregate variables are used in the model, which is estimated in levels, not differences: c_t is quarterly log consumption times 100, k_t is log of investment flow times 100 (gross fixed capital formation across all sectors), e_t is net exports expressed as percent[5] of gross national expenditure (GNE) and u_t is

[5]Since net exports can be negative we can not work with the log level as for the other series.

the unemployment rate expressed in percent. All of the series have been seasonally adjusted. The expenditure components are based on chain volume measures so log differences can be interpreted as real growth rates. The full sample period is December 1977 to March 2018, but the unemployment series only starts in February 1978. A quarterly series of unemployment is extracted from the monthly data. The estimation period will be 1978q2-2018q1. See Appendix 4.A for a more complete description of the data and its sources.

4.3.4 Multivariate FPM with an exogenous recession indicator

The model is similar to the version of FPM used by Kim and Nelson (1999). Let $\boldsymbol{x_t} = (x_{1t}, x_{2t}, \ldots, x_{Nt})'$ represent an $N \times 1$ vector of time series with observations ranging from $1, \ldots, t$. In this application $\boldsymbol{x_t} = (c_t, k_t, e_t, u_t)'$. We specify a basic multivariate form of an unobserved components model developed by Harvey and Koopman (1997), adapted to allow an exogenous indicator of recession to affect both the permanent trend and the mean of the transitory cycle. The first equation is the measurement equation which decomposes each element of $\boldsymbol{x_t}$ into trend, cycle and irregular components:

$$x_{it} = \tau_{it} + \psi_{it} + \delta_i D_{it} + \varepsilon_{it}, \quad \varepsilon_{it} \sim n.i.d.(0, \sigma^2_{\varepsilon i}), \tag{4.1}$$

where τ_{it} is a non-stationary trend and ψ_{it} is a stationary cycle. $\boldsymbol{D_t}$ is a vector of dummy variables which plays a minor role in this model to assist in removing the impact of some observation outliers, and is discussed further in a following section. The evolution of the trend is governed by the state equations:

$$\begin{aligned} \tau_{it} &= \tau_{i,t-1} + \beta_{i,t-1} + \theta_i S_t + \eta_{it}, && \eta_{it} \sim n.i.d.(0, \sigma^2_{\eta i}), \\ \beta_{it} &= \beta_{i,t-1} + r_i R_t + \zeta_{it}, && \zeta_{it} \sim n.i.d.(0, \sigma^2_{\zeta i}). \end{aligned} \tag{4.2}$$

In the most general form this local linear trend specification allows the level and slope of the trend to show random variation. The specification can incorporate a local level model (LOCL, also known as random walk with drift) by restricting the slope innovation variance $\sigma^2_{\zeta i} = 0$, or an integrated random walk model (IRW, smooth trend model) by restricting the level innovation variance $\sigma^2_{\eta i} = 0$. S_t is an exogenous variable structured as an indicator, with $S_t = 1$ indicating that economic activity is in the recessionary phase, and $S_t = 0$ otherwise. A separate loading θ_i is estimated to capture the permanent effect on each series. R_t is dummy variable which allows for a possible permanent structural change in the slope of the trend in 2012, discussed further in a following section.

A trigonometric stochastic cycle is used for the cyclical components. In the following, ψ_{it} is the cyclical component while ψ^*_{it} is used only for construction, using a pair of mutually uncorrelated white noise disturbance terms κ_{it} and κ^*_{it} (this form of cycle was introduced by Harvey (1985)). The mean of the cyclical component for series i will be temporarily shifted by π_i whilst $S_t = 1$.

$$\begin{bmatrix} \psi_{it} \\ \psi^*_{it} \end{bmatrix} = \rho_i \begin{bmatrix} \cos(\lambda_i) & \sin(\lambda_i) \\ -\sin(\lambda_i) & \cos(\lambda_i) \end{bmatrix} \begin{bmatrix} \psi_{i,t-1} \\ \psi^*_{i,t-1} \end{bmatrix} + \begin{bmatrix} \kappa_{it} \\ \kappa^*_{it} \end{bmatrix} + \pi_i \begin{bmatrix} S_{it} \\ S_{it} \end{bmatrix}, \quad (4.3)$$
$$\kappa_{it}, \kappa^*_{it} \sim n.i.d.(0, \sigma^2_{\kappa i}).$$

It is typical to use the similar cycles model of Harvey and Koopman (1997, p. 272) in multivariate business cycle analysis, which restricts the damping factor and cycle frequency to be the same for all cycles. However, the restriction is not applied in this paper, to allow for potential idiosyncratic behaviour amongst the cycles. The model can be used to estimate $\rho_i \in (0, 1)$ and $\lambda_i \in (0, \pi)$ for all $i \in [1, N]$. The central frequency of the relevant cycle is λ_i (with cycle period $= 2\pi/\lambda_i$). The restricted range for ρ_i ensures that the cycle will be stationary. Kim and Nelson (1999) used an autoregressive form for the cyclical component but in the multivariate setting the stochastic cycle model allowed easier estimation and interpretation of the cycle period across the series. The loading π_i is an estimate of the transitory asymmetric effect of the recession indicator.

A literal interpretation of the FPM would require $\pi_i < 0$ (for a procyclical variable), $\theta_i = 0$ and $\sigma^2_{\kappa i} = 0$, i.e. there is a negative and purely transitory shock to the cyclical component during a recession, which is subsequently reversed after the recession. $\sigma^2_{\kappa i} = 0$ means that shocks to the symmetric part of the cycle play no role in the dynamics. That part of the cycle is referred to as symmetric because there is explicitly no difference between responses to positive and negative shocks when $\sigma^2_{\kappa i} > 0$, and fluctuations can be generated both above and below the trend. In direct contrast to FPM, if $\pi_i = 0$ and $\sigma^2_{\kappa i} > 0$ then there is no transitory asymmetric shock and all of the autoregressive behaviour can be explained by the symmetric cycle[6]. The parameter θ_i allows us to test for an additional form of asymmetric permanent response to a recession not explicitly contemplated under FPM. The model also allows stochastic influences on the trend quite separate to the recessionary responses, and it would be anticipated that at least one of $\sigma^2_{\eta i}$ or $\sigma^2_{\zeta i}$ is greater than zero.

