# Optimal Migration

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# Abstract

This study provides a theoretical exploration on the optimality of migration in a two-country world. It provides what determines the optimal stationary state of migration in a two-country dynamic stochastic general equilibrium framework, with endogenous and costly migration. With productive capital also included in the framework, this work provides insights into the substitutability between labour mobility and endogenous capital adjustment.

In general, we study migration under three possible political regimes, which are: 1) a single global planner; 2) two countries with their own central planners; and 3) two free-market economies under perfect and imperfect competition. A single benevolent planner delivers the global optimum welfare and the optimal size of migration. With two planners, we compare the outcomes of Nash and cooperation games, and suggest that a cooperation game is not necessarily superior to a Nash game in determining migration. Comparing planned to free-market economies, we find that a central planner and the perfectly competitive free-market economy are equivalent in terms of welfare maximization with migration. In imperfectly competitive free-market economies, this study shows how efficient wage-employment contracts of all labour groups (home, foreign and migrant labour) interact with each other, with migration jointly determined by all groups' productivity, costs of migration and labour market bargaining in both economies.

A comparison of the derived welfare outcomes shows that migration can improve social welfare for both home and foreign economies, and that the global welfare optimum can be reached by a single global planner, two central planners and perfectly competitive free-market economies. The imperfectly competitive free-market economies cannot achieve the same welfare state due to the presence of bargaining frictions and natural level of unemployment.

Our simulation of the imperfectly competitive free-market model presents the general equilibrium responses to productivity, cost of migration and leisure shocks. A positive productivity shock can increase its origin's output and households' utility, while its effects on the other economy rely on the relative bargaining powers of labour market participants and on the degree of capital adjustment in both countries. Leisure shocks can reduce the utility of their own households, while a foreign leisure shock benefits the home households because it can stimulate migration. A cost of migration shock is at the cost of both home and foreign welfare via migration and capital adjustment. We show that there is a degree of substitutability between the labour mobility of migration and endogenous capital movements in adjusting to the full general equilibrium.

# Declaration

The work presented here has not previously been submitted for any degree or diploma in any university. References are made to acknowledge the work of others and all used information that is not sourced from the work itself. This thesis does not contain materials that need to be approved by the Ethics Committee.

Hao Xu - [redacted]

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# Lists of Abbreviations

| | |
|---|---|
| ABS | Australian Bureau of Statistics |
| DSGE | Dynamic stochastic general equilibrium |
| FOC | First-order conditions |
| RBNZ | Reserve Bank of New Zealand |
| SC | Statistics Canada |
| SNZ | Statistics New Zealand |
| TFP | Total factor productivity |

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# Chapter 1

# Introduction

*"After all that has been said of the levity and inconstancy of human nature, it appears evidently from experience that a man is of all sorts of luggage the most difficult to be transported."*

— Book I, Chapter 8: "Of the Wages of Labour" by Smith (1776)

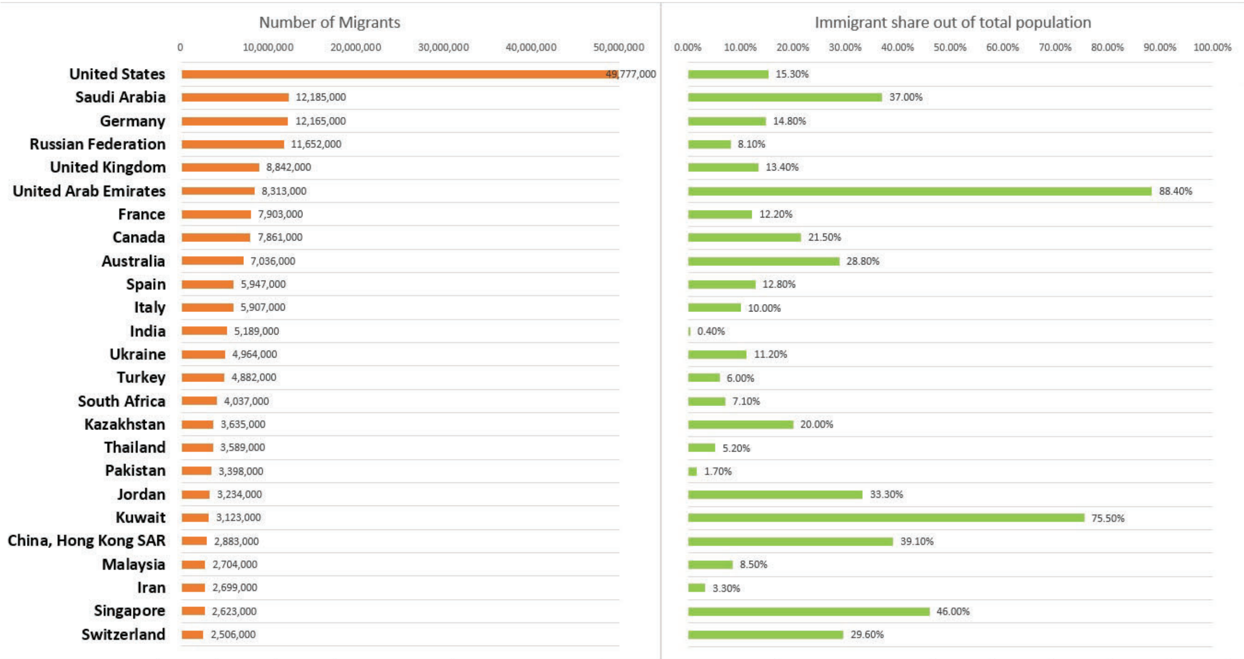
## 1.1 Background

Migration has always been a ubiquitous phenomenon and a heated topic. Since 2000, the average growth rate of migration has been 2.3 per cent per annum globally, and the migration share of the global population within the last several decades has stabilized between 2.2 and 3.5 per cent, despite the difficulties of transporting humans revealed centuries ago by Smith (1776). The estimated population of migrants in 2017 exceeded 258 million (DESA, 2017). Some argue that this estimated size of migration has been reached in a "constrained" age of migration,[1] compared with an average 3 per cent decadal migration rate in sending countries during the first round of globalization from the mid-nineteenth to the early twentieth century (Hatton and Williamson, 2005a; Heitger, 1993).

Since the mid-20th century, migration, voluntary or forced displacement, has been a large and persistent feature of the second and third rounds of globalization. Up to 2015, among 248 million total migrants, UNHCR (2015) has reported 59.5 million displaced individuals across the world. Fig.1.1 displays data on the number and share of international migrants for the top 25 destination countries, from 1960 to 2017. The humanitarian and migrant crisis eased for the European countries only in March 2019 (Rankin, 2019). Such an enormous moving population has generated economic and political controversies such as the

[1] Migration is constrained when the spontaneous pursuit of moving to a more assured location needs to be subordinated to and serve the interests of countries.

Figure 1.1: Top 25 countries of Destination, 2017




NB: Data from MPI (2017).

brain drain and gain in both home and the host economies (Boeri, Brücker, Docquier and Rapoport, 2012; Hagopian, Thompson, Fordyce, Johnson and Hart, 2004; Schiff and Özden, 2005; Stark, Helmenstein and Prskawetz, 1997). Also, the tensions among ethnic groups and native communities have been simultaneously building up due to the rising concerns on job displacement and fiercer competition on public resources (Dahinden, 2009; Decker, Van Gemert and Pyrooz, 2009; Geddes, 2003; Masud-Piloto, 1996; Zimmermann, 1995, 1996).

While numerous economic reports and analyses have explored the possible causes and effects of migration since Jerome (1926), there have been limited explanations and insights to improve our understanding of the size of migration. A main question is whether the share of migration in the global population can be regarded as an equilibrium and an optimal value. What would determine the equilibrium size of migration if it was the free right of every person to decide where s/he lives, and what can free migration bring to the welfare and economy of both home and foreign countries in equilibrium?

The first challenge to address these questions is to construct an effective theoretical framework that can generate endogenous migration and reflect the equilibrium responses of key economic variables of both economies.

Across the history of migration, governments of home and foreign countries of migrants have played a major role in determining the periodical size, selecting the desired inflow groups and shaping domestic views of migrants (Bodvarsson and Van den Berg, 2013; Hatton and Williamson, 2005a). Thus the second challenge requires that the framework can feature different political regimes. By doing so, not only we gain more understanding of the motivations of different stakeholders in choosing migration levels, but we can also derive and compare the optimal size of migration and their welfare implications under different institutional design.

Facing these challenges, this study develops a small scale dynamic stochastic general equilibrium (DSGE) framework, i.e. a two-country model with migration. By deriving the stationary-state equilibrium of this hypothetical global economy, we can observe the long-run interactions between migration and other macroeconomic variables in the two countries. The framework also provides an ideal foundation to study the role of labour mobility and bargaining in the adjustment process of a two-country world toward its global equilibrium and how it can differ from the summed equilibrium state of two economies in autarky. Our model deliberately excludes trade in goods, following Mundell (1957), such trade acts as a substitute for the labour movement to achieve factor-price equalization. Our interest is in understanding how factor mobility as allowing migration delivers factor-price equalization without trade.

## 1.2 What we contribute

This study presents several theoretical possibilities of a two-country world, with the foreign country being the potential recipient of immigrants and stakeholders of the home country making emigration decisions. We build migration theories under different hypothetical political regimes. The actual labour supply decisions can be made by planners in centrally planned economies, as shown in **Chapters 2** and **3**, or else by individual household units who may wish to send some of its members to work abroad in free-market economies, as elaborated in **Chapter 4**. We have included two production factors in both economies, labour and capital. Labour is internationally mobile and may be heterogeneous across countries. Capital is not mobile, and may be fixed or endogenously adjustable. The production technology of the two economies can be dissimilar and the capital-labour ratios may differ across countries.

Our study is a substantial development on the general equilibrium model of migration by Benhabib and Jovanovic (2012), which is a recent theoretical exploration of the study of optimal migration. We thus start with a general comparison between our study and theirs to articulate the major challenges that we have addressed. Some innovative features in our set-up that bring some new perspectives to migration are also discussed with more details and motivations.

### 1.2.1 Migration is not a zero-sum game

Our study allows for differentiated capital-labour ratios so that migration can be a welfare-improvement process for both foreign and home countries. In contrast, Benhabib and Jovanovic (2012) formulate a global welfare function in a binary Markov form, which sums to one so that migration must be a zero-sum game. According to Benhabib and Jovanovic (2012), for a given period of time, any migration flows lead to a transfer of welfare from the home to the foreign economy via human capital movements. However, our study finds that the home country is not necessarily worse-off, though it may be the case in the short run if there are no remittances sent home (Mandelman and Zlate, 2012). Our framework develops the global welfare as a sum for the two countries' welfare in which optimal welfare with migration is always *no-less-than* the sum of two autarkic economies. Specifically, the welfare of the home economy may not be worse off as its capital-labour ratio achieves its stationary state with the adjustment of migration.

### 1.2.2 Capital-labour heterogeneity and production heterogeneity

This thesis provides insight on how migration would affect the welfare of both the home and foreign economies under two possible assumptions of production differentiation between the two economies: namely differentiated capital-labour endowments or heterogeneous production technologies. In this study, how capital-labour ratio differentials and/or production technology heterogeneity induce migration is proven theoretically. Comparing the global welfare optima of these two scenarios, migration is a more significant adjustment for production heterogeneity than capital-labour heterogeneity.

The global misallocation of human capital resolved by migration in Benhabib and Jovanovic (2012) is a variant of our assumption of a possible "differentiated capital-labour ratio". Benhabib and Jovanovic (2012) mainly studies a single labour input factor model from Lucas (1990) and considers the possible misallocation of human capital. We show what would happen if we include capital into production. Comparing with a labour-only production model in which the loss of labour, migration, will certainly lead to a drop in production and output per capita, our framework sheds light on the effects of capital-labour ratio on output per capita and welfare. Furthermore, we compare the effects of fixed capital on optimal migration and its subsequent effects on the global economy with the effects of endogenously adjustable capital.

We also feature a production function that allows a productivity advantage of local foreign residents in comparison to their immigrant counterparts. The local-migrant complementarity in the foreign country/firm production technology assumes that migrants carry less human capital than the locals in the foreign economy - an assumption grounded on real-world observations. Recent empirical evidence suggests that the advanced economies tend to select migrants at a young work age to compensate for their ageing demographics (ABS, 2018a,b; RBNZ, 2019; SC, 2016, 2019; SNZ, 2018).[2] Compared to the senior employees with years or even decades of experiences in the industry grounded on local cultural traits, migrants from different cultural backgrounds tend inevitably to be less productive (at least, initially). Furthermore, it is a universal observation across most advanced countries that migrants, especially the first-year migrants, work at significantly lower average wages than the locals of the foreign economy (Aldashev, Gernandt and Thomsen, 2012; Borjas, 2003; Brenzel and Reichelt, 2017; Breunig, Hasan and Salehin, 2013; Dustmann, 2003; Hatton and Williamson, 1992; Hum and Simpson, 2000; Islam and Parasnis, 2014; Khan, 2016; Lang, 2000; Zhang, Sharpe, Li and Darity, 2016).[3] The most recent evidence from Smith and Thoenissen (2019) shows that the differentiated human capital of migrants compared with locals' can induce business cycle fluctuations. From an aggregate constant elasticity of substitution production technology perspective, migrant labour needs to be considered separately from the local labour force as a new production factor that complements local labour.

Furthermore, empirical studies have presented significant benefits of taking in migrants who are complementary to locals. A significant piece of evidence can be tracked to Card (1990). Although the inflow of less-skilled Cuban workers in the 1980's Mariel Boatlift event suddenly increased the labour force of the Miami metropolitan area by 7 percent, there were no perceived negative effects on wages and unemployment rate. Moreover, Özden and Wagner (2014) find that Malaysian firms employed more native workers in the face of a larger supply of unskilled labour migrant inflow, experiencing a fall in the cost of production and output expansion.

Under these circumstances, our migration equilibrium model provides a theoretical analysis to the benefits of complementary migration that can be brought into the foreign economy. Our two-country general equilibrium framework allows us to determine the changes that migration can bring to households' consumption, investment, labour supply and welfare in the home and foreign countries.

### 1.2.3 Types of migration cost matter

We introduce an explicit account of the cost of migration. Benhabib and Jovanovic (2012) assume the cost of migration as part of household spending. And studies, such as Rendon and Cuecuecha (2010),[4] show that an increase in the cost of migration can significantly reduce

[2]Some more evidence can be found in the US data, as presented in the round table discussion on migration (Ashenfelter, McFadden, Payne, Potts, Gregory and Martin, 2020).

[3]We also need to be aware that there are exception countries like New Zealand, where immigrants tend to have higher human capital than locals.

[4]In their study of US-Mexico migration, doubling the cost of migration can reduce the migration rate by three fourths of the current level.

the level of migration. However, they do not focus on the possible shares of the burden of the cost of migration between the countries and how different elements of the cost might vary the optimal level of migration. In this study, we take a closer look at the migration cost by observing different sharing scenarios and examining the significance of both fixed and variable cost.

Ever since Sjaastad (1962), most published research considers the cost of migration as a fixed lump-sum cost, while a few works such as Clark, Hatton and Williamson (2007) provide a more detailed discussion of the types of costs that cannot be easily analysed in a model. Regarding the variable cost of migration, Konya (2007) has inferred the theoretical effects of individual's assimilation and moving costs and shown their significance by using the 1990 US census.[5] Based on Brazilian labour market conditions, Morten and Oliveira (2016) shows that the mean observed cost of migration could reach $0.8 - 1.2$ times the mean annual wage, and the majority of this cost is a fixed cost. This large expense could be a vital determinant of the stationary-state migration level and in turn the stationary-state welfare levels of the two countries. Our work reconciles both streams of literature.

For a complete understanding of the cost of migration, we rely on the designated four systems. First, in a global centrally planned economy, the cost is studied as an aggregation that is paid by the planner. We have observed that the variable cost appears in determining the equilibrium condition, while the fixed cost is zero when the well-informed planner decides to forbid migration. Second, two individual planners need to bargain or cooperate in terms of migration and sharing the cost of migration. Once the migration decision is made, the fixed cost mainly serves as a corner condition, while the variable cost is vital to determine the size of migration and stationary-state welfare of both economies. Third, to be paid by the households who intend to migrate, the cost of migration eventually shows up in the wage-employment contracts between the labour union and foreign firm cartel. The wages of all labour groups including the foreign and home labour will be directly affected by the cost of migration. At last, the general equilibrium simulations show how migration costs can affect household consumption and utilities. An increase in the fixed cost, such as through government's expenditure to facilitate migration, can increase migration and both countries' welfare, while the increases in variable cost are not necessarily beneficiary.

### 1.2.4 Welfare weights are irrelevant in a cooperative game

This study also reexamine the assumption of welfare weights in a global welfare function under a cooperation game. In contrast to the importance given to the *ex-ante* welfare weights between two countries in Benhabib and Jovanovic (2012)'s study on the Stackelberg

[5] In the studies of illegal migration, Friebel and Guriev (2006) and Tamura (2010) analysed the effects of a unit-based smuggling cost and its interactions with the border enforcement of the destination country.

game between two labour-only production economies, we find that welfare weights have no effect on the optimal size of migration when the two countries cooperate to maximise an aggregate global welfare function in a general equilibrium framework.

### 1.2.5 Efficient contracts in imperfectly competitive free-market economies

We extend Benhabib and Jovanovic (2012)'s study of migration in the context of free-market economies. Here the optimal migration decisions are determined by every household and employment decisions by firm, instead of by the central planners in the centrally planned economies. This study explores the coexistence of the optimal wage-employment contracts of migrants, home and foreign local labour in a free global labour market. McDonald and Solow (1981)'s wage contract model has been developed into a two-country three-labour-market sequential wage bargaining Nash game to examine the effects of optimal migration on both home and foreign wages, employment and unemployment. It is shown that the wage-employment contracts of all labour groups are determined simultaneously by the relative bargaining power, components of cost of migration and marginal productivity of all labour forces.

### 1.2.6 Comparing global optimum welfare among economic systems

Some interesting comparisons of global optimum welfare have been made in this work as we have designed four hypothetical frameworks including a global planner ruling two countries, two individual central planners, two perfectly and imperfectly competitive free-market economies with either endogenous or fixed capital stocks.

It is shown that a global planner with both migration and endogenous capital adjustment always produce the highest global optimum welfare. Coinciding with the most efficient allocation and endowment of labour and capital, the autarkic and fixed capital equilibria can at most equal the highest level.

With domestically adjustable capital, two central planners and the perfectly competitive free-market economy with migration always reach the best solution. However, the imperfectly competitive free-market economy is always suboptimal due to the bargaining frictions and natural levels of unemployment.

If capital is fixed in two economies and immobile, both Nash and cooperation games of migration between two planners can be counter-productive to achieve the global optimum welfare. The perfectly competitive free-market economy can always achieve the optimal migration and thus welfare. Once again, the imperfectly competitive free-market economy is the least preferred.

### 1.2.7 Conclusion

In general, our study presents some key arguments for the economic causation of migration, the welfare improvement to both home and foreign countries brought by migration, the effects of different levels of migration on labour markets of two countries and most importantly, the determinants of the optimal size of migration. We argue that free migration, a spontaneous pursuit of individuals to better living standards under the selfish and rational nature of *homo economicus*, can benefit both home and foreign economies.

Our analysis of optimal migration obviously ignores many social and cultural costs of population expansions- such as public infrastructure, limited essential resources availability (like water) and multicultural tensions. Ignoring these, labour mobility is efficient to adjust for the global general equilibrium and can improve aggregate welfare of isolated countries.

## 1.3 Literature review

### 1.3.1 A brief survey of the time line of migration research

One of the earliest analysis is by Jerome (1926), who studied the effects of early-twentieth-century US migration and its implications on the business cycle. In his magnificent work, Hicks (1932) argued

*"...differences in net economic advantages, chiefly difference in wages, are the main causes of migration."*

which is in line with Smith (1776)'s insights of

*"... the wages of labour vary more from place to place than the price of provisions."*

Their argument soon attracted numerous empirical examinations, of which were confirmatory.

Zipf (1946) hypothesized that migration flow is the numeric product of two communities' population divided by the distance between the two communities. It captures the positive relationships between two communities' population and migration, as well as the negative relationship between migration size and distance of two regions.

A new perspective, brought by Tiebout (1956), stated that a possible motivation of migration is the desire to relocate to the best home that satisfy units' preference for public goods. It raised the significance of political institutions and the importance of preferences of economic agents.

Later, following Hicks (1932); Smith (1776), Sjaastad (1962) and Harris and Todaro (1970) materialized the idea of "distance" as cost of migration, and enriched our understanding of migration from households' economic motivations through differentiated unemployment and wage conditions. This was followed by numerous empirical studies that explored significant country-specific explanatory variables of migration, and described the possible effects of migration on other macroeconomic indicators in the pursuit of guiding and examining migration policies (Borjas, 1987, 1991, 2003; Card, 2009; Heiland and Kohler, 2018; Sequeira, Nunn and Qian, 2019).

Notably, public economists often focus on the effects of regional redistribution policies and other fiscal externalities on the population redistribution (Hercowitz and Pines, 1991, 1997; Huizinga, 1999; Schmidt, Stilz and Zimmermann, 1994). However, the induced labour mobility is always considered as part of the intervention or the responses to adjust the optimal allocation, which provide limited understandings on the non-governed migration phenomenon.

In the more recent literature, Benhabib and Jovanovic (2012) presented how a presumed welfare weight can determine the size of migration between two economies in a Stackelberg game. Studies such as Guerreiro, Rebelo and Teles (2019); Heiland and Kohler (2018); Smith and Thoenissen (2019) and Tombe and Zhu (2019), tended to reconcile both perspectives by featuring migration in a general equilibrium set-up, and explored its interactions with alternative adjustment approaches and equilibrium welfare.

### 1.3.2 Other related literature

Our research is connected with much of the existing literature, including general equilibrium analysis, labour market modelling, both empirical and theoretical explorations on migration and its related featuring and game theory analysis. There are six major themes in our research: the motivation of migration, cost of migration, the general migration equilibrium, Nash bargaining practices, the efficient contract model and the simulated DSGE applications.

Based on Hicks (1932)'s argument on the significance of wage differentials on migration decisions, Sjaastad (1962) suggested that migrants calculate the net present value of lifetime earnings in the two regions (home and each potential foreign), subtracting the cost of moving, and then making the decision to migrate. His paper defines the scope and directions of a major stream of modern economic models of migration from the famous two-sector analysis of Harris and Todaro (1970) to Borjas and all other mainstream migration economists' contributions, which have regarded migration as human capital investment behaviour with a focus on employment and wage differentials as the determinants of migration (Becker, 1974; Borjas, 1987, 1991; Chiswick and Hatton, 2003; Hatton and Williamson, 1992; Katz and Stark, 1986; Klein and Ventura, 2007; Pessino, 1991; Stark, 1984, 1991; Stark and Levhari, 1982; Tunali, 2000). Recently, Kennan and Walker (2011) showed that the movement of US youth could yield about $\$17,000$ higher wages (and more than $\$300,000$ dollars in present value).

Observing the internal migration in developed economies, Tiebout (1956) and Rosen (1974) argued that a decision to migrate can be made due to individuals' and households' preference and utility in an economy under spatial equilibrium, which is called the "equilibrium" perspective of migration. It runs counter to the "disequilibrium" conclusion that the wage differential drives people's willingness to migrate. Greenwood (1997) supported their argument by summarizing empirical findings in the early 1980s that earnings cannot be confirmed as a sufficient determinant of migration. The equilibrium perspective of migration is therefore a major strand of migration theory — see Graves (1976, 1980, 1983), Roback (1982, 1988), Glaeser and Shapiro (2003), and Green et al. (2005, 1996). These papers focused on how climate, rents, opportunities, public policies and quality of life could affect individuals' internal migration decisions within mostly America and other advanced economies. The common ground of the "equilibrium" and "disequilibrium" perspectives of migration is that migration happens if it leads to a welfare improvement, while the so-called "spatial equilibrium" perspective can only denote a particular equivalence of real prices and wages between countries, but not welfare that includes air, water and public facilities.

According to Sjaastad (1962), another determinant of migration is the "cost of moving". Most of the aforementioned literature take this as an exogenous component, while another strand of the literature has provided further insights about the cost, rather than just a given amount of lost output. Clark et al. (2007); Hatton and Williamson (2005b, 2011) have extended the Borjas (1987, 1989)'s models to account for the effects of non-pecuniary costs of migration. In particular, Clark et al. (2007) listed four types of migration cost: individual-specific migration costs that relies on the social network of the particular migrants; direct costs (the material cost of transportation, visa-issuing and cross-border cost); quantitative restrictions on migration (limited permissions to cross the border give an inverse relationship between the cost and the policy cap); and a biased "skill selective" policy (the more preferable the immigrant's skills, the lower the cost of her/his admission of entry). However, this detail in cost components adds difficulty in both its measurement and modelling. Recent literature, by Djajic and Michael (2009); Klein and Ventura (2007); Tombe and Zhu (2019), still considers the migration cost as a fixed portion of the representative agent's income and analyses how it affects economic conditions and policy interactions of the home and foreign economies. A very recent contribution on the significance of the "cost of moving" comes from Guerreiro et al. (2019), who showed how "congestion" effects distort stationary-state optimal immigration and cost the native welfare, which tends to be a lump-sum fixed cost to the foreign economy.

With the DSGE framework becoming the workhorse of modern macroeconomic analyses, the possible effects of migration on other macroeconomic variables in both decentralized and centrally planned economies have been analysed (Battisti, Felbermayr, Peri and Poutvaara, 2018; Chassamboulli and Palivos, 2013, 2014; Lim and Morshed, 2017; Liu, 2010; Mandelman and Zlate, 2008; Meier and Wenig, 1997; Palivos, 2009). These works are mainly conducted to shape policy suggestions for the foreign economy toward general migration or a particular group of migrants.

Palivos (2009), in the spirit of Hazari and Sgro (2003) and Moy and Yip (2006), presented a theoretical analysis on the four effects of illegal migration[6] in a decentralized DSGE model. He analysed assumptions of labour homogeneity or heterogeneity. Other research studies the effects of a remittance tax (Lim and Morshed, 2017; Mandelman and Zlate, 2008). However, these pay little attention to the possible welfare gain of labour mobility though they all treat it as a significant adjustment process. Hamilton and Whalley (1984) measured the welfare improvement of migration by looking at the incremental effects of unrestricted cross-border labour mobility on productivity in a simple labour supply and demand model. Guerreiro et al. (2019) showed how an immigration policy selecting high or low skilled migrants can optimise the welfare of native population through the government's tax discrimination and public good redistributive systems, and thus promote policy suggestions for actively selecting immigrant groups that maximise the benefit of native populations by profiting from skill premia. Whilst intuitively sensible, they do not delve into important details about migration. For example, which side, home and/or foreign, would pursue migration in the first place? Will there be migration if one side desires and another side resists? To what extent can migration play a significant role in equilibrium adjustment? And importantly, is it possible that migration can increase equilibrium productivity?

Our study also features Nash Bargaining for migration between the centrally planned economies as developed by Binmore et al. (1986), which has a long tradition in determining employees' wage contracts. There are two main streams of literature in bargaining wages. In most continuous models that converge to a balanced growth stationary state, the bargain strikes a balance between the present marginal value of being employed for the employee and the present net profit of having an additional worker for the employer (Shimer, 2010). On the other hand, bargaining in stationary models that feature search and matching processes focus on the potential gain for the two sides (Blanchard and Gali, 2010; Diamond, 1982). The employee intends to maximise his/her gain from becoming employed from unemployed after search costs, while firms aim to measure the difference in value between having an additional worker and a vacancy with searching. Both strands of literature are successful in replicating

[6] The four effects are the wage-depressing effect, the job displacement effect, the capital consumption effect and the exploitation effect, which focuses on how consumers could benefit from illegal migrant workers.

domestic wage bargains but with no geographical externalities. Our modelling extends the analysis of wage bargains in a geographic equilibrium.

Moreover, McDonald and Solow (1981, 1985) presented a well-featured efficient contract model based on real-world wage bargaining between unions and firms, assuming an implicit bargaining over employment. The model describes the labour supply and demand decisions in the free-market economy as a positive relationship between wage and employment. This model shows that the set of intersections between isoprofit loci (along which the firms are at the same profit level) and indifference curves (where the representative household obtain identical expected utility) is on an increasing curve in wage and employment space, with the relative bargaining power of unions and firms determining the actual outcome. By explicitly analysing the relative bargaining power of firms in the free-market labour contract, we provide another theoretical proof to Ranjan and Rodriguez-Lopez (2019)'s finding that a larger monopsony power of firms in the labour market is associated with relatively lower wages and thus lower welfare. Extending the efficient contract model in a two-country global economy with migration, we infer the efficient responses of wages and employment levels of three labour groups (domestic labour, foreign labour, and migrants) to exogenous labour market changes from an equilibrium perspective.

We investigate the effects of the total-factor-productivity shocks, migration cost shocks in both frameworks and leisure shocks in the framework that capital is endogenous. Smets and Wouters (2003) revealed the adverse responses between employment and wages to an additive one-standard-deviation productivity shock. It is important to see whether this finding is consistent or reversed with our simulated general equilibrium responses in two labour markets that includes migrants. We ask how the effects of TFP shocks to two economies allowing labour mobility depend on the different levels of relative bargaining powers of both home and foreign labour unions in wage-employment bargains.

A leisure (or preference) shock in representative households' utility functions has been studied in the previous DSGE analysis, however mostly in closed economies (Bhattacharya and Gale, 1985; Harashima, 2014; Smets and Wouters, 2003; Zaheer Malik, Zahid Ali, Imtiaz and Aftab, 2019). Our study extends this work to a two-country economy with labour mobility and we find that though migration can occur, an increase in the home demand of leisure will not affect most macroeconomic variables (and most importantly employment) in equilibrium, and only its own utility level. From a political perspective, people tend to overestimate the effects of immigrants or exaggerate the size of migration (Citrin and Sides, 2008; Hopkins, Sides and Citrin, 2019). However, the economic consequences of labour mobility are vital in improving welfare.

In short, the general trend of economic studies on migration has moved from mainly a descriptive partial labour market analysis and observations on the collected data to more comprehensive general equilibrium modelling. Our study joins the tide and presents a general equilibrium view on modelling migration between two countries.

## 1.4 Thesis outline

In the subsequent chapters, we consider three basic scenarios: 1) a single global dictator; 2) two centrally planned economies; and 3) two free-market economies. Analysing the first scenario provides insights about the internal migration phenomenon, while studying the latter two helps to understand real-world international migration phenomena such as the nearly absolute power of migration permission by foreign countries and the implications of migrants' wage contracts in free economies.

**Chapter 2** sets the general equilibrium framework under a single global planner. It also presents a theoretical solution about how and why the planner will choose to reallocate the labour force between regions of her realm. **Chapter 3** explores what determines the optimal migration agreed by two individual central planners in the home and foreign countries respectively, and how their interactions on migration can affect optimal migration and aggregate welfare. We find that optimal migration can benefit both foreign and home economies. **Chapter 4** presents a general equilibrium framework for both perfectly and imperfectly competitive free-market migration models, extending McDonald and Solow (1981)'s efficient contract model to migrant labour markets within a two-country free-market imperfectly competitive economy. It also gives theoretical inferences on the possible labour market partial equilibrium responses to variations of labour market conditions. **Chapter 5** simulates the general equilibrium responses of **Chapter 4** models to different shocks. We also explore how labour mobility between two countries can interact with domestic capital adjustment in global general equilibrium. **Chapter 6** summarises the research findings of **Chapter 2** to **5**, and provides a welfare comparison of the four designated economic systems analysed a global planner, two separate country planners, a perfectly and an imperfectly competitive free-market world.

In a nutshell, the major motivations of this study are to examine the theoretical causes of migration, whether migration leads to stationary-state welfare improvement and to what extent migration policy is an effective adjustment tool under different political institutions. The actual effects of both fixed and variable migration costs on the optimal level of migration under different regimes, and whether the home or foreign economy should pay for more of the cost, are important questions investigated. Our study presents some interesting and intuitive propositions addressing these issues.

# Chapter 2

# Two-Country Migration under a Global Central Planner

> This chapter studies how a global dictator determines migration. We compare the scenarios of no, full and positive optimal migration in a two-country world with heterogeneity in the capital-labour ratios and in the production, and where physical capital is fixed or endogenously adjustable.

We begin by constructing a two-country general equilibrium framework under a global central planner. The purpose is twofold: to identify what motivates the central planner to choose migration; and to study the global optimum welfare that this two-country economy can achieve under different assumptions.

By listing the conditions of zero, full and positive (but not full) migration, this study shows how heterogeneity between two countries stimulates factor mobility, and thus causes economic migration. Furthermore, this chapter also presents the global welfare optimum with migration, which is always *no-worse-than* the autarkic global equilibrium. Also, we show how endogenous capital adjustment can complement migration to achieve the global welfare optimum.

The basic message of this chapter is that moving labour from the low-productivity country to the higher-productivity one ("migration"), leads to a gross gain in welfare. If this gain is sufficient to cover the migration cost, then there is a net gain in welfare. As this migration is beneficial for the two regions/countries taken as a consolidated entity, this is a policy that the central planner would pursue.

## 2.1 The framework

### 2.1.1 The planners' perspective

Suppose there exists a world with two countries, a single consumption good, given populations that can migrate, capital that is specific to each country and a benevolent planner who rules the two countries.[1] The native population in the two countries is:

$$L^h \geq N_t^h + M_t \tag{2.1.1}$$

$$L^f \geq N_t^f \tag{2.1.2}$$

where $N_t^h$ and $N_t^f$ are numbers of domestic employment and $M_t$ is the number that migrate from the home to the foreign country (where $h$ is for the home country of migrants and $f$ stands for the foreign country). Term $L$ stands for the native population of the two countries. The population of the home country must be no less than its domestic employment plus the supply of migrants. We further assume migrants will be returning to the home country at the end of each period and the home households need to make migrant supply decisions at the beginning of each period.[2]

Assume the global planner maximises

$$\max_{\{C_t^f, N_t^f, C_t^M, C_t^h, M_t, N_t^h\}} \{\sum_{t=0}^{+\infty} \beta^t [U^f(C_t^f, N_t^f) + U^h(C_t^h + C_t^M, N_t^h + M_t)]\}$$

subject to

$$Y_t^f + Y_t^h \geq C_t^f + C_t^h + C_t^M - (1-\delta^h)K_t^h + K_{t+1}^h - (1-\delta^f)K_t^f + K_{t+1}^f + CM_t \tag{2.1.3}$$

$U^i(C_t^f, N_t^f)$ $(i \in (h, f))$ is the period utility of the representative infinitely lived household in each country. $\beta$ is the time discount factor that is adopted by the global planner. $C_t^f, C_t^h$ and $C_t^M$ are the aggregate consumption decisions at time $t$ that the planner makes for the immigrant-receiving (foreign) economy, the immigrant-sending (home) economy and for the migrants, respectively. $N_t^f$, $N_t^h$ and $M_t$ are the foreign, home and migrant aggregate employment, respectively, and full employment will prevail at all times. $K_t^f$ and $K_t^h$ are the foreign and home aggregate capital stocks at time $t$.[3] The depreciation rate of the foreign is $\delta^f$ and of the home is $\delta^h$. $Y_t^f$ and $Y_t^h$ are the foreign and home aggregate production level at

[1] In subsection 3 of this chapter, we will relax this assumption.

[2] Whilst it would be more realistic to model the duration of migration endogenously, our assumption is reasonable since we focus on stationary state outcomes.

[3] As a major adjustment factor, some particular assumptions will be imposed to control the capital variables in the subsequent sections dealing with specific scenarios of migration.

time $t$. $CM_t$ is the aggregate cost of migration at time $t$,[4] which is elaborated as following.

Note that cardinal utility has been widely studied from various perspectives, see Colander (2007); Strotz (1953) and Abdellaoui, Bleichrodt and lHaridon (2008). This thesis follows Ramsey (1928) and Uzawa and Arrow (1989)'s design with a specific focus on the national consumption and labour supply. For example, an isoelastic form of the utility function has been widely applied in many DSGE studies (De Walque, Smets and Wouters, 2005; Kim and Kim, 2003; Li and Spencer, 2016; Merz, 1995; Smets and Wouters, 2003, 2007).

### 2.1.2 Cost of migration in the centrally planned economy

In this study, we explore two aspects of the cost of migration. For the individuals migrating, there would exist a unit-based cost of migration. At the same time for the other country which has to cope with losing or gaining population, a periodical fixed cost is also incurred.

For the globally planned economy, the migration turns into a labour mobility adjustment by the planner. We consider an aggregate cost of migration, $CM_t$, comprising a fixed cost and a variable cost ($\chi$) determined by the number of migrants.[5] Thus, we introduce the following

$$\begin{aligned} CM_t = CM_0 + \chi M_t, &\text{where } M_t > 0,\, CM_0 > 0 \\ &\text{or } M_t = 0,\, CM_0 = 0 \end{aligned} \tag{2.1.4}$$

where $CM_0$ is the overall fixed cost in each period of migration. Note that the migration cost is assumed to not generate income for anyone. It is thus in the nature of "iceberg" costs as seen in the literature on trade with transportation costs as described by Samuelson (1954), Anderson and Van Wincoop (2004) and Fingleton and McCann (2007).

### 2.1.3 Production

We adopt a classical two-factor production function: labour and capital. As we are allowing migration, the foreign country production will be affected by two types of labour: migrants and its domestic labour, which can be homogeneous or heterogeneous.

To explore the causes and effects of migration, two assumptions are made on the two production factors: (1) the capital stocks once installed cannot migrate and (2) labour allocation is what the planner uses to maximise aggregate welfare.

[4]This cost is paid every period if $M_t > 0$. More realistically, this cost may decline over time. However, we ignore this complication.

[5]In a centrally planned world, the fixed cost includes social/government expenditure (e.g. social and cultural cost of foreign inhabitants, or the cost of a department of migration and so on), while the variable cost maybe transportation fees and the cost of issuing visas.

At this stage, we introduce the generalized form of production functions for the two economies.[6]

$$Y_t^f = f(K_t^f, N_t^f, M_t) \tag{2.1.5}$$
$$Y_t^h = f(K_t^h, N_t^h) \tag{2.1.6}$$

For the foreign economy, its production is subject to three input factors: the foreign capital stock ($K_t^f$), its domestic labour supply ($N_t^f$) and its migrant workers ($M_t$), while home production is determined by its domestic labour supply ($N_t^h$) and the home country capital stock ($K_t^h$). We further assume that both functions are twice differentiable as follows, $\{\frac{\partial Y}{\partial N}.\frac{\partial Y}{\partial M}, \frac{\partial Y}{\partial K}\} >> 0$, $\{\frac{\partial^2 Y}{\partial K \partial N}, \frac{\partial^2 Y}{\partial M \partial N}, \frac{\partial^2 Y}{\partial K \partial M}\} >> 0$ and $\{\frac{\partial^2 Y}{\partial N^2}.\frac{\partial^2 Y}{\partial M^2}, \frac{\partial^2 Y}{\partial K^2}\} << 0$.[7]

### 2.1.4 The Kuhn-Tucker conditions

The present period utility function for the foreign economy is assumed to have the following iso-elastic form:

$$U^f(C_t^f, N_t^f) = \frac{(C_t^f)^{1-\eta^f} - 1}{1 - \eta^f} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1 + \frac{1}{\nu^f}} \tag{2.1.7}$$

Following Merz (1995) and Kim and Kim (2003)'s set-up, we assume a constant coefficient of relative risk aversion (CRRA) for the household ($\eta^f$), and a constant Frisch elasticity of labour supply ($\nu^f$). The higher the coefficient of CRRA ($\eta^f$), the more risk averse the individual/households. The Frisch elasticity ($\nu^f \geq 0$) indicates, for a given marginal utility of wealth, how much labour supply would increase in response to a one-unit increase in the real wage.

Analogously, the home economy period utility function is:

$$U^h(C_t^h + C_t^M, N_t^h + M_t) = \frac{(C_t^h + C_t^M)^{1-\eta^h} - 1}{1 - \eta^h} - \frac{(N_t^h + M_t)^{1+\frac{1}{\nu^h}}}{1 + \frac{1}{\nu^h}} \tag{2.1.8}$$

The home household bears the utility of consumption and disutility of labour of both its migrating and remaining members. This is a reflection of the diaspora nature of immigration.

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[6] The production technology will be specified in **Section 2.3.1** and **Section 2.3.2** under different scenarios.

[7] Notation $>> 0$ in the set theory means that every point in the established set is bigger than zero, while $<< 0$ says the opposite.

The global central planner's intertemporal program is:

$$\max_{\{C_t^f, C_t^M, C_t^h, M_t, N_t^h, K_{t+1}^h, K_{t+1}^f\}} \mathcal{L} = \sum_{t=0}^{+\infty} \beta^t [U^f(C_t^f, N_t^f) + U^h(C_t^h + C_t^M, N_t^h + M_t)$$
$$+\zeta_{1,t}(Y_t^f + Y_t^h - C_t^f - C_t^h - C_t^M + (1-\delta^h)K_t^h - K_{t+1}^h + (1-\delta^f)K_t^f - K_{t+1}^f - CM_t)] \tag{2.1.9}$$

The necessary Kuhn-Tucker conditions for an optimum are:

$$\frac{\partial \mathcal{L}}{\partial C_t^f} \leq 0; C_t^f \geq 0; C_t^f \frac{\partial \mathcal{L}}{\partial C_t^f} = 0; \qquad \frac{\partial \mathcal{L}}{\partial C_t^M} \leq 0; C_t^M \geq 0; C_t^M \frac{\partial \mathcal{L}}{\partial C_t^M} = 0$$
$$\frac{\partial \mathcal{L}}{\partial C_t^h} \leq 0; C_t^h \geq 0; C_t^h \frac{\partial \mathcal{L}}{\partial C_t^h} = 0; \qquad \frac{\partial \mathcal{L}}{\partial M_t} \leq 0; M_t \geq 0; M_t \frac{\partial \mathcal{L}}{\partial M_t} = 0$$
$$\frac{\partial \mathcal{L}}{\partial N_t^h} \leq 0; N_t^h \geq 0; N_t^h \frac{\partial \mathcal{L}}{\partial N_t^h} = 0; \qquad \frac{\partial \mathcal{L}}{\partial K_{t+1}^h} \leq 0; K_{t+1}^h \geq 0; K_{t+1}^h \frac{\partial \mathcal{L}}{\partial K_{t+1}^h} = 0$$
$$\frac{\partial \mathcal{L}}{\partial K_{t+1}^f} \leq 0; K_{t+1}^f \geq 0; K_{t+1}^f \frac{\partial \mathcal{L}}{\partial K_{t+1}^f} = 0; \qquad \frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} \geq 0; \zeta_{1,t} \geq 0; \zeta_{1,t} \frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} = 0$$

For a migrant receiving and sending global economy, additional necessary economic conditions are:

$$C_t^f > 0; K_{t+1}^h > 0; K_{t+1}^f > 0$$

which says that foreign consumption ($C_t^f$) and capital stocks of both home and foreign economies at the next period ($K_{t+1}^h$, $K_{t+1}^f$) must be positive. For home labour consumption ($C_t^M$), migrant consumption ($C_t^h$), migrant labour and home labour supply ($M_t$, $N_t^h$), the complementary slackness conditions allow us to consider the following three situations:
1) no migration ($C_t^M = 0$, $M_t = 0$, $C_t^h > 0$, $N_t^h > 0$);
2) full migration ($C_t^M > 0$, $M_t > 0$, $C_t^h = 0$, $N_t^h = 0$);
3) positive migration ($C_t^M > 0$, $M_t > 0$, $C_t^h > 0$, $N_t^h > 0$).

## 2.2 No migration

After imposing the positivity conditions featuring in the no-migration case into the conditions of the Kuhn-Tucker theorem, the complementary slackness conditions are transformed to:

$$C_t^f > 0; \frac{\partial \mathcal{L}}{\partial C_t^f} = 0; \qquad C_t^M = 0; \frac{\partial \mathcal{L}}{\partial C_t^M} \leq 0; \qquad C_t^h > 0; \frac{\partial \mathcal{L}}{\partial C_t^h} = 0;$$
$$M_t = 0; \frac{\partial \mathcal{L}}{\partial M_t} \leq 0; \qquad N_t^h > 0; \frac{\partial \mathcal{L}}{\partial N_t^h} = 0; \qquad K_{t+1}^h > 0; \frac{\partial \mathcal{L}}{\partial K_{t+1}^h} = 0$$
$$K_{t+1}^f > 0; \frac{\partial \mathcal{L}}{\partial K_{t+1}^f} = 0; \qquad \frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} \geq 0; \zeta_{1,t} \geq 0; \zeta_{1,t}\frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} = 0$$

These conditions can be shown as the following system of equations.

$$\{\frac{\partial \mathcal{L}}{\partial C_t^f}\}: \qquad (C_t^f)^{-\eta^f} - \zeta_{1,t} = 0 \tag{2.2.1}$$
$$\{\frac{\partial \mathcal{L}}{\partial C_t^h}; \frac{\partial \mathcal{L}}{\partial C_t^M}\}: \qquad (C_t^h)^{-\eta^h} - \zeta_{1,t} = 0 \tag{2.2.2}$$
$$\{\frac{\partial \mathcal{L}}{\partial M_t}\}: \qquad -(N_t^h)^{\frac{1}{\nu^h}} + \zeta_{1,t}(\frac{\partial Y_t^f}{\partial M_t} - \chi) \leq 0 \tag{2.2.3}$$
$$\{\frac{\partial \mathcal{L}}{\partial N_t^h}\}: \qquad -(N_t^h)^{\frac{1}{\nu^h}} + \zeta_{1,t}\frac{\partial Y_t^h}{\partial N_t^h} = 0 \tag{2.2.4}$$
$$\{\frac{\partial \mathcal{L}}{\partial K_{t+1}^h}\}: \qquad -\zeta_{1,t} + \beta[\zeta_{1,t+1}(\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} + 1 - \delta^h)] = 0 \tag{2.2.5}$$
$$\{\frac{\partial \mathcal{L}}{\partial K_{t+1}^f}\}: \qquad -\zeta_{1,t} + \beta[\zeta_{1,t+1}(\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} + 1 - \delta^f)] = 0 \tag{2.2.6}$$
$$\{\frac{\partial \mathcal{L}}{\partial \zeta_{1,t}}\}: \qquad Y_t^f + Y_t^h = C_t^f + C_t^h - (1-\delta^h)K_t^h + K_{t+1}^h - (1-\delta^f)K_t^f + K_{t+1}^f + CM_t \tag{2.2.7}$$

The stationary-state equilibrium imposes the following conditions:

$$\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} = \frac{\partial Y_t^h}{\partial K_t^h}; \frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} = \frac{\partial Y_t^f}{\partial K_t^f}; \zeta_{1,t+1} = \zeta_{1,t}; M_{t+1} = M_t = 0; \tag{2.2.8}$$

Inserting the above conditions into the system of equation (eqs.2.2.1 - 2.2.7), we derive the following outcomes:

$$\zeta_{1,t} = (C_t^h)^{-\eta^h} = (C_t^f)^{-\eta^f} = \zeta_{1,t+1} = (C_{t+1}^h)^{-\eta_h} = (C_{t+1}^f)^{-\eta^f}; \tag{2.2.9}$$

$$\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} + 1 - \delta^h = \frac{\partial Y_t^h}{\partial K_t^h} + 1 - \delta^h = \frac{1}{\beta}; \tag{2.2.10}$$

$$\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} + 1 - \delta^f = \frac{\partial Y_t^f}{\partial K_t^f} + 1 - \delta^f = \frac{1}{\beta}; \tag{2.2.11}$$

$$\frac{\partial Y_t^f}{\partial M_t} - \chi \leq \frac{\partial Y_t^h}{\partial N_t^h}; \tag{2.2.12}$$

Eqs.2.2.9 - 2.2.11 state the main features of the stationary-state equilibrium. Eq.2.2.9 shows the smooth consumption plan the global dictator makes for its citizens. Eqs.2.2.10 and 2.2.11 state the stationary equilibrium returns of capital in both the foreign and home economies.

Eq.2.2.12 is a condition that specifically stems from our set-up of no migration, which is called the "No Migration Condition". When the marginal productivity of remaining at home remains greater than the marginal profit of migrating to the foreign economy (the difference between the marginal product of migrant labour and the variable cost of migration), there will be no migration.

Under the condition of No Migration, the equilibrium state of welfare is shown as the following utility function:

$$U(C_t^h, N_t^h) + U(C_t^f, N_t^f) = \frac{(C_t^h)^{1-\eta^h} - 1}{1 - \eta^h} + \frac{(C_t^f)^{1-\eta^f} - 1}{1 - \eta^f} - \frac{(N_t^h)^{1+\frac{1}{\nu^h}}}{1 + \frac{1}{\nu^h}} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1 + \frac{1}{\nu^f}} \tag{2.2.13}$$

Global welfare is a simple summation of the two autarkic countries' welfare. Essentially, this equilibrium coincides with the global equilibrium of the completely symmetric two-country world, which involves no migration, and thus no interaction between the two countries in the globally optimal equilibrium.

## 2.3 Full migration

Under the assumptions that $C_t^M > 0$, $M_t > 0$, $C_t^h = 0$, and $N_t^h = 0$, the transformed conditions featuring the case of full migration are:

$$C_t^f > 0; \frac{\partial \mathcal{L}}{\partial C_t^f} = 0; \qquad C_t^M > 0; \frac{\partial \mathcal{L}}{\partial C_t^M} = 0; \qquad C_t^h = 0; \frac{\partial \mathcal{L}}{\partial C_t^h} \leq 0;$$
$$M_t > 0; \frac{\partial \mathcal{L}}{\partial M_t} = 0; \qquad N_t^h = 0; \frac{\partial \mathcal{L}}{\partial N_t^h} \leq 0; \qquad K_{t+1}^h > 0; \frac{\partial \mathcal{L}}{\partial K_{t+1}^h} = 0;$$
$$K_{t+1}^f > 0; \frac{\partial \mathcal{L}}{\partial K_{t+1}^f} = 0; \qquad \frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} \geq 0; \zeta_{1,t} \geq 0; \zeta_{1,t} \frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} = 0$$

The complementary slackness conditions are therefore:

$$\{\frac{\partial \mathcal{L}}{\partial C_t^f}\}: \qquad (C_t^f)^{-\eta^f} - \zeta_{1,t} = 0 \tag{2.3.1}$$
$$\{\frac{\partial \mathcal{L}}{\partial C_t^h}; \frac{\partial \mathcal{L}}{\partial C_t^M}\}: \qquad (C_t^h + C_t^M)^{-\eta^h} - \zeta_{1,t} = 0 \tag{2.3.2}$$
$$\{\frac{\partial \mathcal{L}}{\partial M_t}\}: \qquad -(N_t^h + M_t)^{\frac{1}{\nu^h}} + \zeta_{1,t}(\frac{\partial Y_t^f}{\partial M_t} - \chi) = 0 \tag{2.3.3}$$
$$\{\frac{\partial \mathcal{L}}{\partial N_t^h}\}: \qquad -(N_t^h + M_t)^{\frac{1}{\nu^h}} + \zeta_{1,t}\frac{\partial Y_t^h}{\partial N_t^h} \leq 0 \tag{2.3.4}$$
$$\{\frac{\partial \mathcal{L}}{\partial K_{t+1}^h}\}: \qquad -\zeta_{1,t} + \beta[\zeta_{1,t+1}(\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} + 1 - \delta^h)] = 0 \tag{2.3.5}$$
$$\{\frac{\partial \mathcal{L}}{\partial K_{t+1}^f}\}: \qquad -\zeta_{1,t} + \beta[\zeta_{1,t+1}(\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} + 1 - \delta^f)] = 0 \tag{2.3.6}$$
$$\{\frac{\partial \mathcal{L}}{\partial \zeta_{1,t}}\}: \qquad Y_t^f + Y_t^h = C_t^f + C_t^h + C_t^M - (1-\delta^h)K_t^h + K_{t+1}^h - (1-\delta^f)K_t^f + K_{t+1}^f + CM_t \tag{2.3.7}$$

To present the conditions of full migration in this economy, the positivity conditions ($\zeta_{1,t} \geq 0$) and the stationary-state equilibrium condition in eq.2.2.8 have to be met. The system eqs.2.3.1 - 2.3.7 can be resolved as:

$$\zeta_{1,t} = (C_t^f)^{-\eta^f} = (C_t^M)^{-\eta^h} \geq 0 \tag{2.3.8}$$
$$\zeta_{1,t} = (C_t^f)^{-\eta^f} = (C_t^M))^{-\eta^h} = \zeta_{1,t+1} = (C_{t+1}^f)^{-\eta^f} = (C_{t+1}^M)^{-\eta^h}; \tag{2.3.9}$$
$$\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} + 1 - \delta^h = \frac{\partial Y_t^h}{\partial K_t^h} + 1 - \delta^h = \frac{1}{\beta}; \tag{2.3.10}$$
$$\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} + 1 - \delta^f = \frac{\partial Y_t^f}{\partial K_t^f} + 1 - \delta^f = \frac{1}{\beta}; \tag{2.3.11}$$
$$\frac{\partial Y_t^f}{\partial M_t} - \chi \geq \frac{\partial Y_t^h}{\partial N_t^h}; \tag{2.3.12}$$

Eq.2.3.8 shows that the positivity condition is met as the consumption of migrants is positive under the assumption of full migration. Eqs.2.3.9 - 2.3.11 are the stationary equilibrium conditions: the former gives the smooth consumption plan and the latter two give the equilibrium return of capital in the two countries.

Analogous to **Section 2.1**, eq.2.3.12 is the "Full Migration condition", which says that we have full migration if and only if the marginal profit of migration remains bigger than the marginal productivity of home labour after all home labour migrate.

The global utility at time $t$ can be shown as:

$$U(C_t^h + C_t^M, N_t^h + M_t) + U(C_t^f, N_t^f) = \frac{(C_t^M)^{1-\eta^h} - 1}{1-\eta^h} - \frac{(M_t)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} + \frac{(C_t^f)^{1-\eta^f} - 1}{1-\eta^f} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \tag{2.3.13}$$

with the utility of home country consumption fully determined by the migrants in the foreign economy, since the utility of home domestic consumption and the disutility of home labour supply equal zero.

## 2.4 Positive but not full migration

In this case, all the consumption and labour supply variables are positive. Thus we generate the following complementary slackness conditions.

$$\begin{aligned}
&C_t^f > 0; \frac{\partial \mathcal{L}}{\partial C_t^f} = 0; && C_t^M > 0; \frac{\partial \mathcal{L}}{\partial C_t^M} = 0; && C_t^h > 0; \frac{\partial \mathcal{L}}{\partial C_t^h} = 0;\\
&M_t > 0; \frac{\partial \mathcal{L}}{\partial M_t} = 0; && N_t^h > 0; \frac{\partial \mathcal{L}}{\partial N_t^h} = 0; && K_{t+1}^h > 0; \frac{\partial \mathcal{L}}{\partial K_{t+1}^h} = 0;\\
&K_{t+1}^f > 0; \frac{\partial \mathcal{L}}{\partial K_{t+1}^f} = 0; && && \frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} \geq 0; \zeta_{1,t} \geq 0; \zeta_{1,t}\frac{\partial \mathcal{L}}{\partial \zeta_{1,t}} = 0
\end{aligned}$$

The central planner has to optimise the consumption of all citizens in both foreign and home countries.

$$\{C_t^f\}: \qquad (C_t^f)^{-\eta^f} - \zeta_{1,t} = 0 \tag{2.4.1}$$

$$\{C_t^M, C_t^h\}: \qquad (C_t^h + C_t^M)^{-\eta^h} - \zeta_{1,t} = 0 \tag{2.4.2}$$

Therefore

$$\zeta_{1,t} = (C_t^f)^{-\eta^f} = (C_t^h + C_t^M)^{-\eta^h} \tag{2.4.3}$$

This says the marginal utility of aggregate consumption in each country is set to be the same.

Because the consumption of either foreign or home household is positive in this situation, the positivity condition of $\zeta_{1,t}$ is met.

Secondly, a decision is made to optimally allocate the home economy's labour supply ($L^h$) between the home ($N_t^h$) and foreign economy ($M_t$), see eq.2.1.1. The central planner will set the foreign economy's employment at its maximum ($L^f$).

$$\{M_t\}: \qquad -(N_t^h + M_t)^{\frac{1}{\nu^h}} + \zeta_{1,t}(\frac{\partial Y_t^f}{\partial M_t} - \chi) = 0 \tag{2.4.4}$$

$$\{N_t^h\}: \qquad -(N_t^h + M_t)^{\frac{1}{\nu^h}} + \zeta_{1,t}\frac{\partial Y_t^h}{\partial N_t^h} = 0 \tag{2.4.5}$$

Given the predetermined current capital stocks, the optimality conditions for the capital stocks at $t+1$ are

$$\{K_{t+1}^h\}: \qquad -\zeta_{1,t} + \beta[\zeta_{1,t+1}(\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} + 1 - \delta^h)] = 0 \tag{2.4.6}$$

$$\{K_{t+1}^f\}: \qquad -\zeta_{1,t} + \beta[\zeta_{1,t+1}(\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} + 1 - \delta^f)] = 0 \tag{2.4.7}$$

Solving the system of equations from 2.4.1 to 2.4.7 with the equilibrium conditions in eq.2.2.8,

$$\zeta_{1,t} = (C_t^f)^{-\eta_f} = (C_t^h + C_t^M)^{-\eta^h} = \zeta_{1,t+1} = (C_{t+1}^f)^{-\eta^f} = (C_{t+1}^h + C_{t+1}^M)^{-\eta^h} \tag{2.4.8}$$

$$\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} + 1 - \delta^f = \frac{\partial Y_t^f}{\partial K_t^f} + 1 - \delta^f = \frac{1}{\beta} \tag{2.4.9}$$

$$\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} + 1 - \delta^h = \frac{\partial Y_t^h}{\partial K_t^h} + 1 - \delta^h = \frac{1}{\beta} \tag{2.4.10}$$

$$\frac{\partial Y_t^f}{\partial M_t} - \chi = \frac{(N_t^h + M_t)^{\frac{1}{\nu^h}}}{\zeta_{1,t}} \tag{2.4.11}$$

$$\frac{\partial Y_t^h}{\partial N_t^h} = \frac{(N_t^h + M_t)^{\frac{1}{\nu^h}}}{\zeta_{1,t}} \tag{2.4.12}$$

In the stationary-state equilibrium with positive migration (but not full), the welfare of the global economy will be maximised with migration when eqs.2.4.9 - 2.4.12 are satisfied. The above solutions can be further simplified to two equivalence conditions between home and foreign production factors. Specifically, eqs.2.4.9 and 2.4.10 shows that in an equilibrium with positive migration, the marginal productivity of home and foreign capital (after depreciation) must be equal, while eqs.2.4.11 and 2.4.12 give a similar equivalence between marginal productivity of labour (after migration cost) in foreign and home economies. These "capital and labour equilibrium conditions" are:

$$\frac{\partial Y_t^f}{\partial K_t^f} = \frac{\partial Y_t^h}{\partial K_t^h} + (\delta^f - \delta^h) \tag{2.4.13}$$

$$\frac{\partial Y_t^f}{\partial M_t} - \chi = \frac{\partial Y_t^h}{\partial N_t^h} \tag{2.4.14}$$

In general, migration will maximise welfare when the net marginal productivity of capital in both economies converge and the marginal profit from migration approaches the marginal product of home labour.

An important understanding drawn from **Sections 2.1 to 2.4** is that production technology determines the marginal productivity of production factors, and is a key determinant of migration. At the same time, as shown in next section (eqs.2.4.17 to 2.4.20), the production technology is determined by both its structural parameters and the two economies' capital-labour endowments.

To consider the role of heterogeneity, there are many possible causes that can be drawn from the derived two equilibrium conditions. These include any different parameters (one or several simultaneously) and the capital-labour endowment individually in the two economies.

In the following subsections, we will only explore two different *ex-ante* autarky cases: 1) similar production functions with different *ex-ante* capital-labour ratios; 2) different production functions with the same *ex-ante* capital-labour ratio.[8] In so doing, we hope to capture the key thrusts that determine optimal migration, and to compare the global optimum position with other institution designs apart from a global central planner.

**Proposition 2.4.1** *Heterogeneity in the capital-labour ratios or in the production functions leads to optimal migration.*

### 2.4.1 Similar production functions with different capital-labour ratios

Suppose both countries have the same form of a constant elasticity of substitution (CES) production functions with capital and labour inputs as the following[9]

$$Y_t^f = Z_t^f[\varphi^f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f + M_t)^{\mu^f}]^{\frac{1}{\mu^f}} \tag{2.4.15}$$

$$Y_t^h = Z_t^h[\varphi^h (K_t^h)^{\mu^h} + (1-\varphi^h)(L^h - M_t)^{\mu^h}]^{\frac{1}{\mu^h}} \tag{2.4.16}$$

[8]The cases of the same production functions with the same *ex-ante* capital-labour ratios, and different production functions with different *ex-ante* capital-labour ratios will be ignored because the former case will provide no incentive to migrate and the latter is a combination of the sub-cases.

[9]The CES function provides a positive definite Hessian matrix, which is necessary for maximised welfare.

In autarky, both countries' output ($Y_t^f$,$Y_t^h$) rely only on their own domestic labour supply ($N_t^f$, $L^h$) and capital stock ($K_t^f$, $K_t^h$). Labour homogeneity applies in production so that the marginal product of a migrant would equal the marginal product of foreign domestic labour in the foreign economy production function. $\varphi^f$, $\varphi^h$ are income share parameters that can differ. The elasticity of substitution between labour and capital in the foreign and home countries can differ and are $\frac{1}{1-\mu^f}$ and $\frac{1}{1-\mu^h}$, respectively. The productivity shocks are set as $Z_t^f$ and $Z_t^h$ in the foreign and home countries of migrants. The autarky capital-labour ratios are different $\frac{K_t^f}{N_t^f} \neq \frac{K_t^h}{L^h}$.

The marginal product of capital of both economies can be shown in terms of their capital-labour ratios.

$$\begin{aligned}
\frac{\partial Y_t^f}{\partial K_t^f} &= Z_t^f \varphi^f (K_t^f)^{\mu^f-1}[\varphi_f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f + M_t)^{\mu^f}]^{\frac{1}{\mu^f}-1} \\
&= \varphi^f (Z_t^f)^{\mu^f} (\frac{K_t^f}{N_t^f + M_t})^{\mu^f-1} (\frac{Y_t^f}{N_t^f + M_t})^{1-\mu^f} \qquad (2.4.17) \\
\frac{\partial Y_t^h}{\partial K_t^h} &= \varphi_h (Z_t^h)^{\mu^h} (\frac{K_t^h}{L^h - M_t})^{\mu^h-1} (\frac{Y_t^h}{L^h - M_t})^{1-\mu^h} \qquad (2.4.18) \\
\text{where:} \quad \frac{Y_t^f}{N_t^f + M_t} &= Z_t^f [\varphi^f (\frac{K_t^f}{N_t^f + M_t})^{\mu^f} + (1-\varphi^f)]^{\frac{1}{\mu^f}} \qquad (2.4.19) \\
\text{and:} \quad \frac{Y_t^h}{L^h - M_t} &= Z_t^h [\varphi^h (\frac{K_t^h}{L^h - M_t})^{\mu^h} + (1-\varphi^h)]^{\frac{1}{\mu^h}} \qquad (2.4.20)
\end{aligned}$$

To obtain a positive optimal level of migration ($M^*$),[10] the equilibrium condition of the two economies' rate of return on capital (eqs.2.4.13) has to be met, as well as the equilibrium relation between marginal productivity of a migrant and of home labour (eq.2.4.14):

**Condition A**: From eq.2.4.13,

$$\frac{\partial Y_t^f}{\partial K_t^f} = \frac{\partial Y_t^h}{\partial K_t^h} + (\delta^f - \delta^h)$$

This is a necessary condition for a global optimal equilibrium with positive migration. When there is a difference of the capital-labour ratio in the two countries (see eqs.2.4.17 - 2.4.20), migration will improve productivity and welfare by decreasing the foreign economy capital-labour ratio (with a relative excess endowment of capital) and increasing the capital-labour ratio at home (with a relative excess endowment of labour).

Therefore, by observing the optimal capital equilibrium conditions of each economy (eqs.2.4.9 and 2.4.10), we find the domain of *ex-ante* capital-labour ratios of the two economies that would support positive optimal migration.

[10] Henceforth, a variable with an asterisk " * " represents its optimal equilibrium value.

From eq.2.4.9, $\beta(1 + \frac{\partial Y_t^f}{\partial K_t^f} - \delta^f) = 1$ is met in equilibrium. If the initial marginal return of foreign economy capital is smaller than this equilibrium rate,

$$\frac{\partial Y_t^f}{\partial K_t^f} < \frac{1}{\beta} + \delta^f - 1 \tag{2.4.21}$$

the foreign capital stock is too large relative to its *ex-ante* domestic labour force so that an inflow of migrants can quickly increase the marginal product of foreign capital ($\frac{\partial^2 Y_t^f}{\partial K_t^f \partial M_t} > 0$), eventually achieving the optimal level $M^*$.

From eq.2.4.10, $\beta(1+\frac{\partial Y_t^h}{\partial K_t^h} - \delta^h) = 1$ occurs at the stationary equilibrium state. If the capital endowment is initially scarce in the home country, its marginal product of capital will exceed its equilibrium rate:

$$\frac{\partial Y_t^h}{\partial K_t^h} > \frac{1}{\beta} + \delta^h - 1 \tag{2.4.22}$$

Assuming capital is immobile between two countries and slow to accumulate or decumulate, the adjustment of labour supply is the quickest way to achieve the optimal marginal product of capital. With increasing emigration, the home labour force declines and the marginal productivity of capital decreases ($\frac{\partial^2 Y_t^h}{\partial K_t^h \partial N_t^h} > 0$).

This condition provides a theoretical basis for Borjas (1989)'s observation that some countries have an incentive to regulate their emigrants' departure, with the aim of achieving the optimal efficiency level of the capital-labour ratio. This condition can also be interpreted as immigrants resolving the initially sub-optimal capital-labour ratios for the two economies in the global economy so that both may expect higher welfare.

**Condition B** from eq.2.4.14:

$$\frac{\partial Y_t^f}{\partial M_t} = \frac{\partial Y_t^h}{\partial N_t^h} + \chi \tag{2.4.23}$$

More migration would occur if the net marginal product of a migrant is greater than the marginal cost of a migrant. The indifference between sending another migrant to the foreign economy or remaining at home is another necessary condition of the global equilibrium with positive migration. Together with **Condition A**, the optimal equilibrium of this two-country world is at the point where the two conditions hold that represent the optimal allocation of capital and labour of the two countries.

**Conditions A** and **B** enable a description of how the initial global sub-optimal allocation of capital and labour is corrected by optimal levels of migration.

To observe how different capital-labour ratios would affect the optimal level of migration, we need to derive the two conditions in terms of the equilibrium capital-labour ratios ($\frac{K_t^h}{L^h - M_t}$,

$\frac{K_t^f}{N_t^f+M_t})$.

Based on eqs.2.4.17 - 2.4.20 evaluated at their expected value, **Condition A** gives

$$\varphi^f(\frac{K^{f*}}{L^f+M^*})^{\mu^f-1}\{[\varphi^f(\frac{K^{f*}}{L^f+M^*})^{\mu^f}+(1-\varphi^f)]^{\frac{1}{\mu^f}}\}^{1-\mu^f}$$
$$=\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h-1}\{[\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h}+(1-\varphi^h)]^{\frac{1}{\mu^h}}\}^{1-\mu^h}+(\delta^f-\delta^h)$$

or

$$\frac{K^{f*}}{L^f+M^*}=\{k^f\}_1=\{\frac{\{\frac{\varphi^h[\varphi^h+(1-\varphi^h)(\frac{K^{h*}}{L^h-M^*})^{-\mu^h}]^{\frac{1-\mu^h}{\mu^h}}+(\delta^f-\delta^h)}{\varphi^f}\}^{\frac{\mu^f}{1-\mu^f}}-\varphi^f}{1-\varphi^f}\}^{-\frac{1}{\mu^f}} \tag{2.4.24}$$

Under the property $\{\mu\}\in(-\infty,0)\cup(0,1]$ in the CES production function, $\frac{\partial\{k^f\}_1}{\partial\frac{K^{h*}}{L^h-M^*}}>0$ and $\frac{\partial^2\{k^f\}_1}{\partial(\frac{K^{h*}}{L^h-M^*})^2}<0$ imply that **Condition A** is a concave curve in the $\{\frac{K^{h*}}{L^h-M^*},\frac{K^{f*}}{L^f+M^*}\}$ space. **Condition A** shows the foreign capital labour ratios at different levels of the home capital labour ratio from a capital equilibrium perspective.

If the initial foreign capital-labour ratio (when $M_t=0$) exceeds $\{k^f\}_1$, the planner will allow positive migration. This will occur if 2.4.21 and/or 2.4.22 hold.

Eq.2.4.21, when $M_t=0$, can be shown as

$$\varphi^f(\frac{K_t^f}{N_t^f})^{\mu^f-1}\{[\varphi^f(\frac{K_t^f}{N_t^f})^{\mu^f}+(1-\varphi^f)]^{\frac{1}{\mu^f}}\}^{1-\mu^f}\leq\frac{1}{\beta}+\delta^f-1$$
$$\frac{K_t^f}{N_t^f}>\{k^f\}^*=\{\frac{[\frac{\frac{1}{\beta}-1+\delta^f}{\varphi^f}]^{\frac{\mu^f}{1-\mu^f}}-\varphi^f}{1-\varphi^f}\}^{-\frac{1}{\mu^f}} \tag{2.4.25}$$

$\{k^f\}^*$ in eq.2.4.25 gives the optimal capital-labour ratio of the foreign economy. The *ex-ante* foreign capital-labour ratio (when $M_t=0$) has to be greater than $\{\underline{k^f}\}_1$ for the planner to allow for migration.

Eq.2.4.22, when $M_t=0$, becomes

$$\varphi^h[\varphi^h+(1-\varphi^h)(\frac{L^h}{K_t^h})^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}>\frac{1}{\beta}-1+\delta^h$$
$$\frac{L^h}{K_t^h}>[\frac{(\frac{\frac{1}{\beta}+1-\delta^h}{\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}{1-\varphi^h}]^{\frac{1}{\mu^h}}$$
$$\frac{K_t^h}{L^h}<\{k^h\}^*=[\frac{(\frac{\frac{1}{\beta}+1-\delta^h}{\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}{1-\varphi^h}]^{-\frac{1}{\mu^h}} \tag{2.4.26}$$

By increasing migration, $\frac{K_t^h}{L^h-M_t}$ will converge to $\{k^h\}^*$. The *ex-ante* home capital-labour ratio (when $M_t = 0$) has to be smaller than the equilibrium level to incentivize emigration from the home economy.