Finally, if there is a true underlying business cycle factor which affects all the series then it might be expected that the cyclical innovations are correlated across equations. The cyclical innovations κ_{it} are assumed to be multivariate normal with covariance matrix

$$\Sigma = \begin{bmatrix} \sigma^2_{\kappa 1} & \sigma_{\kappa 12} & \sigma_{\kappa 13} & \sigma_{\kappa 14} \\ \sigma_{\kappa 12} & \sigma^2_{\kappa 2} & \sigma_{\kappa 23} & \sigma_{\kappa 24} \\ \sigma_{\kappa 13} & \sigma_{\kappa 23} & \sigma^2_{\kappa 3} & \sigma_{\kappa 34} \\ \sigma_{\kappa 14} & \sigma_{\kappa 24} & \sigma_{\kappa 34} & \sigma^2_{\kappa 4} \end{bmatrix}.$$

Foreshadowing later results, none of the covariances with the consumption cyclical innovation κ_1 were significant so the covariance parameters $\sigma_{\kappa 1j}$ were restricted to zero for all j. In the results section, correlation coefficients are reported with

[6]Kim and Nelson (1999) also allowed the variance $\sigma^2_{\kappa i}$ to have different values in the normal and recessionary regimes, but that feature has not been implemented here.

the notation $r_{ij} = \sigma_{\kappa ij}/\sigma_{\kappa i}\sigma_{\kappa j}$. In principle, a similar covariance matrix could be specified for a cross equation relationship between the irregular innovations[7]. As it happens, none of the series was found to have a significant irregular component in the uncorrelated framework, so correlated irregular components were not pursued any further. Correlation between trend and cycle innovations has been found to be useful in several univariate macroeconomic studies (such as Morley et al. (2003) and Proietti (2006)). Preliminary analysis for this research suggested that it would not be possible to identify more parameters additional to those already specified in the multivariate model, so an orthogonality restriction was imposed between state innovations. For a detailed explanation of the identification issues see Appendix 5.B in Chapter 5.

The main differences of the model in this paper with the Kim and Nelson (1999) model is that the latter allowed the state indicator variable to evolve as a first-order Markov-switching process. Kim and Nelson applied a univariate model to US GDP whereas a multivariate model is applied in this paper. One of the measures of success for Kim and Nelson was that they found a plausible relationship between the two states identified in that model and the normal and recession states identified by the NBER. In this paper the indicator has been selected to be a plausible global recession indicator, so the more relevant measure of success will be whether the model achieves a materially better statistical fit for the data with the asymmetrical components included compared to a base model without them.

4.3.5 Outliers and structural change

A small number of observations considered to be outliers were managed with dummy variables in $\boldsymbol{D_t}$ in Equation 4.1. Outliers may otherwise interfere with the identification of the cyclical components and have an impact on the normality of the

[7] If there was correlation between the irregular innovations (ε_{it}) and that was the only cross equation relationship then the model would be equivalent to a system of seemingly unrelated time series equations (Commandeur & Koopman, 2007, p. 111).

prediction errors. A dummy variable for the one-off introduction of a goods and services tax in Australia in July 2000, lagged by one period, was used for the investment series. Another was used to deal with an unexplained positive spike in consumption in 1982q2, near the start of the 1982-1983 recession. Finally, a dummy variable was used to indicate an irregular positive spike in exports in 1997q2, associated with the export of a frigate and gold sales by the Reserve Bank of Australia during that quarter[8].

It has been observed by the World Bank (2018, p. 159) that the growth rate of global potential output has been markedly lower in the current decade compared to its long-term average, due in part to lower productivity growth and demographic trends. All of the components of Australian GDP shown in Figure 4.3 show marked turning points in 2012, around the time of the peak in the European sovereign debt crisis. Lower growth rates of output have been sustained since that time but insufficient time has elapsed to determine conclusively whether there has been a structural shift to an environment with lower growth than historic norms, or whether the change is at least partly cyclical. A slope dummy variable R_t in Equation 4.2 (set to one for 2012q1) was used to allow for a possible structural shift to lower growth rates in economic activity around that time (if the coefficient of $R_t \neq 0$ it will have the effect of a permanent slope change). It will be determined empirically for each series whether the slope dummy is useful in explaining the behaviour beyond that which can be explained by a the stochastic trend and cyclical components.

4.3.6 Choosing model components

Analysis of each of the four time series using a univariate equivalent of the model described in Equations 4.1 to 4.3 was used in a preliminary stage to help identify the trend and cycle components which provided the best fit to the data. A number of factors were considered including the goodness of fit according to the Akaike in-

[8]Refer to the June Quarter 1997 release of the National Expenditure, Income and Product report of the Australian Bureau of Statistics, Catalogue 5260, page 11.

formation criterion and statistically significant individual variances for the stochastic components. For example, none of the series could support significant variances for both trend and slope, so a zero restriction was applied to either $\sigma^2_{\eta i}$ or $\sigma^2_{\zeta i}$ in each case. Further, strong preference was given to solutions which generated satisfactory diagnostic test results for the absence of autocorrelation in the prediction residuals, to support the claim that the cyclical component has satisfactorily captured any autoregressive behaviour.

The investment series (k_t) was fitted best by including a local level trend and a cyclical component with a period estimated to be about 29 quarters (about 7 years). Consumption (c_t) was also fitted best using a local level trend without an irregular component. There was only weak evidence of a periodic cycle, although there is apparent autocorrelation in first differences. For the consumption series only, the measurement Equation 4.1 was augmented to include two terms in lagged first differences of c_t, lagged 4 and 8 periods respectively, to remove autocorrelation from prediction errors. Net exports (e_t) and the unemployment rate (u_t) were modelled by a smooth trend (integrated random walk) plus a cycle, with no irregular component. The slope dummy variable R_t was found to be significant at 1% for the trend of e_t, at 12% for k_t, and was insignificant for the other two variables. It was retained for k_t despite its low significance since it contributed to improved goodness of fit and diagnostic test results. The slope dummy coefficient was then restricted to zero for c_t and u_t.

4.3.7 Restrictions

Several parameter restrictions were applied in the multivariate framework to aid identification of the most important components of the model and to improve the precisions of estimates of the remaining parameters. There is only weak evidence of a business cycle in consumption which makes the cycle period difficult to estimate. The period was restricted to be the same as the period for the cycle in investment.

There is value in retaining the same trend and cycle structure for consumption as the other elements so that the same tests for asymmetric responses can be applied. Note that this restriction does not impose a large cycle onto consumption, since the cycle variance and persistence parameters are still freely estimated and, as anticipated, they turn out to be small and not significant. Preliminary testing of the multivariate model with uncorrelated innovations generated an estimated cycle period of about 28 periods for the net exports series. When correlation between cyclical innovations was introduced, it was difficult to obtain convergence with a meaningful cycle period for net exports, indicating that the likelihood function is probably quite flat in the local region. Accordingly, the cycle period for net exports was fixed at 28 periods when cyclical innovation correlations were freely estimated. As noted previously, the correlation between consumption cycle innovations and other cycles was restricted to zero. Finally, preliminary estimates indicated insignificant coefficients for the asymmetric transitory response of consumption (π_1) and for the asymmetric permanent responses of investment and net exports (θ_2 and θ_3), so zero restrictions were applied to those parameters.

4.3.8 Estimation method

The model specified in Section 4.3 was set out in state space form and estimated using a Kalman filter (see Appendix 5.A in Chapter 5 for a description of the state space form). The Kalman filter is a recursive procedure which can be used to find maximum likelihood estimates of the model parameters. A smoothing procedure can be applied to the filtered series using all of the information in the sample period to generate so called smoothed estimates of the unobserved components including the estimated cycles shown in this paper. For a description of filtering and smoothing algorithms see Commandeur and Koopman (2007, pp. 84–89). The model was estimated using EViews 10.