The labour equilibrium condition, **Condition B**, requires the net marginal gain of migration to be equal to the marginal cost. Again, we start by showing the marginal products of foreign and home labour as functions of their capital-labour ratios.

$$\begin{aligned}\frac{\partial Y_t^f}{\partial N_t^f + M_t} =&(1-\varphi^f)(Z_t^f)^{\mu^f}(\frac{Y_t^f}{N_t^f + M_t})^{1-\mu^f}\\ \frac{\partial Y_t^h}{\partial L^h - M_t} =&(1-\varphi^h)(Z_t^h)^{\mu^h}(\frac{Y_t^h}{L^h - M_t})^{1-\mu^h}\end{aligned}$$

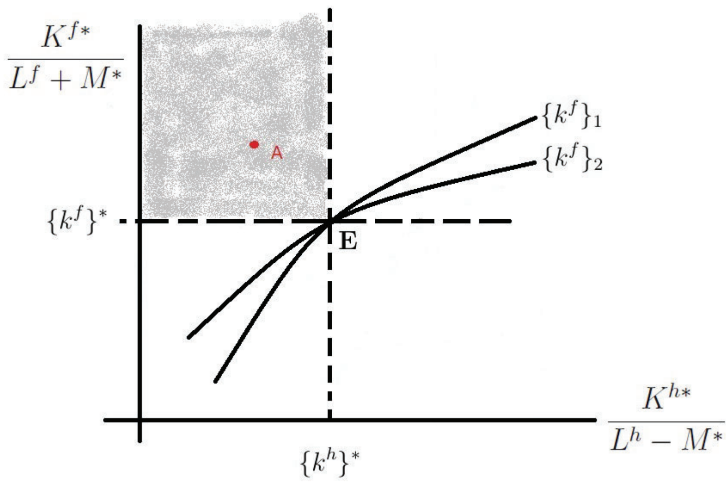
In equilibrium, substituting eqs.2.4.19 and 2.4.20 into these gives:

$$\begin{aligned}&(1-\varphi^f)\{[\varphi^f(\frac{K^{f*}}{L^f+M^*})^{\mu^f}+(1-\varphi^f)]^{\frac{1}{\mu^f}}\}^{1-\mu^f}\\ =&(1-\varphi^h)\{[\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h}+(1-\varphi^h)]^{\frac{1}{\mu^h}}\}^{1-\mu^h}+\chi\end{aligned}$$

$$\frac{K^{f*}}{L^f+M^*} =\{k^f\}_2 = \{\frac{[\frac{(1-\varphi^h)\{[\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h}+(1-\varphi^h)]^{\frac{1}{\mu^h}}\}^{1-\mu^h}+\chi}{(1-\varphi^f)}]^{\frac{\mu^f}{1-\mu^f}}-(1-\varphi^f)}{\varphi^f}\}^{\frac{1}{\mu^f}} \quad (2.4.27)$$

Note under the properties of the CES production function that $\mu \in (-\infty, 1]$, $\frac{\partial\{k^f\}_2}{\partial\frac{K^{h*}}{L^h-M^*}} > 0$ and $\frac{\partial^2\{k^f\}_2}{\partial(\frac{K^{h*}}{L^h-M^*})^2} < 0$ shows that this condition is also a concave curve. The curve of $\{k^f\}_2$ in **Figure 2.1** shows all possible home capital labour ratios ($\frac{K^{h*}}{L^h-M^*}$) at different levels of the foreign capital labour ratio from a labour equilibrium perspective.

Figure 2.1: Migration capital-labour ratios




An initial capital-labour ratio difference (such as at point **A** in **Figure 2.1**) implies a benefit from migration since the foreign and home capital-labour ratios are situated in the grey domain in **Figure 2.1** bounded by $\{k^f\}^*$, $\{k^h\}^*$. If positive migration is optimal, the general equilibrium point must be at **E** where the capital equilibrium and labour equilibrium conditions meet.

The global utility in this case is the sum of the two countries' utility:

$$\begin{aligned}U =& U(C_t^h + C_t^M, N_t^h + M_t) + U(C_t^f, N_t^f) \\ =& \frac{(C_t^h + C_t^M)^{1-\eta^h} - 1}{1-\eta^h} + \frac{(C_t^f)^{1-\eta^f} - 1}{1-\eta^f} - \frac{(N_t^h + M_t)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}}\end{aligned}$$

Substituting eqs.A.1.16 and A.1.17 from **Appendices A.1.1** into this, the optimum indirect utility (with capital adjustment in the short run assumed very slow and negligible) is

$$\begin{aligned}IU^* =& \frac{[\frac{(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h}+(1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}}{(L^h)^{\frac{1}{\nu^h}}}]^{\frac{1-\eta^h}{\eta^h}} - 1}{1-\eta^h} + \frac{[\frac{(1-\varphi^f)[\varphi^f(\frac{K^{f*}}{L^f+M^*})^{\mu^f}+(1-\varphi^f)]^{\frac{1-\mu^f}{\mu^f}}}{(L^f)^{\frac{1}{\nu^f}}}]^{\frac{1-\eta^f}{\eta^f}} - 1}{1-\eta^f} \\ &- \frac{(L^h)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} - \frac{(L^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \qquad (2.4.28)\end{aligned}$$

Remembering that both eq.2.4.24 and 2.4.27 give a positive relationship between the optimal foreign and home capital-labour ratios within the domain of shaded area in **Figure 2.1**, the above stationary equilibrium aggregate indirect utility can be reduced to a positive function of the optimal home capital labour ratio $\frac{K^{h*}}{L^h-M^*}$.

$$IU^* = f(\frac{K^{h*}}{L^h - M^*})$$

A further elaboration on how $\frac{K^{h*}}{L^h-M^*}$ could affect the welfare of two countries can be given by observing its effects on the output per capita in the two countries. In the domain of eq.2.4.25 where $\frac{K_t^f}{N_t^f+M_t}$ decreases to the optimum level $\{k^f\}^*$ with the inflow of migrants, the foreign indirect utility will increase as the foreign output per capita increases to the stationary state.[11] In the domain of eq.2.4.26 where $\frac{K_t^h}{L_t^h-M_t}$ increases to the optimum level $\{k^h\}^*$, the home indirect utility will increase as the home output per capita increases.

In **Figure 2.1**, migration can quickly shorten the distance between the original autarky point (such as point **A**) and the equilibrium point, if the initial point of autarky is situated in the shaded area of **Figure 2.1**. A general conclusion we draw from eq.2.4.28 is that global welfare with $M^* > 0$ would be *superior* than autarky in the short term when capital

[11] Based on eqs. 2.4.17 - 2.4.20 and 2.4.24, the foreign output per capita is

$$\frac{Y^{f*}}{N^{f*}+M^*} = \mathbf{\{}1 - (\varphi^f)^{\frac{1}{1-\mu^f}}\{\varphi^h[\varphi^h + (1-\varphi^h)(\frac{K^{h*}}{L^h - M^*})^{-\mu^h}]^{\frac{1-\mu^h}{\mu^h}} + (\delta^f - \delta^h)\}^{\frac{\mu^f}{\mu^f-1}}\mathbf{\}}^{-\mu^f}$$

which gives the positive relationship between $M_t$ and $\frac{Y^{f*}}{N^{h*}+M^*}$ in the defined domain of eq.2.4.25.

adjustment speed is negligible.

$$U(C^{h*}+C^{M*},N^{h*}+M^{*})+U(C^{f*},N^{f*})\geq U(C_t^h,L^h)+U(C_t^f,N_t^f) \tag{2.4.29}$$

The two sides are equal in the no-migration case (**Section 2.1**) when the capital-labour endowment is situated in the unshaded area in **Figure 2.1**.

However, with the capital stock of both economies being endogenous, the optimum global utility over the long term with capital adjustment can be shown by substituting eqs.A.1.20 and A.1.21 into 2.4.28:

$$IU^{*}=\frac{\left[\frac{(1-\varphi^h)\left[\frac{1-\varphi^h}{\left(\frac{1+\delta^h\beta-\beta}{\beta\varphi^h}\right)^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}\right]^{\frac{1-\mu^h}{\mu^h}}\frac{1+\delta^h\beta-\beta}{\beta\varphi^h}}{(L^h)^{\frac{1}{\nu_h}}}\right]^{\frac{1-\eta_h}{\eta^h}}-1}{1-\eta^h}+\frac{\left[\frac{(1-\varphi^f)\left[\frac{1-\varphi^f}{\left(\frac{1+\delta^f\beta-\beta}{\beta\varphi^f}\right)^{\frac{\mu^f}{1-\mu^f}}-\varphi^f}\right]^{\frac{1-\mu^f}{\mu^f}}\frac{1+\delta^f\beta-\beta}{\beta\varphi^f}}{(L^f)^{\frac{1}{\nu^f}}}\right]^{\frac{1-\eta^f}{\eta^f}}-1}{1-\eta^f}$$
$$-\frac{(L^h)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}}-\frac{(L^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \tag{2.4.30}$$

Sooner or later, the capital-labour ratio will decrease to the equilibrium in the foreign economy while the home capital-labour ratio would self-adjust to its equilibrium. In the stationary state, the indirect utility of this global economy is a constant, indifferent to the level of migration because capital or labour mobility can adjust optimally the capital-labour ratios in the two economies at the same global optimal equilibrium output.

Together with our earlier discussion on the fixed capital cases, when the adjustment speed of capital accumulation is slow in the short term and the two economies have the same production technology, migration could be the better solution to adjust to the global optimum in the short run.

### 2.4.2 Different production functions but the same *ex-ante* capital-labour ratios

In this section, we introduce differentiated production functions for the home and foreign economies of migrants, but assume initial capital-labour ratios are identical. The production functions are:

$$Y_t^f = Z_t^f\{\omega^f(M_t)^{\lambda^f}+(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}} \tag{2.4.31}$$
$$Y_t^h = Z_t^h[\varphi^h(K_t^h)^{\mu^h}+(1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1}{\mu^h}} \tag{2.4.32}$$

According to eq.2.4.31, the foreign country production employs migrant workers ($M_t$), its local labour workers ($N_t^f$) and local capital ($K_t^f$) to produce output ($Y_t^f$). Local labour is assumed to be complementary with local capital. Adhering to Krusell, Ohanian, Ríos-Rull and Violante (2000)'s skill capital complementarity set-up, the foreign aggregate output function in the foreign economy at time $t$ is characterized by a particular locals and migrants production relationship.[12] The set-up is consistent with the fact that the average wage of local workers tends to be higher than the average wage of migrant workers, which represents a local premium.[13] The parameters $(\omega, \varphi)$ represent the income shares. The elasticity of substitution between migrant labour and local capital is assumed to equal the elasticity of substitution between the two types of labour, which is $\frac{1}{1-\lambda^f}$. The foreign elasticity of substitution between local labour and domestic capital is $\frac{1}{1-\mu^f}$. The local labour-capital complementarity in the foreign economy requires $\lambda^f > \mu^f$ so that the elasticity of substitution between migrant and local labour is larger than the elasticity of substitution between local labour and capital, which in turns suggests a higher level of complementarity between local labour and capital.

The local-migrant complementarity in production is motivated by real-world observations.

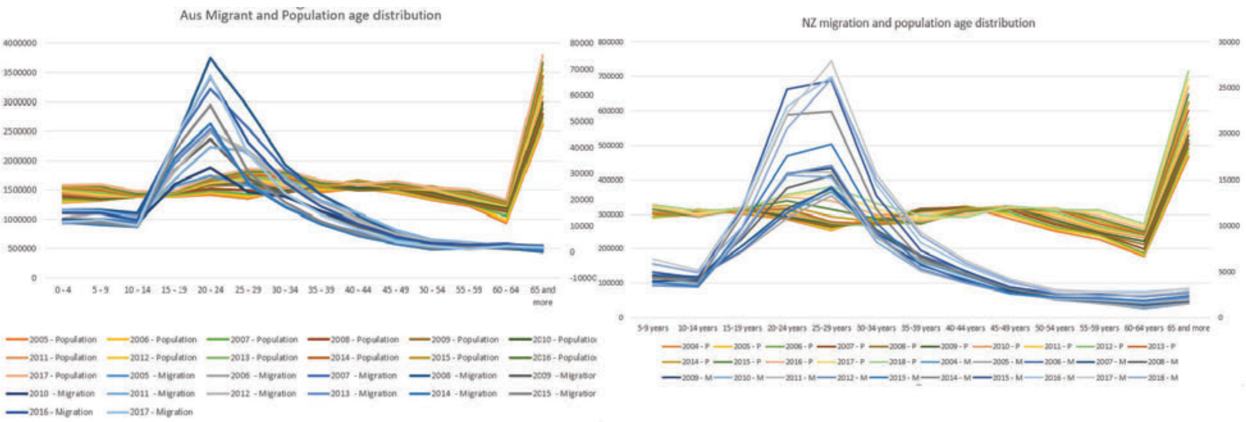
First of all, the evidence on the wage gap between worldwide migrants and locals suggests human capital differences between the two groups (see Hatton and Williamson (1992) for US; Hum and Simpson (2000) for Canada; Brenzel and Reichelt (2017) for German; Breunig, Hasan and Salehin (2013); Islam and Parasnis (2014) for Australia; Khan (2016) for India; and Zhang, Sharpe, Li and Darity (2016) for China). Note that China and India are not traditionally immigrants' destinations. However, internal migration has been substantial in recent decades and the wage gaps between urban citizens and migrants from remote areas have been noted. From an aggregate production function perspective, the difference in wage is captured by different levels of complementarity of the migrant and local labour with local capital.

Second, a common economic challenge for the advanced economies is brought by ageing demographic issues, which can be relieved by technological advancements and/or selective migration. Selective migration has been favoured by foreign advanced economies in recent years. **Figure 2.2** gives the age distribution of Australia and New Zealand population and migrants since 2004-05. It shows that the portion of migrants at a working age (15 - 34 years) has continuously increased in the last decades, while the portion of retirees (aged

[12]Note that Krusell et al. (2000) assume two types of capital: equipment and structures. Our study excludes structures.

[13]The foreign country is assumed to have a more complicated production function than the home country. Compare industrial and agricultural countries, where the former clearly have more diversified demand and more complicated production processes. The complexity of industrial production that delivers higher productivity attracts migrants from agricultural areas through higher wages and better living standards. The inflow of migrants expands the labour market and increases aggregate demand, which further expands production in the industrial country. This intuition can be extended if we add extant theories that study the cost of migration. For example, the cumulative causation, suggested by Georges (1990); Massey (1985, 1990); Myrdal (1957), argues for a declining cost of migration as the size of migration flow increases, which in turn further increases the possible size of migration. Labour mobility itself could initiate a benign cycle in the foreign economy.

Figure 2.2: The age distributions of population and migrants in Australia and New Zealand




NB: The size of population at different age groups is on the left-hand side vertical axis, while the size of migration at the corresponding age groups is on the right-hand side vertical axis.
Data from ABS (2018a,b); RBNZ (2019); SNZ (2018).

60 or more) has picked up in the foreign economies. Compared to senior and experienced domestic workers, the young migrants are carrying relative lower human capital.[14]

As in the previous section, to achieve the global optimum outcome with positive migration, the capital and labour equilibrium conditions, **Condition A and B**, must be satisfied even under the imposed assumption of equal *ex-ante* capital-labour ratios

$$\frac{K_t^f}{L^f} = \frac{K_t^h}{L^h} \tag{2.4.33}$$

where in the centrally planned economies, $L^h = M_t + N_t^h$ and $L^f = N_t^f$.[15]

Using the production functions in eq.2.4.31 and 2.4.32, the capital equilibrium condition (**Condition A**) in eq.2.4.13 becomes

$$\begin{aligned}
&\frac{M^*}{K^{f*}} = \{\overline{m}\}_1 \\
&= \{\frac{\{\frac{\Phi(\frac{K^{h*}}{L^h - M^*})}{\varphi^f(1-\omega^f)[\varphi^f + (1-\varphi^f)(\frac{K^{f*}}{L^f})^{-\mu^f}]^{\frac{\lambda^f - \mu^f}{\mu^f}}}\}^{\frac{\lambda^f}{1-\lambda^f}} - (1-\omega^f)[\varphi^f + (1-\varphi^f)(\frac{K^{f*}}{L^f})^{-\mu^f}]^{\frac{\lambda^f}{\mu^f}}}{\omega^f}\}^{\frac{1}{\lambda^f}}
\end{aligned} \tag{2.4.34}$$

where $\Phi(\frac{K^{h*}}{L^h - M^*}) = \varphi^h[\varphi^h + (1-\varphi^h)(\frac{K^{h*}}{L^h - M^*})^{-\mu^h}]^{\frac{1-\mu^h}{\mu^h}} + (\delta^f - \delta^h)$.

[14]The immigration history has presented an interesting story that from 1881 to 1911, the more urban and educated migrants from the north of Italy tended to land in the agricultural economies of Southern Latin America, while most agricultural northern Italians (60 per cent) migrated to the USA (Hatton and Williamson, 1998).

[15]$M_t$ should be zero in the *ex-ante* scenario.

Now, we turn to **Condition B** (2.4.23) $\frac{\partial Y_t^f}{\partial M_t} = \frac{\partial Y_t^h}{\partial N_t^h} + \chi$, which becomes

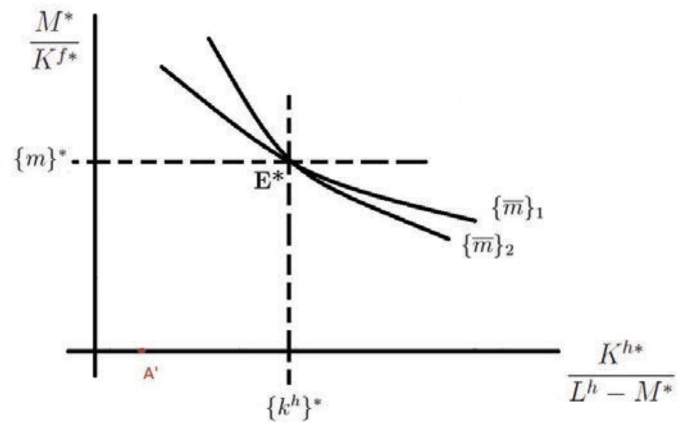
$$\frac{M^*}{K^{f*}} = \{\overline{m}\}_2$$

$$\left\{\frac{\left\{\frac{\left\{\frac{(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h}+(1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}}{\omega^f}+\chi\right\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}\right\}^{\frac{\mu^f}{\lambda^f}} - (1-\varphi^f)(\frac{M^*}{L^f})^{\mu^f}}{\varphi^f}\right\}^{-\frac{1}{\mu^f}} \qquad (2.4.35)$$

Note that $\{\overline{m}\}_1$ and $\{\overline{m}\}_2$ are decreasing convex functions of the *ex-post* home capital per labour $\frac{K_t^h}{L^h-M_t}$ given the CES production function with $\{\mu, \lambda\} \in (-\infty, 1]$.

When the capital and labour equilibrium conditions (2.4.34 and 2.4.35) are both met, the global optimum of this two-country world will occur. Thus solving eqs.2.4.34 and 2.4.35 obtains optimal migration, $\overline{M}$. This is at point **E***($\{k^h\}^*$, $\{m\}^*$) in **Figure 2.3**.

Figure 2.3: Optimal migration in the $\{\frac{M^*}{K^{f*}}, \frac{K^{h*}}{L^h-M^*}\}$ space




When the two economies have the same *ex-ante* capital-labour ratio but different production technologies, migration will be desirable if and only if the *ex-ante* home capital labour ratio is less than its optimum level (e.g. at point **A′**). The local complementarity in the foreign economy's production function will justify immigration as the inflow of immigration will increase the marginal productivity of both foreign capital and domestic labour. However, the upper limit of migration is always constrained by the foreign economy capital stock. The optimal point **E*** will eventually be reached through a positive (but not full) migration.

**Proposition 2.4.2** *If the two economies are only distinguished by their production technologies, migration occurs optimally until the home capital-labour has risen to maximise home productivity and the foreign capital-migrant ratio rises to maximise foreign productivity after accounting for migration costs.*

The global utility function is:

$$
\begin{aligned}
U =& U(C_t^h + C_t^M, N_t^h + M_t) + U(C_t^f, N_t^f) \\
=& \frac{(C_t^h + C_M^t)^{1-\eta^h} - 1}{1-\eta^h} + \frac{(C_t^f)^{1-\eta^f} - 1}{1-\eta^f} - \frac{(N_t^h + M_t)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}}
\end{aligned}
$$

In the optimum derived from eqs.A.1.37 and A.1.39 in **Appendix 1.1.2**, we have

$$
\begin{aligned}
IU^* =& \frac{[\frac{1+\delta^h\beta-\beta}{\beta}\frac{1-\varphi^h}{\varphi^h}\frac{1}{(L^h)^{\frac{1}{\nu^h}}}(\frac{K^{h*}}{L^h-M^*})^{1-\mu^h}]^{\frac{1-\eta^h}{\eta^h}} - 1}{1-\eta^h} + \frac{[\overline{C^f(M^*)}]^{\frac{1}{\eta^f}} - 1}{1-\eta^f} \\
& - \frac{(L^h)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} - \frac{(L^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}}
\end{aligned} \tag{2.4.36}
$$

Where $\overline{C^f(M^*)} = \{(1-\varphi^f)(1-\omega^f)\{\omega_f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f}$
$+ (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(L^f)^{\mu^f-1-\frac{1}{\nu^f}}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}\}^{\frac{1}{\eta^f}}$.

In the short run, when the speed of capital adjustment is slow and/or negligible, eq.2.4.36 shows that both home and foreign households benefit from migration as an increase in $M^*$ leads an increasing $\frac{K^{h*}}{L^h-M^*}$ and $\overline{C^f(M^*)}$.

We have

$$
U(C^{h*} + C^{M*}, N^{h*} + M^*) + U(C^{f*}, N^{f*}) \geq U(C_t^h, L^h) + U(C_t^f, N_t^f) \tag{2.4.37}
$$

When capital adjustment is slow in the short run, migration leads to a *no-worse-than* autarky equilibrium. The two sides are equal in the no migration case (**Section 2.1**) if the capital endowment is larger than $\{k^h\}^*$ in **Figure 2.3** when the home economy is initiated at a point with scarce labour.

Over time, the capital stocks in the two economies will adjust to its stationary level corresponding to their own fixed employment levels, we substitute eqs.A.1.38 and A.1.41 into 2.4.36.

$$
\begin{aligned}
IU^* =& \frac{[\frac{(1-\varphi^h)[\frac{1-\varphi^h}{(\frac{1+\delta^h\beta-\beta}{\beta\varphi^h})^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}]^{\frac{1-\mu^h}{\mu^h}}\frac{1+\delta^h\beta-\beta}{\beta\varphi^h}}{(L^h)^{\frac{1}{\nu^h}}}]^{\frac{1-\eta^h}{\eta^h}} - 1}{1-\eta^h} + \frac{[\overline{C^f}]^{\frac{1}{\eta^f}} - 1}{1-\eta^f} \\
& - \frac{(L^h)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} - \frac{(L^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}}
\end{aligned} \tag{2.4.38}
$$

where $\overline{C^f}$ is only a function of $M^*$ when replacing the $K^{f*}$ from eq.A.1.41. Since the $K^{f*}$ from eq.A.1.41 is also a positive function of $M_t$ and $\overline{C^f(M^*)}$ is also a positive function of $K^{f*}$, it is easy to prove that $\overline{C^f}$ will increase with higher level of migration without showing the gigantic equation. In this case, the global indirect utility is a positive function of $M^*$ because the foreign economy prefers as many immigrants as possible though the home economy's optimum welfare is constant at any level of migration. The global dictator pursuing the highest possible level of welfare will eventually have full migration, in which the marginal cost of migration ($\chi$) is counted in determining the optimal $K^{f*}$ in eq.A.1.41.

In general, when the two countries obtain a similar form of production technology, allowing migration increases the speed of adjustment toward the global optimum. If one of countries obtains a relatively more sophisticated production technology with imposed prerequisites to foreign labour force, migration will improve the optimum in both the short and long run.

## 2.5 Chapter conclusions

**Chapter 2** provides a theoretical exploration on the two-country migration phenomenon under a global central planner. We first construct the benevolent dictator's utility function and its resource constraint. Then, the study introduces a linear form of cost of migration, where the fixed and variable components have jointly built up the aggregate migration cost. On the supply side, the study started with the generalized three-factor (capital, labour and migrants) and two-factor (capital, labour) production functions for the foreign and home countries, respectively.

From this framework, we question under what circumstances, migration can happen. Through presenting *no migration*, *full migration* and *positive migration* conditions, this study provides a perspective to understand what motivates a global utility maximizing planner to pursue migration between two countries s/he controls. The algebraic derivation attributes the economic cause of migration to the marginal productivity of labour and the variable cost of migration. Thus, the specified forms of production functions of the two countries matter. This finding leads to the expansions in **Sections 2.4.1** and **2.4.2**, which asks about the optimal size of migration under two production conditions (similar production functions with different capital-labour endowments and different production functions with the same *ex-ante* capital-labour ratio). The equilibrium sizes of migration are presented with their resulting changes to the stationary-state global welfare under both fixed and endogenous capital assumptions. In short, migration always yields a Pareto improvement, specifically a general equilibrium welfare improvement if capital is not adjusting and a *no-worse-off* global utility levels if capital is adjustable.

In so doing, this chapter clarifies the key determinants in a global economy (or a closed economy) of migration with algebraic derivations of optimal migration, as well as the effects of migration on global stationary-state welfare.

# Chapter 3

# Migration between Two Centrally Planned Economies

This chapter studies how two central planners interact in determining migration. With give labour endowments and migration, the stationary-state welfares are shown under fixed and endogenous capital. We compare the sizes of optimal migration produced in a Nash and a cooperative game between the two country planners.

This chapter starts with modifying the framework to have individual central planners for home and foreign economies. Assuming there exists differences in both capital-labour endowments and production technologies between two countries, our welfare analysis presents the significance of migration to the two autarky economies, with their potential divergence on the stationary-state size of migration.

Then, to study the migration phenomenon if two planners diverge, a Nash and cooperation game on migration are studied and compared. An important finding is that under certain conditions, cooperation can be counter-productive in pursuing optimal migration and the best global optimum welfare. Moreover, the global optimum welfare with optimal migration can be reached by the Nash bargain.

## 3.1 The set-up

Now suppose that the two countries are governed by individual central planners. To study migration, we assume that the migrant receiver, the foreign, is endowed with a relatively larger capital-labour ratio than the home country of migrants.[1] To maximise the efficiency

[1] We will start with an analysis based on a difference in labour inputs, and then fixed labour and capital stocks in both economies so that we can observe what the different types of heterogeneity contribute; see

of the relatively abundant capital in the foreign economy, the planner of the foreign has to attract immigrants through a higher wage.

Following **Proposition 2.4.1**, we start with the positive optimal migration assumption that the foreign economy has a larger capital-labour ratio than the home ($\frac{K_t^f}{L^f} > \frac{K_t^h}{L^h}$) and a more complex production technology as follows.

$$
\begin{aligned}
Y_t^f =& Z_t^f \{\omega_f (M_t)^{\lambda^f} + (1-\omega^f)[\varphi^f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}} && (3.1.1)\\
Y_t^h =& Z_t^h [\varphi^h (K_t^h)^{\mu^h} + (1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1}{\mu^h}} && (3.1.2)\\
CM_t =& CM_0 + \chi M_t && (3.1.3)\\
L^h =& N_t^h + M_t && (3.1.4)\\
L^f =& N_t^f && (3.1.5)
\end{aligned}
$$

The supply side features to the foreign production relying on migrants, foreign labour and immobile foreign capital and the home production relying on the remaining home labour force and immobile home capital. Full employment holds in the labour markets.

On the demand side, the planners choose a migration plan to maximise their aggregate utility subject to their nation-wide resource constraints. Formally, the foreign country planner solves

$$\max_{\{C_t^f, N_t^f\}} \{\sum_0^{+\infty} (\beta^f)^t [U^f(C_t^f, N_t^f)]\}$$

with the same instantaneous utility function as in eq.2.1.7. With full employment optimally adopted for the centrally planned economy,[2] the primary objective of the foreign planner is to optimise the aggregate consumption ($C_t^f$), subject to

$$C_t^f \leq Y_t^f - Y_t^M + (1-\delta^f)K_t^f - K_{t+1}^f - (1-s)CM_t$$

where $Y_t^M$ is the total compensation to migrant labour or aggregate migrant labour income. The migrants' income can be freely repatriated to the home households on time. The foreign aggregate consumption ($C_t^f$) is subject to the current period output ($Y_t^f$), after depreciated stock of current capital ($(1-\delta^f)K_t^f$), migrants' compensation ($Y_t^M$), capital for production in the next period ($K_{t+1}^f$) and its liable share in the aggregate migration cost ($(1-s)CM_t$). Here we assume $Y_t^M = C_t^M$,[3] and the foreign budget constraint becomes

**Section 3.2** for a complete welfare analysis for both cases.

[2]Aiming at maximised output, $\frac{\partial Y_t^i}{\partial N_t^i} > 0, \forall i \in \{h, f\}$ will always dominate the planners' decisions on labour supply.

[3]We aware that literature such as (Bandeira et al., 2018; Mandelman and Zlate, 2012) have been advocating and demonstrating the significance of remittance to the home and foreign economies. However, based on our analysis in **Appendix A.2 The remittance set-up**, the remittance would not affect the optimal size of migration as the representative household do not care which member consumes what, only the total.

$$C_t^f \leq Y_t^f - C_t^M + (1-\delta^f)K_t^f - K_{t+1}^f - (1-s)CM_t$$

Analogously, the home country central planner solves

$$\max_{\{C_t^M, C_t^h, M_t, N_t^h\}} \{\sum_0^{+\infty} (\beta^h)^t [U^h(C_t^h + C_t^M, N_t^h + M_t)]\}$$

subject to

$$C_t^h \leq Y_t^h + (1-\delta^h)K_t^h - K_{t+1}^h - sCM_t \tag{3.1.6}$$
$$C_t^M \leq Y_t^f + (1-\delta^f)K_t^f - C_t^f - K_{t+1}^f - (1-s)CM_t \tag{3.1.7}$$

After paying for the home share of cost of migration ($sCM_t$), the total output ($Y_t^h$) the home planner received would be used for consumption of its workers and net investment. And the latter eq.3.1.7 is just a reformation of the aforementioned foreign households' budget constraint.

The derived Euler equations are as follows:

$$\{K_{t+1}^f\}: \quad 1 = \beta^f[(1 + \frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} - \delta^f)(\frac{C_t^f}{C_{t+1}^f})^{\eta^f}] \tag{3.1.8}$$

$$\{N_t^f\}: \quad (N_t^f)^{\frac{1}{\nu^f}}(C_t^f)^{\eta^f} = \frac{\partial Y_t^f}{\partial N_t^f} \tag{3.1.9}$$

$$\{K_{t+1}^h\}: \quad 1 = \beta^h[(1 + \frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} - \delta^h)(\frac{C_t^h + C_t^M}{C_{t+1}^h + C_{t+1}^M})^{\eta^h}] \tag{3.1.10}$$

$$\{N_t^h\}: \quad \frac{U'(N_t^h)}{U'(C_t^h)} = (N_t^h + M_t)^{\frac{1}{\nu^h}}(C_t^h + C_t^M)^{\eta^h} = \frac{\partial Y_t^h}{\partial N_t^h} \tag{3.1.11}$$

$$\{M_t\}: \quad \frac{U'(M_t)}{U'(C_t^M)} = (N_t^h + M_t)^{\frac{1}{\nu^h}}(C_t^h + C_t^M)^{\eta^h} = \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial CM_t}{\partial M_t} \tag{3.1.12}$$

which go along with eqs.2.4.31 (foreign production), 2.4.32 (home production),[4] 3.1.7 (foreign economy budget constraint), 3.1.6 (home budget constraint) and 2.1.4 (the identification of cost of migration) and 2.1.1 (the home labour market population set-up) that describe the variables $\{CM_t, C_t^f, C_t^h, C_t^M, N_t^f, N_t^h, M_t, Y_t^h, Y_t^f, K_t^f, K_t^h, L^h\}$. Note that we only have eleven equations for twelve endogenous variables, and so we condition the solutions on $M_t$ in order to see how it would affect the optimal welfare of two countries. A full elaboration on what determines the optimal $M^*$ in Nash bargaining between the two countries will be given in **Section 3.2 and 3.3**.

[4]Note that we directly start with the assumption of differentiated production technology in the two economies as the assumption of homogeneous production will lead to a similar conclusion to the one shown in **Section 2.3.1**.

Combining eqs.3.1.11 and 3.1.12 gives the identical labour equilibrium (in **Section 2.3.2**) condition as follows.

$$\frac{\partial Y_t^M}{\partial M_t} = \frac{\partial Y_t^h}{\partial N_t^h} - \chi$$

while eqs.3.1.8 and 3.1.10 with the optimised consumption smoothing plan in the stationary state replicates the capital equilibrium condition adjusted by the differed time discount factors of two planners as follows

$$\frac{\partial Y_t^h}{\partial K_t^h} = \frac{\beta^f}{\beta^h}\left(\frac{\partial Y_t^f}{\partial K_t^f} + 1 - \delta^f\right) - 1 + \delta^h$$

Thus, the limits of the stationary equilibrium level of immigration ($\{\overline{M}\}_f$) and emigration ($\{\overline{M}\}_h$) can be obtained from eqs.3.1.8 and 3.1.10

$$M_t \leq \{\overline{M}\}_f = (\Theta^f)^{\frac{1}{\lambda^f}} K^{f*} \tag{3.1.13}$$

$$M_t \leq \{\overline{M}\}_h = L^h - K^{h*}(\Theta^h)^{-\frac{1}{\mu^h}} \tag{3.1.14}$$

where $\Theta^f = \frac{\{\frac{\frac{1}{\beta^f}-1+\delta^f}{\varphi^f(1-\omega^f)[\varphi^f+(1-\varphi^f)(\frac{K^{f*}}{L^f})^{-\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}}\}^{\frac{\lambda^f}{1-\lambda^f}} - (1-\omega^f)[\varphi^f+(1-\varphi^f)(\frac{K^{f*}}{L^f})^{-\mu^f}]^{\frac{\lambda^f}{\mu^f}}}{\omega^f}$ positively related to the steady state foreign capital stock and $\Theta^h = \frac{(\frac{\frac{1}{\beta^h}+1-\delta^h}{\varphi^h})^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}{1-\varphi^h}$. $\{\overline{M}\}_f$ is the maximum amount of immigrants that the foreign economy could receive to optimise its productivity, and $\{\overline{M}\}_h$ is the maximum labour that the home economy would wish to have emigrate to optimise its domestic capital-labour ratio. If there exists a unique general equilibrium point, we will reach a similar optimum migration-foreign labour ratio as in **Section 2.3.2**. The optimal migration level must meet the limits of eqs.3.1.13 and 3.1.14.

Specifically, eqs.3.1.13 and 3.1.14 say that the optimal level of migration permissible by both economies is subject to the capital stock of each economies. In the foreign economy, optimal migration $\{\overline{M}\}_f$ is a positive function of its equilibrium capital stock, which can be interpreted as a larger equilibrium capital stock in the foreign raising its limit on the demand for immigrants. For the home economy, its migration limits $\{\overline{M}\}_h$ is a negative function of the home equilibrium capital stock, which means a larger equilibrium capital stock at home will lead to a smaller emigration limit.

Moreover, it is important to consider what would happen if the two economies end up desiring different limits of migration possibly due to the different time discount factors ($\beta^f, \beta^h$). These limits are likely to coincide with what we perceive in the real world, with countries being prudent in making migration decisions, putting restrictions on the legal amount of annual immigration and emigration.

## 3.2 Welfare of the centrally planned economies

### 3.2.1 Stationary state welfare with fixed labour supply and endogenous capital

In this section, we investigate the migration phenomenon with only fixed labour supply ($L^f$ and $L^h$), while the capital stock is assumed to be endogenous in both economies.[5]

To study changes in welfare from migration, we shall first observe what endogenous capital means at the boundary of conditions (eqs.3.1.13 and 3.1.14). Eq.3.1.13 states that the foreign optimal migration is a positive function of its own equilibrium capital stock $K^{f*}$. When this is endogenized, it is dependent on the level of optimal migration $M^*$ as follows.[6]

$$K^{f*} = \{\frac{(\Lambda^f)^{\frac{\mu^f}{\lambda^f}} M^{*\mu^f} - (1-\varphi^f)(L^f)^{\mu^f}}{\varphi^f}\}^{\frac{1}{\mu^f}} \tag{3.2.1}$$

where $\Lambda^f = \frac{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h})[\frac{1-\varphi^h}{(\frac{\beta^h\varphi^h}{1+\delta^h\beta^h-\beta^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}$ will be increased with the variable cost of migration $\chi$, with $\frac{\partial K^{f*}}{\partial M^*} > 0$ and $\frac{\partial^2 K^{f*}}{\partial (M^*)^2} < 0$.

This equation states that the optimal capital stock is a positive function of migration.[7] Therefore, when capital is endogenously determined, the foreign economy would accept as many immigrants as possible because the increase of immigrants will increase the foreign capital stock, which itself is a positive driver of optimal migration (as shown in eq.3.1.13). Eq.3.2.1, in effect, favours full migration.

Meanwhile for the home economy, eq.3.1.14 has the endogenous home capital stock in equilibrium determined by its domestic employment as follows.[8]

$$K^{h*} = \{\Lambda^h\}^{-\frac{1}{\mu^h}} N^{h*} \tag{3.2.2}$$

where $\Lambda^h = \frac{[\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}{1-\varphi^h}$. Increases in the number of migrants would reduce home employment, however this would not affect the equilibrium capital-labour ratio as eq.3.2.2 shows. In this case, there would be no binding boundary condition for the home economy to determine the optimal size of migration. It indicates that the home country does not care

[5]Appendix: A.1 provides a complete mathematical analysis.

[6]Detailed derivation is shown in eq.A.3.26 in the Appendix.

[7]The foreign economy labour force $N_t^f$ is fixed and fully employed under the centrally planned regime, and the level of immigration will be determined in **Section 3.3** in a Nash game.

[8]See eq.A.3.24 for the complete derivation.

about the number of emigrants. The central planner would be indifferent between zero and positive migration.

Here we explain the above intuition in the derivations of indirect utilities of the two economies.

By solving the system of equations in Appendix A.1 with endogenous capital ($K_t^f$, $K_t^h$), the equilibrium levels of consumption ($C^{h*}$, $C^{M*}$) can be shown to be functions of the stationary-state labour inputs.[9]

Denoting $[\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h})(L^h)^{-\frac{1}{\nu^h}}]^{\frac{1}{\eta^h}} = \kappa_{A1}$;$[(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h})^{\frac{1}{1-\mu^h}}-\delta^h] = \kappa_{A2}$; $[\frac{1-\varphi^h}{(\frac{\beta^h\varphi^h}{1+\delta^h\beta^h-\beta^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1}{\mu^h}} = \kappa_{A3}$; we get:

$$C^{M*} = \kappa_{A1}(\kappa_{A2})^{\frac{1-\mu^h}{\eta^h}} - \kappa_{A2}\kappa_{A3} + sCM_0 + s\chi M^* \quad (3.2.3)$$

$$C^{h*} = \kappa_{A2}\kappa_{A3} - sCM_0 - s\chi M^* \quad (3.2.4)$$

So

$$C^{h*} + C^{M*} = \kappa_{A1}(\kappa_{A2})^{\frac{(1-\mu^h)}{\eta_h}}$$

where $\kappa_{A1}$, $\kappa_{A2}$ and $\kappa_{A3}$ are constant under the full employment assumption in the centrally planned economy. Note that $C^{h*} + C^{M*}$ is independent of $M^*$ and of any foreign economy parameters.

Together with eq.2.1.8, the stationary-state utility of a home country household can be denoted as a function of labour inputs as follows.

$$U^{h*} = \frac{\kappa_{A1}^{1-\eta^h}(\kappa_{A2})^{\frac{(1-\mu^h)(1-\eta^h)}{\eta^h}} - 1}{1-\eta^h} - \frac{(L^h)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} \quad (3.2.5)$$

In short, since $N^{h*} + M^* = L^h$ which is fixed, the stationary-state aggregate welfare of the home economy is a constant, independent of the migration level. This is somewhat counter-intuitive as migration is modelled as a productive movement between the two centrally planned economies.

The key to why the home country's aggregate indirect utility is a constant is to understand the uniqueness of the equilibrium capital-labour ratio, and the equilibrium capital and labour stock in the home economy. In this section, where capital is endogenously determined by the local labour force, the equilibrium home capital stock is dependent on the remaining labour force $N^{h*}$, which means the unique optimal capital-labour ratio can be achieved at any level of migration. Further, as capital is now determined, there would be no incentive to keep the last person at home if everyone else had migrated, since staying at home with his/her own capital can actually generate the same level of welfare as in the foreign economy. The

[9]See eqs.A.3.18 to A.3.21 with eq.A.3.24.

central planner's stationary-state welfare is indifferent to the size of migration.[10]

**Proposition 3.2.1** *With a fixed level of employment and endogenous capital, the stationary-state equilibrium social welfare of the centrally planned home economy is constant and independent of migration.*

For the foreign country welfare analysis, the stationary-state consumption of foreign households can also be shown as a function of its labour supply. From eq.A.3.22

$$C^{f*} = \{(1-\varphi^f)(1-\omega^f)\{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f}$$
$$+ (1-\varphi_f)(L^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} (L^f)^{\mu^f - 1 - \frac{1}{\nu^f}} [\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f - \mu^f}{\mu^f}}\}^{\frac{1}{\eta^f}}$$

and substituting eq.3.2.1 into this yields:

$$\begin{aligned} C^{f*} = \{&(1-\varphi^f)(1-\omega^f)[(1-\omega^f)\Lambda^f + \omega^f]^{\frac{1-\lambda^f}{\lambda^f}} \\ &\{\Lambda^f\}^{\frac{\lambda^f-\mu^f}{\lambda^f}} (L^f)^{\mu^f - 1 - \frac{1}{\nu^f}} (M^*)^{1-\mu^f}\}^{\frac{1}{\eta^f}} \end{aligned} \tag{3.2.6}$$

Note that $(1-\omega^f)\Lambda^f = \frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h})[\frac{1-\varphi^h}{(\frac{\beta^h\varphi^h}{1+\delta^h\beta^h-\beta^h})^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}]^{\frac{1-\mu^h}{\mu^h}} + \chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}} - \omega^f$ is always larger than zero as the left hand of eq.A.1.40 is always larger than zero. We observe that optimal foreign consumption depends on its labour force and on migration and its marginal cost, $\chi$.

Then substituting this into eq.2.1.7, the stationary-state indirect utility of the foreign economy can be derived as a function of labour used:

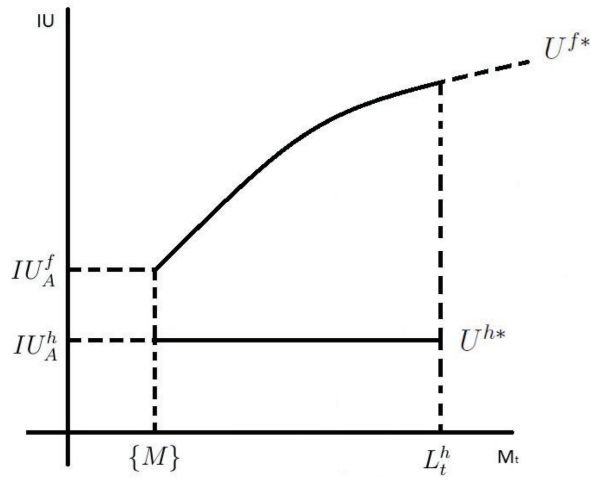
$$\begin{aligned} U^{f*} &= \frac{\{L^f, M^*\}^{1-\eta^f} - 1}{1-\eta^f} - \frac{(L^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \\ &= IU^f\{M^*\} \end{aligned} \tag{3.2.7}$$

The indirect utility of the foreign economy ($U^{f*}$) is a concave function of the optimal level of migration ($M^*$) as $\frac{\partial U^{f*}}{\partial M^*} > 0$ and $\frac{\partial^2 U^{f*}}{\partial (M^*)^2} < 0$.

**Proposition 3.2.2** *Since the marginal welfare of migrant labour is always positive, the foreign economy will pursue full migration.*

[10]In the next section, when the economy is also endowed with a fixed level of capital, there exists a unique level of the home labour force that meets the requirement of the optimal capital-labour ratio. When the size of migration reaches its optimal level at $\{\overline{M}\}_h$ in eq.3.1.14, the capital-labour ratio at home will also reach optimal efficiency, as does output per capita. It is the equilibrium output per capita at home which produces the optimal level of indirect utility in the next section (see eq.3.2.10) giving a different view on how migration affects the welfare of the two economies.

Figure 3.1: Full migration




$NB_1$: $\{M\}$ is a corner condition when the migration benefit exceeds the fixed cost of migration for the two economies. For simplicity, we have shown the minimum permitted levels of migration as the same for both economies. But they should be different, which will be shown in **Section 3.3**.
$NB_2$: $IU_A^h$ and $IU_A^f$ are the autarky levels of indirect utility for the home and foreign economies, respectively.

The equilibrium migration under capital endogeneity as **Figure 3.1**. From the perspective of global welfare, migration from a simple to a complex production area is a global welfare improvement process, and the gain is reaped in the foreign economy. The solution is simply full migration.

### 3.2.2 Stationary state welfare analysis under constant labour and capital

This section explores the effect migration has on aggregate welfare under the assumption of fixed labour and physical capital in both economies. The boundary conditions, eqs.3.1.13 and 3.1.14, will be adopted accordingly. With full information and fixed capital, they are also recognized by both planners.

The stationary-state consumption ($C^{h*}$, $C^{M*}$) can be expressed in terms of stationary-state labour and capital inputs.[11]

When $M^* < \{\overline{M}\}_h$ :

$$C^{h*} + C^{M*} = \{\frac{1}{(L^h)^{\frac{1}{\nu^h}}}(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h - M^*})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}\}^{\frac{1}{\eta^h}} \tag{3.2.8}$$

When $M^* > \{\overline{M}\}_h$ :

$$C^{h*} + C^{M*} = \frac{1}{(L^h)^{\frac{1}{\eta^h \nu^h}}}\{\omega^f\{\omega^f + (1-\omega^f)(M^*)^{-\lambda^f}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} - \chi\}^{\frac{1}{\eta^h}} \tag{3.2.9}$$

Under the assumption of fixed labour and capital, we can denote the indirect utility of the home country as a function of $M^*$, increasing before the critical point ($\{\overline{M}\}_h$) and decreasing after it.

$$U^{h*} = IU^h\{M^*\} \tag{3.2.10}$$

For eq.3.2.8, when $M^* < \{\overline{M}\}_h$, $\frac{\partial U^{h*}}{\partial M^*} > 0$ and $\frac{\partial^2 U^{h*}}{\partial (M^*)^2} > 0$, the aggregate indirect utility of the home country is an increasing and convex function of migration as shown in **Figure 3.2**. Once migration exceeds $\{\overline{M}\}_h$, the home capital-labour ratio exceeds the unique optimal ($E^h$ in **Figure 3.2**), resulting in decreasing aggregate indirect utility ($\frac{\partial U^{h*}}{\partial M^*} < 0$ and $\frac{\partial^2 U^{h*}}{\partial (M^*)^2} > 0$ for eq.3.2.9) in the home economy.

As shown in the **Appendix A.3**, eqs.3.2.8 and 3.2.9 are derived from eqs.3.1.11 and 3.1.12, respectively. Eq.3.1.11 refers to the equilibrium relationship between aggregate home consumption (home and migrant) and the home marginal productivity of labour, while eq.3.1.12 is the equilibrium relationship between aggregate home consumption and the foreign marginal productivity of migrants. The home stationary-state migration defined by the labour market equilibrium conditions (eqs.3.1.11 and 3.1.12) will be maximised when it equals to the $\{\overline{M}\}_h$ that is derived from the home capital equilibrium condition (eq.3.1.10). Consistent with **Chapter 2**, the home maximised stationary-state migration occurs when the capital equilibrium condition meets the labour equilibrium condition for the home economy.

The intuition for this switch of the equations at the boundary point $\{\overline{M}\}_h$ is due to the changes in the determinants of the home aggregate utility. Before reaching $\{\overline{M}\}_h$ that maximises the capital-labour ratio (at optimal $E^h$) at home, the home household utility is determined by the remaining home labour's consumption, which relies on the maximised domestic production. The purpose of optimal migration is to reduce the home labour to its most efficient level. However, for $M_t > \{\overline{M}\}_h$, the stationary-state utility is sub-optimal as the extra migration is no longer desired by the home but benefits the foreign economy production. Thus, home utility depends on foreign production, taking into consideration the

[11]See Eqs.A.3.18 to A.3.21.

variable cost of migration.[12]

**Proposition 3.2.3** *The higher the number of migrants, the larger is the stationary-state home country welfare until the country reaches its most efficient stationary-state equilibrium capital-labour ratio.*

For the foreign country welfare analysis, the stationary-state consumption of foreign households can also be shown as a function of capital and labour supply.[13]

$$\begin{aligned} C^{f*} = \{&(1-\varphi^f)(1-\omega^f)\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} \\ &+ (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(L^f)^{\mu^f-1-\frac{1}{\nu^f}}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}\}^{\frac{1}{\eta^f}} \end{aligned} \tag{3.2.11}$$

Then substituting this into eq.2.1.7, the stationary-state utility of a foreign economy household can be derived as a function of optimal migration.

$$\begin{aligned} U^{f*} &= \frac{\{f(M^*)\}^{1-\eta^f} - 1}{1-\eta^f} - \frac{(L^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \\ &= IU^f\{M^*\} \end{aligned} \tag{3.2.12}$$

Since $\frac{\partial U^{f*}}{\partial M^*} > 0$ and $\frac{\partial^2 U^{f*}}{\partial (M^*)^2} < 0$, the indirect aggregate utility of the foreign economy is a positive and concave function of the level of migration in the stationary state bounded by the $\{\overline{M}\}_f$ in eq.3.1.13. However, the constant elasticity of return characteristic in eq.2.4.31 constrains a maximum level of immigration that the foreign country could intake, which is shown as $\{\overline{M}\}_f$ in eq.3.1.13. Once the size of immigration crosses $\{\overline{M}\}_f$, equilibrium in the foreign economy will be broken and immigration will be terminated. In **Figure 3.2**, the benefit from local-immigrant complementarity drives the increasing trend of $IU^f$ for $M_t < \{\overline{M}\}_f$.

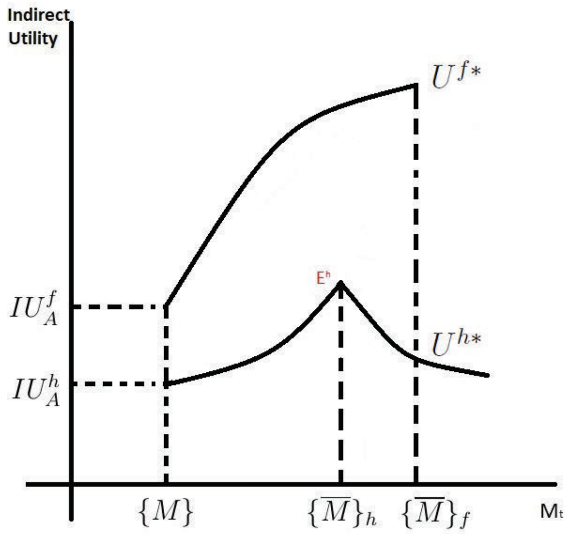
**Proposition 3.2.4** *Immigration will have a positive effect on the optimal stationary-state foreign economy aggregate welfare until the foreign economy reaches its optimal migrant-capital ratio.*

**Figure 3.2** presents the $\{M^*, IU\}$ relationship based on eqs.3.2.10 and 3.2.12. As assumed, the most desired level of immigration $\{\overline{M}\}_f$ in eq.3.1.13 and emigration $\{\overline{M}\}_h$ in eq.3.1.14 are not coincidentally equivalent as shown in **Figure 3.2**.

[12]The fixed cost of migration does not appear in the consumption function in eq.3.2.9. Its role will be studied in the **Section 3.3**.

[13]Detailed derivations are shown in **A1. Stationary state welfare derivation for Section 3.2**, see eq.A.3.22.

Figure 3.2: Equilibrium Indirect Utility and Migration, given ($s$, $\delta^h$ and $\delta^f$)




Note that the home aggregate indirect utility will never reach the same level as the foreign economy's. This is due to our assumption of "local labour-capital complementarity" in the foreign economy's production function, which assumes a permanently lower human capital of home economy labour than for the foreign economy. An algebraic proof of $\{\overline{M}\}_f > \{\overline{M}\}_h$ will be shown in **Section 3.3**.

In general, capital asymmetry would naturally lead to migration as both economies' welfare after migration would be *no-worse-than* without migration. Then, the question is how to determine the optimal level of migration ($M^*$). Due to the possible conflict of interest in choosing migration levels between the two economies, we present two game-theoretic solutions to $M^*$ in the following sections. The solution will lie between $\{\overline{M}\}_f$ and $\{\overline{M}\}_h$, and depend on the share allocation of the cost and relative bargaining powers.

### 3.2.3 Welfare state comparison

An important exploration is whether the welfare states that two economies could reach are comparable under the two afore-studied assumptions: endogenously adjustable capital and labour mobility, and fixed capital stock and labour mobility.

For the home economy, when the stationary-state home indirect utility (under the fixed capital stock assumption) achieves the highest level at $M_t = \{\overline{M}\}_h$, $\frac{K^{h*}}{N^{h*}}$ in eq.3.2.2 (the maximised level of utility that is pursued by the capital and labour adjustment) will be achieved. That means the $U^{h*}$ line on **Figure 3.1** is at the same level as the unique optimal point $E^h$ on **Figure 3.2** as long as the home capital can adjust to its most efficient level relative to the remaining labour force. From the home central planner's viewpoint, the improvement of utility by the home country will be the same with endogenous capital or with migrant mobility.

**Proposition 3.2.5** *Endogenous capital can improve the home stationary-state equilibrium welfare identically to optimal migration.*

If the foreign capital stock is constant, the stationary-state utility of the foreign will achieve its highest at $M_t = \{\overline{M}\}_f$. This positive but not full migration equilibrium welfare would be smaller than the full migration equilibrium welfare derived in eqs.3.2.6 and 3.2.7 in **Section 3.2.1** because full migration from the home economy will increase the equilibrium capital stock in foreign according to eq.3.2.1, and increase the marginal productivity of foreign labour ($\frac{\partial^2 Y_t^f}{\partial N_t^f \partial M_t} > 0$ in the assumed foreign production function, eq.3.1.1).

**Proposition 3.2.6** *Endogenous capital adjustment with migration can achieve a higher stationary level of foreign welfare than the equilibrium welfare only with migration adjustment.*

Combining **Proposition 3.2.5** and **3.2.6** gives the conclusion that the global welfare under labour mobility is improved by allowing endogenous capital adjustment. This result is consistent with our findings in **Chapter 2**.

## 3.3 Nash bargain for the optimal migration

In this section, we adopt a bargaining process between central planners over the migration decision. It is important to note that the bargaining is only meaningful under the assumptions of both a) capital immobility b) negligible capital adjustment. If capital can be quickly adjusted to the varying employment, ***Props. 3.2.1*** and ***3.2.2*** will apply.

Binmore et al. (1986)'s Nash bargain framework has been used. Both economies in this bargaining would prefer full bargaining power, to achieve the migration levels that maximise their own welfare.

From the perspective of the labour-importing (foreign) economy central planner, admitting $M_t$ migrants would generate an aggregate surplus of $\frac{\partial Y_t^f}{\partial M_t} M_t - W_t^M M_t - (1-s)CM_t$,[14] which

[14]Given constant return to scales,

$$Y_t^f = \frac{\partial Y_t^f}{\partial M_t} M_t + \frac{\partial Y_t^f}{\partial N_t^f} N_t^f + \frac{\partial Y_t^f}{\partial K_t^f} K_t^f$$

and so net surplus of migration is

$$Y_t^f - r_t^f K_t^f - W_t^f N_t^f = \frac{\partial Y_t^f}{\partial M_t} M_t$$

says the net gain from migration is generated from the difference between marginal productivity of migrants and the labour-importing country's cost to foreign the migrants (wage and its share of the aggregate migration cost, $1 - s$). Some might argue that the central planners are only bargaining for partial equilibrium in the labour market where the free-market bargaining between households and firms are actually taken from general equilibrium perspective. We advocate this is one of main features of ideal centrally planned economies where central planners can choose to bargain on the migration phenomenon itself, which is the sole cause of disruption in their autarky stationary-state equilibria and which both parties will benefit from.

The labour-exporting (home) economy wishes to maximise its gain, $W_t^M M_t - \frac{\partial Y_t^h}{\partial N_t^h} M_t - sCM_t$. So the labour-exporting economy's net gain comes from the difference between the overall migration compensation (wage income) and its opportunity cost (domestic marginal productivity with having migrants at home and its share of migration cost, $s$).

The Nash bargaining process of the two planners can be formulated as

$$\max_{\{M_t\}}\{[\frac{\partial Y_t^f}{\partial M_t} M_t - W_t^M M_t - (1-s)CM_t]^{b_M}[W_t^M M_t - \frac{\partial Y_t^h}{\partial N_t^h} M_t - sCM_t]^{1-b_M}\}$$
$$b_M \in (0,1)$$

where the $b_M$ is the bargaining power of the foreign country. Assume $\Upsilon$ as the equilibrium share of migrant's income out of migrant's total productivity.

$$W_t^M = \Upsilon \frac{\partial Y_t^f}{\partial M_t}$$

By maximizing the bargaining equation with respect to number of migrants ($M_t$), an optimal level of migration can be found (using eq.2.1.4).

$$\begin{aligned}
&\{[(1-\Upsilon)\frac{\partial Y_t^h}{\partial N_t^h} + (s-\Upsilon)\chi]b_M \frac{\partial^2 Y_t^f}{\partial (M_t)^2} - [(1-b_M)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2}] \\
&\quad [(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi]\}(M_t)^2 \\
&+\{\{[(s-\Upsilon)b_M + (1-s)\Upsilon]\frac{\partial^2 Y_t^f}{\partial (M_t)^2} + (1-b_M)(1-s)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}\}CM_0 \\
&\quad - [(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi][\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi]\}M_t \\
&+\{(s-\Upsilon)b_M \frac{\partial Y_t^f}{\partial M_t} + (1-s)[\Upsilon \frac{\partial Y_t^f}{\partial M_t} - (1-b_M)\frac{\partial Y_t^h}{\partial N_t^h} - s\chi]\}CM_0 = 0
\end{aligned} \tag{3.3.1}$$

Solving the above condition gives multi-equilibria issues and their economic interpretations are impossible to read. Thus, we need to consider some special cases to better understand this condition.

In the subsequent parts of this subsection, we discuss four cases considering extreme values of $s$ and $b_M$ under three different assumptions on the migration cost: zero fixed cost, non-zero fixed cost and only fixed cost. Interestingly, the exploration provides only four possible outcomes under extreme conditions from the twelve cases. The last part of this subsection takes a further step toward investigating the four outcomes.

### 3.3.1 Zero fixed cost

We start by assuming there is no fixed cost of migration $\{CM_0 = 0\}$. The equation 3.3.1 collapses to

$$\begin{aligned}&\{[(1-\Upsilon)\frac{\partial Y_t^h}{\partial N_t^h} + (s-\Upsilon)\chi]b_M\frac{\partial^2 Y_t^f}{\partial (M_t)^2} - [(1-b_M)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}]\\ &[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi]\}(M_t)^2\\ &-[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi][\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi]M_t = 0\end{aligned} \tag{3.3.2}$$

It gives two possible equilibria:

$$\text{Zero or } \frac{[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-(1-s)\chi](\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}-s\chi)}{[(1-\Upsilon)\frac{\partial Y_t^h}{\partial N_t^h}+(s-\Upsilon)\chi]b_M\frac{\partial^2 Y_t^f}{\partial (M_t)^2}-[(1-b_M)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}][(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-(1-s)\chi]}.$$

As long as the wage offered by the labour importing (foreign) economy remains attractive, we would not expect to achieve a stationary-state migration at zero.

Further analyses are carried out for the stationary-state level (second solution) of migration: what if one of the countries obtains an extremely high bargaining power ($b_M = 0$, $b_M = 1$).

In the first case, if the labour-exporting (home) economy possesses the absolute power in migration bargaining, $b_M = 0$ gives

$$M_t = \frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi}{-(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})} \tag{3.3.3}$$

*Existence condition 1: As long as we have a positive marginal gain of migration for the home country, there would exist an equilibrium level of migration.*

When the home economy has all bargaining power in determining the size of migration, the share of home economy in migration cost ($s$) plays a vital role to determine the optimal migration level. Eq.3.3.3 says that a positive equilibrium migration inflow to the foreign economy will only exist when the gap between wage of being a migrant and marginal product of being home labour is outweighing the home country's burden on each individual's migration cost.[15]

Furthermore, extreme conditions on the $s$ value ($s = 1$, $s = 0$), the share of home economy on the world-wide overall migration cost, are also presented

*Case 1*: when $s = 1$, the home economy pays for all costs incurred from migration.

$$M_t = \frac{\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - \chi}{-(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})} \tag{3.3.4}$$

In this case, the foreign economy won't have to pay any cost. The equilibrium outcome of migration is now completely driven by the marginal gain of migration to the home country net of the marginal cost of migration and the growth rate of migrant's wage and home country productivity. The solution provides a meaningful positive stationary-state level of migration due to the diminishing marginal productivity of labour in both economies.

*Case 2*: when $s = 0$, the foreign economy bears all migration cost.

$$M_t = \frac{\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h}}{-(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})} \tag{3.3.5}$$

The equilibrium level of migration is only determined by the marginal gain of the home country and its consideration on the growth rate of marginal productivity of home labour (being a migrant and a home labour).

For the above two outcomes under the opposite conditions on the share value ($s$), the existence of a global optimal level of migration has to be consistent with the *Existence condition 1.* In the scenario where the migration bargain is driven by the home economy, with the increase of the home economy's burden in the overall cost of migration, the stationary-state level of migration will decrease, and the lowest equilibrium level of migration achieves when the home economy bears all cost.

In general, when the home economy entitles the absolute power in migration bargaining, the equilibrium level of migrants will increase with the foreign economy's share of cost of migration.

When $b_M = 1$, the labour-importing takes absolute control on the migration bargaining.

[15] As the set-ups of both countries' production functions meet the law of diminishing marginal productivity, the numerical value of the denominator is positive.

The solution of eq.3.3.2 becomes

$$M_t = \frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{3.3.6}$$

*Existence condition 2: Given the diminishing marginal productivity in the labour-importing economy, if there exists a net marginal gain of migration for the foreign economy, this two-country world will achieve an equilibrium level of migration when the foreign economy has all power in migration bargaining.*

Then, in a bargain that is designed to maximise the foreign economy interest, the following situations are examined.

*Case 3*: when $s = 1$, the home (labour-exporting) economy takes all cost of migration.

$$M_t = -\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{3.3.7}$$

When the foreign economy seizes the absolute power in the migration bargaining and makes the home economy bear all cost of migration, the optimal level of migration completely depends on the migrants' marginal product of labour and its derivative (the growth rate of marginal product of migrants), in order to optimise the desire of the foreign economy social planner, the wage payout is given minimum concern.

*Case 4*: if $s = 0$, the foreign economy takes all cost.

$$M_t = -\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - \chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{3.3.8}$$

When the foreign economy establishes the complete bargaining power in the bargaining and pays for all, the variable cost of migration will have to be taken into account. In pursuing an optimised level of productivity growth, the marginal gain of migration matters the most.

The *Existence condition 2* also applies to the two above cases. A simple subtraction between the two above outcomes tells that the stationary-state migration level when the foreign economy takes all cost is smaller than the equilibrium generated from the opposite case when the foreign economy obtains full bargaining power.[16]

[16]The result of the subtraction is shown as

$$M_t|_{\{b_M=1,s=1\}} - M_t|_{\{b_M=1,s=0\}} = -\frac{\chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} > 0$$

Where $\Upsilon\frac{\partial Y_t^f}{\partial M_t}$ is the wage of migrant.

In general, if the foreign economy obtains the full power on migration bargaining, the highest stationary-state level of migration appears when the home economy bears all cost of migration.

### 3.3.2 Non-zero fixed cost of migration

Suppose the fixed cost is unavoidable for both economies. We will have to start our analyses based on eq.3.3.1.

If $b_M = 0$, the labour-exporting(home) economy has accessed the absolute bargaining power.

$$\begin{aligned}
&\{-(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi]\}(M_t)^2 \\
&+\{[(1-s)(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})]CM_0 - [(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi](\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi)\}M_t \\
&+\{(1-s)(\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi\}CM_0 = 0 \qquad (3.3.9)
\end{aligned}$$

*Case 5*: if $s = 1$, the exporting (home) economy pays for all the migration cost.

$$[\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2}](M_t)^2 + (\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - \chi)M_t = 0 \qquad (3.3.10)$$

The equation gives the optimal levels of migration at $\frac{\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - \chi}{-(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})}$ and 0.

Interestingly, the solutions for **Case 5** and **Case 1** (same conditions on the bargaining power and cost share allocation with and without the fixed cost) are identical.

*Case 6*: If $s = 0$, the exporting economy has successfully made the foreign economy bear the burden of the migration cost.

$$\begin{aligned}
&-\{(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - \chi]\}(M_t)^2 \\
&+\{(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})CM_0 - [(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - \chi](\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})\}M_t \\
&+(\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})CM_0 = 0 \qquad (3.3.11)
\end{aligned}$$

In this case, the bargained outcomes of migration are $\frac{CM_0}{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - \chi}$ and $-\frac{\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$.

There may be a multi-equilibria issue. However, we argue that the first outcome is a corner condition for the foreign economy to allow for migration while the foreign economy bears all the cost.

Firstly, it is unreasonable for the home economy to consider the fixed cost of migration for which it is not responsible. When $b_M = 0$ and $s = 0$, the home economy, exporting its labour force, has all the bargaining power but does not have to bear any cost for labour export.

Secondly, the former outcome can be inferred in two ways. First, given a positive marginal gain of migration for the foreign economy, the larger fixed cost of migration the higher the optimal level of migration. Second, for a given level of fixed cost, a larger marginal gain of having an additional immigrant in the foreign country leads to a smaller amount of migration. Either way, the inference is irrational and counter-intuitive.

Due to the above interpretation, only the second equilibrium point shall be accepted by a rational home social planner, which is identical to **Case 2**. The numerator $(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})$ is the marginal gain of the home economy to allow for an emigrant, and the denominator takes into account the diminishing marginal productivity of the migrant labour force in two countries, which says that the home economy is acting responsibly to maximise its self-interest.

When $b_M = 1$, the labour-importing takes absolute control on their border defence. Eq. 3.3.1 turns into

$$\begin{aligned}
&(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}(\frac{\partial Y_t^h}{\partial N_t^h} - \Upsilon\frac{\partial Y_t^f}{\partial M_t} - s\chi)(M_t)^2 \\
&+\{(1-\Upsilon)s\frac{\partial^2 Y_t^f}{\partial (M_t)^2}CM_0 - [(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi](\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi]\}M_t \\
&+[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi]sCM_0 = 0
\end{aligned} \tag{3.3.12}$$

*Case 7*: if $s = 1$, the importing (foreign) economy obtains full bargaining power without bearing any cost of migration. Above equation can be reduced to

$$\begin{aligned}
&-[(1-\Upsilon)(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - \chi)]\frac{\partial^2 Y_t^f}{\partial (M_t)^2}(M_t)^2 \\
&+\{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}CM_0 - (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - \chi)\}M_t + (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}CM_0 = 0
\end{aligned} \tag{3.3.13}$$

The equilibrium levels of migration are $M_t = -\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$, and/or $\frac{CM_0}{\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - \chi}$.

The former outcome is identical to the ones in **Case 3**. When the foreign obtains full cost and pays nothing for migration, the central planner's concern is to optimise the marginal productivity of migrants.

Similar to **Case 6**, the second algebraic solution of $M_t$ is a required corner condition for the home economy to allow for immigration as the home economy pays for all cost of migration in this case, which is unrelated to the foreign economy.

*Case 8*: if $s = 0$, the foreign economy makes the exporting economy bear all the migration cost.

$$(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}M_t^2 + [(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - \chi]M_t = 0 \tag{3.3.14}$$

The solutions are $M_t = -\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$ and zero.

Once again, the outcomes between **Case 8** and **Case 4** are identical, showing that the fixed cost of migration is irrelevant to the optimal level of migration by two central planners.

**Proposition 3.3.1** *The fixed cost of migration is irrelevant to determine the equilibrium level of migration under the Nash bargain between two central planners, while it features corner conditions for permitting migration.*

**Proposition 3.3.2** *Under the assumption that migration cost is only determined by the size of migration, the central planner will pursue more migrants when the other party takes more cost.*

### 3.3.3 Only fixed cost of immigration

In this section, we explore a restricted case where there is only fixed cost of immigration. So the variable cost ($\chi$) on each migrant is zero in this case.

In this case, eq.3.3.1 collapses to the following form:

$$\begin{aligned}
&\{(1-\Upsilon)\frac{\partial Y_t^h}{\partial N_t^h}b_M\frac{\partial^2 Y_t^f}{\partial (M_t)^2} - (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}[(1-b_M)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}]\}(M_t)^2 \\
&+\{\{[(s-\Upsilon)b_M + (1-s)\Upsilon]\frac{\partial^2 Y_t^f}{\partial (M_t)^2} + (1-b_M)(1-s)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}\}CM_0 \\
&\quad - (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})\}M_t \\
&+\{(s-\Upsilon)b_M\frac{\partial Y_t^f}{\partial M_t} + (1-s)(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - (1-b_M)\frac{\partial Y_t^h}{\partial N_t^h})\}CM_0 = 0
\end{aligned} \tag{3.3.15}$$

Bargaining power ($b_M$) of the foreign and the home share allocation ($s$) of the cost of migration matter so that the four extreme scenarios as *Case 1 to Case 4* can also be applied here.

If $b_M = 0$, the labour-exporting (home) economy has accessed the absolute bargaining power.

$$\begin{aligned}
&-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})(M_t)^2\\
&+\{(1-s)[\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}+\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}]CM_0-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})\}M_t\\
&+(1-s)(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})CM_0=0 \qquad (3.3.16)
\end{aligned}$$

*Case 9*: When $s = 1$, the home economy has the burden of all the cost of migration.

$$\begin{aligned}
&-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})(M_t)^2\\
&+[-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})\}M_t=0 \qquad (3.3.17)
\end{aligned}$$

It gives two solutions: 0 and $-\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$.

Again, we would only accept the second solution as there are always incentives to migration due to the capital asymmetric structure and productivity gain of being a migrant. The stationary-state level of migration for **Case 9** is $-\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$, which coincides with **Case 2,6**.

*Case 10*: when $s = 0$; the foreign economy bears all the migration cost.

$$\begin{aligned}
&-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})(M_t)^2\\
&+[(\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}+\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2})CM_0-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})]M_t\\
&+(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})CM_0=0 \qquad (3.3.18)
\end{aligned}$$

By resolving the above equation, two optimal levels of migration are: $\frac{CM_0}{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}}$ and $-\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$.

The first solution is a corner solution because when the home economy has full bargaining power ($b_M = 0$), the migration size is driven by the home economy welfare. When the home economy central planner has made the foreign economy bear the full burden of the fixed cost of migration, it is irrational for the home planner to send out a stationary-state migration flow that is defined as fixed cost of migration divided by foreign economy marginal gain of migrant labour. The central planner will not care about the cost as s/he is not responsible for. In contrast, s/he would be more likely to pursue the second stationary-state solution, which aims to maximise the gain to the home economy only.

In short, the equilibrium migration for **Case 10** is $M_t = -\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$, as the same as the **Case 2, 6** and most importantly **Case 9**.

It seems that the share allocation becomes irrelevant when the cost of migration is a fixed lump sum, which leads to a great concern on a large existing literature as it is normal that cost of migration is set to be fixed.

If $b_M = 1$, the labour-importing (foreign) economy has possessed the full bargaining power in determining the migration flow.

$$\begin{aligned}
&(1-\Upsilon)[\frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2} - \Upsilon\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}](M_t)^2 \\
&+ [s(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}CM_0 - (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})]M_t \\
&+ s(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}CM_0 = 0
\end{aligned} \tag{3.3.19}$$

*Case 11*: when $s = 1$, the home economy takes on all cost of migration.

$$\begin{aligned}
&(1-\Upsilon)[\frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2} - \Upsilon\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}](M_t)^2 \\
&+ [(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}CM_0 - (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})]M_t \\
&+ (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}CM_0 = 0
\end{aligned} \tag{3.3.20}$$

It can be solved with two outcomes: $M_t = -\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$, and $M_t = \frac{CM_0}{\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h}}$. A similar intuition applies so that only $M_t = -\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$ is accepted for the equilibrium level of migration, an identical outcome as in **Case 3** and **7**.

*Case 12*; when $s = 0$, the foreign economy pays for all cost while the home obtains the full bargaining power.

$$(1-\Upsilon)[\frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}-\Upsilon\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}](M_t)^2$$
$$+[-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})]M_t=0 \tag{3.3.21}$$

Then, the optimal migration can be solved as: $M_t=0$ and $-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$. As 0 is not an applicable answer due to the economic incentive assumed in the framework, the stationary-state migration would be $-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$.

In general, once there is only fixed cost of migration, the Nash bargain generates two solutions. That is when the home economy obtains full power ($M_t=-\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$), and the opposite case ($M_t=-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$).

**Proposition 3.3.3** *When the periodical cost of migration is a fixed lump sum, the share allocation is irrelevant to determine the stationary-state level of migration.*

**Proposition 3.3.4** *The stationary-state equilibrium size of migration is largest when the foreign economy has full bargaining power, regardless of the existence of a variable or fixed cost of migration.*

**Proposition 3.3.5** *If this is no variable cost, the highest stationary level of migration is only generated when the migration bargain is driven by the foreign economy and the cost of migration is irrelevant to determine the optimal level of migration.*

*Proof*: To evaluate the size of four situations, we only need to evaluate the sign of (**Case 3, 7, 11, 12 - Case 2, 6, 9, 10**), which says the largest migration when the foreign economy has the control in migration bargaining minus the largest migration when the home economy has it.

$$-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} - [-\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}]$$

$$= \frac{-\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} - \Upsilon\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^f}{\partial (M_t)^2} + \Upsilon\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^f}{\partial (M_t)^2} - \frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}[\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}]}$$

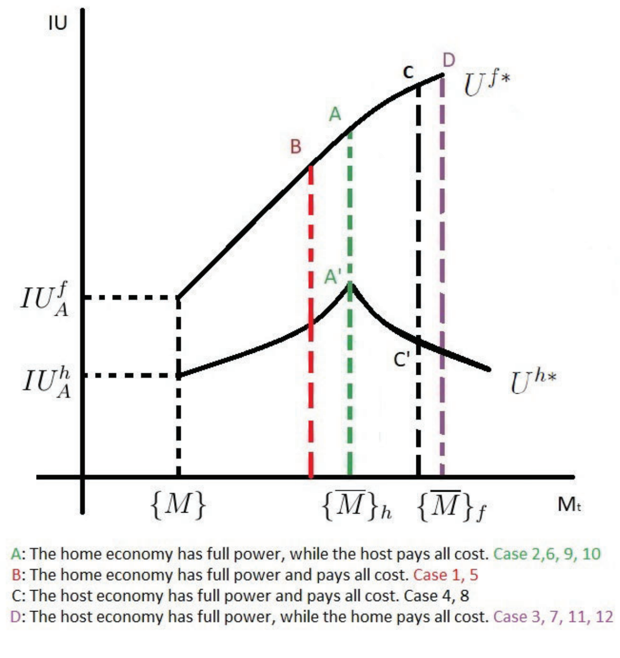
$$= -\frac{\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}[\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}]} > 0 \quad (3.3.22)$$

According to the law of diminishing marginal productivity, the above difference carries a positive sign, which argues that the highest stationary level of migration occurs when the foreign economy has the full bargaining power but pays nothing in all scenarios.

### 3.3.4 Exploration on the four levels of stationary-state welfare

Based on **Section 3.2**, the stationary-state levels of migration will determine the levels of aggregate welfare of the two economies. Relying on the foregoing four equilibrium solutions, the four possible stationary welfare states can be added into **Figure 3.2**, and that leads to the following **Figure 3.3**.

Figure 3.3: Four solutions




where $\{M\}$ is the set of corner conditions.[17] For the foreign and home economies, they would

[17] As both the mark-up power of the foreign economy and the variable cost component of migration have

allow migration if and only if $M_t \geq \frac{CM_0}{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}$ and $M_t \geq \frac{CM_0}{\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}-\chi}$, respectively.

From the perspective of the foreign economy central planner, the welfare of the foreign economy is the highest when it manages to achieve its desirable migration but make the home country bear the cost. To prevent its lowest welfare state, that is when the home achieves the desirable migration of its own and pays all the migration cost (on the red line), the foreign economy would like to pay for migration so that welfare state moves from **Point B** to **Point A** that motivates the home to send more migrants.

To prove this, we have to compare **Case 4, 8** (the foreign economy has the full bargain power and pays for all cost) with the **Case 2, 6, 9, 10** (the home economy makes its desired emigration but takes no burden on the cost).

$$
\begin{aligned}
&-\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}-\left(-\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\right)\\
=&\frac{[-(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}+\chi](\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})+(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})}
\end{aligned}
$$

$$
=\frac{-(1-\Upsilon)(\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2})+\chi(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\Upsilon\frac{\partial^2 Y_t^f}{\partial (M_t)^2})} \tag{3.3.23}
$$

The solution has an ambiguous sign as the denominator is positive, while two components of the numerator carry opposite signs. It seems that the margin parameter ($\Upsilon$) between the marginal product of migrant labour and the migrant's wage determines the superiority of two solutions. The two solutions can be identical if the equilibrium value $\Upsilon$ coincides with

$$
\Upsilon_1=\frac{(\frac{\partial Y_t^f}{\partial M_t}-\chi)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}{\frac{\partial Y_t^f}{\partial M_t}\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2}+\frac{\partial Y_t^h}{\partial N_t^h}\frac{\partial^2 Y_t^f}{\partial (M_t)^2}+\chi\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}<1
$$

which can be seen as a threshold value for **Case 4, 8** = **Case 2,6**. When $\Upsilon < \Upsilon_1$, the inference comes true and **Figure 3.3** will work. The foreign economy has an incentive to obtain the full bargaining power to increase the amount of immigration. However, it is interesting that under certain circumstances ($\Upsilon = \Upsilon_1$), **Point A** and **C** will overlap and two situations equal.

When the foreign loses all the power in this bargain ($b_M = 0$), **Figure 3.3** suggests that a rational foreign planner would, in effect, voluntarily bear all the cost of migration in order

not yet been studied, we cannot distinguish which one of the corner conditions would be larger than the other.

to encourage a larger size of immigration so that the foreign productivity could be further improved, so does the indirect utility.

However, a question occurs in the opposite case ($b_M = 1$), the foreign economy targets on the optimised welfare at **Point D** which only happens while the cost is taken by the home economy. At the same time, the home economy is also trying to avoiding paying any of the cost in the pursuit of a slightly higher welfare state at **Point C$'$**,[18] which allows the home economy to retain more labour force at home to improve its domestic productivity.

This new division of interest is grounds for a second stage of bargaining in which the two economies wish the opposite side to pay for the cost of migration. They would bargain on the home economy's share of cost of migration ($s$) with the given $M_t$, which is essentially a function of $s$. Substituting $b_M = 1$ into eq.3.3.1 gives the eq.3.3.12, which gives

$$M_t = \frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{3.3.24}$$

which will collapse to **Case 3, 7** when $s = 1$, and to **Case 4, 8** when $s = 0$.[19]

In the second stage of bargaining, we should substitute this equation into the original bargaining process, which then transforms as follows.

$$\begin{aligned} max\{[&\frac{\partial Y_t^f}{\partial M_t}\{\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\} - \Upsilon\frac{\partial Y_t^f}{\partial M_t}\{\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\} \\ &- (1-s)(CM_0 + \chi\{\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\})]^{b_M} \\ [&\Upsilon\frac{\partial Y_t^f}{\partial M_t}\{\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\} - \frac{\partial Y_t^h}{\partial N_t^h}\{\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\} \\ &- s(CM_0 + \chi\{\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\})]^{1-b_M}\}, b_M \in (0,1) \end{aligned}$$

The marginal productivity of a migrant and its second derivative are constant as the second stage bargaining is only triggered when there exists divergence in the interest of two planners.

As shown in **Figure 3.3**, the bargaining on the home economy share of migration cost

[18] In fact, if the idea of loss aversion suggested by Tversky and Kahneman (1991) applies, the desire toward **Point C$'$** would only be stronger.

[19] At the same time, it also gives rise to a general corner condition.

$$M_t = \frac{sCM_0}{\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi}$$

($s$) only happens when the bargaining power is under the absolute control of the foreign economy. When we apply $b_M = 1$ to the above bargaining:

$$[-2(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}\chi + 2\chi(1-s)\chi + CM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}]$$
$$\{(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi)[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi] + sCM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}\} = 0$$

A simple way to solve this equation is to examine the $s$ when either of the brackets equals zero. The former square bracket equals zero when the home economy pays the share at

$$s = \frac{2\chi[\chi - (1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}] + CM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}{2\chi^2}$$

Substituting it into eq.3.3.24, the optimal level of migration is

$$M_t = -\frac{CM_0}{2\chi} \tag{3.3.25}$$

which is a meaningless negative number so that $[-2(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}\chi + 2\chi(1-s)\chi + CM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}] = 0$ is a rejected condition.

Then, the latter curly bracket has to equal zero to achieve an accepted bargaining outcome of $s$ so that

$$(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - s\chi)[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi] + sCM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2} = 0 \tag{3.3.26}$$

*Case 13:* When there is only fixed cost of migration, this equation becomes

$$(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} + sCM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2} = 0$$

which gives that $s = -\frac{(\Upsilon\frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h})\frac{\partial Y_t^f}{\partial M_t}}{CM_0\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$. The migration level for this scenario is

$$M_t = -\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{3.3.27}$$

which coincides the outcomes of **Case 3, 7, 11, 12**. In particular, together with **Case 11, 12** (the first stage bargain with given $s = 0$ and $s = 1$), the share allocation of cost between the two economies tends to be irrelevant to determine the optimal level of migration, when there exists only fixed cost of migration.

*Case 14:* When cost of migration is only a variable cost which only incurs with actual migration, eq.3.3.26 is rewritten as

$$(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}-s\chi)[(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-(1-s)\chi]=0$$

In this case, we achieved two possible outcomes: $s=\frac{\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}}{\chi}$ and $s=\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{\chi}$. Therefore, two levels of migration also occur

$$\begin{aligned}\{M_t\}_1=&-\frac{\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}-\chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\\ \{M_t\}_2=&-2\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}\end{aligned} \tag{3.3.28}$$

It is clear that our first solution $\{M_t\}_1$ is perfectly situated between **Point C** when $M_t=\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$ and **Point D** when $M_t=-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$.

However, to have the second solution located into the accepted domain requires that $\{M_t\}_2<-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$ while $\{M_t\}_2$ is surely bigger than **Point C** at $M_t=\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$ (exactly two times). This condition has imposed a specific necessary requirement that is a specific relationship between the variable cost of migration and the mark-up power of the foreign economy at[20]

$$\Upsilon>1-\frac{2\chi}{\frac{\partial Y_t^f}{\partial M_t}}$$

Which permits $\{M_t\}_2$ as a viable result of migration so we engage in a multi-equilibria issue at

$$\{M_t\}_2=-2\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}},\text{when}\Upsilon>1-\frac{2\chi}{\frac{\partial Y_t^f}{\partial M_t}} \tag{3.3.29}$$

*Case 15:* When there exists both fixed and variable parts of cost of migration, we solved the eq.3.3.26. Assuming $\chi(\frac{\partial Y_t^f}{\partial M_t}+\frac{\partial Y_t^h}{\partial N_t^h}-2\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\chi)-CM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}=\Psi_1$, $[\chi(\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h}-\chi)-CM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}]^2=\Psi_2$ and $\chi(\Upsilon\frac{\partial Y_t^f}{\partial M_t}-\frac{\partial Y_t^h}{\partial N_t^h})CM_0(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}=\Psi_3$ and achieve that

$$s=\frac{-\Psi_1+\sqrt{\Psi_2+4\Psi_3}}{2\chi^2}$$

[20]Remind that $\Upsilon$ represents the share of migrant's wage out of his/her own marginal productivity.

which, in effect, produce the optimal level of migration at

$$M_t = -\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} - \frac{2\chi^2 + \Psi_1 - \sqrt{\Psi_2 + 4\Psi_3}}{-2\chi(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{3.3.30}$$

As long as the solution in eq.3.3.30 is bigger than **Point C** at $M_t = \frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t} - \chi}{-(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$, we achieve a viable equilibrium. In fact, this necessary condition will always meet as long as $-\Psi_1 + \sqrt{\Psi_2 + 4\Psi_3}$ stays positive and $2\chi^2 \geq -\Psi_1 + \sqrt{\Psi_2 + 4\Psi_3}$.

In general, after the second stage of bargaining, the equilibrium level of migration is $-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}}$ for the case of "only fixed cost", $\{M_t\}_1$ and $\{M_t\}_2$ for the case of "only variable cost", as well as the outcome of eq.3.3.30 when both costs exist.[21]

## 3.4 Cooperative migration between two central planners

All propositions of Benhabib and Jovanovic (2012) rely on the predetermined welfare weights of the home and foreign countries. The objective of Benhabib and Jovanovic (2012)'s central planner is to maximise the weighted sum of the aggregate welfare, which can also be adopted in a cooperative regime.

To examine the magnitude of the effects of the "welfare weights" $[\theta, 1-\theta]$ on the optimal level of migration in our general equilibrium framework, the two cooperating central planners maximise their welfare as follows

$$\max_{C_t^f, C_t^h, C_t^M, N_t^h, M_t, K_{t+1}^h, K_{t+1}^f} \mathbf{\{}\theta\{\sum_{t=0}^{+\infty}(\beta^f)^t[U^f(C_t^f, N_t^f)]\} + (1-\theta)\{\sum_{t=0}^{+\infty}(\beta^h)^t[U^h(C_t^h + C_t^M, N_t^h + M_t)]\}\mathbf{\}}$$

subject to:

$$\{\text{foreign}\} : (1-s)CM_t \leq Y_t^f + (1-\delta^f)K_t^f - K_{t+1}^f - C_t^M - C_t^f \tag{3.4.1}$$

$$\{\text{home}\} : sCM_t \leq Y_t^h + (1-\delta^h)K_t^h - K_{t+1}^h - C_t^h \tag{3.4.2}$$

The expressions $\beta^f$ and $\beta^h$ are the time discount factors of the foreign and the home, respectively, at time $t$. The two central planners aim to maximise the cooperatively weighted aggregate welfare by making optimal decisions of foreign consumption, home consumption, home labour supply, next period capital levels and most importantly, migrants supply and

[21] It is obvious that the results are all functions of marginal productivity of different labour including migration labour force. A further exploration of these outcomes has been shown in **Appendix A.4**.

their consumption allocations. Note that under the centrally planned regime, full employment is applied for both countries. Although the planners can manipulate home production scales through migration decisions, the foreign labour supply is predetermined.

The Lagrangian is

$$\begin{aligned}
&\theta\Big\{\sum_{t=0}^{+\infty}(\beta^h)^t\{\frac{(C_t^h+C_t^M)^{1-\eta^h}-1}{1-\eta^h}-\frac{(N_t^h+M_t)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}}\\
&\quad+\xi_{1,t}[Y_t^h-C_t^h+(1-\delta^h)K_t^h-K_{t+1}^h-s(CM_0+\chi M_t)]\}\Big\}\\
(1-\theta)&\Big\{\sum_{t=0}^{+\infty}(\beta^f)^t\{\frac{(C_t^f)^{1-\eta^f}-1}{1-\eta^f}-\frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}}\\
&\quad+\xi_{2,t}[Y_t^f+(1-\delta^f)K_t^f-K_{t+1}^f-C_t^M-C_t^f-(1-s)(CM_0+\chi M_t)]\}\Big\}
\end{aligned}$$

The full migration and no migration cases would be similar to those in **Chapter 2**. Here we assume, under cooperation in migration decisions, positivity conditions for the $\{C_t^f$, $C_t^h$, $C_t^M$, $N_t^h$, $M_t$, $K_{t+1}^h$, $K_{t+1}^f\}$. The Kuhn-Tucker conditions are thus

$$\{C_t^f\}: \quad (1-\theta)[(C_t^f)^{-\eta^f}-\xi_{2,t}]=0 \tag{3.4.3}$$

$$\{C_t^h\}: \quad \theta[(C_t^h+C_t^M)^{-\eta^h}-\xi_{1,t}]=0 \tag{3.4.4}$$

$$\{C_t^M\}: \quad \theta(C_t^h+C_t^M)^{-\eta^h}-(1-\theta)\xi_{2,t}=0 \tag{3.4.5}$$

$$\{N_t^h\}: \quad \theta[(N_t^h+M_t)^{\frac{1}{\nu^h}}+\xi_{1,t}\frac{\partial Y_t^h}{\partial N_t^h}]=0 \tag{3.4.6}$$

$$\{M_t\}: \quad \theta[(N_t^h+M_t)^{\frac{1}{\nu^h}}+\xi_{1,t}(-s\chi)]+(1-\theta)\xi_{2,t}[\frac{\partial Y_t^f}{\partial M_t}-(1-s)\chi]=0 \tag{3.4.7}$$

$$\{K_{t+1}^h\}: \quad \theta[-\xi_{1,t}+\beta^h\xi_{1,t+1}(\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h}+1-\delta^h)]=0 \tag{3.4.8}$$

$$\{K_{t+1}^f\}: \quad (1-\theta)[-\xi_{2,t}+\beta^f\xi_{2,t+1}(\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f}+1-\delta^f)]=0 \tag{3.4.9}$$

The complementary slackness conditions are

$$\xi_{1,t}=(C_t^h+C_t^M)^{-\eta^h}\geq 0 \tag{3.4.10}$$

$$\xi_{2,t}=(C_t^f)^{-\eta^f}\geq 0 \tag{3.4.11}$$

From eqs. 3.4.3 - 3.4.5, we get

$$\xi_{1,t} = \frac{1-\theta}{\theta}\xi_{2,t} \tag{3.4.12}$$
$$(C_t^h + C_t^M)^{-\eta^h} = \frac{1-\theta}{\theta}(C_t^f)^{-\eta^f}$$

Most importantly, eqs.3.4.6 and 3.4.7 of the Kuhn-Tucker conditions present the optimal level of migration under the cooperative planners.

$$\theta\xi_{1,t}(\frac{\partial Y_t^h}{\partial N_t^h} + s\chi) - (1-\theta)\xi_{2,t}[\frac{\partial Y_t^f}{\partial M_t} - (1-s)\chi] = 0$$

Substituting eq.3.4.12 into the above equation, the "optimal migration condition" under cooperation equalizes the net marginal product of the two countries:

$$\frac{\partial Y_t^f}{\partial M_t} = \frac{\partial Y_t^h}{\partial N_t^h} + \chi \tag{3.4.13}$$

which is independent of $\theta$.

**Proposition 3.4.1** *When the two dictators cooperate in making migration decisions, the optimal migration is determined by differentiated marginal products of the migrant labour between two countries but is not affected by the welfare weights.*

**Proposition 3.4.2** *The fixed component of the aggregate cost of migration has no effect on the determination of optimal migration, consistent with previous findings.*

Eq.3.4.13 is the same as **Section 2.3**'s "labour equilibrium condition". Now we turn to the "capital equilibrium condition" based on eqs.3.4.8 and 3.4.9. After taking the equilibrium conditions $\frac{\partial Y_{t+1}^h}{\partial K_{t+1}^h} = \frac{\partial Y_t^h}{\partial K_t^h}$, $\frac{\partial Y_{t+1}^f}{\partial K_{t+1}^f} = \frac{\partial Y_t^f}{\partial K_t^f}$, $\xi_{1,t+1} = \xi_{1,t}$, $\xi_{2,t+1} = \xi_{2,t}$, we reach

$$\frac{\partial Y_t^h}{\partial K_t^h} = \frac{\beta^f}{\beta^h}(\frac{\partial Y_t^f}{\partial K_t^f} + 1 - \delta^f) - 1 + \delta^h \tag{3.4.14}$$

This cooperative capital equilibrium condition is similar to eq.2.4.13 when there is only one global dictator, while the key difference is that the two central planners can have a different attitude toward future utility ($\beta^f \neq \beta^h$). We then need to consider how the time discount factors determine optimal migration.

Comparing eq.2.4.13 with eq.3.4.14 and assuming all related parameters of two equations are the same, the scenario $\beta^f < \beta^h$ can produce a smaller than the home marginal product of capital when two factors equal. Since in equilibrium, $\frac{\partial Y_t^h}{\partial K_t^h} = \varphi^h[\varphi^h + (1-\varphi^h)(\frac{L^h - M^*}{K^{h*}})^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}$ gives that migration (decreasing the home labour force) reduces the marginal product of home capital due to the decreased relative scarcity of the home capital. We gain more understanding on the global optimum welfare for both **Chapter 2** and **3**.

**Proposition 3.4.3** *The global planner needs to allow for individual time discount factors for two countries to reach the global welfare optimum of two cooperative central planners.*

When the home economy is more forward looking ($\beta^f < \beta^h$), the planner will permit more migrants to decrease the marginal product of home capital to speed up the adjustment process. Recall that in **Section 2.3.2**: if we have the fixed capital stock, there will be a positive but not full optimal migration, while the full migration would only occur once relaxing the assumption with capital adjustment. So, in the short run when the capital adjustment is slow and negligible, individual time discount factors can produce a higher optimal migration that the assumption of a uniform time discount factor.

Furthermore, we explore whether this cooperation is productive or under what conditions this cooperation between two central planners will always achieve a *no-worse-than* Nash outcome. In other words, we wish to see under which specific conditions co-operation can be counter-productive.

According to eq.2.4.31, the optimal migration from eq.3.4.13 is

$$\overline{M_{Coop}} = [\frac{(\frac{\partial Y_t^h}{\partial N_t^h} + \chi)^{\frac{\lambda^f}{1-\lambda^f}} - \omega^f}{1-\omega^f}]^{-\frac{1}{\lambda^f}}[\varphi^f(K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{1}{\mu^f}} \tag{3.4.15}$$

To be productive, the above outcome must be *no-less* than the optimal migration of Nash Bargaining (eq.A.4.11) so that $\overline{M_{Coop}} \geq \overline{M_{Nash}}$.

$$[\frac{(\frac{\partial Y_t^h}{\partial N_t^h} + \chi)^{\frac{\lambda^f}{1-\lambda^f}} - \omega^f}{1-\omega^f}]^{-\frac{1}{\lambda^f}}[\varphi^f(K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{1}{\mu^f}}$$
$$\geq [\frac{-\lambda^f(1-\omega^f)}{\omega^f}]^{\frac{1}{\lambda^f}}[\varphi^f(K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{1}{\mu^f}}$$

Substituting $\frac{\partial Y_t^h}{\partial N_t^h} = (1-\varphi^h)[\varphi^h(\frac{K_t^h}{N_t^h})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}$ into the above equation gives

$$\frac{K_t^h}{N_t^h} \geq \overline{\frac{K^h}{N^h}} = \{\frac{[\frac{(\frac{(\lambda^f-1)\omega^f}{\lambda^f})^{\frac{1-\lambda^f}{\lambda^f}} - \chi}{1-\varphi^h}]^{\frac{\mu^h}{1-\mu^h}} - (1-\varphi^h)}{\varphi^h}\}^{\frac{1}{\mu^h}} \tag{3.4.16}$$

Note that due to the application of the Law of Diminishing Marginal Returns, the substitutability between migrants and foreign labour force would belong to a particular domain that is $\lambda^f \in (-\infty, 1)$, so as $\mu^f$ and $\mu^h$ according to the local-migrant complementarity set-up. And the share parameter of migrant $\omega^f$ in the foreign country production is always smaller than 1.

We also note the significant role of the variable cost of migration in determining the critical value of the home capital-labour ratio. Thus value would be increased if there is a higher variable cost of migration. As a result, a higher variable cost of migration puts a higher bound for the migration cooperation scheme.

Eq.3.4.16 shows that the predetermined parameters on home capital-labour substitutability, variable cost of migration, share of home capital in production, share of migration in the foreign production and migrants-foreign labour substitutability give a critical value of the *ex-post* capital-labour ratio at home. Cooperation would be more productive than the Nash game if and only if the *ex-post* home capital-labour ratio exceeds this value meaning that the home production will be maintained at least on a corresponding critical level. Full migration might not be happening in a cooperation game.

**Proposition 3.4.4** *When the capital labour ratio at home is less than a critical level, cooperation would be counter-productive.*

To gain a deeper understanding on why cooperation can be counter-productive, we shall take a closer look at eqs.3.4.15 and A.4.11. Eq.3.4.15 says that optimal migration is determined by the marginal productivity of home labour, variable cost of migration, the foreign capital and labour stocks, as well as parameters of the foreign production process explicitly.[22] It indicates that the equilibrium migration in the cooperation game is a joint consideration that sustains the efficient production of both home and foreign economies. However, the optimum of the Nash migration ($\overline{M_{Nash}}$ in eq.A.4.11), **Point D** in Fig.3.3, is drawn from to solely maximise the foreign production.

Comparing two outcomes, it is the pursuit of the home production efficiency in the cooperation game that eventually impedes the optimal size of migration, and thus impacts the home capital labour ratio in eq. 3.4.16.

In a nutshell, to reach the theoretical global optimum welfare in a fixed-capital two-country global economy, the global planner needs to allow for individual time discount factors ($\beta$) for two countries. Two planners can reach this state via the Nash game when the foreign economy has all power and the home pays for the cost (**Point D** in Fig.3.3). Moreover, the migration cooperation between two planners is only productive if the *ex-post* capital-labour ratio at home can reach the critical level in eq.3.4.16.

[22]Note that the home production parameters are also important through the marginal productivity of home labour.

## 3.5 Chapter conclusions

**Chapter 3** considers the decisions on migration of two individual planners. This chapter has presented how the global welfare can be affected by both migration and capital adjustment when both heterogeneity in capital-labour endowment and production technology occur. An non-trivial result are the maximum amounts of migrants that the home and foreign economies can send and accept respectively.

We explore if migration improves the stationary-state welfare of the two economies, individually. When the capital is endogenous, we find a general favouring of migration as migrants, being complementary workers in foreign production, always bring up the foreign productivity despite its marginal productivity has been falling. The foreign utility is always improved with the inflow of migrants, while the stationary-state home utility is a constant as shown in Fig.3.1. However, when the capital adjustment speed is slow or negligible, migration, being the sole adjustment approach, distributes the aggregate gain of adjustment between two countries. Fig.3.2 shows that the foreign would wish to take more migrants to maximise its productivity, while the home needs to achieve the stationary capital-labour ratio that is constrained by the fixed amount of capital stock at home.

The possible conflict between the planners in determining the size of migration leads to the issue whether Nash bargaining is a better strategy for these two central planners than the cooperative game strategy when making two-country migration policies. We show that the cooperative game strategy in migration policy-making does not necessarily generate higher migration than the Nash-game. When the foreign economy dominates the Nash migration bargaining and the home economy takes on all cost, the foreign economy will optimise its own welfare at any cost of the home.

# Chapter 4

# Migration between free-market economies

We study how migration influences labour markets in both perfectly and imperfectly competitive free-market economies. We develop equilibria with migration under efficient labour market contracts that depends on the relative bargaining power of participants.

To understand the possible different welfare implications of migration for both home and foreign economies between free-market and centrally planned economies, we now construct an imperfectly competitive free-market global economy. Given our focus on migration, imperfect competition manifests in the labour market only.

A real-world concern is how migration can affect unemployment and wages in home and foreign economies. To this end, we introduce labour market failure (unemployment) as a consequence of imperfect competitiveness with bargaining. In particular with our extension of the efficient contracts model, capital owners that gain more labour market bargaining power generate more unemployment and lower wages.

This chapter analyses the global optimum welfare of a perfectly competitive free-market global economy and an imperfectly competitive one. In each case, we consider how households in the two countries can benefit from migration when capital stocks are fixed and when endogenously determined.

## 4.1 Welfare implications of migration in a perfectly competitive free market

We analyse the case for a perfectly competitive free-market economy, to compare it with the full information centrally planned economy.

A major difference in modelling migration between centrally planned economies and market economies is who makes the consumption and labour supply decisions. The central planner makes all decisions at an aggregate level for its citizens, while the representative household makes its individual consumption and working plans in a free-market economy. Further, the planner can internalize all negative and positive externalities.

At the beginning of every period, with a given cost of migration, the representative household engages with the firms in the foreign country for a deal that involves both the migrant labour supply of the household and the wage of migrants.[1]

As in the previous chapters (see eq.2.1.4), the cost of migration is

$$CM_t = CM_0 + \chi M_t \tag{4.1.1}$$

We assume the home household bear the burden of all the costs of sending its own members abroad, which will only indirectly affect the foreign economy through migrants' wage setting.[2]

In the free-market economy framework, the welfare analysis is taken from a representative household perspective. The household's objective is to maximise the utility of the whole household subject to their period budget constraint.

$$U^h = \sum_{t=0}^{+\infty} (\beta^h)^t \left[\frac{(C_t^M + C_t^h)^{1-\eta^h} - 1}{1-\eta^h} - \frac{(N_t^h + M_t)^{(1+\frac{1}{\nu^h})}}{1+\frac{1}{\nu^h}}\right]$$
$$\text{s.t.: } C_t^M + C_t^h + K_{t+1}^h \leq W_t^M M_t + W_t^h N_t^h + (1 + r_t^h - \delta^h) K_t^h - CM_t$$

The budget constraint shows that the sum of the representative household's consumption, investment and migration cost (both fixed and variable components) expenditure can be no bigger than its income from labour supply and prior capital investment.[3] The households in the home country make consumption plans for their domestic and migrant labour, as well as their working hours.

[1] Underlying this assumption is an international search and matching process. We abstract from this aspect but it is worth pursuing in future research.

[2] In **Chapter 3**, we allowed for a share of the cost, $s$, to be borne by the home economy and $1 - s$ by the foreign. Here we assume $s = 1$ for simplicity.

[3] It is important to note that we always assume the non-prohibitive cost of migration to analyse a possible positive migration equilibrium. If the cost of migration (either the variable or fixed component) is too large, migration should be zero and the household would refuse to make a migration decision.

The welfare of the households in the foreign economy can be modelled as:

$$
\begin{aligned}
U^f &= \sum_{t=0}^{+\infty}(\beta^f)^t[\frac{(C_t^f)^{1-\eta^f}-1}{1-\eta^f} - \frac{(N_t^f)^{(1+\frac{1}{\nu^f})}}{1+\frac{1}{\nu^f}}] \\
\text{s.t.: } C_t^f &+ K_{t+1}^f \leq W_t^f N_t^f + (1+r_t^f-\delta^f)K_t^f
\end{aligned}
$$

A significant difference between the centrally planned economies and free-market economies is that the foreign country households' budget constraint here does not include the immigrants' earnings, which is a vital part of the aggregate budget constraint (eq.A.2.7). This is because the foreign planner can internalize migrant earnings in foreign consumption decisions.

Then, the Euler equations become

$$
\begin{aligned}
\{M_t\}: &\quad (W_t^M-\chi)U'(C_t^h+C_t^m)+U'(M_t+N_t^h)=0 && (4.1.2)\\
\{N_t^h\}: &\quad W_t^hU'(C_t^m+C_t^h)+U'(M_t+N_t^h)=0 && (4.1.3)\\
\{K_{t+1}^h\}: &\quad -U'(C_t^m+C_t^h)+\beta^h(1+r_{t+1}^h-\delta^h)U'(C_{t+1}^m+C_{t+1}^h)=0 && (4.1.4)\\
\{N_t^f\}: &\quad W_t^fU'(C_t^f)+U'(N_t^f)=0 && (4.1.5)\\
\{K_{t+1}^f\}: &\quad -U'(C_t^f)+\beta^f(1+r_{t+1}^f-\delta^f)U'(C_{t+1}^f)=0 && (4.1.6)
\end{aligned}
$$

On the supply side, we adopt the heterogeneous production processes of the two economies as in **Section 2.4.2**, with migrant complementarity in foreign production.

$$
\begin{aligned}
Y_t^f &= Z_t^f\{\omega^f(M_t)^{\lambda^f}+(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}} && (4.1.7)\\
lnZ_t^f &= \gamma^f lnZ_{t-1}^f+\varepsilon_t^f, \gamma^f\in[0,1] && (4.1.8)\\
Y_t^h &= Z_t^h[\varphi^h(K_t^h)^{\mu^h}+(1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1}{\mu^h}} && (4.1.9)\\
lnZ_t^h &= \gamma^h lnZ_{t-1}^h+\varepsilon_t^h, \gamma^h\in[0,1] && (4.1.10)
\end{aligned}
$$

The home and foreign firms will maximise profits:

$$
\begin{aligned}
&\max\{Y_t^h-r_t^hK_t^h-W_t^hN_t^h\}\\
&\max\{Y_t^f-r_t^fK_t^f-W_t^fN_t^f-W_t^MM_t\}
\end{aligned}
$$

The first-order conditions (FOCs) for wages and capital returns become:

$$
W_t^f=\frac{\partial Y_t^f}{\partial N_t^f}; W_t^M=\frac{\partial Y_t^f}{\partial M_t}; W_t^h=\frac{\partial Y_t^h}{\partial N_t^h} \quad (4.1.11)
$$

$$
r_t^f=\frac{\partial Y_t^f}{\partial K_t^f}; r_t^h=\frac{\partial Y_t^h}{\partial K_t^h} \quad (4.1.12)
$$

Labour market equilibrium with perfectly competitive free markets for the two countries requires:

$$N_t^h + M_t = L^h \tag{4.1.13}$$
$$N_t^f = L^f \tag{4.1.14}$$

$L^f$ and $L^h$ are the workforce population of the two countries' households, and in these perfectly competitive free-market economies, there is nothing to cause unemployment.[4]

Merging eqs.4.1.2 and 4.1.3 gives the same labour equilibrium condition. The capital equilibrium condition can be replicated by combining eqs.4.1.4 and 4.1.6. The free-market economy capital and labour equilibrium conditions are respectively

$$r_t^h = \frac{\beta^f}{\beta^h}(r_t^f + 1 - \delta^f) - 1 + \delta^h \tag{4.1.15}$$
$$W_t^M = W_t^h + \chi \tag{4.1.16}$$

substituting the equilibrium conditions eqs.4.1.11 and 4.1.12, we achieve the perfectly competitive free-market economy equilibrium conditions as

$$\frac{\partial Y_t^h}{\partial K_t^h} = \frac{\beta^f}{\beta^h}(\frac{\partial Y_t^f}{\partial K_t^f} + 1 - \delta^f) - 1 + \delta^h$$
$$\frac{\partial Y_t^f}{\partial M_t} = \frac{\partial Y_t^h}{\partial N_t^h} + \chi$$

which are identical to the conditions in **3.1.2**, this replication gives the same maximal limits of the stationary equilibrium level of migration ($\{\overline{M}\}_f$ and $\{\overline{M}\}_h$) as eqs.3.1.13 and 3.1.14 as follows.

$$\{\overline{M}\}_f = (\Theta^f)^{\frac{1}{\lambda^f}} K^{f*}$$
$$\{\overline{M}\}_h = L^h - K^{h*}(\Theta^h)^{-\frac{1}{\mu^h}}$$

Both perfectly competitive free-market and the centrally planned economies comply with these equilibrium conditions and limits of migration.

Analogous to **Section 3.2.2**, we derive the same indirect utility of the home country using eqs.4.1.2 and 4.1.3, as a function of $M^*$.

$$IU^h\{M^*\} = h(C^{h*} + C^{M*}) \tag{4.1.17}$$

[4] If there are frictions in the labour markets, the natural rate of unemployment may not be zero. In our model here, the actual and natural rates of unemployment would be the same.

$M^* < \{\overline{M}\}_h$ :

$$C^{h*} + C^{M*} = \{\frac{1}{(L^h)^{\frac{1}{\nu^h}}}(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h - M^*})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}\}^{\frac{1}{\eta^h}} \tag{4.1.18}$$

$M^* > \{\overline{M}\}_h$ :

$$C^{h*} + C^{M*} = \frac{1}{(L^h)^{\frac{1}{\eta^h \nu^h}}}\{\omega^f\{\omega^f + (1-\omega^f)(M^*)^{-\lambda^f}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} - \chi\}^{\frac{1}{\eta^h}} \tag{4.1.19}$$

which increases before achieving the maximal limit and then decreases after that. This is because the highest possible equilibrium home household welfare is associated with the most efficient capital-labour ratio. Before reaching this point, greater migration shortens the distance between actual home capital-labour ratio and its most efficient level. After this point, further equilibrium migration is not desired by the home economy (but yields benefit to the foreign economy) because the home capital-labour ratio will fall from its efficient level, which will deteriorate the efficient capital labour ratio. This is shown in **Figure 3.2**.

At the same time, the foreign households' welfare is an indirect utility function of optimal migration.

$$IU^f\{M^*\} = f(C^{f*}) \tag{4.1.20}$$

$$\text{where: } C^{f*} = \{(1-\varphi^f)(1-\omega^f)\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(L^f)^{\mu^f - 1 - \frac{1}{\nu^f}}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(L^f)^{\mu^f}]^{\frac{\lambda^f - \mu^f}{\mu^f}}\}^{\frac{1}{\eta^f}}$$

which is a positively sloped concave function of migration bounded at $\{\overline{M}\}_f$. If $\{\overline{M}\}_f = \{\overline{M}\}_h$, the two economies achieve the maximal equilibrium migration pursued by both economies. Of course, these two maxima are not necessarily equal and the complementarity in foreign production always implies a higher maximal migration limit for the foreign, shown in **Section 3.3**.

When capital and total labour supply are fixed, we achieve the same limits of optimal migration and the same indirect utility functions in the planned economies and the perfectly competitive free-market economies.

**Proposition 4.1.1** *Assuming migration costs are not prohibitive, when capital is fixed in a perfectly competitive economy, both home and foreign household utility will increase as migration increases until their limits. Migration would stop when the foreign achieves its limit.*

If we remove the assumption of fixed capital, the home and foreign firm capital would, respectively, converge to the equilibrium state implied by eq.4.1.4 and 4.1.6 as follows:

$$K^{h*} =(\Lambda^h)^{-\frac{1}{\mu^h}}(N^{h*}) \tag{4.1.21}$$

$$K^{f*} =\{\frac{(\Lambda^f)^{\frac{\mu^f}{\lambda^f}}(\frac{M^*}{L^f})^{\mu^f}-(1-\varphi^f)}{\varphi^f}\}^{\frac{1}{\mu^f}}(L^f) \tag{4.1.22}$$

Noting eqs.4.1.13 and 4.1.17, substituting eq.4.1.21 into eq.4.1.18 and eq.4.1.22 into 4.1.19 shows that the home household's utility becomes a constant with no regard to the level of migration. However, combining eq.4.1.22 and 4.1.20 shows that the foreign household's indirect utility is a function of $M^*$.

$$IU^{h*} =\frac{(\frac{\frac{1}{\beta^h}-1+\delta^h}{(L^h)^{\frac{1}{\nu^h}}})^{\frac{1-\eta^h}{\eta^h}}(\frac{1-\varphi^h}{\varphi^h})^{\frac{1-\eta^h}{\eta^h}}[\frac{(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}{1-\varphi^h}]^{-\frac{(1-\mu^h)(1-\eta^h)}{\mu^h\eta^h}}-1}{1-\eta^h}-\frac{(L^h)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} \tag{4.1.23}$$

$$IU^{f*}(M^*) =f(M^*) \tag{4.1.24}$$

where $\frac{\partial f(M^*)}{\partial M^*}>0$ and $\frac{\partial^2 f(M^*)}{\partial (M^*)^2}<0$,[5] given the CES production functions with $\mu^f<1$. If capital is endogenously adjustable, there is no necessity for households in the home economy to allow migration as capital adjustment will eventually achieve the same constant equilibrium level of utility. However, foreign households benefit from the migration because the capital stock will be larger than in autarky, thanks to migration. As there is no planner in the free-market economies, all home labour can choose to cross the border. With an increase of migration, the home country employment $N_t^h$ will decrease accompanied by a decumulation of home economy capital. If full immigration occurs, the endogenously determined capital stock at home will also become zero.

**Proposition 4.1.2** *Assuming migration costs are not prohibitive, when capital is endogenously adjustable in a perfectly competitive free economy, foreign household utility will increase with migration, while home household utility is a constant. This leads to full migration.*

**Propositions 4.1.1** and **4.1.2** are conformable with our findings in the centrally planned economies (**Section 3.2**). **Propositions 3.2.5** and **3.2.6** are confirmed in the free-market

[5] $f(M^*)$ is

$$\frac{(\frac{\frac{1}{\beta^f}-1+\delta^f}{(L^f)^{\frac{1}{\nu^f}}})^{\frac{1-\eta^f}{\eta^f}}(\frac{1-\varphi^f}{\varphi^f})^{\frac{1-\eta^f}{\eta^f}}\{\frac{\{\frac{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}\}^{\frac{\mu^f}{\lambda^f}}(\frac{M^*}{L^f})^{\mu^f}-(1-\varphi^f)}{\varphi^f}\}^{\frac{(1-\mu^f)(1-\eta^f)}{\mu^f\eta^f}}-1}{1-\eta^f}$$

$$-\frac{(L^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}}$$

economy. The stationary state of home welfare would be indifferent between allowing capital adjustment and assuming constant capital stock at home but with allowing migration. The foreign migration equilibrium welfare would be higher by assuming endogenous capital adjustment due to the migrant's complementarity to local labour in the foreign production.

In general, optimal migration in a perfectly competitive economy would always comply with the labour and capital equilibrium conditions derived in the centrally planned economies. The limits of optimal migration under these different political institutions are the same. Optimal migration in a perfectly competitive economy would be equivalent to a centrally planned economy.

## 4.2 The imperfectly competitive free-market economy

The real-world free-market economy is imperfect due to many possible causes. For an analysis of migration, a major concern is labour market competitiveness, and so here we only consider how migration decisions depend on labour market imperfections.

In our centrally planned economies, there was no explicit form for the wage to individual household, only a bundle of goods allocated by the central planners to them. In a free but imperfectly competitive labour market, groups of households are assumed to bargain with firms in their own interest. To maximise their power in bargaining, households form unions and firms form cartels.[6] In the real world, the primary objective of unions is to maximise the pay and employment conditions of workers (Oswald, 1993), and so here we assume that the primary objective of labour unions is to maximise the wage income of labour, while the firm cartels bargain about labour costs to maximise profits.

We have a labour union and a firm cartel in each country. The home union needs to juggle its labour supply between its bargains with the two firm cartels, home and foreign. We construct Nash bargains between the labour unions and the firms' cartels. $b_M$ denotes the foreign firms' bargaining power over migrant labour, and $b_f$ ($b_h$) the firms' bargaining power in relation to domestic foreign (home) labour.

In these free markets, each household can choose its allocation between work and leisure in each period. This yields the "reservation wage" whereby everyone has to be motivated to leave non-employment by a reward at the exact break-even point between the marginal disutility of labour supply and the marginal utility of consumption$(-\frac{U_E(N_t)}{U_C(C_t)})$, which is denoted as $W_t^{i,A}$ with $i \in \{h, f\}$ in autarky (here, zero migration).[7] To distinguish the migration-

[6]Firms' market power is represented here through a monopsonistic cartel, but it may arise because they are each large enough to exert oligopsonistic power.

[7]Since the migrants' wage must exceed the home wage for migration to occur, it will be indirectly affected by the autarky reservation wage through its effect on the home wage.

adjusted reservation wage and autarky reservation wage, the migration-adjusted reservation wage will be denoted as $W_t^{i,R}$ with $i \in \{h, f\}$.

The assumed imperfection of the economy leads to labour market failure in the form of unemployment (and thus a change in the labour market relations, eqs.4.1.13 and 4.1.14)

$$L^h = N_t^h + M_t + UN_t^h \tag{4.2.1}$$

$$L^f = N_t^f + UN_t^f \tag{4.2.2}$$

where $UN_t^i$ is the level of unemployment of country $i$ at $t$.[8]

To understand the role of capital in the migration phenomenon, we again present the migration labour market analysis under two assumptions: where capital cannot and can adjust endogenously.

### 4.2.1 Migration under fixed capital

When capital stocks in both economies are fixed and immobile, migration mobility of labour yields a reallocation of labour and production. An efficient bargain may help the two economies achieve their optimal capital-labour ratios, in which case the home labour union plays a vital role due to its involvement in the two bargains.

Without a loss in generality, we assume a staged sequence of bargaining, with the home labour union starting a bargain with the home firm cartel for the domestic labour wage, then the foreign firms' cartel bargaining with the foreign union, and finally bargaining between the foreign firms' cartel and the home union. This sequence delivers a conditional home wage ($W_t^h$), a foreign wage ($W_t^f$) and a migrant wage ($W_t^M$).[9] Given the wage outcomes, the employment and the unemployment levels are determined by the established efficient contracts.

#### 4.2.1.1 Stage 1: home union and home cartel

In bargaining for the home wage, the objective of the home firms' cartel is to maximise its profit of $Y_t^h - W_t^h N_t^h - r_t^h K_t^h$. On the other side of the bargain, the home labour union aims to maximise the net benefit of home households that are employed $W_t^h N_t^h + W_t^M M_t -$

[8] $UN_t^i$ is in effect the non-employment of country $i$ at $t$. It is zero along with employment when the actual wage is lower than the reservation wage because of non-participation of the labour force. The labour force participation is one when the reservation wage is offered. As we have no interest in scenarios with zero participation, it denotes unemployment.

[9] It is important to note that the three stages of bargaining actually happen simultaneously. We present it in a three-stage sequence to better understand what is happening.

$W_t^{h,A}(N_t^h + M_t) - CM_t$ where $W_t^{h,A}$ is the home economy's autarky reservation wage. Both the union and cartel take $W_t^M M_t$ as given at this stage.

As we are bargaining for now in the context of fixed capital, we shall rewrite the firms' cartel's profit function as $Y_t^h - W_t^h N_t^h - \underline{r}^h K^h$, where $\underline{r}^h K^h$ shows a given constant return on a fixed amount of capital which we ignore.[10]

The Nash bargain becomes

$$\max_{\{W_t^h, N_t^h\}} \{(Y_t^h - W_t^h N_t^h)^{b_h} [W_t^h N_t^h + W_t^M M_t - W_t^{h,A}(N_t^h + M_t) - CM_t]^{1-b_h}\},$$
$$b_h \in [0,1] \qquad (4.2.3)$$

where $b_h$ is the bargaining power of the home firm cartel relative to the home union. Given Oswald (1985) and Layard and Nickell (1990)'s insights on the effects of union bargain on equilibrium employment, we argue that the firms' cartel and unions can evolve a long-term repeated bargaining relationship, which enables them to achieve the steady state employment and wage decisions between firms and employees, following Blanchard and Fischer (1989).

Our Nash bargain between firms' profit and households' net wage benefit differs from McDonald and Solow (1981)'s bargain which involves firms' profit and households' utility. Our approach brings the following benefits.

Firstly, we have imposed risk neutrality in bargaining for *both* the union and the cartel, while McDonald and Solow (1981) assume risk neutrality for firms but risk aversion for households.[11] McDonald and Solow (1981) follow the standard intuition that firms, as the "agents" for household capital accumulation and income, should be risk neutral and should simply maximise profits, while the representative household, as the fundamental supplier of labour, accounts for risk aversion, reflecting its trade-off between current and future consumption. We argue that we follow through with this intuition because we have the union as an agent for households and so it should maximise net wage income. The bargain is then between two proxy agents operating in this market, the labour union and the firm cartel, and with labour union maximizing net wage income and the firm cartel maximizing profit, both being risk neutral. The wage income bargain will be included in the budget constraint that is used in the equilibrium expenditure analysis for risk-averse households.

Secondly, the main outcomes of McDonald and Solow (1981) remain. They presented the contract curve between a labour union and a large firm, showing how actual wages and employment are determined in an incomplete competitive free-market economy. Our modifications replicate and extend the contract curve model in a two-country scenario with migrant

[10] In the subsequent parts of this section, the firms' (both home and foreign) owners will receive two types of profit: the capital return by leasing the capital to the firm; and the 'abnormal' profit from bargaining with labour union. Here they simply maximise the latter, and $\underline{r}^h K^h$ is ignored without loss of generality.

[11] **Appendix A.5** shows the bargaining outcomes based on McDonald and Solow (1981)'s set-up.

wage and employment features.

Maximizing eq.4.2.3 with respect to $W_t^h$ gives the conditional bargained home labour wage

$$W_t^h = b_h[\frac{-(W_t^M M_t - W_t^{h,A} M_t - CM_t)}{N_t^h} + W_t^{h,A}] + (1-b_h)\frac{Y_t^h}{N_t^h} \tag{4.2.4}$$

which is a relationship between $W_t^h$ and $N_t^h$ for a given $M_t$.[12]

This wage will be situated between the best alternative for the household and the average cost of labour ($\frac{Y_t^h}{N_t^h}$), depending on $b_h$ the relative bargaining power of the firm cartel.

The migration-adjusted reservation wage is $W_t^{h,R} = \frac{-(W_t^M M_t - W_t^{h,A} M_t - CM_t)}{N_t^h} + W_t^{h,A}$, which is the autarky reservation wage $W_t^{h,A}$ net of the average migration benefit. Note that $W_t^M > W_t^{h,A}$, otherwise no-one would migrate. Insofar as there are migrants, their better wage reduces the adjusted reservation wage required for working at home.

If the firm cartel has full power ($b_h = 1$), the wage will be forced to the lowest migration-adjusted reservation wage, $\underline{W_t^h} = W_t^{h,R} = \frac{-(W_t^M M_t - W_t^{h,A} M_t - CM_t)}{N_t^h} + W_t^{h,A}$. $\underline{W_t^h}$ is the minimum bargained wage and will be less than the reservation wage for $M_t > 0$. As the households are making joint labour supply and consumption decisions, the adjusted reservation wage $(-\frac{U_E(N_t^h + M_t)}{U_C(C_t^h + C_t^M)})$ for the representative household is composed of the migration benefit and the home labour wage income. When a migration benefit is obtained, the home household will supply labour to the home economy even though the actual home wage per worker is lower than the reservation level in autarky.

If the labour union has the full power ($b_h = 0$), the wage will be at the highest possible value, the average labour cost $\frac{Y_t^h}{N_t^h}$.

We further resolve the labour market equilibrium state by considering the first-order condition with respect to $N_t^h$.

$$N_t^h = \frac{b_h(\frac{\partial Y_t^h}{\partial N_t^h} - W_t^h)(W_t^M M_t - W_t^{h,A} M_t - CM_t) + (1-b_h)(W_t^h - W_t^{h,A})Y_t^h}{(W_t^h - b_h\frac{\partial Y_t^h}{\partial N_t^h})(W_t^h - W_t^{h,A})} \tag{4.2.5}$$

If $b_h = 1$, the home employment will be at its lowest level, $\underline{N_t^h} = \frac{W_t^M M_t - W_t^{h,A} M_t - CM_t}{W_t^{h,R} - W_t^h}$ in **Figure 4.1**. A given level of migrant wage benefit abroad will be recognised by the home firm cartel and used to further press down the actual wage at home and thus employment. Under such circumstances, only at $M_t = 0$, the actual home wage will be the autarky reservation wage, and offers the minimal labour supply. If $M_t$ is positive under $b_h = 1$, the

[12] This bargain establishes under the assumption of a given $W_t^M M_t$. Later, we will show that $W_t^M$ will be determined on an efficient contract curve when $M_t$ is chosen.

actual home wage will be lower than the autarky reservation level, which gives less incentive for the home household to supply labour.

If $b_h = 0$, the home employment will be at the highest level, $N_t^h = \frac{Y_t^h}{W_t^h}$. The actual home wage will achieve the average labour cost. With zero 'abnormal' profit of firms, all home labour will work. The home wage will be the highest possible and home employment will equal the remaining labour force $L^h - M_t$. There will be no unemployment. Under such circumstances, the home actual wage $W_t^h$ will be less than $W_t^M - \chi$, the migrant's net benefit. Yet full employment will still be achieved.

Combining eqs.4.2.4 and 4.2.5, the equilibrium wage for the home labour (at a given level of migration) is

$$W^{h*} = b_h W_t^{h,A} + \frac{-b_h(W_t^M M_t - W_t^{h,A} M_t - CM_t) + (1-b_h)Y^{h*}}{b_h \frac{(\frac{\partial Y_t^h}{\partial N_t^h} - W^{h*})(W_t^M M_t - W_t^{h,A} M_t - CM_t)}{(W^{h*} - b_h \frac{\partial Y_t^h}{\partial N_t^h})(W^{h*} - W_t^{h,A})} + (1-b_h)\frac{Y^{h*}}{W^{h*} - b_h \frac{\partial Y_t^h}{\partial N_t^h}}} \tag{4.2.6}$$

Note: $x^*$ represents the equilibrium state of variable $x$.

Substituting eq.4.2.6 into eq.4.2.5 yields $N^{h*}$

$$N^{h*} = \frac{b_h(\frac{\partial Y_t^h}{\partial N_t^h} - W^{h*})(W_t^M M_t - W_t^{h,A} M_t - CM_t) + (1-b_h)(W^{h*} - W_t^{h,A})Y^{h*}}{(W^{h*} - b_h \frac{\partial Y_t^h}{\partial N_t^h})(W^{h*} - W_t^{h,A})} \tag{4.2.7}$$

which depends on $W_t^M M_t$ and the home equilibrium wage. To better understand eqs. 4.2.6 and 4.2.7, we consider two extreme cases when $b_h = 0$ or 1.

$$\begin{aligned}
\{b_h = 0\} :& W^{h*} N^{h*} = Y^{h*} \\
\{b_h = 1\} :& W^{h*} = W_t^{h,A}, \text{ for } M_t = 0 \\
& N^{h*} + M_t = \frac{W_t^M - W^{h*}}{W_t^{h,A} - W^{h*}} M_t, \text{ for } M_t > 0
\end{aligned}$$

When the home firms' cartel has no power in bargaining ($b_h = 0$), the wage and employment will be at the highest possible level, and unemployment will be zero. Also, firms' profit will be zero. When there is an increase in the given migration, zero unemployment will remain due to the firms having no bargaining power.

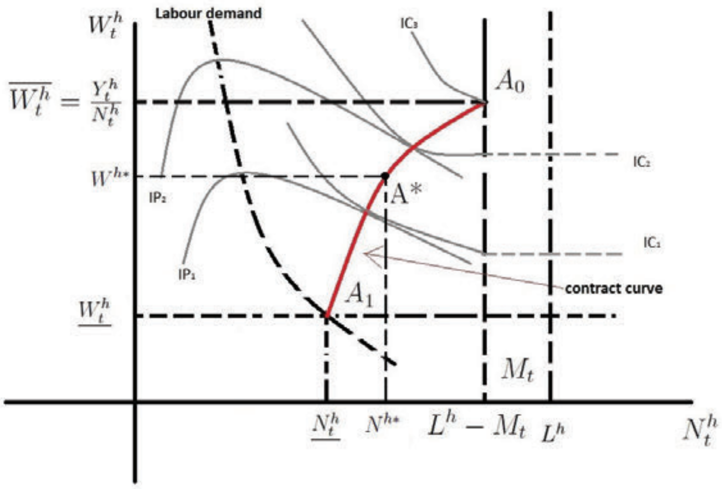
However, when the home firm cartel has all the power ($b_h = 1$), profit will be positive and home domestic labour supply will be incentivized at the lowest possible level for a given level of $W_t^M$ and $M_t$, as will $N_t^h$. Unemployment will be maximised. When there is no migration, the equilibrium wage will be at $W_t^{h,A}$, the reservation level. Home equilibrium employment will be at $\frac{\partial Y_t^h}{\partial N_t^h} = W_t^{h,A}$. With positive migration, the net benefit of migration will be recognised by the cartel and used to further exploit the remaining labour. The equilibrium wage (when $b_h = 1$ and $M_t > 0$) will be lower than the autarky $W_t^{h,A}$, which provides less work incentive to the remaining home labour.

Moreover, at $b_h = 1$, if we have an increase in the given migration, total employment $(N_t^h + M_t)$ will increase, which lowers unemployment.

In short, migration gives a *no-worse-than* autarky unemployment rate when the home firm cartel has full or zero bargaining power.

The foregoing analysis can be represented, as in McDonald and Solow (1981), on an efficient contract curve, which connects points when the home cartel's isoprofit curves ($IP$ in **Figure 4.1**) are tangential to the union's indifference curves ($IC$ in **Figure 4.1**) in wage-home employment space.[13] When the cartel gains more power ($b_h$ increases), the efficient contract outcome shifts from, say, $IP_2$ to $IP_1$ as firms will always push down wages. However, the indifference curve also shifts down showing a lower tangency point on the contract curve with more unemployment.[14]

Figure 4.1: Home wage contract curve




NB: The red line is the efficient contract curve.

[13] All the points on the same indifference curve and isoprofit loci give the same level of households' utility and firms' profit, respectively. The indifference curve and the isoprofit loci are derived from

$$\max_{\{W_t^h, N_t^h\}} \{W_t^h N_t^h + W_t^M M_t - W_t^{h,A}(N_t^h + M_t) - CM_t\}; \max_{\{W_t^h, N_t^h\}} \{Y_t^h - W_t^h N_t^h\}$$

where $\frac{dW_t^h}{dN_t^h} = -\frac{W_t^h - W_t^{h,A}}{N_t^h} - \frac{W_t^M - W_t^{h,A} - \chi}{N_t^h}\frac{dM_t}{dN_t^h} - \frac{M_t}{N_t^h}\frac{dW_t^h}{dN_t^h} < 0$ (at a constant level of $M_t$) and $\frac{d^2 W_t^h}{d(N_t^h)^2} > 0$ for the IC curve and $\frac{dW_t^h}{dN_t^h} = \frac{\frac{dY_t^h}{dN_t^h} - W_t^h}{N_t^h} > 0$ and $\frac{d^2 W_t^h}{d(N_t^h)^2} < 0$ for the IP curve.

Note that the 'labour demand' curve in **Figure 4.1** is the set of solutions that maximise firms' profit. Higher profit and utility can be achieved through moving toward the south-east of the curve to where the contract curve is located. The equilibrium point reached on the contract curve depends, for example, on the relative bargaining power of the two parties.

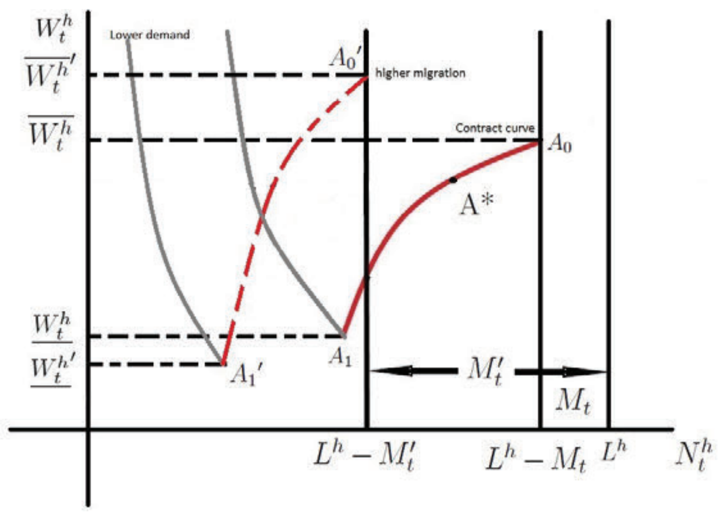
[14] Unemployment increases when home labour employment $N_t^h$ decreases as wages fall from $W_t^h = \frac{Y_t^h}{N_t^h}$ toward $W_t^h = W_t^{h,R}$.

On the contract curve, for every $b_h$, we have $\frac{\partial W_t^h}{\partial N_t^h} > 0$ and $\frac{\partial^2 W_t^h}{\partial (N_t^h)^2} < 0$, as long as the marginal product of home labour $\frac{\partial Y_t^h}{\partial N_t^h}$ is *no-less* than the average labour cost $\overline{W_t^h}$.[15] The home equilibrium is shown as point A* with the home wage at $W^{h*}$ and employment at $N^{h*}$. The home unemployment rate would be the ratio of domestic unemployed to remaining labour, $\frac{L^h - M_t - N^{h*}}{L^h - M_t}$.

When the union gains more power (a decreasing $b_h$) at a given level of $M_t$, households receive a higher wage and are willing to supply a higher level of labour. However, the profit of the firm $(Y_t^h - W_t^h N_t^h)$ gets closer to zero. As $b_h$ goes to zero, the wage will be raised toward its perfectly competitive equilibrium level where $W_t^h = \overline{W_t^h}$, at $A_0$. Full employment $(L^h - M_t)$ is then achieved since the monopsony in labour markets collapses, yielding the equivalent of the perfectly competitive equilibrium, with firms having zero 'abnormal' profit. In the complete monopsony case where the cartel instead has all the bargaining power ($b_h$ goes to 1), the actual wage will be pressed down to the minimum wage, $\underline{W_t^h}$. Employment at home will then be at the lowest level on the contract curve, $A_1$. Unemployment will be at its highest level, and meanwhile firms achieve maximum 'abnormal' profit.

**An increase in given migration**

Figure 4.2: An increase in migration




[15]According to eq.4.2.4, given $W_t^M M_t$, when $\frac{\partial Y_t^h}{\partial N_t^h} > \frac{Y_t^h}{N_t^h}$,

$$\frac{\partial W_t^h}{\partial N_t^h} = [b_h \frac{W_t^M M_t - W_t^{h,A} M_t - CM_t}{N_t^h} + (1-b_h)(\frac{\partial Y_t^h}{\partial N_t^h} - \frac{Y_t^h}{N_t^h})]\frac{1}{N_t^h} > 0$$

and at the same time, assuming $\frac{\partial Y_t^h}{\partial N_t^h} - \frac{Y_t^h}{N_t^h} = \Delta_1$

$$\frac{\partial^2 W_t^h}{\partial (N_t^h)^2} = -\{2b_h \frac{W_t^M M_t - W_t^{h,A} M_t - CM_t}{N_t^h} - (1-b_h)\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + 2(1-b_h)\Delta_1\}(\frac{1}{N_t^h})^2 < 0.$$

Eqs.4.2.4 and A.5.1 give related contract curves. A risk-neutral assumption on the household side will imply a lower level of wage than risk-averse labour suppliers to achieve the same level of welfare.

As the migrants' wage $W_t^M$ is always presented with the size of migration $M_t$ in all stage 1 equations from eq.4.2.3 to 4.2.7, $W_t^M$ and $M_t$ will have the same effects.

When migration $M_t$ increases to $M_t'$, the wage limits move from $\overline{W_t^h}$ and $\underline{W_t^h}$ to $\overline{W_t^h}'$ and $\underline{W_t^h}$, respectively. The highest wage (when $b_h = 0$), $\frac{Y_t^h}{N_t^h}$ based on eq.4.2.4, increases. This is because the average product of labour must rise as home employment falls.[16]

At the same time, the loss of population abroad reduces aggregate demand and labour demand, which results in a leftward shift of the underlying labour demand curve. The lowest (migration-adjusted reservation) wage $\frac{-(W_t^M M_t - W_t^{h,A} M_t - CM_t)}{N_t^h} + W_t^{h,A}$ decreases due to the increased migration benefit, which allows the firm cartel to further exploit the home wage when it has full power in bargaining.

The effect of migration on the home wage in general depends on the size of the migration. A greater dispersion between $\overline{W_t^h}'$ and $\underline{W_t^h}'$ in **Figure 4.2** suggests that migration has an ambiguous effect on the home wage at a given home cartel bargaining power $b_h \in (0, 1)$. Both the positive effects of migration on the average labour cost ($\frac{\partial \frac{Y_t^h}{N_t^h}}{\partial M_t} > 0$) and its negative effects on the migration-adjusted reservation wage ($\frac{\partial \frac{-(W_t^M M_t - W_t^{h,A} M_t - CM_t)}{N_t^h} + W_t^{h,A}}{\partial M_t} < 0$) matter. Note that $\frac{\partial \frac{Y_t^h}{\partial N_t^h}}{\partial M_t} > 0$ is only true if $\frac{Y_t^h}{N_t^h} = \frac{Y_t^h}{L^h - M_t}$ where there is no unemployment. When $b_h < 1$, unemployment arises and the effect of migration on $\frac{Y_t^h}{L^h - M_t - UN_t^h}$ will depend on the negative marginal effect of migration on unemployment. And the most desirable migration occurs when $\frac{\partial \frac{Y_t^h}{L^h - M_t - UN_t^h}}{\partial M_t} = 0$. When migration exceeds this level, the average labour cost will be negatively affected by migration leading to higher unemployment.

At this stage, due to the as yet unknown determinants of $M_t$, we observe that the enlarged dispersion resulting from an increase in migration indicates the effect of migration on equilibrium home wages depends on $M_t$ and $b_h$.

For a given level of migration, if $b_h$ is large, the negative effects of the home cartel exploiting individuals' reservation wage will take over and lead to a lower actual bargained wage and higher home unemployment. The smaller the firm cartel power ($b_h \to 0$), the larger migration is required to have $\frac{\partial W_t^h}{\partial M_t} < 0$ in eq.4.2.4. For a given level of $b_h$, if $M_t$ is large, the same exploitation on $W_t^M M_t$ exerts a bigger negative influence on the actual home wage than a small $M_t$, and so $\frac{\partial W_t^h}{\partial M_t} < 0$ which leads to higher unemployment.

[16] When $b_h = 0$, the home union has all bargaining power, and the wage set at the average labour cost would attract all labour to work so that actual employment must decrease from $L^h - M_t$ to $L^h - M_t'$. Differentiating $\frac{Y_t^h}{L^h - M_t}$ with respect to $M_t$ gives

$$\frac{\partial \frac{Y_t^h}{L^h - M_t}}{\partial M_t} = Z_t^h \varphi^h (K_t^h)^{\mu^h} (L^h - M_t)^{-\mu^h - 1} [\varphi^h (\frac{K_t^h}{L^h - M_t})^{\mu^h} + (1 - \varphi^h)]^{\frac{1-\mu^h}{\mu^h}} > 0$$

when the home capital stock ($K_t^h$) is fixed.

For **Stage 1**, we observe that sufficiently large migration and/or high enough firm power in wage bargains will increase home unemployment.

#### 4.2.1.2 Stage 2: foreign union and foreign cartel

We now consider bargaining between the foreign labour union and the foreign firms' cartel. Similar to **Stage 1**, the foreign labour union maximises its net benefit of working ($W_t^f N_t^f - W_t^{f,R} N_t^f$),[17] while the foreign firms' cartel maximises overall firm profit $Y_t^f - W_t^f N_t^f - W_t^M M_t$, each taking $W_t^M$ and $M_t$ as given (and again ignoring the given return on capital stock). Their bargain becomes:

$$\max_{\{W_t^f, N_t^f\}} \{(Y_t^f - W_t^f N_t^f - W_t^M M_t)^{b_f} (W_t^f N_t^f - W_t^{f,R} N_t^f)^{1-b_f}\}, b_f \in (0,1)$$

The foreign labour wage and employment will be maximised at

$$W_t^f = b_f W_t^{f,R} + (1-b_f) \frac{Y_t^f - W_t^M M_t}{N_t^f} \tag{4.2.8}$$

$$N_t^f = \frac{(1-b_f)(Y_t^f - W_t^M M_t)}{W_t^f - b_f \frac{\partial Y_t^f}{\partial N_t^f}} \tag{4.2.9}$$

The foreign domestic wage is determined by the bargaining power of the foreign labour union ($b_f$), the foreign economy reservation wage $W_t^{f,R}$ and the average foreign labour cost of its firms ($\frac{Y_t^f - W_t^M M_t}{N_t^f}$). The foreign wage contract curve is similar to what was shown in **Figure 4.1** for the home economy, with employment increasing from a monopsony state with high unemployment to the perfectly competitive equilibrium state with full employment and zero 'abnormal' profit. Combining eqs.4.2.8 and 4.2.9 gives

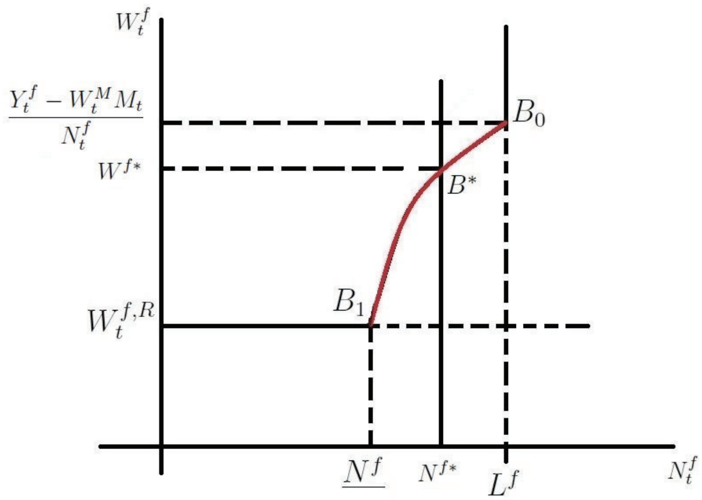
$$W_t^{f,R} = \frac{\partial Y_t^f}{\partial N_t^f} \tag{4.2.10}$$

which determines the equilibrium employment level in the foreign economy, $N^{f*}$. Together with eq.4.2.8, the contract curve can be drawn as **Figure 4.3**.

The wage of foreign labour will be at a point on the contract curve depending on the relative

[17] In the foreign economy, as migration is irrelevant to the foreign representative family from a household's consumption and labour supply perspective. We have $W_t^{f,R} = W_t^{f,A} = -\frac{U_E(N_t^f)}{U_C(C_t^f)}$, which the motivation to have labour supply would be at the same level.