4.4 Results

4.4.1 Parameter estimates

Many of the model parameters are transformed before estimation to ensure that the estimate will lie within a permissible range after the transformation has been reversed. Variances are estimated as the square of a coefficient to ensure the result is non-negative. Cycle frequencies (λ_i) are estimated within a boxed range using a logistic function to ensure the estimated cycle period will lie within a plausible range for a business cycle. The damping coefficients (ρ_i) are boxed within (0, 1) to ensure that the cyclical component will be stationary, and estimated correlation coefficients are boxed within (-1, 1). Parameter estimates in transformed and natural terms (by reversing the transformation) are given in Table 4.1. The results in Table 4.1 show that both investment and net exports have significant symmetrical cyclical components since $\sigma^2_{\kappa 2}$ and $\sigma^2_{\kappa 3}$ are significantly greater than zero, and both show a high level of persistence ($\rho_2 = 0.86$ and $\rho_3 = 0.85$ respectively). As anticipated, the consumption cycle does not have significant variance or persistence. Arguably, the cyclical component would be better described as capturing some short term autoregressive behaviour in consumption rather than a business cycle. The unemployment rate also shows a symmetrical cyclical component with significant variance, with a period of approximately five and half years (22.7 quarters). Cyclical shocks are more persistent in unemployment than any of the other series since the estimate of ρ_4 is highest of all the series. The signs of the estimated cross-series correlation coefficients between the cyclical innovations have the expected signs since the investment cycle is anticipated to be negatively correlated with both net exports and the unemployment rate. Two of the correlations are significant at 10% and the other at 22%, but all three were retained in the model due to their overall contribution to model fit.

Table 4.1: **Estimation results**

	Param.	coef.	std. err.	p.val.	nat-ural coef.	Q(4) (pval)	Q(8) (pval)	Q(20) (pval)	H(48) (pval)	JB (pval)
c_t	λ_1	-2.729	0.634		0.219	1.25	6.34	22.21	2.42	0.41
	$period_1$				28.67	(0.741)	(0.501)	(0.274)	(0.002)	(0.813)
	ρ_1	-4.103	111.87		0.213					
	$\sigma^2_{\xi 1}$	0.180	0.280	0.521	0.032					
	$\sigma^2_{\eta 1}$	0.572	0.122	0.000	0.327					
	θ_1	-0.271	0.114	0.017	-0.271					
	π_1									
k_t	λ_2	-2.729	0.634		0.219	3.22	12.76	21.24	2.18	1.58
	$period_2$				28.67	(0.358)	(0.078)	(0.323)	(0.006)	(0.453)
	ρ_2	1.540	0.411		0.857					
	$\sigma^2_{\xi 2}$	1.775	0.724	0.014	3.149					
	$\sigma^2_{\eta 2}$	1.775	0.849	0.037	3.149					
	θ_2									
	π_2	-1.253	0.555	0.024	-1.253					
e_t	λ_3				0.224	1.95	6.85	15.18	1.49	5.43
	$period_3$				28.00	(0.583)	(0.444)	(0.711)	(0.154)	(0.066)
	ρ_3	1.499	0.334		0.852					
	$\sigma^2_{\xi 3}$	0.442	0.027	0.000	0.195					
	$\sigma^2_{\zeta 3}$	0.027	0.014	0.062	0.00072					
	θ_3									
	π_3	0.163	0.100	0.103	0.163					
u_t	λ_4	-2.185	0.205		0.276	3.39	5.35	11.66	2.57	194.94
	$period_4$				22.72	(0.335)	(0.618)	(0.900)	(0.001)	(0.000)
	ρ_4	2.426	0.456		0.933					
	$\sigma^2_{\xi 4}$	0.186	0.009	0.000	0.035					
	$\sigma^2_{\zeta 4}$	0.023	0.008	0.005	0.00051					
	θ_4	0.270	0.084	0.001	0.270					
	π_4	0.078	0.052	0.134	0.078					
correl.	r_{23}	-0.640	0.379	0.091	-0.309	logl	-738.577	AIC	9.5947	
	r_{24}	-0.419	0.345	0.224	-0.206	nobs.	160	iterat.	112	
	r_{34}	0.385	0.213	0.071	0.190	partial	7	status	Converg. ach.	

Notes: Q(n) is the test statistic for the null hypothesis of zero autocorrelation in one-step ahead prediction errors up to n lags. H(n) is the test statistic for homoscedasticity between the first and final one third (n) of the errors in the sample. JB is the Jarque-Bera test statistic for normality of the errors.

The parameters of most interest for this paper are θ_i and π_i which indicate the presence of permanent and transitory asymmetric responses respectively to the recession indicator. Consumption shows a permanent response of -0.27 to the indicator, significant at 5%. Given the structure of Equation 4.2 this is interpreted as a reduction in the slope of the trend in consumption by 0.27% for the duration of the recession. Investment shows an asymmetric transitory response of -1.25 significant at 5%. The transitory response is interpreted as a reduction in the mean of the cyclical component for the duration of the recession. Net exports show only a transitory asymmetric response of +0.16, not quite significant at 10%. Finally, the unemployment rate also shows a highly significant permanent response of +0.27

and a less significant transitory response of +0.08. Graphs of the smoothed cyclical components of each series are shown in Figure 4.4, which display a higher tendency for sharp downward spikes in investment, and sharp upward spikes in net exports and unemployment. Smoothed estimates of the unobserved trends, cycles and the original series are shown in Figure 4.6 in Appendix 4.B.