Figure 4.3: foreign wage contract curve




NB: $\underline{N^f}$ is the employment level when $W_t^f = W_t^{f,R}$.

bargaining power. The equilibrium $(B^*)$ is given by the following two conditions.

$$N^{f*} : W_t^{f,R} = (1-\varphi^f)(1-\omega^f)[\varphi^f(\frac{K_t^f}{N^{f*}})^{\mu^f} + (1-\varphi^f)]^{\frac{\lambda^f-\mu^f}{\mu^f}}$$
$$\{\omega^f(\frac{M_t}{N^{f*}})^{\lambda^f} + (1-\omega^f)[\varphi^f(\frac{K_t^f}{N^{f*}})^{\mu^f} + (1-\varphi^f)]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} \tag{4.2.11}$$
$$W^{f*} : W^{f*} = b_f W_t^{f,R} + (1-b_f)\frac{Y^{f*} - W_t^M M_t}{N^{f*}} \tag{4.2.12}$$

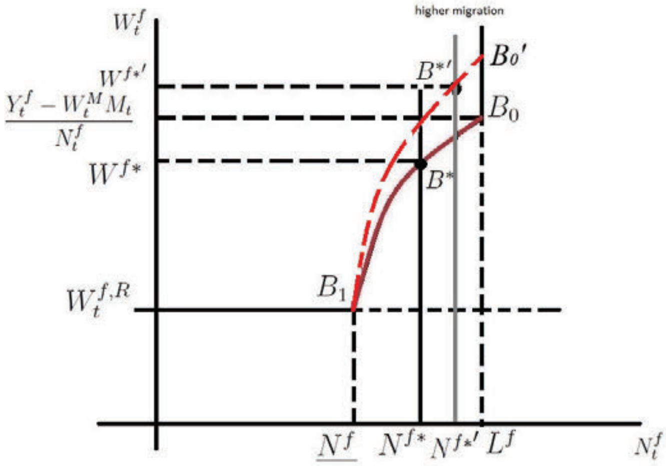
Eq.4.2.11 shows that the equilibrium foreign employment would be a constant for a given level of migration $(M_t)$ and foreign capital stock $(K_t^f)$. Eq.4.2.12 gives an explicit relationship between the equilibrium foreign employment and the foreign equilibrium wage. With the constant foreign employment, greater foreign union power in bargaining $(1-b_f)$ leads to a higher equilibrium foreign wage but has no effect on equilibrium unemployment.

The irrelevancy for the equilibrium foreign unemployment of the foreign cartel bargaining power $(b_f)$ is a significant feature that the efficient contract curve delivers, but only if we maintain a fixed capital stock in the economy. Though we have ignored the rental cost and return of capital in bargaining, the capital endowment will inevitably affect the level of employment in equilibrium through its effect on output. In autarky, equilibrium output in the foreign economy is constrained by the fixed stock of capital and will always imply a constant equilibrium level of foreign labour demand. Rational households will supply this level of labour in equilibrium and accept the bargained wage.

Migration, in this set-up, improves equilibrium foreign employment, and reduces unemployment.

An increase in the given $M_t$ tilts up the wage contract curve moving $B_0$ to $B_0{}'$ in **Figure**

Figure 4.4: foreign wage contract curve




**4.4**. Since $\frac{\partial W_t^f}{\partial M_t} > 0$ (from eq.4.2.8), this encourages higher actual employment. By observing eq.4.2.11, the complementarity in production would increase equilibrium employment (moving from $N^{f*}$ to $N^{f*'}$), and thus reduce equilibrium unemployment ($\frac{L^f - N^{f*}}{L^f}$). The equilibrium wage will be pushed up from $B^*$ to $B^{*'}$ according to eq.4.2.12. In short, an increase in $M_t$, *ceteris paribus*, will benefit the foreign economy via increasing wages and decreasing unemployment.

Differentiating eq.4.2.12 with respect to $W_t^M$ shows that the equilibrium foreign wage decreases with the migrants' wage. However, the equilibrium foreign employment rate in eq.4.2.11 is independent of $W_t^M$, which in turn indicates unchanged equilibrium unemployment in the foreign economy.

From the foreign perspective of **Stage 2**, an increase in migration will increase the domestic labour wage and decrease unemployment (at a constant migrant wage). A higher migrants' wage (at constant $M_t$) will decrease the equilibrium foreign labour wage but leave constant equilibrium foreign unemployment.

#### 4.2.1.3 Stage 3: home union and foreign cartel

In the third stage, we consider the conditional determinants of migrants' wage and employment. The bargain is between the migrant labour union and foreign firm cartel.

As in **Stage 1**, the agent of home country households, the home labour union, maximises the net benefit of employment (both at home and foreign) $W_t^h N_t^h + W_t^M M_t - W_t^{h,A}(N_t^h + M_t) - CM_t$. Here we take the home wage ($W_t^h$) and employment ($N_t^h$) as given.

The foreign firm cartel will maximise foreign firm profits $Y_t^f - W_t^f N_t^f - W_t^M M_t$, as in **Stage 2**, conditioned on given levels of the foreign wage ($W_t^f$) and employment ($N_t^f$), and ignoring the given capital return.

Therefore, given $W_t^h$, $N_t^h$, $W_t^f$, and $N_t^f$, the migration bargain is formulated as

$$\max_{\{W_t^M, M_t\}} \{(Y_t^f - W_t^f N_t^f - W_t^M M_t)^{b_M} [W_t^h N_t^h + W_t^M M_t - W_t^{h,A}(N_t^h + M_t) - CM_t]^{1-b_M}\}, b_M \in [0,1]$$

where $b_M$ denotes the relative bargaining power of the foreign cartel over migrants.

The first-order conditions with respect to $W_t^M$ and $M_t$ are

$$W_t^M = (1-b_M)\frac{Y_t^f - W_t^f N_t^f}{M_t} + b_M \frac{-W_t^h N_t^h + W_t^{h,A}(N_t^h + M_t) + CM_t}{M_t} \tag{4.2.13}$$

$$M_t = \frac{b_M(\frac{\partial Y_t^f}{\partial M_t} - W_t^M)(W_t^h N_t^h - W_t^{h,A} N_t^h - CM_0) + (1-b_M)(Y_t^f - W_t^f N_t^f)(W_t^M - W_t^{h,A} - \chi)}{(W_t^M - b_M \frac{\partial Y_t^f}{\partial M_t})(W_t^M - W_t^{h,A} - \chi)} \tag{4.2.14}$$

The bargained migrants' wage is a weighted average of the average migrant labour cost for the foreign firms ($\frac{Y_t^f - W_t^f N_t^f}{M_t}$) and of the home-employment-adjusted reservation wage of migrants ($\underline{W_t^M} = \frac{-W_t^h N_t^h + W_t^{h,A}(N_t^h + M_t) + CM_t}{M_t}$).

Corresponding to $\underline{W_t^h}$ in **Stage 1** which is the minimum home wage taking migration benefit as given, $\underline{W_t^M}$ is the minimum wage that the foreign cartel targets while bargaining with the migrant labour union, given the home labour income. Both home and foreign cartels aim to press the whole family income down toward the autarky reservation wage ($W_t^{h,A}$) while obtaining full bargaining power.

To gain a deeper understanding of eqs.4.2.13 and 4.2.14, we consider the conditional equilibrium migrants' wage and employment when $b_M$ takes on extreme values.

$$\text{When } b_M = 1: W^{M*}M^* + W_t^h N_t^h = W_t^{h,A}(N_t^h + M^*) - CM^* \tag{4.2.15}$$

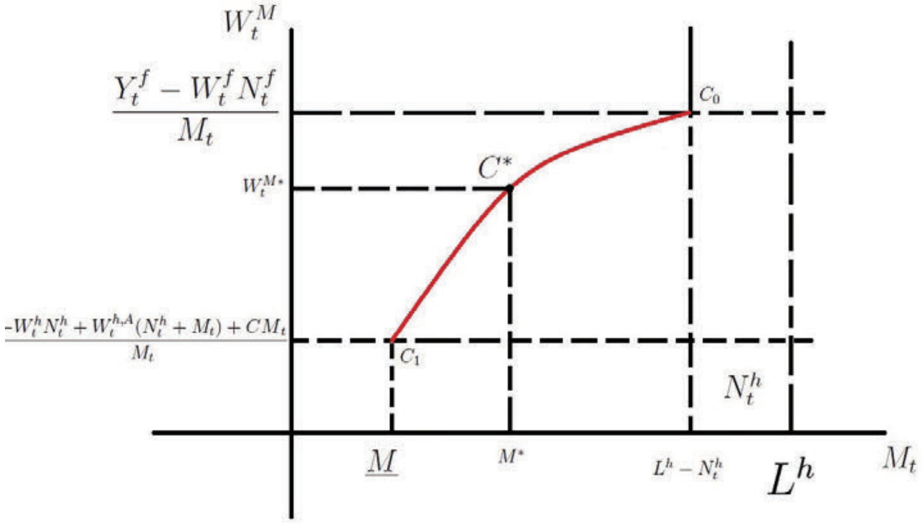
$$\text{When } b_M = 0: W^{M*}M^* = Y^{f*} - W_t^f N_t^f \tag{4.2.16}$$

Similar to **Stage 1**, when the relative bargaining power of foreign cartel to home union rises so that $b_M \to 1$, home wage income allows a heavier exploitation by the foreign cartel toward migrants. In the limit, $b_M \to 0$ produces point $C_1$ in the **Figure 4.5**, which yields minimum migration ($\underline{M}$) with maximum unemployment at the given level of $N_t^h$.

When $b_M \to 0$, the migrant union can deliver a wage bargain at their own average labour cost for given levels of foreign wage and employment, which leaves the foreign firm zero profit from production. We reach the point $C_0$, where all home labour, apart from those assumed employed at home, will choose to migrate and work abroad ($M_t = L^h - N_t^h$), so that unemployment is zero.

For $b_M \in (0,1)$, the conditional equilibrium is at a point like $C^*$, and the unemployment rate would be $L^h - N_t^h - M^*$. The point $C^*$ is the solution from eqs.4.2.13 and 4.2.14, which gives the equilibrium migrant wage $W_t^{M*}$ and equilibrium migration $M^*$.

Figure 4.5: Migrant wage contract curve




NB: $\frac{\partial W_t^M}{\partial M_t} > 0$ and $\frac{\partial^2 W_t^M}{\partial (M_t)^2} < 0, \forall \frac{\partial Y_t^f}{\partial M_t} \geq \frac{Y_t^f - W_t^f N_t^f}{M_t}$

As in stage 1, as long as the marginal product of migrants is *no-less* than the average cost of migrants, the migrant contract curve is positively sloped and concave (shown as the red curve in **Figure 4.5**).

Now, we investigate what would happen to migration and foreign and home unemployment if the given home and foreign labour market conditions ($W_t^h$, $N_t^h$, $W_t^f$, $N_t^f$) change.

**An improvement in the given home labour market conditions**

As the home labour wage and employment are included as home labour income in eq.4.2.13, an increase in $N_t^h$ has the same effect on migrants' wages as the home wage's increase.

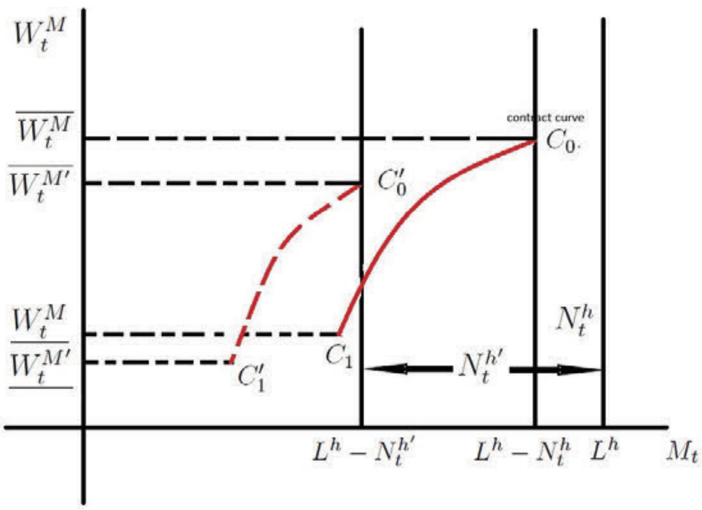
With an improvement in domestic labour market conditions, the red contract curve will shift to $C_0'C_1'$ in **Figure 4.6**.

When $N_t^h$ increases to $N_t^{h'}$, the migrant wage extremes shift downward (from $\overline{W_t^M}$ and $\underline{W_t^M}$ to $\overline{W_t^{M'}}$ and $\underline{W_t^{M'}}$, respectively). Both the highest and lowest possible wages of migrants will fall. When $b_M = 0$, the migrant wage is at the average labour cost and migration is maximised at $M_t = L^h - N_t^{h'}$. Differentiating $\frac{Y_t^f - W_t^f N_t^f}{M_t}$ with respect to $N_t^h$ gives

$$\frac{\partial \frac{Y_t^f - W_t^f N_t^f}{L^h - N_t^h}}{\partial N_t^h} = -\left(\frac{\partial Y_t^f}{\partial N_t^h} - \frac{Y_t^f - W_t^f N_t^f}{L^h - N_t^h}\right)\frac{1}{L^h - N_t^h} < 0.$$

When the foreign cartel has all the power ($b_M = 1$), the wage will be pressed to the minimum

Figure 4.6: A higher $W_t^h$ and/or $N_t^h$




recognizing the increased home wage benefit.

$$\frac{\partial \frac{-W_t^h N_t^h + W_t^{h,A}(N_t^h + M_t) + CM_t}{M_t}}{\partial N_t^h} = \frac{-W_t^h + W_t^{h,A}}{M_t} \leq 0$$

for any given $N_t^h > 0$.

At any given level of $b_M$, both the wage and number of migrants decrease with an increase of home labour as the general migrant wage equation (eq.4.2.13) is a weighted average of both extreme wage outcomes, which are adversely related to home employment ($N_t^h$).

Improved home employment has an ambiguous effect on home unemployment ($L^h - N_t^h - M_t$) due to the negative effect of $N_t^h$ on migration. Based on $\frac{\partial(L^h - N_t^h - M_t)}{\partial N_t^h}$, the actual effect of home employment depends on $\frac{\partial M_t}{\partial N_t^h}$. Since $\frac{\partial M_t}{\partial N_t^h} > -1$,[18] home unemployment will decrease.

Regarding foreign unemployment ($L^f - N_t^f$), an increase in given home employment and/or wage leads to a worsening of foreign labour market conditions through the negative effects of migrants. The reduction in migration decreases the marginal productivity of foreign labour (since $\frac{\partial^2 Y_t^f}{\partial N_t^f M_t} > 0$), and thus the demand for foreign labour. foreign unemployment rises.

In general, improving home labour market conditions will reduce the home unemployment at the cost of the migration benefit, foreign output and thus worsens foreign unemployment.

[18] Differentiating eq.4.2.14 with respect to $N_t^h$ gives

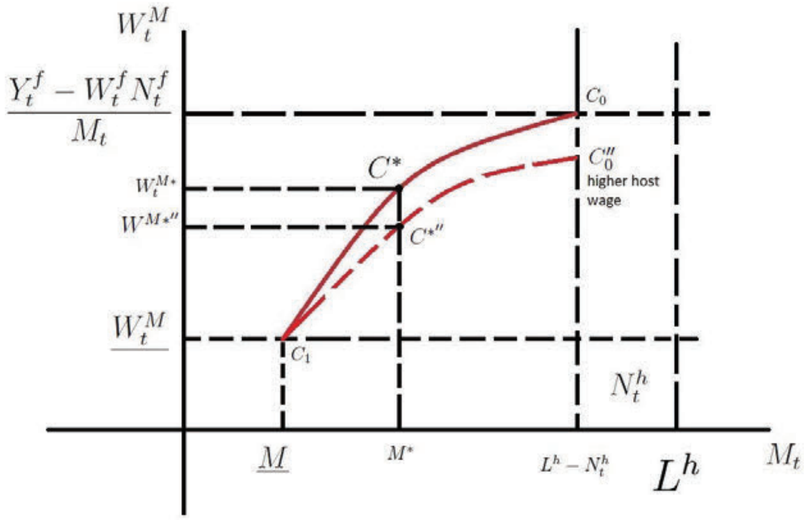
$$\frac{\partial M_t}{\partial N_t^h} = \frac{b_M(\frac{\partial Y_t^f}{\partial M_t} - W_t^M)(W_t^h - W_t^{h,A})}{(W_t^M - b_M \frac{\partial Y_t^f}{\partial M_t})(W_t^M - W_t^{h,A} - \chi)}$$

which can be negative as $b_M \to 1$. However, at $b_M = 1$, the marginal effect of home employment on migration reaches $-\frac{W_t^h - W_t^{h,A}}{W_t^M - W_t^{h,A} - \chi}$. It is always bigger than -1 because migration only incurs if $W_t^M - \chi \geq W_t^h$ when the marginal benefit of migration is higher than the opportunity cost, referring to eq.4.1.16.

### An increase in the given $W_t^f$

At a constant foreign employment ($N_t^f$), an increase in the foreign wage will tilt down the migrant wage contract curve.

Figure 4.7: A higher foreign wage




A higher foreign wage at a fixed $N_t^f$ has no effect on the migrants' wage when the foreign cartel has been able to set the migrants wage at the reservation level since $b_M = 1$. But it will strictly decrease the migrants' wage when $b_M < 1$ since $\frac{\partial W_t^M}{\partial W_t^f} = -(1 - b_M)\frac{N_t^f}{M_t} < 0$ in eq.4.2.13. The contract curve tilts down from $C_1C_0$ to $C_1C_0''$.

Home unemployment for $b_M \in (0, 1)$ will be unchanged as the bargained wage will be still higher than the reservation level. This is consistent with the effect of the migrants' wage (at constant $M_t$) on foreign wage and unemployment in **Stage 2**.

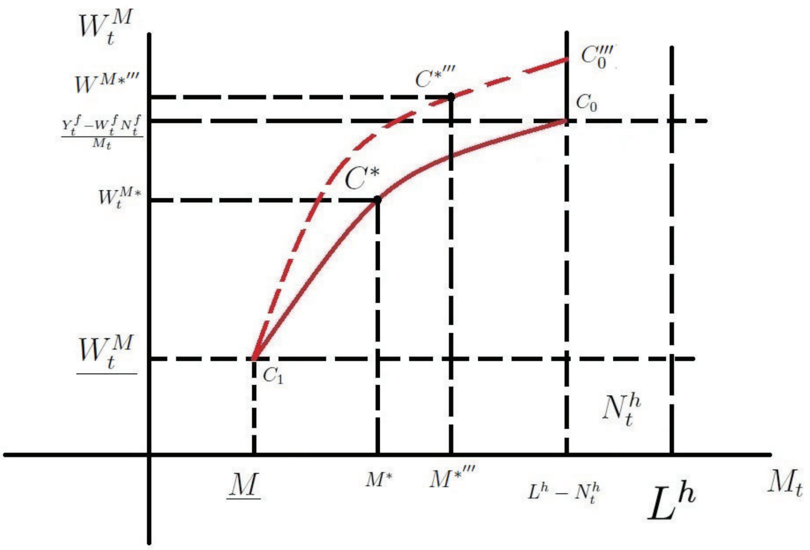
### An increase in the given $N_t^f$

A larger foreign employment, at a constant home country employment, benefits both economies.

The intuition is that a larger foreign employment stimulates more immigration because $\frac{\partial^2 Y_t^f}{\partial N_t^f M_t} > 0$, so that an additional unit of foreign employment has a positive effect on the marginal productivity of migrants. Under the assumption of local-migrant complementarity in the foreign production, one more unit of foreign labour will increase the demand for migrants, which will raise their bargained wage to $W^{M*'''}$ ($\frac{\partial W_t^M}{\partial N_t^f} > 0$ in eq.4.2.13). The migration size will be at $M^{*'''}$ according to eq.4.2.11 with the increased given level of $N_t^f$. A higher migrants' wage is associated with larger migration on the contract curve, which improves foreign and home unemployment for given $N_t^h$. The migrant equilibrium moves from $C^*$ to $C^{*'''}$ in the **Figure 4.8**.

When $b_M = 0$ foreign cartel has no power in migrants' wage bargain, and an increase in foreign employment (at a fixed foreign wage) will increase the migrant wage (moving $C_0$ to

Figure 4.8: A higher foreign employment under given $N_t^h$




$C_0''')$ since $\frac{\partial \frac{Y_t^f - W_t^f N_t^f}{M_t}}{\partial N_t^f} = \frac{\frac{\partial Y_t^f}{\partial N_t^f} - W_t^f}{M_t} > 0$ given constant $L^h - N_t^h$. Under this circumstance, home unemployment is at zero as $L^h - N_t^h$ people choose to migrate, while foreign unemployment declines due to the given rise in $N_t^f$. Both countries benefit.

When $b_M = 1$, the foreign cartel determines the migrants' contract with the migrants' wage subject to the home reservation wage ($W_t^{h,A}$) and the given home labour market bargained outcome ($W_t^h N_t^h$), which are irrelevant to foreign production. Thus, the overall migrants' income is fixed by the foreign cartel at $W_t^{h,A}(N_t^h + M_t) + CM_t - W_t^h N_t^h$. Home households will not increase their migration supply at the lowest migrants' wage. Migration and the migrants' wage will stay at $C_1$ in the graph.

In short, improved foreign labour market conditions will increase migration, and reduce the foreign and home unemployment.

#### 4.2.1.4 Jointly comprehending the three-stage outcomes

To have a complete viewpoint on the global labour market, we combine the wage-employment contract relations from the foregoing three stages since wage bargains are happening simultaneously.

Substituting the home wage (eq.4.2.4) and the foreign wage (eq.4.2.8) into the migrants' wage outcome (eq.4.2.13) gives

$$W_t^M = \frac{(1-b_M)b_f(Y_t^f - W_t^{f,R}N_t^f) + (1-b_h)b_M[W_t^{h,A}(N_t^h + M_t) + CM_t - Y_t^h]}{[b_f(1-b_M) + b_M(1-b_h)]M_t} \tag{4.2.17}$$

showing that the foreign reservation wage ($W_t^{f,R}$) has a negative effect on the migrants' wage, while a higher home autarky reservation wage ($W_t^{h,A}$) and costs of migration ($CM_0$ and $\chi$) will increase the wage of migrants.

Meanwhile, migrants' income is positively related to the foreign output, while being negatively related to the home output. Based on the home and foreign production function (eqs.4.1.7 and 4.1.9), the home and foreign output can be directly affected by the technology shocks ($Z_t^h$, $Z_t^f$) respectively. A booming home technology, thus, brings more home employment and higher home wage so that migrants are less attracted by the foreign economy.[19] And an increase in the foreign production technology will increase the marginal productivity of migrants. As long as the foreign cartel is not at full power in the migration bargain ($b_M \neq 1$) and does not lose all power in the domestic labour bargain ($b_f = 0$), an increase in $Z_t^f$ will always benefit the migrants.

Substituting eq.4.2.17 into eqs.4.2.4 and 4.2.8, the complete derivations of the home and foreign wages are

$$W_t^h = \frac{-b_h b_f(1-b_M)[Y_t^f - W_t^{f,R}N_t^f - W_t^{h,A}(N_t^h + M_t) - CM_t] + (1-b_h)[(1-b_M)b_f + b_M]Y_t^h}{[b_f(1-b_M) + b_M(1-b_h)]N_t^h} \tag{4.2.18}$$

$$W_t^f = \frac{(1-b_M b_h)b_f W_t^{f,R}N_t^f + (1-b_f)(1-b_h)b_M[Y_t^f + Y_t^h - W_t^{h,A}(N_t^h + M_t) - CM_t]}{[b_f(1-b_M) + b_M(1-b_h)]N_t^f} \tag{4.2.19}$$

The first equation suggests that both home and foreign autarky reservation wages ($W_t^{h,A}$, $W_t^{f,R}$) will have a positive effect on the bargained home wage if and only if $M_t > 0$. The home labour wage is positively affected by its own output but negatively by the foreign output. An non-trivial finding by eq.4.2.19 is that the foreign labour wage will be improved as increases in both home and foreign output ($Y_t^h$, $Y_t^f$).

Here provides a more detailed elaboration on the two equations. The home labour income rises with the home technology innovation through its effects on the home output and home labour marginal productivity. Meanwhile, the $W_t^h$ is negatively affected by the foreign technology. A technology booming in the foreign will increase the migration benefit through its incremental effects on the marginal productivity of all labour. At a fixed level of home cartel bargaining power, a larger migration benefit results in a heavier exploitation of the home firm to the remaining home labour.

Furthermore, migration has a negative effect on the home labour wage. As migration in-

[19]This is always true as

$$\frac{\partial^2 Y_t^h}{\partial N_t^h \partial Z_t^h} > 0$$

under the assumptions of the constant return to scale production function.

creases, foreign output increases due to higher labour input and complementarity in the foreign production, while home output decreases due to the loss in aggregate demand. Under such circumstances, the home wage would fall.

In eq.4.2.19, an increase in the foreign reservation wage will increase the foreign wage, opposite to the autarky home reservation wage. The positive relationship between two outputs and the foreign labour wage shows that technology innovations in both economies will benefit the foreign labour income. It has to be through the adjustment of migration. A higher foreign output, *ceteris paribus*, gives a higher foreign wage. At the same time, taking foreign output as given, an increase in the home output will decrease the attractiveness of the foreign economy to *would-be* migrants and reduce total migrants' income in the foreign ($W_t^M M_t$) so that the average foreign labour cost ($Y_t^f - W_t^M M_t$) increases, leading to a higher foreign wage.[20]

**Proposition 4.2.1** *A foreign technology innovation benefits the wages of migrants and its own labour, while home technology changes benefit domestic workers (in both countries) but not migrants.*

The most important outcome from eqs.4.2.17 to 4.2.19 is that the bargaining powers ($b_h$, $b_M$, $b_f$) prevail in all wage equations. Here we present eight extreme scenarios to explore how migration and unemployment of two countries are affected by bargaining powers.

*Scenario 1:* $b_f = 1$, $b_M = 0$, $b_h = 0$

When the foreign cartel has all power over its local labour, the home union has all monopsony power in both migrant and domestic labour supply. Eqs.4.2.17 to 4.2.19 become

$$\begin{aligned} W_t^M M_t =& Y_t^f - W_t^{f,R} N_t^f \\ W_t^h N_t^h =& Y_t^h \\ W_t^f N_t^f =& W_t^{f,R} N_t^f \end{aligned}$$

The migrants' income will be maximised at the highest possible level that equals foreign production minus minimum aggregate labour cost when $b_f = 1$ foreign cartel presses the foreign wage to the bottom. However, the home wage income also equals the average labour productivity due to the union's domination in the home labour market bargaining.

In this case, the size of migration would be determined by the productivities of two economies. As long as $Y_t^f - W_t^{f,R} N_t^f > Y_t^h$,[21] full migration occurs. In opposite, no migration is

[20] However, it is important to note that the foreign output and home output cannot be isolated from each other in a migration economy. A smaller migration induced by higher home output could potentially decrease the foreign output.

[21] We neglected the constant return of capital in this subsection.

generated. A positive equilibrium migration occurs when $Y_t^f - W_t^{f,R} N_t^f = Y_t^h$.

$$M_t = \{\frac{\{\frac{Z_t^h[\varphi^h(K^h)^{\mu^h}+(1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1}{\mu^h}}+W_t^{f,R}N_t^f}{Z_t^f}\}^{\lambda^f} - (1-\omega^f)[\varphi^f(K^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}}{\omega^f}\}^{\frac{1}{\lambda^f}}$$

shows that the size of migration depends on the macroeconomic variables of both economies.

For the foreign economy, though the foreign wage is suppressed to the lowest level, full migration and the complementarity of two labour inputs in the foreign production can motivate the optimal foreign equilibrium employment $N^{f*}$ in the eq.4.2.11.

In general, we will have positive migration. The home labour supply will increase to maxima due to the highest incentive from both home and foreign countries. And the increase of foreign employment relies on the size of migration inflow.

*Scenario 2:* $b_f = 1$, $b_M = 0$, $b_h = 1$

Here the firm cartels obtain the monopsony power with their local labour union but the foreign cartel loses all power in bargaining with the home union. The migrant, home labour and foreign labour incomes are

$$\begin{aligned} W_t^M M_t =& Y_t^f - W_t^{f,R} N_t^f \\ W_t^h N_t^h + (Y_t^f - W_t^{f,R} N_t^f) =& W_t^{h,A}(N_t^h + M_t) + CM_t \\ W_t^f N_t^f =& W_t^{f,R} N_t^f \end{aligned}$$

Incorporating the above outcomes gives

$$\begin{aligned} W_t^M M_t =& Y_t^f - W_t^{f,R} N_t^f \\ W_t^h N_t^h + W_t^M M_t =& W_t^{h,A}(N_t^h + M_t) + CM_t \\ W_t^f =& W_t^{f,R} \end{aligned}$$

Similar to *Scenario 1*, migrants' wage reaches the highest when the foreign wage is at reservation level and the foreign firms are paying the migrants with all their output net of the foreign wage.

Home labour income will be at the lowest when the home labour union loses its bargain with the home cartel. The sole purpose of home employment is to achieve enough income that can meet up with the sum of autarky reservation income and total cost of migration, with the given highest migration benefit.

foreign wage will be at its reservation level due to the full bargaining power of the foreign cartel. Its equilibrium employment will be following eq.4.2.11, which $N_t^f$ is a positive function of $M_t$.

In short, a rational home household will choose full migration. The home unemployment is zero as no one prefers to stay and foreign unemployment can be reduced to its lowest as *Scenario 1.*

**Proposition 4.2.2** *Full migration occurs when the foreign cartel has all power in its local wage bargain but loses in migrants' wage bargain, irrelevant to the home labour bargain.*

*Scenario 3:* $b_f = 1$, $b_M = 1$, $b_h = 0$

Now assume that the foreign cartel has all power over both foreign and migrant labour, while the home cartel has none. The three wage equations are

$$\begin{aligned} W_t^M M_t + Y_t^h =& W_t^{h,A}(N_t^h + M_t) + CM_t \\ W_t^h N_t^h =& Y_t^h \\ W_t^f N_t^f =& W_t^{f,R} N_t^f \end{aligned}$$

The solutions can be reduced to

$$\begin{aligned} W_t^h N_t^h =& Y_t^h \\ W_t^M M_t + W_t^h N_t^h =& W_t^{h,A}(N_t^h + M_t) + CM_t \\ W_t^f =& W_t^{f,R} \end{aligned}$$

Home wage will be at the highest as the home union has full power in bargaining with the home cartel. Full employment will be reached in the home economy.

Because the home union loses all power in the migrants' wage bargain, the foreign cartel can exploit the home labour (migrants) the most due to the high home employment benefit.

The size of migration is determined solely by the migrants' wage equation above. By manipulating the equation, migration only exists if $-\frac{(W_t^h - W_t^{h,A})N_t^h - CM_0}{W_t^M - W_t^{h,A} - \chi} \geq 0$, which is not likely as both numerator and denominator are larger than zero.[22]

In the foreign economy, foreign wage will be suppressed to the reservation level and leads to the lowest autarky employment (eq.4.2.11 with $M_t = 0$).

In this case, a rational home household will choose to not migrate and to make full home labour supply for the average home labour cost. Home and foreign equilibrium employment will be at lowest positions in the contract curve.[23]

[22] The numerator $-[(W_t^h - W_t^{h,A})N_t^h - CM_0]$ can only be positive if the fixed cost of migration ($CM_0$) is large. Under such circumstance, migration is an action to compensate the fixed cost, which is a corner condition.

[23] In autarky, the home and foreign bargains are

$$\max_{\{N_t^h\}} \{(Y_t^h - W_t^h N_t^h)^{b_h} (W_t^h N_t^h - W_t^{h,A} N_t^h)^{1-b_h}\}$$

*Scenario 4:* $b_f = 0$, $b_M = 1$, $b_h = 0$

Assumes both cartels have no power in bargaining with their local labour unions, while the foreign cartel has the monopsony power over the migrants. The wage outcomes are

$$\begin{aligned} W_t^M M_t + Y_t^h =& W_t^{h,A}(N_t^h + M_t) + CM_t \\ W_t^h N_t^h =& Y_t^h \\ W_t^f N_t^f + W_t^{h,A}(N_t^h + M_t) + CM_t =& Y_t^f + Y_t^h \end{aligned}$$

Combining three outcomes gives

$$\begin{aligned} W_t^h N_t^h =& Y_t^h \\ W_t^f N_t^f =& Y_t^f - W_t^M M_t \\ W_t^M M_t + W_t^h N_t^h =& W_t^{h,A}(N_t^h + M_t) + CM_t \end{aligned}$$

The domestic wages in both foreign and home are maximised to the average labour cost, which will motivate full employment in both economies. Migrants' wage will be minimized due to the full bargaining power of the foreign cartel toward home labour union and the high home labour income.

In general, no migration incurs due to the least incentive and the highest home wage income. Both economies will achieve full employment and autarky equilibrium wages (at the marginal product of labour).

**Proposition 4.2.3** *A weak condition of no migration is when the home labour union has no power bargaining toward the foreign cartel while it has full power in the home wage bargain. To prevent overexploitation of migrants, home households might choose to keep all their family members at home.*

*Scenario 5:* $b_f = 1$; $b_M = 1$, $b_h = 1$

When the cartels have all the power in bargaining with all three labour groups, all three comprehended wage equations (eqs.4.2.17 to 4.2.19) are invalidated as both their numerators and denominators turn to zero. We are in a *no-migration* case.

In autarky, as the home cartel has all power, the home household will receive the autarky reservation wage. The home unemployment will be at its highest where $W_t^{h,A} = \frac{\partial Y_t^h}{\partial N_t^h}$. The

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and

$$\max_{\{N_t^f\}} \{(Y_t^f - W_t^f N_t^f)^{b_f} (W_t^f N_t^f - W_t^{f,R} N_t^f)^{1-b_f}\}$$

always generate equilibrium employment at $W_t^{h,A} = \frac{\partial Y_t^h}{\partial N_t^h}$ and $W_t^{f,R} = \frac{\partial Y_t^f}{\partial N_t^f}$.

algebraic form of home unemployment is at

$$L^h - [\frac{(\frac{W_t^{h,A}}{1-\varphi^h})^{\frac{\mu^h}{1-\mu^h}} - (1-\varphi^h)}{\varphi^h}]^{-\frac{1}{\mu^h}} K^h.$$

Meanwhile, foreign households will also only receive the reservation wage. And the foreign employment will be at $W_t^{f,R} = \frac{\partial Y_t^f}{\partial N_t^f}$ in autarky which produces the highest unemployment in the efficient contract model.

*Scenario 6:* $b_f = 0$; $b_M = 0$, $b_h = 0$

At the same time when all unions have full power over the cartels, these three wage equations will also turn to invalid. We again arrive at *no-migration*.

However, as all unions obtain full power in wage bargain, the firms will be paying the average labour costs for all individuals at work and have zero 'abnormal' profits. The unemployment rate in equilibrium will be at zero due to the full incentive of working.

*Scenario 7:* $b_f = 0$; $b_M = 0$, $b_h = 1$

In this case, the foreign cartel loses the bargains with both the foreign local labour and migrants, while the home cartel has the full power in bargaining with the home union. Eqs.4.2.17 to 4.2.19 again invalidates. The two economies will stay in autarky.

For the home economy, as the home firm cartel has the full power in the wage bargain, the home labour will only receive the autarky reservation wage. For the foreign economy, the foreign firm has no power in bargaining, while it delivers the average labour cost of foreign labour as the wage.

In general, the autarky home unemployment will be achieved as *Scenario 5*, while the foreign labour makes full employment.

*Scenario 8:* $b_f = 0$; $b_M = 1$, $b_h = 1$

Now we assume that the home and foreign firms' cartels have full power in bargaining with the home labour union, but the foreign firm cartel has no power in the domestic labour market. *No-migration* incurs.

The home and foreign local labour wages are at the same levels as *Scenario 7*, as well as the equilibrium employment in the two economies.

**Proposition 4.2.4** *The strong no migration condition occurs when either the foreign cartel loses power in all bargaining or the home union loses power in all bargaining.*

When either side in the migrant wage bargain loses all its power, there would be no migration bargaining.

The major difference between *strong* and *weak* conditions of **no migration** is if there exists a valid migrants' wage-employment contract that the contract equations suffice.

After analysing all extreme conditions, here presents the positive but not full migration condition

**Proposition 4.2.5** *The optimal positive but not full migration exists if and only if there is no monopoly and monopsony in labour supply and demand.*

#### 4.2.1.5 Discussions of the findings

The eight extreme scenarios have presented some significant findings on migration in terms of how this decision is made and who benefits from bargaining in the free-market economy framework.

To start, the analysis shows that migration is only desirable when migrants in the foreign labour market have more bargaining power in regard to the foreign firm cartel than does foreign labour. Under this circumstance, full migration will be achieved. Otherwise, the autarky equilibrium is preferred by home households.

This seemingly counter-intuitive finding unveils some important understandings. With full and complete information of all parties, migration is an effective approach to exploit the counterparty for both home and foreign households. In *Scenario 1* and *2* where full migration occurs, the home households can claim the majority of foreign output and leave only the reservation income to the foreign households. And in all other cases where the foreign parties (both cartel and union) have dominant power toward the home households, migration would be zero. This analysis recognises, in a free-market economy, migration is a decision made on the supply side. Without government interventions, foreign labour can only curb migration by increasing their bargaining power.

In addition, our two-country efficient contract model has affirmed the findings of McDonald and Solow (1981), with an interesting extension. In their closed labour market, Fig.4.1 applies such that an improvement in the bargaining power of the labour union brings wages up from the lowest reservation level to the highest average labour cost, and correspondingly, employment increases from the autarky reservation level (highest level of unemployment in the diagram) to full employment. Aggregate welfare goes up with the budget constraint and most importantly, with the bargaining power of labour.

Most of the listed scenarios authenticate this finding in our two-country framework. The increase of the bargaining power of both unions in their domestic labour markets raises wages and employment: for the home labour contracts - *Scenario 3* versus *5*, *4* versus *8*, and *6* versus *7*; for the foreign labour contracts - *Scenario 1* versus *6*, *2* versus *7*, *3* versus *4* and *5* versus *8*.[24]

However, this study has detected a non-trivial exception, by comparing home labour wage-employment outcomes in *Scenario 1* and *2*. When the foreign labour union has no power ($1 - b_f = 0$) and the home union has full power in migration wage-employment bargaining ($1 - b_M = 1$), the increase of the home labour union bargaining power over the home cartel is irrelevant to determine the home households' welfare because of full migration. Although the intuition holds that for any increase of the migrants' bargaining power households' welfare, the home households can make less effort in their domestic labour market if they are better positioned in the migrant wage-employment bargaining.

We conclude that home household labour supply decisions are dominated by their bargaining power and relative positioning in the foreign market. Comparing with a two-country migration economy, the autarky home equilibrium is always suboptimal from the perspective of home households.

### 4.2.2 Migration with adjustable capital

Now we consider when the capital stocks are immediately adjustable endogenously in both economies. The wage bargaining in the domestic labour markets would remain unchanged as the income and profit maximization incentives of both parties are unaffected.

A major difference is the role of migration for the two economies. The home economy would be able to achieve its efficient capital-labour ratio at any level of employment (see eq.4.1.21), which leads to an indifferent attitude toward migration.[25]

On the other side, due to the set-up of local complementarity, the migrants, complementing the local foreign labour force, would effectively increase the foreign productivity. Therefore, migration improves the stationary-state equilibrium output per worker in the foreign economy (see eq.4.1.22). Due to the endogenously determined capital stock, $\frac{\partial^2 Y_t^f}{\partial N_t^f \partial M_t} > 0$ and $\frac{\partial^2 Y_t^f}{\partial K_t^f \partial M_t} > 0$ will dominate the firm's profit maximization strategy. The foreign firm cartels would want as many migrants as possible to increase the overall capital stock the firm can achieve and the foreign domestic labour productivity. Under such circumstances, deliberately pursuing ***Scenario 1*** and ***Scenario 2***, that means giving up the bargaining

[24]We have used $b_h$ and $b_f$ to denote the bargaining power of home and foreign firm cartels. The unions' bargaining powers are, then, $1 - b_h$ and $1 - b_f$.

[25]In the real world, mobile capital only slowly adjusts to the long run. Migration would be desirable for the home economy to increase its catch-up speed to the advanced economies, so that labour mobility becomes a complement of capital mobility during adjustment, see Heiland and Kohler (2018).

to the home labour union ($b_M = 0$) but gaining as much as possible in its domestic bargain ($b_f = 1$), are the best options for the foreign economy.

In this case, **Proposition 4.1.2** can be generalized to

**Proposition 4.2.6** *When capital adjusts, the foreign household utility will increase as migration increases while the home household utility is not affected.*

The proposition is no longer restricted for the perfectly competitive free-market economy but shall be applicable for the general free-market economy.

In this case, the continuous inflow of migrants will deliver better labour market performance with less unemployment for each home household, and the continuous increase of foreign wage ($\frac{\partial^2 Y_t^f}{\partial N_t^f \partial M_t} > 0$) will motivate more foreign labour to work.

### 4.2.3 Welfare comparison between the central planned economy and imperfectly competitive free-market economies

Last, we study the difference of aggregate welfare between the central planned economy (or the perfectly competitive free-market economy) and the unionised imperfectly competitive free-market economies.

The utility functions of two representative households need to be focused

$$\begin{aligned}
U_t^h =& \frac{(C_t^h + C_t^M)^{1-\eta^h} - 1}{1-\eta^h} - \frac{(N_t^h + M_t)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} \\
U_t^f =& \frac{(C_t^f)^{1-\eta^f} - 1}{1-\eta^f} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}}
\end{aligned}$$

where both households value consumption and make prudential labour supply decisions. Incorporating the above utility functions with the following Euler equations

$$\begin{aligned}
W_t^h (C_t^h + C_t^M)^{-\eta^h} =& (N_t^h + M_t)^{1+\frac{1}{\nu^h}} \\
W_t^f (C_t^f)^{-\eta^f} =& (N_t^f)^{1+\frac{1}{\nu^f}}
\end{aligned}$$

in line with eqs.4.1.3 and 4.1.5.

The utility functions can be transformed as

$$U_t^h = \frac{\left[\frac{W_t^h}{(N_t^h+M_t)^{\frac{1}{\nu^f}}}\right]^{\frac{1-\eta^h}{\eta^h}} - 1}{1-\eta^h} - \frac{(N_t^h + M_t)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} \tag{4.2.20}$$

$$U_t^f = \frac{\left[\frac{W_t^f}{(N_t^f)^{\frac{1}{\eta^f}}}\right]^{\frac{1-\eta^f}{\eta^f}} - 1}{1-\eta^f} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \tag{4.2.21}$$

in which the utilities of two households are completely depending on their domestic wage levels and employment status.

In the efficient wage-employment contract model, the wage exhibits a strict concave positive relationship with employment. Full employment is only achieved when the wages (the marginal product of labour $MP_L$) are set at the average product of labour ($AP_L$) at $b = 0$ zero bargaining power of firms. The levels of optimum unemployment for two labour markets occur at the point that wages ($MP_L$) are at the reservation levels ($\overline{W}$). Most importantly, bargained equilibrium outcomes, shown as $A^*$, $B^*$, $C^*$ in Figs.4.1, 4.3, 4.5 respectively, present at neither of the extreme positions and unsurprisingly generate natural levels of unemployment.

Thus, it is possible for an imperfectly competitive free-market economy to achieve the frictionless global optimum welfare. It requires that labour in both economies receives the perfectly competitive equilibrium wages with full employment through the bargain. However, the eight extreme scenarios under the imperfectly competitive free-market assumption have failed to provide such a global equilibrium with optimised migration. Two cases are close but not the same. *Scenario 1* has produced the highest possible home utility and a potential full employment in the foreign but at the foreign reservation wages. The foreign welfare is not at perfectly competitive equilibrium level. *Scenario 4* achieves the marginal product of labour as wages for both the home and foreign labour, while it leads to a **no-migration** case. The benefits of migration to the reallocation of resource are not exploited. The levels of marginal product of both labours are at autarky.[26]

In general, the joint bargaining process of three labour groups has impeded the pursuit of the perfectly competitive equilibrium welfare. When migration is allowed, promoting the migrants' bargaining power in the foreign economies can increase the self-motivated migration given the constant levels of relative bargaining powers of the home and foreign labour.

[26] The rest six extreme cases are differed from the perfectly competitive equilibrium levels because of the natural levels of unemployment and reservation wages in bargaining.

## 4.3 Chapter Conclusions

**Chapter 4** explores migration and its wage-employment issues in a free-market global economy. In this chapter, we start with analysing the welfare implications of migration in a perfectly free-market economy, and the results are no different to a global planner economy. As previous chapters, the possible welfare differences between both fixed and endogenous capital assumptions are also shown.

We explore how migrants' wage and employment are determined if the labour market is imperfect given a labour wedge. First, assuming capital is fixed, we extend McDonald and Solow (1981)'s efficient contract model into a two-household, two-firm and three-group labour market. In so doing, we present a theoretical elaboration of migrants' wage and employment under different levels of relative bargaining powers between unions and firm cartels. A significant result is that positive but not full migration only occurs if there is no monopoly and monopsony in the labour markets. The implications of migration on the established equilibrium in an endogenous capital model are similar to previous chapters, that is endogenous capital adjustment can improve optimal welfare of each country, compared to the fixed capital outcomes.

Furthermore, this chapter has provided a theoretical analysis of the partial equilibrium responses to unexpected changes in home and foreign labour markets and of migration. Eqs.4.2.17 to 4.2.19 show that in general, wages are jointly determined by relative bargaining powers of all labour groups. However a theoretical analysis is very complicated. Instead, in the next chapter, we provide numerical simulations to uncover the general equilibrium responses of wages, employment and migration to various shocks in the complete framework.

# Chapter 5

# General equilibrium responses of optimal migration to productivity, preference and cost shocks

> This chapter simulates how migration contributes to the general equilibrium adjustment in imperfectly competitive free-market economies under various shocks. It presents the responses of migration and other key variables, most importantly, the output of two countries and welfare of two households. By presenting impulse response curves under varying relative bargaining power in the labour markets, we provide further evidence on the substitutability between labour mobility and domestic capital adjustment in adjusting to general equilibrium.

This chapter calibrates and simulates the migration phenomenon in a two-country imperfectly competitive free-market economy. A well-featured empirical simulation can bring the following benefits to our understanding of migration in a heterogeneous labour market structure: 1) It gives the implications of the different relative bargaining powers among workers and firms; 2) it improves our understanding of the general equilibrium features of the efficient wage-employment contract model in a two-country mobile labour free market; 3) it shows how technology, preference and cost shocks affect migration and welfare in the economies; and 4) it provides insights into how shocks in one country can be transmitted to another, and whether migration (and/or migrant wages) propagates or mitigates the transmission.

We first construct a benchmark model that has the following key features. First, we include heterogeneity in the production technologies of the two countries with a local-complementarity production function for the foreign economy and a standard CES two-factor production function for the home. Second, migration could incur both fixed and variable costs. Third, the labour market equilibrium conditions are determined by efficient wage-employment relationships of employers and workers (as in the three-stage bargaining process in **Chapter 4**), which allows an analysis of the implications of the varying bargaining powers of all three types of labour participants. Fourth, capital is fixed and immobile in the benchmark set-up while labour is mobile within and across the countries, just as we had in previous chapters.

In **Section 4.2**, we shed light on how a sudden change in migration size ($M_t$) could alter the wage-employment contract curves of home, foreign and migrants from stages 1 to 3 independently without considering how the changes are induced. However, the possible different causes of migration changes could result in or be provoked by other macroeconomic variables. Further, we also emphasised that labour market bargaining among the parties is made jointly by showing the bargaining outcomes from eqs.4.2.17 to 4.2.19. In this chapter, we use a calibrated model to examine the general equilibrium responses of variables (particularly migration) in the face of various shocks.

We begin by considering total-factor-productivity (TFP) shocks in the two economies. In the designated benchmark model, the changes of migration-related variables are firstly considered as it is the only channel connecting two economies, and followed by the impulse responses of other variables in the two economies. We wish to see if the responses to technology shocks lead to a movement of the wage equilibrium point along the efficient contract curve, consistent with positive wage-employment efficient contract relation. In particular, Smets and Wouters (2003) found that a positive productivity shock increases output and consumption in a one-country economy with falling employment, which they explain by the falling marginal cost due to the rise in productivity (Gali, 1999). These findings are compared with the outcomes of our global economy with migration.

Then, we turn to migration cost shocks. Including fixed- and variable-cost shocks provides insight into the actual roles of these two cost components in a free-market economy. It is interesting that these costs could provoke opposite responses of migration. An increase in the social/government cost of foreign hosting migrants (fixed cost of migration) would increase migration, while an increase in variable cost of migration could decrease migration.

Next, we present the simulation findings under the assumption that capital is adjustable but not internationally mobile (as in preceding chapters). The aim is to compare the simulated general equilibrium responses with theoretically inferred partial equilibrium responses. We contribute to the extant studies on the discussion of the role of the substitutability and complementarity between labour and capital adjustment in a general equilibrium context.[1]

[1]Note that our study should be separated from research in the field of substitutability between capital and labour mobility, i.e. trade and migration. However, the study of the effects of domestic capital adjustment to the general migration equilibrium could give us insights into the interactions between capital adjustment and mobile labour. If the substitutability between free capital and labour adjustment is so difficult to observe, we can decompose the question to **I.** capital adjustment and labour adjustment (non-mobile), **II.** capital adjustment and mobile labour, **III.** capital adjustment and free labour (both mobile and adjustable), **IV.** free capital and labour adjustment, **V.** free capital and mobile labour, **VI.** free capital and free labour, **VII.** mobile capital and mobile labour, **VIII.** mobile capital and labour adjustment, and **IX.** mobile capital and

Adjustable capital inter-temporally allows consideration of a varying preference of households. Smets and Wouters (2003) shows that a positive preference shock has a significant negative crowding-out effect on investment. A leisure shock is then included in the log-linearised system and monitor how the changes of households' preference between consumption and labour supply could affect their life plans and utilities while migration is allowed.

Finally, we compare the impulse responses between fixed-capital and adjustable-capital frameworks for varying relative labour market bargaining powers.

The main results are: 1) In an immobile capital two-country world with free labour mobility, a positive temporary home TFP shock generally decreases migration, while a foreign TFP shock can increase migration; 2) a temporary positive deviation of migration from its steady state benefits both economies, while negative deviations bring temporary harm; 3) an increase in the fixed cost of migration (classified as social/government expenditure of foreign hosting migrants) might increase migration, while the increasing variable cost of migration impedes the flow; and 4) we also show the substitutability between labour mobility and endogenous capital adjustment toward general equilibrium.

mobile labour. Here in this study, we look at the case **II** under varying bargaining power between firms and employees.

## 5.1 The benchmark model and its outcomes

### 5.1.1 Setting up the model

Consistent with the imperfectly competitive free-market set-up in **Chapter 4**, the framework features sixteen endogenous variables $\{C_t^f, C_t^h, U_t^f, U_t^h, C_t^M, N_t^f, N_t^h, M_t, CM_t, Y_t^f, Y_t^h, W_t^f, W_t^h, W_t^M, UN_t^f, UN_t^h\}$ plus two leisure shocks ($Z_t^{fl}$, $Z_t^{hl}$), two TFP shocks ($Z_t^f$, $Z_t^h$) and two cost shocks ($Z_t^{fcm}$, $Z_t^{vcm}$).

The representative households' utilities ($U_t^f$, $U_t^h$) are

$$U_t^f = \frac{(C_t^f)^{1-\eta^f}-1}{1-\eta^f} - Z_t^{fl}\psi\frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \tag{5.1.1}$$

$$U_t^h = \frac{(C_t^M+C_t^h)^{1-\eta^h}-1}{1-\eta^h} - Z_t^{hl}\psi\frac{(M_t+N_t^h)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} \tag{5.1.2}$$

The representative household generates utility from all its members' consumption with considering the disutility from its labour supply. $\psi$ is a positive scalar to rationalize the utility function while calibrating.[2] The parameter $\eta$ denotes the CRRA coefficient of a household and $\nu$ is the Frisch labour supply elasticity (for the home household with a superscript $h$ and for the foreign household with a superscript $f$). In short, the foreign households will consume $C_t^f$ in aggregate, while the home households include two consumption components: the consumption for family members remaining at home ($C_t^h$) and the consumption of migrants ($C_t^M$).[3]

$\{Z_t^{fl}, Z_t^{hl}\}$ could bring additional disutility to the labour supply. They are leisure shocks to capture what happens to the global economy if both households suffer a sudden change in their preference over labour supply.[4] We use them to capture the possible consequences of a preference change. They are set in the AR(1) process as follows.

$$ln(Z_t^{hl}) = \rho^{hl} ln(Z_{t-1}^{hl}) + \zeta_t^{hl} \tag{5.1.3}$$

$$ln(Z_t^{fl}) = \rho^{fl} ln(Z_{t-1}^{fl}) + \zeta_t^{fl} \tag{5.1.4}$$

which parameters $\rho^{hl}$ and $\rho^{fl}$ are the assumed levels of endurance of the shocks. In our study, the persistence parameters are 0.9.

[2]The same scalar is assumed for the two countries.

[3]Sometimes, we included the home households' aggregate consumption as

$$C_t^H = C_t^h + C_t^M$$

to observe a possible aggregate response of the home households.

[4]In the log-linearised form, the shock is equivalent to a risk aversion shock.

**Home demand and supply**

The home household is constrained by the following temporal budget in each period.

$$C_t^M + C_t^h + CM_t = W_t^M M_t + W_t^h N_t^h + (r_t^h - \delta^h)K^h \tag{5.1.5}$$

where expenditures of the household including consumption ($C_t^M$, $C_t^h$) and cost of migration ($CM_t$) equal the overall income from labour supply ($W_t^M M_t$, $W_t^h N_t^h$) and the rental income from the endowed capital stock ($(1 + r_t^h - \delta^h)K^h$).[5]

A significant feature in **Chapter 4** was the existence of 'abnormal' profit of the firms in this incomplete free-market economy. In the theoretical model, we ignored the effects of the 'abnormal' profit in order to explore the partial equilibria in the labour markets (as in the real world, not every labour owns the firms.) In this chapter's dynamic stochastic general equilibrium framework, we assume the production side compensation equations as follows

$$Y_t^h = W_t^h N_t^h + r_t^h K^h \tag{5.1.6}$$
$$Y_t^f = W_t^f N_t^f + W_t^M M_t + r_t^f K^f \tag{5.1.7}$$

which says that the rest of profit of the firms' production after paying their labour supply will be collected by the capital owners. In this case, the representative households are also the owner of the firms. Though the total amount of capital is not adjusting, the rates of return can fluctuate.

The migration cost for period $t$ is set the same as in previous chapters where

$$CM_t = Z_t^{fcm} CM_0 + Z_t^{vcm} \chi M_t \tag{5.1.8}$$

such that the overall cost of migration equals the sum of the fixed and variable costs of migration. Furthermore, to improve our understanding on the roles of both fixed and variable costs of migration in the stationary state and how they interact with the two economies' major indicators, we individually introduce two cost shocks that follow a persistent stochastic process:

$$ln(Z_t^{fcm}) = \rho^{fcm} ln(Z_{t-1}^{fcm}) + \zeta_t^{fcm} \tag{5.1.9}$$
$$ln(Z_t^{vcm}) = \rho^{vcm} ln(Z_{t-1}^{vcm}) + \zeta_t^{vcm} \tag{5.1.10}$$

where both $\rho^{fcm}$ and $\rho^{vcm}$ are between $(0, 1)$ capturing the presumed persistence of the cost shocks. We assume all the persistent parameters are 0.9.

[5] In previous chapters, we simplify our bargaining process to better analyse theoretical outcomes by ignoring the constant return of capital under a fixed capital stock assumption. However, in empirical simulations, we need to add this return of capital back, otherwise we are changing our assumption to a labour-only production process for the two countries.

Furthermore, with utility and budget constraint functions, the optimal home labour and migrant supply decisions made by the home household are thus

$$
\begin{aligned}
(W_t^M - \chi)(C_t^M + C_t^h)^{-\eta^h} &= Z_t^{hl}(N_t^h + M_t)^{\frac{1}{\nu^h}} \\
W_t^h(C_t^M + C_t^h)^{-\eta^h} &= Z_t^{hl}(N_t^h + M_t)^{\frac{1}{\nu^h}}
\end{aligned}
$$

which produce the following equilibrium wage condition

$$W_t^M - \chi = W_t^h \tag{5.1.11}$$

meaning that the home labour wage in equilibrium will always equal the migrants' wage after deducting the marginal variable cost of migration.

On the supply side, the home labour market employment and (voluntary) unemployment are specified

$$UN_t^h = 1 - M_t - N_t^h \tag{5.1.12}$$

which unemployment at home ($UN_t^h$) is the unit-based home labour population minus the home households' labour supply to the home ($N_t^h$) and to the foreign ($M_t$) economies.

The home production technology is in a standard CES form

$$Y_t^h = Z_t^h[\varphi_h(K^h)^{\mu^h} + (1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1}{\mu^h}} \tag{5.1.13}$$

$$ln(Z_t^h) = \rho^h ln(Z_{t-1}^h) + \zeta_t^h \tag{5.1.14}$$

**foreign demand and supply**

Then we turn to the foreign system.

To begin with, the budget constraint of the representative foreign household is

$$C_t^f = W_t^f N_t^f + (r_t^f - \delta^f)K^f \tag{5.1.15}$$

Comparing with the home households, the foreign households only need to make consumption and domestic labour supply decisions. Together with eq.5.1.1, the optimised foreign households' labour supply decision is

$$W_t^f(C_t^f)^{-\eta^f} = Z_t^{fl}(N_t^f)^{\frac{1}{\nu^f}} \tag{5.1.16}$$

which shows how the foreign households juggle between consumption and labour supply according to the bargained wage income.

For the foreign, the labour market condition is

$$UN_t^f = 1 - N_t^f \tag{5.1.17}$$

The local complementarity in the foreign production technology is as specified before

$$\begin{aligned} Y_t^f =& Z_t^f \{\omega^f (M_t)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}} && (5.1.18)\\ ln(Z_t^f) =& \rho^f ln(Z_{t-1}^f) + \zeta_t^f && (5.1.19)\end{aligned}$$

where $\lambda^f > \mu^f$ so that the local labour force is less substitutable to local capital than the foreign labour force, which generates higher complementarity between local production factors. At the same time, the migrants also become complements to the foreign domestic labour force.

After all, the goods market equilibria in both economies are

$$\begin{aligned} Y_t^f =& C_t^f + C_t^M + \delta^f K^f && (5.1.20)\\ Y_t^h =& C_t^h + CM_t + \delta^h K^h && (5.1.21)\end{aligned}$$

where total output per unit equals the consumption and investment. And the investment terms will be fixed at their depreciation levels due to the assumption of constant endowed capital stock in two countries.

Adopting efficiently bargained wage-employment contract relations for all three labour groups gives

$$W_t^M = \frac{(1-b_M)b_f(Y_t^f - r_t^f K^f - W_t^{f,R} N_t^f) + (1-b_h)b_M[W_t^{h,A}(N_t^h + M_t) + CM_t - (Y_t^h - r_t^h K^h)]}{[b_f(1-b_M) + b_M(1-b_h)]M_t} \tag{5.1.22}$$

$$\begin{aligned} W_t^h =& \frac{-b_h b_f (1-b_M)[Y_t^f - r_t^f K^f - W_t^{f,R} N_t^f - W_t^{h,A}(N_t^h + M_t) - CM_t]}{[b_f(1-b_M) + b_M(1-b_h)]N_t^h}\\ &+ \frac{(1-b_h)[(1-b_M)b_f + b_M](Y_t^h - r_t^h K^h)}{[b_f(1-b_M) + b_M(1-b_h)]N_t^h} && (5.1.23)\end{aligned}$$

$$\begin{aligned} &[b_f(1-b_M) + b_M(1-b_h)]N_t^f W_t^f\\ &= (1-b_M b_h) b_f W_t^{f,R} N_t^f +\\ &(1-b_f)(1-b_h)b_M[Y_t^f + Y_t^h - r_t^f K^f - r_t^h K^h - W_t^{h,A}(N_t^h + M_t) - CM_t] && (5.1.24)\end{aligned}$$

which includes the constant capital and its returns based on the Nash bargaining outcomes from eqs.4.2.17 to 4.2.19. In general equilibrium, bargains are happening simultaneously, and relative bargaining powers of the different labour groups and production factors enter every wage outcome. An important feature is the role of the autarky reservation wage of

Table 5.1: Parameters

| Parameters | Value | Descriptions |
|---|---|---|
| $\eta^f$, $\eta^h$ | 0.9 | Gandelman and Hernández-Murillo (2015);<br>the foreign and home households CRRA |
| $\nu^f$, $\nu^h$ | 0.4 | Whalen and Reichling (2017);<br>the foreign and home Frisch elasticity of labour supply |
| $\psi$ | 0.1 | a scalar to rationalize the labour supply disutility |
| $r_t^h$ | 0.12 | Chou, Izyumov and Vahaly (2015);<br>rate of return on the home capital stock; |
| $r_t^f$ | 0.06 | Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019);<br>rate of return on the foreign capital stock; |
| $\delta^h$ | 0.08 | the depreciation rate of home capital stock |
| $\delta^f$ | 0.02 | the depreciation rate of foreign capital stock |
| $\mu^f$, $\mu^h$ | -0.43 | the foreign EOS between capital and labour $\frac{1}{1-\mu}$; extant literature |
| $\lambda^f$ | 0.7 | the EOS between migrants and domestic factors $\frac{1}{1-\lambda^f}$; assumption |
| $\varphi^f$, $\varphi^h$ | 0.5 | capital income shares in foreign and home;<br>Manyika, Mischke, Bughin, Woetzel, Krishnan and Cudre (2019) |
| $\omega^f$ | 0.1 | income share parameter of migrants |
| $b_M$, $b_h$, $b_f$ | $\to 0$ | the bargaining power of firms to migrants, home and foreign labour |

domestic labour supply in determining bargained wages. For example, the negative relationship between migrants' wages and foreign reservation wages can be explained by the negative relationship between foreign employment and reservation wages and the complementarity in production between migrants and foreign labour. It reflects the conventional understanding from the classical wage-setting and price-setting model. Moreover, the positive relationship between migrants' wage and the home reservation wage is also shown in the Nash bargained outcomes, and complies with the concept of the opportunity cost of home labour.

The system of twenty-four equations presents the full picture of a mobile labour and fixed capital two-country global economy. To simulate the model, the system is log-linearised as shown in **Appendix A.6**.

### 5.1.2 Calibration

The parameters and steady-state values for the two countries are calibrated in Tables.5.1 and 5.2. To construct the benchmark model with specific attention on production technology heterogeneity and capital-labour ratio differences, most of the two countries' parameters are set as symmetric. Where available, key parameters comply with published estimates.

For the households' utility functions, both constant relative risk aversion coefficients (CRRA) and the Frisch elasticity of labour supply parameters are taken from established estimations. A country-level estimation, carried out by Gandelman and Hernández-Murillo (2015), suggests that the coefficient of relative risk aversion is around 1 based on seventy-five countries' data on self-reports. In this chapter, we use 0.9 for both home and foreign households.[6] Whalen and Reichling (2017) reviews most existing studies on the Frisch elasticity, and suggests that most micro-based Frisch elasticity estimates range from 0 to 0.8. Here we take the central figure at 0.4.[7]

We assume that the time discount factor is at 0.96 for both countries. This implies 0.04 is the difference between the rate of return on capital and depreciation rate of the two countries per year.[8] The home capital rate of return is 0.12 per annum based on Chou, Izyumov and Vahaly (2015)'s average return on capital of twenty-three transition economies, while the foreign capital rate of return is 0.06 per annum (Jordà, Knoll, Kuvshinov, Schularick and Taylor, 2019).[9]

A main feature of this study is introducing the cost of migration in determining the stationary state of two-country general equilibrium. We calibrate the stationary-state cost of migration as 0.27 for 15 per cent migrants' share of labour supply in the representative home household. According to Morten and Oliveira (2016), fixed cost of migration can account for over 80 per cent of the total cost of migration domestically, which leads to the assumed international fixed cost of migration at approximately 0.22 and the variable cost is calibrated at 0.36.

On the production side, the income share parameter of capital in both economies is set at 0.5, which is advocated by Manyika, Mischke, Bughin, Woetzel, Krishnan and Cudre (2019). To model the local complementarity in production, we assume the elasticity of substitution between domestic capital and labour at 0.7 to yield complementarity. It suggests that the $\mu$ is at $-0.43$ for both economies, a universally applied figure in the established empirical works (Claro, 2003; Goldar, Pradhan and Sharma, 2013; Knoblach, Rößler and Zwerschke, 2016; Mućk, 2017). For the foreign production, we set $\lambda_f$ at 0.7, indicating the elasticity of substitution between migrants and domestic production factors is 3.33.

For the labour market conditions, the benchmark bargaining power for firms toward all labour groups is set to be 0.0001, which assumes a nearly maximised union power.[10] Moreover, we assume the basic household unit labour supply is at 1 for both economies. In the home country, the natural unemployment rate is 5 per cent, while its labour supply to the home and foreign are respectively 80 per cent and 15 per cent. For the foreign, we assume

[6] In Gandelman and Hernández-Murillo (2015), the coefficients are similar across all countries regardless of the degree of development. For example, developed countries like Canada and Germany could have a similar (around 0.9) CRRA to developing countries like India and Cameroon.

[7] Whalen and Reichling (2017) also showed that macro-based estimates tend to be larger than the micro-based ones of the Frisch elasticity.

[8] This set-up indicates different levels of depreciation rates between the foreign and home countries. However it is applicable due to the different industry structures and capital employed between developed and developing countries (Schündeln, 2013).

[9] Jordà et al. (2019) has presented a full account on the return on different capital forms across the time periods from 1870 to 2015. Here we have only employed the return on wealth (the weighted average of equity, bonds, bills and housing) in the post-1980 period.

[10] As most of extreme power scenarios indicate full or no migration, disequilibrium and autarky equilibrium can add little information to the analysis. We assume instead a value close to the extreme scenarios.

Table 5.2: Steady-State values

| Variables | Value | Notes (based on calibration) |
|---|---|---|
| $M$ | 0.15 | migrants' share out of home labour force |
| $CM$ | 0.27 | steady-state cost of migration |
| $CM_0$ | 0.22 | the fixed cost of migration; 80 per cent of the $CM$ |
| $\chi$ | 0.36 | the variable cost of migration |
| $N^h$ | 0.8 | home labour supply |
| $N^f$ | 0.95 | foreign labour supply |
| $Y^h$ | 1.67 | steady-state home production |
| $Y^f$ | 2.6 | steady-state foreign production |
| $C^h$ | 1.01 | home household final consumption (60 per cent of GDP) |
| $C^M$ | 0.18 | steady-state consumption of migrants |
| $C^f$ | 1.82 | foreign household final consumption (70 per cent of GDP) |
| $U^h$ | 0.15 | steady-state home utility |
| $U^f$ | 0.59 | steady-state foreign utility |
| $W^h$ | 1.02 | home labour real wage |
| $W^M$ | 1.39 | migrants' real wage |
| $W^f$ | 1.51 | real wage of foreign labour |
| $W^{h,A}$ | 0.25 | home reservation wage |
| $W^{f,R}$ | 0.75 | foreign reservation wage |
| $K^h$ | 5 | the home capital stock |
| $K^f$ | 30 | the foreign capital stock |

the same natural rate of unemployment with 95 per cent employment rate.

The equilibrium wages of home labour, migrants and foreign labour are calibrated outcomes from the Euler equations. The levels of reservation wages of the foreign and home are set to be one half and a fourth of the equilibrium domestic labour wages.[11]

The capital endowment differential is captured by the assumed capital stocks. The home economy is assumed to be relatively labour-abundant, while the foreign is relatively capital abundant. Thus when the home economy would be set at 5, the foreign economy's capital stock is at 30.

Finally, for the shocks' persistence, we assume 90 per cent of all shocks persist per period, which gives 0.9 for all $\rho$ in eqs.5.1.3, 5.1.4, 5.1.9, 5.1.10, 5.1.14 and 5.1.19. By doing so, the duration of the shocks' effects would diminish within 40 periods in the benchmark fixed capital model. A comparison between the designated benchmark model with capital adjusted models will allows us to uncover the potential substitutability between endogenous capital adjustment and migration.

[11] Note that the foreign (developed economies) reservation wage in the real world is in the vicinity of half of the median wages, while the home (developing countries) reservation wage is often unstated or very small (*Median Income By Country Population*, 2019; *Minimum Wage By Country Population*, 2019). However, the calibrated output difference between developed and developing economies in this hypothetical world is also smaller than the real-world observations.

## 5.2 Outcomes

### 5.2.1 TFP shocks on the benchmark model

Before the detailed analysis for the two shocks, the benchmark (fixed capital with monopolistic labour markets in both countries) simulation has shown some common findings.

First, both countries' output and household utility benefit from the positive TFP shocks when migration is the only channel of transmission. The spill over effects of technology development can be detected in this hypothetical world with only labour mobility.

Second, a domestic productivity shock increases the scarcity of the limited capital stock. When capital stocks are fixed in the two countries, the positive TFP shock will increase the rates of capital return in its origin.

Third, domestic technological unemployment is found, consistent with Smets and Wouters (2003)'s results of a positive productivity shock. In a country with the presumed fixed capital stock and monopolistic labour market, a positive domestic productivity shock has led to higher local unemployment. However, with free migration, the foreign counterpart has experienced higher employment.

#### 5.2.1.1 A positive home TFP shock

Figure 5.1: Responses to a positive home TFP shock

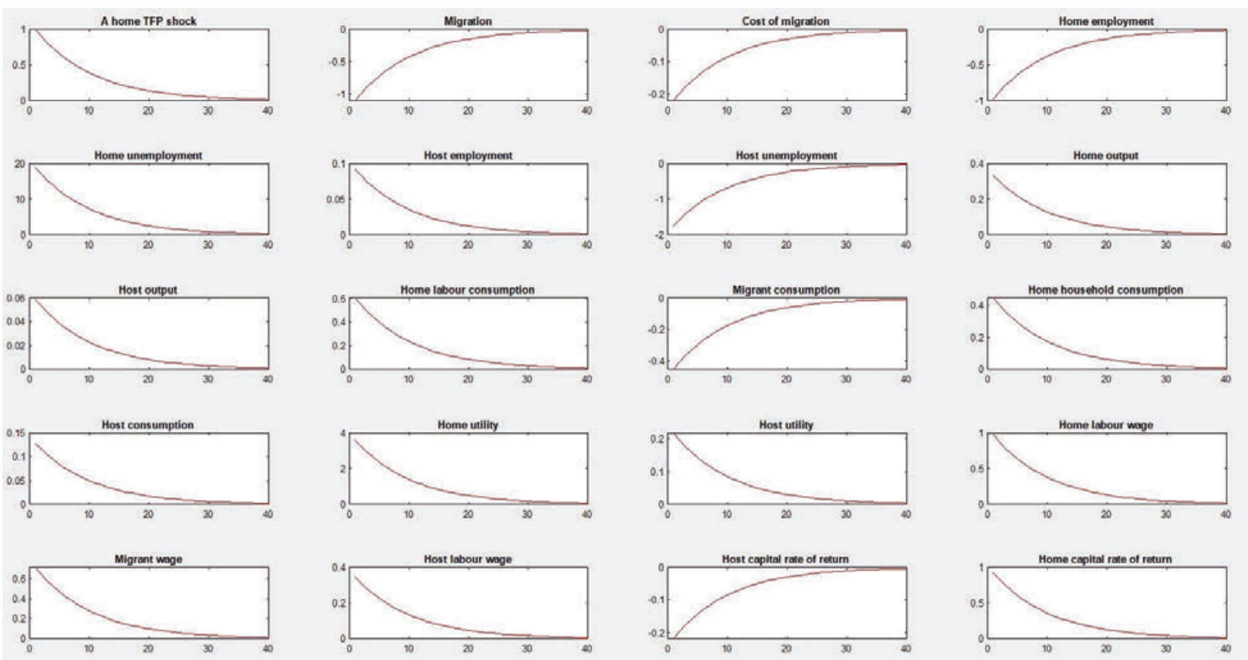


Fig. 5.1 gives the impulse responses of the two-country economy to a positive persistent home productivity shock. In this global economy with fixed and immobile capital, this shock benefits both foreign and home households' consumption and utility. The effects of the shock disappear within 40 years.

First and foremost, an improved home country productivity discourages migration $M_t$ and thus reduces the cost of migration $CM_t$, which inevitably causes a reduced migrant consumption. Temporarily decreased migration has urged the foreign firms to increases its demand for the foreign labour. As a consequence, the wage and employment of foreign labour rise, which confirms the theoretical positive relationship of wage and employment indicated by the efficient contract model.

On the other side, the additive TFP shock increases the home output, which allows a higher compensation for its domestic workers. Home labour income rises, consistent with responses of the home labour consumption. The migrant wage will also increase due to its established equilibrium relationship with the home labour wage, as specified in eq.5.1.11. The positive home TFP shock improves the incomes of all labour market participants, as well as their budget constraints. The utilities of both households increase. The supply side of labour benefits.

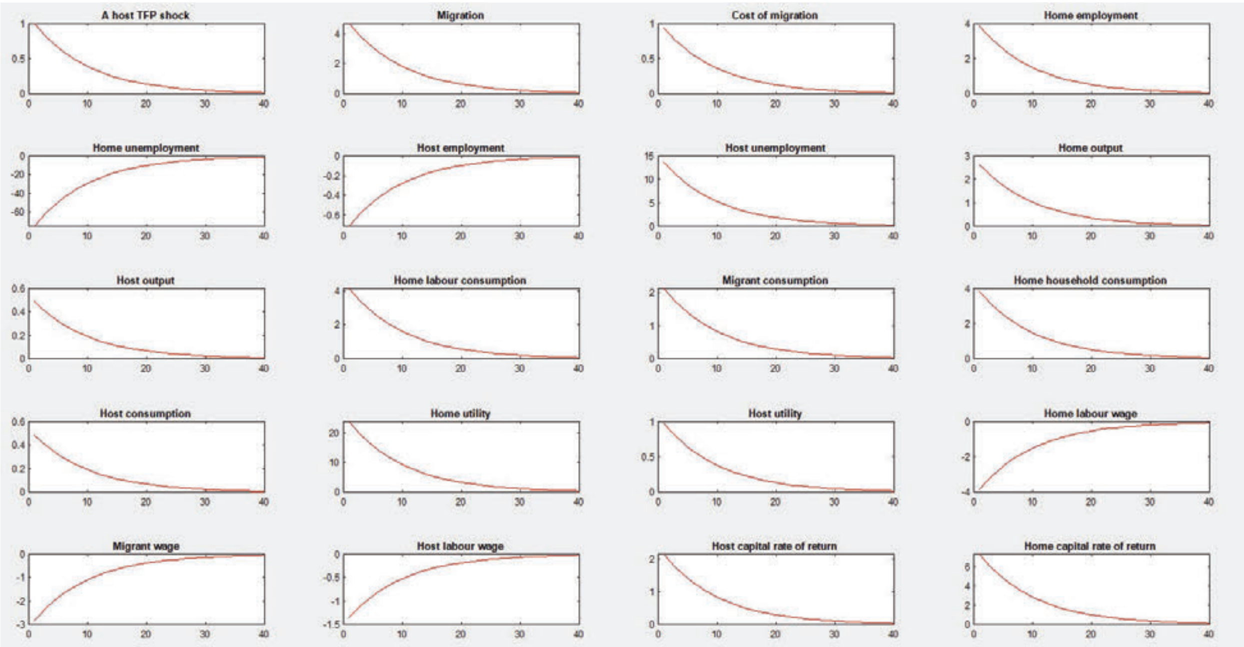
Interestingly, though the efficient contract model is structured in the model, reductions in the home employment and migration present with higher unit wages. It is a joint impact of the equilibrium home labour wage relationship and the rise of marginal product of migrant labour force. For the home households, a rise in the home labour wage gives lower marginal utility of income and higher marginal utility of leisure. With the improvement on the wages brought by the technology improvement, more leisure activities are needed to maximise the household utility. And the same mechanism applies when migrants' wage is also increased due to the rising opportunity cost.

In terms of capital returns, Fig.5.1 shows that the foreign capital rate of return will fall in response to the home additive shock. It is due to the changes in the employment of migrant and foreign labour, as well as the nearly absolute bargaining power of the labour unions toward foreign firms. The falling migration reduces the marginal product of capital, while the rising foreign employment improves it. At the same time, when both foreign labour and migrants can claim their increasing full marginal product, there is not much left of the capital owners considering a very limited increased in the foreign output.

In general, our simulated general equilibrium responses in the nearly absolute monopolistic labour market complies to the classical intuitions in labour economies, though we have presented a much-complicated global labour market with migration. In the home country, where the shock is originated, are consistent with Smets and Wouters (2003)'s, which shows both the technological unemployment and a negative association between real wages and employment. Fig.5.1 also presents a more comprehensive analysis on the responses of the foreign labour market, which has evidenced the efficient contract model.

#### 5.2.1.2 A positive foreign TFP shock

Figure 5.2: Responses to a positive foreign TFP shock




Similar to the home TFP shock, the positive foreign productivity shock benefits both firms and households in the two economies in the benchmark set-up.

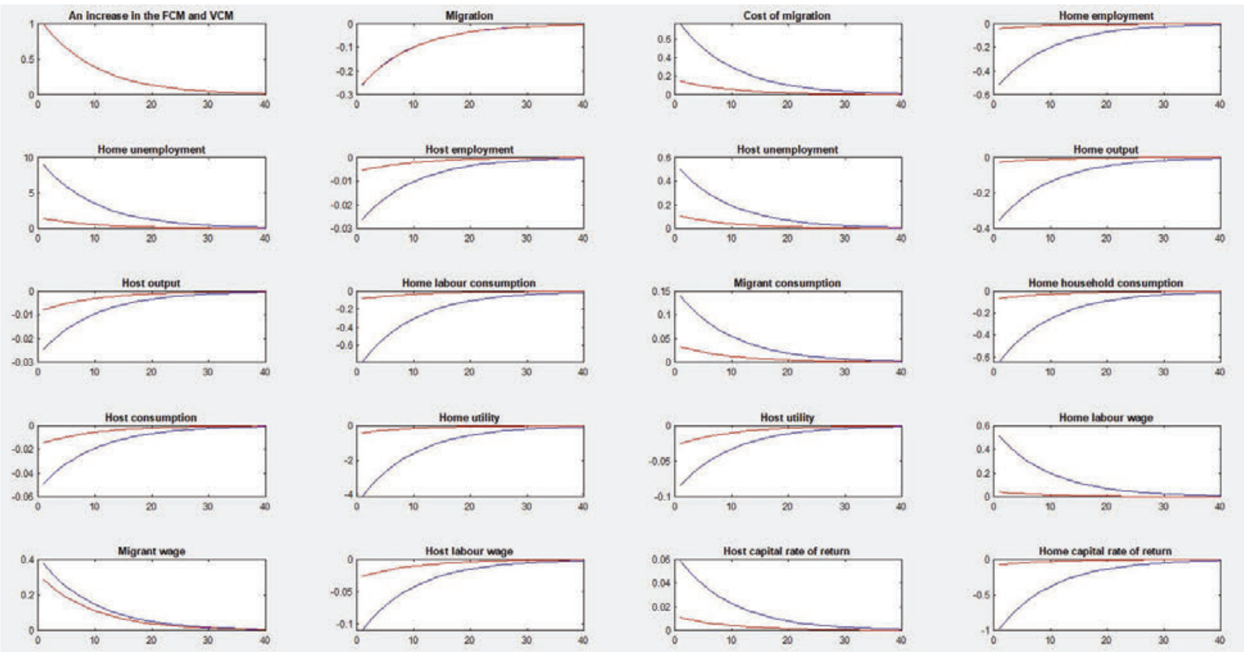
An improved foreign economic environment increases its attractiveness to migrants. Migration is encouraged with the rising cost of migration. A temporarily larger than the steady-state inflow of migrants drags down the marginal product of migrant labour, which results in a lower migrant wage. The falling opportunity cost of home labour supply depresses the home labour wage, which motivates the home households to increase domestic employment. The marginal product of capital tends to increase in response to higher employment. With given capital stock, more employment and unchanged TFP at home improve the home output. Moreover, the home households benefit from the expanded budget constraints and total consumption (assuming a constant ratio of consumption out of output). The home utility then increases.

Taking all above into consideration, the foreign economy is exposed to two changes: a positive migration and TFP shock. The increases in the technological development and migrant labour stimulate the foreign output. However, both wage and employment of the foreign labour fall, with higher capital return. The increase in migration and technology brings up the capital return of the foreign households and increases foreign households' aggregate income. It then allows the foreign households to choose more leisure and reduce their labour supply to maximise their utility. In the monopolistic foreign labour market applying the efficient contract model, a fall in the supply of labour leads to a fall in the wages. This finding complies with Gali (1999), which shows that the USA labour market responded to the identified productivity shock with falling employment.

In a nutshell, a positive TFP shock from either of two countries benefits the two economies when capital is fixed and migration is allowed between monopolistic labour markets. In an unequal two-country world with a fixed amount of mobile labour but immobile capital, a temporary technology innovation from the poor country brings up wages of all, but a shock from the rich side depresses them. Either way, labour mobility has spread the positive effects of additive shocks in terms of countries' output and welfare.

#### 5.2.1.3 Migration cost shocks in the benchmark model

Figure 5.3: Responses to an increase in different costs of migration




NB: Responses to the fixed cost shock are in blue lines, while responses to the variable cost shock are in red lines.

Fig.5.3 presents the general equilibrium responses of the two-country economy to persistent increases of the fixed and variable costs of migration. Although it is assumed that the home household pays the cost of migration, both countries temporarily suffers due to the additional loss of the global equilibrium output when shocks persist.

The same level of decrease has been witnessed in the responses of migration to the shocks on both fixed and variable cost of migration in the designated system with fixed capital stock and monopolistic labour markets. However, as the fixed cost of migration counts 80 per cent of total cost, its increase tends to bring out a larger response in the cost of migration and so forth all other variables.

To compensate the increasing cost in the households' budget, the home households will fully exploit its monopolistic bargaining power in the labour market and thus transfer the additional burden to both home and foreign firms. Without efficient capital adjustment, the home households need to give up some of their labour supply to achieve a higher marginal product of labour. The increases in the home labour and migrant wages are therefore witnessed. Also, the reduced home supply has also decreased the home capital return vein the positive relationship between two factors. After all, the home output suffers due to the loss in both production inputs and home utility falls because of the increasing cost burden and the contracting budget.

On the foreign side, a reduced inflow of migration and an increased migrant wage have contributed to a dipped foreign labour wage through the falling marginal product of foreign labour and the decrease of overall foreign output.[12] With less wage incentive, the foreign employment falls as shown in the efficient wage-employment contract model. Noteworthy is an increase in the foreign capital return. It is jointly determined by the falling output, increasing migrants' income and falling employment of foreign labour (based on eq.5.1.16). When the home union starts claiming higher migrant wages from the foreign firms, the foreign household's utility falls due to the reduced labour and capital incomes given the decreased foreign output.

In general, while capital is fixed in both economies, a suddenly increased cost will temporarily harm the global steady-state output. Further, migration can shift part of home households' cost to the foreign.

### 5.2.2 The endogenous capital general equilibrium model

Now we allow capital in both countries to be domestically adjustable.

This implies that households also have to optimise their investment decisions. Endogenizing $\{K_t^h,\, K_t^f\}$ in eqs.5.1.5 and 5.1.15, both households maximise their investments as follows

$$\beta^h(1 + r_{t+1}^h - \delta^h)(C_{t+1}^M + C_{t+1}^h)^{-\eta^h} = (C_t^M + C_t^h)^{-\eta^h} \tag{5.2.1}$$

$$\beta^f(1 + r_{t+1}^f - \delta^f)(C_{t+1}^f)^{-\eta^f} = (C_t^f)^{-\eta^f} \tag{5.2.2}$$

[12] Again, we need to note that this observation is also partially attributed to the inefficient capital adjustment. Fig.5.8 shows what happens if the capital stock can freely adjust domestically.

while the rates of return on capital from eqs.5.1.13 and 5.1.18 are

$$r_t^h = (Z_t^h)^{\mu^h} \varphi^h (K_t^h)^{\mu^h - 1} (Y_t^h)^{1-\mu^h} \tag{5.2.3}$$

$$r_t^f = (Z_t^f)^{\lambda^f} \varphi^f (1-\omega^f)(K_t^f)^{\mu^f - 1} [\varphi^f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f - \mu^f}{\mu^f}} \tag{5.2.4}$$

where we assume capital returns equal the marginal productivity of capital.

Also, households need to consider the variability of capital returns in their budget constraint equations. Eqs.5.1.5 and 5.1.15 become

$$C_t^M + C_t^h + CM_t + K_{t+1}^h = W_t^M M_t + W_t^h N_t^h + (1 + r_t^h - \delta^h) K_t^h \tag{5.2.5}$$

$$C_t^f + K_{t+1}^f = W_t^f N_t^f + (1 + r_t^f - \delta^f) K_t^f \tag{5.2.6}$$

Finally, the two-country non-traded good markets are cleared individually according to

$$Y_t^h = C_t^h + CM_t + I_t^h \tag{5.2.7}$$

$$Y_t^f = C_t^f + C_t^M + I_t^f \tag{5.2.8}$$

where required investment in the two countries is defined as

$$I_t^h = K_{t+1}^h - (1-\delta^h) K_t^h \tag{5.2.9}$$

$$I_t^f = K_{t+1}^f - (1-\delta^f) K_t^f \tag{5.2.10}$$

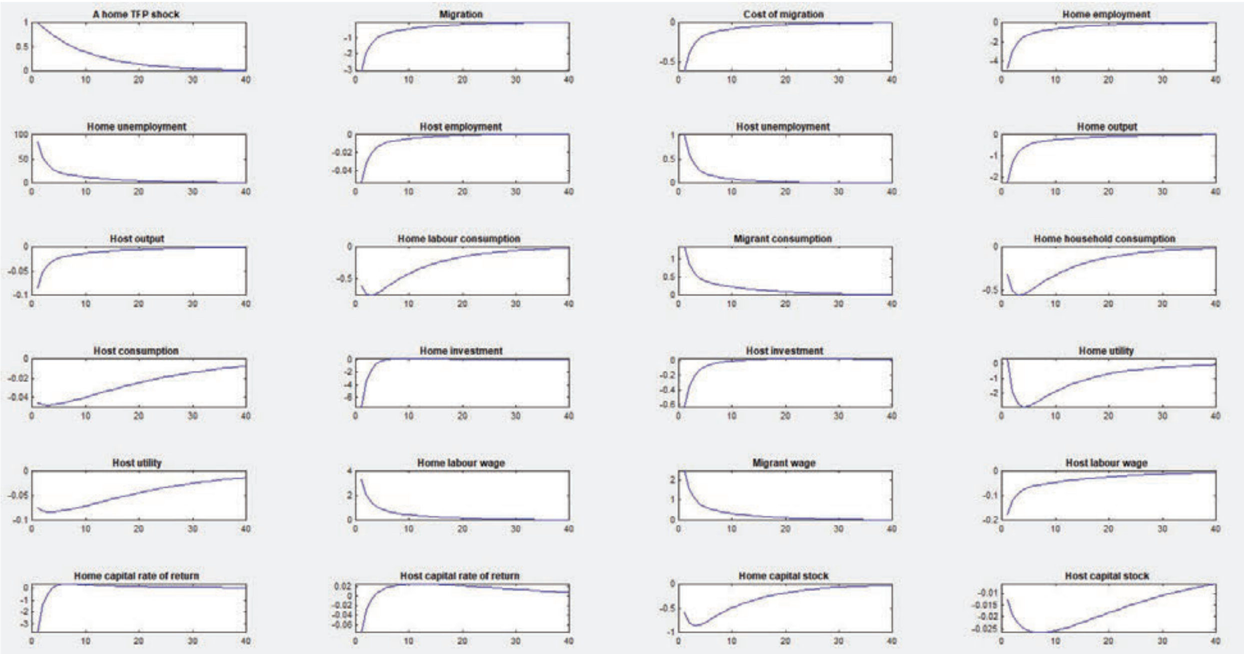
By simulating the log-linearised framework (see **Appendix A.6.2**), we obtain the following results.

#### 5.2.2.1 A positive home TFP shock

Fig.5.4 presents the general equilibrium responses of the capital-adjusted global economy to an additive one-standard-deviation home TFP shock.

For the home economy, endogenous capital adjustment has altered the qualitative results of the fixed-capital benchmark model through aggravating the generated dynamics. Migration will still decrease when the home economic condition has been improved due to the presence of technological advancements. The increase in the productivity, allowing the home labour union to claim higher wages, stimulates both migrant and home labour wages. Again, the rising marginal productivity always comes with lower employment. Then, the fall in the home labour inputs leads to lower marginal product of capital (rate of home capital return), and a drop in the home capital stock. Home output falls with technological unemployment and falling investment.

Figure 5.4: Responses to a positive home TFP shock




The falling investment has made the difference. As shown in the figure, a complete monopolistic labour market has failed to exploit the benefits of a home technological advancement when losses in both capital and labour inputs are shown. The home household is backfired through the contracting income. The fall in the output deteriorates households' income sources and budget constraint, leading to a fall in the home household utility.

Moreover, the positive home TFP shock harms both foreign firms and households. Due to the reduced size of migration, foreign households, encountering decreased marginal products of labour and capital, starts to cut down its investment. The decreases in the foreign capital stock and migrant labour have eventually brought down the foreign labour market performance. All foreign production inputs decrease, and the foreign economy suffers.

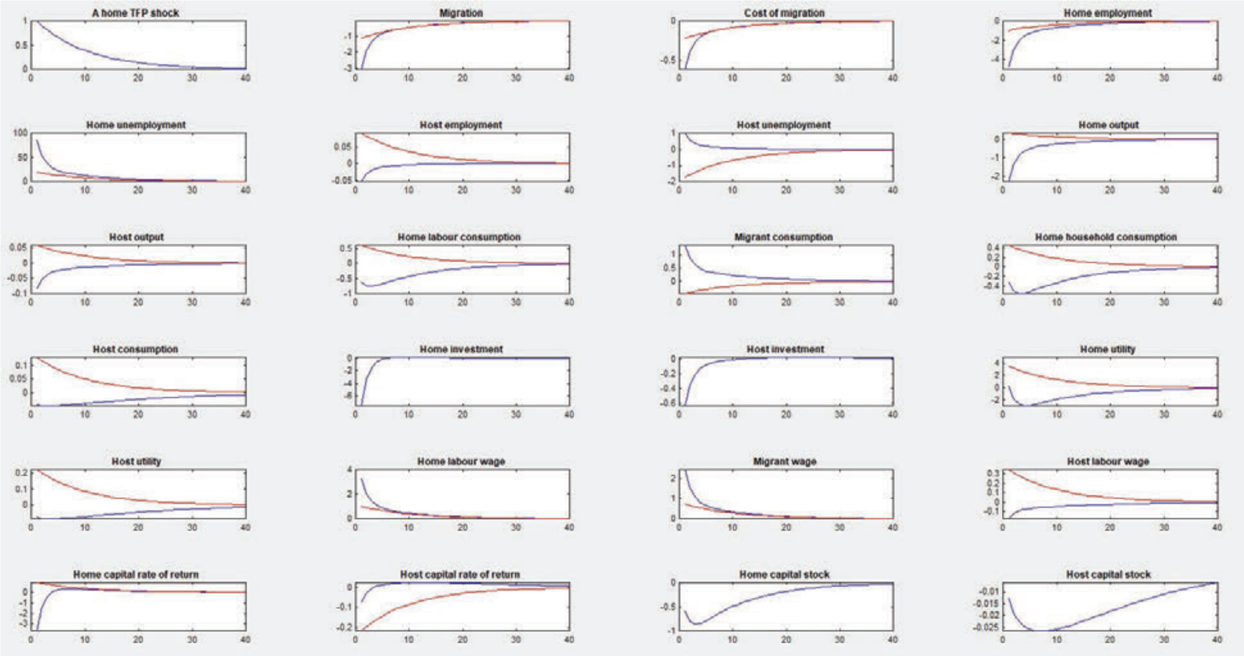
At last, we conclude the effects of a negative change in migration. In this hypothetical global economy with endogenous capital adjustment, the reduced migration harms both home and foreign output, as well as utilities of households. Migration, in a capital-adjustable world, is vital to exploit the benefit of technological advancements.

**Comparing two outcomes**

Fig.5.5 compares the general equilibrium responses of the two frameworks (with fixed and domestically adjustable capital) to the same positive home TFP shock. Domestic capital adjustment in the two nations has amplified the negative effects of dropping migration.

First, with larger responses in the wages, more home workers and migrants are laid off. A lower level of home employment brings down the return rate of capital. The incentive for the home households to invest will be severely harmed. In contrast to the fixed capital model,

Figure 5.5: Comparing responses in the two frameworks




NB: Blue lines represent the responses for the endogenous capital general equilibrium framework, while the red lines give the responses for the fixed-capital framework.

the fall in both home employment and capital inevitably reduce the home output. Also, the fall in capital stock and investment lead to a smaller capital income. As a result, though more leisure is allocated to the home households, the worsening budget constraint results in a decrease in the home household's utility. The monopolistic home labour market cannot help the economy to profit from the positive shock.

Second, the negative migrant wage shock (induced by the home productivity shock) also has a negative effect on the foreign economy. The rising opportunity cost of migration permits a much higher migrant wage. Although there might be still some incentives for the firms to hire more foreign labour in order to sustain its production scale, the falling foreign labour wage and employment shows that the wage competition between two fully-empowered labour groups takes place and its negative effects dominate the general equilibrium responses. The falling foreign investment also has a significant effect on the reduced foreign employment, labour wages and output.

Thirdly, it is important to note that the blue lines (responses of the endogenous-capital model) are fluctuating in a larger scale than the red (responses of the fixed-capital model). It has evidenced the substitutability between domestic capital adjustment and international labour mobility. The generated dynamics in the model with internationally labour mobility and domestic capital adjustment are larger than the ones with only migration.

In general, comparing with the benchmark fixed-capital model, we have no evidence that including an additional adjustment approach other than migration can increase the adjustment speed of the global economy toward the general equilibrium.

#### 5.2.2.2 A positive foreign TFP shock

Figure 5.6: Responses to a positive foreign TFP shock

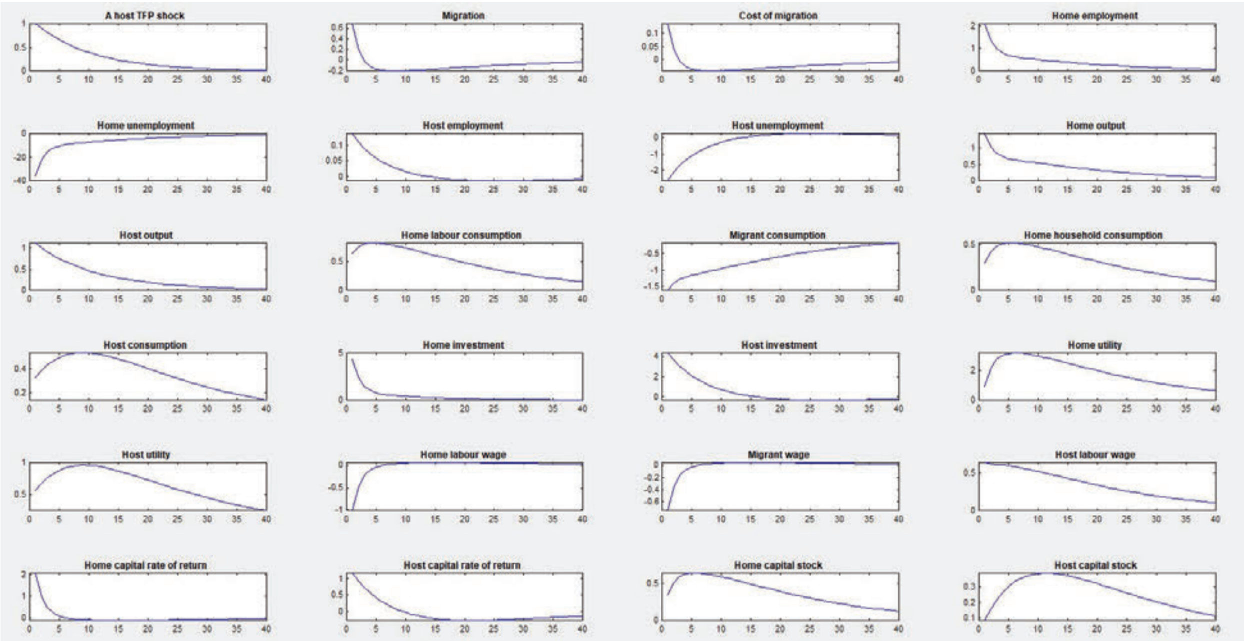



Fig.5.6 shows the general equilibrium responses of the global economy to an additive foreign TFP shock. A positive foreign TFP shock benefits every household in the capital-adjustable global economy. Both home and foreign output will increase.

The positive foreign productivity shock leads to higher marginal products of foreign labour and capital, which in turns motivates more investment and thus encourages foreign employment. Then, the foreign households' budget constraint expands through both higher capital and labour income. The foreign utility increases.

At the same time, the improved foreign economic conditions have exerted a higher degree of attractiveness to the migrants. The increased migration pushes down their marginal product, leading to a falling migrant wage. Due to the home labour equilibrium wage relation, the home economy is in effect exposed to a negative wage shock to the home labour and a loss in its total workforce.

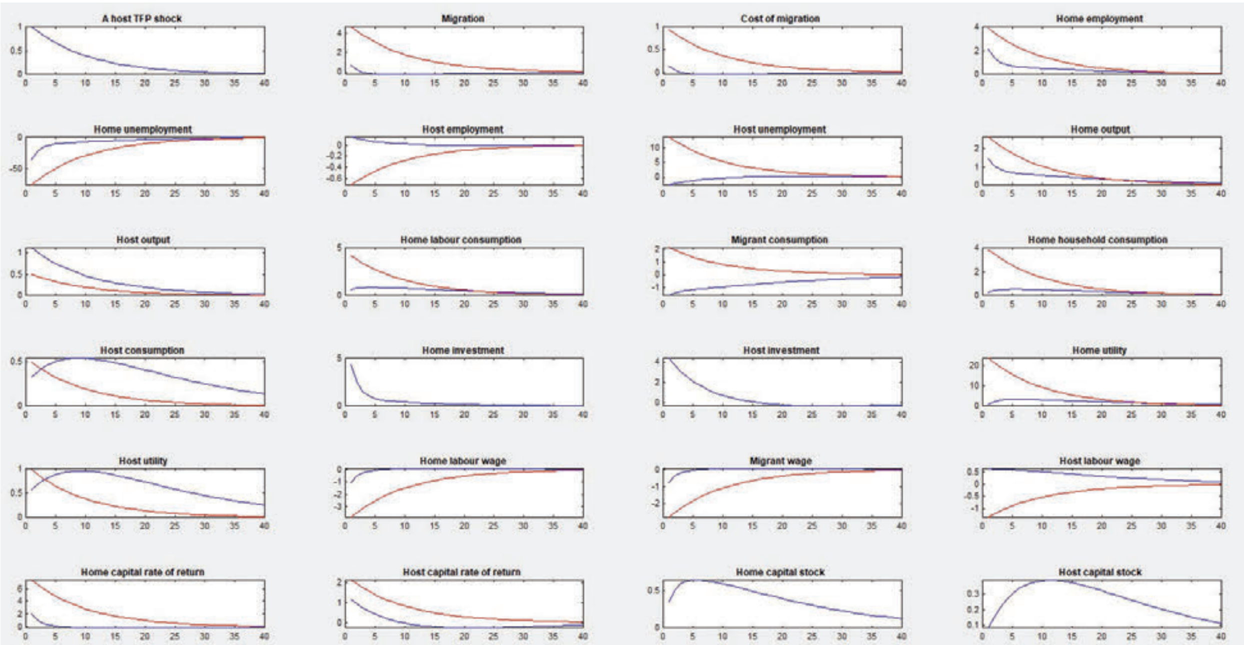
Facing an increased cost of migration, the fall in the home labour wage motivates the home households to supply more labour. And the increasing home employment gives higher marginal product of home capital, which then attracts more home investments and stimulates capital accumulation. A general positive effect has been exerted to the home production and households' income sources. As a result, home utility also increases, coupled with a larger home output.

All in all, a temporary positive deviation of migration from the steady state can stimulate both economies and benefits both households with larger budget constraints. In a global migration economy with the assumptions of adjustable capital and monopolistic labour markets, more migration improves the economic conditions across the world.

### Comparing two outcomes

We then compare the responses of two frameworks in the Fig.5.7. We have noted the following differences.

Figure 5.7: Comparing responses in the two frameworks




NB: Blue lines represent the responses for the capital-adjustable general equilibrium framework, while red lines represent the responses for the fixed-capital framework.

Running counter to Fig.5.5, the above figure shows how a positive migration response has benefited the two-country capital-adjustable economy. Generally, endogenous capital adjustment has reduced the responses of migration and home country variables but notably altered the responses of foreign variables.

The foreign economy and households in the endogenous-capital model receive more benefits than in the fixed model from the foreign TFP shock. The increase in the foreign TFP has brought up the marginal productivity of all foreign inputs. When capital is endogenously determined, a higher return on capital calls more investment. We also note that the increase in investment will moderate the increase of the foreign capital return, comparing with the notable jump in the fixed-capital model (in red lines). At the same time, more foreign capital stock needs to be accommodated by more foreign labour and migrants. The foreign employment is increased due to the increasing needs of investment, a smaller capital return and an improved wage level.

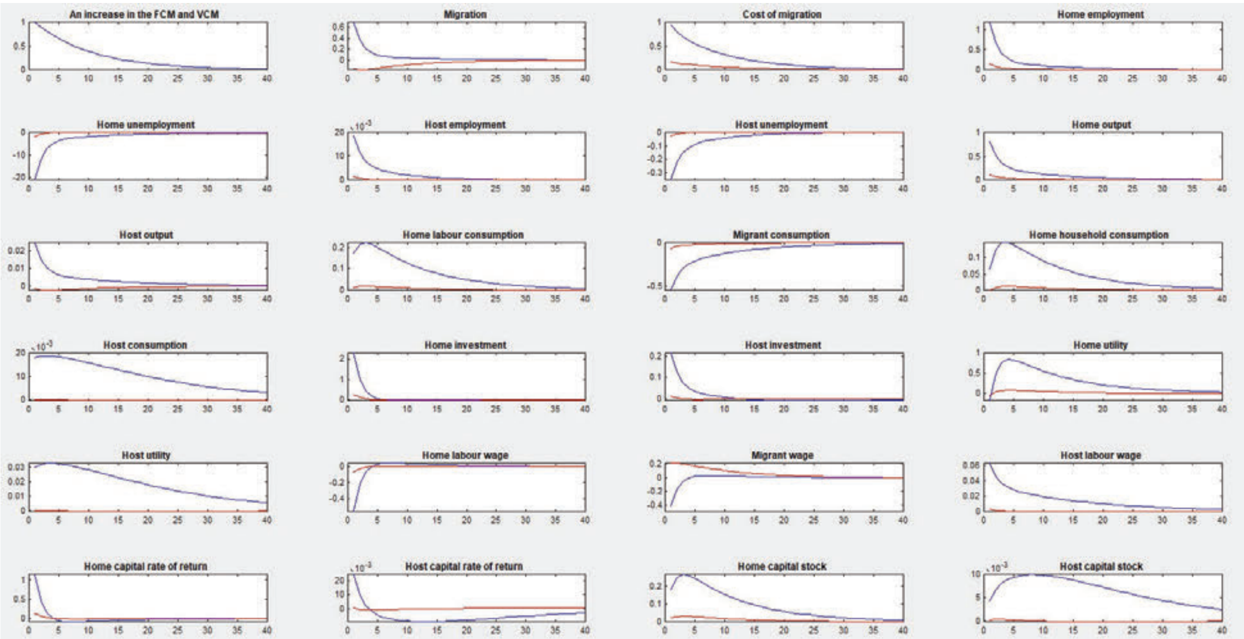
As of migration, the increasing inputs on both foreign labour and capital of the foreign firm have alleviated the demand for migrants, particularly when migrants have full power to claim their marginal productivity. A smaller than the fixed-capital model response of migration is then witnessed. The smaller increase in migration leads to a corresponding smaller drop in migrants' wages. And thus, the home economy is exposed to a relative small shock, which explains the smaller magnitude of all home variables' responses.

At last, the prolonged responses in the home and foreign capital stocks, consumption and utility are shown, especially for the home capital stock, foreign capital return, foreign consumption and utility. The cause of the endured effects is the prolonged periods of capital accumulation, which allows for a longer period of varying returns on capital and of overall income of the households.

In short, business cycle dynamics are more endured in both economies and are amplified in the origin of shocks when we include domestic capital adjustment into a global migration framework.

#### 5.2.2.3 The effects of cost shocks in the endogenous capital model

Figure 5.8: Shocks to the fixed and variable costs of migration




NB: The responses to a shock on the fixed cost of migration are shown in blue lines, while the responses to a shock on the variable cost of migration are shown in red lines.

As in previous sections, we consider how the changes in the costs of migration affect the global economy when capital is adjustable. In contrast to our early outcomes in a fixed-capital model, migration performs as a human capital investment behaviour of which its expenditure stimulates the global economy.

A one-standard-deviation shock on the fixed cost leads to more migration and a higher cost of migration. This is in the opposite of Fig.5.3 where the incremental fixed cost deters migration, but is in consistent with Morten and Oliveira (2016)'s finding. It presents that a higher fixed cost of migration, might be through a larger government expenditure on facilitating migration, can increase the size of migration with capital adjustment.

The increased inflow of migrants has pressed down the marginal product of migrants, and thus the wages of migrants. The fall in the migrant wages has lowered the home labour wage due to the falling opportunity cost. Home households need to supply more to pay for the additional cost of migration. The increase in the home employment then drives the responses of the home production. More capital stocks are accumulated to accommodate the increasing labour input. Home output and households' budget constraint thus expand, which generates a higher level of home utility.

For the foreign economy, a larger migration brings up the marginal product of the foreign labour, which increases the foreign wages and employment. Responding to the increased marginal product of capital brought by the rising domestic employment and migration, a larger foreign investment is then made. foreign output and household utility have all been benefited.

Migration will be lowered while there is an increase in the variable cost of migration as Fig.5.3. The fall in migration and the increase in the variable cost of migration has bumped up the migrant wages through both the channels of marginal productivity of migrants and the home labour equilibrium wage relation. However, a negative response of the home labour wage needs to be discussed. It is motivated by the incremental cost of migration as there would be a need for the home household to put in more employment, which presses down the marginal product of home labour.

For the foreign economy, the negative response of migration the increase in the variable cost of migration has posed slightly positive effects to the households but negatively to the firms. Due to loss in the migrant labour force, the foreign firm increases its demand for the foreign labour through responding a higher foreign labour wage. The foreign investment is thus slightly increased due to the increasing rate of return (marginal product of capital). Both foreign capital and labour incomes are benefited through very limitedly, which raises the utility a bit. However, the increases in the domestic inputs have been insufficient to compensate the loss of migration. foreign output has slightly dipped.

In general, including endogenous capital adjustment has changed the general responses of this benchmark economy toward cost shocks. Most importantly, more migration costs (particularly the fixed cost of migration) can benefit the households, while they bring only loss to the world where capital is not adjusting.

#### 5.2.2.4 The effects of leisure shocks

If capital stocks in two economies are now assumed to be endogenously determined, the leisure shocks matter in the log-linearised system.

Figure 5.9: A home leisure shock

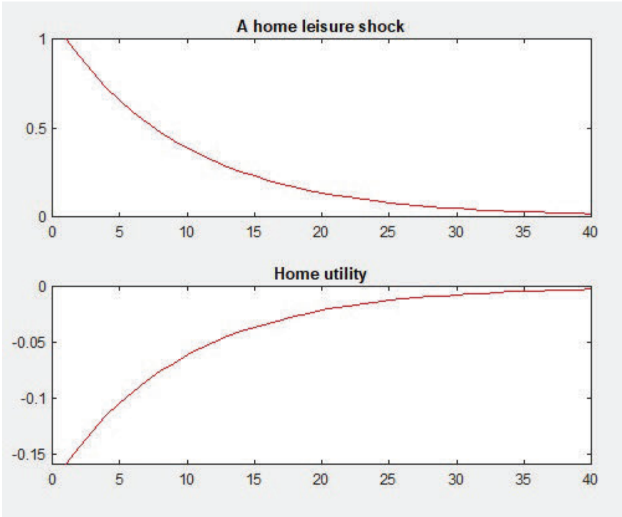



Fig.5.9 shows the two economies' overall responses to the home leisure shock in households' utility functions. It has a negative effect on the home utility. Given equilibrium labour supply and consumption, home households that temporarily increase their preference on leisure utility will suffer.

In this simulation, a home leisure shock has not led to any changes to home employment, unemployment, migration, capital investment or even output. It contradicts almost all the extant literature. It suggests that in a free-migration economy, a sudden change in home labour preference over leisure will not affect the wage of both home and migrant labour, and thus has no impact on the employment nor the actual output.

Our study reckons that it is due to the efficient wage-employment contract and the equilibrium home household wages relation. In a classical closed economy model, the equilibrium wage relation with relative consumption and labour supply is often shown as

$$(C)^{-\eta}W = (N)^{\frac{1}{\nu}}$$

which specifies how the households choose their equilibrium wage by considering both the effects on utility of its increased consumption and the disutility of its increased labour supply. However, in this model, the optimised wage relation is presented as

$$W_t^M - \chi = W_t^h$$

as shown in eq.5.1.11 for the home household. When the households are given two options for their labour supply, they focus on the relative levels of two wages, which is no longer determined by the trade-off between consumption and labour supply but bargained outcomes in the labour markets.

In the stationary state, the wages comply to the equilibrium relationship. The home household labour would be indifferent to work abroad or at home at all levels of labour supply disutility. The established contract between the union and the firms' cartel has associated labour supply with wages. When the equilibrium is successfully established, their labour supply is bound by the contract. In this case, only the externalities to the bargaining process and variable cost can affect the home employment and output.

Although suffering more disutility, home households aggregate labour supply is predominantly determined by the labour market bargaining and variable cost of migration.

Figure 5.10: A foreign leisure shock

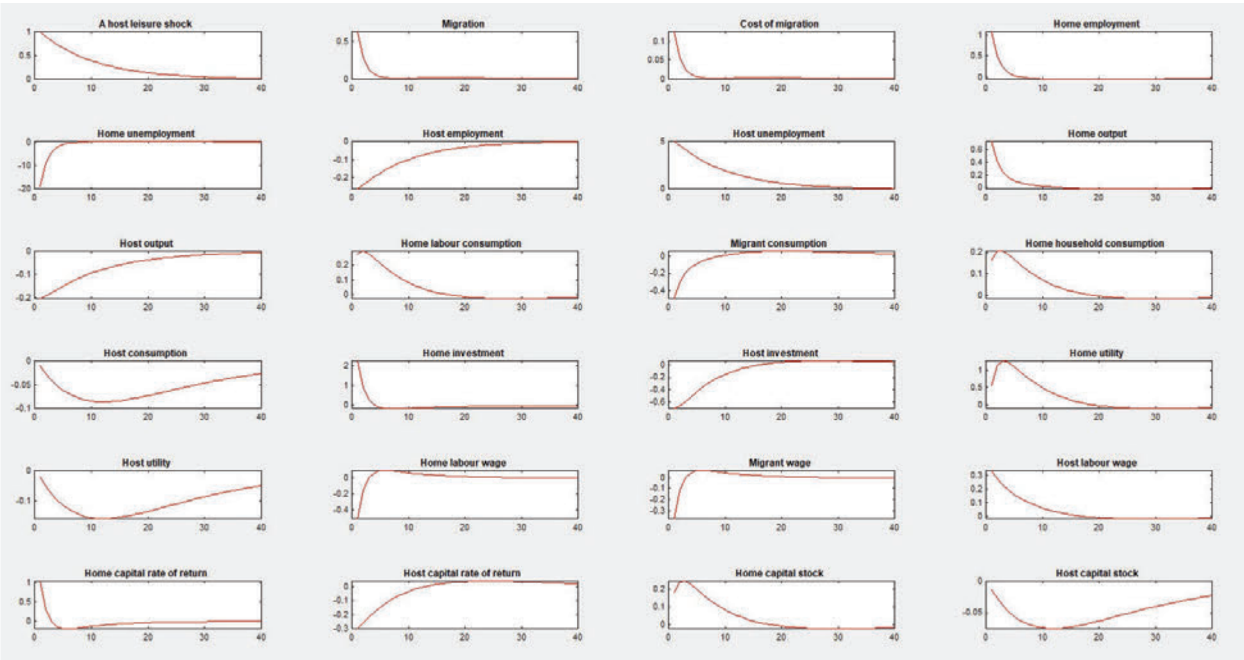



Fig.5.10 gives the impulse responses of the global economy to a foreign leisure shock. When the foreign households become reluctant to work, the foreign countries' production and representative households' utility will fall. But the home households will benefit.

An increased employment disutility will demotivate the foreign households' supply of labour. The foreign employment falls, which gives rise to an increase in the marginal product of foreign labour. Also, due to the positive relation between foreign employment and the marginal product of foreign capital, the foreign capital return and stock decrease. This has confirmed Smets and Wouters (2003)'s finding that a preference shock can crowd out domestic investment. The falling foreign production inputs require the foreign firms to lean on migrant labour. Migration is thus motivated, bringing down the marginal production of migrant in the foreign production. A lower migrant wage is witnessed.

Given eq.5.1.11, a lower wage will be experienced by the home labour, which can motivate more home labour to work. The rising employment in the home firms brings up the returns of the home capital, and thus stimulate the home economy. Via migration, a foreign preference shock boosts the home country investment. The home output rises, which thus generates more income and consumption. At the same time, the increased home households' capital income expands their budget constraints. As a result, the home households' utility rises with an increased home output.

A general comparison of two economies' responses to the leisure shocks provides insight into the actual effects of alternative choices to households facing shocks. In this study, the home economy can remain silent to a domestic leisure shock when its household is endowed with two equilibrium-wage-related labour supply choices, while the foreign leisure shock can affect both economies and the households need to adjust their labour supply decisions.

## 5.3 Varying bargaining power structures

In the previous benchmark model, the home and foreign labour markets are set to be nearly monopolistic as the suppliers (households) have nearly full power in determining wage and employment outcomes.

Apart from the benchmark set-up, seven additional different power structures can be shown. Here we show only the *Case: full monopolistic labour markets*, the other six extreme scenarios can be found in **Appendices A.7**. The detailed comparison study will be presented in the **Section 5.3.2**.

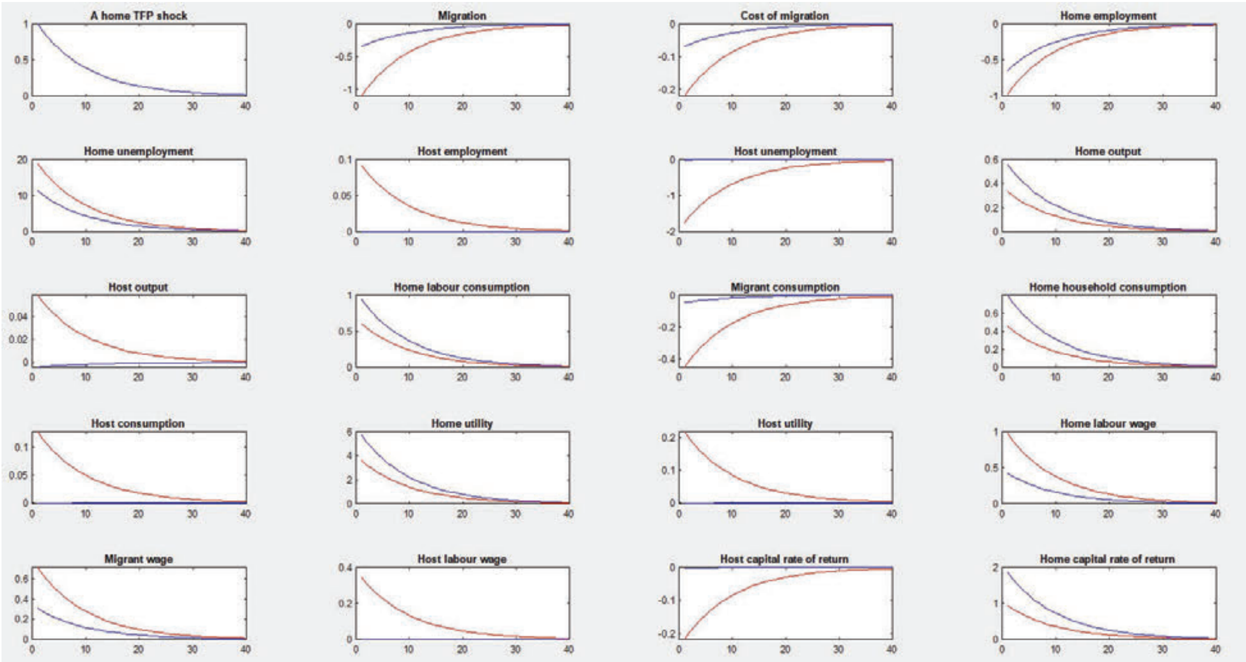
### 5.3.1 Monopsonistic labour markets - $\{b_f = b_h = b_M = 0.9999\}$

#### 5.3.1.1 Responses to productivity shocks

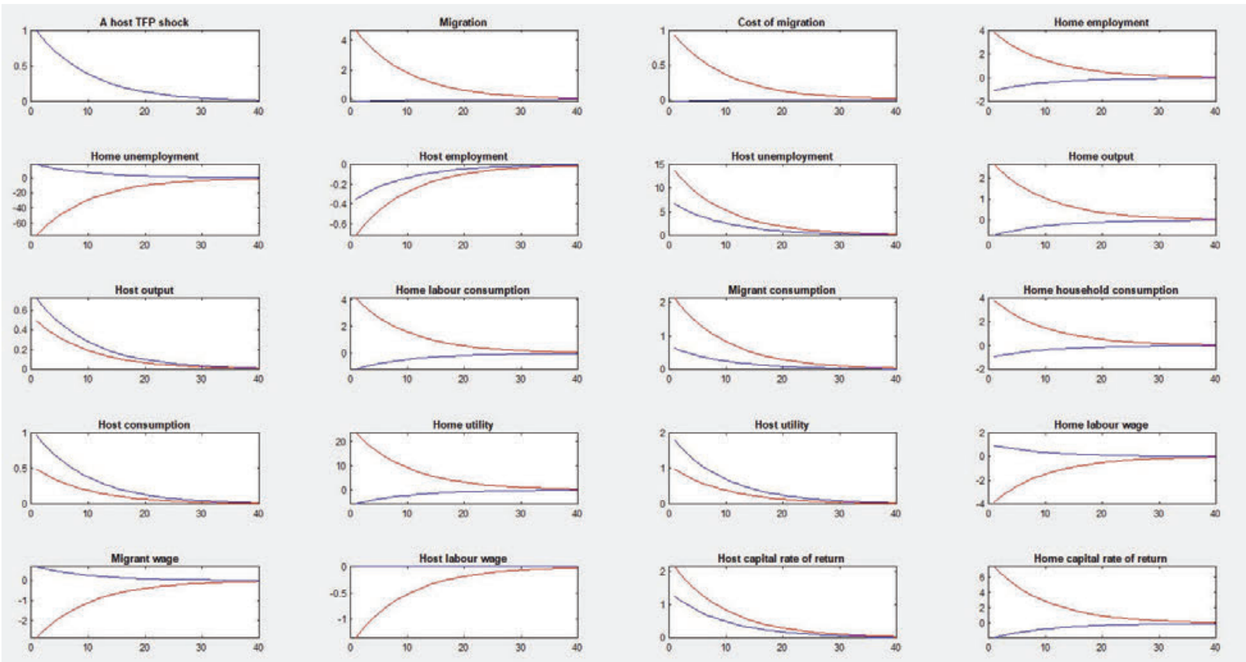
Figs.5.11a and 5.11b show the general equilibrium responses of a global economy with monopsonistic labour markets to the two productivity shocks (in blue lines). The responses of migration and its directly related variables (cost of migration, consumption and wages) have been compared with the responses in the monopolistic model (in red lines).

Experiencing a home TFP shock, migration will drop when the labour markets are monopsonistic, but in a smaller scale than the monopolistic set-up. This might be caused by the empowered firms position in the wage-employment bargaining. The home technological

Figure 5.11: TFP shocks to a global economy with monopsonistic labour markets and migration




(a) A positive home TFP shock




(b) A positive foreign TFP shock

NB: The responses of the benchmark monopolistic model are in red lines, while the responses of monopsonistic labour markets are in blue lines.

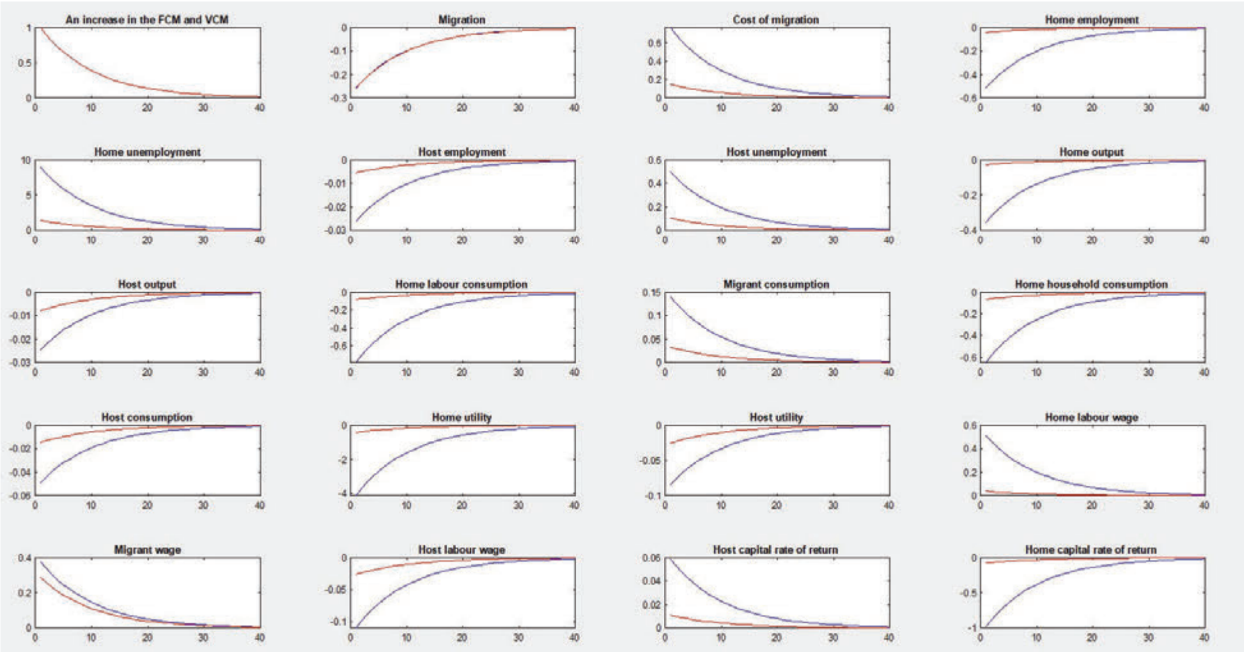
shock increases the wages of its labour force, while firms in a monopsonistic labour market can ask for a smaller increase in the wages. The decrease in the home employment is then moderated, which generates a relatively higher home output at a given amount of capital stocks. Migrant wages are also decreased in a smaller magnitude given the limited changes in migration and foreign employment. Comparing to the red lines, the home output in blue shows a larger positive response to the home TFP shock as home employment stays unchanged in the monoposonistic model than in the monopolistic model. Home utility benefits, while foreign suffers slightly. The foreign firms with full bargaining power would prefer to stabilize the foreign labour market around the steady state by pushing for a optimal migration response, given the rise in migrant wages. All labour and capital-related blue lines of the foreign economy stay nearly unchanged eventually.

In the face of an additive foreign TFP shock, the foreign firm with full power in the labour market can further increase its output by depressing migration from its steady state, instead of admitting more migration with higher migrant wages. With full power in its domestic wage-employment bargaining, the foreign firm has also managed to achieve a smaller decrease in the foreign employment by remaining the steady-state wage. A larger than monopolistic increase of foreign output is achieved by setting out a slightly decrease in migration, a smaller than monopolistic fall in foreign employment, the given fixed stock of capital and most importantly, the additive TFP shock. Note that the foreign capital rate of return has not increased to the level that is reached by the monopolistic model. It is due to the moderated responses of migration and foreign employment. Being the capital and labour supplier simultaneously, the foreign household takes all the benefits of this additive domestic shock.

At last, this study again detects a rise in both countries' unemployment in responding to a productivity shock, perceived by Gali (1999) and Smets and Wouters (2003). Both firms with full bargaining power in the labour markets wish to maximize its output via manipulating migration directly or indirectly. Facing a home TFP shock, the home firms can set out wages of the home labour and thus of migrants through the equilibrium wage relationship to achieve its welfare maximization target. Facing the foreign TFP shock, the foreign firms can directly control migration through the wage-employment contract.

### 5.3.1.2 Responses to the cost shocks

Figure 5.12: responses to an increase in different costs of migration in the monopsonistic labour market




NB: Responses to an increased fixed cost of migration are in blue lines, while responses to an increased variable cost of migration the shock are in red lines.

It is also important to see how the shocks on the migration cost could affect the global economy when all labour markets are monopsonistic. In general, migration responses are consistent with them in the monopolistic case (Fig.5.3). When cost is rising, households without bargaining power are less willing to migrate. Both economies and households suffer from the decreased migration.

The increases in the costs always deter migration in either monopolistic or monopsonistic labour markets. Although foreign firms have all power in bargaining the migrants' wages, a decrease in migration has a positive effect on migrants' productivity and thus wages. Fig.5.12 again shows an adverse relation between the responses of migration and migrant wages.

The response of home labour wages always synchronises with the migrant wages', which has reduced the firm's demand for labour. Give the fixed capital, the home output falls with the reduced labour input. The home budget constraint is thus depressed, which leads to a fall in consumption and utility.

The foreign economy, in Fig.5.12, generally experiences a negative migration shock. The foreign firms, which lost its migrant labour force, also suffers from a lower marginal product of foreign labour. The foreign labour wages fall and the employment decreases in the foreign economy. Foreign output will fall with the reduced employment. As capital is fixed and the cartels have all bargaining power in both economies, the fall in the compensation to labour gives rise to a slightly increased foreign capital return. However, the negative responses of the foreign consumption shows that the aggregate income of the foreign households suffers meaning that the fall in labour income has not been compensated by the rise in the capita income. The foreign utility decreases when less consumption is incurred.

In aggregate, from a global output and welfare perspective, an increasing cost of migration harms two countries' output and utility when the firms have all power in bargaining.

### 5.3.2 Discussions of the responses in fixed-capital models

Here we summarize the findings from the simulated responses in the **A.7**. Relative bargaining powers significantly influence the responses of the two countries by affecting the dynamics of migration and migrant wages.

A positive home TFP shock raises the opportunity cost of moving thus decreasing migration by pushing up the marginal productivity of home labour. A decrease in migration, by keeping more labour at home, temporarily expands the home economy and improves home household utility. However, when the home union has full power in migration bargaining over the foreign cartel ($b_M = 0$), a higher migrant wage is imposed and decreases home employment due to the equilibrium wage relation between home labour and migrants (eq.5.1.11). As shown in Fig.A.1a and A.3a, home output will fall due to the rise in home unemployment, despite the positive home productivity shock.[13] Furthermore, the decrease in migration in the fixed-capital model always harms the foreign economy (in terms of output and welfare).

A positive foreign TFP shock usually raises migration via its incremental effects on the marginal productivity of migrants, unless foreign bargaining power is too great. The foreign output always benefits from its own productivity shock. However, whether the positive effects of the shock can be disseminated to the home economy relies on the foreign cartel not having too much bargaining power over migrants. When having full power in migration bargaining, the foreign cartel will moderate migration, limiting the benefit that home economy receives - see Fig.A.2b, A.4b and A.6b. In addition, the foreign firm cartel may reduce migration (Fig.A.5b and 5.11b) if it has full bargaining over its local labour. Under this circumstance, home output will fall, despite the positive foreign TFP shock.

In this framework with fixed capital, the dynamics of migration matter for the gain or loss of the two economies while facing shocks. Higher migration benefits both economies. Relative

[13] Fig.A.4a shows a positive response of home output at $b_M = 0$ as both firms in this case have full power over their local unions. With home cartel moderating the increase of the home labour wages and foreign cartel stabilizing foreign labour wage, the migrant wage has not been increased to the levels in Fig.A.1a and A.3a.

bargaining powers play a key role in determining the responses of migration.[14]

## 5.4 Varying power structure with endogenous capital

Including the benchmark model, eight nearly extreme bargain power structures can be presented. In order to study how the labour market bargaining could affect the two-country global economy under different capital assumptions, we have observed and compared the effects of four shocks (home and foreign TFP shocks, an increase in either fixed or variable cost of migration).

In this part, we start with manifesting the monopsony labour markets' responses to the shocks and comparing it with the benchmark responses. We will mainly focus on the two main comparisons: 1) the responses of capital stocks and their related variables; and 2) the responses of migration, which is the main channel of the temporal adjustment process between two economies. There will be some interpretations on the length/duration of responding periods of the variables.

Then, we compare and summarize the main findings of all impulse responses curves under different relative bargaining power structures in **Section 5.4.2 Discussion of impulse response curves**. The complete impulse responses have been listed in the **Appendix A.8** and **Appendix A.9**. The responses of the global economy under capital non-adjustment assumption to four shocks are in blue lines, while the responses of the economy under endogenous capital assumption are in red.

### 5.4.1 Monopsonistic labour markets - $\{b_M = b_f = b_h = 0.9999\}$ with endogenous capital

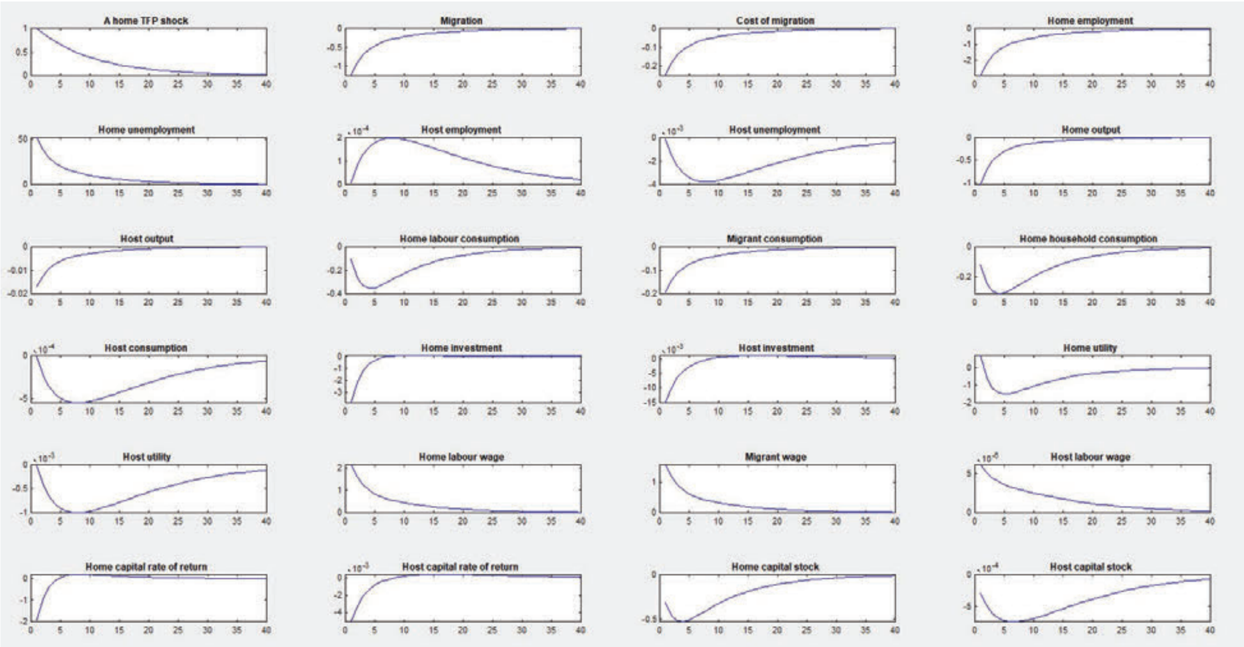
#### 5.4.1.1 A positive home TFP shock

According to Figs.5.13, the endogenous capital assumption has aggravated the damage of the technological unemployment. The positive home TFP shock, featured with monopsonistic labour markets in the two-country migration economy, harms the two economies and households' utility.

Likewise, the additive home TFP shock firstly increases the wages of the home labour. The home firms, thus, decide to drop off more labour. It leads to a large loss in the home

[14] This part only discusses the outcomes of two productivity shock as the cost of migration shocks generate the same intuitions. In the next section, the effects of the cost of migration shocks will be shown in detail through comparing the responses of fixed and adjustable-capital frameworks.

Figure 5.13: Responses to a positive home TFP shock




employment. At the same time, the synchronized migrant wage decreases the foreign firms' willingness to take in more migrants. The home labour supply falls as a whole. Also, the fall in the home employment decreases the marginal product of home capital, which results in a decumulating period of capital stocks. Both home production input suffers and the home utility is reduced.

With full bargaining power, the foreign economy is affected very limitedly in responding to the migration. The magnitudes of its responses are close to zero showing that the priority is to stabilize domestic market and wait for the diminishing of the temporary shocks. In short, the foreign economy's suffering mainly comes from the loss in the migrant labour force alone.

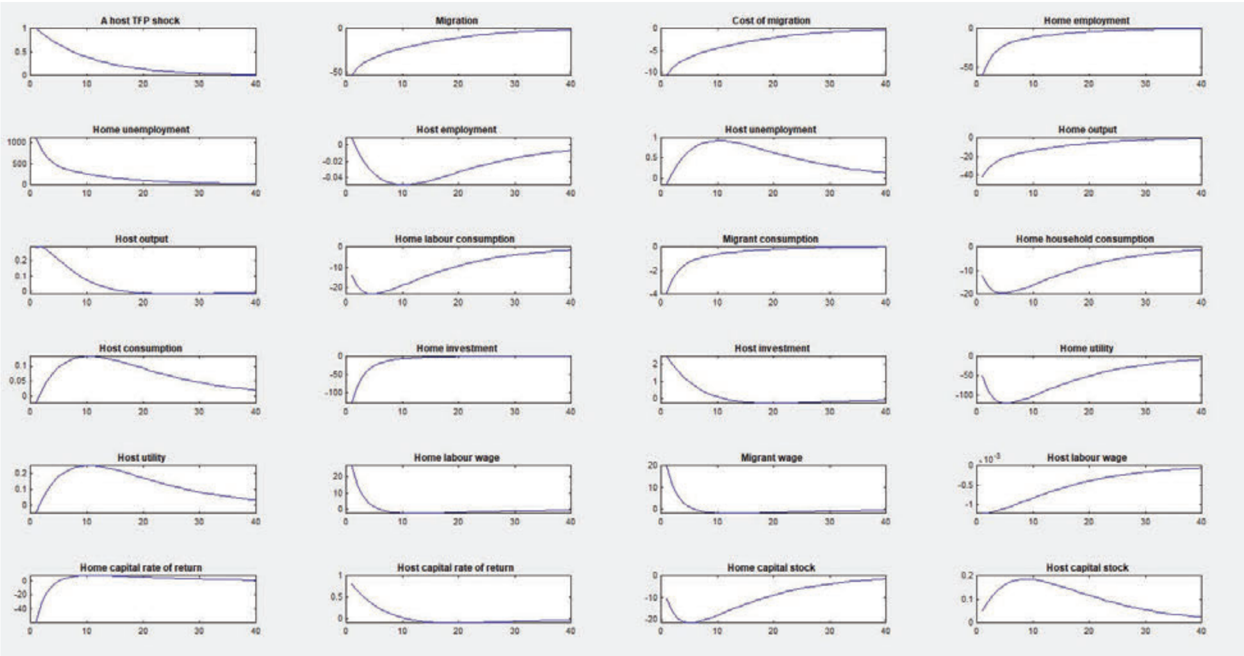
#### 5.4.1.2 A positive foreign TFP shock

Fig.5.14 gives the general equilibrium responses of the adjustable-capital model. In general, the foreign economy allowing domestic capital adjustment will be benefited however at the cost of the home economy.

The foreign firms have chosen to slightly decrease the foreign labour wage in responding to the additive productivity shock, which in turn gives less foreign employment. Higher marginal product of foreign capital is generated as $\frac{\partial^2 Y_t^f}{\partial Z_t^f \partial K_t^f} > 0$, which encourages the firms to invest more on their capital stocks. The migrants wage increases as well due to the shock. The large rise in the migrant wages further restrain the size of migration. The foreign economy, under a monopsonistic labour market assumption, thus enjoys the technology benefit alone.

The home economy, on the other side, suffers from a reduction in migration and an increased

Figure 5.14: Responses to a positive foreign TFP shock




migrant wage. The home labour wage rises with the increased opportunity cost. With full bargaining power, the home firms decide to reduce its cost of hiring by hiring less. Home employment falls, which later brings down the home investment. The home economy has not been benefited from the foreign TFP shock if the migrants has completely no bargaining power in its wage-employment bargain with the foreign firms.

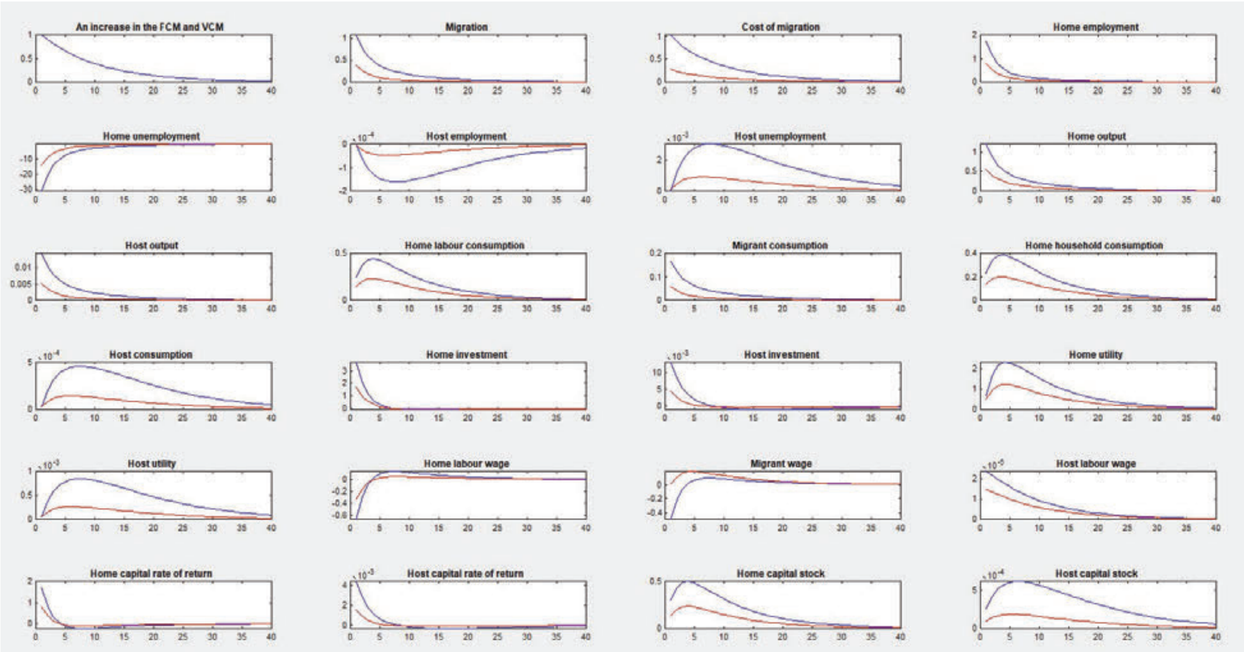
An important message is that no labour group should be endowed with no bargaining power. Allowing the dominance of firm cartels in the labour markets could lead to myopic decisions in production, which significantly impede the contribution of labour adjustment toward the general equilibrium.

#### 5.4.1.3 Responses to cost shocks: both fixed and variable cost of migration

In contrast to Fig.5.12, the assumption of capital-adjustment in Fig.5.15 has reversed the effects of an increase in the migration costs. Higher costs can motivate the economy by encouraging capital accumulation. This finding is consistent with Fig.5.8's outcomes of the additional migration expenditure in the adjustable capital model.

An increase in the fixed cost of migration has increased the migration as Fig.5.3. The increasing migration has pressed down the wages of migrants but pushed up the marginal product of foreign labour and capital stocks, which benefits the foreign economy and household. To the home economy, the lowering migrant wage brings down the home labour wage and allows more employment to be permitted by the firms. It stimulates the investment on home capital and brings up the home output.

Figure 5.15: Responses to the cost shocks




NB: The responses to an increase in the fixed cost of migration are in blue lines, while the responses to an increase in the variable cost of migration are in red.

In response to the increase in the variable cost, the foreign firms with full bargaining power decides to increase migrant wages for more migrant workers. More migrant labour inputs and relatively unchanged foreign production inputs slightly encourage the foreign output. At the same time, home employment rises with the falling home labour wages. Again, increased employment levels give rise to higher return on the home capital. As a result, the home investment increases and output grows.

The comparison between Fig.5.12 and 5.15 has presented the significance of capital adjustment on the simulated general equilibrium responses on different shocks. In a world where capital is fixed and labour is internationally mobile, any additional costs bring an absolute loss that eventually need to be shared by all parties. However, in a world where capital is adjustable, more costs lead to growth due to their positive effects on employment. Moreover, facing cost shocks, firms can increase output of two countries via manipulating migration and capital adjustment.

### 5.4.2 Discussions of impulse response curves

In general, endogenous capital adjustment has brought significant changes compared to the general equilibrium responses of the global economy with migration. Some migration responses are reversed, which leads to opposite responses of the two economies. Relative bargaining powers in general equilibrium adjustment play important roles.

#### 5.4.2.1 The responses of migration to the productivity shocks with or without capital adjustment

In most scenarios, TFP shocks on the marginal productivity of labour lead to a home shock decreasing migration, while a foreign one can increase it. The simulations confirm this in most scenarios but also provide important exceptions.

An additive home shock increases the marginal product of home labour and thus increases the opportunity cost of migration, which should bring down migration. However, if capital is adjustable, Fig.A.27 shows that migration increases in response to a home TFP shock. Specifically, when the home union dominates home labour bargaining and the foreign firm cartel has all bargaining power over foreign and migrant labour ($b_M = b_f \to 1$, $b_h \to 0$), more home labour chooses to migrate despite the increasing home wage. With full power in migration and foreign labour bargaining, the foreign firm moderates the higher migrant wage, encourages foreign employment and through increases in investment. The increases of foreign domestic production factors raise migration through local-migrant complementarity.