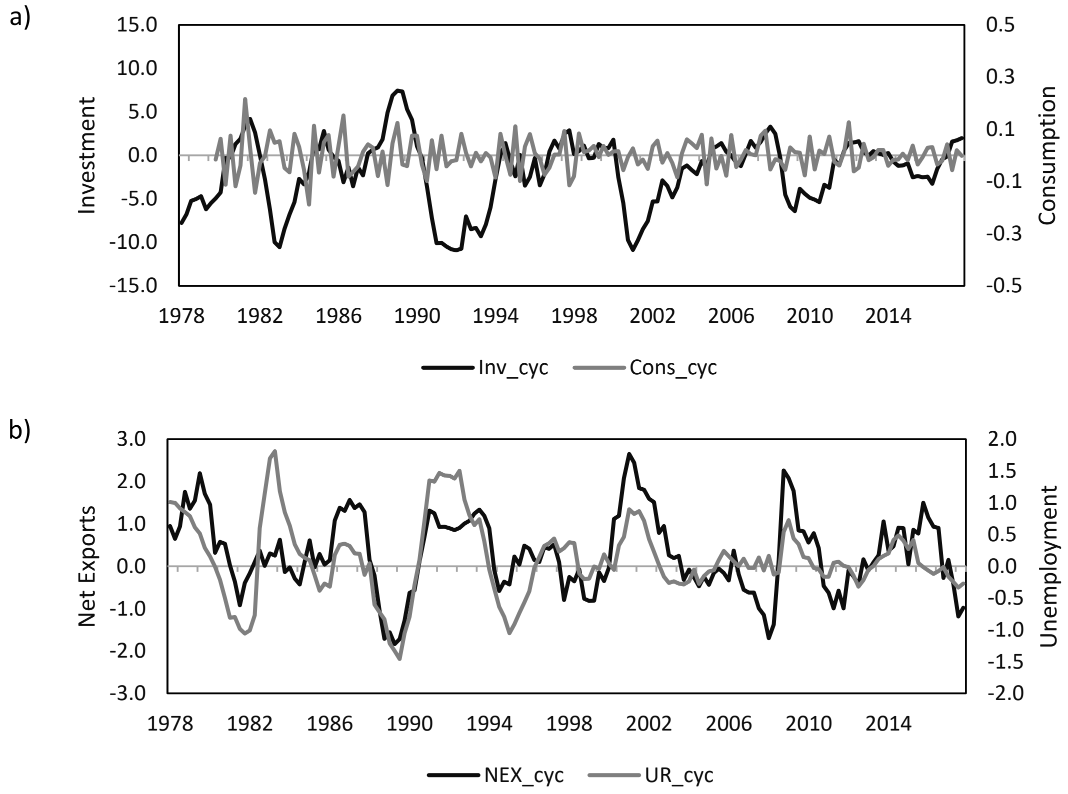

Figure 4.4: **Estimated cyclical components.** a) Investment and Consumption ($100 \times log$). b) Net exports (%GNE) and unemployment rate.

4.4.2 Economic interpretation

The results must be interpreted in the context of the types of asymmetric models described in Section 4.2. The consumption series does not fit the FPM. The recession has what can be described as an "L-shaped" impact on log consumption. The growth rate is reduced for the duration of the recession, and the growth rate increases at the end of it, but there is no bounce back in the *level* of consumption.

The level of consumption is strongly characterised as a random walk with positive drift and low variance, which is subject to the occasional impact of the exogenous recession indicator. There is significant evidence of an asymmetric cyclical pluck to the downside for investment. The transitory nature of the cycle means that this impact will ultimately be erased after the recession. However, it must be observed that investment also has a highly significant *symmetric* cycle, which is not consistent with the notion of a"ceiling" and plucks occurring only in one direction away from that ceiling.

Net exports also has a transitory asymmetric response (p. value of 0.103), but it also has a significant symmetrical cycle (since $\sigma^2_{\kappa 3}$ is highly significant). The positive asymmetric response to a recession reflects a historical association between recessions and a real depreciation of the Australian dollar, which has lead to declining imports and increased exports. It might have been anticipated that net exports would not show any permanent effect from the recession since any initial impacts arising from changes to the real exchange rate can be eroded over time as resources are reallocated towards other industries more favourably impacted by the exchange rate. Finally, the unemployment rate shows significant asymmetric positive spikes during recessions against a relatively smooth trend in normal times. There is some evidence of an asymmetric transitory response, but it is mostly permanent, and can be interpreted as evidence of hysteresis in the unemployment rate. The result does not require a literal interpretation that the permanent positive shock is never reversed. The evidence certainly supports the idea that unemployment tends to decline after a recession, but the characteristic is not well matched econometrically with a transitory cycle, even with a temporary mean-shift. The data supports the idea that the trend will become downward sloping again after the recession, but with no mechanism to suggest that it will necessarily revert to its prior floor level.

The findings that the asymmetric model features are significant and that they provide a substantial improvement in the fit of the model to the data supports the idea that there is a fundamental difference between the economic forces which are operating in recessions compared to normal times. Hamilton (2005, pp. 436–441) emphasises that if the business cycle characteristics of the data require a non-linear dynamic representation then the forces causing the fluctuations should be interpreted as being asymmetric. The results support a conclusion that there is a real phenomenon associated with recessions during which there is some type of short-run failure in the economy to make most efficient use of resources. The empirical results suggest that the short-run failures have a mostly temporary effect on investment and net exports, but that they also have a permanent effect on unemployment and consumption.

4.4.3 Contribution to GDP business cycle

The contribution that each of the key components makes to the cyclical behaviour of GDP can be determined by combining the estimated cyclical components (suitably scaled[9]), to give a proxy for the business cycle in GDP, and to reveal something about the mechanisms by which asymmetric shocks are transmitted into the aggregate measure of output. One of the reasons for retaining the same trend and cycle model for consumption, despite its weak cyclical behaviour, is that it is by far the largest component of GDP in Australia[10]. The amplitude of the cycles in investment and net exports are both large compared to GDP but tend to offset one another to a large degree, as shown in Figure 4.5. While investment and net exports exhibit sharp asymmetric spikes, asymmetry is much less apparent in the cumulative cycle. This suggests that an asymmetric business cycle in GDP would be less evident than the asymmetric cycles in its major components.

[9]The cycles estimated in the model for investment and consumption are for the log quantities and in the case of net exports, as percent of GNE. The cycles can be interpreted as a percentage deviation of the actual series away from its stochastic trend. For each series an equivalent cyclical component was derived expressed in the original terms of real dollars, which were then aggregated to provide an approximate cycle in GDP in real dollars.

[10]As at March 2018, final household consumption expenditure constituted 59% of GDP, general government final expenditure 19%, together 78% of GDP.

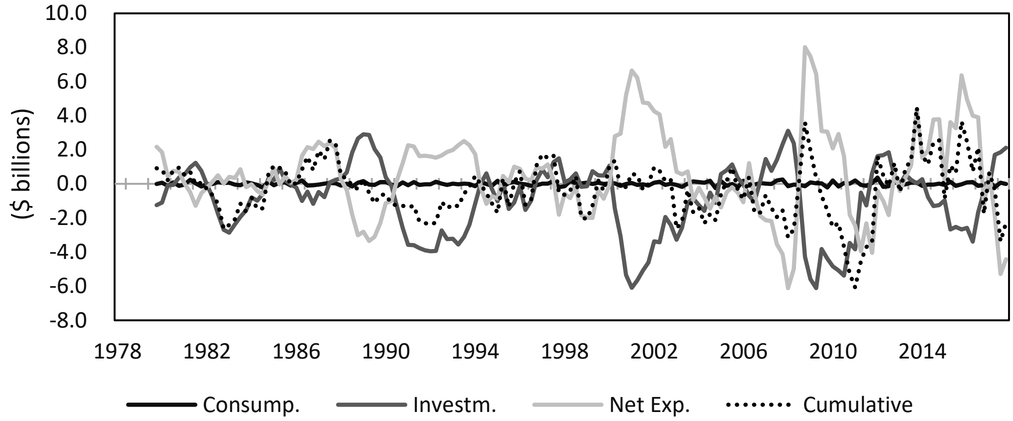

Figure 4.5: **Approximate contributions to the GDP business cycle.**

4.4.4 Diagnostic test results

The columns on the right side of Table 4.1 present diagnostic test results[11] of the one-step ahead prediction errors separately for each of the four series. The number of observations in the error series used in the tests is adjusted to exclude those corresponding to the number of diffuse initial states in the model and for lagged terms in dependent variables. The most important result is that there is no evidence of autocorrelation in the errors at lags of 4, 8 or 20 quarters. There is evidence of heteroscedasticity, most likely reflecting the clearly apparent decline in the volatility of macroeconomic time series after the 1991 recession, so the significance of individual parameter estimates should be treated with caution. The dummy variables were useful in generating plausibly normally distributed errors, except in the case of the unemployment series. Inspection of the unemployment error series reveals that this is due to a couple of outliers during the 1982 recession and in 2009 during the Global Financial Crisis, but it did not seem appropriate to dampen their effect using a dummy variable, given they both occurred during major recessions.