The effects of a foreign TFP shock also rely on the responses of migration. Among the cases featured with $b_M \to 1$ in **Appendix 7** and **8**, an additive foreign productivity shock can lead to a decrease in migration.[15] Among the responses in **Appendix 7** fixed-capital cases, the outcomes of a fall in migration include decreased home output, reducing home households' utility, a smaller than benchmark increase of foreign unemployment, and a larger than benchmark increase in the foreign output and households utility. When capital is adjustable, foreign capital stock strictly increases due to the technological innovation, which exerts benefits to foreign output and utility. However, the home country will be exposed to an unemployment shock, which is followed by capital decumulation, lower output and household utility.

In general, the bargaining power of the foreign firm over the migrants determines the migration responses and eventually the effects on the two economies. By increasing the bargaining power, the foreign cartel can encourage migration to have itself (and eventually both economies) benefiting from the home TFP shock. It can also prohibit the positive effects of a foreign productivity shock on the home economy through reducing migration. Meanwhile, endogenous capital adjustment has a significant role to propagate and prolong migration and its effects.

[15] Except the case **A.7.2**, the foreign TFP shock has incurred a very slightly positive migration response. It is argued that the foreign firms' ultimate target is the perceived nearly doubled increase in the foreign output.

#### 5.4.2.2 How bargaining power and capital adjustment interact with foreign and home TFP shocks

The simulation outcomes in **Appendix A.8** present some interesting interactions between capital adjustment and bargaining power structure in the labour markets.

Facing a home TFP shock, endogenous capital adjustment will amplify the negative responses of migration, sometimes with a longer duration. As mentioned, the incremental loss of migration will always push up migrant wages, which in turn leads to a synchronizing wage response of home labour. Both economies suffer from a loss of production input, which reduces the marginal productivity of capital. Under such circumstances, both firms choose to reduce investment and decumulate capital stocks. A home TFP shock, which raises the wages of migrant and home labour force, has a detrimental effect on the output and utility of both economies.

However, the varying bargaining power structures present one possible exception from the above negative responses. Migration can response positively to a additive home TFP shock if and only if the foreign firm cartel has absolute power, not only over migrant labour but also over its domestic locals. By adjusting foreign investment decisions, foreign labour and migrant wages, Fig.A.27 in **A.8.6** shows that migration responses can be reversed when there are increases in both foreign capital and employment. This is also the only case that both home and foreign capital stocks accumulate after a home TFP shock.[16] When $b_M \to 1$ and $b_f \to 1$, both economies and households benefit from the home TFP shock in the endogenous capital model.

Other interactions between capital adjustment and bargaining power are evidenced among the responses to the foreign TFP shock.

The general responses of the two capital-adjustable economies to the foreign productivity shock are distinguished by the bargaining power of migrant labour force. If $b_M \to 0$ migrants have all power in the bargaining with the foreign firms, the positive responses of migration to the foreign TFP shock in the fixed-capital model will be alleviated considerably. A smaller positive migration has only a correspondingly limited positive effect on the home output. It, thus, allows the foreign economy to benefit more from the domestically originated productivity shock. With accumulation of capital, the foreign output will benefit significantly more than in the fixed-capital cases.

However, if migrants have no power over the foreign firms ($b_M \to 0$), the migration responses are reversed. The negative effects of the decline in migration to the home economy are witnessed. The decrease in migration raises the wages of both migrants and home labour force, which leads to higher unemployment in the home economy. The home capital will

[16] Home capital accumulates due to the increase of home employment when migration increases.

be reduced in responding to this phenomenon. Without positive migration, the home economy suffers. On the other side, the foreign economy can still benefit from the foreign TFP shock. The main channel is through increasing the capital stock as the foreign labour wage-employment is influenced by migrants bargaining power. We have seen increasing capital stocks in Fig.A.16, A.28, A.32 and A.36, with inconsistent responses of the foreign employment status. The foreign economy always benefits from its domestic TFP shock.

An additional observation on the adjustment of capital stocks is that capital adjustment normally takes a fairly longer time than do labour market variables (such as wages and employment) in most scenarios. Very often it can take more than the designated 40 years to disappear. Taking into consideration the role of capital income in the households' budget, capital adjustment can prolong and amplify business cycle dynamics brought by productivity shocks.

#### 5.4.2.3 Cost shocks and capital adjustment

A general result from both the fixed and adjustable-capital framework's responses to a cost shock shows that the dynamics can be transmitted to both economies and the home economy (both output and utility) is not necessarily worse-off, even if it has to pay for all the cost.

An increase in the fixed cost of migration can lead to more migration in most scenarios. Both home and foreign economies and households will benefit from a temporarily increased migration with larger domestic labour, investment, output and utility. As introduced in early chapters, a higher fixed cost of migration, such as an increase in government expenditure on facilitating migration, will increase migration in most scenarios, comply with Morten and Oliveira (2016).

However, if $b_f = b_M \to 1$, $b_h \to 0$ the home labour union has all power in bargaining with the home cartel and the foreign cartel has all power over both home and foreign households (see Fig.A.29), migration can fall in equilibrium if there is an increase in the fixed cost of migration. To shoulder the incremental cost of migration, the home union pushes up the home labour wage as high as possible as it has no bargaining power toward foreign firms. The rising migrant wage due to the equilibrium home labour wage relation decreases the foreign firm's desire to employ migrant labour. Both home and foreign economies are harmed from the fall in migration.

Facing a suddenly increased variable cost of migration, migration will drop in equilibrium.[17] In most cases, the fall in migration and rising unit cost of migration stimulates home employment. With the effective adjustment of capital, home output will eventually increase, as well as home households' utility. However, the negative effects are absorbed by the foreign side

[17] The only exception is the monopsonistic case that has been described earlier in **Section 5.4.1**.

due to the fall in the migrant labour force and higher migrant wages. The foreign output will fall with lower utility of their households. Once again, when $b_f = b_M \to 1$, $b_h \to 0$ the foreign cartel has all power over its employees and the home union determines home employment and wages (see Fig.A.30), and a large fall in migration would occur with a fall in home employment. In this case, both countries suffer.

In a nutshell, capital adjustment reverses the economies' responses to the migration cost shocks. An increase in the fixed cost of migration mostly benefits both economies, and the increase in its variable cost decreases migration with most harm attributed to the foreign.

#### 5.4.2.4 Leisure shocks and relative bargaining powers

Last, this section summarizes the main findings of **A.9**, which shows how two economies with varying bargaining power structures respond to the home and foreign leisure shocks. Fig.A.39 shows an identical to benchmark negative effect of the home leisure shock on home household utility. Replacing the Euler wage condition with the relative equilibrium wage relation between home and migrant labour, we can see that the home household suffers a temporary fall in utility due to the increased disutility of labour supply.[18]

Facing a foreign leisure shock, we affirm the significance of relative bargaining powers. Most importantly, the foreign firm cartel's power over the home union (migrants) matters to the general outcomes. When the foreign cartel has no power over migrants $b_M \to 0$, a foreign leisure preference shock leads to a temporary increase in migration, which will benefit home output and welfare as shown in **Section A.9.2.1**. However, if $b_M \to 1$ as in **A.9.2.2**, a temporary fall in migration is seen in response to the foreign leisure shock. A complete disadvantaged power position of migrants will impede the willingness to migrate, which will be aggravated when the foreign firm also seizes power over its domestic labour which leads to higher foreign unemployment. When migration experiences a temporary decrease, home output and welfare fall. In addition, a foreign leisure shock always leads to temporarily decreased foreign output and welfare because of the reduced foreign employment.

In short, leisure shocks can temporarily affect their originated household utility adversely. The significance of relative bargaining powers (especially over migrants) has been further confirmed.

[18]It needs to be emphasized that the reason why varying relative bargaining power has no effect on this response is that the model has not provided a channel for the shock to affect the employment and output.

#### 5.4.2.5 Summary and limitations

In summary, these simulations suggest that capital adjustment can prolong general equilibrium responses to shocks. Having both internationally labour mobility and domestically capital accumulation does not necessarily speed up the adjustment process toward the general equilibrium in an imperfectly competitive two-country free-market economy. Specifically, capital adjustment often takes more than 30 or 40 years to disappear, which eventually reflects in the fluctuations of utility and output.

Another important finding is that a positive deviation of migration from the steady state normally benefits two economies,[19] while negative deviations bring mostly harm. Moreover, the effects of migration can be propagated and prolonged by domestic capital adjustment.

It is important to note the limits of our study. The first and foremost is the set-up of the world without money. By featuring only goods and labour markets, our framework can only reflect part of the real economy. Ignoring the existence of money presents a discussion of the possible roles of interest and inflation rates on migration phenomenon. Moreover, the set-up of the parameters is hardly universally applicable. As mentioned, we have endowed a high degree of symmetry between two households and firms to explore the theoretical effects of migration to the steady state of economies.

## 5.5 Chapter conclusions

Simulation **Chapter 5** begins by inserting plausible shocks into the established model. Preference, productivity and cost shocks from both origins are included. As there are no intertemporal capital decisions in the benchmark model, the general equilibrium responses of the fixed-capital model include productivity and cost shocks only. The leisure shock is included when capital is adjustable.

Most importantly, positive responses of migration always bring benefits to both home and foreign economies, as well as welfare of two households. The responses of migration can be influenced by bargaining power structure in the two labour markets and feasibility of investment.

Although the theory in **Chapter 4** suggests the co-movement of wage and employment of labour groups, the general equilibrium responses of home wage and employment are often in opposite directions, complying with Smets and Wouters (2003)'s result. This finding also occurs when capital is adjustable. However, it is not always hold for the foreign labour.

Finally, we compare the general equilibrium responses between fixed and endogenous capital

[19]The temporary leisure shock is an exception due to its severe negative effects on foreign employment.

models to the same shocks under varying relative bargaining power structures in the labour markets. First, we find that labour market bargaining powers have significant effects on the overall adjustment process. The relative bargaining powers of firms (particularly to migrants) determine the persistence and magnitude of responses of variables from their steady states. Second, capital adjustment can amplify and extend economic fluctuations in the business cycle.

# Chapter 6

# Conclusions

This paper presents a theoretical exploration on the optimality of migration based on the general equilibrium framework. The study shows how migration might be incentivized, affecting significant macroeconomic variables. A major task is to show how the optimal migration can be generated under different political regimes. We also consider the actual effects of different components of migration cost to the economy.

In general, our model relates to Mundell (1961)'s theory of optimal currency areas showing that migration is a non-trivial adjustment to geographical labour and capital misallocation issues. Our analysis can be generalized to case when the speed of capital adjustment is slow or negligible in the short run. The international labour mobility is a feasible approach to the global welfare optimisation as endogenous capital adjustment.

## 6.1 Research findings

By investigating the occurrence of migration in both centrally planned and free-market economies, this thesis has shown what motivates the planner and households to initiate migration. For centrally planned economies, migration is desirable from the supply side as a rational central planner always aims to optimise the capital-labour ratio in production. In free-market economies, households wish to have some of their members migrate, and the foreign economy facilitates migration as long as there exists wage inequality between the economies net of cost.

The most important result is that migration improves the global stationary-state equilibrium welfare under both centrally planned and free-market economies in a world with production heterogeneity across countries. Under certain circumstances, migration can only benefit one of the countries, however not at the cost of the other (comparing to autarky states).

We present specific conditions when full and no migration would occur under different economic systems. Extending Benhabib and Jovanovic (2012)'s findings, we re-investigate the determinants of migration under a more detailed production model. We also explore what would happen if there was a conflict regarding the desirable sizes of migration between planners in the foreign and home economy. If the bargaining over the share of costs is triggered, the equilibrium will be located between the two optimal sizes.

We show the roles of the fixed cost of migration in determining the optimal level of migration. In **Chapter 2** where the two countries are run by a single global dictator, allowing migration raises welfare, as long as the global budget constraint condition is not violated. Considering two individual central planners in **Chapter 3**, the fixed cost of migration serves as a corner condition of migration that says as long as the total gain of migration outweighs the fixed cost, migration would occur. If the planners cooperate in determining size of migration flows, the fixed cost will only appear in the budget constraints of the planners. Finally, in the imperfectly competitive free-market economy, both variable and fixed costs matter in determining the migrants' wage and size, as well as all other labour groups' wage-employment contracts.

In a cooperation game set-up, the pre-determined share of welfare is irrelevant to the optimal stationary state of migration because the home household can always adjust its share of welfare through manipulating migration decisions.

Furthermore, our extension to the efficient contract model in the free-market economies has successfully replicated to a certain degree real-world migration phenomenon. The explorations on eight scenarios listed the possible consequences of bargaining power differentiation. Noteworthy is that to allow for positive migration, the authorities should prohibit monopsony and monopolistic labour markets. Migrants, as minority groups in the destination labour market, need to be empowered to increase the foreign economy's attractiveness to potential migrants, while giving full bargaining power to migrants will be at the cost of domestic labour.

Finally, the simulated general equilibrium responses have presented some useful results. The comparison of responses under fixed and adjustable capital assumptions has presented some possible associations between capital accumulation and international labour mobility. Relative bargaining powers will determine the labour market outcomes and optimal migration. Then, investment in the capital stock will always amplify the benefits and costs of migration. For the authorities, government expenditure in facilitating migration and promoting international labour mobility benefits both economies in terms of output and welfare.

## 6.2 Welfare comparison and implications

This section summarises the comparison of equilibrium welfare states among four designated economic systems (a single global dictator, two individual central planners, two perfectly competitive free-market economies and imperfectly competitive free-market economies) and what this might indicate.

A global planner (GP) achieves the highest global optimum under the assumptions of endogenous capital adjustment (EK) and fixed capital stocks (FK). However, the welfare levels of global optimum that two assumptions result in are not necessarily equal. Defining the global optimum as the sum of two countries' aggregate welfare $V = U^h + U^f$, we concluded that

$$V_{EK}^{GP} \geq V_{FK}^{GP}$$

where the global optimum welfare under endogenous capital is always no less than the global optimum welfare under fixed capital. The two are equal if and only if the endowments of fixed capital coincide with the optimally adjusted capital stocks in both economies.

In the system with two individual central planners, cooperation is, under most scenarios, preferred by the two planners, which can deliver the global optimum. Under the assumption of free domestic capital adjustment, we have

$$V_{EK}^{2P} \mid_{Coop} = V_{EK}^{GP} = V_{EK}^{2P} \mid_{Nash}$$

where two planners ($2P$) can achieve global optimum via either cooperation or Nash game.

When capital is fixed, we find that

$$V_{FK}^{2P} \mid_{Coop} \leq V_{FK}^{GP}.$$

The two planners' cooperation is counter-productive (less than the global optimum), when the capital-labour ratio at home is less than the derived threshold level as suggested in **Prop. 3.4.4**. For example, countries such as India, which is endowed with labour intensive production and incomplete capital markets, may suffer counter productive cooperation migration. However, the foreign economy, featured by local-migrant complementarity in production, always benefits from labour inflows.

However, a Nash game provides a range of possible outcomes. Comparing with the cooperation outcome, two planners would prefer to have cooperation in making migration decisions. In particular when the foreign economy has full power and the home pays all the migration cost, the Nash game under fixed capital produces a optimum. Put differently, if the capital-labour ratio at home equals the critical value (in eq.3.4.16), Nash and cooperation equilibria coincide.

The welfare optima based on the perfect competitive (PC) free-market economies comply with the global central planner's optimal welfare. We conclude that

$$V_{EK}^{PC} = V_{EK}^{GP}; V_{FK}^{PC} = V_{FK}^{GP}.$$

Migration between two perfectly competitive free-market economies gives an equivalent level of global optimum welfare to migration under a powerful global planner under both assumptions of capital adjustment.

In regard to the imperfectly competitive (IC) free-market economies, the natural level of unemployment and existence of bargaining frictions yield lower welfare than the perfect competitive free-market economy as

$$V_{EK}^{IC} < V_{EK}^{PC} = V_{EK}^{GP}; V_{FK}^{IC} < V_{FK}^{PC} = V_{FK}^{GP}.$$

When bargaining occurs in all market, the wage-employment contract relationships of all labour groups are closely related. Both theoretical partial equilibrium inference and simulated general equilibrium responses recommend limiting both firms and unions' power in bargaining.

## 6.3 Future research

There are three future research directions following from this study.

First of all, this generally stationary framework can be generalized to a more complex dynamic model with growth inequality. We have assumed that the developed country (foreign economy) has a more sophisticated production technology than the developing one (home economy). However, we have not modelled the cause for such an observation. Different possible causes of growth can be included into the model to study the roles migration could play in determining the short-run and long-run inequality between countries. By doing so, this framework can explain what motivates or even enlarges the gap between the home and foreign countries in the current model.

Secondly, a much closer look at the relative bargaining power between planners would enrich our study. The bargaining power is normally treated as an exogenous variable, while in the real world, it is surely endogenous depending for example on human capital. By endogenising the bargaining power, we may explicitly account for the real-world distribution of skills among populations.

A limitation of this study is we have accounted for a limited scope of human capital differentials. This thesis has specified three types of labour: home labour, migrant and foreign labour. In fact, migrant and home labour are the same labour unit from the same household and the difference in the marginal productivity results from their workplace surroundings. This study has only considered the externalized difference that is generated from the working place surroundings. However the difference between locals and migrants in the foreign economy is likely to be also an internalized human capital variety. A detailed study of human capital types, their association with migration, and even their possible link with economic growth may be important for future study on migration selection policies.

Finally, migration also brings a more diversified cultural and social environment. Some studies show that countries with increasing complexity of ethnic groups actually acquire higher levels of economic growth, which is at risk from negative political populism (Ager and Brückner, 2013; Bellini, Ottaviano, Pinelli and Prarolo, 2013; Ely and Thomas, 2001; Ottaviano and Peri, 2006). It will be interesting to reconsider studies on the relationship between social tensions created by migrations and economic growth after Konya (2007), which has only modelled it as a variable cost of migration.

This direction would provide a deeper understanding on how cultural merging, assimilation and divergence can affect innovation, aggregate demand and productivity, as well as institutional effectiveness, which actually determine economic growth in general.

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# Appendix

## A.1 Stationary-state systems of equations for Section 2.3

### A.1.1 Similar production technologies but different initial capital-labour ratio

The equilibrium system of equations of the global central planned economy are:

$$CM^* = CM_0 + \chi M^* \tag{A.1.1}$$
$$Y^{h*} + Y^{f*} = C^{M*} + C^{h*} + C^{f*} + \delta^h K^{h*} + \delta^f K^{f*} + CM^* \tag{A.1.2}$$
$$Y^{f*} = [\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*} + M^*)^{\mu^f}]^{\frac{1}{\mu^f}} \tag{A.1.3}$$
$$Y^{h*} = [\varphi_h (K^{h*})^{\mu^h} + (1-\varphi^h)(L^h - M^*)^{\mu^h}]^{\frac{1}{\mu^h}} \tag{A.1.4}$$
$$\frac{1+\delta^f\beta-\beta}{\beta} = \varphi^f(1-\omega^f)(Y^{f*})^{1-\mu^f}(K^{f*})^{\mu^f-1} \tag{A.1.5}$$
$$\frac{1+\delta^h\beta-\beta}{\beta} = \varphi^h(Y^{h*})^{1-\mu^h}(K^{h*})^{\mu^h-1} \tag{A.1.6}$$
$$(N^{f*})^{\frac{1}{\nu^f}}(C^{f*})^{\eta^f} = (1-\varphi^f)(1-\omega^f)(Y^{f*})^{1-\mu^f}(N^{f*}+M^*)^{\mu^f-1} \tag{A.1.7}$$
$$(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = (1-\varphi^h)(Y^{h*})^{1-\mu^h}(L^h-M^*)^{\mu^h-1} \tag{A.1.8}$$
$$(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = \omega^f(Y^{f*})^{1-\mu^f}(N^{f*}+M^*)^{\mu^f-1} - \chi \tag{A.1.9}$$

Substituting eqs.A.1.1, A.1.3 and A.1.4 into the other equations, the system can be reduced as

$$[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*}+M^*)^{\mu^f}]^{\frac{1}{\mu^f}} + [\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(L^h-M^*)^{\mu^h}]^{\frac{1}{\mu^h}}$$
$$= C^{M*} + C^{h*} + C^{f*} + \delta^h K^{h*} + \delta^f K^{f*} + CM_0 + \chi M^* \quad \text{(A.1.10)}$$
$$\frac{1+\delta^f\beta-\beta}{\beta} = \varphi^f(1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*}+M^*)^{\mu^f}]^{\frac{1-\mu^f}{\mu^f}}(K^{f*})^{\mu^f-1} \quad \text{(A.1.11)}$$
$$\frac{1+\delta^h\beta-\beta}{\beta} = \varphi^h[\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(L^h-M^*)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}(K^{h*})^{\mu^h-1} \quad \text{(A.1.12)}$$
$$(N^{f*})^{\frac{1}{\nu^f}}(C^{f*})^{\eta^f} = (1-\varphi^f)(1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*}+M^*)^{\mu^f}]^{\frac{1-\mu^f}{\mu^f}}(N^{f*}+M^*)^{\mu^f-1} \quad \text{(A.1.13)}$$
$$(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = (1-\varphi^h)[\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(L^h-M^*)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}(L^h-M^*)^{\mu^h-1} \quad \text{(A.1.14)}$$
$$(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = \omega^f[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*}+M^*)^{\mu^f}]^{\frac{1-\mu^f}{\mu^f}}(N^{f*}+M^*)^{\mu^f-1} - \chi \quad \text{(A.1.15)}$$

According to eq.A.1.14, the home aggregate consumption can be shown as

$$C^{h*} + C^{M*} = \{\frac{(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}}{(L^h)^{\frac{1}{\nu^h}}}\}^{\frac{1}{\eta^h}} \quad \text{(A.1.16)}$$

According to eq.A.1.7, the foreign aggregate consumption is

$$C^{f*} = \{\frac{(1-\varphi^f)[\varphi^f(\frac{K^{f*}}{N^{f*}+M^*})^{\mu^f} + (1-\varphi^f)]^{\frac{1-\mu^f}{\mu^f}}}{(N^{f*})^{\frac{1}{\nu^f}}}\}^{\frac{1}{\eta^f}} \quad \text{(A.1.17)}$$

Eq.A.1.11 gives a relationship between capital and labour of the foreign economy in equilibrium as

$$K^{f*} = \{\frac{[\frac{1+\delta^f\beta-\beta}{\beta\varphi^f}]^{\frac{\mu^f}{1-\mu^f}} - \varphi^f}{1-\varphi^f}\}^{-\frac{1}{\mu^f}}(N^{f*}+M^*) \quad \text{(A.1.18)}$$

And eq.A.1.12 gives its home economy counterpart

$$K^{h*} = \{\frac{[\frac{1+\delta^h\beta-\beta}{\beta\varphi^h}]^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}{1-\varphi^h}\}^{-\frac{1}{\mu^h}}(L^h-M^*) \quad \text{(A.1.19)}$$

Substituting eq.A.1.19 into A.1.16, the stationary-state consumption of the home country is a constant as

$$C^{h*} + C^{M*} = \{\frac{(1-\varphi^h)\{\varphi^h[\frac{1-\varphi^h}{(\frac{1+\delta^h\beta-\beta}{\beta\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}] + (1-\varphi^h)\}^{\frac{1-\mu^h}{\mu^h}}}{(L^h)^{\frac{1}{\nu^h}}}\}^{\frac{1}{\eta^h}}$$

$$= \{\frac{(1-\varphi^h)[\frac{1-\varphi^h}{(\frac{1+\delta^h\beta-\beta}{\beta\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}\frac{1+\delta^h\beta-\beta}{\beta\varphi^h}}{(L^h)^{\frac{1}{\nu^h}}}\}^{\frac{1}{\eta^h}} \tag{A.1.20}$$

Substituting eq.A.1.18 into A.1.17 gives

$$C^{f*} = \{\frac{(1-\varphi^f)\{\varphi^f[\frac{1-\varphi^f}{(\frac{1+\delta^f\beta-\beta}{\beta\varphi^f})^{\frac{\mu^f}{1-\mu^f}}-\varphi^f}] + (1-\varphi^f)\}^{\frac{1-\mu^f}{\mu^f}}}{(N^{f*})^{\frac{1}{\nu^f}}}\}^{\frac{1}{\eta^f}}$$

$$= \{\frac{(1-\varphi^f)[\frac{1-\varphi^f}{(\frac{1+\delta^f\beta-\beta}{\beta\varphi^f})^{\frac{\mu^f}{1-\mu^f}}-\varphi^f}]^{\frac{1-\mu^f}{\mu^f}}\frac{1+\delta^f\beta-\beta}{\beta\varphi^f}}{(N^{f*})^{\frac{1}{\nu^f}}}\}^{\frac{1}{\eta^f}} \tag{A.1.21}$$

### A.1.2 Different production technologies but same initial capital-labour ratio

$$CM^* = CM_0 + \chi M^* \tag{A.1.22}$$

$$Y^{h*} + Y^{f*} = C^{M*} + C^{h*} + C^{f*} + \delta^h K^{h*} + \delta^f K^{f*} + CM^* \tag{A.1.23}$$

$$Y^{f*} = \{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}} \tag{A.1.24}$$

$$Y^{h*} = [\varphi^h (K^{h*})^{\mu^h} + (1-\varphi^h)(L^h - M^*)^{\mu^h}]^{\frac{1}{\mu^h}} \tag{A.1.25}$$

$$\frac{1+\delta^f\beta-\beta}{\beta} = \varphi^f(1-\omega^f)(Y^{f*})^{1-\lambda^f}(K^{f*})^{\mu^f-1}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}} \tag{A.1.26}$$

$$\frac{1+\delta^h\beta-\beta}{\beta} = \varphi^h(Y^{h*})^{1-\mu^h}(K^{h*})^{\mu^h-1} \tag{A.1.27}$$

$$(N^{f*})^{\frac{1}{\nu^f}}(C^{f*})^{\eta^f} = (1-\varphi^f)(1-\omega^f)(Y^{f*})^{1-\lambda^f}(N^{f*})^{\mu^f-1}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}} \tag{A.1.28}$$

$$(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = (1-\varphi^h)(Y^{h*})^{1-\mu^h}(L^h-M^*)^{\mu^h-1} \tag{A.1.29}$$

$$(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = \omega^f(Y^{f*})^{1-\mu^f}(M^*)^{\mu^f-1} - \chi \tag{A.1.30}$$

Substituting eqs.A.1.22, A.1.24 and A.1.25 into other equations, the reduced system is

$$\begin{aligned}&\{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}}\\ &+ [\varphi^h (K^{h*})^{\mu^h} + (1-\varphi^h)(L^h - M^*)^{\mu^h}]^{\frac{1}{\mu^h}} = C^{M*} + C^{h*} + C^{f*} + \delta^h K^{h*} + \delta^f K^{f*} + CM^*\end{aligned} \tag{A.1.31}$$

$$\begin{aligned}\frac{1+\delta^f\beta-\beta}{\beta} &= \varphi^f(1-\omega^f)\{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}\\ &(K^{f*})^{\mu^f-1}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}\end{aligned} \tag{A.1.32}$$

$$\frac{1+\delta^h\beta-\beta}{\beta} = \varphi^h[\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(L^h-M^*)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}(K^{h*})^{\mu^h-1} \tag{A.1.33}$$

$$\begin{aligned}(N^{f*})^{\frac{1}{\nu^f}}(C^{f*})^{\eta^f} &= (1-\varphi^f)(1-\omega^f)\{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}\\ &(N^{f*})^{\mu^f-1}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}\end{aligned} \tag{A.1.34}$$

$$(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = (1-\varphi^h)[\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(L^h-M^*)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}(L^h-M^*)^{\mu^h-1} \tag{A.1.35}$$

$$\begin{aligned}(N^{h*}+M^*)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} &= \omega^f\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f}\\ &+ (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(M^*)^{\mu^f-1} - \chi\end{aligned} \tag{A.1.36}$$

As the production technology is taking the same form as **Appendices A.1.1** in the home economy, its aggregate consumption can be shown as

$$C^{M*} + C^{h*} =\{\frac{(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h-M^*})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}}{(L^h)^{\frac{1}{\nu^h}}}\}^{\frac{1}{\eta^h}} \tag{A.1.37}$$

$$=\{\frac{(1-\varphi^h)[\frac{1-\varphi^h}{(\frac{1+\delta^h\beta-\beta}{\beta\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}\frac{1+\delta^h\beta-\beta}{\beta\varphi^h}}{(L^h)^{\frac{1}{\nu^h}}}\}^{\frac{1}{\eta^h}} \tag{A.1.38}$$

According to eq.A.1.34, the foreign aggregate consumption is

$$\begin{aligned}C^{f*} =&\{(1-\varphi^f)(1-\omega^f)\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f}\\ &+ (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(N^{f*})^{\mu^f-1-\frac{1}{\nu^f}}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}\}^{\frac{1}{\eta^f}}\\ &= \overline{C^f(M^*)}\end{aligned} \tag{A.1.39}$$

In this equation, when more migrants come into the foreign economy where the capital stock is exogenous, consumption will increase ($\frac{\partial C^{f*}}{\partial M^*} > 0$).

For the case that capital is endogenous, we need to substitute eq.A.1.24 and A.1.38 into A.1.30:

$$\{(1-\varphi^h)[\frac{1-\varphi^h}{(\frac{1+\delta^h\beta-\beta}{\beta\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}\frac{1+\delta^h\beta-\beta}{\beta\varphi^h}\}^{\frac{1}{\eta^h}}$$

$$=\omega^f(\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}})^{1-\lambda^f}(M^*)^{\lambda^f-1} - \chi$$

$$\omega^f\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(M^*)^{\lambda^f-1}$$

$$=\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h})[\frac{1-\varphi^h}{(\frac{\beta^h\varphi^h}{1+\delta^h\beta^h-\beta^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}} + \chi$$

$$\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} =$$

$$\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}} + \chi}{\omega^f}(M^*)^{1-\lambda^f}$$

$$\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}} =$$

$$\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta^h\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}} + \chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}(M^*)^{\lambda^f}$$

$$(1-\omega^f)[\varphi^f(K^{f*})^{\mu^f}+(1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}=$$

$$\{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f\}(M^*)^{\lambda^f} \tag{A.1.40}$$

$$\varphi^f(K^{f*})^{\mu^f}+(1-\varphi^f)(N^{f*})^{\mu^f}=\{\frac{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}\}^{\frac{\mu^f}{\lambda^f}}(M^*)^{\mu^f}$$

$$\varphi(K^{f*})^{\mu}=\{\frac{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}\}^{\frac{\mu^f}{\lambda^f}}(M^*)^{\mu^f}-(1-\varphi^f)(N^{f*})^{\mu^f}$$

$$K^{f*}=\{\frac{\{\frac{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}\}^{\frac{\mu^f}{\lambda^f}}(M^*)^{\mu^f}-(1-\varphi^f)(N^{f*})^{\mu^f}}{\varphi^f}\}^{\frac{1}{\mu^f}}$$

$$\frac{K^{f*}}{N^{f*}}=\{\frac{\{\frac{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta-\beta}{\beta})[\frac{1-\varphi^h}{(\frac{\beta\varphi^h}{1+\delta^h\beta-\beta})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}\}^{\frac{\mu^f}{\lambda^f}}\frac{M^*}{N^{f*}}^{\mu^f}-(1-\varphi^f)}{\varphi^f}\}^{\frac{1}{\mu^f}} \tag{A.1.41}$$

foreign capital stock would be positively correlated to the migration. The larger the immigration, the more capital in the foreign economy.

## A.2 The remittance set-up

Adhering to recent remittance studies (Bandeira et al., 2018; Mandelman and Zlate, 2012), the income of migrants will be received and allocated by the households as a family decision. The foreign central planner's primary objective is to maximise

$$\max_{\{C_t^f\}}\{\sum_{0}^{+\infty}(\beta^f)^t[U^f(C_t^f, L^f)]\}$$

with the same instantaneous utility function as in eq.2.1.7. With full employment optimally adopted for the centrally planned economy,[1] the primary objective of the foreign planner is to optimise the aggregate consumption ($C_t^f$), subject to

$$C_t^f \leq Y_t^f - Y_t^M + (1-\delta^f)K_t^f - K_{t+1}^f - (1-s)CM_t \tag{A.2.1}$$

where $Y_t^M$ is the total compensation to migrant labour or aggregate migrant labour income. The foreign aggregate consumption ($C_t^f$) is subject to the current period output ($Y_t^f$), after depreciated stock of current capital ($(1-\delta^f)K_t^f$), migrants' compensation ($Y_t^M$), capital for production in the next period ($K_{t+1}^f$) and its liable share in the aggregate migration cost ($(1-s)CM_t$). In this case, the migrant income is used for migrant consumption ($C_t^M$) and their remittance to the home country ($R_t^M$)

$$Y_t^M = C_t^M + R_t^M \tag{A.2.2}$$

At the same time, the home planner's objective is

$$\max_{\{C_t^M, C_t^h, M_t, N_t^h\}}\{\sum_{0}^{+\infty}(\beta^h)^t[U^h(C_t^h + C_t^M, N_t^h + M_t)]\}$$

subject to

$$C_t^h + C_t^M \leq Y_t^h + Y_t^M + (1-\delta^h)K_t^h - K_{t+1}^h - sCM_t \tag{A.2.3}$$

After paying for the home share of cost of migration ($sCM_t$), the total output ($Y_t^h + Y_t^M$) the home planner received would be used for consumption of its workers and net investment. A key aspect of eqs.A.2.1 and A.2.3 is the allocation of the aggregate cost of migration ($CM_t$), with the home country bearing share $s$ and the foreign country taking the rest $(1-s)$.

**Proposition A.2.1** *Remittance only occurs if the stationary equilibrium migration is not*

[1] Aiming at maximised output, $\frac{\partial Y_t^i}{\partial N_t^i} > 0, \forall i \in \{h, f\}$ will dominate the planners' decisions on labour supply at all times.

*optimal.*

**Proof**: The home output per worker is

$$y_t^h = \frac{Y_t^h}{L^h - M_t} = Z_t^h[\varphi^h(\frac{K_t^h}{L^h - M_t})^{\mu^h} + (1-\varphi^h)]^{\frac{1}{\mu^h}} \tag{A.2.4}$$

which is a positive function of size of migration. In the stationary equilibrium, household consumption per worker can be shown as an equilibrium portion of output per worker. In short, the equilibrium home consumption per worker remaining at home ($c_t^h = \frac{C_t^h}{L^h - M_t}$) is a positive function of equilibrium migration.

Meanwhile, as a part of stationary-state output per labour in the foreign, the migrant income ($\frac{Y_t^M}{M_t}$) would increase as the increase of migration following the foreign production function (eq.3.1.1), which gives a positive relationship between consumption per migrant and migration. The aggregate home consumption equilibrium occurs only if

$$\frac{C^{M*}}{M^*} = c^{M*} = c^{h*} = \frac{C^{h*}}{L^h - M^*} \tag{A.2.5}$$

as long as the consumption is set as a utility-adding activity. Therefore, the equilibrium with optimal migration would occur when $c_t^M$ is optimised. In the constraint of eq.A.2.2, the consumption per migrant is optimised when the remittance per labour is zero for a given optimal level of individual migrant labour income.[2]

$$\begin{aligned} y_t^M &= c_t^M \\ Y_t^M &= C_t^M \end{aligned} \tag{A.2.6}$$

To study the general equilibrium issue with optimal migration, we shall adjust the remittance equilibrium framework acknowledging eq.A.2.6. This leads to the following changes in the resource constraints of two economies.

$$C_t^f + C_t^M \leq Y_t^f + (1-\delta^f)K_t^f - K_{t+1}^f - (1-s)CM_t \tag{A.2.7}$$
$$C_t^h \leq Y_t^h + (1-\delta^h)K_t^h - K_{t+1}^h - sCM_t \tag{A.2.8}$$

A possible counterargument from eq.A.2.3 is that the remittance $R_t^M$ might benefit stationary aggregate welfare by increasing capital accumulation, not consumption. However, the

[2]Another proof is by observing $\frac{\partial U_t^h}{\partial R_t^M}$. Substituting eqs.A.2.2 and A.2.3 into the instantaneous utility function (eq.2.1.8) gives

$$\frac{\partial U_t^h}{\partial R_t^M} = -(C_t^h + Y_t^M - R_t^M)^{\eta^h} < 0$$

where $\eta^h$ is the home coefficient of constant relative risk aversion.

stationary-state home capital stock (as in A.3.24) is

$$K^{h*} = \{\frac{[\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}{1-\varphi^h}\}^{-\frac{1}{\mu^h}} N^{h*} \tag{A.2.9}$$

which shows that the only determinant which will permanently increase the stationary capital stock is employment, while remittance has no effect.

**Proposition A.2.2** *Remittance cannot increase the stationary state of capital stock, though it might have a positive effect on the accumulation speed of capital.*

Last, we wish to emphasise that a remittance equilibrium can be optimal. However, it has no effect on the optimal size of migration. A possible future direction of study is to analyse the cost and benefit of capital and labour mobility and investigate the optimised trade-off between the two approaches.

## A.3 Stationary state welfare derivation for Section 3.2

The system of the centrally planned economies are listed as

$$CM^* = CM_0 + \chi M^* \tag{A.3.1}$$

$$Y^{h*} = C^{h*} + \delta^h K^{h*} + sCM^* \tag{A.3.2}$$

$$Y^{f*} = C^{f*} + C^{M*} + \delta^f K^{f*} + (1-s)CM^* \tag{A.3.3}$$

$$Y^{f*} = \{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}} \tag{A.3.4}$$

$$Y^{h*} = [\varphi^h (K^{h*})^{\mu^h} + (1-\varphi^h)(L^h - M^*)^{\mu^h}]^{\frac{1}{\mu^h}} \tag{A.3.5}$$

$$\frac{1+\delta^f\beta^f - \beta^f}{\beta^f} = \varphi^f (1-\omega^f)(Y^{f*})^{1-\lambda^f}(K^{f*})^{\mu^f-1}[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}} \tag{A.3.6}$$

$$\frac{1+\delta^h\beta^h - \beta^h}{\beta^h} = \varphi^h (Y^{h*})^{1-\mu^h}(K^{h*})^{\mu^h-1} \tag{A.3.7}$$

$$(N^{f*})^{\frac{1}{\nu^f}}(C^{f*})^{\eta^f} = (1-\varphi^f)(1-\omega^f)(Y^{f*})^{1-\lambda^f}(N^{f*})^{\mu^f-1}[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}} \tag{A.3.8}$$

$$(L^h)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = (1-\varphi^h)(Y^{h*})^{1-\mu^h}(L^h - M^*)^{\mu^h-1} \tag{A.3.9}$$

$$(L^h)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = \omega^f (Y^{f*})^{1-\lambda^f}(M^*)^{\lambda^f-1} - \chi \tag{A.3.10}$$

Substituting eqs.A.3.1, A.3.4 and A.3.5 into other equations in the above system, the system of ten equations reduce to a system of seven equations as follows:

$$\begin{aligned} &C^{f*} + C^{M*} + \delta^f K^{f*} + (1-s)CM_0 + (1-s)\chi M^* \\ &= \{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1}{\lambda^f}} \end{aligned} \tag{A.3.11}$$

$$C^{h*} + \delta^h K^{h*} + sCM_0 + s\chi M^* = [\varphi^h (K^{h*})^{\mu^h} + (1-\varphi^h)(L^h - M^*)^{\mu^h}]^{\frac{1}{\mu^h}} \tag{A.3.12}$$

$$\begin{aligned} \frac{1+\delta^f\beta^f - \beta^f}{\beta^f} &= \varphi^f (1-\omega^f)\{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} \\ &(K^{f*})^{\mu^f-1}[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}} \end{aligned} \tag{A.3.13}$$

$$\frac{1+\delta^h\beta^h - \beta^h}{\beta^h} = \varphi^h [\varphi^h (K^{h*})^{\mu^h} + (1-\varphi^h)(L^h - M^*)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}(K^{h*})^{\mu^h-1} \tag{A.3.14}$$

$$(L^h)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = (1-\varphi^h)[\varphi^h (\frac{K^{h*}}{L^h - M^*})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}} \tag{A.3.15}$$

$$(L^h)^{\frac{1}{\nu^h}}(C^{h*}+C^{M*})^{\eta^h} = \omega^f \{\omega^f + (1-\omega^f)(M^*)^{-\lambda^f}[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} - \chi \tag{A.3.16}$$

$$\begin{aligned} &(N^{f*})^{\frac{1}{\nu^f}}(C^{f*})^{\eta^f} = (1-\varphi^f)(1-\omega^f)\{\omega^f (M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f (K^{f*})^{\mu^f} \\ &+ (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(N^{f*})^{\mu^f-1}[\varphi^f (K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}} \end{aligned} \tag{A.3.17}$$

According to eq.A.3.15, we suggest that the optimised home consumption is a positive function of the home capital-labour ratio $\frac{K^{h*}}{L^h - M^*}$.

$$C^{h*} + C^{M*} = \{\frac{1}{(L^h)^{\frac{1}{\nu^h}}}(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h - M^*})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}\}^{\frac{1}{\eta^h}} \tag{A.3.18}$$

Which gives a positive relationship between $M^*$ and $C^{h*} + C^{M*}$.

However at the same time, eq.A.3.16 gives a negative relationship between $M^*$ and $C^{h*}+C^{M*}$.

$$C^{h*} + C^{M*} = \frac{1}{(L^h)^{\frac{1}{\eta^h \nu^h}}}\{\omega^f\{\omega^f + (1-\omega^f)(M^*)^{-\lambda^f}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}} - \chi\}^{\frac{1}{\eta^h}} \tag{A.3.19}$$

The questions are in which domain each equation dominates and where the turning point is. In short, eq.A.3.18 is drawn from the home labour supply decision, while eq.A.3.19 is derived from the migrant labour decision. A selfish home central planner, in the face of excess endowment of labour supply, would take the turning point at home migrant boundary $\{\overline{M}\}_h$ established in eq.3.1.14 which would give the largest possible domestic output per capita. The maximum is achieved when the home capital stock is fully utilized at its most efficient level, where home capital equilibrium achieves.

Therefore, we have the functions of aggregate consumption of different individuals, which can all be denoted as functions of $M^*$.

At first, the eq.A.3.18 shall be transformed as a function describing the optimised migration consumption as below.

$$C^{M*} = \{\frac{1}{(L^h)^{\frac{1}{\nu^h}}}(1-\varphi^h)[\varphi^h(\frac{K^{h*}}{L^h - M^*})^{\mu^h} + (1-\varphi^h)]^{\frac{1-\mu^h}{\mu^h}}\}^{\frac{1}{\eta^h}} - C^{h*} \tag{A.3.20}$$

Then, we transform eq.A.3.12 to an intermediate product as below

$$C^{h*} = [\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(L^h - M^*)^{\mu^h}]^{\frac{1}{\mu^h}} - \delta^h K^{h*} - sCM_0 - s\chi M^* \tag{A.3.21}$$

At last, the foreign economy consumption can also be denoted by migration, capital and foreign labour force according to eq.A.3.17.

$$
\begin{aligned}
&(N^{f*})^{\frac{1}{\nu^f}}(C^{f*})^{\eta^f} = (1-\varphi^f)(1-\omega^f)\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu^f} \\
&+ (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(N^{f*})^{\mu^f-1}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}
\end{aligned}
$$

$$
\begin{aligned}
&C^{f*} = \{(1-\varphi^f)(1-\omega^f)\{\omega^f(M^*)^{\lambda^f} + (1-\omega^f)[\varphi^f(K^{f*})^{\mu_f} \\
&+ (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}(N^{f*})^{\mu^f-1-\frac{1}{\nu^f}}[\varphi^f(K^{f*})^{\mu^f} + (1-\varphi^f)(N^{f*})^{\mu^f}]^{\frac{\lambda^f-\mu^f}{\mu^f}}\}^{\frac{1}{\eta^f}} \quad \text{(A.3.22)}
\end{aligned}
$$

Inputting the above three equations into the utility functions in **Section 3.2.2**, we show how migrants could affect the indirect utilities of two economies.

Then, we derive both foreign and home capital as functions of labour inputs.

The algebraic relationship between the home capital and the exogenous variables can be disclosed by having a closer look at eq.A.3.14.

$$
\begin{aligned}
\frac{1+\delta^h\beta^h-\beta^h}{\beta^h} =& \varphi^h\{[\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(N^{h*})^{\mu^h}]^{\frac{1}{\mu^h}}\}^{1-\mu^h}(K^{h*})^{\mu^h-1} \\
\{[\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(N^{h*})^{\mu^h}]^{\frac{1}{\mu^h}}\}^{1-\mu^h} =& \frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}(K^{h*})^{1-\mu^h} \\
[\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(N^{h*})^{\mu^h}]^{\frac{1}{\mu^h}} =& [\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{1}{1-\mu^h}}(K^{h*}) \\
\varphi^h(K^{h*})^{\mu^h} + (1-\varphi^h)(N^{h*})^{\mu^h} =& [\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{\mu^h}{1-\mu^h}}(K^{h*})^{\mu^h} \\
(1-\varphi^h)(N^{h*})^{\mu^h} =& [\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{\mu^h}{1-\mu^h}}(K^{h*})^{\mu^h} - \varphi^h(K^{h*})^{\mu^h} \\
(N^{h*})^{\mu^h} =& \frac{[\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}{1-\varphi^h}(K^{h*})^{\mu^h} \quad \text{(A.3.23)}
\end{aligned}
$$

We would only need to have one more step to transfer the eq.A.3.23. The following shows that the home economy capital is a function of the given supply of home labour force.

$$
\begin{aligned}
N^{h*} =& \{\frac{[\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}{1-\varphi^h}\}^{\frac{1}{\mu^h}}K^{h*} \\
K^{h*} =& \{\frac{[\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}]^{\frac{\mu^h}{1-\mu^h}} - \varphi^h}{1-\varphi^h}\}^{-\frac{1}{\mu^h}}N^{h*} \quad \text{(A.3.24)}
\end{aligned}
$$

Then substituting eq.A.3.24 into A.3.15, the sum of the home household consumption (including both home labour and migrant members) is found.

$$C^{h*} + C^{M*} = \{\frac{(1-\varphi^h)[\frac{1-\varphi^h}{(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}\frac{1+\delta^h\beta^h-\beta^h}{\beta^h\varphi^h}}{(N^{h*}+M^*)^{\frac{1}{\nu^h}}}\}^{\frac{1}{\eta^h}} \tag{A.3.25}$$

Meanwhile, the eqs.3.2.3 and 3.2.4 will be shown when we substitute eq.A.3.24 into eqs.A.3.20 and A.3.21.

In the next stage, an attempt is made to denote the foreign economy aggregate capital as a function of exogenous variables. To start, eq.A.3.25 are substituted into eq.A.3.16.

$$K^{f*} = \{\frac{\{\frac{\{\frac{\frac{1-\varphi^h}{\varphi^h}(\frac{1+\delta^h\beta^h-\beta^h}{\beta^h})[\frac{1-\varphi^h}{(\frac{\beta^h\varphi^h}{1+\delta^h\beta^h-\beta^h})^{\frac{\mu^h}{1-\mu^h}}-\varphi^h}]^{\frac{1-\mu^h}{\mu^h}}+\chi}{\omega^f}\}^{\frac{\lambda^f}{1-\lambda^f}}-\omega^f}{1-\omega^f}\}^{\frac{\mu^f}{\lambda^f}}(M^*)^{\mu^f}-(1-\varphi^f)(N^{f*})^{\mu^f}}{\varphi^f}\}^{\frac{1}{\mu^f}} \tag{A.3.26}$$

At last, the eq.3.2.6 is shown after we substitute eq.A.3.24 and A.3.26 into the eq.A.3.22.

## A.4 More exploration on the four outcomes

According to eqs.2.4.31 and 2.4.32, the marginal productivity of migrant and home labour as well as the second derivatives are derived as the following.

$$\frac{\partial Y_t^f}{\partial M_t} = Z_t^f \omega^f (M_t)^{\lambda^f - 1}[\omega^f (M_t)^{\lambda^f} + (1-\omega^f)(\varphi^f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f})^{\frac{\lambda^f}{\mu^f}}]^{\frac{1-\lambda^f}{\lambda^f}} \tag{A.4.1}$$

$$\frac{\partial^2 Y_t^f}{\partial (M_t)^2} = \omega^f (\lambda^f - 1) Z_t^f (M_t)^{\lambda^f - 2}\{\omega^f (M_t)^{\lambda^f} + (1-\omega^f)[\varphi^f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-\lambda^f}{\lambda^f}}$$
$$\frac{(1-\omega^f)[\varphi^f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}}{\omega^f (M_t)^{\lambda^f} + (1-\omega^f)[\varphi^f (K_t^f)^{\mu^f} + (1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}} \tag{A.4.2}$$

$$\frac{\partial Y_t^h}{\partial N_t^h} = (1-\varphi^h) Z_t^h (N_t^h)^{\mu^h - 1}[\varphi^h (K_t^h)^{\mu^h} + (1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}} \tag{A.4.3}$$

$$\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} = (\mu^h - 1)(1-\varphi^h) Z_t^h (N_t^h)^{\mu^h - 2}[\varphi^h (K_t^h)^{\mu^h} + (1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}$$
$$\frac{\varphi^h (K_t^h)^{\mu^h}}{\varphi^h (K_t^h)^{\mu^h} + (1-\varphi^h)(N_t^f)^{\mu^h}} \tag{A.4.4}$$

We have found that with or without fixed cost of migration, the optimal level of migration when the home economy has all bargaining power and bears all cost is

$$M_t = \frac{\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h} - \chi}{-(\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2})} \tag{A.4.5}$$

When the home economy has all bargaining power and pays no share of the cost of migration, the optimal level of migration is always

$$M_t = -\frac{\Upsilon \frac{\partial Y_t^f}{\partial M_t} - \frac{\partial Y_t^h}{\partial N_t^h}}{\frac{\partial^2 Y_t^h}{\partial (N_t^h)^2} + \Upsilon \frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{A.4.6}$$

Substitute eqs.A.4.1 to A.4.4 into eqs.A.4.5 and A.4.6, two solutions can be further expanded as equations of $M_t$.

$$\{\textbf{Case 1,5}\}: (1-\varphi^h)Z_t^h[\varphi^h(K_t^h)^{\mu^h}+(1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}(N_t^h)^{\mu^h-1}+\chi$$
$$=\Upsilon\omega^f Z_t^f\{\omega^f(M_t)^{\lambda^f}+(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-2\lambda^f}{\lambda^f}}(M_t)^{\lambda^f-1}$$
$$\{\omega_f(M_t)^{\lambda^f}+\lambda^f(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}$$
$$-M_t\varphi^h(1-\varphi^h)(1-\mu^h)Z_t^h[\varphi^h(K_t^h)^{\mu^h}+(1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1-2\mu^h}{\mu^h}}(K_t^h)^{\mu^h}(N_t^h)^{\mu^h-2} \tag{A.4.7}$$

$$\{\textbf{Case 2,6,9,10}\}: (1-\varphi^h)Z_t^h[\varphi^h(K_t^h)^{\mu^h}+(1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1-\mu^h}{\mu^h}}(N_t^h)^{\mu^h-1}$$
$$=\Upsilon\omega^f Z_t^f\{\omega^f(M_t)^{\lambda^f}+(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{1-2\lambda^f}{\lambda^f}}(M_t)^{\lambda^f-1}$$
$$\{\omega^f(M_t)^{\lambda^f}+\lambda^f(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}$$
$$-M_t\varphi^h(1-\varphi^h)(1-\mu^h)Z_t^h[\varphi^h(K_t^h)^{\mu^h}+(1-\varphi^h)(N_t^h)^{\mu^h}]^{\frac{1-2\mu^h}{\mu^h}}(K_t^h)^{\mu^h}(N_t^h)^{\mu^h-2} \tag{A.4.8}$$

At the meantime, when the foreign economy has all bargaining power and pays nothing, the optimal level of migration is found to be

$$M_t=-\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{A.4.9}$$

When the foreign economy has all bargaining power and pays for all cost of migration, the optimal level of migration is always

$$M_t=-\frac{(1-\Upsilon)\frac{\partial Y_t^f}{\partial M_t}-\chi}{(1-\Upsilon)\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} \tag{A.4.10}$$

Together with eqs.A.4.1 and A.4.2, we could show two equations of $M_t$:

$$\{\textbf{Case 3,7,11,12}\}: M_t=(\frac{-\lambda^f(1-\omega^f)}{\omega^f})^{\frac{1}{\lambda^f}}[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{1}{\mu^f}} \tag{A.4.11}$$

$$\{\textbf{Case 4,8}\}: (M_t)^{2\lambda^f-1}=$$
$$\frac{-\lambda^f(M_t)^{\lambda^f-1}[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\{\omega^f(M_t)^{\lambda^f}+(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}}{\omega^f}$$
$$+\frac{\chi(Z_t^f)^{-1}\{\omega^f(M_t)^{\lambda^f}+(1-\omega^f)[\varphi^f(K_t^f)^{\mu^f}+(1-\varphi^f)(N_t^f)^{\mu^f}]^{\frac{\lambda^f}{\mu^f}}\}^{\frac{2\lambda^f-1}{\lambda^f}}}{(\omega^f)^2(1-\Upsilon^f)}$$

The four accepted solutions on the optimal level of migration cannot be reduced to demonstrate obvious positive or negative relationships with other major determinants.

**Proposition A.4.1** *Only the foreign economy technology shock would affect the optimal level of migration when the foreign economy possesses the full power in bargaining. In particular, the direct effect of technology shocks would diminish when the foreign economy bears no cost but has full bargaining power.*

It is interesting to note that when the foreign economy has all the bargaining power in determining the size of migration, the optimal level of migration will only become positive (see {**Case 3, 7, 11, 12**}) when $\lambda^f < 0$, which says that the elasticity of substitution between migrant and local labour ($\frac{1}{1-\lambda^f}$) must be positive but less than 1. It is said that the relative demand for migrants will increase by proportionally less than the relative decreases in the migrants' wage.

It is important to note that $\lambda^f \neq 0$ must also be satisfied for {**Case 3,7,11,12**}.

**Proposition A.4.2** *When the foreign economy obtains an absolute bargaining power in the migration bargain, its planner will only accept the gross complement to the foreign economy local labour.*

*Proof*: When $\lambda^f = 0$, the elasticity of substitution between local and migrant ($\frac{1}{1-\lambda^f}$) is 1, meaning that the nested CES production function becomes a Cobb-Douglas production function. When the production function collapses to a three-factor Cobb-Douglas function, eq A.4.9 cannot be solved for $M_t$ as the following

$$M_t = -\frac{\frac{\partial Y_t^f}{\partial M_t}}{\frac{\partial^2 Y_t^f}{\partial (M_t)^2}} = -\frac{\omega Y_t^f (M_t)^{-1}}{(\omega^f - 1) Y_t^f \omega^f (M_t)^{-2}}$$

so that

$$1 = -\frac{1}{1-\omega^f}$$

There is no solution for $M_t$. Such a finding can be interpreted that to maximise its benefit from migration, the foreign economy would not even accept an equiproportionate elasticity of substitution between locals and migrants.

## A.5 McDonald and Solow (1981)'s bargaining under our framework

Following McDonald and Solow (1981)'s bargaining, both households and firms wish to achieve their objectives in this bargain, which are utility maximization and profit maximization respectively. The home and foreign firms are, respectively, maximizing

$$
\begin{aligned}
&Y_t^h - W_t^h N_t^h \\
&Y_t^f - W_t^f N_t^f - W_t^M M_t
\end{aligned}
$$

while the home and foreign households' objectives are bargaining for the highest possible wage to enlarge their own budget constraints

$$
\begin{aligned}
U_t^h &= \frac{(C_t^h + C_t^M)^{1-\eta^h} - 1}{1-\eta^h} - \frac{(N_t^h + M_t)^{1+\frac{1}{\nu^h}}}{1+\frac{1}{\nu^h}} \\
&\text{s.t} \\
C_t^h + C_t^M &= W_t^h N_t^h + W_t^M M_t + (1-\delta^h)K_t^h - K_{t+1}^h - (CM_0 + \chi M_t) \\
U_t^f &= \frac{(C_t^f)^{1-\eta^f} - 1}{1-\eta^f} - \frac{(N_t^f)^{1+\frac{1}{\nu^f}}}{1+\frac{1}{\nu^f}} \\
&\text{s.t} \\
C_t^f &= W_t^f N_t^f + (1-\delta^f)K_t^f - K_{t+1}^f
\end{aligned}
$$

where the households' budget constraint says that consumption equals total income deducts investment.

The **Stage 1** bargaining between home union and home cartel shall be presented as

$$
\max\{(Y_t^h - W_t^h N_t^h)^{b_h}[U_t^h]^{1-b_h}\}, b_h \in (0,1)
$$

with respect to the wage and employment. $\underline{r}^h K^h$ gives constant return of capital under fixed capital stock assumption.

For given $C_t^M, C_t^h$, the wage and employment of home labour are derived as

$$
\{W_t^h\}: \qquad W_t^h N_t^h = -\frac{b_h U_t^h}{(1-b_h)U_C(C_t^h + C_t^M)} + Y_t^h \tag{A.5.1}
$$

$$
\{N_t^h\}: \qquad W_t^h N_t^h = \frac{b_h(\frac{\partial Y_t^h}{\partial N_t^h} - W_t^h)U_t^h}{(1-b_h)[W_t^h U_C(C_t^h + C_t^M) + U_E(N_t^h + M_t)]} + Y_t^h \tag{A.5.2}
$$

where $U_C(C_t^h + C_t^M) = (C_t^h + C_t^M)^{-\eta_h}$ denotes the first derivative of the home households' utility function $U_t^h$ with respect to consumption (either home labour's or migrant's as they are set to be perfectly substitutable in this study) and $U_E(N_t^h + M_t) = -(N_t^h + M_t)^{\frac{1}{\nu_h}}$ is the first derivative of the utility function with respect to employment.

A simple subtraction between above two equations says that the equilibrium employment of home labour is at $-\frac{U_E(N_t^h+M_t)}{U_C(C_t^h+C_t^M)} = W_t^{h,A} = \frac{\partial Y_t^h}{\partial N_t^h}$, while the wage would be determined by the bargaining power of the home cartel ($b_h$) as follows.

$$W_t^h N_t^h = \frac{b_h}{1-b_h} \frac{U_t^h}{U_E(N_t^h + M_t)} \frac{\partial Y_t^h}{\partial N_t^h} + Y_t^h \tag{A.5.3}$$

This is a modification of McDonald and Solow (1981)'s wage solution that gives the equilibrium wage as a (bargaining power) weighted sum of the marginal product and average cost of labour.

Then, on **Stage 2**, the foreign union bargains with the foreign cartel for the wage and employment conditions of the foreign labour force

$$\max\{(Y_t^f - W_t^f N_t^f - W_t^M M_t)^{b_f} [U_t^f]^{1-b_f}\}, b_f \in (0,1)$$

For given $W_t^M M_t$, the derived foreign labour wage and employment are

$$\{W_t^f\}: \qquad W_t^f N_t^f = -\frac{b_f U_t^f}{(1-b_f) U_C(C_t^f)} + Y_t^f - W_t^M M_t \tag{A.5.4}$$

$$\{N_t^f\}: \qquad W_t^f N_t^f = \frac{b_f(\frac{\partial Y_t^f}{\partial N_t^f} - W_t^f) U_t^f}{(1-b_f)[W_t^f U_C(C_t^f) + U_E(N_t^f)]} + Y_t^f - W_t^M M_t \tag{A.5.5}$$

Similarly, the two equations result in the foreign equilibrium employment at $\frac{\partial Y_t^f}{\partial N_t^f} = -\frac{U_E(N_t^f)}{U_C(C_t^f)}$ $= W_t^{f,R}$, while the wage of foreign labour is determined by the following condition.

$$W_t^f N_t^f = \frac{b_f}{1-b_f} \frac{U_t^f}{U_E(N_t^f)} \frac{\partial Y_t^f}{\partial N_t^f} + Y_t^f - W_t^M M_t \tag{A.5.6}$$

Finally, the **Stage 3** is a bargain between foreign firm cartel and home labour union.

$$\max\{(Y_t^f - W_t^f N_t^f - W_t^M M_t)^{b_M} [U_t^h]^{1-b_M}\}, b_M \in (0,1)$$

The first order conditions with respect to migrant wage and migration are, respectively

$$\{W_t^M\}: \qquad W_t^M M_t = -\frac{b_M U_t^h}{(1-b_M) U_C(C_t^h + C_t^M)} + Y_t^f - W_t^f N_t^f \tag{A.5.7}$$

$$\{M_t\}: \quad W_t^M M_t = \frac{b_M(\frac{\partial Y_t^f}{\partial M_t} - W_t^M)U_t^h}{(1-b_M)[(W_t^M - \chi)U_C(C_t^h + C_t^M) + U_E(N_t^h + M_t)]} + Y_t^f - W_t^f N_t^f \tag{A.5.8}$$

The resulting migrant employment is at $\frac{\partial Y_t^f}{\partial M_t} = \chi + W_t^{h,A}$, and substituting this into eq.A.5.7 yields the migrant wage that relies on $b_M$ and $b_f$.[3]

[3] $W_t^f$ in eq.A.5.7 is determined in the second stage bargain according to eq.A.5.4.

## A.6 Log-linearised system of equations

### A.6.1 Fixed-capital DSGE framework

As a start, steady-state variables will be shown as variables without $t$ and variables shown in $\tilde{x}_t$ are $\frac{x_t - x}{x}$, the percentage deviation of variable $x_t$ at time $t$ from its steady state.

The home households' summed consumption is

$$\tilde{C_t^H} C^H = \tilde{C_t^h} C^h + \tilde{C_t^M} C^M \tag{A.6.1}$$

Then we also linearise the utility functions of two households with the budget constraint for their consumptions.

$$\tilde{U_t^h} = \frac{(C^h + C^M)^{-\eta^h}}{U^h}(C^h \tilde{C_t^h} + C^M \tilde{C_t^M}) - \psi \frac{(N^h + M)^{\frac{1}{\nu^h}}}{U^h}(N^h \tilde{N_t^h} + M\tilde{M}_t) \tag{A.6.2}$$

$$C^M \tilde{C_t^M} + C^h \tilde{C_t^h} + \chi M \tilde{M}_t = W^M M(\tilde{W_t^M} + \tilde{M}_t) + W^h N^h (\tilde{W_t^h} + \tilde{N_t^h}) \tag{A.6.3}$$

$$\tilde{U_t^f} = \frac{(C^f)^{1-\eta^f}}{U^f}\tilde{C_t^f} - \psi \frac{(N^f)^{1+\frac{1}{\nu^f}}}{U^f}\tilde{N_t^f} \tag{A.6.4}$$

$$\tilde{C_t^f} = \tilde{W_t^f} + \tilde{N_t^f} \tag{A.6.5}$$

in which the capital terms are all disappeared due to the set-up of non-changing capital stock and hitherto no deviation of capital variables.

The labour market conditions from eqs.5.1.12 and 5.1.17 are linearised as

$$U\tilde{N_t^h} = -\frac{M\tilde{M}_t + N^h \tilde{N_t^h}}{UN^h} \tag{A.6.6}$$

$$U\tilde{N_t^f} = -\frac{N^f}{UN^f}\tilde{N_t^f} \tag{A.6.7}$$

The equation of the migration cost can be linearised as

$$C\tilde{M}_t CM = CM_0 \tilde{Z_t^{fcm}} + \chi M(\tilde{Z_t^{vcm}} + \tilde{M}_t) \tag{A.6.8}$$

The fixed cost ($CM_0$) is constant over time, it can be increased by an increase in the $\tilde{Z_t^{fcm}}$. $\chi$ is a variable cost of migration and it can be increased while there is an unexpected increase in $\tilde{Z_t^{vcm}}$.

As the cost shocks are already in log forms, we only need to rephrase there here.

$$\tilde{Z_t^{fcm}} = \rho^{fcm} \tilde{Z_{t-1}^{fcm}} + \zeta_t^{fcm} \tag{A.6.9}$$

$$\tilde{Z_t^{vcm}} = \rho^{vcm} \tilde{Z_{t-1}^{fcm}} + \zeta_t^{vcm} \tag{A.6.10}$$

On the production side, we have linearised the production functions with the technology functions.

$$\tilde{Y_t^h} = \tilde{Z_t^h} + (1-\varphi^h)(\frac{N^h}{Y^h})^{\mu^h} \tilde{N_t^h} \tag{A.6.11}$$

$$\tilde{Z_t^h} = \rho^h \tilde{Z_{t-1}^h} + \zeta_h \tag{A.6.12}$$

$$\tilde{Y_t^f} = \tilde{Z_t^f} + \omega^f (\frac{M}{Y^f})^{\lambda^f} \tilde{M}_t + (1-\omega^f) \frac{[\varphi^f (K^f)^{\mu^f} + (1-\varphi^f)(N^f)^{\mu^f}]^{(\lambda^f - \mu^f)/\mu^f}}{(Y^f)^{\lambda^f}} (1-\varphi^f)(N^f)^{\mu^f} \tilde{N_t^f} \tag{A.6.13}$$

$$\tilde{Z_t^f} = \rho^f \tilde{Z_{t-1}^f} + \zeta_f \tag{A.6.14}$$

As the technology equations are already set into log forms, we only need to re-write it.

The three-stage bargaining outcomes of home labour, migrant and foreign labour wages are

$$\begin{aligned}
&(\tilde{W_t^M} + \tilde{M_t})\{(1-b_M)b_f(Y^f - W^{f,R}N^f - r^f K^f) \\
&+ (1-b_h)b_M[W^{h,A}(N^h + M) + CM - Y^h + r^h K^h]\} = (1-b_M)b_f(Y^f \tilde{Y_t^f} - W^{f,R}N^f \tilde{N_t^f} - \tilde{r_t^f} r^f K^f) \\
&+ (1-b_h)b_M[W^{h,A}(N^h \tilde{N_t^h} + M\tilde{M_t}) + \tilde{CM_t}CM - Y^h + \tilde{r_t^h} r^h K^h]
\end{aligned} \tag{A.6.15}$$

$$\begin{aligned}
&(\tilde{W_t^h} + \tilde{N_t^h})\{-b_h b_f(1-b_M)[Y^f - r_t^f K^f - W^{f,R}N^f - W^{h,A}(N^h + M) - CM] \\
&+ (1-b_h)[(1-b_M)b_f + b_M](Y^h - r^h K^h)\} \\
&= -b_h b_f(1-b_M)[Y^f \tilde{Y_t^f} - \tilde{r_t^f} r^f K^f - W^{f,R}N^f \tilde{N_t^f} - W^{h,A}(N^h \tilde{N_t^h} + M\tilde{M_t}) - \tilde{CM_t}CM] \\
&+ (1-b_h)[(1-b_M)b_f + b_M](Y^h \tilde{Y_t^h} - \tilde{r_t^h} r^h K^h)
\end{aligned} \tag{A.6.16}$$

$$\begin{aligned}
&(\tilde{W_t^f} + \tilde{N_t^f})\{(1-b_M b_h)b_f W^{f,R}N^f \\
&+ (1-b_f)(1-b_h)b_M[Y^f + Y^h - r^f K^f - r^h K^h - W^{h,A}(N^h + M) - CM]\} \\
&= (1-b_M b_h)b_f W^{f,R}N^f \tilde{N_t^f} \\
&+ (1-b_f)(1-b_h)b_M[Y^f \tilde{Y_t^f} + Y^h \tilde{Y_t^h} - \tilde{r_t^f} r^f K^f - \tilde{r_t^h} r^h K^h - W^{h,A}(N^h \tilde{N_t^h} + M\tilde{M_t}) - \tilde{CM_t}CM]
\end{aligned} \tag{A.6.17}$$

It is important to specify that the constant capital returns are shown in the denominators due to their essential roles in the steady state.

The optimal wage conditions extracted from the Euler conditions in eqs.5.1.11 and 5.1.16

can be presented as

$$W^M \tilde{W_t^M} = W^h \tilde{W_t^h} \tag{A.6.18}$$

$$\tilde{W_t^f} - \eta_f \tilde{C_t^f} = \frac{1}{\nu_f} \tilde{N_t^f} \tag{A.6.19}$$

At last, the market clearing conditions for two economies are

$$\tilde{Y_t^h} = \frac{C^h}{Y^h} \tilde{C_t^h} + \frac{\chi M}{Y^h} \tilde{M_t} \tag{A.6.20}$$

$$\tilde{Y_t^f} = \frac{C^f}{Y^f} \tilde{C_t^f} + \frac{C^M}{Y^f} \tilde{C_t^M} \tag{A.6.21}$$

### A.6.2 Capital-adjusted DSGE log-linearization

Here we consider our DSGE model with the capital accumulation and adjustment. The labour market conditions (eqs.A.6.6-A.6.7) remain, as well as equations specifying the cost of migration (eq.A.6.8), utility functions of the two households (eqs.A.6.2 and A.6.4), technology shocks (eqs.A.6.12 and A.6.14) and the Euler optimal wage conditions (eqs.A.6.18-A.6.19).

We start by modifying the budget constraints of the two households. Including the home and foreign capital stock gives

$$\begin{aligned} &C^M \tilde{C_t^M} + C^h \tilde{C_t^h} + K^h(\tilde{K_{t+1}^h} - \tilde{K_t^h}) + \delta^h K^h \tilde{K_t^h} + \chi M \tilde{M_t} \\ &= W^M M(\tilde{W_t^M} + \tilde{M_t}) + W^h N^h (\tilde{W_t^h} + \tilde{N_t^h}) + r^h K^h (\tilde{r_t^h} + \tilde{K_t^h}) \end{aligned} \tag{A.6.22}$$

$$C^f \tilde{C_t^f} + K^f[\tilde{K_{t+1}^f} - (1-\delta^f)\tilde{K_t^f}] = W^f N^f (\tilde{W_t^f} + \tilde{N_t^f}) + r^f K^f (\tilde{r_t^f} + \tilde{K_t^f}) \tag{A.6.23}$$

When capital stocks are included, the households have to make prudential decisions between investment and consumption.

The production functions for the two economies are

$$\begin{aligned} \tilde{Y_t^f} =& \tilde{Z_t^f} + \frac{\omega^f (M)^{\lambda^f}}{(Y^f)^{\lambda^f}} \tilde{M_t} \\ &+ \frac{(1-\omega^f)[\varphi^f (K^f)^{\mu^f} + (1-\varphi^f)(N^f)^{\mu^f}]}{(Y^f)^{\lambda^f}} [\varphi^f (K^f)^{\mu^f} \tilde{K_t^f} + (1-\varphi^f)(N^f)^{\mu^f} \tilde{N_t^f}] \end{aligned} \tag{A.6.24}$$

$$\tilde{Y_t^h} = \tilde{Z_t^h} + \frac{1}{(Y^h)^{\mu^h}} [\varphi^h (K^h)^{\mu^h} \tilde{K_t^h} + (1-\varphi^h)(N^h)^{\mu^h} \tilde{N_t^h}] \tag{A.6.25}$$

Note that local complementarity in production requires $\lambda^f > \mu^f$.

Then we linearised the rate of return of two capital stocks.

$$
\begin{aligned}
\tilde{r_t^h} =& \tilde{Z_t^h} + (\mu^h - 1)\tilde{K_t^h} + \frac{1-\mu^h}{(Y^h)^{\mu^h}}[\varphi^h (K^h)^{\mu^h} \tilde{K_t^h} + (1-\varphi^h)(N^h)^{\mu^h} \tilde{N_t^h}] && \text{(A.6.26)} \\
\tilde{r_t^f} =& \tilde{Z_t^f} + (\mu^f - 1)\tilde{K_t^f} + (\lambda^f - \mu^f)\frac{\varphi^f (K^f)^{\mu^f} \tilde{K_t^f} + (1-\varphi^f)(N^f)^{\mu^f} \tilde{N_t^f}}{\varphi^f (K^f)^{\mu^f} + (1-\varphi^f)(N^f)^{\mu^f}} \\
& + \frac{1-\lambda^f}{(Y^f)^{\lambda^f}}\{\omega^f (M)^{\lambda^f} \tilde{M}_t + (1-\omega^f)[\varphi^f (K^f)^{\mu^f} + (1-\varphi^f)(N^f)^{\mu^f}]^{\frac{\lambda^f - \mu^f}{\mu^f}} \\
& [\varphi^f (K^f)^{\mu^f} \tilde{K_t^f} + (1-\varphi^f)(N^f)^{\mu^f} \tilde{N_t^f}]\} && \text{(A.6.27)}
\end{aligned}
$$

The optimal capital investment plans drawn from Euler conditions are

$$
\begin{aligned}
\frac{r^h \tilde{r_t^h}}{1 + r^h - \delta^h} - \eta^h \frac{C^M \tilde{C_{t+1}^M} + C^h \tilde{C_{t+1}^h}}{C^M + C^h} =& - \eta^h \frac{C^M \tilde{C_t^M} + C^h \tilde{C_t^h}}{C^M + C^h} && \text{(A.6.28)} \\
\frac{r^f \tilde{r_t^f}}{1 + r^f - \delta^f} - \eta^f \tilde{C_{t+1}^f} =& - \eta^f \tilde{C_t^f} && \text{(A.6.29)}
\end{aligned}
$$

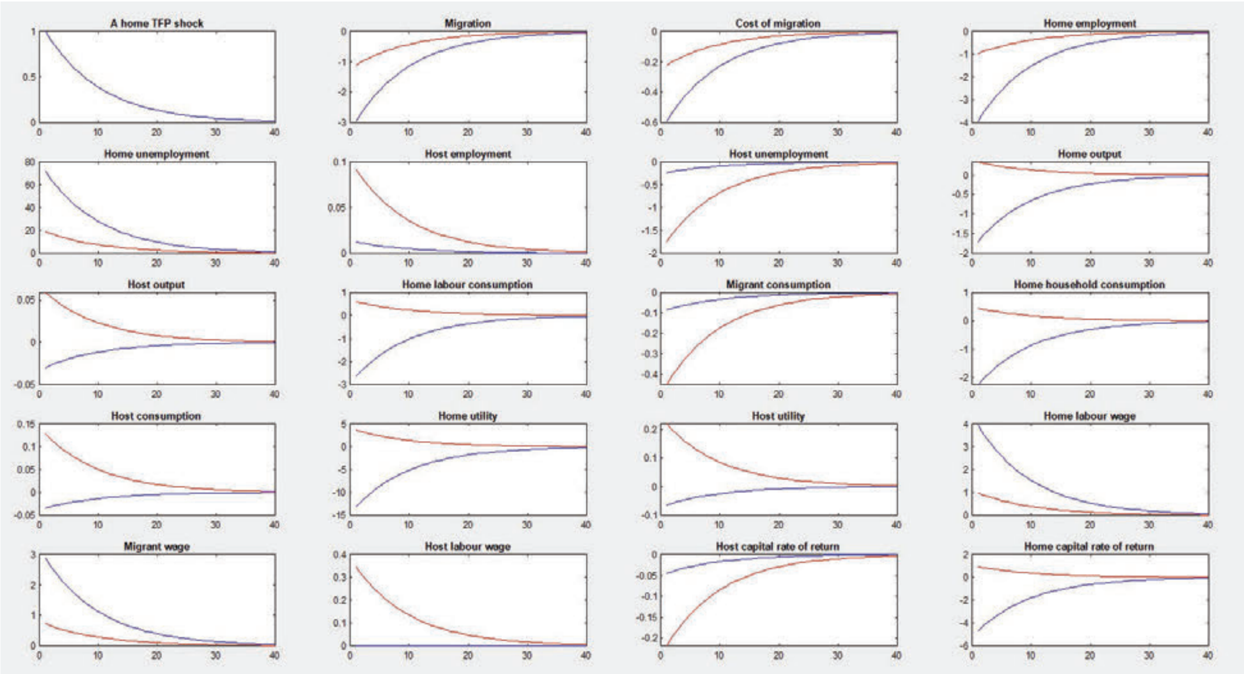
Finally, the new market clearing functions are

$$
\begin{aligned}
Y^f \tilde{Y_t^f} =& C^f \tilde{C_t^f} + C^M \tilde{C_t^M} + K^f \tilde{K_{t+1}^f} - (1-\delta^f) K^f \tilde{K_t^f} && \text{(A.6.30)} \\
Y^h \tilde{Y_t^h} =& C^h \tilde{C_t^h} + \chi M \tilde{M}_t + K^h \tilde{K_{t+1}^h} - (1-\delta^h) K^h \tilde{K_t^h} && \text{(A.6.31)}
\end{aligned}
$$

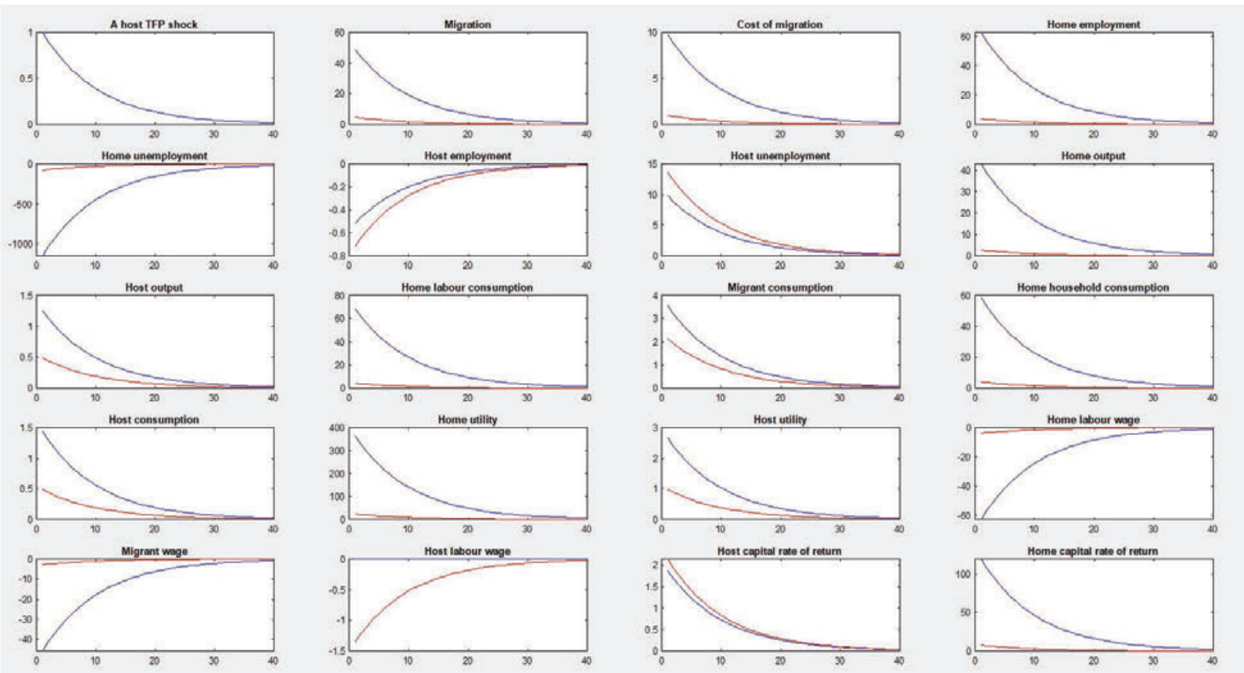
## A.7 All possible relative bargaining power structures

### A.7.1 $b_f = 0.9999$; $b_M = b_h = 0.0001$

Figure A.1: Responses to the TFP shocks




(a) A home TFP shock

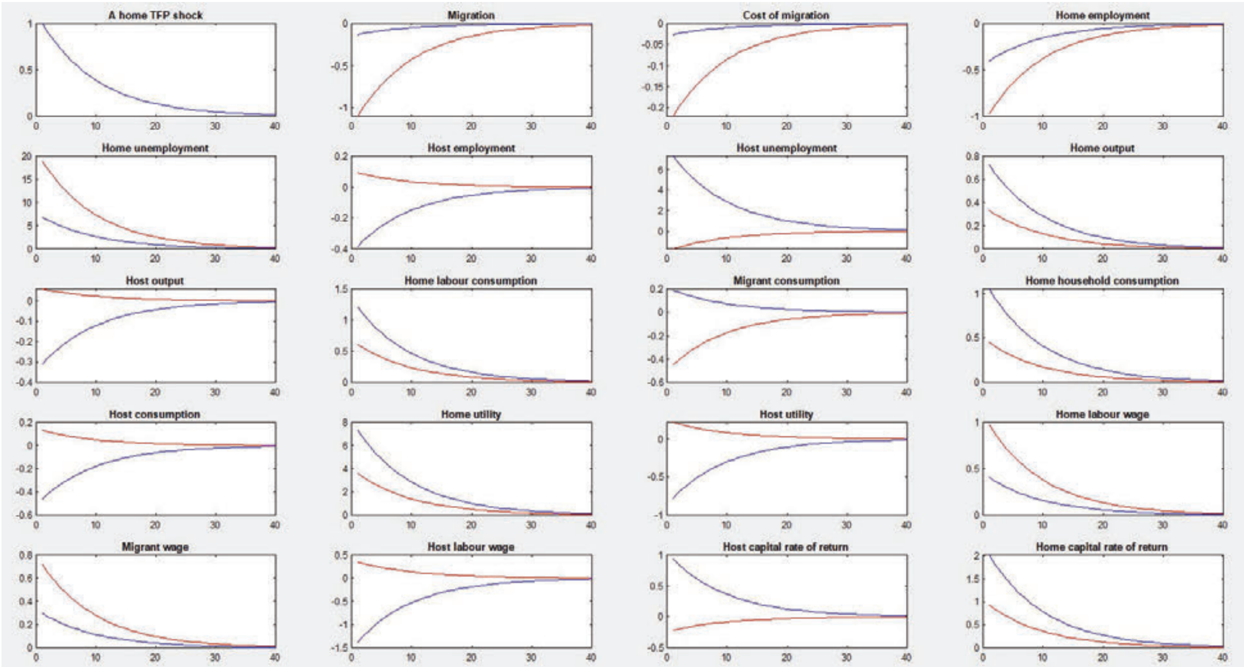



(b) A foreign TFP shock

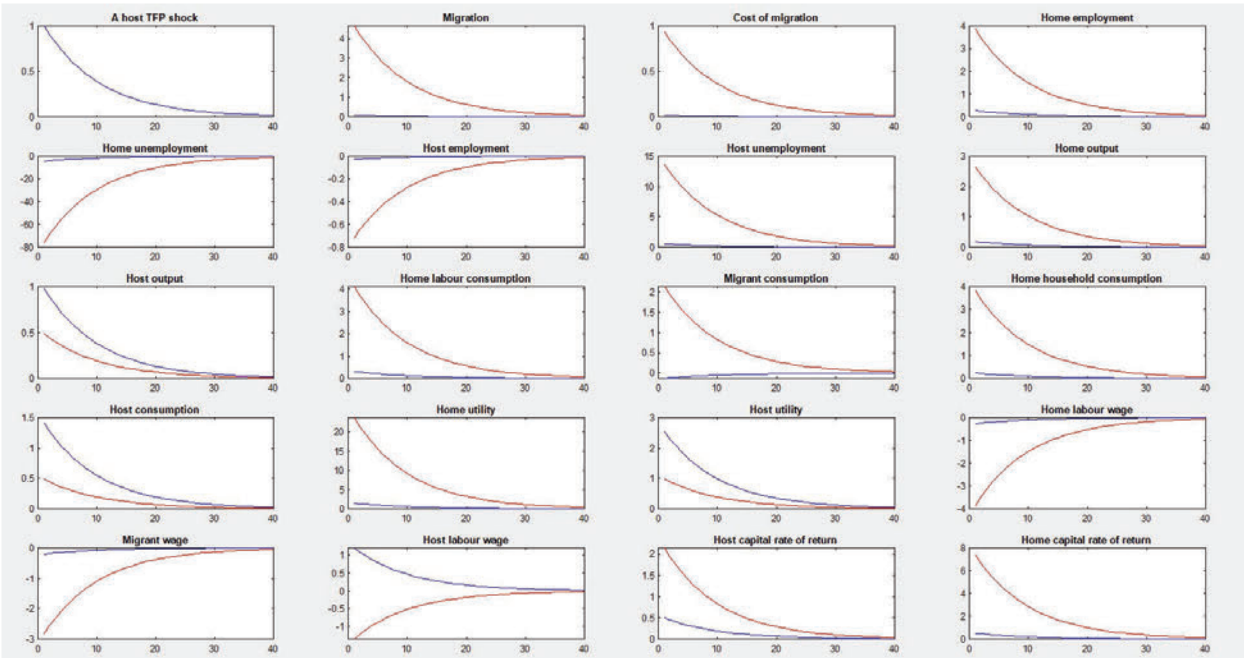
NB: The responses of the monopolistic labour markets are in red lines, while the responses of the foreign-monopsonistic labour markets are in blue lines.

### A.7.2 $b_M = 0.9999$; $b_f = b_h = 0.0001$

Figure A.2: Responses to the TFP shocks




(a) Responses to a positive home TFP shock

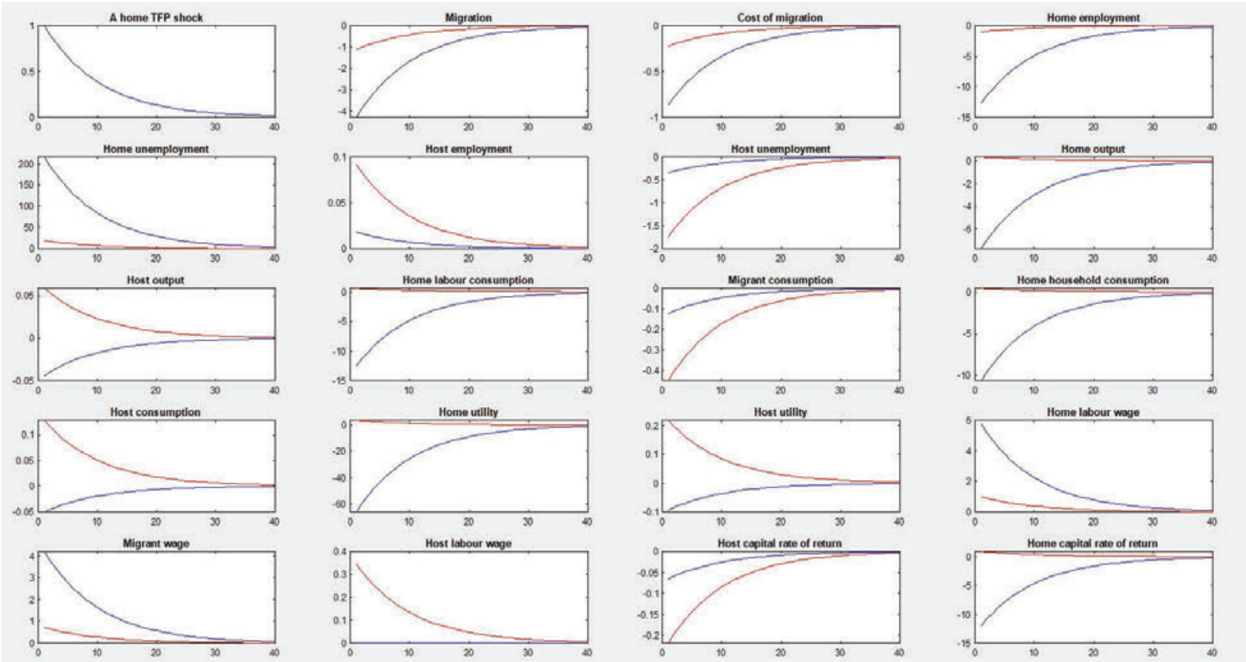



(b) Responses to a positive foreign TFP shock

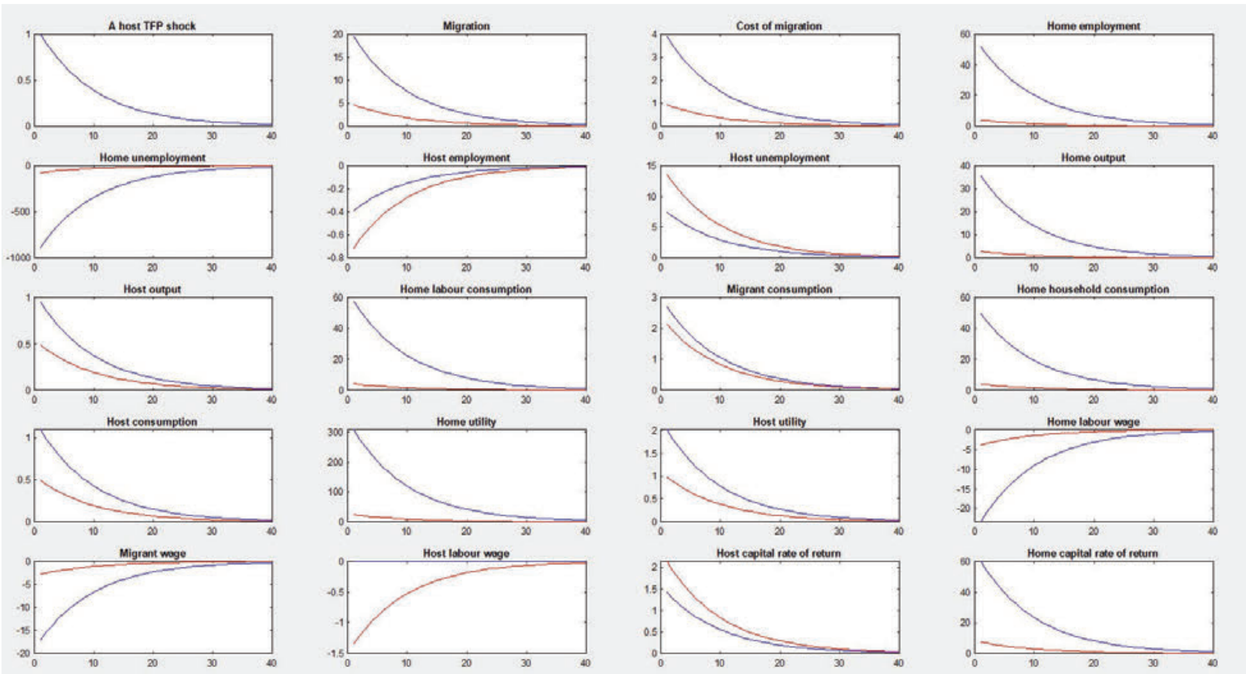
NB: The responses of the benchmark model are in red lines, while the responses of foreign-migrant monopsonistic labour markets are in blue lines.

### A.7.3 $b_h = 0.9999$; $b_f = b_M = 0.0001$

Figure A.3: Responses to the TFP shocks




(a) Responses to a positive home TFP shock

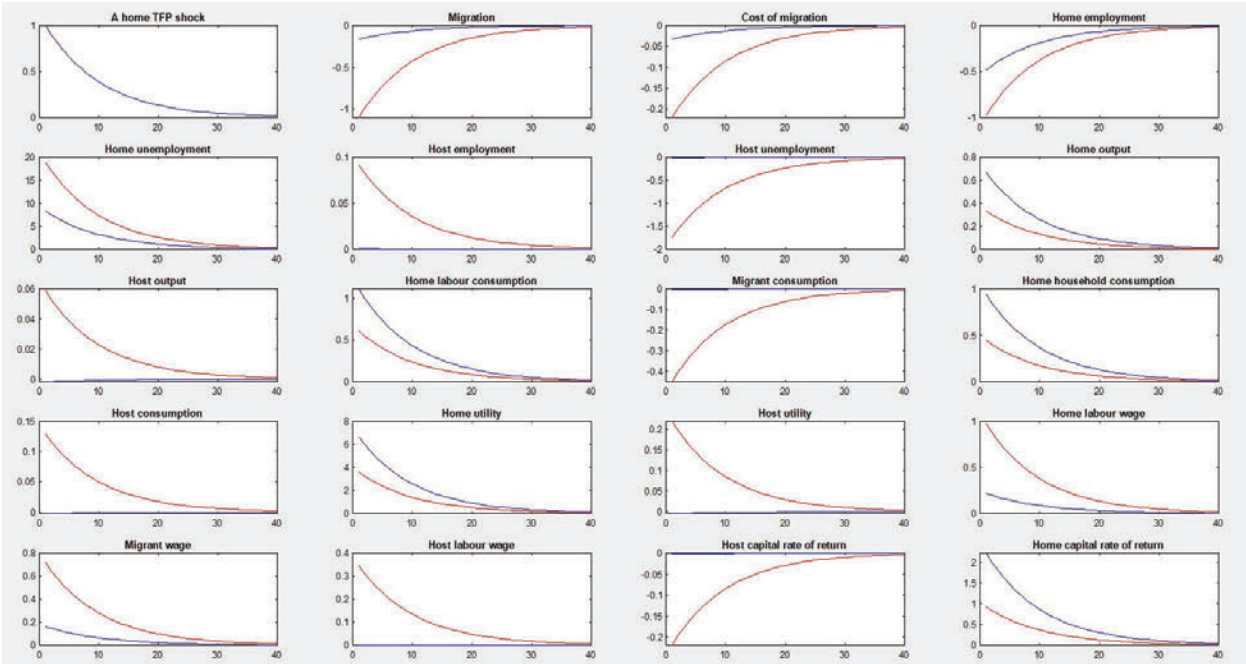



(b) Responses to a positive foreign TFP shock

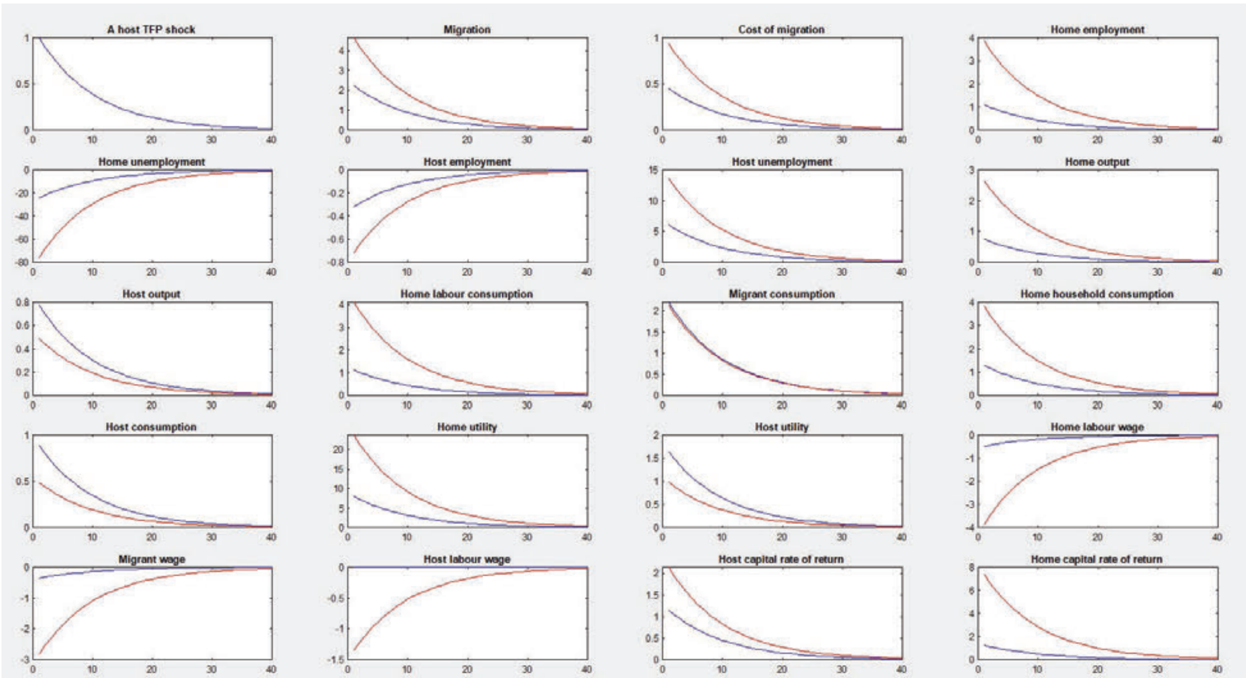
NB: The responses of the benchmark model are in red lines, while the responses of home-monopsonistic labour markets are in blue lines.

### A.7.4 $b_M = 0.0001$; $b_f = b_h = 0.9999$

Figure A.4: Responses to the TFP shocks




(a) Responses to a positive home TFP shock

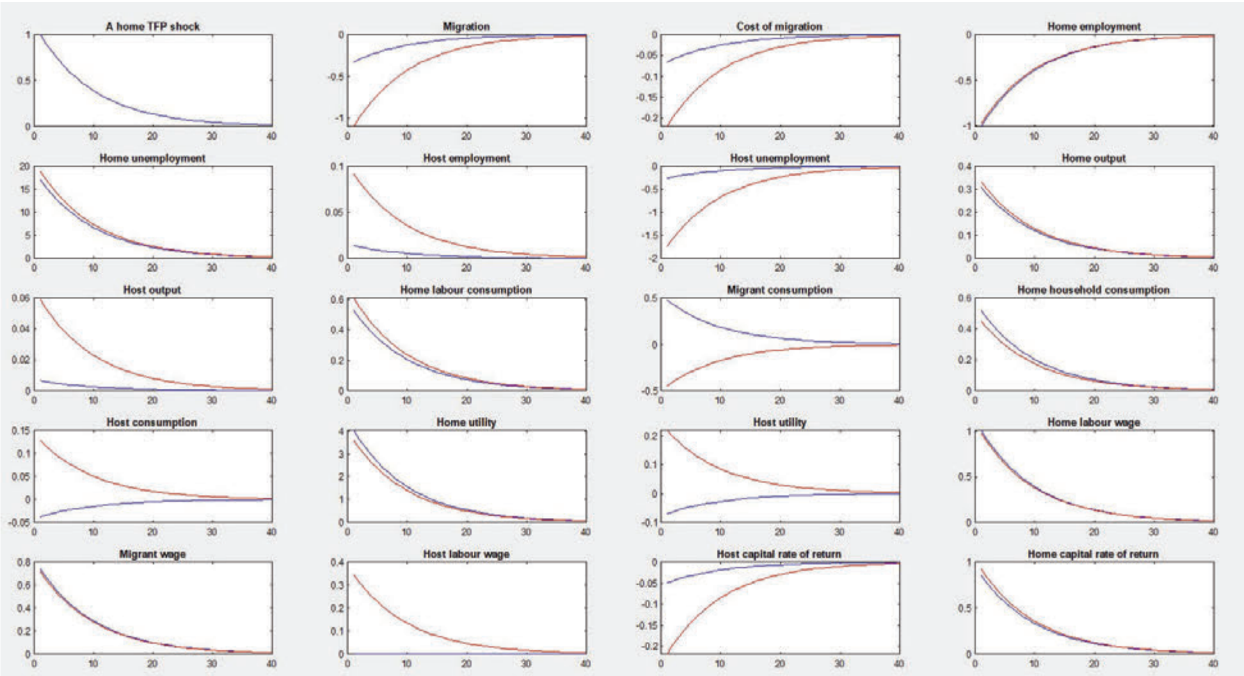



(b) Responses to a positive foreign TFP shock

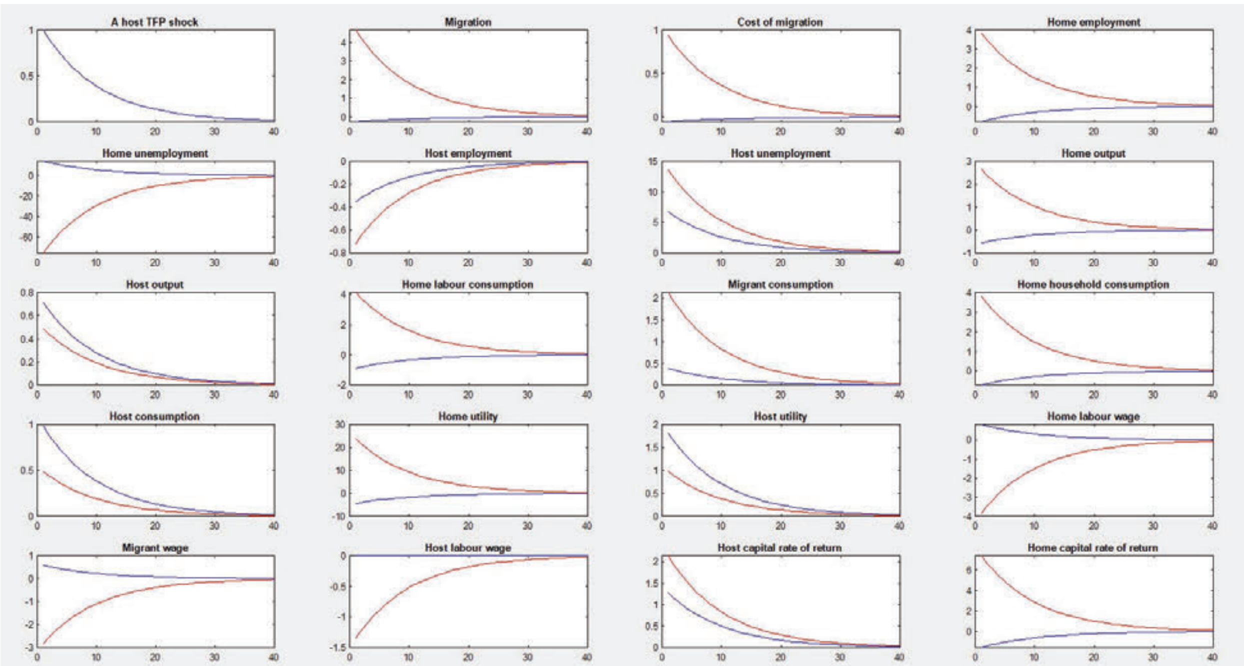
NB: The responses of the benchmark model are in red lines, while the responses of locals-monopsonistic labour markets are in blue lines.

### A.7.5 $b_h = 0.0001$; $b_f = b_M = 0.9999$

Figure A.5: Responses to the TFP shocks




(a) Responses to a positive home TFP shock

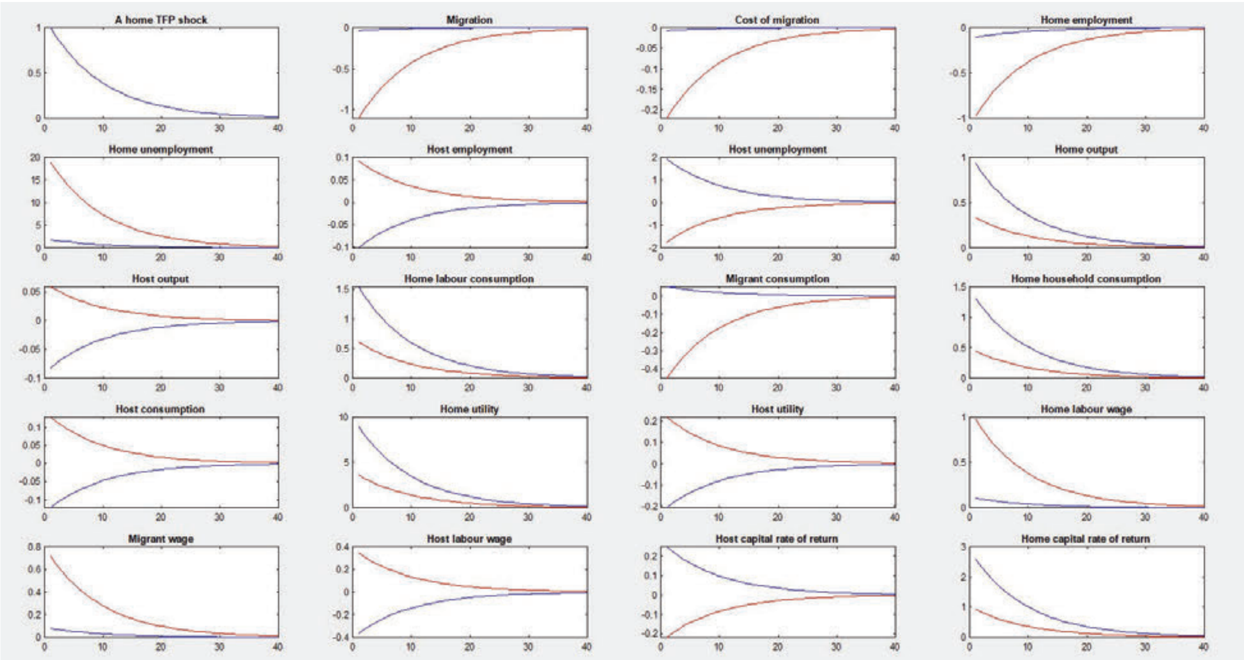



(b) Responses to a positive foreign TFP shock

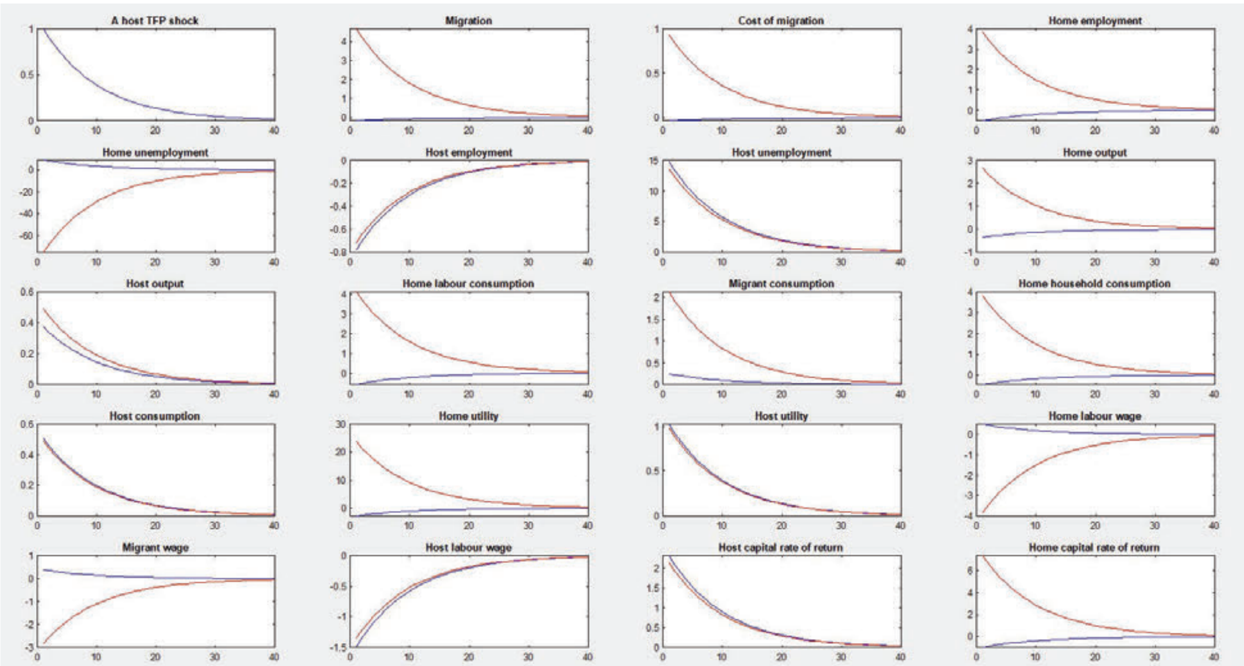
NB: The responses of the benchmark model are in red lines, while the responses of foreign-monopsonistic labour markets are in blue lines.

### A.7.6 $b_f = 0.0001$; $b_h = b_M = 0.9999$

Figure A.6: Responses to the TFP shocks




(a) Responses to a positive home TFP shock



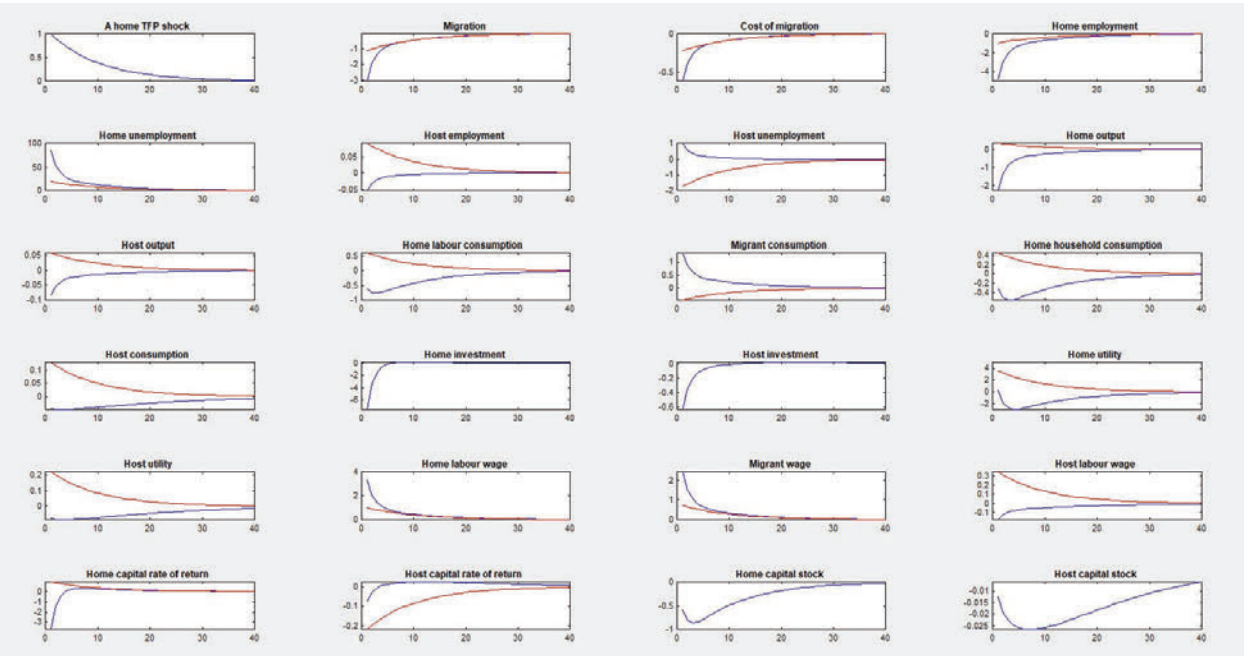

(b) Responses to a positive foreign TFP shock

NB: The responses of the benchmark model are in red lines, while the responses of powerless-home-union labour markets are in blue lines.

## A.8 Impulse responses of the global economy with endogenous capital

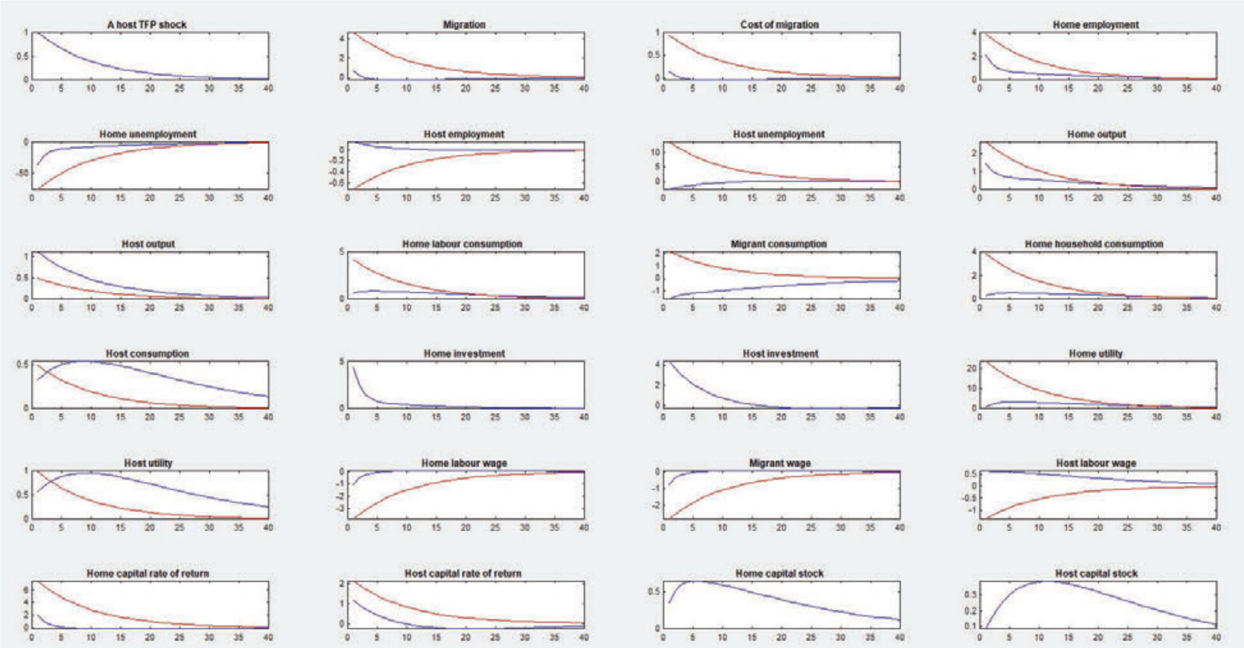
### A.8.1 $b_h = b_M = b_f = 0.0001$

Figure A.7: Responses to a positive home TFP shock



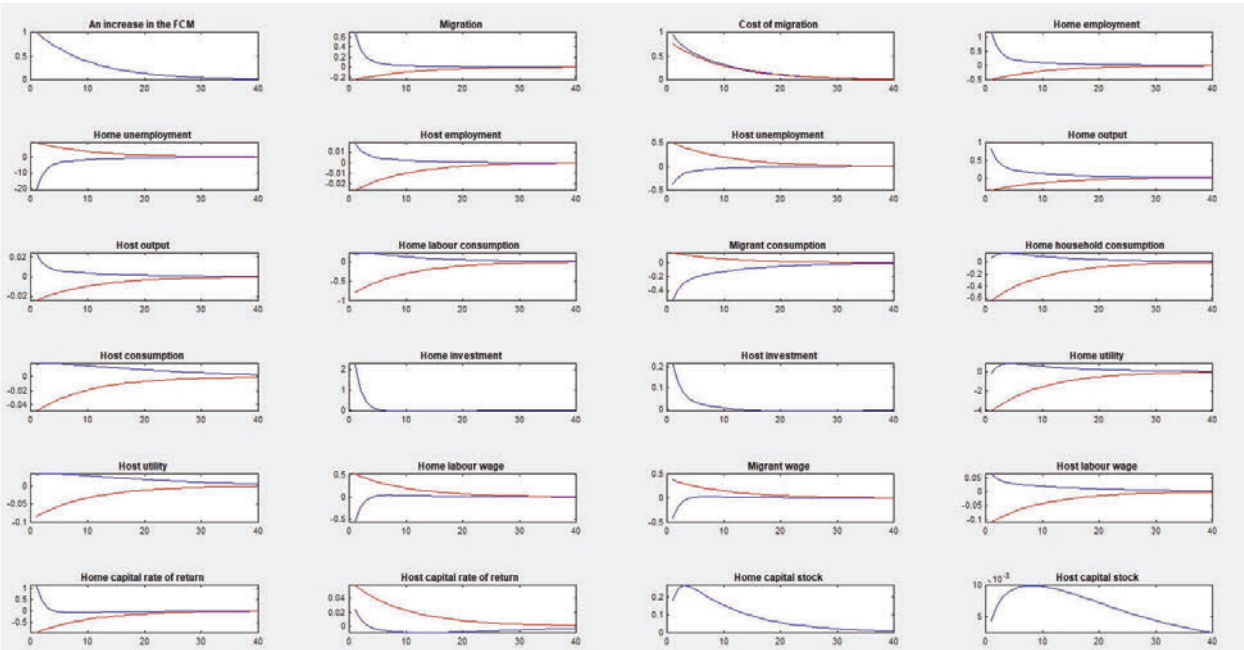

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue lines.

Figure A.8: Responses to a positive foreign TFP shock



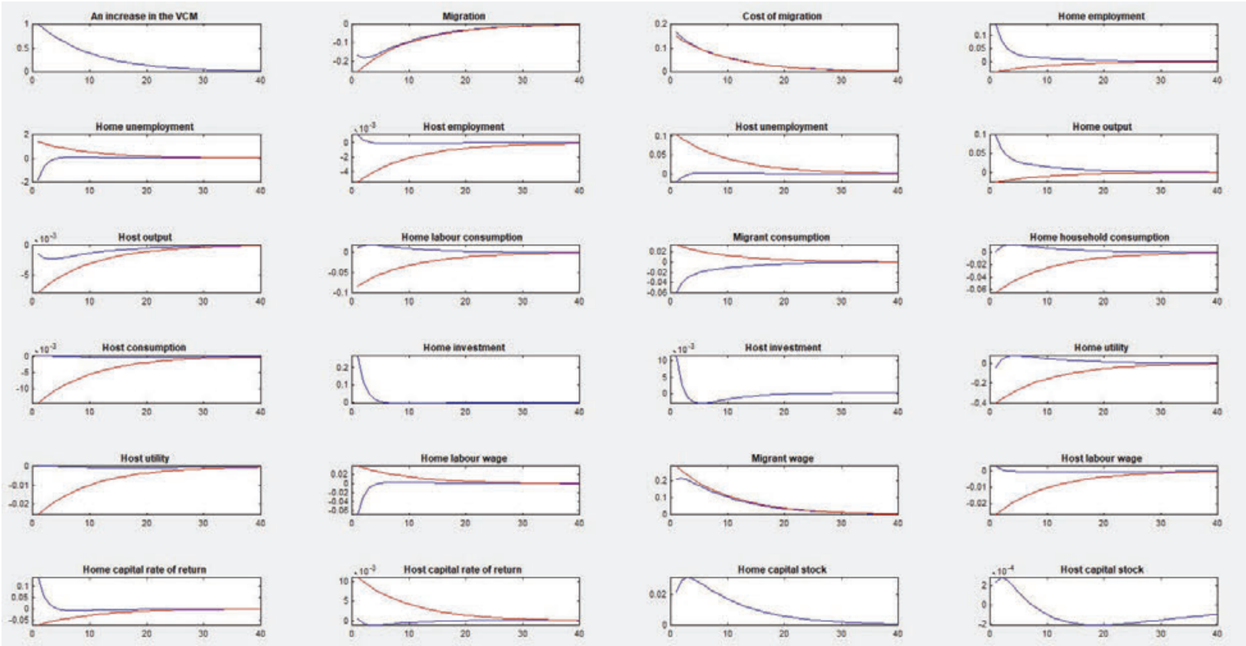

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue lines.

Figure A.9: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue lines.

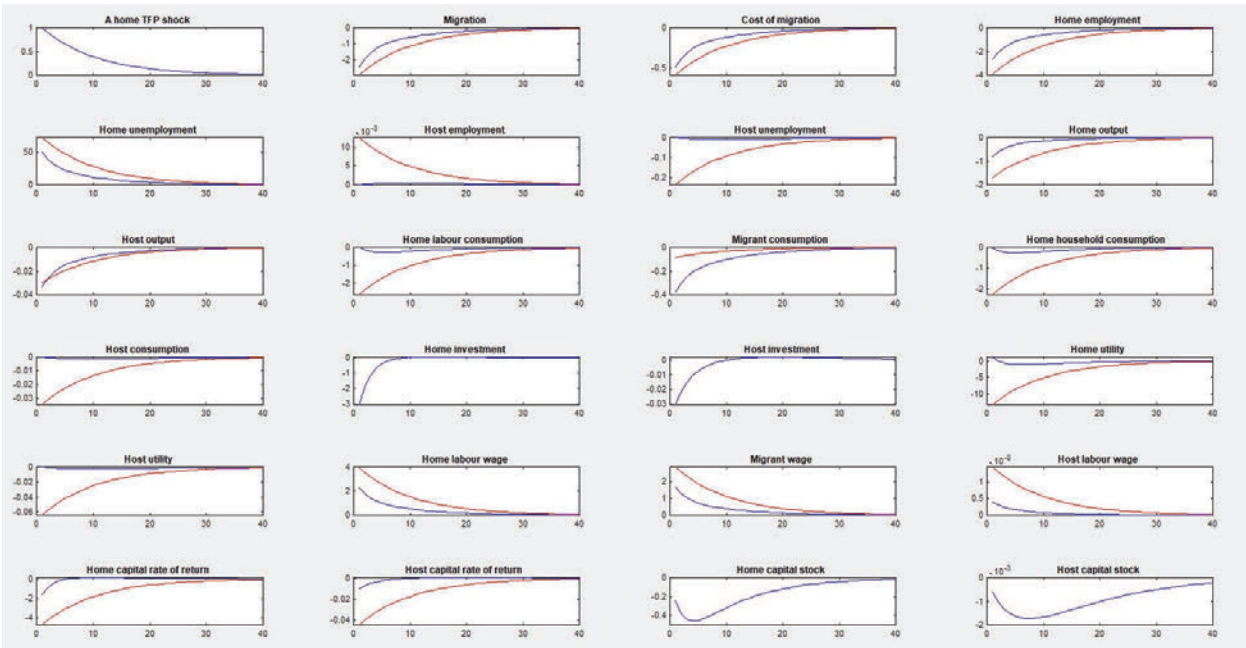
Figure A.10: An increase in the variable cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue lines.

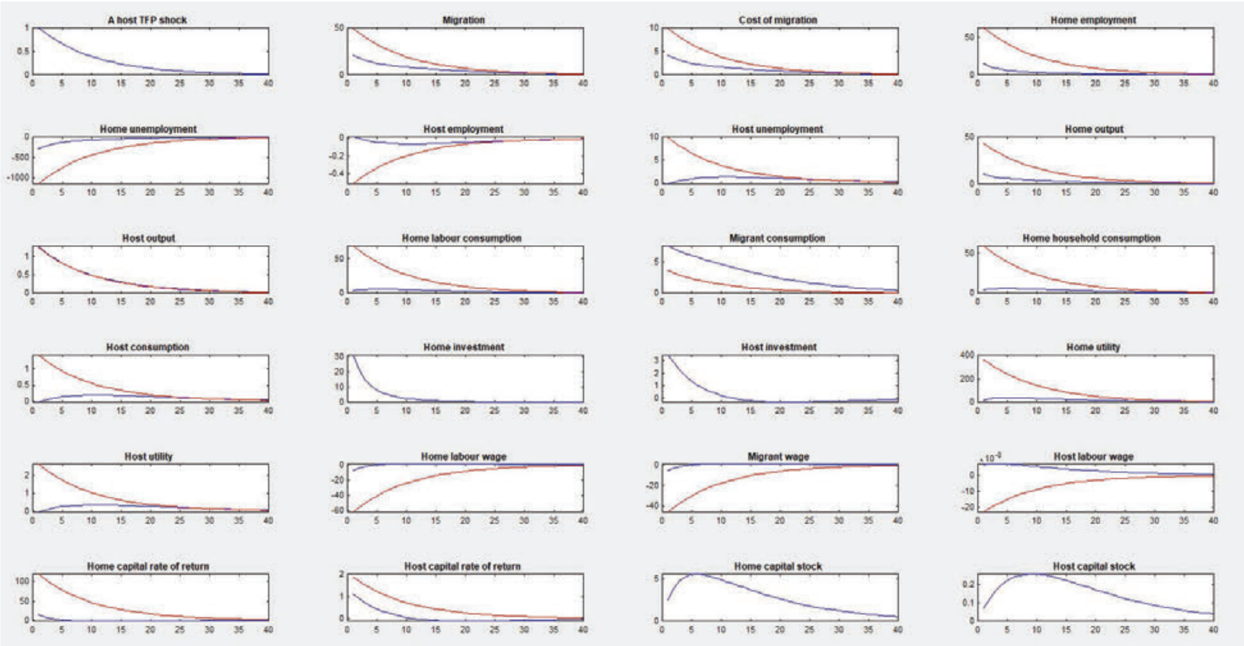
### A.8.2 $b_f = 0.9999$; $b_h = b_M = 0.0001$

Figure A.11: Responses to a positive home TFP shock



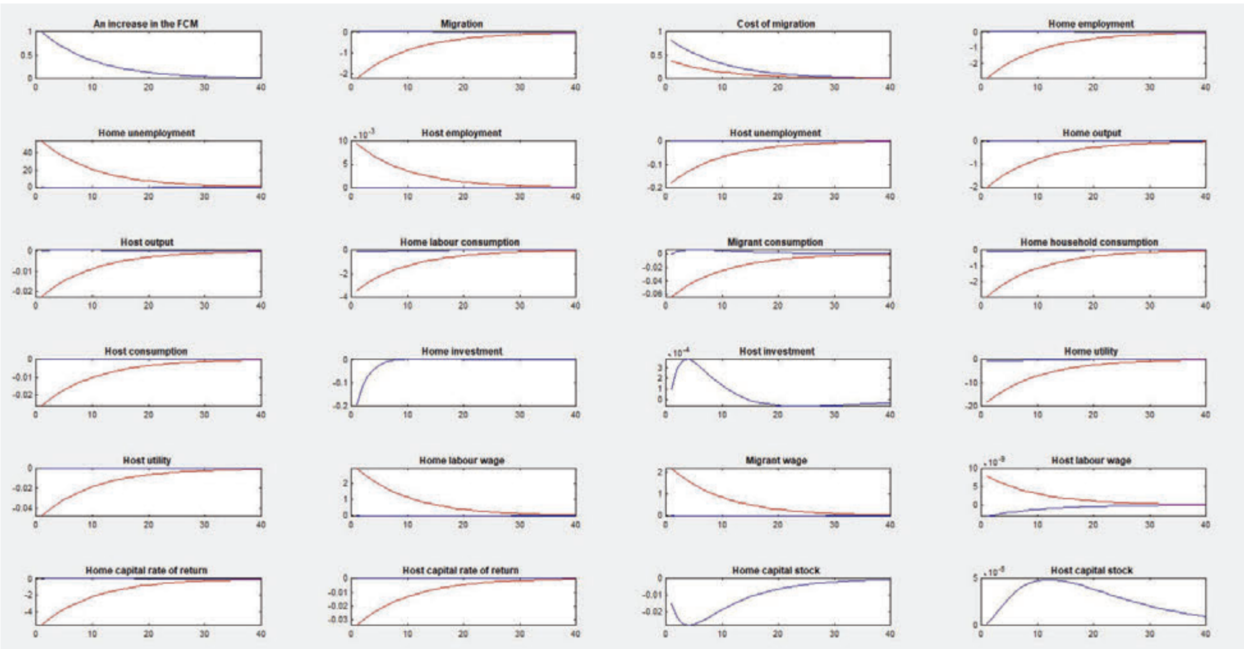

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.12: Responses to a positive foreign TFP shock



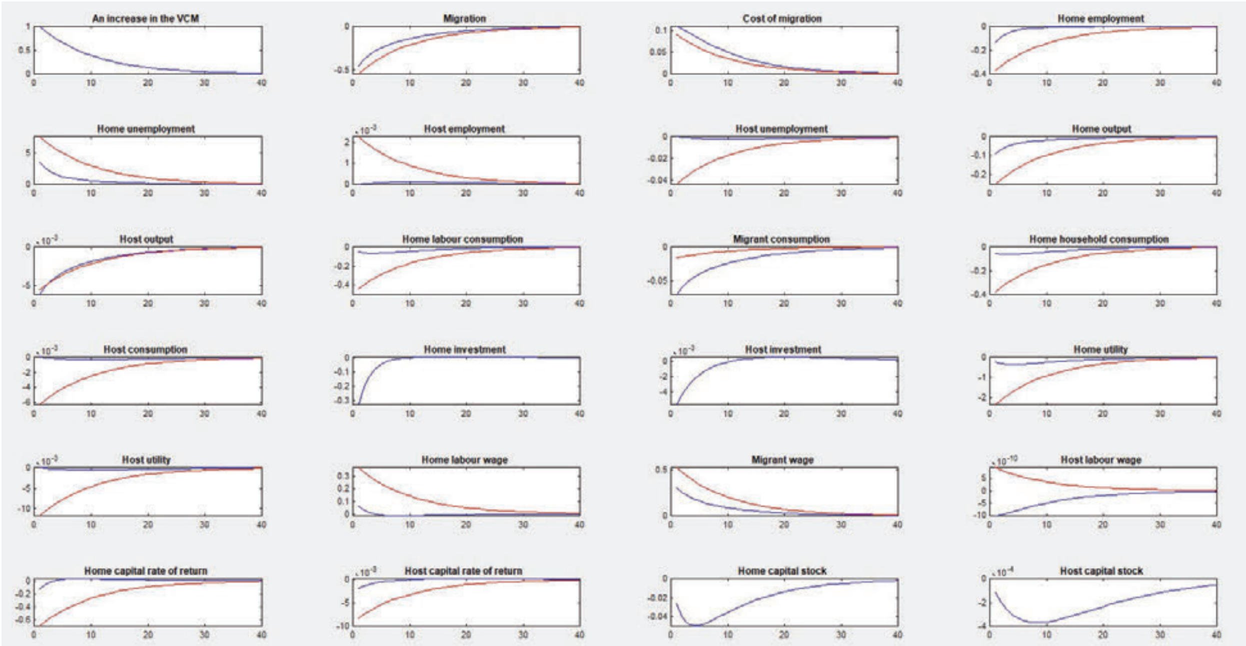

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.13: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

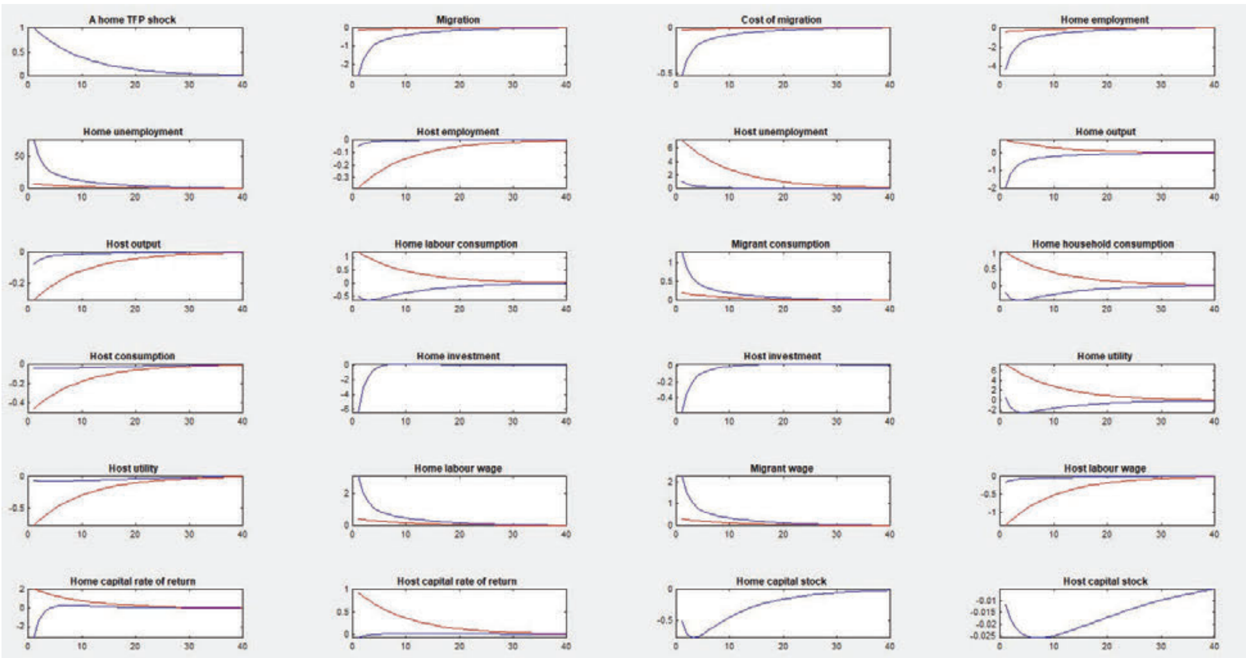
Figure A.14: An increase in the variable cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

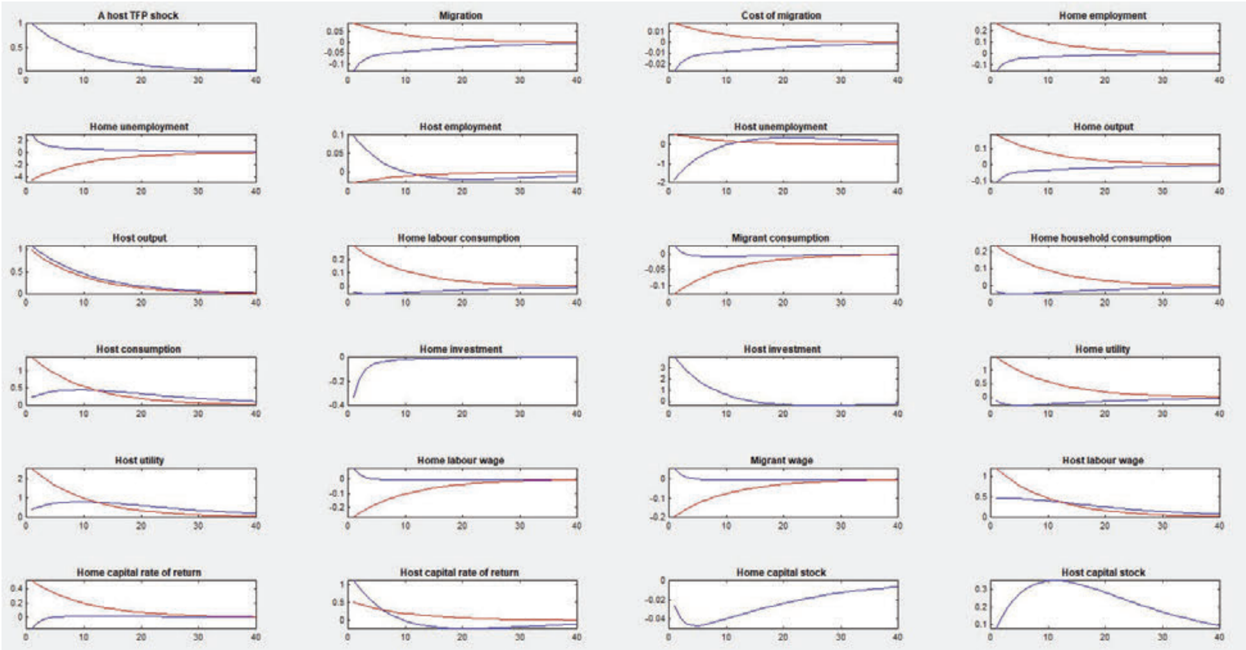
### A.8.3 $b_M = 0.9999$; $b_h = b_f = 0.0001$

Figure A.15: Responses to a positive home TFP shock



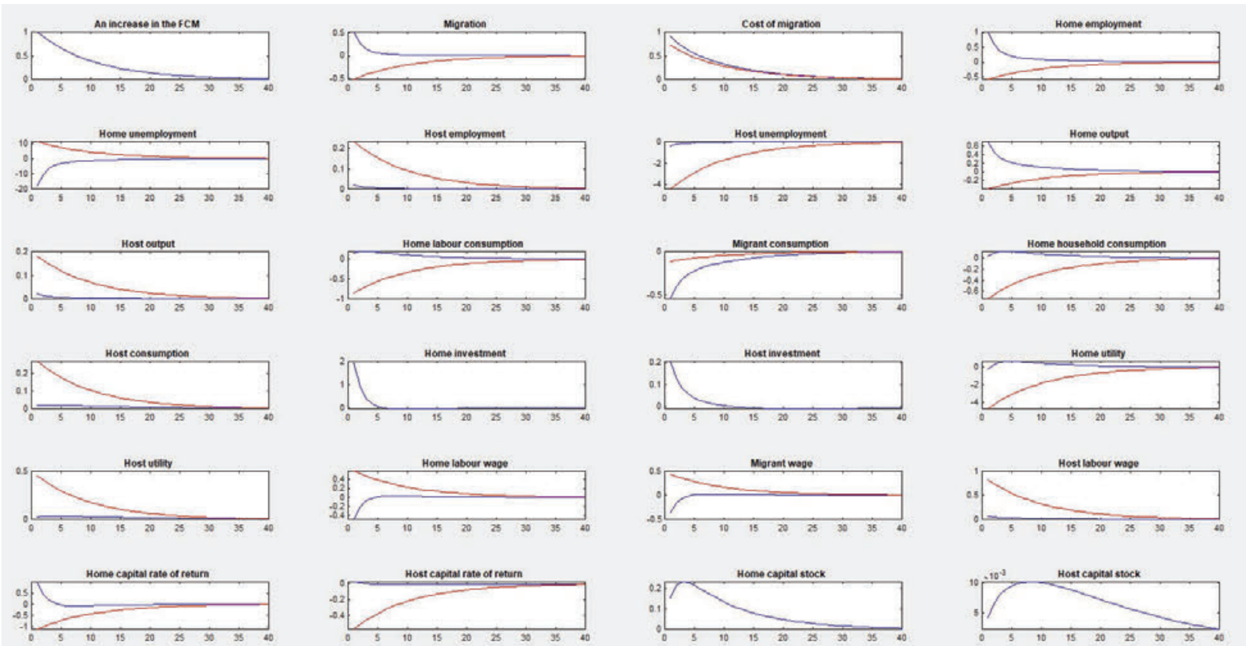

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.16: Responses to a positive foreign TFP shock



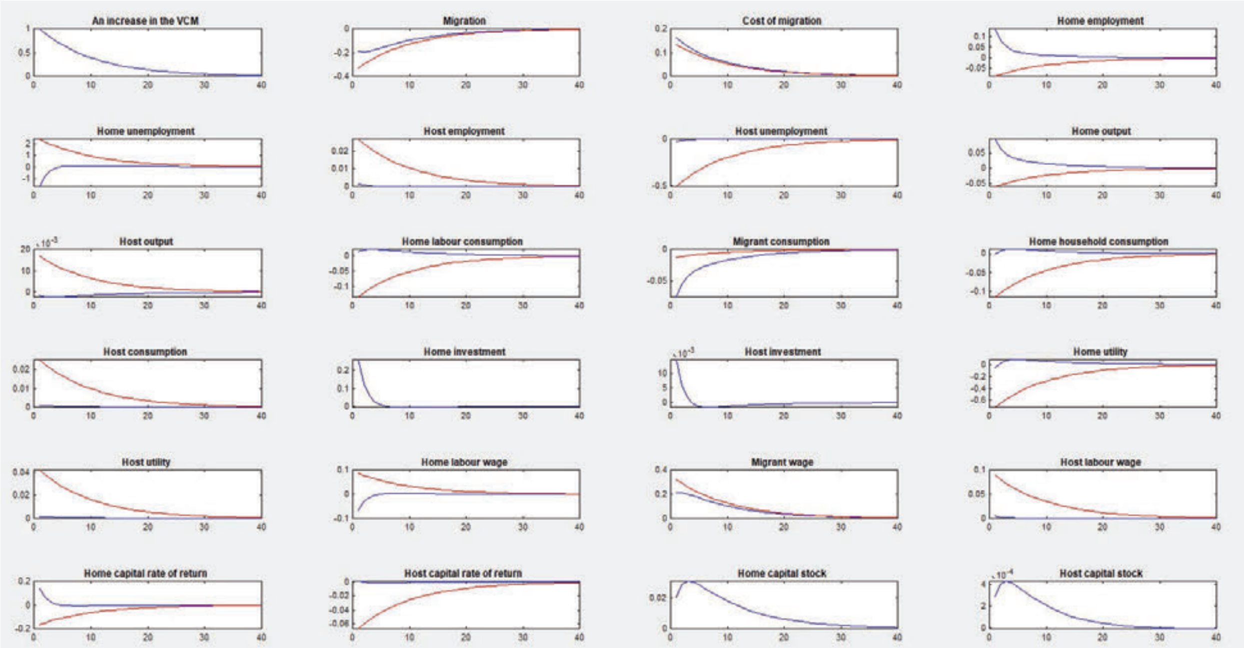

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.17: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

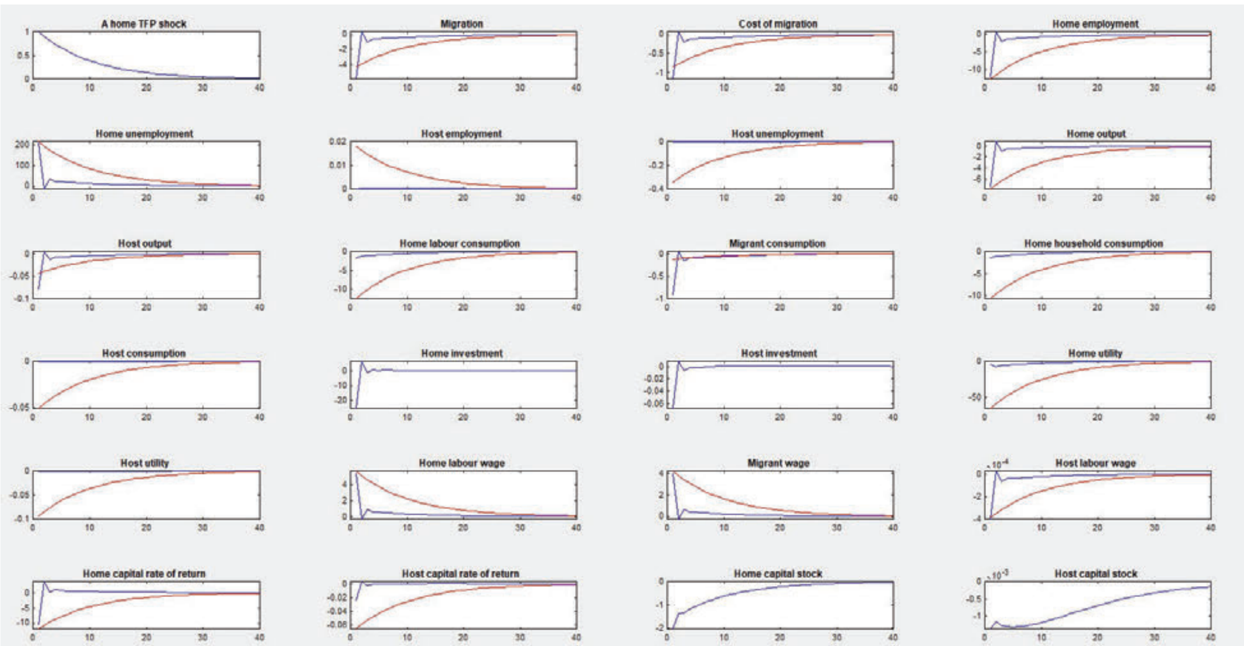
Figure A.18: An increase in the variable cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