[11]The diagnostic tests follow the specification of Commandeur and Koopman (2007, pp. 90–93).

4.5 Conclusion

There is evidence of asymmetric responses to a recession in the unemployment rate and key components of GDP but not in the form contemplated by the Friedman Plucking Model. This conclusion follows from the finding that the symmetric part of the cycle, which generates deviations both upwards and downwards from the trend, plays a significant role in the dynamics for all of the series except consumption (which shows almost no evidence of a business cycle component). So whilst there is significant evidence of asymmetric transitory responses in some of the series, it cannot be held out as the dominant feature of the dynamics. There is also significant evidence of permanent asymmetric shifts in the trend growth rate of consumption and in the trend of the unemployment rate. The permanent asymmetric shocks contribute to L-shaped responses to recessions, rather than the one-sided plucks postulated by the FPM.

The small open economy of Australia exhibits sensitivity to downturns in global demand which have been modelled with an exogenous indicator of recession. The model with asymmetric features provided a materially better fit to the data than a model without the features, which suggests that the dynamics of the processes in recessions are not simply the mirror image of the corresponding processes in normal times. The finding supports a conclusion that recessions are a meaningful phenomenon and that modelling macroeconomic time series can benefit from a non-linear approach.

The business cycle components of investment and net exports tend to make contributions of opposite sign to the cyclical behaviour of GDP. Asymmetric downward spikes in investment may be offset by upward spikes in net exports. The irregular size and timing of responses to recessions in the components means that the GDP business cycle has exhibited less obvious asymmetry than its components.

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Appendix 4.A Data sources

Table 4.2: **Data Sources**

Series	Source	Notes
GDP (Expend. method)	ABS Cat. 5206 Tab.2 A2304402X	Quart., s.adj. chain vol. measure.
GNE[a]	ABS Cat. 5206 Tab.2 A2304113C	Quart., s.adj. chain vol. measure.
Consumption (All sectors;		Incl. household and general govt.
Final consumption expend.)	ABS Cat. 5206 Tab.2 A2304082X	Quart., s.adj. chain vol. measure.
Investment (All sectors;		
Gross fixed capital format.)	ABS Cat. 5206 Tab.2 A2304110W	Quart., s.adj. chain vol. measure.
Exports (goods and services)	ABS Cat. 5206 Tab.2 A2304114F	Quart., s.adj. chain vol. measure.
Imports (goods and services)	ABS Cat. 5206 Tab.2 A2304115J	Quart., s.adj. chain vol. measure.
Unemployment rate (all pers.)	ABS Cat. 6202 Tab.1 A84423050A	Monthly, s.adj. Quart. measure is average of 3 calendar months.
GST Dummy		2000q4=1 (zero otherwise).
Consumption Dummy		1982q2=1; Outlier.
Exports Dummy		1997q2=1; Outlier.
Slope Dummy		2012q1=1; Possible struct. break.
RECNBER	https://fred.stlouisfed.org	USRECQ, NBER recession indicator for the USA
RECUSA, RECOECD	https://data.oecd.org/leadind/composite-leading-indicator-cli.htm	Country groups USA and OECD respectively.

a GNE series used only as denominator of net export measure.

Appendix 4.B Estimated unobserved components

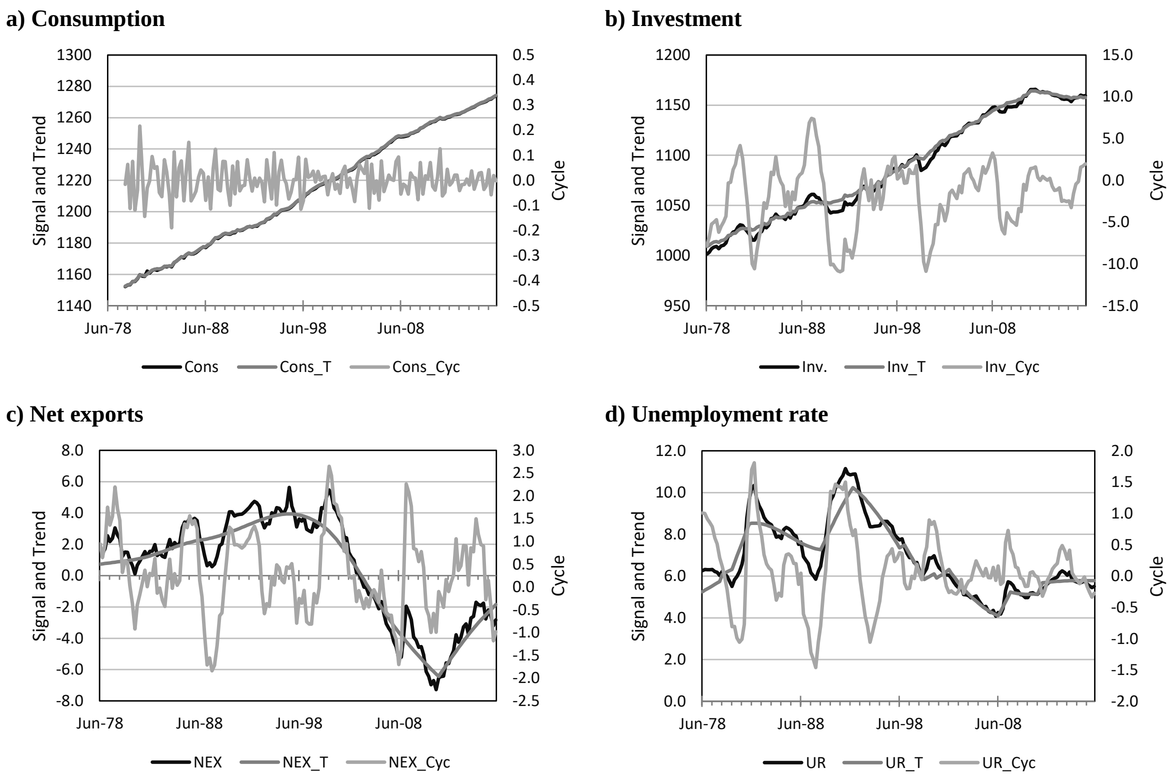

Figure 4.6: **Trend and cycle decomposition: unobserved components.** a) 100 × *log* Consumption; b) 100 × *log* Investment; c) Net Exports (%GNE), d) Unemployment rate.

Appendix 4.C Robustness

Figure 4.2 illustrates that there is some degree of synchronisation of economic activity in Australia with broader indicators of global activity but the question naturally arises whether the selection of the particular indicator RECOECD is a sensible choice, and how it affects the estimation. There can be no claim that it is the "best" possible choice, but the analysis given in Table 4.3 shows that it performs best by a goodness of fit measure from a shortlist of candidate indicators. RECOECD outperforms RECUSA and RECNBER, mostly likely because it better indicates the prolonged nature of the 1991 recession in Australia. In a similar vein it was considered whether the particular threshold (bottom quartile) used to construct the indicator has a material impact on the results. In the lower half of Table 4.3 the goodness of fit is shown for other possible constructions of the indicator from the same OECD leading indicator series but with different threshold values corresponding with various percentiles. Again, the RECOECD (bottom quartile) indicator provides a better fit than those constructed with different threshold values. Closer inspection of the estimated asymmetry parameters associated with different recession indicators revealed that they have the same sign as those estimated in the preferred model in almost every case.

The use of an exogenous indicator of recessions rather than the endogenous estimation from the Australian data may have introduced bias to the estimates of the asymmetry parameters, given the imperfect relationship between recessions in Australia and the rest of the world. Examples of endogenous identification of recessionary sates can be found in univariate models of asymmetry can be found in Morley and Piger (2012) and Morley and Panovska (2016). It has been left to future research to implement an endogenous recession identification regime within the multivariate framework used in this chapter.

Another consideration for robustness is whether the model components for trend, cycle and asymmetric recessionary responses have a meaningful economic interpretation. This cannot be answered with certainty so a guide is taken from the significance of coefficient estimates, and the contribution of key components of goodness of fit according to an information criterion. It has been shown previously in Table 4.1 that trend and cycle components retained in the model all have statistically significant variances, except for consumption. All of the coefficients which represent an asymmetric response have a correctly anticipated sign. Table 4.4 illustrates the contribution made to goodness of fit as the model specification adds complexity, starting with the simplest form equivalent to a set of univariate Hodrick-Prescott (HP) trend and cycle decompositions. Relaxing the HP variance ratio restriction makes the most substantial difference to the fit, even without the recession indicator. Further material improvements are generated by including the asymmetry feature in the model specification, and finally by allowing cross correlation between equations. It is this last feature which separates the analysis from that which could be achieved using univariate analysis.

Table 4.3: **Various recession indicators: Goodness of fit**

Label	Percentile[a]	Log Likelihood	Akaike Info. Crit.
RECOECD	25	-738.577	9.5947
RECUSA	25	-748.455	9.7182
RECNBER	n/a	-750.266	9.7408
ROECD10	10	-749.674	9.7334
ROECD12	12.5	-748.233	9.7154
ROECD17	16.7	-742.680	9.6460
ROECD20	20	-742.435	9.6429
ROECD33	33	-749.361	9.7295
ROECD50	50	-747.478	9.7060

a Approximate percentile (bottom) for the given sample period for the OECD Leading Indicator Series.

Table 4.4: **Model specification: Goodness of fit**

Model specification[a]	Log Likelihood	Akaike Info. Crit.
HP filter[b]	-1212.349	15.2919
Multivariate SSM: Symmetric[c]	-766.795	9.8599
- no recession indicator; no cross correlation		
Multivariate SSM: Asymmetric[d]	-745.254	9.6532
- RECOECD recession indicator; no cross correlation		
Multivariate SSM: Asymmetric[e]	-738.577	9.5947
- RECOECD recession indicator; cyclical cross correl.		

a All specifications included the same dummy variables as used in the preferred model.
b HP filter is equivalent to an IRW trend plus an irregular component interpreted as the cycle, with a fixed ratio between cycle and slope innovation variances.
c Multivariate State Space Model. As for the preferred model, consumption and investment have LOCL trend, while net exports and unemployment have IRW trends. No restriction on variance ratio, unlike the HP filter. No asymmetry features. Restricted equal cycle periods for consumption and investment.
d Inclusion of the recession indicator allows asymmetric responses. With no cross correlation, the results are equivalent to a set of univariate analyses.
e The preferred model as reported in Table 4.1.

Chapter 5

Conclusion

The business cycle is probably a reasonably familiar term even to the layperson, through their direct experience of the labour market, business and trading conditions, or simply through reporting in the popular media. However, it remains an elusive concept to identify or measure with great certainty in econometrics. It is important to attempt it, since many macroeconomic policy settings rely on an understanding of the current state of the cycle in various aggregate indicators and on the relationship between them. This thesis has demonstrated how different variations of the unobserved components model can be used to measure the state of the business cycle and to generate new insights into the cyclical components of economic variables which are of key interest in macroeconomic policy-making, such as unemployment, participation and productivity.

In Chapter 2 it was demonstrated how a common cycle feature can be used as an identification device to extract a common component from a number of related time series from the labour market. The relative magnitude of these jointly estimated cycles allowed the calculation of fresh estimates of the Okun coefficient and corresponding participation coefficients by age and gender for Australia. The estimated coefficients were generally higher than those typically reported in the literature, which was attributed to the methodology used in this article, in preference to more typical methods of filtration that have been used in other research. The results also highlighted the importance of differentiating between permanent trends, which in labour markets will include features such as demographic changes, and transitory cycles, before attempting to frame policy responses.

Chapter 3 shed new light on previously disparate empirical findings of procyclical or countercyclical productivity, against a theoretical framework in which it is usually held to be procyclical. Mechanical measures of average labour productivity are affected by the relative magnitude of cycles in output and hours worked and the lag of hours relative to output. Firms can vary their total employment, the share of full-time and part-time workers and the average number of hours per worker, so it is very difficult to anticipate the behaviour of measures of productivity. An extension of the unobserved components model which allowed direct estimates of phase-shift amongst similar cycles was used to explain the apparent countercyclical and lagging behaviour of average labour productivity relative to the output cycle in Australia. Further, variation at the extensive margin was shown to be much more important than variation at the intensive margin to average labour productivity.

Chapter 4 considered the question whether the recession part of the business cycle is a real macroeconomic phenomenon, in the sense that it must be more than the simple mirror image of the expansionary phase of a cycle, indicating that there are fundamentally different processes at work during a recession, such as a temporary failure of markets to clear. An indirect test of this proposition was made by determining whether a non-linear model with asymmetric responses to a recession provides a better empirical fit than a model with only symmetric responses. A version of Friedman's Plucking Model was developed for a small open economy and applied to the major components of output and the unemployment rate in Australia. Evidence was found in favour of asymmetric characteristics in key series of Australian macroeconomic variables, but not in the form envisaged by the original plucking model. Unemployment and consumption showed evidence of mostly permanent asymmetric responses to recessions while capital investment and net exports showed mostly transitory responses. The existence of significant symmetric cyclical variations does not support the postulated idea of one-sided plucks away from a ceiling which represents an efficient, equilibrium state.

There is scope for further research attempting to combine the econometric features of the models in the three main chapters in a single study, such as testing for asymmetric responses of the Okun coefficients in recessions and normal times, and understanding the impact of the phase shift between output and labour markets on those same coefficients. It is noted, however, that the models estimated in each of the chapters in this thesis had about as many parameters as it was feasible to identify, so adding new features to models would likely have to balanced by new restrictions to achieve identification.