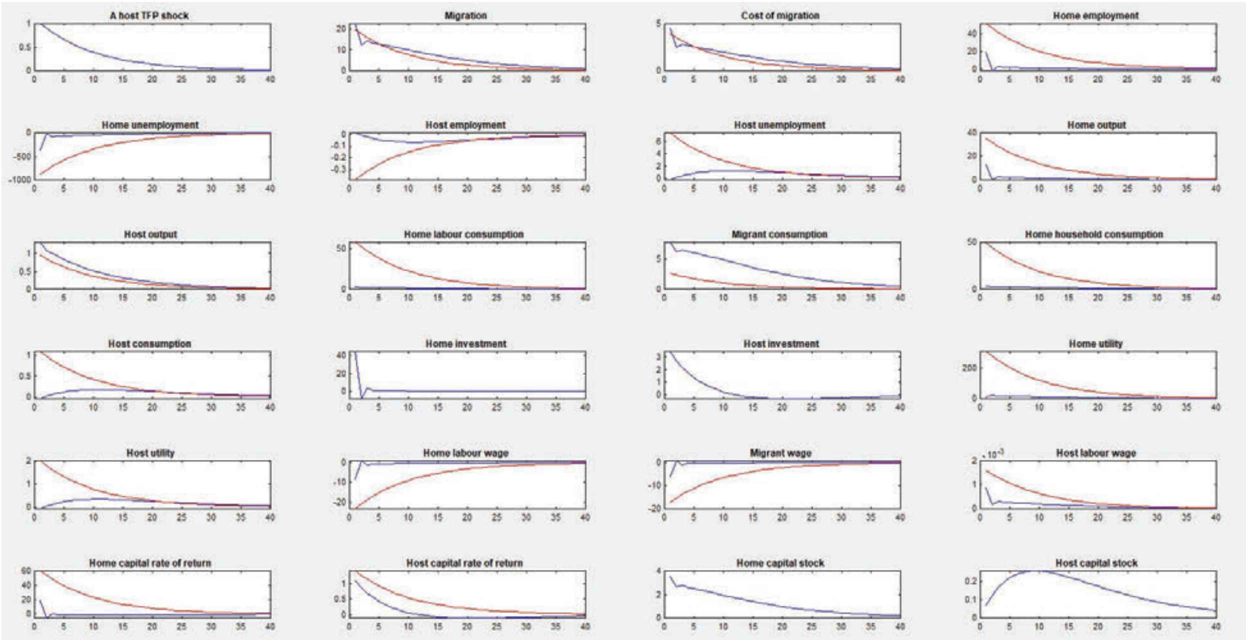
### A.8.4 $b_h = 0.9999$; $b_f = b_M = 0.0001$

Figure A.19: Responses to a positive home TFP shock



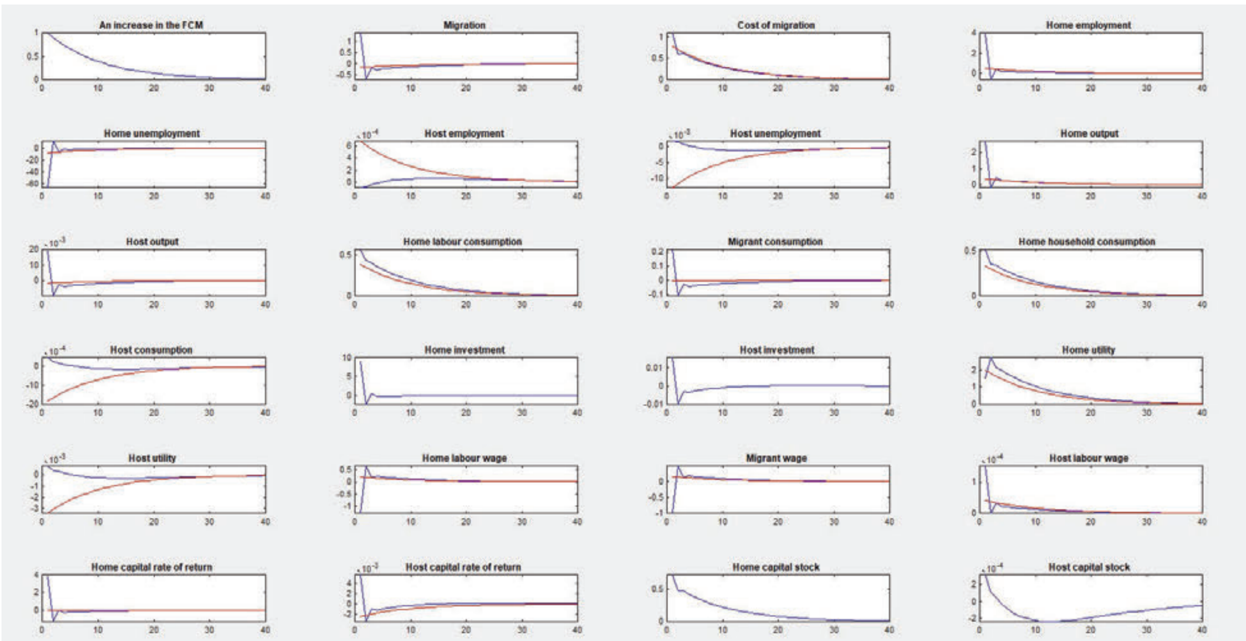

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.20: Responses to a positive foreign TFP shock



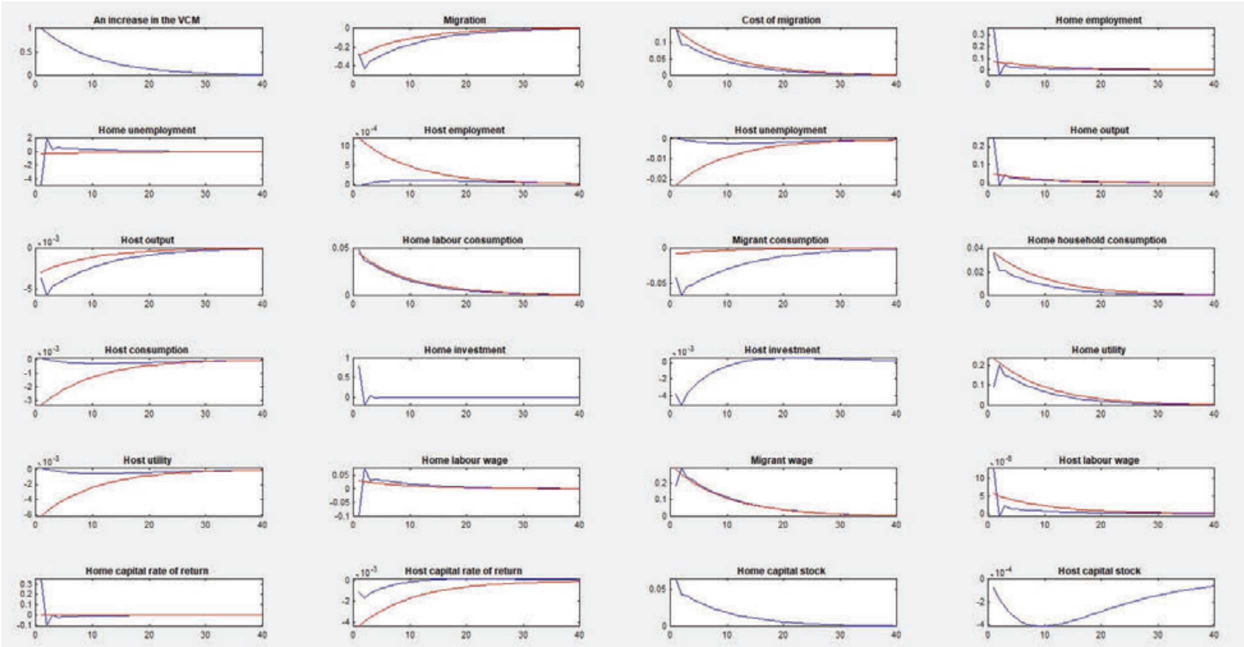

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.21: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

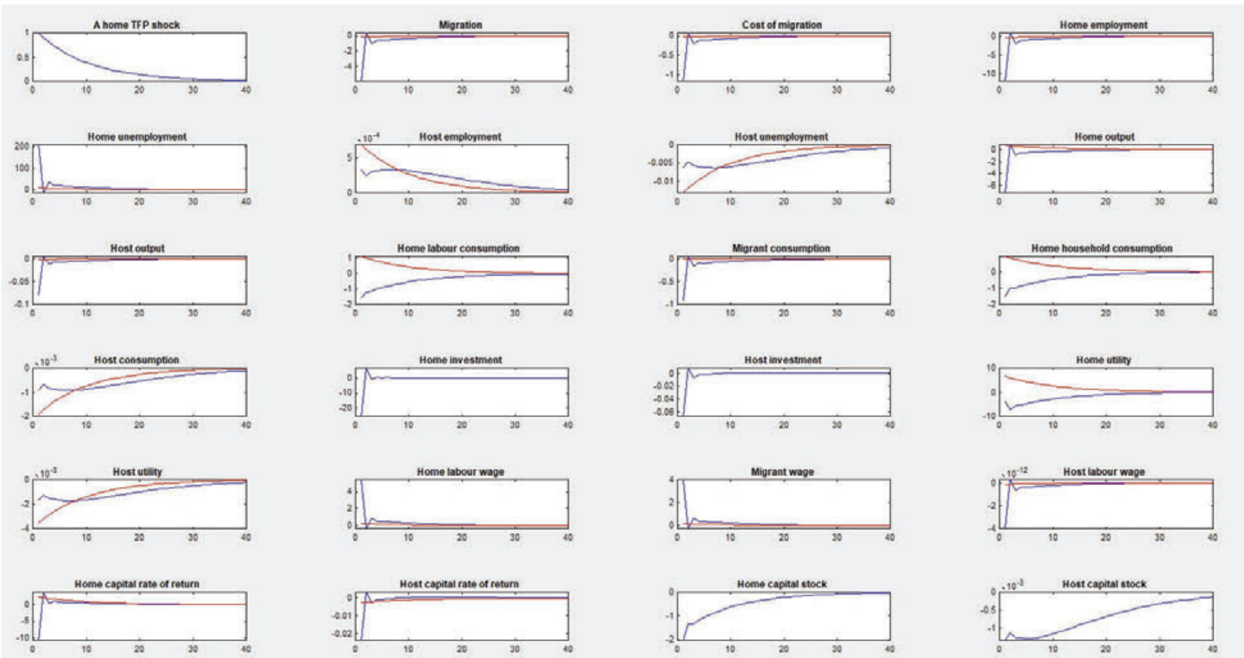
Figure A.22: An increase in the variable cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

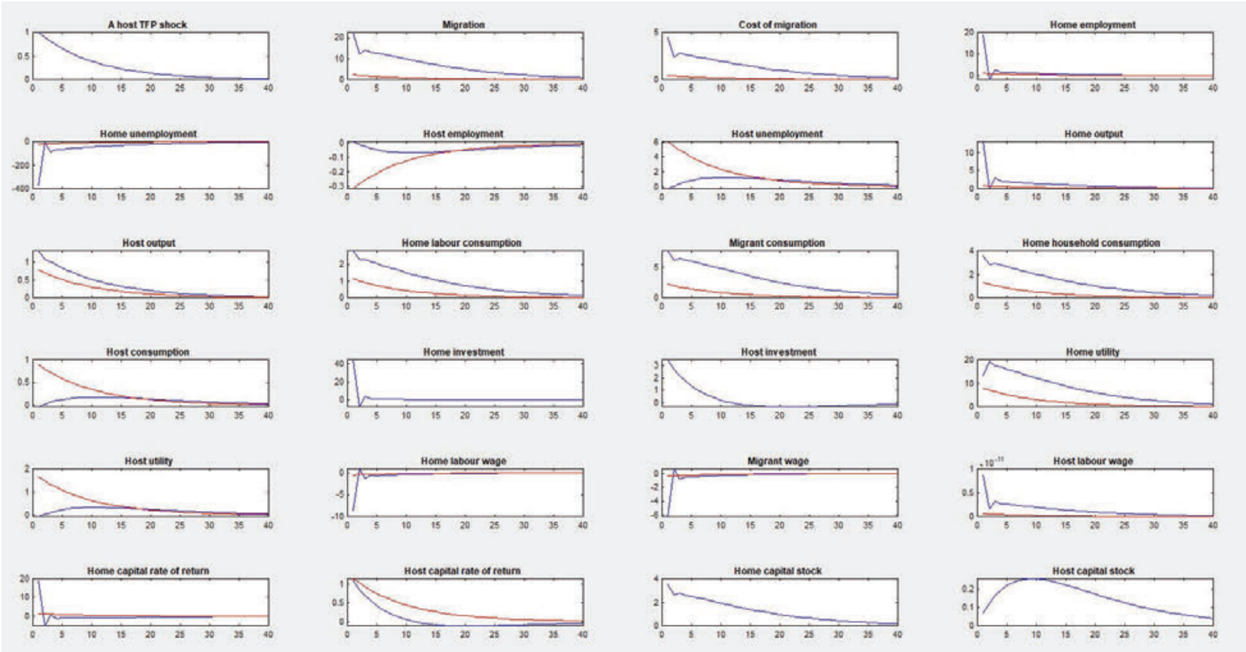
### A.8.5 $b_f = b_h = 0.9999$; $b_M = 0.0001$

Figure A.23: Responses to a positive home TFP shock



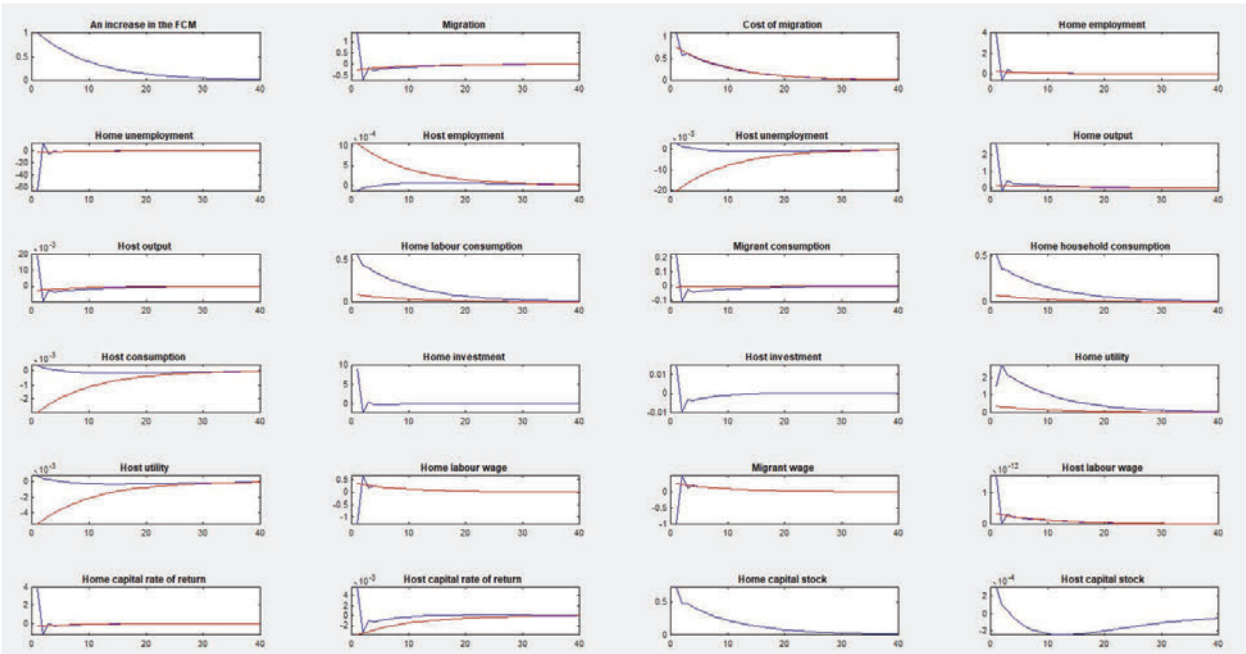

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.24: Responses to a positive foreign TFP shock



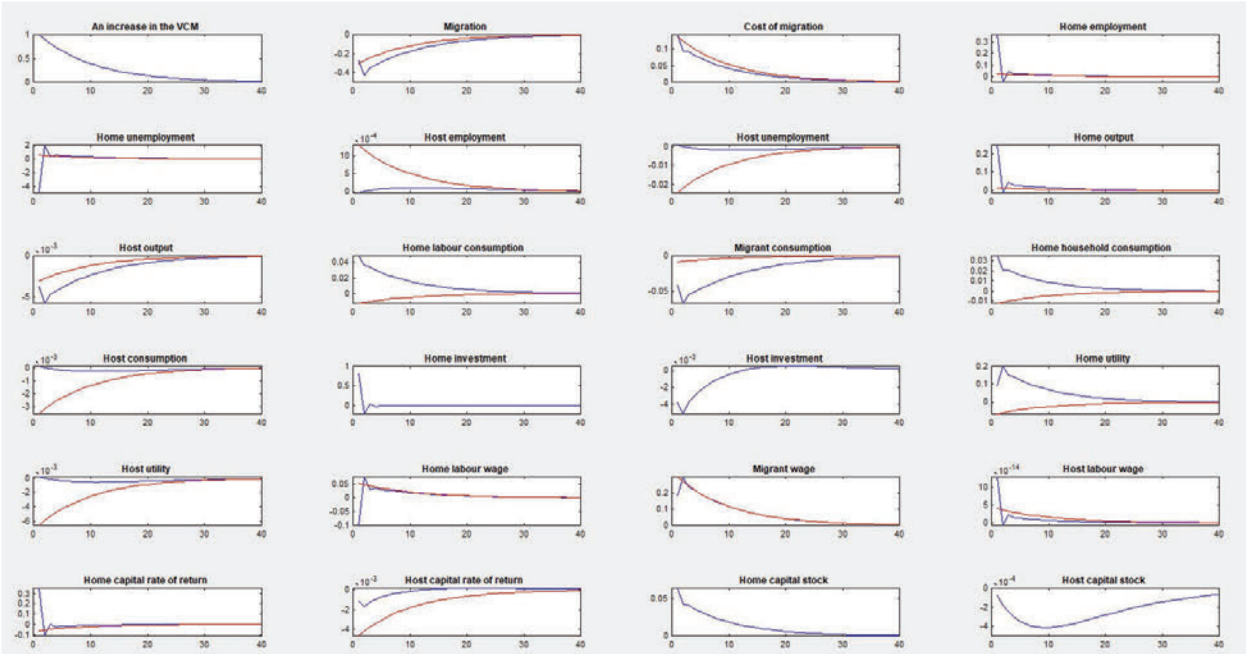

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.25: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

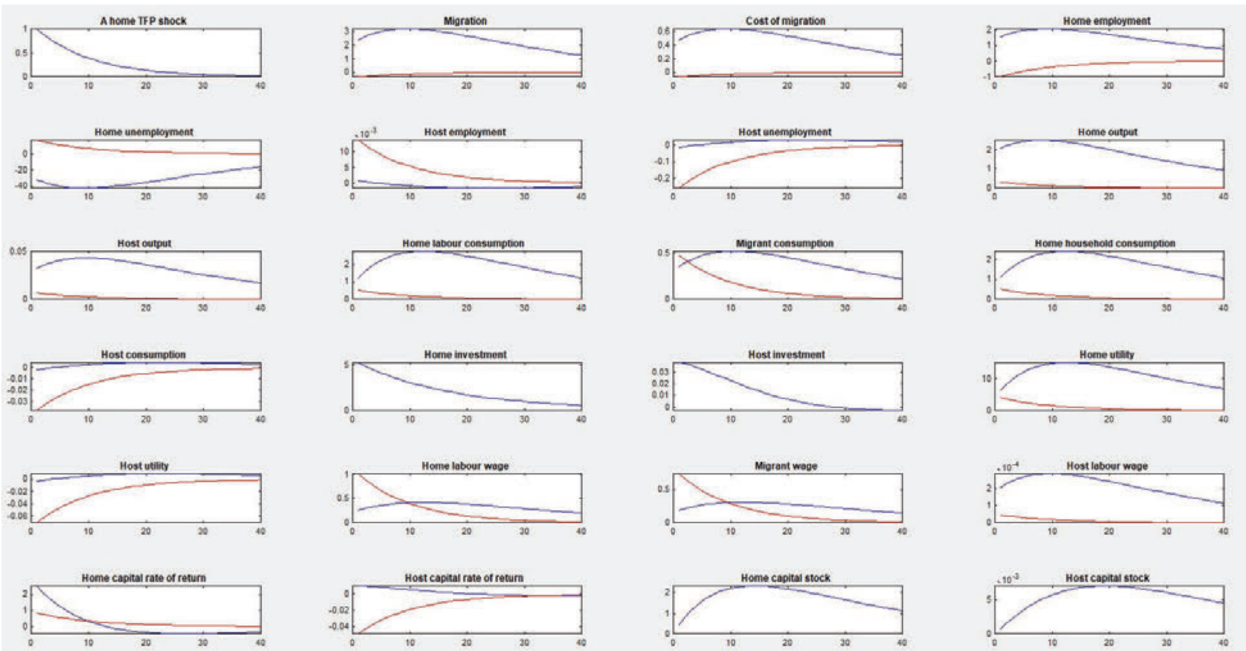
Figure A.26: An increase in the variable cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

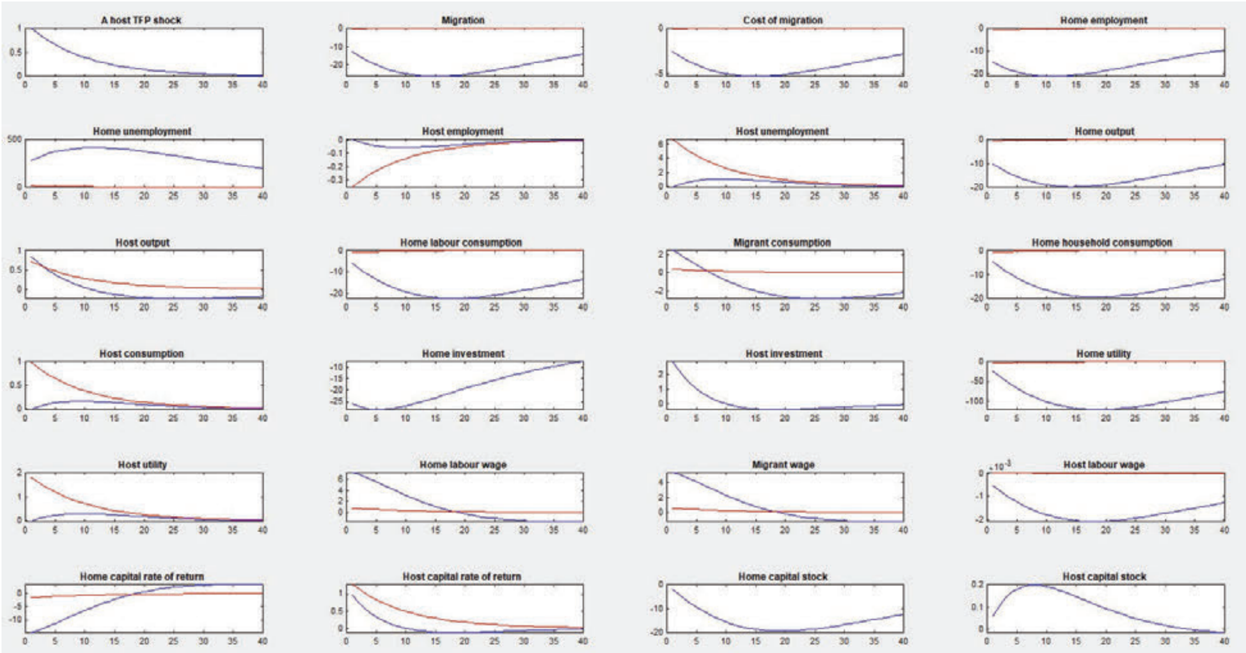
### A.8.6 $b_f = b_M = 0.9999$; $b_h = 0.0001$

Figure A.27: Responses to a positive home TFP shock



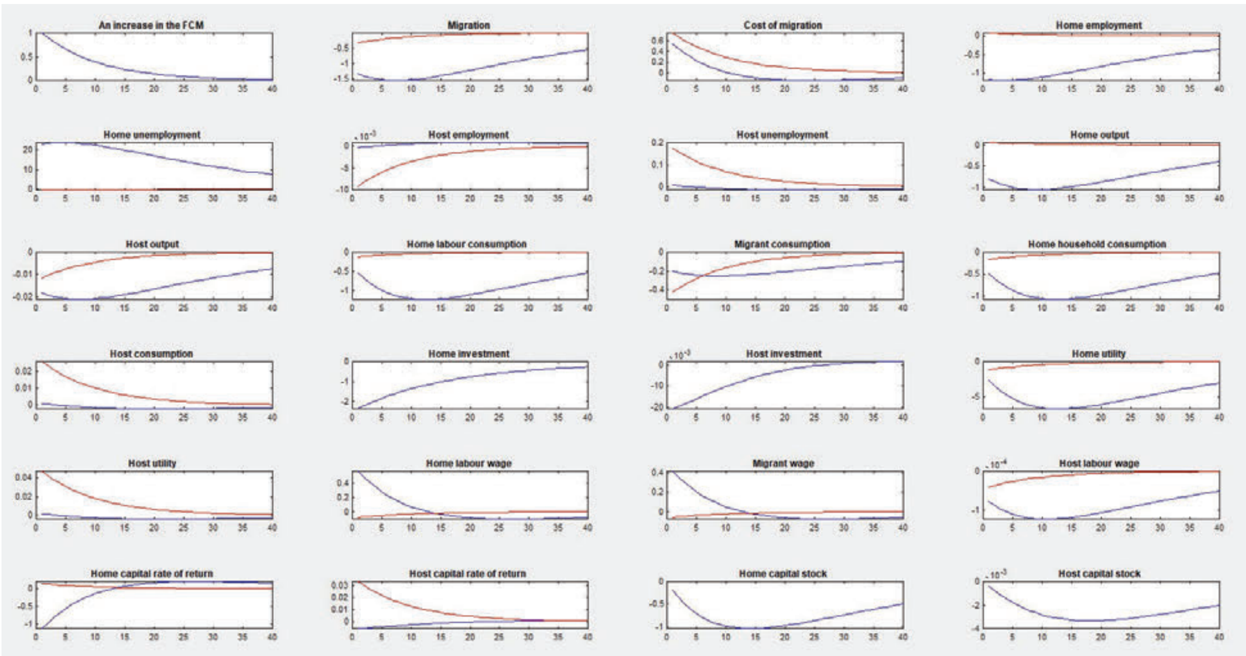

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.28: Responses to a positive foreign TFP shock



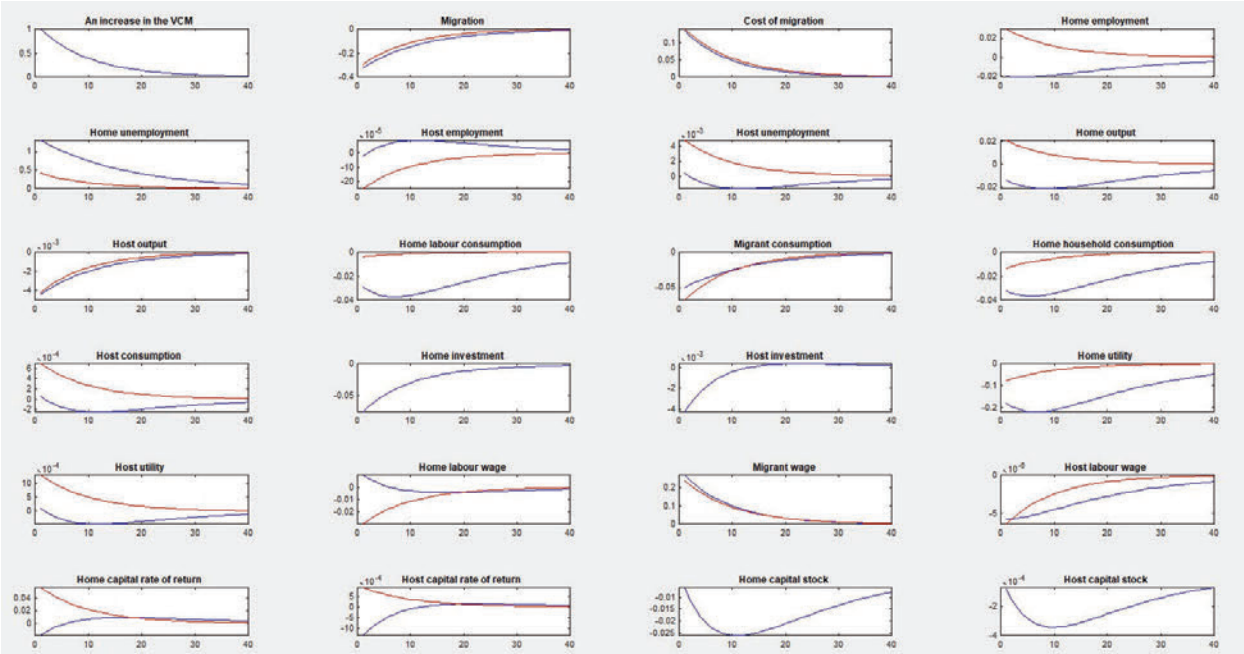

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.29: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

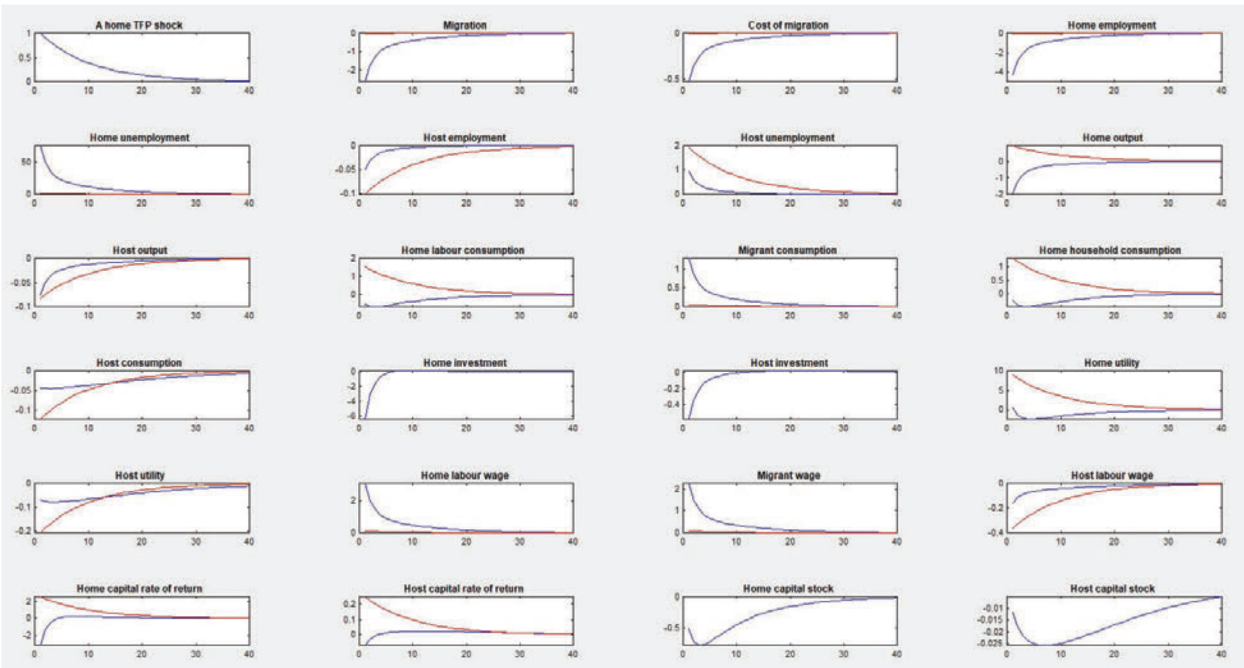
Figure A.30: An increase in the variable cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

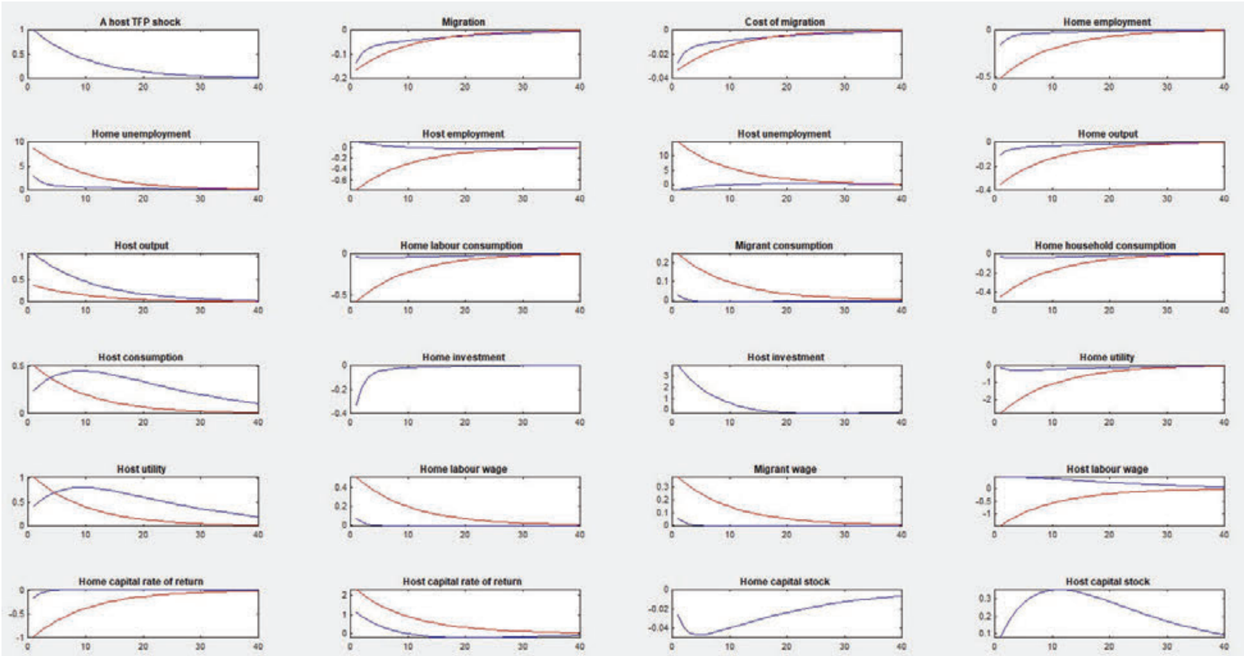
## A.8.7 $b_h = b_M = 0.9999$; $b_f = 0.0001$

Figure A.31: Responses to a positive home TFP shock



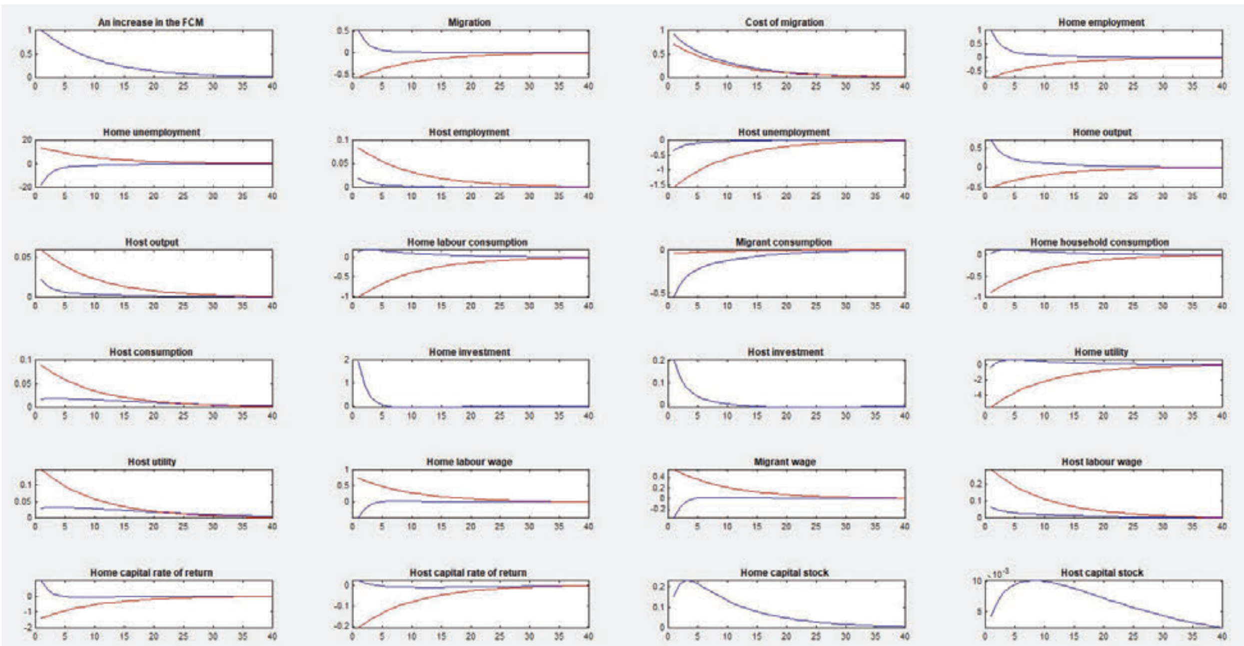

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.32: Responses to a positive foreign TFP shock



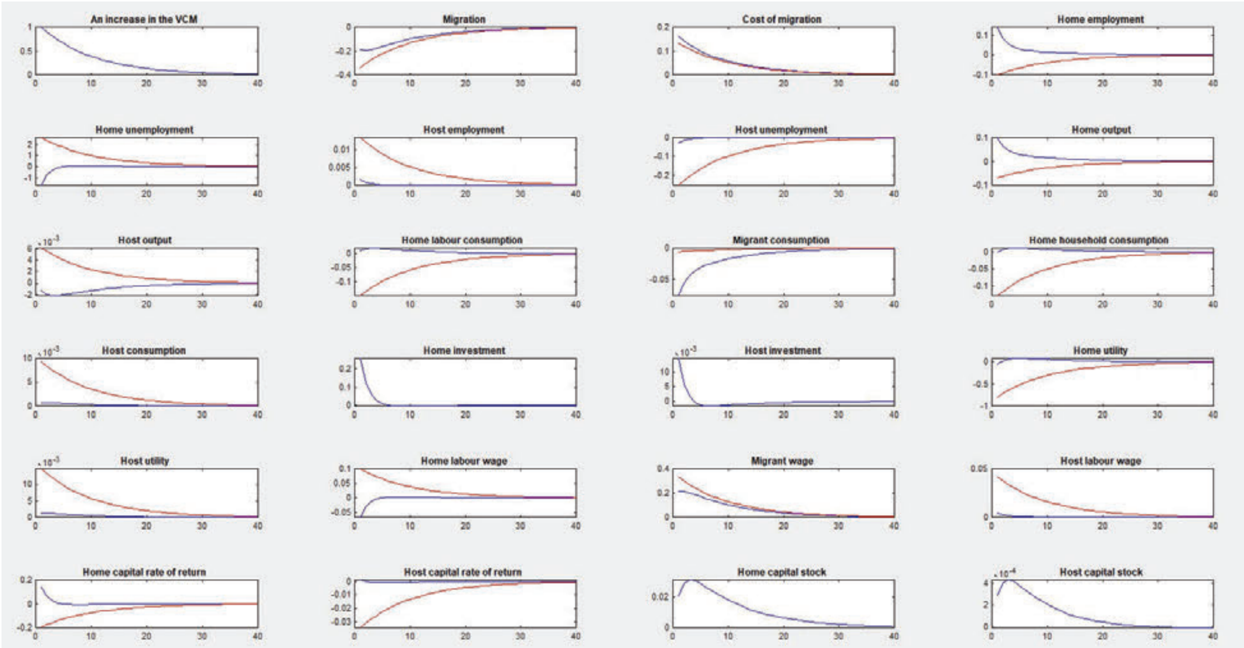

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.33: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

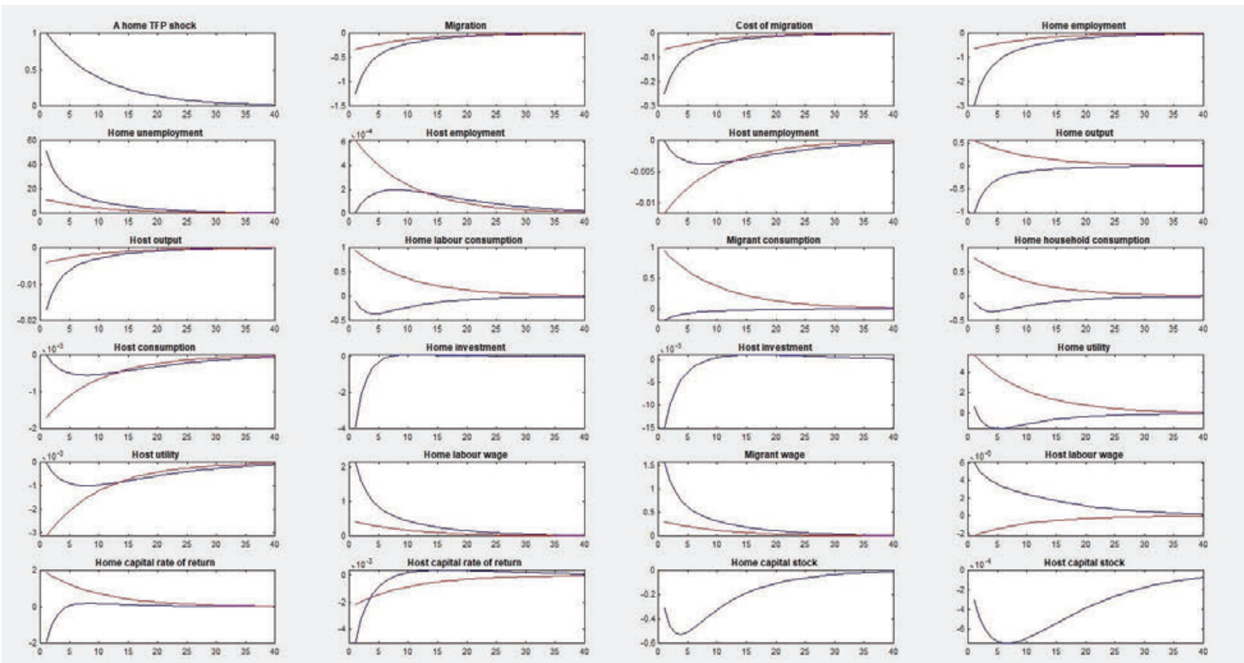
Figure A.34: An increase in the variable cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

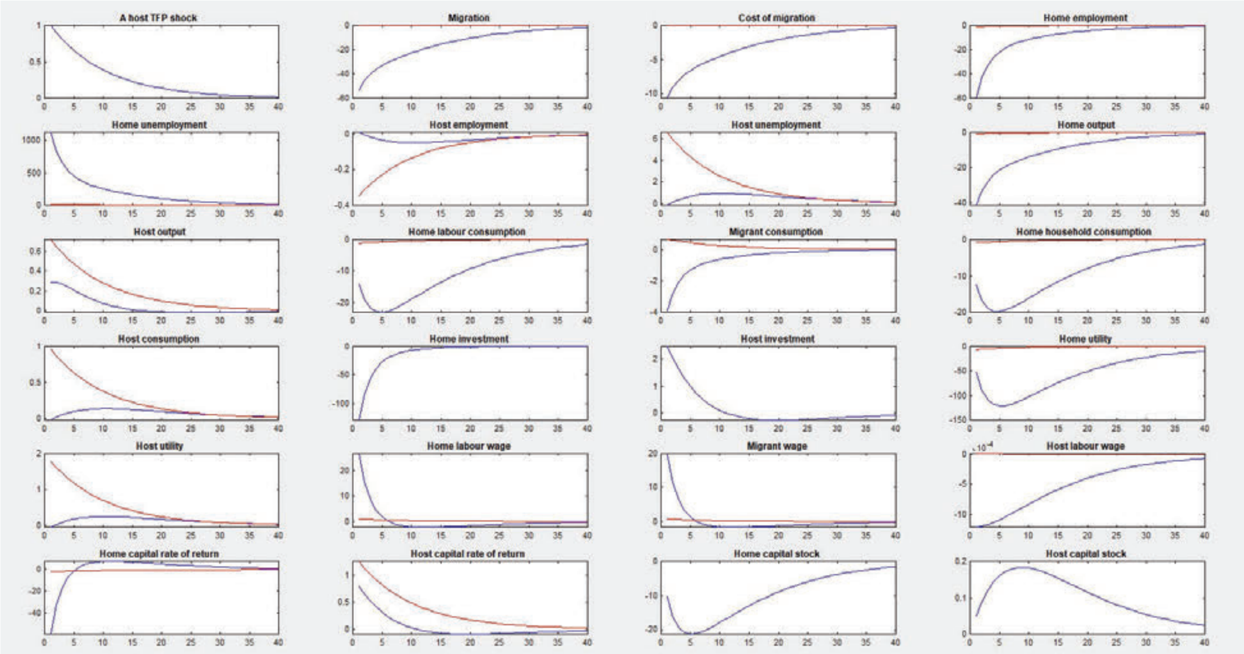
### A.8.8 $b_h = b_M = b_f = 0.9999$

Figure A.35: Responses to a positive home TFP shock



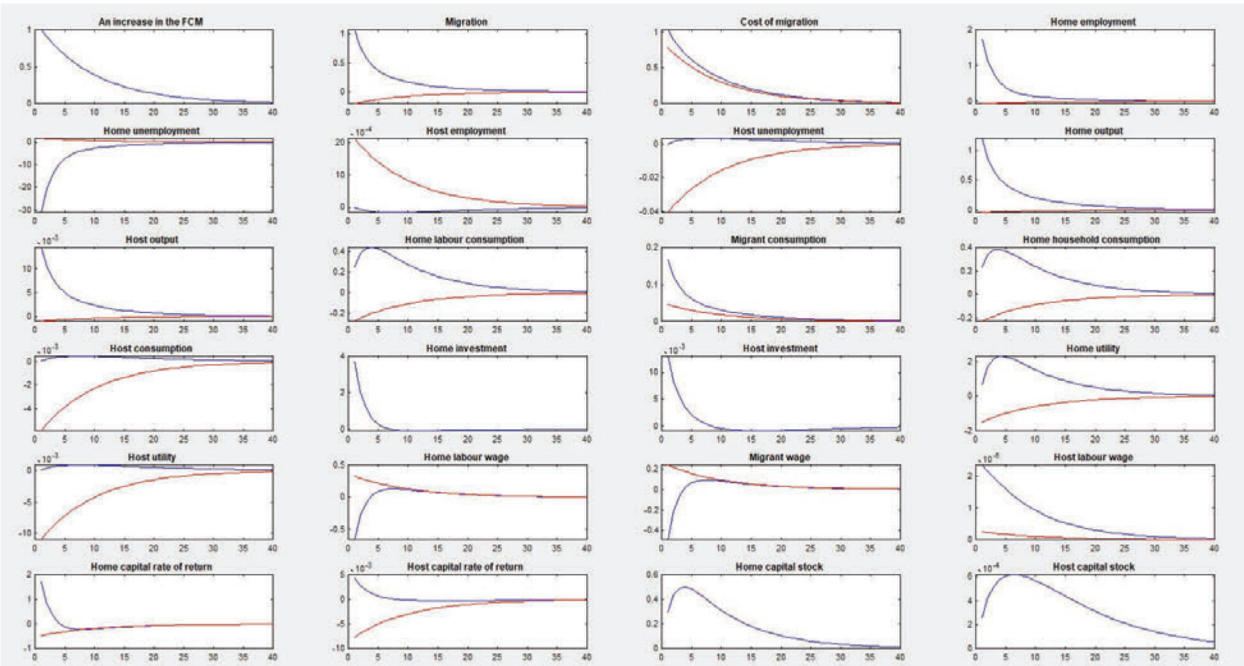

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.36: Responses to a positive foreign TFP shock



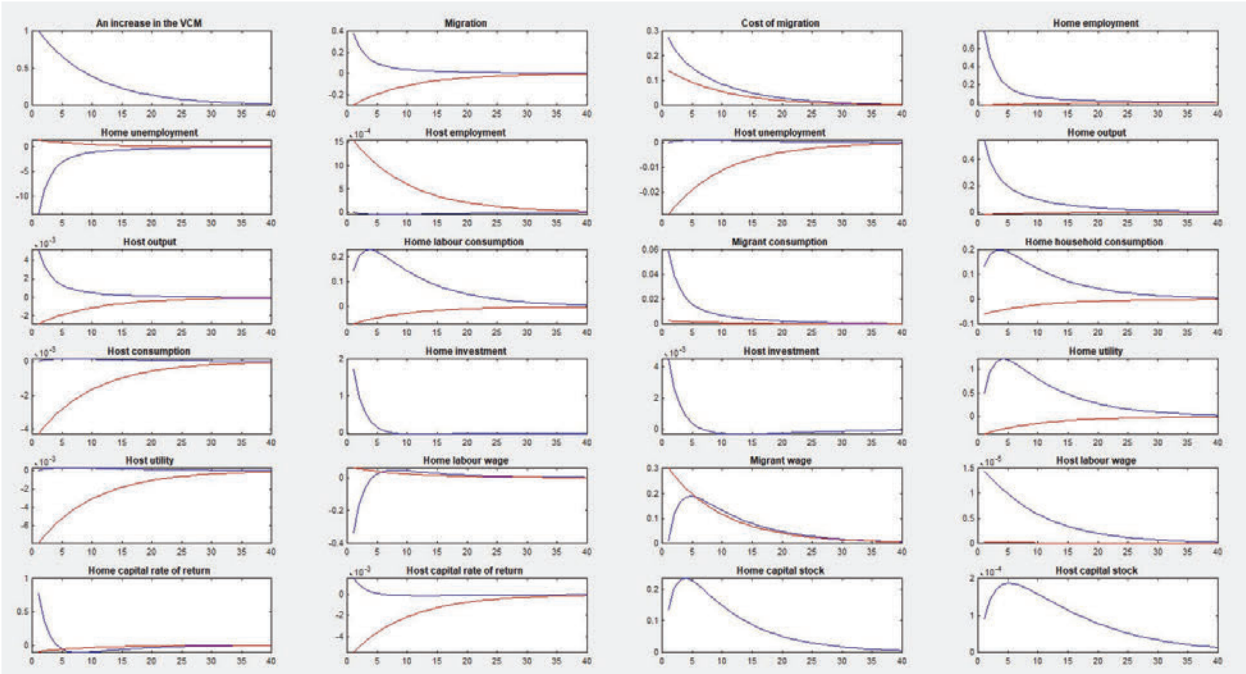

NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.37: An increase in the fixed cost of migration




NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

Figure A.38: An increase in the variable cost of migration

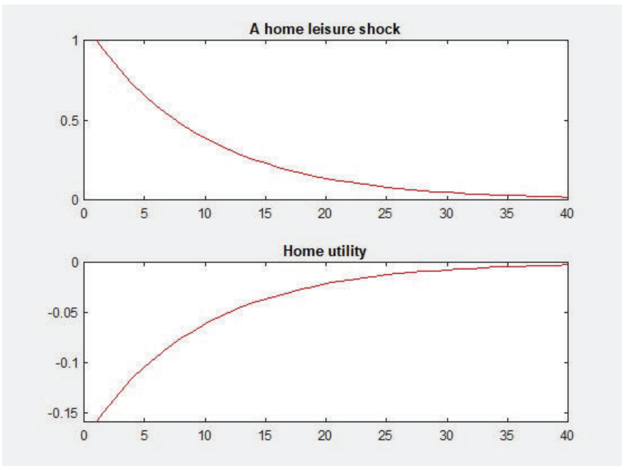



NB: The responses of the fixed-capital model are in red lines, while the responses of adjustable-capital model are in blue.

## A.9 General equilibrium responses to the leisure shocks

### A.9.1 A home leisure shock

Figure A.39: Responses to a positive home leisure shock

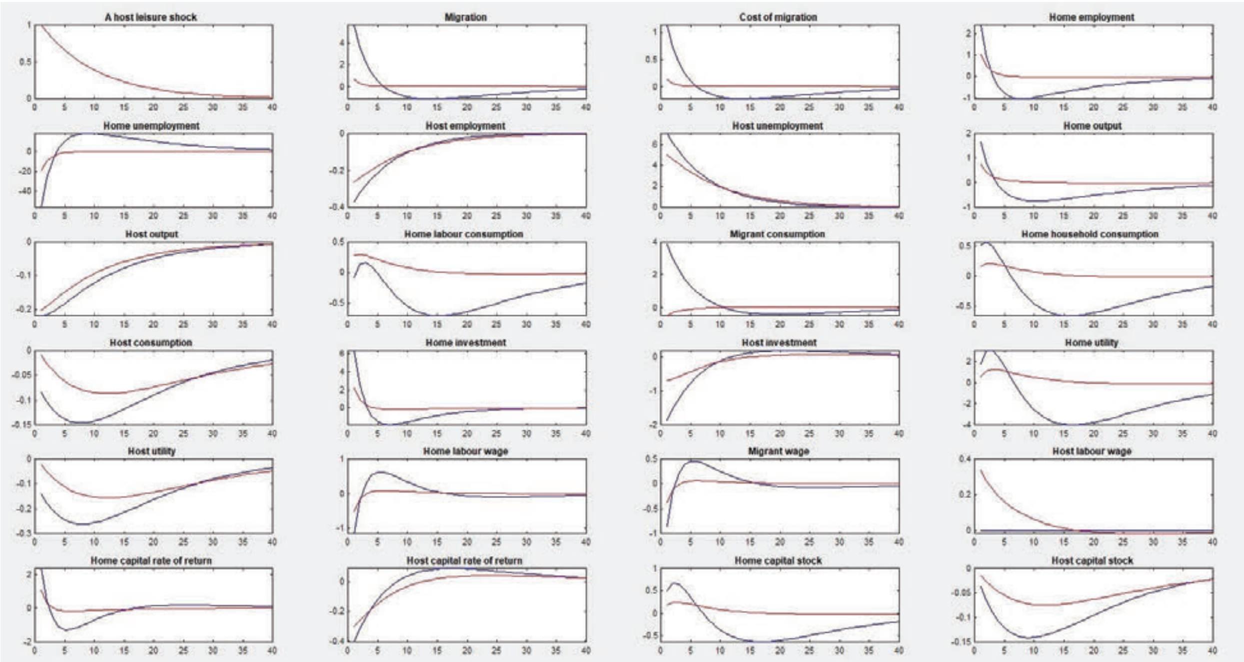



NB: The responses of all seven scenarios of extreme bargaining power structures have overlapped and suggesting a consistent and equivalent fall of the home household utility in response to the home leisure shock in all other aforementioned extreme bargaining power structured labour markets.

## A.9.2 A foreign leisure shock

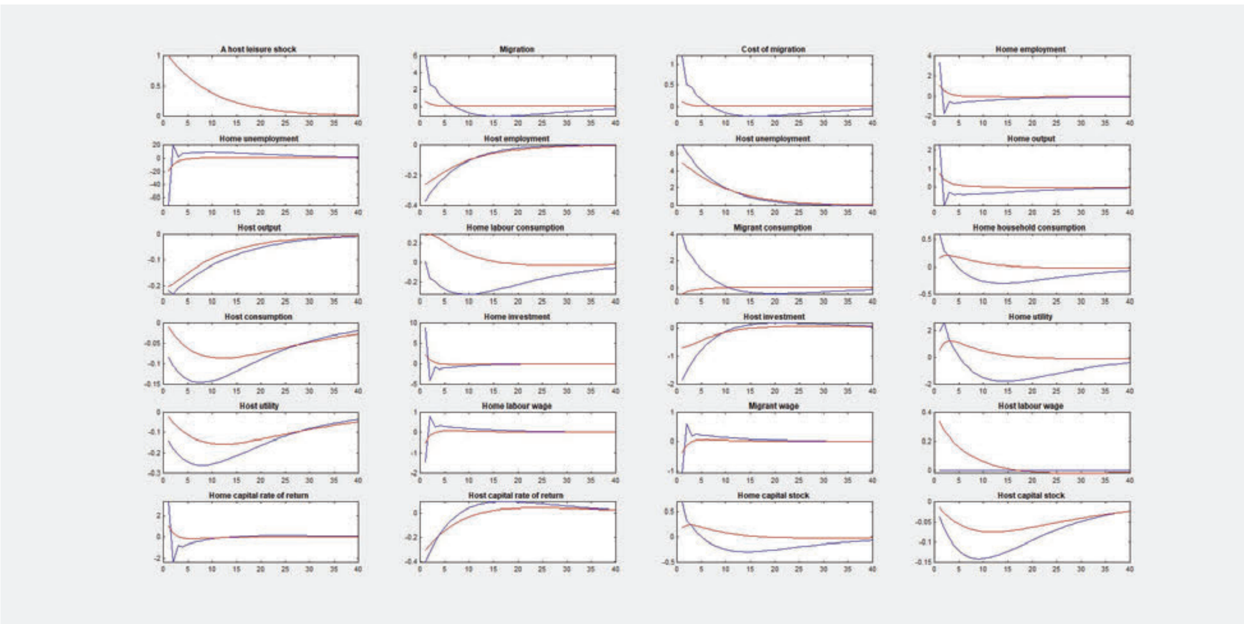
### A.9.2.1 $b_M \to 0$

Figure A.40: Responses of the model with $b_f = 0.9999, b_M = b_h = 0$



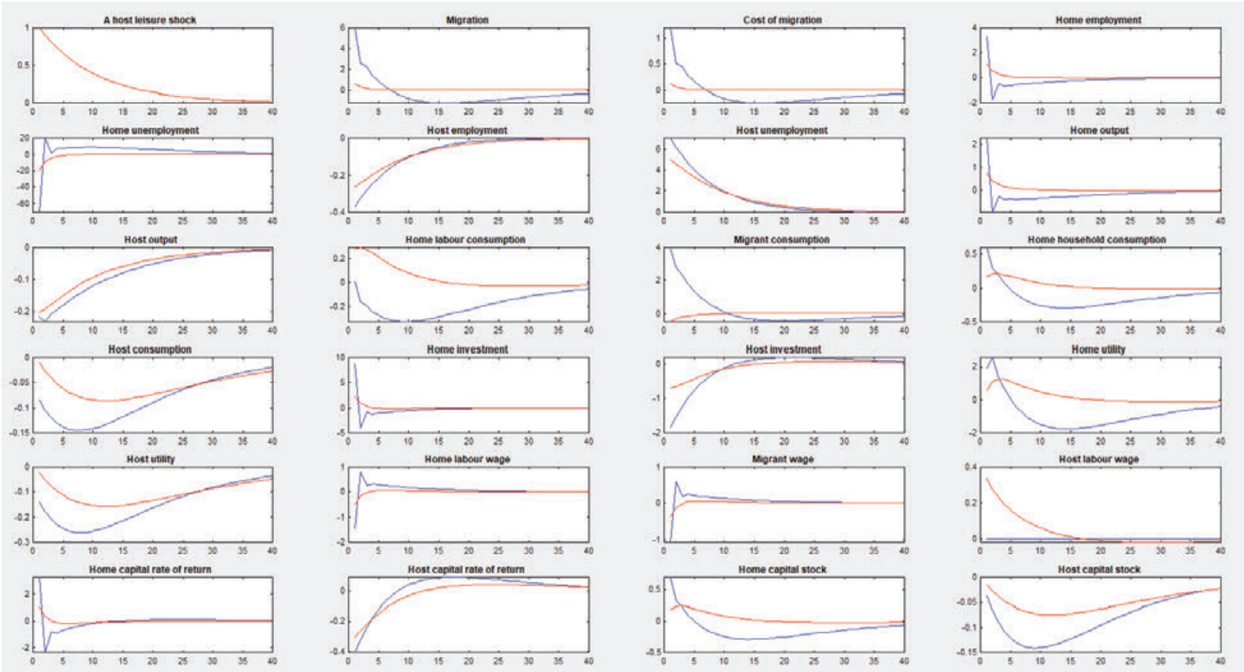

NB: The responses of the model with ($b_f = 0.9999, b_M = b_h = 0$) are in blue lines, while the responses of the benchmark monopolistic model are in red.

Figure A.41: Responses of the model with $b_h = 0.9999$; $b_M = b_f = 0$




NB: The responses of the model with ($b_h = 0.9999$; $b_M = b_f = 0$) are in blue lines, while the responses of the benchmark monopolistic model are in red.

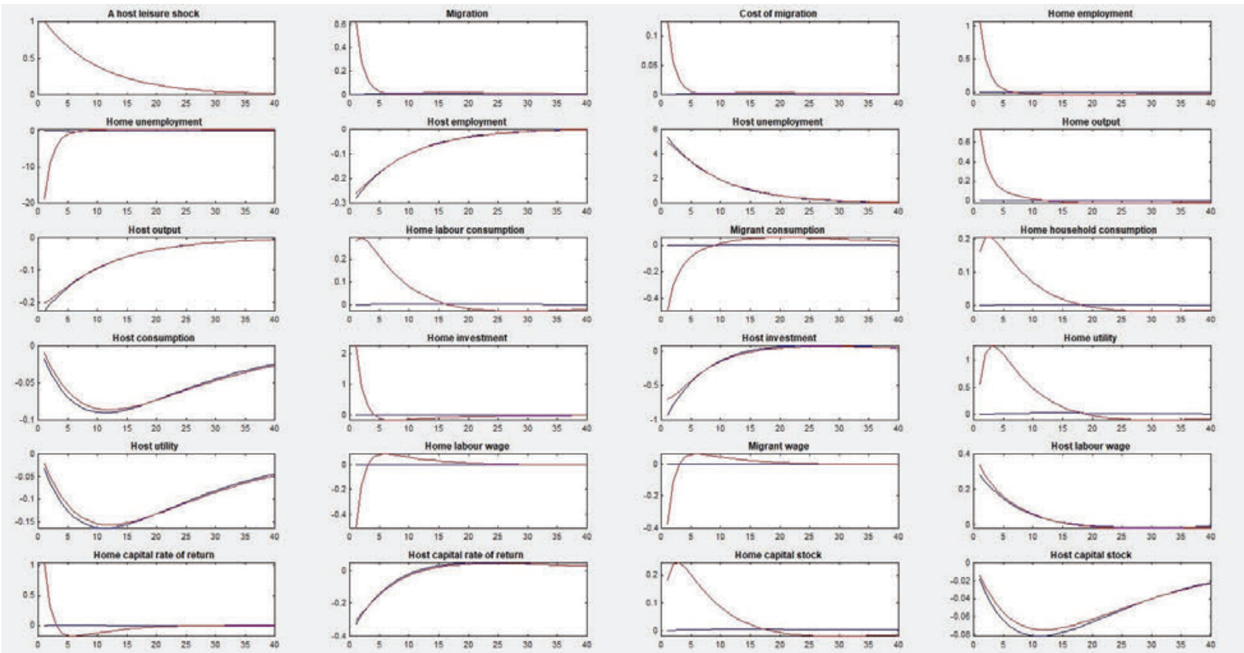
Figure A.42: Responses of the model with $b_M = 0$; $b_h = b_f = 0.9999$




NB: The responses of the model with ($b_M = 0$; $b_h = b_f = 0.9999$) are in blue lines, while the responses of the benchmark monopolistic model are in red.

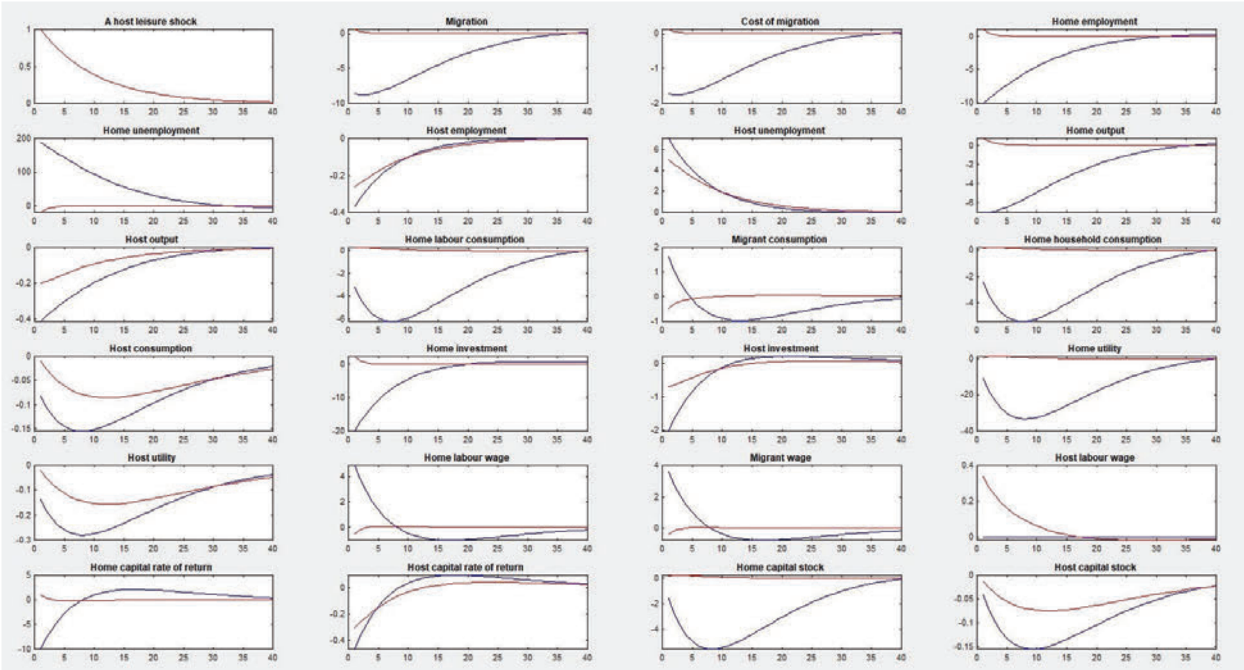
### A.9.2.2 $b_M \to 1$

Figure A.43: Responses of the model with $b_M = 0.9999$; $b_f = b_h = 0$



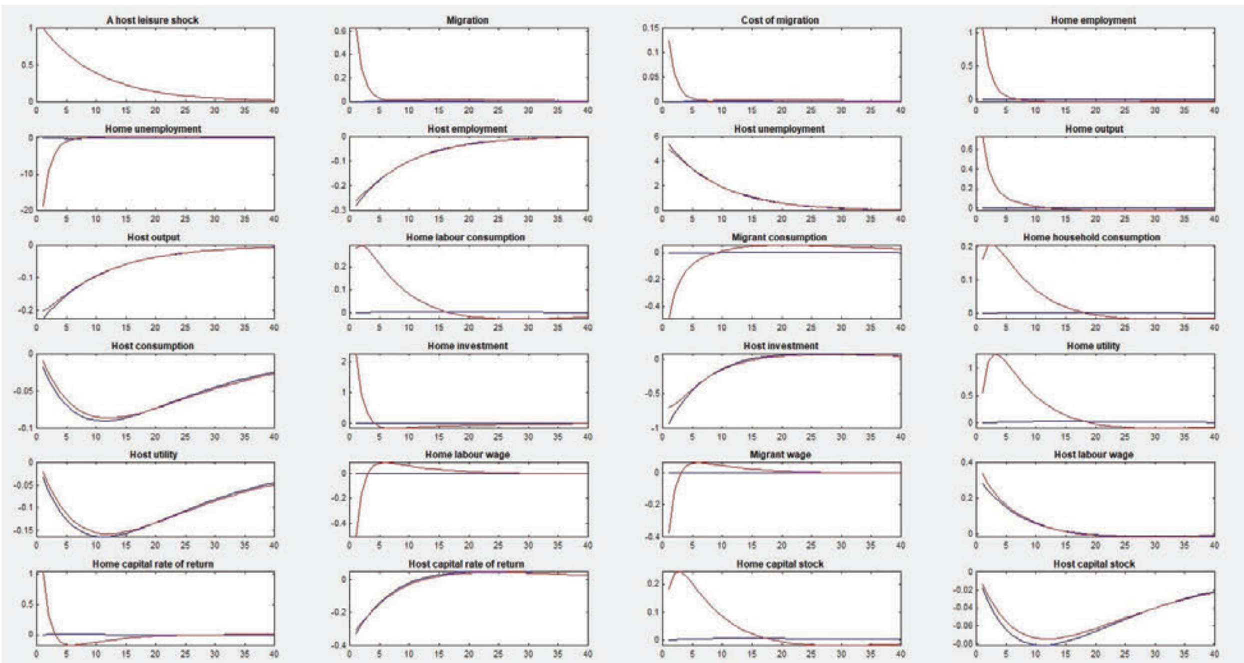

NB: The responses of the model with ($b_M = 0.9999$; $b_f = b_h = 0$) are in blue lines, while the responses of the benchmark monopolistic model are in red.

Figure A.44: Responses of the model with $b_h = 0$; $b_M = b_f = 0.9999$



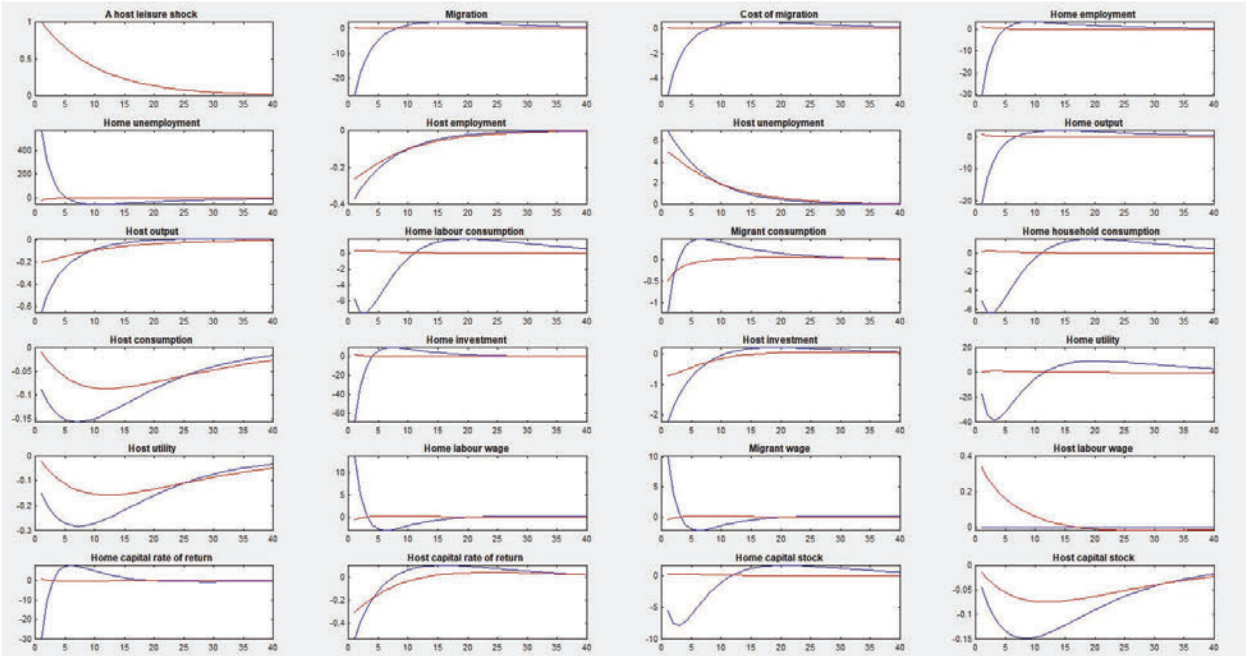

NB: The responses of the model with ($b_h = 0$; $b_M = b_f = 0.9999$) are in blue lines, while the responses of the benchmark monopolistic model are in red.

Figure A.45: Responses of the model with $b_f = 0$; $b_h = b_M = 0.9999$




NB: The responses of the model with ($b_f = 0$; $b_h = b_M = 0.9999$) are in blue lines, while the responses of the benchmark monopolistic model are in red.

Figure A.46: Responses of the model with $b_f = b_h = b_M = 0.9999$




NB: The responses of the model with ($b_f = b_h = b_M = 0.9999$) are in blue lines, while the responses of the benchmark monopolistic model are in red.