The characteristics of an estimated business cycle extracted from an economic time series cannot be completely independent of the filtration method used for the extraction, so care needs to be taken to increase the likelihood that the cycle identified corresponds closely with the true but unknown data generating process. When the purpose of the economic analysis is to understand the comovement of the cyclical components of several variables it is best to make a joint estimation of the cycles in a multivariate framework. To satisfy the aim of providing an accurate statistical characterisation of the business cycle, it has been shown that it is useful to model explicitly data features such as phase differences between series and asymmetry of cycles within series. The three applications of a multivariate unobserved components models in this thesis have demonstrated how such a framework can be used in situations where economic policy making will benefit from a better understanding of the cyclical behaviour of key macroeconomic aggregate variables and the interaction between them.

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Appendix 5.A The state space form

If a time series model has been put into state space form then a general set of algorithms known as the Kalman filter can be used to estimate the model by maximum likelihood and to generate predicted and smoothed estimates of the unobserved state variables. State space form requires a measurement equation, which relates the observed time series to the unobserved states, and state or transition equations which specify the evolution of the state variables through time. Both univariate and multivariate systems can be put into state space form.
Measurement equation:

$$\boldsymbol{y_t} = \boldsymbol{Z_t}\boldsymbol{\alpha_t} + \boldsymbol{\varepsilon_t}, \qquad \boldsymbol{\varepsilon_t} \sim n.i.d.(0, \boldsymbol{H_t}). \tag{5.1}$$

$\boldsymbol{y_t}$ is an $N \times 1$ vector of observed variables, $\boldsymbol{\alpha_t}$ is the $m \times 1$ state vector and $\boldsymbol{Z_t}$ is an $N \times m$ selection matrix which links the observed variables to the unobserved states. $\boldsymbol{\varepsilon_t}$ are independently distributed zero mean random variables with covariance matrix $\boldsymbol{H_t}$. In a Gaussian model $\boldsymbol{\varepsilon_t}$ are also normally distributed (denoted by n.i.d. above).
Transition equation:

$$\boldsymbol{\alpha_{t+1}} = \boldsymbol{T_t}\boldsymbol{\alpha_t} + \boldsymbol{\eta_t}, \qquad \boldsymbol{\eta_t} \sim n.i.d.(0, \boldsymbol{Q_t}). \tag{5.2}$$

$\boldsymbol{T_t}$ is the transition matrix which determines the generation of the unobserved states as a first order Markov process. $\boldsymbol{\eta_t}$ is an $m \times 1$ vector of serially uncorrelated state innovations with covariance matrix $\boldsymbol{Q_t}$. It is typical to express the transition equation in future form with the time index $t+1$ on the left hand side and with the index t for the innovation term on the right hand side. The equation could easily be written in more familiar AR(1) form $\boldsymbol{\alpha_t} = \boldsymbol{T_t}\boldsymbol{\alpha_{t-1}} + \boldsymbol{\eta_t'}$ simply by defining $\boldsymbol{\eta_t'} = \boldsymbol{\eta_{t-1}}$ (Pelagatti, 2016, pp. 92–93). In many applications the system matrices $\boldsymbol{Z_t}, \boldsymbol{T_t}, \boldsymbol{H_t}$ and $\boldsymbol{Q_t}$ will be time invariant (in which case the subscript t can be dropped from the notation for those matrices) and the transition equation is simply a first order vector autoregression process. In typical cases the irregular component $\boldsymbol{\varepsilon_t}$ in the measurement equation is uncorrelated with the state innovations $\boldsymbol{\eta_t}$. An uncorrelated unobserved components model with independent state innovations can be specified with covariance matrix $\boldsymbol{Q_t}$ in diagonal form, whereas a more general form of $\boldsymbol{Q_t}$ would be used to allow for correlation between state innovations (such as between trend and cycle innovations).

We illustrate the state space representation by example with a univariate model with time invariant system matrices, a local linear trend and a trigonometric stochastic cycle component (if an autoregressive form of the stationary component is required then any arbitrarily complex ARMA(p, q) process can be specified in state space form as set out in Pelagatti (2016, pp. 97–98)). It is important to note that there is not a unique state space representation of a system, so the representation can be chosen to aid interpretation and, in some cases, estimation.
Structural time series model:

$$
\begin{aligned}
y_t &= \tau_t + \psi_t + \varepsilon_t, \\
\tau_t &= \tau_{t-1} + \beta_{t-1} + \eta_t, \\
\beta_t &= \beta_{t-1} + \zeta_t, \\
\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} &= \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix},
\end{aligned}
\tag{5.3}
$$

where $\varepsilon_t, \eta_t, \zeta_t, \kappa_t$ and κ_t^* are zero mean n.i.d. disturbances.

State space representation:

$$
\begin{aligned}
y_t &= \boldsymbol{Z\alpha_t} + \varepsilon_t \\
\boldsymbol{\alpha_{t+1}} &= \boldsymbol{T\alpha_t} + \boldsymbol{\nu_t} \\
\boldsymbol{\alpha}_t^T &= \begin{pmatrix} \tau_t & \beta_t & \psi_t & \psi_t^* \end{pmatrix} \\
\boldsymbol{Z} &= \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix} \\
\boldsymbol{T} &= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \rho\cos(\lambda) & \rho\sin(\lambda) \\ 0 & 0 & -\rho\sin(\lambda) & \rho\cos(\lambda) \end{pmatrix} \\
\boldsymbol{\nu}_t^T &= \begin{pmatrix} \eta_t & \zeta_t & \kappa_t & \kappa_t^* \end{pmatrix} \\
\boldsymbol{H} &= \sigma_\varepsilon^2.
\end{aligned}
$$

For an uncorrelated model (no correlation between state variable innovations) the form of the covariance matrix would be:

$$
\boldsymbol{Q} = \begin{pmatrix} \sigma_\eta^2 & 0 & 0 & 0 \\ 0 & \sigma_\zeta^2 & 0 & 0 \\ 0 & 0 & \sigma_\kappa^2 & 0 \\ 0 & 0 & 0 & \sigma_\kappa^2 \end{pmatrix}.
$$

If the covariance between trend and cycle innovations was to be freely estimated then define $\text{cov}(\eta_t, \kappa_t) = \sigma_{\eta\kappa}$ and the covariance matrix would have the form:

$$
\boldsymbol{Q} = \begin{pmatrix} \sigma_\eta^2 & 0 & \sigma_{\eta\kappa} & 0 \\ 0 & \sigma_\zeta^2 & 0 & 0 \\ \sigma_{\eta\kappa} & 0 & \sigma_\kappa^2 & 0 \\ 0 & 0 & 0 & \sigma_\kappa^2 \end{pmatrix}.
$$

The state space representation is completed by specifying the mean and covariance of the initial state vector α_0. Under the assumption of Gaussian disturbances it is straightforward to derive the Kalman filter and construct the likelihood function (Harvey, 1993, pp. 87–92). The Kalman filter is a recursive algorithm which makes a forward pass through the time series to make filtered estimates of the unknown states using only past and current observations and to generate a series of one step ahead forecasts or predictions. The maximum likelihood estimates of the model parameters are those which minimise the predictions errors and their variance. In a backward pass, smoothed estimates of the unobserved components can be constructed using all of the observations. For more detailed description of the Kalman filter see Harvey (1993, pp. 89–94) or Commandeur and Koopman (2007, pp. 84–89).

Appendix 5.B Identification issues for the unobserved components model

Consider a univariate model for the decomposition of a time series into trend, cycle and irregular components in the following general form:

$$\begin{aligned}
y_t &= \tau_t + \psi_t + \varepsilon_t, & \varepsilon_t &\sim i.i.d.(0, \sigma_\varepsilon^2), \\
\tau_t &= \tau_{t-1} + \beta_{t-1} + \eta_t, & \eta_t &\sim i.i.d.(0, \sigma_\eta^2), \\
\beta_t &= \beta_{t-1} + \zeta_t, & \zeta_t &\sim i.i.d.(0, \sigma_\zeta^2), \\
\phi(p)\psi_t &= \nu_t, & \nu_t &\sim i.i.d.(0, \sigma_\nu^2),
\end{aligned} \tag{5.4}$$

where $\phi(p) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p$ is a p^{th} order polynomial in the lag operator. It is convenient to illustrate the identification issues for a model including this autoregressive form of the cyclical component but we will also consider the trigonometric stochastic form of the cycle later. In macroeconomic applications typically $p = 2$. Initially, we impose a restriction that the irregular component ε_t and all of the state variable disturbances are mutually independent so that the specification can be described as an uncorrelated unobserved components model. A zero restriction can be applied to an innovation variance if a stochastic component is not required. The unknown parameters including the variances of the disturbance terms and the autoregressive coefficients are known as the hyperparameters. One way of analysing the identification issue is to compare the number of hyperparameters with the number of coefficients which would appear in the equivalent reduced form of the model. The parameter counting process is illustrated here for a specific case of the model shown in Equation 5.4, where there is a local level trend, constant slope ($\sigma_\zeta^2 = 0$, so $\beta_t = \beta$), an irregular component and a stationary AR(2) cycle.

Reduced form: For exposition, we re-write the system described by Equation 5.4 including lagged instances of some equations.

$$y_t = \tau_t + \psi_t + \varepsilon_t \tag{5.5}$$

$$y_{t-1} = \tau_{t-1} + \psi_{t-1} + \varepsilon_{t-1} \tag{5.6}$$

$$0 = -\tau_t + \tau_{t-1} + \beta + \eta_t \tag{5.7}$$

$$0 = -\phi(2)\psi_t + \nu_t \tag{5.8}$$

$$0 = -\phi(2)\psi_{t-1} + \nu_{t-1}. \tag{5.9}$$

To find the reduced form it is necessary to eliminate the unobserved components from Equation 5.5. Add Equation 5.7 to and subtract 5.6 from 5.5, and multiply the result through by $\phi(2)$ to find

$$\phi(2)\Delta y_t = \mu + \phi(2)\Delta\varepsilon_t + \phi(2)\eta_t + \phi(2)\Delta\psi_t, \tag{5.10}$$

where $\mu = \phi(2)\beta$ is a drift term, noting that the product of a lag polynomial and a constant is a constant. Then add Equation 5.8 and subtract 5.9 from 5.10 to find

$$\phi(2)\Delta y_t = \mu + \phi(2)\Delta\varepsilon_t + \phi(2)\eta_t + \Delta\nu_t, \tag{5.11}$$

The largest number of lags on the right hand side of Equation 5.11 is three (in the term $\phi(2)\Delta e_t$). Since the error terms are independent, and the sum of white noise processes is white noise, we can re-write Equation 5.11 in terms of a new error term:

$$\phi(2)\Delta y_t = \mu + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3}, \tag{5.12}$$

where each of the coefficients θ_i is a function of the original coefficients. Equation 5.12 shows that the reduced form of Δy_t is ARMA(2,3), or equivalently that y_t is ARIMA(2,1,3). The number of parameters in the reduced form is seven

$(\mu, \phi_1, \phi_2, \theta_1, \theta_2, \theta_3, \sigma_e^2)$, but the structural model has only six $(\beta, \phi_1, \phi_2, \sigma_\varepsilon^2, \sigma_\eta^2, \sigma_\nu^2)$, so the structural model is over identified in this example. In essence, this was the criticism by Morley et al. (2003) of the uncorrelated unobserved components model of Clark (1987). Morley et al. showed that if the coefficient of correlation between trend and cycle innovations was freely estimated, adding one more parameter to the model, then it would have the same number of parameters as the reduced form and would be exactly identified.

Trigonometric stochastic cycle: We consider whether the identification issue changes if, instead of the autoregressive form, we use the trigonometric form of a stochastic cycle.

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}. \tag{5.13}$$

It can be shown (Harvey, 1993) that the reduced form of the trigonometric stochastic cycle is ARMA(2,1), which at first sight appears to have four parameters $(\phi_1, \phi_2, \theta, \sigma_\nu^2)$ whereas an AR(2) cycle requires only three. However, the moving average coefficient is strongly restricted and can be expressed in terms of the other parameters so the effective number of free parameters is only three[1]. The structural time series model for the cycle component alone also has three parameters $(\lambda, \rho, \sigma_\kappa^2)$, so that part of the model can be exactly identified.

Multivariate model: In the univariate case it has been shown that the specification may include freely estimated correlation between certain state variable innovations to exactly identify the model depending on the number and structure of

[1] Another way to introduce a free parameter to the model, aside from estimating a coefficient for the correlation between trend and cycle innovations, would be to specify the cycle as an ARMA(2,1) process in the structural model rather than AR(2), which would require the estimation of the moving average coefficient. Morley et al. (2003) and Proietti (2006) show that there is a complex relationship between the parameters in these alternative representations. They show that only the structure with the AR(2) cycle and the free correlation coefficient is consistent with the unrestricted reduced form. The ARMA(2,1) cycle form and the stochastic cycle form imply strong restrictions on the parameter space, which may be rejected by the data.

unobserved components. In a multivariate case there may be a very large number of possible covariances between components (such as trend and cycle) and between series (such as common trends). In short, multivariate models with a completely general covariance matrix for the innovations are likely to be highly under identified, so a large number of restrictions on covariances will have to be imposed to achieve identification. In practice, it is not possible to apply the straightforward parameter counting exercise that has been illustrated here for a univariate case to a multivariate case. Further, it must be acknowledged that decompositions into unobserved components are not unique. As a practical matter, finding maximum likelihood solutions often requires consideration of several possible local maxima. Failure to converge to a plausible solution due to flat regions of the likelihood function, accompanied by high standard errors for parameter estimates, is usually an indication that the model is under identified and that more restrictions are required or that an alternative selection of structural components should be considered.